VIII

THE MORGENSTERN-NEUMANN CARDINAL UTILITY INDEX

A notable feature of the problem of measuring utility which we have discussed so far, is that it concerns consumer choice made under an atmosphere of certainty, i.e. to say, the alternatives open to the individual are "sure prospects". Under certainty, the individual is assumed rationally to choose or prefer among several collection of goods, that collection which yields him the highest level of satisfaction. However, under situations of risk, the theory of consumer choice has to be modified to take into account the risk factor. For example, if 3 utility situations A,B and C are given and the individual prefers A to B and B to C. If given the choice of either the certainty of B or the uncertainty of getting A or C, the individual may prefer the certainty of B even though A is more preferred to B. This is because there is the probability of getting C which yields less satisfaction than B. In the event of facing such choices, the individual will consider 2 factors:

(i) the probability of his getting A and the probability of his getting C and,

(ii) the relative strength of his preference for A over B and for B over C.

The behaviourist cardinal utility theory of Morgenstern and Neumann aims primarily at rationalising the choice of the consumer under such risk situations. The hypothesis is that even under risk, the consumer's choice will still be guided by some sort of utility maximising expectation. How this expectation is calculated will be discussed later. We will show that such choices made under risk, find no place in Marshall's theory and are excluded because he finds it impossible to reconcile them with his Cardinal Utility Theory. But for the present, it is of importance to distinguish the meaning of cardinalism in the Morgenstern-Neumann context, from that of Marshall's.

Difference between Marshallian Cardinalism and the Morgenstern-Neumann Cardinalism

Morgenstern and Neumann have devised their cardinal utility index, not with the intention of reviving Marshallian cardinalism. The originators do not believe in the quantifiability of utility, and more important, their theory provides no support to the theory of aggregation of individual welfare into group welfare.

On the contrary, they are interested in utility measurement only in the operational sense. Whereas to Marshall, utility measurement corresponds to a measurement of some psychic quantity, to Morgenstern and Neumann, it has meaning only as an aid to predicting consumer behaviour under conditions of risk. The important difference is that while Marshall treats the measure as indicating the property of things, the latter considers it of value only as a predictive procedure. It is for this reason that the word 'Cardinal' which is used to describe the index is unfortunate, as it may wrongly be interpreted to mean the orthodox cardinalism of Marshall. Utility measurability, employed to derive their index refers to measurability up to just a linear transformation in contrast to Marshallian quantification.

The emphasis of the M-N Index is on prediction of consumer behaviour, and it is mainly this point which makes the theory behaviouristic.[1]

Marshall's Failure to Explain Choice Under Risk

An interesting question for enquiry is why Marshall

[1] J.V. Neumann & O. Morgenstern, "The Theory of Games and Economic Behaviour", 3rd Edition, 1953, Princeton University Press.

has not integrated the problem of consumer choice under risk, into his theory of utility maximisation. The fundamental reason is that risky choices contradict his principle of maximisation of utility. It will be remembered that Marshall places great reliance on the concept of diminishing marginal utility and that the consumer is assumed to be consciously aware of the law and to maximise his utility under it. Both, the principle of maximisation of satisfaction and the law of diminishing marginal utility will preclude a consumer from bearing risk or engaging in fair games of chances, unless he is paid a premium for it. Such a conclusion is inevitable and obvious under the Marshallian theoretical structure, because under the operation of diminishing marginal utility, if an individual gambles on an even chance, the utility which he gains from winning, say a dollar, is always less than the utility he loses from losing a dollar. On this principle, therefore, an individual who wishes to maximise his utility will never gamble on a 50-50 chance. For this reason, Marshall concludes that choices involving risk cannot be explained by the principle of utility maximisation, and since his theory is essentially one underlying the principle of maximisation, such choices are excluded from his theoretical apparatus.

