II

CARDINAL MEASURABILITY IN MARSHALLIAN CARDINALISM

Marshall's central proposition is that utility is measurable in terms of an external phenomenon, which is the monetary price a consumer is prepared to pay for the satisfaction of a desire. Therefore, utility measurement is an indirect measurement of desire. The desire is reflected in the monetary price a consumer is prepared to pay - so that the more intense the desire, the greater he values the utility and the more he is willing to pay and vice-versa.

Utility measurement, however, cannot be accurately described as a measurement of realised satisfaction. This, Marshall carefully distinguishes, because the utility so derived, may or may not be able to satisfy the desire fully. As Marshall says, " realised satisfactions and desires might differ considerably."[1] The practical difficulty lies in measuring realised satisfactions and comparing them with the desires. As there is no means of doing this, and since Marshall 's interest is centered on a quantitative measurement of satisfaction, he assumes that realised satisfaction corresponds roughly to desire (though in actual fact, it may not be the case), so that the money units paid which measure the intensity of the desire, measure also at the same time, the satisfaction resulting from it.

On the question of utility measurability, it is important to define what is meant by measurability. To say that utility is measurable is meaningless unless the different degrees of measurability are specified. Utility

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[1] Alfred Marshall, "Principles of Economics", 8th Edition, 1956, p.78.

measurability may be divided into 3 classes on the basis of their uniqueness or precision:

(i) measurability up to a monotonic transformation which applies to the case of ordinalism.

(ii) measurability up to a linear transformation which applies to neo-classical cardinalism, and

(iii) quantifiability which characterises Marshall's concept of measurement.

To discuss the meaning and significance of these, let us suppose that 5 utility situations, A,B,C,D, and E are given. Suppose the individual ranks them as follows:

Table 1

	Number assigned (a)	(b)	Sign of 1st. difference	Sign of 2nd. difference
A	19	150		
B	16	82	(A-B) = +ve	
C	14	80	(B-C) = +ve	(A-B) - (B-C) = +ve
D	5	60	(C-D) = +ve	(B-C) - (C-D) = -ve
E	1	50	(D-E) = +ve	(C-D) - (D-E) = +ve

where A is more preferred to B, B to C, C to D, and D to E. This implies that the individual is able to distinguish the sign of the 1st.difference of a change in welfare. This is because if A is greater than B, B is greater than C, C is greater than D, and D is greater than E, it follows that A - B, B - C, C - D, and D - E, will give a positive difference, which the individual can distinguish, though by how much, he is not able to tell. Where the individual can only order his preferences, he is not measuring utility absolutely but relatively, and this is known as ordinal measurability.

Since the preferences A,B,C,D, and E decrease with the rank, we can designate numbers to each preference, but with the provision that a higher number must be assigned to a higher preference. Thus, as illustrated, 2 sets of numbers (a) and (b) may be assigned. These 2 sets of numbers have only one property which is that their sequence decreases with decrease in the sequence of the preferences. That is to

say, they move in the same direction as the preferences, and their characteristic is, therefore, in actual fact a transformation of the characteristic of the preferences. Since this property of the sequence is known as monotonic, therefore, ordinal measurability may be defined as measurability up to a monotonic transformation. A characteristic in the order of the preferences is that it is transitive, that is, if $A > B$, and $B > C$, therefore $A > C$.

However, if the individual is able to compare the differences of 2 utility situations, say, if he is able to say that $A - B \gtrless B - C$, then this implies the ability to distinguish a 2nd. difference. As shown in the table, A - B, B - C, will give the 1st. difference, while the comparison of the magnitude of A - B with the magnitude of B - C will give the 2nd. difference. Mathematically speaking, the degree of measurability in this instant, is up to a linear transformation. The word linear means unity or first degree power.

The Marshallian theory of diminishing marginal utility is in fact, based on utility measurability up to a linear transformation. The reason is that if the individual is able to compare differences between magnitudes, what is actually being compared are increments of utility. The important point to note is that since in Marshall's view, marginal utility diminishes, the sign of the 2nd. difference is always negative. This point is best illustrated by a similar table:

Table 2

	Total utility	Sign of 1st. difference	Sign of 2nd. difference
A	100		
B	96	(A–B) = 4 = +ve	
C	90	(B–C) = 6 = +ve	(A–B)–(B–C) = –2 = –ve
D	82	(C–D) = 8 = +ve	(B–C)–(C–D) = –2 = –ve
E	73	(D–E) = 9 = +ve	(C–D)–(D–E) = –1 = –ve

As shown, A,B,C,D, and E represent 5 total utility situations derived from the consumption of different amounts of the same commodity and are ranked in the order of their magnitude. The 1st. differences which are all positive will give the marginal utilities. The 2nd differences are comparisons between the successive increments of utilities and are always negative due to the operation of diminishing marginal utility.

Table 2, differs from Table 1 in that in the latter Table, A,B,C,D, and E represent total utilities derived from the consumption of different combinations of a given variety of commodities. If, however, the 5 utility situations refer to total utilities derived from consuming different bundles of the same commodity, then the sign of the 2nd. difference still need not be negative throughout, unless we accept Marshall's psychological law of diminishing marginal utility.

