CHAPTER 4

RESULTS AND DISCUSSIONS

4.1 Numerical examples and discussions

The problem formation and the development of the state estimator using the maximum likelihood criterion has been discussed in Chapter 2, and the proposed orthogonal decomposition via Householder transformation method together with the suggested solution algorithm has been presented in Chapter 3. In this chapter, the proposed solving method will be applied on test cases. Specifically, the state estimation program will be implemented in a Matlab environment to solve the 6-bus test system, IEEE 30-bus test system, and the practical 30-bus Sumatera Barat power system. Moreover, comparisons with other solving methods will be carried out to evaluate the performance of the proposed method in terms of numerical stability and computational efficiency. All tests will be run on a personal computer having a 2.99 GHz Intel ® Core ™ 2 Duo CPU and 3.48 GB of RAM.

4.1.1 The 6-bus test system

The 6-bus test system with P+jQ measurements on each end of each transmission line and at each load and generator, and bus voltage measurement at each system bus is shown in Figure. 4.1. The test information and the measurement configurations are shown in Table 4.1. The redundancy shown in Table 4.1 is defined as the ratio of number of measurements and number of state variables.

System measurements are obtained from the load flow result by using fast decoupled method with Gaussian noise added. Noise was generated by a random number generating algorithm so as to be representative of values drawn from a set of numbers having a normal probability density function with zero mean and variance as specified for each measurement type. Symbol for each measurement type: M_{ij}, M_{Vi}, M_{Li}, and M_{Gi} for ***i***,***j***= 1,2,3,4,5,6 is shown in the list of symbols and abbreviations (pp. xii).

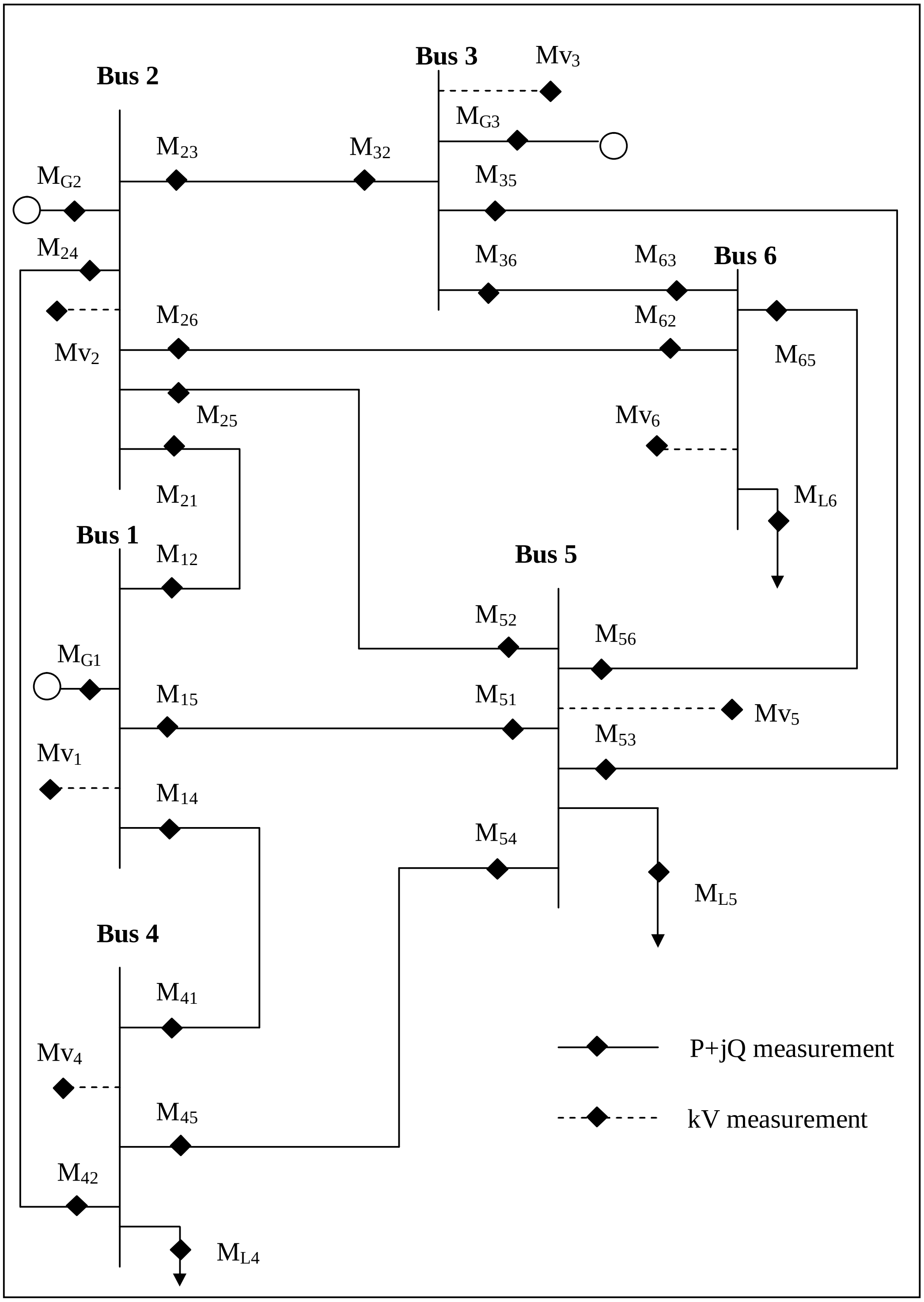

Figure 4.1: 6-bus test system with measurements

Table 4.1: 6-bus test system measurement configurations

Measurement configuration	6-bus system
Number of buses	6
Number of lines	11
Number of voltage measurements	6
Number of injection measurement P/Q	12/12
Number of flow measurement P/Q	22/22
Number of zero injection	0/0,1/1
Total Measurements	62
Redundancy	5.167

The measurement standard deviations used are as stated in Table 4.2.

Table 4.2: 6-bus test system measurements standard deviation

Type of measurement	Standard deviation, s
Real power, P	5 MW
Reactive power, Q	5 MVAR
Voltage, $\|E\|$	3.83 kV

Here, let the base case values be the measurements obtained from the load flow result by using power flow fast decoupled method. The solution procedure started with $\mathbf{x}^0$ being initially set to 1.0 pu for voltage magnitude and 0 rad for phase angle at each bus. Table 4.3 presents the estimated values from the state estimator by applying the proposed method.

Table 4.3: 6-bus test system state estimation solution

Measurement	Base Case Value			Measured Value			Estimated Value		
	kV	MW	MVAR	kV	MW	MVAR	kV	MW	MVAR
M_{V1}	241.5			238.4			240.3		
M_{G1}		107.9	16.0		113.1	20.2		112.01	19.21
M_{12}		28.7	-15.4		31.5	-13.2		30.4	-14.15
M_{14}		43.6	20.1		38.9	21.2		44.79	21.32
M_{15}		35.6	11.3		35.7	9.4		36.82	12.4

