CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

In portfolio selection theory, a rational investor who seeks to maximize his expected utility encounters the optimization problem of choosing a portfolio from a feasible set of competing alternatives that maximize his end-of-period wealth. The mean-variance (MV) analysis (Markowitz, 1952) provides investors the principle of the portfolio selection theory. To achieve the MV efficient portfolios, the bi-objective optimization problem is solved by maximizing the expected returns and minimizing the variance simultaneously. Among a set of MV efficient portfolios, the portfolio choices of investors depend on their trade–off between risk and returns. In other words, an investor chooses his preferred portfolio based upon individual preference for risk and returns which is theoretically described by the utility function.

However, unrealistic assumptions that the MV analysis relies on have been widely criticized in the literature. The major shortcoming is its ability to handle the asymmetric distribution of asset returns. It is assumed by the MV model that asset returns are normally distributed during the period of analysis and parameterized by only the mean and variance of the return distributions. However, ample evidences reveal that the return distributions are non-normal (Amaya & Vasquez, 2010; Arditti, 1967; Bates, 1996; Bekaert & Harvey, 2002; Black, 1976; Campbell & Hentschel, 1992; Canela & Collazo, 2007; Chen, Hong, & Stein, 2001; Christie, 1982; Chunhachinda, Dandapani, Hamid, & Prakash, 1997; Fielitz, 1976; French, Schwert, & Stambaugh, 1987; Gennotte & Leland, 1990; Gibbons, Ross, & Shanken, 1989; Grossman, 1989; Hwang & Satchell, 1999; Jorion, 1988; Prakash, Chang, & Pactwa, 2003; Simkowitz & Beedles, 1978; Singleton & Wingender, 1986; Wang, Meric, Liu, & Meric, 2009). Although, Levy and Markowitz (1979) argued that the MV model might maximize the expected utility, even when return distributions are not normal, if the variation in rate of returns is relatively small. This idea is acceptable but very costly in practice because portfolio rebalancing is required frequently.

In addition, the MV model assumes that an investor's preference can be described by a preference function over the mean and the variance of the portfolio return. This implies that the utility of the investor is approximated by the quadratic function. Although, the quadratic utility function satisfies the non-satiation and risk aversion property, the critical invalidity is that it exhibits an increasing absolute risk aversion for all level of wealth (Bierwag, 1974; Blume & Friend, 1975; Borch, 1974; Friend & Blume, 1975). In the context of portfolio selection, decreasing absolute risk aversion for wealth implies that when an agent experiences an increase in wealth, he will increase the portion of risky asset in his portfolio. In a contrary, an investor with increasing absolute risk aversion for wealth will reduce the proportion of risky asset in portfolio when facing the same situation. The latter case is an unreasonable behavior in the real economy. Besides, another shortcoming of a quadratic utility is the bounded range of possible outcome. Since positive marginal utility is acknowledged as a desirable property of any utility function, quadratic utility is consistent with this property in some range of wealth (Hanoch & Levy, 1970; Levy, 1974). These evidences suggest that the use of quadratic utility should be restricted.

Moreover, the MV analysis assumes that asset prices follow a diffusion or stochastic process in an investor's decision. As a consequence, based on Itõ's lemma which is an identity used in Itõ calculus for finding the differential of a time-dependent function of a stochastic process, moments higher than the second order are not relevant in the investment decision. However, Samuelson (1970) demonstrated that an investor's decision is actually restricted to a discrete time horizon, therefore, efficiency of the MV model could be inadequate for making an investment decision and higher moments should not be neglected in portfolio selection.

1.2 WHY SKEWNESS PREFERENCE MATTERS?

Statistically, skewness is a measure of the asymmetry of the probability distribution of a random variable. Its value can be either unbounded positive or negative. In the case of a normal distribution, skewness is zero. For a distribution with positive skewness, the tail on the right side of the probability density function is longer than that on the left and the mass of the distribution is concentrated on the left side of the distribution. For the distribution with negative skewness, the left tail of the probability density function is longer and the mass is concentrated on the right side of the distribution. In the context of investment alternative, a security exhibiting positive skewness in its return distribution has the chances of gaining extreme positive returns. On the other hand, the probabilities of potential large loss are inherited in an asset with a return distribution of negative skewness. Hence, intuitively, rational investors would prefer a security that possesses a return distribution with higher skewness, if the mean and the variance are equal.

Theoretically, the preference for positive skewness can be examined, if investor utility is described by a preference function over the mean, standard deviation, and skewness of the portfolio return distributions. The majority of researchers developed the skewness preference theory based upon the third-order Taylor's series approximation of the expected utility. This approach is favored among researchers and economists because it is consistent with the desirable properties of utility function described by Arrow (1964) and Pratt (1964). These properties include (a) positive marginal utility for wealth, i.e., non-satiety, (b) decreasing marginal utility for wealth, i.e., risk aversion, and (c) non-increasing absolute risk aversion for wealth, i.e., risky assets are not inferior good.

Among the pioneer works on skewness preference theory, Aditti (1967) demonstrated that an investor whose utility can be approximated by the third-order Taylor's expansion around the expected value will prefer positive skewness in asset returns. He theoretically explained that skewness preference is not a gambling behavior but is a common trait for a rational risk-averse investor. Subsequently, Levy (1969) demonstrated that skewness preference can be proved even when the risk aversion assumption in Arditti's work is relaxed. He showed that at a low level of wealth, utility of an investor is concave but will converge to convex when his wealth has accumulated beyond a certain level. In addition, various forms of utility function such as negative exponential, constant elastic function, hyperbolic absolute risk aversion and log function were analyzed using the Taylor's series expansion to establish the skewness preference theory (Ingersoll, 1975; Kane, 1982; Tsiang, 1972). Moreover, Scott and Horvath (1980) showed that investors who exhibit positive marginal utility, consistent a risk aversion for all wealth levels and strictly consistent moment preference will prefer assets with positive skewness in the return distributions.

Apart from the expected utility model, a collection of empirical papers on asset pricing provides the evidences that contribute to the skewness preference theory. Following the influential contributions by Arditti (1967), Samuelson (1970), and Tsiang (1972), several authors developed the expected return models that incorporate the skewness variable within the context of capital asset pricing. Rubinstein (1973) and Kraus and Litzenberger (1976) established an equilibrium three-moment capital asset pricing model (CAPM) to show that systematic co-skewness is priced. Subsequently, a number of papers verified the model of Kraus and Litzenberger (1976) in different perspectives (Friend & Westerfield, 1980; Galagedera & Brooks, 2007; Lim, 1989; Sears & Wei, 1985, 1988). In addition, Harvey and Siddique (2000) proposed a three-moment CAPM that is focused upon the conditional co-skewness. They demonstrated that co-skewness of portfolios is priced and the average annualized skewness premium of monthly U.S. equities for the period of July 1963 to December 1993 is 3.60 percent. Subsequently, their model was applied to the U.S. stock market (Smith, 2007) and the data of Australian stock market (Doan, Lin, & Zurbruegg, 2010). It was found that the rates of returns of securities were significantly explained by a conditional co-skewness factor. These results reinforce the implication of skewness preference on asset pricing by confirming that investors do trade expected return for skewness.

1.3 IMPACT OF SKEWNESS PREFERENCE ON PORTFOLIO CHOICE

While the preference for skewness of return distributions in portfolio choice is evidently pronounced and its implication on asset pricing is widely documented, the impact of skewness preference on finding the optimal portfolio choice remains undisclosed. In fact, an introduction of skewness to portfolio selection brings about a new research direction in portfolio optimization problem, leading to the mean-variance-skewness (MVS) analysis. In this framework, an investor whose utility that can be approximated by a third-order Taylor's series expansion constructs a portfolio to maximize expected return, minimize risk and maximize skewness simultaneously (Arditti & Levy, 1975; Jean, 1971, 1973). The MVS portfolio optimization problem (hereafter, MVS-POP) is tri-objective and consists of two non-linear objective functions, i.e. variance and skewness, whose objectives compete and conflict with each other. From the optimization point of view, a portfolio that optimizes three objectives at the same time does not exist. Instead, analogous to the MV-POP, a set of trade-off portfolios is usually searched in the multi-dimension moment spaces.

When the number of objectives to be optimized increases from two (MV) to three (MVS), the searching mechanism of an optimization technique becomes more complex because the search space has expanded from two-dimension (2D) to three-dimension (3D). In the early stage of solving the MVS-POP, various mathematical derivations were proposed for attaining MVS efficient portfolios and defining the shape of MVS efficient surface, but, experiment on the real data remained unattainable due to computation restriction (Arditti & Levy, 1975; Jean, 1971, 1973). In the last few decades, various techniques have been applied for solving this problem. These techniques can be categorized into two approaches, i.e. single-objective optimization approach (Konno, Shirakawa, & Yamazaki, 1993; Konno & Suzuki, 1995; Ryoo, 2006) and aggregating approach (Canela & Collazo, 2007; Chunhachinda et al., 1997; Lai, 1991; Prakash et al., 2003; Yu, Wang, & Lai, 2008). However, disadvantages of using these approaches for solving a multi-objective optimization problem are widely acknowledged in the field of operation research (Athan & Papalambros, 1996; Das & Dennis, 1997; Marler & Arora, 2004; Messac, Puemi-Sukam, & Melachrinoudis, 2000).

The most critical shortcoming of these approaches is that there is no guarantee that the obtained solutions represent the global optima or efficient solutions, especially when dealing with a class of non-concave maximization problem, such as maximizing the skewness function of the MVS-POP. It is severe because the notion of efficiency is central to portfolio theory. Basically, when choosing among a feasible set of competing portfolio choices, rational investors who act to maximize their expected utility will consider only the portfolios in the efficient set. Investors with different demand functions and degree of preference may select different choice of efficient portfolios, but none of them will choose the inefficient portfolios. Briec and Kerstens (2010) argued that a clear idea on portfolio choices and relative preference can be developed by investors when they can view the efficient portfolios graphically displayed on the MVS space. In the similar vein, Mitton and

Vorkink (2007, p. 1274) suggested that “to assess MVS efficiency of a portfolio, ideally, MVS efficient frontier would be constructed using return characteristics of available stocks at times of portfolio formation. However, the large number of computation required to construct this frontier in three dimensions makes this approach intractable”. It is observed that, the ability of the techniques that were proposed and implemented in the research in finance is limited for performing this ideal procedure. As a result, the impact of skewness on portfolio choice cannot be accurately analyzed if the MVS-POP is solved by using either the single-objective optimization approach or aggregating approach.

1.4 THE APPLICATION OF MVS ANALYSIS IN ELECTRICITY MARKET

In a deregulated electricity market, the power generation company (Genco) has to decide how to allocate its production among various types of trading instruments for profit maximization. Since a Genco has limited production capacity, generation asset allocation among the trading instruments is an important decision. During the last decade, the MV analysis of Markowitz (1952) was applied for solving the optimal electricity allocation problems (Donghan, Deqiang, Jin, & Yixin, 2007; Hatami, Seifi, & Sheikh-El-Eslami, 2011; Liu & Wu, 2006; Liu & Wu, 2007a, 2007b; Xiaohong, Jiang, Feng, & Guoji, 2008). The problem involves allocation of the generated electricity to several customers in different markets to maximize the Genco’s utility. In their works, it was assumed that a Genco of interest operates within a deregulated energy market by trading its electricity through physical trading instruments, i.e. spot market, day-ahead market and forward contract.

However, the finance literature suggests that skewness of return distributions cannot be neglected in an asset allocation decision unless there is a reason to believe that the distributions are symmetric and investor utility is a function of only the mean and variance (Harvey & Siddique, 2000; Kraus & Litzenberger, 1976; Samuelson, 1970). Besides, many theoretical papers suggested that utility functions that satisfy the non-increasing absolute risk aversion condition (Arrow, 1964; Pratt, 1964) exhibit a preference for skewness in return distributions (Arditti, 1967; Kane, 1982; Levy, 1969; Tsiang, 1972). In the electricity market, several authors provided the evidences that the distribution of electricity spot prices is not normal but skewed (Benth, Cartea, & Kiesel, 2008; Bessembinder & Lemmon, 2002; Cartea & Villaplana, 2008; Hajiabadi & Mashhadi, 2013; Longstaff & Wang, 2004; Lucia & Torró, 2011; Redl, Haas, Huber, & Böhm, 2009).

In addition, the preference for positive skewness can be observed from the agents in the electricity markets. A number of studies showed that the forward premium is positively related with skewness of the spot price distribution (Bessembinder & Lemmon, 2002; Douglas & Popova, 2008; Longstaff & Wang, 2004; Lucia & Torró, 2011; Parsons & De Roo, 2008; Redl et al., 2009; Viehmann, 2011). As a result, electricity allocation under the MV analysis may no longer be sufficient to obtain efficient solutions. According to these observations, the electricity allocation problem should be established under the MVS portfolio model. In other words, the electricity allocation problem should be formulated as a MVS-POP where the expected return, variance and skewness are optimized simultaneously.

1.5 RESEARCH QUESTIONS

This research has the following questions:

1. How to solve the MVS efficient portfolio allocation problem in the multi-dimension space?
2. How does the introduction of skewness into the portfolio selection problem affect the risk-return trade-off?
3. What are the impacts of skewness preference on the portfolio choice?
4. What are the implications for the application of the MVS analysis when the number of trading choices is small?

1.6 RESEARCH OBJECTIVES

To find the answers to the research questions stated above, this research has the following objectives:

1. To demonstrate how multi-objective evolutionary algorithms are applicable for solving problems in MVS efficient portfolio allocation in the multi-dimension space.
2. To examine the risk-return trade-off and the characteristics of MVS efficient portfolios.

3. To investigate the impacts of skewness preference on the efficient portfolio choice by developing a single-period model that allows for a heterogeneous degree of risk aversion and skewness preference.
4. To examine the applicability of the MVS analysis for solving portfolio selection problems when the number of trading choices is small.

1.7 ORGANIZATION OF THE THESIS

This thesis consists of seven chapters. This chapter discusses the background and motivations of the thesis. The second chapter provides a literature review on the skewness preference theory and its implications with the aim to highlight three research gaps. Chapter 3 explains the conceptual framework, formulated optimization problem, and methodology used to achieve the research objectives as well as provides a discussion on the data used. Chapter 4 and Chapter 5 demonstrated how the MVS efficient portfolios are obtained from the proposed techniques in the three-dimension space with an application to the stock market. In addition, the characteristics of MVS efficient portfolios are examined and discussed in Chapter 4. Then, the impacts of skewness preference on the efficient portfolio choice are investigated and the results are reported in Chapter 5. Chapter 6 demonstrates the application of the MVS analysis for solving the electricity allocation problem, where the number of trading choices is small. The results are discussed in the context of a Generation Company in the electricity market. Finally, Chapter 7 summarizes the major findings of the thesis and discusses their implications.

CHAPTER 2

LITERATURE REVIEW

2.1 INTRODUCTION

The literature review of this thesis is divided into six sections. The development of the skewness preference theory is documented in Section 2.2. Section 2.3 reviews the studies that discuss the impact of skewness preference on the decision of portfolio holding. The explanation of the underlying causes of skewness in a stock market is given in Section 2.4. Section 2.5 surveys the application of portfolio models for solving the electricity allocation problem of generation companies in the electricity market. A brief discussion on the techniques used by other researchers for solving multi-objective portfolio optimization problems is given in Section 2.6. Finally, the research gaps in the literature are highlighted in Section 2.7.

2.2 SKEWNESS PREFERENCE THEORY

Empirical evidence reveals that skewness preference plays an importance role in understanding various risk-taking behaviors of an economic agent. A number of researchers have attempted to develop the theoretical explanation from different perspective and different approaches.

2.2.1 Expected Utility Theory

In economics and finance, the expected utility maximization is the most popular approach to the problem of choice under uncertainty. The principle of expected utility maximization explains that in considering a set of completing feasible investment alternatives, a rational decision maker (DM) selects a choice that maximizes his expected utility of wealth.

In the context of portfolio, suppose that a DM at time 0 has to make decision about his portfolio choice that will be held until the end of period 1, and there are N available assets whose returns are denoted by R_i where $i = 1, \dots, N$. If initial wealth is denoted by W_0, his wealth at the end of period 1 (W_1) is defined as:

$$W_1 = (1 + \sum_{i=1}^{N} w_i R_i) W_0 = \left(1 + R_p(\mathbf{x})\right) W_0 \tag{2.1}$$

where $R_p(\mathbf{x}) = \sum_{i=1}^{N} x_i R_i$ and $\mathbf{x} = [x_1 \quad x_2 \quad \cdots \quad x_N]^{\mathrm{T}}$ represents the vector of allocation proportions. According to the expected utility maximization, a rational DM with utility function U is dealing with the optimization problem for solving a portfolio choice $\mathbf{x}$:

$$\max_{\mathbf{x}} \mathrm{E}[U(W_1)] = \mathrm{E}\left\{U\left[\left(1 + R_p(\mathbf{x})\right) W_0\right]\right\} \tag{2.2}$$

It is generally acknowledged that an important property of the utility function is that it is a continuously differentiable function of wealth $U(W)$ defined for $W > 0$. The economic implication of this property is that $U(W)$ has the property of non-satiation, i.e. positive marginal utility for wealth ($U'(W) > 0$)[1], and of risk aversion, i.e. decreasing marginal utility for wealth ($U''(W) < 0$) (Friedman & Savage, 1948). For the non-satiation property, utility is an increasing function of wealth. This means that an investor prefers more wealth to less wealth. In other words, an investor's preference for wealth has never satiated. For the risk aversion property, marginal utility is diminishing with the accumulation of more wealth which implies that the utility function is concave. A general example of diminishing marginal utility with wealth is that an additional gain of one dollar will increase utility of an investor whose initial endowment is one dollar but will be meaningless for another investor who owns an initial wealth of a million dollar. Economists, in general, acknowledge that the increase in marginal utility caused by obtaining an additional dollar decreases as an individual owns more wealth.

Although the expected utility approach is widely used to analyze the decision making process of an economic agent under uncertainty, it will be controversial to conclude on the type of function that best describe the behavior of the majority of investors because different investors generally have different utility functions. However, any choices of function used in the analysis should satisfy at least two properties, namely, non-satiation and risk aversion.

2.2.2 Expected Utility and Mean-variance Analysis

In the seminal work of Markowitz (1952) on the parameter-preference approach, the so called "mean-variance analysis" denoted by "*E*-*V* analysis" in his article is proposed as the

[1] Throughout, $U^{(n)}(\cdot)$ is n^{th}-order derivative of the utility function.

approximation of the expected utility maximization approach. He assumed that an investor's preference can be described by a function over the mean and the variance of the portfolio return, i.e. $U = U(E, V)$. To make *E-V* analysis reconcilable with expected utility maximization, he managed the quantitative part based upon the use of quadratic utility function. Stated formally, suppose that the utility function of an investor is given by:

$$U(W) = W - \frac{b}{2}W^2, b > 0 \tag{2.3}$$

where *W* refers to the wealth owned by the investor and b is a coefficient relating wealth preference to utility. This quadratic utility function satisfies non-satiation and risk aversion property since $U'(W) = 1 - bW > 0$ for $W < 1/b$ and $U''(W) = -b < 0$ for $b > 0$, which guarantee the non-satiation and risk aversion conditions. By taking expectation on both sides, the expected utility is

$$\begin{aligned} \mathrm{E}[U(W)] &= \mathrm{E}(W) - \frac{b}{2}\mathrm{E}(W^2) \\ &= \mathrm{E}(W) - \frac{b}{2}\left[\mathrm{Var}(W) + \mathrm{E}^2(W)\right] \\ &= \mu - \frac{b}{2}[\sigma^2 + \mu^2] \end{aligned} \tag{2.4}$$

where μ and σ^2 are respectively the mean and the variance of *W*. The expected utility of Equation (2.4) is an increasing function of mean and a decreasing function of variance since $\frac{\partial U}{\partial \mu} = 1 - b\mu > 0$ and $\frac{\partial U}{\partial \sigma} = -b\sigma < 0$. Thus, in the *E-V* analysis, a set of efficient portfolios comprises of those with the lowest variance for any given level of expected return and those with the highest expected return for a given level of variance. It should be noted that risk aversion in the context of *E-V* analysis is referred to as the aversion to the dispersion of the probability of the outcome since variance is considered as the risk measure.

Although the portfolio solution obtained from the *E-V* analysis may not be equivalent to that of expected utility approach, it gives an economical approximation with the advantage of less information is required for the analysis. As a result, the mean-variance analysis is extensively discussed in the liquidity preference theory (Tobin, 1958) and the capital asset pricing model (Lintner, 1965; Sharpe, 1964). However, the limitation of using quadratic utility to represent utility of investor was widely criticized (Bierwag, 1974; Blume & Friend, 1975; Borch, 1974; Friend & Blume, 1975; Hanoch & Levy, 1970; Levy, 1974; Simkowitz & Beedles, 1978; Tsiang, 1974), especially after the Arrow-Pratt measure of the degree of risk aversion was introduced (Arrow, 1964; Pratt, 1964).

In general, a risk aversion DM dislikes zero-mean risk. In other words, this DM will always refuse to play a fair game with zero expected return. The degree of risk aversion of a DM can be generally defined by quantifying how much a DM is willing to pay to eliminate the zero-mean risk. This measure is called the "risk premium", denoted as π, and can be defined by:

$$\mathrm{E}[U(W+\tilde{z})] = U(W-\pi) \tag{2.5}$$

where $\tilde{z}$ is zero-mean risk, i.e. $\mathrm{E}(\tilde{z}) = 0$. Pratt (1964) made an analysis of the utility function expressed in Equation (2.5) using the second- and first-order Taylor's series expansion to approximate the left hand side (LHS) and right hand side (RHS) of Equation (2.5), respectively. This approximation yields:

$$\mathrm{E}\left[U(W) + \tilde{z}U'(W) + \frac{\tilde{z}^2}{2}U''(W)\right] = U(W) - \pi U'(W)$$

$$U(E[W]) + \frac{\sigma_{\tilde{z}}^2}{2}U''(W) = U(E[W]) - \pi U'(W)$$

$$\pi \cong \frac{\sigma^2}{2}A(W) \tag{2.6}$$

where $A(W) = -\frac{U''(W)}{U'(W)}$ is the Arrow-Pratt measure of absolute risk aversion. This measure quantifies the degree of individual risk aversion which can be regarded as a measure of the degree of the concavity of the utility function of an individual.

According to the Arrow-Pratt notion of risk aversion, the critical invalidity of quadratic utility can be explained in two aspects. Firstly, Arrow (1964), Pratt (1964), and Friend and Blume (1975) argued that the degree of an investor's risk aversion is supposed to decrease with more wealth held by the investor. The economic implication of this argument is straightforward. For example, consider a coin flipping game where the possible outcome is a gain or loss of 100. The loss of 100 is severe for an agent whose initial wealth is 100 but has a trivial impact on another agent with an initial wealth of a million dollar. Thus the first agent is willing to accept the risk only if he is compensated by a higher premium compared to the second agent. In the context of portfolio selection, decreasing absolute risk aversion with wealth implies that when an agent experiences an increase in wealth, he will increase the portion of risky asset in his portfolio. But many researchers demonstrated that the quadratic utility function exhibits an increasing absolute risk aversion for all level of wealth

(Bierwag, 1974; Blume & Friend, 1975; Borch, 1974; Friend & Blume, 1975) which is irrational in the implicational aspect. Secondly, another shortcoming of quadratic utility comes from the bounded range of possible outcome. Since positive marginal utility is considered as a desirable property of any utility function, quadratic utility is consistent with this property only in some range of wealth, i.e. $W < 1/b$. These evidences suggest the limits to the use of quadratic utility.

In addition to the quadratic utility function assumption, the mean-variance analysis is restricted to the case where an asset return has a multivariate normal distribution. Therefore, the distribution of returns of an asset will differ from others only by the mean and the variance. In contrast, a collection of empirical studies showed that the return distributions are asymmetric at not only the market level but also firm level (Amaya & Vasquez, 2010; Arditti, 1967; Bates, 1996; Bekaert & Harvey, 2002; Black, 1976; Campbell & Hentschel, 1992; Canela & Collazo, 2007; Chen et al., 2001; Christie, 1982; Chunhachinda et al., 1997; Fielitz, 1976; French et al., 1987; Gennotte & Leland, 1990; Gibbons et al., 1989; Grossman, 1989; Hwang & Satchell, 1999; Jorion, 1988; Liu, Margaritis, & Wang, 2012; Prakash et al., 2003; Simkowitz & Beedles, 1978; Singleton & Wingender, 1986; Wang et al., 2009). Hence, this assumption is not realistic in the real economy.

2.2.3 Skewness Preference in Expected Utility Model

According to the Arrow (1964) and Pratt (1964), the desirable properties of utility function are (a) positive marginal utility for wealth, i.e., non-satiety, (b) decreasing marginal utility for wealth, i.e., risk aversion, and (c) non-increasing absolute risk aversion for wealth, i.e., risky assets are not inferior good. The utility functions that satisfy these conditions are, for instance, the cubic function, the negative exponential function, the family of constant elastic utility functions, and the log function. In general, the Taylor's series approximation of the expected utility is widely used in the analysis of utility functions. The general Taylor series expansion for a function $f(x)$ can be written as follows:

$$f(x) = f(x_0) + \frac{f'(x_0)}{1!}(x - x_0) + \frac{f''(x_0)}{2!}(x - x_0)^2 + \cdots$$
$$+ \frac{f^{(n)}(x_0)}{n!}(x - x_0)^n + \frac{f^{(n+1)}(x_0)}{(n+1)!}(x - x_0)^{n+1} \quad (2.7)$$

A rational investor will maximize the expected value $E[U(\cdot)]$ of $U(\cdot)$ where $U(\cdot)$ is the utility function of an investor. Suppose that $U(\cdot)$ is a function of wealth, i.e. $U(W)$ and can be approximated by the third-order Taylor's series expansion around its expected value, thus we obtain:

$$U(W) = U(E[W]) + U'(W)(W - E[W]) + \frac{U''(W)}{2!}(W - E[W])^2$$
$$+ \frac{U'''(W)}{3!}(W - E[W])^3 \quad (2.8)$$

By taking expectation on both sides, the second term on the RHS of Equation (2.8) is $E\big[W - E[W]\big] = 0$. Thus, we obtain:

$$E[U(W)] = U(E[W]) + \frac{U''(W)}{2!}E[(W - E[W])^2] + \frac{U'''(W)}{3!}E[(W - E[W])^3] \quad (2.9)$$

It can be seen that the expected utility presented in Equation (2.9) is a function of those risks associated with higher moments of the wealth distributions. However, if the distribution of wealth is symmetric, the third term on the RHS of Equation (2.9) is zero and therefore an investor utility is a function of solely the mean and variance of the wealth distributions. According to Pratt (1964), risk premium decreases with increasing wealth. Arditti (1967) proved that:

$$\frac{\partial}{\partial W}\left[-\frac{\sigma^2}{2}\frac{U''(W)}{U'(W)}\right] = \frac{\sigma^2}{2}\frac{-U'(W)U'''(W)+\left[U''(W)\right]^2}{[U'(W)]^2} < 0 \tag{2.10}$$

His argument is based on the desirable properties of an investor's utility of (i) positive marginal utility for wealth, $U'(W) > 0$ and (ii) decreasing marginal utility for wealth, $U''(W) < 0$. Thus, Equation (2.10) holds if and only if $U'''(W) > 0$. Since $U'''(W)$ is the coefficient of third moment of the distribution of wealth, the positive value of this coefficient implies that the investor utility can be partly maximized by increasing the skewness of the wealth distribution.

In fact, Markowitz (1952) discussed that the mean-variance analysis will not yield the optimal solutions to investors if either their utilities depend on the first three moments or the preference for the third moment exists. He explained that if an investor's preference is described by a preference function over the mean, the variance, and the skewness of the portfolio return, which is denoted by $U = U(E, V, M)$ in his article, the return skewness is relevant for portfolio selection. He pointed out that skewness preference is connected to a propensity to gamble which should be avoided in investment practice. Therefore, he suggested that skewness should be dropped in portfolio selection.

However, a number of theoretical studies provided results to support Arditti's argument that an investor whose utility can be approximated by the third-order Taylor's expansion around the expected value will prefer positive skewness in asset returns (Hanoch & Levy, 1970; Ingersoll, 1975; Kane, 1982; Levy, 1969; Rubinstein, 1973; Samuelson, 1970; Scott & Horvath, 1980; Tsiang, 1972). For instance, Levy (1969) demonstrated that skewness preference can be proved even when the risk aversion assumption is relaxed. He showed that at a low level of wealth, the utility of an investor is concave but will converge to convexity when his wealth is higher than a certain level. The implication of his results is that an investor can be both risk-averse and skewness-lover which depend on the level of wealth at a given time. In addition, various forms of utility function such as, negative exponential, constant elastic function, hyperbolic absolute risk aversion and log function were analyzed using the Taylor's series expansion to establish the skewness preference theory (Ingersoll, 1975; Kane, 1982; Tsiang, 1972). These studies demonstrated that investors with utility functions that satisfy the Arrow-Pratt's desirable properties for utility function have a preference for positive skewness. Moreover, Scott and Horvath (1980) showed that investors exhibiting positive marginal utility, consistent with risk aversion for all wealth and strict consistency of moment preference will prefer positive skewness in the distribution of returns. These results support the fact that skewness preference is not a mark of gambler or bettor but a common trait of a rational investor who acts to maximize his expected utility.

In the expected utility model, it is unanimous to define skewness preference by a positive sign of the third-order derivative of utility function, i.e. $U''' > 0$. However, the measure of the degree of skewness preference, in the similar vein as the Arrow-Pratt measure of the degree of absolute risk aversion, is still contradictory. There are a few measures that were proposed and discussed in the finance literature. Simkowitz and Beedles (1978) discussed the measure of individual's concern for skewness vis-à-vis dispersion in their proposed utility model and named it as "skewness/variance awareness". However, the application of this measure is restricted only for risk averse and risk lover investors because this measure is not defined if investors are risk neutral.[2] Another measure is "speculation ratio" which is quantified by $\frac{U''}{U'''}$ (Conine & Tamarkin, 1981). The authors suggested that it is a combination of investor trade-off of risk aversion for return skewness and is independent among investors. Nevertheless, economists suggest that a proper measure of skewness preference should have a global property that is comparable to Arrow-Pratt's measure of absolute risk aversion. The global property of Arrow-Pratt's index of absolute risk aversion states that if $V(W) = s\big(U(W)\big)$ with $s' > 0$ and $s'' < 0$, an agent with utility $V(W)$ has a greater risk aversion than one with utility $U(W)$. However, the proof of this property has yet to be theoretically addressed.

In the finance literature, skewness preference is usually linked to either gambling or speculating behavior. In contrast, economists look at the positive third-order derivative of utility function in different perspectives. After the concept of downside risk aversion (DRA) was introduced (Menezes, Geiss, & Tressler, 1980), skewness preference is linked to prevention behavior of the agents who try to protect themselves from negative extreme returns. Kimball (1990) proposed the local index of $-\frac{U'''}{U''}$ to measure the degree of precautionary-saving motive which is named as "prudence measure". This measure

[2] Their utility model is expressed as $U_i = a_i R - b_i(\sigma_R^2) + c_i(m_R^3)$ where σ_R^2 and m_R^3 is respectively the second and the third central moment of return. They defined the skewness/variance awareness as $\theta_i \triangleq c/b$. θ_i is undefined if $b = 0$, i.e. for a risk neutral investor.

suggests the propensity to prepare in the face of future uncertainty. Chui (2000, 2005) demonstrated that the prudence measure can be interpreted as measuring the degree of DRA against one's own risk aversion. He indicated that the prudence premium that a DM is willing to pay for self-protection depends on the degree of his skewness preference relative to his risk aversion. Jindapon and Neilson (2007) verified the results of the previous studies using the comparative statistic approach. They explained that agents with greater prudence have higher willingness to forgo their expected utility in exchange for a reduction in downside risk. From the above arguments, many theoretical papers suggest that $-\frac{U'''}{U''}$ is a good measure of the degree of DRA due to the fact that DRA and prudence are treated as similar. In addition, the local and global comparison properties of the measure as well as its decreasing prudence property make this measure very useful in various applications.

Another potential measure of the degree of skewness preference was proposed by Modica and Scarsini (2005). Based on Ross's notion of stronger risk aversion (Ross, 1981), they demonstrated that the degree of DRA should be measured by the local index of $\frac{U'''}{U'}$ which decreases with wealth, i.e. satisfying the second-order derivative condition.[3] Their result demonstrated that an agent with a greater index of $\frac{U'''}{U'}$ is willing to pay more to insure against a risk with higher negative skewness if the mean and variance of risk is similar. However, they failed to prove that the local comparison of the proposed measure can be translated into a global comparison (Modica & Scarsini, 2005, p. 270). Based on the results of Modica and Scarsini (2005), Jindaporn and Neilson (2007) analyzed the monetary cost

[3] For DRA, the property of decreasing aversion with wealth explains that when an agent is wealthier, his aversion to downside risk decreases. This is analogous to the property of absolute risk aversion measure which describes that an agent become less risk-averse with more wealth accumulation.

problem of agents who choose to shift from the initial probability distribution toward the preferred one, i.e. a lower downside risk. They showed that agents with greater DRA are willing to pay higher cost for protecting themselves from downside risk. Subsequently, Crainich and Eeckhoudt (2008) convincingly argued in favor of the local index $\frac{U'''}{U'}$ as a measure of the intensity of DRA in that the global property of this index can be simply proved in a similar way as the obtained global property of the Arrow-Pratt's index of absolute risk aversion. Meanwhile, the global property of the local index $\frac{U'''}{U''}$ proposed by Kimball (1990) is not easy to obtain unless some restrictions are imposed on the utility function. Finally, their result implies that the greater the index of DRA ($\frac{U'''}{U'}$), the larger is the compensation required for accepting an investment alternative with negative skewness in its return distribution.

2.2.4 Skewness Preference and Decision to Gamble

In general, an individual with diminishing marginal utility and seeking to maximize his utility dislikes a zero-mean risk and always refuses to participate in a fair game with an expected return of zero, unless a premium is paid. However people in reality always participate in fair games, for example, purchasing lottery and horse-race ticket. This risk-taking behavior of an agent is the first economics implication of skewness preference. This is because these investment alternatives exhibit a large chance of losing a small investment, i.e. the lottery ticket price, and a small chance of winning a large return, i.e. the prize. In other words, these fair bets possess a positive skewness in the probability distribution of returns. The plausible explanation of this irrational behavior was developed based on the expected utility model (Friedman & Savage, 1948; Kwang, 1965). Friedman and Savage (1948) argued that an agent with utility function depending on the first three moments of return is a risk lover, and therefore, he will choose an investment alternative that exhibits positive skewness in the probability distribution of returns.

Golec and Tamarkin (1998) provided empirical evidence from the horse track to support the preference for skewness. They used the racetrack data to estimate the coefficients of the proposed expected utility model. They proposed a cubic utility model as a function of a winning the prize of a horse race. They found that the utility function is concave for the low winning prizes and convex for the big prizes. This result supports the previous finding that bettors globally prefer positive skewness, however they are not globally risk lover. In addition, Garrett and Sobel (1999) used the U.S. state lottery data to estimate the utility function proposed by Golec and Tamarkin (1998). Their finding is consistent with that of Golec and Tamarkin (1998). They showed that a lottery buyer exhibits a global skewness preference.

2.2.5 Skewness Preference and Pricing Implication

Following the influential contributions by Arditti (1967), Samuelson (1970), and Tsiang (1972), their models predict that rational investors prefer among others an investment alternative hat has a positive skewness if the mean and the variance are identical. They argued that if expected return is not normally distributed, skewness should be considered as a risk measure and relevant for explaining risk-return relationship. Subsequently, several authors developed the expected return models incorporating skewness as a variable of the capital asset pricing. Rubinstein (1973) and Kraus and Litzenberger (1976) established an equilibrium three-moment capital asset pricing model to show that systematic co-skewness is priced.[4] Systematic co-skewness is defined as a measure of co-movement between individual assets and the aggregate market portfolio. The empirical data of the NYSE from January 1926 through December 1970 experimented by Kraus and Litzenberger (1976) reveals that investors have a preference for positive skewness. As a result, they are willing to trade some expected return for an asset with positive co-skewness with the aggregate market portfolio. Subsequently, a number of papers verified the model of Kraus and Litzenberger (1976) in different perspectives (Friend & Westerfield, 1980; Galagedera & Brooks, 2007; Lim, 1989; Sears & Wei, 1985, 1988).

Friend and Westerfield (1980) extended the work of Kraus and Litzenberger (1976) by including bonds into the market portfolio and constructing the portfolio based on value-weighted index, which is more practical in the real world, instead of equal-weighted index used in previous studies. They found some evidences to support the previous finding that investors are willing to pay a premium for a positively skewed asset. However, the intercept term of their regression is insignificant, i.e. the estimated risk free rate of return is higher than the actual value, which is inconsistent with the results of Kraus and Litzenberger (1976). Some plausible causes of the contradictory results are the difference in market portfolio composition, estimation procedure, and testing methods. Subsequently, Lim (1989) proposed the GMM test method which is more suitable for estimating multivariate models. After setting the conditions according to the model of Friend and Westerfield (1980), he provided empirical evidences to support the result that systematic skewness is

[4] In both works, the utility function is restricted to the first three moments. Rubinstein (1973) relied on a cubic utility function, while Kraus and Litzenberger (1976) approximated investors' utility using a Taylor's series expansion up to third order.

priced. Galagedera and Brooks (2007) tested the systematic co-skewness in the context of downside risk measure and compared it to the downside beta. Using emerging market data, they suggested that downside co-skewness is significantly priced and may be a better explanatory variable of asset price variation than downside beta.

Unlike co-skewness used by the previous studies, Harvey and Siddique (2000) proposed a three-moment CAPM that is focused upon the conditional co-skewness. In other words, their model is allowed to be time-varying. In addition, a procedure to obtain a new systematic skewness factor is introduced. At the end of each month, stocks were categorized into three different portfolios according to the presence of co-skewness and then the return spread between the lowest and highest co-skewness portfolio was computed. This methodology is analogous to the three factors model of Fama and French (1993) that classified stocks into quintile portfolios according to size premium, SMB, i.e. small (market capitalization) minus big, and, premium on book value to market value, HML, i.e. high (book-to-market ratio) minus low. Harvey and Siddique (2000) demonstrated that the co-skewness of portfolios is priced and the average annualized skewness premium of U.S. equities for the period of July 1963 to December 1993 is 3.60 percent. The conditional three-moment CAPM of Harvey and Siddique (2000) was tested using the data of S&P 500 from July 1993 through December 1997 (Smith, 2007) and Australian S&P/ASX 300 from January 2001 through July 2007 (Doan et al., 2010). They suggested that the rates of returns of securities are significantly explained by a conditional co-skewness factor. Smith (2007) found that a preference for co-skewness varies over time with the extent of market skewness. He showed that when the market returns are positively skewed, investors are willing to sacrifice 7.87 percent per unit of gamma, i.e. a standardized measure of coskewness risk. When the market returns are negatively skewed, the annualized premium demanded for bearing gamma risk is 1.80 percent over the period of study.

Another approach that is widely adopted for testing pricing implication of skewness is to use sorting procedure (Amaya & Vasquez, 2010; Boyer, Mitton, & Vorkink, 2010; Chen et al., 2001; Xu, 2007). For this approach, securities are sorted according to their skewness value and grouped to form a number of portfolios. Thus the risk-return tradeoff as well as the relationship between skewness and other variables can be examined. For instance, Chen et al. (2001) estimated the coefficient of skewness from other variables such as past returns, standard deviation, firm size, and monthly turnover, and then sorted the securities into quintile portfolios based on the size of skewness. Boyer et al. (2010) sorted stocks into quintile portfolios based on the predicted "idiosyncratic skewness". In their work, idiosyncratic skewness of stocks is extracted from their idiosyncratic volatility computed from the residuals of time series regressions of the pricing model. Then the predicted idiosyncratic skewness of stocks is forecasted from lagged idiosyncratic volatility, lagged idiosyncratic skewness, and other exogenous variables. In recent years, Amaya and Vasquez (2010) exploited the availability of high frequency data by sorting firms into quintile portfolios based on the "realized skewness" of their returns. The concept of realized skewness is analogous to that of the "realized volatility" (Andersen, Bollerslev, Diebold, & Ebens, 2001; Andersen, Bollerslev, Diebold, & Labys, 2001) which is said to be more suitable for explaining intraday data. Regardless of the calculation method for obtaining skewness and the data set, these studies consistently asserted that the negative relation between skewness and expected return is significantly pronounced. This result reinforces the implication of skewness preference on asset pricing by confirming that investors do trade expected return for skewness.

2.3 SKEWNESS PREFERENCE AND DECISION ON PORTFOLIO CHOICE

In general, complete diversification is a rational investment strategy for a risk-averse agent in a homogeneous stock market, who considers the risk-return tradeoff based only on the mean and variance of return distribution. It is generally accepted that the main purpose of holding a diversified portfolio is to eliminate firm-specific risk, the so called “unsystematic risk” or “idiosyncratic risk”. Some investors construct portfolios according to the rule of thumb by randomly including 30 to 50 stocks in their portfolios with the hope that the idiosyncratic risk will be reduced (Campbell, Lettau, Burton, & Xu, 2001; Statman, 1987). However, the observed behavior of the market participants shows that the majority of individual investors hold only a few stocks in their portfolio which is known as an “undiversified portfolio” (Blume & Friend, 1975; Elton & Gruber, 1977; Evans & Archer, 1968; Goetzmann & Kumar, 2008; Kelly, 1995; Mitton & Vorkink, 2007; Odean, 1999; Polkovnichenko, 2005). This undiversified behavior has attracted attention from many researchers to give an explanation for this phenomenon. For instance, Shefrin and Statman (2000) and Statman (2004) argued that undiversified investors construct portfolios as layers with the bottom layer consisting of defensive stocks for downside protection, and the top layer including offensive stocks for upside gain. The strategy normally considers positively skewed stocks as the offensive component.

Simkowitz and Beedles (1978) pointed out that diversification is not necessarily desirable for an agent whose decision is based on the first three moments of return distribution. They showed empirically that increasing the number of risky assets in a portfolio not only reduces portfolio variance, but also portfolio skewness. Thus they suggested that a progressive loss of portfolio skewness caused by increasing diversification is a plausible explanation of the observed phenomenon of undiversified portfolio held by individual investors (Blume & Friend, 1975). Conine and Tamarkin (1981) derived the optimal number of risky assets for portfolio holding based upon a homogeneous securities universe and equally-weighted portfolio. They demonstrated that the utility with skewness preference decreases with an increase in the number of risky assets in portfolio. Therefore a rational investor may choose to remain undiversified since only a few risky assets are required to construct an optimal portfolio. Mitton and Vorkink (2007) developed a model in which investor preference for the first two moments is homogeneous but for the third moment is heterogeneous. Their model predicts that investors with skewness preference are willing to earn lower returns and accept higher volatility in order to increase portfolio skewness. Using a data set of household accounts from a large discount brokerage in the U.S., they found that the undiversified portfolios have higher skewness and lower Sharp's ratio than the diversified ones. They argued that undiversified investors do not randomly select the small number of stocks in their portfolios but rather intentionally choose stocks that will be most likely to increase the skewness of their portfolios. Using a data set of about 60,000 individual accounts at a large U.S. discount brokerage house during 1991 to 1996, Goetzmann and Kumar (2008) revealed that the highly skewed stocks are favored among undiversified investors and skewness preference is more pronounced for investors who are younger, male, and less affluent.

