CHAPTER 3

THE METHOD OF DEA, DDF AND MLPI

3.1 Introduction

As has been discussed in the literature review chapter, in any organization, technical efficiency can be determined in order to measure the performance. Technical efficiency focuses on the ability to increase the output while keeping the input constant or the ability to reduce the input while keeping the output constant. When incorporating undesirable output, such as pollutants, the measurement is essentially on eco-efficiency. The concept of eco-efficiency can be classified as a measurement of efficiency with the integration of environmental pollution that is regarded as undesirable output together with desirable output. Eco-efficiency can also be interpreted as the efficiency measurement of the economic efficiency that produces desirable output, and ecological efficiency which produces undesirable output. The techniques to measure these two efficiencies will be presented in detail in this chapter.

Data Envelopment Analysis (DEA), which can be considered as a popular technique, has been chosen in this study to measure the technical efficiency. Another approach that has gained popularity, called the Directional Distance Function (DDF) approach, is also employed in this study to evaluate the eco-efficiency. The underlying characteristics of these methods are described further in order to highlight their strengths and weaknesses.

DEA is a well-known technique that has been utilized for efficiency measurement. This technique is able to figure out the efficiency score of organizations and estimate the input that needs to be reduced or output that needs to be increased in relation to the efficiency score. Nevertheless this conventional DEA model accounts for only two categories of variable which are the input and the desirable output variables. When undesirable output is present, the DEA model is no longer applicable. Therefore, another approach that of DDF which treats the separation of undesirable output in the model is employed in this study. To complete the analysis, it would be an advantage to extend the understanding of the productivity change over the years through the Malmquist Luanberger (ML) productivity index which is calculated by the DDF model. Efficiency and productivity measurement are widely used and can be put to work together to complement each other.

The remainder of this chapter is organized in the following manner. This chapter will start with a brief overview on the production possibility set in Section 3.2. Next, Section 3.3 discusses the DEA model. It includes the earlier fractional program of DEA as well as the input and output orientations for variable return to scale (VRS) and constant return to scale (CRS) models in DEA. In addition, the slack-based measure approach as a fundamental of non-radial approach is briefly introduced. Further, Section 3.4 explains the model when it incorporates desirable and undesirable outputs. The core of this chapter provides the inclusion of undesirable output in the efficiency measurement with the DDF model in Section 3.5 which is within the DEA framework. In addition, this chapter also discusses the Malmquist Luenberger productivity index (MLPI) in Section 3.6 in order to study the productivity change over the study period. Section 3.7 summarizes the chapter.

3.2 Production Possibility Set (PPS)

In the production system, the inputs and outputs are two things that are very interrelated. Inputs can be considered as goods that are used in production, while outputs are goods that are produced. For example, in the paper and pulp industry, wood fibre, energy as well as labour are needed as inputs to produce the outputs including net pulp output, newsprint and paperboards. To achieve the optimal production, the amount of inputs should be estimated appropriately so that the outputs can be produced efficiently. To relate between inputs and outputs, the production function has been employed. The production function is the relationship between the inputs and outputs given some technology.

If the combination of inputs and outputs is technically feasible, it can be represented as a 'production possibility set' (PPS). Figure 3.1 below describes the production possibility set.

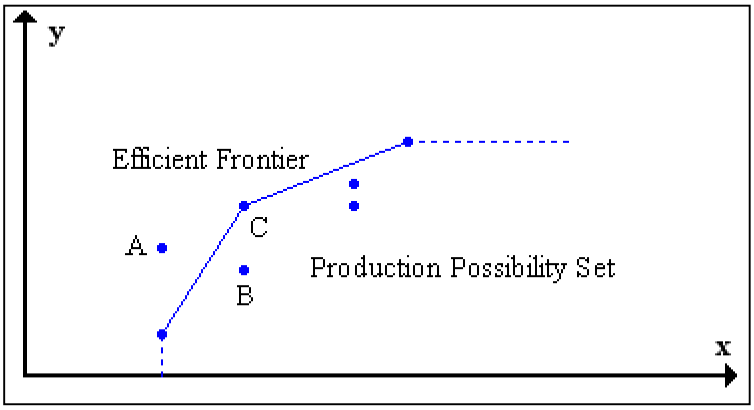

Figure 3.1: Production Possibility Set

Source: Thanassoulis (2001)

The boundary that connects the points is called the production possibility frontier or efficient frontier. Any point within the set is feasible. For example, point B is feasible but not efficient enough. While point A is not possible at all since the point lies outside the boundary of PPS. Point C is efficient since the point lies on the efficient frontier. Hence, only the points that lie on the efficient frontier only can be ascertained as efficient. The PPS will be represented on the production technology (S) as:

$$S = \{(x, y) : x \text{ can produce } y\} \quad (3.1)$$

In the expression, y represents an output vector and x represents an input vector. So, the above definition simply defines the production possibilities as the set of input-output vectors that are attainable given the production technology S. Following Shephard (1970), the input possibility set $L(y)$ for each y can be defined as below:

$$L(y) = \{x : (x, y) \in S\} \quad (3.2)$$

While for the output possibility set $D(x)$ for each x as below:

$$D(x) = \{y : (x, y) \in S\} \quad (3.3)$$

3.3 Data Envelopment Analysis (DEA)

DEA is a linear programming technique for measuring the relative efficiency of a set of decision making units (DMUs) or units of assessment in their use of multiple inputs to produce multiple outputs. DEA identifies a subset of efficient 'best practice' DMUs, and, for the remaining DMUs, their efficiency level is derived by comparison to a frontier constructed from the 'best practice' DMUs. Each of the DMU is analysed separately to examine whether the DMU under consideration could improve its performance by increasing its output and decreasing its input. The best performing DMU is assigned an efficiency score of 100 percent while the performance of other DMUs may vary between 0 and 100 percent relative to the best performance (Thanassoulis, 2001).

Beyond the efficiency measure, DEA also provides other sources of managerial information relating to the performance of the DMUs. DEA identifies the efficient peers for each inefficient DMU. Therefore, DEA can be viewed as a benchmarking technique, as it allows decision makers to locate and understand the nature of the inefficiencies of a DMU by comparing it with a selected set of efficient DMUs with a similar profile.

This technique, originated from the seminal work by Charnes et al. (1978) and has been developed in the Operation Research/Management Science field, which uses mathematical programming techniques and models to solve the problem.

3.3.1 DEA Fractional Program

To begin this model, some notations have been made. Let $x \in R_{+}^{I}$ represent an input vector and $y \in R_{+}^{J}$ represent an output vector while subscripts *i* and *j* represent particular inputs and outputs. Thus x_i represents the i^{th} input, and y_j represents the j^{th} output of a DMU. Then, let the total number of inputs and outputs be represented by *I* and *J* with *I* and *J* > 0. In DEA, multiple inputs and outputs are linearly aggregated using weights. The optimal weights may vary from one DMU to another DMU. Therefore, in the equation below, a_i is the weight assigned to input x_i and b_j is the weight assigned to output y_j during the aggregation.

$$\text{Efficiency} = \frac{\text{Output}}{\text{Input}} = \frac{\sum_{j=1}^{J} b_j\, y_j}{\sum_{i=1}^{I} a_i\, x_i} \tag{3.4}$$

Then the ratio concept above was transformed into a linear programming model. Assume there are *N* DMUs, which have to be compared for the efficiency. Let *m* be one of the DMUs to maximize the efficiency. The following equation gives the ratio form of the basic DEA model, with an output orientation (Ramanathan, 2003).

$$\text{Max} = \frac{\sum_{j=1}^{J} b_{jm}\, y_{jm}}{\sum_{i=1}^{I} a_{im}\, x_{im}}$$

Subject to

$$0 \leq \frac{\sum_{j=1}^{J} b_{jm}\, y_{jn}}{\sum_{i=1}^{I} a_{im}\, x_{in}} \leq 1\,;\, n = 1,2,\dots,\mathrm{N}$$

$$b_{jm}, a_{im} \geq 0\,;\, i = 1,2,\dots,\mathrm{I}\,;\;\; j = 1,2,\dots,\mathrm{J} \tag{3.5}$$

Where

b_{jm} = weight of j^{th} output

y_{jm} = j^{th} output of the m^{th} DMU

a_{im} = weight of i^{th} input

x_{im} = i^{th} input of the m^{th} DMU

y_{jn} = j^{th} output of the n^{th} DMU

x_{in} = i^{th} input of the n^{th} DMU

The above mathematical program, which is considered as a fractional program, when solved, will give the values of weights a_i and b_j, which will maximize the efficiency of DMU *m*.

