METAHEURISTIC ALGORITHMS FOR SOLVING LOT-SIZING AND SCHEDULING PROBLEMS IN SINGLE AND MULTI-PLANT ENVIRONMENTS

MARYAM MOHAMMADI

FACULTY OF ENGINEERING
UNIVERSITY OF MALAYA
KUALA LUMPUR

2015

METAHEURISTIC ALGORITHMS FOR SOLVING LOT-SIZING AND SCHEDULING PROBLEMS IN SINGLE AND MULTI-PLANT ENVIRONMENTS

MARYAM MOHAMMADI

THESIS SUBMITTED IN FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

FACULTY OF ENGINEERING
UNIVERSITY OF MALAYA
KUALA LUMPUR

2015

UNIVERSITY OF MALAYA

ORIGINAL LITERARY WORK DECLARATION

Name of Candidate: **Maryam Mohammadi** I.C/Passport No: **I95756000**

Registration/Matric No: **KHA120059**

Name of Degree: **Doctor of Philosophy**

Title of Thesis:

Metaheuristic Algorithms for Solving Lot-Sizing and Scheduling Problems in Single and Multi-Plant Environments

Field of Study: **Manufacturing Systems (Engineering and Engineering Trades)**

I do solemnly and sincerely declare that:

(1) I am the sole author/writer of this Work;
(2) This Work is original;
(3) Any use of any work in which copyright exists was done by way of fair dealing and for permitted purposes and any excerpt or extract from, or reference to or reproduction of any copyright work has been disclosed expressly and sufficiently and the title of the Work and its authorship have been acknowledged in this Work;
(4) I do not have any actual knowledge nor do I ought reasonably to know that the making of this work constitutes an infringement of any copyright work;
(5) I hereby assign all and every rights in the copyright to this Work to the University of Malaya ("UM"), who henceforth shall be owner of the copyright in this Work and that any reproduction or use in any form or by any means whatsoever is prohibited without the written consent of UM having been first had and obtained;
(6) I am fully aware that if in the course of making this Work I have infringed any copyright whether intentionally or otherwise, I may be subject to legal action or any other action as may be determined by UM.

Candidate's Signature Date: 23/09/2015

Maryam Mohammadi

Subscribed and solemnly declared before,

Witness's Signature Date: 23/09/2015

Name:

Designation:

ABSTRACT

Economic lot scheduling problem (ELSP) is related to lot-sizing and scheduling several items in a single production facility. This project addresses the ELSP where multiple items have shelf life restrictions and planned backorders. For some products, shelf life might be less than the production cycle time, which leads to product spoilage before the end of the cycle. In order to achieve a feasible schedule, the production cycle time needs to be reduced to less than or equal to the shelf life duration. While the cost-minimization cycle time causes the spoilage of products, due to shelf life restrictions, appropriate decisions can be made based on one of three options: production rate reduction, cycle time reduction, or the simultaneous production rate and cycle time reduction. For each option, the optimal cycle time and production rate are appraised, which satisfy the shelf life constraints. On the other hand, backorders incur shelf life constraint alteration, which affects the corresponding inventory models. Accordingly, appropriate modifications are applied to the related mathematical inventory models.

Further, a mixed integer non-linear model for the ELSP is developed which allows each product to be produced more than once per cycle. However, production of each item more than one time may result in an infeasible schedule due to the overlapping production times of various items. To eliminate the production time conflicts, adjustments must be made through advancing or delaying the production start time of some or all the items. The objective is to find the optimal production rate, lot size, production frequency, cycle time, as well as a feasible manufacturing schedule for the family of items, in addition to minimizing the total cost including production, setup, holding, backordering, and adjustment costs.

Lot-sizing problems are more complicated in multi-facility systems because of interdependency between facilities. Therefore, the multi-item multi-period capacitated lot-sizing problem in a multi-stage system composed of multiple suppliers, plants, and

distribution centers is addressed in order to investigate the effectiveness of coordinating production and distribution planning. Combinations of several functions such as purchasing, production, storage, backordering, and transportation between suppliers, plants and distribution centers are considered. The objective is to simultaneously determine the optimal raw material order quantity, production and inventory levels, and the transportation amount so that the demand can be satisfied with the lowest possible cost over a given planning horizon without violating the capacity restrictions of the plants and suppliers. Transfer decisions between plants are made when demand observed at a plant can be satisfied by other production sites to cope with under-capacity of a given plant. Furthermore, the model also allows for sales at distribution centers.

Numerical examples are presented to illustrate the effectiveness and efficiency of the proposed models. Metaheuristic approaches namely genetic algorithm, particle swarm optimization, artificial bee colony, simulated annealing, and imperialist competitive algorithm are adopted for the optimization procedures. To offer more efficiency, Taguchi method is utilized to calibrate the various parameters of the proposed algorithms. The statistical optimization results show the efficiency, effectiveness and robustness of the applied methods in solving the proposed optimization models.

ABSTRAK

Masalah penjadualan lot ekonomi (ELSP) adalah berkait rapat dengan pensaizan lot dan penjadualan beberapa item di dalam sebuah fasiliti pengeluaran tunggal. Projek ini menangani ELSP yang mempunyai beberapa item dengan batasan jangka hayat dan tunggakan tempahan terancang. Bagi sesetengah produk, jangka hayatnya mungkin kurang daripada tempoh kitaran pengeluarannya, menyebabkan kepada kerosakan produk sebelum tiba ke pengakhiran kitaran. Untuk mencapai penjadualan yang boleh dilaksanakan, tempoh kitaran pengeluaran perlu dikurangkan kepada kurang daripada atau sama dengan tempoh jangka hayatnya. Sementara tempoh kitaran dengan kos yang diminimakan menyebabkan kerosakan produk-produk, dengan melihat kepada batasan jangka hayat, keputusan yang sesuai boleh dibuat berdasarkan tiga pilihan: pengurangan kadar pengeluaran, pengurangan tempoh kitaran, atau pengurangan kedua-duanya sekali. Bagi setiap pilihan, tempoh kitaran optimum dan kadar pengeluaran dinilai, yang mana ianya memenuhi batasan jangka hayat. Sementara itu, tempahan tertunggak mendorong perubahan kepada batasan jangka hayat, yang kemudiannya memberikan kesan kepada model inventorinya. Oleh itu, pengubahsuaian yang sewajarnya dibuat kepada model inventori matematik yang berkaitan.

Seterusnya, satu model integer campuran yang bersifat tidak linear bagi ELSP dibangunkan untuk membolehkan setiap produk dapat dihasilkan lebih daripada sekali bagi setiap kitaran. Walaubagaimanapun, pengeluaran setiap item lebih daripada sekali memungkinkan penjadualan yang tidak dapat dilaksanakan disebabkan adanya pertindihan tempoh pengeluaran pelbagai item. Untuk menyingkirkan percanggahan tempoh pengeluaran, pelarasan hendaklah dibuat dengan cara mempercepatkan atau melambatkan masa permulaan bagi pengeluaran bagi sebahagian item atau kesemuanya sekali. Objektifnya adalah untuk mengoptimumkan kadar pengeluaran, saiz lot, kekerapan pengeluaran, tempoh kitaran serta penjadualan pembuatan yang dapat

dilaksanakan bagi kelompok item-item, di samping meminimakan kos keseluruhan termasuk kos-kos pengeluaran, persediaan, induk, tempahan tertunggak dan kos pelarasan.

Masalah pensaizan lot adalah lebih rumit bagi sistem dengan pelbagai fasiliti disebabkan oleh pergantungan antara fasiliti-fasiliti tersebut. Oleh itu, permasalahan pensaizan lot dengan kapasiti pelbagai item dan pelbagai tempoh yang merangkumi pelbagai pembekal, loji dan pusat pengedaran diambil perhatian untuk mengkaji keberkesanan koordinasi pengeluaran dan rancangan pengedaran. Kombinasi pelbagai fungsi seperti pembelian, pengeluaran, penstoran, tempahan tertunggak dan pengangkutan di antara pembekal, loji dan pusat pengedaran diambil kira. Tujuannya adalah untuk menentukan jumlah tempahan bahan mentah yang optimum, tahapan pengeluaran, tahapan inventori, dan jumlah pengangkutan supaya permintaan dapat dipenuhi dengan kos terendah pada satu-satu ufuk perancangan tanpa melanggar had kapasiti loji dan pembekal. Keputusan pemindahan di antara loji dibuat apabila permintaan dalam satu-satu loji dapat dipenuhi di tapak pengeluaran yang lain untuk mengatasi permasalahan loji-loji yang berada di bawah kapasiti. Seterusnya, model tersebut juga membenarkan penjualan di pusat-pusat pengedaran.

Contoh-contoh berangka dipersembahkan untuk memaparkan keberkesanan dan kecekapan model yang dicadangkan. Pendekatan metaheuristik seperti algoritma genetik, pengoptimuman kawanan zarah, koloni lebah buatan, simulasi penyepuhlindapan dan algoritma kompetitif imperialis digunapakai dalam prosedur pengoptimuman. Untuk meningkatkan keberkesanan, kaedah Taguchi digunakan untuk menentukurkan parameter-parameter di dalam algoritma yang dicadangkan. Keputusan pengoptimuman statistikal menunjukkan kecekapan, keberkesanan dan keteguhan metod yang digunakan untuk menyelesaikan pengoptimuman model yang dicadangkan.

ACKNOWLEDGEMENTS

I would like to give all glory to the almighty God for providing me the opportunity, patience and aptitude to accomplish this work. He has been the source of hope and strength in my life and has granted me with countless benevolent gifts.

I would like to express deep gratitude to my supervisors Professor Dr. Ardeshir Bahreininejad and Dr. Siti Nurmaya Binti Musa, for all their benign guidance and invaluable support during this research, and for having faith in me to be able to achieve this accomplishment.

I would also like to extend my appreciation to staffs of Centre of Advanced Manufacturing and Material Processing, and Faculty of Mechanical Engineering, University of Malaya, Kuala Lumpur, Malaysia for their encouragement and kind assistance rendered to me throughout my study. My gratitude also goes to Ministry of Higher Education of Malaysia with High Impact Research (HIR) fund for their financial support.

My appreciation goes as well to Dr. Sabariah Binti Julai as Internal Examiner, and Professor Dr. Rubén Ruiz García and Professor Dr. Azizollah Memariani as External Examiners, for their insightful and constructive comments and suggestions.

Lastly but not the least, I tend to take this opportunity to appreciate the indispensable and perpetual support from my beloved family who has always confided in me and heartened me throughout all that I choose to pursue, and whose words have always instilled incentives in me during my life.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF ABBREVIATIONS

ABC	:	Artificial Bee Colony
ACO	:	Ant Colony Optimization
ANOVA	:	Analysis of Variance
CLSD	:	Capacitated Lot-Sizing Problem with Sequence Dependent Setups
CLSP	:	Capacitated Lot-Sizing Problem
CLSPL	:	Capacitated Lot-Sizing Problem with Linked Lot Sizes
CSLP	:	Continuous Setup and Lot-Sizing Problem
DH	:	Dobson's Heuristic
DLSP	:	Discrete Lot-Sizing and Scheduling Problem
DP	:	Dynamic Programming
ELSP	:	Economic Lot Scheduling Problem
EOQ	:	Economic Order Quantity
GA	:	Genetic Algorithm
GLSP	:	General Lot-Sizing and Scheduling Problem
HGA	:	Hybrid Genetic Algorithm
ICA	:	Imperialist Competitive Algorithm
JRP	:	Joint Replenishment Problem
LP	:	Linear Programming
LR	:	Lagrangian Relaxation
MATLAB	:	Matrix Laboratory
MLCLSP	:	Multi-Level Capacitated Lot-Sizing Problem
MLDLSP	:	Multi-Level Discrete Lot-Sizing and Scheduling Problem
MLPLSP	:	Multi-Level Proportional Lot-Sizing and Scheduling Problem
MLULSP	:	Multi-Level Uncapacitated Lot-Sizing Problem

MPCLSP	:	Multi-Plant Capacitated Lot-Sizing Problem
NFEs	:	Number of Function Evaluations
NP	:	Non Deterministic Polynomial Time Problems
NS	:	Neighborhood Search
PLSP	:	Proportional Lot-Sizing and Scheduling Problem
PSO	:	Particle Swarm Optimization
SA	:	Simulated Annealing
SCM	:	Supply Chain Management
TS	:	Tabu Search
VNS	:	Variable Neighborhood Search
WW	:	Wagner-Whitin

LIST OF APPENDICES

CHAPTER 1: INTRODUCTION

1.1 Research Background

Production planning is the determination, acquisition and arrangement of all facilities necessary for future production of products (Wild, 1974). Production planning and control is needed to achieve the production objectives with respect to quality, quantity, cost and timeliness of delivery. It helps a company to utilize the available resources effectively and gain the uninterrupted production flow in order to minimize production costs and times, and meet customers varied demands with respect to quality and committed delivery schedule (Kumar & Suresh, 2009).

Planning horizon in production planning can be classified into three periodic ranges: long-term (strategic), medium-term (tactical), and short-term (operational) (Bitran & Tirupati, 1993). Long-term planning uses aggregated demand forecasts and makes strategic decisions such as aggregate resource planning to mainly achieve financial targets. Medium-term planning is more detailed and uses partially disaggregated demand to often determine material requirements plan and production quantities over planning horizon in order to optimize both operational and financial criteria while satisfying capacity limitations.

Short-term planning uses totally disaggregated or actual demands to make day-to-day decisions on lot-sizing, scheduling and loading problems (Heizer & Render, 2004; Karimi et al., 2003). Lot-sizing models can be classified either as medium-term or short-term models based on their level of aggregation and decision horizon (Jans & Degraeve, 2008; Clark et al., 2011).

Lot size refers to the quantity to be ordered or produced. Lot sizes generally vary with the type of manufacturing process used. For instance, in job shops, lot sizes tend to be much smaller than line production. If lot sizes become very small, then the need for frequent setup of production facilities or placing several orders with suppliers increases. This may lead to increased setup or order costs, but reduces inventory buildup and costs associated with inventory holding (Swamidass, 2000). On the other hand, a large lot size reduces setup or ordering frequency and hence setup or ordering cost, but requires holding a larger average inventory, which increases the holding cost.

Therefore, the aim of lot-sizing is to determine the optimal timing and level of production so as to achieve the best plausible trade-off between setup and holding costs and satisfy demand over the defined planning horizon (Jans & Degraeve, 2007). A manufacturing firm which seeks to compete in the market must make the right decisions in terms of lot-sizing that has direct effect on the system performance and productivity. This necessitates the formulation and development of appropriate models and solution methods for lot-sizing problems.

Lot-sizing problems can be classified as single stage (with one planning stage), and multi-stage (with several planning stages) (Bahl et al., 1987). In single stage systems, the final products are made directly from raw materials through a single process with no intermediate subassembly (Rizk & Martel, 2001). Demand for a product is obtained from customer orders and/or market forecasts. In multi-stage systems, there is a parent-component relationship between the items. In such production systems, end products are assembled from intermediate products (subassemblies), which might in turn require raw materials or parts to manufacture. The output of one stage is thus the input for the next stage. A stage may also entail an operation such as purchasing of raw materials, production of parts, or assembly (Crowston & Wagner, 1971). This research deals with both single stage lot-sizing and scheduling problems in single facility systems, and multi-stage lot-sizing problems in multi-facility environments.

1.2 Single Facility Lot-Sizing Problem

The most basic and oldest of all mathematical lot-sizing models is the economic order quantity (EOQ) model developed by Harris (1913), which considers a single item with a constant demand rate, continuous time period, and infinite planning horizon. The objective is to obtain the optimal production or order quantity with the lowest cost, based on the tradeoff between setup and inventory costs. The economic lot scheduling problem (ELSP) can be considered as an extension to multiple items sharing a single resource with limited capacities.

Lot-sizing decision is critical to the efficiency of production and inventory systems. In the literature, researchers have been addressing their efforts to research problems on the optimal lot-sizing strategies for different decision-making scenarios. The ELSP is one of the most representative topics as it combines lot-sizing and production scheduling decisions.

The ELSP is related to lot-sizing and scheduling the production of multiple items on a single facility in a cyclic pattern with the aim of meeting demand without backorders and minimizing the setup and holding costs (Rogers, 1958). The ELSP typically imposes a restriction that one item can be produced at a time, so that the machine has to be stopped before commencing the production of a different item. Therefore, a production scheduling problem appears due to the need for incorporating the setups and production runs of various items (I. Moon et al., 2002b).

Most studies investigating the different aspects of the ELSP assumed that every product is manufactured only one time in the rotational production cycle. Goyal (1994) and Viswanathan (1995) implied that manufacturing of every item more than once per cycle might be more economical. Although this policy may result in a solution with a lower cost, it might however bring about an infeasible production schedule due to the overlapping production time of various items. Therefore, to generate a feasible manufacturing schedule, the production cycle of the items requires to be modified in order to prevent overlaps in the schedule. Consequently, modifications in the pertinent cost function and constraints are necessary.

In the real world, it often happens that shortages occur in products or spare parts due to reasons such as machine failure, insufficient inventory to meet demand, fluctuations in demand in excess of inventory or inaccurate demand forecast, low production due to inadequate resources, and so forth. It is thus clear that shortage is a natural phenomenon which happens in such systems and an accurate model should take this into account. As a matter of fact, it is beneficial to concern having shortages when the inventory holding cost is high compared with the shortage cost (Aliyu & Andijani, 1999). When demand in a period is not fully satisfied, the units of end items in shortage can carry over to subsequent periods considered as backordering.

In industry, items are kept as stock in storage facilities to be consumed during the production phase. In literature on inventory systems, product shelf life is often considered unlimited. However, some products have limited life-spans during which the quality and applicability of such products deteriorate over time (Kazaz & Sloan, 2008). By definition, shelf life is the duration for which a product remains unspoiled (Lütke Entrup et al., 2005). The wastage and sales rate losses as well as on-hand inventory are highly affected by the shelf life specifications. Storing products for longer than specified shelf life durations may cause product deterioration or diminution. It might also lead to loss of profitable or fruitful lives of manufactured goods in a developing market for new and competitive merchandise (Xu & Sarker, 2003).

In a multifarious product manufacturing environment where lots have diverse sizes and production times while sharing a common facility, the main objective is to determine an optimal cycle time in which all the products are manufactured. Once the optimum cycle time exceeds the life time restriction for an item, the corresponding inventory model needs to be modified to prevent product spoilage. Shelf life restriction is examined in such condition by implementing the three options of cycle time reduction, production rate reduction, and simultaneous cycle time and production rate reduction. Shelf life constraint appends another feature to the ELSP. Moreover, considering backorders incur shelf life constraint variation, which affects the corresponding inventory models.

So far, however, there has been little research on the ELSP with multiple products having unknown production frequencies, backorders and shelf life constraints. Due to nonlinearity and complexity of the ELSP, it is known as NP-hard problem (Hsu, 1983). Thus, metaheuristic methods can be used to find the optimal or near optimal solutions for the ELSP within a reasonable computation time.

1.3 Multi-Facility Lot-Sizing Problem

The multi-plant structure is a complex multi-stage manufacturing system, where each plant itself denotes a multi-stage system in which the flow of products may be serial, parallel, assembly or general (Billington et al., 1983). In this case, lot-sizing problems become more complicated because of the interdependency between plants. If no interactions exist between the facilities and transportation costs are not considered, then solving a multiple facility problem is equivalent to solving a set of independent single facility problems.

Enterprises are facing highly competitive and fast-changing business environments. Traditionally, companies will usually expand the size of and number of production plants to cater for the increase in production capacity. However, over-increased capacity may results in unwarranted effects such as price reductions of products. To meet customer demands in a timely fashion, companies have used the strategy of outsourcing as a method to increase production capacity. (De Kok, 2000; Tukel & Wasti, 2001; McCarthy & Anagnostou, 2004; Ruiz-Torres et al., 2006). In a global scale, companies cannot compete on their own in the market (Conklin & Perdue, 1994), thus requiring support from other partners by developing a multi-plant manufacturing supply chain to maximize competitive advantages of supply chain members (Chen, 2010).

A large integrated company may possess a hierarchy of production plants, in which the production and assembly processes for manufacturing a product can be dispersed at different plants established in geographically scattered locations (Lin & Chen, 2007). Though, once a job is allocated to a plant, it is usually inefficient to transfer it to other factories (Chan et al., 2005), unexpected circumstances such as machine breakdowns or lack of sufficient operators may vindicate the reallocation of jobs at other plants in real time (Alvarez, 2007).

For many organizations, the shift from the conventional single plant to multi-plant manufacturing environment may bring about difficulties in the production planning. Thus, the production decisions at plants must be re-coordinated to prevent problems such as excessive inventories, ineffective capacity consumptions, and unsatisfactory customer services. The move towards incorporated multi-plant configurations would bring a wide range of opportunities in terms of cost reduction in manufacturing and logistics activities as well as competitive advantages in the global economic arena (Alvarez, 2007; Junqueira & Morabito, 2012). In addition, it allows the company to establish reliable commitments with customers as efficiently as possible and maximize the customer service level.

Although much consideration has been devoted to develop the mathematical models for solving supply chain and production planning problems, specifically in manufacturing and goods distribution, most of these models have considered them as discrete problems. In reality, for most manufacturing environments, these problems are interconnected. Thus, there is a need for developing an integrated model. In a multi-plant production system with scattered customers, the assignment of productions to plants and plants to customers determines the production and distribution performance. Integrating these two functions could lead to significant savings in global costs in addition to an enhancement in pertinent service by exploiting scale economies of production and transportation, balancing production lots and vehicle loads, and decreasing inventory and stock out (Fumero & Vercellis, 1999).

In a multi-plant scenario, a crucial managerial concern is the determination of production quantities (lot size) for each item in each plant and period, such that the total costs at all factories are minimized (Bhatnagar et al., 1993). As stated by Nascimento and Toledo (2008), the multi-plant capacitated lot-sizing problem (MPCLSP) with multiple time periods and products consists of several manufacturing centers that produce identical items, and allows the inter-plant transfers of the products. A few studies have considered the MPCLSP and limited solution approaches have been recommended.

Florian et al. (1980) proved that the single plant multi-item capacitated lot-sizing problem is NP-Hard, so is the respective multi-plant version. Therefore, metaheuristic approaches can be used to efficiently tackle such complex problems and offer good solutions within a reasonable computation time.

1.4 Optimization in Lot-Sizing Problems

Optimization is the process which is executed iteratively for finding the value of variables for which objective function can be either minimized or maximized by satisfying some constraints. For a given problem domain, the main goal is to provide the mode of obtaining the best value of objective function (Gupta & Jain, 2015).

The range of techniques that have been applied to tackle combinatorial optimization problems can be classified into two general categories, the exact methods and the approximate (heuristic) methods. Exact methods seek to solve a problem to guaranteed optimality but their execution on large real world problems usually require too much computation time. Consequently, resolution by exact methods is not realistic for large-sized problems, justifying the use of powerful heuristic and metaheuristics methods (Dhingra, 2006).

A heuristic is a problem-dependent algorithm that exploits problem dependent information to find a sufficiently good solution (not necessarily optimal) to a specific problem (Saka et al., 2013). As such, they usually are adapted to the problem at hand and try to take full advantage of the particularities of the problem. However, because they are often too greedy, they usually get trapped in a local optimum and thus fail, in general, to obtain the global optimum solution.

Metaheuristics are a class of heuristic techniques that have been successfully applied to solve a wide range of combinatorial optimization problems over the years as they provide ways to escape the local optimum solutions (Osman & Laporte, 1996; Voß et al., 2012). They are also often claimed to be able to solve larger instances of a problem and/or to obtain faster results than pure enumerative exact approaches. Moreover, metaheuristics are general purpose algorithms that can be applied to almost any type of optimization problem (Boussaïd et al., 2013). They do not take advantage of any specificity of the problem, and generally they are not greedy. In fact, they may even accept a temporary worsening of the solution (for example, simulated annealing technique), which allows them to explore more thoroughly the solution space and thus to get a better solution (that sometimes will coincide with the global optimum). Although a metaheuristic is a problem-independent technique, it is nonetheless necessary to do some fine-tuning of its intrinsic parameters in order to adapt the technique to the problem at hand.

The drawbacks (efficiency and accuracy) of existing numerical methods have encouraged researchers to rely on metaheuristic algorithms based on the simulations and nature inspired methods to solve engineering optimization problems. Metaheuristic algorithms commonly operate by combining rules and randomness to imitate natural phenomena (Lee & Geem, 2005). These phenomena may include the biological evolutionary process such as genetic algorithm (GA) proposed by Holland (1975), animal behavior such as particle swarm optimization (PSO) proposed by Kennedy and Eberhart (1995), or the physical annealing which is generally known as simulated annealing (SA) proposed by Kirkpatrick et al. (1983).

There are several advantages of using metaheuristic algorithms such as (Madić et al., 2013):

1. Broad applicability: they can be applied to any problems that can be formulated as function optimization problems. The problem can be continues or discrete.
2. Hybridization: they can be combined with more traditional optimization techniques.
3. Ease of implementation: typically easier to understand and implement.
4. Efficiency and flexibility: they can solve large-sized problems faster. Moreover, they are simple to design and implement, and are very flexible.

The use of metaheuristics can be justified due to: (i) complexity of the internal problem that prevents the application of exact techniques, and (ii) a very large quantity of possible solutions that prevent the use of exhaustive algorithms (Gendreau & Potvin, 2005; Talbi, 2009).

It is known that the decision making associated with the lot-sizing and scheduling problem belongs to the category of combinatorial optimization problems. The difficulty to find a general approach for the lot-sizing and scheduling problem is considered in complexity theory as a NP-hard problem (França et al., 1997). Therefore, metaheuristic solution methods must be developed in order to find near optimal solution by exploring the search space efficiently. Metaheuristics has become a great choice for solving NP-hard problems because of their multi-solution and strong neighborhood search capabilities in a reasonable computational time.

As it has been reported in the literature, three types of metaheuristic-based search algorithms namely GA, SA and PSO have been mostly applied in the domain of the lot-sizing and scheduling optimization problems. However, in recent years there is also an increasing trend in the application of newly developed metaheuristic algorithms such as artificial bee colony (ABC) and imperialist competitive algorithm (ICA) for solving lot-sizing and scheduling problems. Therefore, these metaheuristic algorithms are selected as they are tested vastly in plenty of combinatorial optimization problems.

Wolpert and Macready (1997) introduced “No Free Lunch Theory” and concluded that every metaheuristic algorithm has different searching abilities and has its own advantage to deal with the problem domain. So no single algorithm is able to offer satisfactorily results for all problems. In other words, a specific algorithm may show very promising results on a set of problems, but may show poor performance on a different set of problems (Gupta & Jain, 2015).

A classification of different solution methods is shown in Figure 1.1.

Figure 1.1: Classification of common search methodologies and common metaheuristics

1.5 Research Objectives

The objectives of this research are as follow:

i. To formulate and develop mathematical models for a combination of economic lot scheduling problem, backordering and shelf life by applying three options of "production rate reduction", "cycle time reduction", and "simultaneous production rate and cycle time reduction".
ii. To formulate and develop a mathematical model for a multi-product economic lot scheduling problem with shelf life restrictions, backordering, and multiple setups in a production cycle.
iii. To formulate and develop a mathematical model for a multi-period multi-product multi-plant capacitated lot-sizing problem with inter-plant transfers in an integrated supply chain network.
iv. To carry out optimization procedures in order to obtain the optimum or near-optimum solutions for the proposed models by employing well-known metaheuristic algorithms.
v. To compare the performances of the applied metaheuristic algorithms.

1.6 Scope of the Research

This work mainly expands in two directions. The first part of research focuses on the modeling of the multi-item lot-sizing and scheduling problem in a single stage single facility system with a continuous time scale, deterministic static demand and infinite time horizon which is known as the ELSP with integration of multiple setups, backordering, and shelf life. The aim is to determine the optimal lot size, production rate, production frequency, cycle time, as well as a feasible manufacturing schedule for the family of items, and to minimize the total pertinent cost.

The second part is concerned with the multi-item lot-sizing problem in a multi-stage multi-facility system having a discrete time scale, deterministic dynamic demand and finite time horizon. The aim is to find order, production, and shipment quantities in an integrated production-distribution network that are optimal from a system's perspective, in addition to minimizing the cost of the whole supply chain.

Numerical examples are used to illustrate the features and validities of the proposed mathematical models. To solve the models, metaheuristic algorithms namely GA, PSO, SA, ABC, and ICA are utilized. This research aims at comparing the performance of these metaheuristics when applied to the ELSP and MPCLSP. Applied optimizers were written and coded in MATLAB software version R2012a (7.14.0.739) and were run on a laptop with 2.5-GHz AMD and 4GB RAM.

1.7 Organization of the Thesis

Chapter 1 explains the background of the study, problem definition, objectives and scope of the research.

Chapter 2 presents a critical review of available literature on single and multi-level lot-sizing problems in single and multi-facility systems.

Chapter 3 explains the methodology of the research and research frameworks. This chapter also provides a brief explanation of various optimization algorithms used in this study.

Chapter 4 indicates examining and comparing three options namely "production rate reduction", "cycle time reduction", and "simultaneous production rate and cycle time reduction" in the ELSP considering shelf life and backordering.

Chapter 5 encompasses the proposed model formulation and development for optimization of the ELSP with multiple setups, shelf life, and backordering using calibrated metaheuristic algorithms. The computational results and comparisons of the applied algorithms are also presented.

Chapter 6 describes the proposed model formulation for optimization of the MPCLSP in an integrated supply chain network composed of multiple supplier, plants, and distribution centers employing calibrated metaheuristic algorithms. Computational experiments are also presented to compare the performance of the applied metaheuristics and obtained solutions.

Chapter 7 provides the final conclusions and gives a brief summary of the study and recommendations for future research.

CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

In this chapter, the literature related to lot-sizing problems is reviewed. This chapter also provides discussions on five areas related to this project: (1) Single facility lot-sizing problems; (2) Single level lot-sizing problems; (3) Economic lot scheduling problems; (4) Multi-level lot-sizing problems; and (5) Multi-facility lot-sizing problems.

2.2 Lot-Sizing

There are several hierarchical levels of decisions which should be made by a manufacturing company with its production-related activities. Strategic decisions have a long-term scope and address questions such as what to offer on the market (product mix), where to build plants and warehouses (location), or whether to acquire new equipment (investment). Tactical decisions cover problems with a medium-range impact, such as the design of facilities (layout), contracts with suppliers, and adequate workforce levels. As for strategic choices, tactical decisions rely on aggregate data which are demand for product families rather than single products and capacities of entire production lines rather than particular machines (Lang, 2010).

A planning horizon of several years for strategic choices and of several months to one year for tactical considerations makes it impossible to use detailed information. Inputs for such decisions are therefore based on aggregated forecasts with a smaller margin of error. As a third level, operational production planning is concerned with the short-term implementation and execution of plans to reach the goals previously settled on at higher levels. Establishing sequences of operations for each machine and determining exact start and end times of operations are carried out at this level. Operational decisions use detailed information and a finite time grid.

Lot-sizing problems can arise at several points in medium to short-term production planning. Determining the production quantities for end products in the course of master production scheduling usually covers a time span of several weeks and is based on forecasted demand. The lot sizes of end products directly affect the demand for the components from which they are assembled. In the course of the subsequent material requirements planning, lots for subassemblies and parts, as well as orders for raw materials, can be coupled, which thus gives rise to lot-sizing problems farther down the product structure. Lot-sizing decisions can also be integrated with sequencing and scheduling decisions. The time span considered in such a case is very short, and the resulting production plan usually covers about a week period. Figure 2.1 (based on Bahl et al., 1987; Tempelmeier, 1997; Lang, 2010) provides a quick overview of production planning decisions with an emphasis on lot-sizing.

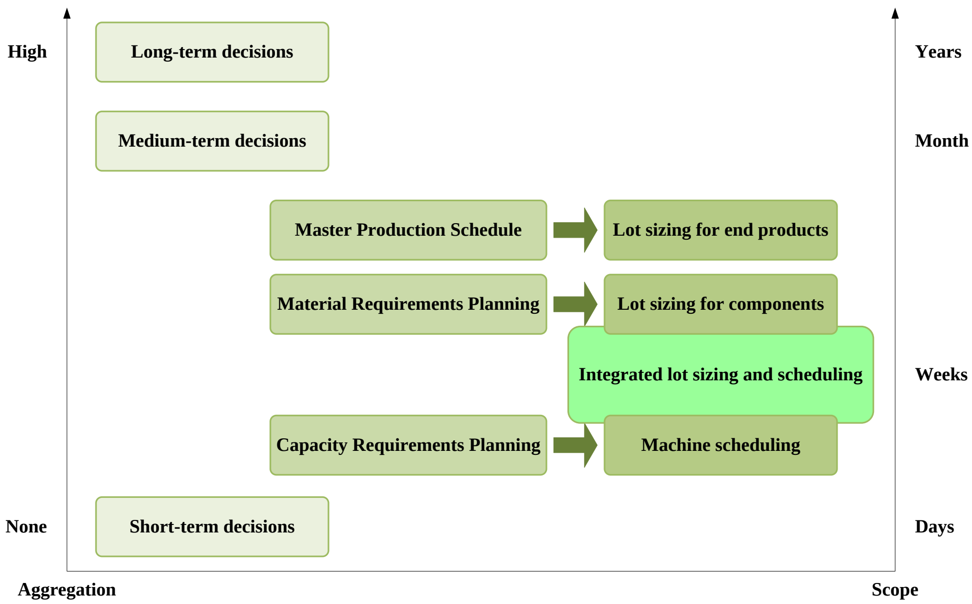


Figure 2.1: Lot-sizing decisions in production planning

Bahl et al. (1987) classified lot-sizing problems into four categories:

i. Single level unconstrained resources
ii. Single level constrained resources
iii. Multi-level unconstrained resources
iv. Multi-level constrained resources

Levels denote the different stages in a bill of material structure where there are dependencies of requirements, and constrained resources stand for production capacity restrictions (Woarawichai et al., 2010).

2.3 Characteristics of Lot-Sizing Models

Lot-sizing models differ in their underlying assumptions and in the details they incorporate. Figure 2.2 illustrates a classification of lot-sizing problems in which each characteristic strongly impacts the modeling and complexity of the problem.

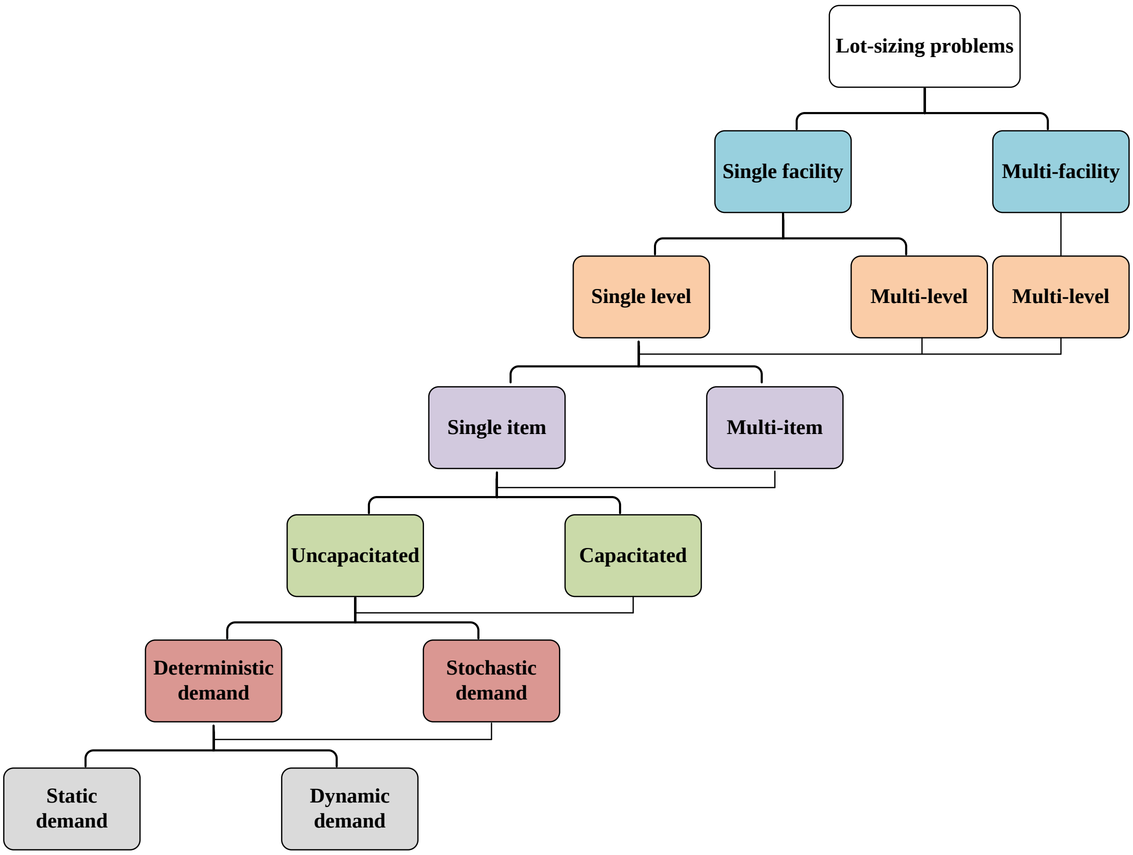


Figure 2.2: A category of lot-sizing problems

Lot-sizing problems can be characterized by a variety of aspects and classification criteria, which are explained in the following subsections.

2.3.1 Planning Horizon

A planning horizon is the length of time into the future for which plans are made. The length of the horizon can be finite or infinite. A finite planning horizon is typically accompanied by a time-varying demand and an infinite planning horizon by a constant demand rate. Furthermore, the time horizon can be divided into discrete or continuous time periods.

As defined by Belvaux and Wolsey (2000), lot-sizing problems can be either small bucket or big bucket. Big bucket problems allow for the production of many items at the same time period without taking into account sequencing issues. Small bucket models considers short time periods in order to be able to model start-ups, switch-offs and/or changeovers. The small bucket models are then split further into those in which only one item can be setup per period, and those with possibly two setups per period.

2.3.2 Number of Products

Single item models consider one type of product at a time. Multi-item models consider a number of products simultaneously. These products must have at least one interrelating or binding factor such as budget, capacity constraint, or a common setup.

2.3.3 Number of Levels

If multiple items are considered, they can either be from a single level of the product structure, i.e. multiple independent final products are considered, or they can be on different levels, i.e. parent-component relationships between the items are present. In such multi-level production systems, end products are assembled from intermediate products (sub-assemblies), which might in turn require raw materials or parts for production. The output of one stage is thus the input for the next stage.

2.3.4 Capacity Constraints

Resources in a manufacturing system contain manpower, equipment, machines, budget, and so forth. If the models assume unlimited capacities of resources, they are considered as uncapacitated problem. Capacitated models recognize that some resources are given in a limited number or amount so that planning and scheduling systems need to avoid over utilizing these resources.

In some cases, it is essential to consider capacity utilization more accurately in order to achieve a feasible production plan. For instance, the capacity utilized when a machine begins or finishes a production batch, or when a machine shifts from one product to another, may need to be considered. In such cases, models deal with setup times, changeover times, or sequencing restrictions.

2.3.5 Setup Structure

A particular setup is often necessary to prepare a machine for the production of a specific product if this machine produces different types of products. Whenever this changeover causes setup times and/or cost, a lot-sizing problem arises.

Setup times implies the capacity consumed because of cleaning, warming, machine adjustments, calibration, inspection, test runs, tool changes, and so forth, when the production for a new product begins. Setup times can be included explicitly in a model. However, due to the complexity in such a case, they are often incorporated indirectly via the setup costs (Jans & Degraeve, 2008). Setup costs and setup times, are generally modeled by considering zero-one variables in the mathematical models and make the problem solving harder.

2.3.6 Demand

Another important characteristic of lot-sizing problem is the nature of demand. Static demand models assume that parameter’s value does not vary over the planning horizon, while dynamic demand models allow for variation over time. If the demand value is known in advance, the demand stream is considered as deterministic. If the demand is based on distribution or a measure of uncertainty, it is considered as stochastic.

Independent demand refers to the demand for a product which is independent of demand for other items. Independent demand for end products is triggered by the market. Dependent demand for components is triggered by scheduled production on superior levels.

2.3.7 Inventory Shortage

Shortage occurs when demand exceeds the available inventory for an item, and can be divided into two categories, namely backordering and lost sale. Backordering occurs when it is probable to fulfill demand of the current period in the next time period. If demand cannot be satisfied at all, lost sale can happen. The combination of backordering and lost sale is also plausible. However, both cases incur penalty cost as they have a negative impact on customer satisfaction.

2.3.8 Deterioration

Another aspect that affects the problem complexity is deterioration or shelf life constraint where items can be held only for a limited lifetime. Deterioration refers to a process in which inventories undergo a change in storage over time, such that they become partially or completely unsuited for consumption and therefore, may impose additional costs for inventory storage. Ignoring deterioration of the items may bring about misleading replenishment policies and shortage in demand which in turn incurs additional shortage cost.

2.4 Single Facility Problems

The classical concept of a single facility can be considered as a single machine or may be defined as a complete assembly line that would in essence form the whole physical plant (Aras & Swanson, 1982). According to Kreipl and Pinedo (2004), a short-term detailed scheduling model is generally only concerned with a single facility, or, at most, with a single stage. Such a model usually takes more detailed information into account than a planning model.

Based on Bruggeman et al. (1982) and Glock et al. (2014), when there is only a single facility or production line, only one product can be in production at a time. For a multi-item environment, production needs to be scheduled in such a way that the machine is never required to produce more than a single product at a time. Therefore, to generate a feasible manufacturing schedule, the production cycle of the items requires to be modified in order to prevent overlaps in the schedule.

Single facility problems have been reviewed broadly in both single and multi-level systems, which are explained as follow.

2.4.1 Single Level Lot-Sizing Problems

In single level structures (also known as single stage), demand for an item is obtained directly from customer orders and/or market forecasts. Single stage lot-sizing problems in procurement/distribution environments typically concern only purchasing and holding costs and avoid transportation cost. Anderson et al. (1997) defined single stage production systems as those which require one operation for each job involving either a single machine, or more than one machine operating in parallel. Nevertheless, each of the parallel machines has exactly the same function.

In the following subsections, the literature related to uncapacitated and capacitated single level lot-sizing problems is reviewed.

2.4.1.1 Uncapacitated Single Item Problem

The single level single item problems with no capacity constraint were at the advent of developments in the lot-sizing and scheduling arena. The EOQ model was introduced by Harris (1913), which assumes a constant demand rate for a single item, infinite planning horizon, and continuous time scale with the aim of minimizing the sum of ordering and inventory holding costs. Wagner and Whitin (WW) (1958) investigated the lot-sizing problem for a single item with unlimited capacities over a finite planning horizon divided into discrete periods. Demand and costs were accordingly time-varying.

Numerous model formulations and solution procedures have been proposed for the uncapacitated single item lot-sizing problems.

Zangwill (1969) improved the WW basic model to include backlogging of demand. Approximate solutions to the single item, single stage uncapacitated lot-sizing problem were suggested by DeMatteis (1968) and Silver and Meal (1973). The major advantage of these approaches is that they are computationally much more efficient than the exact solutions. Hax and Candea (1984) extended the EOQ model by allowing backlogging, lost sales, and quantity discounts. Fordyce and Webster (1985) modified the WW algorithm for situations in which unit cost price is not constant over the planning horizon, and included quantity discounts. Lev and Weiss (1990) and Gascon (1995) presented solutions for the finite horizon EOQ model where costs are time-dependent.

Discounts are a primary marketing mechanism for inducing customers to increase the size of their purchases. Quantity discounts from suppliers and freight discounts from shippers are commonly encountered by organizations. Tersine and Barman (1991) structured quantity and freight discounts into the order size decision in a deterministic EOQ system. Optimum lot-sizing algorithms were derived for the dual discount situations of all-units or incremental quantity discounts and all-weight or incremental freight discounts.

Gupta and Brennan (1992) introduced an easy alternative to the WW backorder algorithm. The performance of the model was compared with several of the traditional lot-sizing rules (lot for lot, EOQ, period order quantity, least unit cost, least total cost, part period algorithm, Silver-Meal algorithm, and WW algorithm) as well as the backorder versions of WW and EOQ. It was concluded that the proposed algorithm is sufficiently robust and relatively easy to apply. Most of the dynamic lot-sizing models assume that production is performed on reliable machines. Kuhn (1997) analyzed the effects of setup recovery with machine breakdowns and corrective maintenance for the single item uncapacitated lot-sizing problem. In a first case, the assumption was made that the setup is totally lost after a breakdown. In a second case, the costs of resuming production of the same item after a breakdown was lower compared to the original setup cost.

Agra and Constantino (1999) examined the single item uncapacitated lot-sizing problem with backlogging and start-up costs where WW costs were assumed. Hernandez and Suer (1999) presented a GA approach to obtain the order quantities for a single item, single level uncapacitated lot-sizing problem. In the experimentation, different strategies were presented to evaluate the behavior of the GA under different parameters sets. The results showed that the proposed procedure generated satisfactory solutions to the considered problem. Richter and Sombrutzki (2000) studied the reverse WW dynamic production planning and inventory control model. In such reverse (product recovery) models, used products arrive to be stored and to be remanufactured at minimum cost. It was assumed that the demand can be met either from newly manufactured products or from return products which have been remanufactured.

Lee et al. (2001) discussed the single item, uncapacitated dynamic lot-sizing problem with a demand time window, where for each demand an earliest and latest delivery date is specified and the demand can be satisfied in the defined period without penalty. It was shown that there exists an optimal solution in which demand is not split, where the complete demand for a specific order can be covered by production from the same period. Loparic et al. (2001) proposed valid inequalities for solving a variant of the single item uncapacitated lot-sizing model of the WW problem involving sales instead of fixed demands and lower bounds on the stock variables. Aksen et al. (2003) introduced a profit maximization version of the WW model for the deterministic single item uncapacitated lot-sizing problem with lost sales. It was assumed that demand cannot be backlogged, and costs and selling prices are time-variant. A forward recursive dynamic programming (DP) algorithm was developed to solve the problem optimally.

Teunter and Flapper (2003) examined a single stage single product production system, where produced units can be non-defective, reworkable defective, or non-reworkable defective. De Toledo and Shiguemoto (2005) proposed an efficient implementation of a forward DP algorithm for solving the lot-sizing problems in a single production center. Brahimi et al. (2006) reviewed various solution methods for solving the single item uncapacitated lot-sizing problem. Chiu (2008) presented a simple algebraic method to replace the use of calculus for determining the optimal lot size. Gutiérrez et al. (2008) addressed the dynamic lot-sizing problem with time-varying storage capacities with the aim of minimizing the total cost including setup, holding, and production/ordering costs.

Gaafar et al. (2009) applied the SA algorithm to find the solution of the deterministic dynamic lot-sizing problem with batch ordering and backorders, and compared the performance of the proposed SA with GA and modified Silver-Meal heuristic. Results indicated that SA algorithm had the best performance, followed by the GA, in terms of the frequency of obtaining the optimum solution and the average deviation from the optimum solution. It was also shown that SA was the most robust of the investigated heuristics as its performance was only affected by the length of the planning horizon. Hwang and van den Heuvel (2009) proposed a DP algorithm to optimally solve the classical uncapacitated single item lot-sizing problem with lost sales, upper bounds on stocks and concave costs. Vargas (2009) presented an algorithm for determining the optimal solution for the stochastic version of the WW dynamic lot-sizing problem.

Sana (2010) investigated an economic production lot size model in an imperfect production system in which the production facility may shift from an “in-control state” to an “out-of-control” state at any random time. In long-run process, the process shifts from the in-control state to the out-of-control state after certain time due to higher production rate and production run time. The proposed model was formulated assuming that a certain percent of total product is defective (imperfect), in out-of-control state, which varies with production rate and production-run time. The objective was to minimize the total cost including manufacturing cost, setup cost, holding cost, and reworking cost of imperfect quality products.

Senyiğit (2010) proposed a heuristic approach to solve the dynamic lot-sizing problem with demand and purchasing price uncertainties. Well-known least unit cost and Silver-Meal algorithms were also modified for both time-varying purchasing price and rolling horizon. The proposed heuristic was basically based on a cost-benefit evaluation at decision points. Absi et al. (2011) considered the single item uncapacitated lot-sizing problem with production time windows, lost sales, early productions, and backlogs. Several properties of the optimal solution for different variants of the problem when production time windows are non-customer specific were presented. The DP algorithm was used to solve the proposed problem.

2.4.1.2 Uncapacitated Multi-Item Problem

The principal concern of this category is to obtain production or order quantity for multiple products so as to minimize the long run average cost for the family of items and meet demand over the defined horizon. The total cost typically involves setup/ordering, inventory, and production/purchasing costs. Based on the absence of capacity constraints and parent-component relationships between products, decisions can be made for each item independently.

However, the existence of joint setup/ordering costs causes the interdependency between products which is known as the economic order quantity with joint replenishment problem. In a multi-item environment with a joint setup cost structure considerable savings may be realized by coordinating the replenishments (Aksoy & Erenguc, 1988). In the joint replenishment problem (JRP), major setup occurs if production is started, and minor setups are required if the processor switches from one item to the next (Graves, 1981). The aim of the model is to find the joint frequency of production/order cycles and the frequency of producing/purchasing each product in addition to minimizing the total cost. An overview of the JRP and solution methods can be found in Goyal and Deshmukh (1992). Later, Goyal and Deshmukh (1993) proposed heuristic procedures for solving the JRP.

Aksoy and Erenguc (1988) differentiated between the deterministic and stochastic and between the static and dynamic JRP. They developed a DP approach for small size static-dynamic JRP and a heuristic method for large family sizes. Van Eijs et al. (1992) distinguished between two different types of strategies, namely direct grouping strategies and indirect grouping strategies, for the multi-item inventory systems having constant deterministic demand and joint replenishment costs. The performances of the strategies were measured as the percentage cost savings of a joint replenishment strategy relative to an independent strategy, and were quantified through simulation.

Federgruen and Tzur (1994) addressed the JRP, where in the presence of joint setup costs, dynamic lot-sizing schedules need to be determined for multiple items over a finite planning horizon with general time-varying cost and demand. They developed a partitioning heuristic for the proposed problem, which partitions the complete horizon of periods into several relatively small intervals, and solved the problem via an efficient branch and bound method.

Hariga (1994) studied the effects of inflation and time value of money on the replenishment policies of items with time continuous non-stationary demand over a finite planning horizon. He developed the DP models for the commonly used replenishment policies in the inventory lot-sizing literature with time-varying demand and shortages. The results showed that the initial cycle length is virtually insensitive to the type of replenishment policy. They also extended the developed models to more practical inventory situations with exponentially deteriorating items and perishable products having fixed life time.

Kirca (1995) considered the multi-item dynamic lot-sizing problem with joint set-up costs. A tight formulation of the problem and the dual of the linear relaxation of this formulation were presented. A procedure to solve the dual problem was developed, where the solution provided a strong lower bound for the proposed problem. The computational experiments revealed that the proposed approach outperforms the branch and bound algorithm.

2.4.1.3 Capacitated Single Item Problem

Capacitated restrictions enhance the complexity of lot-sizing problems. The objective of this group of lot-sizing problem is to obtain the optimal production quantity or order size for a single product that minimizes the total cost including setup/ordering, inventory, and production/purchasing costs, while meeting the known demands and satisfying capacity constraints over the planning horizon.

Lambrecht and Vanderveken (1979) developed a computationally efficient algorithm for solving a single item dynamic lot-sizing problem with capacity constraints in order to obtain an optimum production schedule that minimizes the total production and inventory costs. Reviews of the literature for this class of problem can be found in Drexl and Kimms (1997) and Karimi et al. (2003), who offered overviews of optimal and heuristic solution procedures.

