CHAPTER 4

METHODOLOGY AND ANALYTICAL FRAMEWORK

4.1 Introduction

Economists have long understood that the quantity of money (or any other financial variables) or its growth rate can play a useful role in the monetary policy process only if the fluctuations in money (or other financial variables) over time, reliably corresponds to the fluctuation in income and prices. In other words, the effectiveness of monetary policy highly depends on the choice of intermediate targets. Bearing this in mind, this chapter proceeds to assess the effectiveness of monetary policy in Malaysia by examining the roles of money, credit, and interest rates in the monetary policy transmission mechanism during the pre and post liberalization periods.

The main focus of the study is to show how financial liberalization process has altered the familiar empirical relationship that supported the central role for credit and money. Subsequently, the focus is to explore if there are other intermediate targets such as interest rates that can be used for the effective implementation of monetary policy.

\(^{20}\) Apart from monetary aggregates, other financial variables to be included in the study are credit and interest rates.
4.2 Empirical Framework

The study mainly assesses the effectiveness of monetary policy in Malaysia over the period of 1973 fourth quarter to 2000 first quarter. Empirical framework is broken down into three main sections and they are as follows:

I. It seeks to investigate the long-run relationship between money (and other financial variables) and income or prices. The focus here is to test the stability and the long run equilibrium amongst the variables. The Johansen co-integration technique is used to test the long run relationship amongst the variables.

II. It seeks to investigate the information content of money with respect to the future movements of prices and real income. In the context of the information-variable approach to monetary policy, the error correction model (ECM) is used. This ECM model is used not only to test whether money can predict the future movements in output and prices but whether money can predict the future movement of income or prices that are not predictable on the basis of the fluctuation of income and prices.

III. It seeks to investigate the dynamic interactions between the variables in the VAR system. The forecast error variance decomposition method is applied to investigate the dynamic effect of output or prices to monetary policy shocks. The variance decomposition is used mainly to identify the appropriate channel of monetary policy transmission mechanism.
Prior to the three investigations mentioned above, a stationary test needs to be carried out on the selected variables. The conventional Augmented Dicky-Fuller (ADF) statistics for unit root test and Hylleberg, Engel, Granger and Yoo (HEGY) procedure for seasonal unit root test will be applied. These tests are used mainly to examine the time-series properties of the selected variables.

4.3 Break Down Of The Sample Periods

The analysis of the study will be based on the quarterly data. In order to see the impact of financial liberalization on the effective implementation of monetary policy, the period of study, which covers from 1973, fourth quarter to 2000 first quarter will be broken down as follows:

1. 1973 Fourth Quarter to 1989 Fourth Quarter

This sub-period represents the period before the major shift in the implementation of monetary policy in Malaysia. During this period, the Malaysian Financial System has been subjected to the widely practiced mechanism of financial repression such as interest rate regulation and preferential credit scheme.

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21 Sources of data can be obtained from chapter 1 under the sub-section 1.5.
II. 1979 First Quarter to 2000 First Quarter

This sub-period represents the post-liberalization period in Malaysia. End of 1978 interest rates was deregulated in Malaysia and thus indicated the beginning of the era of financial reforms. The international economic and financial environment in the post liberalization period posed new challenges for national economic management and the operation of monetary policy. During this time Bank Negara undertook series of reforms to expand and strengthen the financial base in the country.\textsuperscript{22}

III. 1990 First Quarter to 2000 First Quarter

This sub-period represents the period after the major shift in the implementation of monetary policy. It was during this period that financial reform made a most dramatic impact on the conduct of monetary policy. In fact, the major shift in the implementation of the monetary policy began in 1989, whereby the Central Bank embarked on a series of financial reforms to improve and modernize the financial system.\textsuperscript{23}

4.4 Unit Root Test

Prior to assessing the relationship amongst variables based upon the notion of vector autoregression and co-integration, their univariate time series properties

\textsuperscript{22} During 1978-79, the Central Bank of Malaysia introduced a package of measures as a concrete step towards a market-oriented financial system. Among the measures were freeing the interest rate controls and reforming the liquidity requirement of the financial institutions.
have to be examined that is their order of integration. Any time series data can be thought of as being generated by a stochastic or random process. A stochastic process is said to be stationary if its mean and variance are constant over time and the value of covariance between two time periods depends only on the distance or lag between the two periods. In other words, as shown in Maddala (1992), if a time series is stationary, its mean, variance and auto-covariance remain the same no matter at what time we measure them.