Measurement	Base Case Value			Measured Value			Estimated Value		
	kV	MW	MVAR	kV	MW	MVAR	kV	MW	MVAR
M_{V2}	241.5			237.8			239.41		
M_{G2}		50.0	74.4		48.4	71.9		47.61	70.45
M_{21}		-27.8	12.8		-34.9	9.7		-29.42	11.75
M_{23}		2.9	-12.3		8.6	-11.9		3.1	-12.35
M_{24}		33.1	46.1		32.8	38.3		32.27	45.01
M_{25}		15.5	15.4		17.4	22.0		15.65	14.89
M_{26}		26.2	12.4		22.3	15.0		26.01	11.14
M_{V3}	246.1			250.7			244.12		
M_{G3}		60.0	89.6		55.1	90.6		59.31	87.03
M_{32}		-2.9	5.7		-2.1	10.2		-3.05	5.93
M_{35}		19.1	23.2		17.7	23.9		19.11	22.81
M_{36}		43.8	60.7		43.3	58.3		43.26	58.29
M_{V4}	227.6			225.7			225.68		
M_{L4}		70.0	70.0		71.8	71.9		70.1	69.85
M_{41}		-42.5	-19.9		-40.1	-14.3		-43.62	-20.75
M_{42}		-31.6	-45.1		-29.8	-44.3		-30.81	-44.14
M_{45}		4.1	-4.9		0.7	-17.4		4.32	-4.97
M_{V5}	226.7			225.2			224.76		
M_{L5}		70.0	70.0		72.0	67.7		71.78	69.53
M_{51}		-34.5	-13.5		-36.6	-17.5		-35.66	-13.81
M_{52}		-15.0	-18.0		-11.7	-22.2		-15.16	-17.48
M_{53}		-18.0	-26.1		-25.1	-29.9		-18.02	-25.65
M_{54}		-4.0	-2.8		-2.1	-1.5		-4.28	-2.62
M_{56}		1.6	-9.7		-2.1	-0.8		1.33	-9.97
M_{V6}	231.0			228.9			229.47		
M_{L6}		70.0	70.0		72.3	60.9		69.03	66.07
M_{62}		-25.7	-16.0		-19.6	-22.3		-25.45	-14.73
M_{63}		-42.8	-57.9		-46.8	-51.1		-42.3	-55.62
M_{65}		-1.6	3.9		1.0	2.9		-1.28	4.28

‘Table 4.3, continued’

The sum of the measurement residuals J($\mathbf{x}$) are calculated at the beginning of each iteration, while the maximum of $\Delta|\mathbf{E}|$, and the maximum of Δq are calculated at the end of each iteration. J($\mathbf{x}$) represents a measure of the overall fit of the estimated values to the measurement values. The value of J($\mathbf{x}$) would be zero if all the measurements were without error. Thus, the estimated values fit the measurement the "best" when J($\mathbf{x}$) is small. With tolerance $\Delta\|\mathbf{x}\|_\infty < 10^{-4}$, the iterative steps for the 6-bus test system produced the results given in Table 4.4.

Table 4.4: 6-bus test system iterative results

Iteration	J($\mathbf{x}$) at the beginning of the iteration (pu)	Largest $\Delta\lvert\mathbf{E}\rvert$ at end of iteration (pu V)	Largest Δq at end of iteration (rad)
1	3663.552	0.05804639	0.11224237
2	39.70989	0.00518528	0.00451475
3	37.57728	0.00005885	0.00001216

Results in Table 4.3 illustrate the effectiveness of the proposed method in calculating quantities that are the "best" possible estimates of the true bus voltages and generator, load, and transmission line MW and MVAR values even with measurement errors. Observe that, active power flow measurement, P on bus line 4-5 shows a value of 0.7 MW whereas the base case value is 4.1 MW. The estimator has done a good job by estimating the value of P as 4.32 MW, which is closer to the base case value.

Since the state estimation program has been developed in Matlab, the Matlab optimization solver, 'lsqnonlin' is also utilized to solve the 6-bus nonlinear least squares state estimation problem for comparison purpose. The Matlab lsqnonlin solver solves the nonlinear lest-squares problem by performing the trust-region-reflective algorithm. This algorithm is a subspace trust-region method and is based on the interior-reflective Newton's method. The Matlab optimization solver is found to require more computation time to converge to the same precision as the proposed method and more computational effort for its execution. Thus, the Matlab optimization solver has less computational efficiency compared to the proposed orthogonal decomposition via Householder transformation method.

Moreover, the 6-bus test system is also tested with standard matrix inversion routine, pseudo inverse approach, Peters Wilkinson method (refer to section 1.2; pp. 7), Givens rotations method (Trefethen and Bau, 1997) and the Hybrid method (refer to section 1.2; pp. 6). The standard matrix inversion routine and pseudo inverse approach failed to converge due to the singularity of the gain matrix. The numerical stability of Peters Wilkinson method depends on the matrix $\mathbf{L}^T\mathbf{L}$ being well-conditioned. $\mathbf{L}^T\mathbf{L}$ is found to have a large condition number. Therefore, the Peters Wilkinson method failed to converge for the 6-bus test system as well.

Table 4.5 shows the convergence comparison between orthogonal decomposition using Householder transformation and Givens rotation. Both methods utilize QR factorization; their difference is the ordering method. The result in Table 4.5 illustrates that convergence is faster with Householder transformation for our 6-bus test system. Hence, it is in general more convenient and efficient to use Householder transformation when solving small bus system. Givens rotation comes into its own, however, when Jacobian matrix $\mathbf{H}$ has many leading zeros in its rows.

Table 4.5: Convergence comparison and residual of Householder transformation and Givens rotation for the 6-bus test system

Iteration	$J(\mathbf{x})$ at the beginning of the iteration (pu)	
	Householder transformation	Givens rotation
1	3663.55	3663.55
2	39.71	39.71
3	37.58	40.39
4	-	37.58

Finally, the numerical stability of the proposed orthogonal decomposition via Householder transformation method in critical condition is tested on the 6-bus test system. Virtual measurements are a kind of information that does not require metering, for example, zero injection at a switching station. The virtual measurements are treated as measurements in the tested system. Hence, to evaluate the capability of the proposed method to satisfy zero injection constraints in the presence of bad measurements, the power injection at bus 6 has been set to zero and the flows on line M_{63} in the six bus system were reversed to create bad measurements, where the active power flow, ***P***, was set to 46.8 MW and reactive power flow, *Q*, was set to 51.1 MVAR. By using the standard deviations stated in Table 4.2, the estimator has not forced the bus with zero injection to be exactly zero. This may not seem like such a big error. However, if there are many buses with zero injections and they all have errors of this magnitude, then the state estimator will have a large amount of load allocated to the buses that are known to be zero. Hence, the estimated values will be meaningless.

The solution to this dilemma is simply forcing the standard deviation values to be a very small number, by increasing the weighting factor, $W = \frac{1}{s}$ for the zero injection buses. This would force the state estimator to make the zero injections so dominant that it would result in correct zero values being produced by the estimator. It has been observed that the assignment of large weighting factors to virtual measurements may cause numerical ill-conditioning of the system.

The ill-conditioning due to giving large weights to enforce zero injection constraints is simulated by increasing weights to measurements at bus 6. With tolerance $\mathbf{\Delta}\|\mathbf{x}\|_{\infty} < 10^{-4}$, we note that the injection at bus 6 is estimated to be zero (correct to 5 decimal places) with standard deviation 0.0005 MW for real power, *P* and 0.0005 MVAR for reactive power, *Q*. Observe that, the larger the weighting factors, the closer the measurements of zero injection buses to zero values.

The changed 6-bus test system was tested with the proposed orthogonal decomposition via Householder transformation method and the Hybrid method with different weighting factors. The numbers of iterations required for convergence of the proposed method and the Hybrid method due to the changes of the weighting factor are shown in Figure 4.2.