Apart from the expected utility model, the behavior of undiversified portfolio holders is explained theoretically by the cumulative prospect theory (Barberis & Huang, 2007) and the optimal expectation framework (Brunnermeier, Gollier, & Parker, 2007; Brunnermeier & Parker, 2005). According to Barberis and Huang (2007), undiversified behavior can be described based on cumulative prospect theory which is modified from the prospect theory (Tversky & Kahneman, 1992). According to this theory, an agent evaluates risk using a value function that is defined from transformed probabilities rather than objective probabilities. The transformed probabilities can be obtained from applying a weighting function to the objective probabilities. Thus, the cumulative prospect theory investors will have the tendency to put a higher weight on the tail of the distribution to capture his preference, i.e. skewness. In other words, they intentionally construct their portfolios by collecting the high skewness assets in order to make their portfolios a lottery-like asset. As a result, the optimal portfolio can be constructed by including only a few assets. Brunnermeier and Parker (2005) developed a non-expected utility model in which agents optimize based on beliefs of outcomes. In their model, agents have higher level of beliefs about the probabilities of a positive payoff than a negative one. And the felicity of agents which is analogous to utility is an increasing function of the beliefs. Thus, an implication of their model is that agents prefer an investment alternative having positive skewness in the return distribution. Based on this framework, Brunnermeier et al. (2007) contributed to explain undiversified behavior that investors in equilibrium will allocate their wealth to positively skewed assets to increase their felicity, although they may earn a lower average return.

2.4 CAUSE OF SKEWNESS IN STOCK PRICES

Skewness in a stock market has a significant impact on an investor decision making, capital asset pricing as well as some economic phenomenon such as stock market crash, speculation, and bubble burst in many stock markets. A number of researchers in the field of finance especially behavioral finance have provided the explanations to rationalize this phenomenon.

The first explanation is the “leverage effects” (Black, 1976; Christie, 1982). After a large decrease in stock prices, operating leverage and financial leverage will increase resulting in higher volatility of stock returns. A large volatility increases the risk premium that subsequently enhances the selling activities and causes negative skewness in the return distribution. The second explanation is “volatility feedback mechanism” developed by Campbell and Hentschel (1992) and French et al. (1987). Based on the assumption of asymmetry information, stock market volatility increases when news arrives at the market resulting in an increase in risk premiums. The benefits of good news are compensated by an increased risk premium. In contrast, the impacts of bad news are strengthened by an additional risk premium that drives up the skewness of the distribution of returns. With the homogeneous expectation assumption, however the volatility feedback can cause positive skewness in stock market returns in the case where good news are interpreted by homogeneous investors. This good news will push up the stock price as well as the trading volume, thus causing skewness to be pronouncedly positive.

According to the third explanation, the asymmetry in return distributions is due to stochastic bubble generated by emotional and cognitive biases. The stock market bubble can be rational (De Long, Shleifer, Summers, & Waldmann, 1990; Shleifer & Summers, 1990), speculative (Blanchard & Watson, 1983), intrinsic (Froot & Obstfeld, 1991) and contagious (Topol, 1991). When the price of a particular stock rises, some investors are interested to study its fundamental factors. However, for others, an upward movement in price is a good-enough driving force to invest. In some cases of rational bubble, large traders are powerful enough to rapidly drive up the stock price far beyond its fundamental value, resulting in bubble generation. This bubble building process generates positive skewness in the distribution of stock returns. On the other hand, bubble burst causes the distribution of return to be negatively skewed (Blanchard & Watson, 1983).

Additional explanation is developed based on investor heterogeneity theory that argues that investors' opinions about the fundamental values of individual securities and stock market are diverse. Due to differences in opinions, private and hidden information can be revealed through the trading process by other investors (Harris & Raviv, 1993; Kandel & Pearson, 1995; Odean, 1998). The reactions from observing the trading process can drive stock price movements, although in reality, there is no new information on changes of the firm's fundamental value. Several hypotheses related to investors' heterogeneity were tested. For example, the large trade volume of insurance portfolios and/or dynamic hedgers who strategically track their portfolios with movements of the market portfolio regardless of whether any information is received might be misinterpreted by other traders. In the case of bear market, the reaction from aggregate investors, who trade their wealth by observing the trading process of other investors, will create price pressure that results in negative skewness in stock market return distributions and eventually a crash in stock market (Gennotte & Leland, 1990; Grossman, 1989). On the other hand, the large buy volume from insurance portfolios in a bull market can stimulate investors aggregately to drive up the stock market returns. As a result, positive skewness in stock market returns can be generated.

Furthermore, investor heterogeneity coupled with the hypothesis of short sale constraints of investors in a bearish market will lead to asymmetry in the distribution of returns due to the revelation of private signals of relatively pessimistic investors during market declines. Such revelation could lead other investors to increase their expected risk premium and to enlarge the sale forces (Diamond & Verrecchia, 1987; Hong & Stein, 2003). Moreover, Xu (2007) developed the price convexity function of information by observing that investors react with different extent to the same signal. In his model, disagreement over information precision and short sale constraints are the causes to price convexity. His empirical findings are consistent with several other studies (Bris, Goetzmann, & Zhu, 2007; Chang, Cheng, & Yu, 2007) which reveal that heterogeneity combined with precision of signals increases skewness of individual stock returns rather than reduces skewness.

2.5 ASSET ALLOCATION PROBLEM IN THE ELECTRICITY MARKET

In a deregulated electricity market, a power generation company (Genco) has various types of trading choices available for profit maximization. Due to the fact that a Genco has limited capacity to produce the electricity, the allocation of generated electricity among the multiple energy markets is an important decision for a Genco. In the financial market, investors allocate their wealth to securities such as stock, bond and derivative instruments to form an investment portfolio. The number of assets available for investment is usually large. In the electricity market, on the other hand, a Genco trades its generated electricity via various trading choices including transaction in the spot market, transaction in the forward market and contractual instruments such as forward contract, future contract, options and swaps. The number of trading choices, hence, is limited. Although the form of trading choices in the financial market and electricity market seems different, a number of similarities can be observed. For example, each trading instrument has its own return characteristic associated with a particular sort of risk. The properties of risky and risk-free asset in finance are also present in the trading choices in the electricity market. From this sense, asset allocation concept in finance can be easily extended to energy allocation in the electricity market.

2.5.1 Trading Environment

The deregulated electricity market facilitates its price efficiency and liquidity by offering several types of trading choices not only to the Gencos but also other market participants. In the electricity market, market participants can be electricity producers, i.e. Gencos, electricity distributors, retail users, power trading firms and investors. Trading choices in an electricity market can be generally categorized into three main groups. In the first group, Gencos trade their electricity in the real-time market or the so called “spot market”. In this market, participants can enter to sell their offers or to purchase other participants’ bids on real-time basis. A market operator does matching between bids and offers then determines the market clearing price (MCP). The MCP will be announced on an hourly basis; hence 24 spot prices are announced daily. It should be noted that spot prices have high fluctuations because the demand for electricity is uncertain and electricity is a non-storable product.

Transaction in day-ahead market or the so called “forward market” is the second group of trading alternatives for the Gencos. In this market, a Genco submits its offer to sell electricity in terms of price ($/MWh) and quantity (MWh) for the next delivery day to the pool market system. The market operator clears the orders by matching bids and offers and then informs the Genco of interest whether its offers are accepted or rejected. It should be noted that there is no guarantee that all Genco’s offers will be matched. Thus, the bidding strategy is another importance issue for the Gencos participating in this market.

The third group of trading choice is contractual instruments such as forward contract, future contract, options and swaps. For a forward contract, the buyer and the seller agree, in advance, on a pre-specified price of electricity that will be transmitted from the seller to the buyer at a fixed amount and for a certain period of time. Similarly, a future contract has the same features as the forward contract except that the quantity of electricity in this contract is standardized. Additionally, physical delivery is not necessary for closing the contract’s position since the future contract can be settled by financial payment at any time before its maturity date. Options represent the rights but not obligation to buy or sell electricity at a pre-specified price at a certain time in the future. Again, options are usually settled by financial payment rather than physical delivery.

In general, electricity futures and options are traded at an organized exchange such as the Power Pool and Independent System Operator (ISO), meanwhile forward contract is transacted over-the-counter (OTC) or between two parties. The forward contract between two parties is technically known as "bilateral forward contract". The physical delivery transactions, namely, transactions in the spot market, transactions in the day-ahead market and forward contracts are basically exchanged at a physical electricity market such as Pennsylvania-New Jersey-Maryland (PJM) market and California Independent System Operator (CAISO).

2.5.2 Studies on the Generation Asset Allocation Problem

During the last decade, the MV analysis in finance was widely applied for solving the optimal electricity allocation problems (Donghan et al., 2007; Hatami et al., 2011; Liu & Wu, 2006; Liu & Wu, 2007a, 2007b; Xiaohong et al., 2008). The problem involves allocation of the generated electricity to several customers in different markets to maximize the Genco's expected return and minimize risk measured by the variance of return. It is generally assumed that the Genco of interest operates within a deregulated energy market by trading its electricity through physical trading instruments, i.e. spot market, day-ahead market and forward contract.

According to their research framework, a Genco can sell its generated electricity in the spot market on real-time basis. However, electricity spot price is highly volatile due to non-storability of the product and uncertainty in its demand. Therefore, trading in the spot market is regarded as a risky trading choice. In the case of forward contract, a Genco bilaterally makes a trade agreement in terms of price ($/MWh), quantity (MWh), delivery time and location with a counterparty in advance. A Genco can sign forward contracts with customers located in the same pricing zone, known as “local forward contract”, and different pricing zones, known as “non-local forward contract”. According to the locational marginal pricing (LMP) scheme where the location of buyer and seller has an influence on the Genco’s revenue, a local forward contract can be considered as risk-free trading choice because the transmitted quantity and contract price are established beforehand, and therefore no uncertainty is involved. Although the details of a non-local bilateral contract is agreed in advance as the case of a local bilateral contract, it bears risk since the congestion charge is levied on the Genco when there is line congestion during the delivery. Since line congestion is unpredictable, non-local forward contract is considered as a risky trading choice.

In their methodology, these authors computed the expected returns and variance-covariance matrix of available trading choices and solved the optimal electricity allocation problem by maximizing the Genco’s utility function. In their works, the Genco’s utility was formulated based on quadratic function, thus only the first two moments of the return distribution were considered. Although the distribution of electricity spot prices exhibits asymmetry, the skewness of returns was neglected by these studies.

2.6 SOLVING THE MVS PORTFOLIO OPTIMIZATION PROBLEM

As stated earlier, many researchers experience the empirical and theoretical difficulties of using the MV analysis due to its restrictive assumptions. This is true for application in the portfolio optimization problem both in the financial and electricity markets. For instance, MV portfolio model assumes that asset returns are normally distributed during the period of analysis and can be characterized by mean and variance of returns distribution. In reality, the asset returns are not normally distributed (Amaya & Vasquez, 2010; Arditti, 1967; Bates, 1996; Bekaert & Harvey, 2002; Black, 1976; Campbell & Hentschel, 1992; Canela & Collazo, 2007; Chen et al., 2001; Christie, 1982; Chunhachinda et al., 1997; Fielitz, 1976; French et al., 1987; Gennotte & Leland, 1990; Gibbons et al., 1989; Grossman, 1989; Hwang & Satchell, 1999; Jorion, 1988; Liu, Margaritis, & Tourani-Rad, 2012; Prakash et al., 2003; Simkowitz & Beedles, 1978; Singleton & Wingender, 1986; Wang et al., 2009). However, Levy and Markowitz (1979) argued that the MV model might maximize the expected utility, even when return distributions are not normal, if the deviation in rate of returns is relatively small. This idea is costly for a real application since portfolio rebalancing is required very frequently.

Besides, the MV model relies on a continuous time distribution assumption where asset prices follow a diffusion or stochastic process. Thus, based on Ito's lemma, any moments higher than the second order are not relevant to investors' investment decisions. However, it has been proved that an investor's decision making is restricted to a discrete time interval. Therefore, MV efficiency becomes inadequate and higher moments become relevant to portfolio selection (Samuelson, 1970). Furthermore, a number of authors questioned on the usefulness of the quadratic utility function (Blume & Friend, 1975; Hanoch & Levy, 1970; Levy, 1974) and derived skewness preference theory using various forms of the utility function (Arditti, 1967; Kane, 1982; Levy, 1969; Tsiang, 1972). A number of empirical papers have shown that skewness, regardless of its form, is priced meaning that investors are willing to pay a premium for holding an asset with positive skewness (Harvey & Siddique, 2000; Kraus & Litzenberger, 1976; Xu, 2007). However, investigations on the impact of skewness preference on efficient portfolio selection are very limited because of the difficulties in obtaining a set of MVS efficient portfolios from the search in the multi-dimension space.

2.6.1 The Existing Approaches

According to the fact that a rational investor favors an investment exhibiting positive skewness in the return distribution when the mean and the variance of two portfolios are similar, portfolio selection under the MV model may no longer satisfy the portfolio holders. In the MVS analysis, an investor whose utility is a function of the first three moments of return distributions constructs a portfolio so as to maximize expected return, minimize risk and maximize skewness simultaneously (Arditti & Levy, 1975; Jean, 1971, 1973). Thus, the MVS-POP is a tri-objective optimization problem whose objectives compete and conflict with each other. From the optimization point of view, a portfolio that optimizes three objectives at the same time does not exist. In the last few decades, various techniques have been applied for solving this problem. These techniques can be categorized into two approaches including single-objective optimization approach and aggregating approach.

For the single-objective optimization approach, a tri-objective MVS-POP is converted to a single-objective optimization problem where portfolio skewness is optimized with respect to the constraints in the expected return and variance. Thus MVS-POP is commonly expressed in single-objective framework as follows:

$$\begin{aligned} \text{Maximize} \quad & S_p(\mathbf{x}) \\ \text{Subject to} \quad & R_p(\mathbf{x}) = R_C \\ & V_p(\mathbf{x}) = V_C \\ & \textstyle\sum_{i=1}^{N} x_i = 1 \,,\, x_i \geq 0 \end{aligned}$$

where $R_p(\mathbf{x})$, $V_p(\mathbf{x})$, and $S_p(\mathbf{x})$ denote respectively the expected return, variance, and skewness of a portfolio. Thus a portfolio solution, i.e. $\mathbf{x} = [x_1 \quad x_2 \quad \cdots \quad x_N]^{\mathrm{T}}$, is one that satisfies the constraint $\sum_{i=1}^{N} x_i = 1$, $x_i \geq 0$ and maximizes the portfolio skewness at given values of R_C and V_C. According to the optimization theory, however, maximizing the skewness function is a class of non-concave maximization problems where the global maximum cannot be solved by the conventional linear programming. Therefore, several studies focused upon derivations of a linear function that can be used to approximate skewness function (Konno et al., 1993; Konno & Suzuki, 1995; Ryoo, 2006). Then the MVS-POP can be solved using a mathematic programing technique. For instance, Konno et al. (1993) developed the mean-absolute deviation-skewness (MADS) portfolio model. In their model, the use of absolute deviation which can be written in a linear form was suggested instead of the quadratic function. Then, the skewness function was approximated by the lower semi-third moment derived from a piecewise linear function. Thus, the MVS-POP can be solved by maximizing the lower semi-third moment with respect to the mean and absolute deviation constraints. This concept was further improved by Konno and Suzuki (1995). They derived and optimized the piecewise linear function of skewness subject to given values of mean and the piecewise linear function of variance. Based on the concept of fuzzy variable, Bhattacharyya, Kar, and Majumder (2011) and Li, Qin, and Kar (2010) applied the fuzzy variable technique to analyze a MVS portfolio model. They optimized the skewness of a fuzzy return variable at given values of its mean and variance using the fuzzy simulation.

In the aggregating approach or the so called "weighted-sum approach", a MVS-POP is reformulated into a single-objective. However, the single-objective optimization approach, this approach combines all objectives into an aggregate objective with the use of relative importance coefficients assigned to each optimized objectives. In the aggregating approach, a MVS-POP can be stated as follows:

Problem $$\min_{\mathbf{x}} F(\mathbf{x}) = \left[-\beta_1 R_p(\mathbf{x}) + \beta_2 V_p(\mathbf{x}) - \beta_3 S_p(\mathbf{x})\right] \quad (2.11)$$

Subject to $\sum_{i=1}^{N} x_i = 1,\ x_i \geq 0$

where β_i is a relative importance coefficient that represents a preference for objective *i.* A portfolio solution that satisfies the constraint $\sum_{i=1}^{N} x_i = 1,\ x_i \geq 0$ and minimizes the aggregate function $F(\mathbf{x})$ can be obtained for a given value of β_i. A higher value assigned to the coefficient of an objective implies the higher preference for that objective. By varying the values of these coefficients, a set of solutions can be obtained by optimizing the aggregate objective function. For instance, polynomial goal programming (PGP), introduced by Tayi and Leonard (1988) to solve problems in the field of operations research, has been widely applied for solving the MVS-POP in the finance literature. This technique allows the levels of preference toward an objective of individual investors to be taken into account in portfolio selection. For PGP, a new objective, i.e. deviation from the goal (best) value of return and skewness, is formulated and minimized with the use of parameters that represent investor preferences for return and skewness. Then, by restricting the variance to be equal to one, the optimal portfolio can be solved at any given parameter values. The PGP was applied for solving MVS-POP with different data sets. For instance, Lai (1991) used five US stocks to prove this model. Chunhachinda et al. (1997) tested Lai's model with 14 country indices, while Prakash et al. (2003) extended the work of Chunhachinda et al. (1997) by adding the emerging Latin American country indices. The industrial indices of emerging markets constructed by MSCI Global Industrial Classification Standard were used by Canela and Collazo (2007). Although different data sets were used, they consistently concluded that the incorporation of skewness preference of individual investors in portfolio selection lead to major changes in optimal portfolio selection. In recent years, the neural network-based method was utilized for solving a MVS-POP based on the weighted-sum approach. Yu et al. (2008) demonstrated that a set of MVS optimal portfolios can be obtained by changing the values of objective coefficients.

2.6.2 Shortcomings of the Existing Approaches

Although a portfolio solution can be obtained by these approaches, many concerns were widely discussed in the field of operations research. Firstly, prior knowledge about the preference for each objective and the given value of constraints which are subjective to a DM is required. In many cases, these predetermined values unfavorably guided search mechanism of an algorithm to the wrong direction. As a result, the obtained solution is most likely to be a sub-optimal solution (Athan & Papalambros, 1996; Messac et al., 2000). Secondly, only one solution is obtained from one optimization run. Thus a collection of portfolio solutions has to be generated by performing a series of separate runs of an algorithm by changing the given values of preference coefficients and constraints. However, a large number of time-consuming computations are required which makes solving a MVS-POP intractable within reasonable time in practice. Thirdly, since a MVS-POP belongs to the class of non-concave maximization problems, Das and Dennis (1997), Marler and Arora (2004), and Messac et al. (2000) demonstrated that even when the set values of preference coefficients or constraints are fractionally and continuously varied, there is no guarantee that an obtained solution will be the global optimum.

Among the observed shortcomings, the possibility of not achieving the global optimum or efficient solutions is very serious because the notion of efficiency is central to portfolio theory and it is one of the main concerns of this thesis. In general, a utility maximization agent facing a decision to choose a portfolio among a feasible set of investment portfolios, will consider only a choice within a set of efficient portfolios. Among a group of agents, portfolio choice varies based on individual preference or utility function. However, none of them will choose an inefficient portfolio. As a consequence, the portfolio choices obtained from the approaches discussed above may be far from the choices that maximize investor utility.

2.6.3 Multi-objective Evolutionary Algorithms (MOEAs)

In the MV model, investors determine a set of feasible portfolios by using the mean, the variance, and the covariance. The efficient portfolios are those among a feasible set with minimum variance at a given expected return, or maximum expected return for a given variance. In the MVS model, Ingersoll (1975) suggested that the MVS efficient set is a collection of portfolios with:

Maximum $R_p(\mathbf{x})$ for given $V_p(\mathbf{x})$ and $S_p(\mathbf{x})$,

Minimum $V_p(\mathbf{x})$ for given $R_p(\mathbf{x})$ and $S_p(\mathbf{x})$,

Maximum $S_p(\mathbf{x})$ for given $R_p(\mathbf{x})$ and $V_p(\mathbf{x})$,

Due to the conditions above it can be seen that to obtain a set of MVS efficient portfolios requires a large number of computations in the multi-dimension spaces. Firstly, a sufficient number of plausible solutions that satisfy the problem constraints have to be generated. Secondly, the evaluating and the screening process are performed to keep only the efficient solutions and eliminate the inefficient ones. Then, investors can choose from a set of MVS efficient portfolios, a portfolio that is most likely to maximize his utility.

Evolutionary algorithm (EA) is a generic term for a class of stochastic search techniques, whose search mechanisms mimic the natural evolution and the Darwinian concept of survival of the fittest (Goldberg, 1989). EA embodies the techniques of genetic algorithms (GAs), evolutionary strategies (ES) and evolutionary programming (EP). Originally, EA is proposed for solving single-objective optimization problems (Goldberg, 1989; Holland, 1975). Subsequently, it was developed for solving MOOPs. In addition, EA is considered as a derivative-free approach, thus it is suitable for solving a class of non-concave maximization problems such as the MVS-POP. Its ability for solving a complex MOOPs significantly attracts researchers to implement MOEAs for solving MOOPs in different areas of research.

As widely recognized in the literature, an MOEA has the ability for searching partially ordered spaces for several alternative trade-offs and for dealing with discontinuous efficient surface. Thus, a set of resulting solutions can be obtained within a single run of simulation instead of performing a series of separate runs as in the case of the single-objective approach and the aggregating approach. Importantly, the selection process of MOEAs which is performed based on Pareto dominance relation (Deb, 2001) ensures that the resulting solutions are efficient portfolios. Unlike other stochastic search techniques whose solutions are randomly generated until a number of iterations is satisfied, the solutions for a new iteration (or the new generation) of MOEAs are developed from the efficient solutions found in the current iteration. Therefore, the approximation of multiple efficient solutions can be effectively executed within a short CPU time.

In the past decades, many MOEAs using the Pareto-based approach were introduced to solve this problem. The initial proposed Pareto-based MOEAs are non-elitist[5] MOEAs such as the multi-objective genetic algorithm (MOGA) (Fonseca & Fleming, 1993), the Niched Pareto genetic algorithm (NPGA) (Horn & Nafpliotis, 1993), and the non-dominated sorting genetic algorithm (NSGA) (Srinivas & Deb, 1994). Meanwhile, the modern Pareto-based MOEAs incorporate the elitist process into the GA operators such as the strength Pareto evolutionary algorithm (SPEA) (Zitzler & Thiele, 1999a), the Pareto envelope-based selection algorithm (PESA) (Corne, Knowles, & Oates, 2000), the Pareto-archive evolution strategy (PAES) (Corne et al., 2000), the non-dominated sorting genetic algorithm II (NSGA-II) (Deb, Pratap, Agarwal, & Meyarivan, 2002), and improved strength Pareto evolutionary algorithm (SPEA-II) (Zitzler, Laumanns, & Thiele, 2002b).

2.6.4 Application of MOEAs for Solving Portfolio Optimization Problem

The most well-known MOOP in finance is portfolio selection problem or portfolio optimization problem. The MV model of Markowitz (1952) explains that investors optimize their portfolio by minimizing the portfolio's risk and maximizing its expected return, simultaneously. This problem can be efficiently resolved using either linear programming (LP) or quadratic programming (QP). Due to the successfulness of MOEAs for solving MOOPs in the field of engineering and science, several papers introduced MOEAs for solving the MV-POP. For instance, Shoaf and Foster (1998) applied MOGA for solving the MV-POP with five assets. They demonstrated two advantages of uing MOGA over QP. Firstly, MOGA can simultaneously optimize two objectives, whereas QP sets expected return as a constraint while risk is minimized. Therefore, QP returns only one

[5] Elitist is a process designed to ensure that the good solutions will be reserved and passed through the next iteration (generation).

solution for a set value of expected return, while MOGA produced many solutions within a single run of algorithm. Secondly, the solutions of MOGA are superior than those of QP. Their result shows that, at a given value of risk, the portfolios obtained from MOGA offer higher expected return than those obtained from QP. Subsequently, (Vedarajan, Chan, & Goldberg, 1997) authenticated the findings by experimenting NSGA for solving portfolio selection problem with five stocks in the US stock market, namely, Boeing, Disney, Exxon, McDonald, and Microsoft. Their results confirmed the superiority of the solutions solved by NSGA over those attained from QP.

The progressive capabilities of MOEAs for solving the MV-POPs attracted the attention of researchers to utilize MOEAs for the complex portfolio selection problems. The literature has developed in two aspects. Firstly, the practical constraints and objectives that exist in the real-world trading situation are incorporated into the MV-POPs. The well-known constraints in the literature are cardinality constraint, i.e. number of assets held in portfolio (Anagnostopoulos & Mamanis, 2010; Chang, Meade, Beasley, & Sharaiha, 2000; Felix Streichert , Holger Ulmer , & Zell, 2003; Soleimani, Golmakani, & Salimi, 2009), lower and upper bounds, i.e. the minimum and maximum allocation ratio to an asset (Skolpadungket, Dahal, & Harnpornchai, 2007), transaction cost constraint (Lin & Liu, 2008; Soleimani et al., 2009), roundlots constraint, i.e. roundup number of stocks bought, and non-negative constraint, i.e. disallowing of short sale. In addition, some objectives such as, tracking error and other firm’s fundamental data are incorporated into the standard portfolio optimization problem (Beasley, Meade, & Chang, 2003; Branke, Scheckenbach, Stein, Deb, & Schmeck, 2009; Ghandar, Michalewicz, Zurbruegg, & Chee, 2010; Adam Ghandar, Michalewicz, & Zurbruegg, 2012). In this aspect, the ability of MOEAs compared to the traditional mathematic programming is strengthened because it is widely recognized that when these constraints are incorporated in POPs, the Pareto front or efficient frontier is most likely to be discontinuous. Therefore, the traditional mathematic programming such as QP and LP are no longer suitable for solving these problems.

Secondly, various MOEAs with some modifications in the GA operator were proposed for dealing with the MO-POP. For instance, Chang et al. (2000) compared the performance of three heuristic stochastic search techniques including MOGA, tabu search, and simulated annealing for solving the POP with cardinality constraint. They asserted that GA outperforms tabu search and simulated annealing. Ehrgott, Klamroth, and Schwehm (2004) reinvestigated the performance of MOGA, tabu search, and simulated annealing in dealing with the five-objective POP. Their results confirm the superiority of MOGA among the three algorithms. The performances of the PESA, the NSGA-II and the SPEA-II were evaluated for solving the standard MV-POP (Diosan, 2005). She found that the performance of PESA dominates the other two algorithms. In recent year, Anagnostopoulos and Mamanis (2010) reassessed the performance of the three MOEAs as used in the work of Diosan (2005). But the problem was set as the tri-objective POP including maximizing expected return and minimizing both risk and cardinality, at the same time. They argued that the SPEA-II outperforms in the sense of the closeness to the true Pareto front and the diversity of solutions along the true Pareto front. Subsequently, Anagnostopoulos and Mamanis (2011) confirmed these findings by investigating the performance of five MOEAs for solving their proposed tri-objective POP with a different data set.

According to a recent comprehensive literature review on the application of MOEAs for portfolio management (Metaxiotis & Liagkouras, 2012), the authors pointed out that the majority of papers experimented with MOEA for solving bi-objective optimization problems where the expected return is maximized and variance is minimized. Several works proposed the use of other risk measure such as value at risk (VaR), expected shortfall (ES), and semi variance instead of variance used in the traditional MV analysis. Although the importance of skewness in asset pricing is widely documented and recognized, it is surprising that there are only two papers that considered skewness as an objective in portfolio selection problem (Chang, Yang, & Chang, 2009; Lin, 2012). However, the problem in their work was formulated based on weighted sum approach whose disadvantages are discussed in the previous sub-section.

2.7 RESEARCH GAPS

The research gaps found from the literature can be categorized into three parts as discussed in the following sub-sections.

2.7.1 MVS Efficient Portfolios in Multi-dimension Moment Space

As stated in the paper of Tsiang (1972, p. 359), a pioneering work on skewness preference, "if we regard the phenomenon of increasing absolute risk aversion as absurd, we must acknowledge that a normal risk-averter individual would have a preference for skewness, in addition to an aversion to dispersion of the probability distribution of returns". Many studies suggest the use of skewness in portfolio selection in the case where the return distribution is not normal and the investor utility cannot be approximated by only mean and variance (Jean, 1971, 1973; Kane, 1982; Samuelson, 1970). With the assumptions that investors are single-period maximizers of expected utility of terminal wealth, and utility function is either a cubic utility or a Taylor's series expansion of terminal wealth, the efficient portfolio is one that maximizes investor utility. However, maximizing this convex utility function, if possible, is very cumbersome. Most of the studies employed a parameter-preference model[6], MVS model, to investigate the impact of skewness preference on portfolio selection.

However, solving the MVS-POP is not easy due to the complexity of the problem itself and the procedure for determining the MVS efficient portfolios. To obtain the MVS efficient portfolios, ideally, a feasible set of portfolios that satisfy the constraints is firstly generated. Then, efficiency comparison between portfolios is conducted in order to retain only the efficient portfolios and to remove the inefficient ones out of the efficient set. Although various techniques were proposed in the literature as discussed in Section 2.6.1, their ability is restricted to perform these mentioned processes. In addition, the disadvantages of using these techniques for solving the MOOPs such as the MVS-POP are well recognized in the field of operations research (Athan & Papalambros, 1996; Das & Dennis, 1997; Marler & Arora, 2004; Messac et al., 2000), especially their unwarranted ability to provide efficient solutions. As a result, the impact of skewness on portfolio selection may be inaccurately

[6] Rubinstein (1973) explained that a parameter-preference model is a model in which probability data are summarized by "parameter", ordinarily the central moments. Due to the difficulties the expected utility model encounters in empirical studies, this model is widely employed. The MV model is a well-known parameter-preference model where investors make a portfolio decision based on the parameters such as mean, variance, and covariance.

interpreted. To close this research gap, Chapter 4, Chapter 5, and Chapter 6 employ the MOEAs for solving the MVS-POP and obtaining the MVS efficient portfolios in the multi-dimension moment space.

2.7.2 Skewness Preference and Portfolio Choice

Due to the fact that elimination of the firm-specific risk or unsystematic risk can be achieved through diversification, this task is crucial for risk-averse investors whose utility can be described by the mean and variance of return distribution. However, observed behavior from economic agents reveals that portfolio holding of investors is far from diversification since there are only very few assets held in their portfolios (Blume & Friend, 1975). One plausible theoretical explanation of this behavior is linked to skewness preference. Based upon the expected utility model, Simkowitz and Beedles (1978) explained that diversification may not be desirable for investors who base their decisions on the first three moments of return distributions. Conine and Tamarkin (1981) proved mathematically that the demand function of investors who homogeneously prefer skewness can be optimized by holding a small number of assets in their portfolio.[7]

Subsequently, Mitton and Vorkink (2007) developed a model in which the preference of investors for the first two moments is the same but the preference for skewness is heterogeneous across agents. They argued that by allowing heterogeneous preference for skewness, their model allows investors, in equilibrium, to hold undiversified portfolio. The empirical finding of Goetzmann and Kumar (2008) reveals that undiversified portfolios of

[7] The derivation is conducted by assuming that each asset in a portfolio has equal weight, i.e. investment proportion.

household accounts obtained from a large discount brokerage house in the U.S. are less efficient in the MV analysis and have higher skewness than the diversified ones.

In the literature, a collection of studies examined the impacts of skewness preference on portfolio choice using different approaches but none of them conducted the investigation in the context of MVS efficient portfolios due to computation restriction. For example, Mitton and Vorkink (2007) pointed out that undiversified portfolios which are inefficient in MV analysis may be efficient under MVS analysis. However, "to assess MVS efficiency of a portfolio, ideally, MVS efficient frontier would be constructed using return characteristics of available stocks at times of portfolio formation. However, the large number of computation required to construct this frontier in three dimensions makes this approach intractable" (Mitton & Vorkink, 2007, p. 1274). In addition, the model proposed by Mitton and Vorkink (2007) requires economic agents to be classified into two groups including the *Traditional investors* with concave utility function and the *Lotto investors* with convex utility function.[8] This assumption seems unreasonable due to the fact that skewness preference, i.e. $U''' > 0$, is an inheriting result of risk aversion, i.e. $U'' < 0$ (Arditti, 1967). Therefore, skewness preference is a common trait for a rational risk-averse investor and not only for the *Lotto investor*s.

To close the research gap, Chapter 5 of this thesis develops a single-period model that allows for heterogeneity in the degree of risk aversion and skewness preference. The MVS

[8] In their work, *Traditional investor* is used to represent an investor whose utility is described by the quadratic function. In other words, a *Traditional investor* acts to maximize their utility based on the MV analysis. The term *Lotto investor* is used to describe an investor whose demand function can be approximated by the third-order Taylor's series expansion. Thus, the utility of a *Lotto investor* can be maximized by maximizing expected return and skewness, and minimizing variance of the return distribution.

efficient portfolios obtained from the MOEAs are used to examine the impact of skewness preference on efficient portfolio choice.

2.7.3 MVS Analysis for Solving an Electricity Allocation Problem

A number of studies examined how a Genco allocates its generated electricity to available trading choices in order to maximize its utility under the MV framework (Donghan et al., 2007; Hatami et al., 2011; Liu & Wu, 2006; Liu & Wu, 2007a, 2007b; Xiaohong et al., 2008). However, the use of the MV analysis is limited only for the case that the utility of an agent is represented by quadratic function and the return of the asset follows the normal distribution (Harvey & Siddique, 2000; Kraus & Litzenberger, 1976; Samuelson, 1970). Many theoretical papers suggested that utility functions that satisfy the non-increasing absolute risk aversion condition (Arrow, 1964; Pratt, 1964) exhibit a preference for skewness in return distributions (Arditti, 1967; Kane, 1982; Levy, 1969; Tsiang, 1972). Besides, several authors provided the evidence that the distribution of electricity spot prices is not normal but skewed (Benth et al., 2008; Bessembinder & Lemmon, 2002; Cartea & Villaplana, 2008; Hajiabadi & Mashhadi, 2013; Longstaff & Wang, 2004; Lucia & Torró, 2011; Redl et al., 2009). Moreover, the preference for positive skewness can be observed from agents in the electricity markets. Many studies showed that the forward premium is positively related with skewness of the spot price distribution (Bessembinder & Lemmon, 2002; Douglas & Popova, 2008; Longstaff & Wang, 2004; Lucia & Torró, 2011; Parsons & De Roo, 2008; Redl et al., 2009; Viehmann, 2011). As a result, the electricity allocation problem should be established under the MVS model. Thus, the asymmetry in the distribution of electricity spot prices which was assumed to be symmetric in previous works is taken into account in this thesis.

In addition, the generation asset allocation problem between the spot and physical forward markets is generally analyzed based on known forward prices and forecasted spot prices. Since spot prices during the decision making period are unknown, the reliability of the optimization solution depends crucially on the accuracy of their forecasts. Given the fact that electricity spot prices are characterized by non-constant mean and variance (Garcia, Contreras, van Akkeren, & Garcia, 2005; Jun Hua, Zhao Yang, Zhao, & Kit Po, 2008), multiple seasonalities and high volatility (Aggarwal, Saini, & Kumar, 2009), omission of these effects in the forecasting model may result in erroneous Pareto-optimal solutions (Suksonghong & Goh, 2012). In previous studies, spot prices were forecasted using simple forecasting methods (Liu & Wu, 2006; Liu & Wu, 2007a, 2007b), while seasonality was determined from the mean value of historical spot prices of a similar day and month (Donghan et al., 2007).

Moreover, the number of available trading choices in an electricity market is small compared to a stock market. Steuer, Qi, and Hirschberger (2007) observed that when an investment universe is significantly small, the allocation of investment of the resulting portfolio solutions obtained from an optimization is clustered in only a few assets. In other words, the efficient portfolio solutions consist of a small number of assets. In the finance point of view, diversification helps reduce firm-specific risk, thus allocation of investment to only a few investment alternatives may not be a desirable strategy for a Genco.

Constraints such as cardinality constraint, ceiling limit constraint and class constraint are typically incorporated in the optimization problem to increase the number of assets in the portfolio and/or to prevent excessive investment in a small number of assets. However, the value of constraint is subjectively predetermined by a DM leading to unnecessary restriction of the search space of an algorithm. As a result, the obtained solutions may not be efficient (Steuer et al., 2007).

To close the research gap, the electricity allocation problem is formulated as the MVS-POP in Chapter 6 where expected return, variance and skewness are optimized simultaneously. Besides, the generalized autoregressive conditional heteroscedastic (GARCH) model with seasonality dummies (Suksonghong & Goh, 2012) is employed to capture the seasonality patterns and time varying volatility of electricity spot prices. The forecasts are used for computing the variables such as expected return, variance, skewness, variance-covariance matrix and skewness-coskewness matrix. These variables are the input variables for solving the MVS-POP.

In addition, we propose an additional objective to increase the number of trading choices included in the portfolio solutions of a Genco. The proposed objective is set to minimize the difference between the highest and the lowest proportion of investment within a portfolio solution.

CHAPTER 3

METHODOLOGY

3.1 INTRODUCTION

This chapter is divided into five sections. In the first section, after this introduction, the conceptual framework is developed and the MVS portfolio model is formulated. The proposed approach for solving the formulated MVS-POP is discussed in the second section. The third, the fourth, and the fifth section explain respectively the methodology for Chapter 4, Chapter 5, and Chapter 6 including the analytical framework, procedure, the data, and the optimization methodology.

3.2 CONCEPTUAL FRAMEWORK

This section illustrates the conceptual model for investors whose utility can be described over the first three moments of the return distributions. This utility function is the Taylor's series expansion up to the third order. Let $U(R)$ represent the investor utility function where R is the random return. Using Taylor's series expansion around the mean value of R and taking expected value on both sides, we obtain the following:

$$U(R) = U(E[R]) + U'(R)(R - E[R]) + \frac{U''(R)}{2!}(R - E[R])^2 + \frac{U'''(R)}{3!}(R - E[R])^3 \tag{3.1}$$

$$E[U(R)] = U(E[R]) + \frac{U''(R)}{2!}E[(R - E[R])^2] + \frac{U'''(R)}{3!}E[(R - E[R])^3] \tag{3.2}$$

where $E(\cdot)$ is the expected value operator, $E[R]$ is the expected return, and $E[(R - E[R])^n]$ is the n^{th} moment of the return distribution. The desirable properties of the investor utility are (i) positive marginal utility, i.e. $U' > 0$, (ii) decreasing marginal utility, i.e. $U'' < 0$, and (iii) non-increasing absolute risk aversion, i.e. $U''' > 0$ (Pratt, 1964; Samuelson, 1970). Therefore, the expected utility can be optimized by simultaneously maximizing the expected return and skewness and minimizing the variance of the return distribution. Since the parameter-preference based approach is used in our study, we adopt the MVS portfolio model. The purpose is to develop a multi-objective framework for a single-period MVS model for solving the portfolio optimization problem. Thus, it is noted that all considered variables are deterministic as assumed by Markowitz (1952). In other words, the expected return, variance-covariance matrix, and skewness-coskewness matrix are known before making a decision. Suppose that a single investment holding period where N securities are available for investment is considered. At the beginning of the holding period, an investor determines the proportion of his initial investment that will be allocated to each available security. Suppose that $\mathbf{x}$, which represents a portfolio solution, is a $N \times 1$ vector of investment allocation proportion to N securities. Let vector $\mathbf{R}$ with size $N \times 1$ represent the expected returns of N securities. Matrix $\mathbf{\Lambda}$ is a non-singular $N \times N$ variance-covariance matrix. The theoretical model of Conine and Tamarkin (1981) suggests that all joint product moments between assets are relevant for portfolio selection, thus the total skewness should be decomposed as idiosyncratic skewness, curvilinear, and triplicate product moments. As a result, matrix Ω with size $N \times N \times N$ is used to represents skewness-coskewness matrix. We transform matrix Ω size $N \times N \times N$ to $N \times N^2$ matrix by slicing each $N \times N$ layer and then placing each layer in the same order, adjacent to each other.

According to the literature, a large class of investors whose utility can be approximated by the third order Taylor's series expansion around the expected value will maximize the expected return and skewness of portfolio as well as minimize the portfolio variance simultaneously. Suppose that the expected return, variance, and skewness of a portfolio are denoted by $R_p(\mathbf{x})$, $V_p(\mathbf{x})$, and $S_p(\mathbf{x})$, respectively. Thus, the three objectives of a MVS-POP can be expressed as follows:

$$\text{Maximize} \quad R_p(\mathbf{x}) = \mathbf{x}^{\mathrm{T}}\mathbf{R} = \textstyle\sum_{i=1}^{N} x_i R_i \tag{3.3}$$

$$\text{Minimize} \quad V_p(\mathbf{x}) = \mathbf{x}^{\mathrm{T}}\mathbf{\Lambda}\,\mathbf{x} = \textstyle\sum_{i=1}^{N} x_i x_j \sigma_{i,j} \tag{3.4}$$

$$\text{Maximize} \quad S_p(\mathbf{x}) = \mathbf{x}^{\mathrm{T}}\mathbf{\Omega}\left(\mathbf{x} \otimes \mathbf{x}\right) = \textstyle\sum_{i=1}^{N}\sum_{j=1}^{N}\sum_{k=1}^{N} x_i x_j x_k \gamma_{i,j,k} \tag{3.5}$$

where $\mathbf{x}^{\mathrm{T}}$ is the transpose of vector $\mathbf{x}$, x_i is the proportion of investment allocated to security *i*, and R_i is the expected return of security *i*, $\sigma_{i,j}$ is covariance between securities *i* and *j*, and $\gamma_{i,j,k}$ represents the coskewness between securities *i*, *j* and *k*. The sign $\otimes$ is the Kronecker product. Thus, the proposed MVS-POP can be formulated under the multi-objective optimization framework as follows:

$$\textbf{Prob. 1} \qquad \min_{\mathbf{x}} F(\mathbf{x}) = \left[-R_p(\mathbf{x}), V_p(\mathbf{x}), -S_p(\mathbf{x})\right] \tag{3.6}$$

It is implied in the **Prob.1** that all objectives are optimized at the same time without any priori information about the preference for any particular objective. In the other words, these three objectives are equally important in the optimization process.

3.3 THE PROPOSED TECHNIQUE

For a multi-objective optimization problem (MOOP) such as that of MVS-POP whose objectives are completing and conflicting with each other, a solution that optimizes all objectives at the same time does not exist. Therefore, the task of DM is to find a set of compromising (non-dominated) solutions in the multi-dimensional feasible space and then select the best solution that matches with the preference of a DM.

3.3.1 The Multi-objective Optimization Framework

Basically, a MOOP contains at least two objectives that will be optimized, i.e. minimized or maximized. The general form of a MOOP is usually stated with objective functions and constraint functions as follows:

$$
\begin{aligned}
\text{Optimize} \quad & f_k(\mathbf{x}) && k = 1, 2, \ldots, K \\
\text{Subject to} \quad & g_l(\mathbf{x}) = 0; && l = 1, 2, \ldots, L \\
& h_m(\mathbf{x}) \geq 0; && m = 1, 2, \ldots, M \\
& x_i^L \leq x_i \leq x_i^U; && i = 1, 2, \ldots, N
\end{aligned}
\tag{3.7}
$$

where $\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{bmatrix}$ is a solution vector of N decision variables which can be conveniently written as $\mathbf{x} = [x_1 \quad x_2 \quad \cdots \quad x_N]^{\mathrm{T}}$, and K represents the number of objective functions to be optimized. $g_l(\mathbf{x})$ and $h_m(\mathbf{x})$ are the equality and inequality constraint functions respectively, where L and M represent the number of equality and inequality constraints. x_i^L and x_i^U are the lower and upper bound of the value of x_i which constitute the boundary of the solution space (S). Noted that the inequality function, $h_m(\mathbf{x})$, is normally treated as greater than or equal to, however, it can be represented a constraint function of less than or equal to by multiplying the function with (-1).

3.3.2 Multi-dimension Search Space

In single-objective optimization problems, there is only one objective to be optimized. Thus, an algorithm searches for a solution vector $\mathbf{x} \in S$ that satisfies all constraints and optimizes (minimizes or maximizes) a scalar $f(\mathbf{x})$. The search process of an algorithm takes place in the solution space $\mathbb{R}^N$ restricted in the feasible region S. In other words, the search directions are guided by the setting values of the constraints.

