CHAPTER 4

METHODOLOGY

4.1 Introduction

In this study, several financial econometric methods will be used for exploring the relationships or the long-run equilibrium and short-run dynamics among the stock prices, real interest rate, real economic activity and real money balances. These include the unit root test, cointegration test, causality test and impulse response function. Our analysis is primarily based on multivariate framework.

4.2 Unit Root Tests

In order to examine the existence of unit root or to establish the order of integration of the time series data, two tests are conducted, namely augmented Dickey-Fuller (ADF) and Phillips-Perron (PP) unit root tests. A time series variable $Y_t$ is said to be integrated of order $d$ or $Y_t \sim I(d)$, if it is stationary after differencing $d$ times.
4.2.1 Augmented Dickey-Fuller Unit Root Test

A widely used test is the ADF unit root test. The test was proposed by Dickey and Fuller (1979). We first consider the Dickey-Fuller (DF) test with an autoregressive process of order one, AR(1):

\[ Y_t = \mu + \rho Y_{t-1} + \varepsilon_t \]  \hspace{1cm} (4.1)

\( \varepsilon_t \) is assumed to be white noise. This can be rewritten as:

\[ \Delta Y_t = \mu + \gamma Y_{t-1} + \varepsilon_t \]  \hspace{1cm} (4.2)

where \( \gamma = \rho - 1 \).

If series is correlated at higher order lags, the assumption of white noise disturbance is violated. The ADF test makes a parametric correction for higher order correlation by assuming that the \( Y_t \) series follows an AR(m) process. This approach controls for higher order correlation by adding lagged difference terms of the dependent variable \( Y_t \) to the right-hand side of the regression. Specifically, the ADF test is based on the following regression:

\[ \Delta Y_t = \mu + \beta t + \gamma Y_{t-1} + \sum_{i=1}^{m} \alpha_i \Delta Y_{t-i} + \varepsilon_t \]  \hspace{1cm} (4.3)

where

\( Y_t \) is a time series variable,

t = 1, 2, ..., n,

n is the sample size,
\( m \) is the number of lags of \( \Delta Y_t \) included.

We include both the drift and deterministic time trend in the regression since all the series have non-zero means and some series may also contain a deterministic trend. This augmented specification is used for testing the null and alternative hypotheses:

\[
H_0 : \gamma = 0 \\
H_1 : \gamma < 0
\]

We carry out the test by performing a t-test on the estimated \( \gamma \). The t-statistic under the null hypothesis of a unit root does not follow the conventional t-distribution. Dickey and Fuller (1979) simulated the critical values for selected sample sizes and this was further improved by MacKinnon (1991) by implementing a much larger set of simulations than that tabulated by Dickey and Fuller. If \( H_0 \) is rejected, \( Y_t \) does not contain a unit root and is stationary. It is said to be integrated of order zero or I(0). If the rejection is not found then we test the unit root for the first differences of the series by using same procedure. The rejection of the null hypothesis means that \( Y_t \) contains a unit root and is integrated of order one, \( Y_t \text{ l}(1) \) or \( \Delta Y_t \text{-l}(0) \). If \( H_0 \) still not rejected, then the process is to be repeated with the next higher order of differencing until a rejection is found.

We fit equation (4.3) with \( Y_t = \ln(IIPS_t), \ln(SP_t), \ln(RIR_t), \ln(RB1_t) \) and \( \ln(RB2_t) \) to determine the order of integration of each series. We run the regressions by using \( m = 1, 3 \) and \( 6 \) to ensure that the results are robust to different lag lengths.
4.2.2 Phillips-Perron Unit Root Test

Phillips and Perron (1988) proposed a non-parametric method to control for higher-order serial correlation in the time series. Consider an AR(1) process:

\[ \Delta Y_t = \mu + \gamma Y_{t-1} + \epsilon_t \]  \hspace{1cm} (4.4)

The PP test makes a correction to the t-statistic of the \( \gamma \) coefficient from the AR(1) regression to take into consideration of serial correlation in \( \epsilon_t \). This test uses the Newey-West estimator, which takes a weighted average of the sample autocorrelation of \( \epsilon_t \). The following is to be computed:

\[ \hat{\gamma}^2_w = \gamma_0 + 2 \sum_{v=1}^{q} \left( 1 - \frac{v}{q+1} \right) \hat{\gamma}_j \]  \hspace{1cm} (4.5)

where

\[ \hat{\gamma}_j = \frac{\sum_{t=v+1}^{n} \frac{\hat{\epsilon}_t \hat{\epsilon}_{t-j}}{n}} \]  \hspace{1cm} (4.6)

and \( q \) is the truncation lag for the Newey-West corrections.