Examples of problems and related modeling approaches for dynamic capacitated lot-sizing in a single level system with discrete time representation are the ELSP (Rogers, 1958), continuous setup and lot-sizing problem (CSLP) (Karmarkar & Schrage, 1985), capacitated lot-sizing problem (CLSP) (Günther, 1987), discrete lot-sizing and scheduling problem (DLSP) (Fleischmann, 1990; Salomon et al., 1991), proportional lot-sizing and scheduling problem (PLSP) (Haase, 1994; Drexl & Haase, 1995), and capacitated lot-sizing problem with sequence-dependent setups (CLSD) (Haase, 1996).

The DLSP allows for the production of only one item in each period. The production is further assumed to be "all or nothing", and the total capacity available per period is used for the production of the scheduled item. The CSLP is equivalent to the DLSP without the "all or nothing" requirement, which can lead to periods with some slack capacity. The PLSP goes one step further and allows for the production of a second item to avoid excessive idle time on the resource. The CLSD assumes a fixed lead time offset of one (macro) period in order to secure a feasible material flow between production stages while small bucket models usually only require a micro period as a fixed lead time offset.

Fleischmann and Meyr (1997) integrated all mentioned models (ELSP, CSLP, CLSP, DLSP, PLSP, and CLSD) within the general lot-sizing and scheduling problem (GLSP). They used a two-fold time structure, where each macro-period is divided into several micro-periods of variable length. A complete sequence of items was established. All mentioned models commonly consider that setup times can only be considered if they do not exceed the length of a period. However, Koçlar and Süral (2005), through a simple modification of the GLSP, showed that setup times exceeding the length of a period can also be incorporated. The capacitated lot-sizing problem with linked lot sizes (CLSPL) (Suerie & Stadtler, 2003), extends the CLSP with the possibility of setup carryover. The CLSPL belongs to the class of large bucket problems, which allow many setup operations within a single period.

Florian et al. (1980) and Chen and Thizy (1990) proved that the single item CLSP is NP-hard. To deal with the intricacy of the problem and find the optimal solution in reasonable amount of time, numerous studies have applied heuristic and metaheuristic algorithms. Gavish and Johnson (1990) proposed a fully polynomial approximation scheme for solving the single item CLSP. However, their approach is more suitable for continuous models. Sandbothe and Thompson (1990) included backordering into the single item CLSP, and presented a polynomial algorithm for solving the case of constant capacities and a heuristic algorithm for solving the variable production capacity.

Kirca (1990) developed a DP-based algorithm for the single item lot-sizing problem with concave costs and arbitrary capacities. The performance of the algorithm was compared with the performance of the existing procedures in the literature for the general, the constant capacity, and the constant unit cost problems. The computational results demonstrated that proposed algorithm is at least three times faster than the other procedures for all problem types considered. Chen et al. (1994) developed a DP method for the single item capacitated dynamic lot size model with non-negative demands and no backlogging. The proposed approach produced the optimal value function in piecewise linear segments.

Chung (1994) studied a deterministic single product capacitated dynamic lot size model with linear production and holding costs where the setup costs, unit production costs, and capacities are arbitrary functions of the period, and the unit production costs satisfy the constraint. To solve the problem, the DP algorithm was combined with branch and bound approach. Lotfi and Yoon (1994) considered a multi-period single item production scheduling problem with a deterministic time-varying demand pattern and concave cost functions. Optimal production lot sizes were determined subject to dynamic production capacity and no backlogs in addition to minimizing the total costs of production, setup, and inventory. The proposed algorithm was tested extensively by solving several randomly generated problems with varying degrees of complexity, and showed quite good performance for practical applications.

Hindi (1995a) considered a capacitated single item lot-sizing model where a startup cost is incurred for switching the production facility on, and a separate reservation cost is incurred for keeping the facility on whether it is used for production or not. A tabu search (TS) scheme was developed for solving the problem which was capable of reaching the optimal solution for a large number of varied problem instances. Hardin et al. (2007) analyzed the quality of lower and upper bounds provided by a range of fast algorithms for single item CLSP with time-varying demands.

Akbalik and Pochet (2009) provided valid inequalities for the single item CLSP with step-wise production costs. Constant-sized batch production was carried out with a limited production capacity in order to satisfy the customer demand over a finite horizon. They suggested a cutting plane algorithm for different classes of the proposed valid inequalities. Computational results showed the efficiency of the proposed algorithm compared to the existing methods. Hellion et al. (2012) examined the single item CLSP with concave production and storage costs, and minimum order quantity. They proposed a polynomial time algorithm to solve the problem optimally, and computationally tested the algorithm on various instances.

2.4.1.4 Capacitated Multi-Item Problem

The multi-item CLSP is the extension of the WW problem to multiple items and consequently limited capacities in each period. The CLSP is referred to as a big bucket problem since several items may be produced per period. The aim of the classical single level CLSP is to determine the quantities and timing of production batches in order to satisfy external requirements while incurring minimum costs. No backlogging is allowed and sequencing decisions are not included into this problem. The overall model of this category of lot-sizing problem is presented below (Based on Drexl & Kimms, 1997; Pochet, 2001; Karimi et al., 2003).

(a) ***Assumptions of the CLSP***

i. The planning horizon is finite and divided into big time buckets.
ii. The lots are determined for multiple items without any interrelationships. The model thus considers a single production level.
iii. The external demand for the items is dynamic and deterministic, and has to be satisfied immediately and completely.
iv. The standard model allows no backordering (delaying fulfillment) or lost sales (no fulfillment).
v. There is a single resource with limited capacity that is shared by all items.
vi. Overtime decisions are not considered in the standard version.
vii. For every item produced in a given period, one setup takes place. The current setup state of the resource has no influence on setting it up for the next item. The setups are sequence-independent, postponing the scheduling decision.
viii. The setup state of the resource at the end of a period does not extend to the beginning of the subsequent period. This means that there is no possibility of a setup carryover, and not even a partial sequence is established.

ix. Setup times are not included as such. Instead of modeling them explicitly, they are hidden in the setup costs, represented as opportunity costs accounting for the time lost to actual production.

x. Variable production costs, holding costs and setup costs can either be modeled to vary with time, or they can be modeled static throughout the planning horizon. The time-invariant production costs and the respective term in the objective function can be omitted from the model as the total amount of production for each item is predetermined by the sum of its external demand. The objective of the CLSP is to minimize all costs incurred throughout the planning horizon.

(b) ***Indices***

i index for item ($i = 1, \dots, N$)

t index for period ($t = 1, \dots, T$)

(c) ***Parameters***

d_{it} External demand for item i in period t

C_t Available capacity in time units in period t

R_i Capacity in time units needed to produce one unit of item i

O_{it} Variable production cost for one unit of item i in period t

S_{it} Setup cost for item i in period t

H_{it} Holding cost for one unit of item i in period t

M_{it} Upper bound on the production quantity of item i in period t

(d) ***Decision Variables***

Q_{it} Production quantity (lot size) of item i in period t

I_{it} Inventory level of item i at the end of period t

$$Y_{it} = \begin{cases} 1 & \text{if item } i \text{ is produced in period } t \text{ (i.e. } Q_{it} > 0) \\ 0 & \text{otherwise} \end{cases}$$

(e) ***Mathematical Formulation***

Using the above notations, the mathematical model for the CLSP is presented below.

$$\text{Minimize} \sum_{i=1}^{N} \sum_{t=1}^{T} S_{it} Y_{it} + O_{it} Q_{it} + H_{it} I_{it} \tag{2.1}$$

Subject to:

$$\sum_{i=1}^{N} R_i Q_{it} \le C_t \tag{2.2}$$

$$I_{it} = I_{i(t-1)} + Q_{it} - d_{it} \tag{2.3}$$

$$Q_{it} \le M_{it} Y_{it} \tag{2.4}$$

$$M_{it} = \min \left\{ C_t \big/ R_i , \sum_{t'=t}^{T} d_{it'} \right\} \tag{2.5}$$

$$I_{i1} = 0 \tag{2.6}$$

$$X_{it}, I_{it} \ge 0 \tag{2.7}$$

$$Y_{it} = \{0,1\} \tag{2.8}$$

The objective function in Eq. (2.1) minimizes the total cost that includes setup, production, and inventory costs. Equation (2.2) represents the capacity constraint. The overall consumption for production must be lower than or equal to the available capacity. Equation (2.3) concerns the inventory balance equation. Equation (2.4) relates the binary setup variable Y_{it} to the production variable Q_{it}. It means that when there is production in period t, Y_{it} must be equal to 1. In other words, production of an item is only possible if the resource has been setup for that item. If no setup takes place (Y_{it} = 0), no production can take place (i.e. Q_{it} = 0). The upper bound on the production, M_{it}, is either given by the remaining unfulfilled demand or by the available capacity as shown in Eq. (2.5). Equation (2.6) shows that the inventory of item *i* for period 1 is zero. Equation (2.7) indicates that all decision variables are non-negative. Equation (2.8) shows Y_{it} is defined as the binary variable.

Many heuristic techniques have been developed for solving the variations of single level CLSP. Dixon and Silver (1981) developed a heuristic for solving the multi-item single level lot-sizing problem with limited capacity in a single facility production system, where the time required to setup the facility was avoided. The objective was to determine lot sizes so that costs are minimized, no backlogging occurs, and capacity is not exceeded. The results indicated that the proposed heuristic will usually generate a very good solution with a relatively small amount of computational effort.

Thizy and van Wassenhove (1985) designed a Lagrangian relaxation (LR) approach, in which capacity constraints are relaxed, in an attempt to decompose the problem into some uncapacitated single item lot-sizing sub-problems, solvable by the WW algorithm. The proposed method incorporated a primal partitioning scheme with a network flow sub-problem to obtain good feasible solutions.

A review of the CLSP can be found in Maes and van Wassenhove (1988), which was focused on heuristic solution procedures compared in extensive numerical studies. The authors classified the existing approaches into single resource and mathematical programming-based heuristics. De Souza and Armentano (1994) presented a multi-item CLSP with setup times for production of items, which was constrained by a limited regular time and a limited overtime as well as a limitation on production level of any item in a given period. The proposed problem was tackled by a cross decomposition based algorithm, which can provide optimal or near optimal solutions if computation time is restricted.

Millar and Yang (1994) developed both the LR and Lagrangian decomposition approaches to solve the single level CLSP with backordering. Computational analysis showed that both algorithms are quite effective, particularly when item setup and unit backorder costs are high. Hindi (1995b) addressed the problem of multi-item dynamic lot-sizing in the presence of a single capacitated resource. A model based on variable redefinition was developed leading to a solution strategy based on a branch and bound search with sharp low bounds. The resulting solution scheme was very efficient in terms of computation time. The standard CLSP was tackled by Hindi (1996), who combined the TS algorithm on the setup pattern for solving a reformulation of the problem as an uncapacitated transshipment problem.

Özdamar and Bozyel (2000) extended the CLSP to include overtime decisions and capacity consuming setups with the objective of minimizing inventory holding and overtime costs. Heuristic approaches such as hierarchical production planning approach, a GA approach based on the transportation-like formulation of the single item production planning model with dynamic demand, and a SA algorithm based on shifting family lot sizes among consecutive periods were developed to deal with the proposed problem. Computational results demonstrated that the SA approach produced high quality schedules and was computationally more efficient.

Özdamar et al. (2002) integrated the GA with TS and SA algorithms (GATA) to solve the CLSP with overtime and setup times. It was assumed that setups do not incur costs other than lost production capacity and therefore, setups contribute to total costs implicitly via overtime costs whenever capacity bottlenecks occur. The proposed GATA integrated the powerful characteristics of all three search algorithms, and the results demonstrated that GATA outperformed other heuristics reported in Özdamar and Bozyel (2000). Xie and Dong (2002) proposed a GA for solving the CLSP by designing a domain-specific encoding scheme for the lot sizes and by providing a heuristic shifting procedure as the decoding schedule. They designed the presentation technique that encoded only the binary variables for the setup patterns, but derives other decision variables by making use of the problem-specific knowledge.

Hindi et al. (2003) examined the multi-item single level CLSP with setup times. A lower bound on the value of the objective function was calculated by the LR approach with sub-gradient optimization. During the process, attempts were made to obtain feasible solutions through a smoothing heuristic, followed by a local search with a variable neighborhood search (VNS). Solutions found were further optimized by solving a capacitated transshipment problem.

Liu et al. (2004) studied single item inventory capacitated lot size model with lost sales. Gupta and Magnusson (2005) examined the capacitated lot-sizing and scheduling problem with sequence-dependent setup costs and non-zero setup times, with the additional feature that setups may be carried over from one period to the next, and that setups are preserved over idle periods. They developed a heuristic for solving large problem instances, and coupled with a procedure for obtaining a lower bound on the optimal solution. It was shown that the heuristic is more effective when there are many more products than there are planning periods.

Song and Chan (2005) investigated a single item lot-sizing problem with backlogging on a single machine at a finite production rate. The objective was to minimize the total cost of setup, stockholding and backlogging to satisfy a sequence of discrete demands. Both varying demands over a finite planning horizon and fixed demands at regular intervals over an infinite planning horizon were considered. A DP algorithm was proposed for the computation of an optimal production schedule for the varying demands case and a simpler one for the fixed demands case.

Jodlbauer (2006) developed a non-time discrete approach for an integrated planning procedure, and applied to a multi-item capacitated production system with dynamic demand. The objective was to minimize the total costs, which consist of holding and setup costs for one period. The proposed approach was based on a specific property of the setup cost function, which allows for replacement of the integer formulation for the number of setup activities in the model.

Federgruen et al. (2007) developed a progressive interval heuristic for the multi-item CLSP with deterministic demand, joint and item-dependent setup cost with the aim of finding a lot-sizing strategy that satisfies the demands for all items over the entire horizon without backlogging, and minimizing the sum of inventory-carrying costs, fixed-order costs, and variable-order costs. Marinelli et al. (2007) proposed a solution approach for a capacitated lot-sizing and scheduling real problem with parallel machines and shared buffers, arising in a packaging company producing yoghurt. An effective two-stage optimization heuristic based on the decomposition of the problem into a lot-sizing problem and a scheduling problem was developed. The proposed heuristic showed near-optimal solutions, all obtained in a short computation time.

Jans and Degraeve (2008) presented an overview of developments in the field of modeling of the deterministic single level dynamic lot-sizing problems, where focus was on the modeling of various industrial extensions rather than the solution approaches. Quadt and Kuhn (2008) provided reviews of extensions of the basic CLSP including parallel machines, backorders, and setup times, and illustrated model formulations for each of the extensions. Absi and Kedad-Sidhoum (2009) addressed a multi-item CLSP with setup times, safety stock and demand shortages. It was assumed that demand cannot be backlogged, but can be totally or partially lost. A LR procedure for the resource capacity constraints and a DP algorithm to solve the induced sub-problems were developed. Some experimental results showed the effectiveness of the proposed approaches.

Anily et al. (2009) examined a multi-item lot-sizing problem with joint setup costs and constant capacities. Apart from the usual per unit production and storage costs for each item, a setup cost was incurred for each batch of production. Computational results were presented to test the effectiveness of using the tight linear programs in strengthening the basic mixed integer programming formulations of the joint setup problem both when the storage cost conditions are satisfied, and also when they are violated.

Pan et al. (2009) addressed the capacitated dynamic lot-sizing problem arising in closed-loop supply chain where returned products are collected from customers. It was assumed that returned products can either be disposed or be remanufactured to be sold as new ones again; hence the market demands can be satisfied by either newly produced products or remanufactured ones. The proposed problem was analyzed and solved using DP algorithms under different scenarios. It was shown that the problem with only disposal or remanufacturing can be converted into a traditional CLSP and be solved by a polynomial algorithm if the capacities are constant. A pseudo-polynomial algorithm was proposed for the problem with both capacitated disposal and remanufacturing. It was indicated that the proposed algorithms perform well when solving problems of practical sizes.

Buschkühl et al. (2010) differentiated solution procedures for the CLSP into mathematical programming-based approaches, Lagrangian heuristics, decomposition and aggregation heuristics, metaheuristics, and problem specific greedy heuristics. They have also discussed both different modeling approaches to the optimization problems and different algorithmic solution approaches. Zhang et al. (2012) proposed a LR-based solution approach to solve a mixed integer CLSP in a closed-loop supply chain considering setup costs, product returns, and remanufacturing. Numerical experiments using synthesized data demonstrated that proposed approach can find quality solutions efficiently.

Ramezanian et al. (2013) presented a mathematical model for integrating lot-sizing and scheduling problem in capacitated flow shop environments. Two mixed integer programming-based approaches with rolling horizon framework were used to solve the proposed model. Mehdizadeh and Fatehi Kivi (2014) proposed a new mixed integer programming model for multi-item CLSP with setup times, safety stock and demand shortages in closed-loop supply chains. The returned products from customers can either be disposed or be remanufactured to be sold as new ones again. Due to the complexity of problem, three metaheuristics algorithms namely SA, vibration damping optimization algorithm and harmony search algorithm were used to solve the model. The results confirmed the efficiency of the harmony search algorithm against the other methods.

Chan et al. (2015) explored the multi-item CLSP by addressing the backlogging and associated high penalty costs incurred. At the same time, penalty cost for exceeding the resource capacity was also taken into account. To solve this computationally complex problem, a less explored algorithm biased random key GA was applied. The results showed that the proposed algorithm is an efficient tool to tackle such complex problems.

De Reyck (2015) developed period decompositions for the CLSP with setup times. Based on two strong reformulations of the problem, a transformed reformulation and valid inequalities were presented that speed up column generation and the LR approach. An efficient hybrid scheme was proposed that combines column generation and the LR in a novel way. Computational experiments showed that the proposed solution method for finding lower bounds is competitive with the available approaches in the literature.

Hajipour et al. (2015) considered a multi-item CLSP with setup times, safety stocks, and demand shortages plus lost sales and backorder considerations for various production methods (i.e., job shop, batch flow, and continuous flow among others). Two novel Pareto-based multi-objective metaheuristic algorithms were proposed, namely multi-objective vibration damping optimization and multi-objective harmony search algorithm. The proposed algorithms were compared with two well-known evolutionary algorithms called the non-dominated sorting the GA and multi-objective SA to demonstrate the efficiency and effectiveness of the proposed methods.

Tempelmeier and Hilger (2015) proposed the stochastic dynamic lot-sizing problem with multiple items and limited capacity under two types of fill rate constraints. They proposed linear programming (LP) models, where the non-linear functions of the expected backorders and the expected inventory on hand were approximated by piecewise linear functions. The models were solved with a variant of the fix-and-optimize heuristic.

2.4.2 Economic Lot Scheduling Problem

The ELSP is concerned with scheduling the production of multiple items in a single facility on a periodical basis with the restriction that one item is produced at a time. The objective of the ELSP is to determine the lot size and the schedule of production of each product so as to minimize the long run average costs incurred per unit time, namely the setup and holding costs (Rogers, 1958). This problem exists in many production systems such as metal forming, plastic injection, weaving and assembly lines, etc.

Feasible solutions for the ELSP (production schedules and lot sizes) should satisfy one by one production (single machine), no shortage, and the capacity constraints. Throughout the past half century, a considerable amount of research on this problem has been published with several directions of extensions. Subsequently, various heuristic approaches have been suggested using any of the basic period (Brander & Forsberg, 2006; Nilsson & Segerstedt, 2008; Salvietti & Smith, 2008), common cycle (Khoury et al., 2001; Torabi et al., 2005; Tang & Teunter, 2006; Teunter et al., 2009), or time-varying lot size methods (I. Moon et al., 2002b; Giri et al., 2003; Raza et al., 2006).

Dobson (1987) developed a heuristic for finding feasible schedules for the ELSP having the time-varying lot sizes and cycle times. Lopez and Kingsman (1991) provided an excellent review for the ELSP and the solution approaches. Zipkin (1991) examined a version of the ELSP in which items can be produced several times in different amounts during a cycle. It was shown how to compute the optimal lot sizes and cycle length, given the sequence of items in a cycle. The proposed procedure was designed to be used along with a heuristic for selecting the sequence of items in a cycle. The two algorithms together comprised a simple and plausible heuristic for the ELSP as a whole.

Bourland and Yano (1994) developed an optimization-based model that considers capacity slack, safety stock, and overtime explicitly in the ELSP with stochastic demand. The objective was to minimize the expected cost per unit time of inventory, overtime, and where applicable, setup costs. The solution was a continuous-time production plan that consists of a time-dependent inventory trajectory for each of the parts, including the placement of planned idle time in the schedule. The results on the relative merits of capacity slack and safety stock indicated that capacity slack in the form of planned idle time is not a cost-effective hedge against demand uncertainty in this context.

Gallego and Joneja (1994) extended the traditional model of the ELSP by considering various issues associated with the management of the raw materials for production of several items. In the presence of setup and holding costs for the raw materials, a planning model was formulated which provides a sharp lower bound on the cost of any policy for the problem. The solution was used to obtain policies which are guaranteed to be very close to optimal in the worst case. It was attempted to obtain good feasible schedules for both machine and raw materials.

Shaw (1998) considered the capacitated ELSP with piecewise linear production costs and general holding costs, which is a NP-hard problem but solvable in pseudo-polynomial time. The computational experiments indicated that the algorithm is capable of solving quite large problem instances within a reasonable amount of time. Bollapragada and Rao (1999) focused on simultaneous resource allocation, lot-sizing and scheduling in a multi-machine deterministic ELSP environment, with the objective of minimizing the long-run average cost including the production, setup, inventory, and shortage penalty costs.

Salvietti and Smith (2008) extended the ELSP to include price optimization with the objective of maximizing profits. A solution approach based on column generation was provided, which is able to produce very close to optimal results with short solution times. Bollapragada et al. (2011) investigated a discrete-time dynamic demand ELSP for multiple non-identical production lines. In particular, the problem of apportioning item production to distinct manufacturing lines with different costs (production, setup and inventory) and capabilities was considered. The computational results showed that the best of the developed approaches is able to handle the proposed problem outperforming general-purpose solvers and other randomized approaches.

Zanoni et al. (2012) addressed the multi-product ELSP with manufacturing and remanufacturing. It was assumed that manufacturing and remanufacturing operations are performed on the same production line, and both have the same quality, thus, they fulfill the same demand stream. A simple and easy to implement algorithm was proposed to solve the model using a basic period policy by relaxing the constraint of common cycle time and a single (re)manufacturing lot for each item in each cycle. Numerical examples showed the applicability of the algorithm and the cost savings.

Adelman and Barz (2013) formulated the ELSP with sequence-dependent setup times and costs as a semi-Markov decision process. Using an affine approximation of the bias function, a semi-infinite linear program was obtained determining a lower bound for the minimum average cost rate. Under a very mild condition, the proposed problem was reduced to a relatively small convex quadratically constrained linear problem by exploiting the structure of the objective function and the state space. Horng and Yang (2013) addressed the stochastic ELSP considering the make-to-stock production of multiple standardized products on a single machine with limited capacity, possibly random setup times under random demands, and possibly random processing times. A method was proposed that combines the ABC algorithm and ordinal optimization theory to find a good solution. Test results demonstrated that the proposed method is promising in the aspects of solution quality and computational efficiency.

Abdelsalam and Elassal (2014) extended the work of Ben-Daya et al. (2013) and relaxed the assumption of deterministic demand and constant holding and ordering costs for the joint ELSP in a three-layer supply chain. Computational intelligent algorithms were adopted to solve the proposed mixed integer problem, and performance comparisons were conducted to find the best solution. Löhndorf et al. (2014) conducted simulation optimization for the stochastic ELSP with sequence-dependent setup times. Salvietti et al. (2014) presented the stochastic version of the ELSP with pricing. The control variables of the stochastic problem were the production quantities and cycle lengths for each product. A solution method based on simulation, decomposition, and column generation was proposed, and tested using a number of designed experiments. The method was found to produce very close to optimal solutions quickly.

The ELSP is categorized as NP-hard (Hsu, 1983), which leads to difficulty of checking every feasible schedule in a reasonable amount of computation time. Most researchers have focused on the generation of near optimal repetitive schedules. Recently, metaheuristic algorithms have been implemented effectively to solve the ELSP.

Khouja et al. (1998) solved the ELSP with consideration of basic period approach using the GA, and showed that the GA is preferably appropriate for solving the problem. I. Moon et al. (2002b) utilized a hybrid genetic algorithm (HGA) to solve the single facility ELSP based on the time-varying lot size method, and compared the performance of the HGA with the well-known Dobson's heuristic (DH) (1987). Numerical experiments showed that the proposed algorithm outperformed the DH. Yao and Huang (2005) applied the HGA to solve the ELSP with deteriorating items using the extended basic period approach under the power-of-two policy.

Chatfield (2007) developed a genetic lot scheduling procedure to solve the ELSP under the extended basic period approach. The procedure was applied to the well-known Bomberger's benchmark (1966) problem, and compared with the proposed GA by Khouja et al. (1998). It was shown that genetic lot scheduling produces regularly lowers cost solutions than the GA method suggested in Khouja et al. (1998). Jenabi et al. (2007) solved the ELSP in a flow shop setting utilizing the HGA and SA algorithms. Their computational results indicated the superiority of the proposed HGA compared to the SA with respect to the solution quality. However, the proposed SA outperformed the HGA in terms of the required computation time.

Chandrasekaran et al. (2007) investigated the ELSP with the time-varying lot size approach and sequence-independent/sequence-dependent setup times of parts, and applied the GA, SA, and ACO algorithms. The computational performance analyses revealed the effectiveness of the proposed metaheuristic methods. Raza and Akgunduz (2008) examined the ELSP with time-varying lot size approach, and conducted a comparative study of heuristic methods, namely the DH, HGA, TS, SA, and NS on Bomberger's (1966) and Mallya's (1992) problems. Their results showed that the SA outperformed DH, HGA, and NS. The SA algorithm also indicated faster convergence than the TS algorithm, but resulted in solutions of a similar quality.

Sun et al. (2009) solved the ELSP in a multiple identical machines environment using the GA under the extended basic period and power-of-two policy. Tasgetiren et al. (2011) proposed a discrete ABC algorithm for the ELSP. Bulut et al. (2012) proposed a GA for the ELSP under the extended basic period approach and power-of-two policy. The experimental results showed that the proposed GA is highly competitive to the best-performing algorithms from the existing literature. Chung and Chan (2012) proposed a two-level GA to determine production frequencies for the ELSP. Kayvanfar and Zandieh (2012) solved the ELSP with deteriorating items and shortage using the ICA approach.

Peixin (2012) proposed an improved PSO algorithm for the ELSP under the power-of-two policy. Tasgetiren et al. (2012) presented a discrete harmony search algorithm for the ELSP with power-of-two policy. Ganguly et al. (2013) proposed a hybrid discrete differential evolution algorithm for the ELSP with time variant lot-sizing. Babaei et al. (2014) studied the capacitated lot-sizing and scheduling problem with sequence-dependent setups, setup carryover, and backlogging, and applied the GA to solve the model. To test the accuracy of the algorithm, a lower bound was developed and compared against the GA. In computational experiments, proposed GA performed extremely well. Bulut and Tasgetiren (2014) applied a discrete ABC algorithm for the ELSP with returns under the basic period policy with power-of-two multipliers. It was shown that the proposed algorithm performs well under the applied policy and it has the potential of improving the best known solutions when the applied policy is relaxed.

2.4.2.1 Shelf Life

Many industrial products have very short life cycles as well as shelf life constraints. Shelf life is the length of time that a product may be stored without becoming unfit for use, consumption, or sale. Shelf life does not only reflect the physical condition of a product, it may also reflect the productive or marketable life of a product as well in a competitive emerging market.

Silver (1989) considered the shelf life constraint for a multi-item single facility system while disallowing production cost under the postulation that production rate variation does not impose any further expenses. Two options of decreasing cycle time and production rate were investigated. It was concluded that if the shelf life constraint is flawed, it is more cost-efficient to reduce the production rate. Silver (1989) also stated that if the production rate decreases, the manufacturing process should be performed for a longer period.

However, Sarker and Babu (1993) implied that associated costs will increase as the production time is increased. This means that extra expenses are encountered by the manufacturing plant when the production rate increases. Hence, production time cost must be taken into account to incorporate the impact of production time length. Sarker and Babu (1993) modified the model proposed by Silver (1989) by considering production time cost. They found that when production cost is included in the model, it may be more efficient to reduce the cycle time rather than the manufacturing rate. They also implied that storage time can be lowered through regular restocking of items, subsequently decreasing the inventory maintained in stock.

Goyal (1994) investigated the results obtained by Sarker and Babu (1993), and suggested that their proposed model can be improved by allowing the production of items more than one time in a cycle. Viswanathan (1995) stated that although Goyal's suggestion (1994) can incur a lower inventory cost, his method does not assure a feasible production schedule. Yan et al. (2013) indicated that advancing or delaying the manufacturing start times of some items can lead to a feasible production plan. Accordingly, costs associated to the adjustment schedule must be taken into account. They suggested a two-stage heuristic algorithm. Initially, their model was simplified by omitting the schedule adjustment constraints and costs. Then, in the case of an infeasible schedule a modification procedure was employed using a greedy heuristic of sequentially selecting the activities, one every time, for either moving forward or postponing the manufacturing start time, until a practicable schedule is achieved.

However, the solution of the large scale proposed ELSP model seems to be out of reach using the suggested approach by Yan et al. (2013) due to its complexity and computational effort. Furthermore, In Yan et al. (2013), the items' production frequencies were restricted to three in order to make the problem practical, and limit the computational effort. Thus, efficient heuristic methods are required to solve the proposed model for large problems usually found in real-world situations.

Silver (1995) proposed the simultaneous adjustment of “cycle time” and “production rate” when the shelf life constraint is violated for one item. It was shown that after eliminating the infeasibility, the results obtained through the suggested option can be at least as good as adjusting only one of these two parameters. Viswanathan (1995) implied that Sarker and Babu’s model (1993) offers a feasible schedule only when all the items produced have the same frequency. Goyal (1996) implied that in some circumstances the models suggested by Silver (1989, 1995) and Sarker and Babu (1993) may result in an infeasible production schedule, since the feasibility condition for the cycle time was not considered in their studies.

Viswanathan and Goyal (1997) improved the model proposed by Silver (1995) by determining the optimum cycle time and production rate for a group of items, along with binding the shelf life constraint for multiple items. Later, Viswanathan and Goyal (2000) enhanced their previous model by considering planned backorders. They demonstrated that in models having backorders, some changes in shelf life constraint occur. However, Viswanathan and Goyal (2000) did not examine the three options to obtain the pertinent cost functions and the optimal cycle time if the shelf life constraint is violated.

Chowdhury and Sarker (2001) addressed the raw material inventory planning for a family of items having limitations on the shelf lives of items stored in inventory in addition to a generalized manufacturing cost of processes, where the cost of the process may increase or reduce, depending on the production system. Viswanathan and Goyal (2002) revealed that the model proposed by Chowdhury and Sarker (2001) causes the flaw of a shelf life constraint when adjusting both cycle time and production rate. Viswanathan and Goyal (2002) modified their model due to such flaw to obtain the optimum production rate and cycle time. Gupta and Karimi (2003) studied scheduling a two-stage multi-product process with limited product shelf life in an intermediate storage.

Xu and Sarker (2003) developed a model by considering the effects of production, holding, setup, and shortage costs in the inventory system while incorporating the shelf life constraint. They examined the three options in order to decide which option offers the lowest yearly cost for inventory system operations. However, Xu and Sarker (2003) did not assess the feasibility condition for cycle time after adjusting the production rate. Evidently, some of the proposed mathematical formulations for calculating the optimal cycle time and related costs are inconsistent and inaccurate.

Sharma (2004) examined a multi-product manufacturing environment by allowing shortages to be completely backordered and considering the shelf life constraint for one item. Mathematical formulations were provided to obtain the optimum cycle time and shortages. Nonetheless, Xu and Sarker (2003) and Sharma (2004) eliminated the effect of shortage on the shelf life constraint previously reported by Viswanathan and Goyal (2000). Therefore, their proposed models do not offer appropriate solutions when the shelf life constraint is violated (Goyal & Viswanathan, 2006).

Chakravarthy and Daniel (2004) studied an inventory system in which items have shelf lives, and demand is according to a Markovian arrival process that can be backordered up to a specific level. Soman et al. (2004) applied the basic period approach to solve the ELSP with shelf life considerations. Lütke Entrup et al. (2005) developed a mixed-integer LP model which incorporates shelf life limitations and production planning in addition to scheduling for perishable items. Sharma (2006) revised his previous work (Sharma, 2004) by incorporating fractional backordering in a multiple item production situation with shelf lives and using the common cycle time approach to solve the problem. Gürler and Özkaya (2008) examined perishable products by considering a continuous review (s, S) type policy with random shelf life and replenishment batch demand. Liu et al. (2008) incorporated shelf life considerations in the ELSP and solved the proposed model using the time-varying lot size method.

2.4.3 Multi-Level Lot-Sizing Problems

The multi-level (also known as multi-stage) lot-sizing problem is concerned with determining the lot size for producing or procuring an item at each level in order to satisfy the demand for end items at the right time and if possible at the lowest cost (Dellaert et al., 2000). In a multi-stage system, the production of final product requires completion of a number of operations or stages. A fixed sequence of operations is assumed, so that the output from one stage serves as the input to an immediate successor stage.

End items are made up with a number of intermediate products which, in turn, consist of combinations of components (purchased parts and raw materials). Each end item is therefore described by a bill of materials, which is the product recipe. When the issue of satisfying the demand for end items emanating from customers is considered, the right quantity of each sub-component has to be available at the right time. As products are associated with holding and setup costs, different inventory policies lead to different costs, and determining an optimal policy is the main concern.

Brüggemann and Jahnke (1994) extended the standard mixed-integer linear model formulation for the multi-item discrete lot-sizing and scheduling problem by additional partially nonlinear constraints for the case of two-stage batch production, known as (MLDLSP). A SA approach was suggested for computing production schedules on both stages. Kimms (1996) presented two heuristic approaches to solve a multi-level proportional lot-sizing and scheduling problem (MLPLSP). The first one was a variant of a so-called randomized regret based heuristic which is assumed to be the fastest available method for this particular class of problems. The second approach was a TS technique that was superior with respect to both the run-time performance and the average deviation from the optimum objective function values.

Population-based methods dealing with the multi-level uncapacitated lot-sizing problems (MLULSP) were implemented in the following studies. Dellaert and Jeunet (2000), Dellaert et al. (2000), and Prasad and Chetty (2001) employed the GAs; Tang (2004) and Homberger (2010) applied the SA; Pitakaso et al. (2007), Homberger and Gehring (2009), and Buer et al. (2013) employed the ant colony optimization (ACO) algorithm; and Han et al. (2009) applied the PSO method with flexible inertial weight.

An extensive study of stochastic local search procedures, in particular the SA, was presented in Jeunet and Jonard (2005), tackling the MLULSP. Berretta et al. (2005) enhanced their procedure consisting of a smoothing, improvement and perturbation step with elements of the TS and SA methods, which improved the obtained solution quality.

The multi-level capacitated lot-sizing problem (MLCLSP) is considered as an extension of the big bucket single level CLSP. There are constraints that must be taken into account such as demand satisfaction, inventory balance, limited capacity of resources, and setup time of products. The dynamic MLCLSP was introduced by Billington et al. (1983). The problem to determine a feasible solution for a MLCLSP with non-zero setup times is NP-complete (Maes et al., 1991), where exact methods fail in solving large problem instances.

Kuik et al. (1993) investigated the multi-level lot-sizing problem for assembly production systems with a bottleneck. They developed heuristics based on the LP, and compared the performance of these heuristics with the performance of the SA and TS algorithms. The results showed that SA and TS perform well compared to pure LP-based heuristics, but the effectiveness of the latter heuristics can be improved by combining them with elements from SA and TS. Stadtler (1997) reformulated the shortest route model to solve the dynamic multi-item MLCLSP. A model formulation was also introduced in which finding a tradeoff between model size and tightness of the lower bound obtained by the LP relaxation was enabled.

Özdamar and Barbarosoğlu (1999) addressed the multi-stage capacitated lot-sizing and loading problem, in which it deals with the issue of determining the lot sizes of items in serially-arranged manufacturing stages and loading them on parallel facilities in each stage to satisfy dynamic demand over a given planning horizon. To solve the proposed problem, the SA and GA were integrated to enhance their individual performances. These global optimization methods were further incorporated into a LR scheme, hence creating a hybrid solution methodology. Numerical results confirmed the efficiency of integrating the solution techniques.

Barbarosoğlu and Özdamar (2000) described an analysis of different neighborhood transition schemes and their effects on the performance of a general purpose SA procedure for solving the dynamic MLCLSP with general product structures. The results indicated that the performance of SA was highly dependent on the definition and the tightness of the search space. Furthermore, the increase in the number of search moves carried out by SA was shown to improve the results significantly with linearly increasing computation times. Hung and Chien (2000) compared the performance of the SA, TA and GA methods for the MLCLSP with multiple demand classes. Their findings showed that the TS and SA performed best in the confirmed order demand class and forecast demand class, respectively.

Özdamar and Barbarosoğlu (2000) examined the multi-item MLCLSP with general product structures. The difficulty in solving the MLCLSP is to provide capacity-feasible lot sizes while maintaining the non-negativity of the inventories belonging to the items in the lower levels of the product structures. It was attempted to resolve this issue by combining the capability of the LR to decompose the hard-to-solve problems into smaller sub-problems and the intensive search capability of the SA.

Chen and Chu (2003) developed a heuristic approach based on combining the LR with local search for supply chain planning modeled as multi-item MLCLSP. Numerical experiments showed that the proposed approach can find good solutions for problems of realistic sizes in a short computation time. Dellaert and Jeunet (2003) provided a detailed overview of heuristics for solving the MLULSP with time-invariant cost structure in material requirements planning systems. Berretta and Rodrigues (2004) developed methods based on evolutionary metaheuristics, more specifically a memetic algorithm for solving the MLCLSP. The proposed heuristics were evaluated using randomly generated instances and well-known examples in the literature. Pitakaso et al. (2006) combined the ACO algorithm with the fix and relax heuristics to solve the MLCLSP.

Inderfurth et al. (2007) studied the problem of scheduling manufacturing of work and rework processes on a single facility under deterioration of reworkables. The processing of a batch contains two stages, where setup time as well as setup cost are required to start batch processing and switch from production to rework. The objective was to find batch sizes such that all demands are satisfied and total setup, rework and inventory holding cost is minimized. Polynomial time algorithms were presented to solve two realistic special cases of this problem.

Fakhrzad and Zare (2009) examined the lot size determination problems in a complex multi-stage production scheduling problems with production capacity constraint. By determining the decision variables, machinery production capacity and customer's demand, an integer linear program was provided with the objective of minimizing the total cost of setup, inventory and production. Through combining the GA with one of the neighborhood search (NS) techniques, a new approach was developed for solving the problem. Mohammadi et al. (2009) considered the problem of multi-product multi-level capacitated lot-sizing and sequencing problem with sequence-dependent setups. A mathematical model was developed to quickly find feasible solutions for non-small instances. Hybrid methods that mix rolling-horizon approach and heuristic were developed, and accuracy of hybrid methods was tested in order to find the trade-offs between objective values and computing times.

Caserta et al. (2010) presented a metaheuristic called corridor method for solving the MLCLSP with setup carryover. The algorithm iteratively built new corridors around the best solution found within each corridor and, therefore, explored adjacent portions of the search space. The algorithm was tested on instances of a standard benchmark library and the reported results showed the robustness and effectiveness of the proposed scheme.

Wu and Shi (2011) demonstrated theoretically the relationships between several mathematical formulations for solving the MLCLSP with linked lot sizes in order to investigate the comparative efficiencies associated with these models. It was attempted to provide the theoretical and numerical results in order to find significant guidelines for choosing an effective formulation in different situations. Wu et al. (2011) presented an optimization framework for solving MLCLSP with backlogging.

Ramezanian and Saidi-Mehrabad (2013) addressed the lot-sizing and scheduling problem of a flow shop system with capacity constraints, sequence-dependent setups, uncertain processing times and uncertain multi-product and multi-period demand. Due to the complexity of problem, two MIP-based heuristics with rolling horizon framework named non-permutation heuristic and permutation heuristic were performed to solve the model. Also, a hybrid metaheuristic based on a combination of SA, firefly algorithm and proposed heuristic for scheduling was developed to solve the problem. Computational results on a set of randomly generated instances showed the efficiency of the hybrid metaheuristic against exact solution algorithm and heuristics.

Toledo et al. (2013) applied a hybrid multi-population GA to solve the MLCLSP with backlogging. The proposed method combined the multi-population based metaheuristic using fix-and-optimize heuristic and mathematical programming techniques. The results showed that presented algorithm had a better performance for most of the test sets solved compared with those available in literature, especially when longer computing time was given.

Hajipour et al. (2014) investigated a multi-level problem of lot-sizing with capacity constraints in a finite planning horizon in order to determine the economical lot size value of each product in each period, so that besides fulfilling all the needs of customers, the total cost of the system is minimized. A combination of ACO and a heuristic method called shifting technique was proposed to solve the problem, and the results were compared with TS, SA, and GA. The computational results indicated the efficiency of the proposed method in comparison to other metaheuristics.

Toledo et al. (2014) applied a GA approach embedded with mathematical programming techniques to solve a synchronized and integrated two-level lot-sizing and scheduling problem motivated by a real-world problem that arises in soft drink production. A production process compounded by raw material preparation/storage and soft drink bottling was considered. The proposed GA deals with sequencing decisions for production lots, so that an exact method can solve a simplified LP model, responsible for lot-sizing decisions. The computational results showed that the evolutionary/mathematical programming approach outperforms the literature methods in terms of production costs and run times.

Chen (2015) employed two approaches namely fix-and-optimize and VNS method for the dynamic MLCLSP considering both without and with setup carryover. Numerical experiments on benchmark instances showed that both applied approaches can obtain a better solution for most instances compared with that found by the fix-and-optimize approach proposed by Helber and Sahling (2010).

Figure 2.3 illustrates an overview of the mentioned single and multi-level lot-sizing models.

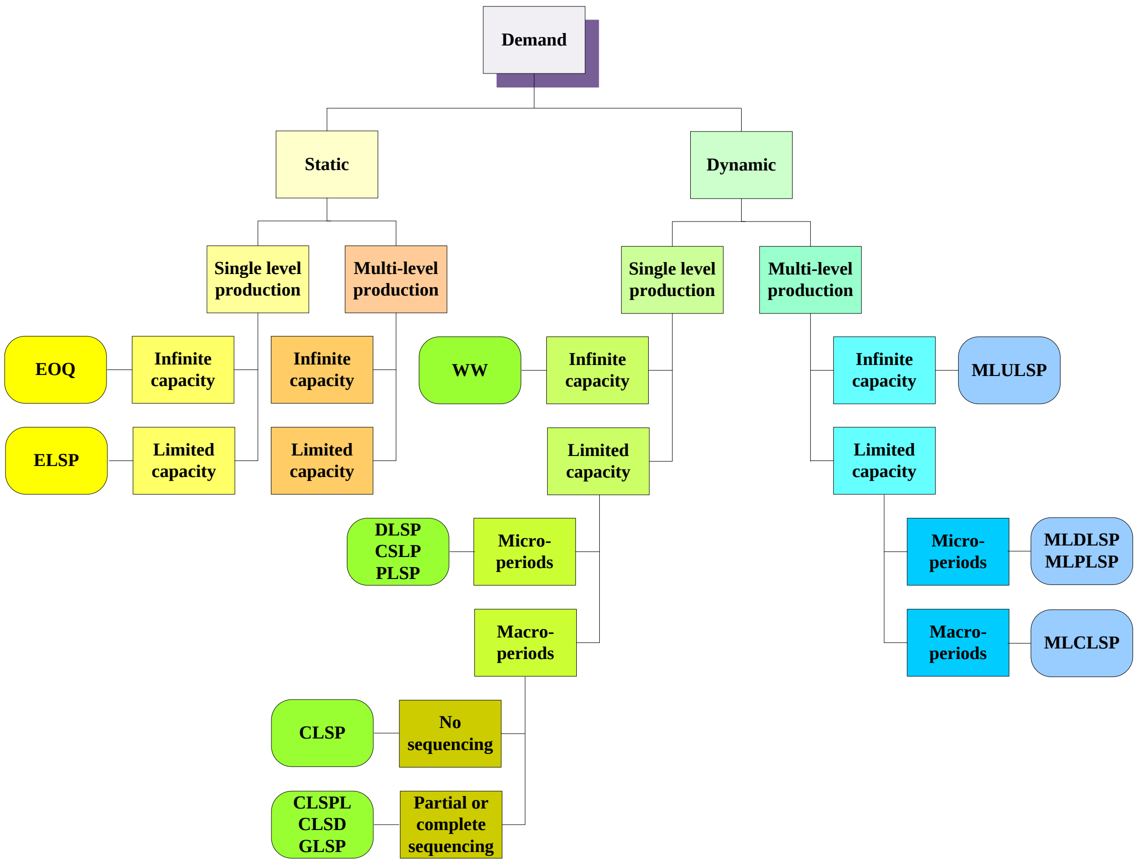


Figure 2.3: Classification of single and multi-level lot-sizing models

2.5 Multi-Facility Problems

Dantzig (1955) introduced the concept of multi-facility systems in which production of items at one facility requires inputs from other facilities. If transportation costs are not taken into account in multi-facility models, then the considered system is a multi-machine or multi-stage single facility system. Due to the complexities of solving multi-facility problems, the interdependencies between the facilities in the system are often not taken into account, and generally multi-facility problems are solved by optimizing the costs of each facility individually (Rizk & Martel, 2001).

Numerous studies investigating the lot-sizing and scheduling problem have concentrated on how to effectively schedule production operations within the confines of a single production facility. However, from the perspective of minimizing the total cost in a supply chain, companies usually acknowledge that the cost of a product is not only determined with the amount of factory resources used to convert the raw material into a finished product, but also with the amount of resources used to deliver the product to the customer. Hence, concentrating only on lot-sizing and scheduling of production operations within plants may not be sufficient to obtain the desired low levels in the production and logistics costs of the supply chain.

Supply chain management (SCM) problems are also connected with the solution of lot-sizing problems in procurement-production-distribution networks. It helps to find the optimal order, production and shipment quantities and minimize the cost including purchasing, production and transportation flows for a set of commodities from a set of production facilities to a set of customers. Goyal and Deshmukh (1992) reviewed models of integrated procurement-production systems, which study combined decisions on the optimal procurement lot size of raw material and the optimal production lot size of finished products.

Since the early 1980s the integration of company-overlapping aspects in terms of logistics has become manifest. In this regards, the term SCM was stated for the first time by Oliver and Webber (1982). A supply chain is defined as a connected series of activities, which is concerned with planning, coordinating and controlling functions including procurement of materials, transformation of materials and intermediate products into intermediate and finished products, in addition to the distribution of finished products to customer (Ganeshan et al., 1999; Goetschalckx, 2011). SCM as defined by Vorst (2000) is the integrated planning, coordination and control of all material and information flows in the supply chain to deliver superior consumer value at lower cost to the supply chain as a whole whilst satisfying requirements of other stakeholders in the chain.

The term supply chain may also imply that only one player is involved at each stage. In reality, a manufacturer may receive materials from several suppliers and then supply several distributors. Thus, most supply chains are in fact considered as networks. It might be more accurate to use the term supply network or supply web in order to describe the structure of most supply chains (Chopra & Meindl, 2001), as shown in Figure 2.4. Each stage in a supply chain is connected through the flow of products, information, and funds.

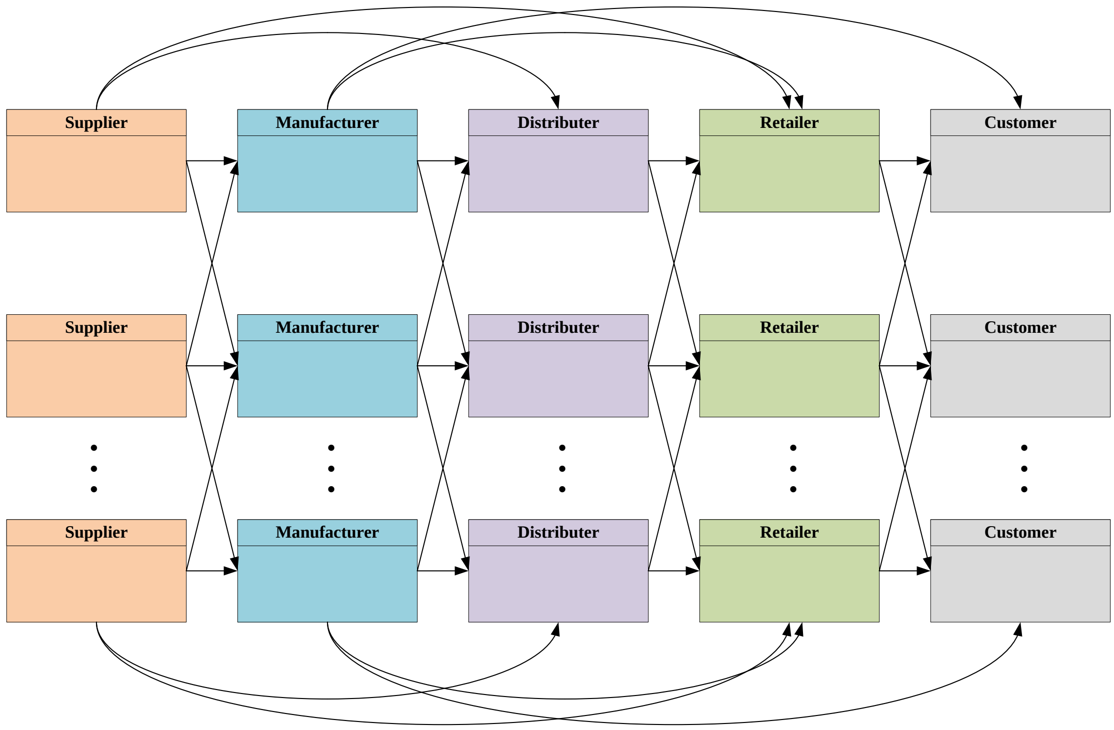


Figure 2.4: Supply chain network (Chopra & Meindl, 2001)

Improving the cooperation among supply chain partners and efficiently integrate the flow of goods, services and information can help the chain lessen extra inventory in the system, reduce pertinent costs from the logistic network, and enhance customer service level dramatically. In today's competitive market environment, SCM has been promoted as a key strategic component of companies that can help them achieve or maintain their competitive edge.

According to Cooper et al. (1997), SCM as a management philosophy takes a system approach to view the supply chain as a single entity. This means that the partnership concept is extended into a multi-firm effort to manage the flow of goods from suppliers to the ultimate customer. Each firm in the supply chain directly or indirectly affects the performance of the other supply chain members, as well as the overall performance of the supply chain.

Dhaenens-Flipo (2000) presented a hierarchical heuristic which decomposes a multi-facility multi-item production and distribution problem into several sub-problems, based on an analogy of the vehicle routing problem, to determine the plant, production line, and production sequence for the items in an integrated way. Dhaenens-Flipo and Finke (2001) modeled the multi-facility, multi-item, multi-period production and distribution problem in the form of a network flow problem and applied it successfully to a real industrial problem.

As stated by Hwang et al. (2013), the multi-stage lot-sizing problem with production capacities deals with a supply chain that consists of a manufacturer with stationary production capacity and intermediaries (distribution centers or wholesalers) and a retailer to face the deterministic demand. An optimal supply chain plan for this problem specifies when and how many units each organization of the supply chain should produce or transport to ultimately fulfill the demand at the retailer with the objective of minimizing the total supply chain costs.

The importance of the coordination between production and distribution operations have been studied in Chandra and Fisher (1994) and Fumero and Vercellis (1999). It has been shown that integrated scheduling of production and distribution operations perform substantially better than unsynchronized scheduling of such operations. Hence, it is important for the companies to recognize that a reduction in total cost of the supply chain and an increase in customer satisfaction can be realized through integrated scheduling of production and distribution operations.