Meanwhile, in the multi-objective optimization approach, not only a solution vector, but also a vector of objective functions located in the objective space $\mathbb{R}^K$ is taken into account. The elements of the objective-value vector $\mathbf{z} \in Z = \begin{bmatrix} f_1(\mathbf{x}) \\ f_2(\mathbf{x}) \\ \vdots \\ f_K(\mathbf{x}) \end{bmatrix}$ or $\mathbf{z} \in Z = \begin{bmatrix} z_1 \\ z_2 \\ \vdots \\ z_K \end{bmatrix}$ represent the values of objectives contributed by a solution vector $\mathbf{x}$ where K is the number of considered objectives. Thus, a vector of solution $\mathbf{x} \in S$, in the solution space, associates with a vector of objective function $\mathbf{z} \in Z$ in the objective value space. This mapping takes place between an *N*-dimension solution space to a *K*-dimension objective space.

3.3.3 Determination of Non-dominated Solutions (Efficient Solutions)

In general, the optimization procedure is started by randomly generating a set of solutions that satisfy the problem constraints. The proposed technique detects the efficient solutions by evaluating the objective functions of the candidate solutions. For example, consider **Prob.1** with five assets as an investment universe. The value of expected return, variance, and skewness of the candidate portfolio solution $\mathbf{x}_{\mathrm{A}} = [x_1 \quad x_2 \quad \cdots \quad x_5]^{\mathbf{T}}$ is computed. Then, these objective values are saved as an objective-value vector, i.e. $\mathbf{z}_{\mathrm{A}} = [z_1 \quad z_2 \quad z_3]^{\mathbf{T}}$, where z_1, z_2, and z_3 represent respectively the value of expected return, variance, and skewness. Then, the objective-value vector $\mathbf{z}_{\mathrm{A}}$ is stored in the objective space. In the other words, one solution vector has one corresponding objective-value vector in the objective space. Many of such objective-value vectors can be obtained.

In order to determine a set of efficient solutions, the objective-value vectors are compared. In general, for MOOP whose objectives are equally important, a comparison can be conducted based on the Pareto dominance relation to obtain solutions that achieve "Pareto optimality". Let K^+ be a set of maximized objectives and K^- be a set of minimized objectives.

Definition 1. Objective-value vector $\mathbf{z}^*$ is said to be better than (dominates) vector $\mathbf{z}$ if and only if $\forall i \in K^+, z_i^* \geq z_i \wedge \exists i \in K^+, z_i^* > z_i$ and $\forall i \in K^-, z_i^* \leq z_i \wedge \exists i \in K^-, z_i^* < z_i$

Performing a comparison based on **Definition 1**, the set of non-dominated objective-value vectors can be obtained. From each non-dominated objective vector, we trace back to its corresponding solution vector. These are Pareto optimal solutions.

Definition 2. A solution vector $\mathbf{x}^*$ is said to be the efficient (non-dominated) solution if and only if its corresponding objective-value vector $\mathbf{z}^*$ is the non-dominated objective-value vector. Otherwise, $\mathbf{x}^*$ is the inefficient (dominated) solution.

Figure 3.1: Mapping between Solution Space and Objective Space

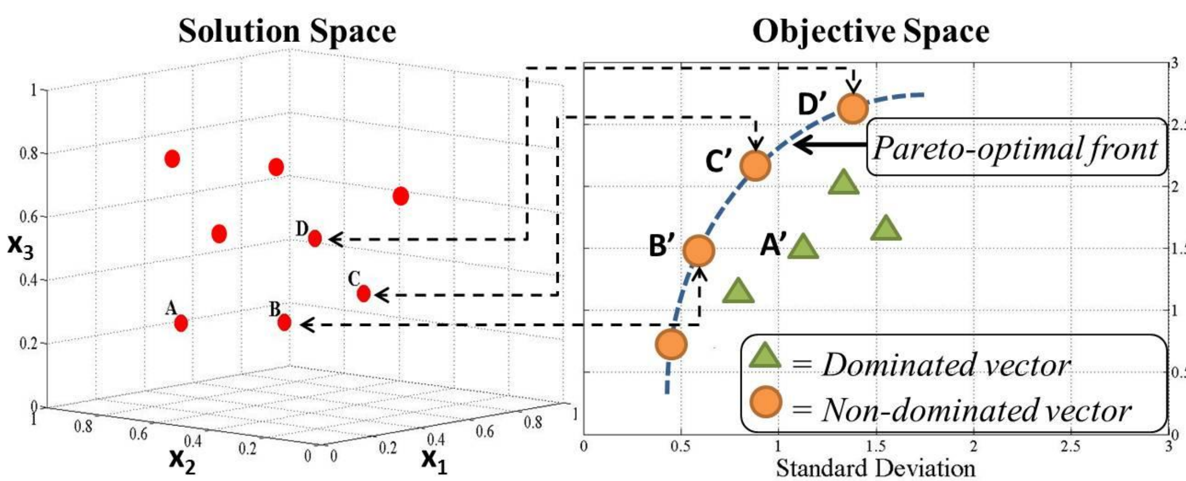


Figure 3.1 is an illustration of the portfolio optimization problem where the expected return is maximized and variance is minimized simultaneously. It is assumed that there are three available securities, i.e. $\mathbf{x} = [x_1 \quad x_2 \quad x_3]^{\mathbf{T}}$, as an investment universe. The solution space is shown on the left panel while the objective space is presented on the right panel. The figure demonstrates that the portfolio solution A, B, C, and D can be traced back to its corresponding objective-value vector A′, B′, C′ and D′, respectively. According to **Definition 1**, vectors A′ and B′ have the same expected return. However, vector B′ has a lower standard deviation than vector A′. Thus, vector A′ is dominated by vector B′. By comparing vector B′ with vector C′, vector B′ is better than C′ for one objective (standard deviation), but it is worse than C′ for another objective (expected return). Therefore, vector A′ does not dominate vector C′ and vice versa. Similar results can be observed when comparing vector B′ to D′ and C′ to D′. Thus, by comparing all pairs of vectors, vectors B′, C′ and D′ are considered as the non-dominated objective-value vectors. According to **Definition 2**, by tracing back to their corresponding portfolio solutions, portfolios B, C and D are the non-dominated portfolio solutions which are generally referred to as the “efficient portfolios”. The boundary or front that is formed by the non-dominated objective-value vectors is named as the “Pareto optimal front” or “efficient frontier”.

In the context of the MVS-POP formulated as **Prob.** 1, the searching process takes place within the *N*-dimensional solution space, where *N* is the number of available securities, and 3-dimentional objective space, i.e. the expected return, variance, and skewness. Therefore, we can say that portfolio A ($\mathbf{x}_A$) dominates portfolio B ($\mathbf{x}_B$) if and only if $R_p(\mathbf{x}_A) \geq R_p(\mathbf{x}_B)$, $V_p(\mathbf{x}_A) \leq V_p(\mathbf{x}_B)$ and $S_p(\mathbf{x}_A) \geq S_p(\mathbf{x}_B)$ with at least one strict inequality. A portfolio that is not dominated by any other portfolios is the MVS efficient portfolios and is a member of the MVS efficient surface.

As discussed in Section 2.6.3, MOEAs have the ability to perform the search in the multi-dimension space, in parallel with the ability to evaluate the efficiency of the solutions based on the concept of Pareto optimality. Thus, a set of MVS efficient portfolios can be effectively obtained within a single run of simulation. In the past decade, many MOEAs were developed and proposed for solving complex MOOPs in various disciplines. The non-dominated sorting genetic algorithm II (NSGA-II) (Deb et al., 2002) and the improved strength Pareto evolutionary algorithm (SPEA-II) (Zitzler, Laumanns, & Thiele, 2002a) are regarded as the most efficient and well-established MOEAs. However, the result of performance comparison between these two algorithms is incoherent. In some problems the NSGA-II outperforms the SPEA-II, and vice versa. Therefore, these two algorithms are employed in this study,[9] and a comparison is made in Chapter 4.

3.4 PORTFOLIO SELECTION WITH SKEWNESS PREFERENCE

Chapter 4 examines portfolio selection of financial assets within the MVS framework. The proposed MOEAs techniques, specifically NSGA-II and SPEA-II are applied for solving MVS-POP in the multi-dimension space. Besides, the characteristics of MVS efficient portfolios in terms of risk-return trade-of as well as the efficient surface are examined using

[9] The detailed explanation of the NSGA-II and SPEA-II is provided in APPENDIX A.

the data of emerging stock market indices. The impact of different investment horizons on the characteristics of MVS efficient portfolios is discussed.

3.4.1 The Optimization Problem

The MVS-POP is as stated in **Prob. 1** with two standard constraints. The optimization problem is as follows:

Prob. 1 $\min_{\mathbf{x}} F(\mathbf{x}) = \left[-R_p(\mathbf{x}), V_p(\mathbf{x}), -S_p(\mathbf{x})\right]$

Subject to $\sum_{i=1}^{N} x_i = 1$

$x_i \geq 0$

The first constraint is applied to make sure that all investment is fully allocated to available securities. The second constraint implies that the short selling is not allowed. As pointed out by Levy (1972) that different investment horizons can result in different efficient portfolio set to investors. He argued that the portfolio performance evaluated using the yearly rate of returns differs significantly from that using the monthly data set. In addition, Prakash, De Boyrie, and Hamid (1997) demonstrated that the estimates of the moments of the return distribution can be biased by the choice of an investment horizon. Therefore, to investigate whether different investment horizons has an impact on the MVS portfolio performance and composition, **Prob. 1** is solved using two sets of data including weekly and monthly rate of stock market returns.

3.4.2 Data

The data set consists of the indices for sixteen emerging stock markets including ten in Asia and six in Latin America.[10] The weekly and monthly price indices of these countries were obtained from the Morgan Stanley Capital International (MSCI) database which is available from DataStream. It should be noted that the MSCI converts the international price indices into US dollars using the spot foreign exchange rate for a particular period. The data range from January 2009 to December 2012. Therefore, there are 209 and 48 observations for the weekly and monthly data set, respectively.

This study covers the emerging markets in Asia that include China, India, Indonesia, Malaysia, Pakistan, Philippines, South Korea, Sri Lanka, Taiwan, and Thailand. The emerging markets of Latin America examined are Argentina, Brazil, Chile, Columbia, Mexico, and Peru. The skewness of return distributions of emerging markets is generally more pronounced because these markets are relatively small in size (in terms of market capitalization) (Doan et al., 2010; Harvey & Siddique, 2000).

3.4.3 Procedure

The procedure adopted for the analysis in Chapter 4 is divided into three parts. Firstly, the weekly and monthly returns from January 2009 to December 2012 for sixteen market indices are calculated. As a conventional approach for a single-period model, the rate of returns is computed from dividing the difference between closing price of today and the previous trading day by the closing price of the previous trading day as below.

[10] All the stock exchanges categorized by MSCI as emerging markets in Asia and Latin America are used in this chapter.

$$R_{i,t} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}} \quad (3.8)$$

where $R_{i,t}$ is the rate of returns for market *i* at time *t* and $P_{i,t}$ is the index closing price at time *t* for market *i*. The vectors are annualized for the different investment horizons.[11] Then, the following statistics are computed from the historical rate of returns of the market indices:

$$R_i = \frac{1}{T}\sum_{t=1}^{T} R_{i,t} \quad (3.9)$$

$$\sigma_{i,j} = \frac{1}{T}\sum_{t=1}^{T}\left(R_{i,t} - \bar{R}_i\right)\left(R_{j,t} - \bar{R}_j\right) \quad (3.10)$$

$$\gamma_{i,j,k} = \frac{1}{T}\sum_{t=1}^{T}\frac{\left(R_{i,t}-\bar{R}_i\right)}{\sqrt{\sigma_{i,i}}}\frac{\left(R_{j,t}-\bar{R}_j\right)}{\sqrt{\sigma_{j,j}}}\frac{\left(R_{k,t}-\bar{R}_k\right)}{\sqrt{\sigma_{k,k}}} \quad (3.11)$$

where *i, j,* and *k* = 1, 2, 3,…, 16, and *T* is total number of observations. The set of sixteen values of R_i is stored in the 16×1 vector **R.** The covariance $\sigma_{i,j}$ is kept in the variance-covariance matrix **Λ** with dimensions 16×16 and the coskewness $\gamma_{i,j,k}$ is organized in the skewness-coskewness matrix **Ω** of size 16×256.

Secondly, the normality test is conducted to test whether the return distributions of the sixteen market indices are normal. Since we assumed that the investor utility function can be expanded in the Taylor's series, the investors will prefer assets having a probability of large upside gain. Nevertheless, if the distribution of assets returns is symmetric and can be sufficiently characterized by mean and variance, investors are not capable to form

[11] For weekly annualized returns, the values obtained from Equation (3.8) are multiplied by 52. Meanwhile, the value of 12 is used as a multiplier in the case of the monthly annualized returns.

portfolios having high skewness in the return distribution. As a result, the normality test is regarded as a prerequisite step for constructing MVS portfolios.

There are various methods for conducting the normality test. However, which test should be used is an important question due to the fact that each test works well under different conditions. For example, with small sample sizes, the normality tests have small power to reject the null hypothesis of normal distribution. As a result, the test statistic always shows that small sample data come from the normal distribution, even though they are actually non-normally distributed. In the literature of statistics, the Shapiro-Wilk test (Shapiro & Wilk, 1965) and the Jarque-Bera test (Jarque & Bera, 1987) are popular as they are powerful normality tests. The former has good power especially when we anticipate that the distribution is asymmetric (Yap & Sim, 2011). The latter is suitable when we suspect that the data come from normal distribution, but it does not work well when dealing with small sample data (less than 50 observations) (Yazici & Yolacan, 2006). Since we did not have a priori information on the shape of distribution, we conducted the normality test using both the Shapiro-Wilk and Jarque-Bera tests. The hypothesis to be tested is stated as follows:

H_0: The parent population is normally distributed.

H_1: The parent population is not normally distributed.

To determine whether to reject the null hypothesis, the probability associated with the test statistic is computed. If the probability or the P-value is less than the significance level, i.e. $\alpha = 0.10$, the null hypothesis is rejected in the favor of the alternative hypothesis that the data distribution is not normal.

Thirdly, the proposed MOEAs together with the Pareto dominance relation described in Section 3.3 are utilized for searching the MVS efficient portfolios in 3D space. The non-dominated sorting genetic algorithm II (NSGA-II) (Deb et al., 2002) and the improved strength Pareto evolutionary algorithm (SPEA-II) (Zitzler et al., 2002a), regarded as the most efficient and well-established MOEAs, are applied. The summary of the parameter setting for the NSGA-II and SPEA-II is exhibited in Table 3.1.

Table 3.1: Parameter Setting for the NSGA-II and SPEA-II

Parameter	**Setting and Values for NSGA-II**	**Setting and Values for SPEA-II**
Chromosome coding	Real-number coding with 16-bit chromosome	Real-number coding with 16-bit chromosome
Problem setting	**Prob. 1** with 16 assets as a universe	**Prob. 1** with 16 assets as a universe
Crossover method	SBX crossover with probability = 1.0	SBX crossover with probability = 1.0
Mutation method	Variable-wise polynomial mutation with probability = 1/number of decision variable	Variable-wise polynomial mutation with probability = 1/number of decision variable
Population size	200	200
Archive size	N/A	200
Number of generations	500	500
Number of repeated runs	5	5

3.5 THE IMPACT OF SKEWNESS PREFERENCE ON PORTFOLIO CHOICE

In Chapter 5, the impact of different degree of risk aversion and skewness preference on portfolio choice and portfolio holding is investigated. We develop a single-period model in which investors exhibit a homogeneous preference for the first three moments of return distributions but possess a heterogeneous degree of risk aversion and skewness preference. Our proposed model is built upon the model of Mitton and Vorkink (2007) with several key extensions. Firstly, in Mitton and Vorkink (2007), agents in the economy are classified as either *Traditional investors* or the *Lotto investors.* As discussed in Section 2.7.2, this assumption seems unreasonable since skewness preference is recognized as a common trait for a rational risk-averse investor, not only for a *Lotto investor*. In addition, Barberis and Huang (2007) discussed a potential pitfall of Mitton and Vorkink (2007)'s model that the global optimum portfolio for a *Lotto investor* may involve only a highly skewed stock with the investor taking an infinite position in this one asset portfolio. In the contrary, a different degree of risk aversion and skewness preference in our proposed model not only allows risk-averse agents to rationally hold portfolios with return skewness, but also allows skewness preference investors to take into account the return dispersion in portfolio decisions.

Secondly, in contrast to the coefficient that governs preference for skewness used by Mitton and Vorkink (2007), the local index ($\frac{U'''}{U'}$) in our model that measures the degree of skewness preference, as discussed in Section 2.2.3, demonstrates the local and the global properties in the similar way as the global property of Arrow-Pratt's measure of degree of absolute risk aversion.[12] Finally, the implications of the model of Mitton and Vorkink (2007) were tested by a case study of three assets as an investment universe. However, their findings may be of limit because Konno and Yamazaki (1991) demonstrated that the behavior of a model with a small number of assets is often different from that with a large set universal. In contrast, the optimization techniques that we proposed to use allow us to overcome this problem. The implications of our proposed model can be tested from a set of MVS efficient portfolios obtained from a large number of assets that forms the investment universe.

3.5.1 The Model

The proposed model is developed based on the approach of Arrow-Pratt in deriving the measure of risk premium. Basically, a risk-averse agent will always refuse any risky investment ($\tilde{z}$) with an expected payoff of zero unless it is compensated by a risk premium (π). His expected utility can be stated as:

$$\mathrm{E}[U(W+\tilde{z})] = U(W-\pi) \tag{3.12}$$

where *W* represents the wealth of the agent. The value of *certainty equivalent* (CE) is generally used to measure risk premium for a risk-averse agent. CE is the minimum amount of money that a risk-averse agent will accept where he is indifferent between taking up the risky investment and having this certain monetary amount of CE. For a risk-neutral agent, the CE of a risky investment is equal to the expected monetary value of the investment. Meanwhile, the CE of any risky investment for a risk-averse agent is less than the expected

[12] For the theoretical derivation, please refer to Modica and Scarsini (2005) and Crainich and Eeckhoudt (2008).

monetary value of the investment. Thus, the risk premium is the difference between the expected monetary value of the investment and CE which can be stated mathematically as:

$$\pi = E(W + \tilde{z}) - \mathrm{CE} \tag{3.13}$$

Pratt (1967) explained that the risk premium is a function of the distribution of risky investment, initial wealth, and utility function. Using the third-order Taylor's series expansion to approximate the LHS of Equation (3.12), we get:

$$\begin{aligned} E[U(W + \tilde{z})] \quad &\cong E\left[U(W) + \frac{\tilde{z}U'(W)}{1!} + \frac{\tilde{z}^2 U''(W)}{2!} + \frac{\tilde{z}^3 U'''(W)}{3!}\right] \\ &= U(E[W]) + \frac{U'(W)}{1!}E(\tilde{z}) + \frac{U''(W)}{2!}E(\tilde{z}^2) + \frac{U'''(W)}{3!}E(\tilde{z}^3) \\ &= U(E[W]) + \frac{\sigma^2}{2!}U''(W) + \frac{S}{3!}U'''(W) \end{aligned} \tag{3.14}$$

where $E(\tilde{z}) = 0$, $E(\tilde{z}^2) = \sigma^2$ is the variance of the return distributions of the risky investment $\tilde{z}$, and $E(\tilde{z}^3) = S$ is the skewness of the return distribution of risky investment $\tilde{z}$. The Taylor's series first-order expansion of the RHS of Equation (3.12) yields:

$$U(W - \pi) \quad = U(E[W]) - \pi U'(W) \tag{3.15}$$

Then, by replacing Equation (3.14) and Equation (3.15) into Equation (3.12), we obtain:

$$U(E[W]) + \frac{\sigma^2}{2!}U''(W) + \frac{S}{3!}U'''(W) = U(E[W]) - \pi U'(W)$$

$$\pi \quad = -\frac{\sigma^2}{2!}\frac{U''(W)}{U'(W)} - \frac{S}{3!}\frac{U'''(W)}{U'(W)} \tag{3.16}$$

In the expected utility theory, it is well recognized that when the utility is an increasing function of the random variable, maximizing the utility of CE, i.e. $U(CE)$, is equivalent to maximizing the expected utility, i.e. $E[U(W+\tilde{z})]$. Replacing Equation (3.16) into Equation (3.13) and rearranging terms, we obtain:

$$\text{Max } U(CE) = \text{Max } \left[U(E[W]) + \frac{\sigma^2}{2!}\frac{U''(W)}{U'(W)} + \frac{S}{3!}\frac{U'''(W)}{U'(W)}\right]$$

or $$\text{Max } U(CE) = \text{Max } \left[U(E[W]) - \frac{\sigma^2}{2!}A + \frac{S}{3!}P\right] \quad (3.17)$$

where A is Arrow-Pratt's absolute risk aversion coefficient, i.e. $-\frac{U''}{U'}$, and P represents the degree of skewness preference, i.e. $\frac{U'''}{U'}$, which was proposed by Modica and Scarsini (2005) and Crainich and Eeckhoudt (2008). Given the desirable properties of investor utility of (i) positive marginal utility, i.e. $U' > 0$, (ii) decreasing marginal utility, i.e. $U'' < 0$, and (iii) non-increasing absolute risk aversion, i.e. $U''' > 0$, thus coefficient $A > 0$ and $P > 0$. It implies that a rational investor averts to dispersion and favors positive skewness of return distributions.

In this model, rational investors exhibit a homogeneous preference for the first three moments of return distributions. However, the model allows investors to possess a different degree of risk aversion and skewness preference. Therefore, the proposed model has three major implications. Firstly, for a given level of variance, the model predicts that investors with greater skewness preference will accept lower expected returns to enjoy the benefit of increased skewness. The benefit of investing in an asset with a larger skewness of the return distribution can be thought of as chasing after the possibilities of positive extreme outcomes or to avoid downside risks. Secondly, at a given level of expected return, the model predicts that investors with greater degree of skewness preference are willing to expose themselves to larger return dispersions in order to stretch the right tail of the return distribution. On the other hand, the greater absolute risk-averse investors will hold portfolios with lower variance at the expense of the return skewness. Observed from the first two implications, investors with greater risk aversion and a lower degree of skewness preference has a greater tendency to select a portfolio located nearby the MV efficient set, but those with greater skewness preference and who are less risk-averse will sacrifice MV efficiency in exchange for a portfolio that has either higher possibilities of achieving extreme returns or lower downside risks. Finally, our model predicts that investors with greater skewness preference tend to hold less number of assets in their portfolio, i.e. undiversified portfolio, in order to increase their exposure to positive skewness of the return distribution. In contrast, the greater risk-averse investors reduce portfolio variance mainly through diversification, thus they tend to hold higher number of assets in their portfolio.

To investigate the empirical implications of the proposed model, there are two steps to be accomplished. Firstly, the MVS efficient portfolios are searched in the multi-dimension space. This step can be done by solving the MVS-POP formulated as **Prob. 1** in Section 3.2 using the MOEAs suggested earlier. Secondly, using the resulting MVS efficient portfolios from step one, the impact of the degree of skewness preference on portfolio selection can be examined.

3.5.2 The Optimization Problem

In Chapter 5, **Prob. 1** is optimized with respect to two standard constraints. The first constraint is applied to make sure that all initial investment is fully allocated to available securities. The second constraint implies that short selling is not allowed. The analytical framework can be expressed as follows:

Prob. 1 $\min_{\mathbf{x}} F(\mathbf{x}) = \left[-R_p(\mathbf{x}), V_p(\mathbf{x}), -S_p(\mathbf{x})\right]$

Subject to $\sum_{i=1}^{N} x_i = 1$

$x_i \geq 0$

3.5.3 Data

We expand the analysis on the application of the MVS framework in the stock market by using firm-level data in Chapter 5. The monthly closing prices of the component securities of Dow Jones Industrial Average index (DJIA) were collected. These data were extracted from DataStream. The sample period is from January 2004 to December 2011. It should be noted that DJIA is comprised of 30 large publicly owned companies based in the U.S. However, the stock of the VISA Inc. was dropped in this study because it was firstly traded in March 19, 2008. Thus, 29 securities were considered in our analysis. The list of the companies and their trading symbols are exhibited in Table 3.2

Table 3.2: The List of Securities for Analysis on the Impact of Skewness Preference

Symbol	**Company**	**Symbol**	**Company**
U:AXP	AMERICAN EXPRESS	U:MCD	MCDONALD'S
U:BA	BOEING	U:MMM	3M
U:CAT	CATERPILLAR	U:MRK	MERCK & CO.
U:CVX	CHEVRON	@:MSFT	MICROSOFT
@:CSCO	CISCO SYSTEMS	U:NKE	NIKE
U:DD	DU PONT	U:PFE	PFIZER
U:DIS	WALT DISNEY	U:PG	PROCTER & GAMBLE
U:GE	GENERAL ELECTRIC	U:T	AT&T
U:GS	GOLDMAN SACHS	U:TRV	TRAVELERS COS.
U:HD	HOME DEPOT	U:UNH	UNITEDHEALTH GP.
U:IBM	IBM	U:UTX	UNITED TECHNOLOGIES
@:INTC	INTEL	U:VZ	VERIZON COMMUNICATIONS
U:JNJ	JOHNSON & JOHNSON	U:WMT	WAL MART STORES
U:JPM	JP MORGAN CHASE & CO.	U:XOM	EXXON MOBIL
U:KO	COCA COLA		

Note: U: and @: indicate that a security is traded in the New York Stock Exchange (NYSE) or NASDAQ, respectively.

3.5.4 Procedure

The procedure for Chapter 5 is divided into four parts. The first two parts are adopted from Chapter 4. We firstly calculated the monthly returns of the 29 securities using the data from January 2004 to December 2011. The rate of return is computed from:

$$R_{i,t} = \frac{P_{i,t} - P_{i,t-1}}{P_{i,t-1}} \tag{3.18}$$

Then, monthly rate of returns were annualized by using 12 as a multiplier. The following statistics are computed from the rates of return of the 29 stocks:

$$R_i = \frac{1}{T}\sum_{t=1}^{T} R_{i,t} \tag{3.19}$$

$$\sigma_{i,j} = \frac{1}{T}\sum_{t=1}^{T}\left(R_{i,t} - \bar{R}_i\right)\left(R_{j,t} - \bar{R}_j\right) \tag{3.20}$$

$$\gamma_{i,j,k} = \frac{1}{T}\sum_{t=1}^{T} \frac{\left(R_{i,t}-\bar{R}_i\right)}{\sqrt{\sigma_{i,i}}} \frac{\left(R_{j,t}-\bar{R}_j\right)}{\sqrt{\sigma_{j,j}}} \frac{\left(R_{k,t}-\bar{R}_k\right)}{\sqrt{\sigma_{k,k}}} \tag{3.21}$$

Since the data include 29 stocks, i, j, and k = 1, 2, 3,…, 29. T is the total number of observations. The 29 values of average return R_i are stored in the 29×1 vector **R.** The covariance $\sigma_{i,j}$ is saved in the variance-covariance matrix $\mathbf{\Lambda}$ with dimensions 29×29 and the coskewness $\gamma_{i,j,k}$ is stored in the skewness-coskewness matrix $\mathbf{\Omega}$ with dimensions 29×841.

Secondly, we perform the normality test to test whether the return distributions of the 29 stocks are symmetric. Our time series data for each stock consists of 96 observations. The sample size is reasonably large, thus the Jarque-Bera test is employed. The hypothesis to be tested is stated as follows:

H_0: The parent population is normally distributed.

H_1: The parent population is not normally distributed.

Thirdly, MOEAs are utilized for searching the MVS trade-off solutions in the 3D space. As is shown in Chapter 4, the optimization results suggest the superiority of the SPEA-II over the NSGA-II. Therefore, the SPEA-II is implemented for finding the MVS efficient portfolios in Chapter 5. The summary of parameter setting for the SPEA-II is given in

Table 3.3. Lastly, the resulting set of MVS efficient portfolios is used to examine the implication of heterogeneous degree of risk aversion and skewness preference on portfolio choices of investors. Two methods are adopted for this investigation. First, we sort the resulting MVS efficient portfolios into quintile portfolios according to two considered factors, i.e. portfolio skewness and number of assets included in portfolios. Then the average expected return, SD, and skewness for all the portfolios in each of the quintiles were compared. Second, the MVS efficient portfolios that maximize $U(CE)$ were identified for given values of degree of risk aversion (A) and skewness preference (P) that represent an investor's preference. From these efficient portfolios, the three implications of the model (3.17) stated in Section 3.5.1 are examined.

Table 3.3: Parameter Setting for the SPEA-II

Parameter	**Setting and Values for SPEA-II**
Chromosome coding	Real-number coding with 29-bit chromosome
Problem setting	**Prob. 1** with 29 stocks as a universe
Crossover method	SBX crossover with probability = 1.0
Mutation method	Variable-wise polynomial mutation with probability = 1/number of decision variable
Population size	200
Archive size	200
Number of generations	500
Number of repeated runs	5

3.6 ELECTRICITY ALLOCATION PROBLEM

In deregulated electricity markets, generation companies are faced with the problem of how to allocate their electricity output to different trading options in order to maximize profit. The problem is very much similar to the asset allocation problem in stock markets. In Chapter 6, the use of the MVS portfolio model is proposed in the presence of skewness preference in the electricity market (Bessembinder & Lemmon, 2002; Douglas & Popova, 2008; Longstaff & Wang, 2004; Lucia & Torró, 2011; Parsons & De Roo, 2008; Redl et al., 2009; Viehmann, 2011) and asymmetric distribution of the electricity spot prices (Benth et al., 2008; Bessembinder & Lemmon, 2002; Cartea & Villaplana, 2008; Hajiabadi & Mashhadi, 2013; Longstaff & Wang, 2004; Lucia & Torró, 2011; Redl et al., 2009). The MVS-POP formulated in Section 3.2 is applied to the generation asset allocation problem in the electricity market. Besides, given that the number of trading options is not as many as that for stock markets, an additional objective is proposed to prevent under-diversification and to promote diversification benefits.

3.6.1 The Optimization Problem

This analytical chapter focuses on electricity allocation between the spot and forward markets as presented in Liu and Wu (2007a and 2007b). In the spot market, a Genco can sell the electricity it generates on real-time basis. However, electricity spot prices are highly volatile due to non-storability of the product and uncertainty in its demand. Therefore, trading in the spot market is considered a risky choice. In the case of a forward contract, the Genco makes a bilateral trade agreement on price ($/MWh), quantity (MWh) as well as delivery time and location with its customers in advance. A Genco can sign forward contracts with customers located in the same pricing zone (known as local forward contract) as well as in different pricing zones (known as non-local forward contract). The local forward contract is risk free because the transmitted quantity and contract price are agreed beforehand, and no uncertainty is involved. Although a non-local bilateral contract is also established in advance, it bears risk because according to the locational marginal pricing (LMP) scheme, a congestion charge is levied on the Genco if there is line congestion during the delivery to the customer located in a different pricing zone.

In Chapter 5, the asset allocation problem of the Genco is formulated as a MVS-POP. According to the fact that the distribution of electricity spot prices is asymmetric and skewness preference can be observed from the agents in the electricity market, the MVS portfolio model is proposed for solving the electricity allocation problem. Thus, **Prob. 1** with two standard constraints formulated in Section 3.2 is implemented.

Prob. 1 $\min_{\mathbf{x}} F(\mathbf{x}) = \left[-R_p(\mathbf{x}), V_p(\mathbf{x}), -S_p(\mathbf{x})\right]$

Subject to $\sum_{i=1}^{N} x_i = 1$

$x_i \geq 0$

In addition, as explained in the research gaps in Section 2.7.3, an additional objective is proposed to enhance the diversification of the solutions. The diversification benefit can be achieved by increasing the number of trading options in the portfolio and preventing excessive allocation to a small number of options. This process can be completed using a constraint such as cardinality constraint, ceiling limit constraint and class constraint. However, the value of constraint in these approaches is subjectively predetermined by a DM causing the searching space of an algorithm to be unnecessarily restricted. As a result, the obtained solutions may not be optimal in the context of optimization (Steuer et al., 2007). In this study, under-diversification is avoided by adding an objective to minimize the difference between the highest and the lowest proportion of electricity allocation within a solution vector **x**. Let this difference be denoted as $D(\mathbf{x})$. The fourth objective can be stated as follows:

Minimize $D(\mathbf{x}) = \mathrm{Max}\,\mathbf{x} - \mathrm{Min}\,\mathbf{x}$

where $\mathrm{Max}\,\mathbf{x}$ represents the maximum proportion of electricity allocated to an option in a solution vector **x**, whereas the minimum proportion is indicated by $\mathrm{Min}\,\mathbf{x}$. Therefore, **Prob. 2** has four objectives to be optimized. Denoted as the MVS-D model, the problem is formulated as follows:

Prob. 2 $\min_{\mathbf{x}} F(\mathbf{x}) = \left[-R_p(\mathbf{x}), V_p(\mathbf{x}), -S_p(\mathbf{x}), D(\mathbf{x})\right]$

Subject to $\sum_{i=1}^{N} x_i = 1$

$x_i \geq 0$

3.6.2 Data

The analysis in Chapter 6 is based on electricity spot prices quoted in the Pennsylvania-New Jersey-Maryland (PJM) electricity market, which is the largest deregulated electricity market in the U.S. Since the LMP scheme is adopted in this market, the electricity spot prices are different from location to location. The considered areas of this market cover nine pricing zones or pricing nodes, including AEGO, BGE, DPL, METED, PECO, PENELEC, PEPCO, PPL, and PSEG. The electricity spot prices of nine pricing zones are downloaded from the PJM website.[13] There are daily data for the period of 1 August 1998 to 30 July 2006.

3.6.3 Procedure

To solve **Prob. 1** and **Prob. 2**, the expected return for each trading choice are firstly calculated. Then the variance, skewness, covariance, and coskewness are computed using historical data of the electricity spot prices. **Prob. 1** and **Prob. 2** involve different input variables as discussed below. Suppose that *N* trading choices are available and the Genco of interest is located in zone 1. The following notations are used in the discussion:

$E(\cdot)$	expected value
P_{G}	power generation output (MWh)
h	trading time of each trading interval
$\lambda_{i,t}^{s}$	spot price of zone *i* at t^{th} trading interval (\$/MWh)
a, b, c	fuel consumption coefficients of the power plant
λ_{t}^{c}	coal price at t^{th} trading interval (\$/MBtu)
$\lambda_{i,t}^{B}$	bilateral forward price of zone *i* at t^{th} trading interval (\$/MWh)

[13] Available at http://www.pjm.com/

(a) Return, variance and skewness for spot market

The rate of return to the Genco is calculated from its profit, i.e. revenue minus production cost, divide by the production cost. Revenue is the product of selling price ($/MWh) and selling quantity (MWh). We conventionally assume that the production cost of the Genco of interest is a quadratic function of energy output, trading time and energy price (Donghan et al., 2007; Liu & Wu, 2007a). The expected return, variance and skewness for trading in the spot market are as follows:

$$E(R_S) \quad = \frac{\sum_{t=1}^{T} P_G h(E(\lambda_{1,t}^S)) - \sum_{t=1}^{T}(a + bP_G + cP_G^2)h\lambda_t^C}{\sum_{t=1}^{T}(a + bP_G + cP_G^2)h\lambda_t^C}$$

$$= K\sum_{t=1}^{T} P_G h(E(\lambda_{1,t}^S)) - 1, \text{ where } K = \frac{1}{\sum_{t=1}^{T}(a + bP_G + cP_G^2)h\lambda_t^c} \tag{3.22}$$

$$V_S \quad = K^2 \sum_{t=1}^{T}(P_G h)^2 \left[V(\lambda_{1,t}^S)\right] \tag{3.23}$$

$$S_S \quad = K^3 \sum_{t=1}^{T}(P_G h)^3 \left[S(\lambda_{1,t}^S)\right] \tag{3.24}$$

where $t = 1, \dots, T$ represents the period when the trading decision has to be made, and $(a + bP_G + cP_G^2)h\lambda_t^c$ represents the production cost of the Genco. The expected return $E\left(\lambda_{1,t}^S\right)$ is obtained by forecasting the spot prices (see discussion below). The terms $V\left(\lambda_{1,t}^S\right)$ and $S\left(\lambda_{1,t}^S\right)$ are variance and skewness respectively, calculated using historical data of the same date.

(b) Return, variance and skewness for forward market

Of the N trading choices that are available, there are N-1 non-local bilateral forward contracts. The expected return, variance and skewness for non-local bilateral forward trading in zone i are as follows:

$$E(R_i) \quad = \quad K\sum_{t=1}^{T} P_G h\left[\lambda_{i,t}^{B} - (E(\lambda_{i,t}^{S}) - E(\lambda_{1,t}^{S}))\right] - 1 \tag{3.25}$$

$$V_i \quad = K^2 \sum_{t=1}^{T} \left(P_G h\right)^2 \left[V(\lambda_{1,t}^{S}) + V(\lambda_{i,t}^{S}) - 2\sigma(\lambda_{1,t}^{S}, \lambda_{i,t}^{S})\right] \tag{3.26}$$

$$S_i \quad = K^3 \sum_{t=1}^{T} \left(P_G h\right)^3 [S(\lambda_{1,t}^{S}) + S(\lambda_{i,t}^{S}) - 3\gamma(\lambda_{1,t}^{S}, \lambda_{1,t}^{S}, \lambda_{i,t}^{S}) + 3\gamma(\lambda_{1,t}^{S}, \lambda_{i,t}^{S}, \lambda_{i,t}^{S})] \tag{3.27}$$

where i = 2 to N , and $E\left(\lambda_{i,t}^{S}\right) - E\left(\lambda_{1,t}^{S}\right)$ represents the expected congestion charge.

(c) Covariance between trading choices

The covariance of returns to trading in zone 1 and zone i is:

$$\sigma_{1,i} \quad = K^2 \sum_{t=1}^{T} (P_G h)^2 \left[V(\lambda_{1,t}^{S}) - \sigma(\lambda_{1,t}^{S}, \lambda_{i,t}^{S})\right] \tag{3.28}$$

The covariance of returns to trading in zone i and zone j is:

$$\sigma_{i,j} \quad = K^2 \sum_{t=1}^{T} (P_G h)^2 \left[V(\lambda_{1,t}^{S}) - \sigma(\lambda_{1,t}^{S}, \lambda_{i,t}^{S}) - \sigma(\lambda_{1,t}^{S}, \lambda_{j,t}^{S}) + \sigma(\lambda_{i,t}^{S}, \lambda_{j,t}^{S})\right] \tag{3.29}$$

where i = 2 to N and $i \neq j$.

(d) Coskewness between trading choices

To reduce the computation complexity, we considered only curvilinear product moments, i.e. $\gamma_{i,i,j}$, as coskewness between trading choices. The coskewness coefficients for returns to trading in different pricing zones are given by the following:

$$\gamma_{1,1,i} = K^3 \sum_{t=1}^{T} (P_G h)^3 \left[S(\lambda_{1,t}^S) - \gamma(\lambda_{1,t}^S, \lambda_{1,t}^S, \lambda_{i,t}^S) \right] \tag{3.30}$$

$$\gamma_{i,i,1} = K^3 \sum_{t=1}^{T} (P_G h)^3 \left[S(\lambda_{1,t}^S) + \gamma(\lambda_{i,t}^S, \lambda_{i,t}^S, \lambda_{1,t}^S) - 2(V(\lambda_{1,t}^S) + \sigma(\lambda_{i,t}^S, \lambda_{1,t}^S)) \right] \tag{3.31}$$

$$\gamma_{i,i,j} = K^3 \sum_{t=1}^{T} (P_G h)^3 [S(\lambda_{1,t}^S) - \gamma(\lambda_{1,t}^S, \lambda_{1,t}^S, \lambda_{j,t}^S) + \gamma(\lambda_{i,t}^S, \lambda_{i,t}^S, \lambda_{1,t}^S) \tag{3.32}$$

$$-\gamma(\lambda_{i,t}^S, \lambda_{i,t}^S, \lambda_{j,t}^S) - 2(\gamma(\lambda_{1,t}^S, \lambda_{1,t}^S, \lambda_{i,t}^S) + \gamma(\lambda_{1,t}^S, \lambda_{i,t}^S, \lambda_{j,t}^S))]$$

where i = 2 to N and $i \neq j$.

(e) Forecasting of expected spot prices

To compute the expected returns of trading in the spot market and the non-local bilateral forward contracts as shown in Equations (3.22) and (3.25), the expected spot prices during the decision-making period of all the pricing zones have to be forecasted. Since the daily spot prices for the decision making period are unknown, the reliability of the optimization solutions depends crucially on the accuracy of their forecasts. Given the fact that electricity spot prices are characterized by non-constant mean and variance, multiple seasonalities and high volatility (Aggarwal et al., 2009; Garcia et al., 2005; Jun Hua et al., 2008), omission of these effects in forecasting will not produce accurate solutions (Suksonghong & Goh, 2012).

The GARCH model with seasonality (Suksonghong & Goh, 2012) is employed for forecasting the daily spot prices in the decision making horizon. The model is given as follows:

$$\begin{aligned}
\lambda_{i,t}^{s} &= \omega . tr_t + \sum_{j=1}^{J} \theta_j \lambda_{i,t-j}^{s} + \sum_{k=1}^{7} \alpha_k D_{k,t} + \sum_{l=1}^{12} \rho_l M_{l,t} + u_t \\
u_t &= z_t . \sqrt{h_t} \qquad (3.33) \\
h_t &= \beta_0 + \sum_{q=1}^{Q} \beta_q u_{t-q}^2 + \sum_{p=1}^{P} \delta_p h_{t-p} + \phi_1 D_{Mon,t} + \cdots + \\
&\quad \phi_6 D_{Sat,t} + \phi_7 M_{Jan,t} + \cdots + \phi_{17} M_{Nov,t}
\end{aligned}$$

where	tr_t	deterministic time trend variable
	$D_{k,t}$	dummy variable for day-of-the-week effect
	$M_{l,t}$	dummy variable for month-of-the-year effect
	u_t	noise at time t and $u_t \mid \Omega_{t-1} \sim (0, h_t)$, Ω_{t-1} is the information set available at time t-1
	h_t	conditional variance of the spot prices at time t
	z_t	an independently and identically distributed random variable with mean 0 and variance 1

The day-of-the-week dummy variable is $D_{k,t}$= 1 for day-k and 0 otherwise, k = Monday, Tuesday,…, Sunday. The month-of-the-year dummy variable is $M_{l,t}$= 1 for month-l and 0 otherwise, l = January, February,…, December. Lags of the electricity spot prices $\lambda_{i,t-j}^{S}$ are included in the equation to allow for a lag dependent structure up to order J. The coefficients $\omega, \theta, \alpha, \rho, \beta, \delta,$ and ϕ are estimated using the maximum likelihood estimation method on the historical spot prices. Only the day-of-the-week and month-of-the-year factors that are significant are included in the conditional variance (h_t) equation to preserve parsimony of the model. The autocorrelation and partial autocorrelation functions of the squared residuals are used as guidance to determine the order of P and Q in Equation (3.33) (Garcia et al., 2005).