The PP t-statistic is:

\[ t_{pp} = \gamma_0^{1/2} t_\gamma - \frac{\hat{\gamma}^2_w}{\hat{\gamma}^2_w + \frac{(w - \gamma_0) n s^2}{2 w s}} \]  \hspace{1cm} (4.7)
where

t_γ and s_γ denote the t-statistic and standard error of γ,

s is the standard error of the test regression of (4.4).

The asymptotic distribution of the PP t-statistic is same as the ADF t-statistic. The critical values are tabulated by MacKinnon. The procedure to determine the presence of unit root in series is similar to that for the ADF test.

4.2.3 Unit Root Test with a Structural Break

The standard ADF and PP tests do not allow for the existence of a structural break in the time series. We consider the unit root test with a structural break in the series proposed by Perron (1989). Perron conducted the unit root test with a break in trend occurring at the Great Crash of 1929 and oil-price shock in 1973. He employed an adjusted ADF type unit root test strategy. The results had revealed that the standard test of unit root could not reject the unit root hypothesis while the rejection was found for most data series with the inclusion of a break in the trend function.

Zivot and Andrews (1992) reanalyzed the data series used by Perron by treating the breakpoint as an endogenous occurrence. They found that there was less evidence against the unit root hypothesis than Perron's findings for most variables.
We follow Perron's method in treating the breakpoint to be exogenous. From Figures 3.1 to 3.4, there was a significant change in the trend of each data series that occurred in the third quarter of 1998. Thus, we denote August 1998 as the breakpoint (T_B). We consider possible changes in both the intercept and slope in the trend function and the regression equation to test for a unit root is as below:

\[ Y_t = \mu + \beta t + \theta_1 DU_t + \theta_2 DT_t + \gamma Y_{t-1} + \sum_{i=1}^{\alpha} \phi_i \Delta Y_{t-i} + \epsilon_t \]  \hspace{1cm} (4.8)

where

- \( DU_t = 1 \text{ if } t > T_B, 0 \text{ otherwise}, \)
- \( DT_t = t \text{ if } t > T_B, 0 \text{ otherwise}. \)

The asymptotic distribution of the t-statistic for testing \( H_0: \gamma = 0 \) in equation (4.8) is presented by Perron (1989). To find the percentage points from the distribution, the value of break fraction is defined to be \( \lambda = T_B/n. \)

### 4.3 Cointegration and Vector Error Correction Model

Engle and Granger (1990) introduced a two-step estimation for detecting the existence of cointegration between non-stationary variables. Stock and Watson (1991) presented their tests for multiple cointegrating vectors. Finally, Johansen (1991) successfully developed a maximum likelihood approach to estimation and testing for multiple cointegrating vectors. Cointegration implies that the system
follows an error correction representation and conversely an error correction system has cointegrated variables.

The vector error correction (VEC) model includes cointegrating relations in the model, as it restricts the long-run behaviour of the endogenous variables to converge to their cointegrating relationships while allowing for short-run dynamic adjustments. The deviation from long-run equilibrium is corrected gradually through the error correction term, which is incorporated in the VEC model. In conclusion, the VEC model allows for evaluation of the short-run dynamics and long-run adjustment in the system.

