4.1 Image Reduction and Calibration

4.1.1 Reduction Process

Every raw image of stellar spectra taken directly from CCD camera consists of many effects layered on top of the raw data due to the detector’s nature and also the optical imperfections in the components of the instruments used such as vignetting. Thus, it is necessary to remove these undesirable layers by doing reduction process. The basic procedures can be simplified as Figure 4.1.

![Figure 4.1: The process procedure on the raw images in order to get clean data.](image)

This process also includes the transformation from the raw data into some measure of intensity or energy flux as a function of wavelength. The reduction was conducted fully by using IRAF.
Prior to the reduction process, there are several operations that need to be performed. Firstly is to make a master frame. This can be done by taking the average of 5 to 8 frames for each bias frame, flat-field frame and dark frame, thus averaging each frame respectively in order to reduce the random noise. For simply averaging the bias, flat-field and dark frame, ‘imsum’ task can be used.

> imsum [file1, file2, … , file5] [outputnamefile] optione=average verb+

The term ‘verb+’ at the end of the command is to list out the images and its operation involved. Thus, it is an optional.

Secondly, subtract the raw image with master bias and master dark frame. This part can be neglected if the CCD is very efficient to cool down to -40°C below surrounding temperature. Additionally, subtracting the bias frame is to correct the pixel value due to the offset value of the CCD.

The subtracting can be done by using ‘imarith’ task in IRAF. The task is basically binary image arithmetic which simply does the arithmetic operation on the images just like numbers. Whatever the number of noise read at a pixel, the same amount will be subtracted from the data image.

> imarith [operandfile1] [operation +,-,*,/] [operandfile2] [outputfilename]

Alternatively, we could list out the parameter of the command and do the operation by key in the images file name and the operation needed for the arithmetic to be completed. Press “:go” to run the command.
4.1.2 Data Extraction

The clean reduced data are still containing dark background above and below the spectrum which is unnecessary for the data analysis. Hence, it is vital to extract the two-dimensional image to one-dimensional image. This task can be done under ‘noao’ group and ‘apextract’ package with task name ‘apsum’.

For each input image, the two dimensional spectra are extracted to one dimensional spectra by summing the pixels across the dispersion axis at each wavelength within a set of defined apertures. It consists of an aperture number, a beam number, a title, a center, limits relative to the center, a curve describing shifts of the aperture center across the dispersion axis as a function of the wavelength, and parameters for background fitting and subtraction.

After we assign which aperture to be extracted (basically it is automatically chosen if there is only one), the data need to be fitted according to a suitable function such as ‘legendre’ or ‘spline3’ prepared in the task. The data needs to be fitted by adjusting the order, deleting anomalies and reducing the root mean square value to get a good extraction. However, by overdoing the deleting part, there is a possibility for the spectra to be cut.
The output image root names are specified by the output list. If the list is empty or shorter than the input list the missing names are taken to be the same as the input image names. Because the root names have extensions added, it is common to default to the input names in order to preserve a naming relation between the input dimensional spectra and the extracted spectra.

![Figure 4.3](image)

4.1.3 Wavelength Calibration

Spectral wavelength calibration consists of establishing the dispersion relationship of pixel to wavelength of the profile. This technical procedure is needed to make possible of the corrections, calculations and comparisons of the spectral processing. Basically, the operation identified the wavelength of the object of interest by using certain comparison spectrum. By assuming the law of dispersion is linear, whatever domain the wavelength is, the spectrum always has the same calibration.
Figure 4.4: The two-dimensional neon profile lines image for wavelength reference.

Figure 4.5: The neon profile lines seen in three dimensions.

Figure 4.6: The extracted spectrum of neon profile lines before the calibration.
In order to do the calibration, it is necessary to have a spectral reference for calibration lamps used since every element has its own unique spectral identification. The calibration is being done on the extracted spectrum of calibration by using same aperture as the stellar spectrum data.

Then, the profile lines should be identified so that each frame can be known of representing which range of wavelength. In this part, “identify” task is being used.