3.3.2 Fractional Program to Linear Program

Mathematical programs can be transformed to linear programs, which are a simpler formulation than fractional programs. The simplest way to convert fractional programs to linear programs is to normalize either the numerator or the denominator of the fractional programming objective function. The weighted sum of inputs is unity (equal to 1) in the linear programming constraint. Since the weighted sum of outputs that has to be maximized is the objective function, this formulation is considered as the output maximization DEA program. On the other hand, if the weighted sum of outputs is unity, the formulation is considered as the input minimization DEA program (Ramanathan, 2003). The above fractional program when transformed to the linear program is as the follows.

$$\begin{aligned}
&\text{Max } \sum_{j=1}^{J} b_{jm}\, y_{jm} \\
&\text{Subject to} \\
&\quad \sum_{i=1}^{I} a_{im} x_{im} = 1 \\
&\quad \sum_{j=1}^{J} b_{jm}\, y_{jn} - \sum_{i=1}^{I} a_{im}\; x_{in} \leq 0\,;\ \mathrm{n} = 1,2,\dots,\mathrm{N} \\
&\quad b_{jm}, a_{im} \geq 0\,;\ i = 1,2,\dots,\mathrm{I}\,;\ \ j = 1,2,\dots,\mathrm{J} \qquad (3.6)
\end{aligned}$$

3.3.3 CCR and BCC Models

There are two classical DEA models, CCR (Charnes, Cooper, & Rhodes, 1978) and BCC (Banker, Charnes, & Cooper, 1984). Both models can be orientated in two different ways, which are output maximization or input minimization. For input orientation, the assessment is on the movement of input level towards the frontier through proportional reduction while the output level remains unchanged. The objective of the input orientated model is to minimize inputs while producing at least the given output levels. This input orientation is contrary to output orientation where the movement of output level towards the efficiency frontier through the proportional increase while input level remains unchanged. The objective of the output orientated model is to maximize outputs while using not more than the observed amount of any input (Charnes et al., 1994). The choice between an input and an output orientation can be based upon the consideration of which factors are more easily controlled by the DMU. For instance, if producers are required to meet market demand, and can freely adjust input usage, then an input orientation model is appropriate (Ramanathan, 2003).

The CCR model is referred to as the constant return to scale (CRS) model while the BCC is referred to as the variable return to scale (VRS) model. Banker et al. (1984) extended the CRS model by relaxing the assumption of CRS to VRS. The VRS model differs from the CRS model in that it envelops the data more closely, thereby producing technical efficiency estimates greater than or equal to those from the CRS model (VRS $\geq$ CRS).

To differentiate between CRS and VRS, the CRS model estimates the gross efficiency of a DMU while the VRS model takes into account the variation of efficiency with respect to the scale of operation, and hence, measures pure technical efficiency. The

CRS and VRS frontier can be illustrated in Figure 3.2. From Figure 3.2, only DMU A is considered as 100 percent efficient through CRS model while all the DMUs are assigned 100 percent efficient through VRS model. Thus, this illustration exhibits that DMU A, B, and C through VRS model are purely efficient due to their scales of operation.

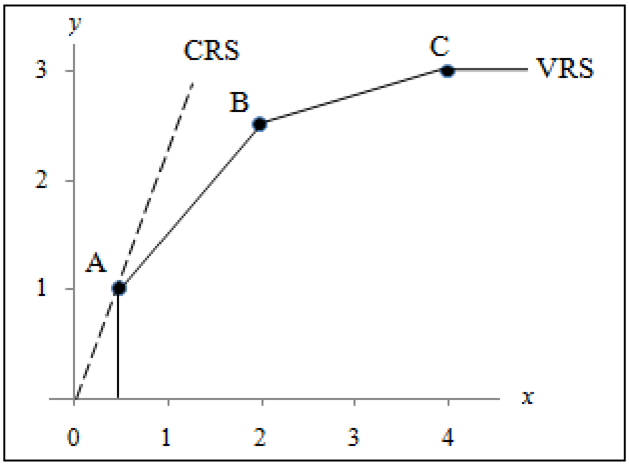

Figure 3.2: CRS and VRS technology frontier

Source: Thanassoulis (2001)

With regards to the choice of CRS or VRS model, Dyson et al. (2001) recommended running the return to scale test in which the data should be tested separately for scale effect. The VRS model is appropriate only when scale effects can be demonstrated.

For the CCR model, a formal definition of the PPS to this model can be made by four postulations as below (Thanassoulis, 2001):

Postulate 1: Strong free disposability of input and output

If $(x', y') \in S$ and $x \geq x'$, then $(x, y') \in S$ where $x \geq x'$ means that at least one element of x is greater than the corresponding element x'. If $(x', y') \in S$ and $y \leq$

y', then $(x', y) \in S$ where $y \leq y'$means that at least one element of y is less than the corresponding element y'.

This can informally be referred to as a phenomenon of inefficient production.

Postulate 2: No output can be produced without some input

$(x', 0) \in S$; but if $y' \geq 0$ then $(0, y') \notin S$.

Postulate 3: Constant return to scale

If $(x', y') \in S$ then for each positive real value $\lambda > 0$, thus $(\lambda x', \lambda y') \in S$.

Postulate 4: Minimum extrapolation

All observed DMUs $\{(x_n, y_n) : n = 1, 2 \dots \mathrm{N}\} \in S$ and S is the smallest closed and bounded set satisfying postulate 1 – 3.

Following Färe, Grosskopf and Lovell (1994a) the connection between DEA efficiency measurement and the representation of the production technology (S) is given by:

$$\begin{aligned} S = \{(x, y)\colon & \sum_{n=1}^{N} z_n x_{in} \leq x_i \,, i = 1,2, \dots, I; \\ & \sum_{n=1}^{N} z_n y_{jn} \geq y_j \,, j = 1,2, \dots, J; \\ & z_n \geq 0 \,; \;\; n = 1,2, \dots, N\} \end{aligned} \tag{3.7}$$

where z_n are the intensity variables or weights assigned to each observation of $n = 1,2, \dots, N$ in constructing the production possibility frontier for input x and output y.

Tables 3.1 and 3.2 represent the four different DEA models in CCR and BCC. These four models are output maximizing and input minimizing for primal model and dual model. The primal model is also referred to as the multiplier formulation while the dual model is referred to as the envelopment formulation of the DEA model. In the primal model, a_i and b_j are the weights for the input and output, respectively, and treated as variables in the model. The input and output weights at the optimal solution can be used to indicate the relative importance of the inputs and outputs in determining the efficiency level of the DMU. The BCC model differs from the basic CCR model while assessing efficiency because BCC includes the convexity constraint $\sum_{n=1}^{N} z_n = 1$ in the dual formulation.