3.6.4 The Setting of the Numerical Case Studies

In the field of power systems research, a numerical case study is generally used to demonstrate the application of the proposed model. Suppose that the Genco trades the generated electricity in PJM electricity market where the LMP scheme is adopted. It is assumed that the Genco is making decisions on the optimal electricity allocation solutions for the month of August 2006. Therefore, the expected spot prices from 1 to 31 August 2006 of nine pricing zones need to be forecasted. The daily electricity spot prices between 1 August 1998 and 30 July 2006 were collected from the PJM market for estimating Equation (3.33), and for computing the variance, covariance, skewness and coskewness of the different trading choices. We assume that the Genco of interest is located in PEPCO and is making the portfolio optimization decision by considering to trade in the spot market and non-local bilateral forward contracts as the investment universe. Following Liu and Wu (2007a), the two case studies of non-local bilateral forward contract prices ($/MWh) during the period of August 2006 are given as follows:

Case study 1:[14]

AEGO:	$\lambda_{2,t}^{B} = 40.9$	METED:	$\lambda_{5,t}^{B} = 39.0$	PPL:	$\lambda_{8,t}^{B} = 38.3$
BGE:	$\lambda_{3,t}^{B} = 40.2$	PECO:	$\lambda_{6,t}^{B} = 39.6$	PSEG:	$\lambda_{9,t}^{B} = 41.4$
DPL:	$\lambda_{4,t}^{B} = 40.7$	PENELEC:	$\lambda_{7,t}^{B} = 37.0$		

Case study 2:

AEGO:	$\lambda_{2,t}^{B} = 40.5$	METED:	$\lambda_{5,t}^{B} = 40.5$	PPL:	$\lambda_{8,t}^{B} = 40.5$
BGE:	$\lambda_{3,t}^{B} = 40.5$	PECO:	$\lambda_{6,t}^{B} = 40.5$	PSEG:	$\lambda_{9,t}^{B} = 40.5$
DPL:	$\lambda_{4,t}^{B} = 40.5$	PENELEC:	$\lambda_{7,t}^{B} = 40.5$		

It is further assumed that the Genco has a cost function of $C(P_G, \lambda_t^c) = \left(a + bP_G + cP_G{}^2\right)\lambda_t^c$. Suppose that this Genco has a 350-MW fossil generator. Therefore, the fuel consumption coefficients are: a = 647.0865 MBtu/h, b = 14.8661 MBtu/MWh and c = 0.0065 MBtu/MW^{2}h (Wood & Wollenberg, 1996). The cost of coal is assumed constant at 1.29 \$/MBtu during the period of analysis (EIA, 2011). For the considered trading constraints, the two standard constraints presented in **Prob. 1** imply respectively that all the generated capacity must be traded, and short sales, i.e. selling contracts borrowed from a broker with an obligation to buy these contracts back for returning to the broker, are not allowed.

[14] The average spot prices of the individual zones were used in this case study.

3.6.5 Optimization Method and Setting

In the literature, the robustness of the NSGA-II and SPEA-II for solving a MOOP with three objectives or less to be optimized, as is the case of **Prob. 1**, is widely documented. However, in **Prob. 2**, the MOOP is formulated as MVS-D portfolio model. This adds to the complexity of the optimization problem by increasing the number of objectives from three to four. In optimization problems, an increase in the number of conflicting objectives significantly raises the difficulty in the use of an algorithm to find the optimal solutions (Deb, Thiele, Laumanns, & Zitzler, 2005). According to the dominance relation explained by **Definition 1** and **Definition 2** in Section 3.1.2, the chance that no one solution can dominate the other is expectably high if the number of objectives to be optimized is large. Therefore, in order for algorithms to provide a good approximation of the true Pareto front, a large number of non-dominated solutions have to be screened using suitable techniques (Pierro, Soon-Thiam, & Savic, 2007; Purshouse & Fleming, 2007).

Since we formulated the electricity allocation problem as the MVS-D portfolio model expressed in **Prob. 2**, which is a four-objective optimization problem, the NSGA-II and SPEA-II may not yield a good approximation of the true Pareto solutions. Chapter 6 experimentd the use of a newly developed MOEA, the compressed objective genetic algorithm II (COGA-II) (Boonlong, Chaiyaratana, & Maneeratana, 2010), which is purposively designed for the optimization problem with a large number of objectives. The performance of the NSGA-II, SPEA-II, and COGA-II for solving **Prob.1** and **Prob. 2** is compared using two standard performance comparison methods, namely, the average distance to the true Pareto-optimal front (M_1) and the Hypervolume (HV). In addition, we propose a method which is named as “Contribution Ratio to Artificial true Pareto solutions” (*CRA*).

The M_1 criterion (Zitzler, Deb, & Thiele, 2000) can be evaluated in the solution space or objective space. In this chapter, the metric M_1 is measured in the objective space by computing a distance of a solution i to the true Pareto-optimal front, d_i, which is the Euclidean distance of the solution i to its nearest solution j on the true Pareto-optimal front. The Euclidean distance is given by:

$$d_i = \sqrt{\sum_{k=1}^{m}\left(\frac{f_{ik} - f_{jk}}{(f_k)_{\max} - (f_k)_{\min}}\right)^2} \tag{3.34}$$

where f_{ik} and f_{jk} are the values of objective k for solutions i and j respectively, while $(f_k)_{\min}$ and $(f_k)_{\max}$ are respectively the minimum and maximum values of objective k for the true Pareto-optimal solutions. Since the true Pareto front of a tested problem is not known, the artificial true Pareto front which is obtained from the merged non-dominated individuals from all runs of the three MOEAs was used instead in the evaluation of M_1. The distance, d_i, of a solution i is estimated as the Euclidean distance of the solution i to its nearest solution j on the artificial true Pareto-optimal front. M_1 is the average of d_i for all individuals in a set of non-dominated solutions. However, it should be noted that M_1 obtained from the artificial true Pareto-optimal front is not the exact value of that obtained from the true Pareto front. It can only be used to compare the closeness of solutions from the employed MOEAs to the Pareto front.

The *HV* criterion, a maximum criterion, was originally proposed and employed in Zitzler and Thiele (1999b). It measures not only the closeness to the true Pareto front but also the diversity of solutions. The *HV* refers to the area (two objectives), volume (three objectives) or hypervolume (four or more objectives) between a given reference point and a non-dominated front to be evaluated. The *HV* is a popular criterion especially for a problem with unknown true Pareto solutions.

Since the true Pareto solutions are unknown, M_1 is probably not sufficient for evaluating the performance of different MOEAs. To overcome this weakness, we propose the criterion termed as the contribution ratio to the artificial true Pareto solutions (*CRA*). The artificial true Pareto solutions were firstly determined from the combination of output sets from the three algorithms. *CRA* refers to the ratio of the number of solutions that are members of the artificial true Pareto solutions to the total number of solutions in the output set. Therefore, it ranges in values between 0 and 1. If *CRA* is equal to 1.0, all solutions in the output set contribute to the forming of the artificial true Pareto solutions. On the other hand, if *CRA* is equal to 0.0, none of the solutions in the output set are members of the artificial true Pareto-optimal solutions. The summary of parameter setting for the three experimented algorithms for solving **Prob. 1** and **Prob. 2** is given in Table 3.4.

Table 3.4: Parameter Setting for the NSGA-II, SPEA-II, and COGA-II for Solving the Numerical Case Studies of **Prob. 1** and **Prob. 2**

Parameter	**Setting and Values**
Chromosome coding	Real-number coding with 9-bit chromosome
Problem setting	**Prob. 1** and **Prob. 2** with 9 trading choices as a universe
Crossover method	SBX crossover with probability = 1.0
Mutation method	Variable-wise polynomial mutation with probability = 1/number of decision variable
Population size	100
Archive size[15]	100
Number of generations	600
Number of repeated runs	30

[15] It should be noted that, according to the algorithm procedure, the archive is used in the COGA-II and SPEA-II but not in NSGA-II.

CHAPTER 4

PORTFOLIO SELECTION WITH SKEWNESS PREFERENCE: EVIDENCE FROM THE EMERGING STOCK MARKETS

4.1 INTRODUCTION

This chapter demonstrates the implementation of MOEAs for the search of MVS efficient portfolios in the multi-dimension space. The MVS-POP formulated as **Prob. 1** in Section 3.2 is solved by using the NSGA-II and SPEA-II. The results obtained are used to examine the risk-return characteristics of MVS efficient portfolios as well as the shape of MVS efficient surface. Besides, two different investment horizons are used to empirically investigate the impact of investment horizon on risk-return relationship of the MVS efficient portfolios. Section 4.2 reports the summary statistics and the normality test results for weekly and monthly data of the sixteen emerging market indices. It also documents the estimated value of the input variables required for solving **Prob. 1**. Section 4.3 illustrates graphically the MVS efficient portfolios plotted in the 3D MVS space. The risk-return characteristics of the MVS efficient portfolios together with the impact of different investment horizons are discussed in Section 4.4. The concluding remarks of the chapter are stated in Section 4.5.

4.2 SUMMARY STATISTICS AND THE NORMALITY TEST RESULTS

Table 4.1 and Table 4.2 exhibit the summary statistics together with the normality tests statistics for the annualized weekly and monthly rates of returns of the 18 emerging markets, respectively.[16] The second column of Table 4.1 reveals that Sri Lanka has the highest average weekly rate of returns (0.3187) followed by Thailand (0.2892) and Indonesia (0.2791). For the average monthly rate of returns, Table 4.2 shows that Sri Lanka also recorded the highest value (0.3064), followed by Indonesia (0.2864) and Taiwan (0.2809). Meanwhile, Argentina, China, and Brazil have the lowest average weekly and monthly rate of returns. It is observed that only Argentina exhibits an average rate of returns that is negative in the period of study.

For the average volatility, regardless of the investment horizon, Argentina and Sri Lanka are the most volatile markets, whereas Malaysia and Philippines have the lowest standard deviation. The evidence in the fourth column of Table 4.1 indicates that for weekly rate of returns, only five market indices of Sri Lanka, India, Indonesia, China, and Colombia exhibit positive skewness, whereas the rest of the market indices have negative skewness. Interestingly, monthly rate of returns shows a different picture. Table 4.2 reveals that the indices of five markets namely Malaysia, Philippines, Thailand, Argentina, and Chile have negative skewness. The contrast in results of the higher moments computed from the different investment horizons are consistent with the findings of Chunhachinda, Dandapani,

[16] Hereafter, for short, the terms “weekly rate of returns” and “monthly rate of returns” refer to “annualized weekly rate of returns” and “annualized monthly rate of returns”, respectively.

Hamid, and Prakash (1994), Chunhachinda et al. (1997), Prakash et al. (1997), and Prakash et al. (2003). They referred to this observation as the “intervalling effect”

The results of the normality test of the return distributions of the 16 indices are presented in the last four columns of Table 4.1 and Table 4.2. The probability value (P-value) associated with the W-statistic of the Shapiro-Wilk test and JB-statistic of the Jarque-Bera test indicates the significance level of the rejection of the null hypothesis. For weekly rate of returns, Table 4.1 reveals that the majority of the market indices have skewness that is significant. The Shapiro-Wilk test fails to reject the null hypothesis only for Taiwan and Colombia, while the Jarque-Bera test fails to reject the null hypothesis for Sri Lanka, Thailand, and Mexico. In all cases, at least one of the tests rejected the normality. There is also strong evidence against normality when monthly were used, but there are more cases where the null hypothesis cannot be rejected compared to weekly data. The market indices of Indonesia, Philippines, Argentina, Colombia, and Peru do not exhibit evidence against normality, as indicated by both the Shapiro-Wilk and Jarque-Bera tests. Overall, the results support the argument that non-zero skewness is present and therefore it should not be neglected in portfolio decision.

To solve **Prob. 1** formulated in Section 3.2.1, the input variables explained in Equation (3.3) to Equation (3.5) are required. To obtain these variables, computations were performed according to Equation (3.8) to Equation (3.11) using the data of the 16 emerging market indices. The input variables including the return matrix **R** and variance-covariance

Table 4.1: Summary Statistics and the Normality Test Results of the Annualized Weekly Rate of Returns for Sixteen Emerging Markets

Market Index	Mean	Standard Deviation	Skewness	Kurtosis	W-statistic	P-value	JB-statistic	P-value
China	0.1062	1.7962	0.0248	3.9744	0.9767	0.0016***	6.4064	0.0406**
India	0.1434	2.0696	0.3668	5.0282	0.9671	0.0001***	8.6167	0.0135**
Indonesia	0.2791	1.9558	0.0595	5.2986	0.9593	0.0000***	24.1349	0.0000***
Malaysia	0.1829	1.0894	-0.3694	3.4381	0.9869	0.0531*	4215.1826	0.0000***
Pakistan	0.1844	1.8587	-0.3883	10.6545	0.8801	0.0000***	8.0923	0.0175**
Philippines	0.2657	1.5941	-0.1596	3.9448	0.9824	0.0106***	15.5225	0.0004***
South Korea	0.1980	2.0587	-0.4197	4.4429	0.9636	0.0000***	62.1303	0.0000***
Sri Lanka	0.3187	2.2935	2.8051	24.3271	0.7756	0.0000***	0.4488	0.7990
Taiwan	0.1456	1.5987	-0.0711	2.9019	0.9955	0.7942	55.2406	0.0000***
Thailand	0.2892	1.7725	-0.3478	3.6711	0.9865	0.0446**	3.8632	0.1449
Argentina	-0.0044	2.6046	-0.7420	5.3087	0.9557	0.0000***	8.2297	0.0163**
Brazil	0.1252	2.2456	-0.2952	4.2016	0.9755	0.0011***	40.3435	0.0000***
Chile	0.1866	1.6325	-0.5705	5.4230	0.9563	0.0000***	45.8236	0.0000***
Colombia	0.2775	1.5953	0.0159	3.2254	0.9920	0.3163	512.6170	0.0000***
Mexico	0.1857	1.9417	-0.4312	5.3732	0.9547	0.0000***	0.2574	0.8792
Peru	0.1988	2.2124	-0.1911	3.5482	0.9881	0.0791*	65.3109	0.0000***

Note: *, **, and *** denote significance at the 0.10, 0.05, and 0.01 levels, respectively.

W-statistic is the Shapiro-Wilk test statistic.

JB-statistic is the Jarque-Bera test statistic.

Table 4.2: Summary Statistics and the Normality Test Results of the Annualized Monthly Rate of Returns for Sixteen Emerging Markets

Market Index	Mean	Standard Deviation	Skewness	Kurtosis	W-statistic	P-value	JB-statistic	P-value
China	0.0828	0.9854	0.0584	4.6371	0.9370	0.0137**	3.5593	0.1687
India	0.1447	1.2198	0.9543	4.1272	0.9351	0.0116**	8.6700	0.0131**
Indonesia	0.2864	1.0514	0.2613	3.7228	0.9633	0.1452	1.0710	0.5854
Malaysia	0.1738	0.5664	-0.8463	5.3450	0.9325	0.0093***	13.2712	0.0013***
Pakistan	0.1851	0.8720	1.6840	9.2627	0.8417	0.0000***	81.2604	0.0000***
Philippines	0.2672	0.8035	-0.2276	3.4519	0.9773	0.4860	0.5595	0.7560
South Korea	0.1962	1.0810	0.6612	7.6678	0.8780	0.0002***	35.7171	0.0000***
Sri Lanka	0.3064	1.2430	1.7048	8.2497	0.8375	0.0000***	63.8927	0.0000***
Taiwan	0.2809	0.8700	0.4949	4.5790	0.9429	0.0230**	6.4547	0.0397**
Thailand	0.1483	1.0069	-0.6382	4.3553	0.9501	0.0436**	4.2465	0.1196
Argentina	-0.0625	1.2828	-0.5605	3.3888	0.9699	0.2632	2.5590	0.2782
Brazil	0.0942	1.1278	0.4687	4.2067	0.9566	0.0792*	3.5145	0.1725
Chile	0.1764	0.9217	-0.5851	5.1985	0.9306	0.0080***	9.3585	0.0093***
Colombia	0.2604	0.8841	0.0683	3.7929	0.9608	0.1160	0.7091	0.7015
Mexico	0.1719	1.0710	0.0204	4.7164	0.9212	0.0037***	3.9196	0.1409
Peru	0.1862	1.1362	0.4317	4.0281	0.9609	0.1166	2.7032	0.2588

Note: *, **, and *** denote significance at the 0.10, 0.05, and 0.01 levels, respectively.

W-statistic is the Shapiro-Wilk test statistic.

JB-statistic is the Jarque-Bera test statistic.

Table 4.3: The Return Matrix **R** and Variance-covariance Matrix **Λ** of the Annualized Weekly Rate of Returns for Sixteen Emerging Markets

Input Variable	China	India	Indonesia	Malaysia	Pakistan	Philippines	South Korea	Sri Lanka	Taiwan	Thailand	Argentina	Brazil	Chile	Colombia	Mexico	Peru
R	0.106	0.143	0.279	0.183	0.184	0.266	0.198	0.319	0.146	0.289	-0.004	0.125	0.187	0.278	0.186	0.199
Λ																
China	**3.226**	2.663	2.260	1.452	0.782	1.654	2.728	0.542	2.000	2.074	2.238	2.999	1.840	1.564	2.569	2.394
India	2.663	**4.283**	2.263	1.458	0.790	1.692	2.732	1.778	1.990	2.063	2.372	3.024	1.970	1.728	2.540	2.363
Indonesia	2.260	2.263	**3.825**	1.446	0.519	1.728	2.380	0.855	1.743	1.819	2.266	2.271	1.546	1.374	2.128	1.924
Malaysia	1.452	1.458	1.446	**1.187**	0.571	1.123	1.489	0.467	1.068	1.152	1.469	1.539	1.015	0.890	1.247	1.217
Pakistan	0.782	0.790	0.519	0.571	**3.455**	0.430	0.786	-0.293	0.807	0.434	0.365	0.653	0.298	0.273	0.546	0.425
Philippines	1.654	1.692	1.728	1.123	0.430	**2.541**	1.667	0.517	1.307	1.534	1.716	1.742	1.077	1.071	1.216	1.268
South Korea	2.728	2.732	2.380	1.489	0.786	1.667	**4.238**	0.876	2.366	2.003	2.388	3.205	1.964	1.649	2.625	2.117
Sri Lanka	0.542	1.778	0.855	0.467	-0.293	0.517	0.876	**5.260**	0.603	0.863	0.545	0.644	0.761	0.540	0.558	0.415
Taiwan	2.000	1.990	1.743	1.068	0.807	1.307	2.366	0.603	**2.556**	1.403	1.807	2.166	1.358	1.087	1.777	1.676
Thailand	2.074	2.063	1.819	1.152	0.434	1.534	2.003	0.863	1.403	**3.142**	1.600	1.948	1.256	1.359	1.529	1.705
Argentina	2.238	2.372	2.266	1.469	0.365	1.716	2.388	0.545	1.807	1.600	**6.784**	3.373	2.159	1.742	2.753	2.594
Brazil	2.999	3.024	2.271	1.539	0.653	1.742	3.205	0.644	2.166	1.948	3.373	**5.043**	2.729	2.320	3.523	3.474
Chile	1.840	1.970	1.546	1.015	0.298	1.077	1.964	0.761	1.358	1.256	2.159	2.729	**2.665**	1.512	2.276	2.007
Colombia	1.564	1.728	1.374	0.890	0.273	1.071	1.649	0.540	1.087	1.359	1.742	2.320	1.512	**2.545**	1.934	1.927
Mexico	2.569	2.540	2.128	1.247	0.546	1.216	2.625	0.558	1.777	1.529	2.753	3.523	2.276	1.934	**3.770**	2.942
Peru	2.394	2.363	1.924	1.217	0.425	1.268	2.117	0.415	1.676	1.705	2.594	3.474	2.007	1.927	2.942	**4.895**

Table 4.4: The Return Matrix **R** and Variance-covariance Matrix **Λ** of the Annualized Monthly Rate of Returns for Sixteen Emerging Markets

Input Variable	China	India	Indonesia	Malaysia	Pakistan	Philippines	South Korea	Sri Lanka	Taiwan	Thailand	Argentina	Brazil	Chile	Colombia	Mexico	Peru
R	0.083	0.145	0.286	0.174	0.185	0.267	0.196	0.306	0.148	0.281	-0.063	0.094	0.176	0.260	0.172	0.186
Λ																
China	**0.971**	0.876	0.799	0.480	0.238	0.627	0.770	0.330	0.657	0.768	0.739	0.953	0.643	0.613	0.892	0.854
India	0.876	**1.488**	1.013	0.462	0.300	0.713	0.969	0.901	0.811	0.951	0.919	1.115	0.787	0.800	1.006	0.875
Indonesia	0.799	1.013	**1.106**	0.483	0.245	0.594	0.868	0.535	0.647	0.884	0.735	0.901	0.622	0.692	0.859	0.764
Malaysia	0.480	0.462	0.483	**0.321**	0.110	0.342	0.453	0.190	0.355	0.447	0.510	0.531	0.385	0.346	0.476	0.414
Pakistan	0.238	0.300	0.245	0.110	**0.760**	0.030	0.497	0.038	0.414	0.158	0.241	0.329	0.074	0.132	0.331	0.265
Philippines	0.627	0.713	0.594	0.342	0.030	**0.646**	0.521	0.352	0.455	0.645	0.489	0.657	0.532	0.504	0.581	0.619
South Korea	0.770	0.969	0.868	0.453	0.497	0.521	**1.168**	0.439	0.784	0.768	0.888	0.966	0.594	0.601	0.971	0.710
Sri Lanka	0.330	0.901	0.535	0.190	0.038	0.352	0.439	**1.545**	0.443	0.559	0.579	0.555	0.567	0.575	0.368	0.377
Taiwan	0.657	0.811	0.647	0.355	0.414	0.455	0.784	0.443	**0.757**	0.620	0.663	0.771	0.510	0.481	0.710	0.638
Thailand	0.768	0.951	0.884	0.447	0.158	0.645	0.768	0.559	0.620	**1.014**	0.701	0.852	0.679	0.698	0.769	0.763
Argentina	0.739	0.919	0.735	0.510	0.241	0.489	0.888	0.579	0.663	0.701	**1.645**	0.973	0.672	0.507	0.916	0.760
Brazil	0.953	1.115	0.901	0.531	0.329	0.657	0.966	0.555	0.771	0.852	0.973	**1.272**	0.831	0.705	1.017	0.949
Chile	0.643	0.787	0.622	0.385	0.074	0.532	0.594	0.567	0.510	0.679	0.672	0.831	**0.849**	0.555	0.650	0.605
Colombia	0.613	0.800	0.692	0.346	0.132	0.504	0.601	0.575	0.481	0.698	0.507	0.705	0.555	**0.782**	0.645	0.617
Mexico	0.892	1.006	0.859	0.476	0.331	0.581	0.971	0.368	0.710	0.769	0.916	1.017	0.650	0.645	**1.147**	0.901
Peru	0.854	0.875	0.764	0.414	0.265	0.619	0.710	0.377	0.638	0.763	0.760	0.949	0.605	0.617	0.901	**1.291**

matrix $\mathbf{\Lambda}$ for weekly and monthly data are exhibited, respectively, in Table 4.3 and Table 4.4. The skewness-coskewness matrix $\mathbf{\Omega}$ of the weekly and monthly rates of returns are presented in Table B.1 and Table B.2 of APPENDIX B, respectively.

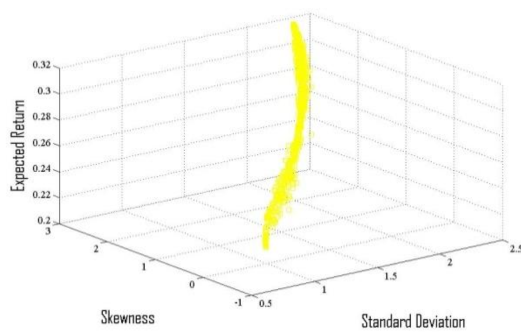
4.3 MVS EFFICIENT PORTFOLIOS AND THE EFFICIENT SURFACE

In this section, the characteristics of the non-dominated portfolio solutions of **Prob. 1** obtained from the NSGA-II and SPEA-II are examined. According to the parameter setting exhibited in Table 3.1, 200 portfolio solutions were obtained in a single run of each algorithm. Each algorithm was run for five times, thus 1,000 portfolio solutions were obtained. Then, the comparison based on the Pareto dominance relation explained in Section 3.3.3 was performed to these 1,000 portfolio solutions in order to screen for only the non-dominated portfolio solutions. Finally, for the weekly investment horizon, the NSGA-II and SPEA-II yields 753 and 748 non-dominated portfolio solutions, respectively. For the monthly investment horizon, 688 and 729 non-dominated portfolio solutions are produced, respectively, by the NSGA-II and SPEA-II.

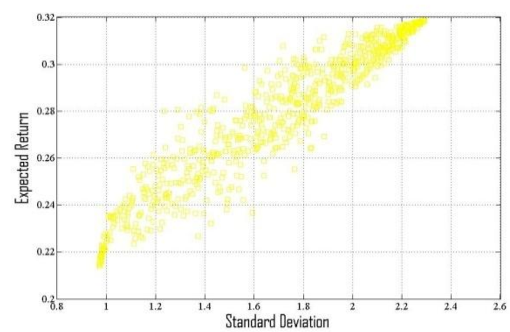
The non-dominated portfolio solutions or the so called "MVS efficient portfolios" are plotted in the 3D objective space based on the information of their corresponding objective-value vectors and illustrated in four different points of view. In Figure 4.1(a) and Figure 4.2(a), the efficient portfolios of weekly investment horizon obtained from the NSGA-II and SPEA-II, respectively, are illustrated in the MVS space. Panels (b), (c), and (d) of

Figure 4.1 and Figure 4.2 show the resulting MVS efficient portfolios on the mean-SD diagram (front view), mean-skewness diagram (side view), and skewness-SD diagram (top view), respectively. Figure 4.1 and Figure 4.2 reveal that the efficient surface of the efficient solutions seems comparatively the same for the two implemented algorithms. The similarity of the results accords with our expectation and ensures the robustness of our results. Although there are differences in the process and operation between the NSGA-II and SPEA-II, both algorithms are regarded as the most efficient MOEAs in the literature. Therefore, only slightly differences in results are generally observed when they are used to solve an identical problem.

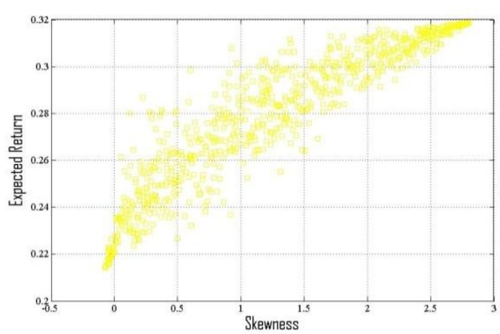
Figure 4.1: The MVS Efficient Solutions from NSGA-II (Weekly Data)



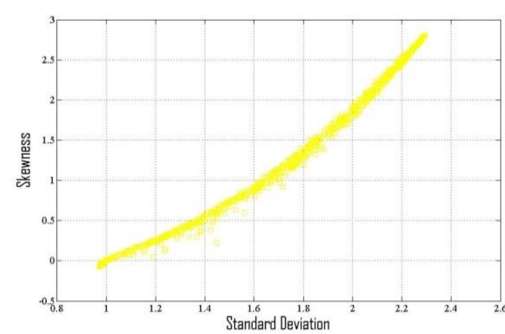
Panel (a)



Panel (b)

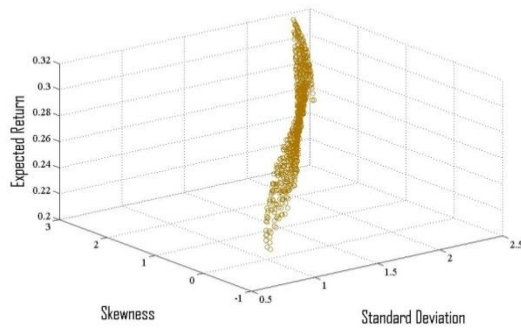


Panel (c)

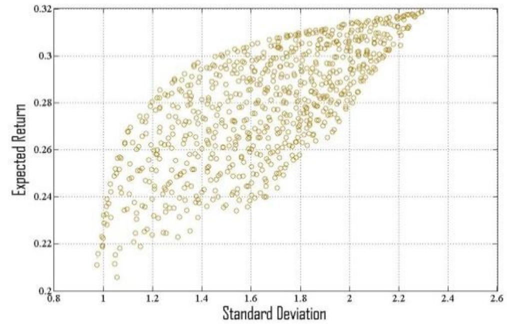


Panel (d)

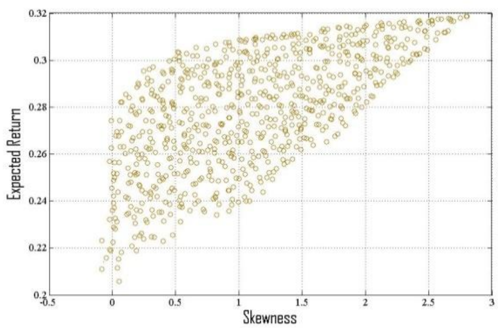
Figure 4.2: The MVS Efficient Solutions from SPEA-II (Weekly Data)



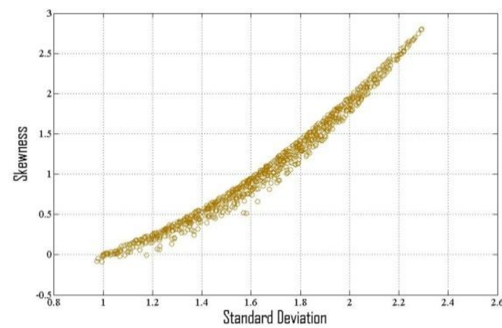
Panel (a)



Panel (b)



Panel (c)



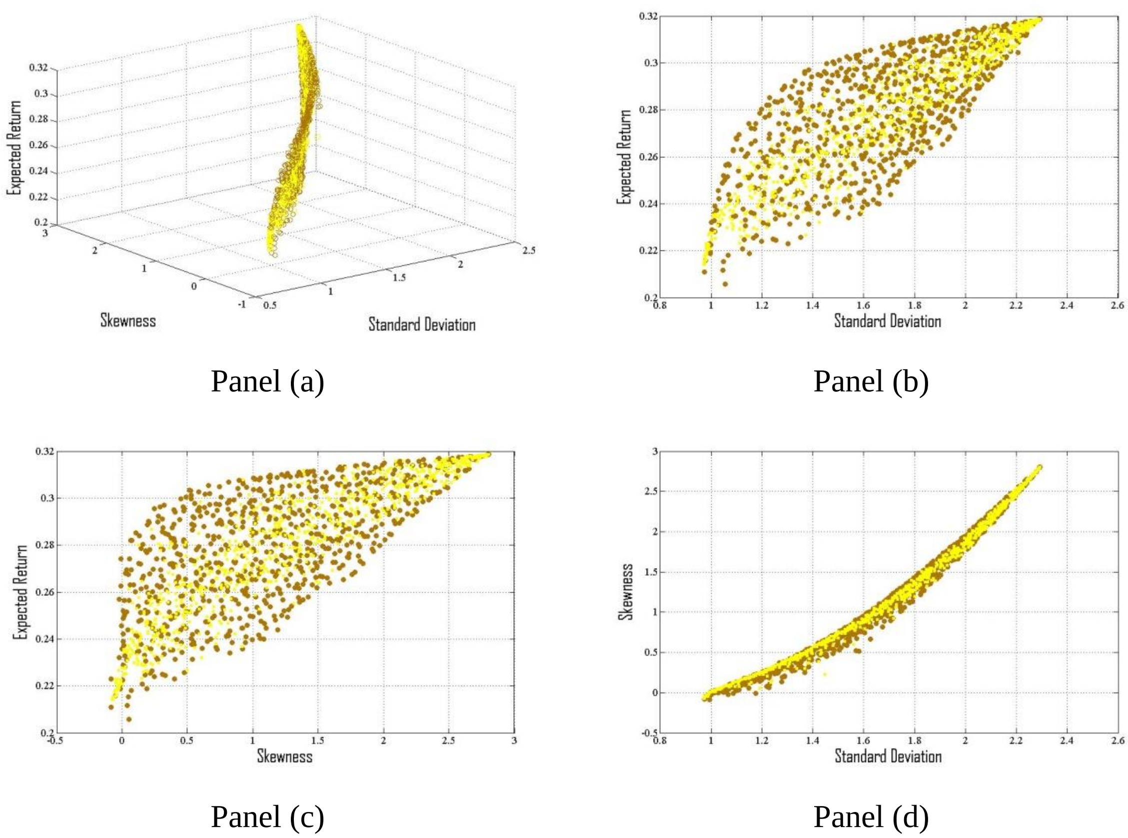
Panel (d)

Figure 4.3 compares the efficient portfolios obtained from both algorithms in the same diagram. The efficient portfolios from the NSGA-II and SPEA-II are marked by yellow and brown, respectively. Figure 4.3 reveals that both algorithms are able to provide good solutions of the efficient portfolios, evident from the solutions that reside on the efficient frontier.[17] For instance, in panel (b) of Figure 4.3, the north-west boundary of the mean-SD diagram is the target of optimal portfolios, i.e. where expected return is maximized and SD is minimized. It can be seen that both algorithms found the solutions that are on the convex

[17] As explained in Section 3.2, the efficient frontier is the boundary or frontier that is formed by the efficient portfolios.

curve of the mean-SD frontier. Similar results can be observed from the other panels of Figure 4.3. However, we can observe from Figure 4.3 that the efficient solutions of SPEA-II are less clustered than those of NSGA-II. Technically, it can be said that SPEA-II has a better “diversity preservation of the solutions” than NSGA-II. Better diversity preservation will have solutions that cover a larger space. The diversity of preservation is widely used as a criterion for performance comparison of algorithms.[18]

Figure 4.3: Comparative Results of the Implemented Algorithms (Weekly Data)

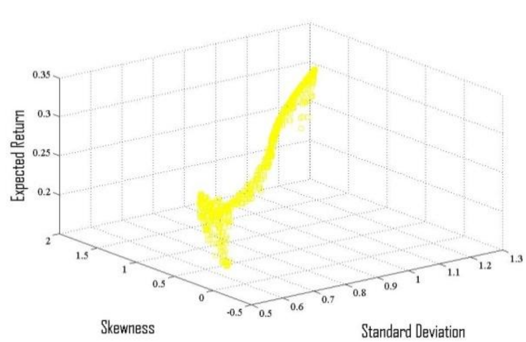


Note: Yellow is for NSGA-II. Brown is for SPEA-II.

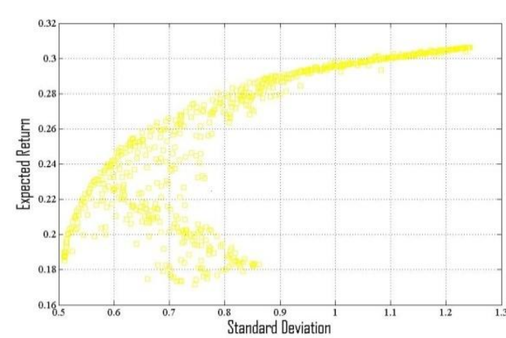
[18] The hypervolumn (*HV)* measure (Zitzler & Thiele, 1998) explained in Section 3.5.3 is an example of an indicator that measures the diversity preservation of the solutions.

In Figure 4.4 and Figure 4.5, the efficient portfolios of monthly investment horizon obtained from the NSGA-II and SPEA-II, respectively, are displayed. Panels (a), (b), (c), and (d) of Figure 4.4 and Figure 4.5 show the resulting MVS efficient portfolios on the mean-SD-skewness diagram (aggregate view), mean-SD diagram (front view), mean-skewness diagram (side view), and skewness-SD diagram (top view), respectively. It is clear from Figure 4.4 and Figure 4.5 that the efficient surface of solutions from both algorithms looks similar. The similarity of the results is consistent with the results of Figure 4.1 and Figure 4.2. In Figure 4.6, the yellow and brown markers represent the efficient portfolios from the NSGA-II and SPEA-II, respectively, in the mean-SD-skewness diagram. Consistent with the weekly investment horizon, Figure 4.6 shows that the efficient portfolios given by both algorithms are located on the efficient frontier. For example, in panel (b) of Figure 4.6, the north-west surface of the mean-SD diagram is favorable. It is found that both algorithms provide the solutions that lie on the convex curve of the mean-SD frontier. Similar results are observed from the other panels of Figure 4.6. In addition, we found from Figure 4.6 that SPEA-II has a better diversity preservation of the solutions than NSGA-II. Therefore, in the next section, only the results of the weekly and monthly efficient portfolios from the solutions of the SPEA-II are used for further analysis.

Figure 4.4: The MVS Efficient Solutions from NSGA-II (Monthly Data)

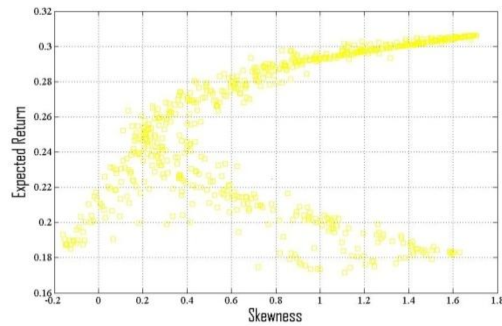


Panel (a)

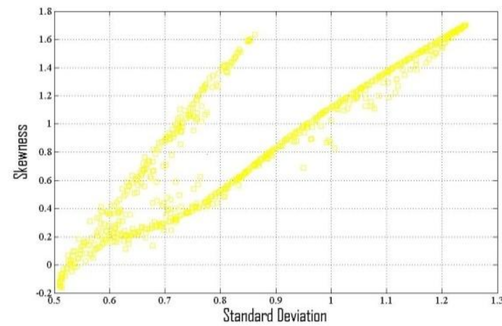


Panel (b)

Figure 4.4 (continued): The MVS Efficient Solutions from NSGA-II (Monthly Data)

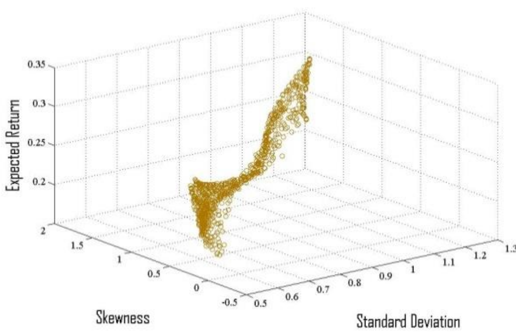


Panel (c)

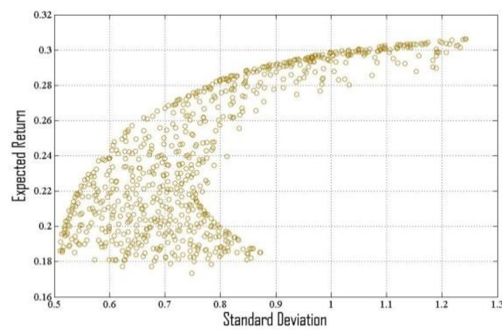


Panel (d)

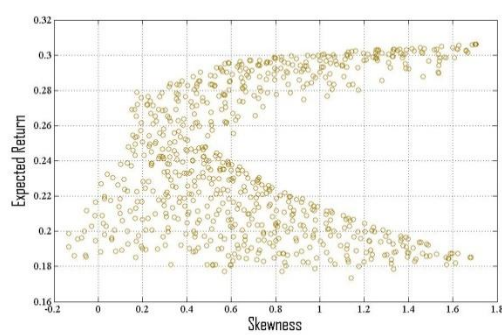
Figure 4.5: The MVS Efficient Solutions from SPEA-II (Monthly Data)



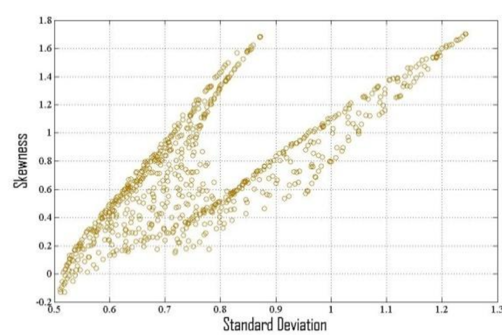
Panel (a)



Panel (b)

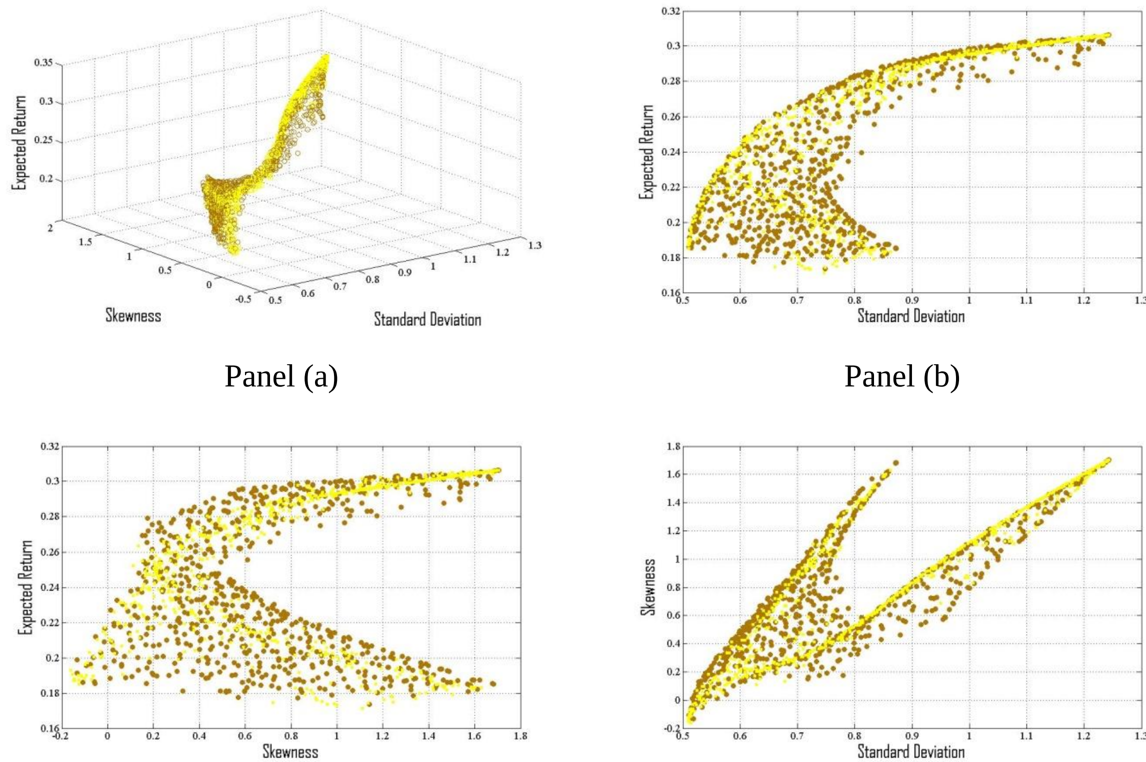


Panel (c)



Panel (d)

Figure 4.6: Comparative Results of the Implemented Algorithms (Monthly Data)



Panel (a) Panel (b)

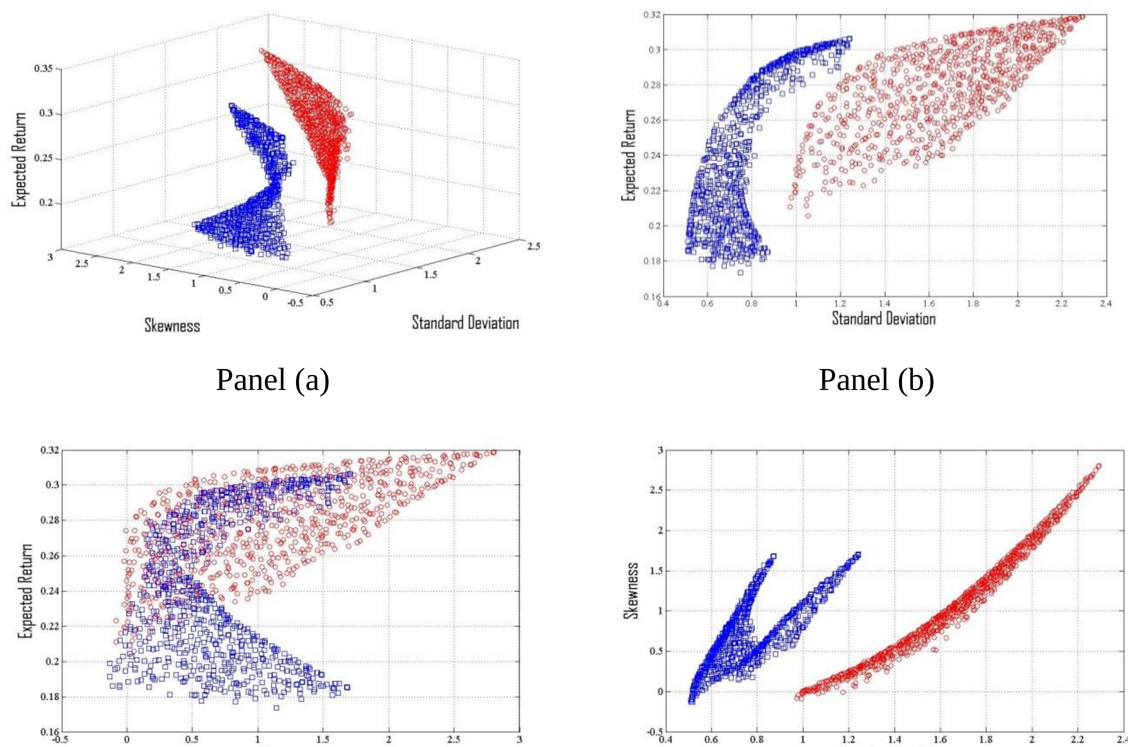
Panel (c) Panel (d)

Note: Yellow is for NSGA-II. Brown is for SPEA-II.