Armentano et al. (2011) studied the problem of integrating production and distribution planning with a capacity constrained plant that produces a number of items distributed by a fleet of homogenous vehicles to customers having known demand. Two TS variants for this problem were proposed, one that involves construction and a short-term memory, and one that incorporates a longer term memory used to integrate a path relinking procedure to the first variant. Yan et al. (2011) integrated a production-distribution model for a deteriorating inventory item in a two-echelon supply chain. A procedure for determining the optimal supply chain decisions was outlined with the objective of minimizing the total system cost.

Amorim et al. (2012) presented a multi-objective integrated production and distribution planning for perishable products having a fixed and a loose shelf life. The results showed that the economic benefits derived from using an integrated approach are much dependent on the freshness level of products delivered. Pal et al. (2012) presented a three-layer multi-item production-inventory model for multiple suppliers and retailers. An integrated profit of the supply chain was optimized by optimal ordering lot sizes of the raw materials.

Ben-Daya et al. (2013) examined the joint economic lot-sizing problem in the context of a three-stage supply chain consisting of a single supplier, single manufacturer and multi-retailers. The objective was to specify the timings and quantities of inbound and outbound logistics for all parties involved such that the chain-wide total ordering, setup, raw material and finished product inventory holding costs are minimized. The derivative-free methods was employed to derive a near closed form solution for the developed model.

Sarker et al. (2013) examined a tree-type three-echelon production-distribution supply chain system with allowable backorder. It was attempted to improve service rate by reducing the backorder at the retailer level. A branching search process was utilized to obtain the solutions. Chen and Sarker (2014) utilized the ACO algorithm for solving an integrated inventory lot-sizing and vehicle-routing model in a multi-supplier single assembler system with just-in-time delivery. The results showed that integrated model can reduce the total cost and highlighted the importance of cooperation between suppliers and manufacturers in just-in-time production practices.

Sana et al. (2014) studied the replenishment size/production lot size problem both for perfect and imperfect quality products in a three-layer supply chain consisting of multiple suppliers, manufacturers and retailers. Further, the expected average profits of suppliers, manufacturers and retailers were formulated by trading off setup costs, purchasing costs, screening costs, production costs, inventory costs and selling prices. In a numerical illustration, the optimal solution of the collaborating system showed a better optimal solution than the existing related models in literature.

Yazdani et al. (2015) improved the mathematical models for the multi-factory parallel machine problems, and compared them with the available models in both size and computational complexities. They solved the models by the ABC algorithm and compared them against the available algorithms on both small and large instances. It was shown that proposed metaheuristic performed much more effectively.

2.5.1 Multi-Plant Systems

To lower the production cost and meet customer demands in time, companies have to examine alternative solutions for their logistics network. One of these solutions can be shifting from one plant manufacturing facility to multi-plant enterprise. Bhatnagar et al. (1993) distinguished supply chain coordination planning into two broad categories: coordination in terms of incorporating decisions of various functions, including production planning, distribution, and marketing, and coordination of associating decisions within the same operation through several echelons of the corporation. The authors refer to the latter level of coordination as “multi-plant coordination”. Each plant here refers to a manufacturing facility that is centered around related production processes.

Shifting from single plant to multi-plant organization can bring several advantages such as being proximate to low cost raw materials, closeness to market, flexibility in producing various products and specialization in activities, and so forth (Maritan et al., 2004). However, decision making in multi-plant systems has to attempt towards integration of several manufacturing plants' activities in such a way that they align their tasks in direction of improving overall performance of the enterprise. Each plant's internal function is as important as it's relation with other plants since each plant is a part of the network.

Sambasivan and Schimidt (2002) provided a heuristic approach based on transfers of production lots between the periods and the plants to solve the single stage MPCLSP. Sambasivan and Yahya (2005) obtained better results for the problem using the LR approach. Nascimento et al. (2010) embedded the setup carry-over to the proposed MPCLSP, and developed a greedy randomized adaptive search procedure as well as a path relinking intensification procedure that outperformed the LR approach proposed by Sambasivan and Yahya (2005).

Lin and Chen (2007) presented a mathematical model for a multi-stage multi-site production planning problem in a thin film transistor-liquid crystal display factory, and combined two different time scales of daily and monthly time buckets. Guimarães et al. (2012) studied a real-case scenario of a beverage industry in order to produce a long-term plan of assigning and scheduling production lots in a multi-plant environment.

Martin et al. (1993), Chandra and Fisher (1994), and Thomas and Griffin (1996) presented substantiations of the possible economic advantages resulting from production-distribution integration. Pirkul and Jayaraman (1998) formulated a mixed integer programming model for a multi-commodity, multi-plant capacitated facility location problem, which was then solved via the LR approach to minimize the total operating costs for the distribution network. Jolayemi and Olorunniwo (2004) proposed a deterministic model for a multi-plant and multi-warehouse problem with extensible capacities while permitting subcontracting in the case of unavailable sufficient resources.

Park (2005) presented a heuristic solution to solve a multi-plant, multi-retailer, multi-item problem over a multi-period horizon. The model addressed the coordination of production and distribution planning with the aim of maximizing the total net profit. Aghezzaf (2007) discussed the problem of production capacity and warehouse management in a supply network in which each plant can be enabled to produce any product type through inter-plant mold transfers.

It is proven that the single plant multi-item capacitated lot-sizing problem is NP-Hard (Florian et al., 1980; Bitran & Yanasse, 1982), so is the respective multi-plant version. Therefore, metaheuristic approaches can be used to efficiently tackle such complex problems and offer good solutions within a reasonable computation time. C. Moon et al. (2002) proposed a GA for finding high quality approximate solutions in an integrated process planning and scheduling model for the multi-plant supply chain. Chan et al. (2008) suggested a cooperative multiple PSO procedure to decrease the overall tardiness in a multi-plant supply chain scenario, and attempted to resolve the production planning and scheduling problem.

Tseng et al. (2010a, b) applied the GA and PSO to solve an integrated assembly sequence planning and plant assignment problem where products are assembled in a multi-plant system with the objective of minimizing the total of assembly and multi-plant costs. Yang et al. (2010) proposed a quasi-transportation problem for multi-plant order allocation to minimize the total cost under the capacity load constraint, and solved it using the GA. Behnamian and Ghomi (2013) developed a heuristic algorithm and the

GA approach for sequencing and scheduling of distributed multi-factory production network problem with parallel machine in order to minimize the maximum completion time of jobs.

2.6 Conclusions

In this chapter, a detailed explanation of some of the issues related to this study, such as single and multi-level lot-sizing problems in single facility systems, economic lot scheduling problems, and multi-facility lot-sizing problems were given. The current literature was reviewed and discussed. Moreover, the characteristic, aspects and classification criteria affecting the modeling and complexity of the lot-sizing problems were extracted from the literature and categorized.

Based on the literature review performed in this study, little attention has been devoted to economic lot scheduling problem considering multiple setups, shelf life, and backordering. Likewise, little attention has been paid to the multi-plant capacitated lot-sizing problem in integrated production-distribution systems. Therefore, this study attempts to address these shortcomings by developing comprehensive mathematical models and solution approaches for such problems arising in reality.

CHAPTER 3: METHODOLOGY

3.1 Introduction

This chapter explains the methodology used in this research. The research frameworks and a brief overview of the applied metaheuristic algorithms are also described in this chapter.

3.2 Research Methodology

At the first step, a literature review on different aspects of lot-sizing problems in single stage single facility and multi-stage multi-facility systems was gathered. Based on the literature, the gaps and problems were determined. Next, the objectives and scope of the project were identified. The first part of research focuses on modeling the multi-item lot-sizing and scheduling problem in a single stage single facility system with a continuous time scale, deterministic static demand and infinite time horizon which is known as ELSP, while considering the effect of shelf life, backordering, and multiple production frequencies for each product. The aim is to determine the optimal lot size, production rate, production frequency, cycle time, as well as a feasible manufacturing schedule for the family of items, and to minimize the total pertinent cost.

The second research direction emphases on lot-sizing problem in an integrated production-distribution system. It focuses on the formulation and modeling the multi-item capacitated lot-sizing problem in a multi-stage multi-facility system with discrete time scale, deterministic dynamic demand and finite time horizon which is known as MPCLSP. The objective is to find the optimal order, production, and shipment quantities that minimize the cost of the whole supply chain.

Then, based on parameters and variables affecting the defined problems, the objectives and scope of the study, research frameworks were created. Next, mathematical models for the considered systems were developed. Numerical examples were selected to test the effectiveness of the proposed models. The proposed mathematical models were coded in MATLAB programming software. MATLAB is a high performance language for technical computing integrates computation, visualization, and programming in an easy-to-use environment where problems and solutions are expressed in familiar mathematical notation. MATLAB interface comes with abundant help for the user, and simple mechanisms for passing and storing commonly used options in configuration/preset files. It is also a comprehensive tool designed to develop and solve optimization models faster, easier and more efficient.

The coded objective functions and constraints were then applied in metaheuristic optimizers such as GA, PSO, ABC, SA, and ICA approaches. In the next step, data analysis and comparative study were conducted using the obtained statistical results such as mean, worst, best and standard deviation. Finally, conclusions present the objectives, contributions, and achievements of this research.

A flow chart that summarizes the overall process of this project is shown in Figure 3.1.

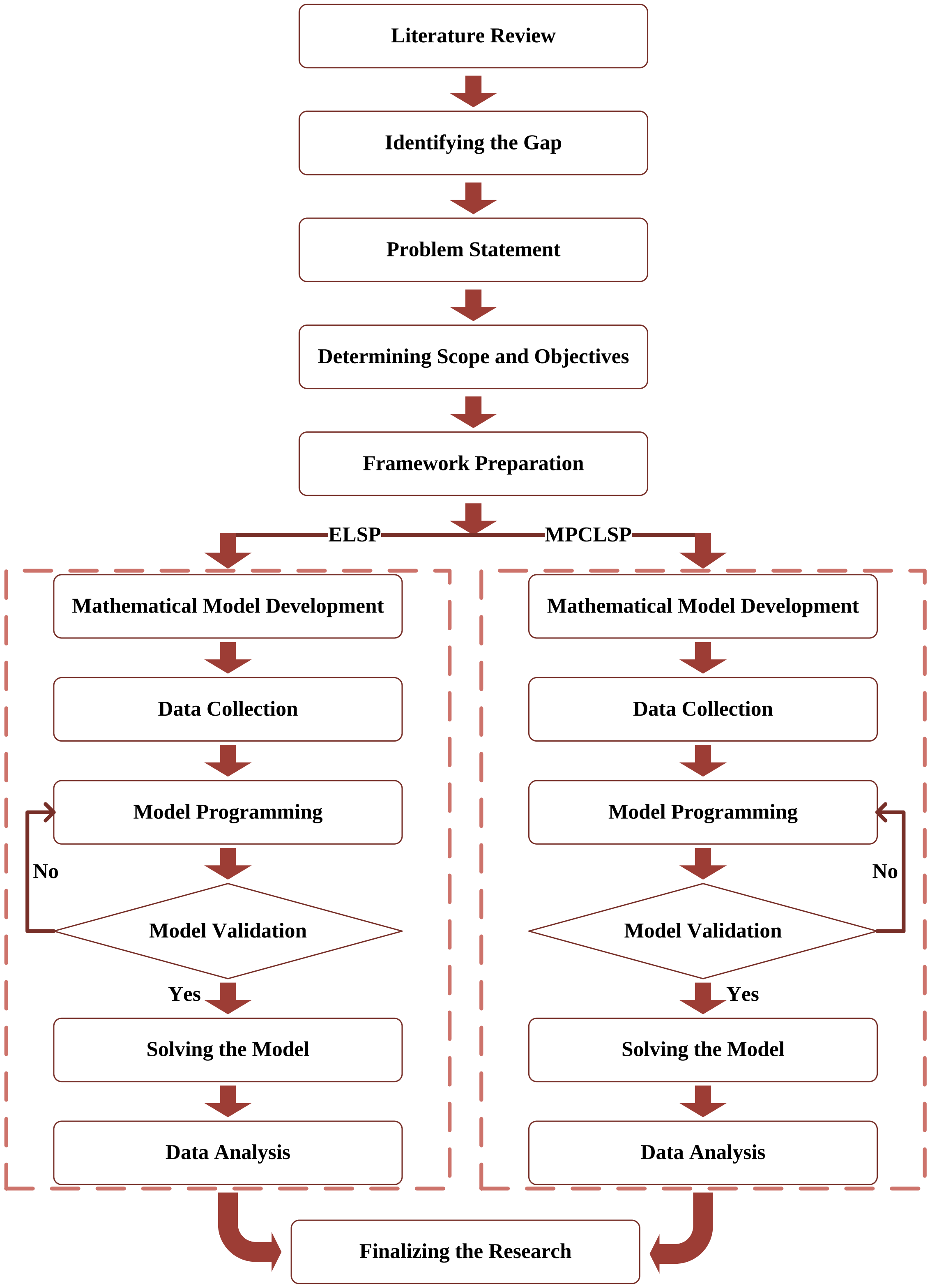


Figure 3.1: Research methodology process

3.3 Research Framework

Based on the characteristics of lot-sizing problem given in Section 2.3, different aspects of lot-sizing in both single facility and multi-facility systems are taken into consideration. The research framework used in this project is shown in Figure 3.2.

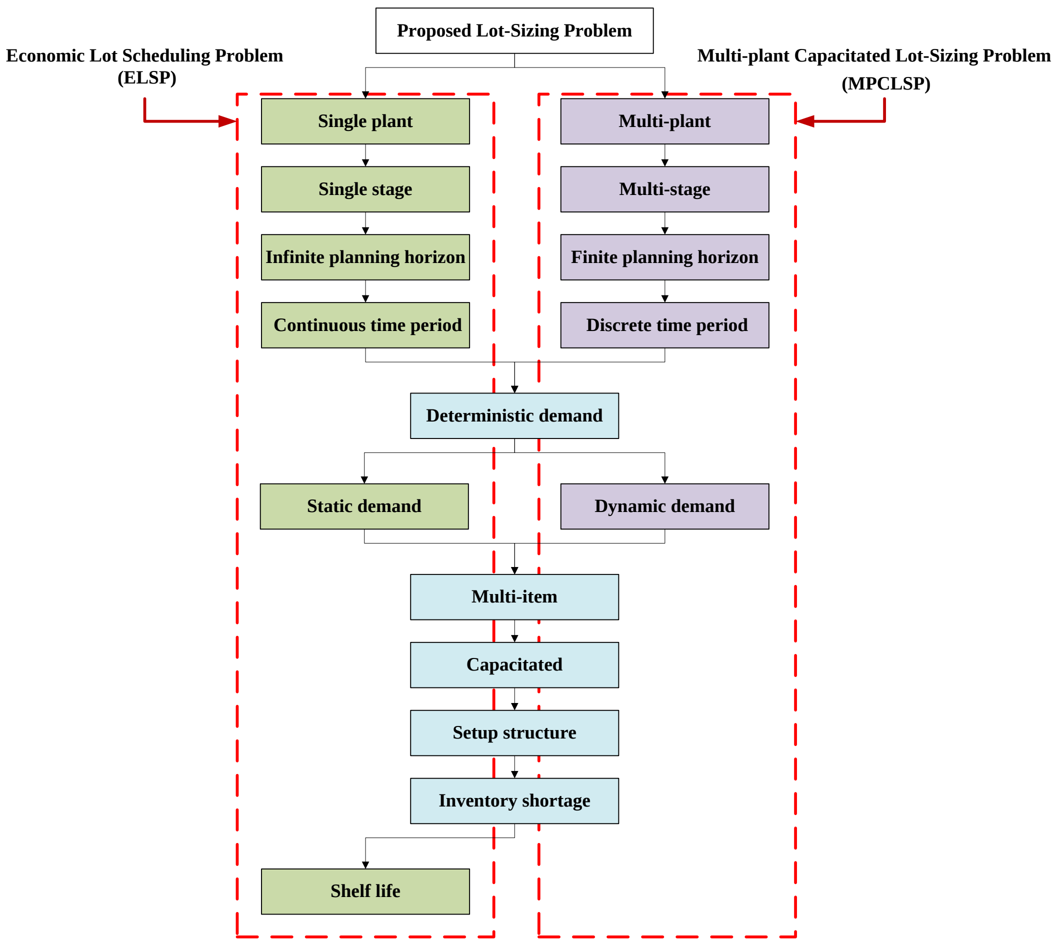


Figure 3.2: Research framework

3.4 The Proposed ELSP

The ELSP is concerned with lot-sizing and scheduling several items in a single production facility. The facility is such that only one item can be produced at a time and the demand rate for each item is constant over the planning horizon. Because scarce resources are usually shared in common by several items, the ELSP is a single stage multi-item problem. The ELSP is a continuous time model and the planning horizon is infinite. To solve the problem, common cycle strategy is used for all products. The common cycle approach imposes a condition that products' cycle times should be of equal length, with the cycle time of a product defined in terms of the duration between starts of two consecutive runs of the product.

Generally, the inventory systems assume implicitly unlimited shelf lives for the stored items. However, some items can be stored in the inventory only for a certain shelf life period which may be shorter than the production cycle time. There has been little research on conventional ELSP models including the shelf time factor, although many items, especially in the food industries, usually deteriorate or can be held only for a limited lifetime. Therefore, the motivation was to devise an effective solution approach to solve the ELSP with shelf life. Moreover, the basic ELSP does not allow backordering. Hence, the impact of incorporating planned backorders into the system is also investigated. Furthermore, the capacity constraint is taken into account to check whether or not the available machining time is sufficient for setups and production.

While the cost-minimizing cycle time causes the spoilage of an item on account of shelf life limitations, the cycle time period must be decreased to less than or equal to the shelf life to ensure a feasible schedule. This objective may be achieved by reducing either the total cycle time or the production rate, or by using a mix of reduced cycle time and production rate. Consequently, the total annual costs for these three options are appraised and compared under specific constraints. The objective is to select the option that yields the minimum yearly cost, in addition to obtaining an optimum cycle time that satisfies shelf life constraint and accommodates a feasible production schedule.

The detailed explanations of pertinent mathematical models and three options are given in Chapter 4. The framework of the proposed model is shown in Figure 3.3.

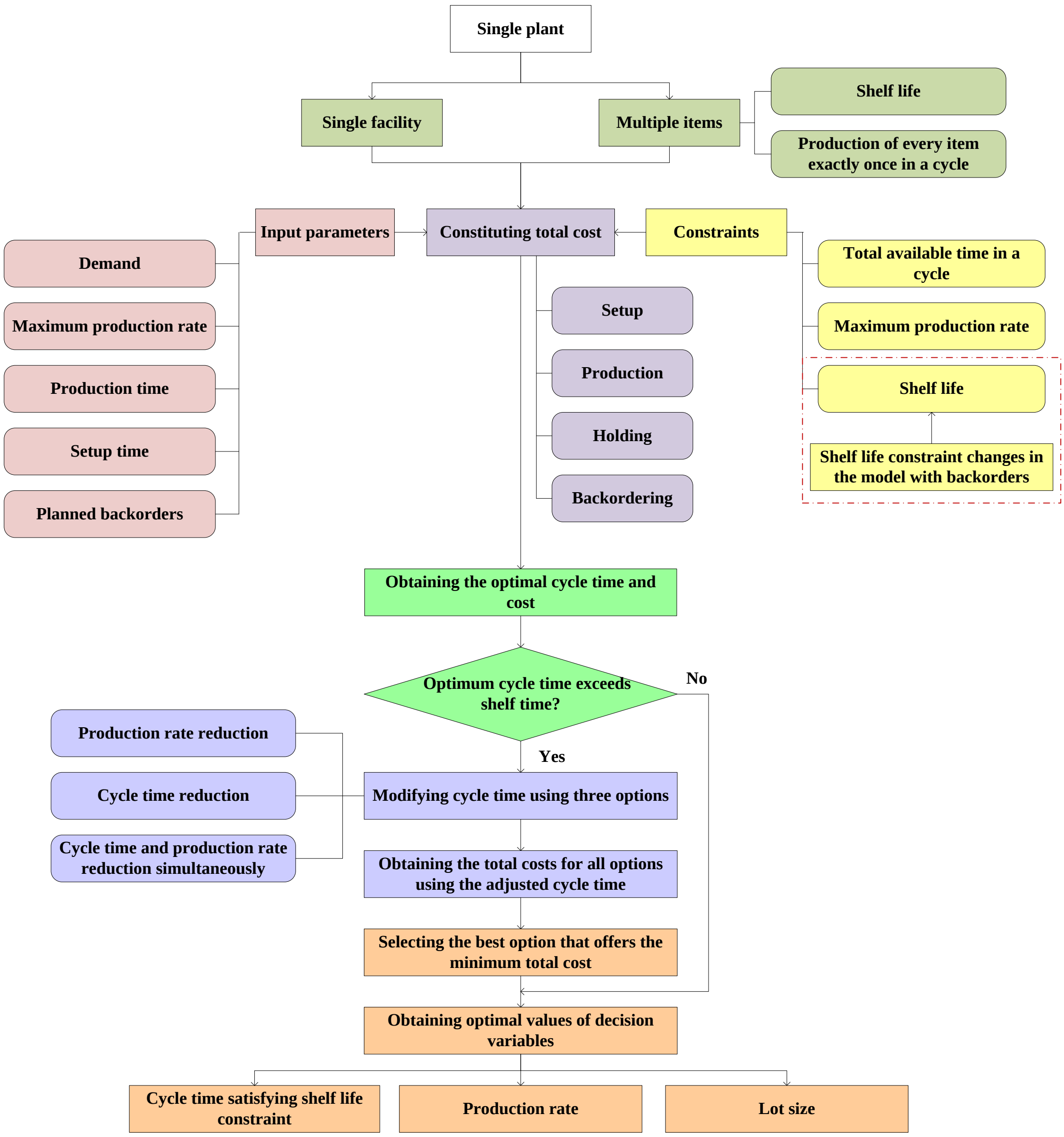


Figure 3.3: Modified ELSP considering shelf life and backordering

Most studies investigating the various aspects of the ELSP assumed that each item is manufactured exactly one time in the rotational production cycle. However, manufacturing of every item more than once per cycle might be more economical. Though this strategy may produce a lower cost, it may cause an infeasible schedule because of overlapping production time of several products. The problem of schedule infeasibility, when the production of each item more than one time in every cycle is permissible, can be tackled by rescheduling the manufacturing start times of some items. Therefore, modification in the pertinent cost function and constraints are required in order to obtain a feasible schedule and minimize the total cost.

The ELSP is categorized as NP-hard problem, which leads to difficulty of checking every feasible schedule in a reasonable amount of computation time. To deal with this intricacy and obtaining optimal or near-optimal results, metaheuristic methods such as the GA, PSO, ABC, and SA are utilized. The descriptions of the proposed problem and algorithms are explained in detail in Chapter 5. The framework of the proposed model is shown in Figure 3.4.

Figure 3.4: The proposed ELSP allowing the production of items more than once in a cycle

3.5 The Proposed MPCLSP

The WW model is one of the first models for a dynamic demand problem where a finite planning horizon is subdivided into several discrete periods and demand is given per period and might change over time. The WW problem is considered as single level and single item without capacity constraints. The CLSP can be considered as the extension of the WW problem to capacity constraints and multi-item problem where its objective is to determine the optimal lot size which minimizes production, setup, and inventory costs.

Multi-facility systems are complex networks in the supply chain, where each facility in the network signifies a multi-stage system. Lot-sizing problems, in this situation, are complex due to the interdependency existing between different facilities. Therefore, further research was carried out to examine the multi-item multi-period CLSP in a multi-plant system. Moreover, the combination of several functions such as purchasing, production, storage, backordering, and transportation is considered.

The nature of the multi-plant problem is closely related to the MLCLSP. In the MLCLSP, the lot sizes need to be obtained for multi-level production inventory systems with capacity constraints on the production facilities. In the multi-plant problem, there is no common resource between sites, and therefore, the production is controlled independently in each site. The MPCLSP with multiple products and time periods is comprised of multiple production centers that manufacture all the same products and allow transfers among the plants.

Since the MPCLSP is considered as NP-hard problem, using the exact methods may encounter difficulties for solving medium to large size instances. Furthermore, both deterministic and heuristic optimization methods may not be able to solve such problem efficiently. Therefore, metaheuristic algorithms as solution methods were adopted to find cost effective and quality solutions for the proposed problem. In this study, metaheuristic approaches namely the GA, PSO, ABC and ICA are applied to solve the proposed MPCLSP model.

The description of the proposed MPCLSP and applied algorithms are given in detail in Chapter 6. The framework of the proposed model is shown in Figure 3.5.

Figure 3.5: The proposed multi-period multi-item MPCLSP

3.6 Solution Methods

Metaheuristic algorithms have been extensively used in solving complex optimization problems in various fields of scientific and engineering disciplines, since classical and traditional techniques may not efficiently find the global optimum solutions.

In the literature, diverse classes of metaheuristic algorithms inspired by various models of biological species or physical phenomena have been developed for solving complex optimization problems. A common feature behind such metaheuristic algorithms is that they go through three general phases: initiate, neighborhood search, and terminate. That is, these algorithms start from one or a population of initial incumbent solution(s) and then iteratively search for better solutions from the neighborhood of the incumbent solution(s) by following specific mechanisms until the terminating condition is met. The specific mechanisms of these algorithms are in fact types of NS techniques in analogy with natural phenomena such as biology evolution, biological behaviors, metal annealing processes, etc.

Since the capacitated lot-sizing and scheduling problem is considered as NP-hard in nature, a number of widely used metaheuristic methods are selected to solve the proposed mathematical problems in this research. The overall procedures of the applied algorithms are explained in the following subsections.

3.6.1 Genetic Algorithm

The GA is considered as an evolutionary algorithm and a population-based method that attempts to finds the optimal or near-optimal solutions through conducting a random search. Fundamental of the GA was primarily instated by Holland (1975). The GA method has been effectively used for solving continuous and discrete combinatorial problems (Mitchell, 1996). Simplicity and capability of finding quick reasonable solutions for intricate searching and optimization problems have brought about a growing interest over the GA. This algorithm is based upon "survival of the fittest" principles by Darwin Theory of Evolution and simulates the process of natural evolution.

A GA contains a set of individuals that constitute the population. Every individual in the population is represented by a particular chromosome which refers to a plausible solution to the existing problem. Throughout consecutive repetitions, called generations, the chromosomes evolve through reproduction process. During each generation, the fitness value of each chromosome is evaluated. Upon the selection of some chromosomes from the existing generation as parents, offspring will be produced by either crossover or mutation operators. The algorithm will be stopped when a termination condition is reached. The overall flowchart of the GA approach is shown in Figure 3.6.

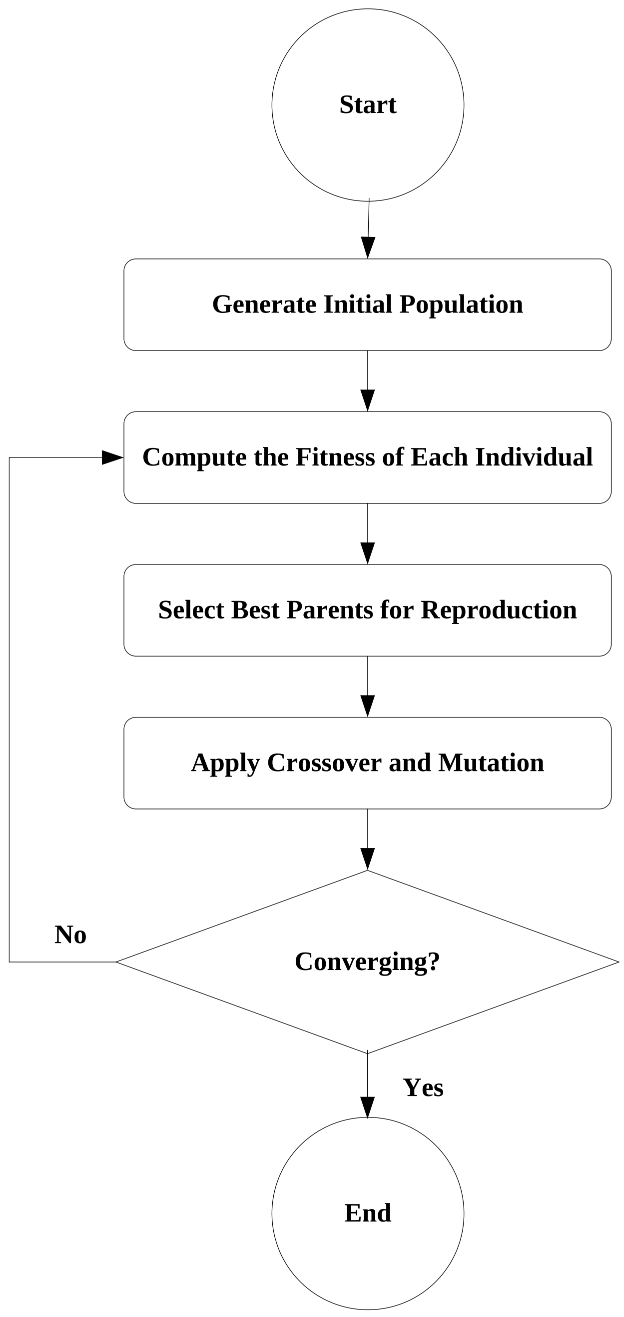


Figure 3.6: Overall procedure of GA approach (Köksoy & Yalcinoz, 2008)

3.6.2 Particle Swarm Optimization

The PSO algorithm is a population-based evolutionary method which was initially introduced by Kennedy and Eberhart (1995). The idea of the procedure was inspired by the social behavior of fish schooling or bird flocking choreography. Similar to the GA, the PSO begins its search process using a population of individuals positioned on the search space, and explores for an optimum solution by updating generations.

Unlike the GA, PSO has no genetic operators such as crossover and mutation to operate the individuals of the population, and the members of the whole population are kept during the search process. Instead, it relies on the social behavior of the individuals to create new solutions for future generation. The PSO exchanges the information among individuals (particles) and the population (swarm). Every particle continuously updates its flying path based on its own best previous experience in which the best previous position is acquired by all members of particle’s neighborhood. Moreover, in the PSO, all particles assume the whole swarm as their neighborhood. Therefore, there occurs social sharing of information between particles of a population, and particles benefit from the neighboring experience or the experience of the whole swarm in the searching procedure (Chen & Lin, 2009).

In a PSO algorithm, the initial population is initiated randomly with particles and evaluated to compute fitness together with finding the particle best (best value of each individual so far) and global best (best particle in the whole swarm). Initially, each individual with its dimensions and fitness value is assigned to its particle best. On the other hand, the best individual among particle best population with its dimension and fitness value is assigned to the global best. Then, a loop starts to converge to an optimum solution. In the loop, particle and global bests are determined to update the velocity first. Then the current position of each particle is updated with the current velocity. Evaluation is again performed to compute the fitness of the particles in the swarm. This loop is terminated with a stopping criterion predetermined in advance. The overall flowchart of the PSO algorithm is shown in Figure 3.7.

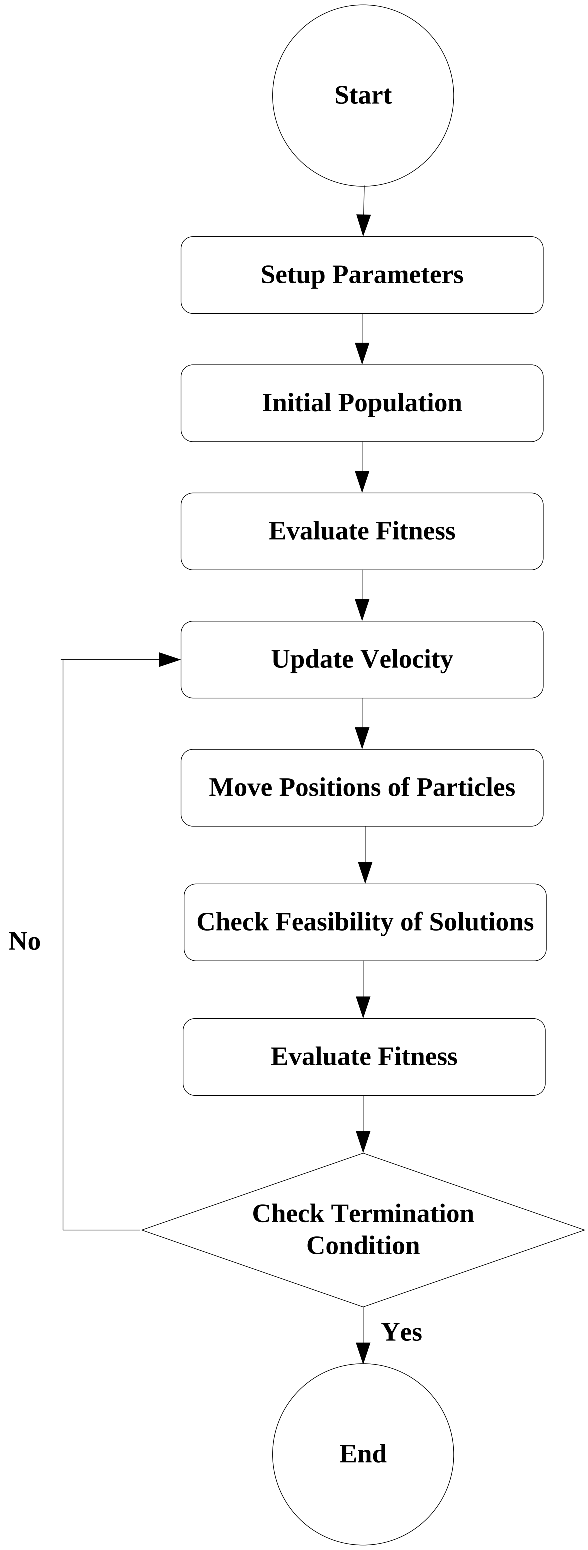


Figure 3.7: Overall procedure of PSO algorithm (Tseng et al., 2010b)

3.6.3 Artificial Bee Colony

The ABC algorithm was proposed by Karaboga (2005) which mimics the intelligent behavior of the honey bee swarm in foraging foods. It is a population-based search algorithm suitable for multi-variable and continuous multi-modal optimization problems. The ABC algorithm includes four main components, namely food sources, employed bees, onlooker bees, and scout bees.

A food source position represents a possible solution to the problem to be optimized. The amount of nectar of a food source corresponds to the quality of the solution represented by that food source (Karaboga & Basturk, 2008). A bee which has found a food source to exploit is called an employed bee. Onlookers are those waiting in the hive to receive the information about the food sources from the employed bees. An onlooker bee appraises the food information obtained from employed bees, and chooses a food source based on the probability related to their nectars' amounts. For this purpose the greedy selection method is applied, so that if the amount of nectar of a new source is higher than the previous one in their memory, onlooker bees update the new position and forget the previous one.

Scouts are the bees which are randomly searching for new food sources around the hive. The employed bee whose its food source has been exhausted becomes a scout. In the basic ABC process, in every cycle at most one scout goes outside for exploring a new food source. After the new position is specified, a new algorithm cycle begins. After each cycle, the finest solution will be memorized. The same process iterates until the termination condition is reached. The overall flowchart of the ABC algorithm is shown in Figure 3.8.

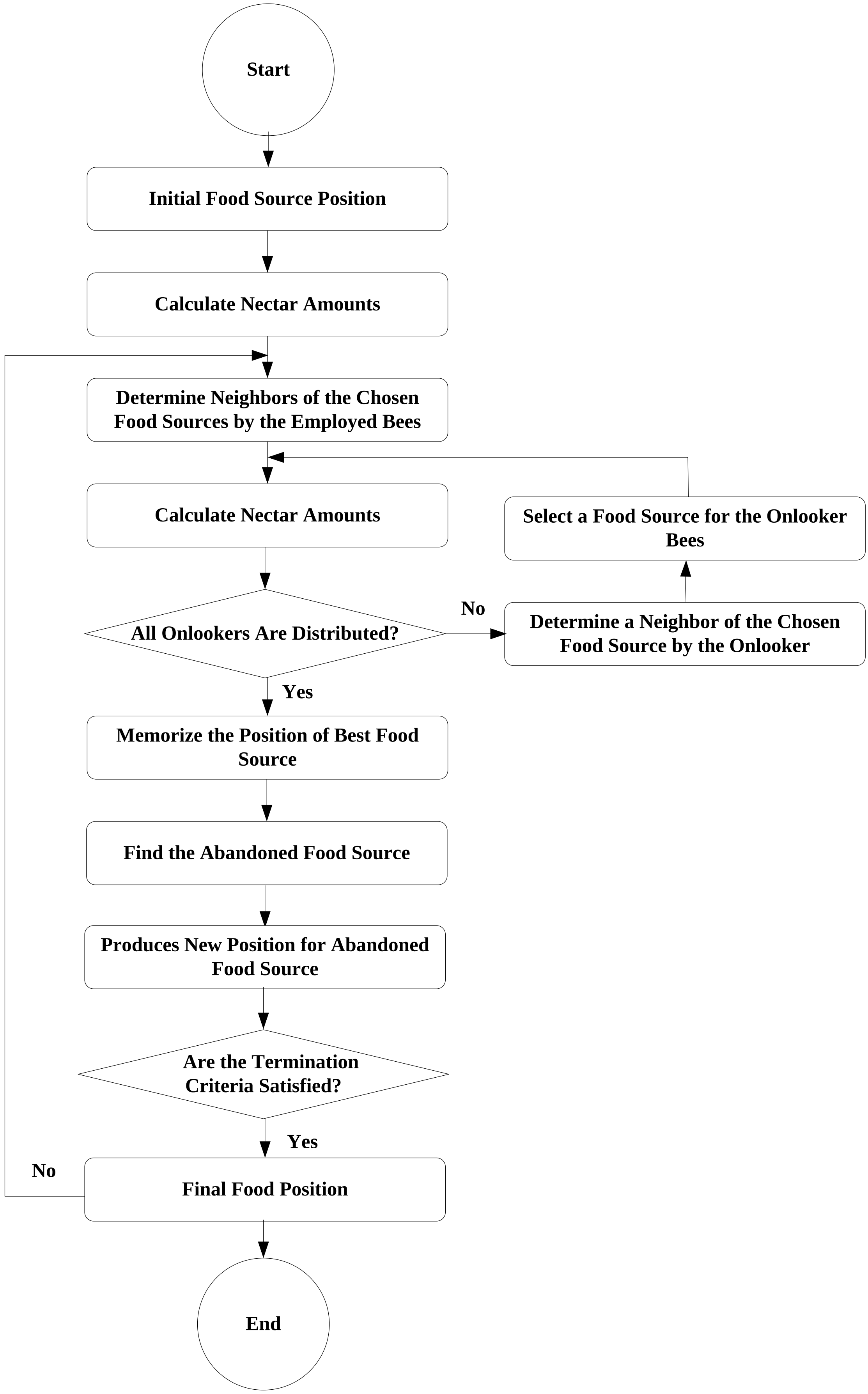


Figure 3.8: Overall procedure of the ABC algorithm (Pan et al., 2011)

3.6.4 Simulated Annealing

The SA algorithm is an effective stochastic search method for solving combinatorial and global optimization problems proposed by Kirkpatrick et al. (1983). The basic idea is inspired from the physical process of cooling molten material to solid form. Based on this procedure, the SA explores different areas of the solution space of a problem by annealing from a high to a low temperature. During the search process both good solutions as well as low quality solutions are accepted with a nonzero probability related to the temperature in the cooling schedule at that time. This feature can prevent getting trapped in local minima. In the beginning, this probability is large, and it will be reduced during the execution with a positive parameter such as temperature (Yaghini & Khandaghabadi, 2013)

The SA approach consists of initial temperature, number of iterations at each temperature, temperature reduction function, and final temperature. SA procedure starts at an initial temperature specified by user. It must be adequately high to allow the process an escape mechanism at early stages of the algorithm procedure. Hence, the SA algorithm begins with an initial high temperature where most of the moves are accepted. Each iteration consists of generating a random neighbor of the incumbent and subjecting it to an acceptance criterion. Cooling schedule determines the functional form of the change in temperature required in the SA. The overall flowchart of the SA algorithm is shown in Figure 3.9.

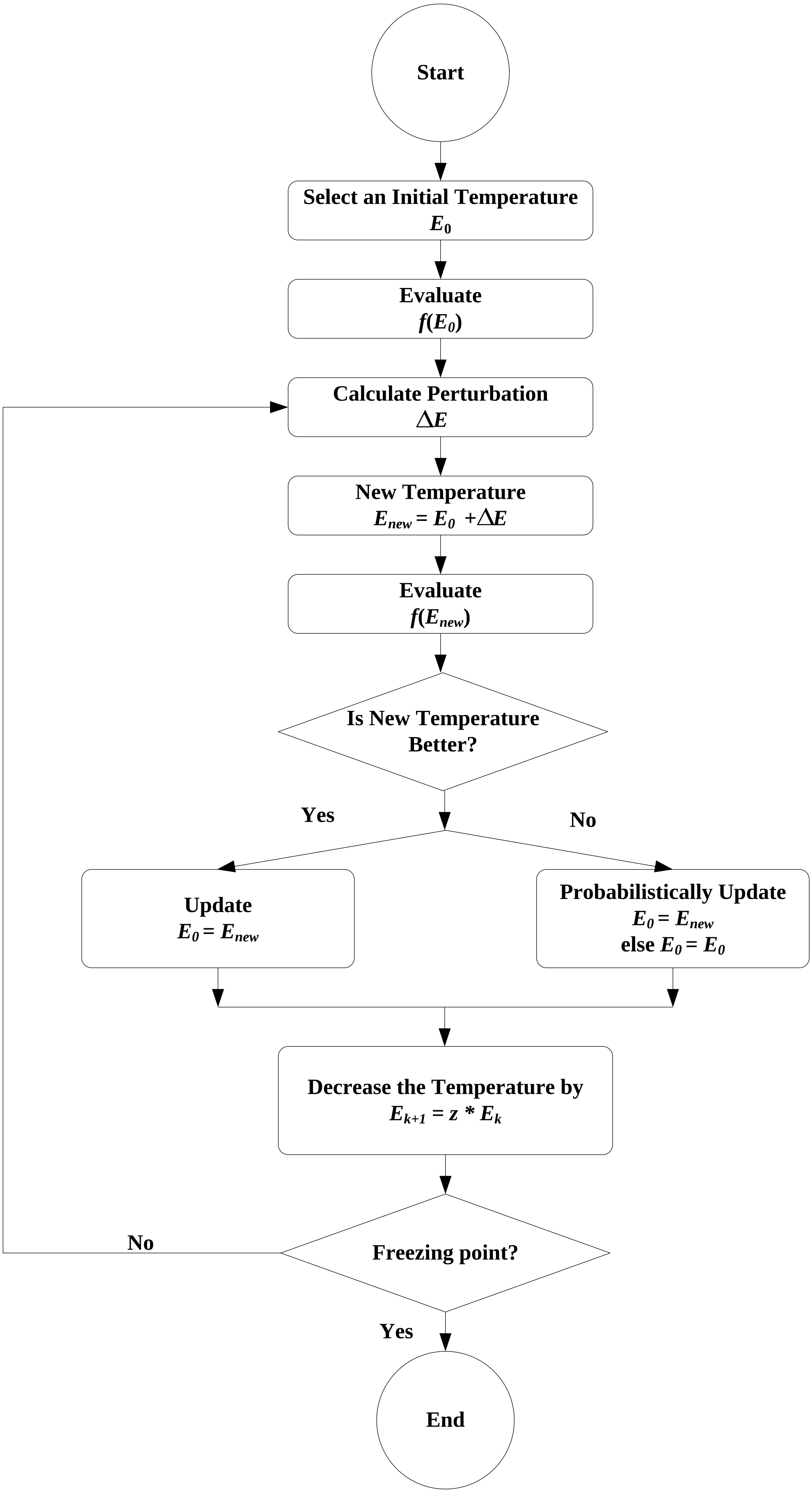


Figure 3.9: Overall procedure of the SA algorithm (Zhong & Pan, 2007)

3.6.5 Imperialist Competitive Algorithm

The ICA approach is a type of population-based algorithms that is inspired by imperialist competition (Atashpaz-Gargari & Lucas, 2007). The ICA defines the term ‘country’ (Parsapoor & Bilstrup, 2013) to encode the optimization problems. An initial population of the ICA is a set of countries, where each country is a vector of the optimization parameters. The population is classified into two groups: the colonies and the imperialists. The countries that have the higher power are considered as the imperialists and start to take possession of the countries with the lower power, which are stated as colonies. In this way, each imperialist creates its empire. The steps of creating empires are explained as follow:

i. First, the cost of each country (it is equivalent to the fitness function in the GA) is calculated. The countries with minimum values of cost (in minimization problems) are chosen as the imperialists.
ii. The imperialists take procession of the colonies based on the normalized power.
iii. The normalized cost for each imperialist is calculated.
iv. After forming the empires, the movement of colonies toward the imperialists is started (assimilation operator). If a colony reaches to a higher power than its imperialist, the position of the colony and its imperialist must be exchanged (evolution operator).
v. Finally, the imperialists start a competition to take the possession of the weakest colonies of the weakest empires. During the competition, the weakest colony from the weakest empire is picked and joined to the most powerful imperialist. The weakest empires, whose colonies are joined to other empires, will be eliminated. The algorithm is converged to a global optimum when there is one empire.

The evolutionary operators of the ICA are summarized as follow:

i. Assimilation operator: This operator updates the cost function of the colonies by moving them to their corresponding imperialists.
ii. Revolution operator: This operator updates the cost function of colonies by changing the elements of colonies. The goal of the revolution operator is to change some parameters of the individual in order to prevent the algorithm from falling into local suboptimal solutions.
iii. Exchange operator: It updates the position state of colonies and imperialists.
iv. Competition operator: It updates the position of the colonies by picking it from one imperialist and joining it to another.

The overall flowchart of the ICA approach is shown in Figure 3.10.

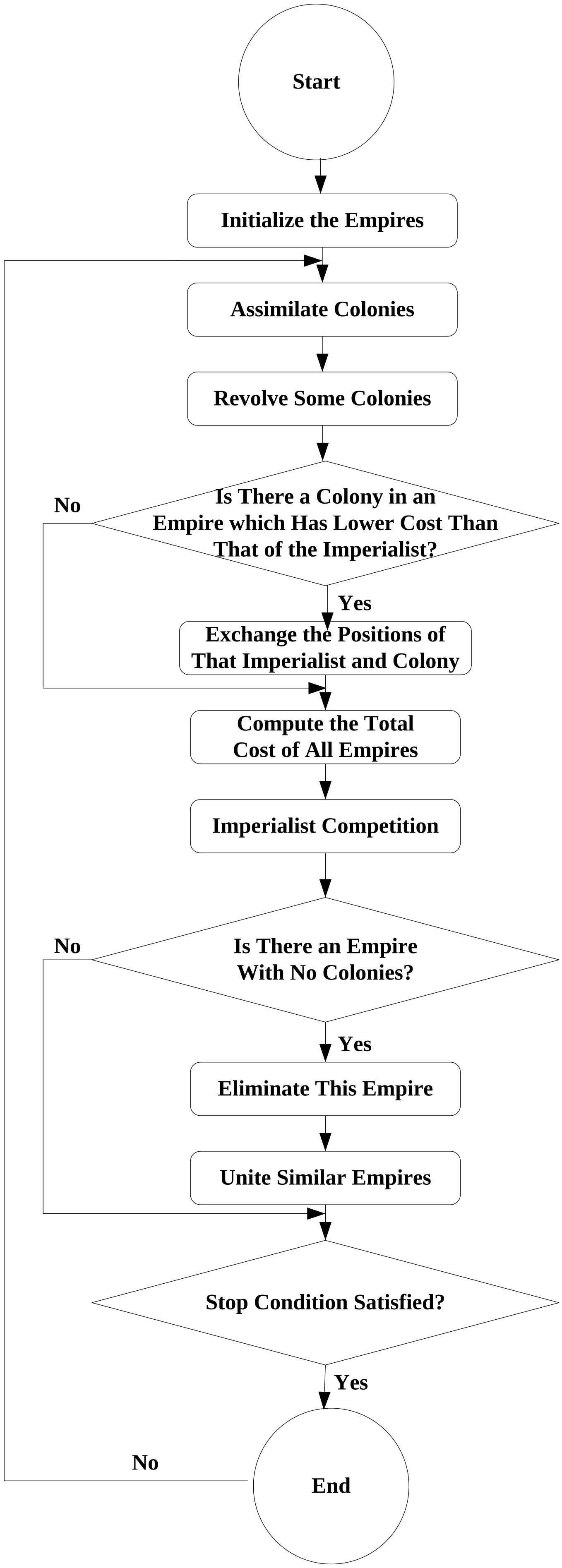


Figure 3.10: Overall procedure of the ICA approach (Nazari-Shirkouhi et al., 2010)

3.7 Conclusions

In this chapter, the methodology used in the study was explained and illustrated by a flowchart. The proposed research frameworks were also demonstrated. The proposed frameworks were divided into three main phases, namely the ELSP considering shelf life and backordering, the ELSP allowing the production of items more than once in a cycle, and MPCLSP in an integrated production-distribution system.

A brief overview of the mechanisms of a number of metaheuristic approaches, namely GA, PSO, ABC, SA, and ICA was also presented, which are used to deal with the intricacy of the proposed models and obtain near-optimal to optimal solutions in a reasonable computation time.

CHAPTER 4: OPTIMAL CYCLE TIME FOR PRODUCTION-INVENTORY SYSTEMS CONSIDERING SHELF LIFE AND BACKORDERING

4.1 Introduction

This chapter investigates a manufacturing system considering the production of a family of items in a single facility environment. Total backordering is allowed for any of the products, and each item has a specified shelf life. Nevertheless, in existing literature, when backordering is included, the shelf life constraint variation is not taken into account. While the cost-minimizing cycle time causes the spoilage of an item due to shelf life limitation, the cycle time period must be decreased to less than or equal to the shelf life to ensure a feasible schedule. Therefore, three options namely “cycle time reduction,” “production rate reduction” and “cycle time and production rate reduction simultaneously” are examined in order to obtain an optimum cycle time that satisfies the shelf life constraint. The cost functions for three options are also modified because of the shelf life constraint adjustment after considering backordering.

The rest of this chapter is organized as follows: Section 4.2 presents a detailed discussion on the cost functions and problem constraints by examining the three mentioned options. Section 4.3 demonstrates numerical examples and reports the obtained results. Finally, conclusions are provided in Section 4.4.

4.2 Problem Description and Mathematical Formulations

In this study, a single machine is considered that produces N types of items in manufacturing cycle time of T. Total backordering is permitted for each item while each product has a certain shelf life. The total cost, including setup, holding, production, and shortage cost are expressed through mathematical formulation for the cycle time T. Using the total cost function under the given constraints, the optimal cycle time and cost are obtained while disallowing the shelf life constraint.

For the case where the optimum cycle time T causes spoilage of an item, three options are investigated: 1) production rate reduction, 2) cycle time reduction, and 3) cycle time and production rate reduction simultaneously. Next, the adjusted cycle times for the three options are estimated by considering the shelf life constraint. Using the adjusted cycle times, the total costs for all options are found and compared in order to select the best option that offers the minimum total cost.

The mathematical model studied throughout this chapter is based on the following assumptions and notations:

(a) ***Assumptions***

i. There is a constant demand rate for each product.

ii. The production rate for each product is finite.

iii. The facility can produce only one item at a time.

iv. All items are produced on each manufacturing cycle.

v. Stock is used on a first-in-first-out basis.

