APPENDIX A

NEWTON'S METHOD

This section describes the Newton's method for multi-dimensional problems. Suppose we wish to find $\mathbf{x}$ such that $\mathbf{g}(\mathbf{x}) = \mathbf{0}$. The function $\mathbf{g}$ and the unknowns $\mathbf{x}$, are vectors. Then, to use Newton's method, we observe:

$$\mathbf{g}(\mathbf{x} + \mathbf{\Delta x}) = \mathbf{g}(\mathbf{x}) + [\mathbf{g}'(\mathbf{x})]\mathbf{\Delta x} = \mathbf{0},$$

where we have expanded $\mathbf{g}(\mathbf{x} + \mathbf{\Delta x})$ in a Taylor's series about $\mathbf{x}$ and ignored all higher-order terms. If we let $\mathbf{g}(\mathbf{x})$ be defined as:

$$\mathbf{g}(\mathbf{x}) = \begin{bmatrix} g_1(x_1, x_2, x_3) \\ g_2(x_1, x_2, x_3) \\ g_3(x_1, x_2, x_3) \end{bmatrix},$$

then the Jacobian matrix of the first derivatives of $\mathbf{g}(\mathbf{x})$ is

$$\mathbf{g}'(\mathbf{x}) = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \frac{\partial g_1}{\partial x_2} & \frac{\partial g_1}{\partial x_3} \\ \frac{\partial g_2}{\partial x_1} & \frac{\partial g_2}{\partial x_2} & \frac{\partial g_2}{\partial x_3} \\ \frac{\partial g_3}{\partial x_1} & \frac{\partial g_3}{\partial x_2} & \frac{\partial g_3}{\partial x_3} \end{bmatrix}.$$

The adjustment at each step is then:

$$\mathbf{\Delta x} = [\mathbf{g}'(\mathbf{x})]^{-1}[-\mathbf{g}(\mathbf{x})].$$