Unit root test is carried out to test the stationarity of a time series. Consider the following model that is generated by AR (1) process:

\[ Y_t = \alpha Y_{t-1} + \varepsilon_t \]  

To test for the presence of unit root, the following null hypothesis has to be evaluated:

\[ H_0 : \alpha = 1 \quad \quad \quad H_1 : |\alpha| < 1 \]

If \(|\alpha| < 1\), \(Y_t\) is integrated of order zero that is I (0). In this situation the time series has no unit problem and the series is said to be stationary. On the other hand if the \(\alpha = 1\), \(Y_t\) is integrated of order one that is I (1). Unit root problem exists and the time series is said to be non-stationary.

Assuming the time series is differenced once as follows:

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23 The Central Bank introduced a package of reforms to broaden and deepen the process of financial intermediation. During this period the interest rates were largely liberalized.
\[ \Delta Y_t = Y_t - Y_{t-1} \]
\[ \Delta Y_t = \alpha Y_{t-1} - Y_{t-1} + \epsilon_t \]
\[ \Delta Y_t = (\alpha - 1)Y_{t-1} + \epsilon_t \]
\[ \Delta Y_t = \beta Y_{t-1} + \epsilon_t \]  \hspace{1cm} (4.2)

where \( \beta = \alpha - 1 \) and \( \Delta \) is the first difference operator. If the \( \beta = 0 \), then the first difference of the time series is stationary and the original series which is non-stationary is said to be integrated of order one.\(^{24}\)

The purpose of testing for unit root test is to know whether the time series belongs to the trend-stationary process (TSP) class or whether it belongs to the difference-stationary process (DSP) class. If the error term in equation (4.1) is stationary then it represents the TSP while if the error term in equation (4.2) is stationary then it represents the DSP. As stated in Gujarati (1995), the practical significance of TSP and DSP is for the purpose of long term forecasting whereby the forecast made from TSP is said to be more reliable while forecast made from a DSP will be less reliable.

Two widely applied tests namely the ADF statistics and the HEGY procedures were used to conduct the null hypothesis of a unit root. If the error term \( \epsilon \) is autocorrelated, ADF test is appropriate as it uses the lagged difference terms so that the error term is serially independent. The ADF based approach basically involves the running of the following univariate regression:

\(^{24}\) If a time series has to be difference \( d \) times before it becomes stationary, the original series is integrated of order \( d \) (I(d)).
\[ \Delta Y_t = \beta_0 + \beta_1 Y_{t-1} + \beta_2 t + \sum_{i=1}^{S} \theta \Delta Y_{t-i} + \sum \alpha_i S_i + \varepsilon_t \]  \hspace{1cm} (4.3)

where \( Y_t \) is the natural logarithm of the variable of interest and \( \varepsilon_t \) is a white noise stationary error term. \( \beta_0 \) is a drift while \( t \) is a linear time trend and \( \sum \alpha_i S_i \) are seasonal dummy variables. The equation is designed for testing whether a series is I(0) or I(1). The important coefficient to look at is the \( \beta_1 \).

To test for the presence of unit root, the following null hypothesis has to be evaluated:

\[ H_0 : \beta_1 = 0 \]

If the test shows that \( \beta_1 = 0 \), then \( Y_t \) is non-stationary and it is deemed to have a unit root problem. Since the distribution of ADF does not follow the usual student's t, the approximate critical values of this statistics need to be obtained from Fuller(1976). The number of lags (\( n \)) has to be selected such that the regression yields non-serially correlated errors.

Apart from conducting the usual unit root tests, seasonal unit root tests must be carried out to ascertain whether each of the series in question possess a unit root at some frequency other than the usual zero frequency such as biannual and or annual frequencies. Since quarterly data are used in the study, such test is deemed important. HEGY procedure is used to test the hypothesis of constant seasonal versus varying seasonal movements in the data. The HEGY's test result will indicate whether the unit root for each variable can be appropriately removed through first order differencing (Tan, 1996).
The HEGY test may be administered to a series $Y_t$ estimating the following equation:

$$\Omega(B)x_{4t} = \Pi_1 x_{1t-1} + \Pi_2 x_{2t-1} + \Pi_3 x_{3t-2} + \Pi_4 x_{3t-1} + \mu_t + \varepsilon_t \quad (4.4)$$

where

$$x_{1t} = (1 + L + L^2 + L^3) Y_t$$
$$x_{2t} = -(1 - L + L^2 - L^3) Y_t$$
$$x_{3t} = -(1 - L^3) Y_t$$
$$x_{4t} = (1 - L^4) Y_t$$