(b) ***Notations***

For the entire family:

N Total number of items

T Production cycle time (years)

T_O Optimum production cycle time (years)

O Production cost for operating the machine (including setup times) (dollars/year)

C Total cost (dollars/year)

For item i ($i = 1, 2, \ldots, N$):

d_i Demand for item i (units/year)

p_i Production rate for item i (units/year)

r_i Ratio of demand to production rate for item *i*

A_i Setup time (loading and unloading) for item *i* (years)

S_i Machine setup cost for item *i* (dollars/unit/year)

b_i Backorder amount for item *i* (units/year)

B_i Shortage cost for item *i* (dollars/unit/year)

H_i Inventory holding cost for item *i* (dollars/unit/year)

Q_i Production quantity for item *i* (units)

L_i Shelf life of item *i* (years)

4.2.1 General Cost Function

The annual setup cost for the products and machines is given by:

$$\frac{1}{T}\sum_{i=1}^{N}(S_i + OA_i) \tag{4.1}$$

The annual production cost is given by:

$$\frac{1}{T}\sum_{i=1}^{N} O(\frac{d_i}{p_i})T \tag{4.2}$$

The annual holding cost considering backordering is given by:

$$\sum_{i=1}^{N}\frac{H_i\left(Q_i(1-\frac{d_i}{p_i})-b_i\right)^2}{2Q_i(1-\frac{d_i}{p_i})} \tag{4.3}$$

The annual shortage cost for a group of *N* items is given by:

$$\sum_{i=1}^{N}\frac{B_i b_i^2}{2Q_i(1-\frac{d_i}{p_i})} \tag{4.4}$$

Therefore, the total annual cost *C*(*T*) is obtained by:

$$C(T)=\frac{1}{T}\sum_{i=1}^{N}(S_i+OA_i)+\frac{1}{T}\sum_{i=1}^{N}O(\frac{d_i}{p_i})T+\sum_{i=1}^{N}\frac{H_i\left(Q_i(1-\frac{d_i}{p_i})-b_i\right)^2}{2Q_i(1-\frac{d_i}{p_i})}$$
$$+\sum_{i=1}^{N}\frac{B_i b_i^2}{2Q_i(1-\frac{d_i}{p_i})} \tag{4.5}$$

By substituting $Q_i = Td_i$ and $d_i / p_i = r_i$ in Eq. (4.5) followed by a simplification, the total yearly cost without the shelf life consideration can be obtained by Eq. (4.6):

$$C(T)=\frac{1}{T}\sum_{i=1}^{N}(S_i+OA_i)+O\sum_{i=1}^{N}r_i+\frac{T}{2}\sum_{i=1}^{N}H_i d_i(1-r_i)-\sum_{i=1}^{N}H_i b_i$$
$$+\frac{1}{2T}\sum_{i=1}^{N}\frac{(H_i+B_i)b_i^2}{d_i(1-r_i)} \tag{4.6}$$

4.2.2 Ignoring the Shelf Life Constraint

For a possible solution, the total time for the setup and production to generate *N* items per cycle cannot exceed the cycle time *T* (Silver, 1989), that is:

$$\sum_{i=1}^{N}(A_i+T\frac{d_i}{p_i})\le T \tag{4.7}$$

Equation (4.7) can be rearranged as:

$$\frac{\sum_{i=1}^{N}A_i}{1-\sum_{i=1}^{N}r_i}\le T \tag{4.8}$$

It must be noted that $(1-\sum_{i=1}^{N}r_i)$ indicates the long-run proportion time that is available for setups. For an infinite horizon problem $(1-\sum_{i=1}^{N}r_i)>0$ is necessary in order to have a feasible solution (I. Moon et al., 2002a). Hence, it is required that:

$$\sum_{i=1}^{N} r_i < 1 \tag{4.9}$$

By neglecting the shelf life constraint, solution $dC(T)/dT = 0$ in Eq. (4.6) yields the optimum cycle time T_O since $C(T)$ is a convex function of T. Therefore, T_O is obtained as follows:

$$T_O = \left(\frac{2\sum_{i=1}^{N}(S_i + OA_i) + \sum_{i=1}^{N} \frac{(H_i + B_i)b_i^2}{d_i(1-r_i)}}{\sum_{i=1}^{N} H_i d_i (1-r_i)} \right)^{1/2} \tag{4.10}$$

The obtained T_O in Eq. (4.10) must be greater than the total time available for setup and production in a cycle as shown in Eq. (4.8). Otherwise, T_O should be considered equivalent to $\sum_{i=1}^{N} A_i \Big/ (1 - \sum_{i=1}^{N} r_i)$.

By substituting T_O from Eq. (4.10) into Eq. (4.6), the optimum yearly cost $C(T_O)$ for processing the inventory system can be obtained as shown in Eq. (4.11).

$$\begin{aligned} C(T_O) = &\left[\left(\sum_{i=1}^{N} H_i d_i (1-r_i) \right) \left(2\sum_{i=1}^{N}(S_i + OA_i) + \sum_{i=1}^{N} \frac{(H_i + B_i)b_i^2}{d_i(1-r_i)} \right) \right]^{1/2} \\ &+ O\sum_{i=1}^{N} r_i - \sum_{i=1}^{N} H_i b_i \end{aligned} \tag{4.11}$$

4.2.3 Incorporating the Shelf Life Constraint

Each product i hypothetically has L_i years of shelf life. When the L_j (considered as the shelf life of item j) exceeds the cycle time T, the existing inventory model does not require modifications. However, when the optimal cycle time exceeds the time restriction of shelf life for that item, the inventory model should be reformulated to avoid product spoilage. It is assumed that the inventory is used on a first-in-first-out basis. Accordingly, item j, which is kept for the maximum length in stock, is produced at the later stage of the manufacturing period. Thus, the maximum time for storing product j is equivalent to $T(1-d_j/p_j)$. Hence, the constraint for shelf life can be written as $T(1-d_j/p_j) \le L_j$ (Silver, 1989). That is to say:

$$T \le \frac{L_j}{(1-d_j/p_j)} \tag{4.12}$$

However, the shelf life constraint alters in the model allowing for backorders (Viswanathan & Goyal, 2000).

4.2.4 Shelf Life Constraint Adjustment with Planned Backorders

The inventory-time relationship over the manufacture cycle for item j is shown in Figure 4.1.

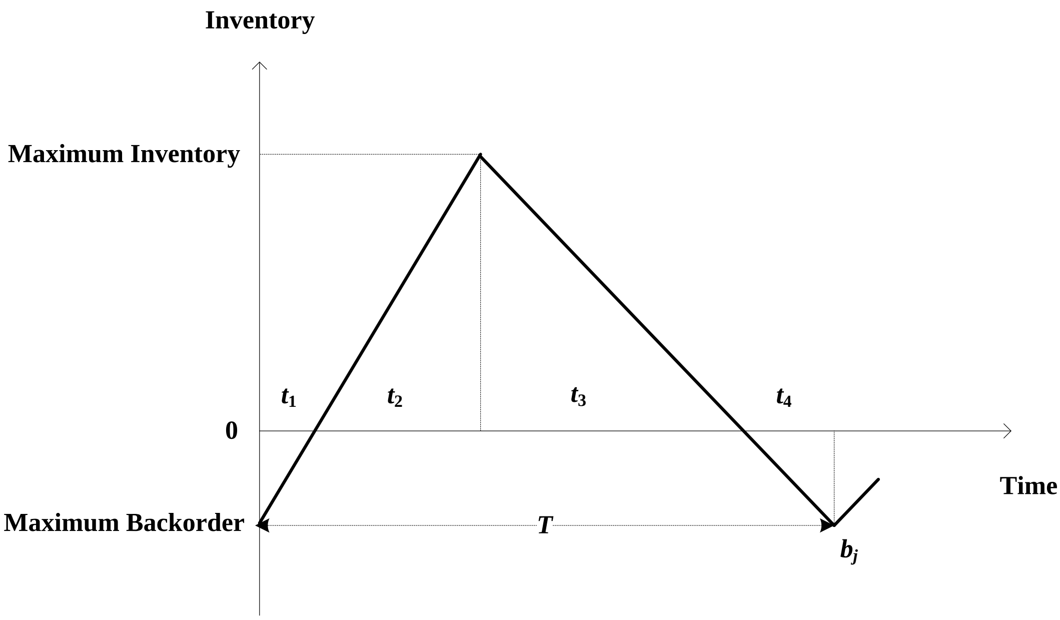


Figure 4.1: Inventory level of an item during a production cycle (Viswanathan & Goyal, 2000)

At the commencement of the cycle, there is a backorder of b_j for item j. The total production cycle time T consists of four stages as follows: t_1, the time in which the inventory level is negative because of backorders; t_2, the time that inventory level is greater than zero and increasing; t_3, the time which production has ended and the inventory level is being decreased from the maximum level to zero; and t_4, the time that inventory level lowers from zero to the total backorder at the end of the cycle. Since the demand rate for item j is denoted as d_j and the production rate is denoted as p_j, we have:

$$t_4 = \frac{b_j}{d_j} \tag{4.13}$$

$$t_1 d_j + b_j = t_1 p_j \tag{4.14}$$

$$t_1 + t_4 = \frac{b_j}{d_j (1 - d_j / p_j)} \tag{4.15}$$

since:

$$t_1 + t_2 = T \frac{d_j}{p_j} \tag{4.16}$$

and:

$$t_1 + t_2 + t_3 + t_4 = T \tag{4.17}$$

Hence:

$$t_2 = T \frac{d_j}{p_j} - \frac{b_j}{p_j (1 - d_j / p_j)} \tag{4.18}$$

and:

$$t_3 = T(1 - d_j / p_j) - \frac{b_j}{d_j} \tag{4.19}$$

The optimum b_j is obtained by equating the first derivative of the total cost given in Eq. (4.6) to zero, which gives:

$$b_j = d_j T(1 - d_j / p_j) \frac{H_j}{(H_j + B_j)} \tag{4.20}$$

The maximum amount of time that an item stays in inventory is equal to t_3. Therefore, the shelf life constraint for item *j* becomes as Eq. (4.21).

$$t_3 = T(1 - d_j / p_j) - \frac{b_j}{d_j} \le L_j \tag{4.21}$$

Which it can be re-written as:

$$T(1 - d_j / p_j) - \frac{d_j T(1 - d_j / p_j) \frac{H_j}{(H_j + B_j)}}{d_j} \le L_j \tag{4.22}$$

It can be rearranged again as:

$$T \le \frac{L_j (\frac{H_j + B_j}{B_j})}{(1 - d_j / p_j)} \tag{4.23}$$

In Eq. (4.23), the variable parameters are the production rate p_j and cycle time T which can be adjusted in order to satisfy the shelf life constraint. Next, the three options are examined to adjust these variables individually and simultaneously. Moreover, the available related inventory models are modified in order to find a feasible solution following shelf life adjustment.

4.2.5 Option 1: Production Rate Reduction

The reduction of production rate lowers the average inventory and thus items in the inventory are used up at a quicker rate. Hence, products are not kept longer than their shelf life duration.

In this option, the production rate p is adjusted to satisfy the constraint for shelf life along with the cycle time T which is considered as a fixed measure (T is the same as the optimal cycle time obtained by Eq. (4.10)) as illustrated in Figure 4.2.

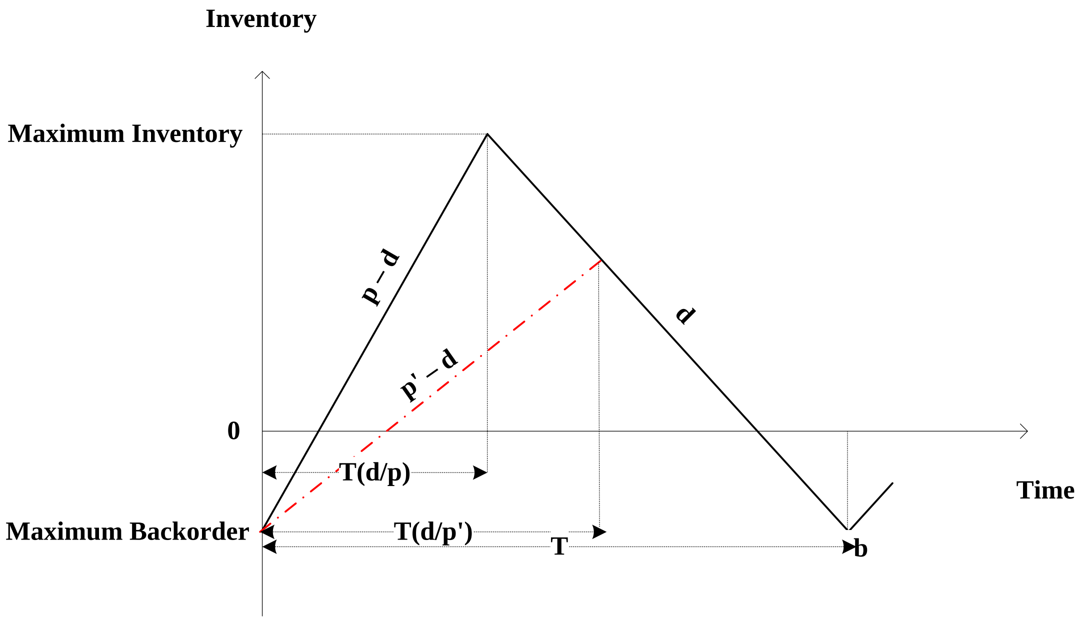


Figure 4.2: Reduction in the production rate with unchanged cycle time

Now, having the reduced production rate (p'_j), Eq. (4.23) is modified to:

$$T_1 \le \frac{L_j(\frac{H_j + B_j}{B_j})}{(1 - d_j / p'_j)} \tag{4.24}$$

By replacing T with T_1 equivalent to $L_j(\frac{H_j + B_j}{B_j}) \Big/ (1 - d_j / p'_j)$ and p_j with p'_j in Eq. (4.6), the total cost for option 1, $C(T_1)$, is found as:

$$\begin{aligned} C(T_1) = {} & \frac{B_j(1-r'_j)}{L_j(B_j+H_j)}\left(\sum_{i=1}^{N}(S_i + OA_i)\right) + O\sum_{\substack{i=1\\ i\neq j}}^{N} r_i + Or'_j \\ & + \frac{L_j(B_j+H_j)}{2B_j(1-r'_j)}\left(\sum_{\substack{i=1\\ i\neq j}}^{N} H_i d_i(1-r_i)\right) + \frac{L_j(B_j+H_j)H_j d_j}{2B_j} - \sum_{i=1}^{N} H_i b_i \\ & + \frac{B_j(1-r'_j)}{2L_j(B_j+H_j)}\left(\sum_{\substack{i=1\\ i\neq j}}^{N} \frac{(H_i+B_i)b_i^2}{d_i(1-r_i)}\right) + \frac{B_j b_j^2}{2L_j d_j} \end{aligned} \tag{4.25}$$

where $r_i = d_i / p_i$ and $r'_j = d_j / p'_j$

The production capacity constraint can be written as:

$$\sum_{i=1}^{N} A_i / T + \sum_{\substack{i=1 \\ i \neq j}}^{N} d_i / p_i \leq (1 - d_j / p'_j) \tag{4.26}$$

This option is applicable only when the optimum cycle time T_O given by Eq. (4.10) satisfies Eq. (4.27) as follows:

$$T \leq \frac{L_j \left(\frac{H_j + B_j}{B_j}\right) - \sum_{i=1}^{N} A_i}{\sum_{\substack{i=1 \\ i \neq j}}^{N} d_i / p_i} \tag{4.27}$$

4.2.6 Option 2: Cycle Time Reduction

In option 2, the cycle time is reduced to the shelf life duration of the item which prevents the possibility of product being kept further than its shelf life. As shown in Figure 4.3, cycle time T is altered from the optimal cycle time T_O in Eq. (4.10) to T_2, and the production rate p_j is kept fixed.

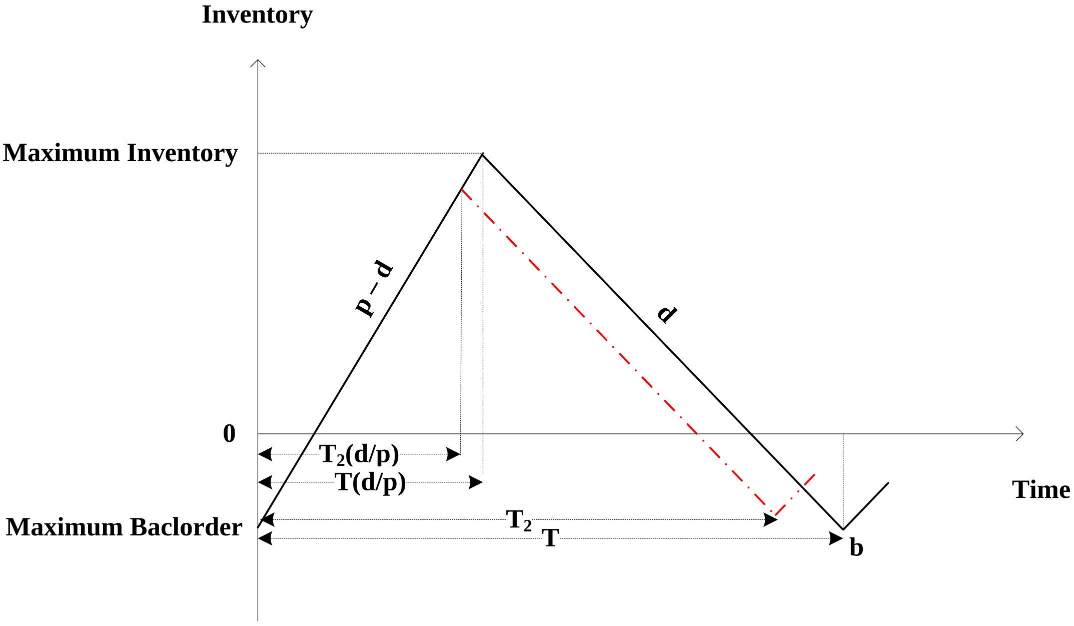


Figure 4.3: Modified cycle time with unchanged production rate

The adjusted cycle time T_2 is given by:

$$T_2 = \frac{L_j(\frac{H_j + B_j}{B_j})}{(1-d_j/p_j)} \tag{4.28}$$

Subject to:

$$\frac{\sum_{i=1}^{N} A_i}{(1-\sum_{i=1}^{N} r_i)} \le T_2 \tag{4.29}$$

By replacing the new cycle time, T_2, from Eq. (4.28) into Eq. (4.6), the total cost for option 2, $C(T_2)$, can be obtained as:

$$\begin{aligned} C(T_2) = & \frac{B_j(1-r_j)}{L_j(B_j + H_j)}\left(\sum_{i=1}^{N}(S_i + OA_i)\right) + O\sum_{i=1}^{N} r_i \\ & + \frac{L_j(B_j + H_j)}{2B_j(1-r_j)}\left(\sum_{i=1}^{N} H_i d_i(1-r_i)\right) - \sum_{i=1}^{N} H_i b_i \\ & + \frac{B_j(1-r_j)}{2L_j(B_j + H_j)}\left(\sum_{i=1}^{N}\frac{(H_i + B_i)b_i^2}{d_i(1-r_i)}\right) \end{aligned} \tag{4.30}$$

The total costs in Eqs. (4.25) and (4.30) must be compared in order to identify which option (1 or 2) is more cost-effective. Therefore, the difference in the total yearly inventory costs for the two options is given by $\Delta C = C(T_1) - C(T_2)$, which can be calculated as follows:

$$\begin{aligned} \Delta C = & O(r_j' - r_j) + \frac{B_j(r_j - r_j')}{L_j(B_j + H_j)}\left(\sum_{i=1}^{N}(\mathrm{S}_i + OA_i)\right) \\ & + \frac{L_j(B_j + H_j)}{2B_j}\sum_{\substack{i=1 \\ i \ne j}}^{N} H_i d_i(1-r_i)\left(\frac{r_j' - r_j}{(1-r_j')(1-r_j)}\right) \\ & + \frac{B_j}{2L_j(B_j + H_j)}\sum_{\substack{i=1 \\ i \ne j}}^{N}\frac{(H_i + B_i)b_i^2}{d_i(1-r_i)}\left(r_j - r_j'\right) \end{aligned} \tag{4.31}$$

A positive value for ΔC implies that the cost of option 1 is greater than option 2. Hence, it is efficient to use option 2.

4.2.7 Option 3: Simultaneous Adjustment of the Production Rate and Cycle Time

In this option, the production rate p and cycle time T are adjusted simultaneously to prevent flawing the shelf life constraint. As shown in Figure 4.4, cycle time and production rate are reduced to T_3 and p''_j respectively.

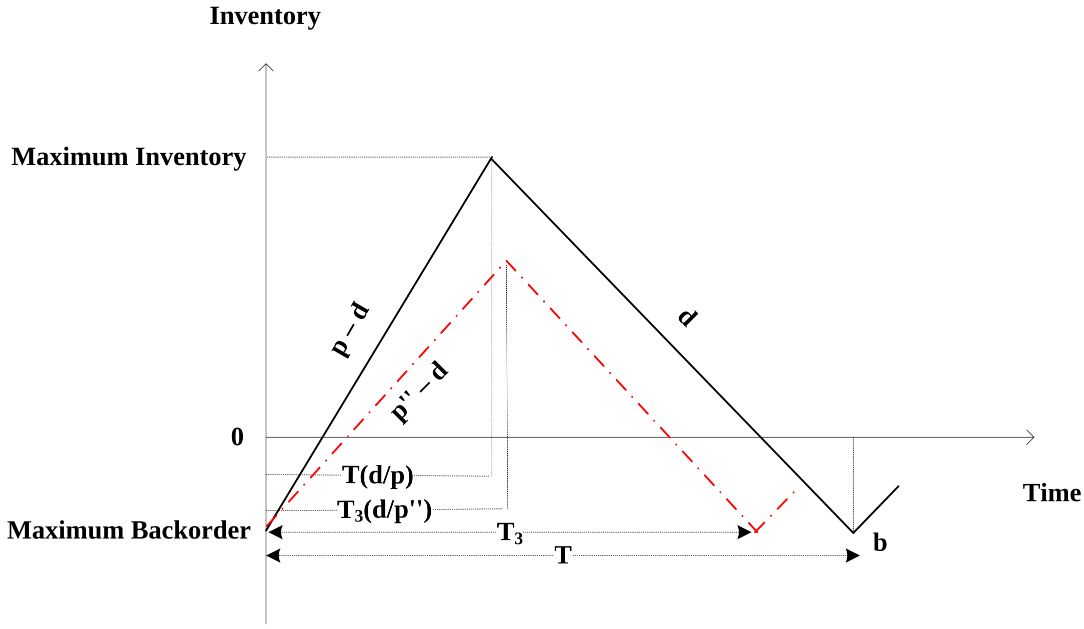


Figure 4.4: Simultaneous reduction of cycle time and production rate

Then the shelf life constraint will be:

$$(1 - d_j / p''_j) \le \frac{L_j \left(\frac{H_j + B_j}{B_j} \right)}{T_3} \tag{4.32}$$

The reduced cycle time T_3 can be outlined in terms of the production rate p''_j through the following equation:

$$T_3 = \frac{L_j \left(\frac{H_j + B_j}{B_j} \right)}{(1 - d_j / p''_j)} \tag{4.33}$$

Equation (4.33) is equivalent to:

$$r_j'' = 1 - \frac{L_j\left(\frac{H_j + B_j}{B_j}\right)}{T_3} \tag{4.34}$$

where $r_j'' = d_j / p_j''$

Subsequently, the capacity constraint can be written as:

$$\frac{\sum_{i=1}^{N} A_i + T_3 \sum_{\substack{i=1 \\ i \neq j}}^{N} r_i}{T_3} \leq (1 - d_j / p_j'') \tag{4.35}$$

The constraints represented in Eqs. (4.32) and (4.35) are feasible only when:

$$\sum_{i=1}^{N} A_i + T_3 \sum_{\substack{i=1 \\ i \neq j}}^{N} d_i / p_i \leq L_j\left(\frac{H_j + B_j}{B_j}\right) \tag{4.36}$$

or:

$$T_3 \leq \frac{L_j\left(\frac{H_j + B_j}{B_j}\right) - \sum_{i=1}^{N} A_i}{\sum_{\substack{i=1 \\ i \neq j}}^{N} d_i / p_i} \tag{4.37}$$

By changing T in Eq. (4.6) to T_3 in Eq. (4.33), the total cost is given as:

$$\begin{aligned} C(T_3) = {} & \frac{1}{T_3} \sum_{i=1}^{N} (\mathrm{S}_i + OA_i) + O \sum_{\substack{i=1 \\ i \neq j}}^{N} r_i + Or_j'' + \frac{T_3}{2} \sum_{\substack{i=1 \\ i \neq j}}^{N} H_i d_i (1 - r_i) \\ & + \frac{T_3}{2} H_j d_j (1 - r_j'') - \sum_{i=1}^{N} H_i b_i + \frac{1}{2T_3} \sum_{\substack{i=1 \\ i \neq j}}^{N} \frac{(H_i + B_i) b_i^2}{d_i (1 - r_i)} \\ & + \frac{1}{2T_3} \times \frac{(H_j + B_j) b_j^2}{d_j (1 - r_j'')} \end{aligned} \tag{4.38}$$

Substituting Eq. (4.34) into Eq. (4.38), we have:

$$C(T_3) = \frac{1}{T_3}\sum_{i=1}^{N}\left(S_i + OA_i\right) - \frac{1}{T_3}O\frac{L_j(H_j + B_j)}{B_j} + \frac{1}{T_3}\sum_{\substack{i=1\\ i\neq j}}^{N}\frac{(H_i + B_i)b_i^{\,2}}{2d_i(1-r_i)}$$
$$+\frac{T_3}{2}\sum_{\substack{i=1\\ i\neq j}}^{N}H_i d_i(1-r_i) - \sum_{i=1}^{N}H_i b_i + \frac{H_j d_j L_j(H_j + B_j)}{2B_j} + \frac{B_j b_j^{\,2}}{2d_j L_j} \tag{4.39}$$
$$+O\sum_{\substack{i=1\\ j\neq 1}}^{N} r_i + O$$

The objective is to find the optimum T_3 that yields the minimum total cost given in Eq. (4.39).

There are two cases for this option that must be examined:

(a) ***Case1:***

$$\sum_{i=1}^{N}(S_i + OA_i) + \sum_{\substack{i=1\\ i\neq j}}^{N}\frac{(H_i + B_i)b_i^{\,2}}{2d_i(1-r_i)} \leq OL_j\left(\frac{H_j + B_j}{B_j}\right) \tag{4.40}$$

In this case, Eq. (4.39) is minimized by selecting the smallest amount of T_3. Decreasing T_3 in Eq. (4.34) means reducing r_j'' or increasing the production rate p_j''. When $T = L_j\left(\frac{H_j + B_j}{B_j}\right)$ or r_j'' = 0, an infinite value of p_j'' is achieved. It should be noted that raising p_j may incur costs not reflected in the production cost. However, if p_j is permitted to be only decreased (not raised), then the current production rate of item j is at its highest possible level. Therefore, the best solution is to select option 2.

(b) ***Case2:***

$$\sum_{i=1}^{N}(S_i + OA_i) + \sum_{\substack{i=1\\ i\neq j}}^{N}\frac{(H_i + B_i)b_i^{\,2}}{2d_i(1-r_i)} > OL_j\left(\frac{H_j + B_j}{B_j}\right) \tag{4.41}$$

For almost all practical purposes, the third option may be more applicable when case 2 takes place. From Eq. (4.39), it can be seen that the total cost $C(T_3)$ is a convex function of T_3. Hence, the optimal value for T_3 minimizes the total cost that can be obtained by solving $dC(T_3)/dT_3 = 0$, which is:

$$T^* = \left(\frac{2\sum_{i=1}^{N}(S_i + OA_i) - \frac{2OL_j(H_j + B_j)}{B_j} + \sum_{\substack{i=1 \\ i \neq j}}^{N} \frac{(H_i + B_i)b_i^2}{d_i(1-r_i)}}{\sum_{\substack{i=1 \\ i \neq j}}^{N} H_i d_i (1-r_i)} \right)^{1/2} \quad (4.42)$$

If $T^* \leq L_j(\frac{H_j + B_j}{B_j})$, then T^* is not applicable as a negative value of r_j'' will be obtained given in Eq. (4.34). Under such conditions, due to the convexity of total cost $C(T_3)$, the lowest acceptable value of T_3 can be used ($L_j(\frac{H_j + B_j}{B_j})$), provided that r_j'' is allowed to be decreased to a value of 0. If p_j'' cannot be raised from its current value, then option 2 must be used.

Alternatively, if $T^* > L_j(\frac{H_j + B_j}{B_j})$, then T^* is acceptable as it produces a feasible value for p_j''. Otherwise, option 2 must be used again. Furthermore, it should be noted that for very small production cost values (not practical in practice), T^* can be greater than T_O, which could cause shelf life violation.

By substituting Eq. (4.42) into Eq. (4.39), $C(T^*)$ will be obtained as:

$$\begin{aligned} C(T^*) = {} & \left[\left(\sum_{\substack{i=1 \\ i \neq j}}^{N} H_i d_i (1-r_i) \right) \left(2\sum_{i=1}^{N}(S_i + OA_i) - \frac{2OL_j(H_j + B_j)}{B_j} + \sum_{\substack{i=1 \\ i \neq j}}^{N} \frac{(H_i + B_i)b_i^2}{d_i(1-r_i)} \right) \right]^{1/2} \\ & - \sum_{i=1}^{N} H_i b_i + \frac{H_j d_j L_j (H_j + B_j)}{2B_j} + \frac{B_j b_j^2}{2d_j L_j} + O\sum_{\substack{i=1 \\ j \neq 1}}^{N} r_i + O \end{aligned} \quad (4.43)$$

In order to search for the cost minimization of T_3, the lower and upper bounds are also practical. The upper bound is provided from Eq. (4.37). In Eq. (4.34), as T_3

decreases, r''_j decreases (i.e. p''_j increases). When the maximum feasible production rate value is considered as the primary p_j, the lower bound will be given as:

$$T_3 = \frac{L_j\left(\frac{H_j + B_j}{B_j}\right)}{(1 - d_j / p_j)} \tag{4.44}$$

4.3 Numerical Examples and Discussions

The input data is presented in Table 4.1, which was initially provided by Silver (1989) and subsequently used in various literature works (Sarker & Babu, 1993; Silver, 1995; Viswanathan & Goyal, 2000; Xu & Sarker, 2003; Sharma, 2004). The additional data for backorder amounts and shortage cost are taken from Sharma (2004).

Table 4.1: Input data

Item i	d_i	p_i	A_i	S_i	H_i	b_i	B_i	L_i	$L_i\left(\frac{H_i+B_i}{B_i}\right) \Big/ \left(1-\frac{d_i}{p_i}\right)$
1	1000	3000	0.0005	70	10	11	100	0.20	0.33
2	500	2500	0.0010	80	12	5	150	0.11	0.15
3	700	2500	0.0015	135	15	6	200	0.20	0.30

Equations (4.10) and (4.11) are used to obtain T_O and $C(T_O)$ respectively. The results are presented in Table 4.2.

Table 4.2: Optimal cycle time and total cost ignoring the shelf life constraint

Production Cost ($/year)	T_O **(year)**	$C(T_O)$ **($/year)**
O = 5000	0.1842	7311.05
O = 2500	0.1820	5236.76
O = 1000	0.1807	3991.95
O = 500	0.1803	3576.97
O = 100	0.1799	3244.98
O = 0	0.1799	3161.97

Accordingly, T_O must satisfy constraint (1) given in Eq. (4.8), i.e.:

$$T_O \geq \frac{\sum_{i=1}^{3} A_i}{\left(1 - \sum_{i=1}^{3} r_i\right)} \text{ or } T_O \geq 0.0161 \text{years}$$

Values of T_O show that for all production costs, constraint (1) in Eq. (4.8) is satisfied. Moreover, T_O must meet the shelf life constraint given in Eq. (4.23). However, T_O does not satisfy this constraint for item 2 meaning that item 2 violates the shelf life condition. Thus, either one of three options must be applied for which item 2 can satisfy the constraint for shelf life.

(a) ***Option 1: Production Rate Reduction***

Using Eq. (4.24), the new cycle time, T_1, is calculated as:

$$T_1 = T_O = \frac{L_2 (\frac{H_2 + B_2}{B_2})}{(1 - d_2 / p_2')}$$

Thus, the new production rate, p_2', and the total cost for option 1, $C(T_1)$, can be calculated using Eqs. (4.24) and (4.25). The results for various production cost values are shown in Table 4.3.

Table 4.3: Reduced production rate and the corresponding cost

Production Cost ($/year)	p_2' **(units/year)**	***C(T₁)* ($/year)**
O = 5000	1409	8006.96
O = 2500	1439	5530.97
O = 1000	1459	4063.34
O = 500	1466	3577.22
O = 100	1472	3189.46
O = 0	1473	3092.67

By exchanging p_2 with p_2', the shelf life constraint is fulfilled for the second item. Then the feasibility condition for the cycle time must be examined using Eq. (4.27):

$$T_O \le \frac{\left(L_2(\frac{H_2 + B_2}{B_2}) - \sum_{i=1}^{3} A_i \right)}{\sum_{\substack{i=1 \\ i \ne j}}^{3} r_i} \text{ or } T_O \le 0.1888 \text{ years}$$

Option 1 is feasible as the optimum cycle time T_O for all production costs mentioned in Table 4.2 satisfies this constraint.

(b) ***Option 2: Cycle Time Reduction***

From Eq. (4.28), the adjusted cycle time T_2 is obtained as follows:

$$T_2 = \frac{L_2(\frac{H_2 + B_2}{B_2})}{(1 - d_2/p_2)}$$

In option 2, the total cost $C(T_2)$ can be obtained using Eq. (4.30). The results for different production costs are shown in Table 4.4.

Table 4.4: Reduced cycle time and the corresponding cost

Production Cost ($/year)	T_2 **(year)**	$C(T_2)$ **($/year)**
O = 5000	0.1485	7392.62
O = 2500	0.1485	5308.78
O = 1000	0.1485	4058.48
O = 500	0.1485	3641.71
O = 100	0.1485	3308.30
O = 0	0.1485	3224.94

This option is applicable as T_2 satisfies the capacity constraint as follows:

$$T_2 \ge \frac{\sum_{i=1}^{3} A_i}{1 - \sum_{i=1}^{3} r_i} \text{ or } T_2 \ge 0.0161 \text{ years}$$

The comparisons of total cost values obtained from two options are indicated in Table 4.5. Figure 4.5 represents the trend of cost function values found by two options for various production cost amounts.

Table 4.5: Comparisons of total costs obtained by option 1 and option 2

Production Cost ($/year)	$C(T_1)$ **($/year)**	$C(T_2)$ **($/year)**	$\Delta C = C(T_1) - C(T_2)$ **($/year)**
O = 5000	8006.96	7392.62	614.34
O = 2500	5530.97	5308.78	222.19
O = 1000	4063.34	4058.48	4.86
O = 500	3577.22	3641.71	-64.49
O = 100	3189.46	3308.30	-118.84
O = 0	3092.67	3224.94	-132.27

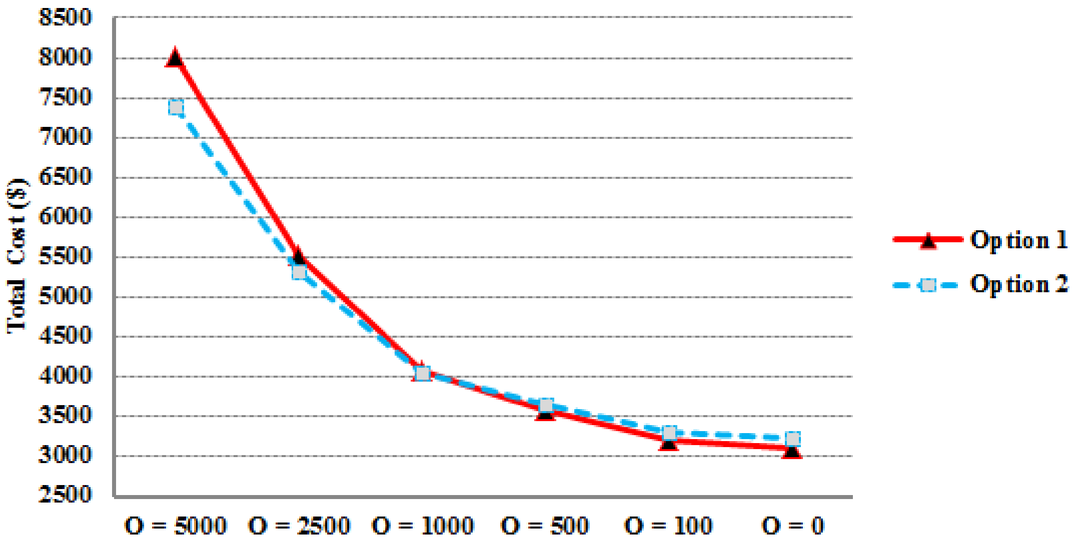


Figure 4.5: Graphical representation of the performance comparison between options 1 and 2 in terms of cost function values

The total cost values in Table 4.5 indicate that decreasing cycle time leads to producing lower cost than decreasing production rate due to rather high production cost. This conclusion was achieved earlier by Sarker and Babu (1993) and later by Yan et al. (2013). Adversely, when machine operating cost decreases, the production rate decline produces a lower cost than a cycle time reduction. Sarker and Babu (1993) found $8258.12 and $7556.57 for options 1 and 2 respectively when O = $5000.

Silver (1989) showed that in the absence of production cost (i.e. O = $0), reducing the production rate proves to be more economical than cycle time reduction. Silver (1989) obtained $3208.01 and $3378.06 for option 1 and option 2 correspondingly, which indicates that the cost of option 2 is about 5% higher than the cost obtained by option 1. The model presented in this study offers lower cost for both options compared to Silver (1989) (3.60% and 4.53% reductions in cost attained by options 1 and 2 respectively).

Clearly, the production cost has significant influence on determining which option to be selected. According to Sarker and Babu (1993), the preference toward one of these two options depends on various parameters such as setup time, setup cost, production cost, holding cost, shortage cost, and shelf time.

It should be noted that slowing down the production rate decreases inventory levels consistent with a Just-in-Time manufacturing philosophy, hence lowers the inventory holding cost. However, production rate reduction causes the production to be carried out for a longer time. If the production time is raised then all related costs such as labor, machine operating and probably some other associated costs are augmented. Hence, production rate increase causes some further costs to the manufacturer (Sarker & Babu, 1993). For this reason, production time cost is taken into account in order to include the effect of the length of production time.

Sharma (2004) inserted shortages into his model, and for production cost of \$5000, total costs \$8232.46 and \$7454.20 for options 1 and 2 were obtained respectively. The present model produced lower costs after adjustments to the shelf life constraint and corresponding mathematical models.

Models with planned backorders have the total related costs reduced, and the items are stored in inventory for a shorter period of time leading to a less restrictive shelf life constraint. Furthermore, in this situation, the average inventory level is lower, resulting in lower inventory holding costs. Negative inventory known as shortage arises when demand is higher than the production capacity and available inventory. Unsatisfied demand can be backordered and fulfilled in the next period. In a highly competitive situation in emerging markets, unsatisfied demand will usually be lost. However, this lost sale usually happens in retail sales than in manufacturing environment. Hence anticipation for penalty for not fulfilling demand in time (i.e., shortage) is vital in competitive developing markets (Xu & Sarker, 2003).

(c) ***Option 3: Adjustment of Production Rate and Cycle Time Concurrently***

For this option, two cases should be taken into account as shown in Table 4.6.

Table 4.6: Feasibility assessment of production rate and cycle time reduction simultaneously

Production Cost ($/year)	$\sum_{i=1}^{3}(S_i + OA_i) + \sum_{\substack{i=1 \\ i \neq j}}^{3} \frac{(H_i + B_i) b_i^2}{2d_i(1-r_i)}$	$OL_2(\frac{H_2 + B_2}{B_2})$
O = 5000	267.34	594.00
O = 2500	259.84	297.00
O = 1000	255.34	118.80
O = 500	253.84	59.40
O = 100	252.64	11.88

Table 4.6 indicates that for *O* = $5000 and $2500, the inequality below applies:

$$\sum_{i=1}^{3}(S_i + OA_i) + \sum_{\substack{i=1 \\ i \neq j}}^{3} \frac{(H_i + B_i) b_i^2}{2d_i(1-r_i)} < OL_2(\frac{H_2 + B_2}{B_2})$$

Thus, to obtain the minimum total cost, the smallest possible value of T_3 should be chosen (i.e. the smallest possible r_2''). If r_2'' can be decreased to a very small value without imposing extra cost, then $T_3 = L_2(\frac{H_2 + B_2}{B_2})$ or T_3 = 0.1188 years should be used, which results in $C(T_3)$ = $6716.14 and $5119.67 for *O* = $5000 and $2500 respectively. In this case, the adjusted production rate p_2'' is infinite. However, if r_2'' cannot be reduced (i.e. the production rate cannot exceed its defined maximum value), then option 2 should be chosen with T_2 = 0.1485 years, p_2 = 2500 units/year, and $C(T_2)$ = $7392.62 and $5308.78 for *O* = $5000 and $2500 respectively.

Similarly, in order to find the optimal T_3, the lower and upper bounds are applicable. The upper bound T_3 is given as:

$$T_3 \le \frac{\left(L_2 (\frac{H_2 + B_2}{B_2}) - \sum_{i=1}^{3} A_i \right)}{\sum_{\substack{i=1 \\ i \ne j}}^{3} d_i / p_i} \text{ or } T_3 \le 0.1888 \text{ years}$$

The lower bound T_3 is:

$$T_3 = \frac{L_2 (\frac{H_2 + B_2}{B_2})}{(1 - d_2 / p_2)} = 0.1485 \text{ years}$$

The search becomes appropriate for $0.1485 \le T_3 \le 0.1888$. A simple procedure may be conducted by reducing T_3 from 0.1888 to 0.1485 in a small step, for which the optimum T_3 will be found. By substituting the input data from Table 1 into Eq. (4.39) and considering that O = \$5000, $C(T_3)$ is given as:

$$C(T_3) = -\frac{276.34}{T_3} + 7113.33 T_3 + 8197.16$$

In line with the search procedure, T_3 = 0.1485 years produces the minimum cost, that is $C(T_3)$ = \$7392.62. In this case, p_2'' = 2500 units/year, for which the results are similar to the findings obtained by option 2 because:

$$\sum_{i=1}^{3} (S_i + OA_i) + \sum_{\substack{i=1 \\ i \ne j}}^{3} \frac{(H_i + B_i) b_i^2}{2 d_i (1 - r_i)} < OL_2 (\frac{H_2 + B_2}{B_2}).$$

For O = \$1000, \$500 and \$100, $\sum_{i=1}^{3} (S_i + OA_i) + \sum_{\substack{i=1 \\ i \ne j}}^{3} \frac{(H_i + B_i) b_i^2}{2 d_i (1 - r_i)} > OL_2 (\frac{H_2 + B_2}{B_2})$; thus, using Eqs. (4.42) and (4.43), T^* and $C(T^*)$ can be calculated, and p_2'' can be obtained from Eq. (4.34). The results are shown in Table 4.7.

Table 4.7: Simultaneous reduction of the production rate and cycle time and the corresponding cost

Production Cost ($/year)	T^* **(year)**	p_2'' **(units/year)**	$C(T^*)$ **($/year)**
O = 1000	0.1621	1873	4049.65
O = 500	0.1855	1391	3576.15
O = 100	0.2023	1211	3169.71

As Table 4.7 demonstrates, $T^* > L_2(\frac{H_2 + B_2}{B_2})$ or $T^* > 0.1188$ years; thus T^* can be used since Eq. (4.34) produces a feasible value for p_2''. However, T^* must satisfy the following capacity constraint:

$$T^* \leq \frac{L_2(\frac{H_2 + B_2}{B_2}) - \sum_{i=1}^{3} A_i}{\sum_{\substack{i=1 \\ i \neq j}}^{3} d_i / p_i} \text{ or } T^* \leq 0.1888 \text{ years}$$

For O = $100, T^* exceeds 0.1888 years, and it is higher than T_O (0.1799 years), which leads to shelf life constraint violation. According to Silver (1995), this option is not applicable for small production cost values, and either option 1 or 2 must be used. Based on the results, if O = $100, option 1 is suitable.

For O = $1000, the comparisons of total cost values given in Table 4.8 shows that option 3 is more cost-effective. The production quantity (Q) in this case is 126, 104, and 113 units for items 1, 2, and 3 respectively.

Table 4.8: Comparisons of total costs obtained by three options for production cost $1000

Option	***P* (units/year)**	***T* (year)**	***C*(*T*) ($/year)**
1	1459	0.1807	4063.34
2	2500	0.1485	4058.48
3	1873	0.1621	4049.65

Comparisons of three options for O = $1000 are also presented graphically in Figures 4.6 to 4.8.

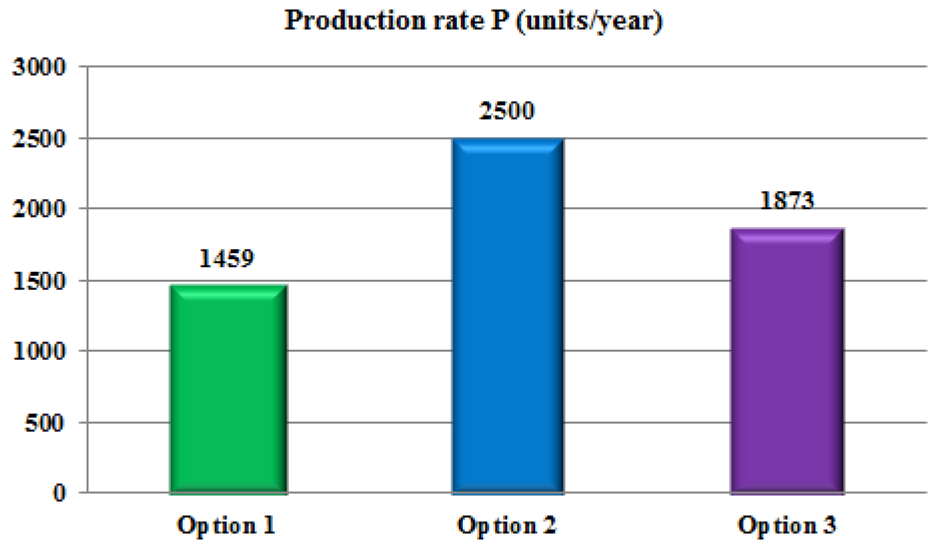


Figure 4.6: Comparisons of production rates obtained by three options for production cost $1000

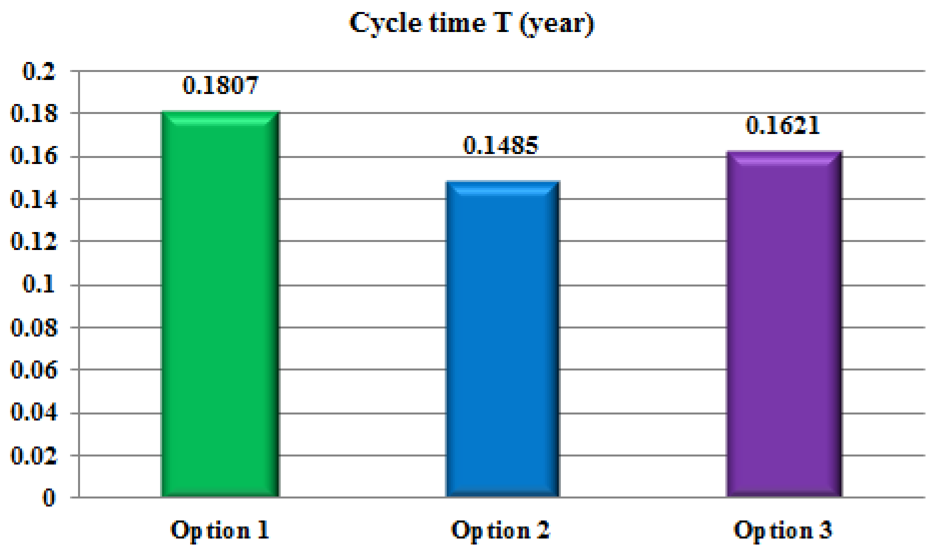


Figure 4.7: Comparisons of cycle times obtained by three options for production cost $1000

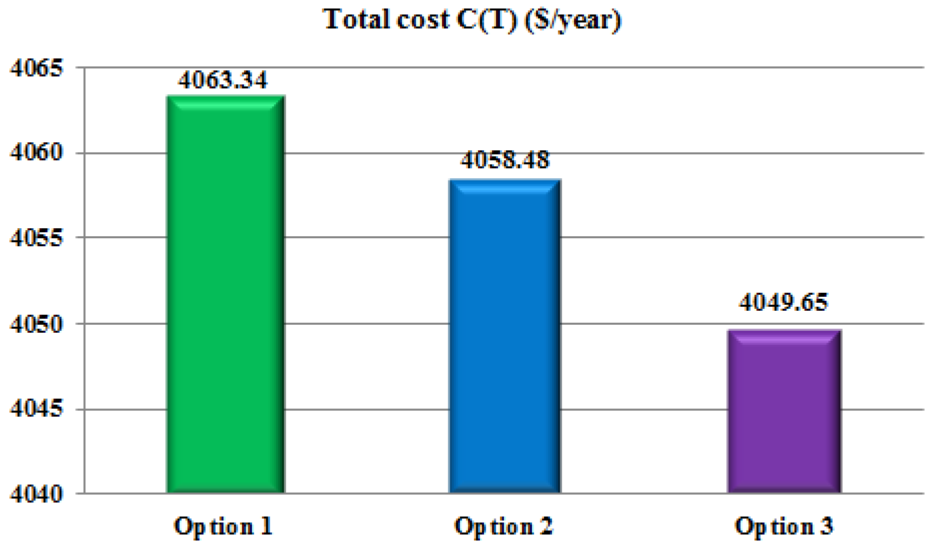


Figure 4.8: Comparisons of total costs obtained by three options for production cost $1000

Silver (1995) examined O = $1000 applying option 3, and obtained T^*= 0.158 years, p_2'' = 1639 units/year, and $C(T^*)$ = $4193.82. Eventually, a lower total cost is obtained by option 3 proposed in this work (3.44% cost reduction).

4.4 Conclusions

Recently, shelf life during which items remain safe within production plants is being increasingly considered in production optimization models. This chapter addressed a manufacturing system, whereby multiple items produced in a single facility have shelf life restrictions and planned backorders. For some products, shelf life may be lower than the production cycle time, which can lead to item spoilage before reaching the end of the cycle. In such a case, in order to achieve a feasible schedule, the production cycle time needs to be reduced to less than or equal to the shelf life duration. On the other hand, backorders incur shelf life constraint alteration, which affects the corresponding inventory models. Accordingly, appropriate modifications were applied to the related mathematical inventory models.

The aim was to determine the optimal cycle time in addition to minimizing the long-run average costs including production, setup, holding, and shortage costs. While the cost-minimization cycle time caused the spoilage of products due to shelf life restrictions, appropriate decisions were made based on one of the three options: production rate reduction, cycle time reduction, and the simultaneous production rate and cycle time reduction. As a result, for each option the optimal cycle time and production rate were estimated, which satisfy the shelf life constraints. Numerical examples were presented to illustrate the influence of production cost, backorders, and shelf life on total annual cost.

The superiority of this work stems from the integration of the backorders and shelf life constraint, and its significant effect on the corresponding inventory models. The computational results indicated the dominance of the modified models with respect to producing lower total costs compared to other related models existing in the literature.

CHAPTER 5: OPTIMIZATION OF ECONOMIC LOT SCHEDULING PROBLEM CONSIDERING MULTIPLE SETUPS, BACKORDERING AND SHELF LIFE USING CALIBRATED METAHEURISTIC ALGORITHMS

5.1 Introduction

This chapter addresses the optimization of the ELSP, where multiple items are produced on a single machine in a cyclical pattern. It is assumed that each item can be produced more than once in every cycle, each product has a shelf life restriction, and backordering is permitted. The objective is to determine the optimal production rate, production quantity, production frequency, cycle time, as well as a feasible manufacturing schedule for the family of items, and to minimize the long-run average costs. Efficient search procedures are presented to obtain the optimum solutions by employing well-known metaheuristic algorithms.

The rest of this chapter is organized as follows: Section 5.2 presents the proposed mixed integer non-linear ELSP model. In Section 5.3, metaheuristic algorithms namely the GA, PSO, ABC, and SA are explained to solve the proposed model. Section 5.4 describes the Taguchi method employed to tune various parameters of the applied algorithms. Section 5.5 demonstrates the numerical example and discusses the computational results. Finally, conclusions are given in Section 5.6.

