

THE HICKSIAN ORDINAL INDIFFERENCE-PREFERENCE THEORY

Hicks and Allen, who worked on Pareto's ideas, were careful enough to exclude diminishing marginal utility from their ordinal indifference-preference theory. Nevertheless, their analytical apparatus of indifference curves was borrowed from Edgeworth who originally devised it. Since the Hicksian-Allen theory is the best representative of an ordinal utility theory, we shall assess the cardinal-ordinal controversy by primarily concentrating on the Marshallian and Allen-Hicksian theories. We shall first point out similarities. Firstly, both theories are based on introspective grounds. In other words, each act of choice by a consumer, is interpreted as the result of some mental process within the consumer. Secondly, both theories seek to explain consumer demand and consumer equilibrium.

It will be seen that if the criteria for judging a superior theory is that it should be meaningful, valid and of greater operational significance (i.e. of wider application in the solution of problems), then the conclusion is that Hicks' theory, built on ordinal measurability of utility, represents an advancement over Marshall's cardinal utility theory. There are 2 grounds for this:-

Firstly, the ordinal theory is able to state a more general and realistic demand theory than the cardinal theory and,

Secondly, it does so on the basis of fewer assumptions.

Though Marshall's theory is valid if given all the assumptions, it is of less general application. The most important assumption Marshall makes, is the constancy of the

marginal utility of money. The assumption is a necessity in order to preserve money as a common numeraire for measuring utility. In view of the assumption of constant marginal utility of money, the theory is inevitably restricted for 3 reasons.

(i) It is only applicable to a one-commodity model.

(ii) It oversimplifies the composite effect of a price-change and

(iii) It is at variance with the theory of welfare economics where income redistribution is advocated on the assumption that the marginal utility of income diminishes as income increases.

In the case of the first restriction, the Marshallian cardinal utility theory breaks down if the consumer has more than one commodity to spread his expenditure over. The reason is that the assumption of constant marginal utility of money is incompatible with a two-or-more-commodity model. This restriction is so obviously serious that Hicks was led to conclude that Marshall's "Cardinalism has eliminated itself"¹.

The argument can be presented in this way. Assuming that for a given income, a consumer allocates it over his several purchases to arrive at the equilibrium position where his utilities are maximised:

$$\text{i.e. } \frac{\text{mux}}{\text{Px}} = \frac{\text{muy}}{\text{Py}} = \frac{\text{muz}}{\text{Pz}} \quad (\text{assuming 3 commodities, } x, y \text{ and } z.)$$

Let us now consider commodity x, and assumed prices to be constant for the other commodities y and z. At equilibrium, q_1 of x is purchased at price p_1 . The outlay on x, therefore, will be equal to $p_1 q_1$.

Applying Marshallian measurability, the marginal utility of x is equal to the price of x multiplied by the

¹J.R. Hicks, "A Revision of Demand Theory", Oxford University Press, 1956, p.15.

constant unit of measurement (which is the marginal utility of money).

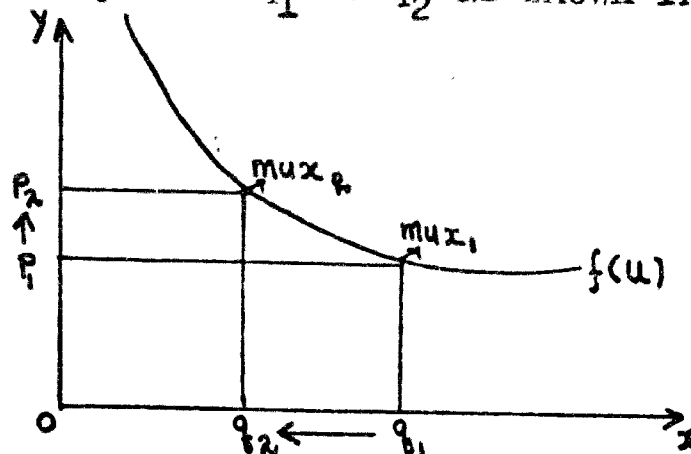
$$\text{i.e. } \text{mux}_1 = p_1 \times \text{mum}$$

Suppose now, the price of x rises from p_1 to p_2 .

Since $p_1 < p_2$, therefore,

$$\text{mux}_1 < p_2 \times \text{mum} \text{ (assuming mum to be constant)}$$

Holding mum $\times p_2$ constant, mux_1 must rise in order to maintain the equality. Since the marginal utility function $f(u)$ is negatively sloped, the equality is achieved only by a contraction of the quantity from q_1 to q_2 as shown in the diagram.