$Y_t =$ variable of interest

$L =$ lag operator

$\mu_t =$ consist of deterministic terms such as seasonal dummies, time trend and constant

Based upon the estimates of the above, the following null hypothesis will be evaluated:

I. $H_0 : \Pi_1 = 0 \quad H_1 : \Pi_1 < 0$

II. $H_0 : \Pi_2 = 0 \quad H_1 : \Pi_2 < 0$

III. $H_0 : \Pi_3 = \Pi_4 = 0 \quad H_1 : \Pi_3 \neq 0 \quad H_1 : \Pi_4 \neq 0$

The first null hypothesis corresponds to a test that the series in question has a unit root at zero frequency. The second null hypothesis corresponds to a test that the series possesses a unit root at biannual frequency while the third null hypothesis
deals with test for the presence of unit root at annual frequency. If \( \Pi_2 \) and either \( \Pi_3 \) or \( \Pi_4 \) are different from zero, the series in question are said to be free from seasonal unit root problem (Tan, 1996).

4.5 The Co-integration Of Financial Variables With Income And Prices

For a practical conduct of monetary policy, especially of those that are in a multiyear context, the long run relationship between the financial variables and income or prices is important. The long run relationship actually depends on whether the relationship between the respective levels of financial variable and income or prices is stationary. If money and income are both individually non-stationary, this may not imply that the ratio of one to the other is also non-stationary. In statistical terms, two series are co-integrated whenever they are individually integrated but their linear combination is stationary (Greene, 1997).

For example:

\[
y_t = \beta m_t + \epsilon_t,
\]

(4.5)

If the two series are both \( I(1) \), there may be a \( \beta \) such that

\[
\epsilon_t = y_t - \beta m_t,
\]

(4.6)

whereby the disturbance \( \epsilon_t \) is stationary that is \( I(0) \). The implication would be that if the series are drifting upward together at roughly the same rate, they are said to be co-integrated and the vector \([1, -\beta]\) is a co-integrating vector.
In this study, an attempt has been made to estimate the long run relationship between the financial variables and income or price using the co-integration technique advocated by Johansen (1988). As demonstrated in Greene (1997) and Tan (1996), the advantage of using the Johansen's maximum likelihood approach to co-integration is that it allows for the possible existence of multiple co-integrating vectors and their identifications particularly in regressions involving more than two variables.

To carry out the Johansen co-integration test, first need to formulate the following VAR:

$$Y_t = \Gamma_1 Y_{t-1} + \Gamma_2 Y_{t-2} + \ldots + \Gamma_p Y_{t-p} + e_t$$  \hspace{1cm} (4.7)

The order of the model $p$ must be determined in advance. If the above model involves $N$ variables then $Y_t = [y_1, y_2, \ldots, y_n]$, which are individually $I(1)$ and as a result $N \times 1$ vector can be formed and $\Gamma_1$ is an $N \times N$ matrix of parameters. Re-parameterising the system of equations above in an error correction representation yields the following:

$$\Delta Y_t = \Pi_p Y_{t-p} + \Pi_1 \Delta Y_{t-1} + \ldots + \Pi_{p-1} \Delta Y_{t-p+1} + e_t$$

or

$$\Delta Y_t = \Pi_p Y_{t-p} + \sum_{i=1}^{p} \Pi_i \Delta Y_{t-i} + e_t$$ \hspace{1cm} (4.8)

where $i = 1, 2, \ldots, (p - 1)$.

The long run relationship among the variables in the VAR is embodied in the matrix $\Pi_p$ and thus this matrix controls the co-integration properties. As mentioned before,
in the above model, \( Y \) is a vector of \( I(1) \) variables while \( \Delta Y_t, \Sigma_{i=1}^{\Pi} \Pi_i \Delta Y_{t-1} \) and \( e_t \) are stationary. All the possible combinations of the levels of \( Y \) that yield high correlation with the \( I(0) \) elements in (4.8) are then estimated and these combinations are referred as the co-integrating vectors.

The rank of the matrix \( \Pi_p \) is determined by the number of co-integrating vectors \( r \) among the elements of \( Y \). There are three possible cases:

**Case 1:** \( \Pi_p \) is a full rank \( N \), then any linear combination of \( Y_{t-1} \) is stationary and by implication, the elements in vector \( Y \) are not \( I(1) \).