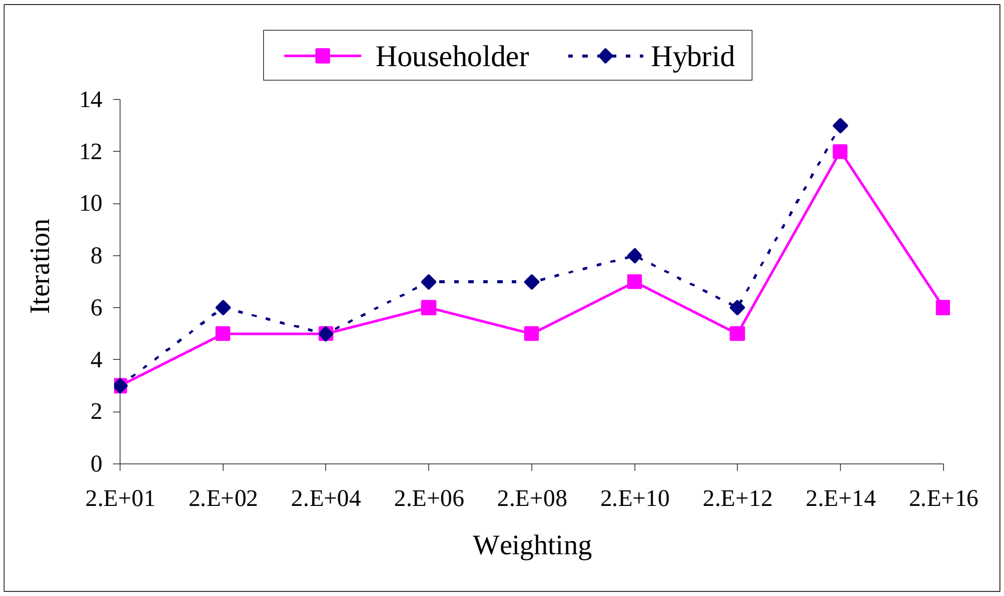

Figure 4.2: Convergence comparison of orthogonal decomposition via Householder transformation method versus the Hybrid method for the 6-bus test system

Observe that the Hybrid method takes more iteration to converge than the proposed method. Moreover the Hybrid method diverges when the weighting factor greater than $2.0\text{x}10^{16}$ while the proposed method is still converging.

Both the proposed method and the Hybrid method utilize numerically stable QR decomposition. The difference between them lies in the solution of $\mathbf{U}^T\mathbf{U}\Delta\mathbf{x} = \mathbf{H}^T\mathbf{R}^{-1}[\mathbf{z} - \mathbf{f}(\mathbf{x})]$ for the Hybrid method and $\mathbf{U}\,\Delta\mathbf{x} = \mathbf{Q}^T\mathbf{R}^{-1/2}[\mathbf{z} - \mathbf{f}(\mathbf{x})]$ for the proposed method. Although the Hybrid method does not require storage for matrix $\mathbf{Q}$, but as the weighting factor increases, the Hybrid method needs more iteration to converge and may even fail to converge. The proposed method has the advantage that the measurement weights can be adjusted to extreme values as demonstrated by the numerical example.

In conclusion, the Hybrid method is not numerically stable if the weighting factors of the virtual measurements (zero injections) are set very high. On the other hand, the proposed method is insensitive to changes in weighting factors when compared to the Hybrid method.

Next, the state estimation program will be run in a larger IEEE 30-bus test system to evaluate the capability of the proposed method in solving larger power system state estimation problem.

4.1.2 The IEEE 30-bus test system

The IEEE 30-bus test case represents a portion of the American Electric Power System (in the Midwestern US) as of December, 1961. The data was kindly provided by Iraj Dabbagchi of AEP and entered in IEEE Common Data Format by Rich Christie at the University of Washington in August 1993. Figure 4.3 shows the power system network for the IEEE 30-bus test system. The test information and measurement configurations are shown in Table 4.6.

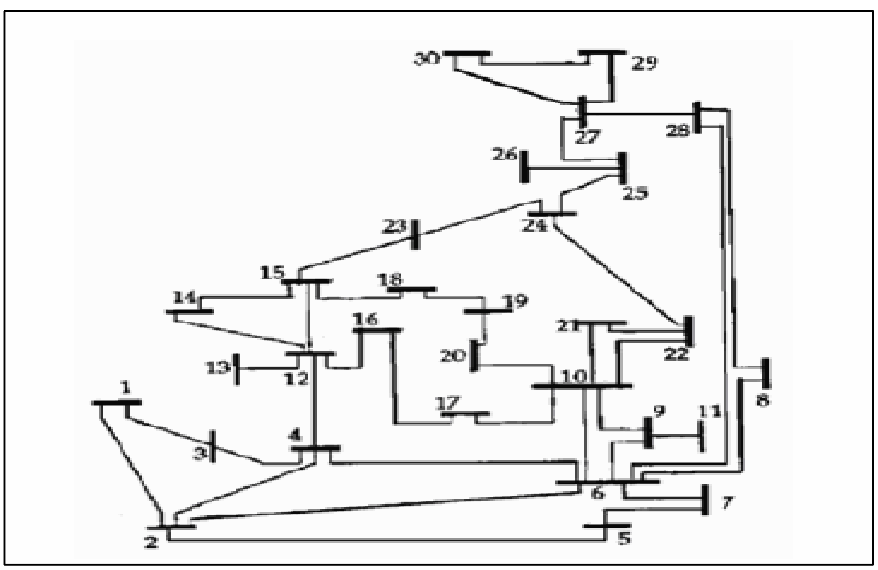

Figure 4.3: IEEE 30-bus test system

Table 4.6: IEEE 30-bus test system measurement configurations

Measurement configuration	IEEE 30-bus system
Number of buses	30
Number of lines	41
Number of voltage measurements	30
Number of injection measurement P/Q	30/30
Number of flow measurement P/Q	82/82
Number of zero injection	6/6
Total Measurements	254
Redundancy	4.2333

As in the 6-bus test system, the system measurements are obtained from the load flow result by using power flow fast decoupled method with Gaussian noise added. The measurements standard deviations used are as stated in Table 4.7.

Table 4.7: IEEE 30-bus test system measurements standard deviations

Type of measurement	Standard deviation, s
Real power, P	5 MW
Reactive power, Q	5 MVAR
Voltage, $\lvert E \rvert$	3.83 kV

The IEEE 30-bus test system contains features that present a good test for the proposed state estimation program, such as having 6 zero injection buses in the system, where the power injection measurements at buses 6, 9, 22, 25, 27 and 28 are zeroes and zero power flows from bus 9 to bus 11 and from bus 12 to bus 13. As in the 6-bus test system, large weighting factors will be assigned to the zero injection buses and zero power flow measurements so that correct zero values will be produced by the estimator for the zero injection buses and zero power flow measurements. For this purpose, the standard deviation, s , for the zero injection buses have been set to smaller values, i.e. 0.005 MW for real power, *P*, and 0.005 MVAR for reactive power, *Q*. This leads to large weighting factors and subsequently, numerical ill-conditioning of the system.

As usual, let the base case values be the measurements obtained from the load flow result by using fast decoupled method and the solution procedure started with $\mathbf{x}^0$ being initially set to 1.0 pu for voltage magnitude and 0 rad for phase angle at each bus. Table 4.8 and Table 4.9 show the estimated voltage magnitudes, power injections and power flows respectively from the state estimator after applying the proposed method for the IEEE 30-bus test system.

