CHAPTER III

THEORETICAL FRAMEWORK

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3.1 Introduction

This chapter of the thesis states the theoretical framework, which acts as a benchmark to ensure that methods applied in this study are in with line theoretical arguments. A perennial question in macroeconomic theory is the reason for the observed real effects of changes in monetary policy. A famous answer to this question is that people are not perfectly informed about changes in market conditions. The central ideal of the Lucas-Phelps model is that when a producer observes a change in the price of his product he does not know whether it reflect a change in the good's relative price or a change in the aggregate price level. A change in the relative price alters the optimal amount to produce. A change in the aggregate price level, on the other hand, leaves optimal production unchanged.

This study will discuss if monetary shocks have real effects for the reasons described by the Lucas imperfect-information model.

3.2 The Case of Perfect Information

3.2.1 Producer Behavior

There are many different goods in the economy. A representative producer of a typical good, good i, produces according to the production function:

$$Q_i = L_i \qquad (3.1)$$

Where L_i is the amount the individual works and Q_i is the amount he produces. Real consumption, Ci, nominal income divided by a price index:

$$C_i = \frac{P_i Q_i}{P}.$$

Utility depends positively on consumption and negatively on hours worked:

$$U_i = C_i - \frac{1}{\gamma} L_i^{\gamma}, \quad \gamma > 1 \qquad (3.2)$$

Thus, there is constant marginal utility of consumption and increasing marginal disutility of work.

When the aggregate price level P is know, the individual's maximization problem is simple. Substituting the previous equations into (3.2) gives:

$$U_i = \frac{P_i L_i}{P} - \frac{1}{\gamma} L_i^{\gamma}. \qquad (3.3)$$

Taking prices as given (competitive markets), an individual maximizes utility by selecting L_i to satisfy the first order condition:

$$\frac{P_i}{P} - L_i^{\gamma-1} = 0. \tag{3.4}$$

Rearranging, we get:

$$L_i = \left(\frac{P_i}{P}\right)^{\frac{1}{\gamma-1}} \tag{3.5}$$

Letting lowercase letters denote logs:

$$l_i = \frac{1}{\gamma-1}(p_i - p) \tag{3.6}$$

This is a labor supply function (and, indirectly, an output supply function) in which an individual's hours depend on the relative price of the individual has output price. If $P_i = P$, then $L_i = 1$, $Q_i = 1$, $l_i = 0$, $q_i = 0$, then the utility function was designed so that this would be the result.

3.2.2 Demand

Assume that the demand for good i has the following form[3.1]:

$$q_i = y + z_i - \eta(p_i - p), \quad \eta > 0, \tag{3.7}$$

Where y is the log of a measure of aggregate income, z_i is a shock to demand for good i (with mean zero across goods)[3.2], and η is the demand elasticity. More specifically, y is defined to be the average of the q_i's across goods, and p is defined to be the average of the p_i's across goods:

$$y = \bar{q}_i \quad (3.8) \qquad \text{and} \qquad p = \bar{p}_i. \quad (3.9)$$

Aggregate demand is given by:

$$y = m - p \tag{3.10}$$

This is just a simple way of modelling aggregate demand; the essential property is that the price level and output are inversely related. While m can be literally interpreted as the log money supply, it might be thought of more generally as any aggregate demand shifter.

3.1 This is not derived from a utility maximization problem.

3.2 That is, the total (log) demand for good i is $\ln N + y + z_i - \eta(p_i - p)$, where N is the number of producers of each good.

3.2.3 Equilibrium

This equilibrium requires that quantities demanded equal quantities supplied in each market i. From (3.6) and (3.7) we obtain:

$$\frac{1}{\gamma-1}(p_i) = y + z_i - \eta(p_i - p). \qquad (3.11)$$

Solving for p_i yields:

$$p_i = \frac{\gamma-1}{1+\eta\gamma-\eta}(y+z_i) + p \qquad (3.12)$$

Next, average the left- and right-hand sides of (3.12):

$$p = \frac{\gamma-1}{1+\eta\gamma-\eta} y + p \qquad (3.13)$$

Solve for y[3.3]:

$$y = 0. \qquad (3.14)$$

Recall equation (3.10):

$$y = m - p$$

If $y = 0$, then (3.10) implies:

$$p = m \qquad (3.15)$$

[3.3] The result that equilibrium log output is zero implies that equilibrium level of output is 1. This results from the $1/\gamma$ term multiplying L_i^γ in the utility function, (3.2).

Thus, money is neutral in this model. An increase in m leads to a proportional increase in p. In addition, since p is observable and markets clear, the average level of log output is zero. An increase in aggregate demand does not lead to higher aggregate output in the perfect information version of the model.

