Chapter 6

The evolutionary study of δ-Scorpii

6.1 SOME PREVIOUS WORK ON THE EVOLUTIONARY STAGE OF BE STARS

In general, the evolutionary stage of a Be star at the centre is uncertain owing to the presence of the surrounding circumstellar envelope, which forms at the equatorial region of the star. There are two general phenomenological mechanisms of envelope formation; firstly, in the case of a binary star and secondly, for a single star.

For binary stars, which had a potential of mass transfer, the Be phenomenon could arise after a Roche-lobe overflow event, when one of the components gains mass and angular momentum from the donor (Packet, 1981; Harmanec, 1987; Gies, 2000). The envelope formation in some interacting binaries such as a Be/X-ray binary is well explained by this mechanism (Coe, 2000). In this case, the evolutionary stage of the Be star would depend on the evolutionary stage of the mass-losing star as well as on the properties of the circumstellar envelope (Harmanec, 2000).

Not all Be stars are binaries and not all Be binaries are interacting binaries that enable the Roche-lobe overflow to occur. Thus, the Be phenomenon not only occur in binaries but also in single stars. In the case of a single Be star, the circumstellar envelope is formed by the process of mass ejection at the equatorial region of the stars at the centre. For the mass to be ejected equatorially, certain velocities need to be attained in order to launch the material into orbit, which demands that the star's rotation rate reaches a certain limit. Be stars have long been known as fast rotators; rotating at least 1.5 to 2 times faster than normal B stars (Slettebak, 1979; Zorec, 2004). Much work has been done on determining the rotation rate of Be stars. Slettebak (1949, 1966) and Slettebak, Collins &Truax (1992) have scrutinised the hypothesis of the critical velocity of the star by Struve (1931), stating that the rotation of Be stars is supposed at a subcritical rate with $v_e/v_c \approx$ 0.7–0.8 (Porter, 1996; Chauville, et al., 2001). Townsend, et al. (2004) suggest that the rotation must be much closer to the critical value than $v_e/v_c \approx 0.7$, if rotation does play a direct causal role in the Be phenomenon. In fact, the rotation rate of $v_e/v_c \approx 0.95$ should be reached for launching material into the orbit. The study of rotational effects on fundamental stellar parameters by Frèmat et al. (2005) shows that Be stars rotate at a higher average of rotational rate, $\omega \approx 0.9$. Owing to the fast rotation, the stars will be flattened at the polar region and bulge at equatorial regions, which in turn induces the gravity darkening effect (Zeipel, 1924). This non-uniform surface gravity and temperature distribution reduces the central temperature and total bolometric luminosity that causes the star to burn its available nuclear fuel more slowly and the duration of the main sequence evolutionary phase is longer than that of a slowly rotating star of the same mass (Sackmann, 1070; Clement, 1979; Smith, 1986; Meynet&Maeder, 2000).

Previous work of the evolutionary status of Be stars suggested that the stars start to appear in cluster age around 10 Myr and show the highest frequency in cluster age 13–25

Myr (Fabregat&Torrejon, 2000). After this age, the frequency of Be stars decreased. The decreased frequency was found consistent with the dependence of the Be stars abundance to the spectral type, which reaches a maximum at the spectral type of B1–B2 (Zorec&Briot, 1997). Feinstain (1990) also studied Be stars in open clusters and noticed that the frequency of these stars was maximum near the middle age of the MS phase. Fabregat&Torrejon (2000) found that clusters younger than 10 Myr and without associated nebulosity are almost completely lacking of Be stars. They had suggested that the Be phenomenon is an evolutionary effect that appears in the second half of the main sequence lifetime of a B star.

6.2 AIM OF THE EVOLUTIONARY STUDY

The evolutionary stage of δ-Scorpii is studied through the model of stellar evolution process of this star created using EV stellar evolution code developed by Eggleton (2002). The main purpose is to find a correlation between the evolutions of the star's rotation rate from zero age main sequence (ZAMS) to terminal age main sequence (TAMS) and the variation of the observed *FWHM* of the photospheric line, *HeII*λ4686 for this study. The rotational velocity of δ-Sco, $v \sin i$ = 165 km/s (Abt, etal., 2002) was adopted for this study. From the speckle interferometric observations by Miroshnichencko et al. (2001), the inclination angle i = 38°±5°. If the inclination angle and rotational velocity of this star are true, then the rotation speed v of this star on its axis is about 268 km/s. This value will then be compared with the rotation calculated from the evolutionary model.

6.3 THE PARAMETERS

In this section, we discuss the parameters that have been used in the evolution process. There are two types of parameters: the initial input parameters and the output parameters. The initial input parameters are the main constituents that generate the evolution process where the values are varied randomly, namely the initial mass and initial rotation at a given metallicity. There are many other main constituents involved in the evolution process; however, their values were maintained when the evolution process began.