Table 3.1: CCR models with input and output orientation

Input Orientation	
Primal Model	Dual Model
Max $\sum_{j=1}^{J} b_{jm}\, y_{jm}$ Subject to $\sum_{i=1}^{I} a_{im}\, x_{im} = 1$ $\sum_{j=1}^{J} b_{jm}\, y_{jn} - \sum_{i=1}^{I} a_{im}\, x_{in} \le 0\,; n = 1,2,\dots,N$ $b_{jm}, a_{im} \ge 0\,;\; i = 1,2,\dots,I\,;\; j = 1,2,\dots,J$ (3.8)	Min θ_m Subject to $\sum_{n=1}^{N} z_n x_{in} \le \theta_m x_{im}\,; i = 1,2,\dots,I$ $\sum_{n=1}^{N} z_n y_{jn} \ge y_{jm};\; j = 1,2,\dots,J$ $z_n \ge 0\,;\; n = 1,2,\dots,N$ θ_m unrestricted (free) (3.9)
Output Orientation	
Primal Model	Dual Model
Min $\sum_{i=1}^{I} a'_{im}\, x_{im}$ Subject to $\sum_{j=1}^{J} b'_{jm}\, y_{jm} = 1$ $\sum_{j=1}^{J} b'_{jm}\, y_{jn} - \sum_{i=1}^{I} a'_{im}\, x_{in} \le 0\,; n = 1,2,\dots,N$ $b'_{jm}, a'_{im} \ge 0\,;\; i = 1,2,\dots,I\,;\; j = 1,2,\dots,J$ (3.10)	Max $\emptyset_m$ Subject to $\sum_{n=1}^{N} z_n x_{in} \le x_{im}\,; i = 1,2,\dots,I$ $\sum_{n=1}^{N} z_n y_{jn} \ge \emptyset_m y_{jm}\,;\; j = 1,2,\dots,J$ $z_n \ge 0\,;\; n = 1,2,\dots,N$ $\emptyset_m$ unrestricted (free) (3.11)

Table 3.2: BCC models with input and output orientation

Input Orientation	
Primal Model	Dual Model
Max $\sum_{j=1}^{J} b_{jm}\, y_{jm} - \rho_m$ Subject to $\sum_{i=1}^{I} a_{im}\, x_{im} = 1$	Min θ_m Subject to $\sum_{n=1}^{N} z_n x_{in} \le \theta_m x_{im}\,; i = 1,2,\dots,I$ $\sum_{n=1}^{N} z_n y_{jn} \ge y_{jm}\,;\; j = 1,2,\dots,J$

$\sum_{j=1}^{J} b_{jm}\, y_{jn} - \sum_{i=1}^{I} a_{im}\, x_{in} - \rho_m \le 0\, ; n = 1,2,\dots,N$ $b_{jm}, a_{im} \ge 0\, ;\ i = 1,2,\dots,I\, ;\ \ j = 1,2,\dots,J$ (3.12)	$\sum_{n=1}^{N} z_n = 1$ $z_n \ge 0\, ;\ n = 1,2,\dots,N$ θ_m unrestricted (free) (3.13)
Output Orientation	
Primal Model	Dual Model
Min $\sum_{i=1}^{I} a'_{im}\, x_{im} - \rho_m$ Subject to $\sum_{j=1}^{J} b'_{jm}\, y_{jm} = 1$ $\sum_{j=1}^{J} b'_{jm}\, y_{jn} - \sum_{i=1}^{I} a'_{im}\, x_{in} - \rho_m \le 0\, ; n = 1,2,,N$ $b'_{jm}, a'_{im} \ge 0\, ;\ i = 1,2,\dots,I\, ;\ \ j = 1,2,\dots,J$ (3.14)	Max $\emptyset_m$ Subject to $\sum_{n=1}^{N} z_n x_{in} \le x_{im}\, ; i = 1,2,\dots,I$ $\sum_{n=1}^{N} z_n y_{jn} \ge \emptyset_m y_{jm}\, ; j = 1,2,\dots,J$ $\sum_{n=1}^{N} z_n = 1$ $z_{\mathrm{n}} \ge 0\, ;\ n = 1,2,\dots,N$ $\emptyset_m$ unrestricted (free) (3.15)

(The term ρ_m in primal model was interpreted by BCC as an indicator of returns to scale)

The dual model that involves θ and $\emptyset$ measure the efficiency of a DMU in terms of the radial contraction factor with contraction to its input levels or expansion to its output levels under efficient operation. The model that involves θ aims to produce the observed outputs with minimum inputs. That is the reason why inputs are multiplied by efficiency, according to its constraint rules. Because of this characteristic, this model is classified as an input oriented envelopment model. Another model that involves $\emptyset$ is aimed to maximize output production, subject to a given resource level. Therefore, this model is classified as an output oriented envelopment model. Each model is in the form of a pair of dual linear programs. This means that the dual of the output maximizing multiplier model is the input oriented envelopment model. Similarly the dual of the input minimizing multiplier model is the output oriented envelopment model (Ramanathan, 2003). The difference between multiplier and envelopment in DEA model is that the multiplier version is utilized when the input and output are emphasized in an application since the solution for the multiplier model will provide weights of input and outputs. While the envelopment version is used when the relations among the DMUs are emphasized since the solution will provide weights of DMUs.

In this study, the production process of the manufacturing sector is assumed to exhibit a Constant Return to Scale (CRS). This study will look at the overall technical efficiency measurement rather than pure technical efficiency and scale efficiency. The CRS assumption will also be utilized for the entire analysis in this study in order to compare the general performance among the states. Furthermore, the CRS model is assumed because the efficiency measure is obtained without controlling the scale size of the DMU. In other words, by using the CRS assumption, the scale size of the DMU does not impact on the efficiency score of the DMU (Thanassoulis, 2001). In addition, this study will also observe the productivity growth through the Malmquist Lunberger productivity index, which will be discussed later. According to Grifell-Tatjé and Lovell (1995), the CRS technology must however be imposed to get a more accurate calculation of the Malmquist index. Following these arguments, CRS is plausible to be used in this study.

The mathematical formulation, which is used in this study in order to measure the technical efficiency is the output oriented CRS model (3.11) in Table 3.1. The DEA output oriented envelopment model (3.11) seeks a set of *z* values, which maximize the $\emptyset_m$ and identifies a point within the production possibilities set whereby output levels of DMU *m* can be increased to the highest possible while inputs remain at the current level. The efficiency scores of DMUs in this model are bounded between zero and one. The best performing DMUs are assigned an efficiency score of one while the performances of other DMUs that score less than one are considered inefficient.

To describe the efficient frontier by using the output oriented DEA approach, Figure 3.3 exhibits five DMUs, which are A, B, C, D and E. Assume that all DMUs use a similar quantity of a single input (x) level and two different quantity of output (y_1, y_2) levels. The output oriented DEA identifies A, B, C and D as the best practice units whereby this line is also known as the efficient frontier. DMU E lies below the efficient frontier, thus DMU E is regarded inefficient. Point E' is the benchmarking standard for DMU E. The efficiency score for DMU E can be computed by 0E/0E', which is the ratio of radial distances. This implies that DMU E can improve its efficiency by as much as EE'/0E' to hit the target E'.