**Case 2:** \( \Pi_p \) has rank between 0 and \( N \), then there exits \( r \) co-integrated vectors which are identifiable and incorporatable into error correction model. Since the maximum number of co-integrating vectors can only be \( N - 1 \), \( r \) must be smaller than \( N \).

**Case 3:** \( \Pi_p \) has rank zero and thus no linear combinations of \( Y_{t-1} \) are stationary implying that the elements in vector \( Y \) are \( I(1) \) but not co-integrated.

If \( \Pi_p \) has less than full rank, we can express it as follows:

\[
\Pi_p = \gamma \alpha'
\]  

(4.9)

The above equation involves estimating the matrix \( \alpha \) that contain all the possible co-integrating vectors and the \( \gamma \) matrix containing the corresponding set of error
correction coefficients. If there are r co-integrating vectors, then \(\alpha\) and \(\gamma\) each have r columns (Cochrane, 1997).

Rewriting equation (4.8) with \(\alpha\) and \(\gamma\), shows the error correction representation:

\[
\Delta Y_t = -\gamma \alpha' Y_{t-p} + \Pi_1 \Delta Y_{t-1} + \ldots + \Pi_{p-1} \Delta Y_{t-p+1} + e_t
\]  

(4.10)

\(\alpha' Y_{t-p}\) must be stationary so that \(\gamma \alpha' Y_{t-p}\) will also be stationary. Thus \(\alpha\) is the matrix of co-integrating vectors. If elements in \(Y_t\) do co-integrate then at least one of the \(\alpha_i\) vectors will be statistically significant. Thus by virtue of Granger representation theorem, \(\gamma_i\) must also contain at least one non-zero element.

The rank of the matrix \(\Pi_p\) can be determined by referring to the eigenvalues \(\lambda_i\) derived from the maximization of the concentrated likelihood function of equation (4.10). Hence, the number of co-integrating vectors \(r\) can be determined by the using the following likelihood ratio test (Tan, 1996).

Trace Statistic:

\[
\eta_r = -T \sum_{i=r+1}^{\infty} \ln (1 - \lambda_i)
\]

where the null hypothesis is that there is at most \(r\) co-integrating vectors against the alternative hypothesis that there are \(r\) or more co-integrating vectors.

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\(\alpha\) is an \(N \times r\) matrix.
An alternative test statistics to the above is the maximal eigenvalue statistics computed as follows:

$$\zeta_r = -T \sum_{i=r+1}^{n} \ln(1 - \lambda_i)$$

where $r=0,1,2,3,\ldots,n-2, n-1$

The null hypothesis is that there are at most $r$ co-integrating vectors against the alternative hypothesis of $r+1$ co-integrating vectors (Dekle & Pradhan, 1997).

4.6 Error Correction Model

The methodology reported in the previous section focuses on long-run relationships between financial variables and income or prices. However, in this section, the focus will be on the short-run relationships connecting the growth rate of financial variables to the growth rates of income or prices.

The modern analysis of the association between money growth and future output growth begins with Sim's introduction of the debatable Granger tests. In this study, Sim-type auto-regression model is used with an error correction (EC) term formed by the relevant estimated co-integration vector embedded in it. The co-integration relationship is interpreted as deviations from the long run equilibrium. The Granger test will be used to determine whether the selected financial variable is useful in forecasting economic activity, as demonstrated in Maddala (1992) and Friedman & Kuttner (1992)).
The error correction model specification is as follows:26

I. Real Income Equations

Three-variable system which includes financial variables, price index and real income:

\[ \Delta y_t = \alpha EC_{t-1} + \sum_{i=1}^{n} \beta_i \Delta x_{t-i} + \sum_{i=1}^{n} \gamma_i \Delta p_{t-i} + \sum_{i=1}^{n} \delta_i \Delta y_{t-i} + \sum \theta_i S_i + \epsilon_i \]  
(4.11)

Four-variable system which includes financial variables, price index, fiscal variable and real income:

\[ \Delta y_t = \alpha EC_{t-1} + \sum_{i=1}^{n} \beta_i \Delta x_{t-i} + \sum_{i=1}^{n} \gamma_i \Delta p_{t-i} + \sum_{i=1}^{n} \lambda_i \Delta f_{t-i} + \sum_{i=1}^{n} \delta_i \Delta y_{t-i} + \sum \theta_i S_i + \epsilon_i \]  
(4.12)

II. Price Equations

Three-variable system which includes financial variables, price index and real income:

\[ \Delta p_t = \alpha EC_{t-1} + \sum_{i=1}^{n} \beta_i \Delta x_{t-i} + \sum_{i=1}^{n} \gamma_i \Delta p_{t-i} + \sum_{i=1}^{n} \delta_i \Delta y_{t-i} + \sum \theta_i S_i + \epsilon_i \]  
(4.13)