3.3 Imperfect Information

3.3.1 Producer Behaviour

This study also considers the case where producers observe the price of the good they produce, but not the aggregate price level.

Define the relative price of good i as $r_i = p_i - p$, we get:

$$\begin{aligned} p_i &= p + (p_i - p) \\ p_i &= p + r_i \end{aligned} \tag{3.16}$$

Individuals' supply choices are motivated by relative prices, but relative prices are not observed; p_i is observed, but the individual must make a forecast regarding r_i.

Assume that individuals calculate the expectation of r_i given p_i, and then act as if this expected value were known with certainty (This implicitly have been making this assumption of *certainty equivalence* in all of our work with rational expectations). This implies that with uncertainty, equation (3.6) is modified to give:

$$l_i = \frac{1}{\gamma - 1} E[r_i \mid p_i] \tag{3.17}$$

This model must next describe the process generating values for m. Assume that m is normally distributed with mean $E[m]$ and variance V_m (this is a bit different and more general than the demand process specified in the paper by Lucas that was assigned earlier).

This model also assume that the z_i's, which we earlier assumed had mean zero, are normally distributed, and that the z_i and m shocks are independent.

The next will invoke our solution to the signal extraction problem. We wish to forecast r_i using our knowledge of p_i. Recall that $p_i = p + r_i$; i.e. we observe a sum, but wish to forecast a component of the sum. Writing our "synthetic" regression forecast in a form where variables are expressed as deviations from means we get:

$$E[r_i \mid p_i] = \frac{V_r}{V_r + V_p}(p_i - E[p]), \qquad (3.19)$$

Where V_r is the variance of r_i and V_p is the variance of p. Use of this formula requires that r_i and p be independent normal variables. Now this model has not derived expressions for V_r and V_p, but its will be able to do so eventually (here that the r_i's have unconditional mean zero, and the p_i's have unconditional mean $E[p]$).

Recall equation (3.17):

$$l_i = \frac{1}{\gamma - 1} E[r_i \mid p_i]$$

Substituting (3.19) into (3.17) yields:

$$l_i = \frac{1}{\gamma - 1} \frac{V_r}{V_r + V_p}(p_i - E[p]) \qquad (3.20a)$$

or

$$l_i \equiv b(p_i - E[p]). \qquad (3.20b)$$

Averaging across producers yields:

$$y = b(p - E[p]) \tag{3.21}$$

This is the Lucas supply function. It states that the departure of output from its normal level (which is zero in the model) is an increasing function of the surprise in the price level.

3.3.2 Equilibrium

Now combine aggregate demand, equation (3.10) and the Lucas aggregate supply curve, (3.21):

$$m - p = b(p - E[p]).$$

Solve for p:

$$p = \frac{1}{1+b}m + \frac{b}{1+b}E[p]. \tag{3.22}$$

This equation can be solving from this point by using the method of undetermined coefficients. However, Romer used a trick that often (but not always) to solve rational expectations models more quickly. Take expectations on both sides of (3.22):

$$E[p] = \frac{1}{1+b}E[m] + \frac{b}{1+b}E[p] \tag{3.24}$$

This equation can be solved for $E[p]$:

$$E[p] = E[m]. \tag{3.25}$$

Substituting into (3.22):

$$p = \frac{1}{1+b} m + \frac{b}{1+b} E[m]$$

or , using the fact that $m = E[m] + (m - E[m])$:

$$p = E[m] + \frac{1}{1+b}(m - E[m]). \tag{3.26}$$

Recall equation (3.21):

$$y = b(p - E[p])$$

Now substituting (3.25) and (3.26) into (3.21) gives a solution for output:

$$y = b\left(E[m] + \frac{1}{1+b}(m - E[m]) - E[m]\right)$$

$$y = \frac{b}{1+b}(m - E[m]). \tag{3.27}$$

Equations (3.26) and (3.27) illustrate the basic features of the Lucas model. The component of aggregate demand that is observed, $E[m]$, affect only prices, but the component that is not observed, $m - E[m]$, has real effects. Consider for concreteness, an unobserved increase in m, that is, a higher realization of m given its distribution. This increase in the money supply raises aggregate demand, and thus produces an outward shift in the demand curve for each good. Since the increase is not observed, each supplier's best guess is that some portion of the rise in demand for his product reflects a relative prices shock. Thus, producers increase their output.

In conclusion, Expected variations in money affect prices in proportion. Money surprises affect both prices and output, with the division of the impacts depending on underlying variances of relative and general prices.

3.4 Conclusion

Section 3.2, the case of perfect information where the money stock is publicly observed. In this situation, money is neutral. Section 3.3, then turns to the case of imperfect information where the money stock is not observed. In this situation, money is non-neutral.