5.2 Problem Description and Mathematical Formulations

The ELSP in this study is considered in a single machine environment with production of N items in the manufacturing cycle time of T, where backordering is permitted for any of the items, and each of which has a specified shelf life. Moreover, the restricted assumption of production of every item exactly once in a cycle considered in previous studies is removed, and it is allowed to produce each item more than once in every cycle.

The objective is to find the optimal production rate, production frequency, cycle time, and batch size of each item as well as a feasible manufacturing schedule for the family of items. Furthermore, it is attempted to minimize the long-run average cost including setup, production, holding, and backorder costs, in addition to the adjustment cost if production time conflicts are occurred between the products in a cycle.

The mathematical model studied throughout this chapter is based on the following assumptions and notations:

(a) ***Assumptions***

i. Each item has a deterministic and constant demand.

ii. Each item has a deterministic and constant setup time.

iii. The first-in-first-out rule is considered for the inventory transactions.

(b) ***Indices***

i	Product (i = 1, 2, … , N)
N	Total number of products
j	Batch number (j = 1, 2, …, f_i)
w, w'	Production batch (w, w' = 1, 2, … , $F = \sum_{i=1}^{N} f_i$)

(c) ***Parameters***

d_i	Demand rate for item i (units/year)
$P_i^{\max}$	Maximum possible production rate for item i (units/year)
$R_i^{\min}$	Ratio of demand to maximum production rate for item i
L_i	Shelf life of item i (years)
A_i	Setup time for item i (years)

S_i Setup cost for item i (dollars/unit/year) (machine operating cost is excluded during setup)

H_i Inventory holding cost for item i (dollars/unit/year)

B_i Backordering cost for item i (dollars/unit/year)

O Machine operating cost (dollars/year)

(d) ***Variables***

p_i Production rate for item i

r_i Ratio of demand to production rate for item i

f_i Production frequency for item i per cycle

t_i Cycle time for item i

k_i Production start time for item i

τ_i Production time for item i

α_i^j Production start time advancement for item i in its j^{th} production batch

β_i^j Production start time delay for item i in its j^{th} production batch

ϑ_i Machine time for item i

Q_i Production batch size of item i

b_i Amount of item i backordered in each cycle

m_i Maximum backorder level for item i

T Entire production cycle time

$C(T)$ Total cost for the entire production cycle time

$$\psi_i^w \begin{cases} 1 & \text{if item } i \text{ is produced in the } w^{th} \text{ batch} \\ 0 & \text{otherwise} \end{cases}$$

Using the above notations, the mathematical model for the ELSP with various production frequencies, backordering, and shelf life constraints is presented as follows:

5.2.1 Cost Function

The annual cost for setting up the machine and products, and production is given by:

$$\frac{1}{T}\sum_{i=1}^{N}(S_i + OA_i) + \frac{1}{T}\sum_{i=1}^{N}O(\frac{d_i}{p_i})T \tag{5.1}$$

The annual cost for holding the products with considering backorders, is given by:

$$\sum_{i=1}^{N}\frac{H_i\left(Q_i(1-\frac{d_i}{p_i})-b_i\right)^2}{2Q_i(1-\frac{d_i}{p_i})} \tag{5.2}$$

The annual backordering cost is given by:

$$\sum_{i=1}^{N}\frac{B_i b_i^{\,2}}{2Q_i(1-\frac{d_i}{p_i})} \tag{5.3}$$

The adjustment cost in case of overlapping production times of items is given by (Yan et al., 2013):

$$\sum_{i=1}^{N}\frac{H_i + B_i}{2}d_i\left(\sum_{j=1}^{f_i}(\alpha_i^j)^2 + \sum_{j=1}^{f_i}(\beta_i^j)^2\right)(1-\frac{d_i}{p_i}) \tag{5.4}$$

Adding Eqs. (5.1) to (5.4), the total annual cost, $C(T)$, is given by:

$$\begin{aligned} C(T) = {} & \frac{1}{T}\sum_{i=1}^{N}(S_i + OA_i) + \frac{1}{T}\sum_{i=1}^{N}O(\frac{d_i}{p_i})T + \sum_{i=1}^{N}\frac{H_i\left(Q_i(1-\frac{d_i}{p_i})-b_i\right)^2}{2Q_i(1-\frac{d_i}{p_i})} \\ & + \sum_{i=1}^{N}\frac{B_i b_i^{\,2}}{2Q_i(1-\frac{d_i}{p_i})} + \sum_{i=1}^{N}\frac{H_i + B_i}{2}d_i\left(\sum_{j=1}^{f_i}(\alpha_i^j)^2 + \sum_{j=1}^{f_i}(\beta_i^j)^2\right)(1-\frac{d_i}{p_i}) \end{aligned} \tag{5.5}$$

Substituting $Q_i = t_i d_i$, $d_i / p_i = r_i$, $T = t_i f_i$, and $b_i = d_i t_i (1-r_i)\dfrac{H_i}{H_i + B_i}$ in Eq. (5.5), and then simplifying, the total yearly cost for a group of *N* items, where there are f_i production batches for item *i*, can be obtained by:

$$C(T) = \frac{1}{T}\sum_{i=1}^{N}\left[S_i + OA_i f_i + Or_i t_i f_i + t_i^2 f_i \frac{d_i H_i}{2}(1-r_i)\frac{B_i}{H_i + B_i} \right. \\ \left. + \frac{H_i + B_i}{2} d_i \left(\sum_{j=1}^{f_i} (\alpha_i^j)^2 + \sum_{j=1}^{f_i} (\beta_i^j)^2 \right)(1-r_i) \right] \tag{5.6}$$

5.2.2 Constraints

For a feasible solution, the total setup time and production time for *N* products cannot go beyond the cycle time *T*. Therefore,

$$\frac{\sum_{i=1}^{N} A_i f_i}{1 - \sum_{i=1}^{N} r_i} \le T \tag{5.7}$$

$1 - \sum_{i=1}^{N} r_i$ is the long-run proportion of time available for setups. For infinite horizon problem, $\left(1 - \sum_{i=1}^{N} r_i\right) > 0$ is necessary in order to have a feasible solution.

Therefore, it is necessary that:

$$\sum_{i=1}^{N} r_i < 1 \tag{5.8}$$

The adopted production rate for each item should not exceed the maximum possible production rate. Hence:

$$p_i \le P_i^{\max} \qquad \text{for } i = 1, 2, \ldots, N \tag{5.9}$$

or,

$$R_i^{\min} \le r_i \qquad \text{for } i = 1, 2, \ldots, N \tag{5.10}$$

Where $R_i^{\min} = d_i / P_i^{\max}$

It is assumed that:

$$\sum_{i=1}^{N} R_i^{\min} \le 1 \tag{5.11}$$

Otherwise, there would not be any feasible production schedule.

It is supposed that each item i has a shelf life of L_i years, and the inventory is used on first-in-first-out basis. Accordingly, item i with the longest keeping period will be produced at the later section of the manufacturing cycle. Thus, the maximum time that product i is stored is $T(1-d_i / p_i)$ (Silver, 1989). However, the shelf life constraint amends in the model allowing the backorders. Thus, the shelf life constraint in this condition is:

$$t_i(1-r_i)\frac{B_i}{H_i + B_i} \le L_i \qquad \text{for } 1 \le i \le N \tag{5.12}$$

When the optimal cycle time goes beyond the time restriction of life for an item, spoilage of the product might occur that in turn leads to a loss to the manufacturer. The storage time for an item can be lowered by producing that item more frequently in a manufacturing cycle (Goyal, 1994). If item i is produced more than once in a production cycle, the shelf life constraint considering the production start time advancement or delay for the j^{th} batch of item i (in case of an infeasible schedule) will be changed to:

$$t_i(1-r_i)\frac{B_i}{H_i + B_i} + \alpha_i^j - \beta_i^j \le L_i \qquad \text{for } i = 1, 2, \ldots, N, \; j = 1, 2, \ldots, f_i \tag{5.13}$$

The required machine time for production of item i in every cycle time t_i, ϑ_i, is the total of setup time and production time of that item:

$$\vartheta_i = A_i + \tau_i \qquad \text{for } i = 1, 2, \ldots, N \tag{5.14}$$

where $\tau_i = t_i \dfrac{d_i}{p_i}$.

Since, item i is allowed to be produced more than once every t_i years, the machine time available for other items in every t_i cycle is $t_i - \vartheta_i$. If the required machine time for other products exceeds the available time, it causes production times conflicts between some or all the items. Therefore, the constraints given in Eqs. (5.15) to (5.21) must be met to avoid the schedule infeasibility.

The production of an item can be commenced only after the completion of production of its former batch. Hence:

$$\begin{aligned} &k_{I(w)} + (J(w)-1)t_{I(w)} - \alpha_{I(w)}^{J(w)} + \beta_{I(w)}^{J(w)} \geq \\ &k_{I(w-1)} + (J(w-1)-1)t_{I(w-1)} - \alpha_{I(w-1)}^{J(w-1)} + \beta_{I(w-1)}^{J(w-1)} + r_{I(w-1)}t_{I(w-1)} \quad \text{for } w = 2, \ldots, F \end{aligned} \tag{5.15}$$

Where $F = \sum_{i=1}^{N} f_i$

In Eq. (5.15), $I(w)$ represents that w^{th} production batch within a manufacturing cycle belongs to which item. Therefore:

$$I(w) = \sum_{i=1}^{N} \psi_i^w i \qquad \text{for } w = 1, 2, \ldots, F \tag{5.16}$$

Where

$$\psi_i^w = \begin{cases} 1 & \text{if item } i \text{ is produced in the } w^{th} \text{ batch} \\ 0 & \text{otherwise} \end{cases} \tag{5.17}$$

In Eq. (5.15), $J(w)$ shows the item's batch number. Hence:

$$J(w) = \sum_{w'=1}^{w} \psi_{I(w)}^{w'} \qquad \text{for } w = 1, 2, \ldots, F \tag{5.18}$$

Equation (5.19) shows the total number of batches or production frequency for each item in a cycle:

$$\sum_{w=1}^{F} \psi_i^w = f_i \qquad \text{for } i = 1, 2, \ldots, N \tag{5.19}$$

To prevent the production of different items from overlap in a cycle, Eq. (5.20) must be used:

$$\sum_{i=1}^{N} \psi_i^w = 1 \qquad \text{for } w = 1, 2, \ldots, F \tag{5.20}$$

Equation (5.21) restricts the completion time of the last batch so that it cannot go beyond the entire production cycle time:

$$k_{I(F)} + (J(F) - 1)t_{I(F)} - \alpha_{I(F)}^{J(F)} + \beta_{I(F)}^{J(F)} + r_{I(F)} t_{I(F)} \le T \tag{5.21}$$

It should be noted that for attaining production feasibility an item's production start time can be either advanced or delayed, but both cannot occur. Therefore:

$$\alpha_i^j . \beta_i^j = 0 \qquad \text{for } i = 1, 2, \ldots, N,\ j = 1, 2, \ldots, f_i \tag{5.22}$$

The optimum backorder level for each item can be expressed as Eq. (5.23):

$$m_i = t_i d_i (1 - r_i) \frac{H_i}{H_i + B_i} \qquad \text{for } i = 1, 2, \ldots, N \tag{5.23}$$

Equation (5.24) can be used to obtain the lot size for each item:

$$Q_i = d_i t_i \qquad \text{for } i = 1, 2, \ldots, N \tag{5.24}$$

Constraints (5.25) are the non-negativity constraints:

$$\begin{aligned}
&T \geq 0 \\
&r_i \geq 0 && \text{for } i = 1, 2, \ldots, N \\
&\alpha_i^j, \beta_i^j \geq 0 && \text{for } i = 1, 2, \ldots, N, \; j = 1, 2, \ldots, f_i \\
&f_i > 0 \text{ integer} && \text{for } i = 1, 2, \ldots, N
\end{aligned} \tag{5.25}$$

5.3 Solution Algorithms

The formulation given in Section 5.2 is a nonlinear mixed integer problem. These characteristics justify the model to be adequately difficult to solve using the exact methods. To deal with the intricacy and obtain near-optimal results in a reasonable computation time, metaheuristic approaches are widely used for which the GA, PSO, ABC, and SA methods are explained in the following subsections. Related codes are presented in Appendices A to D.

5.3.1 GA Approach

The GA is a type of stochastic optimization method that randomly searches the solution space to find a solution (Parsapoor & Bilstrup, 2012). The general steps of the GA can be summarized as follow (Gen & Cheng, 2008):

i. Encoding solutions of problem into chromosomes.
ii. Creating initial population of solutions randomly.
iii. Evaluating chromosomes in terms of their fitness in order to select parents.
iv. Applying genetic operators (crossover and mutation) in order to reproduce new chromosomes (offspring).
v. Evaluating the new population.
vi. Maintain the best chromosomes among parents and offspring.
vii. If stopping criteria is met, then stop. Otherwise, go to step iii.

The required steps to solve the proposed model by a GA are explained below.

(a) ***Initial Conditions***

The initial information required to begin a GA includes:

1. The number of chromosomes kept in each generation that is named population size, and indicated by 'N_{pop}'.
2. The probability of operating crossover known as crossover rate represented by 'P_c'.
3. The probability of operating mutation called mutation rate represented by 'P_m'.
4. Maximum number of iterations denoted by 'max iter'.

Both the crossover and mutation rates alter in the range [0.1, 1]. The computational results show that the effect of P_c on the total cost value is positive. Therefore, the smaller the crossover probability, the lesser the total cost will be. However, $C(T)$ decreases as P_m increases.

N_{pop}, P_c, and P_m are calibrated using the Taguchi method descried in Section 5.4.

(b) ***Chromosome Representation***

One of the most important factors for effective application of the GA is creating an appropriate chromosomal structure. A GA starts with encoding the variables of the problem as finite-length strings. These strings are called chromosomes. The number of genes in a chromosome is equal to decision variables. As proposed model in this study is a non-linear problem containing three different types of variables (discrete, continuous, and binary), a real number representation is applied to reduce this complexity. Matrices X_1, X_2, and X_3 present the general form of chromosomes:

$$X_1 = \begin{pmatrix} p_1 & p_2 & \cdots & p_N \\ t_1 & t_2 & \cdots & t_N \end{pmatrix} \tag{5.26}$$

Matrix X_1 contains two rows and N columns. The two elements of each column represent the adopted production rate p (integer) and cycle time t (floating point) for each item respectively.

$$X_2 = \begin{pmatrix} \psi_1^1 & \psi_1^2 & \cdots & \psi_1^F \\ \psi_2^1 & \psi_2^2 & \cdots & \psi_2^F \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ \psi_N^1 & \psi_N^2 & \cdots & \psi_N^F \end{pmatrix} \tag{5.27}$$

X_2 is a $N \times F$ matrix of binary values for the variable ψ_i^w. N shows the total number of products, and F indicates the total number of production frequencies for all items. To have a feasible schedule for each column only one non-zero value of 1 must be generated, and the rest of elements are 0. The mechanism of generating 1 and 0 is random (1 and 0 indicate that the item is produced or not during cycle time T). Summation of values in each row shows the production frequency for each item.

When the GA generates a random initial population for variable ψ, the matrix may have more than one value of 1 in each column. Therefore, the columns of matrix ψ must be rechecked. If the number of value of 1 in per column is greater than one, the values of 1 must be changed to 0, and only one value of 1 should be kept. In order to decide which of those 1s must be converted to 0, a cost for each element is considered in the form of $\left(O \times d_i / p_i\right)$. The 1s with higher costs will be changed to 0, and only one element with the lowest cost is kept.

$$X_3 = \begin{pmatrix} \alpha_1^1, \beta_1^1 & \alpha_1^2, \beta_1^2 & \dots & \alpha_1^F, \beta_1^F \\ \alpha_2^1, \beta_2^1 & \alpha_2^2, \beta_2^2 & \dots & \alpha_2^F, \beta_2^F \\ . & . & . & . \\ . & . & . & . \\ . & . & . & . \\ \alpha_N^1, \beta_N^1 & \alpha_N^2, \beta_N^2 & \dots & \alpha_N^F, \beta_N^F \end{pmatrix} \tag{5.28}$$

As products are allowed to have more than one setup per cycle, it might cause an infeasible production plan. In order to attain feasibility, the production start time of some items can be advanced (α_i^j) or delayed (β_i^j). However, the start time for a product can be adjusted by either advancing or delaying, but not both. Matrix X_3, containing N rows and F columns, represents the chromosome for the floating point variables α_i^j and β_i^j.

(c) ***Initial Population***

The GA generates a randomly initial population of g chromosomes, where g denotes the size of population. Let $X_g = \{X_{g1}, X_{g2}, \dots, X_{gd}\}$ indicates the g^{th} chromosome in the population, and each solution X_g is a D-dimensional vector, where D refers to the optimization variables. Then, the GA updates boundaries for each variable using Eq. (5.29):

$$X_{gn} = lb_n + y \times (ub_n - lb_n) \quad \text{for } g = 1, 2, ..., N_{pop} \text{ and } n = 1, 2, ..., D \tag{5.29}$$

Where y is a random number in the range [0, 1], and lb_n and ub_n are the lower and upper bounds for the dimension n, respectively. For the integer variables, X_{gn} is rounded.

(d) ***Evaluation and Constraint Handling***

When chromosomes are produced, a fitness value must be assigned to chromosomes of each generation in order to evaluate them. This evaluation is achieved by the objective function given in Eq. (5.6) to measure the fitness of each individual in the population.

As shown in subsection 5.2.2, the presented mathematical model contains various constraints, which may lead to the production of infeasible chromosomes. In order to deal with infeasibility, the penalty policy is applied, which is the transformation of a constrained optimization problem into an unconstrained one. It can be attained by adding or multiplying a specific amount to/by the objective function value according to the amount of obtained constraints' violations in a solution. When a chromosome is feasible, its penalty is set to zero, while in case of infeasibility, the coefficient is selected sufficiently large (Pasandideh & Niaki, 2008). Therefore, the fitness function for a chromosome will be equal to the sum of the objective function value and penalties as shown in Eq. (5.30), where *s* represents a solution, and *C*(*s*) is the objective function value for solution *s*. The penalty policy is employed for all the metaheuristic algorithms presented in this research.

$$\begin{aligned} & fitness(s) = C(s) + Penalty(s) \\ & Penalty = 0 \qquad \text{if } s \text{ is feasible} \\ & Penalty > 0 \qquad \text{otherwise} \end{aligned} \tag{5.30}$$

(e) ***Selection***

Selection in a GA determines the evolutionary process flow. In each generation, individuals are chosen to reproduce offspring for the new population. Therefore, it provides the selection of the individual and the number of its copies which will be chosen as parent chromosomes. Usually, the fittest individuals will have a larger probability to be selected for the next generation.

In this research, the ''roulette wheel" method has been applied for the selection process. The basic thought behind this method is that every individual is provided an opportunity to become a parent proportional to its fitness value. All individuals have a chance of being selected to reproduce the next generation. Clearly, individuals with the larger fitness have a higher chance of being selected to form the mating pool for the next generation. The selection probability, a_g, for individual g with fitness C_g, is calculated by Eq. (5.31) (Gen et al., 2008):

$$a_g = \frac{C_g}{\sum_{g=1}^{Npop} C_g} \tag{5.31}$$

The selection procedure is based on spinning the wheel N_{pop} times, each time selecting a single chromosome for the new process.

(f) ***Crossover***

The crossover is the main operator of generating new chromosomes. It applies on two parent chromosomes with the predetermined crossover rate (P_c), and produces two offspring by mixing the features of parent chromosomes. It causes the offspring to inherit favorable genes from parents and creates better chromosomes. In this research, the arithmetic crossover operator that linearly combines parent chromosome vector is used to produce offspring. The two offspring are obtained using Eqs. (5.32) and (5.33).

$$offspring_{(1)} = y \times parent_{(1)} + (1-y) \times parent_{(2)} \tag{5.32}$$

$$offspring_{(2)} = y \times parent_{(2)} + (1-y) \times parent_{(1)} \tag{5.33}$$

Where y is a random number in the range [0, 1]. For variables p and t, the size of random numbers must be equal to N, and for variable F, it is equal to one, since F is an individual element. For variables ψ, α, and β, the random number returns an $N \times F$ matrix, where here $F = \min(offspring_{(1)}^{F}, offspring_{(2)}^{F})$, when the produced offsprings for variable F have different sizes. For the integer variables, the values of generated offsprings by Eqs. (5.32) and (5.33) are rounded to the nearest integer amount. Figure 5.1 shows how the crossover operator for the continuous variables t_i, α_i^j, and β_i^j works.

Parent 1	0.36	0.12	0.20

Parent 2	0.16	0.34	0.31

$y = 0.9$

Offspring 1	0.34	0.14	0.21

Offspring 2	0.18	0.32	0.30

Figure 5.1: An example of a crossover operation

(g) ***Mutation***

Mutation exerts stochastic change in chromosome genes with probability P_m. It is considered as a background operator that keeps genetic diversity within the population. Mutation may contribute in preventing the algorithm to get stuck at a local minimum as well as reaching an untimely convergence. It ensures that the irreversible loss of genetic information does not take place. First, a random number in range [0, 1] is generated. Then, if random number is less than 0.5, Eq. (5.33), and if it is greater than 0.5, Eq. (5.34) will be used to mutate the selected genes. Suppose a particular gene such as X_k is chosen for mutation; then the value of X_k will be changed to the new value X'_k using Eqs. (5.34) and (5.35):

$$X'_k = X_k - y \times \left(1 - \frac{\rho}{\text{max iter}}\right) \times (\mathrm{X}_k - lb_k) \tag{5.34}$$

$$X'_k = X_k + y \times \left(1 - \frac{\rho}{\text{max iter}}\right) \times (ub_k - X_k) \tag{5.35}$$

Where k is an integer value in range [1, N], lb_k and ub_k are the lower and upper bounds of the specific gene, y is a random variable in the range [0, 1], and ρ is the number of current iteration. For the integer variables, the values obtained by Eqs. (5.34) and (5.35) are rounded.

For the binary variable ψ, an integer number in the range [1, (N×F)] is generated in order to select the element of matrix ψ. Then, if the chosen element is 1, it will be replaced with 0, and vice versa. For variables α and β, first, an integer number in the range [1, (N×F)] is produced to select the element. Then, a random number in the range [0, 1] is generated. If it is less than 0.5, Eq. (5.36), and if it is greater than 0.5, Eq. (5.37) will be used to mutate the selected element.

$$X'_k = X_k - y \times \left(1 - \frac{\rho}{\text{max iter}}\right) \times (X_k - 0) \tag{5.36}$$

$$X'_k = X_k + y \times \left(1 - \frac{\rho}{\text{max iter}}\right) \times (1 - X_k) \tag{5.37}$$

(h) ***New Population***

Fitness function value of all members, including parents and offspring are assessed in this stage. Next, the chromosomes with higher fitness scores are selected to create a new population. To attain a better solution, the fittest chromosomes must be maintained at the end of this stage. Note that the number of selected chromosomes must be equal to N_{pop}.

(i) ***Termination***

The selection and reproduction of parents will be continued until the algorithm reaches a stopping criterion. The procedure can be ended after a predetermined number of iterations, or when no substantial improvement during any iteration is achieved. In this study, the procedure terminates when the algorithm reaches the maximum number of iterations which is set to 3000. Moreover, in order to have fair comparisons, the same number of iteration is used for all applied optimizers presented in this research.

5.3.2 PSO Algorithm

Every particle in the swarm has five individual properties: (i) position, (ii) velocity, (iii) objective function value related to the position, (iv) the best position explored so far by the particle, and (v) objective function value related to the best position of the particle. In any iteration of the PSO, the velocity and position of particles are updated according to Eqs. (5.38) and (5.39):

$$V_g(k+1) = WV_g(k) + c_1 y_1 \left(\lambda_g(k) - \chi_g(k)\right) + c_2 y_2 \left(\gamma_g(k) - \chi_g(k)\right) \tag{5.38}$$

$$\chi_g(k+1) = \chi_g(k) + V_g(k+1) \tag{5.39}$$

Where $g = 1, 2,\ldots, N_{pop}$; k denotes the iteration; V_g is the velocity of g^{th} particle, W is the inertia weight that controls the impact of the previous velocity of the particle on its current velocity, and it plays an important role in balancing global and local search ability of PSO; c_1 is the cognitive parameter, and c_2 is the social parameter; y_1 and y_2 are random numbers within the range [0, 1]; λ_g is the own best position found by particle g; χ_g is the current value of particle g; and γ_g is global best particle explored so far by the whole swarm.

For each particle in swarm, its fitness value will be evaluated. Then, each particle's fitness evaluation will be compared with the current particle's own best. If current value is better than own best, own best value will be set to the current value, and the own best location to the current location. Next, the fitness evaluation with the population's overall previous best will be compared. If the current value is better than global best, then global best will be set to the current particle's array index and value. The new velocity of each particle g is calculated using Eq. (5.38), and the position of particle g is updated by Eq. (5.39), which is adding its new velocity to its current position. This process continues until stopping condition is satisfied, which is reaching the maximum number of iterations equal to 3000.

5.3.3 ABC Algorithm

The ABC algorithm was inspired by the honey bee foraging behavior. The ABC method includes four main components, namely food sources, employed bees, onlooker bees, and scout bees. The main steps of the ABC procedure are described below.

(a) *Initialization of the Parameters*

The main parameters of the ABC algorithm are the colony size (*NB*), number of food sources (*NS*), number of trials after which a food source is supposed to be discarded (*limit*), and maximum number of cycles of the search process (*MCN*). Several combinations of *NB* and *limit* have been implemented by applying Taguchi method described in Section 5.4 The number of food sources (*NS*) is considered as *NB*/2. *MCN* is set to 3000.

The first half of the colony consists of the employed artificial bees and the second half includes the onlookers. For every food source, there is only one employed bee, i.e. the number of employed bees is equal to the number of food sources around the hive (Karaboga, 2005).

(b) *Initialization of the Population*

The ABC algorithm generates a randomly initial population of g solutions (g = 1, 2, ... , *NS*). Then the algorithm updates boundaries for each variable using Eq. (5.29). After initialization, the population of the food source positions is commanded to iterate the cycle of search processes for the employed, onlooker and scout bees (cycle = 1, 2, . . . , *MCN*).

(c) ***Employed Bee Phase***

Employed bees exploit the neighborhood of their location to select a random solution to be perturbed. In every cycle of the search process, they revise the positions of the food source in their memory according to the visual information, and measure the nectars' values (fitness) of the new positions (reformed solutions). Once all the employed bees finish the search process, they share the information such as nectar amounts of the food source, distance, and their positions with the onlooker bees. Then, every employed bee explores its neighbourhood food source (X_g) to generate a new food source (X_{new}) according to Eq. (5.40):

$$X_{new(n)} = X_{gn} + y \times \left(X_{gn} - X_{en} \right) \quad \text{for } e = 1, 2, ..., NS; n = 1, 2, ... ,D; \; e \neq g \tag{5.40}$$

Where X_g is a D-dimensional vector, and D refers to the optimization parameters; and y is a uniformly distributed random number in the range [-1, 1]. For the integer variables $X_{new(n)}$ is rounded. Once X_{new} is determined, it will be appraised and compared with X_g. If the quality of X_g is worse than X_{new}, X_g will be substituted with X_{new}; otherwise, X_g is kept. This means that a greedy selection process is utilized between the new candidate and old solutions.

(d) ***Onlooker Bee Phase***

Onlooker bees wait at the dance area around the hive to make decision for selecting food sources. The length of a dance is related to the nectar's quality (fitness value) of the food sources currently being utilized by the employed bees. They evaluate the nectars' information acquired from all the employed bees, and select food sources with probability related to their nectars' amounts. Subsequently, they produce new food information, discard the one with inferior quality compared to the old one, and share their information on the dance area. An onlooker bee appraises the food information obtained from employed bees, and chooses a food source (X_g) based on the probability (a_g) pertinent to the nectar's quality. a_g is determined using Eq. (5.41):

$$a_g = \frac{C_g}{\sum_{g=1}^{NS} C_g} \tag{5.41}$$

Where C_g is the fitness value of the g^{th} food source X_g. Once the onlooker bee has chosen its food source, it makes an adjustment on X_g using Eq. (5.40). The same process applied on the employed bees for the selection and replacement adjustment is also applied on the onlooker bees. If solution X_g cannot be improved, its counter holding trials is incremented by one; otherwise, the counter is reset to zero. This process is repeated until all onlookers are distributed into food source sites.

(e) ***Scout Bee Phase***

Scout bees perform a random search for new food sources, and substitute the discarded ones. If a food source (X_g) cannot be further improved by a predetermined number of trials (*limit*), associated employed bee will abandon the food source, and that employed bee will become a scout. The counters, which are updated during search, are utilized in order to decide if a source is to be abandoned. If the value of the counter is greater than the *limit*, then the source associated with this counter is assumed to be exhausted and is abandoned. The food source abandoned by its bee is replaced with a new food source discovered by the scout (Akay & Karaboga, 2012).

The scout randomly generates a food source. Equation (5.29) is used for this purpose. In the basic ABC process, during per cycle at most one scout goes outside for exploring a new food source. After the new position is specified, a new algorithm cycle begins. After each cycle, the finest solution will be memorized. The same processes are iterated until the termination condition is reached.

5.3.4 SA Algorithm

The SA algorithm is a local search method inspired by the physical annealing process studied in statistical mechanics and was initially proposed for combinatorial optimization problems. The SA algorithm repeats an iterative neighbor generation procedure and follows search directions that aim to improve the objective function value towards the global optimum. The main steps of the SA algorithm are described below.

(a) ***Initialization***

In this step, the input parameters of the SA algorithm are initialized. The parameters are:

i. Initial Temperature

The initial temperature (E_0) is the starting point of temperature computation in every iteration. E_0 should be adequately high to escape a premature convergence. Basically, the SA algorithm starts with an initial temperature where almost all worsening moves are accepted regardless of the objective function value.

ii. Iteration

It shows the number of iteration in each temperature.

iii. Final Temperature

The temperature is remained fixed once it reaches the lowest temperature limit (E_f).

(b) ***Cooling Schedule***

System temperature determines the degree of randomness towards solution, and it is reduced with a known plan in accordance with the progress of solution procedure. In reality, system temperature is a solution subspace of the problem accepted in each iteration. As the algorithm progresses and the temperature decreases, inappropriate solutions have smaller chance of being accepted. Cooling schedule determines the functional form of the change in temperature required in the SA.

A geometric temperature reduction rule, which is the most commonly utilized decrement rule, is applied for this study. If the temperature at k^{th} iteration is E_k, then the temperature at $(k+1)^{th}$ iteration is given by (Kirkpatrick, 1984):

$$E_{k+1} = z \times E_k \tag{5.42}$$

Where z denotes the cooling factor in the range [0, 1].

(c) ***Neighborhood Representation***

The neighborhood search structure is a procedure that generates a new solution that slightly changes the current solution. To delineate the neighborhood configuration, the following process is used in order to prevent the fast convergence of the SA. The number of neighborhood searches in each temperature level (epoch length) is considered to be 10. It is the number of solutions which are accessible in an immediate move from the current solution.

Suppose a particular vector such as X_k is selected; then the value of X_k will be changed to the new value X'_k using Eqs. (5.43) and (5.44).

$$X'_k = X_k - y \times 0.1 \times (X_k - lb_k) \tag{5.43}$$

$$X'_k = X_k + y \times 0.1 \times (ub_k - X_k) \tag{5.44}$$

Where k is an integer value in range [1, N], lb_k and ub_k are the lower and upper bounds of the specific vector, and y is a random variable in the range [0, 1]. For the integer variables the values obtained by Eqs. (5.43) and (5.44) are rounded.

For the binary variable ψ, two integer numbers y_1 and y_2 are generated, where y_1 is in range [1, $(N \times F)$ -1] and y_2 is in range [y_1+1, $N \times F$] in order to select the elements of matrix ψ. Then, if the elements in the selected columns are 1s, they will be replaced with 0s, and vice versa.

For variables α and β, the same integer numbers y_1 and y_2 are used to select the elements. Then the following equations are employed to change the chosen elements:

$$X'_{(y_1:y_2)} = X_{(y_1:y_2)} - y \times 0.1 \times \left(X_{(y_1:y_2)} - 0 \right) \tag{5.45}$$

$$X'_{(y_1:y_2)} = X_{(y_1:y_2)} + y \times 0.1 \times \left(1 - X_{(y_1:y_2)} \right) \tag{5.46}$$

Where y is a random number in the range [0, 1]. The quantity of generated random numbers must be equal to $(y_2 - y_1 + 1)$.

For selecting which equation to be used, a random number in the range [0, 1] is generated. Then, if random number is less than 0.5, Eqs. (5.43) and (5.45), and if it is greater than 0.5, Eqs. (5.44) and (5.46) will be used.

(d) ***Main Loop of the SA***

The SA begins with a high temperature and selects initial solutions (s_0) randomly. Next, a new solution (s_n) within the neighbourhood of the current solution (s) is computed in each iteration. In the minimization problem, if the value of the objective function, $C(s_n)$, is smaller than the previous value, $C(s)$, the new solution is accepted. Otherwise, the SA algorithm uses a stochastic function given in Eq. (5.47) for accepting the new solution in order to prevent the local optimum trap.

$$a = \exp(-\Delta C / E) \tag{5.47}$$

Where $\Delta C = C(s_n) - C(s)$, and E is the current state temperature.

(e) ***Termination Condition***

The algorithm ends after a pre-set number of iterations without refining the current best solution.

5.4 Parameter Tuning

The trial and error method are traditionally used to determine parameters' values of metaheuristic algorithms. However, it cannot determine optimal parameter settings and hence cannot solve problems efficiently. This method also consumes a considerable amount of time (Lin et al., 2012). In metaheuristic algorithms the parameters are controllable factors, the problem being solved is the process input, and the fitness function is the process output. Therefore, instead of employing suggested values by other researchers or using a trial and error procedure, it seems reasonable to adjust the parameters using statistical methods based on a set of experiments such as Taguchi method (Sadeghi et al., 2013).

The motivation to utilize the Taguchi method in this research is that it has been recognized as a cost-effective method that can simultaneously scrutinize several factors and distinguish quickly the factors with principal impacts on final solution by carrying out the minimal number of possible experiments. Taguchi method is a fractional factorial experiment as an efficient alternative for full factorial experiments. Taguchi divides the factors affecting the performance (response) of a process into two groups: noise factors (N) that cannot be controlled and controllable factors (S) such as the parameters of a metaheuristic algorithm which can be controlled by designers. Taguchi method focuses on the level combinations of the control parameters to minimize the effects of the noise factors.

In Taguchi's parameter design phase, an experimental design is used to arrange the control and noise factors in the inner and outer orthogonal arrays respectively. The orthogonal arrays are used to study a large number of decision variables with a small number of experiments. Afterward, the signal-to-noise (*S*/*N*) ratio is computed for each experimental combination. After the calculation of the *S*/*N* ratios, these ratios are analyzed to determine the optimal control factor level combination (Mousavi et al., 2014). Taguchi categorizes objective functions into three groups: First, smaller is better for which the objective function is a minimization type. Second, nominal is the best; for which the objective function has modest variance around its target. Third, bigger is better, where the objective function is a maximization type (Sadeghi et al., 2013).

Since the objective function of the model of this research is the minimization type, ‘‘the smaller the better’’ category is appropriate, where *S*/*N* is given by Eq. (5.48):

$$S/N = -10\log_{10}\left(\frac{1}{n}\sum_{e=1}^{n} C_e^2\right) \tag{5.48}$$

Where C_e is the objective function value of a given experiment *e*, and *n* is the number of times the experiment is performed.

In the Taguchi method, the parameters that might have considerable effects on the process output are initially chosen for tuning. The parameters of the GA that require calibration are P_c, P_m, and N_{pop}. In the PSO, c_1, c_2, *W*, and N_{pop} are the parameters to be tuned. In the SA, the parameters to be tuned are E_0, E_f, and *z*. *NB* and *limit* are the ABC parameters to be calibrated. Afterward, by a trial and error method, the ranges that produce satisfactory fitness function are chosen to employ the experiments.

Table 5.1 shows the algorithms’ parameters, each at three levels with nine observations. Figures 5.2 to 5.5 shows the mean *S*/*N* ratio plots for different parameter levels of the proposed algorithms. According to Figures 5.2 to 5.5, the best parameter levels are the highest mean of *S*/*N* values. Table 5.2 shows the optimal levels of the parameters for all algorithms.

Table 5.1: The GA, PSO, ABC, and SA parameters' levels

Algorithm	Parameters	Levels		
		1	**2**	**3**
GA	P_c	0.1	0.2	0.3
	P_m	0.8	0.9	1
	N_{pop}	100	150	200
PSO	*c1 & c2*	0.5,4	4,0.5	4,4
	W	0.55	0.75	0.95
	N_{pop}	100	150	200
ABC	*NB*	50	100	200
	Limit	2	50	100
SA	E_0	10	20	30
	E_f	0.01	0.01	0.001
	z	0.85	0.90	0.95

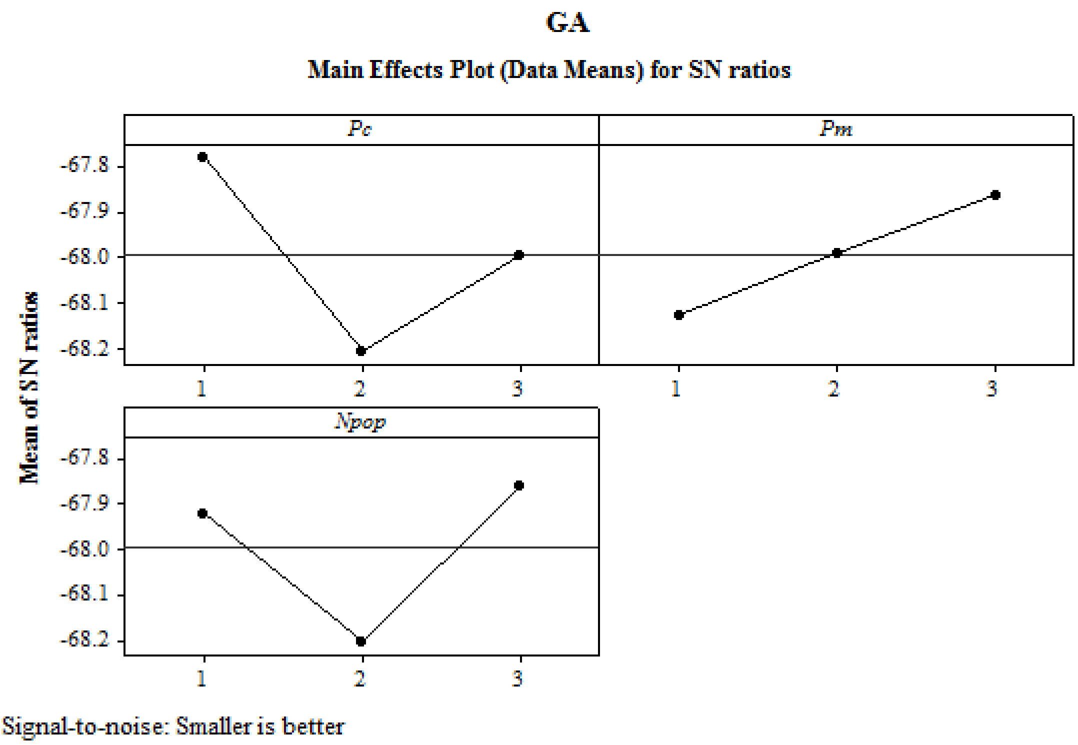


Figure 5.2: The mean *S*/*N* ratio plot for each level of the factors of the GA approach

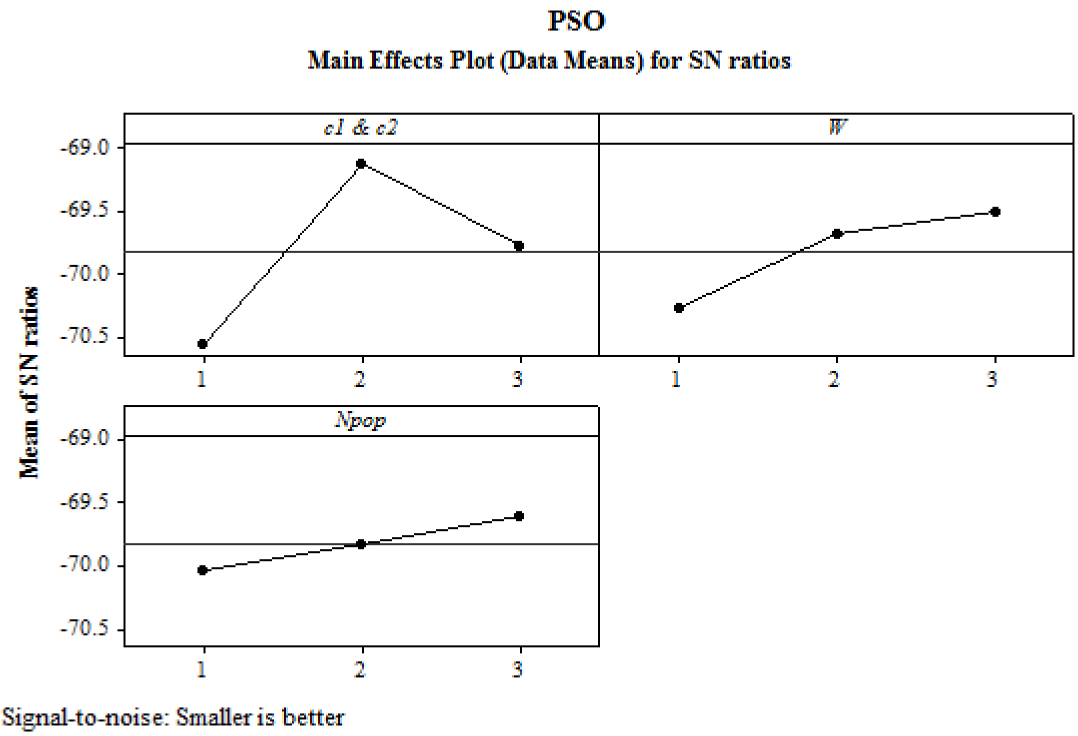


Figure 5.3: The mean *S*/*N* ratio plot for each level of the factors of the PSO algorithm

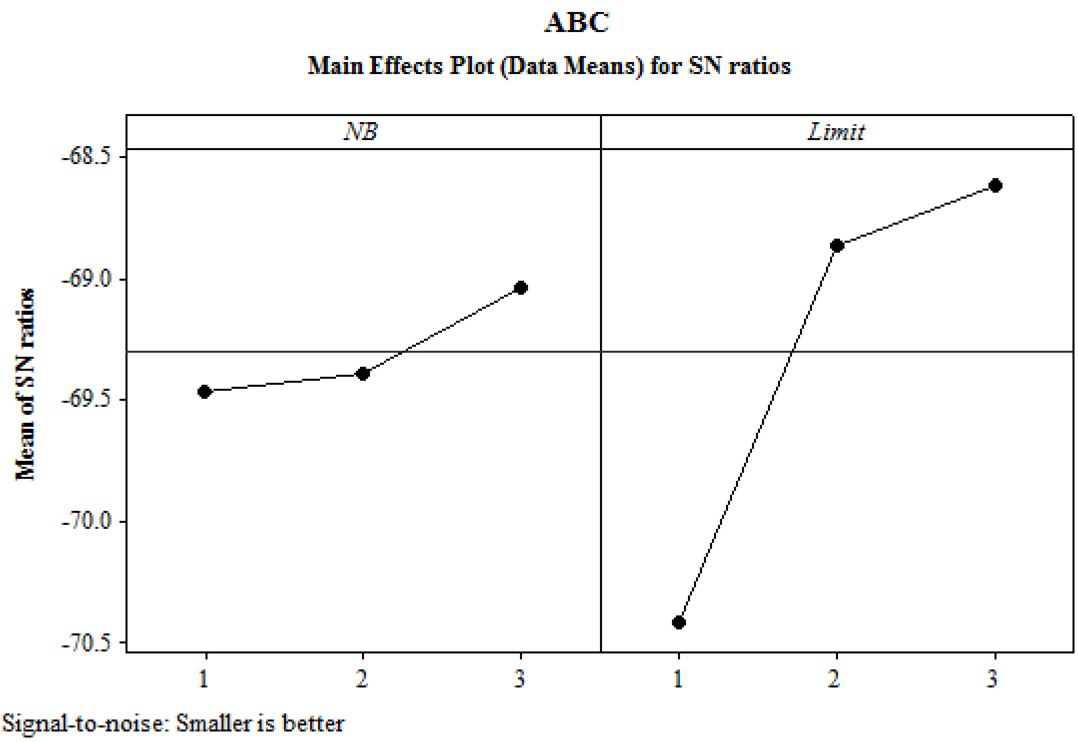


Figure 5.4: The mean *S*/*N* ratio plot for each level of the factors of the ABC algorithm

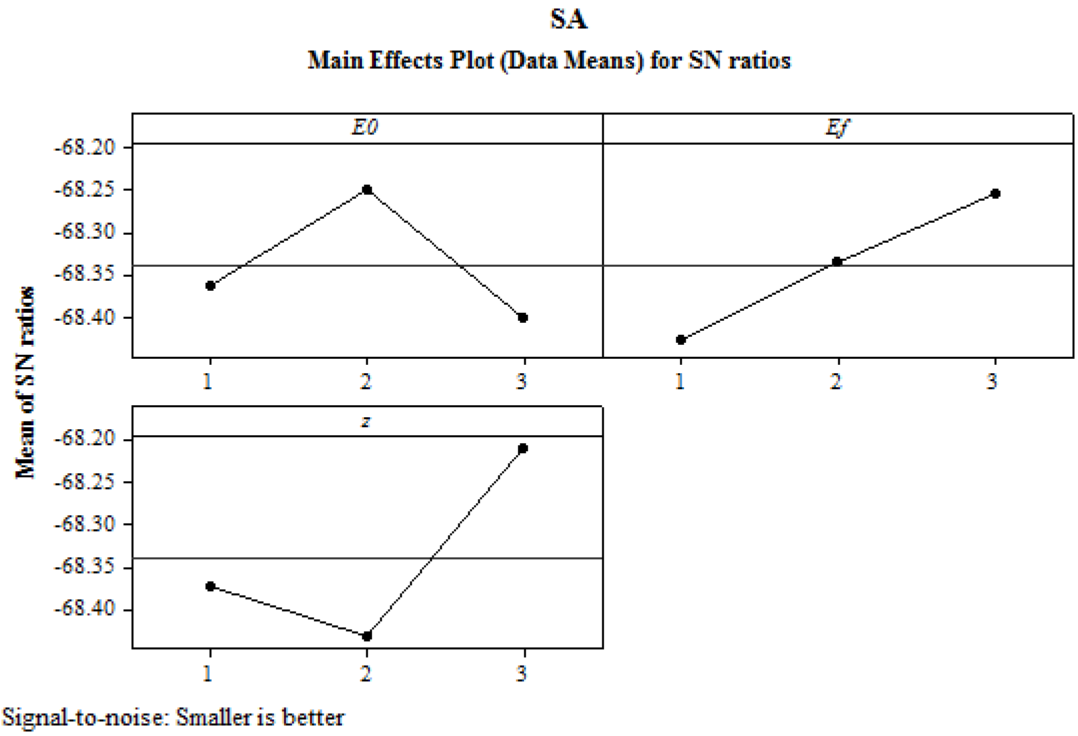


Figure 5.5: The mean *S*/*N* ratio plot for each level of the factors of the SA algorithm

Table 5.2: Optimal values of the algorithms' parameters

Algorithm	Parameters	Optimal Values
GA	P_c P_m N_{pop}	0.1 1 200
PSO	$c1$ & $c2$ W N_{pop}	4, 0.5 0.95 200
ABC	*NB* *Limit*	200 100
SA	E_0 E_f z	20 0.001 0.95

5.5 Results and Discussions

In order to illustrate the performance of the four metaheuristic approaches on the proposed ELSP model, a three-product inventory problem is investigated using the data given in Table 5.3. Applied optimizers were written and coded in MATLAB programming software.

Table 5.3: Input data for the ELSP model

Item *i*	d_i	P_i^{max}	A_i	S_i	H_i	B_i	L_i
1	1000	3000	0.0005	125	3	5	0.20
2	400	2500	0.0010	25	25	50	0.11
3	700	2500	0.0015	75	15	25	0.20

In order to validate the proposed model, three similar models presented by Silver (1995), Viswanathan and Goyal (2000), and Yan et al. (2013) were selected. The same input data used in the previous models as well as their obtained results for the decision variables were tested on the new ELSP model. In addition, for the variables which were missing in their studies, the value of zero was assigned. After running the proposed ELSP model, the same results for the three objective functions were obtained as acquired by the previous offered models. This approach was used to verify the correctness of the developed ELSP model.

In order to compare the performances of the four algorithms, 10 different optimization runs have been carried out with the parameters settings given in Table 5.2. The statistical optimization results for the ELSP along with the required CPU time for machine operating costs of $1000, $750 and $500 using the GA, PSO, ABC, and SA methods are reported in Tables 5.4 to 5.6 respectively.

Table 5.4: Objective function values for machine operating cost $1000

Problem No.	**GA**		**PSO**		**ABC**		**SA**	
	Fitness ($)	CPU (s)	Fitness ($)	CPU (s)	Fitness ($)	CPU (s)	Fitness ($)	CPU (s)
1	2396.73	176.94	2490.24	161.95	2394.06	124.17	2498.21	108.76
2	2398.66	162.21	2498.24	159.46	2399.17	137.39	2504.15	102.31
3	2398.84	167.83	2500.24	171.18	2400.73	125.27	2506.99	100.43
4	2399.24	168.35	2594.25	160.33	2401.22	143.12	2513.32	104.94
5	2399.35	179.11	2609.94	160.13	2401.73	121.14	2517.9	110.23
6	2400.75	169.84	2621.92	158.88	2444.51	128.65	2518.36	106.73
7	2401.02	176.41	2634.34	181.38	2469.86	129.98	2528.85	109.65
8	2406.10	147.90	2639.11	158.79	2476.08	141.93	2530.66	104.72
9	2417.43	172.10	2644.21	158.99	2492.79	129.43	2532.44	119.63
10	2424.24	167.27	2644.23	158.31	2492.87	115.33	2536.62	101.17

Table 5.5: Objective function values for machine operating cost $750

Problem No.	GA		PSO		ABC		SA	
	Fitness ($)	CPU (s)	Fitness ($)	CPU (s)	Fitness ($)	CPU (s)	Fitness ($)	CPU (s)
1	2243.23	158.99	2138.37	166.37	2038.73	179.59	2238.49	103.29
2	2295.15	145.05	2237.71	159.27	2137.48	126.90	2275.81	103.76
3	2298.74	110.71	2305.74	158.54	2142.79	146.28	2301.82	104.53
4	2300.22	147.02	2305.74	158.99	2167.39	166.39	2306.48	104.49
5	2303.54	142.34	2308.37	167.42	2172.52	140.82	2328.97	103.41
6	2306.81	151.67	2328.78	159.66	2192.55	167.53	2336.58	104.86
7	2314.24	142.64	2365.45	158.64	2193.29	155.43	2341.56	102.97
8	2343.53	144.17	2432.56	166.25	2222.07	162.86	2353.86	96.92
9	2377.65	150.14	2432.56	165.81	2228.68	158.49	2376.73	105.36
10	2390.32	111.38	2603.10	160.33	2300.10	166.13	2435.55	109.21

Table 5.6: Objective function values for machine operating cost $500

Problem No.	GA		PSO		ABC		SA	
	Fitness ($)	CPU (s)	Fitness ($)	CPU (s)	Fitness ($)	CPU (s)	Fitness ($)	CPU (s)
1	1839.20	183.57	1862.94	129.48	1822.02	137.63	1877.17	103.57
2	1845.08	175.29	1862.94	129.44	1824.71	133.10	1879.16	98.89
3	1846.07	173.45	1865.78	123.34	1830.80	127.20	1880.56	102.53
4	1854.96	177.88	1869.82	128.96	1831.72	138.68	1888.96	100.97
5	1858.05	181.62	1878.10	129.34	1837.27	125.17	1892.31	97.12
6	1859.20	164.85	1880.13	160.20	1851.69	149.43	1898.11	102.32
7	1869.27	187.43	1881.90	164.11	1856.47	132.57	1900.46	102.34
8	1884.38	186.49	1887.09	129.58	1867.34	141.26	1902.28	103.26
9	1895.32	173.80	1892.13	158.04	1887.28	140.43	1904.28	105.64
10	1896.42	172.92	1901.17	130.01	1899.74	139.81	1908.28	103.56

Furthermore, the data set was also applied on the ELSP models proposed by Silver (1995), Viswanathan and Goyal (2000), and Yan et al. (2013) in order to compare the performance of the developed model with previous reported models in literature. Table 5.7 represents the minimum cost found by each algorithm accompanied with obtained results by previous presented models for different values of annual machine operating cost.