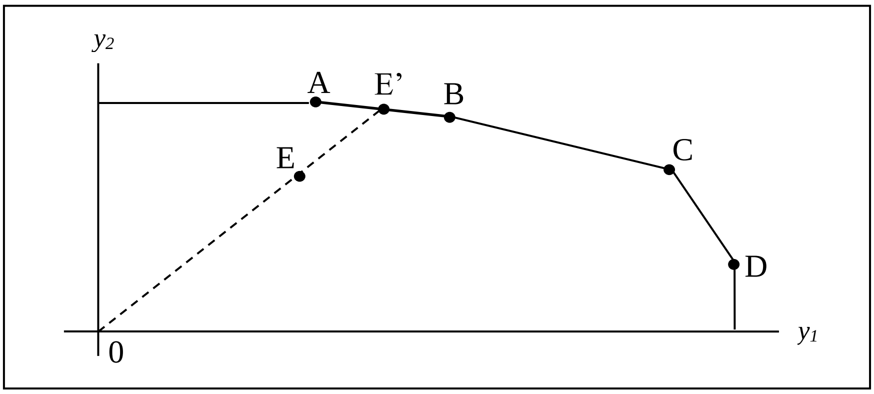

Figure 3.3: The efficiency frontier for output oriented DEA model

Source: Thanassoulis (2001)

Note that this conventional DEA model accounts for only two categories of variable, which are the input and the desirable output variables. When undesirable outputs are present, the model of DEA is no longer applicable. For instance, in Figure 3.3, DMU E is inefficient and its efficiency can be evaluated by referring to the frontier lines on DMU E'. This evaluation implies that DMU E needs to increase both y_1 and y_2 in order to improve the efficiency. If y_1 axis is substituted by undesirable output (u), then the concept of undesirable output is erroneous using the model of DEA. This is because the concept of desirable output contradicts with the undesirable output. The desirable output needs to be increased while the undesirable output needs to be decreased. Therefore, another approach that treats the separation of desirable and undesirable outputs will be discussed further in Sections 3.4 and 3.5 to overcome the erroneous of undesirable output concept in the output oriented DEA model.

3.3.4 A slack-based measure in DEA

Before continuing with the model incorporating the desirable and undesirable outputs, let us understand another model in the DEA approach, which is the slack-based measure. In the previous section, the DEA model, specifically the CRS model is a radial efficiency measure because the CRS model optimizes the inputs and outputs of the DMU at a certain proportion. The optimal objective value for the CRS model is called the ratio (or radial) efficiency. The optimal solution obtained will disclose the existence, if any, of excesses in inputs and shortfalls in outputs which are known as slacks (Tone, 2001). However, using the radial measure fails to take into account the non-zero input and output slacks in the efficiency measurement. On the other hand, the non-radial measures (i.e. slack-based measure) take into consideration the input and output slacks.

The slack-based measure model introduced by Tone (2001) is defined as follows:

$$
\begin{aligned}
&\text{Min} \frac{1-\frac{1}{I}\sum_{i=1}^{I}\frac{s_i}{x_{im}}}{1+\frac{1}{J}\sum_{j=1}^{J}\frac{s_j}{y_{jm}}} \\
&\text{Subject to} \\
&\qquad \sum_{n=1}^{N} z_n x_{in} + s_i = x_{im}\,;\ \ i = 1,2,\dots,I \\
&\qquad \sum_{n=1}^{N} z_n y_{jn} - s_j = y_{jm}\,;\ \ j = 1,2,\dots,J \\
&\qquad z_n,\ s_i,\ s_j \geq 0\,;\ \ n = 1,2,\dots,N
\end{aligned} \tag{3.16}
$$

Where s_i is the slack value of the i^{th} input, and s_j is the slack value of the j^{th} output. The slack s_i and s_j indicate the input excess and the output shortfall, respectively. The objective function in model (3.16) satisfies the properties of unit invariant and monotone whereby the measure should be invariant with respect to the unit of data and should be monotone decreasing in each slack in input and output, respectively.

Figure 3.4 illustrates the SBM model with a simple example using single input and single output. Using the CRS assumption, it can be seen that DMU B is inefficient. The efficiency score for DMU B based on output orientation can be computed by 3/6.67 which is the ratio of the radial distance with the score of 44.9 percent. The efficiency score for DMU B based on input orientation can be computed by 2.25/5, which is the ratio of the radial distance with the score of 45 percent. The non-radial model yields the same frontier as the CRS model, but may yield a different efficiency score. Using the slack-based measure model, DMU B may be projected to any point on the frontier between B' and B".

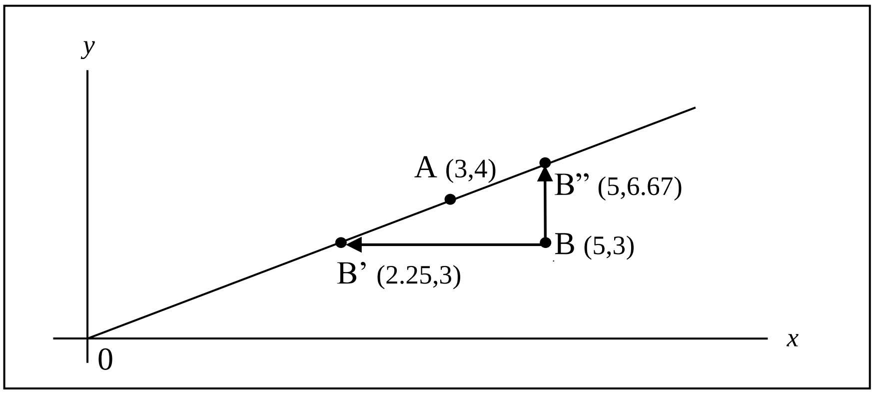

Figure 3.4: Illustration of the SBM model

Source: Fried et al. (2008)

3.4 Model incorporating the desirable and undesirable outputs

To continue with the model incorporating the desirable and undesirable outputs, additional notations have been added to expression (3.1). The notations used in the following are similar to the ones used in previous DEA models as to avoid confusion in the model development. Let $x \in R_+^I$ represents an input vector, $y \in R_+^J$ represents a desirable output vector while $u \in R_+^K$ represents an undesirable output vector. Thus, the above definition simply defines the "environmental output set" for production technology (*T)* as:

$$T = \{(x, y, u) : x \text{ can produce } (y, u)\} \qquad (3.17)$$

To specify and model the production technology when desirable and undesirable outputs are jointly produced, Färe et al.'s (2005) assumptions that have been denoted in the form of postulates as below have been followed:

Postulate 1: Inputs are strongly disposable

If $x' \geq x$ then $P(x') \supseteq P(x)$

This equation implies that if inputs are increased (or not reduced), then the outputs set will not shrink. In other words, inputs are not congesting outputs.

Postulate 2: Desirable and undesirable outputs are null-jointness

If $(y, u) \in P(x)$ and $u = 0$ then $y = 0$

This equation implies that if desirable and undesirable outputs are null-joint, then if no undesirable outputs are produced, it is not possible to produce any desirable outputs, or conversely, if desirable outputs are produced then some undesirable byproducts must also be produced.

Postulate 3: Desirable and undesirable outputs are weakly disposable

If $(y, u) \in P(x)$ and $0 \leq \theta \leq 1$ then $(\theta y, \theta u) \in P(x)$

This equation implies that both desirable and undesirable outputs are weakly disposable whereby any proportional contraction of desirable and undesirable outputs together is feasible, i.e. for given inputs x, reductions in undesirable outputs are always possible if desirable outputs are reduced in proportion. The idea is that, it is costly to reduce undesirable outputs, since to do so at the margin one must also reduce desirable outputs in order to guarantee that the new output vector (y, u) is feasible. Weak disposability is complemented by the assumption that desirable outputs by themselves are strongly disposable, which is defined as below.

Postulate 4: Desirable outputs are strongly disposable

If $(y, u) \in P(x)$ then for $y' \leq y,\ (y', u) \in P(x)$

This equation implies that the desirable outputs are freely disposable, but are not a maintained condition for the undesirable outputs. In other words, the desirable outputs can be reduced without cutting down the undesirable outputs, which means that some of the desirable outputs can always be 'freely' disposed without any cost.

