CHAPTER V

APPLICATION OF MATHEMATICAL CONCEPTS TO ECONOMICS

Mathematical Method

It is the introduction of the mathematical method which marks a stage of growth in economics, the entrance of economics on a scientific era. Economists today will agree that mathematical treatment in economics is relatively useful as the relations become relatively complicated.

Before Jevons, however, all the many attempts of mathematical approach fell flat. Professor Irving Fisher points out that those writers before Jevons suffered complete oblivion in mathematical treatment because they misconceived the application of mathematics and that the world was not prepared for it, as the movement was too advanced and premature.[1]

Mathematical method really began with Jevons

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[1] I. Fisher, Mathematical Investigations in the Theory of Value and Prices, New Haven, Yale University Press, p. 109.

in 1871.[2] Jevons' pursuit of the mathematical method in economics is quite evident in his Theory of Political Economy, in which he expressed most of his economic reasonings in mathematical terms, though there is still plenty of scope for his mathematical treatment in it.

In trying to arrive at accurate quantitative notions about utility, value, labour, capital, etc., Jevons came to the conclusion that even the most puzzling of notions value rendered the possibility of mathematical analysis and expression. Jevons' theory of utility and its applications are in many respects the most striking outcome of mathematical economics.

Application of the Law of Substitution of Similars

The Law of Substitution of Similars expresses the capacity of mutual replacement existing in any two objects which are alike or equivalent to a sufficient degree. For instance, in the equation A = B, A might mean "Iron", and B "the most useful of metals." The principle behind this Law simply involves the substitution of the term on one side of an identity for the other term.

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[2] Others who have pursued or admired the mathematical path, dependently or independently, are Whewell, Cournot Gossen, Walras, Newcomb, Wicksteed, Foxwell, Edgeworth, etc.

Jevons applied this mathematical notion of the Law of Substitution of Similars to his pure economic theory in the form of the Law of Indifference. For instance, in his "Theory of Exchange" he expressed the Law of Indifference as follows: "that in the same open market, at any one moment, there cannot be two prices for the same kind of article."[3] In other words, when two objects or commodities are alike or equivalent to a sufficient degree, they will either of them be taken instead of the other with perfect indifference by a purchaser. Take, for example, two sacks of rice which appear to be exactly the same quality and are of equal contents. A purchaser will be indifferent as to which sack of rice he should purchase since both sacks of rice are sufficiently alike.

Every such act of indifferent choice gives rise to an equation of degrees of utility. Thus whereas in logic the Substitution of Similars applies to the substitution of terms having a similar meaning, in economics the Law of Indifference refers to an equivalence of utilities. The principle involved in its application, however, is essentially the same.

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[3] W.S. Jevons, Theory of Political Economy, 5th.ed., p. 91.

Similarly, Jevons' "Theory of Rent"[4] also rests upon the principle of the Law of Indifference. Accordingly, if different qualities of land yield different amounts of produce for the same labour, there must be an excess of profit in some over others.

Application of the Theory of Probability

Jevons considered the theory of probability as absolutely essential for the precise study of any empirical science. In his conception of the scientific method, he pointed out the impossibility of expounding the methods of induction without resting them upon the theory of probability.

As we have no choice but to be satisfied with partial knowledge, since perfect knowledge would be infinite knowledge beyond our capacities, we have to rely on the theory of probability to acquire some measure of precision.

Thus Jevons contended that, if the statistical methods in economics might have some degree of reliability, they need to be grounded upon the theory of probability.

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[4] Jevons' chapter on the "Theory of Rent" in The Theory of Political Economy, follows closely that of McCulloch and James Mill, being based on differences in the fertility of different pieces of land, and on the law of diminishing returns.

Consequently, he urged the extensive use of probability techniques in economics, as well as in other social sciences.

He cited the examples of the Means and the Laws of Divergence from an Average as statistical concepts founded on the theory of probability, though they are simply "the alpha and omega of the scientific method."

An important area of application of the theory of probability is in his applied economics. According to him, one of the chief difficulties in any investigation of value arises from the independent variations of value. If the value of an article, A (gold, for instance), falls in comparison with several other articles, B, C, D, E (copper, lead, tin, silver, for instance) then the rise or fall of that article under investigation will be difficult and thus we have to fall back upon mere probabilities.

As, for example, Jevons applied the ordinary methods of the theory of probabilities to the results stated in his pamphlet on the value of gold.[5] Only by

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[5] Jevons' article on "The Serious Fall in the Value of Gold" was reprinted in Investigations in Currency and Finance, 2nd. ed., ed. by H. S. Foxwell, London, Macmillan and Co., 1884, p. 13-118.

doing so he was able to establish in greater certainty that gold had depreciated in some degree.