In our analysis, we first perform the cointegration test on the four variables in the first system, which are \(\ln(\text{IIPS}_t), \ln(\text{SP}_t), \text{RIR}_t\) and \(\ln(\text{RB1}_t)\), and then \(\ln(\text{IIPS}_t), \ln(\text{SP}_t), \text{RIR}_t\) and \(\ln(\text{RB2}_t)\) in the second system of equations. We conduct the cointegration test using the methodology developed by Johansen. Consider \(y_t\) as a vector of 4 non-stationary \(I(1)\) variables which are \(Y_{1t}, Y_{2t}, Y_{3t}\) and \(Y_{4t}\). A vector autoregression of order \(p\), \(\text{VAR}(p)\) is given by:

\[
y_t = A_1 y_{t-1} + A_2 y_{t-2} + \ldots + A_p y_{t-p} + \epsilon_t \tag{4.9}
\]

We rewrite this VAR as:

\[
\Delta y_t = \Pi y_{t-1} + \sum_{i=1}^{p-1} \Gamma_i \Delta y_{t-i} + \epsilon_t \tag{4.10}
\]
where

\[
Y_t = \begin{bmatrix} Y_{1t} \\ Y_{2t} \\ Y_{3t} \\ Y_{4t} \end{bmatrix}, \quad \Pi = \begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \Pi_{14} \\ \Pi_{21} & \Pi_{22} & \Pi_{23} & \Pi_{24} \\ \Pi_{31} & \Pi_{32} & \Pi_{33} & \Pi_{34} \\ \Pi_{41} & \Pi_{42} & \Pi_{43} & \Pi_{44} \end{bmatrix}, \quad \Gamma_j = \begin{bmatrix} c_{1j,1} & c_{1j,2} & c_{1j,3} & c_{1j,4} \\ c_{2j,1} & c_{2j,2} & c_{2j,3} & c_{2j,4} \\ c_{3j,1} & c_{3j,2} & c_{3j,3} & c_{3j,4} \\ c_{4j,1} & c_{4j,2} & c_{4j,3} & c_{4j,4} \end{bmatrix}, \quad e_t = \begin{bmatrix} e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \end{bmatrix}
\]

where

\[e_t \sim iid(0, \Sigma_{ee})\]

\[
\Sigma_{ee} = \begin{pmatrix} \operatorname{var}(e_{1t}) & \operatorname{cov}(e_{1t}, e_{2t}) & \operatorname{cov}(e_{1t}, e_{3t}) & \operatorname{cov}(e_{1t}, e_{4t}) \\ \operatorname{cov}(e_{2t}, e_{1t}) & \operatorname{var}(e_{2t}) & \operatorname{cov}(e_{2t}, e_{3t}) & \operatorname{cov}(e_{2t}, e_{4t}) \\ \operatorname{cov}(e_{3t}, e_{1t}) & \operatorname{cov}(e_{3t}, e_{2t}) & \operatorname{var}(e_{3t}) & \operatorname{cov}(e_{3t}, e_{4t}) \\ \operatorname{cov}(e_{4t}, e_{1t}) & \operatorname{cov}(e_{4t}, e_{2t}) & \operatorname{cov}(e_{4t}, e_{3t}) & \operatorname{var}(e_{4t}) \end{pmatrix}
\]

The Granger representation theorem asserts that if the coefficient matrix \(\Pi\) has reduced rank \(r < k\), where \(r\) refers to the number of cointegrating relations and \(k\) is the number of endogenous variables in the system, then there exist \(k \times r\) matrices \(\alpha\) and \(\beta\) each with rank \(r\), such that \(\Pi = \alpha \beta'\) and \(\beta'y_t\) is \(I(0)\). Each column of \(\beta\) is the cointegrating vector. Elements of \(\alpha\) are adjustment parameters in the VEC model. Thus, \(\alpha \beta'y_{t,1}\) express the error correction term (ECT). Therefore, equation (4.10) is the VEC model, which is the VAR model with the inclusion of ECT.
Johansen's methodology is to estimate \( \Pi \) from an unrestricted VAR and test whether the restrictions implied by reduced rank of \( \Pi \) can be rejected. If \( r = 0 \), then \( \Pi = 0 \) and the components of \( y_t \) is not cointegrated and the VEC model then takes the form of a VAR in first difference:

\[
\Delta y_t = \sum_{i=1}^{r-1} \Gamma_i \Delta y_{t-i} + e_t
\]  

(4.11)

In our analysis, if \( r = 4 \), the \( \Pi \) matrix has full rank. The related model is same as equation (4.10), which may reduce to equation (4.9). This may be due to the low power of the cointegration test with small sample sizes or specification error. If \( 0 < r < 4 \), the model is:

\[
\Delta y_t = \alpha \beta' y_{t-1} + \sum_{i=1}^{r-1} \Gamma_i \Delta y_{t-i} + e_t
\]  

(4.12)

which means that \( y_t \) is cointegrated with \( r \) cointegrating relations or \( r \) long-term equilibrium relationships.