To satisfy the properties of null jointness and weak disposability in the postulate above, it can be represented by the technology. The technology constructed that joins both desirable and undesirable outputs can be called an environmental DEA technology because the set is formulated in the DEA framework (Färe & Grosskopf, 2004). Assume that there are $n = 1,2,..., N$ DMUs and for DMU$_n$ the observed data on the vectors of inputs, desirable outputs and undesirable outputs are $x_n = (x_{1n}, x_{2n},...,x_{In})$, $y_n = (y_{1n}, y_{2n},...,y_{Jn})$ and $u_n = (u_{1n}, u_{2n},...,u_{Kn})$, respectively. The environmental DEA technology exhibiting CRS can be depicted as below:

$$T = \{(x, y, u): \sum_{n=1}^{N} z_n x_{in} \leq x_i \, , i = 1,2, \dots, I;$$
$$\sum_{n=1}^{N} z_n y_{jn} \geq y_j \, , j = 1,2, \dots, J;$$
$$\sum_{n=1}^{N} z_n u_{kn} = u_k \, , k = 1,2, \dots, K;$$
$$z_n \geq 0 \, ; \;\; n = 1,2, \dots, N\} \tag{3.18}$$

In equation (3.18), the inequality constraint on input (*x*) and desirable output (*y*) have been imposed with free disposability. As for equality constraint on undesirable output (*u*), it has been imposed by weak disposability, i.e., both desirable and undesirable outputs can be scaled down together.

To explain equation (3.18) further, Figure 3.5 illustrates the production possibilities set constructed for a technology *T* while assuming that all DMUs use a similar quantity of a single input (*x*) to produce a dissimilar quantity of a single desirable (*y*) and a single undesirable (*u*) output. The production possibility set of weak disposability ($P^W(x)$) is bounded by the 0ABCD0 while the production possibility set of strong disposability ($P^S(x)$) is bounded by the 0EBCD0. $P^W(x)$ satisfies weak disposability of outputs since any element (*y*, *u*) in $P^W(x)$ can be proportionally contracted (scaled toward the origin) and still remain in the set. On the other hand, $P^W(x)$ does not satisfy strong disposability since a point like E represents an output vector smaller than the output vector B (which belongs to $P^S(x)$), yet E does not belong to $P^W(x)$. Therefore, if a technology satisfies strong disposability, it also satisfies weak disposability. But, if a technology satisfies weak disposability, it would not necessarily satisfy strong disposability (Färe & Grosskopf, 2004).

Both weak and strong disposability are assumed with respect to the disposal of undesirable output. It is also assumed that under weak disposability of undesirable output firms face environmental regulation and operate under regulated technology. Strong disposability of undesirable output, on the other hand, implies that disposal of undesirable is cost free and firms operate under unregulated technology.

In addition, Figure 3.5 also exhibits that for the $P^W(x)$ technology, if u = 0 then the only feasible production of good output is y = 0. This technology illustrates the null-jointness concept in postulate 2 above where desirable and undesirable outputs are null-jointness.

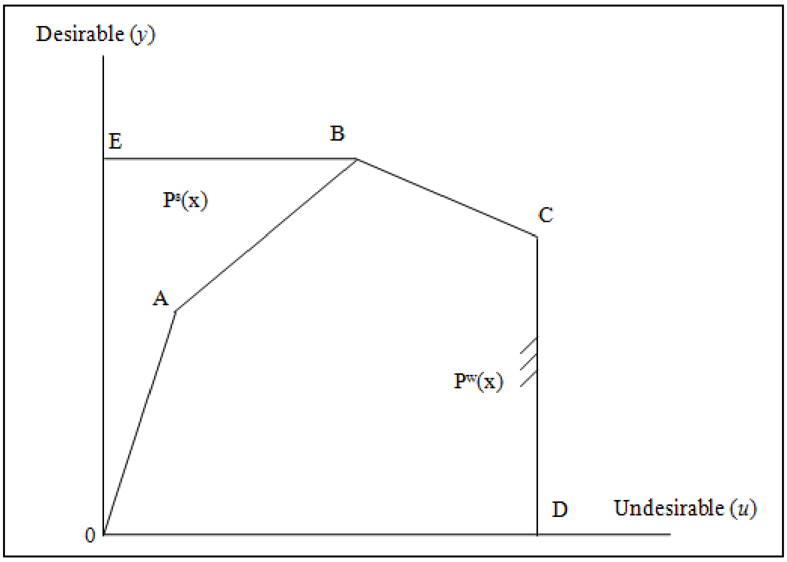

Figure 3.5: A graphical representation for environmental production function

Source: Färe and Grosskopf (2004)

3.5 Directional Distance Function (DDF)

In the conventional DEA model, efficiency is measured by maximizing the production (desirable) of outputs with a restricted amount of inputs. However, when there is joint production of the desirable and undesirable outputs, the efficiency measurement is best defined by increasing desirable outputs and simultaneously decreasing undesirable outputs (Färe et al., 1989). To handle this situation, the Directional Distance Function (DDF) approach was introduced by Chung et al. (1997) to measure eco-efficiency.

The DDF idea is to expand desirable outputs and reduce inputs and/or undesirable outputs simultaneously based on a given direction vector (Chung et al., 1997). This approach is based on Luenberger's shortage function (1992) to obtain a technical efficiency measurement from the potential of increasing outputs and simultaneously reducing inputs. The purpose of this approach is to provide measures of performance that directly account for the reductions in undesirable outputs. This approach can split the output variables by increasing the desirable output and decreasing the undesirable output simultaneously.

The DDF approach is more appropriate than the conventional DEA approach when desirable and undesirable outputs are jointly produced. The DEA approach measures the output-oriented efficiency based on the assumption of strong disposability. Strong disposability allows any output to be produced without any cost. This assumption is improper for the technologies when undesirable outputs such as carbon dioxide (CO_2) emissions are disposed of simultaneously with marketed outputs. The CO_2 emissions cannot be disposed freely and some cost of abatement is required depending on the regulation. To express this situation, the DDF approach assumes weak disposability for the undesirable outputs. The weak disposability assumption implies that the disposal of undesirable outputs is costly, and therefore, the undesirable outputs can only be reduced when desirable outputs are reduced simultaneously (Färe & Grosskopf, 2005).

To measure the inefficient of DMU, the Directional Distance Function (DDF) has been employed. The DDF model on the technology *T* can be defined as below:

$$\vec{D}_T(x, y, u; g_y, g_u) = Max\{\beta : (y + \beta g_y, u - \beta g_u \in T\} \qquad (3.19)$$

The distance function on the technology T ($\vec{D}_T$) above tries to look for the extension of desirable output in the g_y direction and the reduction of undesirable output in the g_u direction. In other words, proportion β seeks to increase the desirable output and reduce the undesirable output simultaneously. For example, if β is equivalent to 10%, all desirable outputs will be expanded by 10% while concurrently all undesirable outputs will be contracted by 10% as well. This measurement expands desirable output and reduces undesirable output given by the direction vector of g.