26 The error correction model noted by Engle & Granger (1987) is a model by which it forces gradual adjustments of the dependent variable towards some long run values with explicit allowance made for the short run dynamics.
Four-variable system which includes financial variables, price index, fiscal variable and real income:

\[ \Delta p_t = \alpha EC_{t-1} + \sum_{i=1}^{n} \beta_i \Delta x_{t-i} + \sum_{i=1}^{n} \gamma_i \Delta p_{t-i} + \sum_{i=1}^{n} \lambda_i \Delta g_{t-i} + \sum_{i=1}^{n} \delta_i \Delta y_{t-i} + \sum \theta_i S_i + \varepsilon_t \]  (4.1)

where \( y, p, x, g, EC \) and \( S_i \) are respectively real income, price index, financial variable, government expenditure, error correction term and seasonal dummy variables. The \( \alpha, \beta_i, \gamma_i, \lambda_i, \delta_i \) and \( \theta_i \) are coefficients to be estimated while \( \varepsilon_t \) is a disturbance terms.

The error correction model incorporates the following variables:27

I. Income

The difference of log of real gross domestic product (RGDP) is used to represent the real income. It is denoted as \( \Delta y_t \). Real GDP is measured at 1987 price level. Prior to 1991, the quarterly real GDP is obtained from Tilak & Lee (1996).

II. Price Level

The difference of log consumer price index (CPI) is used to represent the price level. It is denoted as \( \Delta p_t \). The CPI is measured in 1994 price level.

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27 The descriptions and sources of the variables can be obtained from Table 1.1 in Chapter 1.
III. Fiscal Variable

The difference of log total Federal government expenditure is used to represent the fiscal variable. It is denoted as $\Delta g_t$. Total government expenditure is obtained by adding current expenditure and development expenditure. Changes in government spending have been used in the model as a control variable because government spending is not subject to influences or feedback from the dependent variable. Moreover, it is included in the error-correction model in order to assess whether its presence has any effect on the significance of the financial variables indicated.

IV. Financial Variables

A spectrum of competing financial aggregates, namely the credit aggregates, monetary aggregates and interest rates are used for the relative comparisons of the information content of these variables in forecasting economic activities. The financial variables are denoted as $\Delta x_t$.

- Credit Aggregate

The difference of log of three alternative credit aggregates is used to represent the credit aggregate. They are total domestic credit (CR1), monetary survey’s claims on private sectors (CR2) and commercial bank’s claim on the private sectors (CR3).
• **Monetary Aggregates**

The difference of log of three alternative monetary aggregates is used to represent the monetary aggregates. They are $M_1$, $M_2$ and $M_3$. $M_1$ represents narrow money supply while $M_2$ and $M_3$ represent broad money supply.

• **Interest rates**

Difference of three alternative interest rates will be used to represent the interest rate variable. They are three-month Treasury bill rates (TBR3), three-month inter-bank rates (IBR3) and commercial banks average lending rates (ALR).

Before conducting the Granger test, need to determine the lag lengths in the multivariate ECM model. Number of previous studies such as Friedman & Kuttner (1992), Mulayana (1995), Farizah (1999) used four lags in their studies. However, in this study the optimal lag length is determined by using the Autoregressive Conditional Heteroscedasticity (ARCH) serial correlation test and the results showed that three lags are appropriate for the study.

To conduct a multivariate Granger test of whether the selected financial variable forecasts income or price level, the above error correction models can be written as follows:\textsuperscript{28}

\textsuperscript{28} Only equation (4.12) is shown and the other three equations - (4.11), (4.13) and (4.14) can also be written in the similar form.
\[ \Delta y_i = \alpha EC_{r-1} + A(L)\Delta x_{i-1} + B(L)\Delta p_{r-1} + C(L)\Delta g_{r-1} + D(L)\Delta y_{r-1} + \sum \theta_j S_j + \varepsilon_i \] (4.15)

\( A(L),..D(L) \) are polynomials in the lag operator, a symbol that indicates lagged values of the variable to the right of the \( L \). For example:

\[ L\Delta x_i = \Delta x_{i-1} \] (4.16)

and powers of \( L \) indicate the number of times to apply the lag operator:

\[ L^2\Delta x_i = L(L(L\Delta x_i)) = L(L\Delta x_{i-1}) = L\Delta x_{i-2} = \Delta x_{i-3} \] (4.17)

and if three lags is used the polynomials are expanded as follows:

\[ A(L)\Delta x_{r-1} = (\beta_1 + \beta_2 L + \beta_3 L^2)\Delta x_{r-1} \]

\[ = \beta_1\Delta x_{r-1} + \beta_2\Delta x_{r-2} + \beta_3\Delta x_{r-3} \] (4.18)

