APPENDIX C

JACOBIAN MATRIX: H(x)

$$\mathbf{H(x)} = \begin{bmatrix} \frac{\partial|E_i|}{\partial|E_i|} & \mathbf{L} & \frac{\partial|E_i|}{\partial|E_j|} & \mathbf{L} & \frac{\partial|E_i|}{\partial q_i} & \mathbf{L} & \frac{\partial|E_i|}{\partial q_j} \\ \mathbf{M} & \mathbf{L} & \mathbf{M} & \mathbf{L} & \mathbf{M} & \mathbf{L} & \mathbf{M} \\ \frac{\partial P_i}{\partial|E_i|} & \mathbf{L} & \frac{\partial P_i}{\partial|E_j|} & \mathbf{L} & \frac{\partial P_i}{\partial q_i} & \mathbf{L} & \frac{\partial P_i}{\partial q_j} \\ \mathbf{M} & \mathbf{L} & \mathbf{M} & \mathbf{L} & \mathbf{M} & \mathbf{L} & \mathbf{M} \\ \frac{\partial Q_i}{\partial|E_i|} & \mathbf{L} & \frac{\partial Q_i}{\partial|E_j|} & \mathbf{L} & \frac{\partial Q_i}{\partial q_i} & \mathbf{L} & \frac{\partial Q_i}{\partial q_j} \\ \mathbf{M} & \mathbf{L} & \mathbf{M} & \mathbf{L} & \mathbf{M} & \mathbf{L} & \mathbf{M} \\ \frac{\partial P_{ij}}{\partial|E_i|} & \mathbf{L} & \frac{\partial P_{ij}}{\partial|E_j|} & \mathbf{L} & \frac{\partial P_{ij}}{\partial q_i} & \mathbf{L} & \frac{\partial P_{ij}}{\partial q_j} \\ \mathbf{M} & \mathbf{L} & \mathbf{M} & \mathbf{L} & \mathbf{M} & \mathbf{L} & \mathbf{M} \\ \frac{\partial Q_{ij}}{\partial|E_i|} & \mathbf{L} & \frac{\partial Q_{ij}}{\partial|E_j|} & \mathbf{L} & \frac{\partial Q_{ij}}{\partial q_i} & \mathbf{L} & \frac{\partial Q_{ij}}{\partial q_j} \\ \mathbf{M} & \mathbf{L} & \mathbf{M} & \mathbf{L} & \mathbf{M} & \mathbf{L} & \mathbf{M} \end{bmatrix}$$

Voltage magnitudes:

$$\frac{\partial|E_i|}{\partial|E_i|} = 1 \qquad \frac{\partial|E_i|}{\partial q_i} = 0$$

$$\frac{\partial|E_i|}{\partial|E_j|} = 0 \qquad \frac{\partial|E_i|}{\partial q_j} = 0$$

Real power injections:

$$\frac{\partial P_i}{\partial|E_i|} = |E_i|G_{ii} + \frac{P_i}{|E_i|}$$

$$\frac{\partial P_i}{\partial|E_j|} = |E_i|\left[G_{ij}\cos(q_i - q_j) + B_{ij}\sin(q_i - q_j)\right]$$

$$\frac{\partial P_i}{\partial q_i} = -Q_i - |E_i|^2 B_{ii}$$

$$\frac{\partial P_i}{\partial q_j} = |E_i||E_j|\left[G_{ij}\sin(q_i - q_j) - B_{ij}\cos(q_i - q_j)\right]$$

Reactive power injections:

$$\frac{\partial Q_i}{\partial|E_i|} = -|E_i|B_{ii} + \frac{Q_i}{|E_i|}$$

$$\frac{\partial Q_i}{\partial|E_j|} = |E_i|\left[G_{ij}\sin(q_i - q_j) - B_{ij}\cos(q_i - q_j)\right]$$

$$\frac{\partial Q_i}{\partial q_i} = P_i - |E_i|^2 G_{ii}$$

$$\frac{\partial Q_i}{\partial q_j} = -|E_i||E_j|\left[G_{ij}\cos(q_i - q_j) + B_{ij}\sin(q_i - q_j)\right]$$

Real power flows:

$$\frac{\partial P_{ij}}{\partial |E_i|} = -2|E_i|G_{ij} + |E_j|\left[G_{ij}\cos(q_i - q_j) + B_{ij}\sin(q_i - q_j)\right]$$

$$\frac{\partial P_{ij}}{\partial |E_j|} = |E_i|\left[G_{ij}\cos(q_i - q_j) + B_{ij}\sin(q_i - q_j)\right]$$

$$\frac{\partial P_{ij}}{\partial q_i} = |E_i||E_j|\left[-G_{ij}\sin(q_i - q_j) + B_{ij}\cos(q_i - q_j)\right]$$

$$\frac{\partial P_{ij}}{\partial q_j} = |E_i||E_j|\left[G_{ij}\sin(q_i - q_j) - B_{ij}\cos(q_i - q_j)\right]$$

Reactive power flows:

$$\frac{\partial Q_{ij}}{\partial |E_i|} = 2|E_i|\left[B_{ij} - B_{cap\,i}\right] + |E_j|\left[G_{ij}\sin(q_i - q_j) - B_{ij}\cos(q_i - q_j)\right]$$

$$\frac{\partial Q_{ij}}{\partial |E_j|} = |E_i|\left[G_{ij}\sin(q_i - q_j) - B_{ij}\cos(q_i - q_j)\right]$$

$$\frac{\partial Q_{ij}}{\partial q_i} = |E_i||E_j|\left[G_{ij}\cos(q_i - q_j) + B_{ij}\sin(q_i - q_j)\right]$$

$$\frac{\partial Q_{ij}}{\partial q_j} = -|E_i||E_j|\left[G_{ij}\cos(q_i - q_j) + B_{ij}\sin(q_i - q_j)\right]$$