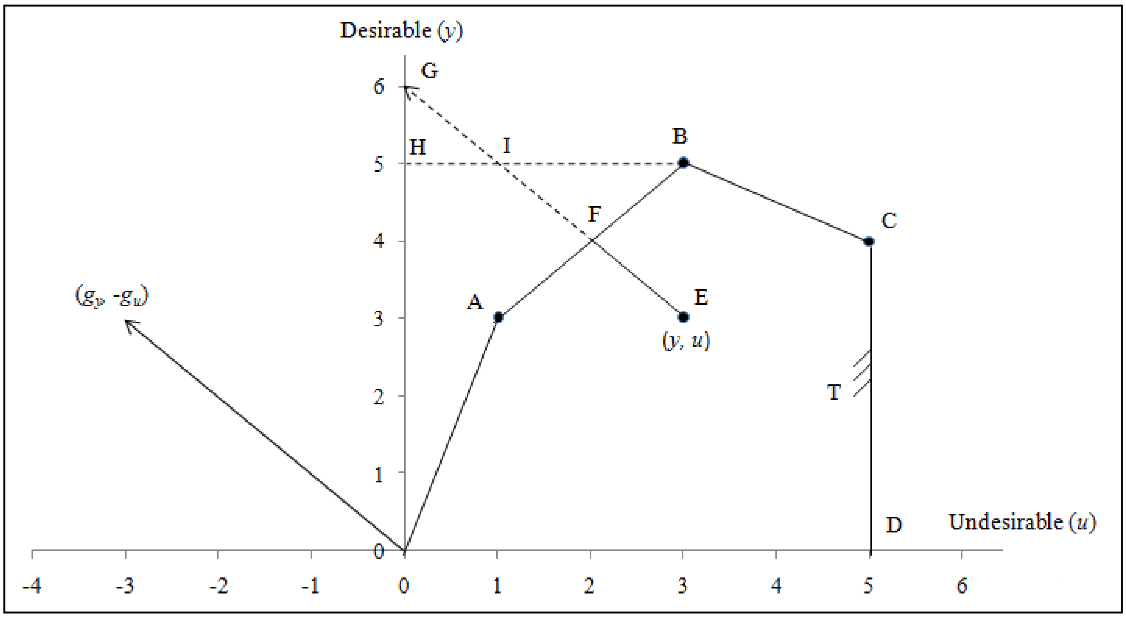

Figure 3.6: The efficiency frontier for DDF model

Source: Domazlicky and Weber (2004)

Referring to Figure 3.6, the efficient frontier is represented by the line 0, A, B, C and D. There are four DMUs under observation. The set of four DMUs (y, u) are A = (3, 1), B = (5, 3), C = (4, 5) and E = (3, 3). DMU A, B, and C are all on the efficient frontier of T, thus it can be categorized as efficient DMUs. However, DMU E is below the efficient frontier thus it can be categorized as inefficient DMU. DMU E is evaluated relative to the point F on the frontier line. Using DDF model for DMU E, g = (y, -u) =

(3,-3) and $\vec{D}_T(x, y, u; g_y, g_u) =$ EF/EG = 0.33, a value which implies that if DMU E adopted the best practice methods of production (in this case a linear combination of DMU A and B production methods) the desirable output will be expended by 0.33 while concurrently the undesirable output will be contracted by 0.33 as well, giving equal emphasis to the expansion of desirable output and the reduction of undesirable output. Therefore, in Figure 3.6, the directional output distance function will expand the output bundle (y, u) at E, along the g direction until it hits the production boundary of $y + \beta g_y, u - \beta g_u$ at F. To explain the freely disposable for undesirable outputs, consider Figure 3.6 again. As shown above, the set T is bounded by 0HBCD0 for strong disposability and only DMU B and C are the best practice on the frontier. To evaluate the DMU E, using the same direction vector, g = (3, -3) it could operate at point I with $\vec{D}_T(x, y, u; g_y, g_u) =$ EI/EG = 0.66.

The DDF uses linear programming to compute eco-efficiency of the DMU m under CRS and weak disposability of undesirable outputs assumptions is formulated as below (see Chung et al., 1997):

$$
\begin{aligned}
&\text{Max}\ \beta_m \\
&\text{Subject to} \\
&\qquad \sum_{n=1}^{N} z_n x_{in} \leq x_{im}\ ; \quad i = 1,2,\ldots,I \\
&\qquad \sum_{n=1}^{N} z_n y_{jn} \geq y_{jm}(1 + \beta_m)\ ; \quad j = 1,2,\ldots,J \\
&\qquad \sum_{n=1}^{N} z_n u_{kn} = u_{km}(1 - \beta_m)\ ; \quad k = 1,2,\ldots,K \\
&\qquad z_n \geq 0\ ; \quad n = 1,2,\ldots,N
\end{aligned} \tag{3.20}
$$

Where

z_n = intensity variables

x_{in} = i^{th} input of the n^{th} DMU

x_{im} = i^{th} input of the m^{th} DMU

y_{jn} = j^{th} desirable output of the n^{th} DMU

y_{jm} = j^{th} desirable output of the m^{th} DMU

u_{kn} = k^{th} undesirable output of the n^{th} DMU

u_{km} = k^{th} undesirable output of the m^{th} DMU

Where $0 \leq \beta_m \leq 1$ is the inefficiency score of the DMU m. The direction vector of g is taken as (y, $-u$) along which the desirable outputs to be extended and the undesirable outputs contracted. A score of zero indicates an efficient DMU while any positive values denote inefficiency.

Since β_m is the inefficiency scores, to obtain the eco-efficiency score using DDF model (∂_m), is formulated as follows:

$$\partial_m = 1 - \beta_m \tag{3.21}$$

Note that β_m is between 0 and 1, thus, ∂_m also falls into 0 and 1 closed interval.

The DDF model has been employed in this study because it is simple, intuitive and can be easily put into practice. In fact, many published papers have used this approach (Refer Appendix A for examples of articles that used DDF in their studies). Furthermore, the DDF is flexible as it allows for the evaluation of efficiency using a single direction vector from the observed points.

Nevertheless, with the conventional DDF approach as explained in this section, it can be seen that this approach has its drawbacks. There are no standard techniques on how to determine the direction vector in the modelling. The direction to expand desirable output and reduce undesirable output is made subjectively, in other words, user specified. This arbitrary direction (g = (y,-u)) may be inappropriate for every output bundle (Bian, 2008). In addition, the DDF model leaves out for the non-zero input and output slacks in the efficiency measurement and thus fails to account for the non-radial excesses in input and shortfalls in output (Jahanshahloo et al., 2012). Therefore, some modifications on the original DDF model need to be considered to ensure accuracy in the eco-efficiency score.

3.6 Malmquist Luenberger Productivity Index (MLPI)

The measures of efficiency of DMU provided in the DEA and DDF models only present the efficiency of static performance. Concentrating only on static efficiency estimates provides an incomplete view of DMU performance over time. For this reason, the Malmquist Productivity Index will be utilized to measure the movement of DMUs with regards to technical changes and efficiency changes. The measurement of technical change determines the shift of the efficient frontier. On the other hand, the measurement of efficiency change determines the change in output efficiency between the two periods, i.e. it measures how far an observation is from the frontier of technology (Färe et al., 2001). The Malmquist Productivity Index has been utilized tremendously by the researchers due to its ability to provide an explanation of productivity growth.

An alternative measure of productivity change is the Malmquist Luenberger Productivity Index (MLPI), which measures the environmental sensitivity of productivity growth. Malmquist Luenberger (ML) is different from the Malmquist Index since this measure is constructed from the directional technology distance functions, which simultaneously adjust desirable and undesirable outputs in a direction chosen by the decision maker (Fried et al., 2008). The ML index changes the desirable outputs and undesirable outputs proportionally because it chooses the direction to be g =

(y^t, $-u^t$), i.e. increases desirable outputs and decreases undesirable outputs. As a similar concept to the DDF approach, ML also seeks to increase the desirable outputs while simultaneously decreasing undesirable outputs (Chung et al., 1997).

Following model (3.20), the DDF, given g = $(g_y, -g_u)$ as a direction vector with respect to technology t is defined as below:

$$\vec{D}_O^t\left(x^t, y^t, u^t; g_y, -g_u\right) = sup\left\{\beta: \left(y^t + \beta g_y, u^t - \beta g_u\right) \epsilon\, P^t(x^t)\right\} \quad (3.22)$$

Using DDF model, the MLPI is defined with the technology of period t as the reference technology; where g_y^*= y^t and $-g_u^*$ = $-u^t$.