4.4 RISK-RETURN CHARACTERISTICS OF MVS EFFICIENT PORTFOLIOS

As pointed out by Levy (1972), different investment horizons can result in different efficient portfolio sets to investors. Both weekly and monthly datasets were employed to examine the risk-return characteristics of MVS efficient portfolios. We firstly plotted the weekly and the monthly MVS efficient portfolios in red and blue, respectively, in Figure 4.7. It illustrates that the different investment horizons have an impact on the characteristics of the efficient portfolios. Figure 4.7(b) shows that the monthly MVS efficient portfolios dominate the majority of the weekly ones since they possess lower SD at any given value of expected return. However, the maximum expected return of a monthly MVS efficient portfolio can achieve is 30.64 percent meanwhile a weekly one can achieve 31.87 percent. This surplus in expected return can be achieved by selecting a portfolio with high SD and skewness. For example, Figure 4.7(b) and Figure 4.7(c) show that the expected return of weekly MVS efficient portfolios are higher than those of the monthly ones when portfolio SD is higher than 1.2428 and skewness is more than 1.7043. In addition, it can be seen from Figure 4.7(d) that the SD of efficient portfolios becomes larger with an increase in portfolio skewness.

Figure 4.7: Comparative Results of Different Investment Horizons



Panel (a) Panel (b)

Panel (c) Panel (d)

Note: Red is for weekly data. Blue is for monthly data.

4.4.1 Risk-return Characteristics for Fixed Standard Deviation

The risk-return characteristics can be examined since it is possible to plot the MVS efficient portfolios graphically. An investigation of the trade-off between expected return and skewness is conducted by searching among the efficient portfolios for those whose SD matches with a given value. Table 4.5 and Table 4.6 give respectively the weekly and monthly MVS efficient portfolios with the value of SD fixed at 1.00, 1.10, and 1.20.

Table 4.5 shows the six efficient portfolios that are obtained when SD is fixed at 1.00. They are labelled as Port.1, Port.2 and so on in the table. The information from Table 4.5 reveals that, for a given value of portfolio SD, there are many MVS efficient portfolios with different values of expected return and skewness. These portfolios are considered efficient since they are non-dominated solutions, i.e. no other solutions dominate them. For example, with portfolio SD of 1.2, Port.1 has larger expected return than Port.2 but Port.2 has higher skewness than Port.1. Thus they cannot dominate each another. The same result can be observed when comparing Port.1 and Port.3, Port.1 and Port.4, and so on. In addition, the expected returns of the MVS efficient portfolios decrease with an increase in portfolio skewness. This result explains the expected return-skewness trade-off where the investors have to forgo expected return for a portfolio with larger skewness for either reducing a probability of experiencing a large loss or raising the chance of gaining an extreme return.

Considering the portfolio composition, we found that Malaysia and Sri Lanka indices are the choices in which the MVS efficient portfolios mainly invest. For efficient portfolios with SD equal 1.0, the investment proportion allocated to Malaysia and Sri Lanka is in the range of 29 to 44 percent and 19 to 25 percent, respectively. However, the investments in Sri Lanka rises up to between 38 and 42 percent while the allocations to Malaysia reduces to between 3 and 31 percent for the case that SD is set to 1.2. The result is coherent with the parameter values reported in Table 4.1 since Malaysia index possess the lowest SD with the negative skewness and moderate expected return, while Sri Lanka index offers the highest expected return and skewness and prominent value of SD.

For the monthly investment horizon, the proposed technique is still able to provide many MVS efficient portfolios at a given value of SD, although the number of MVS efficient portfolios with the SD value of 1.1 and 1.2 reported in Table 4.6 is less than those of the weekly investment horizon. This result is expected because the summary statistics reported in Table 4.2 show that the values of SD of indices are smaller than those exhibited in Table 4.1. Besides, the trade-off between expected return and skewness can be coherently observed from Table 4.6. We found that, for three different values of SD, the negative relation between expected return and skewness still holds. The expected return of the MVS efficient portfolios falls with a rise in portfolio skewness.

An analysis of portfolio holding shows that majority of the MVS efficient portfolios reported in Table 4.6 heavily allocate the investments to Sri Lanka and Philippines indices. From Table 4.2, Sri Lanka index offers the highest expected return with very high value of SD and skewness while Philippines index has comparatively low SD with moderate expected return and skewness. It can be seen that investments in Sri Lanka index range from 68 to 72 percent for SD equal to 1.0, while these investment proportions increase up to 96 percent when SD is set to 1.2. However, this larger investment proportion offers the efficient portfolios with higher portfolio expected return and skewness.

By comparing the results from Table 4.5 and Table 4.6 in terms of portfolio holding, Sri Lanka index is chosen as a portfolio component for both investment horizons and govern the characteristics of the efficient portfolios. Besides, we notice that, at any given level of SD, the MVS efficient portfolios formed by using weekly data include a significantly larger number of assets to eliminate unsystematic risk, in contrast to those constructed based on monthly data which allocate the investments to only less than four assets to achieve the same level of risk reduction.

Our finding on expected return-skewness trade-off is consistent with the previous studies including Lai (1991), Chunhachinda, et al. (1997), Prakash, et al. (2003), and Canela and Collazo (2007). However, our methodology is more robust leading to a stronger validity of the results. In the previous studies, the MVS portfolios are solved by firstly setting portfolio variance equal to 1.0 as a problem constraint. Next a maximum expected return portfolio and a maximum skewness portfolio that satisfy this variance constraint are identified. Then another comprising a portfolio that best balances the two objectives, i.e. expected return and skewness, is solved. The resulting portfolios are used to examine the trade-off between expected return and skewness. In contrast, the methodology we adopted provides a few portfolios to examine this trade-off, without the need to constrain the SD value in the optimization problem. Besides, the variance constraint in the previous studies causes a DM, at best, to find only a couple members of MVS efficient portfolios but, at worst, the obtained solutions are not efficient. In contrast, our technique always ensures that the resulting portfolios are MVS efficient.

Table 4.5: The MVS Efficient Portfolios of Weekly Investment Horizon with SD Value of 1.00, 1.10, and 1.20

	Investment Allocation Proportion					
	Port.1	Port.2	Port.3	Port.4	Port.5	Port.6
China	-	-	-	0.02	-	-
India	-	-	-	-	-	-
Indonesia	-	-	-	-	0.02	-
Malaysia	0.33	0.29	0.29	0.33	0.44	0.31
Pakistan	0.12	0.18	0.15	0.18	0.16	0.20
Philippines	0.09	0.14	0.14	0.01	-	0.03
South Korea	-	-	-	-	-	-
Sri Lanka	0.14	0.19	0.19	0.21	0.25	0.23
Taiwan	-	0.04	0.06	0.03	-	0.08
Thailand	0.11	-	-	-	-	0.02
Argentina	-	-	-	-	-	-
Brazil	-	-	0.01	-	-	-
Chile	0.01	-	-	-	0.01	-
Colombia	0.19	0.14	0.14	0.22	0.12	0.11
Mexico	-	-	-	-	-	-
Peru	-	0.01	0.01	-	-	-
Expected Return	23.99%	23.32%	23.19%	23.03%	22.99%	22.65%
Standard Deviation	**1.0**	**1.0**	**1.0**	**1.0**	**1.0**	**1.0**
Skewness	-0.026	-0.004	-0.001	0.024	0.034	0.037

Table 4.5: (continued) The MVS Efficient Portfolios of Weekly Investment Horizon with SD Value of 1.00, 1.10, and 1.20

	Investment Allocation Proportion						
	Port.1	Port.2	Port.3	Port.4	Port.5	Port.6	Port.7
China	-	-	0.02	-	-	-	-
India	-	-	-	-	-	-	-
Indonesia	0.03	0.01	-	-	-	-	-
Malaysia	0.26	0.20	0.21	0.21	0.28	0.28	0.28
Pakistan	0.12	0.20	0.14	0.15	0.12	0.12	0.12
Philippines	-	-	0.06	-	0.07	0.04	0.04
South Korea	-	-	-	-	-	-	-
Sri Lanka	0.30	0.31	0.31	0.31	0.34	0.34	0.34
Taiwan	0.12	0.11	0.13	0.20	0.15	0.16	0.16
Thailand	-	0.03	0.02	-	-	-	-
Argentina	-	-	-	-	0.01	0.01	-
Brazil	-	-	-	-	-	-	-
Chile	0.02	0.01	0.04	-	0.02	0.02	0.01
Colombia	0.14	0.08	0.06	0.13	-	-	-
Mexico	-	-	-	-	-	-	-
Peru	-	0.04	-	-	-	0.03	0.03
Expected Return	23.57%	23.40%	23.24%	22.99%	22.88%	22.66%	22.66%
Standard Deviation	**1.1**	**1.1**	**1.1**	**1.1**	**1.1**	**1.1**	**1.1**
Skewness	0.120	0.129	0.137	0.140	0.182	0.186	0.187

Table 4.5 (continued): The MVS Efficient Portfolios of Weekly Investment Horizon with SD Value of 1.00, 1.10, and 1.20

	Investment Allocation Proportion						
	Port.1	Port.2	Port.3	Port.4	Port.5	Port.6	Port.7
China	-	-	-	0.01	0.01	-	-
India	-	-	-	-	-	-	0.02
Indonesia	0.02	0.05	0.05	0.01	-	0.09	-
Malaysia	0.03	0.21	0.21	0.26	0.31	0.24	0.23
Pakistan	0.16	0.07	0.11	0.13	0.11	0.06	0.10
Philippines	0.15	0.12	0.13	0.05	0.06	0.01	0.01
South Korea	-	-	-	-	-	-	-
Sri Lanka	0.40	0.38	0.39	0.42	0.41	0.39	0.41
Taiwan	0.11	0.07	0.07	0.01	0.07	0.16	0.14
Thailand	-	0.06	0.03	0.01	-	-	-
Argentina	-	-	-	-	-	-	-
Brazil	-	-	-	-	-	-	-
Chile	0.02	0.02	-	-	0.01	-	0.06
Colombia	0.09	-	-	-	0.01	0.03	0.03
Mexico	-	0.01	-	0.03	-	-	-
Peru	0.01	-	-	0.06	-	0.01	-
Expected Return	25.73%	25.38%	25.20%	24.53%	24.25%	24.22%	23.63%
Standard Deviation	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**	**1.2**
Skewness	0.247	0.249	0.252	0.266	0.271	0.289	0.294

Table 4.6: The MVS Efficient Portfolios of Monthly Investment Horizon with SD Value of 1.00, 1.10, and 1.20

	Investment Allocation Proportion						
	Port.1	Port.2	Port.3	Port.4	Port.5	Port.6	Port.7
China	-	-	-	-	-	-	0.03
India	-	-	-	-	-	-	-
Indonesia	0.16	0.12	0.06	0.02	-	-	-
Malaysia	-	-	-	-	-	-	-
Pakistan	-	-	-	-	-	-	-
Philippines	0.16	0.19	0.23	0.27	0.27	0.28	0.25
South Korea	-	-	-	-	-	-	-
Sri Lanka	0.68	0.69	0.71	0.71	0.71	0.71	0.72
Taiwan	-	-	-	-	-	-	-
Thailand	-	-	-	-	-	-	-
Argentina	-	-	-	-	-	0.01	-
Brazil	-	-	-	-	-	-	-
Chile	-	-	-	-	-	-	-
Colombia	-	-	-	-	0.01	-	-
Mexico	-	-	-	-	-	-	-
Peru	-	-	-	-	0.01	-	-
Expected Return	29.68%	29.65%	29.57%	29.50%	29.39%	29.19%	28.96%
Standard Deviation	**1.0**	**1.0**	**1.0**	**1.0**	**1.0**	**1.0**	**1.0**
Skewness	0.961	1.002	1.058	1.075	1.087	1.088	1.103

Table 4.6 (continued): The MVS Efficient Portfolios of Monthly Investment Horizon with SD Value of 1.00, 1.10, and 1.20

	Investment Allocation Proportion								
	Port.1	Port.2	Port.3	Port.4	Port.5		Port.1	Port.2	Port.3
China	-	-	-	-	-		-	-	-
India	-	-	-	-	-		-	-	-
Indonesia	0.18	0.08	-	0.01	-		0.04	-	-
Malaysia	-	-	-	-	-		-	-	-
Pakistan	-	-	-	-	-		-	-	-
Philippines	-	0.04	0.12	0.13	0.15		-	-	0.04
South Korea	-	-	-	-	-		-	-	-
Sri Lanka	0.82	0.81	0.85	0.86	0.85		0.96	0.95	0.96
Taiwan	-	-	-	-	-		-	-	-
Thailand	-	0.06	0.03	-	-		-	0.04	-
Argentina	-	-	-	-	-		-	-	-
Brazil	-	-	-	-	-		-	-	-
Chile	-	-	-	-	-		-	-	-
Colombia	-	-	-	-	-		-	-	-
Mexico	-	-	-	-	-		-	-	-
Peru	-	-	-	-	-		-	-	-
Expected Return	30.24%	30.12%	30.09%	30.09%	30.02%		30.56%	30.51%	30.45%
Standard Deviation	**1.1**	**1.1**	**1.1**	**1.1**	**1.1**		**1.2**	**1.2**	**1.2**
Skewness	1.200	1.237	1.373	1.379	1.380		1.574	1.597	1.607

4.4.2 Risk-return Characteristics for Fixed Expected Return

The risk-return characteristics are now analyzed when the rate of return is fixed at a given level. Table 4.7 and Table 4.8 show the weekly and monthly MVS efficient portfolios, respectively, when the expected return is fixed at 28.0 percent. It is revealed from Table 4.7 and Table 4.8 that the proposed technique is able to search and identify many portfolios at a given value of expected return and these portfolios are efficient in the MVS framework. In both the tables, the MVS efficient portfolios with expected return of 28.0 percent are labelled as Port.1, Port.2 and so on. With portfolio return fixed at a constant, it can be demonstrated from the results that the value of SD of MVS efficient portfolios decreases with a diminishing value of skewness. This result is examined from the characteristics of 20 and 16 MVS efficient portfolios in the case of weekly and monthly investment horizon, respectively. The implication of this result is that investors need to expose themselves to a larger return dispersion in order to increase the probability of exposure to the extreme expected returns. In the similar vein, they trade the chances of large positive expected returns for the reduction of portfolio risk measured by SD. This finding makes a significant contribution to the literature since it has not been addressed by the previous studies due to the limitation of the techniques they used. Although Simkowitz and Beedles (1978) found that portfolio SD increases with an increasing portfolio skewness, the trade-off was examined from the household accounts of a large brokerage firm but not from the perspective of efficient portfolios.

Additionally, it is revealed from Table 4.7 that at a given level of expected return, the MVS efficient portfolios can be ranked in an order from the highest SD (with the highest skewness), i.e. Port.1, to the lowest SD (with the lowest skewness), i.e. Port.20. In the MV portfolio model, an optimization will search for the portfolio with the global minimum variance (or SD) at a given value of expected return, while the skewness objective is not taken into consideration. A collection of the global minimum variance portfolios at different values of expected return is known as "the MV efficient frontier". Port.20 in Table 4.7 is not only an MVS efficient portfolio, but it is also MV efficient due to the fact that it has attained the global minimum SD with an expected return of 28.0 percent, and it is not dominated by any other MVS efficient portfolios. This result is also similar for Port.16 in Table 4.8. This finding is the second contribution of this chapter that the MV efficient portfolios are a subset of the MVS efficient portfolios.

Table 4.7: The MVS Efficient Portfolios of Weekly Investment Horizon with Expected Return Value of 28.0 Percent

	Investment Allocation Proportion									
	Port.1	Port.2	Port.3	Port.4	Port.5	Port.6	Port.7	Port.8	Port.9	Port.10
China	-	-	-	-	-	-	-	-	-	-
India	0.16	0.22	0.21	0.10	0.05	0.10	0.15	-	-	-
Indonesia	-	-	-	-	-	-	-	-	0.04	-
Malaysia	-	-	0.01	-	-	-	-	0.13	0.08	-
Pakistan	-	-	-	-	-	-	0.05	-	-	-
Philippines	-	-	-	0.01	-	0.07	-	-	0.06	0.15
South Korea	-	-	-	-	-	-	-	-	-	-
Sri Lanka	0.80	0.78	0.78	0.76	0.77	0.75	0.73	0.75	0.69	0.67
Taiwan	-	-	-	0.09	0.14	0.06	0.02	0.12	0.13	0.18
Thailand	-	-	-	-	-	-	0.04	-	-	-
Argentina	0.03	-	-	-	-	0.02	-	-	-	-
Brazil	-	-	-	-	-	-	-	-	-	-
Chile	-	-	-	-	0.03	-	-	-	-	-
Colombia	-	-	-	-	-	-	-	-	-	-
Mexico	-	-	-	-	-	-	-	-	-	-
Peru	-	-	-	0.02	-	-	-	-	-	-
Expected Return	**28.0%**	**28.0%**	**28.0%**	**28.0%**	**28.0%**	**28.0%**	**28.0%**	**28.0%**	**28.0%**	**28.0%**
Standard Deviation	2.011	2.002	1.988	1.890	1.875	1.870	1.847	1.797	1.697	1.685
Skewness	1.978	1.974	1.929	1.642	1.596	1.571	1.495	1.366	1.122	1.098

Table 4.7 (continued): The MVS Efficient Portfolios of Weekly Investment Horizon with Expected Return Value of 28.0 Percent

	Investment Allocation Proportion									
	Port.11	Port.12	Port.13	Port.14	Port.15	Port.16	Port.17	Port.18	Port.19	Port.20
China	-	-	-	-	-	-	-	-	-	-
India	-	-	-	-	-	-	-	-	-	-
Indonesia	-	-	-	-	-	-	-	-	-	-
Malaysia	0.04	0.14	0.22	0.05	0.01	0.03	0.02	-	-	0.06
Pakistan	0.06	0.03	0.01	0.11	-	0.12	0.13	0.12	0.11	0.04
Philippines	0.12	0.02	0.12	0.07	0.19	0.10	-	0.12	0.18	0.24
South Korea	-	-	-	-	-	-	-	-	-	-
Sri Lanka	0.65	0.65	0.64	0.61	0.57	0.58	0.42	0.42	0.38	0.34
Taiwan	0.06	0.07	-	0.06	0.12	-	0.02	-	-	-
Thailand	-	0.10	-	0.09	0.08	0.10	0.13	0.03	0.26	0.07
Argentina	-	-	-	-	-	-	-	-	-	-
Brazil	-	-	-	-	-	-	-	-	-	-
Chile	0.05	-	-	-	-	0.07	-	-	0.03	-
Colombia	-	-	-	-	-	-	0.28	0.30	0.02	0.25
Mexico	-	-	-	-	-	-	-	-	-	-
Peru	-	-	-	-	0.02	-	-	-	-	-
Expected Return	**28.0%**	**28.0%**	**28.0%**	**28.0%**	**28.0%**	**28.0%**	**28.0%**	**28.0%**	**28.0%**	**28.0%**
Standard Deviation	1.607	1.614	1.597	1.533	1.518	1.477	1.281	1.257	1.278	1.222
Skewness	0.913	0.910	0.876	0.738	0.715	0.615	0.287	0.249	0.241	0.173

Table 4.8: The MVS Efficient Portfolios of Monthly Investment Horizon with Expected Return Value of 28.0 Percent

	Investment Allocation Proportion							
	Port. 1	Port.2	Port. 3	Port. 4	Port. 5	Port. 6	Port. 7	Port. 8
China	0.05	0.05	-	-	-	-	-	-
India	-	-	-	-	0.01	-	-	-
Indonesia	-	-	-	-	-	-	-	0.01
Malaysia	-	-	-	0.02	0.02	0.01	0.02	-
Pakistan	-	-	-	-	-	0.03	-	0.03
Philippines	0.29	0.37	0.36	0.42	0.42	0.38	0.48	0.45
South Korea	-	-	-	-	-	-	-	-
Sri Lanka	0.66	0.58	0.56	0.52	0.51	0.52	0.49	0.50
Taiwan	-	-	0.07	-	-	-	-	-
Thailand	-	-	-	-	-	0.02	-	-
Argentina	-	-	-	-	-	-	-	-
Brazil	-	-	-	-	-	-	-	-
Chile	-	-	-	-	-	-	-	-
Colombia	-	-	-	0.01	0.01	0.04	-	-
Mexico	-	-	-	-	-	-	-	-
Peru	-	-	-	-	-	-	-	-
Expected Return	**28.0%**	**28.0%**	**28.0%**	**28.0%**	**28.0%**	**28.0%**	**28.0%**	**28.0%**
Standard Deviation	0.953	0.895	0.888	0.859	0.854	0.847	0.841	0.836
Skewness	0.985	0.815	0.793	0.696	0.679	0.653	0.640	0.628

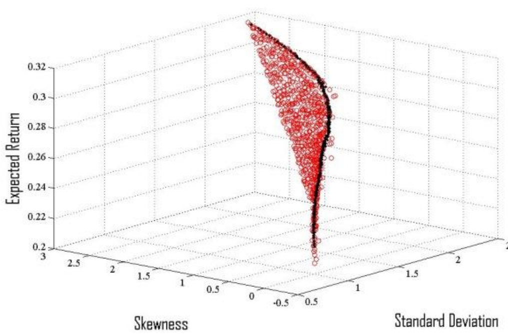
Table 4.8 (continued): The MVS Efficient Portfolios of Monthly Investment Horizon with Expected Return Value of 28.0 Percent

	Investment Allocation Proportion							
	Port. 9	Port. 10	Port. 11	Port. 12	Port. 13	Port. 14	Port. 15	Port. 16
China	-	-	-	-	-	-	-	-
India	-	-	-	-	-	-	-	-
Indonesia	-	-	-	-	-	-	0.18	0.13
Malaysia	-	0.04	-	-	-	-	0.02	-
Pakistan	0.05	-	-	-	-	-	0.02	0.01
Philippines	0.43	0.47	0.50	0.51	0.51	0.44	0.37	0.41
South Korea	-	-	-	-	-	-	-	-
Sri Lanka	0.51	0.48	0.46	0.45	0.44	0.38	0.38	0.33
Taiwan	-	-	-	-	-	-	0.01	-
Thailand	-	-	0.02	0.02	-	-	-	-
Argentina	-	-	-	-	-	-	-	-
Brazil	-	-	-	-	-	-	-	-
Chile	-	-	-	-	-	-	-	-
Colombia	-	-	-	-	0.04	0.18	-	0.11
Mexico	-	-	-	-	-	-	-	-
Peru	-	-	-	-	-	-	-	-
Expected Return	**28.0%**	**28.0%**	**28.0%**	**28.0%**	**28.0%**	**28.0%**	**28.0%**	**28.0%**
Standard Deviation	0.834	0.832	0.828	0.825	0.818	0.805	0.801	0.791
Skewness	0.62	0.61	0.59	0.58	0.56	0.49	0.42	0.38

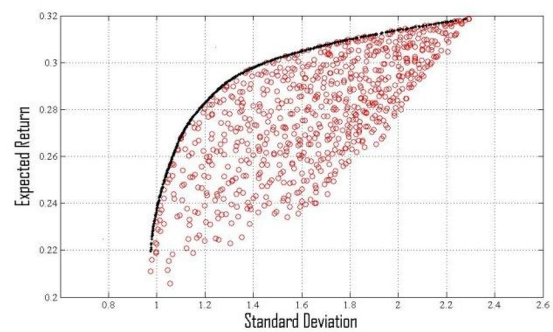
An additional analysis is conducted to verify the above argument. The MVS efficient portfolios of **Prob. 1** for the weekly and monthly investment horizons generated using SPEA-II are plotted in the MVS space and presented in Figure 4.8(a) and Figure 4.9(a), respectively. Panels (b), (c), and (d) of Figure 4.8 and Figure 4.9 illustrate the resulting MVS efficient portfolios on mean-SD diagram, mean-skewness diagram, and skewness-SD diagram, respectively. The black curve represents the MV efficient frontier that is obtained by optimizing **Prob. 1** without the skewness objective. The results suggest that the members of the MVS efficient portfolios with minimum SD at different levels of expected return are on the MV efficient frontier. Meanwhile, the rest located outside the MV efficient frontier are considered as inefficient portfolios under the MV portfolio model (see, in particular Panel (b) of Figure 4.8 and Figure 4.9). Therefore, it can be confirmed that the

MV efficient portfolios are a subset of the MVS efficient portfolios. In addition, we argue that the portfolios considered as inefficient in the MV framework are actually efficient when skewness is taken into portfolio decision. Although these portfolios have higher SD at a given level of expected return, they offer larger skewness than that of the MV efficient portfolios (see, in particular Panel (d) of Figure 4.8 and Figure 4.9). This finding gives an explanation of the phenomenon of why investors hold portfolios with larger SD although their expected returns may be same as the other portfolios with smaller SD. They do so because these portfolios are MVS efficient.

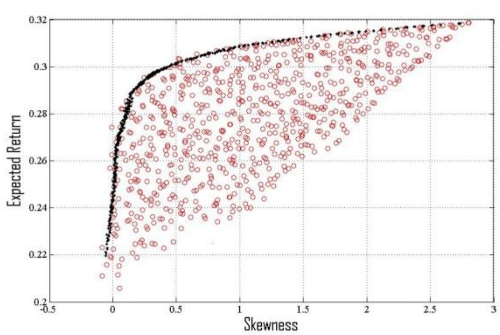
Figure 4.8: Impact of Skewness on MV Efficiency – Weekly



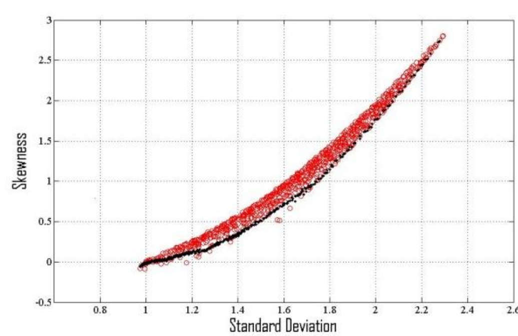
Panel (a)



Panel (b)



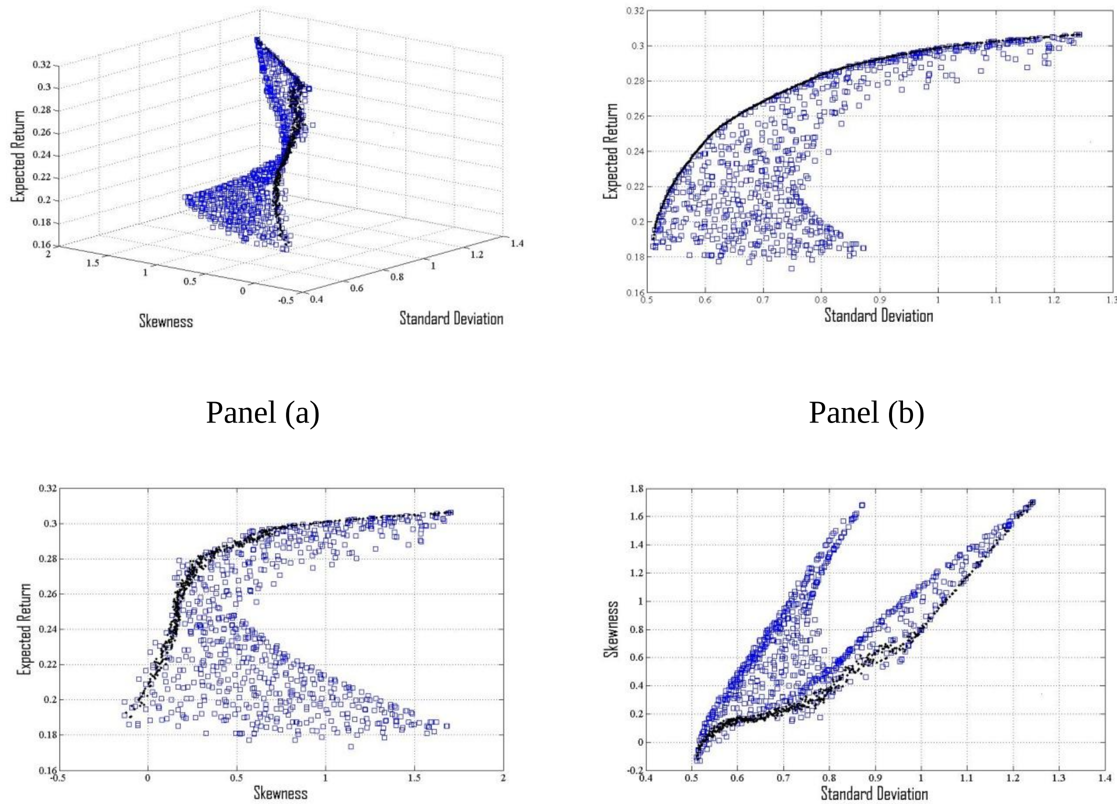
Panel (c)



Panel (d)

Note: Black is for MV efficient portfolios. Red is for MVS efficient portfolios.

Figure 4.9: Impact of Skewness on MV Efficiency – Monthly



Panel (a) Panel (b)

Panel (c) Panel (d)

Note: Black is for MV efficient portfolios. Blue is for MVS efficient portfolios.

For the analysis of portfolio composition, the results from Table 4.7 and Table 4.8 demonstrate that the incorporation of skewness into portfolio selection causes a major change in portfolio holdings. For example, in Table 4.7, Port.20 which is also MV efficient mainly allocates the investment on the indices of Sri Lanka, Philippines, and Colombia whereas Port.1 which is the portfolio with the highest skewness invests heavily on the indices of Sri Lanka and India. Although, the indices of Philippines and Colombia offer higher levels of expected returns with a moderate value of SD, their skewness value is comparatively lower for Colombia and negative for Philippines. Thus, the allocations on investment on the index of the Philippines are small for the first nine MVS efficient portfolios, while only four MVS efficient portfolios included the index of the Colombia.

Analogous results are observed for the monthly investment horizon. Port.1 and Port.16 in Table 4.8 have the highest and the lowest value of skewness, respectively. Most of the MVS efficient portfolios in Table 4.8 mainly consist of in Sri Lanka and Philippines in the investment, whereas Port.16 which is also MV efficient allocates some investments to other indices such as Indonesia, Colombia, and Pakistan. Interestingly, it is showed in Table 4.2 that the market of the Philippines offers high expected return but has negative skewness. However, the coskewness between the returns of the Sri Lanka and Philippines market as reported in Table 4.6 is considerably high. This evidence suggests that not only the skewness of individual assets but also the coskewness between assets are important for constructing the MVS efficient portfolios.

In addition, the results from Table 4.7 and Table 4.8 show the advantage of the techniques used in this study for solving the MVS-POP. It can be seen that our methodology provides many MVS efficient solutions for a given level of expected return. In this case, the preferences of or other information received by an investor with a targeted level of expected return can be used for making portfolio decisions. For example, a fund manager whose target return is 28.0 percent may receive good news on the Malaysia and Thailand stock markets. Therefore this information can be used for selecting the investment portfolio among a set of MVS efficient solutions. Among the MVS efficient portfolios in Table 4.7, Port.12, Port.14, Port.15, Port.16, and Port.17 will be considered as the investment alternatives for this fund manager. In contrast, by using the other techniques adopted in the previous studies, only one or limited solutions can be obtained from the optimization process. As a result, a DM can only choose among these limited solutions without any efficient alternatives for taking other preferences or information into consideration.

4.5 CONCLUDING REMARKS

In this chapter, we illustrated the methodology for solving the efficient portfolios based upon the MVS analysis. The MVS-POP is formulated for the case in which the asset returns are not normally distributed but skewed. The investors, whose utility can be approximated by a third-order Taylor's series expansion, act to maximize their utility by choosing the efficient portfolios that maximize expected return and skewness, while minimizing SD simultaneously. We proposed and demonstrated the use of selected MOEAs for searching the MVS efficient portfolios in a three-dimension moment space. Then, the characteristics of MVS efficient portfolios were examined.

The empirical investigation was conducted using the indices of sixteen emerging markets in Asia and Latin America. The evidences indicate that the majority of the market indices have significant non-zero skewness in the distributions for both annualized weekly and annualized monthly returns. This finding affirms that skewness should not be neglected in portfolio selection. Nevertheless, the results reveal that, on the majority of the annualized weekly returns exhibit negative skewness. In contrast, most of the distributions of the annualized monthly returns are positively skewed. This different in the sign of skewness between the two investment horizons reaffirms the “intervalling effect” previous addressed by Chunhachinda et al. (1994), Chunhachinda et al. (1997), Prakash et al. (1997), and Prakash et al. (2003).

A salient characteristic of the MVS efficient portfolios is that their expected returns are smaller for portfolios with larger skewness at a given level of SD. This finding is consistent with the previous studies including Lai (1991), Chunhachinda, et al. (1997), Prakash, et al. (2003), and Canela and Collazo (2007). This result implies that, among a set of MVS efficient portfolios, investors need to trade expected return to invest in portfolios with larger skewness. On the other hand, at a given value of expected return, the SD of MVS efficient portfolios increases with an increase in portfolio skewness. This result implies that investors need to expose themselves to a larger return dispersion in order to increase the probability of extreme positive expected returns. In another context, they are willing to trade the chances of receiving a large positive expected return for a reduction of portfolio risk measured by SD. This finding makes a significant contribution to the literature since it has not been addressed by the previous studies due to the limitation of the techniques applied.

Lastly, regardless of investment horizon, the MVS efficient portfolios have the lowest SD for a given level of expected return and skewness. By plotting the MV and MVS efficient portfolios together, the result confirms that the MV efficient portfolios are a subset of the MVS efficient portfolios. In addition, the results suggest that some inefficient portfolios under the MV portfolio model are actually efficient when skewness is taken into portfolio decision. This finding gives an explanation of the phenomenon of why investors hold portfolios that are not MV efficient. They do so because these portfolios are efficient under the MVS portfolio model.

CHAPTER 5

THE IMPACT OF SKEWNESS PREFERENCE ON EFFICIENT PORTFOLIO CHOICES

5.1 INTRODUCTION

This chapter discusses the impact of different degree of skewness preference on portfolio choice and portfolio holding. The MVS-POP formulated as **Prob. 1** in Section 3.2 is firstly solved to identify the set of MVS efficient portfolios. The resulting efficient portfolios are used to test the three implications of the proposed model developed in Section 3.5.1. Section 5.1 reports the summary statistics and the normality test results for 29 securities listed in the DJIA. Section 5.2 gives the results of the estimated input variables used for solving MVS efficient portfolios. Section 5.3 demonstrates the MVS efficient portfolios obtained from the SPEA-II in the three-dimension space. The impacts of different degree of skewness preference on efficient portfolio choices are discussed in Section 5.4. The conclusion of the chapter is made in Section 5.5.

5.2 SUMMARY STATISTICS AND THE NORMALITY TEST RESULTS

Table 5.1 exhibits the summary statistics together with the Jarque-Bera normality test statistics for the monthly annualized rate of returns[19] of 29 securities listed in the DJIA. The second column of Table 5.1 reveals that during the period of January 2004 to December 2011, only MCD and NKE show positive average rate of return of 0.174 and 0.129, respectively. The lowest average rate of return is GE (-0.076), followed by PFE (-0.064) and CSCO (-0.050). The third column of Table 5.1 shows that CAT has the highest standard deviation of 1.350, followed by AXP (1.265) and GE (1.095). GE has the lowest standard deviation of 0.475, followed by PG (0.537) and MCD (0.557). The values of skewness are reported in the fourth column of the table. The majority of the DJIA listed stocks exhibit negative skewness during the period of the study. We found that only two stocks, namely, JPM and XOM, have positive skewness of 0.809 and 0.007, respectively. Meanwhile, IBM presents the highest negative skewness of -2.198. The other two stocks that have high negative skewness are CAT (-2.147) and DIS (-1.472).

The result of the Jarque-Bera normality test is presented in the fifth and sixth column of Table 5.1. The probability value (P-value) associated with the JB-statistic indicates the significance level of the rejection of the null hypothesis. The majority of the DJIA listed stocks, i.e. 24 from 29, exhibit significant skewness. The Jarque-Bera test only fails to reject the null hypothesis for 5 stocks, namely, JNJ, JPM, MSFT, WMT, and XOM. This

[19] Hereafter in this chapter, the term “rate of returns” refers to “annualized monthly rate of returns”.

result supports the argument that the return distributions of securities are not empirically normal, and violates the standard assumption of the MV portfolio model.

Table 5.1: Summary Statistics and the Normality Test Results of the Monthly Annualized Rate of Returns of 29 Securities Listed in the DJIA

Symbol	Mean	Standard Deviation	Skewness	JB-statistic	P-value
U:AXP	0.012	1.265	0.809	546.328	0.000***
U:BA	0.072	1.003	-0.645	6.600	0.037**
U:CAT	0.099	1.350	-2.147	442.593	0.000***
U:CVX	0.114	0.917	-0.825	36.558	0.000***
@CSCO	-0.050	0.974	-1.029	49.871	0.000***
U:DD	0.010	0.993	-0.941	85.863	0.000***
U:DIS	0.058	0.915	-1.472	145.062	0.000***
U:GE	-0.076	1.095	-1.036	55.806	0.000***
U:GS	-0.007	1.086	-0.818	27.796	0.000***
U:HD	0.020	0.898	-0.571	39.116	0.000***
U:IBM	0.080	0.789	-2.198	631.089	0.000***
@INTC	-0.033	1.057	-0.863	20.661	0.000***
U:JNJ	0.028	0.475	-0.189	1.389	0.499
U:JPM	-0.021	0.999	0.007	3.786	0.151
U:KO	0.042	0.579	-0.901	50.209	0.000***
U:MCD	0.174	0.557	-0.645	12.220	0.002***
U:MMM	0.000	0.757	-0.440	7.118	0.028**
U:MRK	-0.024	1.017	-0.844	41.625	0.000***
@MSFT	-0.011	0.828	-0.212	1.341	0.512
U:NKE	0.129	0.858	-1.044	53.974	0.000***
U:PFE	-0.064	0.789	-0.612	33.792	0.000***
U:PG	0.038	0.537	-0.613	10.001	0.007***
U:T	0.016	0.662	-0.790	15.507	0.000***
U:TRV	0.044	0.743	-0.308	6.326	0.042**
U:UNH	0.069	1.075	-0.748	16.260	0.000***
U:UTX	0.057	0.714	-1.209	57.166	0.000***
U:VZ	0.022	0.737	-0.421	33.444	0.000***
U:WMT	0.013	0.629	-0.290	1.448	0.485
U:XOM	0.092	0.802	-0.077	0.682	0.711

Note: **, and *** denote significance at the 0.05 and 0.01 levels, respectively.

JB-statistic is for the Jarque-Bera test.

To solve the MVS-POP formulated as **Prob. 1** in Section 3.2.1, the input variables explained in Equation (3.3) to Equation (3.5) are required. To obtain these variables, a computation of Equation (3.14) to Equation (3.17) is performed using the returns of the 29 securities. The input variables, including the return matrix **R** and variance-covariance matrix **Λ** are reported in Table 5.2. The skewness-coskewness matrix **Ω** is exhibited in Table C.1 of APPENDIX C.