Thus, after the adjustment,

$$\text{mux}_2 = p_2 \times \text{mum}$$

The total outlay on x will now be $p_2 q_2$. At the increased price and a reduced quantity,

$$p_2 q_2 \cong p_1 q_1 .$$

If $p_2 q_2 = p_1 q_1$, no additional adjustment is required. The equilibrium will still be:

$$\frac{\text{mux}}{P_x} = \frac{\text{muy}}{P_y} = \frac{\text{muz}}{P_z}$$

But the equality is extremely rare unless the marginal utility function $f(u)$ is a rectangular hyperbola, i.e. if it is of unitary elasticity.

If the new outlay on x is less than the old outlay, then (assuming no increase in the consumer's income), the consumer will have more money to spend on commodities y and z. But if the new outlay is greater than the old outlay, then less money will be available for commodities y and z.

So long as $p_2q_2 \geq p_1q_1$, a fresh complication in the attainment of equilibrium is introduced. In both cases, adjustment must be made to the consumer's expenditure on y and z . This is attainable only by revising the unit of utility measurement, i.e. changing the marginal utility of money.

To demonstrate, if $p_2q_2 < p_1q_1$, then the excess money in the consumer's possession will be allocated to both y and z . To achieve this necessitates lowering the marginal utility of money. This can be shown thus,

since $muy_1 = P_y \times m_{um}$, an increase in the quantity of y , will lower the marginal utility of y .

$$\text{i.e. } muy_2 < P_y \times m_{um}$$

To maintain equality, m_{um} must be adjusted downward.

$$\text{i.e. } muy_2 = P_y \times m_{um_2}$$

$$\text{where } m_{um_2} < m_{um}$$

Similarly for commodity z , the marginal utility of money is lowered. If $p_2q_2 > p_1q_1$, that is, when less money is available for commodities y and z , then the ~~money~~^{marginal} utility of money must be adjusted upward.

This analysis shows clearly that the assumption of constant m_{um} , is compatible with the Marshallian Law of Demand only in a one-commodity model. In a situation where the consumer can spread his expenditure on more than one commodity, the Marshallian demand theory breaks down, unless m_{um} is varied. The conclusion is that varying the m_{um} means varying the unit of utility measurement. When the measuring rod of money fluctuates, it is therefore, entirely meaningless to assert the measurability of utility or its quantifiability in terms of some fixed constant unit.

The second restriction of constant m_{um} , as have stated, obscures the composite character of a price effect. Under the assumption, the income effect of a price change is assumed to be zero or negligible. The implication of this is that it necessarily narrows down the commodities to be treated. Under

the assumption, only those commodities which are divisible and which account for a small portion of the consumer's expenditure are cardinally measurable, for only then is it justified to neglect the income effect. At this juncture, we may point out several inconsistencies in Marshall, who cites commodities like wheat and house-room, which being necessities, and therefore, forming a large proportion of a consumer's income, will have a large income effect.¹

The consequence of ignoring the income effect, leads to the failure of the Marshallian Law to explain the Giffen case. In this instant, the negative income effect of a price fall, more than offset the positive substitution effect, so that the normal inverse-relationship between price and quantity fails to hold. For Giffen cases, therefore, price varies directly with the quantity demanded. Thus, the demand curve will be positively sloped, which is contrary to the central Marshallian proposition of a declining marginal utility function which underlies his Law of Demand.

From the point of view of welfare theory and policy formulation, the third restriction of constant marginal utility of money is that it contradicts the basic assumption for the redistribution of income. Income redistribution through progressive taxation is based on the assumption that the larger the money income, the lower will be its marginal utility. If marginal utility of income is constant, then there is no basis for progressive taxation.

We see, therefore, that in order to measure utility by an invariant unit of measurement, Marshall has to assume the constancy of the marginal utility of money. Such an assumption is made at the expense of a more general and operationally significant theory.

The Hicksian theory, on the other hand, which

¹Marshall, op.cit., pp. 89-90.

dispenses with the assumption, is able to state a more satisfactory theory of demand and consumer equilibrium. The quantitative concept of utility which features in the Marshallian theory, is regarded by Hicks to be "unnecessary in explaining market phenomena."² To remove all quantitative traces of utility, the notion of marginal utility is replaced by the marginal rate of substitution, while the diminishing marginal rate of substitution is substituted for diminishing marginal utility. The Hicksian ordinal theory is judged to be superior on the grounds that with fewer assumptions, it is able to enunciate a theory of demand which takes into account both the substitution and income effects of a price change, thus filling an important gap in Marshall's cardinal theory. Besides, it is also applicable to a multi-good model. One of the 2 axes in the 2-dimensional diagram, may represent the commodity to be dealt with, while the other axis under general purchasing power, may represent claims on other commodities and services.