Though Jevons did not specifically make use of the probability techniques or the theory of games in his discussion on "Anticipated Feeling," or "Uncertainty of Future Events,"[6] it is nevertheless not unreasonable to assume that he had something of interest to say about the problem of uncertainty in economic behaviour, since Jevons thought that probability theory ought to form the basis of the social as well as the natural sciences.

Opposition to Jevons' Mathematical Treatment in Economics

While many have pursued or admired the mathematical path, a few have seen nothing good in the mathematical method. A writer in the Saturday Review, quoted by Professor Edgeworth says of Jevons:

> "The equations .....assuming them to be legitimate, seem to us to be simply useless as long as the functions are obviously indeterminable."

Professor Fisher, in defence of Jevons, comments

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6 Discussions on"Anticipated Feeling" and "Uncertainty of Future Events" are found in Chapter II of The Theory of Political Economy, 5th. ed.

that mathematics studies relations as well as calculations and that numerical indeterminability is common even in mathematical physics.

Professor Cairnes was of the opinion that economic truths are not discoverable through the instru instrumentality of mathematics. Though he did not refute the possibility of employing geometrical diagrams or mathematical formulae for the purpose of exhibiting economic doctrines reached by other paths, he categorically denied Jevons' doctrine that economic knowledge could be extended by such means and that mathematics could be applied to the development of economic truth, as it had been applied to the development of mechanical and physical truth. He further criticised that unless we could obtain premises capable of being stated with numerical accuracy, the employment of such methods was necessarily barren. Here he was indeed following the footsteps of Mill, who had remarked that mathematical principles were "manifestly inapplicable, where causes on which any class of phenomena depends are so imperfectly accessible to our observation, that we cannot ascertain, by a proper induction, their numerical laws."[7]

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[7] J.S. Mill, Logic, Bk. III, Ch. 24.

Professor Cliffe Leslie[8] argued similarly against the application of mathematics to economics on the ground that economic premises were not capable of exact quantitative determination. He was in favour of the "historical method of investigation," which he carried over from the special sphere of jurisprudence to that of political economy. To Professor Leslie, political economy is not a body of natural laws in the true sense, or of universal and immutable truths, but an assemblage of speculations and doctrines which are the result of a particular history, coloured even by the history and character of its chief writers.

Last but not least, Dr. Ingram had expressed no confidence in mathematical approach to economics. "Mathematics can indeed formulate ratios of exchange," said Dr. Ingram, "when they have once been observed; but it cannot by any process of its own determine those ratios; for quantitative conclusions imply quantitative premises and these are wanting. There is then no future

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[8] T.E. Cliffe Leslie, an orthodox English political economist, besides questioning both the possibility and the expediency of using mathematical methods in economics, had also expressed his regret that "much of Mr. Jevons' own reasoning is put into a mathematical form, because it is one unintelligible or unattractive to many students of considerable intellectual power and attainments." (Essays in Political Economy, 2nd. edition, Dublin, 1888, p.66-72.

for this kind of study, and is only waste of intellectual power to pursue it."[9] However, it is unfortunate that Dr. Ingram did not discuss what mathematics had done or attempted, but merely complained that it could not do everything and therefore had no future.

In passing, it has to be pointed out that what is essentially the same theory advanced by Jevons has been independently worked out by Menger[10] and the Austrian school, without the explicit use of diagrams or algebraical formulae. It therefore appears that the utility of mathematical method is purely relative. It helps greatly some persons, slightly others, or even be a hindrance to some. If Menger and the Austrian School could advance the same theory without being much involved with mathematics as they had done, there is reason to doubt whether Jevons' claim that economics must be purely a mathematical science need to be taken seriously.

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[9] J.K. Ingram, History of Political Economy, New York, 1888, p.182.

[10] Menger's work was published in 1871, the same year as that in which Jevons' Theory of Political Economy appeared. It may be said that the work of Menger and his followers is mathematical in tone, though not in language.

I would, therefore, tend to agree with Dr. Kaufamann that the mathematical method can neither be shown "on philosophical grounds" to be the only scientific method in economics, nor can it be rejected on other philosophical grounds as in principle inadequate.[11] One can only try to get evidence by careful analysis as to the extent of its usefulness.

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[11] F. Kaufamnn, "On the Subject-Matter and Method of Economic Science," in Economica, Vol. XIII, 1933, p.401.