Table 4.8: State estimation solution of power injection for the IEEE 30-bus test system

Bus	Base Case Value			Measured Value			Estimated Value		
	kV	MW	MVAR	kV	MW	MVAR	kV	MW	MVAR
1	243.80	260.98	-21.26	241.62	270.98	-22.61	250.93	261.29	-18.54
2	240.35	18.30	42.41	237.82	26.30	42.50	247.46	21.61	41.43
3	235.22	-2.40	-1.20	227.10	-3.25	-21.20	241.43	-3.78	-7.74
4	233.29	-7.60	-1.60	250.47	-5.56	-2.56	239.87	-7.18	3.72
5	232.30	-94.20	15.96	230.00	-96.80	11.65	239.96	-91.64	15.44
6	232.74	0.00	0.00	225.35	0.00	0.00	239.55	0.00	0.00
7	230.78	-22.80	-10.90	238.49	-22.80	-10.90	238.37	-26.53	-6.12
8	232.30	-30.00	2.41	293.25	-40.00	13.41	239.78	-32.28	11.60
9	241.94	0.00	0.00	240.95	0.00	0.00	244.65	0.00	0.00
10	240.60	-5.80	-2.00	246.10	-6.55	-3.40	244.72	-4.53	11.66
11	248.86	0.00	15.68	231.01	0.00	15.29	244.65	0.00	0.00
12	243.39	-11.20	-7.50	241.50	-14.20	-8.50	247.66	-14.94	0.45
13	246.33	0.00	9.78	227.56	0.00	9.69	247.66	0.00	0.00
14	239.98	-6.20	-1.60	252.24	-6.80	-2.80	244.93	-4.63	1.40
15	238.92	-8.20	-2.50	243.50	-7.34	-13.50	242.49	-4.65	-5.59
16	240.56	-3.50	-1.80	237.45	-6.79	-10.00	244.42	-8.82	-3.24
17	239.38	-9.00	-5.80	235.64	4.00	-2.20	244.38	-0.36	-2.06
18	236.74	-3.20	-0.90	229.13	-24.50	-25.40	238.14	-12.34	-9.62
19	236.14	-9.50	-3.40	231.54	-16.80	-4.00	240.10	-7.70	5.93
20	237.08	-2.20	-0.70	232.25	-6.30	-7.00	240.83	-3.43	-2.49
21	237.77	-17.50	-11.20	286.49	-14.30	-13.30	242.70	-17.82	-3.86
22	237.89	0.00	0.00	248.15	0.00	0.00	242.70	0.00	0.00
23	236.51	-3.20	-1.60	237.94	-2.00	-7.70	240.37	-3.16	-2.37
24	235.22	-8.70	-6.70	238.14	-9.90	-16.70	239.89	-6.99	-4.66
25	234.28	0.00	0.00	232.28	0.00	0.00	240.67	0.00	0.00

Bus	Base Case Value			Measured Value			Estimated Value		
	kV	MW	MVAR	kV	MW	MVAR	kV	MW	MVAR
26	230.21	-3.50	-2.30	236.67	-3.00	-3.80	240.07	-2.47	0.96
27	235.66	0.00	0.00	286.58	0.00	0.00	241.29	0.00	0.00
28	231.86	0.00	0.00	238.51	0.00	0.00	238.72	0.00	0.00
29	231.10	-2.40	-0.90	233.40	-5.60	-1.79	237.75	-2.85	6.46
30	228.46	-10.60	-1.90	196.26	-14.50	-28.90	228.94	-9.67	-12.12

'Table 4.8, continued'

Table 4.9: State estimation solution of power flow for the IEEE 30-bus test system

From bus	To bus	Base Case Value		Measured Value		Estimated Value	
		MW	MVAR	MW	MVAR	MW	MVAR
1	2	177.801	-26.309	175.641	-21.609	177.187	-25.332
1	3	83.178	2.605	74.848	-2.355	84.101	6.792
2	1	-172.310	30.300	-171.680	31.360	-172.043	34.539
2	4	45.724	1.246	47.164	2.436	47.264	4.396
2	5	82.974	0.633	77.244	-4.407	83.042	1.485
2	6	61.918	-1.558	67.868	-5.268	63.344	1.007
3	1	-80.377	-0.624	-74.427	4.786	-81.383	-0.331
3	4	77.975	-3.867	77.785	-4.527	77.605	-7.411
4	2	-44.619	-6.063	-42.979	-4.113	-46.143	-5.112
4	3	-77.206	4.132	-76.336	4.572	-76.877	8.580
4	6	70.192	-22.430	69.262	-25.610	69.670	-20.447
4	12	44.035	-16.527	47.665	-19.327	46.168	-12.562
5	2	-79.987	2.475	-82.927	4.695	-80.224	5.660
5	7	-14.210	9.496	-3.290	4.746	-11.413	9.781
6	2	-59.875	-0.727	-60.555	3.183	-61.325	0.926
6	4	-69.601	5.966	-69.031	8.806	-69.112	12.612
6	7	37.530	-2.774	42.860	-6.884	38.418	-6.301
6	8	29.553	-4.400	29.853	-5.730	31.440	-11.736
6	9	27.792	-19.112	27.312	-25.052	26.723	-10.692
6	10	15.884	-5.781	11.724	-16.791	15.426	-3.777
6	28	18.718	-0.001	20.188	4.929	18.431	0.531
7	5	14.362	-13.207	7.682	-15.797	11.518	-11.721
7	6	-37.162	0.356	-33.592	1.996	-38.048	5.603
8	6	-29.449	2.908	-21.329	4.078	-31.316	11.191
8	28	-0.551	-3.241	-4.011	-3.131	-0.961	0.410
9	6	-27.792	21.372	-23.502	16.352	-26.723	12.245
9	10	27.792	5.840	34.062	1.100	26.723	-0.009
9	11	0.000	-15.246	0.000	0.000	0.000	0.000
10	6	-15.884	7.284	-23.084	1.354	-15.426	5.029
10	9	-27.792	-5.038	-24.932	-10.318	-26.723	0.704
10	17	5.412	4.648	3.412	12.008	1.899	1.268
10	20	9.061	3.758	12.511	4.038	12.957	2.976
10	21	15.786	10.125	19.866	4.035	15.455	5.422
10	22	7.619	4.654	11.179	4.444	7.312	2.776
11	9	0.000	15.683	0.000	0.000	0.000	0.000
12	4	-44.035	21.657	-40.695	14.907	-46.168	17.584
12	13	0.000	-9.660	0.000	0.000	0.000	0.000
12	14	7.843	2.522	1.833	7.292	7.205	1.582
12	15	17.826	7.048	17.726	7.688	16.672	10.166