The evolving output parameters that we considered in this study are actually the star's physical characteristics, particularly the temperature, luminosity, age, mass, radius, rotation and radiation pressure. In each step of the evolutionary track models, there exist eighty-eight (88) parameters used for subsequent analysis. We show some parts of the output file that lists the output values of every step in the evolution process in Appendix C. A simple script in Fortran (Appendix D) has been written to tabulate a few parameters from several thousand of generated data. We will explain our interest in the parameters of the evolutionary process in the following sections.

6.3.1 Mass

In this study, the initial mass of the star is the mass when it starts burning hydrogen and the value is randomly determined. The initial mass is supposed slightly higher than its current mass as δ-Sco is still at the main sequence phase. A value in the range of 12.5 to 13.2 $M_\odot$ was chosen for the initial mass, which is slightly higher than its current mass, 12.4 ± 0.8 $M_\odot$(Tycner, 2011).

6.3.2 Rotation

Rotation is one of the important characteristics for distinguishing between Be and normal B stars. In this case, rotation is referred to the surface rotation of the star and the value can be increased or decreased up to certain limits depending on the stellar mass. The evolution process at ZAMS of a star can be run with some acceptable rotation speeds at the mass given, as long as the angular momentum of the star is conserved during the evolution. The initial rotation speed would be decreased if we found that the evolutionary process cannot be initialised or run for the given stellar mass for conserving the angular momentum. This value can be changed in the configuration file, fort.23 at *P1* parameter (refer to Appendix B). The rotation speed is given in day units that can be converted into km/s unit by the following relation:

$$v_{rot} = \frac{2\pi R}{P\_rot} \tag{6.1}$$

where v_{rot} and P_rot are the rotation speed in km/s and days or seconds per rotation, respectively and R is the radius of the star in km.

In this study, we used code that produces a star that is in uniform rotation, i.e., the rotation period is the same throughout the star at a given time. However, this uniform rotation rate varies with time as the star evolves because the moment of inertia increases (mostly but not always) with time and because the angular momentum drops as the stellar wind carries off some of the angular momentum.

Our program star δ-Sco, is an early type star that is still burning hydrogen in its core; thus, we only extend the evolution process up to the core helium-burning phase, as we are focusing only on the changes of the parameters understudy, particularly the star's rotation rate from ZAMS to TAMS. The initial rotation has been chosen in such a way that the rotation rates of the star could not exceed its critical ratio $v_{rot}/v_{cr} \leq 1$ during the main sequence phase.

6.3.2.1 Critical velocity

At critical velocity, the modulus of the centrifugal force becomes equal to the modulus of the gravitation attraction at the equator. The maximum angular velocity reached gives the effective gravity $g_{eff} = 0$ at the equator and thus,

$$\Omega_{crit}^2 = \frac{GM}{R_{e,crit}^3} \quad (6.2)$$

where $R_{e,crit}$ is the equatorial radius at break-up, which is also equal to 1.5 times the polar radius at critical $R_{p,crit}$. For a star with a highly condensed centre, a solid body rotation can thus be applied in eq.(6.2). Therefore, the critical velocity at the equator of the star is deduced as follows,

$$v_{crit}^2 = \Omega_{crit}^2 R_{crit}^2 = \frac{GM_*}{R_{e,crit}} = \frac{2GM_*}{3R_{p,crit}} \quad (6.3)$$

For the case of rotating stars with a high radiation pressure, the break-up velocity is written as:

$$v_{crit}^2 = \frac{GM}{R}(1 - \Gamma) \quad (6.4)$$

where Γ is the Eddington factor. This is the ratio of the stellar to the Eddington luminosity, which can be expressed as

$$\Gamma = \frac{\kappa L}{4\pi c G M} \tag{6.5}$$

where the Eddington luminosity is $L_{Edd} = \frac{4\pi c G M}{\kappa}$ (6.6)

and κ is the opacity or absorption coefficient.