Table 5.7: Comparisons of the best obtained solutions using four optimization engines and previous models

Operating Cost	GA	PSO	ABC	SA	Silver (1995)	Viswanathan and Goyal (2000)	Yan et al. (2013)
O = $1000	$2396.73	$2490.24	$2394.06	$2498.21	$3678	$3091	$2788
O = $750	$2243.23	$2138.37	$2038.73	$2238.49	$3427	$2884	$2590
O = $500	$1839.20	$1862.94	$1822.02	$1877.17	$3177	$2666	$2454

Figure 5.6 shows the optimization performance of the four metaheuristic methods for the best run compared to the values found by other models in terms of objective function values for different machine operating costs.

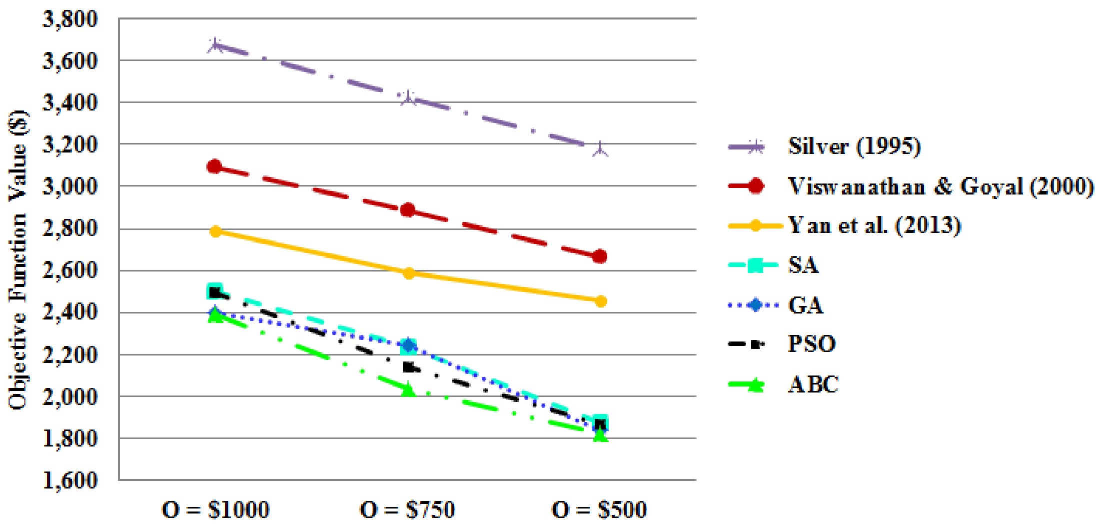


Figure 5.6: Graphical representation of the performance comparison between the applied metaheuristic algorithms and previous methods in terms of objective function value

It can be interpreted from Table 5.7 that all metaheuristic methods found the minimum total cost among all methods reported in the literature. It is notable that the ABC algorithm found the best known solutions, and outperformed the proposed GA, SA, and PSO methods for all machine operating costs. The ABC algorithm is also competitive to the GA approach. The ABC obtained approximately 14%, 21%, and 26% reduction in total cost for machine operating costs $1000, $750, and $500 respectively compared to the two-stage heuristic algorithm proposed by Yan et al. (2013).

It is evident that if a product is allowed to be produced more than once per cycle a lower total cost will be generated. Hence, the results approve the findings pointed out earlier by Goyal (1994), Viswanathan (1995), and Yan et al. (2013) regarding obtaining a lower cost while production of items more than once in a cycle. Assumption made by Silver (1995) and Viswanathan and Goyal (2000) to produce every item exactly once per cycle led to higher total costs. However, backordering was ignored in Silver (1995). According to Viswanathan and Goyal (2000), incorporating backordering in the model generates a lower cost compared to the model ignoring backorders.

Figure 5.7 represents the trend of best objective function values obtained by the applied metaheuristic algorithms for all machines operating costs.

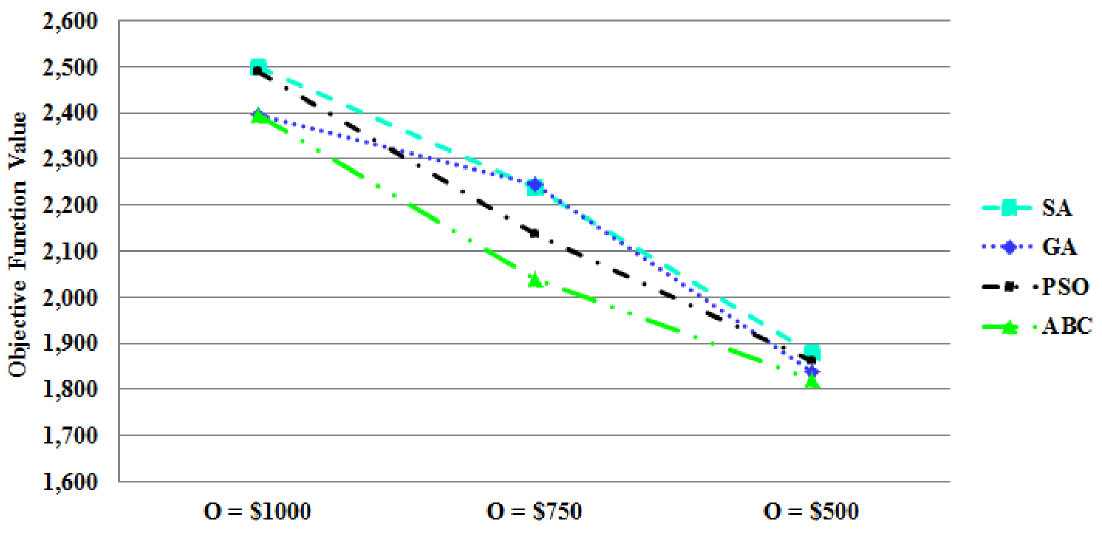


Figure 5.7: Trend of best objective function values obtained by applied algorithms for different machine operating costs

Figure 5.8 shows the performance of four metaheuristic approaches in terms of objective function values for machine operating cost $1000 in 10 runs. The figure shows that the ABC and GA performed better than PSO and SA.

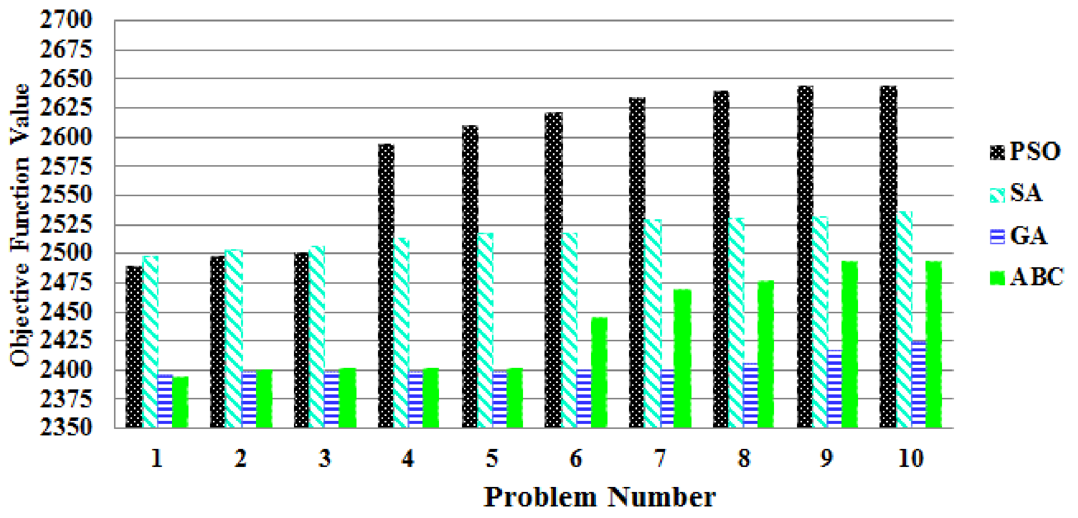


Figure 5.8: Comparison of applied metaheuristics in terms of objective function value for machine operating cost $1000

The computation time (CPU time) of applied metaheuristics for machine operating cost $1000 in 10 runs is illustrated in Figure 5.9.

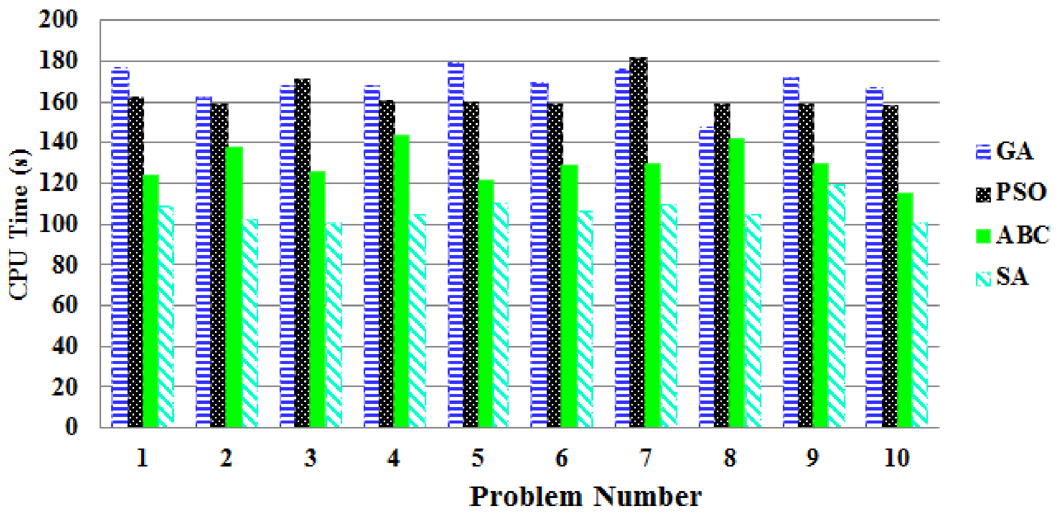


Figure 5.9: Comparison of applied metaheuristics in terms of CPU time for machine operating cost $1000

As it can be observed, the SA algorithm performed better in terms of CPU time. The ABC method stands at second rank in terms of convergence speed. However, GA took a longer computation time than the other methods.

The model's variables obtained by the ABC method for the best solutions presented in Table 5.7 for all machine operating costs are summarized in Table 5.8.

Table 5.8: Summary of optimization results obtained by the ABC algorithm

Item i	p_i	r_i	f_i	t_i	Q_i	m_i	T
				\$1000			
1	3000	0.33	1	0.30	300	75	
2	2500	0.16	3	0.10	40	11	0.30
3	2500	0.28	2	0.15	105	28	
				\$750			
1	3000	0.33	2	0.165	165	41	
2	2500	0.16	3	0.11	44	12	0.33
3	2500	0.28	3	0.11	77	21	
				\$500			
1	2800	0.36	2	0.20	200	48	
2	2000	0.20	4	0.10	40	11	0.40
3	2000	0.35	3	0.133	93	23	

The results also showed that for each item the storage period has not exceeded than its shelf life, which avoids the spoilage of products.

Sarker and Babu (1993) revealed that when machine operating cost is considered, reduction of the cycle time can be more cost effective. Likewise, the results in this research indicated that with rather high machine operating cost, decreasing cycle time yields a lower cost than decreasing the production rates. Adversely, when the machine operating cost decreases, production rate reduction produces a lower cost. For instance, when the machine operating cost is \$1000, decreasing cycle time yields a lower cost than reduction in the production rates. For machine operating cost \$500, the production rate of item 1 is reduced from 3000 to 2800, and for items 2 and 3, the production rates have been diminished from 2500 to 2000; while, cycle time *T* is increased. It shows that production rate reduction generates a lower cost than the cycle time reduction. Silver

(1989) declared that if the manufacturing rate is decreased, the production should be performed for a longer period.

Silver (1989) also stated that in the absence of production cost, i.e O = \$0, the reduction in production rate is more cost-effective than decreasing the cycle time. For instance, O = \$0 was examined by the ABC method, and the production rate for item 1, 2, and 3 decreased to 2000, 800, and 1400 respectively. Furthermore, total cost of \$1040 was obtained, which is much less than the results reported in previous works (45% reduction in total cost compared to Yan et al. (2013)). However, as machine operating cost \$0 is not probable in practice, it is not tested on the problem instances.

The order of production of items, and their frequency for machine operating costs \$1000, \$750, and \$500 obtained by ABC method are shown in matrices X_4, X_5, and X_6 respectively.

$$X_4 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

$$X_5 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$X_6 = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \end{bmatrix}$$

As the number of production frequency increases, production time conflicts may occur between production start times of some products. If the schedule is feasible, there is no need to adjust the production times as there is no overlapping between production times of the products. However, if conflicts exist in production times, some adjustments are necessary to attain feasibility. Figure 5.10 shows the production schedule over the first production cycle for three items with machine operating cost $1000.

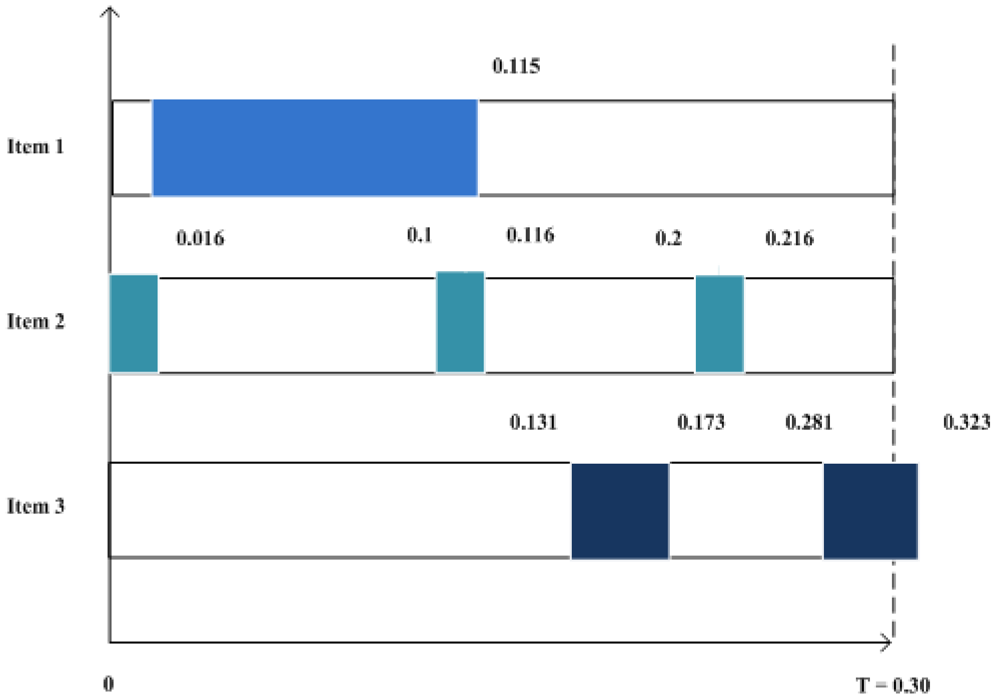


Figure 5.10: Production schedule before adjustment for machine operating cost $1000

It can be seen that the second batch of item 2 has manufacturing time conflict with the first batch of item 1, showing the impracticality of the existing schedule. Item 1 has to be produced from time 0.016 to 0.115, while second batch of item 2 must be produced from 0.1 to 0.116. Therefore, for achieving a feasible schedule, the start time for the second batch of item 2 is delayed from 0.1 to 0.115. Moreover, second batch of item 3 has to be manufactured during the period 0.281 to 0.323. However, according to Eq. (5.21), the last batch cannot surpass the overall cycle time. Hence, production start time of the second batch of item 3 is advanced from 0.281 to 0.258.

The advancement and delay of production start times (time units) in each cycle time T for each instance are shown in Tables 5.9 to 5.14.

Table 5.9: Production start time advancements for machine operating cost $1000

α	***j* = 1**	***j* = 2**	***j* = 3**	***j* = 4**	***j* = 5**	***j* = 6**
i = 1	0	0	0	0	0	0
i = 2	0	0	0	0	0	0
i = 3	0	0	0	0	0	0.023

Table 5.10: Production start time delays for machine operating cost $1000

β	***j* = 1**	***j* = 2**	***j* = 3**	***j* = 4**	***j* = 5**	***j* = 6**
i = 1	0	0	0	0	0	0
i = 2	0	0	0.015	0	0	0
i = 3	0	0	0	0	0	0

Table 5.11: Production start time advancements for machine operating cost $750

α	***j* = 1**	***j* = 2**	***j* = 3**	***j* = 4**	***j* = 5**	***j* = 6**	***j* = 7**	***j* = 8**
i = 1	0	0	0	0	0	0.0218	0	0
i = 2	0	0	0	0	0	0	0	0
i = 3	0	0	0	0	0.0520	0	0	0

Table 5.12: Production start time delays for machine operating cost $750

β	***j* = 1**	***j* = 2**	***j* = 3**	***j* = 4**	***j* = 5**	***j* = 6**	***j* = 7**	***j* = 8**
i = 1	0	0	0	0	0	0	0	0
i = 2	0	0	0	0	0	0	0	0
i = 3	0	0	0	0	0	0	0	0

Table 5.13: Production start time advancements for machine operating cost $500

α	***j* = 1**	***j* = 2**	***j* = 3**	***j* = 4**	***j* = 5**	***j* = 6**	***j* = 7**	***j* = 8**	***j* = 9**
i = 1	0	0	0	0	0	0.0414	0	0	0
i = 2	0	0	0	0	0.0414	0	0	0	0
i = 3	0	0	0	0	0	0	0	0	0.0045

Table 5.14: Production start time delays for machine operating cost $500

β	***j* = 1**	***j* = 2**	***j* = 3**	***j* = 4**	***j* = 5**	***j* = 6**	***j* = 7**	***j* = 8**	***j* = 9**
i = 1	0	0	0	0	0	0	0	0	0
i = 2	0	0	0	0.0385	0	0	0	0	0
i = 3	0	0	0	0	0	0	0.0255	0	0

Furthermore, to display the convergence of the applied GA, PSO, ABC, and SA methods, the graphic illustrations of convergence path corresponding to fitness function in terms of the iteration number for machine operating cost $1000 are shown in Figures 5.11 to 5.14 respectively.

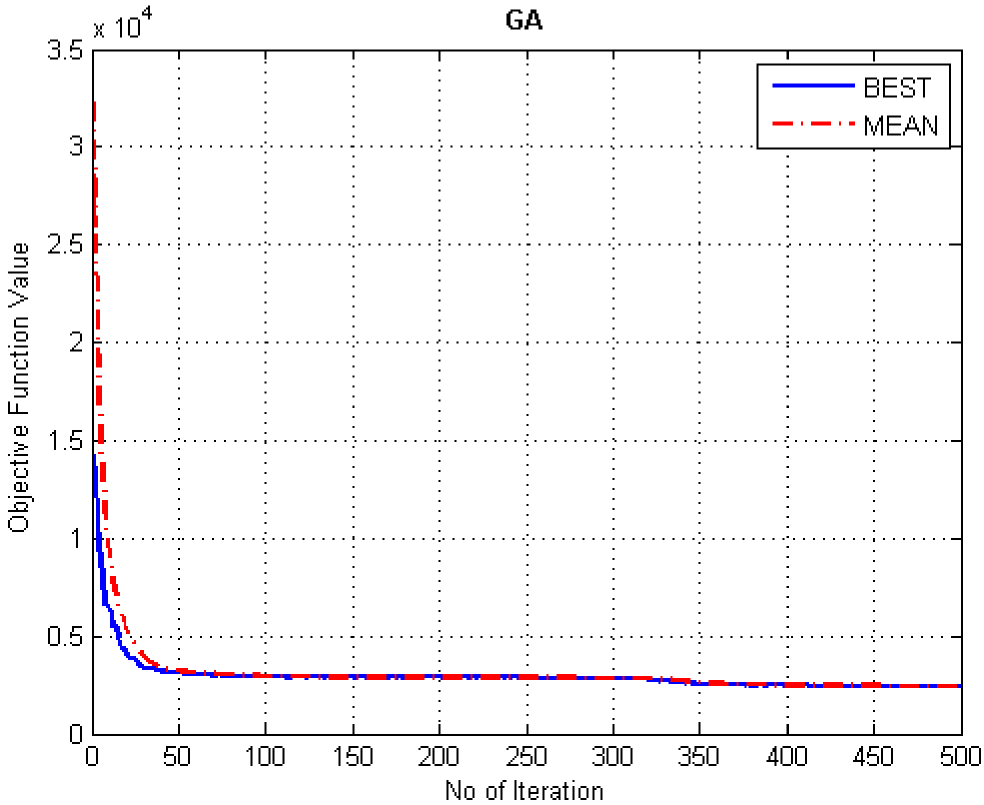


Figure 5.11: Convergence path of fitness function for machine operating cost $1000 by GA approach

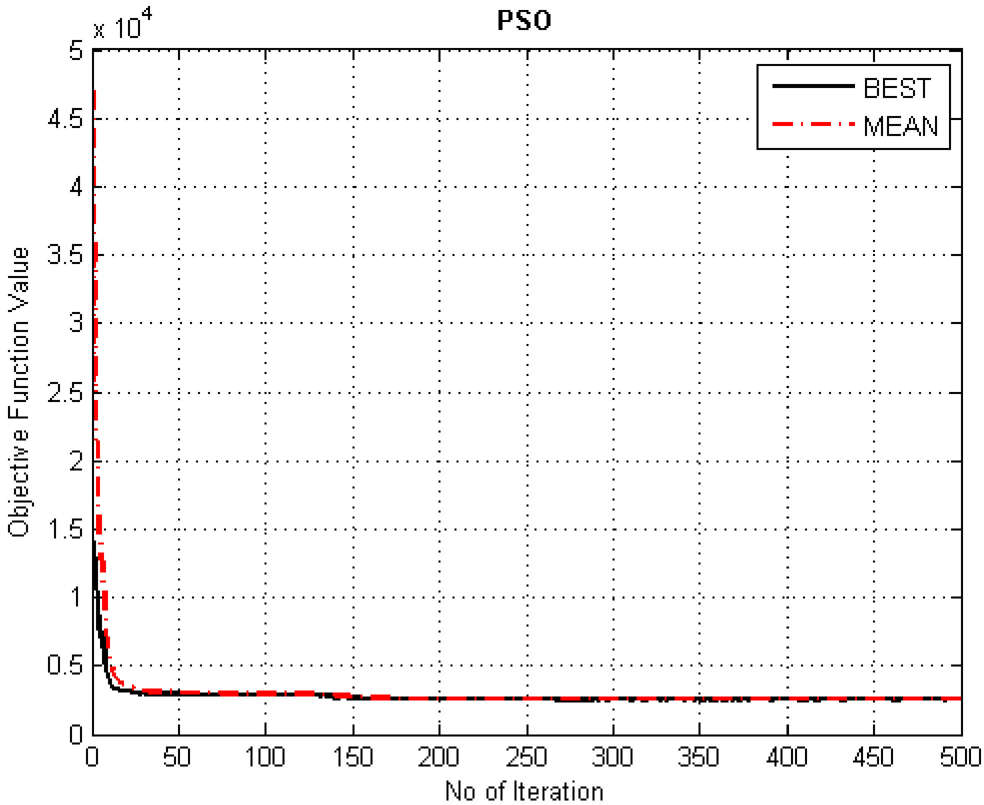


Figure 5.12: Convergence path of fitness function for machine operating cost $1000 by PSO algorithm

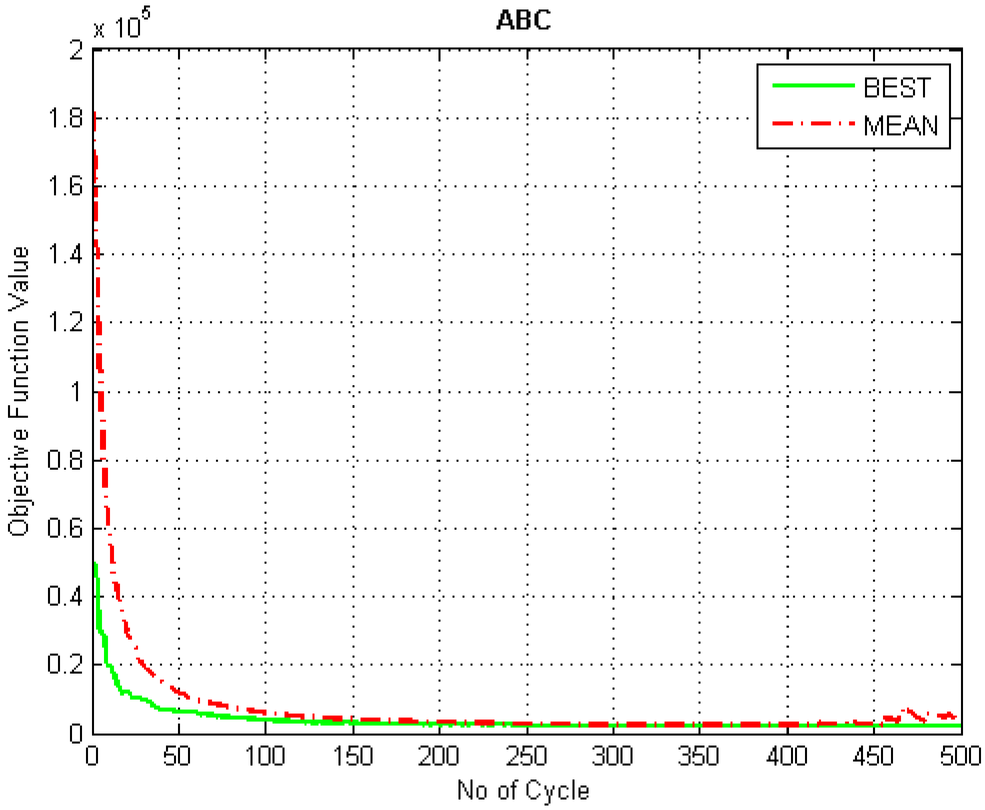


Figure 5.13: Convergence path of fitness function for machine operating cost \$1000 by ABC algorithm

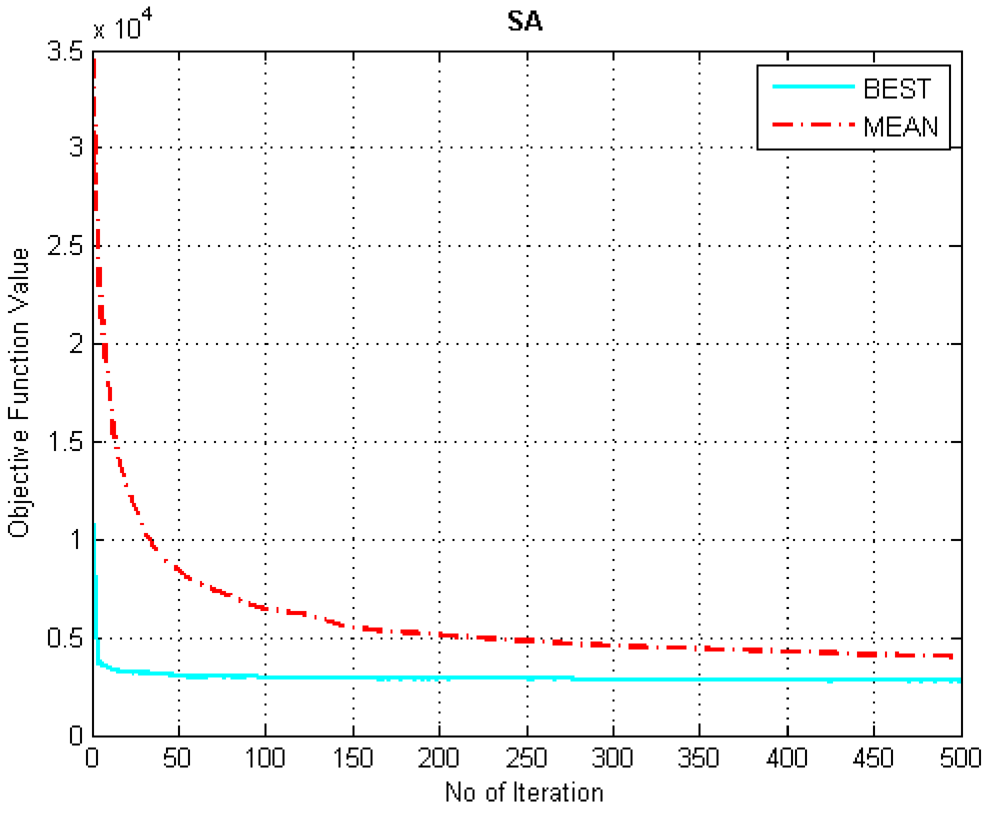


Figure 5.14: Convergence path of fitness function for machine operating cost \$1000 by SA algorithm

All methods are iterated until the fitness value of the “best-so-far” chromosome stabilizes and does not change for many generations. This means the algorithm has converged to a solution(s). As it is shown in Figures 5.11 to 5.14, all algorithms reached to around \$3000 (near optimum point) in almost 200 iterations. However, from iteration 200 to 500, the cost is decreased to around \$2400, which this amount of cost reduction has remarkable impact on the profit gained for industries. Therefore, as the problem is cost minimization, the iterations have been continued until the search reaches the lowest possible cost. The changes in the latest iterations are not noticeable because the scales of vertical axis in figures start from a very high value (e.g. 3.5×10^4).

Furthermore, the time taken to reach the optimal solution depends on the computer used. Hence, it is worth to utilize a high performance computer such as parallel computers in order to reach better solutions in reasonable computation time.

To compare the performance of the metaheuristic algorithms statistically, the one-way analysis of variance (ANOVA) is utilized based on the objective function values of 10 experiments obtained for machine operating cost \$1000. This process is performed using Minitab software. Table 5.15 shows the ANOVA results.

Table 5.15: The ANOVA results for objective function values of machine operating cost \$1000

Source	**Degree of freedom (DF)**	**SS**	**MS**	**F-test**	***p*-value**
Optimization Engines	3	204627	68209	43.54	0.000
Error	36	56398	1567		
Total	39	261025			

The *p*-value is a measure of how unusual the value of the test statistic is given that the null hypothesis is true. Here, the hypothesis is there is no difference in the average costs of the algorithms. The null hypothesis is rejected when the *p*-value turns out to be less than a predetermined significance level. The *p*-value is a number between 0 and 1, and for 95% confidence level it is interpreted in the following way:

i. A small *p*-value (typically ≤ 0.05) indicates strong evidence against the null hypothesis, so the null hypothesis is rejected.
ii. A large *p*-value (> 0.05) indicates weak evidence against the null hypothesis, so it is failed to reject the null hypothesis.
iii. *p*-values very close to the cutoff (0.05) are considered to be marginal (could go either way).

The *p*-value obtained from the results is 0.000, which indicates the null hypothesis is rejected at 95% confidence level, meaning that the mean values of total cost of four algorithms are not all the same.

5.6 Conclusions

In this chapter, a mixed-integer non-linear model was addressed which considers the practical characteristics including backordering, shelf life, and multiple setups for each product in a manufacturing cycle. The problem of obtaining the optimum production rate, lot size, and production frequency for each item, the optimal production cycle time for all the products, in addition to a feasible manufacturing schedule was investigated. However, the assumption of production of items more than once in a cycle may cause an infeasible schedule due to the overlapping production times of various items. To eliminate the production time conflicts and achieve a feasible schedule, the production start time of some items was modified by either advancing or delaying.

The solution of the large scale proposed ELSP model may be out of reach using the existing approaches due to the complexity and the required computational efforts associated with the model. Thus, efficient heuristic methods are required to solve the proposed NP-hard model. Accordingly, effective solution approaches based on real-coded GA, PSO, ABC, and SA algorithms for integer, non-integer, and binary variables were presented to solve the model. Furthermore, to make the algorithms more effective, Taguchi method was employed to tune various parameters of the applied algorithms.

Each of such methods was applied to a set of problem instances taken from literature and the performances were compared against other existing models in the literature. The results indicated the efficiency of the applied metaheuristic algorithms in solving the proposed model. Comparisons were based on the percentage improvement in the total cost. All the applied methods showed an impressive performance and excellent solution qualities. The metaheuristic algorithms can also efficiently handle large-sized instances in a moderate computation time. Based on the results, the ABC algorithm produced the lowest cost, which may indicate its superiority in searching for optimal solutions of similar problems.

CHAPTER 6: OPTIMIZATION OF MULTI-PLANT CAPACITATED LOT-SIZING PROBLEM IN AN INTEGRATED PRODUCTION-DISTRIBUTION NETWORK USING CALIBRATED METAHEURISTIC ALGORITHMS

6.1 Introduction

In this chapter, the multi-item, multi-period, multi-plant capacitated lot-sizing problem with inter-plant interactions, multiple suppliers and distribution centers is addressed, which can cover a variety of problems arising in the literature and in practice. The fundamental concept is to simultaneously optimize decision variables of different functions in a supply chain that have been conventionally optimized individually due to the complexity in their integration. The combinations of several functions such as purchasing, production, storage, backordering, and transportation are considered to evaluate the impact of coordination on the cost performance of a multi-plant firm. A number of well-known metaheuristic algorithms are utilized to solve the proposed model.

The subsequent sections of this chapter are organized as follow: Section 6.2 presents the mathematical formulation for the proposed problem. In Section 6.3, metaheuristic algorithms namely the GA, PSO, ABC, and ICA are presented to solve the proposed model. Section 6.4 demonstrates the numerical example. Section 6.5 explains the parameter calibration procedure using the Taguchi method. In Section 6.6 the computational results are discusses. Finally, the conclusions are shared in Section 6.7.

6.2 Problem Description and Mathematical Formulations

The considered production and distribution planning problem consists of M suppliers, J plants, and W distribution centers, as shown in Figure 6.1. It is assumed that production takes place in a multi-plant manufacturing company, where the plants are geographically distributed in different locations of a country. Each product is made of K

raw materials which are provided by the suppliers. It is designated that for each raw material type there is only one particular supplier. Each plant is characterized by its own inventory and production capacities. It is possible to store excess production at the plant storage which has capacity limit, but no storage is possible for end products at distribution centers.

Any of the products produced in each plant can be transported to any of the distribution centers that are located in different areas. Obviously, the demand in a distribution centre is served by the closest plant. Transfer decisions between plants are made when demand observed at a plant can be satisfied by other production sites to cope with under-capacity of that particular plant. It should be noted that the customer would pay only for the transportation from the nearest plant. The transportation cost from other plants to the plant where demand has been placed, has to be borne by the company.

Since all factories, suppliers and distribution centers are spread out geographically, the transportation cost can vary. Homogenous vehicles of a given capacity are stationed at each supplier and plant to deliver products from suppliers to plants, between the production plants, and from plants to distribution centers. The model is developed with the assumption that sales are made at the distribution centers. In addition, backordering is allowed when demand at a distribution centre cannot be entirely satisfied.

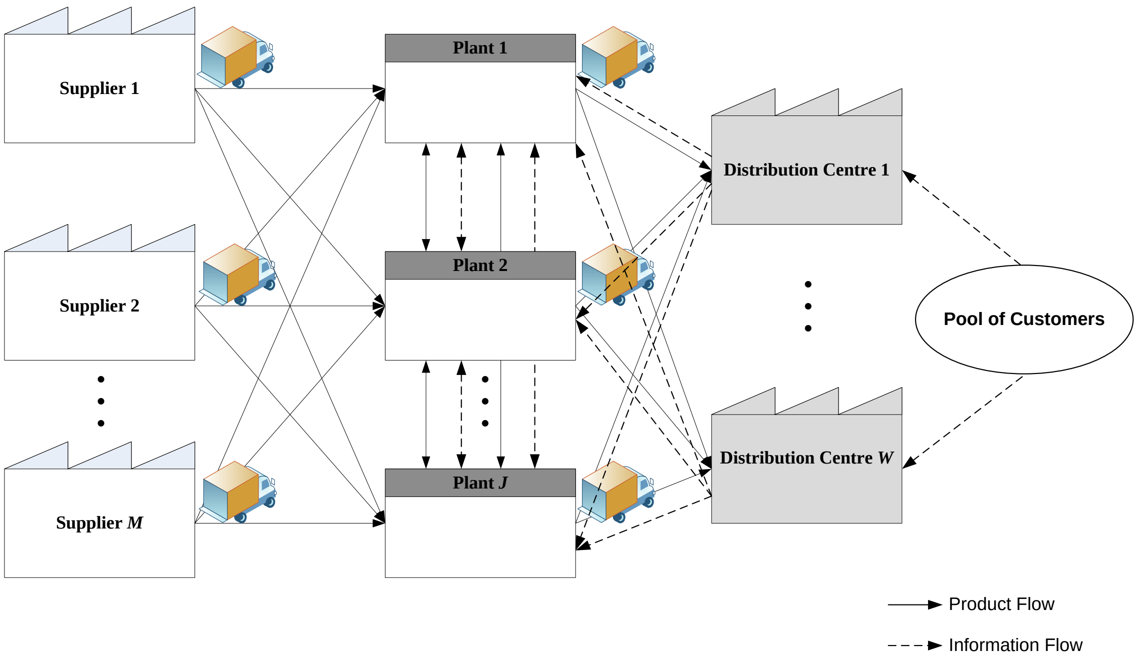


Figure 6.1: A schematic representing the proposed multi-plant problem

The following notations are used in the development of the mathematical model presented in this chapter.

(a) ***Indices***

i Product, $i \in \{1,2,...,N\}$

k Raw material, $k \in \{1,2,...,K\}$

v Resource, $v \in \{1,2,...,V\}$

m Supplier, $m \in \{1,2,...,M\}$

j, l, l' Plant, $j,l,l' \in \{1,2,...,J\}$

w Distribution centre, $w \in \{1,2,...,W\}$

t Period, $t \in \{1,2,...,T\}$

(b) ***Parameters***

d_{iwt} Demand of product i at distribution centre w in period t

U_{iwt} Selling price of product i at distribution centre w in period t

A_{ijt} Setup time of product *i* at plant *j* in period *t*

P_{ijt} Production time of product *i* at plant *j* in period *t*

F_{jt} Total available production time at plant *j* in period *t*

x_{ki} Amount of raw material *k* required to produce a unit of product *i*

E_{kmt} Number of raw material *k* that can be provided by supplier *m* in period *t*

λ_{kmjt} Percentage of rejected raw material *k* delivered by supplier *m* to plant *j* in period *t*

R_{vij} Amount of resource *v* required to produce a unit of product *i* at plant *j*

N_{vjt} Total amount of resource *v* available at plant *j* in period *t*

π_{kmt} Ordering cost of raw material *k* at supplier *m* in period *t*

τ_{kmt} Purchasing cost of raw material *k* at supplier *m* in period *t*

S_{ijt} Setup cost for product *i* at plant *j* in period *t*

O_{ijt} Production cost of product *i* at plant *j* in period *t*

H_{kjt} Holding cost of raw material *k* at plant *j* in period *t*

H'_{ijt} Holding cost of product *i* at plant *j* in period *t*

B_{iwt} Backordering cost of product *i* at distribution centre *w* in period *t*

ϖ_{mj} Distance between supplier *m* and plant *j*

μ_{jl} Distance between plant *j* and plant *l*

ζ_{jw} Distance between plant *j* and distribution centre *w*

σ_{kj} Storage capacity for raw material *k* at plant *j*

σ'_{ij} Storage capacity for item *i* at plant *j*

ς_{k} Vehicle available capacity respect to raw material *k*

ς'_{i} Vehicle available capacity respect to product *i*

η^{F} Fixed transportation cost of vehicle

η^V Variable transportation cost of vehicle per trip

ρ_k Safety stock coefficient with respect to raw material *k*

ε_j Performance percentage of available time at plant *j*

ξ_{vj} Productivity percentage of resource *v* at plant *j*

δ A very large number

δ' A very large number

(c) ***Decision Variables***

Q_{ijt} Quantity of product *i* produced at plant *j* in period *t*

α_{kmjt} Purchase amount of raw material *k* shipped from supplier *m* to plant *j* in period *t*

I_{kjt} Inventory level of raw material *k* stored at plant *j* at the end of period *t*

I'_{ijt} Inventory level of product *i* stored at plant *j* at the end of period *t*

C_{ijwt} Quantity of product *i* that is available to be shipped from plant *j* to distribution centre *w* in period *t*

Z_{ijlt} Quantity of product *i* transferred from plant *j* to plant *l* in period *t*

Y_{iwt}^{j} Total number of product *i* shipped from plant *j* to distribution centre *w* in period *t*

b_{iwt} Shortage amount of product *i* at distribution centre *w* in period *t*

ϕ_{mjt} Number of vehicles required to ship products from supplier *m* to plant *j* in period *t*

v_{jlt} Number of vehicles required to transfer products from plant *j* to plant *l* in period *t*

Ω_{jwt} Number of vehicles required to ship products from plant *j* to distribution centre *w* in period *t*

$$\chi_{ijt} \begin{cases} 1 & \text{If there is a setup for product } i \text{ at plant } j \text{ in period } t \\ 0 & \text{Otherwise} \end{cases}$$

$$\varphi_{kmjt} \begin{cases} 1 & \text{if an order for raw material } k \text{ is allocated to supplier } m \text{ at plant } j \text{ in period } t \\ 0 & \text{Otherwise} \end{cases}$$

6.2.1 Cost Function

(a) ***Procurement Cost***

$$\sum_{k}\sum_{m}\sum_{j}\sum_{t}\varphi_{kmjt}\pi_{kmt} + \alpha_{kmjt}\tau_{kmt} \tag{6.1}$$

Equation (6.1) shows the total procurement cost. It consists of the ordering cost that depends on whether procurement has taken place or not, and the purchasing cost of raw materials over the planning horizon.

(b) ***Production Cost***

$$\sum_{i}\sum_{j}\sum_{t}\chi_{ijt}S_{ijt} + Q_{ijt}O_{ijt} \tag{6.2}$$

Equation (6.2) expresses the total production cost. The first term represents the setup cost and the second term shows the production cost. It must be noted that the setup cost depends on whether production takes place or not; therefore, binary variable χ_{ijt} is used in expression of setup cost.

(c) ***Inventory Cost***

$$\sum_{k}\sum_{j}\sum_{t}I_{kjt}H_{kjt} + \sum_{i}\sum_{j}\sum_{t}I'_{ijt}H'_{ijt} \tag{6.3}$$

Equation (6.3) shows the inventory costs of raw materials and finished items in plants.

(d) ***Transportation Cost***

$$\eta^F \sum_m \sum_j \sum_t \phi_{mjt} + \eta^F \sum_j \sum_{l \neq j} \sum_t \nu_{jlt} + \eta^F \sum_j \sum_w \sum_t \Omega_{jwt} + \\ \eta^V \sum_m \sum_j \sum_t \varpi_{mj} \phi_{mjt} + \eta^V \sum_j \sum_{l \neq j} \sum_t \mu_{jl} \nu_{jlt} + \eta^V \sum_j \sum_w \sum_t \zeta_{jw} \Omega_{jwt} \tag{6.4}$$

Equation (6.4) shows the transportation cost between the suppliers and plants, inter-plants, and from plants to distribution centers. The cost depends on the associated fixed and variables costs of vehicles. The movement of a vehicle incurs a fixed cost associated to vehicle's depreciation and insurance, cost of capital, and driver wages, and the variable transportation cost relates to the transported item, its quantity, and the path taken for each route travelled.

Without loss of generality, it is assumed that transfer cost from plant *j* to plant *l* is smaller than transfer cost from plant *j* to plant *l'* plus transfer cost from plant *l'* to plant *l*.

(e) ***Shortage Cost***

$$\sum_i \sum_w \sum_t b_{iwt} B_{iwt} \tag{6.5}$$

Equation (6.5) shows the shortage cost at distribution centre *w*. Demand at a distribution centre during any period can be satisfied by direct transfer of items from the nearest plant. If that plant cannot fully satisfy the orders, it can be fulfilled by the transfer of items from other production plants to address the under-capacity of a given plant. Transfers among plants occur within the same time period. In the case when the demand cannot be fulfilled by any other plants, then a shortage would occur, and the demand at distribution centre must be satisfied in the next period.

(f) ***Sales Income***

$$\sum_i \sum_w \sum_t Y_{iwt}^{j} U_{iwt} \tag{6.6}$$

Equation (6.6) expresses the total income over the planning horizon. The total revenue is the total selling income from the sales of products shipped to the distribution centers.

$$\begin{aligned}
\text{Min } f = \Bigg(& \sum_k \sum_m \sum_j \sum_t \varphi_{kmjt}\pi_{kmt} + \alpha_{kmjt}\tau_{kmt} \\
& + \sum_i \sum_j \sum_t \chi_{ijt}S_{ijt} + Q_{ijt}O_{ijt} \\
& + \sum_k \sum_j \sum_t I_{kjt}H_{kjt} + \sum_i \sum_j \sum_t I'_{ijt}H'_{ijt} \\
& + \eta^F \sum_m \sum_j \sum_t \phi_{mjt} + \eta^F \sum_j \sum_{l \neq j} \sum_t \nu_{jlt} + \eta^F \sum_j \sum_w \sum_t \Omega_{jwt} \\
& + \eta^V \sum_m \sum_j \sum_t \varpi_{mj}\phi_{mjt} + \eta^V \sum_j \sum_{l \neq j} \sum_t \mu_{jl}\nu_{jlt} + \eta^V \sum_j \sum_w \sum_t \zeta_{jw}\Omega_{jwt} \\
& + \sum_i \sum_w \sum_t b_{iwt}B_{iwt} \Bigg) \\
& - \sum_i \sum_w \sum_t Y_{iwt}^{j} U_{iwt}
\end{aligned} \tag{6.7}$$

Equation (6.7) is the objective function of the proposed model, where the sum of procurement, production, inventory, shortage, and transportation costs over the time horizon should be minimized from which the total sale is deducted.

6.2.2 Constraints

(a) ***Raw Material Purchasing Constraint***

$$\alpha_{kmjt} = \max\left\{0, \sum_i \rho_k (x_{ki}Q_{ijt}) - I_{kj(t-1)}\right\} \qquad \forall k, m, j, t \tag{6.8}$$

Equation (6.8) shows the required amount of raw material *k* that plant *j* must purchase from supplier *m* in period *t*. If total amount of raw material *k* used in production of all items multiplied by a safety stock coefficient (ρ_k) is less than the existing inventory of raw material *k*, then the factory does not need to order any raw material. The safety stock coefficient is considered to protect the firm in uncertain conditions, i.e. if a supplier fails to deliver the raw material at the required time, or the supplier's quality is found to be substandard upon inspection, which would leave the plant without the required raw materials.

(b) ***Inventory Constraints for Raw Materials***

$$I_{kj(t-1)} = 0 \qquad \forall k, j, t = 1 \tag{6.9}$$

Initial inventory level of raw materials is considered to be zero as shown in Eq. (6.9).

$$I_{kjt} = I_{kj(t-1)} + \alpha_{kmjt} - \lambda_{kmjt}\alpha_{kmjt} - \sum_i x_{ki} Q_{ijt} \qquad \forall k, m, j, t \tag{6.10}$$

Equation (6.10) represents the balance equation for the inventory of raw materials at plants at the end of period *t*. It must be noted that there is only one supplier for each type of raw material. Furthermore, plants do not pay for the rejected raw materials, and their associated cost is paid by the respective supplier.

(c) ***Charging Ordering Cost Constraint***

$$\alpha_{kmjt} \le \delta \varphi_{kmjt} \qquad \forall k, m, j, t \tag{6.11}$$

Equation (6.11) describes that a plant cannot place a procurement order without charging an ordering cost. φ_{kmjt} is a binary variable with value of 1 if an order is allocated to supplier *m* at time *t*, otherwise, it is 0. The symbol δ is defined as a sufficiently large number to ensure that it is greater than each α_{kmjt}.

(d) ***Supplier Capacity Constraint***

$$\sum_{j} \alpha_{kmjt} \leq E_{kmt} \qquad \forall k, m, t \tag{6.12}$$

Equation (6.12) ensures that the order size of raw materials released for each supplier is limited by its capacity.

(e) ***Inventory Constraints for Finished Items at Plants***

$$I'_{ij(t-1)} = 0 \qquad \forall i, j, t = 1 \tag{6.13}$$

Eq. (6.13) shows the initial inventory level of products at the beginning of planning horizon.

$$I'_{ijt} = I'_{ij(t-1)} + Q_{ijt} + \sum_{l' \neq j} Z_{il'jt} - Y^{j}_{iwt} - \sum_{l \neq j} Z_{ijlt} \quad \forall i, j, w, t \tag{6.14}$$

Equation (6.14) is the inventory balance equation for finished items at plants.

It is supposed that if during period *t* there is a transfer into plant *j*, there cannot be any transfer out from plant *j* to other plants during that period. Hence:

$$Z_{ijlt} \times Z_{il'jt} = 0 \qquad \forall i, j, l \;\&\; l' \neq j, t \tag{6.15}$$

(f) ***Setup Forcing Constraint***

$$Q_{ijt} \leq \delta' \chi_{ijt} \qquad \forall i, j, t \tag{6.16}$$

Equation (6.16) forces χ_{ijt} to be nonzero if Q_{ijt} is nonzero. Since each χ_{ijt} is constraint to be 0 or 1 the only nonzero value is 1. Thus, if there is positive production of product *i* in period *t*, i.e. $Q_{ijt} > 0$ then $\chi_{ijt} = 1$ and the fixed cost of S_{ijt} is charged. The symbol δ' is defined as a sufficiently large number to ensure that it is greater than each Q_{ijt}.

(g) ***Total Available Time Constraint***

$$\sum_i (P_{ijt} Q_{ijt} + A_{ijt} \chi_{ijt}) \leq F_{jt} \varepsilon_j \qquad \forall j, t \tag{6.17}$$

Equation (6.17) limits the production time available at a plant during period *t*. The overall time consumptions for production and setup in each plant for all products must be lower than or equal to the available time capacity. It also considers the limitations associated with the capacity of available time.

(h) ***Resource Constraint***

$$\sum_i R_{vij} Q_{ijt} \leq N_{vjt} \xi_{vj} \qquad \forall v, j, t \tag{6.18}$$

Equation (6.18) ensures that a manufacturer does not plan beyond the available resources (machine or human) of each plant in each period. It also considers resources' productivity.

(i) ***Transportation Limitation Constraint***

$$C_{ijwt} + \sum_{l \neq j} Z_{ijlt} \leq I_{ijt-1} + Q_{ijt} + \sum_{l' \neq j} Z_{il'jt} \qquad \forall i, j, w, t \tag{6.19}$$

Equation (6.19) shows that the number of products available to be transferred from plant *j* to distribution centre *w* and other plants in period *t* should not exceed the previous period inventory and production quantity in plant *j* as well as the transferred products to plant *j* in period *t*.

$$\sum_{l' \neq j} Z_{il'jt} \leq \max\left\{ (b_{iw(t-1)} + d_{iwt}) - C_{ijwt}, 0 \right\} \qquad \forall i, j, w, t \tag{6.20}$$

Equation (6.20) restricts the transfer quantity from other plants to plant *j* during period *t*. It implies that if total amount of item *i* available at plant *j* to be transferred to distribution centre *w* in period *t* is greater than backorder amount from previous period and demand at distribution *w* in period *t*, then plant *j* does not need outsourcing. In this condition C_{ijwt} will be equal to Y_{iwt}^{j}. Otherwise, plant *j* needs to request the shortage amount of item *i* from other plants.