---

**Figure 4.7:** The “identify” task which is to identify the reference wavelength in the calibration process.

In this task, characteristics in the input images are recognized interactively and assigned user coordinates. A "coordinate function" mapping pixel may be determined from the identified features. A user coordinate list may be defined to automatically identify additional features. This task is also used to assess positions of features and determine spectral dispersion solutions.
At this stage, by referring to the list of element line profile of the calibration lamps, two lines are needed at least to be marked with exact referred value of wavelength. Wrong value can cause inaccuracy in the later process and analysis.

As shown in Figure 4.8, it is clearly that the raw spectral image is horizontally flipped from the real spectrum due to the reflection process in the optical instruments. That is one of the important reasons for doing wavelength calibration.

![Figure 4.8: The wavelength that has been marked and labelled.](image)

Then, the clean data that has been reduced and extracted in previous steps need to be assigned to “refspectra” task in order to allocate wavelength reference spectra. The calibration profile that we have calibrated before will be linked as reference to that particular spectral frame. From here, the spectrum will be given the wavelength before it could be analyzed.

```
> refspectra [extractedfilename] referen=[calibrationfile]
```
After wavelength calibration has been assigned on the spectrum image, ‘dispcor’ task is useful at this stage. The main purpose if this task is to do dispersion correction and resample spectra. In addition, we will also know the specific range of the wavelength in the particular frame and the difference from one end to the other end of the frame will be shown as well. Hence, the number of angstroms per pixel will be known.

> dispcor [filename] [outputfilename]
4.1.3 Normalization

The normalization is adjusting the relative intensity of the profile with reference continuum value of 1. This is due to the imbalance value of each spectrum because of the difference in exposure time, noise value and spectral characteristics. Hence, it is fair to do the analysis when they are all normalized to the same value.

The normalization process can be done also in IRAF under the task ‘continuum’. In this task, a one-dimensional function is fit to the spectral continuum in an option of multispec, echelle, or onedspec format images and then divided into the spectrum to produce continuum normalized spectra. The first two formats will normalize the spectra or lines in each image.
The parameters of the fitting may differ from line to line within images and between images. There are options of function to be chosen in order to have a very nice fitting which are legendre or chebyshev polynomial, linear or cubic spline of a given number or order of spline pieces.

<table>
<thead>
<tr>
<th>PACKAGE</th>
<th>onedspec</th>
</tr>
</thead>
<tbody>
<tr>
<td>TASK</td>
<td>continuum</td>
</tr>
</tbody>
</table>

- **input**: input images
- **output**: output images
- **lines**: Image lines to be fit
- **bands**: Image bands to be fit
- **type**: Type of output
- **replace**: Replaces rejected points by fit?
- **wavescale**: Scales the X axis with wavelength?
- **logscale**: Takes the log (base 10) of both axes?
- **override**: Overrides previously fit lines?
- **liston**: List fit but don’t modify any images?
- **logfile**: List of log files
- **interactive**: Set fitting parameters interactively?
- **sample**: Sample points to use in fit
- **naverage**: Number of points in sample averaging
- **function**: Fitting function
- **order**: Order of fitting function
- **low rej**: Low rejection in sigma of fit

**Figure 4.11**: The “continuum” task for spectrum normalization under “onedspec” package.
4.2 Spectral Analysis

This part of the chapter will consider about the analysis of the processed data including the measurements on the Hα line profile, the existence of TiO and the variation of spectral shift.

All of the analysis is done by using IRAF especially in determining the equivalent width (EW) and full width at half maximum (FWHM) especially for Hα, Ca I and Fe I. Hence, based on this information, radial velocity can be obtained as well.

4.2.1 Hα 6562 Å spectral line profile

Line profile is actually the intensity distribution $I(\lambda, \theta)$ or flux $F(\lambda)$ as a function of wavelength while $\theta$ denotes the angle between the normal layer to the emitting layer and the line of sight (Schatzman and Praderie, 1993). $I(\lambda, \theta)$ can be measured if the surface angle, such as surface of the sun, can be measured. For stars, however, is hard because its disk cannot be resolved like sun.
Hence, the energy $f(\lambda)$ received from the star’s surface can be written as below:

$$f(\lambda) = \frac{L(\lambda)}{4\pi rd^2} \quad \text{(4.1)}$$

while,

$$L(\lambda) = 4\pi R^2 \int_0^{\infty} F(\lambda) \, d\lambda \quad \text{(4.2)}$$

hence,

$$= \frac{R^2}{d^2} F(\lambda) \quad \text{(4.3)}$$

where

- $F(\lambda)$ = the flux emitted at the stars surface with wavelength $\lambda$
- $L(\lambda)$ = the luminosity function
- $d$ = distance from earth to the star
- $R$ = effective radius of the star