The wider application of the theory is demonstrated by its ability to analyse the Giffen case which Marshall dismisses as a special phenomenon. Also, through ordinal indifference preference, the equilibrium of the consumer is capable of being established as in the Marshallian. In equilibrium, the marginal rate of substitution is equal to the price ratio: i.e. $M.R.S. = \frac{m_{ux}}{m_{uy}} = \frac{P_x}{P_y}$, which is similar

to Marshall's proportionality of marginal utilities to prices: i.e. $\frac{m_{ux}}{P_x} = \frac{m_{uy}}{P_y}$, which when cross-multiplied gives:- $\frac{m_{ux}}{m_{uy}} = \frac{P_x}{P_y}$.

At this point, the ordinal indifference-preference

²J.R.Hicks, "Value and Capital", 2nd. Edition, Oxford University Press, 1946, p.18.

theory might be criticised for employing cardinal measurability of utility unconsciously, thus relapsing back into the cardinalist world. The crucial point is that the M.R.S. given by the ratio of 2 marginal utilities is indeterminate, unless the 2 marginal utilities are at least quantifiable in principle.

T. Majumdar, however, strongly objected to the criticism which, in his opinion, is based on a misunderstanding of the character of the indifference curves. Briefly, his argument is that the M.R.S. defined as a ratio of 2 marginal utilities is the result of believing that indifference relation is one of weakness. That is to say, if 2 points, A and B lie on an indifference curve, B just weakly compensates A at the margin of substitution. Such a view tends to obscure the real nature of indifference relations. In his opinion, instead of being weakly related, all points on an indifference curve bear a strong relation with one another. For example, point B is indifferent to point A by virtue of it being strongly compensated after a movement from A to B. Hence, rather than not being able to choose between A and B as is implied by a weak relation, an individual, on the principle of compensated-indifference, can name B as compensation for loss of A. Majumdar's contention, therefore, "is that the M.R.S. can be so defined as to make its meaning independent of the meaning of marginal utility.....from the logic of the compensation-principle alone."³

Majumdar's defence is valid only provided that the Hicksian theory can still arrive at the equilibrium point for the consumer without making use of marginal utility. Hicks himself realises the cardinalist traces implied in the derivation of the indifference curves and his latest book, "A Revision of Demand Theory", (Oxford University Press, 1956), marks an effort to remove all traces of cardinalism. In the

³Tapas Majumdar, Op. cit., p.76.

book, Hicks revises the presentation of the indifference-preference hypothesis. The analytical tool of indifference curves is completely abandoned and the analysis is made in terms of verbal description and strict logic.

Robertson's Criticism

Hicks' original indifference preference theory came under the sharp criticism of Robertson whose views on measurability follow the Marshallian tradition. Robertson's criticism is that an ordinalist should be contented with only comparing magnitudes and not the differences between magnitudes. Once a comparison between differences in magnitude is made, then we are committed to cardinalism. In other words, it is permissible for an ordinalist to say that $A > B > C$, but once he begins to say that $(A-B) > (B-C)$, then he cannot remain an ordinalist. This is because it is then possible to place B in such a position that a movement from A to B is equivalent to a movement from B to C. According to Robertson, it will then follow that the interval AC is twice that of AB thus implying cardinal measurability.⁴

It will be noticed that Robertson's main objection is that an ordinalist should not claim the ability to distinguish a 2nd difference which will denote measurability up to a linear transformation. Even if so, the Marshallian case of quantification cannot be rehabilitated for a linear transformation need not imply addibility.

In opposition to Robertson's objection, L. Robbins made the pertinent argument that an ordinalist can still judge differences between 2 movements without actually being committed to cardinal measurement. Robbins' view is that it is possible for an ordinalist to say that his preference for welfare situation A over B is rated more highly than his

⁴D.H. Robertson, "Utility and All That", Manchester School of Economic and Social Studies, Vol. 19, 1951, p. 117 & pp. 124-25

preference for B over C, and that such judgement of preference intensities is in itself ordinal and not cardinal.⁵

Such dispute shows that on the question of measurability, the division between cardinalism and ordinalism on the basis of monotonic and linear transformations is not sharp.

⁵L.C. Robbins, "Robertson on Utility and Scope",
Economica, 1953, p.103.