In order to determine the number of cointegrating relations, Johansen and Juselius (1990) developed two test statistics, which are trace statistic and maximum eigenvalue statistic. The critical values are obtained from Osterwald-Lenum (1992). We use the likelihood ratio trace statistic in our analysis. The trace statistic for a null hypothesis of \( r \) cointegrating relation is:

\[
Q_r = -n \sum_{i=r+1}^{k} \log(1 - \hat{\lambda}_i)
\]  

(4.13)
where

\[ \lambda_i \] is the i-th largest eigenvalues of \( \Pi \).

We determine the number of cointegrating relations \( r \) by proceeding sequentially from \( r = 0 \) to \( r = 3 \) until a rejection is found. The null and alternative hypotheses are

\[
H_a : r = 0 \\
H_1 : r > 0
\]

The null hypothesis means no cointegrating relation versus the alternative hypothesis of at least one cointegrating relation. If we reject the null hypothesis, then we proceed to test \( H_0 : r = 1 \) versus \( H_1 : r > 1 \). If we do not reject the null hypothesis, it implies that the system has one cointegrating relation. However, if we fail to reject \( H_0 \), we then test for \( H_0 : r = 2 \) against \( H_1 : r > 2 \). Since our analysis has two systems with four variables, the test is conducted up to \( r = 3 \). We perform the test for the full sample period and both sub-sample periods, from one lag to six lags before the optimal lag length is determined. The lag length that minimizes the Bayesian Schwarz Criterion (BIC) in VEC model will be the value chosen for \( m \) and \( p = 1 \). We use the BIC criterion because both the Akaike Information Criterion (AIC) and Final Prediction Error (FPE) criteria overestimate the order of an AR(p) time series model asymptotically. To avoid this overestimation, a
stronger penalty term is needed which is included in the BIC criterion.\(^2\) BIC is sometimes referred to as the Schwartz Criterion (SC). The SC criterion is defined as:

\[
SC = -2 \left( \frac{L}{n} \right) + \frac{k \log n}{n} \tag{4.14}
\]

where

- \(k\) is the number of parameters,
- \(n\) is the number of observations,
- \(L\) is the log likelihood.

Each system of equation may consist of one, two or three error correction terms in the VEC model.

### 4.4 Adjustments to Disequilibrium and Granger Causality Test

The error correction equation may be interpreted as the disequilibrium mechanism which guides the system to the equilibrium. We now demonstrate the test to determine which variable responds to disequilibrium in the error correction model (ECM). We carry out join tests on the error correction terms to find out which variable adjusts to disequilibrium in the system. As an example, consider a VEC model for \(Y_{1t}\) with two error correction terms:

\(^2\) Seeingle and White (1999) for details.
\[ \Delta Y_t = \eta_t + \rho_{11} \Delta Y_{1,t-1} + \rho_{12} \Delta Y_{2,t-1} + \sum_{i=1}^{4} \sum_{j=1}^{m} \delta_j \Delta Y_{yt} + \epsilon_t \quad (4.15) \]

If \( Y_{1t} \) adjusts to disequilibrium, then the error correction terms should be jointly significant. We then apply the F-test that needs the value of the sum of squared errors from equation (4.15), denoted as \( \text{RSS}_{\text{ui}} \), and the sum of squared errors from the same equation but with the exclusion of error correction terms, denoted as \( \text{RSS}_{\text{R}} \). The F-statistic is computed as follows:

\[ F = \frac{(\text{RSS}_{\text{R}} - \text{RSS}_{\text{ui}}) / r}{\text{RSS}_{\text{R}} / df} \quad (4.16) \]

where

- \( r \) = number of cointegrating relation,
- \( df \) = degree of freedom in the unrestricted equation.

The statistic has \( F(r, df) \) distribution under the null hypothesis that the coefficients of the error correction terms are both zero. If the F-statistic is larger than the critical value, it implies that the dependent variable adjusts to disequilibrium.