(j) ***Vehicles Constraints***

$$\sum_{k}\frac{\alpha_{kmjt}}{\varsigma_{k}} \le \phi_{mjt} \qquad \forall m, j, t \tag{6.21}$$

Equation (6.21) calculates the number of vehicles used for transportation of raw materials from suppliers to plants.

$$\sum_{i}\frac{Z_{ijlt}}{\varsigma'_{i}} \le \nu_{jlt} \qquad \forall j, l \ne j, t \tag{6.22}$$

$$\sum_{i}\frac{Y_{iwt}^{j}}{\varsigma'_{i}} \le \Omega_{jwt} \qquad \forall j, w, t \tag{6.23}$$

Equations (6.22) and (6.23) determine the number of vehicles required for delivery of products from a plant to other plants and distribution centers respectively.

(k) ***Backordering Constraint at Distribution Centers***

$$b_{iwt} = \max\left\{(b_{iw(t-1)} + d_{iwt}) - Y_{iwt}^{j}, 0\right\} \qquad \forall i, j, w, t \tag{6.24}$$

Equation (6.24) limits the backorder quantity in period *t* by the current demand plus the backorder amount from the previous period. The shortage in period *t* will be zero if the amount of demand of item *i* at distribution centre *w* in period *t* plus its previous backorder is equal or smaller than total quantity of item *i* transferred to distribution centre *w*.

(l) ***Storage Capacity Constraints***

$$I_{kjt} \le \sigma_{kj} \qquad \forall k, j, t \tag{6.25}$$

$$I'_{ijt} \le \sigma'_{ij} \qquad \forall i, j, t \tag{6.26}$$

Equations (6.25) and (6.26) determine the upper limit of inventory level for each type of raw material and product in plants respectively.

(m) ***Non-Negativity and Binary Constraints***

$$
\begin{aligned}
&Q_{ijt}, \alpha_{kmjt}, I_{kjt}, I'_{ijt}, b_{iwt}, Z_{ijlt}, C_{ijwt}, Y_{iwt}^{j} \geq 0 && \forall k, i, m, j, l \neq j, w, t \\
&\phi_{mjt}, \nu_{jlt}, \Omega_{jwt} \geq 0, \text{integer} && \forall m, j, l \neq j, w, t \\
&\chi_{ijt}, \varphi_{kmjt} \in \{0,1\} && \forall k, i, m, j, t
\end{aligned}
\tag{6.27}
$$

Equation (6.27) enforces the restrictions of non-negativity and binary nature on the decision variables.

6.2.3 Assignment of Demand to Plants

The following procedure is employed to assign demand of each distribution centre to the plants.

i. Supply as much as possible of demand of product *i* at distribution centre *w* from the nearest plant *j* as long as there is inventory from previous period and enough capacity for production at plant *j* in period *t*. This model allows outsourcing from other plants only when the demand cannot be met thoroughly at the current plant. After the assignment, inventory at plant *j* is updated.

ii. If demand of product *i* in period *t* is not fully satisfied by plant *j*, the remaining demand will be supplied from the second nearest plant subject to the available capacity and inventory at that plant.

iii. Steps 1 and 2 are repeated until all distribution centers have satisfied their demands for all the products.

iv. If the demand in period *t* cannot be fully satisfied by the current inventory, production, and inter-plant transfers, it will be backordered, but the backorder demand must be fulfilled in the next time period.

6.3 Solution Algorithms

The MPCLSP presented in Section 6.2 is a NP-hard problem. To deal with the intricacy and obtain near-optimal to optimal solutions in a reasonable computation time, metaheuristic approaches are widely used for which the GA, PSO, ABC, and ICA approaches are explained in the following subsections. Related codes are presented in Appendices E to H.

6.3.1 GA Approach

The required steps to solve the proposed model by the GA are explained below.

(a) ***Parameters***

The initial information required to begin a GA includes the number of chromosomes kept in each generation called population size, 'N_{pop}', the probability of operating crossover, 'P_c', the probability of operating mutation, 'P_m', and maximum number of generations, '*max generation*'.

(b) ***Chromosome Representation***

A GA starts with encoding the variables of the problem as finite-length strings or *chromosomes*. The chromosomes are considered as strings of the quantities of the produced items (lot size Q) with $N \times J \times T$ dimensions where N shows the total number of products, J indicates total number of plants, and T denotes total number of periods. The representation of a chromosome is illustrated in Figure 6.2.

Q_{111}	Q_{112}	…	Q_{ijt}	…	Q_{NJT-1}	Q_{NJT}

Figure 6.2: The Structure of a chromosome

(c) ***Initial Population***

The GA generates a randomly initial population of g chromosomes within the boundary of the component. Let Q_g represent the gth chromosome in the population. Then, each chromosome is generated by:

$$Q_g = \text{round}\left[\text{lb}\,Q(i, j, t), \text{ub}\,Q(i, j, t)\right] \tag{6.28}$$

Where g denotes the size of population (g = 1, 2, … , N_{pop}), and lb$Q$ and ub$Q$ are the lower and upper bounds for variable Q respectively. Therefore, Eq. (6.28) produces integer random numbers for variable Q within the predetermined limits.

The population size depends only on the nature of problems and must be balanced between time complexity and search space measure. Larger population size may increase the probability of finding optimal solution, but would correspondingly increase the computation time, in addition to an increase in the number of function evaluations (NFEs). The NFEs determines the speed (computational effort) and the robustness of the algorithm. Smaller NFEs would result in a shorter time to reach the global optimum (Sadollah et al., 2013).

(d) ***Selection***

In each generation, a collection of offspring chromosomes is generated through a recombination process of parents using the roulette wheel procedure. The selection process is based on spinning the roulette wheel N_{pop} times. The following process is used to choose two parents:

i. The fitness value of the population is obtained.
ii. A particular population member to be a parent with a probability is selected. The selection probability, a_g, for individual g with objective function value f_g, is calculated by Eq. (6.29):

$$a_g = \frac{f_g}{\sum_{g=1}^{Npop} f_g} \tag{6.29}$$

iii. Cumulative normalized fitness value for each chromosome is then calculated.

iv. A real random number r in range [0, 1] is generated.

v. Two chromosomes are selected whose cumulative probabilities are greater than r.

Although all individuals in the population have a chance of being selected to reproduce the next generation, those with higher fitness value are more likely to be selected for the mating pool.

(e) ***Crossover***

In a crossover process, it is essential to mate pairs of chromosomes to produce offspring. This is carried out by a random selection of a pair of chromosomes from the generation with probability P_c. The number of chromosomes for carrying out the crossover operator is obtained by Eq. (6.30):

$$N_{crossover} = N_{pop} \times P_c \tag{6.30}$$

In this study, the arithmetic crossover operator that linearly combines the parent chromosome vector is used to produce offspring based on Eqs. (6.31) and (6.32).

$$offspring_{(1)} = y \times parent_{(1)} + (1 - y) \times parent_{(2)} \tag{6.31}$$

$$offspring_{(2)} = y \times parent_{(2)} + (1 - y) \times parent_{(1)} \tag{6.32}$$

Where y is a random vector in range [0, 1], and has a dimension equal to the size of the selected part (say the first part) of the chosen parent. Because variable Q is integer, the amounts of produced offsprings are rounded. The crossover process will be repeated $N_{crossover}/2$ times.

(f) ***Mutation***

The mutation operator injects diversity in the population of solutions by perturbing some of them with probability P_m. The solution spaces that are not discovered by the crossover operator are found using the mutation operator.

The expected number of chromosomes which undergoes the mutation operation is obtained using Eq. (6.33):

$$N_{mutation} = N_{pop} \times P_m \tag{6.33}$$

The steps involved in the mutation operation are as follow:

i. An integer random number in range [1, N_{pop}] is generated in order to select a chromosome (Q_g). The total numbers of elements selected for mutation are $\mathrm{n}Q = N \times J \times T$.

ii. Two integer random numbers r_1 and r_2 are produced in order to select the elements of the chromosome for mutation. The considered range for r_1 is [1, nQ-1] and r_2 is [r_1+1, nQ].

iii. The value of selected elements of chromosome Q_g is changed using Eq. (6.34):

$$Q'_g\left(r_1 : r_2\right) = Q_g\left(r_1 : r_2\right) + \mathrm{distance}Q_g\left(r_1 : r_2\right) \tag{6.34}$$

The distance is the amount an element can be changed as shown in Eq. (6.35):

$$\mathrm{distance}Q_g = 0.1 \times r \times \left(\mathrm{ub}Q_g - \mathrm{lb}Q_g\right) \tag{6.35}$$

Where r is a random number generated from the continuous uniform distribution within the range of [-1, 1]. The size of generated random numbers is to be equal to the upper bound Q_g. Equation (6.35) causes the value of chosen elements exchange with uniform values randomly selected between the upper and lower range. The result obtained by Eq. (6.34) is rounded to attain an integer value.

However, there is a possibility that the calculated value by Eq. (6.34) exceeds the upper and lower bounds of Q_g. Therefore:

$$\begin{cases} \text{if } Q'_g > \text{ub}Q_g \ \text{ then } Q'_g = \text{ub}Q_g \\ \text{if } Q'_g < \text{lb}Q_g \ \text{ then } Q'_g = \text{lb}Q_g \end{cases} \tag{6.36}$$

In this study, various combinations of the crossover and mutation rates in the range [0.1, 1] are examined. The computational results show that as P_c and P_m increases, the total cost will decrease.

(g) ***Evaluation and Constraint Handling***

As the chromosomes are produced, a fitness value is assigned to chromosomes of each generation for their evaluation. This evaluation is achieved by the objective function given in Eq. (6.7) which measures the fitness of each individual in the population.

As shown in subsection 6.2.2, the proposed model contains various constraints, which may lead to the production of infeasible chromosomes. In order to deal with infeasibility, a penalty value is assigned to chromosomes that are not feasible. It can be attained by adding a specific amount to the objective function value according to the amount of constraints' violations obtained in a solution.

When a chromosome is feasible, its penalty is set to zero, whereas in case of infeasibility, the coefficient is selected sufficiently large. Therefore, the fitness function for a chromosome will be equal to the sum of the objective function value and penalties as shown in Eq. (6.37), where s represents a solution, and $f(s)$ is the objective function value for solution s. The penalty policy is employed for all metaheuristic algorithms utilized in this research.

$$fitness(s)=\begin{cases} f(s); & \text{if } s \text{ is feasible} \\ f(s)+\text{Penalty}(s); & \text{otherwise} \end{cases} \tag{6.37}$$

(h) ***New Population***

The fitness function values of all chromosomes are evaluated in this stage. The chromosomes with higher fitness scores are then selected to generate a new population. Note that the number of chosen chromosomes must be equal to N_{pop}.

(i) ***Termination***

The selection and reproduction of parents will be continued until the algorithm reaches a stopping criterion. The procedure can be ended after a predetermined number of generations, or when no substantial improvement over a successive generation is achieved.

6.3.2 PSO Algorithm

In any iteration of PSO, the velocity and position of particles are updated according to Eqs. (6.38) and (6.39):

$$V_g^{(e+1)}=\omega V_g^e+c_1 y_1\left(best_g-\psi_g^e\right)+c_2 y_2\left(global_g-\psi_g^e\right) \tag{6.38}$$

$$\psi_g^{(e+1)}=\psi_g^e+V_g^{(e+1)} \tag{6.39}$$

Where g = 1, 2,…, N_{swarm} and e denotes the iteration (e = 1, 2, …, max iteration). V_g is the velocity of gth particle. ω is the inertia weight that controls the impact of the previous velocity of the particle on its current velocity, which plays an important role in balancing global and local search ability of the PSO. Applying a large inertia weight at the start of the algorithm and making it decay to a small value through the PSO execution makes the algorithm search globally at the beginning of the search, and search locally at the end of the execution (Coelho & Sierakowski, 2008). The inertia weight is updated in each iteration by $\omega^{(e+1)} = \omega^{e} \times (1-\beta)$ where in this study β is considered 0.01.

c_1 is the cognitive parameter, and c_2 is the social parameter; y_1 and y_2 are random numbers within the range [0, 1], which should be the same size as V_g; $best_g$ is the best searching experience so far by particle g; ψ_g is the current position of particle g; and $global_g$ is the position in parameter space of the best fitness returned for the entire swarm. The search procedure of the PSO is summarized as follows:

i. Generate an initial population of g particles (solutions) with random positions and velocities within the boundary of the component according to Eq. (6.28), where g denotes the size of swarm.
ii. Evaluate the fitness value of each particle in the swarm.
iii. Compare each particle's fitness with the current particle's own best. If current value is better than own best, own best value will be set to the current value, and the own best location to the current location.
iv. Compare the fitness value with the population's overall previous best. If the current value is better than global best, then global best will be set to the current particle's array index and value.
v. Update the velocity of each particle g using Eq. (6.38), and the position of particle g using Eq. (6.39). The values obtained by Eq. (6.38) for the velocity is rounded to the nearest integer amount. If the value obtained by Eq. (6.39) exceeds upper and lower bounds of the particle, Eq. (6.36) is used to set the value to its boundaries.
vi. Terminate the procedure if the termination criterion is satisfied, otherwise go to step (ii).

6.3.3 ABC Algorithm

The main steps of the ABC algorithm are as follow.

(a) ***Initialization of the Parameters***

The main parameters of the ABC algorithm are the total number of bees (*NB*), number of food sources (*NS*) which is equal to the number of the employed bees or onlooker bees, number of trials after which a food source is supposed to be discarded (*limit*), and maximum number of cycles of the search process (*max cycle*). *NS* is considered equal to *NB*/2.

(b) ***Initialization of the Population***

The ABC algorithm generates a randomly initial population of g solutions (g = 1, 2, ... , *NS*). Each solution is generated using Eq. (6.28). Their fitness is then evaluated, and the best fitness is considered as global food source. After initialization, the population of the food sources (solutions) is subjected to repeat cycles of the search processes of the employed, onlooker and scout bees.

(c) ***Employed Bee Phase***

For every food source position, only one employed bee is allocated denoting that the number of food source positions around the hive is equal to the number of employed bees (E_b). An employed bee makes a modification on the position of the food source in its memory and finds a neighboring food source, and then evaluates its nectar amount (fitness) of the associated food source.

In order to determine a neighboring food source position to the previous one in memory, the algorithm changes some randomly selected parameter and keeps the remaining parameters unchanged. To select a neighborhood an integer random number in the range [1, *NS*] is generated. Within the neighbourhood of every food source site represented by Q_g, a new food source Q'_g is determined by changing some parameter of Q_g. Steps of creating a change are given below:

i. Find the total numbers of parameters that can be selected for change and store it in nQ where $\mathrm{n}Q = N \times J \times T$.

ii. Generate an integer random number n in range $\left[0,\ \mathrm{round}(0.1 \times \mathrm{n}Q) + 1\right]$.

iii. Generate two integer random numbers r_1 and r_2. The considered range for r_1 is [1, nQ-n] and r_2 is r_1+n.

iv. Create the neighbor food source position by adding the difference between the selected parameter value and other random solution parameter value to the current selected parameter value as shown in Eq. (6.40):

$$Q'_g\left(r_1 : r_2\right) = Q_g\left(r_1 : r_2\right) + y \times \left[Q_g\left(r_1 : r_2\right) - Q_p\left(r_1 : r_2\right)\right] \tag{6.40}$$

Where y is a matrix 1×n+1 containing uniformly distributed real random numbers in the range [-1, 1], and $p \in \{1, 2, ..., E_b\} \wedge p \neq g$. To have an integer value the amount produced by Eq. (6.40) is rounded.

v. If a parameter value produced by Eq. (6.40) exceeds predetermined limit of food source, the parameter can be set to an acceptable value. Hence, the value of the parameter exceeding its upper and lower bounds is set to its boundaries as shown in Eq. (6.36).

vi. Assign a fitness value to the solution Q'_g. Afterwards, a greedy selection is applied between Q_g and Q'_g, and an improved one is chosen based on fitness values indicating the nectar amount of the food sources at Q_g and Q'_g. If the fitness of Q'_g is equal to or better than Q_g, Q'_g will be replaced with Q_g and will become a new member of the population; otherwise Q_g is kept. If Q_g cannot be improved, its counter holding the number of trials is incremented by 1, otherwise, the counter is reset to 0.

(d) ***Onlooker Bee Phase***

After all employed bees complete their searches, they share their information related to the nectar amounts and the positions of their sources with the onlooker bees on the dance area. An onlooker bee evaluates the nectar information taken from all the employed bees and chooses a food source Q_g depending on its probability value a_g calculated by Eq. (6.41):

$$a_g = \frac{f_g}{\sum_{g=1}^{NS} f_g} \tag{6.41}$$

Where f_g is the fitness value of the *g*th food source. Obviously, the higher the f_g, the more probability that the *g*th food source is selected.

To select a food source a real random number within the range [0, 1] is generated. If the cumulative probability a_g associated with that source is greater than this random number then the onlooker bee produces a modification on the position of this food source site using Eq. (6.40).

To select a neighborhood an integer random number in range [1, *NS*] is generated. After the new food source is evaluated, greedy selection is applied and the onlooker bee either memorizes the new position by forgetting the old one or keeps the old one. If solution Q_g cannot be improved, its counter holding trials is incremented by 1, otherwise, the counter is reset to 0. This process is repeated until all onlookers are distributed onto food source sites.

(e) ***Scout Bee Phase***

In a cycle, a food source that cannot be improved through limit cycle is abandoned. In order to select a food source to be abandoned, a control parameter called *limit* is used. If a solution representing a food source is not improved by a predetermined number of trials, then that food source is abandoned by its employed bee and the employed bee is converted to a scout. The number of trials for releasing a food source is equal to the value of limit. Then a scout bee is employed to search new food source randomly using Eq. (6.28) to replace the abandoned one.

It is supposed that only one source can be exhausted in each cycle, and only one employed bee can be a scout. If more than one counter exceeds the limit value, one of the maximum ones might be chosen programmatically. After every cycle, the best solution is memorized.

The process is repeated until a termination criterion is reached which can be reaching the maximum number of iterations, or when the algorithm does not seem to converge in its initial phase.

6.3.4 ICA Approach

The ICA approach is a population based evolutionary algorithm, which has been used extensively to solve various kinds of combinatorial optimization problems. This method is based on socio-political process of imperialistic competition. The main steps of the ICA are described below.

(a) ***The Initialization Mechanism***

Similar to other optimization methods, the ICA first creates the initial population. Each individual of the population is named a 'country'. The word 'country' corresponds to the 'chromosome' in the GA terminology. This array is shown in Eq. (6.42).

$$\text{country} = [L_1, L_2, \ldots, L_N] \tag{6.42}$$

Where *L*s are the variables to be optimized. The power of a country is inversely proportional to its fitness function value which is obtained by evaluation of cost function *f* at variables as shown in Eq. (6.43).

$$\text{cost} = f(\text{country}) = f([L_1, L_2, \ldots, L_N]) \tag{6.43}$$

Equation (6.28) is used to create the initial population of size $N_{country}$ and Eq. (6.7) is used to evaluate the fitness of variable. Based on cost values, a certain number of countries that have the lowest costs are selected as imperialist (N_{imp}) and the rest, known as colonies ($N_{colony} = N_{country} - N_{imp}$), are divided among these imperialists. Each imperialist and its allocated colonies form an empire.

(b) ***Assimilation of Colonies***

The imperialist countries would try to absorb their colonies toward themselves. For this purpose the assimilation policy is considered in the ICA. Based on this concept each colony moves toward its imperialist by X units as shown in Eq. (6.44).

$$Q_g^{e+1} = Q_g^e + X \tag{6.44}$$

Where Q_g^e and Q_g^{e+1} are the current and new position of gth colony of each empire respectively, e is the number of iteration (knows as decade in the ICA), and X is:

$$X = \beta \times y \times D \tag{6.45}$$

Where D is the distance between the initial position of colony and its imperialist (D = imp. Q_g^e - colony. Q_g^e). The position of the colony after movement is defined by the random parameter y in range [0, 1], which must have the dimension equal to the size of D. Parameter $\beta > 1$ causes the colony to get closer to its imperialist from different directions. $\beta = 2$ results in good convergence of countries to the global minimum in most of the implementations. If the value of Q_g^{e+1} exceeds its predetermined limit, it must be set to its upper and lower bounds as shown in Eq. (6.36). Then, fitness of Q_g^{e+1} will be evaluated.

(c) ***Revolution***

Revolution is a fundamental change in power or organizational structures that takes place in a relatively short period of time. The revolution increases the exploration of the algorithm and avoids the early convergence of countries to local minimum with a revolution rate. A very high value of revolution rate reduces the exploitation power of algorithm as well as the convergence rate (Nazari-Shirkouhi et al., 2010). This mechanism is similar to mutation process in the GA for creating diversification in solutions.

Mutation increases the variety in the population, so this operator is used for creating a revolution in variables same as the GA. For this purpose random number *r* in range [0, 1] is generated. If *r* is smaller than revolution rate (θ), a change will be applied on the selected colony of the particular imperialist. Steps of creating a change are as follow:

i. Find the total numbers of elements of the colony that can be chosen for change and store it in nQ where $\mathrm{n}Q = N \times J \times T$.
ii. Generate an integer random number *n* in range $\left[0,\ \mathrm{round}(0.01 \times \mathrm{n}Q) + 1\right]$.
iii. Generate two integer random numbers r_1 and r_2. The considered range for r_1 is [1, nQ-n] and r_2 is r_1+n.
iv. Create the change using Eq. (6.46):

$$Q_g^{e+1}\left(r_1 : r_2\right) = \mathrm{lb}Q_g^e\left(r_1 : r_2\right) + y \times \left[\mathrm{ub}Q_g^e\left(r_1 : r_2\right) - \mathrm{lb}Q_g^e\left(r_1 : r_2\right)\right] \tag{6.46}$$

Where y is a matrix $1 \times n+1$ containing real random numbers in the range [0, 1], and $\mathrm{lb}Q_g^e$ and $\mathrm{ub}Q_g^e$ are the lower and upper bounds of *g*th colony in decade *e*. Values obtained by Eq. (6.46) are rounded to the nearest integers. Then, fitness of new colony is evaluated.

(d) ***Exchange the Colony with Imperialist***

After moving toward the imperialist, a colony may reach a position with lower cost than its imperialist. In this case, the colony will become the imperialist in the current empire and vice versa. In the next decades, colonies in the empire will move towards the new imperialist.

(e) ***Total Power of an Empire***

Based on total power of empires, an imperialistic competition takes place between empires. Total power of an empire is mainly affected by the power of its imperialist and slightly by its colonies. Hence, total power of an empire is calculated by Eq. (6.47):

$$Tf_g = f(\text{imp}_g) + \gamma \times \text{mean}[f(\text{colony of empire}_g)] \tag{6.47}$$

Where Tf_g is the total cost of the gth empire and γ is a positive number in the range of [0, 1]. The small value of γ makes the total power of the empire to be determined by almost only its imperialist and increasing it will enhance the role of the colonies in determining the total power of an empire.

(f) ***Imperialistic Competition***

Every empire tries to take over the colonies of other empires. The imperialistic competition gradually causes reduction in power of weaker empires and growth in power of powerful ones. The imperialistic competition is modeled by selecting the weakest colonies of the weakest empire in every iteration and making a competition among all empires to take over this colony. The likelihood of possession of the colony for each empire is proportionate to its total power.

The normalized total cost of an empire can be obtained by Eq. (6.48):

$$NTf_g = \max(Tf_{g'}) - Tf_g + \varepsilon \tag{6.48}$$

ε is a small number to avoid the value of NTf_g to become exactly zero. NTf_g and Tf_g are the normalized total cost and total cost of gth empire respectively. Having the normalized total cost, the possession probability of gth empire is obtained by Eq. (6.49):

$$a_g = \frac{NTf_g}{\sum_{g'=1}^{N_{imp}} NTf_{g'}} \tag{6.49}$$

Powerful empires have higher chance of possessing the colony. To distribute the mentioned colony among empires vector c is formed as follows:

$$c = \left[c_1, c_2, ..., c_{N_{imp}} \right] \tag{6.50}$$

Then, vector h with uniformly distributed random numbers in range [0, 1] is created which has the same size as c:

$$h = \left[h_1, h_2, ..., h_{N_{imp}} \right] \tag{6.51}$$

Vector z is formed by subtracting h from c:

$$z = c - h = \left[z_1, z_2, ..., z_{N_{imp}} \right] \tag{6.52}$$

Referring to vector z, the mentioned colony is handed to an empire whose corresponding index in z is maximum (Atashpaz-Gargari & Lucas, 2007).

(g) ***Elimination of Powerless Empires***

Powerless empires will collapse in the imperialistic competition and their colonies will be divided among other empires. It is assumed that an empire would collapse and be removed when it loses all of its colonies. This process will be continued and causes the countries to converge to the global minimum of the cost function. At the end, all the empires will collapse except the most powerful one, and all the colonies would be handled by this unique empire. At this stage, the imperialist and colonies would have the same position and power (Atashpaz-Gargari & Lucas, 2007).

6.4 Numerical Example

A typical company is willing to plan its production and distribution planning. This company owns three plants which are spread geographically around a country, and two distribution centers located in two different cities. The planning time horizon is assumed to be six periods. The number of product families is assumed to be three. There are three types of raw materials, each supplied by a different supplier.

The total available time for each plant is assumed to be 10560 min per period $(22 \text{ days} \times 8 \text{ hours} \times 60 \text{ minutes})$. There are 47, 44 and 30 workers at plant 1, 2 and 3 respectively, with no hiring and firing of workers during the considered planning horizon. Safety stock coefficient for each raw material type is considered 1.5 per period. It is supposed that the chance of rejecting each type of raw material by plant 1, 2 and 3 is 0.02, 0.01, and 0.02 respectively in each period. Performance for total available time and worker's productivity at each plant are assumed to be 85 and 80 percent respectively.

The fixed transportation cost of a vehicle is considered to be $1000 and the variable cost is $2 per trip. The capacity of each vehicle is set to be 100 units for each product type and 250 units for raw materials. Storage capacity for raw material 1, 2 and 3 in each plant is considered to be 4000, 3500, and 3000 units and for product types 1, 2 and 3 is set to be 4500, 4000 and 3500 units respectively. Supplier capacity to provide each raw material type for three plants is assumed to be 15000 units per period. The values for the remaining data are presented in Tables 6.1 to 6.9.

Table 6.1: Demand

	d_{iwt}											
	Distribution centre 1						Distribution centre 2					
Product	Period t						Period t					
	1	2	3	4	5	6	1	2	3	4	5	6
1	680	600	550	520	650	630	630	550	530	500	550	675
2	550	530	600	500	600	550	700	680	570	680	530	530
3	500	700	600	550	590	630	590	680	530	660	540	500

Table 6.2: Selling price ($/unit period)

	U_{iwt}											
	Distribution centre 1						Distribution centre 2					
Product	Period t						Period t					
	1	2	3	4	5	6	1	2	3	4	5	6
1	53	53	53	53.5	53.5	53.5	51	51	51	51.5	51.5	51.5
2	55	55	55	55.5	55.5	55.5	52.5	52.5	52.5	53	53	53
3	57	57	57	57.5	57.5	57.5	53	53	53	53.5	53.5	53.5

Table 6.3: Setup time and production time (min)

	A_{ijt}			P_{ijt}		
Plant	Product i			Product i		
	1	2	3	1	2	3
1	6.06	4.90	5.89	2.48	3.25	4.03
2	7.18	5.20	5.75	2.77	3.53	4.17
3	7.73	5.27	6.62	2.84	3.85	4.21

It is assumed that setup time and production time remain fixed during the planning horizon.

Table 6.4: Required resource for each product (manpower-min)

	R_{vij}								
Resource	Plant 1			Plant 2			Plant 3		
	Product i			Product i			Product i		
	1	2	3	1	2	3	1	2	3
1	13.8	12	9.6	15	13.2	11.2	16.6	15.8	14

Table 6.5: Ordering cost and purchasing cost of raw material ($/unit period)

	π_{kmt}			τ_{kmt}		
Supplier	Raw material k			Raw material k		
	1	2	3	1	2	3
1	0.64	0	0	2.40	0	0
2	0	0.37	0	0	1.5	0
3	0	0	0.76	0	0	3.48

It is assumed that ordering and purchasing costs remain fixed during the planning horizon.

Table 6.6: Setup cost and production cost ($/unit period)

Plant	S_{ijt}			O_{ijt}		
	Product *i*			Product *i*		
	1	2	3	1	2	3
1	2.55	1.58	3.60	3.50	4.25	6.20
2	2.50	1.50	3.00	3.77	4.85	6.19
3	2.80	1.70	3.38	3.80	5.00	6.71

It is assumed that setup and production costs remain fixed during the planning horizon.

Table 6.7: Raw material and end product inventory costs in plants ($/unit period)

Plant	H_{kjt}			H'_{ijt}		
	Raw material *k*			Product *i*		
	1	2	3	1	2	3
1	3	3	3	4	5	3
2	2	3	2	3	4	2
3	4	4	4	5	6	4

It is assumed that inventory costs remain fixed during the planning horizon.

Table 6.8: Raw material consumption rate and backordering cost in distribution centers ($/unit period)

Product	x_{ki}			B_{iwt}	
	Raw material k			Distribution centre *w*	
	1	2	3	1	2
1	2	1	1	27	26
2	3	2	2	28	27
3	1	1	3	29	27

It is assumed that backordering cost remains fixed during the planning horizon.

Table 6.9: Distances between supply chain entities (km)

Plant	ϖ_{mj}			μ_{jl}			ζ_{jw}	
	Supplier *m*			Plant *j*			Distribution centre *w*	
	1	2	3	1	2	3	1	2
1	150	760	670	0	810	555	110	1000
2	800	850	155	810	0	1220	950	175
3	415	750	1000	555	1220	0	670	1300

6.5 Parameter Calibration

Since the quality of the solutions obtained by metaheuristic algorithms depends on the values of their parameters, Taguchi method is utilized to calibrate the parameters of the four proposed algorithms. The objective function of the proposed model is the minimization type, therefore, “smaller is better” category of Taguchi method is used, where *S*/*N* ratio is given by Eq. (6.53) (Phadke, 1989):

$$S/N = -10\log_{10}\left(\frac{1}{n}\sum_{e=1}^{n} f_e^2\right) \tag{6.53}$$

Where f_e is the objective function value of a given experiment *e*, and *n* is the number of times the experiment is performed.

In the Taguchi method, the parameters that have considerable effects on the process output are initially chosen for tuning. The parameters that require calibration are N_{pop}, P_c, and P_m in the GA, N_{swarm}, c_1, c_2, and ω in the PSO, *NB* and *limit* in the ABC, and $N_{countries}$, N_{imp}, θ, and γ in the ICA. The ranges for the parameters that produce satisfactory fitness function values are chosen by trial and error methods.

Table 6.10 shows the algorithms' parameters, each at three levels with nine observations. Figures 6.3 to 6.6 show the mean *S*/*N* ratio plot of the applied algorithms. The best parameters' levels are the highest mean of *S*/*N* values. Table 6.11 shows the optimal levels of the parameters for all algorithms.

Table 6.10: The GA, PSO, ABC, and ICA parameters' levels

Algorithm	**Parameters**	**Levels**		
		1	**2**	**3**
GA	N_{pop}	50	100	150
	P_c	0.85	0.9	0.95
	P_m	0.70	0.80	0.90
PSO	N_{swarm}	50	100	150
	c1	1	1.5	2
	c2	1	1.5	2
	ω	0.80	1	1.2
ABC	*NB*	50	100	150
	Limit	10	50	100
ICA	$N_{country}$	50	100	150
	N_{imp}	5	10	15
	θ	0.10	0.20	0.30
	γ	0.15	0.25	0.35

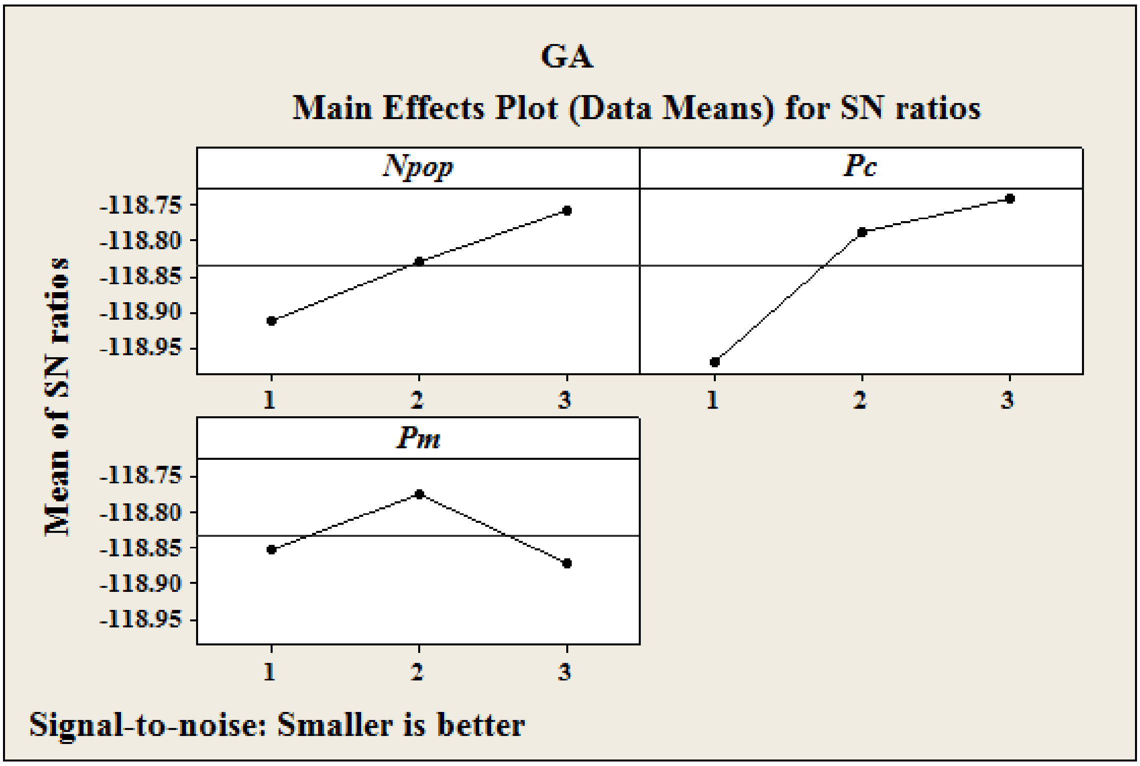
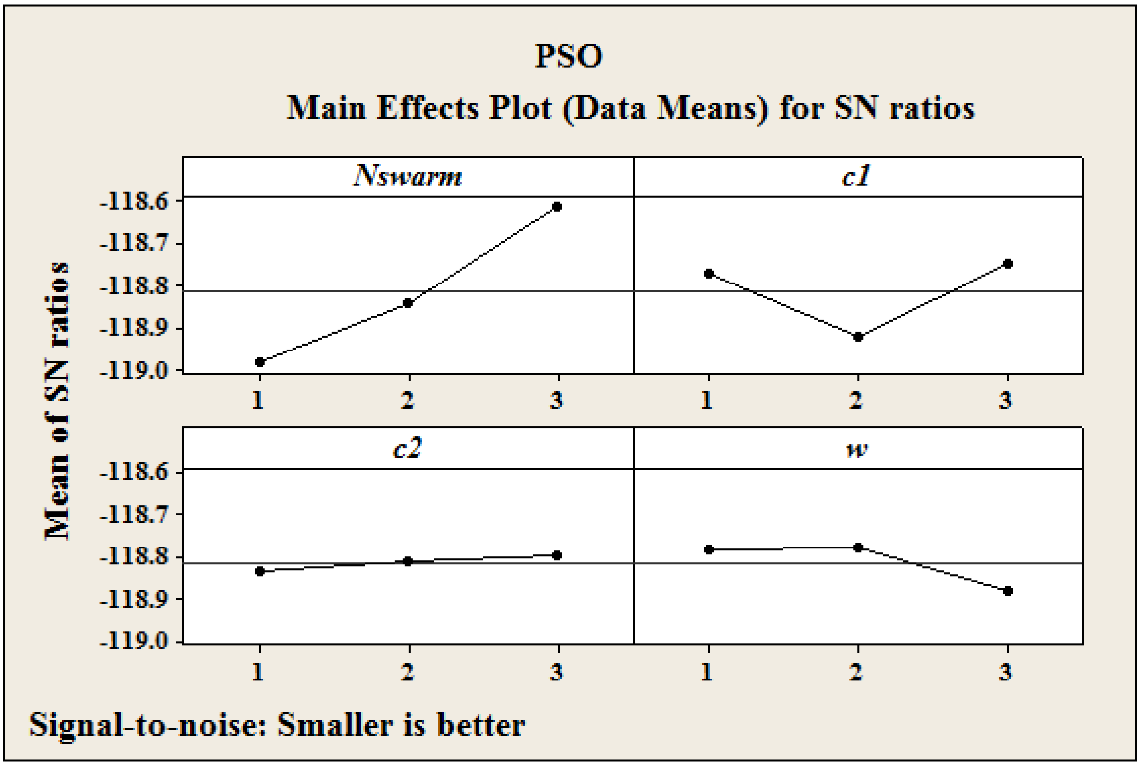


Figure 6.3: The mean *S*/*N* ratio plot for each level of the factors of the GA approach



* *w* stands for ω as indicated in Table 6.10.

Figure 6.4: The mean *S*/*N* ratio plot for each level of the factors of the PSO approach

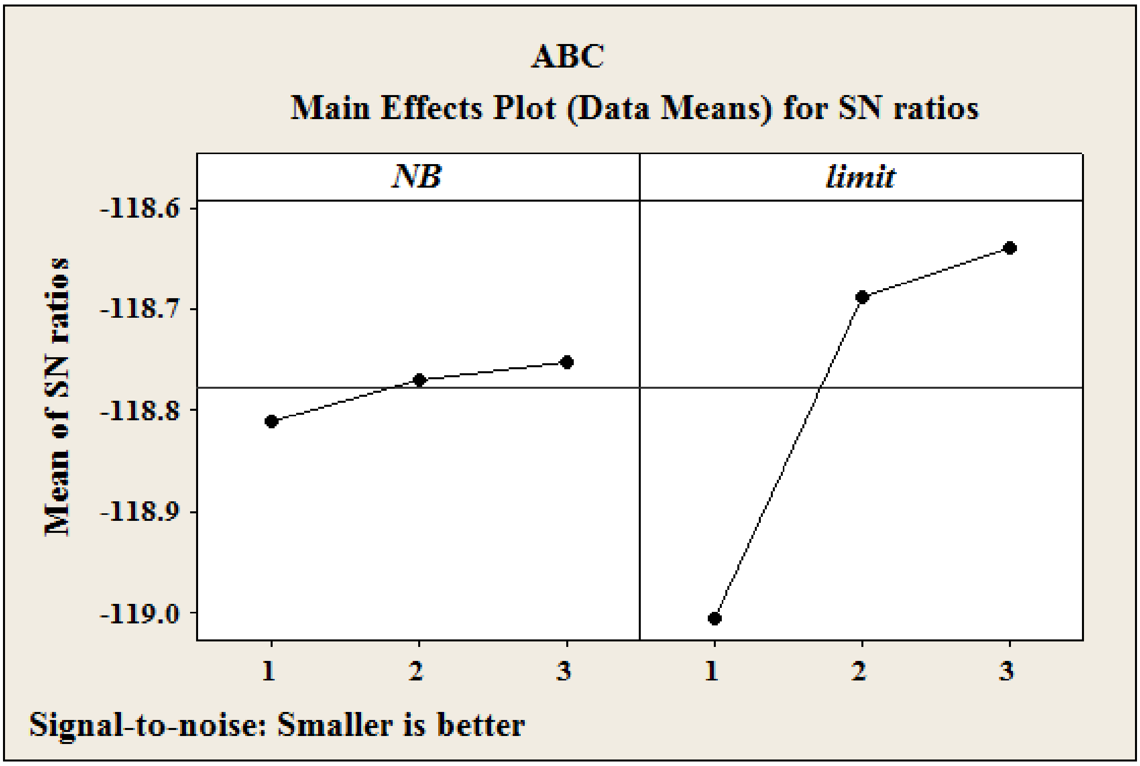
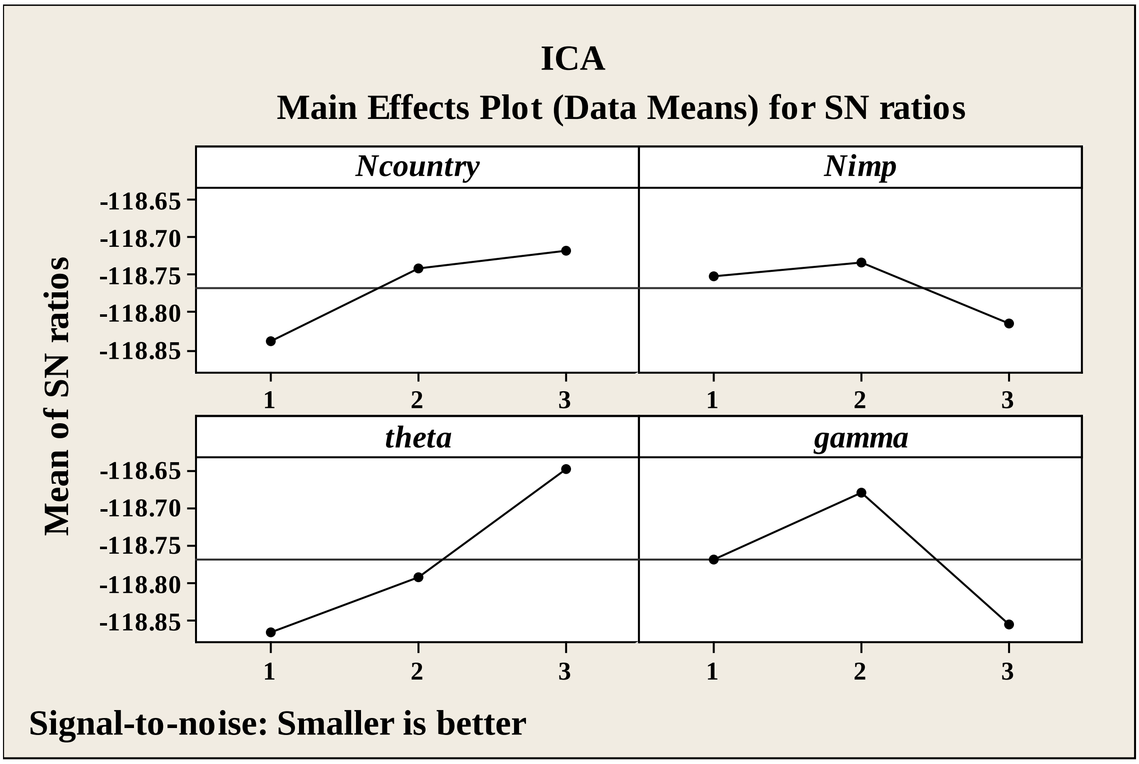


Figure 6.5: The mean *S*/*N* ratio plot for each level of the factors of the ABC approach



* *theta* stands for θ, and *gamma* stands for γ as indicated in Table 6.10.

Figure 6.6: The mean *S*/*N* ratio plot for each level of the factors for the ICA approach

Table 6.11: Optimal values of the algorithms' parameters

Algorithm	Parameters	Optimal values
GA	N_{pop}	150
	P_c	0.95
	P_m	0.80
	max generation	200
PSO	N_{swarm}	150
	c1	2
	c2	2
	ω	1
	max iteration	200
ABC	*NB*	150
	Limit	100
	max cycle	200
ICA	$N_{countries}$	150
	N_{imp}	10
	θ	0.30
	γ	0.25
	max decade	200

6.6 Results and Discussions

The applied optimizers were written and coded in MATLAB software. In order to validate the results obtained by all four algorithms and to investigate the performance of methods in terms of the solution quality and the required CPU time, 20 independent runs were carried out using the parameters settings given in Table 6.11. The objective function values for all runs are given in Table 6.12.

Table 6.12: Objective function values obtained by applied metaheuristic algorithms

Run Number	GA	PSO	ABC	ICA
1	848,067.30	849,176.44	858,616.09	855,034.49
2	850,082.43	854,445.35	859,592.11	857,533.07
3	852,541.96	855,613.04	862,548.10	858,755.59
4	853,761.39	865,922.48	863,291.50	860,029.49
5	854,389.84	867,406.98	864,694.67	861,616.09
6	855,894.30	869,235.74	865,798.47	862,392.66
7	857,554.22	870,423.49	866,145.27	863,818.04
8	858,196.94	871,761.29	867,811.40	869,408.87
9	859,963.66	872,441.48	868,823.49	870,034.35
10	863,933.85	876,301.24	869,949.71	871,176.70
11	864,624.39	877,615.81	871,688.26	872,509.70
12	865,428.18	878,972.55	872,020.54	873,723.78
13	868,335.94	881,333.33	876,388.01	874,842.02
14	868,412.71	883,587.86	878,941.68	875,282.19
15	871,364.99	883,804.79	879,374.95	876,521.90
16	872,345.41	884,404.79	879,990.59	877,019.07
17	873,241.66	885,163.58	883,889.28	878,407.67
18	876,193.48	887,855.43	886,060.76	880,179.76
19	876,932.03	888,041.52	890,523.11	883,934.97
20	877,381.70	896,651.68	891,951.89	886,178.31

As it can be seen in Table 6.12, all methods produced similar results. It approves that if the MPCLSP model be solved by any metaheuristic method, similar results will be obtained, which it verifies the correctness of the developed model.

Figure 6.7 shows the performance of four metaheuristic approaches in terms of objective function values. It illustrates that for all runs GA performed better for the proposed problem.

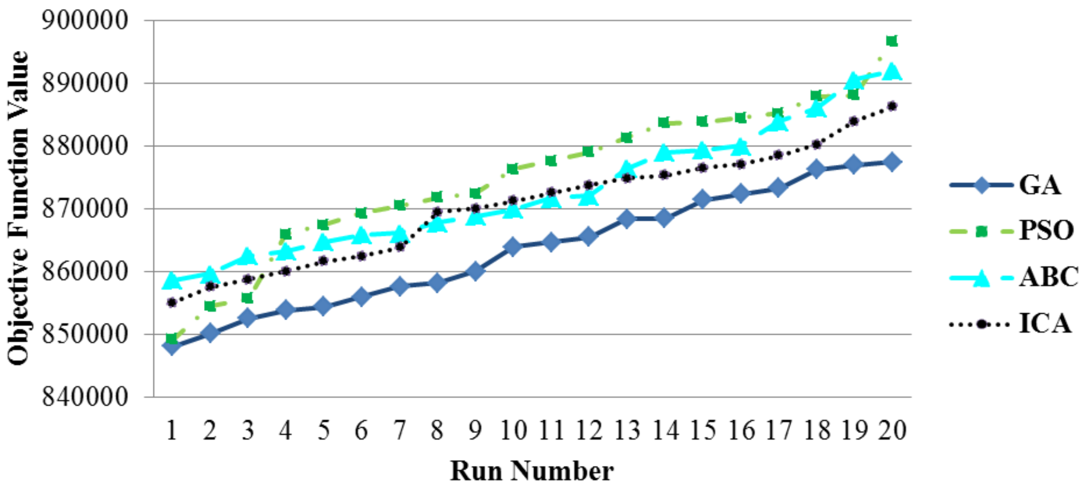


Figure 6.7: Graphical comparison of applied methods in terms of objective function value

Furthermore, using the Minitab software one-way ANOVA was conducted to compare the performance of the algorithms based on the results obtained for 20 runs. Table 6.13 presents the ANOVA results. The *p*-value of the test-statistics on the equality of the mean of the objective function values is 0.004. This means that the null hypothesis of the test is rejected at 95% confidence level, i.e. there is difference between the mean of objective function values obtained by four algorithms. Figure 6.8 supports this conclusion as well.

Table 6.13: One-way ANOVA results for objective function values

Source	Degree of freedom (DF)	SS	MS	F-test	p-value
Optimization Engines	3	1521005841	507001947	4.80	0.004
Error	76	8029063317	105645570		
Total	79	9550069158			

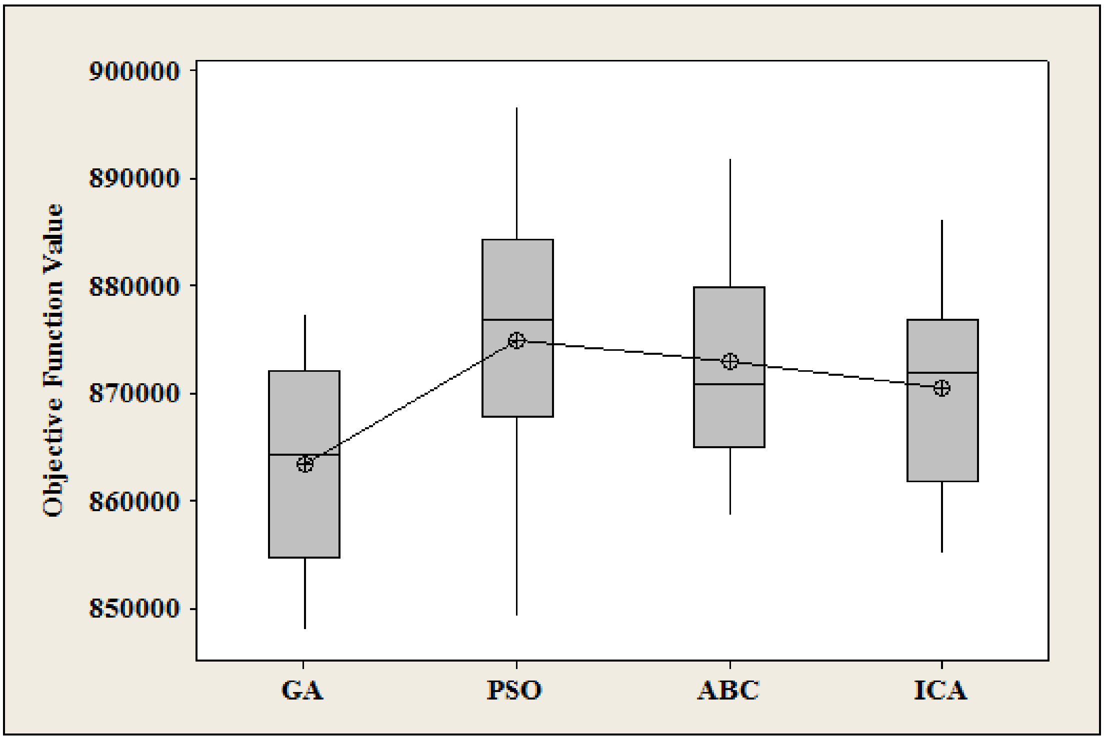


Figure 6.8: Box-plot of objective function values

The statistical optimization results obtained from four algorithms including the worst, mean, best solutions, standard deviation (SD), and NFEs are listed in Table 6.14.

Table 6.14: Statistical results obtained from four metaheuristic algorithms

Methods	Worst solution	Mean solution	Best solution	SD	NFEs
GA	877381.70	863432.32	848067.30	9380.23	37616
PSO	896651.68	875007.94	849176.44	12328.95	20550
ABC	891951.89	872904.99	858616.09	10015.37	29779
ICA	886178.31	870419.94	855034.49	9071.00	36643

Based on Table 6.14, the GA is found to be superior to other methods and surpassed the PSO, ABC, and ICA in terms of function value (accuracy). The GA method has found the best solution (848067.30) in 37616 function evaluations (143 generations). Although the PSO offered modest solution quality with a smaller number of function evaluations (20550), its average cost and SD in 20 experiments are inferior to the other methods. The performances of the ABC and ICA are almost similar in terms of objective function value.