From bus	To bus	Base Case Value		Measured Value		Estimated Value	
		MW	MVAR	MW	MVAR	MW	MVAR
12	16	7.166	3.269	6.386	6.549	7.350	4.162
13	12	0.000	9.777	0.000	0.000	0.000	0.000
14	12	-7.769	-2.403	-6.479	-4.703	-7.147	-1.462
14	15	1.569	0.675	-3.711	-0.635	2.514	2.861
15	12	-17.611	-6.726	-10.531	-12.796	-16.454	-9.737
15	14	-1.563	-0.672	-5.593	-7.272	-2.486	-2.835
15	18	5.981	1.688	8.621	6.348	10.833	3.884
15	23	4.992	2.971	6.092	3.031	3.460	3.098
16	12	-7.114	-3.185	-11.724	-6.415	-7.292	-4.039
16	17	3.614	1.308	-7.236	5.338	-1.531	0.802
17	10	-5.397	-4.616	-5.697	-3.456	-1.897	-1.264
17	16	-3.603	-1.287	-8.653	-6.237	1.533	-0.797
18	15	-5.943	-1.628	-2.873	5.072	-10.705	-3.624
18	19	2.743	0.677	5.283	2.127	-1.638	-5.993
19	18	-2.738	-0.669	5.722	6.721	1.661	6.040
19	20	-6.762	-2.822	-3.802	2.868	-9.356	-0.107
20	10	-8.979	-3.614	-12.199	-7.034	-12.811	-2.650
20	19	6.779	2.848	8.679	-3.612	9.384	0.162
21	10	-15.675	-9.935	-20.725	-10.295	-15.373	-5.244
21	22	-1.825	-1.468	-1.925	-3.118	-2.450	1.389
22	10	-7.566	-4.569	-7.806	-8.789	-7.273	-2.695
22	21	1.826	1.469	1.826	3.959	2.451	-1.387
22	24	5.740	3.060	4.150	10.500	4.822	4.082
23	15	-4.961	-2.922	0.519	-5.652	-3.441	-3.059
23	24	1.761	1.280	-7.609	-2.950	0.281	0.685
24	22	-5.695	-3.008	-3.555	-4.238	-4.018	-4.018
24	23	-1.755	-1.270	2.725	2.050	-0.280	-0.684
24	25	-1.250	1.987	2.400	-2.283	-1.930	0.037
25	24	1.260	-1.966	4.150	-7.976	1.937	-0.026
25	26	3.544	2.367	3.744	1.767	2.483	-0.938
25	27	-4.805	-0.423	-1.415	-0.753	-4.420	0.964
26	25	-3.500	-2.317	-0.660	0.114	-2.467	0.962
27	25	4.829	0.459	3.549	-2.521	4.440	-0.925
27	28	-18.110	5.008	-20.000	4.258	-17.412	3.617
27	29	6.190	1.679	4.710	-0.491	6.250	0.669
27	30	7.092	1.645	-0.288	1.245	6.722	5.827
28	6	-18.660	-2.464	-19.830	5.217	-18.378	-1.748
28	8	0.552	-5.471	1.142	-8.501	0.966	-5.026
28	27	18.110	-3.719	19.680	-10.459	17.412	-2.516
29	27	-6.104	-1.558	1.116	0.792	-6.171	-0.520
29	30	3.704	0.596	1.954	-3.924	3.317	6.977
30	27	-6.930	-1.419	-3.810	-1.239	-6.491	-5.394
30	29	-3.670	-0.549	0.330	-3.689	-3.183	-6.724

‘Table 4.9, continued’

Results in Table 4.8 and Table 4.9 illustrate the effectiveness of the proposed method in solving the power system state estimation problem of the larger IEEE 30-bus test system with measurement errors and numerical ill-conditioning due to large weighting factor assigned to the zero injection buses. Observe that the zero injection buses and zero power flows measurements are estimated to be zero (correct to 4 decimal places). Moreover, the reactive power measurement, Q on bus 24 shows a value of -16.7 MVAR whereas the base case value is -6.7 MVAR. The estimator has done a good job by estimating the value of Q as -4.66 MVAR, which is closer to the base case value.

With tolerance $\Delta\|\mathbf{x}\|_{\infty} < 10^{-4}$, the iterative steps for the IEEE 30-bus test system produced the results given in Table 4.10. The presence of ill-conditioning due to giving large weights to zero injection buses to enforce zero injection constraints does not prevent the estimator from converging, but only increases the value of the measurement residuals $\mathrm{J}(\mathbf{x})$.

Table 4.10: Iterative results for the IEEE 30-bus test system

Iteration	$\mathrm{J}(\mathbf{x})$ at the beginning of the iteration (pu)	Largest $\Delta\lvert\mathbf{E}\rvert$ at end of iteration (pu V)	Largest Δq at end of iteration (rad)
1	11847970	0.09542755	0.31939581
2	3018381	0.00834751	0.03742904
3	891.2458	0.00034716	0.00026147
4	349.6367	0.00000346	0.00000642

The results above showed that the proposed state estimation program is applicable to a large numerical ill-conditioning system and is found to be numerically stable for such a system.

The IEEE 30-bus test system is also tested with the Matlab lsqnonlin solver. This solver converges more slowly than the proposed method and fails to give zero estimation for zero injection equality constraints in the IEEE 30-bus system state estimation problem.

Moreover, the IEEE 30-bus test system is also tested with standard matrix inversion routine, pseudo inverse approach, Peters Wilkinson method (refer to section

1.2; pp. 7), Givens rotations method (Trefethen and Bau, 1997) and the Hybrid method (refer to section 1.2; pp. 6).

The standard matrix inversion routine and pseudo inverse approach failed to converge due to the singularity of the gain matrix. The numerical stability of Peters Wilkinson method depends on the matrix $\mathbf{L}^T\mathbf{L}$ being well-conditioned. $\mathbf{L}^T\mathbf{L}$ is found to have a large condition number. Therefore, the Peters Wilkinson method failed to converge for the IEEE 30-bus test system as well.

To compare the stability of the proposed method with the Hybrid method (a well known numerically stable method) and the Givens rotation method in solving the IEEE 30-bus system state estimation problem, the system is tested with several weighting factors. With tolerance $\Delta\|\mathbf{x}\|_\infty < 10^{-4}$**,** the numbers of iterations required for convergence of the proposed method and the Hybrid method are shown in Figure 4.4. The Givens rotation method has the same numbers of iterations required for convergence as the proposed method.

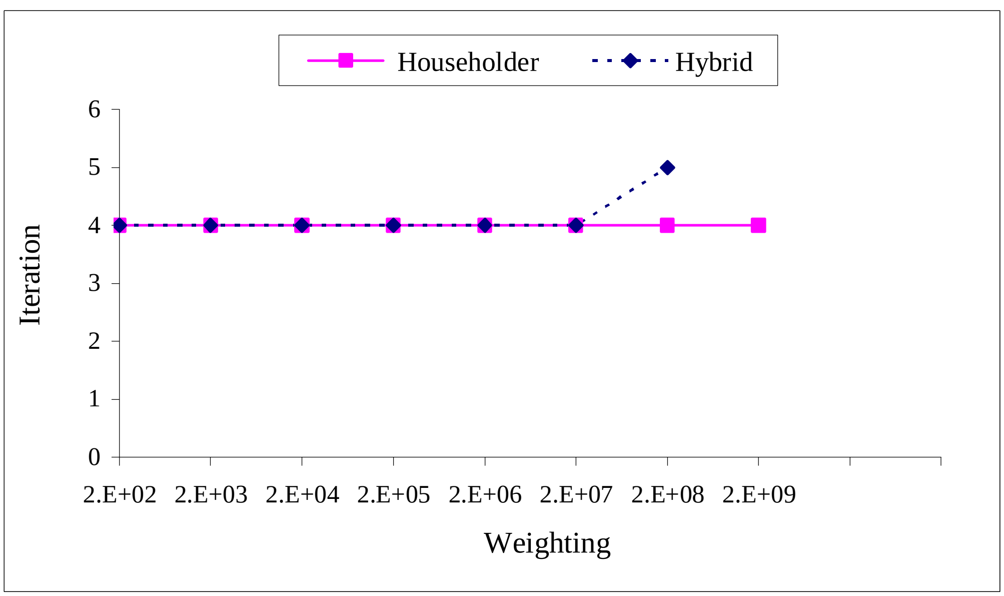

Figure 4.4: Convergence comparison of orthogonal decomposition via Householder transformation method versus the Hybrid method for the IEEE 30-bus test system

The convergence result in Figure 4.4 shows that both methods perform almost the same for weighting factors that are less than 2.0×10^8. However, for weighting factors larger than that, the Hybrid method has slower convergence and eventually diverges, while the proposed method still performs well. The result also shows that the orthogonal decomposition via Householder transformation method has higher numerical stability compared with the Hybrid method for a large system with numerical ill-conditioning.