Here, we may quiry whether, if the risk of a gamble is reduced so as to make the probability of winning very likely, it would not be compatible with Marshall's theory. The answer is that, if the probability of winning is very likely, then the gamble will be in the nature of a sure prospect and will involve no contradiction. If, however, there is a likely chance of losing, then the risky choice will involve a contradiction. Hence, Marshall's conclusion is that whenever an individual does engage in risk, it will then be for non-economic reasons, either that he is ignorant of the odds or is unduly optimistic, or because he derives pleasure from being subjected to uncertainty, for example, in gambling.

In his view, gambling always involves an economic loss, while insurance against risk, on the other hand, is always an economic gain, since the collective uncertainties of all individuals can be reduced to a certainty through premium payments.[2] Marshall's theory, therefore, explains only the taking of insurance, while gambling is left to psychological and irrational motives.

Such a view does not seem to be in accordance with observed facts, judging from the extensive indulgence in gambling and risk-taking of every sort. As a matter of fact, governments in most countries have found that an effective means of raising revenues is through the organisation of national lotteries.

The truth is that both gambling and insurance are extensively carried out by individuals. There is no division between the two activities and an individual may indulge in both of them. In the Marshallian theory, these two activities undertaken by the same individual will definitely involve an irreconciable contradiction. Morgenstern and Neumann have deviced their Utility Index principally to rationalise the consumer's behaviour under just this type of situation. The Utility Index offers the explanation that an individual may conceivably take up insurance or prefer to gamble, depending upon his mathematical expectation of the utility derived from each. Their hypothesis is thus, at variance with Marshall's, since according to the latter, maximisation of utility cannot explain a preference for risk.

However, the M-N Index is internally consistent with itself, since it is calculated not with reference to the law of diminishing marginal utility. We will now discuss the construction and operation of the M-N Utility Index in predicting consumer choice under risk.

[2] Marshall, Op. Cit., p.111 footnote 2.

Construction and Operation of the M-N Index

Previously, we have stated that the M-N Utility Index is strictly operational in sense, with the main purpose of predicting choice among several risky alternatives. We will discuss how this is done, the assumptions involved and the limitations imposed on the theory.

The basic hypothesis of the M-N Cardinal Utility Theory, is that when confronted by several risky alternatives, the consumer will choose that which gives the highest mathematical expectation of utility. Suppose a lottery ticket promises 2 prizes A and B. The mathematical expectation of utility for the lottery ticket is calculated by the formula:-

$$U(L) = P.U(A) + (1-P).U(B)$$

where, U(L) is the mathematical expectation of utility for the lottery, U(A) and U(B), the respective utility of prizes A and B, and P the probability of getting the prizes. The probability value is always 1, so that if the probability of winning A equals P, therefore, (1-P) is the probability of winning B. Thus, if P is given as 2/5, U(A) and U(B) as 10 and 5 respectively, therefore,

$$U(L) = 2/5.(10) + 3/5.(5)$$
$$= 7.$$

This means that the mathematical expectation of utility for the lottery ticket is 7. In this case, we have assumed that the probability and utility measures are given. However, the most important procedure is to derive the numerical measures of probability and utility for without them, the mathematical expectation of utility cannot be calculated.

The process of prediction, which is the major purpose of the theory is conducted on the basis of the same formula. Suppose an individual is confronted with the choice between 2 risky alternatives, represented say, by 2 different lottery tickets, L_1 offering prizes A and B, and L_2 offering

prizes C and D. Assuming that the probability values P_1 of winning the prizes in ticket L_1 and P_2 in ticket L_2, are calculated and given as : $P_1 = 2/3$, $P_2 = 1/5$.
Thus, the mathematical expectation of utility for ticket L_1, will be equal to :-

$$U(L_1) = P_1.(UA) + (1-P_1).U(B)$$
$$= 2/3.U(A) + 1/3.U(B)$$

and for ticket L_2 will be equal to:-

$$U(L_2) = P_2.U(C) + (1-P_2).U(D)$$
$$= 1/5U(C) + 4/5.U(D)$$

Before the mathematical expectations of each ticket can be calculated, the Utility Indices for prizes A,B,C and D have first to be calculated. To do this, necessitates the choice of a utility scale. Using the standard formula of Morgenstern and Neumann, we can assume a 3rd lottery ticket, L_3 with prizes X and Y, which is to serve as the initial scale. Thus:

$$U(L_3) = P.U(X) + (1-P).U(Y)$$

In order to construct a scale, an arbitrary origin and a unit of measurement have to be chosen just as in the case of a temperature scale. In the same way, we can fix our scale so as to make $U(Y) = 0$ (i.e. as the arbitrary origin) and $U(X)=1$ (i.e. as the unit of measure in the equation).