Table 5.2: The Return Matrix **R** and Variance-covariance Matrix **Λ** of 29 Securities Listed in the DJIA

Input Variable	U:AXP	U:BA	U:CAT	U:CVX	@CSCO	U:DD	U:DIS	U:GE	U:GS	U:HD
R	0.012	0.072	0.099	0.114	-0.050	0.010	0.058	-0.076	-0.007	0.020

Λ	U:AXP	U:BA	U:CAT	U:CVX	@CSCO	U:DD	U:DIS	U:GE	U:GS	U:HD
U:AXP	**1.583**	0.608	1.097	0.474	0.649	0.853	0.723	0.846	0.468	0.586
U:BA	0.608	**0.996**	0.740	0.407	0.456	0.602	0.581	0.666	0.445	0.374
U:CAT	1.097	0.740	**1.803**	0.755	0.780	0.959	0.898	0.989	0.606	0.742
U:CVX	0.474	0.407	0.755	**0.832**	0.359	0.388	0.468	0.418	0.257	0.351
@CSCO	0.649	0.456	0.780	0.359	**0.939**	0.488	0.497	0.552	0.438	0.432
U:DD	0.853	0.602	0.959	0.388	0.488	**0.976**	0.610	0.746	0.585	0.479
U:DIS	0.723	0.581	0.898	0.468	0.497	0.610	**0.828**	0.662	0.459	0.401
U:GE	0.846	0.666	0.989	0.418	0.552	0.746	0.662	**1.186**	0.493	0.494
U:GS	0.468	0.445	0.606	0.257	0.438	0.585	0.459	0.493	**1.168**	0.222
U:HD	0.586	0.374	0.742	0.351	0.432	0.479	0.401	0.494	0.222	**0.799**
U:IBM	0.485	0.381	0.626	0.370	0.451	0.465	0.450	0.421	0.415	0.374
@INTC	0.633	0.467	0.735	0.351	0.641	0.483	0.484	0.586	0.510	0.361
U:JNJ	0.198	0.177	0.232	0.149	0.161	0.193	0.196	0.231	0.172	0.096
U:JPM	0.711	0.444	0.763	0.299	0.398	0.647	0.540	0.716	0.581	0.406
U:KO	0.274	0.207	0.375	0.246	0.234	0.196	0.284	0.300	0.197	0.172
U:MCD	0.244	0.207	0.340	0.225	0.187	0.207	0.264	0.233	0.159	0.206
U:MMM	0.541	0.389	0.589	0.324	0.396	0.452	0.405	0.444	0.252	0.278
U:MRK	0.233	0.401	0.384	0.259	0.284	0.255	0.316	0.347	0.316	0.095
@MSFT	0.569	0.282	0.477	0.316	0.430	0.329	0.304	0.454	0.424	0.280
U:NKE	0.665	0.377	0.703	0.380	0.468	0.509	0.475	0.526	0.374	0.410
U:PFE	0.233	0.337	0.485	0.272	0.234	0.339	0.314	0.385	0.132	0.162
U:PG	0.322	0.208	0.335	0.176	0.252	0.169	0.261	0.320	0.089	0.180
U:T	0.334	0.264	0.508	0.324	0.298	0.250	0.274	0.347	0.175	0.266
U:TRV	0.359	0.405	0.485	0.314	0.287	0.341	0.396	0.347	0.308	0.241
U:UNH	0.321	0.466	0.427	0.227	0.135	0.354	0.305	0.389	0.167	0.176
U:UTX	0.524	0.490	0.740	0.320	0.393	0.490	0.434	0.564	0.356	0.303
U:VZ	0.382	0.245	0.556	0.335	0.308	0.269	0.289	0.357	0.175	0.361
U:WMT	0.182	0.137	0.298	0.150	0.194	0.185	0.239	0.232	0.164	0.275
U:XOM	0.306	0.349	0.556	0.604	0.236	0.304	0.371	0.329	0.266	0.193

Table 5.2 (continued): The Return Matrix **R** and Variance-covariance Matrix **Λ** of 29 Securities Listed in the DJIA

Input Variable	U:IBM	@INTC	U:JNJ	U:JPM	U:KO	U:MCD	U:MMM	U:MRK	@MSFT	U:NKE
R	0.080	-0.033	0.028	-0.021	0.042	0.174	0.000	-0.024	-0.011	0.129
Λ	U:IBM	@INTC	U:JNJ	U:JPM	U:KO	U:MCD	U:MMM	U:MRK	@MSFT	U:NKE
U:AXP	0.485	0.633	0.198	0.711	0.274	0.244	0.541	0.233	0.569	0.665
U:BA	0.381	0.467	0.177	0.444	0.207	0.207	0.389	0.401	0.282	0.377
U:CAT	0.626	0.735	0.232	0.763	0.375	0.340	0.589	0.384	0.477	0.703
U:CVX	0.370	0.351	0.149	0.299	0.246	0.225	0.324	0.259	0.316	0.380
@CSCO	0.451	0.641	0.161	0.398	0.234	0.187	0.396	0.284	0.430	0.468
U:DD	0.465	0.483	0.193	0.647	0.196	0.207	0.452	0.255	0.329	0.509
U:DIS	0.450	0.484	0.196	0.540	0.284	0.264	0.405	0.316	0.304	0.475
U:GE	0.421	0.586	0.231	0.716	0.300	0.233	0.444	0.347	0.454	0.526
U:GS	0.415	0.510	0.172	0.581	0.197	0.159	0.252	0.316	0.424	0.374
U:HD	0.374	0.361	0.096	0.406	0.172	0.206	0.278	0.095	0.280	0.410
U:IBM	**0.616**	0.494	0.155	0.315	0.224	0.199	0.315	0.195	0.267	0.361
@INTC	0.494	**1.105**	0.231	0.384	0.312	0.217	0.386	0.364	0.467	0.470
U:JNJ	0.155	0.231	**0.223**	0.147	0.135	0.108	0.123	0.227	0.114	0.116
U:JPM	0.315	0.384	0.147	**0.988**	0.165	0.156	0.313	0.263	0.365	0.409
U:KO	0.224	0.312	0.135	0.165	**0.332**	0.167	0.150	0.287	0.190	0.211
U:MCD	0.199	0.217	0.108	0.156	0.167	**0.307**	0.103	0.188	0.160	0.247
U:MMM	0.315	0.386	0.123	0.313	0.150	0.103	**0.567**	0.175	0.267	0.328
U:MRK	0.195	0.364	0.227	0.263	0.287	0.188	0.175	**1.022**	0.260	0.205
@MSFT	0.267	0.467	0.114	0.365	0.190	0.160	0.267	0.260	**0.678**	0.325
U:NKE	0.361	0.470	0.116	0.409	0.211	0.247	0.328	0.205	0.325	**0.728**
U:PFE	0.146	0.203	0.151	0.293	0.158	0.141	0.167	0.393	0.148	0.084
U:PG	0.148	0.251	0.130	0.172	0.178	0.157	0.168	0.222	0.158	0.211
U:T	0.240	0.271	0.114	0.275	0.202	0.164	0.200	0.292	0.215	0.275
U:TRV	0.220	0.407	0.117	0.360	0.183	0.194	0.245	0.305	0.226	0.304
U:UNH	0.160	0.278	0.154	0.186	0.081	0.152	0.169	0.370	0.148	0.161
U:UTX	0.289	0.425	0.146	0.403	0.162	0.171	0.341	0.270	0.338	0.337
U:VZ	0.271	0.306	0.116	0.205	0.213	0.201	0.217	0.295	0.255	0.313
U:WMT	0.216	0.207	0.114	0.207	0.126	0.157	0.119	0.156	0.122	0.216
U:XOM	0.277	0.292	0.158	0.243	0.194	0.180	0.222	0.273	0.235	0.258

Table 5.2 (continued): The Return Matrix **R** and Variance-covariance Matrix **Λ** of 29 Securities Listed in the DJIA

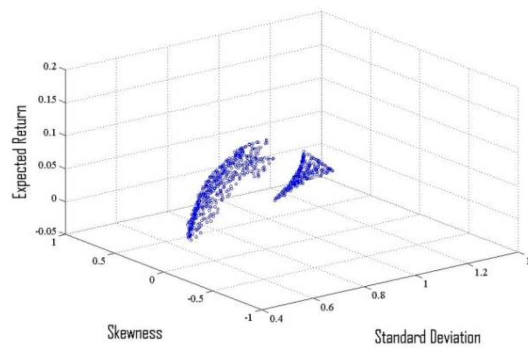
Input Variable	U:PFE	U:PG	U:T	U:TRV	U:UNH	U:UTX	U:VZ	U:WMT	U:XOM
R	-0.064	0.038	0.016	0.044	0.069	0.057	0.022	0.013	0.092
Λ	U:PFE	U:PG	U:T	U:TRV	U:UNH	U:UTX	U:VZ	U:WMT	U:XOM
U:AXP	0.233	0.322	0.334	0.359	0.321	0.524	0.382	0.182	0.306
U:BA	0.337	0.208	0.264	0.405	0.466	0.490	0.245	0.137	0.349
U:CAT	0.485	0.335	0.508	0.485	0.427	0.740	0.556	0.298	0.556
U:CVX	0.272	0.176	0.324	0.314	0.227	0.320	0.335	0.150	0.604
@CSCO	0.234	0.252	0.298	0.287	0.135	0.393	0.308	0.194	0.236
U:DD	0.339	0.169	0.250	0.341	0.354	0.490	0.269	0.185	0.304
U:DIS	0.314	0.261	0.274	0.396	0.305	0.434	0.289	0.239	0.371
U:GE	0.385	0.320	0.347	0.347	0.389	0.564	0.357	0.232	0.329
U:GS	0.132	0.089	0.175	0.308	0.167	0.356	0.175	0.164	0.266
U:HD	0.162	0.180	0.266	0.241	0.176	0.303	0.361	0.275	0.193
U:IBM	0.146	0.148	0.240	0.220	0.160	0.289	0.271	0.216	0.277
@INTC	0.203	0.251	0.271	0.407	0.278	0.425	0.306	0.207	0.292
U:JNJ	0.151	0.130	0.114	0.117	0.154	0.146	0.116	0.114	0.158
U:JPM	0.293	0.172	0.275	0.360	0.186	0.403	0.205	0.207	0.243
U:KO	0.158	0.178	0.202	0.183	0.081	0.162	0.213	0.126	0.194
U:MCD	0.141	0.157	0.164	0.194	0.152	0.171	0.201	0.157	0.180
U:MMM	0.167	0.168	0.200	0.245	0.169	0.341	0.217	0.119	0.222
U:MRK	0.393	0.222	0.292	0.305	0.370	0.270	0.295	0.156	0.273
@MSFT	0.148	0.158	0.215	0.226	0.148	0.338	0.255	0.122	0.235
U:NKE	0.084	0.211	0.275	0.304	0.161	0.337	0.313	0.216	0.258
U:PFE	**0.616**	0.169	0.207	0.187	0.410	0.253	0.237	0.068	0.272
U:PG	0.169	**0.286**	0.168	0.140	0.164	0.161	0.184	0.100	0.150
U:T	0.207	0.168	**0.433**	0.182	0.181	0.231	0.388	0.131	0.202
U:TRV	0.187	0.140	0.182	**0.546**	0.281	0.268	0.166	0.170	0.266
U:UNH	0.410	0.164	0.181	0.281	**1.143**	0.317	0.249	0.015	0.253
U:UTX	0.253	0.161	0.231	0.268	0.317	**0.504**	0.247	0.138	0.267
U:VZ	0.237	0.184	0.388	0.166	0.249	0.247	**0.537**	0.168	0.233
U:WMT	0.068	0.100	0.131	0.170	0.015	0.138	0.168	**0.392**	0.103
U:XOM	0.272	0.150	0.202	0.266	0.253	0.267	0.233	0.103	**0.637**

5.3 MVS EFFICIENT PORTFOLIOS AND THE EFFICIENT SURFACE

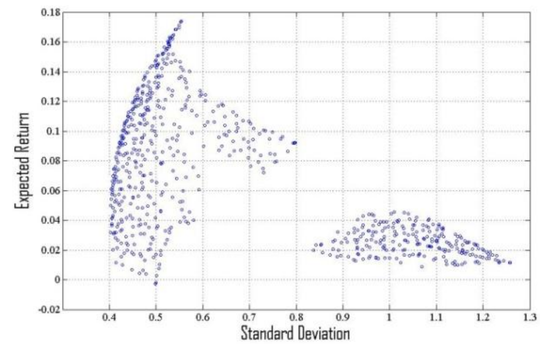
The results of performance comparison between two optimization algorithms in Chapter 4 suggest the superiority of SPEA-II for solving the MVS-POP. Following the parameter setting of the SPEA-II given in Table 3.3, 200 portfolio solutions were obtained from each run of the algorithm. The SPEA-II was run for five times, thus 1,000 portfolio solutions were obtained. Then, the comparison based on the Pareto dominance relation explained in Section 3.3.3 was performed to these 1,000 portfolio solutions in order to identify the non-dominated portfolio solutions or efficient portfolios. Finally, 660 MVS efficient portfolios were attained from SPEA-II.

In Figure 5.1, the MVS efficient portfolios obtained are displayed in four different views. The MVS efficient portfolios solved from using 29 DJIA listed securities (given in Table 5.1) are plotted in the MVS space and presented in Figure 5.1(a). Panels (b), (c), and (d) of Figure 5.1 illustrate the resulting MVS efficient portfolios on mean-SD diagram (front view), mean-skewness diagram (side view), and skewness-SD diagram (top view), respectively. It can be seen that the shape of the MVS efficient surface is discontinuous. This result may be caused by the fact that the feasible portfolios located in this discontinuous area either do not satisfy the optimization constraints or they are dominated by other efficient portfolios.

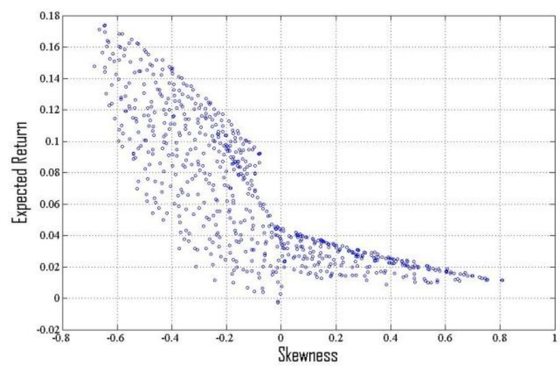
Figure 5.1: The MVS Efficient Solutions from a Universe of DJIA Listed Securities



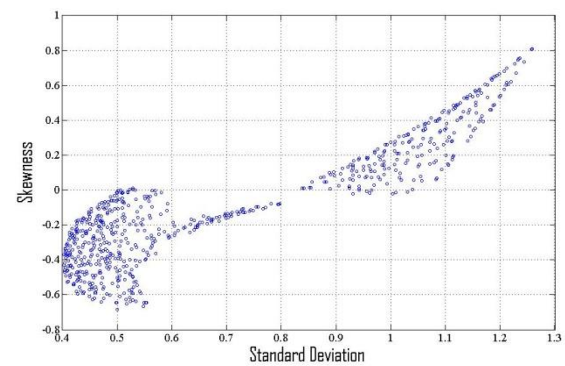
Panel (a)



Panel (b)



Panel (c)



Panel (d)

5.4 THE IMPACT OF SKEWNESS PREFERENCE ON PORTFOLIO CHOICE

5.4.1 Sorting of Efficient Portfolio Choices

To investigate the pricing implications of the MVS efficient portfolios, we sorted the MVS efficient portfolios obtained from Section 5.3 into quintiles according to the expected return. Since a total of 660 MVS efficient portfolios were obtained from SPEA-II, each quintile comprises of 132 efficient portfolios. Then, we computed the average value of expected return, SD, and skewness of the portfolios in each quintile. Table 5.3 exhibits the averages of these three statistics for each quintile.

Table 5.3: The Average Expected Return, SD, and Skewness of MVS Efficient Portfolios Sorted into Quintiles by the Level of Expected Return

Quintile	Average Expected Return	Average Standard Deviation	Average Skewness
Q5 (High-Expected Return)	13.50%	0.515	-0.456
Q4	9.68%	0.540	-0.293
Q3	5.91%	0.551	-0.196
Q2	3.10%	0.873	0.099
Q1 (Low-Expected Return)	1.64%	0.941	0.288

In Table 5.3, Quintile 1 contains the portfolios with the bottom 20% lowest expected returns portfolios, while Quintile 5 includes the portfolios with the top 20% highest expected returns. Table 5.3 clearly indicates that the average SD and skewness of Quintile 5 are much lower than the corresponding statistics for Quintile 1. The differences between the average SD and skewness of Quintile 1 and Quintile 5 are 0.426 and 0.744, respectively. The implication of the result is that investors in the MVS framework need to trade the average expected return of 11.86 percent in order to increase the average skewness of portfolios from -0.456 to 0.288. This trade-off is consistent with our finding discussed in Chapter 4. Thus, investors can reduce their portfolio risk at the cost of having to accept low expected returns. The MV model has the notion that a portfolio with high expected return is inherited with high level of SD. Interestingly, our result in Table 5.3 reveals that quintile with the lowest average expected return has a larger average SD than that of the quintile with the highest average expected return. This result can be explained by the fact that the SD is an appropriate risk measure for the MV model when the return distribution is symmetric. However, if the skewness is significantly non-zero as is the case of our empirical results, investors may consider this statistic as a better measure of risk. As a result, the risk-return trade-off will only be suitably characterized by considering also the skewness of portfolios. Table 5.3 shows that investors have to increase their average portfolio SD by about 0.426 for achieving an average positive skewness of 0.288 from the original of -0.456.

Next, the MVS efficient portfolios were sorted into quintiles according to the value of skewness. Then, we computed the average value of expected return, SD, and skewness of portfolios in each quintile, as well as the average size of portfolio holding, i.e. the average number of securities included in a portfolio, of the portfolios in each quintile. Table 5.4 reports the average of these four statistics for each quintile. Quintile 1 includes portfolios with the bottom 20% lowest skewness, while Quintile 5 has portfolios with the top 20% highest skewness. The result demonstrates that the average expected return of Quintile 1 is higher than that of Quintile 5 by about 10.04 percent. But the average SD of Quintile 1 is less than the corresponding statistic of Quintile 5 by 0.634. For the average portfolio size, a decreasing number of securities held in a portfolio can be observed from Quintile 1 to Quintile 5.

The first implication of the results is that high-skewness portfolios generally have lower average expected returns than low-skewness portfolios. It affirms that the trade-off between expected return and skewness has a negative relation. Secondly, high-skewness portfolios have larger average dispersions of returns than the low-skewness ones. This larger dispersion of the return distributions represents a longer right tail which implies the possibilities of gaining an extreme expected return. As a result, investors are willing to accept lower average expected returns for their preference of the right skewed return distributions. Finally, we found that an increase in average portfolio size reduces not only portfolio SD but also portfolio skewness.

Table 5.4: The Average Expected Return, SD, Skewness, and Portfolio Holding of MVS Efficient Portfolios Sorted into Quintiles by the Value of Skewness

Quintile	Average Skewness	Average Expected Return	Average Standard Deviation	Average Portfolio Holding
Q5 (High-Skewness)	0.402	2.10%	1.104	3.2
Q4	0.037	3.31%	0.812	4.6
Q3	-0.162	7.18%	0.561	5.7
Q2	-0.318	9.10%	0.473	6.1
Q1 (Low-Skewness)	-0.518	12.14%	0.470	5.2

To investigate further, we sorted the MVS efficient portfolios into quintiles based on the number of securities included in a portfolio. Then, we computed the average expected return, SD, skewness, and portfolio size of the portfolios in each quintile. The averages of these statistics are reported in Table 5.5. The efficient portfolios in Quintile 1 are of the smallest portfolio holding, while those in Quintile 5 have portfolios with the largest number of securities. The average skewness of the portfolios in Quintile 1 is 0.269 while that in Quintile 5 is -0.270. This implies that investors with a greater skewness preference are willing to hold relatively undiversified portfolios, i.e. two to three securities. In contrast, investors with greater risk aversion tend to include a larger number of securities into their portfolios in order to reduce risk. This result is consistent with the third implication of our model which expects that investors with a greater skewness preference hold less number of assets in their portfolios in order to increase their exposure to positive skewness of the return distributions.

Table 5.5: The Average Expected Return, SD, Skewness, and Portfolio of MVS Efficient Portfolios Sorted into Quintiles by Portfolio Holding

Quintile	Average Portfolio Holding	Average Expected Return	Average Standard Deviation	Average Skewness
Q5 (High-Portfolio Holding)	8.0	5.93%	0.460	-0.270
Q4	5.8	9.07%	0.510	-0.315
Q3	4.7	8.44%	0.621	-0.214
Q2	3.7	6.48%	0.808	-0.027
Q1 (Low-Portfolio Holding)	2.6	3.91%	1.021	0.269

5.4.2 Portfolio Choices for the Different Degree of Skewness Preference

To illustrate the impact of heterogeneous degree of absolute risk aversion (A) and skewness preference (P) on portfolio choices, we computed and then identified, among the MVS efficient portfolios, the efficient portfolio that maximizes the expected utility for given values of parameter A and P in the following equation. Let investor utility be a function of portfolio return, i.e. $U(R_p)$, from Equation (3.17), the maximization of utility is given as follows:

$$\begin{aligned} \text{Max E}\left[U(R_p)\right] &= \text{Max } U(CE) \\ &= \text{Max } \left[E\left(R_p(\mathbf{x})\right) - \frac{1}{2!}A.V_p(\mathbf{x}) + \frac{1}{3!}P.S_p(\mathbf{x})\right] \end{aligned} \tag{5.1}$$

It can be seen from Equation (5.1) that each efficient portfolio **x** is defined by three statistics including expected return, variance, and skewness. The contribution of each efficient portfolio **x** to expected utility does not only depend on these statistics, but also the parameters *A* and *P*. Equation (5.1) implies that the expected utility increases with an increasing value of expected return and skewness, but decreases with an increasing value of variance. As explained in Section 3.5.1, larger parameter values of *A* and *P* are associated with greater risk aversion and greater skewness preference. A larger parameter value of *A* will bring about a greater penalty to the expected utility. Meanwhile, a larger parameter value of *P* will increase the expected utility.

From the 660 efficient portfolios obtained from SPEA-II as explained in Section 5.3, we identified the efficient portfolio choices that maximize the expected utility expressed in Equation (5.1) for selected values of parameter *A* and *P*. The resulting portfolio choices are reported in Table 5.6. The parameters *A* and *P* were varied from the values of two to five and zero to five, respectively. Larger parameter values of *A* and *P* are associated with greater risk aversion and greater skewness preference, respectively. It can be observed from the results that, at a given degree of skewness preference, SD of portfolio choices tend to reduce with an increasing degree of risk aversion. For example, in the column three in which parameter *A* is fixed at one, SD of efficient portfolio choices decreases from 0.453 to 0.418 when the degree of risk aversion increases from two to five. This result is consistent with the Arrow-Pratt's notion of absolute risk aversion. Besides, at a given degree of risk aversion, we found that the skewness of portfolio choices increases with an increasing degree of skewness preference. For example, with the degree of risk aversion is fixed at four, the skewness of the efficient portfolio choices increases from -0.460 to 0.807 when degree of skewness preference increases from zero to four. It can be validated from this result that the parameter P is a reasonable measure of the degree of skewness preference as argued by Modica and Scarsini (2005) and Crainich and Eeckhoudt (2008).

In addition, Table 5.6 demonstrates that the average expected return of efficient portfolio choices tends to drop when the degree of skewness preference increases. For instance, with the degree of risk aversion fixed at four, the expected return of efficient portfolio choices decline from 6.10 percent to 0.26 percent as the degree of skewness preference increases from zero to three. Although the expected return increases to 1.17 percent when parameter value of P is equal to four and five, the general trend of decreasing expected return with increasing parameter values of P is obvious. This result asserts the validity of the first implication of our model. The general claim of this result is that investors with greater skewness preference are willing to accept lower average expected returns to enjoy the benefit of portfolios with larger skewness.

Furthermore, the result reveals that the dispersion of the return distributions of efficient portfolio choices is larger when the degree of skewness preference increases. For instance, consider the results with the degree of risk aversion fixed at four. The SD of efficient portfolio choices increases from 0.404 up to 1.258 when the degree of skewness preference rises from zero to five. This result is consistent with the model's implication that investors with greater degree of skewness preference are willing to expose themselves to larger return dispersions that will stretch the right tail of the return distributions to increase the possibilities of gaining.

Table 5.6: The Summary Statistics of Portfolio Choices that Maximize the Expected Utility for Given Values of Absolute Risk Aversion (A) and Degree of Skewness Preference (P)

Parameter A,P	2,0	2,1	2,2	2,3	2,4	2,5
Expected Return	7.44%	2.37%	1.17%	1.17%	1.17%	1.17%
Standard deviation	0.409	0.453	1.258	1.258	1.258	1.258
Skewness	-0.460	-0.107	0.807	0.807	0.807	0.807
Parameter A,P	3,0	3,1	3,2	3,3	3,4	3,5
Expected Return	6.10%	3.35%	0.26%	1.17%	1.17%	1.17%
Standard deviation	0.404	0.426	0.501	1.258	1.258	1.258
Skewness	-0.460	-0.189	-0.003	0.807	0.807	0.807
Parameter A,P	4,0	4,1	4,2	4,3	4,4	4,5
Expected Return	6.10%	3.35%	2.37%	0.26%	1.17%	1.17%
Standard deviation	0.404	0.426	0.453	0.501	1.258	1.258
Skewness	-0.460	-0.189	-0.107	-0.003	0.807	0.807
Parameter A,P	5,0	5,1	5,2	5,3	5,4	5,5
Expected Return	6.10%	2.67%	2.37%	0.26%	0.26%	1.17%
Standard deviation	0.404	0.418	0.453	0.501	0.501	1.258
Skewness	-0.460	-0.221	-0.107	-0.003	-0.003	0.807

To examine the model implication on portfolio holding, Table 5.7 exhibits the composition of the efficient portfolio choices reported in Table 5.6. It is revealed from the result that, for all values of degree of risk aversion, the investment allocation tends to concentrate on a single stock (AXP) when the degree of skewness preference increases. For investors with a lower degree of risk aversion, i.e. A = 2, a single stock is the optimal portfolio choice if their degree of skewness preference reaches the value of two. In contrast, the degree of skewness preference at the value of five will induce investors with a higher degree of risk aversion, i.e. A = 5, to hold a single stock as the optimal choice. This result verifies our model implication that investors with a greater preference for skewness hold less number of assets in their portfolio i.e. underdiversified portfolio, in order to increase their exposure to positive skewness of the return distributions.

Table 5.7: The Composition of Efficient Portfolio Choices that Maximized the Expected Utility at a Given Value of Absolute Risk Aversion (A) and Degree of Skewness Preference (P)

U:AXP	-	-	1.00	1.00	1.00	1.00	-	-	-	1.00	1.00	1.00
U:BA	-	-	-	-	-	-	-	-	-	-	-	-
U:CAT	-	-	-	-	-	-	-	-	-	-	-	-
U:CVX	-	-	-	-	-	-	-	-	-	-	-	-
@CSCO	-	-	-	-	-	-	-	-	-	-	-	-
U:DD	-	-	-	-	-	-	-	-	-	-	-	-
U:DIS	-	-	-	-	-	-	-	-	-	-	-	-
U:GE	-	-	-	-	-	-	-	-	-	-	-	-
U:GS	-	-	-	-	-	-	-	-	-	-	-	-
U:HD	-	-	-	-	-	-	-	-	-	-	-	-
U:IBM	-	-	-	-	-	-	-	-	-	-	-	-
@INTC	-	-	-	-	-	-	-	-	-	-	-	-
U:JNJ	0.38	0.28	-	-	-	-	0.31	0.42	0.01	-	-	-
U:JPM	-	-	-	-	-	-	-	-	0.01	-	-	-
U:KO	-	-	-	-	-	-	-	-	-	-	-	-
U:MCD	0.31	-	-	-	-	-	0.25	0.06	-	-	-	-
U:MMM	-	-	-	-	-	-	-	0.03	-	-	-	-
U:MRK	-	-	-	-	-	-	-	-	-	-	-	-
@MSFT	-	-	-	-	-	-	-	-	-	-	-	-
U:NKE	-	-	-	-	-	-	-	-	-	-	-	-
U:PFE	-	0.09	-	-	-	-	-	0.07	0.25	-	-	-
U:PG	0.10	-	-	-	-	-	0.11	-	-	-	-	-
U:T	-	-	-	-	-	-	-	-	-	-	-	-
U:TRV	-	0.18	-	-	-	-	-	0.15	0.26	-	-	-
U:UNH	-	0.13	-	-	-	-	-	0.08	0.01	-	-	-
U:UTX	-	-	-	-	-	-	-	-	-	-	-	-
U:VZ	-	-	-	-	-	-	-	-	-	-	-	-
U:WMT	0.16	0.30	-	-	-	-	0.18	0.16	0.46	-	-	-
U:XOM	-	0.01	-	-	-	-	-	0.01	-	-	-	-
Parameter A,P	2,0	2,1	2,2	2,3	2,4	2,5	3,0	3,1	3,2	3,3	3,4	3,5
Expected Return	7.44%	2.37%	1.17%	1.17%	1.17%	1.17%	6.10%	3.35%	0.26%	1.17%	1.17%	1.17%
Standard deviation	0.409	0.453	1.258	1.258	1.258	1.258	0.404	0.426	0.501	1.258	1.258	1.258
Skewness	-0.460	-0.107	0.807	0.807	0.807	0.807	-0.460	-0.189	-0.003	0.807	0.807	0.807

Table 5.7 (continued): The Composition of Efficient Portfolio Choices that Maximized the Expected Utility at a Given Value of Absolute Risk Aversion (*A*) and Degree of Skewness Preference (*P*)

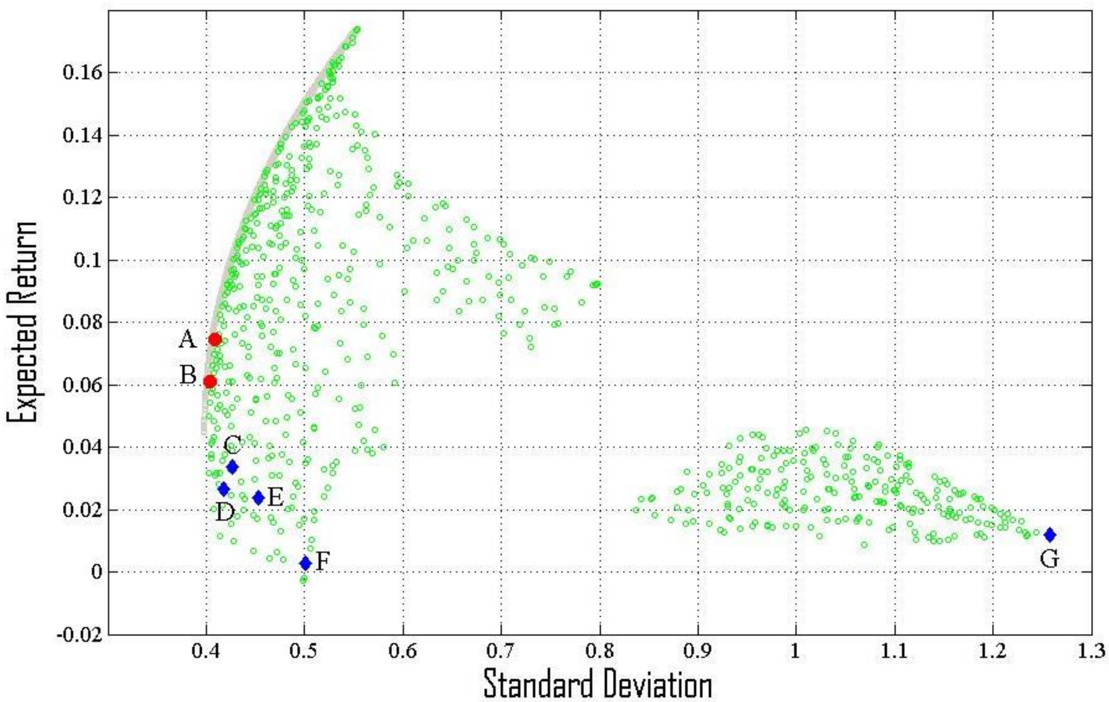
U:AXP	-	-	-	-	1.00	1.00	-	-	-	-	-	1.00
U:BA	-	-	-	-	-	-	-	-	-	-	-	-
U:CAT	-	-	-	-	-	-	-	-	-	-	-	-
U:CVX	-	-	-	-	-	-	-	-	-	-	-	-
@CSCO	-	-	-	-	-	-	-	-	-	-	-	-
U:DD	-	-	-	-	-	-	-	-	-	-	-	-
U:DIS	-	-	-	-	-	-	-	-	-	-	-	-
U:GE	-	-	-	-	-	-	-	-	-	-	-	-
U:GS	-	-	-	-	-	-	-	-	-	-	-	-
U:HD	-	-	-	-	-	-	-	0.01	-	-	-	-
U:IBM	-	-	-	-	-	-	-	0.01	-	-	-	-
@INTC	-	-	-	-	-	-	-	-	-	-	-	-
U:JNJ	0.31	0.42	0.28	0.01	-	-	0.31	0.31	0.28	0.01	0.01	-
U:JPM	-	-	-	0.01	-	-	-	-	-	0.01	0.01	-
U:KO	-	-	-	-	-	-	-	-	-	-	-	-
U:MCD	0.25	0.06	-	-	-	-	0.25	-	-	-	-	-
U:MMM	0.08	0.03	-	-	-	-	0.08	0.03	-	-	-	-
U:MRK	-	-	-	-	-	-	-	-	-	-	-	-
@MSFT	-	-	-	-	-	-	-	-	-	-	-	-
U:NKE	-	-	-	-	-	-	-	-	-	-	-	-
U:PFE	-	0.07	0.09	0.25	-	-	-	0.07	0.09	0.25	0.25	-
U:PG	0.11	-	-	-	-	-	0.11	0.15	-	-	-	-
U:T	0.05	-	-	-	-	-	0.05	-	-	-	-	-
U:TRV	-	0.15	0.18	0.26	-	-	-	0.08	0.18	0.26	0.26	-
U:UNH	-	0.08	0.13	0.01	-	-	-	0.09	0.13	0.01	0.01	-
U:UTX	-	-	-	-	-	-	-	-	-	-	-	-
U:VZ	-	-	-	-	-	-	-	-	-	-	-	-
U:WMT	0.18	0.16	0.30	0.46	-	-	0.18	0.22	0.30	0.46	0.46	-
U:XOM	0.01	0.01	0.01	-	-	-	0.01	0.03	0.01	-	-	-
Parameter A,P	4,0	4,1	4,2	4,3	4,4	4,5	5,0	5,1	5,2	5,3	5,4	5,5
Expected Return	6.10%	3.35%	2.37%	0.26%	1.17%	1.17%	6.10%	2.67%	2.37%	0.26%	0.26%	1.17%
Standard deviation	0.404	0.426	0.453	0.501	1.258	1.258	0.404	0.418	0.453	0.501	0.501	1.258
Skewness	-0.460	-0.189	-0.107	-0.003	0.807	0.807	-0.460	-0.221	-0.107	-0.003	-0.003	0.807

The analysis includes a zero degree of skewness preference for the case in which investors make portfolio decision without any preference for skewness of return distribution. These investors basically choose their portfolio choices based solely on the mean and the variance of the return distributions. Thus their portfolio choices are those having the lowest SD at a given value of expected return, i.e. the MV efficient portfolios. However, this behavior is theoretically unacceptable since we showed that, *ceteris paribus*, investors prefer return distributions with positive skewness. In contrast, risk-averse investors who exhibit preference on skewness of return distributions are willing to sacrifice MV efficiency in order to increase portfolio skewness.

In Figure 5.2, the optimal portfolio choices exhibited in Table 5.7 are displayed together with all MVS efficient portfolios (green dots) and MV efficient portfolios (gray dots). The MV efficient portfolios were solved by optimizing **Prob. 1** with the skewness objective omitted. Portfolios represented by the red dots are the MVS efficient portfolio choices with a zero degree of skewness preference. Those represented by the blue diamonds are the MVS efficient portfolio choices for an investor who exhibits a preference for skewness. Figure 5.2 illustrates that the optimal portfolio choices for a risk-averse investor who exhibits a preference for skewness are not MV efficient. In contrast, the portfolio choices of a risk-averse investor with a zero degree of skewness preference reside on the MV efficient frontier.

We observe from Figure 5.2 and Table 5.7 that the reason to sacrifice MV efficiency for investors whose utility is a function of the first three moments of return distributions is that they evaluate risk based not only on the dispersion but also the skewness. It can be seen that the efficient portfolio choices for zero skewness preference, which are MV efficient portfolios, have the largest negative skewness compared to those with non-zero skewness preference. Therefore, increasing preference for skewness comes at a cost of higher SD, where the larger distribution arise from a flatter right tail of the return distributions in order to either avoid large losses or obtain large gains.

Figure 5.2: Portfolio Choices for Different Degree of Risk-aversion and Skewness Preference



Note: The degree of risk aversion and skewness preference (*A*,*P*) of portfolio choices are as follows:

A (2,0)

B (3,0); (4,0); (5,0)

C (5,1)

D (3,1); (4,1)

E (2,1); (4,2); (5,2)

F (3,2); (4,3); (5,3); (5,4)

G (2,2); (2,3); (2,4); (2,5); (3,3); (3,4); (3,5); (4,4); (4,5); (5,5)

This finding sheds light on the extensive application of our proposed model for explaining behavior of investors. First of all, the proposed model has the property of non-increasing absolute risk aversion for all wealth levels, while the quadratic utility model, in the MV analysis, satisfies this property only in some range of random variable. Secondly, although skewness preference is regarded as a common trait for rational investors, some investors may make a portfolio decision independently on the skewness objective. Thus, although the analysis of efficient portfolio choices is conducted based on the MVS model, the proposed model can also provide the portfolio choices that are MV efficient as different degrees of skewness preference, including zero degree, can be accommodated. Finally, our model does not require the assumption of normal distribution on the expected returns, rendering it suitable for real-world applications.

5.5 CONCLUDING REMARKS

The main objective of this chapter is to investigate the impact of different degree of skewness preference on efficient portfolio choices. We developed a model in which an investor has utility that can be approximated by the third-order Taylor's series expansion. In our model, investors have homogeneous preference function for mean, variance, and skewness of the return distributions, but they possess heterogeneous degree of risk aversion and skewness preference. As a consequence, the efficient portfolios are solved using the MVS-POP formulated as **Prob. 1**. Then, the choice of investment portfolio is made among the MVS efficient portfolios, depending on the preferred degree of risk aversion and skewness preference.

To test the implications of the proposed model, we computed and then identified, among the MVS efficient portfolios, the portfolio that maximizes the expected utility for given values of degree of absolute risk aversion (*A*) and skewness preference (*P*). We validated the use of parameters *A* and *P* by showing that, at any given degree of skewness preference, SD of portfolio choices tend to reduce with an increasing degree of risk aversion which is consistent with the Arrow-Pratt's notion of absolute risk aversion. We demonstrated that skewness of portfolio choices increases with an increasing degree of skewness preference at a given degree of risk aversion.

The first implication of the model is verified by the result that the average expected return of portfolio choices tends to decrease when the degree of skewness preference increases. This implies that the investors with greater skewness preference accept lower average expected returns to enjoy the benefits of higher skewness. The benefits can be thought of as avoiding large losses from return distributions with negative skewness or gaining larger returns from return distributions with positive skewness. Next, the result reveals that the dispersion of return distributions of portfolio choices is larger when investors have higher degrees of skewness preference. This result supports the second implication of the model which implies that investors with greater degrees of skewness preference are willing to expose themselves to larger return dispersion due to a flatter right tail of the return distributions.

For the third implication of the model, the result reveals that investment allocations tend to concentrate on very few securities when the degree of skewness preference increases at a given level of degree of risk aversion. This result suggests that investors with greater skewness preference tend to hold a small number of assets in their portfolio, i.e.

underdiversified portfolio, in order to increase their exposure to return distributions with positive skewness.

CHAPTER 6

MVS ANALYSIS FOR SOLVING AN ELECTRICITY ALLOCATION PROBLEM

6.1 INTRODUCTION

This chapter demonstrates the application of MVS analysis for solving an asset allocation problem where the number of trading choices is limited. The application chosen is an electricity allocation problem of a generation company (Genco). The problem is formulated as **Prob. 1** and solved using the NSGA-II and the SPEA-II. Then the usefulness of the additional objective which is proposed to enhance the benefit of diversification of the solution is examined. This newly proposed problem, i.e. MVS-D-POP, formulated as **Prob. 2** in Section 3.5.1 is optimized by three algorithms, namely, NSGA-II, SPEA-II, and COGA-II. The COGA-II is selected for the implementation since it is specifically developed for solving an optimization problem with more than three objectives. Finally, the results obtained from different algorithms are compared using the methods explained in Section 3.5.3. In Section 6.2, the summary statistics and the normality test results of historical electricity spot prices for the selected pricing zones in the PJM market are reported. Besides, it also provides the statistics on the returns of nine trading choices for two case studies. This section also gives the results of the estimated input variables used for solving MVS efficient portfolios. In Section 6.3, the performance of the three different algorithms is compared. The MVS and MVS-D efficient portfolios obtained from the COGA-II are plotted in the three-dimension space for further analysis. The conclusion of the chapter is made in Section 6.4.

6.2 SUMMARY STATISTICS AND THE NORMALITY TEST RESULTS

As described in Section 3.6.2, the electricity spot prices of nine pricing zones, namely, AEGO, BGE, DPL, METED, PECO, PENELEC, PEPCO, PPL, and PSEG were quoted from the PJM market. In our case studies, the electricity allocation problem for the Genco of interest is solved for the month of August 2006. Thus the daily electricity spot prices for the period of August 1998 to July 2006 were collected for the analysis.

Table 6.1 reports the summary statistics together with the normality test results of the daily electricity spot prices of nine pricing zones from the period of August 1998 to July 2006. Table 6.1 shows that the average spot prices of all pricing zones are slightly different. The highest mean price is observed in the PSEG zone, while the PENELEC zone offers the lowest mean price during the period of study. Besides, it indicates that the SD of the historical spot prices is considerably high where the coefficient of variation is more than 50%. This result suggests that the electricity spot prices are highly volatile, as argued by previous studies. In addition, it is revealed that the electricity spot prices of all the pricing zones are not normally distributed but positively skewed. The null hypothesis of a normal distribution is rejected for all pricing zones according to the *p*-values that are smaller than the 0.001 level. This evidence supports our argument that skewness cannot be neglected in the electricity allocation problem.

Table 6.1: Summary Statistics and the Normality Test Results of the Historical Daily Electricity Spot Prices of Nine Pricing Zones

Pricing zone	AECO	BGE	DPL	METED	PECO	PENELEC	PEPCO	PPL	PSEG
Mean	40.76	40.00	40.57	38.86	39.45	36.90	40.33	38.17	41.30
Standard deviation	27.87	28.21	27.51	26.38	26.89	22.81	28.65	25.77	27.65
Coefficient of variation	0.68	0.71	0.68	0.68	0.68	0.62	0.71	0.68	0.67
Skewness	3.97	3.53	4.42	4.16	4.32	4.58	3.26	4.37	3.82
JB- statistic	157,128	88,418	230,567	183,256	210,118	272,674	60,893	222,644	134,649
P-value	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

According to the procedure described in Section 3.6.3, a computation of the statistics on returns, namely, expected return, variance, covariance, skewness, and coskewness for all trading choices is a prerequisite process before solving **Prob. 1** and **Prob. 2**. As explained in the methodology chapter (Section 3.6.3), the expected electricity spot prices during the decision making period are firstly forecasted before the expected return of a trading choice in electricity market can be computed. Then the forecasts are used to calculate the expected return of the considered trading choices according to Equation (3.22) and Equation (3.25). This study adopted the GARCH with seasonality dummies expressed in Equation (3.33) to forecast the daily electricity spot prices. Meanwhile, the other statistics on returns are computed based on the historical electricity spot prices for the period of August 1998 to July 2006.

Based on the parameter setting described in Section 3.6.3, it is assumed that the Genco of interest has a 350-MW fossil generator and therefore its cost function is $(647.087 + 14.866P_G + 0.0065{P_G}^2)\lambda_t^c$, where P_G is the output power (MWh) and λ_t^c is the fuel price

(\$/MBtu). Suppose that the cost of coal is constant at 1.29 \$/MBtu during the decision making period. The expected returns of all trading choices for two case studies are reported in Table 6.2. Table 6.3 exhibits the variance-covariance matrix, i.e. matrix **Λ**, which is computed from Equation (3.28) and Equation (3.29). Table 6.4 illustrates the skewness-coskewness matrix, i.e. matrix **Ω**, which is calculated from Equation (3.30) to Equation (3.32).

Table 6.2: The Return Matrix **R** of the Nine Trading Choices for the Two Case Studies

	PEPCO	AECO	BGE	DPL	METED	PECO	PENELEC	PPL	PSEG
R - Case 1	0.684	0.002	0.286	0.339	0.156	0.086	0.333	0.407	0.018
R - Case 2	0.684	-0.013	0.299	0.331	0.216	0.124	0.475	0.497	-0.020

Table 6.3: The Variance-covariance Matrix **Λ** of the Nine Trading Choices

Λ	PEPCO	AECO	BGE	DPL	METED	PECO	PENELEC	PPL	PSEG
PEPCO	**1.384**	0.133	0.040	0.173	0.161	0.175	0.391	0.200	0.157
AECO	0.133	**0.178**	0.037	0.159	0.103	0.149	0.112	0.105	0.150
BGE	0.040	0.037	**0.037**	0.043	0.036	0.040	0.037	0.039	0.037
DPL	0.173	0.159	0.043	**0.224**	0.115	0.163	0.128	0.119	0.155
METED	0.161	0.103	0.036	0.115	**0.110**	0.109	0.119	0.100	0.104
PECO	0.175	0.149	0.040	0.163	0.109	**0.171**	0.130	0.116	0.149
PENELEC	0.391	0.112	0.037	0.128	0.119	0.130	**0.267**	0.137	0.136
PPL	0.200	0.105	0.039	0.119	0.100	0.116	0.137	**0.124**	0.109
PSEG	0.157	0.150	0.037	0.155	0.104	0.149	0.136	0.109	**0.206**

Table 6.4: The Skewness-coskewness Matrix **Ω** of the Nine Trading Choices

Ω	PEPCO (i=1)	AECO (i=2)	BGE (i=3)	DPL (i=4)	METED (i=5)	PECO (i=6)	PENELEC (i=7)	PPL (i=8)	PSEG (i=9)
$\gamma_{1,1,i}$	**5.030**	0.232	0.006	0.260	0.254	0.266	0.975	0.334	0.325
$\gamma_{2,2,i}$	9.677	**-0.374**	-0.045	-0.406	-0.123	-0.339	0.084	-0.085	-0.305
$\gamma_{3,3,i}$	9.877	-0.022	**-0.013**	-0.027	-0.016	-0.021	0.013	-0.015	-0.018
$\gamma_{4,4,i}$	9.695	-0.474	-0.076	**-0.646**	-0.173	-0.444	0.096	-0.133	-0.381
$\gamma_{5,5,i}$	9.583	-0.031	-0.014	-0.039	**0.009**	-0.022	0.116	0.021	-0.010
$\gamma_{6,6,i}$	9.628	-0.316	-0.041	-0.352	-0.097	**-0.297**	0.105	-0.063	-0.256
$\gamma_{7,7,i}$	8.513	0.168	0.040	0.187	0.180	0.188	**0.344**	0.211	0.201
$\gamma_{8,8,i}$	9.478	0.002	-0.010	-0.004	0.032	0.012	0.150	**0.045**	0.024
$\gamma_{9,9,i}$	9.523	-0.252	-0.024	-0.272	-0.055	-0.222	0.123	-0.020	**-0.229**

6.3 RESULTS AND DISCUSSIONS

6.3.1 Comparison of the Implemented MOEAs

First of all, we discuss the result of performance comparison between the three experimented algorithms. Then the efficient solutions of **Prob. 1** and **Prob. 2** generated by the algorithm that performs the best are used for further discussions. The mean and standard deviation of the running time of the three MOEAs are exhibited in Table 6.5. Besides, the average and standard deviation of the performance criteria explained in Section 3.6.5 such as, M_1, *HV*, and *CRA* for the three algorithms are reported in Table 6.6 to Table 6.8, respectively. For the parameter setting shown in Table 3.4, 60,000 efficient solutions were generated from 30 repeated runs. The statistics were computed from the results of these 60,000 solutions.