In addition to numerical stability, the efficiency of a solution method is also reliant on computation time. For the comparison of computation efficiency, the proposed orthogonal decomposition via Householder transformation method, the Givens rotation method and the Hybrid method (both utilized QR factorization) are tested on the IEEE 30-bus test system. The state estimation algorithm was implemented in MATLAB 7.7 and run on a personal computer having a 2.99 GHz Intel ® Core ™ 2 Duo CPU and 3.48 GB of RAM. Table 4.11 shows the comparison of the three aforementioned methods in terms of their average computation time per iteration for the IEEE 30-bus test system. The computation time per iteration includes the time spent on factorization and back substitutions.

Table 4.11: Average computation time per iteration for the IEEE 30-bus test system

Method	No. of Iteration	Average computation time per iteration (second)
Householder transformation	4	0.015475
Givens rotation	4	0.457425
Hybrid method	4	0.0111

The result in Table 4.11 illustrates that all methods have the same number of convergence iterations but with different amount of computation time. Among the three, the Givens rotation method requires the longest computation time. The numerical stability of the Givens rotation method is comparable to the proposed method. Yet, the computation time needed by the former is about three times that of the latter. Therefore, the Givens rotation method is costlier than the proposed method albeit having comparable numerical stability. Although the computation time of the Hybrid method is slightly less than the proposed method, it has inferior numerical stability.

In conclusion, the orthogonal decomposition via Householder transformation method is found to have good performance in terms of numerical stability and computation efficiency for a large system with numerical ill-conditioning.

4.1.3 The 30-bus Sumatera Barat power system

The 30-bus Sumatera Barat power system is a comprehensive test system since it is a practical power system used by Perusahaan Listrik Negara (PLN) in Indonesia. PLN is the only vertically integrated electricity utility in Indonesia. It is a monopoly operator of transmission and distribution networks of over 32,000 km and 580,000 km in length respectively. It is the Indonesia's largest electricity producer with a generation capacity of over 22,000 MW, accounting for 85% of the market. Hence, the power system it uses is a good test system that can be used to gauge the performance of the proposed method in solving the power system state estimation problem. Figure 4.5 shows the power system network for the 30-bus Sumatera Barat power system. The system information is shown in Table 4.12 and the measurement standard deviations are shown in Table 4.13.

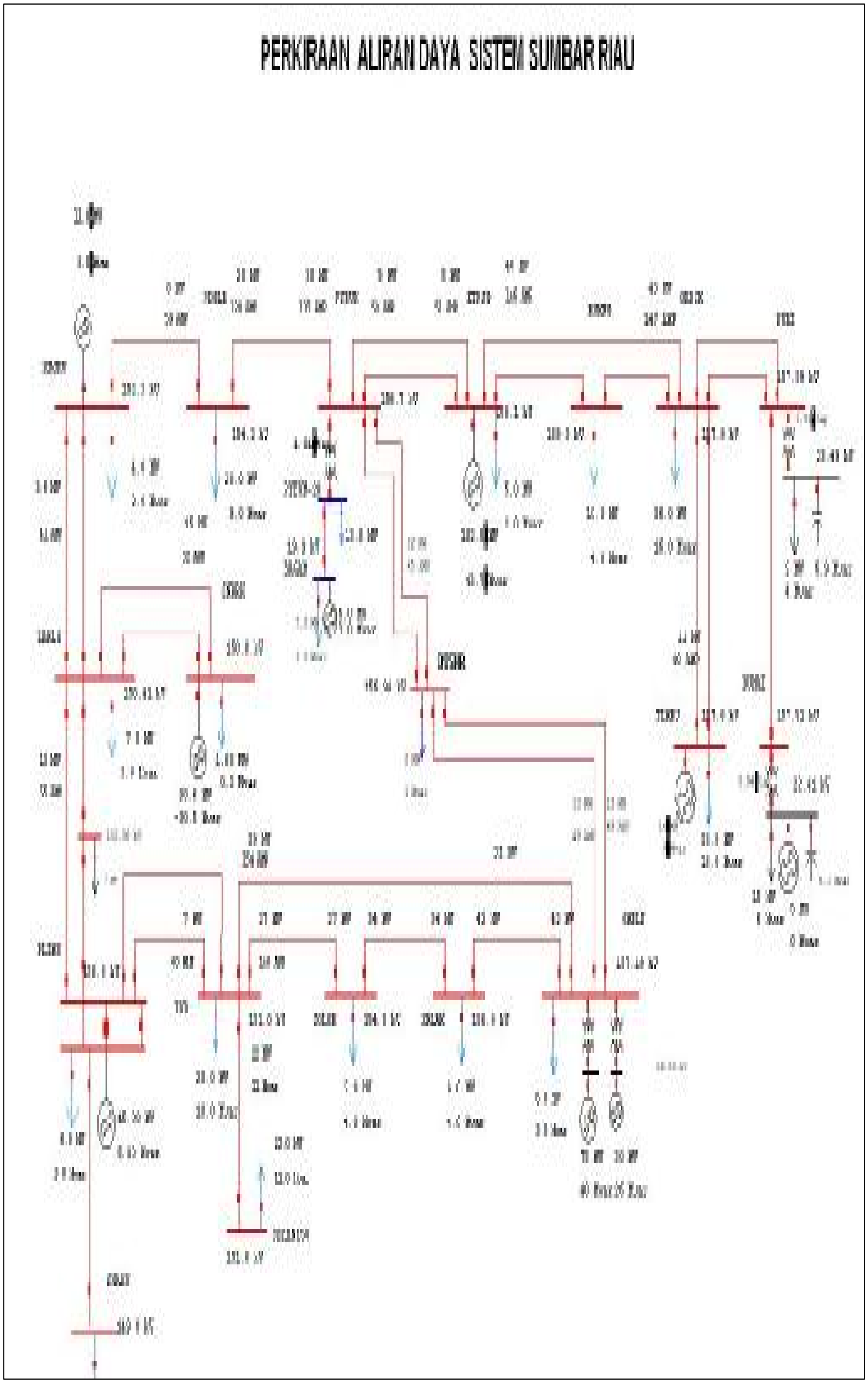

Figure 4.5: 30-bus Sumatera Barat power system

Table 4.12: System information for the 30-bus Sumatera Barat power system

Measurement configuration	30-bus Sumatera Barat power system
Buses	30
Generators	9
Loads	21
Switched shunts	2
AC. Trans. lines	33
LTCs (control volt)	6
Slack bus	SNKRK(9)
Number of voltage measurements	30
Number of injection measurement P/Q	30/30
Number of flow measurement P/Q	82/82
Number of zero injection	6/6
Total Measurements	214
Redundancy	3.5667

Table 4.13: 30-bus Sumatera Barat power system measurements standard deviation

Type of measurement	Standard deviation, s
Real power, P	1 MW
Reactive power, Q	1 MVAR
Voltage, $\lvert E \rvert$	3.16 kV

The 30-bus Sumatera Barat power system consists of 6 zero injection buses. The power injection measurements at buses 12, 18, 19, 21, 23 and 24 are zeroes and zero active power flows from bus 6 to bus 21. Hence, large weighting factors are assigned to the zero injection buses so that correct zero values will be produced by the estimator for the zero injection buses and power flow measurements. Due to this, the standard deviation, s for the zero injection buses have been set to smaller values, i.e. 0.0001 MW for real power, P, and 0.0001 MVAR for reactive power, Q. These large weighting factors will lead to numerical ill-conditioning of the system.

The 30-bus Sumatera Barat power system state estimation problem was solved using the proposed method and the results are shown in Table 4.14 and Table 4.15. The base case values are the system measurements obtained from the load flow result by using the PowerWorld Simulator (http://www.powerworld.com/products.asp), whereas the measured values were obtained by adding Gaussian noise.