Thus, $$U(L_3) = P.1 + (1-P).0$$
$$= P$$

To arrive at a utility measure for A, we have to ask the individual to state his preference between the prize A and the 3rd or standard lottery L_3. If he prefers A to L_3, it will mean that the utility of A is higher than the utility of L_3. The reason why L_3 is not chosen, therefore, is because the probability of winning the highly valued prize is low. Hence, it follows that we can vary the probability value so as to make the lottery ticket L_3 more attractive. The probability can be adjusted to a point where the individual becomes indifferent between A and L_3. At this point, the Utility Index for A is derived. Thus, if the probability value $P = 1/3$ which will make the individual indifferent; therefore, from the formula:

$U(L_3) = 1/3.$

Since A is indifferent to L_3, therefore, the Utility Index of $A = 1/3$.

In a similar way, we can derive the measures of utility for the remaining prizes B, C and D, by varying the probability in ticket L_3 to a stage where the individual becomes indifferent in each case. Suppose that by this procedure, we find that $U(B) = 1/3$, $U(C) = 4/5$ and $U(D) = 2/5$. We can then substitute these utility measures into the 2 lottery tickets L_1 and L_2.

Thus, from the previous formula:

<u>Ticket L_1</u>

$$
\begin{aligned}
U(L_1) &= 2/3.U(A) + 1/3.U(B) \\
&= 2/3.1/3 + 1/3.1/3 \\
&= 2/9 + 1/9 = 3/9 = 1/3 = 0.333.
\end{aligned}
$$

<u>Ticket L_2</u>

$$
\begin{aligned}
U(L_2) &= 1/5.U(C) + 4/5.U(D) \\
&= 1/5.4/5 + 4/5.2/5 \\
&= 4/25 + 8/25 = 12/25 = 0.48
\end{aligned}
$$

Therefore, the mathematical expectation of utility

for $L_1 = 0.333$

and for $L_2 = 0.48$

Since $L_2 > L_1$, the prediction will be that the individual will choose ticket L_2. In the illustration, lottery tickets are used to represent risky alternatives but the alternatives can be anything, for example, buying industrial shares, or choosing an occupation or venturing into some form of business enterprise. The illustration shows that a great advantage of the M-N Theory is that it is scientific and the results can be verified or tested by experiment or empirical observations. The prediction will be true provided all the assumptions of Morgenstern and Neumann are fulfilled.

The assumptions are that the individual's preferences must be consistent and transitive and that he is motivated by the mathematical expectation of utility. If the individual fails to choose L_2, then the explanation is that either his system of preferences do not satisfy the assumptions or that his behaviour contradicts the hypothesis of mathematical expectation.

In the calculation of the Index, the individual is assumed to be able to make comparisons between the utilities of various alternatives. Thus, in contrast to ordinalism, utility measurement in the Morgenstern-Neumann case is more precise, that is, up to a linear transformation. But the important point to note is that the predictions themselves are ordinal. That is to say, no attempt or claim is made as to how much ticket L_2 is preferred to ticket L_1 in our example. The basis of prediction still lies in ranking.[3]

However, though it may be a scientific theory with considerable empirical content, there are some obvious limitations. The great disadvantage is that it is too mathematical, and the conditions required to calculate the Utility Index are too restrictive, and above all, it assumes rather unrealisticly that the consumer has the power or the endurance to calculate his utility on the basis of mathematical expectation. However, in contrast to Marshall, the Utility Index does not claim the addibility of utility.

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[3] W.J. Baumol, "Economic Theory and Operations Analysis", Prentice-Hall, 1961, p.342.