From the results of running time demonstrated in Table 6.5, NSGA-II is obviously outstanding in terms of computation time. It only spent, on average, 2.5 seconds and 3 seconds for solving **Prob. 1** and **Prob.** 2, respectively. COGA-II ranked second while SPEA-II has an average running time almost double of that for COGA-II. For all the three algorithms, the running time does not vary much between the two different case studies. But COGA-II and SPEA-II required substantially longer running time when the number of optimized objectives increases from three (MVS) to four (MVS-D), whereas the running time of NSGA-II increased only slightly from solving **Prob. 1** to **Prob. 2**. However, the mean running time of all the three MOEAs is remarkably short, at less than 20 seconds on average.

Table 6.5: Mean and Standard Deviation (SD) of Running Time (Seconds)

Optimized Problem	Case Study	COGA-II		NSGA-II		SPEA-II	
		Mean	SD	Mean	SD	Mean	SD
MVS	1	7.9617	0.1272	2.5482	0.0854	14.8048	0.1404
MVS	2	7.9690	0.1140	2.5648	0.0765	14.8444	0.1480
MVS-D	1	10.2635	0.1458	2.9751	0.0072	19.4637	0.1370
MVS-D	2	10.4689	0.0833	3.0071	0.0620	19.7350	0.1690

Table 6.6: Mean and Standard Deviation (SD) of M_1

Optimized Problem	Case Study	COGA-II		NSGA-II		SPEA-II	
		Mean	SD	Mean	SD	Mean	SD
MVS	1	0.008645	0.002505	0.012545	0.003550	0.016276	0.002911
MVS	2	0.014418	0.005326	0.017278	0.003404	0.022765	0.005879
MVS-D	1	0.007901	0.001661	0.022788	0.006305	0.016384	0.002741
MVS-D	2	0.009622	0.001773	0.025145	0.006171	0.020339	0.003728

Before conducting the comparison between the MOEAs based on the M_1, *HV*, and *CRA* criteria, the set of true Pareto front needs to be identified. We conducted the standard method to obtain the artificial true Pareto front for the case where the true Pareto front is unknown. The artificial true Pareto front can be constructed by, firstly, combining all the portfolio solutions obtained from the three MOEAs. Then, among the combined portfolio solutions, we determined the non-dominated portfolio solutions by comparing them based on the Pareto dominance relation explained in Section 3.3.3. The artificial true Pareto front is given by the non-dominated portfolio solutions.

According to the M_1 criterion, the lower the value of M_1, the shorter the distance between a solution and the artificial true Pareto front. Thus, M_1 is a minimum criterion. Table 6.6 shows that, for both **Prob. 1** and **Prob.** 2, the solutions obtained from COGA-II have shorter distance to the artificial true Pareto front compared to those from NSGA-II and SPEA-II. As claimed earlier, COGA-II is developed for handling a MOOP with high number of optimized objectives. Table 6.6 reveals that the performance of COGA-II based on M_1 is even better when the number of optimized objectives increases from three to four.

In Table 6.7, the mean and SD of the *HV* criterion is exhibited. As explained in Section 3.6.5, *HV* is a maximum criterion whose values represent the level of diversity of the solutions. The higher value of diversity indicates that the solutions are well distributed along the artificial true Pareto front, whereas a lower *HV* denotes that the solutions are clustered in some particular areas. Table 6.7 reveals that, for both **Prob. 1** and **Prob. 2**, COGA-II outperforms SPEA-II and NSGA-II in the aspect of *HV* criterion. For both case studies of each optimization problem, the mean values of *HV* for COGA-II are higher than those of NSGA-II and SPEA-II. However, regardless of the MOEAs applied and the case studies, the solutions of **Prob. 1** are better distributed along the artificial true Pareto front than those of **Prob. 2**.

Table 6.7: Mean and Standard Deviation (SD) of *HV*

Optimized Problem	Case Study	COGA-II		NSGA-II		SPEA-II	
		Mean	SD	Mean	SD	Mean	SD
MVS	1	51.02478	0.148316	50.06765	0.245983	50.68226	0.150190
MVS	2	56.43273	0.201165	54.91343	0.334048	56.03167	0.212187
MVS-D	1	21.36057	0.181966	20.62899	0.316303	21.05846	0.175315
MVS-D	2	22.87187	0.190710	21.65327	0.397034	22.48699	0.192791

Table 6.8: Mean and Standard Deviation (SD) of *CRA*

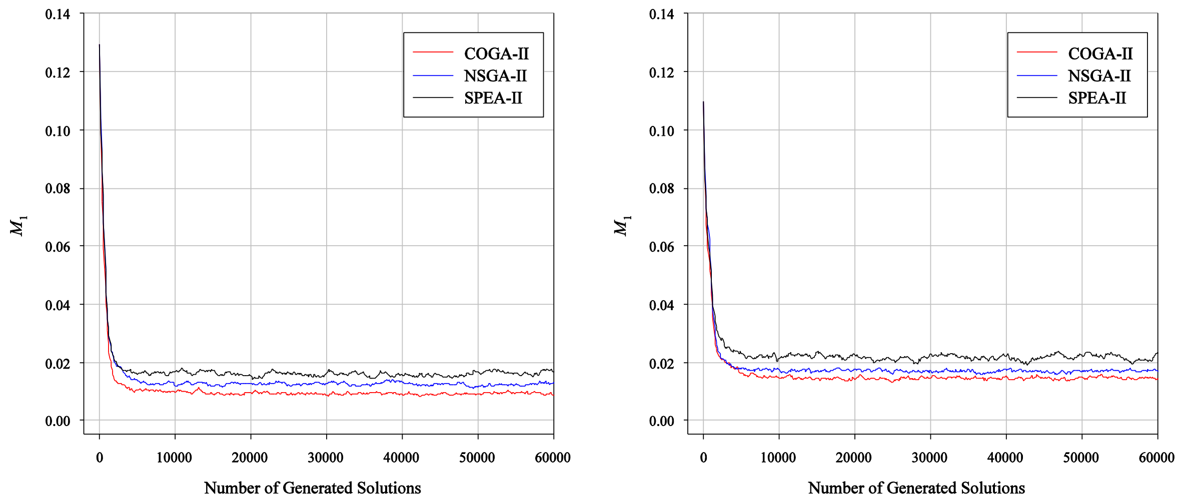
Optimized Problem	Case Study	COGA-II		NSGA-II		SPEA-II	
		Mean	SD	Mean	SD	Mean	SD
MVS	1	0.545333	0.045541	0.377667	0.067091	0.321667	0.055961
MVS	2	0.494000	0.059283	0.358000	0.059504	0.305333	0.042405
MVS-D	1	0.634000	0.046875	0.389667	0.058515	0.459000	0.053585
MVS-D	2	0.635000	0.046665	0.385333	0.052702	0.445333	0.048971

According to the results in Table 6.8, the superiority of COGA-II is more pronounced when the performance is compared based on the *CRA* criterion. As explained in Section 3.6.5, the values of CRA for an algorithm represent the ratio of the number of solutions obtained from the algorithm that are members of the artificial true Pareto front. Thus CRA is a maximum criterion whose maximum value is one. Table 6.8 demonstrates that, COGA-II outperforms both NSGA-II and SPEA-II. We found that about 50 percent of the solutions obtained from COGA-II are members of the artificial true Pareto front of **Prob. 1** for both case studies. This ratio increases to about 63 percent when COGA-II is used for solving **Prob. 2** for both case studies.

In addition, the results of the performance criteria from a pair of algorithms can be compared statistically using the paired *t*-test. We found that COGA-II performs significantly better than NSGA-II and SPEA-II in this analysis (*p*-value less than 0.01 for almost all pairs of MOEAs). Similar results hold for the *CRA* criterion, where COGA-II outperforms NSGA-II and SPEA-II statistically with *p*-value less than 0.001. The results based on *HV* also reveal that COGA-II has better performance than that of NSGA-II and SPEA-II. Furthermore, we found that the superiority of COGA-II performance is strengthened when dealing with the optimization problem with 4 objectives, i.e. **Prob. 2** for the M_1 and *CRA* criteria.

The averages of M_1, *HV*, and *CRA* versus the number of generated solutions from all 30 repeated runs are plotted in Figure 6.1 to Figure 6.6. The main objective of the plots is to investigate the speed of convergence of the performance criterion values for each algorithm. The better algorithm is the one whose performance values, i.e. M_1, *HV*, or *CRA*, converge at a low number of generations. The results are evident in Figure 6.1 to Figure 6.6 which discloses the superiority of COGA-II compared to NSGA-II and SPEA-II. The figures show that the solutions of all the three experimented algorithms rapidly achieved convergence within about 10,000 generated solutions which is equivalent to only 100 generations (number of generated solutions = population size × number of generations). Except for the two cases of MVS-D when the *HV* criterion is applied, the other criteria have a higher tendency to converge faster for COGA-II compared to NSGA-II and SPEA-II.

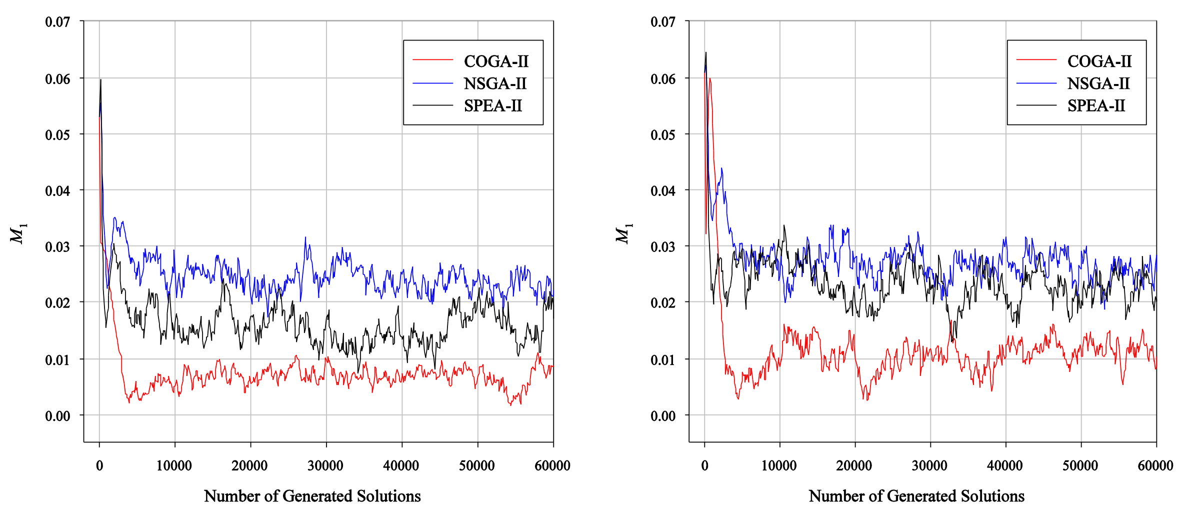
Figure 6.1: Mean M_1 versus Number of Generated Solutions of Case Study 1 and 2 for MVS-POP (3 Optimized Objectives)



(a) MVS – case study 1

(b) MVS – case study 2

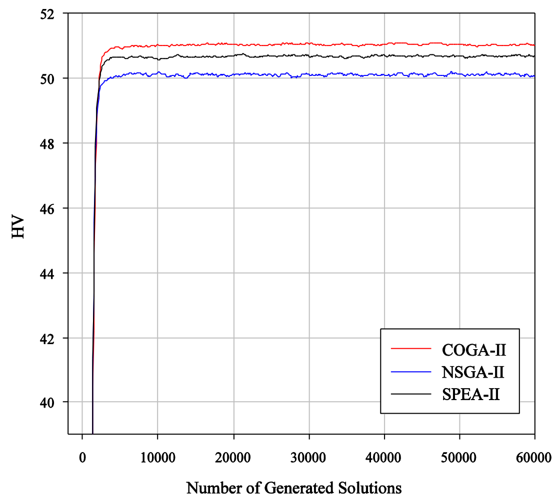
Figure 6.2: Mean M_1 versus Number of Generated Solutions of Case Study 1 and 2 for MVS-D-POP (4 Optimized Objectives)



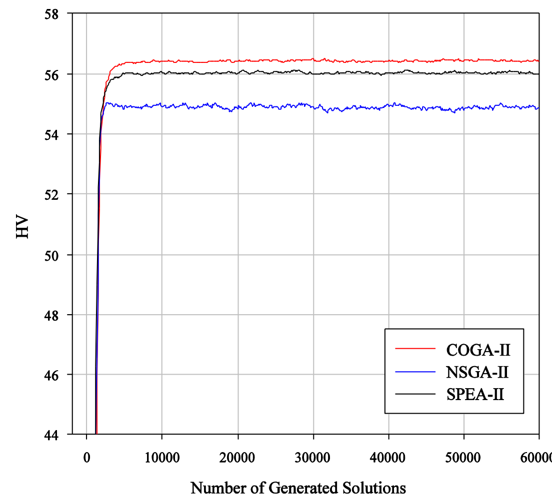
(a) MVS-D – case study 1

(b) MVS-D – case study 2

Figure 6.3: Mean *HV* versus Number of Generated Solutions of Case Study 1 and 2 for MVS-POP (3 Optimized Objectives)

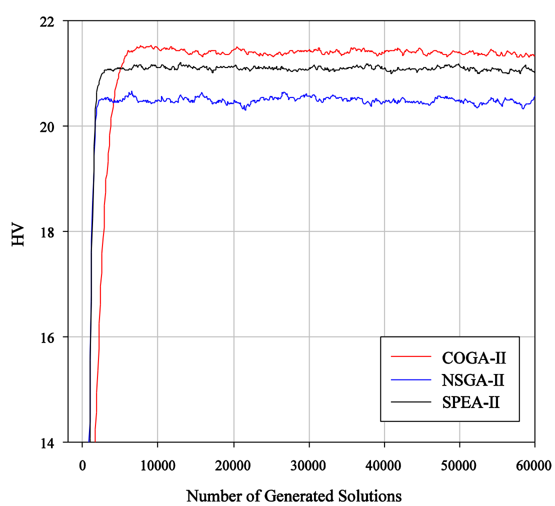


(a) MVS – case study 1

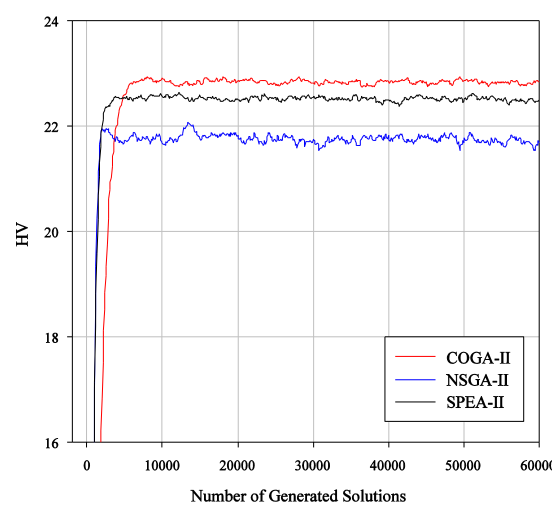


(b) MVS – case study 2

Figure 6.4: Mean *HV* versus Number of Generated Solutions of Case Study 1 and 2 for MVS-D-POP (4 Optimized Objectives)

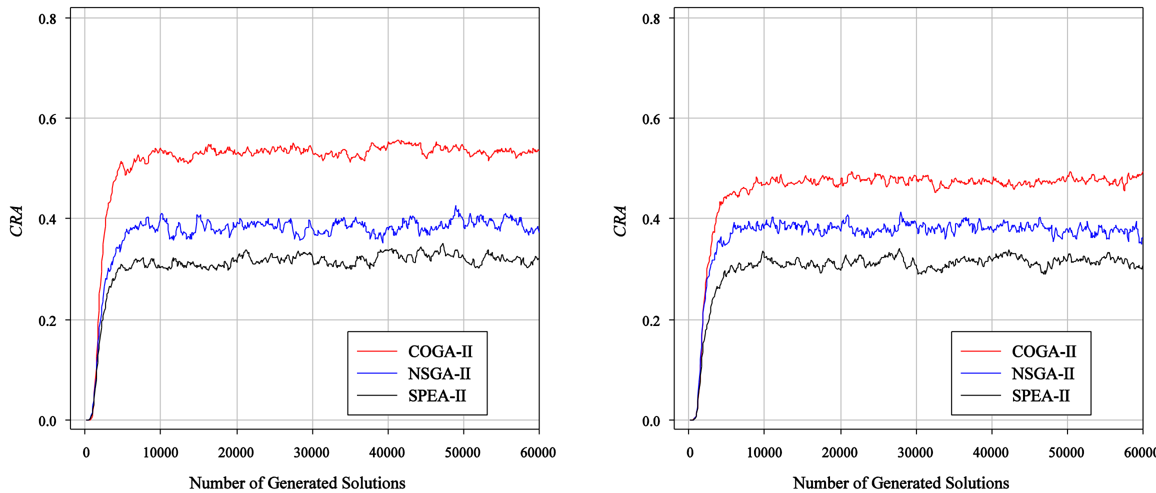


(a) MVS-D – case study 1



(b) MVS-D – case study 2

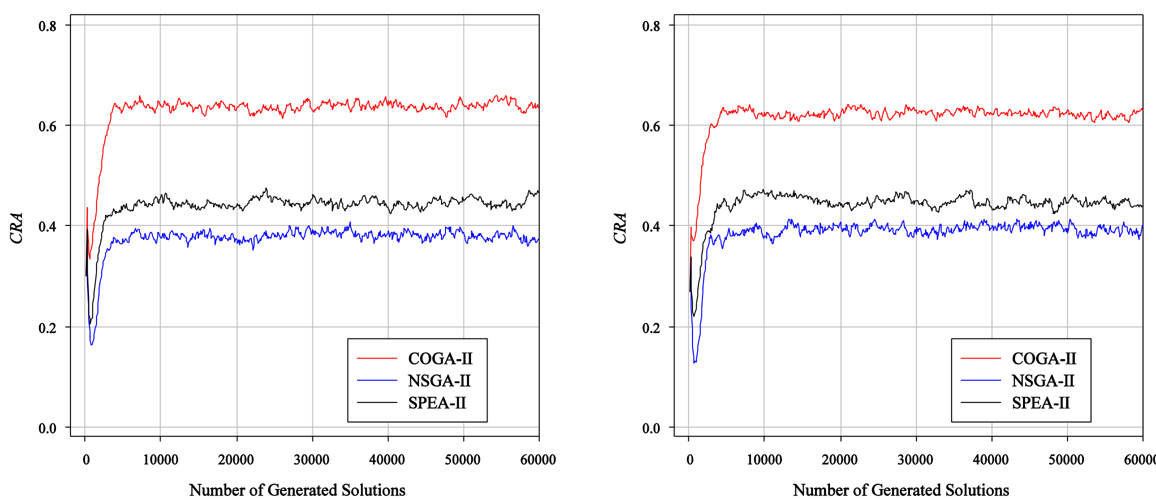
Figure 6.5: Mean *CRA* versus Number of Generated Solutions of Case Study 1 and 2 for MVS-POP (3 Optimized Objectives)



(a) MVS – case study 1

(b) MVS – case study 2

Figure 6.6: Mean *CRA* versus Number of Generated Solutions of Case Study 1 and 2 for MVS-D-POP (4 Optimized Objectives)



(a) MVS-D – case study 1

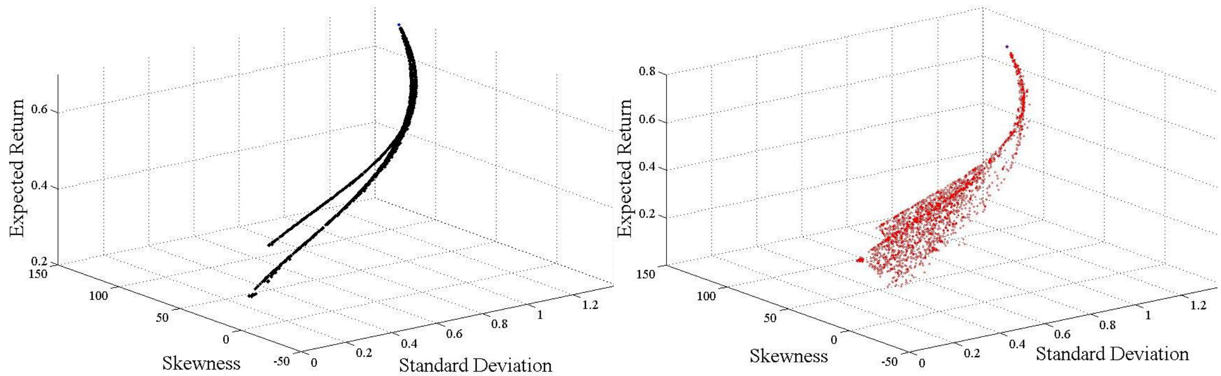
(b) MVS-D – case study 2

6.3.2 Optimal Electricity Allocation

The analysis in this section is based on the results from COGA-II that is found to have the best performance. The efficient portfolios of the MVS and MVS-D problems obtained from COGA-II are presented in Figure 6.7 and Figure 6.8, respectively. The results show that the Pareto fronts are discontinuous in shape that logically represents the nature of problem with completing and conflicting objectives. The fourth objective of minimizing $D(\mathbf{x})$ that we propose to add to the MVS portfolio model is to avoid allocation proportions that are overly focused on very few trading choices, i.e. x_i, in order to increase investment diversification. Figure 6.7 and Figure 6.8 reveal that MVS-D efficient portfolios achieve the diversification benefit since a mass of non-dominated solutions reside in the low-standard deviation space. It implies, from a Genco point of view, that the allocation of electricity to more trading choices helps to reduce trading risk measured by standard deviation. In fact, this additional objective poses more challenge to the use of MOEAs for the optimization problem as the increased objective dimension could compromise the performance of the algorithm. However, the results of the M_1 and *CRA* criteria suggest that the superiority of the COGA-II performance over the other two algorithms is even higher for the MVS-D problem compared to the MVS problem. To support this finding further, the solutions from COGA-II for the optimization of the MVS and MVS-D problems are further compared.

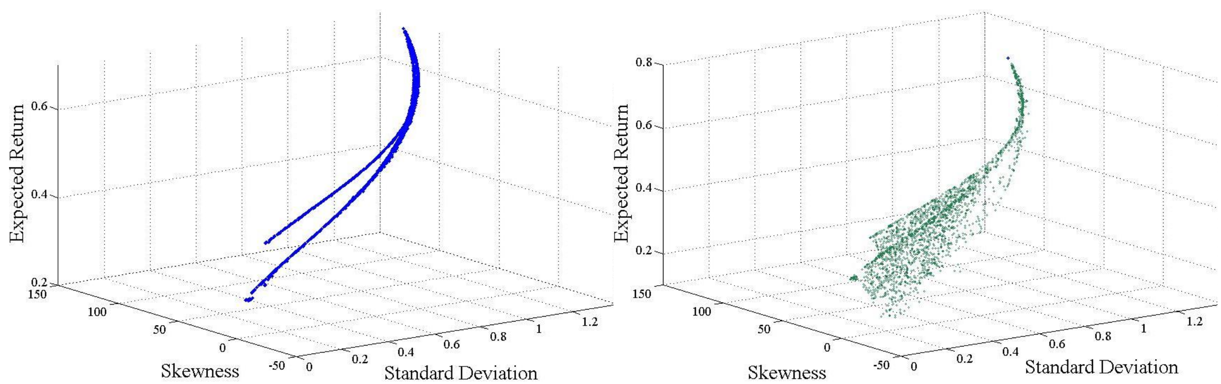
This comparison is performed on the efficient portfolios, the so called “Pareto fronts”, obtained from solving the MVS and the MVS-D portfolio optimization problems. Figure 6.9 illustrates the comparison of MVS and MVS-D Pareto fronts plotted in the MVS

Figure 6.7: Pareto Fronts of Case Study 1 using COGA-II for Optimization Problem of (a) MVS and (b) MVS-D



(a) MVS (b) MVS-D

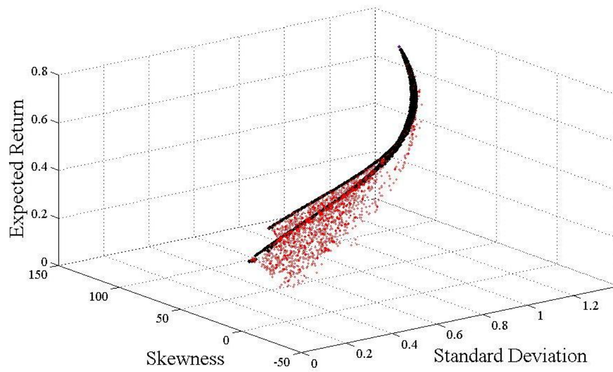
Figure 6.8: Pareto Fronts of Case Study 2 using COGA-II for Optimization Problem of (a) MVS and (b) MVS-D



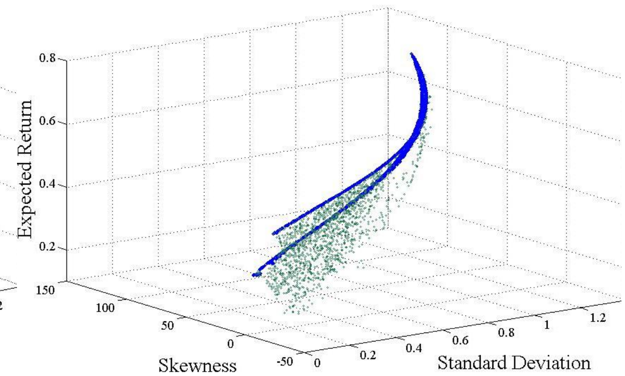
(a) MVS (b) MVS-D

objective space. It is clear that the Pareto front of MVS-D fully envelopes the MVS Pareto front. Given this finding, addition to the MVS efficient portfolios, the solutions for the MVS-D model offers more efficient portfolio choices for the consideration of the Genco. Theoretically, the MVS efficient portfolios should be a subset of the MVS-D efficient portfolios. This suggests that the performance of COGA-II does not deteriorate when the number of objectives is increased because the algorithm can still retain the efficient solutions of the problem with a lesser number of objectives.

Figure 6.9: Comparison of Pareto Fronts of MVS and MVS-D Efficient Portfolios Generated by COGA-II



(a) Pareto fronts of MVS (black) vs MVS-D (red) – Case study 1



(b) Pareto fronts of MVS (blue) vs MVS-D (green) – Case study 2

Table 6.9 gives an example of electricity allocation solutions of the MVS and MVS-D problems for case study 1. These solutions were selected to facilitate comparison of the MVS efficient portfolio to the MVS-D efficient portfolio at given levels of expected return and SD. The choice of MVS efficient portfolios depends on the Genco's trade-off between return, risk and skewness. Meanwhile, the additional objective related to the number of trading choices included in the portfolios, stems on the Genco's consideration in the case of

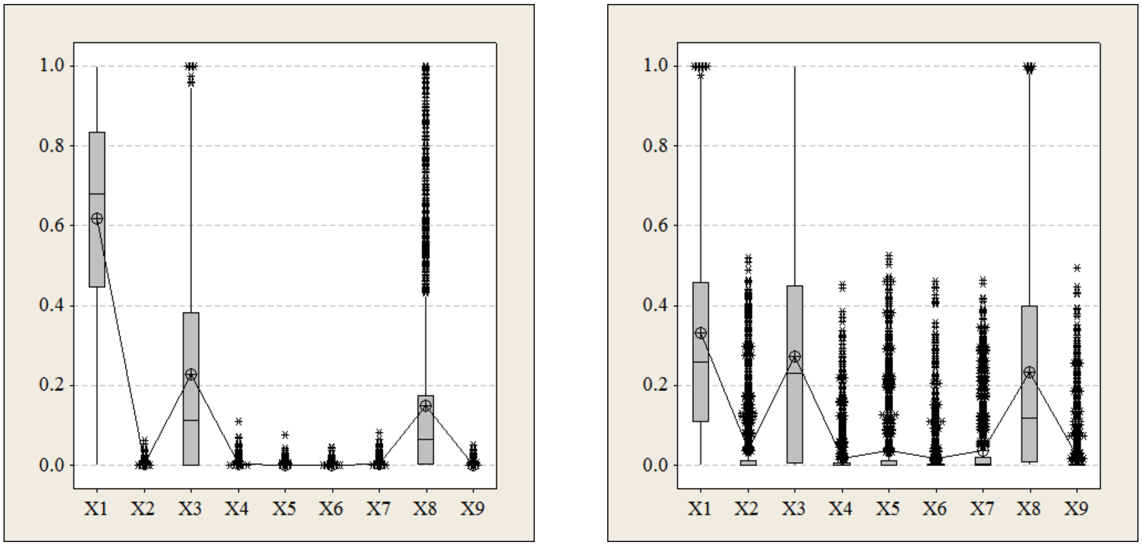
Table 6.9: Selected MVS and MVS-D Efficient Portfolios

Problem	Electricity allocation proportion (%)									Expected Return	Standard Deviation	Skewness
	z_1	z_2	z_3	z_4	z_5	z_6	z_7	z_8	z_9			
MVS	31.19	-	-	-	-	-	-	68.81	-	0.49	0.53	12.39
	43.63	-	-	56.37	-	-	-	-	-	0.49	0.68	18.75
	49.64	-	47.04	3.32	-	-	-	-	-	0.49	0.71	22.12
	49.63	-	40.99	-	-	-	-	9.37	-	0.49	0.71	20.79
MVS-D	32.01	-	8.04	-	-	-	0.33	59.62	-	0.49	0.53	10.89
	44.62	-	20.57	-	0.08	-	-	32.04	2.69	0.49	0.68	15.18
	47.45	0.18	17.85	33.66	-	0.14	0.61	0.12	-	0.49	0.71	17.72

MVS-D efficient portfolios. Suppose that the return target of the Genco is 49 percent, the efficient portfolio choices can be screened from a vast number of non-dominated portfolios displayed in Figure 6.7. It is revealed from Table 6.9 that for a given level of expected return, portfolio skewness is enlarged by increasing portfolio SD. This result can be observed from both the MVS and MVS-D models. In addition, at a given value of expected return and SD, the MVS-D efficient portfolios allocate electricity to a higher number of trading choices compared to the MVS ones. However, the MVS efficient portfolios offer a higher level of skewness than those of MVS-D model with the same expected return and SD. It is obvious that the reduction of risk in the MVS-D solutions is the result of the additional objective in the model to promote diversification.

To highlight the benefit of the proposed diversification enhancing objective, we present in Figure 6.10 the box plots of the allocation proportion, i.e. x_i, of the MVS and MVS-D efficient portfolios obtained from COGA-II for case study 1. The symbol '⊕' represents the

Figure 6.10: Box Plot of the Allocation Proportions of the MVS and MVS-D Efficient Portfolios Obtained from COGA-II for Case Study 1



(a) MVS – Case 1 (b) MVS-D – Case 1

Note: ⊕ is the mean of the allocation proportion

mean of x_i. The box in the plot contains 50 percent of the data points from the 25th to 75th percentile, and the line drawn across the box is the median of x_i. Observations outside the interquartile range are plotted on the whiskers of the box. Figure 6.10(a) shows that the MVS efficient portfolios allocate an average of 62 percent of the electricity to trading in the spot market (x_1), 23 percent to the bilateral contract with the customer located in AEGO zone (x_2), and 15 percent to the bilateral contract with the customer located in PPL zone (x_8). It can be observed that the efficient portfolios of this three-objective model excessively allocate the electricity to a particular trading choice. Figure 6.10(b) presents the MVS-D case. These portfolio solutions exhibit a reduction in the electricity allocation to the trading in spot market (x_1) to an average of 33 percent, while the allocation to the bilateral contract with the customers located in AEGO zone (x_2) increases to an average of 27 percent and the allocation to the bilateral contract with the customers located in PPL zone (x_8) raises to 23 percent. The remaining 17 percent is allocated to bilateral contracts with customers located in other zones. The inclusion of the fourth objective has not only allocated the generated electricity more uniformly, but it has also spread out the investment to more trading choices that will increase the diversification benefit.

6.4 CONCLUDING REMARKS

In this chapter, the application of the MVS portfolio model is extended to the trading of electricity in a deregulated market by a Genco, where the number of trading choices is small. To overcome the potential weakness of the MVS framework where optimized solutions have a tendency to limit the scope of investment, an additional objective is proposed to increase diversification benefits. The shift from the three-objective problem to one with four objectives increased the complexity of the optimization process. To deal with this, we experiment the use of the newly proposed COGA-II, of which the results were compared to the widely accepted algorithms of NSGA-II and SPEA-II. The results suggest the superiority of performance in COGA-II in dealing with high dimensional multi-objective optimization problems in terms of not only proximity to the Pareto-optimal solutions but also diversity of its solution. In addition, COGA-II also produces solutions where the non-dominated front of a problem with more objectives envelopes those of a problem with a lesser number of objectives.

In the context of the application, solving the electricity allocation problem based on the MVS framework together with the proposed additional objective (MVS-D) is particularly useful because COGA-II can provide solutions with Pareto fronts that also cover those based on the traditional MVS framework. As a result, the approach avoided over-concentration of investment in a few trading choices. The electricity was more uniformly allocated among a larger number of trading choices that promote diversification benefit for the power generation companies.

CHAPTER 7

CONCLUSION

7.1 INTRODUCTION

This thesis is organized to achieve four objectives in order to close the research gaps mentioned in Section 2.7 of Chapter 2. The first objective is to demonstrate how multi-objective evolutionary algorithms are applied for solving problems in MVS efficient portfolio allocation in the multi-dimension space. The second objective is to examine the risk-return trade-off and the characteristics of MVS efficient portfolios. The third objective is to investigate the impacts of skewness preference on the efficient portfolio choice. The fourth objective is to apply the MVS analysis in finance for solving asset allocation problems where the number of trading choices is small.

This chapter presents the conclusion of the thesis. The next section recapitulates the findings from the analysis in Chapter 4, Chapter 5, and Chapter 6. The implications of the findings are discussed in Section 7.3. The contributions of the thesis are explained in Section 7.4. The last section outlines the limitations of the study and provides some directions for future research that could be extended from the present study.

7.2 SUMMARY OF FINDINGS

With the ample evidences that the distribution of portfolio returns is not normal but skewed, portfolio selection with a consideration of skewness in addition to the mean and variance could yield superior results to an investor whose utility is a function of the first three moments of the return distributions. In our analysis, the result of normality test shows that the distribution of firm and market returns exhibits skewness that is significantly different from zero. The majority of the component securities of the DJIA have negative skewness. There is also strong evidence against normality when the data of emerging markets in Asia and Latin America were used. These results support the argument that non-zero skewness is present. As a result, skewness should not be omitted from the decision on portfolio selection.

To solve the MVS portfolio optimization problem in which three considered objectives are optimized at the same time, the NSGA-II and SPEA-II which are regarded as the most efficient MOEAs were applied. The algorithms firstly generated the candidate solutions that satisfy the problem constraints. The efficient portfolios among the candidate solutions were compared based on the Pareto dominance relation. The selection process of the implemented MOEAs ensures that the obtained solutions are Pareto efficient since these solutions are not dominated by any other candidate solutions in the feasible set. Regardless of the data set, the results demonstrated that both algorithms can provide a set of MVS efficient portfolios within a short computation time with slightly different algorithm performance. However, SPEA-II tends to perform better than NSGA-II in terms of providing better diversity of the solutions.

In addition to the MVS portfolio model, the four-objective optimization problem was formulated for searching the efficient set of MVS-D portfolios. The problem involves the optimization of four objectives simultaneously. The additional objective increases the complexity to the algorithms in searching and identifying the efficient solutions. We experimented with the COGA-II, which is deliberately designed for solving a problem with more than three objectives to be optimized, for solving the MVS-D portfolio optimization problem. The result suggests the superiority of performance of the COGA-II over the NSGA-II and SPEA-II. Despite this superiority, all the algorithms can considerably provide a good approximation of the efficient set of both MVS and MVS-D portfolios.

To examine the risk-return characteristics of MVS efficient portfolios, the efficient portfolios obtained from the MOEAs were plotted graphically on the three-dimension mean-SD-skewness diagram. As expected, the MVS efficient surface is not continuous due to the competing and conflicting objectives of the problem. The investigation of the trade-off between expected return and skewness was done by searching among the MVS efficient portfolios for those whose SD matches with a selected value. The result shows that, at a given value of SD, expected returns of the MVS efficient portfolios are smaller for those with larger skewness. This implies that investors have to forgo expected return for a portfolio with larger skewness.

When the rate of return of the MVS efficient portfolios is fixed at a constant, the trade-off between skewness and SD can be analyzed. The result reveals that, at a given value of expected return, SD of the MVS efficient portfolios decreases with a diminishing value of skewness. This implies that, on one hand, investors are required to expose themselves to a larger return dispersion if they need to increase the chance of gaining extreme expected returns. On the other hand, they have to forgo the chance for large gains if they desire to reduce the dispersion of returns.

In addition, at a given value of expected return, MVS efficient portfolios with the lowest SD are also MV efficient portfolios because these MVS efficient portfolios achieve the global minimum SD for a given value of expected return. Further analysis was conducted by plotting the MV and MVS efficient portfolios together in the same mean-SD diagram. The result illustrated that MVS portfolios with the global minimum SD for a given value of expected return reside on the MV efficient frontier. This implies that the MV efficient portfolios are a subset of the MVS efficient portfolios. The result also reveals that some inefficient portfolios under the MV portfolio model are actually efficient in the MVS portfolio model. This explains why investors hold portfolios that are not MV efficient. The underlying reason of this phenomenon is that these portfolios are MVS efficient portfolios.

To investigate the impacts of different degree of skewness preference on efficient portfolio choice, a single-period model that allows for a heterogeneous degree of risk aversion (A) and skewness preference (P) was developed. As stated earlier, when choosing among a feasible set of competing portfolio choices, rational investors who act to maximize their expected utility will consider only the portfolios in the efficient set. Thus, MVS efficient portfolios from a universe of 29 securities of DJIA were generated by the SPEA-II and the results were used for further analysis. Among the MVS efficient portfolios, the portfolio that maximizes the expected utility for given values of degree of risk aversion (A) and skewness preference (P) was identified.

The results suggest that the average expected return of portfolios tends to decrease when investors have higher degrees of skewness preference. This implies that investors with greater skewness preference are willing to accept lower expected returns for holding a portfolio with higher skewness. Besides, the dispersion of return distribution of portfolio choices is larger when the degree of skewness preference increases. This implies that investors with greater skewness preference are willing to accept larger return dispersion in exchange for a flatter right tail of the return distributions.

For the size of portfolio choices, the result shows that investment allocations tend to concentrate on very few securities when the degree of skewness preference increases at a given level of degree of risk aversion. This result gives an explanation to why investors hold a small number of securities in their portfolios, i.e. underdiversified portfolios. They do so in order to increase their exposure to return distributions with positive skewness.

The MVS portfolio model is also investigated for solving the asset allocation problem where the number of trading choices is small. The electricity allocation problems in the electricity market were examined and it is extended to the MVS-D model to promote diversification in portfolio choices. The MVS and MVS-D efficient portfolios were obtained from the COGA-II whose performance is more superior to the NSGA-II and SPEA-II for solving problems with a high number of objectives. It is revealed from both MVS and MVS-D efficient portfolios that portfolio SD is higher when skewness increases at a given value of expected return. This implies that a generation company (Genco) has to accept larger dispersion of the return distribution if it demands higher portfolio skewness.

A comparison between the MVS and MVS-D models, at given levels of expected return and SD shows that the portfolio solutions of MVS model allocate electricity to a smaller number of trading choices than those of MVS-D but their skewness are higher. This implies that Gencos need to forgo skewness of return distribution in order to avoid excessive investment in a small number of trading choices. The reduction of over-concentration of investment promotes diversification since electricity is more uniformly allocated among trading choices and eventually it helps to reduce portfolio risk.

7.3 IMPLICATIONS OF THE STUDY

Skewness plays an important role in explaining the distribution of returns of financial assets at both firm level and market level. There are ample evidences that non-zero skewness is present regardless of country of investigation, period of study, and investment horizon. This is a big challenge to the traditional financial theories such as the mean-variance analysis and the CAPM model because the assumption of zero skewness is required by these theories.

Based on the utility theory, preference for positive skewness is derived using expected utility functions that can be approximated by the third-order Taylor's series expansion. The derivation of skewness preference allows researchers to theoretically elucidate investor behaviors that seem irrational in the framework of traditional theory. Some puzzles that are related to skewness preference are, for instance, why investors buy lottery, why investors hold portfolio that is not MV efficient, and why investors hold underdiversified portfolio.

A plausible answer for all these three questions is associated to individual preference for positive skewness. Logically, rational investors prefer, among two investment alternatives, one with larger skewness of return distributions if the mean and variance are similar. In the context of financial asset pricing, there are abundant studies that support the skewness preference of investors. These studies showed that investors are willing to pay premium for securities with positive skewness. As shown in this study, skewness has an impact on portfolio decision making. By extension, the pricing of financial assets will also be affected by skewness. Asset pricing models, therefore, should cover factors beyond the second moment, and consider also skewness.

In the MV framework where skewness is neglected, portfolio selection for rational investors involves maximizing mean and minimizing variance. Portfolio choices that achieve global minimum variance at a given level of expected returns are MV efficient portfolios, while portfolio choices that reside away and under the MV efficient frontier are inferior investment alternatives. In reality, mutual funds, market portfolios, and/or investment portfolios being managed by professional fund managers may not be always MV efficient. These portfolios could be located away from the MV efficient frontier.

This study exploits an innovative methodology from the field of engineering to provide an explanation to these puzzles. The ability to search for the feasible solutions in multi-dimension space, and in parallel, evaluate the efficiency of the solutions in another multi-dimension space allows us to construct the efficient set of MVS portfolios within a single run of algorithms. This make a clear understanding of why investors hold portfolios that are not MV efficient. Since skewness preference is a common trait for rational risk-averse investors, they rationally make a portfolio selection decision based not only on the mean and variance, but also on the skewness of return distributions. Therefore, they may discard MV efficiency in order to increase the skewness of their portfolios. In fact, these portfolios are efficient in the MVS framework. The implication is clear that portfolio choice selection cannot be based on the first two moments of distributions of asset returns, as suggested in the MV framework. Portfolio efficiency should be defined clearly, whether it refers to only MV efficiency, or MVS efficiency.

However, the extent to which investors will forgo expected returns or expose themselves to larger dispersion of portfolio returns depends on the degree of skewness preference. This means that investors with higher degree of skewness preference are willing to trade higher expected return and to accept larger SD in their investment decision to optimize for portfolios with a flatter right tail of the return distributions. Besides, diversification strategy may not be relevant to investors in the MVS framework since well-diversified portfolios not only have smaller SD but also distributions that are less skewed compared to underdiversified portfolios at a given value of expected return. Thus, investors in the MVS framework are willing to hold underdiversified portfolios. In general, investors with greater skewness preference tend to hold a smaller number of securities in their portfolios to increase portfolio skewness. Diversification strategies, therefore, must take preference for skewness into consideration.