The spectral resolution of the spectrograph $\lambda/\Delta\lambda$ and the signal-to-noise ratio determines the amount of information contained in a spectral line. A good data should have spectral resolution of 50,000 and signal-to-noise ratio of 100 (Schatzman and Praderie, 1993). The data in this work are having signal-to-noise ratio in the range of 10-200.

### 4.1.1.1 Equivalent width (EW)

A measure of strength or intensity of a spectral line is equivalent width. An equivalent line is expressed in unit of wavelength which is angstrom. On a plot of intensity against wavelength, a spectral line appears as a curve with a shape defined by the line profile. It characterized the power of a line with respect to the continuum. The equivalent width is the width of a rectangle centred in a spectra line. For a plot of
intensity against wavelength, it has the same area as the line itself. This can be specifically defined as the section of surface measured between the level of continuum, normalized to one, and reference zero, then having a surface identical to line profile (Schatzman and Praderie, 1993). It is an integral quantity where it can be expressed as:

\[
W = \int_{\lambda_1}^{\lambda_2} \frac{F_c(\lambda) - F_1(\lambda)}{F_c(\lambda)} d\lambda
\]

where
- \(F_c\) = the continuum flux which equals to 1
- \(F_1\) = the line flux
- \(d\lambda\) = the sampling in wavelength unit

Figure 4.12: The illustration of equivalent width.

The equivalent width is measured by using ‘splot’ command in IRAF where it detects a spectral curve and assign a measurement on it. Shown in table 4.1 is the value of equivalent width (EW) of the observed stars. The values of the equivalent width are arranged according to the decimal subdivision of spectral type.
The amount of energy or intensity depends on the number of atoms giving rise to the line profile because of the intensity which is proportional to the number of photons emitted or absorbed by this particular atom. The more atoms present to absorb the photons, the stronger the line is.

4.1.1.2 Full width at half maximum (FWHM)

The terms already tells what it is. Basically, it is width measured at half deep between the continuum level to the topmost peak or bottommost dip of spectral line. Same method is used in order to measure this value.

The FWHM of spectral line profile is assumed to have a resemblance with a Gaussian curve, hence the expression is:

$$e^{\left(\frac{-\lambda^2}{2\sigma^2}\right)}$$

where $\sigma =$ the value of standard deviation
$\lambda =$ the center wavelength value

![Figure 4.13: The illustration of full width at half maximum.](image)
From this value, another parameter can be calculated which is the expansion or disk speed. It can be calculated by using this relation:

\[ v_e = \frac{\text{FWHM}}{\lambda} \times c \]  

\[ \text{----- (4.6)} \]

where \( c = \) the light speed constant in km/s  
\( \lambda = \) the center wavelength value

The expanding of line profile is called line broadening cause by the surrounding or the environment wherever the emission and the absorption took place. This can be influenced by many factors such as Doppler Effect, gas turbulence, pressure broadening, rotation of celestial body, gas turbulence of the Zeeman Effect. This part will be discussed in the next chapter.

IRAF software contains a package to find the value of FWHM as well as EW and other information. Included in ‘splot’ package, the software has its own algorithm that can determine the value automatically by recognizing the curve shape that analyst wanted.
4.1.1.3 Line depth ($R_c$)

Line depth is defined as maximum depth of line. It can be measured empirically from the observed spectrum and used to determine the source function in the star’s atmosphere where the spectral line comes from (Eaton, 1995). Table 4.1 shows the $R_c$ values obtained from the observed spectrum. Just like equivalent width, the deeper the line, the stronger the spectral profile is.
4.1.2 Hα line position and Doppler shifts

The position of Hα in the star’s spectrum is determined by the wavelength calibration. Then, the apparent or observed wavelength of Hα is being compared with the value of wavelength at rest (λ6562.852 Å).