After the regression is estimated, an F-test is performed for the hypothesis:

\[ H_0 : A(L) = 0 \]

Or it can be written as follows:

\[ H_0 : \beta_1 = \beta_2 = \beta_3 = 0 \]

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\({}^{29}\) It shows that all of the coefficients on the lagged financial variable (\( \Delta x \)) are zero.
If the hypothesis is rejected, the financial variable as a predictor variable is able to forecast the income even after the history of the other variables (government expenditure and price levels) are taken into account. This implies that, the selected variable contains information about future output beyond the information contained in these additional variables. The information contents of the selected financial variables are assessed by testing the significance of $\beta$s in equation (4.11), (4.12), (4.13) and (4.14).

### 4.7 Variance Decomposition

The forecast error variance decomposition (VDs) display information on the role played by different structural shocks in explaining the variability of the series at different horizon. In this study, the VDs are used to investigate the dynamic effect of output and price to monetary policy shocks especially during the post liberalization period. Accordingly, the Sim-type forecast error variance decomposition is used. It is suitable as it gives an economic measure of feedback that is simply quantifying how much feedback exists from one variable to another (Pindyak & Rubinfeld, 1991).

To carry out the forecast error VD, the following Choleski decomposition approach is used (Cochrane, 1997):

$$ x_t = C(L) \eta_t, \quad E(\eta_t \eta_t') = I $$

$C(L)$ gives the response of $x_t$ to the new shocks $\eta_t$, whereby $\eta_t = Q \varepsilon_t$. 

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\[ C(L) = C_0 + C_1L + C_2L^2 + \ldots \text{ and the elements for } C(L) \text{ is represented as } c_{yy,0} + c_{yy,1}L + c_{yy,2}L^2 + \ldots \]

The one step ahead forecast error variance is:

\[ \varepsilon_{t+1} = x_{t+1} - E_r(x_{t+1}) = C_0 \eta_{t+1} = \begin{bmatrix} c_{yy,0} & c_{yz,0} \\ c_{zy,0} & c_{zz,0} \end{bmatrix} \begin{bmatrix} \eta_{y,t+1} \\ \eta_{z,t+1} \end{bmatrix} \]  (4.19)

And the variance:

\[ \text{var}(y_{t+1}) = c_{yy,0}^2 \sigma^2(\eta_{y}) + c_{yz,0}^2 \sigma^2(\eta_{z}) = c_{yy,0}^2 + c_{yz,0}^2 \]  (4.20)

whereby, \(c_{yy,0}^2\) gives the amount of the one step ahead forecast error variance of \(y\) due to the \(\eta_y\) shock and \(c_{yz,0}^2\) gives the amount due to the \(\eta_z\) shock and is reported in fractions.\(^{30}\)

Thus, the total variance of the one step ahead forecast errors:

\[ \text{var}(x_{t+1}) = (C_0C_0') \]  (4.21)

If there are \(\tau\) shocks, then:

\[ C_0C_0' = C_0I_1C_0' + C_0I_2C_0' + \ldots + C_0I_{\tau}C_0' \]  (4.22)

\(^{30}\) The one step ahead forecast error variance is only shown for \(y\), as for \(z\), the approach is also similar.
whereby, the part of the one step ahead forecast error variance due to the first shock is $C_0^0C_0^0$ and the part due to $\tau$ shock is $C_0^\tau C_0^\tau$.

As for $k$ steps ahead forecast error variance:

$$x_{t+k} - E_t(x_{t+k}) = C_0^0\eta_{t+k} + C_1^0\eta_{t+k-1} + \ldots + C_{k-1}^0\eta_{t+1}$$ (4.23)

$$\text{var}_t(x_{t+k}) = C_0^0C_0^0 + C_1^0C_1^0 + \ldots + C_{k-1}^0C_{k-1}^0 = \sum_\tau \nu_{k,\tau}$$ (4.24)

whereby the variance of $k$ step ahead forecast errors due to the $\tau^{th}$ shock is:

$$\nu_{k,\tau} = \sum_{j=0}^{k-1} C_j^\tau C_j^\tau$$ (4.25)