The implication for the electricity market is how Gencos should reinvent their electricity allocation strategies in the presence of asymmetric distribution of electricity spot prices and skewness preference. In deregulated electricity markets, electricity allocation strategy of Gencos is generally formulated based on the MV portfolio optimization model. However, the results suggest that better outcome could be achieved by increasing the skewness of portfolio since this strategy increases the chance of obtaining extreme positive returns and decreases the probability of large losses. It is found empirically that the distribution of returns of transactions in the spot market is highly skewed to the right. Other trading instruments such as transaction in the day-ahead market and future contracts have return distributions that are also highly skewed. These instruments should be considered in the portfolio selection decision of a Genco. In addition, the results reveal that skewness has an impact on electricity allocation decision. Therefore, pricing of electricity spot prices should consider skewness as a factor in the electricity pricing model in addition to other variables.

7.4 CONTRIBUTIONS OF THE STUDY

The contributions of this thesis in closing the research gaps mentioned in Section 2.7 of Chapter 2 are discussed in this section. In the aspect of innovation of methodology, MOEAs that were successfully used to solve complex multi-objective optimization problems in other fields of research were applied to solve the MVS portfolio optimization problem, that are otherwise different because of its multi-objective nature. Unlike other techniques adopted in the previous studies, this proposed method allows us to independently optimized three competing and conflicting objectives at the same time. By doing so, a set of MVS efficient portfolios can be generated in a single run of algorithms and it can be graphically plotted in the three-dimension MVS space. Besides, search and evaluation processes of algorithms ensure that the resulting portfolios are MVS efficient. The proposed methodology allows investors to have a clear picture about MVS efficient surface that helps them make a portfolio selection decision in an effective way. The risk-return characteristics of MVS efficient portfolios can be examined based on a substantial number of portfolios, especially in the aspect of trade-off between skewness and SD that has not been addressed by previous studies.

In the context of theoretical contribution, a single-period model that allows for a heterogeneous degree of risk aversion and skewness preference was developed for explaining the impacts of skewness preference on efficient portfolio choices. This model provides the explanations to two puzzles, i.e., why investors hold portfolio that is not MV efficient and why investors hold underdiversified portfolio. The model reveals that skewness preference is a common trait for rational risk-averse investors. Investors consider skewness in addition to the mean and variance of the return distributions in their portfolio selection. A portfolio that is not MV efficient may be efficient in the MVS framework. Besides, complete diversification may no longer be a favorite strategy since diversification reduces portfolio skewness. We also illustrated that MV efficient portfolios are actually a subset of MVS efficient portfolios. Among the MVS efficient set, a portfolio that attains the global minimum SD at a given value of expected return is an MV efficient portfolio.

An extensive application of the MVS framework to solve the electricity allocation problem in the electricity market where the number of trading choices is small makes a contribution in terms of cross-discipline application. We demonstrated that the MVS portfolio model is suitable for an asset allocation problem in the electricity market since the distribution of electricity spot prices is skewed and skewness preference can be observed in this market. An additional objective, incorporated in the MVS portfolio model, is also proposed to increase the number of trading choices included in the portfolio solutions. This is necessary because unlike the financial market, the number of trading choices available in the electricity market is a lot lesser. It is shown that the MVS-D portfolio model is useful for a Genco since efficient solutions of the MVS model are retained in the set of MVS-D efficient solutions. As a result, a set of solutions with different trade-off between skewness and the number of trading choices in portfolios, at given levels of expected return and SD, are available for the Genco to make a portfolio selection decision.

7.5 LIMITATIONS OF THE STUDY AND DIRECTION FOR FUTURE RESEARCH

Several limitations remain in this study. Firstly, the terms of higher moments considered in the study are restricted up to the third moment (skewness). The fourth moment (kurtosis) is not considered since theoretical explanation on its application for portfolio selection is still ambiguous. The derivation of kurtosis preference based on utility theory is at a developing stage. An investigation on the impact of kurtosis preference on portfolio choice will be interesting if a parameter that measures preference for kurtosis can be theoretically validated. By using our proposed approach, kurtosis can be incorporated in the MVS model as a fourth objective in the portfolio optimization problem.

Secondly, the portfolio selection process is examined with in-sample approach. Application of our method to out-of-sample portfolio selection has not been executed. The performance of efficient portfolios obtained from MV and MVS framework could be compared based on out-of-sample data. In addition, an evaluation of the performance of MVS efficient portfolios across different market conditions such as the bull market, bear market, and/or during the crisis can be conducted. This will allow investors to evaluate how MVS efficient portfolios perform in different market conditions.

Lastly, our approach to portfolio optimization is performed within a static asset allocation framework where all variables are known before running the optimization. In the last decade, however, an increasing number of studies on dynamic portfolio selection have been conducted and this brings more challenge and complexity to the portfolio optimization problem. In this aspect, variables such as expected return vector and variance-covariance matrix are allowed to change over time. Portfolio weights as well as the efficient frontier will also inevitably change with the investment period. Thus, the application of the MVS model to dynamic portfolio selection will be another interesting direction for future research.

REFERENCES

Aggarwal, Sanjeev Kumar, Saini, Lalit Mohan, & Kumar, Ashwani. (2009). Electricity price forecasting in deregulated markets: A review and evaluation. *International Journal of Electrical Power & Energy Systems, 31*(1), 13-22.

Amaya, Diego , & Vasquez, Aurelio (2010). Skewness from High-Frequency Data Predicts the Cross-Section of Stock Returns. *Working Paper*, *HEC Montreal*

Anagnostopoulos, K. P., & Mamanis, G. (2010). A portfolio optimization model with three objectives and discrete variables. *Computers & Operations Research*, *37*(7), 1285-1297.

Anagnostopoulos, K. P., & Mamanis, Georgios. (2011). Multiobjective evolutionary algorithms for complex portfolio optimization problems. *Computational Management Science, 8*(3), 259-279. doi: 10.1007/s10287-009-0113-8

Andersen, Torben G., Bollerslev, Tim, Diebold, Francis X., & Ebens, Heiko. (2001). The distribution of realized stock return volatility. *Journal of Financial Economics, 61*(1), 43-76.

Andersen, Torben G., Bollerslev, Tim, Diebold, Francis X., & Labys, Paul. (2001). The Distribution of Realized Exchange Rate Volatility. *Journal of the American Statistical Association, 96*(453), 42-55.

Arditti, Fred D. (1967). Risk and the Required Return on Equity. *The Journal of Finance*, *22*(1), 19-36.

Arditti, Fred D., & Levy, Haim. (1975). Portfolio Efficiency Analysis in Three Moments: The Multiperiod Case. *The Journal of Finance, 30*(3), 797-809.

Arrow, K. J. (1964). The Role of Securities in the Optimal Allocation of Risk-bearing. *The Review of Economic Studies, 31*(2), 91-96. doi: 10.2307/2296188

Athan, Timothy Ward, & Papalambros, Panos Y. (1996). A Note on Weighted Criteria Methods for Compromise Solutions in Multi-objective Optimization. *Engineering Optimization, 27*(2), 155-176. doi: 10.1080/03052159608941404

Barberis, Nicholas, & Huang, Ming. (2007). Stocks as Lotteries: The Implications of Probability Weighting for Security Prices. *National Bureau of Economic Research Working Paper Series, No. 12936*.

Bates, David S. (1996). Jumps and Stochastic Volatility: Exchange Rate Processes Implicit in Deutsche Mark Options. *The Review of Financial Studies, 9*(1), 69-107.

Beasley, J. E., Meade, N., & Chang, T. J. (2003). An evolutionary heuristic for the index tracking problem. *European Journal of Operational Research, 148*(3), 621-643. doi: http://dx.doi.org/10.1016/S0377-2217(02)00425-3

Bekaert, Geert, & Harvey, Campbell R. (2002). Research in emerging markets finance: looking to the future. *Emerging Markets Review, 3*(4), 429-448.

Benth, Fred Espen, Cartea, Álvaro, & Kiesel, Rüdiger. (2008). Pricing forward contracts in power markets by the certainty equivalence principle: Explaining the sign of the market risk premium. *Journal of Banking & Finance, 32*(10), 2006-2021. doi: http://dx.doi.org/10.1016/j.jbankfin.2007.12.022

Bessembinder, Hendrik, & Lemmon, Michael L. (2002). Equilibrium Pricing and Optimal Hedging in Electricity Forward Markets. *The Journal of Finance, 57*(3), 1347-1382. doi: 10.1111/1540-6261.00463

Bhattacharyya, Rupak, Kar, Samarjit, & Majumder, Dwijesh Dutta. (2011). Fuzzy mean-variance-skewness portfolio selection models by interval analysis. *Computers & Mathematics with Applications, 61*(1), 126-137.

Bierwag, G. O. (1974). The Rationale of the Mean-Standard Deviation Analysis: Comment. *The American Economic Review, 64*(3), 431-433.

Black, Fischer. (1976). *Studies of stock price volatility changes.* Paper presented at the Meetings of the American Statistical Association, Business and Economics Statistics Section.

Blanchard, Olivier J., & Watson, Mark W. (1983). *Bubbles, Rational Expectations and Financial Markets*. Working Paper Series. National Bureau of Economic Research. Retrieved from http://www.nber.org/papers/w0945

Blume, Marshall E., & Friend, Irwin. (1975). The Asset Structure of Individual Portfolios and Some Implications for Utility Functions. *The Journal of Finance, 30*(2), 585-603.

Boonlong, Kittipong , Chaiyaratana, Nachol, & Maneeratana, K. (2010, 2010). *Improved Compressed Objective Genetic Algorithm : COGA-II.* Paper presented at the The International Conference on Evolutionary Computation, Valencia, Spain.

Borch, Karl. (1974). The Rationale of the Mean-Standard Deviation Analysis: Comment. *The American Economic Review, 64*(3), 428-430.

Boyer, Brian, Mitton, Todd, & Vorkink, Keith. (2010). Expected Idiosyncratic Skewness. *Review of Financial Studies, 23*(1), 169-202. doi: 10.1093/rfs/hhp041

Branke, J., Scheckenbach, B., Stein, M., Deb, K., & Schmeck, H. (2009). Portfolio optimization with an envelope-based multi-objective evolutionary algorithm. *European Journal of Operational Research, 199*(3), 684-693. doi: http://dx.doi.org/10.1016/j.ejor.2008.01.054

Briec, Walter, & Kerstens, Kristiaan. (2010). Portfolio selection in multidimensional general and partial moment space. *Journal of Economic Dynamics and Control, 34*(4), 636-656. doi: http://dx.doi.org/10.1016/j.jedc.2009.11.001

Bris, Arturo, Goetzmann, William N., & Zhu, Ning. (2007). Efficiency and the Bear: Short Sales and Markets around the World. *The Journal of Finance, 62*(3), 1029-1079.

Brunnermeier, Markus K., Gollier, Christian, & Parker, Jonathan A. (2007). Optimal Beliefs, Asset Prices, and the Preference for Skewed Returns. *The American Economic Review, 97*(2), 159-165.

Brunnermeier, Markus K., & Parker, Jonathan A. (2005). Optimal Expectations. *American Economic Review, 95*(4), 1092-1118. doi: doi: 10.1257/0002828054825493

Campbell, John Y., & Hentschel, Ludger. (1992). No news is good news: An asymmetric model of changing volatility in stock returns. *Journal of Financial Economics, 31*(3), 281-318.

Campbell, John Y., Lettau, Martin, Burton, G. Malkiel, & Xu, Yexiao. (2001). Have Individual Stocks Become More Volatile? An Empirical Exploration of Idiosyncratic Risk. *The Journal of Finance, 56*(1), 1-43.

Canela, Miguel Ángel, & Collazo, Eduardo Pedreira. (2007). Portfolio selection with skewness in emerging market industries. *Emerging Markets Review, 8*(3), 230-250.

Cartea, Álvaro, & Villaplana, Pablo. (2008). Spot price modeling and the valuation of electricity forward contracts: The role of demand and capacity. *Journal of Banking & Finance, 32*(12), 2502-2519. doi:http://dx.doi.org/10.1016/j.jbankfin.2008.04.006

Chang, Eric C., Cheng, Joseph W., & Yu, Yinghui. (2007). Short-Sales Constraints and Price Discovery: Evidence from the Hong Kong Market. *The Journal of Finance, 62*(5), 2097-2121.

Chang, T. J., Meade, N., Beasley, J. E., & Sharaiha, Y. M. (2000). Heuristics for cardinality constrained portfolio optimisation. *Computers & Operations Research, 27*(13), 1271-1302.

Chang, Tun-Jen, Yang, Sang-Chin, & Chang, Kuang-Jung. (2009). Portfolio optimization problems in different risk measures using genetic algorithm. *Expert Systems with Applications, 36*(7), 10529-10537.

Chen, Joseph, Hong, Harrison, & Stein, Jeremy C. (2001). Forecasting crashes: trading volume, past returns, and conditional skewness in stock prices. *Journal of Financial Economics, 61*(3), 345-381.

Chiu, W. Henry. (2000). On the Propensity to Self-Protect. *The Journal of Risk and Insurance, 67*(4), 555-577. doi: 10.2307/253850

Chiu, W. Henry. (2005). Degree of downside risk aversion and self-protection. *Insurance: Mathematics and Economics, 36*(1), 93-101. doi: http://dx.doi.org/10.1016/j.insmatheco.2004.10.005

Christie, Andrew A. (1982). The stochastic behavior of common stock variances : Value, leverage and interest rate effects. *Journal of Financial Economics, 10*(4), 407-432.

Chunhachinda, Pornchai, Dandapani, Krishnan, Hamid, S., & Prakash, A. J. (1994). Efficacy of portfolio performance measures: an evaluation. *Quarterly Journal of Business and Economics, 33*, 74-87.

Chunhachinda, Pornchai, Dandapani, Krishnan, Hamid, Shahid, & Prakash, Arun J. (1997). Portfolio selection and skewness: Evidence from international stock markets. *Journal of Banking & Finance, 21*(2), 143-167.

Coello, Carlos A Coello. (1999). A Comprehensive Survey of Evolutionary-Based Multiobjective Optimization Techniques. *Knowledge and Information Systems, 1*(3), 129-156.

Coello, Carlos A Coello. (2001). A Short Tutorial on Evolutionary Multiobjective Optimization *Evolutionary Multi-Criterion Optimization* (Vol. 1993, pp. 21-40): Springer Berlin / Heidelberg.

Coello, Carlos A Coello, Lamont, Gary B., & Van Veldhuizen, David A. (2007). *Evolutionary Algorithms for Solving Multi-Objective Problems*. NJ, USA: Springer-Verlag New York, Inc.

Conine, Thomas E., Jr., & Tamarkin, Maurry J. (1981). On Diversification Given Asymmetry in Returns. *The Journal of Finance, 36*(5), 1143-1155.

Corne, DavidW, Knowles, JoshuaD, & Oates, MartinJ. (2000). The Pareto Envelope-Based Selection Algorithm for Multiobjective Optimization. In M. Schoenauer, K. Deb, G. Rudolph, X. Yao, E. Lutton, J. Merelo & H.-P. Schwefel (Eds.), *Parallel Problem Solving from Nature PPSN VI* (Vol. 1917, pp. 839-848): Springer Berlin Heidelberg.

Crainich, David, & Eeckhoudt, Louis. (2008). On the intensity of downside risk aversion. *Journal of Risk and Uncertainty, 36*(3), 267-276. doi: 10.1007/s11166-008-9037-x

Das, I., & Dennis, J. E. (1997). A closer look at drawbacks of minimizing weighted sums of objectives for Pareto set generation in multicriteria optimization problems. *Structural optimization, 14*(1), 63-69. doi: 10.1007/BF01197559

De Long, J. Bradford, Shleifer, Andrei, Summers, Lawrence H, & Waldmann, Robert J. (1990). Noise Trader Risk in Financial Markets. *Journal of Political Economy 98*(4), 703-738.

Deb, Kalyanmoy. (1997). Mechanical component design using genetic algorithms. In D. Dasgupta & Z. Michalewicz (Eds.), *Evolutionary Algorithms in Engineering Applications* (pp. 495-512): Springer Berlin / Heidelberg.

Deb, Kalyanmoy. (2001). *Multi - objective optimization using evolutionary algorithms*. New York: John Wiley & Sons, Inc. .

Deb, Kalyanmoy. (2005). Multi-Objective Optimization. In E. Burke & G. Kendall (Eds.), *Search Methodologies* (pp. 273-316): Springer US.

Deb, Kalyanmoy, & Agrawal, R. B. (1995). Simulated binary crossover for continuous search space. *Complex Systems, 9*(2), 115-148.

Deb, Kalyanmoy, & Gulati, Surendra. (2001). Design of truss-structures for minimum weight using genetic algorithms. *Finite Elements in Analysis and Design, 37*(5), 447-465.

Deb, Kalyanmoy, Pratap, A, Agarwal, S, & Meyarivan, T. (2002). A fast and elitist multiobjective genetic algorithm: NSGA-II. *Evolutionary Computation, IEEE Transactions on, 6*(2), 182-197. doi: 10.1109/4235.996017

Deb, Kalyanmoy, Thiele, Lothar, Laumanns, Marco, & Zitzler, Eckart. (2005). Scalable Test Problems for Evolutionary Multiobjective Optimization. In A. Abraham, L. Jain & R. Goldberg (Eds.), *Evolutionary Multiobjective Optimization* (pp. 105-145): Springer Berlin Heidelberg.

Diamond, Douglas W., & Verrecchia, Robert E. (1987). Constraints on short-selling and asset price adjustment to private information. *Journal of Financial Economics, 18*(2), 277-311.

Diosan, L. (2005, 28-30 Nov. 2005). *A multi-objective evolutionary approach to the portfolio optimization problem.* Paper presented at the International Conference on Computational Intelligence for Modelling, Control and Automation, 2005 and International Conference on Intelligent Agents, Web Technologies and Internet Commerce.

Doan, Phuong, Lin, Chien-Ting, & Zurbruegg, Ralf. (2010). Pricing assets with higher moments: Evidence from the Australian and us stock markets. *Journal of International Financial Markets, Institutions and Money, 20*(1), 51-67.

Donghan, Feng, Deqiang, Gan, Jin, Zhong, & Yixin, Ni. (2007). Supplier Asset Allocation in a Pool-Based Electricity Market. *Power Systems, IEEE Transactions on, 22*(3), 1129-1138.

Douglas, Stratford, & Popova, Julia. (2008). Storage and the electricity forward premium. *Energy Economics, 30*(4), 1712-1727. doi: http://dx.doi.org/10.1016/j.eneco.2007.12.013

Ehrgott, Matthias, Klamroth, Kathrin, & Schwehm, Christian. (2004). An MCDM approach to portfolio optimization. *European Journal of Operational Research, 155*(3), 752-770.

EIA. (2011). *Annual Energy Outlook 2011 with Projections to 2035*. (DOE/EIA-0383(2011)). U.S. Energy Information Administration Retrieved from www.eia.gov/forecasts/aeo/.

Elton, Edwin J., & Gruber, Martin J. (1977). Risk Reduction and Portfolio Size: An Analytical Solution. *The Journal of Business, 50*(4), 415-437. doi: 10.2307/2352173

Evans, John L., & Archer, Stephen H. (1968). Diversification and The Reduction of Disperson: An Empirical Analysis. *The Journal of Finance, 23*(5), 761-767. doi: 10.1111/j.1540-6261.1968.tb00315.x

Fama, Eugene F., & French, Kenneth R. (1993). Common risk factors in the returns on stocks and bonds. *Journal of Financial Economics, 33*(1), 3-56.

Felix Streichert , Holger Ulmer , & Zell, Andreas. (2003). Evolutionary algorithms and the cardinality constrained portfolio selection problem *Operations Research Proceedings 2003, Selected Papers of the International Conference on Operations Research, 2003*.

Fielitz, Bruce D. (1976). Further Results on Asymmetric Stable Distributions of Stock Price Changes. *The Journal of Financial and Quantitative Analysis, 11*(1), 39-55.

Fonseca, C. M., & Fleming, P. J. . (1993). Genetic algorithms for multi-objective optimization: Formulation, discussion and generalization. *Proceedings of the Fifth International Conference on Genetic Algorithms*, 416-423.

French, Kenneth R., Schwert, G. William, & Stambaugh, Robert F. (1987). Expected stock returns and volatility. *Journal of Financial Economics, 19*(1), 3-29.

Friedman, Milton, & Savage, L. J. (1948). The Utility Analysis of Choices Involving Risk. *Journal of Political Economy, 56*(4), 279-304.

Friend, Irwin, & Blume, Marshall E. (1975). The Demand for Risky Assets. *The American Economic Review, 65*(5), 900-922.

Friend, Irwin, & Westerfield, Randolph. (1980). Co-Skewness and Capital Asset Pricing. *The Journal of Finance, 35*(4), 897-913.

Froot, Kenneth A, & Obstfeld, Maurice. (1991). Intrinsic Bubbles: The Case of Stock Prices. *American Economic Review 81*(5), 1189-1214.

Galagedera, Don U. A., & Brooks, Robert D. (2007). Is co-skewness a better measure of risk in the downside than downside beta?: Evidence in emerging market data. *Journal of Multinational Financial Management, 17*(3), 214-230.

Garcia, R. C., Contreras, J., van Akkeren, M., & Garcia, J. B. C. (2005). A GARCH forecasting model to predict day-ahead electricity prices. *Power Systems, IEEE Transactions on, 20*(2), 867-874.

Garrett, Thomas A., & Sobel, Russell S. (1999). Gamblers favor skewness, not risk: Further evidence from United States' lottery games. *Economics Letters, 63*(1), 85-90. doi: http://dx.doi.org/10.1016/S0165-1765(99)00012-9

Gennotte, Gerard, & Leland, Hayne. (1990). Market Liquidity, Hedging, and Crashes. *The American Economic Review, 80*(5), 999-1021.

Ghandar, A., Michalewicz, Z., Zurbruegg, R., & Chee, Cheong. (2010, 18-23 July 2010). *Index tracking fund enhancement using evolving multi-criteria fuzzy decision models.* Paper presented at the Evolutionary Computation (CEC), 2010 IEEE Congress on.

Ghandar, Adam, Michalewicz, Zbigniew, & Zurbruegg, Ralf. (2012). Enhancing Profitability through Interpretability in Algorithmic Trading with a Multiobjective Evolutionary Fuzzy System. In C. C. Coello, V. Cutello, K. Deb, S. Forrest, G. Nicosia & M. Pavone (Eds.), *Parallel Problem Solving from Nature - PPSN XII* (Vol. 7492, pp. 42-51): Springer Berlin Heidelberg.

Gibbons, Michael R., Ross, Stephen A., & Shanken, Jay. (1989). A Test of the Efficiency of a Given Portfolio. *Econometrica, 57*(5), 1121-1152.

Goetzmann, William N., & Kumar, Alok. (2008). Equity Portfolio Diversification. *Review of Finance, 12*(3), 433-463. doi: 10.1093/rof/rfn005

Goldberg, David E. (1989). *Genetic Algorithms in Search, Optimization, and Machine Learning*. Reading, MA: Addison-Wesley.

Golec, J., & Tamarkin, M. (1998). Bettors love skewness, not risk, at the horse track. *Journal of Political Economy, 106*(1), 205-225.

Grossman, Sanford J. (1989). *An Analysis of the Implications for Stock and Futures Price Volatility of Program Trading and Dynamic Hedging Strategies*. Working Paper Series. National Bureau of Economic Research.

Hajiabadi, M. E., & Mashhadi, H. R. (2013). Analysis of the Probability Distribution of LMP by Central Limit Theorem. *Power Systems, IEEE Transactions on, 28*(3), 2862-2871. doi: 10.1109/TPWRS.2013.2252372

Hanoch, Giora, & Levy, Haim. (1970). Efficient Portfolio Selection with Quadratic and Cubic Utility. *The Journal of Business, 43*(2), 181-189.

Harris, M, & Raviv, A. (1993). Differences of opinion make a horse race. *Review of Financial Studies, 6*(3), 473-506. doi: 10.1093/rfs/6.3.473

Harvey, C.R., & Siddique, A. (2000). Conditional Skewness in Asset Pricing Tests. *The Journal of Finance, 55*(3), 1263-1295. doi: 10.1111/0022-1082.00247

Hatami, A., Seifi, H., & Sheikh-El-Eslami, M. K. (2011). A Stochastic-Based Decision-Making Framework for an Electricity Retailer: Time-of-Use Pricing and Electricity Portfolio Optimization. *Power Systems, IEEE Transactions on, 26*(4), 1808-1816. doi: 10.1109/tpwrs.2010.2095431

Holland, John H. (1975). *Adaptation in Natural and Artificial Systems*. MI: University of Michigan Press.

Hong, Harrison, & Stein, Jeremy C. (2003). Differences of Opinion, Short-Sales Constraints, and Market Crashes. *Review of Financial Studies, 16*(2), 487-525. doi: 10.1093/rfs/hhg006

Horn, J., & Nafpliotis, N. (1993). *Multi-objective optimization using the Niched Pareto genetic algorithm*. Illinois Genetic Algorithms Laboratory, University of Illinois, US.

Hwang, Soosung, & Satchell, Stephen E. (1999). Modelling emerging market risk premia using higher moments. *International Journal of Finance & Economics, 4*(4), 271-296. doi: 10.1002/(sici)1099-1158(199910)4:4<271::aid-ijfe110>3.0.co;2-m

Ingersoll, Jonathan. (1975). Multidimensional Security Pricing. *The Journal of Financial and Quantitative Analysis, 10*(5), 785-798.

Jarque, Carlos M., & Bera, Anil K. (1987). A Test for Normality of Observations and Regression Residuals. *International Statistical Review / Revue Internationale de Statistique, 55*(2), 163-172. doi: 10.2307/1403192

Jean, William H. (1971). The Extension of Portfolio Analysis to Three or More Parameters. *The Journal of Financial and Quantitative Analysis*, *6*(1), 505-515.

Jean, William H. (1973). More on Multidimensional Portfolio Analysis. *The Journal of Financial and Quantitative Analysis, 8*(3), 475-490.

Jindapon, Paan, & Neilson, William S. (2007). Higher-order generalizations of Arrow–Pratt and Ross risk aversion: A comparative statics approach. *Journal of Economic Theory, 136*(1), 719-728. doi: http://dx.doi.org/10.1016/j.jet.2006.03.010

Jorion, Philippe. (1988). On Jump Processes in the Foreign Exchange and Stock Markets. *The Review of Financial Studies*, *1*(4), 427-445.

Jun Hua, Zhao, Zhao Yang, Dong, Zhao, Xu, & Kit Po, Wong. (2008). A Statistical Approach for Interval Forecasting of the Electricity Price. *Power Systems, IEEE Transactions on, 23*(2), 267-276.

Kandel, Eugene, & Pearson, Neil D. (1995). Differential Interpretation of Public Signals and Trade in Speculative Markets. *Journal of Political Economy*, *103*(4), 831-872. doi: 10.2307/2138584

Kane, Alex. (1982). Skewness Preference and Portfolio Choice. *The Journal of Financial and Quantitative Analysis*, *17*(1), 15-25.

Kelly, Morgan. (1995). All their eggs in one basket: Portfolio diversification of US households. *Journal of Economic Behavior & Organization*, *27*(1), 87-96.

Kimball, M. S. (1990). Precautionary Saving in the Small and in the Large. *Econometrica*, *58*(1), 53-73.

Konno, Hiroshi, Shirakawa, Hiroshi, & Yamazaki, Hiroaki. (1993). A mean-absolute deviation-skewness portfolio optimization model. *Annals of Operations Research*, *45*(1), 205-220. doi: 10.1007/bf02282050

Konno, Hiroshi, & Suzuki, Ken-ichi. (1995). A Mean-Variance-Skewness Portfolio Optimization Model. *Journal of the Operations Research Society of Japan, 38*(2), 173-187.

Konno, Hiroshi, & Yamazaki, Hiroaki. (1991). Mean-Absolute Deviation Portfolio Optimization Model and Its Applications to Tokyo Stock Market. *Management Science, 37*(5), 519-531.

Kraus, Alan, & Litzenberger, Robert H. (1976). Skewness Preference and the Valuation of Risk Assets. *The Journal of Finance, 31*(4), 1085-1100.

Kwang, Ng Yew. (1965). Why do People Buy Lottery Tickets? Choices Involving Risk and the Indivisibility of Expenditure. *Journal of Political Economy, 73*(5), 530-535.

Lai, Tsong-Yue. (1991). Portfolio selection with skewness: A multiple-objective approach. *Review of Quantitative Finance and Accounting, 1*(3), 293-305. doi: 10.1007/bf02408382

Levy, H. (1969). A Utility Function Depending on the First Three Moments. *The Journal of Finance, 24*(4), 715-719.

Levy, H. (1972). Portfolio Performance and the Investment Horizon. *Management Science, 18*(12), B645-B653.

Levy, H. (1974). The Rationale of the Mean-Standard Deviation Analysis: Comment. *The American Economic Review, 64*(3), 434-441.

Levy, H., & Markowitz, H. M. (1979). Approximating Expected Utility by a Function of Mean and Variance. *The American Economic Review, 69*(3), 308-317.

Li, Xiang, Qin, Zhongfeng, & Kar, Samarjit. (2010). Mean-variance-skewness model for portfolio selection with fuzzy returns. *European Journal of Operational Research, 202*(1), 239-247.

Lim, Kian-Guan. (1989). A New Test of the Three-Moment Capital Asset Pricing Model. *Journal of Financial and Quantitative Analysis, 24*(2), 205.

Lin, Chang Chun, & Liu, Yi-Ting. (2008). Genetic algorithms for portfolio selection problems with minimum transaction lots. *European Journal of Operational Research, 185*(1), 393-404.

Lin, Ping Chen. (2012). Portfolio optimization and risk measurement based on non-dominated sorting genetic algorithm. *Journal of Industrial and Management Optimization, 8*(3), 549-564.

Lintner, John. (1965). The Valuation of Risk Assets and the Selection of Risky Investments in Stock Portfolios and Capital Budgets. *The Review of Economics and Statistics, 47*(1), 13-37.

Liu, Min, & Wu, F. F. (2006). Managing Price Risk in a Multimarket Environment. *Power Systems, IEEE Transactions on, 21*(4), 1512-1519. doi: 10.1109/tpwrs.2006.882455

Liu, Min, & Wu, Felix F. (2007a). Portfolio optimization in electricity markets. *Electric Power Systems Research, 77*(8), 1000-1009.

Liu, Min, & Wu, Felix F. (2007b). Risk management in a competitive electricity market. *International Journal of Electrical Power & Energy Systems, 29*(9), 690-697.

Liu, Ming-Hua, Margaritis, Dimitris, & Tourani-Rad, Alireza. (2012). Risk appetite, carry trade and exchange rates. *Global Finance Journal, 23*(1), 48-63. doi: http://dx.doi.org/10.1016/j.gfj.2012.01.004

Liu, Xinyi, Margaritis, Dimitris, & Wang, Peiming. (2012). Stock market volatility and equity returns: Evidence from a two-state Markov-switching model with regressors. *Journal of Empirical Finance, 19*(4), 483-496. doi: http://dx.doi.org/10.1016/j.jempfin.2012.04.011

Longstaff, Francis A., & Wang, Ashley W. (2004). Electricity Forward Prices: A High-Frequency Empirical Analysis. *The Journal of Finance, 59*(4), 1877-1900. doi: 10.2307/3694881

Lucia, Julio J., & Torró, Hipòlit. (2011). On the risk premium in Nordic electricity futures prices. *International Review of Economics & Finance, 20*(4), 750-763. doi: http://dx.doi.org/10.1016/j.iref.2011.02.005

Marion, R., Michel, R., & Faye, C. (2006). Atmospheric correction of hyperspectral data over dark surfaces via simulated annealing. *Geoscience and Remote Sensing, IEEE Transactions on, 44*(6), 1566-1574. doi: 10.1109/tgrs.2006.870408

Markowitz, Harry. (1952). Portfolio Selection. *The Journal of Finance, 7*(1), 77-91.

Marler, R. T., & Arora, J. S. (2004). Survey of multi-objective optimization methods for engineering. *Structural and Multidisciplinary Optimization, 26*(6), 369-395. doi: 10.1007/s00158-003-0368-6

Masoum, M. A. S., Ladjevardi, M., Jafarian, A., & Fuchs, E. F. (2004). Optimal placement, replacement and sizing of capacitor Banks in distorted distribution networks by genetic algorithms. *Power Delivery, IEEE Transactions on, 19*(4), 1794-1801. doi: 10.1109/tpwrd.2004.835438

Menezes, C.F., Geiss, C., & Tressler, J. (1980). Increasing downside risk. *American Economic Review, 70*(5), 921-932.

Messac, Achille, Puemi-Sukam, Cyriaque, & Melachrinoudis, Emanuel. (2000). Aggregate Objective Functions and Pareto Frontiers: Required Relationships and Practical Implications. *Optimization and Engineering, 1*(2), 171-188. doi: 10.1023/A:1010035730904

Metaxiotis, K., & Liagkouras, K. (2012). Multiobjective Evolutionary Algorithms for Portfolio Management: A comprehensive literature review. *Expert Systems with Applications, 39*(14), 11685-11698.

Mitton, Todd, & Vorkink, Keith. (2007). Equilibrium Underdiversification and the Preference for Skewness. *The Review of Financial Studies, 20*(4), 1255-1288.

Modica, Salvatore, & Scarsini, Marco. (2005). A note on comparative downside risk aversion. *Journal of Economic Theory, 122*(2), 267-271. doi: http://dx.doi.org/10.1016/j.jet.2004.06.008

Odean, Terrance. (1998). Volume, Volatility, Price, and Profit When All Traders Are Above Average. *The Journal of Finance, 53*(6), 1887-1934. doi: 10.1111/0022-1082.00078

Odean, Terrance. (1999). Do Investors Trade Too Much? *The American Economic Review, 89*(5), 1279-1298.

Parsons, John E, & De Roo, Guillaume. (2008). *Risk premiums in electricity forward prices—data from the ISO New England Market.* Paper presented at the Electricity Market, 2008. EEM 2008. 5th International Conference on European.

Pierro, F. D., Soon-Thiam, Khu, & Savic, D. A. (2007). An Investigation on Preference Order Ranking Scheme for Multiobjective Evolutionary Optimization. *Evolutionary Computation, IEEE Transactions on, 11*(1), 17-45. doi: 10.1109/tevc.2006.876362

Polkovnichenko, Valery. (2005). Household Portfolio Diversification: A Case for Rank-Dependent Preferences. *Review of Financial Studies, 18*(4), 1467-1502. doi: 10.1093/rfs/hhi033

Prakash, A. J., De Boyrie, M. E., & Hamid, S. (1997). Effect of intervaling on rate of return probability distribution. *International Journal of Finance, 9*(4), 820-833.

Prakash, Arun J., Chang, Chun-Hao, & Pactwa, Therese E. (2003). Selecting a portfolio with skewness: Recent evidence from US, European, and Latin American equity markets. *Journal of Banking & Finance, 27*(7), 1375-1390.

Pratt, John W. (1964). Risk Aversion in the Small and in the Large. *Econometrica, 32*(1/2), 122-136.

Purshouse, R. C., & Fleming, P. J. (2007). On the Evolutionary Optimization of Many Conflicting Objectives. *Evolutionary Computation, IEEE Transactions on, 11*(6), 770-784. doi: 10.1109/tevc.2007.910138

Redl, Christian, Haas, Reinhard, Huber, Claus, & Böhm, Bernhard. (2009). Price formation in electricity forward markets and the relevance of systematic forecast errors. *Energy Economics, 31*(3), 356-364. doi: http://dx.doi.org/10.1016/j.eneco.2008.12.001

Ross, S. A. (1981). Some stronger measures of risk aversion in the small and the large with applications. *Econometrica, 49*(3), 621-638.

Rubinstein, Mark E. (1973). The Fundamental Theorem of Parameter-Preference Security Valuation. *The Journal of Financial and Quantitative Analysis, 8*(1), 61-69.

Ryoo, H. S. (2006). A compact mean-variance-skewness model for large-scale portfolio optimization and its application to the NYSE market. *J Oper Res Soc, 58*(4), 505-515.

Samuelson, Paul A. (1970). The Fundamental Approximation Theorem of Portfolio Analysis in terms of Means, Variances and Higher Moments. *The Review of Economic Studies, 37*(4), 537-542.

Scott, Robert C., & Horvath, Philip A. (1980). On the Direction of Preference for Moments of Higher Order than the Variance. *The Journal of Finance, 35*(4), 915-919.

Sears, R. Stephen, & Wei, K. C. John. (1985). Asset Pricing, Higher Moments, and the Market Risk Premium: A Note. *The Journal of Finance, 40*(4), 1251-1253.

Sears, R. Stephen, & Wei, K. C. John. (1988). The Structure Of Skewness Preferences In Asset Pricing Mode. *The Financial Review, 23*(1), 25.

Shapiro, S. S., & Wilk, M. B. (1965). An Analysis of Variance Test for Normality (Complete Samples). *Biometrika, 52*(3/4), 591-611. doi: 10.2307/2333709

Sharpe, William F. (1964). Capital Asset Prices: A Theory of Market Equilibrium under Conditions of Risk. *The Journal of Finance, 19*(3), 425-442.

Shefrin, Hersh, & Statman, Meir. (2000). Behavioral Portfolio Theory. *The Journal of Financial and Quantitative Analysis, 35*(2), 127-151.

Shleifer, Andrei, & Summers, Lawrence H. (1990). The Noise Trader Approach to Finance. *The Journal of Economic Perspectives, 4*(2), 19-33. doi: 10.2307/1942888

Shoaf, J., & Foster, J. A. (1998, 4-9 May 1998). *The efficient set GA for stock portfolios.* Paper presented at the The 1998 IEEE International Conference on Evolutionary Computation, ICEC'98

Simkowitz, Michael A., & Beedles, William L. (1978). Diversification in a Three-Moment World. *The Journal of Financial and Quantitative Analysis, 13*(5), 927-941.

Singleton, J. Clay, & Wingender, John. (1986). Skewness Persistence in Common Stock Returns. *The Journal of Financial and Quantitative Analysis, 21*(3), 335-341.

Skolpadungket, P., Dahal, K., & Harnpornchai, N. (2007, 25-28 Sept. 2007). *Portfolio optimization using multi-obj ective genetic algorithms.* Paper presented at the IEEE Congress on Evolutionary Computation, CEC 2007.

Smith, Daniel R. (2007). Conditional coskewness and asset pricing. *Journal of Empirical Finance, 14*(1), 91-119.

Soleimani, Hamed, Golmakani, Hamid Reza, & Salimi, Mohammad Hossein. (2009). Markowitz-based portfolio selection with minimum transaction lots, cardinality constraints and regarding sector capitalization using genetic algorithm. *Expert Systems with Applications, 36*(3, Part 1), 5058-5063.

Srinivas, N., & Deb, Kalyanmoy. (1994). Muiltiobjective Optimization Using Nondominated Sorting in Genetic Algorithms. *Evolutionary Computation, 2*(3), 221-248. doi: 10.1162/evco.1994.2.3.221

Statman, Meir. (1987). How Many Stocks Make a Diversified Portfolio? *The Journal of Financial and Quantitative Analysis, 22*(3), 353-363. doi: 10.2307/2330969

Statman, Meir. (2004). The Diversification Puzzle. *Financial Analysts Journal, 60*(4), 44-53. doi: 10.2307/4480587

Steuer, Ralph, Qi, Yue, & Hirschberger, Markus. (2007). Suitable-portfolio investors, nondominated frontier sensitivity, and the effect of multiple objectives on standard portfolio selection. *Annals of Operations Research, 152*(1), 297-317. doi: 10.1007/s10479-006-0137-1

Suksonghong, Karoon, & Goh, Kim Leng. (2012). *Forecasting Electricity Spot Prices: GARCH with Seasonality*. Paper presented at the 2012 IEEE International Conference on Engineering Technology and Economic Management.

Tayi, Giri Kumar, & Leonard, Paul A. (1988). Bank Balance-Sheet Management: An Alternative Multi-Objective Model. *The Journal of the Operational Research Society, 39*(4), 401-410.

Tobin, J. (1958). Liquidity Preference as Behavior Towards Risk. *The Review of Economic Studies, 25*(2), 65-86.

Topol, Richard. (1991). Bubbles and Volatility of Stock Prices: Effect of Mimetic Contagion. *The Economic Journal, 101*(407), 786-800. doi: 10.2307/2233855

Tsiang, S. C. (1972). The Rationale of the Mean-Standard Deviation Analysis, Skewness Preference, and the Demand for Money. *The American Economic Review, 62*(3), 354-371.

Tsiang, S. C. (1974). The Rationale of the Mean-Standard Deviation Analysis: Reply and Errata for Original Article. *The American Economic Review, 64*(3), 442-450.

Tversky, Amos, & Kahneman, Daniel. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty, 5*(4), 297-323. doi: 10.1007/bf00122574

Vedarajan, G., Chan, L. C., & Goldberg, D. E. (1997). *Investment Portfolio Optimization using Genetic Algorithms.* Paper presented at the The Genetic Programming 1997 Conference, Stanford University, California.

Viehmann, Johannes. (2011). Risk premiums in the German day-ahead Electricity Market. *Energy Policy, 39*(1), 386-394. doi: http://dx.doi.org/10.1016/j.enpol.2010.10.016

Wang, Jia, Meric, Gulser, Liu, Zugang, & Meric, Ilhan. (2009). Stock market crashes, firm characteristics, and stock returns. *Journal of Banking & Finance, 33*(9), 1563-1574.

Wood, A.J., & Wollenberg, B.F. (1996). *Power Generation, Operation, and Control* (2 ed.). New York: McGraw-Hill.

Xiaohong, Guan, Jiang, Wu, Feng, Gao, & Guoji, Sun. (2008). Optimization-Based Generation Asset Allocation for Forward and Spot Markets. *Power Systems, IEEE Transactions on, 23*(4), 1796-1808. doi: 10.1109/tpwrs.2008.2002184

Xu, Jianguo. (2007). Price Convexity and Skewness. *The Journal of Finance, 62*(5), 2521-2552.

Yap, B. W., & Sim, C. H. (2011). Comparisons of various types of normality tests. *Journal of Statistical Computation and Simulation, 81*(12), 2141-2155. doi: 10.1080/00949655.2010.520163

Yazici, Berna, & Yolacan, Senay. (2006). A comparison of various tests of normality. *Journal of Statistical Computation and Simulation, 77*(2), 175-183. doi: 10.1080/10629360600678310

Yu, Lean, Wang, Shouyang, & Lai, Kin Keung. (2008). Neural network-based mean–variance–skewness model for portfolio selection. *Computers & Operations Research, 35*(1), 34-46. doi: http://dx.doi.org/10.1016/j.cor.2006.02.012

Zitzler, E., Laumanns, M., & Thiele, L. (2002a). SPEA-II: Improving the strength Pareto evolutionary algorithm (Vol. Technical Report no.103): Swiss Federal Institute of Technology (ETH), Zurich, Switzerland.

Zitzler, E., Laumanns, M., & Thiele, L. (2002b). *SPEA-II: Improving the strength Pareto evolutionary algorithm*. Computer Engineering and Networks Laboratory (TIK), Swiss Federal Institute of Technology (ETH), Zurich, Switzerland.

Zitzler, E., & Thiele, L. (1999a). Multiobjective evolutionary algorithms: a comparative case study and the strength Pareto approach. *Evolutionary Computation, IEEE Transactions on, 3*(4), 257-271. doi: 10.1109/4235.797969

Zitzler, E., & Thiele, L. (1999b). Multiobjective evolutionary algorithms: a comparative case study and the strength Pareto approach. *IEEE Transactions on Evolutionary Computation, 3*(4), 257-271. doi: 10.1109/4235.797969

Zitzler, Eckart, Deb, Kalyanmoy, & Thiele, Lothar. (2000). Comparison of Multiobjective Evolutionary Algorithms: Empirical Results. *Evolutionary Computation, 8*(2), 173-195. doi: 10.1162/106365600568202

Zitzler, Eckart, & Thiele, Lothar. (1998). Multiobjective optimization using evolutionary algorithms — A comparative case study. In A. Eiben, T. Bäck, M. Schoenauer & H.-P. Schwefel (Eds.), *Parallel Problem Solving from Nature — PPSN V* (Vol. 1498, pp. 292-301): Springer Berlin / Heidelberg.