The ML index defined by Chung et al., (1997) using DDF can be formulated as below:

$$ML_t^{t+1} =$$

$$\left[\frac{(1+\vec{D}_O^{t+1}(x^t, y^t, u^t; y^t, -u^t))}{(1+\vec{D}_O^{t+1}(x^{t+1}, y^{t+1}, u^{t+1}; y^{t+1}, -u^{t+1}))} \frac{(1+\vec{D}_O^{t}(x^t, y^t, u^t; y^t, -u^t))}{(1+\vec{D}_O^{t}(x^{t+1}, y^{t+1}, u^{t+1}; y^{t+1}, -u^{t+1}))}\right]^{\frac{1}{2}} \quad (3.23)$$

Where ‘o’ indicates that output-oriented approach is used. (x^t, y^t, u^t) is a production point where desirable and undesirable outputs $(y^t, -u^t)$ are produced using input (x^t) in period t. Similarly, $(x^{t+1}, y^{t+1}, u^{t+1})$ denotes that with the use of input (x^{t+1}), one can produce desirable output (y^{t+1}) and undesirable output (u^{t+1}) at the period $t+1$. This concept is illustrated in Figure 3.7.

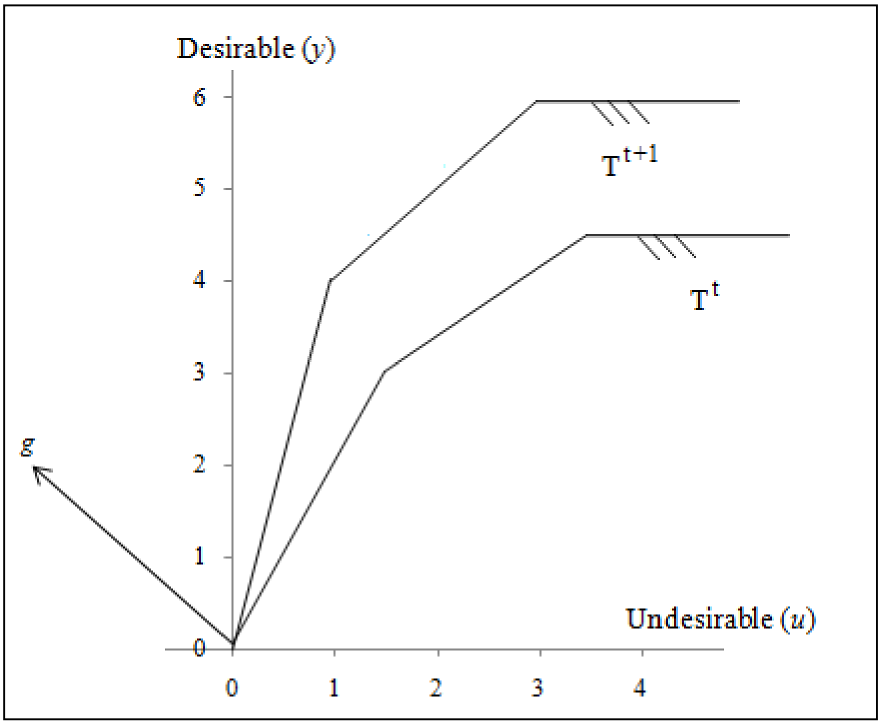

Figure 3.7: Malmquist Luenberger productivity indicator

Source: Chambers, Färe and Grosskopf (1996)

The notation $\vec{D}_o^t(x^{t+1}, y^{t+1}, u^{1+t})$ represents the distance from the period *t+1* observation to the period *t* technology. If the value of ML_t^{t+1} is greater than one, it indicates a positive Total Factor Productivity (TFP) change between period *t* and *t+1*, while if the value is less than one, it indicates a TFP decline. The value of $ML_t^{t+1} = 1$ indicates that there have been no changes in inputs and outputs over two time periods. Equation (3.23) can be further decomposed into two measured components of productivity change, which are eco-efficiency change (MLEFFC) and technical change (MLTC). MLEFFC represents a movement towards the best practice frontier while MLTC represents a shift in technology between *t* and *t+1*.

$$MLEFFC_t^{t+1} = \left[\frac{(1+\vec{D}_o^t(x^t, y^t, u^t; y^t, -u^t))}{(1+\vec{D}_o^{t+1}(x^{t+1}, y^{t+1}, u^{t+1}; y^{t+1}, -u^{t+1}))}\right] \tag{3.24}$$

$MLTC_t^{t+1} =$

$$\left[\frac{(1+\vec{D}_o^{t+1}(x^t,y^t,u^t;y^t,-u^t))}{(1+\vec{D}_o^{t}(x^t,y^t,u^t;y^t,-u^t))}\ \frac{(1+\vec{D}_o^{t+1}(x^{t+1},y^{t+1},u^{t+1};y^{t+1},-u^{t+1}))}{(1+S\vec{D}_o^{t}(x^{t+1},y^{t+1},u^{t+1};y^{t+1},-u^{t+1}))}\right]^{\frac{1}{2}} \tag{3.25}$$

For each observation, four distance functions must be calculated in order to measure the MLPI. Two distance functions use observation and technology for time period *t* and *t+1* i.e. $\vec{D}_o^t(x^t, y^t, u^t; y^t, -u^t)$ and $\vec{D}_o^{t+1}(x^{t+1}, y^{t+1}, u^{t+1}; y^{t+1}, -u^{t+1})$, while another two use the mixed period of *t* and *t+1*, i.e. $\vec{D}_o^t(x^{t+1}, y^{t+1}, u^{t+1}; y^{t+1}, -u^{t+1})$ and $\vec{D}_o^{t+1}(x^t, y^t, u^t; y^t, -u^t)$. Using the DDF approach in model (3.20), the solution of the four distance functions can be solved as follows:

$$\begin{aligned}
&\vec{D}_o^t(x^t, y^t, u^t; y^t, -u^t) = \text{Max}\, {\beta_m}^t \\
&\text{Subject to} \\
&\qquad \sum_{n=1}^{N} z_n^t x_{in}^t \le x_{im}^t\ ;\ \ i = 1,2,\dots,I \\
&\qquad \sum_{n=1}^{N} z_n^t y_{jn}^t \ge y_{jm}^t(1+\ \beta_m)\ \ ;\ \ j = 1,2,\dots,J \\
&\qquad \sum_{n=1}^{N} z_n^t u_{kn}^t = u_{km}^t(1+\ \beta_m)\ ;\ \ k = 1,2,\dots,K \\
&\qquad z_n^t \ge 0\ ;\ \ n = 1,2,\dots,N
\end{aligned} \tag{3.26}$$

$$\begin{aligned}
&\vec{D}_o^{t+1}(x^{t+1}, y^{t+1}, u^{t+1}; y^{t+1}, -u^{t+1}) = \text{Max}\, {\beta_m}^{t+1} \\
&\text{Subject to} \\
&\qquad \sum_{n=1}^{N} z_n^{\mathrm{t+1}} x_{in}^{\mathrm{t+1}} \le x_{im}^{\mathrm{t+1}}\ ;\ \ i = 1,2,\dots,I \\
&\qquad \sum_{n=1}^{N} z_n^{\mathrm{t+1}} y_{jn}^{\mathrm{t+1}} \ge y_{jm}^{\mathrm{t+1}}(1+\ \beta_m)\ \ ;\ \ j = 1,2,\dots,J \\
&\qquad \sum_{n=1}^{N} z_n^{\mathrm{t+1}} u_{kn}^{\mathrm{t+1}} = u_{km}^{\mathrm{t+1}}(1+\ \beta_m)\ ;\ \ k = 1,2,\dots,K \\
&\qquad z_n^{\mathrm{t+1}} \ge 0\ ;\ \ n = 1,2,\dots,N
\end{aligned} \tag{3.27}$$