Table 4.14: State estimation solution of power injection for the 30-bus Sumatera Barat power system

Bus	Base Case Value			Measured Value			Estimated Value		
	kV	MW	MVAR	kV	MW	MVAR	kV	MW	MVAR
1	157.19	-6.00	-3.00	157.19	-5.87	-3.21	143.90	-5.57	-3.32
2	156.90	-8.00	-4.00	152.13	-8.22	-4.10	143.58	-7.42	-3.28
3	154.02	-7.00	-4.00	170.45	-7.26	-4.06	140.39	-7.54	-3.98
4	150.96	-20.00	-10.00	122.85	-20.23	-10.08	136.92	-23.04	-9.10
5	150.96	-22.00	-11.00	157.38	-22.41	-10.68	136.92	-16.67	-12.07
6	150.63	15.00	5.00	164.07	15.38	5.45	136.53	11.92	6.57
7	149.83	-46.00	-22.00	160.80	-46.38	-21.71	-135.63	-47.30	-22.11
8	150.41	-7.00	-3.00	159.08	-6.62	-2.31	136.32	-6.91	-3.43
9	150.00	29.55	-31.02	150.60	29.66	-31.10	135.84	30.95	-33.13
10	152.32	7.00	3.50	162.48	7.11	3.06	138.63	7.26	3.05
11	154.18	-20.00	-9.00	162.71	-19.49	-9.00	140.81	-20.77	-9.17
12	156.74	0.00	0.00	152.90	0.00	0.00	143.67	0.00	0.00
13	20.05	-13.00	-6.00	19.29	-12.67	-5.68	18.06	-14.26	-8.80
14	19.93	-1.00	0.00	19.34	-0.76	0.50	18.38	-1.06	1.68
15	160.13	97.00	41.70	138.00	97.21	41.38	147.75	96.93	37.59
16	159.34	-10.00	-4.00	155.83	-10.17	-3.30	147.05	-9.12	-3.19
17	157.94	-36.00	-16.00	159.71	-36.07	-15.18	145.68	-35.19	-17.18
18	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
19	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
20	150.52	-6.00	-3.00	145.26	-5.98	-3.40	136.41	-1.93	-7.53
21	22.32	0.00	0.00	23.56	0.00	0.00	20.23	0.00	0.00
22	157.61	-21.00	-6.00	169.60	-21.61	-6.22	145.25	-21.17	-9.03
23	157.89	0.00	0.00	172.00	0.00	0.00	146.12	0.00	0.00
24	157.52	0.00	0.00	142.64	0.00	0.00	145.97	0.00	0.00
25	156.98	-2.00	-1.00	160.16	-2.19	-0.50	143.81	-1.79	-2.87
26	23.49	-9.00	-4.00	23.96	-8.80	-4.69	21.74	-8.59	2.60
27	22.41	-10.00	-6.00	20.40	-9.72	-5.66	20.86	-10.18	1.60
28	12.42	75.00	40.00	11.56	74.50	39.30	11.42	74.57	39.80
29	12.30	30.00	26.00	13.55	29.54	25.18	11.28	28.15	23.29
30	150.36	-7.00	-3.00	148.39	-6.98	-3.60	136.19	-8.12	-5.07

Table 4.15: State estimation solution of power flow for the 30-bus Sumatera Barat power system

From bus	To bus	Base Case Value		Measured Value		Estimated Value	
		MW	MVAR	MW	MVAR	MW	MVAR
1	2	38.845	33.918	40.045	35.418	41.178	32.715
1	4	30.529	25.996	30.829	24.096	29.856	27.410
1	25	12.964	-1.856	10.964	-2.356	13.059	-3.334
1	28	-75.069	-34.373	-76.869	-35.773	-74.568	-33.228
1	29	-29.981	-24.752	-28.581	-25.052	-28.150	-22.033
2	1	-38.814	-33.996	-41.814	-29.096	-41.140	-32.736
2	3	34.245	28.445	36.745	26.545	33.724	29.457
3	2	-33.977	-29.316	-32.177	-33.616	-33.401	-29.828
3	4	26.878	24.425	24.278	25.925	25.861	25.849
4	1	-30.006	-27.804	-30.806	-30.704	-29.224	-28.202
4	3	-26.646	-25.519	-30.446	-25.419	-25.579	-26.432
4	5	15.096	15.086	8.096	13.086	16.672	12.052
4	6	7.164	15.519	10.264	17.219	7.544	16.851
5	4	-15.096	-15.106	-16.696	-13.406	-16.671	-12.068
6	4	-7.154	-15.745	-6.154	-16.345	-7.530	-17.015
6	8	-12.931	5.051	-12.231	-1.449	-13.065	4.917
6	20	36.654	36.553	36.754	36.953	40.046	35.895
6	21	0.000	-22.433	0.000	-17.433	0.000	-18.429
7	20	-45.887	-22.129	-48.887	-18.929	-47.301	-22.106
8	6	12.966	-6.795	11.766	-11.795	13.108	-6.299
8	9	-15.022	15.316	-18.822	15.116	-15.447	16.294
8	10	-3.556	-11.764	-6.856	-13.964	-3.316	-12.764
8	30	16.621	-3.947	21.321	-7.147	17.513	-2.639
9	8	15.043	-15.544	15.243	-19.444	15.475	-16.429
10	8	3.596	10.327	-0.204	11.227	3.373	11.666
10	11	-0.203	-17.889	-0.303	-19.289	0.519	-18.956
11	10	0.260	15.103	-3.340	15.303	-0.437	16.756
11	12	-20.270	-25.443	-24.570	-26.643	-20.337	-25.930
12	11	20.428	23.651	19.628	22.151	20.534	24.652
12	13	13.997	8.503	16.997	9.703	15.349	10.141
12	15	-5.036	-15.340	-4.636	-13.040	-5.833	-16.538
12	25	-12.264	-0.394	-10.164	6.306	-12.109	1.183
13	12	-13.997	-7.131	-17.697	-11.431	-15.349	-8.081
13	14	1.008	0.001	-0.492	-3.199	1.085	-1.637
14	13	-1.002	-0.003	-1.802	3.297	-1.059	1.683
15	12	5.102	13.151	1.302	11.951	5.929	14.881
15	16	46.622	8.899	42.422	4.499	45.640	6.192
15	17	39.907	4.947	42.807	5.947	39.434	3.817
16	15	-46.513	-9.469	-46.513	-4.569	-45.520	-6.512
16	17	36.661	4.067	34.661	6.267	36.399	3.318
17	15	-39.629	-8.421	-37.129	-2.221	-39.118	-6.301

From bus	To bus	Base Case Value		Measured Value		Estimated Value	
		MW	MVAR	MW	MVAR	MW	MVAR
17	16	-36.491	-7.224	-35.791	-5.624	-36.203	-5.701
17	22	10.507	1.903	7.407	7.803	10.600	3.137
17	23	9.597	-3.262	13.797	-4.362	9.468	-4.524
20	6	-36.627	-36.728	-35.727	-40.328	-40.011	-36.026
20	7	45.982	21.989	50.682	21.289	47.423	22.150
20	30	-9.834	5.724	-6.234	9.524	-9.341	6.342
21	6	0.000	24.925	0.000	21.425	0.000	20.477
22	17	-10.494	-2.986	-14.594	-0.986	-10.584	-4.039
23	17	-9.543	1.759	-9.743	-0.141	-9.400	3.317
23	24	10.037	-1.503	9.037	0.198	10.215	-2.965
23	26	8.964	-25.324	6.264	-28.824	8.585	-21.733
24	23	-10.008	-0.549	-8.408	-0.249	-10.180	1.244
24	27	10.006	-21.177	14.706	-27.477	10.180	-19.444
25	1	-12.939	0.133	-14.639	-1.467	-13.029	1.925
25	12	12.284	-0.725	9.584	0.775	12.133	-2.098
26	23	-8.964	28.600	-9.764	27.600	-8.585	24.628
27	24	-10.006	22.903	-7.906	26.803	-10.180	21.208
28	1	75.069	40.003	72.369	38.003	74.568	39.797
29	1	29.981	26.000	26.181	18.600	28.150	23.292
30	8	-16.596	3.104	-16.996	-0.796	-17.479	1.992
30	20	9.846	-6.627	9.646	-3.327	9.355	-7.067