For a fast rotating star of mass *M*, the upper layers of the star are no longer bound if the Eddington luminosity is reached at the surface. This would cause high mass loss but it still depends on the mass and rotational velocities. At critical velocity, the total gravity somewhere on the stellar surface is zero. From eq.(6.7)

$$g_{tot} = g_{eff}\,[1 - \Gamma(\Omega, \vartheta)] \tag{6.7}$$

Thus, $$g_{eff}\,[1 - \Gamma_{\Omega}(\vartheta)] = 0 \tag{6.8}$$

There are two solutions given for eq.(6.8): 1) g_{eff}= 0 and 2) $\Gamma_{\Omega}(\vartheta)$=1. The former implies that the centrifugal acceleration is equal to the Newtonian gravity at the equator, i.e.,

$$\Omega^2 R_{e,crit}^3 = \mathrm{GM} \tag{6.9}$$

Thus, this gives the critical velocity at the equator as

$$v_{crit,1} = \Omega R_{e,crit} = \left(\frac{2}{3}\frac{GM}{R_{p,crit}}\right)^{1/2} \tag{6.10}$$

where the polar radius at the critical velocity $R_{p,crit} = \frac{2}{3} R_{e,crit}$

It is clear from eq.(6.10) that the critical velocity is independent of the Eddington factor, except for Γ values above 0.639 (Maeder, 2009a) owing to von Zeipel's theorem. This theorem says that the radiative flux at the surface of a rotating star is proportional to the local effective gravity at the considered colatitude, which means that when the rotation of the star increases, the radiative flux at the equator decreases, the same way as does the effective gravity. Therefore, the decrease of the effective temperature at the equator prevents significant radiation pressure effects for stars with moderate rotation and is only true if the value of Γ is small.

The other solution of eq.(6.8) is $\Gamma_\Omega(\vartheta)$=1. This solution is for the case of critical velocity near the Eddington limit. $\Gamma(\Omega,\vartheta)$ is defined as the local Eddington ratio at the surface of a rotating star where

$$\Gamma(\Omega,\vartheta) = \frac{\kappa(\Omega,\vartheta)L(P)}{4\pi cGM\left(1-\Omega^2/2\pi G\,\overline{\varrho_M}\right)} \qquad (6.11)$$

For $\Gamma_\Omega(\vartheta)$=1, eq.(6.11) can then be written as

$$\frac{\kappa(\Omega,\vartheta)L(P)}{4\pi cGM} = 1 - \frac{\Omega^2}{2\pi G\,\bar{\varrho}_M} \qquad (6.12)$$

Substituting eq.(6.5) in eq.(6.12), it can then be simplified as

$$\frac{\Omega^2}{2\pi G\,\bar{\varrho}_M} = 1 - \Gamma \qquad (6.13)$$

where $\bar{\varrho}_M = \frac{M}{V}$; thus,

$$\frac{\Omega^2 V}{2\pi GM} = 1 - \Gamma \qquad (6.14)$$

Given that $V^{'}(\omega) = \frac{V(\omega)}{\frac{4}{3}\pi R_{p,crit}^{3}}$ and $\omega^2 = \frac{\Omega^2 R_{e,crit}^3}{GM}$, the term $\frac{\Omega^2 V}{2\pi GM}$ in eq.(6.14)

can be written as

$$\frac{16}{81}\omega^2 V^{'}(\omega) = 1 - \Gamma \tag{6.14}$$

From eq.(6.14), the rotation parameter ω *(=* Ω/Ω_c*)* can be determined for a given value of Γ. The star had a maximum Γ ratio when it reached break-up velocity. The maximum value of $\frac{16}{81}\omega^2 V^{'}(\omega)$ is 0.3607 when ω=1 (Meader, 2009b). If Γ happened to be larger than 0.6393, then the maximum value of ω is smaller than 1.0. Thus, the critical velocity of Γ is larger than 0.6393 and $v_{crit,2}$ is given by

$$v_{crit,2}^2 = \Omega^2 R_e^2(\omega) = \frac{8}{27}\frac{GM\omega^2}{R_{p,crit}^3} R_e^2(\omega) \tag{6.16}$$

where $\Omega^2 = \frac{8}{27}\frac{GM\omega^2}{R_{p,crit}^3}$ and $R_e(\omega)$ is the equatorial radius of the corresponding rotation parameter. The above expression of $v_{crit,2}$ can be correlated with $v_{crit,1}$ (eq. (6.10)) and an Eddington factor (eq. (6.15)) as follows

$$v_{crit,2}^2 = \frac{9}{4} v_{crit,1}^2 \frac{R_e^2(\omega)}{R_{p,crit}^2}\frac{1-\Gamma}{V^{'}(\omega)} \tag{6.17}$$

Eq.(6.17) shows the critical velocity of stars having an Eddington factor larger than 0.6393. The stellar mass at this condition was calculated to be much higher than 100 $M_{\odot}$ (Maeder, 2009c). Based on this relation, it shows that stars with high Γ have a lower value of critical velocity compared with $v_{crit,1}$. In the extreme condition, if Γ = 1, the stars might reach their critical velocity even if the surface velocity is zero, which means that the surface is unbound. However, the critical velocity of this second solution of eq. (6.8) was not the aim of this study. As the mass of δ-Sco is 12.4 ± 0.8 $M_\odot$(Tycner, 2011), the value of Γ is calculated within the range of 0.039 to 0.060 (Meader, 2009c) and therefore, the critical velocity at the equator with condition of $g_{eff} = 0$ is applied in this study.