From this discussion, it will be seen that Cardinalism denotes at least measurability up to a linear transformation. On this basis, therefore, a dividing line between ordinalism and cardinalism may be drawn. The distinction, however, is not sharp , being one of degree. As we have seen, the distinction is based on distinguishing the sign of the 1st. difference in the case of ordinalism and on distinguishing the sign of the 2nd. difference in the case of cardinalism. As an evidence of the unclear demarcation, T. Majumdar has defined measurability up to a linear transformation as " ordinal measurability of relative preferences."[1]

But the Marshallian concept of measurabilility denotes even greater precision. Marshallian cardinalism denotes quantifiability, besides measurability up to a linear transformation. This postualates that utility can be added together for an individual, subject to certain assumptions. Obviously, basing on Marshall's proposition, an

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[1]Tapas Majumdar, "The Measurement of Utility", Macmillan & Company Ltd; 1958, p.134.

individual is able to perceive a welfare situation in isolation. This conclusion follows Marshall's assertion that utility is quantifiable, whereas in the case of the ordinalist, welfare perceptions are made only relatively, that is, an individual can only discern his welfare comparatively since he is not capable of absolute measurement. Since the essential basis of a theory of welfare, lies in that the individual should be able at least, to make "better or worse" comparisons between welfare situations, we see that the ordinalist has an operational definition of utility measurability. That is to say, without the assumption of Marshallian quantification which implies absolute perception of welfare, the ordinalist's stand of relative perception of welfare need not exclude him from a theory of welfare.

Marshall's Assumptions of Quantifiability

The Marshallian proposition that utility for an individual can be added together, not only for one commodity, but for the utility derived from all other commodities making up the consumer's budget, is based on the following assumptions:

(i) The constancy of the marginal utility of money for the individual.

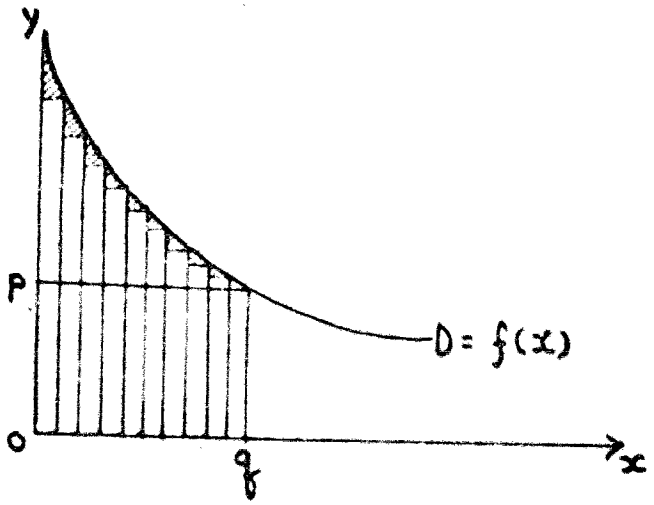
(ii) That the utility of the good to be measured is independent of that derived from another. This excludes the effects of cross-elasticities exerted by rival as well as complementary goods.

(iii) Measurement must proceed on a one-commodity at a time basis.

(iv) Prices of all other commodities are assumed to be given, and

(v) finally, the individual's income, tastes and preferences are constant.

Given all these conditions, total utility of a commodity is measured by the integral under the individual's demand function from zero to the quantity purchased.



i.e. Total utility (of good x) = $\int_0^q f(x)dx$

It is evident from the diagram that the total utility is derived from the summation of the individual strips. If the strips are infinitely narrow, the shaded areas may be neglected as they will then be very insignificantly small. For example, the area of each is obtained by $\Delta y . \Delta x$, and if Δy and Δx are very small fractionally, say $\frac{1}{100}$ and $\frac{1}{200}$ respectively, their product will be of negligible magnitude.

In equilibrium, the position of utility maximisation for the individual is given by the proportionality between marginal utilities and the respective prices thus:

$$\frac{mux}{Px} = \frac{muy}{Py} = \frac{muz}{Pz}$$ (assuming only 3 goods)

Therefore, the total utility of all the goods according to Marshall, when the individual utility is added together is given by:

Total of all utilities = Ux + Uy + Uz .....

Marshall Justifies the 'addibility' aspect of utility for the individual on the assumption that the marginal utility of money is constant and that the individual utility is independent of one another.

That utility is quantifiable in this sense is questionable. In practice, it is not justifiable to add the utility say, from tea to that of coffee or the utility say, from bread to that of butter. The objection lies in the fact

that cross-elasticities exist between the former competitive goods and between the latter complementary goods, so that the utility derived from one commodity is not a function of its own quantity only, but is also affected by the prices and quantity consumed of rival and complementary commodities.

For this reason mainly, the Marshallian assertion of independent utilities is untenable in practice. The further assumption of constant marginal utility of money is justified only when the proportion of income spent on each good is small in comparison with the total income. This, as we shall soon discuss, restricts the application and significance of the all important Demand Theory of Marshall. However, on the basis of the objection against the assumption of independent utilities, it is not justifiable to sum up utilities derived from a collection of goods which have cross-elasticity effects.