‘Table 4.15, continued’

Results in Table 4.14 and Table 4.15 show that the state estimation program is feasible on the practical 30-bus Sumatera Barat power system with measurement errors and numerical ill-conditioning due to large weighting factors assigned to the zero injection buses. Observe that the zero injection buses and zero power flows measurements are estimated to be zero (correct to 4 decimal places). Moreover, the reactive power flow from bus 17 to bus 22 shows a value of 7.803 MVAR whereas the base case value is 1.903 MVAR. The estimator has done a good job by estimating 3.137 MVAR, which is closer to the base case value.

With tolerance $\Delta\|\mathbf{x}\|_{\infty} < 10^{-4}$, the iterative steps for the 30-bus Sumatera Barat power system produced the results given in Table 4.16. The presence of ill-conditioning due to giving large weights to zero injection buses to enforce zero injection constraints does not prevent the estimator from converging, but only increases the value of the measurement residuals $J(\mathbf{x})$.

Table 4.16: Iterative result for the 30-bus Sumatera Barat power system

Iteration	$J(\mathbf{x})$ at the beginning of the iteration (pu)	Largest $\Delta\lvert\mathbf{E}\rvert$ at end of iteration (pu V)	Largest Δq at end of iteration (rad)
28	5048.6	0.00000287	0.00000276

The 30-bus Sumatera Barat power system is also tested with the Matlab lsqnonlin solver. This solver converges more slowly than the proposed method and fails to give zero estimation for zero injection equality constraints in the 30-bus Sumatera Barat power system state estimation problem.

Again, to compare the numerical stability of the proposed method in solving the 30-bus Sumatera Barat power system with other methods, the system is tested with the standard matrix inversion routine, pseudo inverse approach, Peters Wilkinson method (refer to section 1.2; pp. 7), Givens rotations method (Trefethen and Bau, 1997) and the Hybrid method (refer to section 1.2; pp. 6).

The standard matrix inversion routine and pseudo inverse approach failed to converge due to the singularity of the gain matrix. The numerical stability of Peters Wilkinson method depends on the matrix $\mathbf{L}^T\mathbf{L}$ being well-conditioned. $\mathbf{L}^T\mathbf{L}$ is found to have a large condition number. Therefore, the Peters Wilkinson method failed to converge for the 30-bus Sumatera Barat power system as well.

To compare the stability of the proposed method with the Hybrid method (a well known numerically stable method) and the Givens rotation method in solving the 30-bus Sumatera Barat power system state estimation problem, the system is tested with several weighting factors._With tolerance $\Delta\|\mathbf{x}\|_\infty < 10^{-4}$**,** the numbers of iterations required for convergence of the proposed method and the Hybrid method are shown in

Figure 4.6. The Givens rotation method has the same numbers of iterations required for convergence as the proposed method.

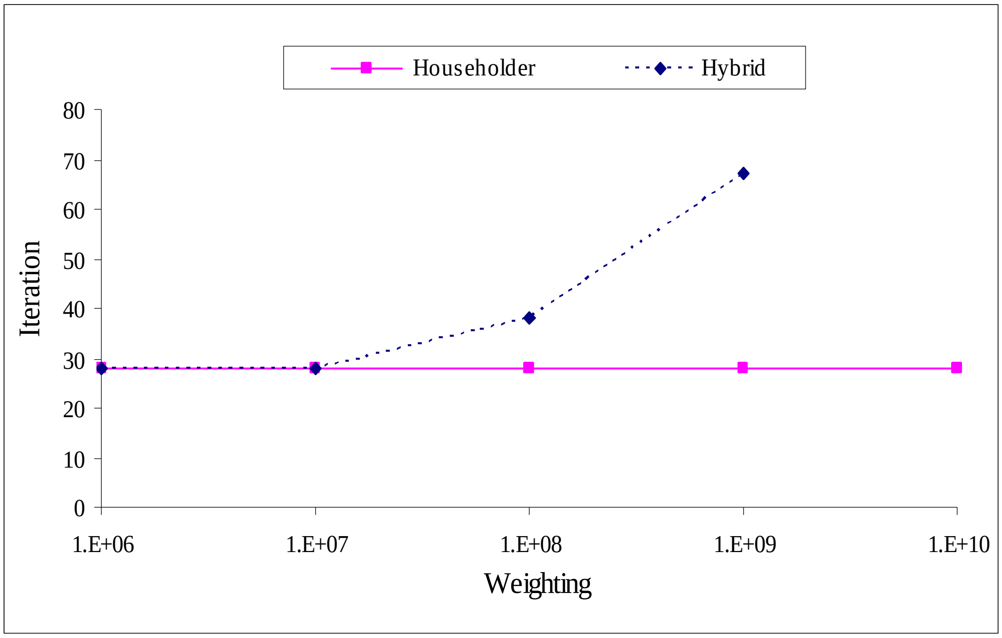

Figure 4.6: Convergence comparison of orthogonal decomposition via Householder transformation method versus the Hybrid method for the 30-bus Sumatera Barat power system

The convergence comparison result in Figure 4.6 shows that both methods perform similarly for weighting factor less than $1x10^8$. However, the Hybrid method converges slower and eventually diverges when the weighting factor is set to be more than $1x10^9$ while the proposed method is still able to converge. This shows that the orthogonal decomposition via Householder transformation method has higher numerical stability compared to the Hybrid method for a large practical system with numerical ill-conditioning.

Apart from numerical stability, the efficiency of a solution method is also influenced by computation time. For the comparison of computational efficiency, the 30-bus Sumatera Barat power system is tested with three methods: the proposed orthogonal decomposition via Householder transformation method, Givens rotation method and the Hybrid method (both utilize QR factorization). The state estimation algorithm was implemented in MATLAB 7.7 and run on a personal computer having a 2.99 GHz Intel ® Core ™ 2 Duo CPU and 3.48 GB of RAM. Table 4.17 shows the comparison of the computation times of the three aforementioned methods. Note that the computation times include the time spent on factorization and back substitutions.

Table 4.17: Computation time for the 30-bus Sumatera Barat power system

Method	No. of Iteration	Total computation time (second)	Average computation time per iteration (second)
Householder transformation	28	0.3063	0.0109
Givens rotation	28	10.0387	0.3585
Hybrid method	28	0.2420	0.0086

The result in Table 4.17 illustrates that all three methods have the same number of convergence iteration but with different computation times. Among the three, the Givens rotation method requires the longest computation time. The numerical stability of the Givens rotation method is comparable to that of the proposed orthogonal decomposition via Householder transformation method. However, the computation time needed by the Givens rotation method is about 33 times greater than that of the proposed method. In other words, the Givens rotation method is costlier than the proposed method, albeit having similar numerical stability. Whereas the computation time for Hybrid method is slightly less than the proposed method. The computation efficiency of the proposed method is very competitive with the Hybrid method.

Consequently, the proposed orthogonal decomposition via Householder transformation method is considered to be a good solution method for practical power system state estimation due to numerical stability and computation efficiency.