The most massive stars experience high mass loss that will also remove a lot of angular momentum, which causes a huge decrease in their surface velocities during the main sequence phase. However, for stars with masses ≤12$M_\odot$ with an initial high rotation speed, they might easily reach break-up velocities near the end of the main sequence phase (Meynet&Maeder, 2000). This circumstance could be related to the appearance of Be stars during the main sequence phase.

6.3.3 Effective temperature T_{eff}

Effective temperature is one of the output parameters that have been used to estimate the position of the star on its evolutionary track. This temperature is actually the surface temperature of the star label as *log T* in Table 6.2. In this study, the surface temperature of 27,000 K (*log T* = 4.4314) for δ-Sco has been adopted from Grigsby et al. (1992).

6.3.4 Radius

The other output parameter that we considered in locating the position of the star on the evolutionary track is that of the star's radius. The radius parameter *log R* (as in Table 6.2) generated from the evolution process was an average of the star's surface radius. As for the estimation of the current radius of the star, one of the methods derived from the definition of the effective temperature and the distance modulus has been employed, as explained below.

The radius of δ-Sco in this study is deduced based on its effective temperature and distance via the bolometric flux method, which operates using the following equation (Harmanec, 2000),

$$\log R/R_{\odot} = 7.474 - 2\log T_{eff} - 0.2\, BC - 0.2\, m_v - \log \pi \qquad (6.18)$$

where R is the radius of the star, T_{eff} is the estimated effective temperature, BC is the bolometric correction, m_v is the observed visual magnitude and π is parallax (in seconds of arc). The value of the associated BC is derived from the relationship between the corrected T_{eff} and BC (Flower, 1996), as depicted in Figure 6.1.

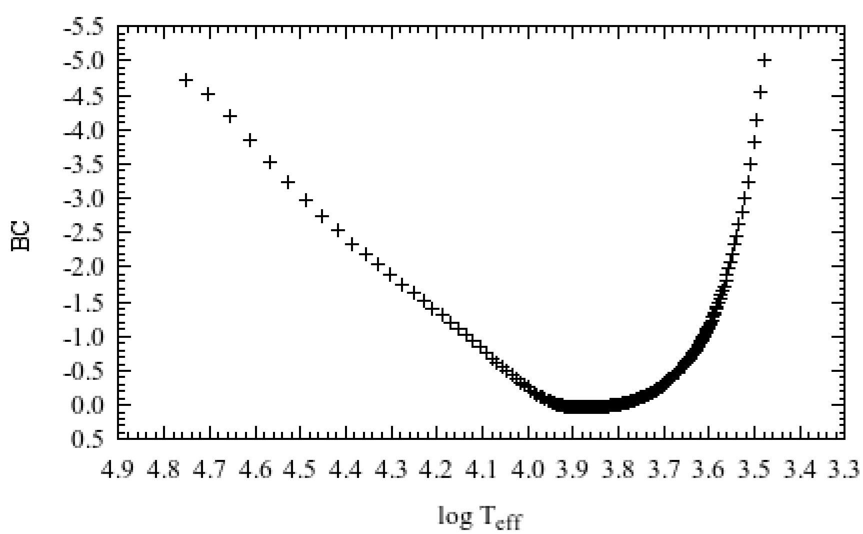

Figure 6.1 – Correlation of BC and log T_{eff}(Flower, 1996)

Because of the specific curve of BC versus log T_{eff}, we divided the curve into three regions: log $T_{eff} \geq 3.9$, $3.6 <$ log $T_{eff} < 3.9$ and log $T_{eff} \leq 3.6$. The estimated T_{eff} of δ-Scorpii of 27,000K (Grigsby et al., 1992) was employed in this study. The correlation of log T_{eff} and BC in the region of log $T_{eff} \geq 3.9$ is expressed in eq. (6.19), as follows:

$$BC = 1.1919 \log T_{eff}^{3} - 17.119 \log T_{eff}^{2} + 75.244 \log T_{eff} - 103.66 \qquad (6.19)$$

From eq.(6.19), the corresponding BC for log T_{eff} of 4.43 is -2.67, which gives 5.99 $R_{\odot}$(*log R* = 0.77) as the radius of δ-Sco by eq. (6.18) with π= 8.12 mas and m_V= 2.29 (Perryman, 1997). Hence, this radius value has been used as a guide or clue to estimate the current position of the star on the evolutionary track.