$$\vec{D}_o^t(x^{t+1}, y^{t+1}, u^{t+1}; y^{t+1}, -u^{t+1}) = \text{Max}\, {\beta_m}^{t+1}$$
Subject to
$$\sum_{n=1}^{N} z_n^{\mathrm{t}} x_{in}^{\mathrm{t}} \le x_{im}^{\mathrm{t+1}}\ ;\ \ i = 1,2, \dots, I$$
$$\sum_{n=1}^{N} z_n^{\mathrm{t}} y_{jn}^{\mathrm{t}} \ge y_{jm}^{\mathrm{t+1}}(1 + \beta_m)\ \ ;\ \ j = 1,2, \dots, J$$
$$\sum_{n=1}^{N} z_n^{\mathrm{t}} u_{kn}^{\mathrm{t}} = u_{km}^{\mathrm{t+1}}(1 + \beta_m)\ ;\ \ k = 1,2, \dots, K$$
$$z_n^{\mathrm{t}}, \ge 0\ ;\ \ n = 1,2, \dots, N \qquad (3.28)$$

$$\vec{D}_o^{t+1}(x^{t}, y^{t}, u^{t}; y^{t}, -u^{t}) = \text{Max}\, {\beta_m}^{t}$$
Subject to
$$\sum_{n=1}^{N} z_n^{\mathrm{t+1}} x_{in}^{\mathrm{t+1}} \le x_{im}^{\mathrm{t}}\ ;\ \ i = 1,2, \dots, I$$
$$\sum_{n=1}^{N} z_n^{\mathrm{t+1}} y_{jn}^{\mathrm{t+1}} \ge y_{jm}^{\mathrm{t}}(1 + \beta_m)\ \ ;\ \ j = 1,2, \dots, J$$
$$\sum_{n=1}^{N} z_n^{\mathrm{t+1}} u_{kn}^{\mathrm{t+1}} = u_{km}^{\mathrm{t}}(1 + \beta_m);\ \ k = 1,2, \dots, K$$
$$z_n^{\mathrm{t+1}} \ge 0\ ;\ \ n = 1,2, \dots, N \qquad (3.29)$$

It should be noted that the above four linear programming problems must be solved for each firm in the sample and for each period, adding to the number of linear programming problems.

In the MLPI, the issue of infeasibility has also been discussed by other researchers (Färe et al., 2001; Jeon & Sickles; 2004; Oh, 2010). The infeasibility solution may occur for MLPI when utilizing the DDF approach for two distance functions of mixed period, i.e. *t* and *t*+1. According to Färe et al. (2001), the production possibilities frontier constructed from observations in period *t* may not contain an observation from period *t*+1 (and vice versa). This happens because a set of inputs and outputs is outside the production set and the movement along the direction vector *g* does not intersect the production frontier. Figure 3.8 illustrates the potential for an infeasible problem with mixed period in MLPI. For instance, recall distance function model (3.28), i.e.

$\vec{D}_o^t(x^{t+1}, y^{t+1}, u^{t+1}; y^{t+1}, -u^{t+1})$. This model denotes the distance function under the frontier at t using t+1 data. As illustrated in Figure 3.8, the frontier of T^t is bounded by 0ABC while the frontier of T^{t+1} is bounded by 0EFG. It is expected that the t+1 data at point H would be outside the frontier of the previous year (t). Thus, the mix period distance function in model (3.28) cannot be calculated since the movement along the direction vector g does not intersect the production frontier. This will happen if the data at t+1 is located outside the current frontier (Jeon & Sickles, 2004).

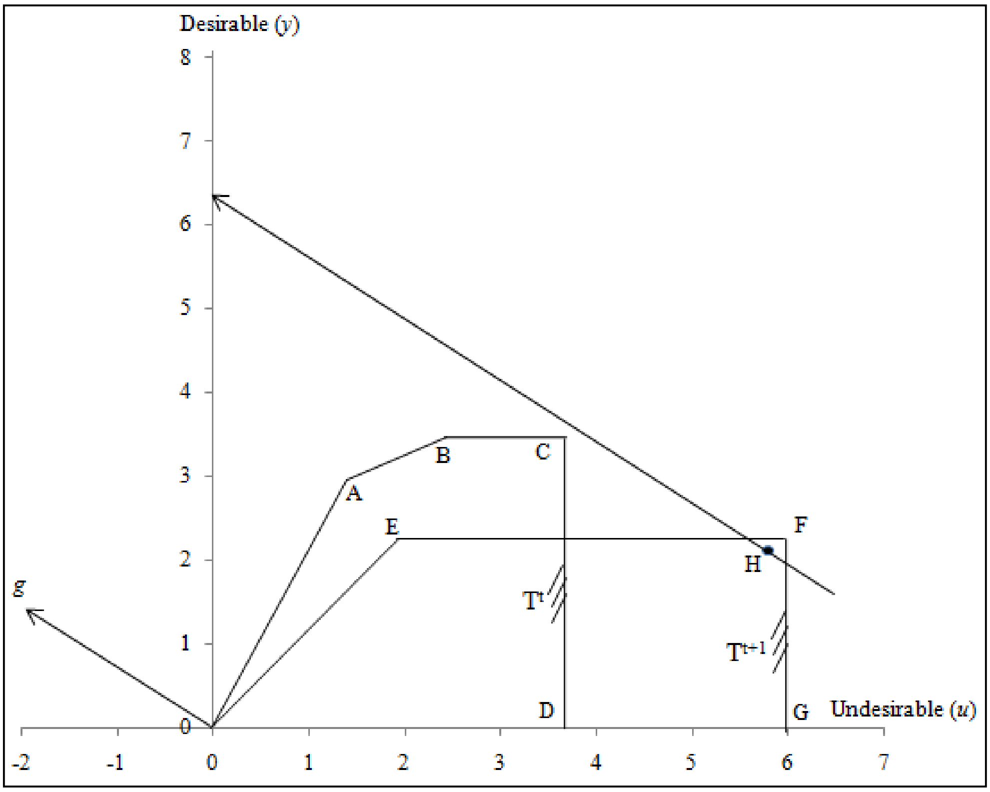

Figure 3.8: Infeasible problem with mix period in MLPI

Source: Oh (2010)

To overcome the infeasibility problem stated above, Färe et al. (2001) used multiple year windows of data as the reference technology. Jeon and Sickles (2004) on the other hand used the index number approach to determine estimates of productivity growth and its decomposition while Oh (2010) employed the concepts of the global Malmquist productivity growth index of Pastor and Lovell (2005) with the DDF of Luenberger (1992).

3.7 Conclusion

In general, this chapter provides the theoretical foundations of a comprehensive efficiency model that integrates the indicators for environmental as well as industrial activities. The DEA approach has been introduced first as a fundamental technique to measure the technical efficiency without the incorporation of undesirable output. A basic DEA model with both CRS and VRS technologies, as well as its orientation (output-oriented or input-oriented) has been explained. In addition, the slack-based measure approach as a fundamental of non-radial approach is briefly introduced. Nevertheless, the conventional DEA model accounts for only two categories of variable, which are input and desirable output variables. When undesirable outputs are present, the DEA model is no longer applicable. Therefore the DDF approach is discussed next to handle the situation when desirable and undesirable outputs are produced simultaneously. As noted in the previous section, the efficiency measurement provided in the DEA and DDF models only present the efficiency of static performance. For this reason, the MLPI has been discussed to measure the movement of DMUs with regards to technical changes and efficiency changes.

It can be seen that the DDF model is an appropriate efficiency measurement approach for the manufacturing sector, as industrial activities release pollutant. This model allows one to expand the desirable outputs while simultaneously contract the undesirable outputs. However, as has been discussed in the literature, there are some drawbacks in using this model as there are no standard techniques on how to determine the direction vector in the modelling. In addition, the DDF model leaves out for the non-zero input and output slacks from the efficiency measurement and thus, fails to account for the non-radial excesses and shortfalls. Therefore, some modifications to the original DDF approach need to be implemented to ensure that an accurate eco-efficiency score can be obtained. The modification on the DDF model will be discussed further in the next chapter as a new development of eco-efficiency measures.