Granger (1969) introduced the concept of Granger causality. \( Y_{1t} \) is said to be Granger-caused by \( Y_{2t} \) if the lagged values of \( Y_{2t} \) can improve the explanation of the current \( Y_{1t} \) apart from the past values of \( Y_{1t} \) itself and other explanatory variables. In other words, the coefficients of the lags of \( Y_{2t} \) are statistically significant. There are four possible directions of causality:
(a) Unidirectional causality from \( Y_{2t} \) to \( Y_{1t} \).

(b) Unidirectional causality from \( Y_{1t} \) to \( Y_{2t} \).

(c) Bi-directional causality between \( Y_{1t} \) and \( Y_{2t} \).

(d) Independence or no causality between \( Y_{1t} \) and \( Y_{2t} \).

To conduct the Granger causality test, we use the VEC model rather than the VAR model if the variables are cointegrated. For the four variables in the system, we consider a VEC model with \( m \) lags and one ECT:

\[
\Delta Y_{1t} = \eta_1 + \rho_1 \hat{z}_{1,t-1} + \sum_{j=1}^{m} \delta_{11,j} \Delta Y_{1t-j} + \sum_{j=1}^{m} \delta_{12,j} \Delta Y_{2t-j} + \sum_{j=1}^{m} \delta_{13,j} \Delta Y_{3t-j} + \sum_{j=1}^{m} \delta_{14,j} \Delta Y_{4t-j} + \varepsilon_{1t}
\]

(4.17)

\[
\Delta Y_{2t} = \eta_2 + \rho_2 \hat{z}_{2,t-1} + \sum_{j=1}^{m} \delta_{21,j} \Delta Y_{1t-j} + \sum_{j=1}^{m} \delta_{22,j} \Delta Y_{2t-j} + \sum_{j=1}^{m} \delta_{23,j} \Delta Y_{3t-j} + \sum_{j=1}^{m} \delta_{24,j} \Delta Y_{4t-j} + \varepsilon_{2t}
\]

(4.18)

\[
\Delta Y_{3t} = \eta_3 + \rho_3 \hat{z}_{3,t-1} + \sum_{j=1}^{m} \delta_{31,j} \Delta Y_{1t-j} + \sum_{j=1}^{m} \delta_{32,j} \Delta Y_{2t-j} + \sum_{j=1}^{m} \delta_{33,j} \Delta Y_{3t-j} + \sum_{j=1}^{m} \delta_{34,j} \Delta Y_{4t-j} + \varepsilon_{3t}
\]

(4.19)

\[
\Delta Y_{4t} = \eta_4 + \rho_4 \hat{z}_{4,t-1} + \sum_{j=1}^{m} \delta_{41,j} \Delta Y_{1t-j} + \sum_{j=1}^{m} \delta_{42,j} \Delta Y_{2t-j} + \sum_{j=1}^{m} \delta_{43,j} \Delta Y_{3t-j} + \sum_{j=1}^{m} \delta_{44,j} \Delta Y_{4t-j} + \varepsilon_{4t}
\]

(4.20)
If we want to check whether $Y_{2i}$ Granger-causes $Y_{1i}$, we proceed with the Granger causality test by testing the null and alternative hypotheses as follows:

\[ H_0 : \rho_1 = \delta_{ij} = 0 \]

\[ H_1 : \text{at least one equation above is not true} \]

If we cannot reject the null hypothesis, then $Y_{2i}$ does not Granger-cause $Y_{1i}$. Inversely, $Y_{2i}$ is said to lead or Granger-cause $Y_{1i}$ if the rejection is found. We repeat the test with all possible causality directions and for the other variables in a similar manner. To implement the Granger causality test, the F-statistics are calculated under the null hypothesis of no causality.

4.5 Impulse Response Function

A commonly applied tool used in macroeconomic policy is the impulse response function (IRF). A shock to the $i$-th variable not only directly affects the $i$-th variable but is also further transmitted to all of the other endogenous variable through the dynamic structure of the VAR. An IRF traces the effect of a one standard deviation shock to one of the innovations on current and future values of all the other endogenous variables.

This study examines the responses among the four variables due to one-time shock in any of the other variables. We use the Cholesky factor from the residual
covariance matrix with the adjustment of degrees of freedom, which is available in the EViews programme. We apply this test on both the sub-samples for comparison.