Every object in the universe is moving and even the observed stars are moving in a relative motion towards or away from us. This movement will cause the wavelength of the light from the star having a shift.

This special occurrence is known as Doppler Effect while the magnitude of this change is called the Doppler shift where it can be used in determining the radial velocity of a star. This can be expressed as:

\[ v_r = \frac{\lambda - \lambda_0}{\lambda_0} \times c \]  

where  
\[ c = \text{the constant speed of light in km/s} \]  
\[ \lambda = \text{the observed wavelength of the line} \]  
\[ \lambda_0 = \text{the rest wavelength of the line} \]  

--------

(4.7)

4.1.3 Photospheric Ca I and Fe I lines profile

These two prominent photospheric lines is also a feature that can be seen in late type stars as they cool down through the spectral type (Eaton 1995). The existence of these can be studied by measuring the EW and FWHM for both of Fe I λ6546 Å and Ca I λ6572 Å. These two lines also happen to be spectral companion for Hα line profile since they both located before and after the Hα wavelength.
4.1.4 TiO molecular band

Titanium oxide bands are easily to be seen in M-type star spectra. In this work, the existence of this molecular band at 7055 Å (Howard, 2008) is highlighted and to see which M-type subclasses have the particular feature in their spectra.

![Spectrum of HD 139006 (A0 V) from 6950 to 7700 Å.](image)

**Figure 4.15:** Spectrum of HD 139006 (A0 V) from 6950 to 7700 Å.
Figure 4.16: Spectrum of HD 80493 (M0 IIIvar) from 6950 to 7700 Å.

Figure 4.17: Spectrum of HD 141477 (M1 III) from 6950 to 7700 Å.
Figure 4.18: Spectrum of HD 148478 (M1.5Iab) from 6950 to 7700 Å.

Figure 4.19: Spectrum of HD 39801 (M2 Iab) from 6950 to 7700 Å.
Figure 4.20: Spectrum of HD 167618 (M2 III) from 6950 to 7700 Å.

Figure 4.21: Spectrum of HD 44478 (M3 III) from 6950 to 7700 Å.
Figure 4.22: Spectrum of HD 112300 (M3 III) from 6950 to 7700 Å.

Figure 4.23: Spectrum of HD 133216 (M3/M4III) from 6950 to 7700 Å.
Figure 4.24: Spectrum of HD 156014 (M5 IIvar) from 6950 to 7700 Å.

Figure 4.25: Spectrum of HD 148783 (M6 IIIvar) from 6950 to 7700 Å.
Table 4.1: The results of the analysis based on the observed stars spectra.

<table>
<thead>
<tr>
<th>Star name</th>
<th>HD</th>
<th>Spectral type</th>
<th>B-V Index Colour</th>
<th>Hα (Å) ± 1.076</th>
<th>v_r (km/s) ± 0.001</th>
<th>FWHM M Hα (Å) ± 0.001</th>
<th>EW Hα (Å) ± 0.001</th>
<th>R_e ± 0.001</th>
<th>Δλ_o (Å) ± 0.001</th>
<th>Fe I M Fe I (Å) ± 0.001</th>
<th>EW Fe I (Å) ± 0.001</th>
<th>R_e ± 0.001</th>
<th>Ca I M Ca I (Å) ± 0.001</th>
<th>EW Ca I (Å) ± 0.001</th>
<th>R_e ± 0.001</th>
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</thead>
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<td>A0V</td>
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<td>6564.176</td>
<td>60.481</td>
<td>388</td>
<td>8.472</td>
<td>8.49</td>
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<td>3.532</td>
<td>0.044</td>
<td>3.251</td>
<td>0.013</td>
<td>6574.180</td>
<td>0.400</td>
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<td>0.409</td>
</tr>
</tbody>
</table>

Notes:
- v_r: observed radial velocity
- v_e: expansion speed
- FWHM: full width at half maximum
- EW: equivalent width
- R_e: line depth
- Δλ_o: Doppler width