The convergence paths of applied methods in terms of best and mean costs for the best run are plotted in Figures 6.9 to 6.12.

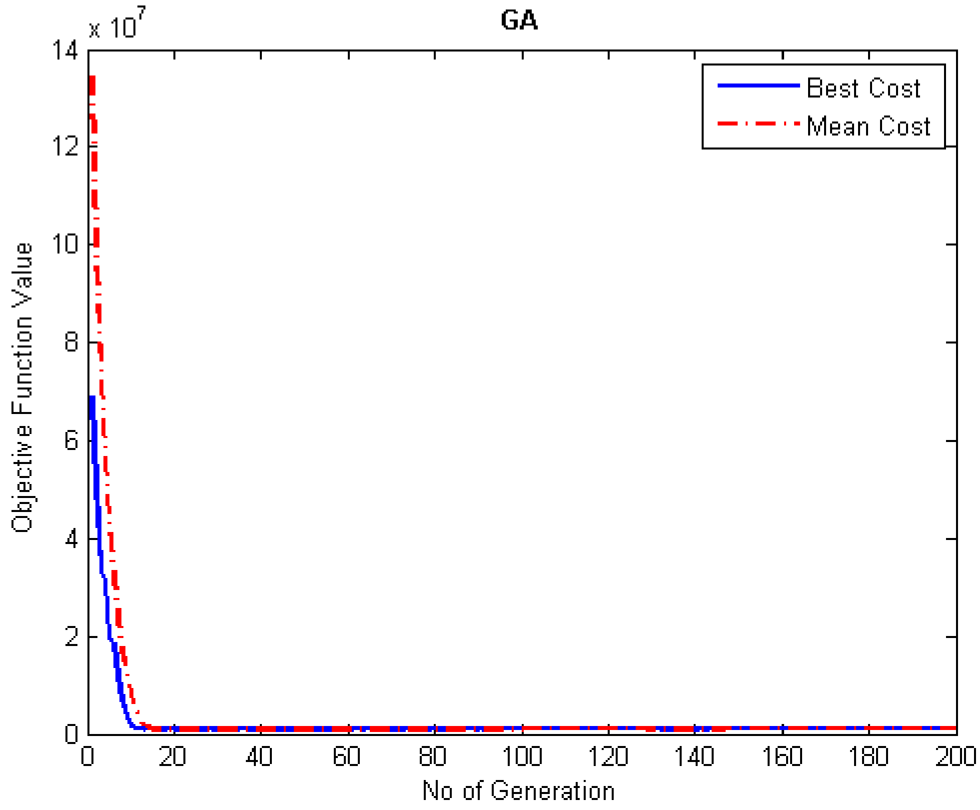


Figure 6.9: The convergence path of fitness function for the best run of the GA approach

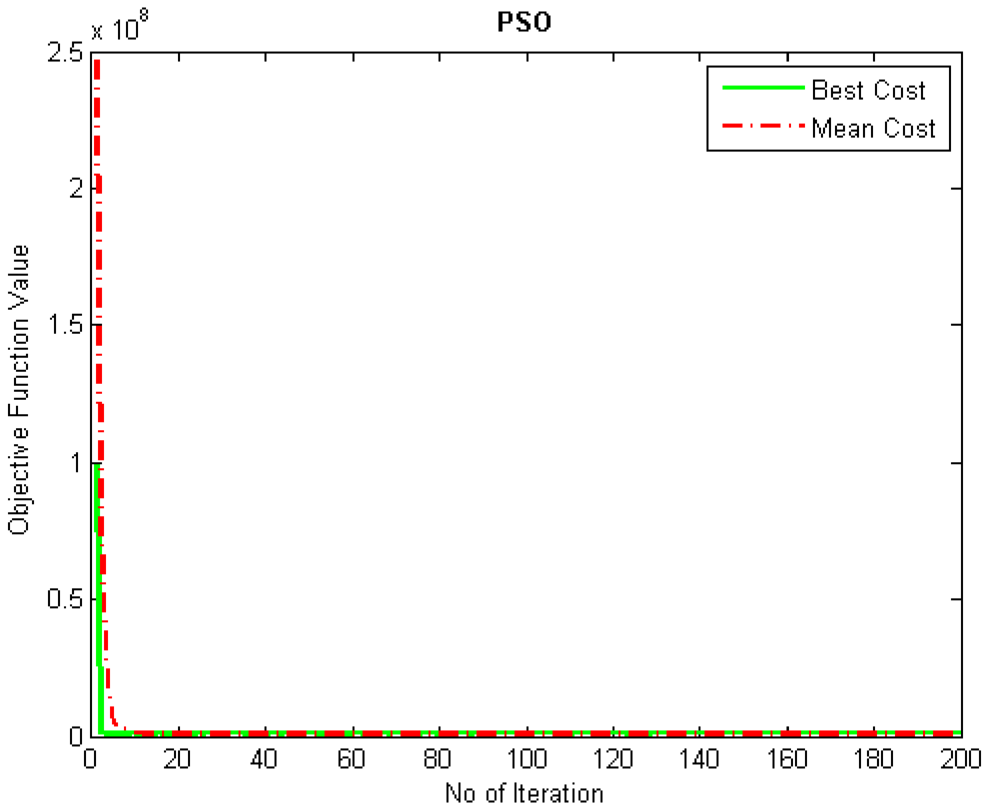


Figure 6.10: The convergence path of fitness function for the best run of the PSO algorithm

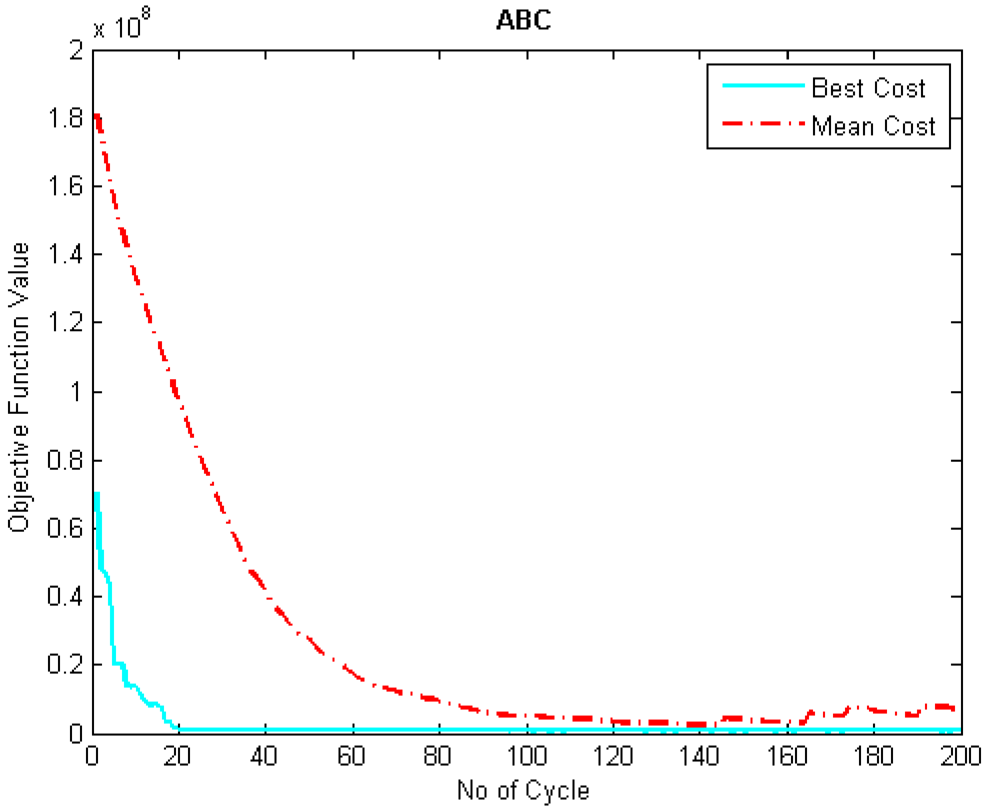


Figure 6.11: The convergence path of fitness function for the best run of the ABC algorithm

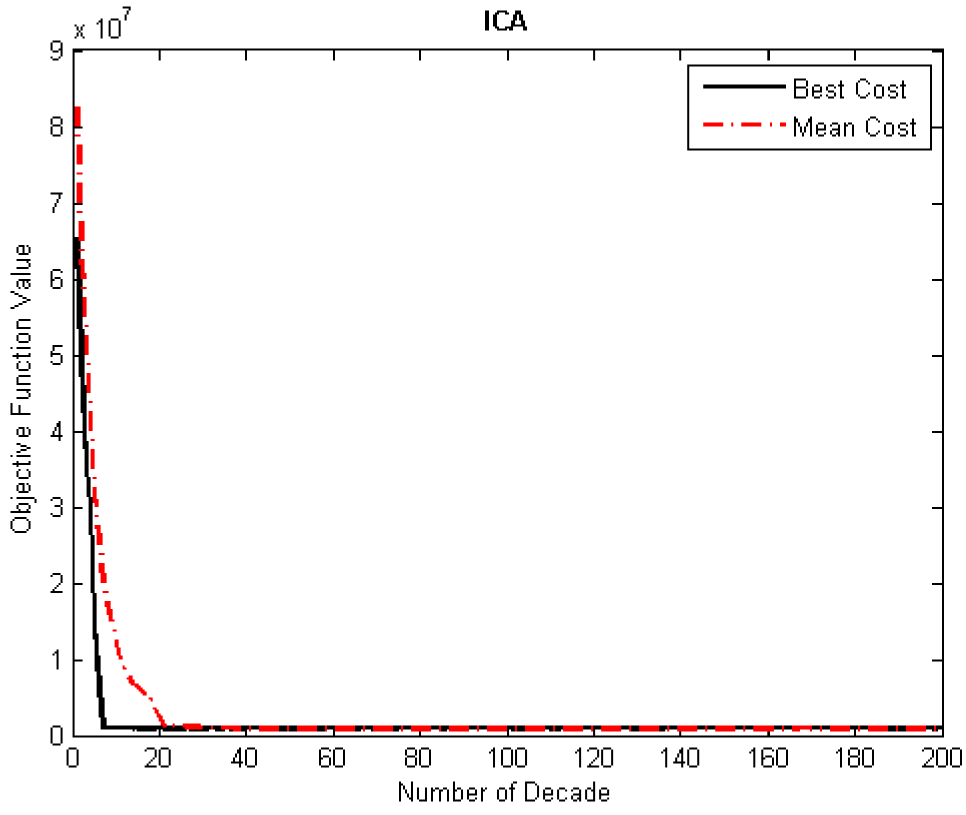


Figure 6.12: The convergence path of fitness function for the best run of the ICA approach

In order to have fair comparisons, the same number of iteration was used for all applied optimizers, which was set to 200. As shown in Figures 6.9 to 6.12, there was no significant improvement in the fitness function values attained for higher number of successive iterations. However, as the problem is cost minimization, it is worth to continue the iterations until the search reaches the lowest possible cost. Using the parallel computers can reduce the computational time dramatically.

Figure 6.13 compares the function values for the best run of all methods versus the number of iterations. The figure shows that the convergence rate of the GA and PSO is almost similar, both converged to near optimum point quickly in the early iterations.

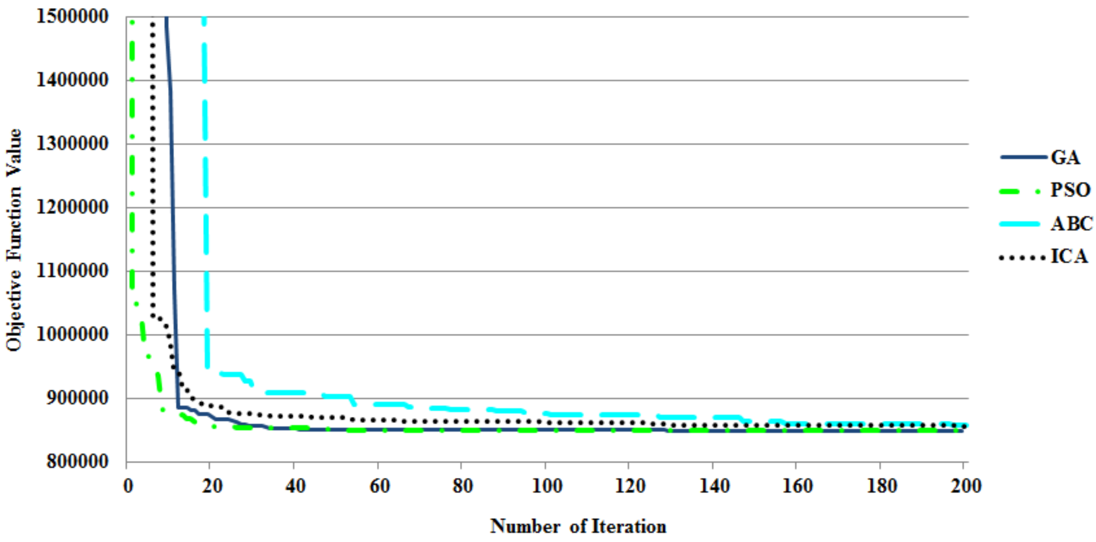


Figure 6.13: Function values versus number of iterations for all applied algorithms

Based on the results, the GA is subsequently used to find the optimal solution of decision variables. The results are reported in Tables 6.15 to 6.23.

Table 6.15: Purchase amount of raw materials

Raw material *k*	Supplier *m*	Plant *j*	Period *t* 1	2	3	4	5	6
			α	α	α	α	α	α
1	1*	1	5145	3427	3477	2351	3173	2155
2	2		3351	2369	2235	1591	1951	1464
3	3		4830	3874	3110	2834	2469	2226
1	1	2	5658	3447	3437	3943	3238	2942
2	2		3779	2302	2321	2624	2097	1943
3	3		5567	3813	3634	3897	3107	2951
1	1	3	980	572	711	706	366	435
2	2		647	356	480	472	246	271
3	3		977	473	801	729	446	365

*there is only one supplier for each type of raw material, i.e. raw materials 1, 2, and 3 are supplied by suppliers 1, 2, and 3 respectively.

Table 6.16: Inventory level of raw materials

Raw material *k*	Plant *j*	Period *t* 1	2	3	4	5	6
		I	*I*	*I*	*I*	*I*	*I*
1	1	1612	1611	1626	1279	1421	1149
2		1083	1127	1099	881	924	781
3		1513	1719	1548	1404	1242	1111
1	2	1773	1671	1634	1780	1608	1458
2		1222	1152	1135	1227	1087	991
3		1745	1777	1731	1798	1573	1449
1	3	307	282	317	327	224	211
2		210	185	217	225	155	139
3		306	251	335	340	253	199

Table 6.17: Production quantity and inventory level of products

Plant *j*	Product *i*	Period *t* 1		2		3		4		5		6	
		Q	*I'*	*Q*	*I'*	*Q*	*I'*	*Q*	*I'*	*Q*	*I'*	*Q*	*I'*
1	1	651	0	460	0	550	0	487	0	619	0	352	0
	2	545	0	598	0	601	0	371	0	461	0	440	0
	3	493	0	645	0	489	0	564	14	347	0	360	0
2	1	583	0	591	41	511	22	541	63	569	82	504	0
	2	670	0	540	0	579	9	671	0	560	30	509	3
	3	596	6	678	4	646	120	623	83	527	70	498	0
3	1	123	47	112	19	124	143	112	222	87	278	90	1
	2	99	64	97	89	95	135	111	117	61	39	65	0
	3	110	103	71	119	129	137	125	262	105	138	64	0

Table 6.18: Quantity of available products to be shipped from plants to distribution centers and total number of products transferred from plants to distribution centers

Product *i*	Plant *j*	Distribution center *w*	Period *t* 1 *C*	1 *Y*	2 *C*	2 *Y*	3 *C*	3 *Y*	4 *C*	4 *Y*	5 *C*	5 *Y*	6 *C*	6 *Y*
1			651	680	460	600	550	550	487	520	619	650	352	630
2	1	1	545	550	530	530	601	650	371	500	461	600	440	550
3			493	500	645	700	489	600	550	550	361	590	360	630
1			583	630	550	550	530	530	500	500	550	550	586	675
2	2	2	670	700	540	680	570	570	680	680	530	530	530	530
3			590	590	680	680	530	530	660	660	540	540	500	500

Table 6.19: Shortage amount before inter-plant transaction

Plant *j*	Product *i*	Period *t* 1	2	3	4	5	6
	1	29	140	0	33	31	278
1	2	5	0	49	129	139	110
	3	7	55	111	0	229	270
	1	47	0	0	0	0	89
2	2	30	140	0	0	0	0
	3	0	0	0	0	0	0

Table 6.20: Quantity of transported products between plants

Product *i*	Plant *j*	Plant *l*	Period *t* 1	2	3	4	5	6
1			0	0	0	0	0	0
2	1	2	0	68	0	0	0	0
3			0	0	0	0	0	0
1			0	0	0	0	0	0
2	2	1	0	0	0	0	0	6
3			0	0	0	0	0	68
1			29	140	0	33	31	278
2	3	1	5	0	49	129	139	104
3			7	55	111	0	229	202
1			47	0	0	0	0	89
2	3	2	30	72	0	0	0	0
3			0	0	0	0	0	0

Table 6.21: Number of vehicles required to ship products from suppliers to plants

Supplier *m*	Plant *j*	Period *t*					
		1	**2**	**3**	**4**	**5**	**6**
1		21	14	14	10	13	9
2	1	14	10	9	7	8	6
3		20	16	13	12	10	9
1		23	14	14	16	13	12
2	2	16	10	10	11	9	8
3		23	16	15	16	13	12
1		4	3	3	3	2	2
2	3	3	2	2	2	1	2
3		4	2	4	3	2	2

Table 6.22: Number of vehicles required for inter-plant transportation

Plant *j*	Plant *l*	Period *t*					
		1	**2**	**3**	**4**	**5**	**6**
1	2	0	1	0	0	0	0
2	1	0	0	0	0	0	1
3	1	1	2	2	2	4	6
	2	1	1	0	0	0	1

Table 6.23: Number of vehicles required to ship products from plants to distribution centers

Plant *j*	Distribution center *w*	Period *t*					
		1	**2**	**3**	**4**	**5**	**6**
1	1	18	19	18	16	19	19
2	2	20	20	17	19	17	18

Since distribution centre 1 is close to plant 1 and distribution centre 2 is close to plant 2, demand at distribution centers 1 and 2 are served by plants 1 and 2 respectively. If the demand of an item could not be met completely by the given plant, the rest of demand was supplied by another nearby plant. Therefore, when plant 1 experienced under-capacity and insufficient inventory problems, it would request plant 3 to transfer the rest of demand. If there were not enough end products available at plant 3, the request was sent to plant 2. Similarly, plant 2 would request plant 1 to supply unmet demands and in the case of inadequate availability of products in plant 1, the request was then sent to plant 3. The results show that after all inter-plant transfers, demand at both distribution centers could be entirely met during all periods.

Based on Table 6.19, the shortage cost incurred for distribution centers 1 and 2 in the considered planning horizon is \$53507, while the transportation cost is \$51430. As shortage cost is higher than the transportation and inventory costs, it indicates that bearing some inventories or/and outsourcing from other plants is preferable than having backorders. The reason lies in the fact that the cost of having backorder is far higher than keeping inventory. In addition, meeting demand in a timely fashion increases customer satisfaction.

The results also show that the binary variables χ_{ijt} and φ_{kmjt} have value 1 for all periods, indicating that the three different products were produced in each of three plants and all plants have ordered all types of raw materials throughout the six-period planning horizon.

6.7 Conclusions

This chapter investigated the effectiveness of coordinating production and distribution planning by addressing a multi-item multi-period capacitated lot-sizing problem in a multi-stage system composed of multiple suppliers, plants, and distribution centers. The combinations of several functions were considered, such as purchasing, production, storage, backordering, and transportation. The objective was to simultaneously determine the optimal raw material order quantity, production and inventory levels, and the transportation amount so that the demand can be satisfied with the lowest possible cost over a given planning horizon without violating the capacity restrictions of the plants and suppliers. Transfer decisions between plants were made when demand observed at a plant was satisfied by other production sites to cope with under-capacity of a given plant. Furthermore, sale at distribution centers was allowed.

A numerical example was described to demonstrate the validity of the proposed model. Efficient search procedures were presented to obtain the optimum solutions by employing four well-known metaheuristic algorithms, namely GA, PSO, ABC, and ICA. In addition, Taguchi method was utilized to calibrate the various parameters of the proposed algorithms. The GA method resulted in the best known solutions and generated lower costs, and in general was found to be superior to other three optimization approaches. In terms of number of function evaluations (computational cost), the PSO was superior to the other methods. The results indicated that the proposed model can provide a promising approach to fulfill an efficient production and distribution planning in such integrated supply chain situations.

CHAPTER 7: CONCLUSIONS

7.1 Concluding Remarks

In this research, attempts were made to evaluate the efficiencies and benefits of using a number of metaheuristic approaches for solving the lot-sizing and scheduling problems. The implications of adding constraints to the economic lot scheduling problem were discussed throughout this research. Generally, constraints regarding the time-modeling increase the complexity of the problem leading to NP-hard situation when such constraints are related to startup times between products. Capacity restrictions also add more complexity to the lot-sizing problems.

Moreover, the capacitated lot-sizing problem was considered in the field of integrated supply chain. The efficient management of supply chain has grown in importance with the realization that it represents a major opportunity for organizations to improve operational performance and overall margins.

In any manufacturing firm, the inventory management plays an essential role in controlling the raw materials, work-in-processes and finished goods. If proper attention is not given to choose efficient inventory policies, a significant amount of the investment may be blocked in the inventory. Because of the heavy losses due to excessive inventory or stockouts, more attention should be given in selecting an effective inventory control system.

In this thesis, mathematical models were formulated and presented for solving the single facility multiple item production problem, where products have shelf life restrictions and can be backordered. The objective was to obtain the optimum cycle time and to minimize the total related cost, including production, setup, holding, and backordering costs. For the case where the optimal cycle time violated the shelf life constraint, three adjustment options were investigated: production rate reduction, cycle time reduction, and production rate and cycle time reduction simultaneously. It was shown that parameters such as production cost, holding cost, setup cost, backorders, and shelf life influence the decision making process to select the appropriate option that offers the minimum yearly cost, and yields an optimum cycle time that satisfies shelf life constraints.

In general, the production time cost exceeds the setup cost in many manufacturing systems. Hence, it can be deduced that lowering cycle time is more cost-effective than reducing the production rate. It was also revealed that in some circumstances, the option of simultaneous reduction of the cycle time and production rate may offer the minimum total cost comparable with adjusting only one of these two factors. Moreover, considering planned backorders contributed to the lower total cost rather than models with no backorders.

Thereafter, the scheduling optimization of a family of items for the economic lot scheduling problem with shelf life restrictions, backordering, and multiple setups in a production cycle was presented. The distinguishing feature of this work, compared to previous studies, was allowing production of each item more than once in a cycle, which brings about a significant reduction in the long run average cost. However, this assumption may lead to an infeasible production schedule. Hence, to eliminate the production time conflicts and to achieve feasibility, the schedule was adjusted by advancing or delaying production start times of some batches of products. The objective was to find the optimum production rate, production frequency, production quantity, and cycle time for the family context so that the total cost including setup, holding, backordering, and adjustment costs are minimized, and a feasible production schedule is accommodated. It was shown that allowing multiple setups leads to improving solutions.

Then, lot-sizing problem was investigated in a multi-plant environment. A multi-plant supply chain is one in which a core exists simultaneously within several manufacturing plants. Manufacturing facilities of the same supply chain should coordinate planning and scheduling tasks and share the flow of information among plants, upstream suppliers, and downstream distributors in order to enhance the whole chain performance. The vital challenge is to define the interactions between different levels of the manufacturing network at the planning and scheduling stage. In an extended production network it is more difficult to manage all the necessary interactions to ensure that disruptions and changes in one plant are taken into account by other plants in order to prevent excessive inventory or stockouts.

Therefore, a mathematical model was presented for the multi-item multi-period capacitated lot-sizing problem for an integrated production and distribution planning in a multi-supplier multi-plant multi-distribution centre logistic environment. The products were distributed to a number of distribution centers at which the demand for each product was known for every period of the planning horizon. The bulk of operational constraints were also represented in order to optimize the supply chain such as resource utilization, demand fulfillment, production capacity, inventory storage capacity, supplier capacity, and vehicle utilization. The goal was to minimize the total cost of the supply chain including procurement, production, setup, inventory, backordering, and transportation, and to meet demand in time. It was indicated that the proposed model can provide a promising approach to fulfill an efficient production and distribution planning in such integrated supply chain.

Since the proposed models are considered as NP-hard, the exact methods may fail to solve them optimally. Hence, well-known metaheuristic algorithms namely genetic algorithm, particle swarm optimization, artificial bee colony, simulated annealing, and imperialist competitive algorithm were used to find the optimal or near-optimal solutions within a moderate computation time. Metaheuristic methods have shown great potentials for solving optimization problems as they conduct global stochastic search. The reason of applying different algorithms to the proposed mathematical models was to validate and to assess the quality of the obtained optimal solutions. It was shown that for both ELSP and MPCLSP models, all applied methods obtained similar results, which it confirms the correctness of the proposed models. In addition, Taguchi method was used to calibrate the parameters of the metaheuristic methods. To compare the performance of the proposed algorithms, the one-way ANOVA was conducted. The statistical results showed that all presented algorithms can efficiently solve the proposed models, while providing promising solutions with respect to solution quality compared to those available in the literature. The metaheuristic algorithms can also handle large-sized instances efficiently in a moderate CPU time.

However, for the proposed economic lot scheduling problem with multiple setups, backordering, and shelf life considerations, ABC algorithm performed better than the GA, PSO, and SA algorithms in terms of objective function value. In terms of CPU time, SA was superior to the other algorithms. For the proposed multi-plant capacitated lot-sizing problem, GA approach offered better results compared to PSO, ABC, and ICA approaches in terms of objective function value. However, PSO algorithm offered modest solution quality in less number of function evaluations for the proposed problem.

7.2 Contributions and Applications

The developed mathematical models for the ELSP and MPCLSP incorporated a variety of practical assumptions, and can cover a variety of problems arising in the literature and in practice particularly scheduling, production and distribution.

It is common in industry to produce several products on a single facility (or machine) due to economies of scale. Typically, these facilities can only produce one product at a time, and have to be stopped and prepared (i.e. setup) at a cost of time and money, before the start of the production run of a different product. A production scheduling problem arises because of the need to coordinate the setups and the production runs of the products. The ELSP is the problem of scheduling production of several products on a single facility, so that demands are met without stockouts or backorders, and the long run average inventory and setup costs are minimized. This problem occurs in many production situations such as:

i. Metal forming and plastics production lines (press lines, and plastic and metal extrusion machines), where each product requires a different die to be set up on the machine.
ii. Assembly lines, which produce several products and/or different product models (electric appliances, motor cars, etc.).
iii. Blending and mixing facilities (for paints, beverages, food, etc.), in which different products are poured into different containers.
iv. Weaving production lines (for textiles, carpets, etc.), in which the main products are manufactured in different colors, widths and grades.

Typically, it is more economical to purchase one high-speed machine capable of producing a number of products than to purchase many dedicated machines.

The proposed MPCLSP model integrated the manufacturing and goods distribution which mostly were considered as discrete problems in the previous studies available in literature. However, in reality for most manufacturing environments, these problems are interconnected. Shifting from single plant to multi-plant organization offers several advantages such as saving on transportation cost and time, improving the customer service by locating the plant close to the customer, being close to low cost raw materials, flexibility in producing several products and specialization in activities, substantial saving in global costs, improvement in relevant service by exploiting scale economies of production and transportation, balancing production lots and vehicle loads, reducing the inventory and stockouts, and so forth.

The MPCLSP can be observed in several industries such as automotive factories, steel corporations, electric power generating industries, food and chemical process industries, and beverage industry, where multiple plants producing the same products are located at different geographical locations in a country or scattered around the world.

Moreover, from the metaheuristic viewpoint, the contribution of the thesis was to find out how metaheuristics performs for the MPCLSP, as previously they have been applied mostly to other production related problems, in particular scheduling, but not to this exact lot-sizing problem. The presented algorithms were computationally effective and beneficial for obtaining the optimal solution for the proposed lot-sizing problems.

In any type of industry, the basic goal remains the same: to identify the most cost effective or profitable way of getting the right product to the right place at the right time in order to satisfy turbulent market demands.

7.3 Recommendations for Future Research

Future research may investigate the possibility of solving the proposed lot-sizing and scheduling problems in stochastic manufacturing environments in which the uncertainty and dynamic nature of some parameters are taken into account.

It may also be of interest to incorporate more realistic characteristic to the proposed economic lot scheduling problem, such as machine failures in analysis of cyclic schedules for products, unequal batches for each product as well as dissimilar production cycles.

Moreover, in the proposed multi-plant capacitated lot-sizing problem, zero lead time for production/replenishments was assumed in order to simplify the problem. Further research may examine the relationship between lot sizes and lead times. Additionally, safety stock for the products can also be integrated to the proposed lot-sizing problem as it can have a significant impact on the firm's performance.

REFERENCES

Abdelsalam, H. M., & Elassal, M. M. (2014). Joint economic lot-sizing problem for a three-layer supply chain with stochastic demand. *International Journal of Production Economics*, *155*, 272-283.

Absi, N., & Kedad-Sidhoum, S. (2009). The multi-item capacitated lot-sizing problem with safety stocks and demand shortage costs. *Computers & Operations Research*, *36*(11), 2926-2936.

Absi, N., Kedad-Sidhoum, S., & Dauzère-Pérès, S. (2011). Uncapacitated lot-sizing problem with production time windows, early productions, backlogs and lost sales. *International Journal of Production Research*, *49*(9), 2551-2566.

Adelman, D., & Barz, C. (2013). A unifying approximate dynamic programming model for the economic lot scheduling problem. *Mathematics of Operations Research*, *39*(2), 374-402.

Aghezzaf, E. H. (2007). Production planning and warehouse management in supply networks with inter-facility mold transfers. *European Journal of Operational Research*, *182*(3), 1122-1139.

Agra, A., & Constantino, M. (1999). Lot-sizing with backlogging and start-ups: The case of Wagner-Whitin costs. *Operations Research Letters*, *25*(2), 81-88.

Akay, B., & Karaboga, D. (2012). A modified artificial bee colony algorithm for real-parameter optimization. *Information Sciences*, *192*, 120-142.

Akbalik, A., & Pochet, Y. (2009). Valid inequalities for the single-item capacitated lot-sizing problem with step-wise costs. *European Journal of Operational Research*, *198*(2), 412-434.

Aksen, D., Altinkemer, K., & Chand, S. (2003). The single-item lot-sizing problem with immediate lost sales. *European Journal of Operational Research*, *147*(3), 558-566.

Aksoy, Y., & Erenguc, S. S. (1988). Multi-item inventory models with co-ordinated replenishments: A survey. *International Journal of Operations & Production Management*, *8*(1), 63-73.

Aliyu, M. D. S., & Andijani, A. A. (1999). Multi-item-multi-plant inventory control of production systems with shortages/backorders. *International Journal of Systems Science*, *30*(5), 533-539.

Alvarez, E. (2007). Multi-plant production scheduling in SMEs. *Robotics and Computer-Integrated Manufacturing*, *23*(6), 608-613.

Amorim, P., Günther, H. O., & Almada-Lobo, B. (2012). Multi-objective integrated production and distribution planning of perishable products. *International Journal of Production Economics*, *138*(1), 89-101.

Anderson, E. J., Glass, C. A., & Potts, C. N. (1997). Machine scheduling. *Local search in combinatorial optimization*, *11*, 361-414.

Anily, S., Tzur, M., & Wolsey, L. A. (2009). Multi-item lot-sizing with joint set-up costs. *Mathematical programming*, *119*(1), 79-94.

Aras, Ö. A., & Swanson, L. A. (1982). A lot-sizing and sequencing algorithm for dynamic demands upon a single facility. *Journal of Operations Management*, *2*(3), 177-185.

Armentano, V. A., Shiguemoto, A. L., & Løkketangen, A. (2011). Tabu search with path relinking for an integrated production–distribution problem. *Computers & Operations Research*, *38*(8), 1199-1209.

Atashpaz-Gargari, E., & Lucas, C. (2007, September). Imperialist competitive algorithm: an algorithm for optimization inspired by imperialistic competition. In *Evolutionary computation, 2007. CEC 2007. IEEE Congress on* (pp. 4661-4667). IEEE.

Babaei, M., Mohammadi, M., & Ghomi, S. F. (2014). A genetic algorithm for the simultaneous lot sizing and scheduling problem in capacitated flow shop with complex setups and backlogging. *The International Journal of Advanced Manufacturing Technology*, *70*(1-4), 125-134.

Bahl, H. C., Ritzman, L. P., & Gupta, J. N. (1987). OR practice-determining lot sizes and resource requirements: A review. *Operations Research*, *35*(3), 329-345.

Barbarosoğlu, G., & Özdamar, L. (2000). Analysis of solution space-dependent performance of simulated annealing: the case of the multi-level capacitated lot-sizing problem. *Computers & Operations Research*, *27*(9), 895-903.

Behnamian, J., & Ghomi, S. F. (2013). The heterogeneous multi-factory production network scheduling with adaptive communication policy and parallel machine. *Information Sciences*, *219*, 181-196.

Belvaux, G., & Wolsey, L. A. (2000). Bc-prod: A specialized branch-and-cut system for lot-sizing problems. *Management Science*, *46*(5), 724-738.

Ben-Daya, M., As'ad, R., & Seliaman, M. (2013). An integrated production inventory model with raw material replenishment considerations in a three layer supply chain. *International Journal of Production Economics*, *143*(1), 53-61.

Berretta, R., & Rodrigues, L. F. (2004). A memetic algorithm for a multistage capacitated lot-sizing problem. *International Journal of Production Economics*, *87*(1), 67-81.

Berretta, R., França, P. M., & Armentano, V. A. (2005). Metaheuristic approaches for the multilevel resource-constrained lot-sizing problem with setup and lead times. *Asia-Pacific Journal of Operational Research*, *22*(02), 261-286.

Bhatnagar, R., Chandra, P., & Goyal, S. K. (1993). Models for multi-plant coordination. *European Journal of Operational Research*, *67*(2), 141-160.

Billington, P. J., McClain, J. O., & Thomas, L. J. (1983). Mathematical programming approaches to capacity-constrained MRP systems: Review, formulation and problem reduction. *Management Science*, *29*(10), 1126-1141.

Bitran, G. R., & Tirupati, D. (1993). Hierarchical production planning. *Handbooks in operations research and management science*, *4*, 523-568.

Bitran, G. R., & Yanasse, H. H. (1982). Computational complexity of the capacitated lot size problem. *Management Science*, *28*(10), 1174-1186.

Bollapragada, R., & Rao, U. (1999). Single-stage resource allocation and economic lot scheduling on multiple, non-identical production lines. *Management Science*, *45*(6), 889-904.

Bollapragada, R., Della Croce, F., & Ghirardi, M. (2011). Discrete-time, economic lot scheduling problem on multiple, non-identical production lines. *European Journal of Operational Research*, *215*(1), 89-96.

Bomberger, E. E. (1966). A dynamic programming approach to a lot size scheduling problem. *Management Science*, *12*(11), 778-784.

Bourland, K. E., & Yano, C. A. (1994). The strategic use of capacity slack in the economic lot scheduling problem with random demand. *Management Science*, *40*(12), 1690-1704.

Boussaïd, I., Lepagnot, J., & Siarry, P. (2013). A survey on optimization metaheuristics. *Information Sciences*, *237*, 82-117.

Brahimi, N., Dauzere-Peres, S., Najib, N. M., & Nordli, A. (2006). Single item lot-sizing problems. *European Journal of Operational Research*, *168*(1), 1-16.

Brander, P., & Forsberg, R. (2006). Determination of safety stocks for cyclic schedules with stochastic demands. *International Journal of Production Economics*, *104*(2), 271-295.

Bruggeman, W., Vandendaele, P., & Muller, H. (1982). Diagnostic analysis of the potential savings of a multi-product/single facility production planning system. *Engineering Costs and Production Economics*, *6*, 33-41.

Brüggemann, W., Jahnke, H. (1994). DLSP for two-stage multi-item batch production. *International Journal of Production Research*, *32*(4), 755-768

Buer, T., Homberger, J., & Gehring, H. (2013). A collaborative ant colony metaheuristic for distributed multi-level uncapacitated lot-sizing. *International Journal of Production Research, 51*(17), 5253-5270.

Bulut, O., & Tasgetiren, M. F. (2014, July). A discrete artificial bee colony algorithm for the Economic Lot Scheduling problem with returns. In *Evolutionary Computation (CEC), 2014 IEEE Congress on* (pp. 551-557). IEEE.

Bulut, O., Tasgetiren, M. F., & Fadiloglu, M. M. (2012). A genetic algorithm for the economic lot scheduling problem under extended basic period approach and power-of-two policy. In *Advanced Intelligent Computing Theories and Applications. With Aspects of Artificial Intelligence* (pp. 57-65). Springer Berlin Heidelberg.

Buschkühl, L., Sahling, F., Helber, S., & Tempelmeier, H. (2010). Dynamic capacitated lot-sizing problems: a classification and review of solution approaches. *OR Spectrum*, *32*(2), 231-261.

Caserta, M., Ramirez, A., & Voß, S. (2010). A math-heuristic for the multi-level capacitated lot-sizing problem with carryover. In *Applications of Evolutionary Computation* (pp. 462-471). Springer Berlin Heidelberg.

Chakravarthy, S. R., & Daniel, J. K. (2004). A Markovian inventory system with random shelf time and back orders. *Computers & Industrial Engineering*, *47*(4), 315-337.

Chan, F. T. S., Chung, S. H., Chan, P. L. Y., Finke, G., & Tiwary, M. K. (2005). Solving distributed scheduling problem with alternative routings by genetic algorithms. In *15th International Conference on Flexible Automation and Intelligent Manufacturing FAIM 2005* (pp. 391-399).

Chan, F. T. S., Kumar, V., & Mishra, N. (2008, September). Resolving multi plant supply chain problem: A novel swarm intelligence based approach. In *Management of Innovation and Technology, 2008. ICMIT 2008. 4th IEEE International Conference on* (pp. 1066-1071). IEEE.

Chan, F. T. S., Tibrewal, R. K., Prakash, A., & Tiwari, M. K. (2015). A biased random key genetic algorithm approach for inventory-based multi-item lot-sizing problem. *Proceedings of the Institution of Mechanical Engineers, Part B: Journal of Engineering Manufacture*, *229*(1), 157-171.

Chandra, P., & Fisher, M. L. (1994). Coordination of production and distribution planning. *European Journal of Operational Research*, *72*(3), 503-517.

Chandrasekaran, C., Rajendran, C., Krishnaiah Chetty, O., & Hanumanna, D. (2007). Metaheuristics for solving economic lot scheduling problems (ELSP) using time-varying lot-sizes approach. *European Journal of Industrial Engineering, 1*(2), 152-181.

Chatfield, D.C. (2007). The economic lot scheduling problem: A pure genetic search approach. *Computers & Operations Research, 34*(10), 2865-2881.

Chen, C. L. (2010, July). A heuristic model for justifying the acceptance of rush orders in multi-plants manufacturing supply chain with outsourcing. In *Industrial Informatics (INDIN), 2010 8th IEEE International Conference on* (pp. 607-611). IEEE.

Chen, H. (2015). Fix-and-optimize and variable neighborhood search approaches for multi-level capacitated lot-sizing problems. *Omega*, *56*, 25-36.

Chen, H. D., Hearn, D. W., & Lee, C. Y. (1994). A new dynamic programming algorithm for the single item capacitated dynamic lot size model. *Journal of Global Optimization*, *4*(3), 285-300.

Chen, H., & Chu, C. (2003). A Lagrangian relaxation approach for supply chain planning with order/setup costs and capacity constraints. *Journal of Systems Science and Systems Engineering*, *12*(1), 98-110.

Chen, W. H., & Thizy, J. M. (1990). Analysis of relaxations for the multi-item capacitated lot-sizing problem. *Annals of operations Research*, *26*(1), 29-72.

Chen, Y. Y., & Lin, J. T. (2009). A modified particle swarm optimization for production planning problems in the TFT Array process. *Expert Systems with Applications*, *36*(10), 12264-12271.

Chen, Z., & Sarker, B. R. (2014). An integrated optimal inventory lot-sizing and vehicle-routing model for a multi-supplier single-assembler system with JIT delivery. *International Journal of Production Research, 52*(17), 5086-5114.

Chiu, S. W. (2008). Production lot size problem with failure in repair and backlogging derived without derivatives. *European Journal of Operational Research*, *188*(2), 610-615.

Chopra, S., & Meindl, P. (2001). *Supply chain management: strategy, planning, operation* (1st Ed.). NJ, USA: Prentice Hall, Upper Saddle River.

Chowdhury, M. R., & Sarker, B. R. (2001). Manufacturing batch size and ordering policy for products with shelf lives. *International Journal of Production Research*, *39*(7), 1405-1426.

Chung, C. S., Flynn, J., & Lin, C. H. M. (1994). An effective algorithm for the capacitated single item lot size problem. *European Journal of Operational Research*, *75*(2), 427-440.

Chung, S. H., & Chan, H. K. (2012). A two-level genetic algorithm to determine production frequencies for economic lot scheduling problem. *Industrial Electronics, IEEE Transactions on*, *59*(1), 611-619.

Clark, A., Almada-Lobo, B., & Almeder, C. (2011). Lot-sizing and scheduling: industrial extensions and research opportunities. *International Journal of Production Research*, *49*(9), 2457-2461.

Coelho, L., & Sierakowski, C. A. (2008). A software tool for teaching of particle swarm optimization fundamentals. *Advances in Engineering Software*, *39*(11), 877-887.

Conklin, J. M., & Perdue, B. A. (1994, May). Extending capabilities through contract manufacturing. In *Electro/94 International. Conference Proceedings. Combined Volumes*. (pp. 145-153). IEEE.

Cooper, M. C., Lambert, D. M., & Pagh, J. D. (1997). Supply chain management: more than a new name for logistics. *The International Journal of Logistics Management*, *8*(1), 1-14.

Crowston, W. B., & Wagner, M. (1971). *Lot size determination in multi-stage assembly systems*. (Unpublished doctoral dissertation). Sloan School of Management, M. I. T., USA.

Dantzig, G. B. (1955). Optimal solution of a dynamic Leontief model with substitution. *Econometrica: Journal of the Econometric Society*, 295-302.

De Kok, T. G. (2000). Capacity allocation and outsourcing in a process industry. *International Journal of Production Economics*, *68*(3), 229-239.

De Reyck, B. (2015). Period decompositions for the capacitated lot-sizing problem with setup times. *Informs Journal on Computing*.

De Souza, K. X., & Armentano, V. A. (1994). Multi-item capacitated lot-sizing by a cross decomposition based algorithm. *Annals of Operations Research*, *50*(1), 557-574.

De Toledo, F. M. B., & Shiguemoto, A. L. (2005). Lot-sizing problem with several production centers. *Pesquisa Operacional*, *25*(3), 479-492.

Dellaert, N. P., & Jeunet, J. (2003). Randomized multi-level lot-sizing heuristics for general product structures. *European Journal of Operational Research*, *148*(1), 211-228.

Dellaert, N., & Jeunet, J. (2000). Solving large unconstrained multilevel lot-sizing problems using a hybrid genetic algorithm. *International Journal of Production Research*, *38*(5), 1083-1099.

Dellaert, N., Jeunet, J., & Jonard, N. (2000). A genetic algorithm to solve the general multi-level lot-sizing problem with time-varying costs. *International Journal of Production Economics*, *68*(3), 241-257.

DeMatteis, J. J. (1968). An economic lot-sizing technique, I: The part-period algorithm. *IBM Systems Journal*, *7*(1), 30-38.

Dhaenens-Flipo, C. (2000). Spatial decomposition for a multi-facility production and distribution problem. *International Journal of Production Economics*, *64*(1), 177-186.

Dhaenens-Flipo, C., & Finke, G. (2001). An integrated model for an industrial production–distribution problem. *IIE Transactions*, *33*(9), 705-715.

Dhingra, A. K. (2006). *Multi-objective flow shop scheduling using metaheuristics*. (Unpublished doctoral dissertation). National Institute of Technology Kurukshetra.

Dixon, P. S., & Silver, E. A. (1981). A heuristic solution procedure for the multi-item, single-level, limited capacity, lot-sizing problem. *Journal of Operations Management*, *2*(1), 23-39.

Dobson, G. (1987). The economic lot-scheduling problem: achieving feasibility using time-varying lot sizes. *Operations Research*, *35*(5), 764-771.

Drexl, A., & Haase, K. (1995). Proportional lot-sizing and scheduling. *International Journal of Production Economics*, *40*(1), 73-87.

Drexl, A., & Kimms, A. (1997). Lot-sizing and scheduling-survey and extensions. *European Journal of Operational Research*, *99*(2), 221-235.

El-Alfy, E. S. M., Thampi, S. M., Takagi, H., Piramuthu, S., & Hanne, T. (2015). *Advances in Intelligent Informatics*. Springer.

Fakhrzad, M. B., & Zare, H. K. (2009). Combination of genetic algorithm with Lagrange multipliers for lot-size determination in multi-stage production scheduling problems. *Expert Systems with Applications*, *36*(6), 10180-10187.

Federgruen, A., & Tzur, M. (1994). The joint replenishment problem with time-varying costs and demands: Efficient, asymptotic and ε-optimal solutions. *Operations Research*, *42*(6), 1067-1086.

Federgruen, A., Meissner, J., & Tzur, M. (2007). Progressive interval heuristics for multi-item capacitated lot-sizing problems. *Operations Research*, *55*(3), 490-502.

Fleischmann, B. (1990). The discrete lot-sizing and scheduling problem. *European Journal of Operational Research*, *44*(3), 337-348.

Fleischmann, B., & Meyr, H. (1997). The general lot-sizing and scheduling problem. *Operations-Research-Spektrum*, *19*(1), 11-21.

Florian, M., Lenstra, J.K., & Kan, A.R. (1980). Deterministic production planning: Algorithms and complexity. *Management Science*, *26*(7), 669-679.

Fordyce, J. M., & Webster, F. M. (1985). Nonconstant unit cost/price with the Wagner-Whitin algorithm. *Production and Inventory Management*, *26*(1), 71-80.

França, P. M., Armentano, V. A., Berretta, R. E., & Clark, A. R. (1997). A heuristic method for lot-sizing in multi-stage systems. *Computers & Operations Research*, *24*(9), 861-874.

Fumero, F., & Vercellis, C. (1999). Synchronized development of production, inventory, and distribution schedules. *Transportation science*, *33*(3), 330-340.

Gaafar, L.K., Nassef, A.O., & Aly, A.I. (2009). Fixed-quantity dynamic lot-sizing using simulated annealing. *The International Journal of Advanced Manufacturing Technology*, *41*(1-2), 122-131.

Gallego, G., & Joneja, D. (1994). Economic lot scheduling problem with raw material considerations. *Operations Research*, *42*(1), 92-101.

Ganeshan, R., Jack, E., Magazine, M. J., & Stephens, P. (1999). A taxonomic review of supply chain management research. In *Quantitative models for supply chain management* (pp. 839-879). Springer US.

Ganguly, S., Chowdhury, A., Mukherjee, S., Suganthan, P. N., Das, S., & Chua, T. J. (2013). A Hybrid Discrete Differential Evolution Algorithm for Economic Lot Scheduling Problem with Time Variant Lot-sizing. In *Soft Computing Models in Industrial and Environmental Applications* (pp. 1-12). Springer Berlin Heidelberg.

Gascon, A. (1995). On the finite horizon EOQ model with cost changes. *Operations Research*, *43*, 716-717.

Gavish, B., & Johnson, R. E. (1990). A fully polynomial approximation scheme for single-product scheduling in a finite capacity facility. *Operations Research*, *38*(1), 70-83.

Gen, M., Cheng, R., & Lin, L. (2008). *Network models and optimization: Multi-objective genetic algorithm approach*. Springer.

Gendreau, M., & Potvin, J. Y. (2005). Metaheuristics in combinatorial optimization. *Annals of Operations Research*, *140*(1), 189-213.

Giri, B. C., Moon, I., & Yun, W. Y. (2003). Scheduling economic lot sizes in deteriorating production systems. *Naval Research Logistics*, *50*(6), 650-661.

Glock, C. H., Grosse, E. H., & Ries, J. M. (2014). The lot-sizing problem: A tertiary study. *International Journal of Production Economics*, *155*, 39-51.

Goetschalckx, M. (2011). *Supply chain engineering*. Springer Science & Business Media.

Goyal, S. K. (1994). A note on effect of production cost on shelf -life. *International Journal of Production Research*, 32(9), 2243-2245.

Goyal, S. K. (1996) A note on dealing with a shelf life constraint in cyclic scheduling by adjusting both cycle time and production rate. *International Journal of Production Research*, *34*(10), 2967-2969.

Goyal, S. K., & Deshmukh, S. G. (1992). Integrated procurement-production systems: A review. *European Journal of Operational Research*, *62*(1), 1-10.

Goyal, S. K., & Deshmukh, S. G. (1993). Discussion a note on ‘the economic ordering quantity for jointly replenishing items’. *International Journal of Production Research*, *31*(12), 2959-2961.

Goyal, S. K., & Viswanathan, S. (2006). On ‘Optimal production policy with shelf life including shortages. *Journal of Operational Research Society*, *57*, 747-747.

Graves, S. C. (1981). A review of production scheduling. *Operations Research*, *29*(4), 646-675.

Guimarães, L., Klabjan, D., & Almada-Lobo, B. (2012). Annual production budget in the beverage industry. *Engineering Applications of Artificial Intelligence*, *25*(2), 229-241.

Günther, H. O. (1987). Planning lot sizes and capacity requirements in a single stage production system. *European Journal of Operational Research*, *31*(2), 223-231.

Gupta, C., & Jain, S. (2015). New Approach for Function Optimization: Amended Harmony Search. In *Advances in Intelligent Informatics* (pp. 643-653). Springer International Publishing.

Gupta, D., & Magnusson, T. (2005). The capacitated lot-sizing and scheduling problem with sequence-dependent setup costs and setup times. *Computers & Operations Research*, *32*(4), 727-747.

Gupta, S. M., & Brennan, L. (1992). Heuristic and optimal approaches to lot-sizing incorporating backorders: an empirical evaluation. *The International Journal of Production Research*, *30*(12), 2813-2824.

Gupta, S., & Karimi, I. A. (2003). Scheduling a two-stage multiproduct process with limited product shelf life in intermediate storage. *Industrial & Engineering Chemistry Research*, *42*(3), 490-508.

Gürler, Ü., and Özkaya, B. Y. (2008) Analysis of the (s, S) policy for perishables with a random shelf life. *IIE Transactions*, *40*, 759-781.

Gutiérrez, J., Sedeño-Noda, A., Colebrook, M., & Sicilia, J. (2008). An efficient approach for solving the lot-sizing problem with time-varying storage capacities. *European Journal of Operational Research*, *189*(3), 682-693.

Haase, K. (1994). *Lotsizing and scheduling for production planning,* number 408 in *Lecture Notes in Economics and Mathematical Systems*, Springer, Berlin.

Haase, K. (1996). Capacitated lot-sizing with sequence dependent setup costs. *Operations-Research-Spektrum*, *18*(1), 51-59.

Hajipour, V., Fattahi, P., & Nobari, A. (2014). A hybrid ant colony optimization algorithm to optimize capacitated lot-sizing problem. *Journal of Industrial and Systems Engineering*, *7*(1), 1-20.

Hajipour, V., Kheirkhah, A., Tavana, M., & Absi, N. (2015). Novel Pareto-based meta-heuristics for solving multi-objective multi-item capacitated lot-sizing problems. *The International Journal of Advanced Manufacturing Technology*, 1-15.

Han, Y., Tang, J., Kaku, I., & Mu, L. (2009). Solving uncapacitated multilevel lot-sizing problems using a particle swarm optimization with flexible inertial weight. *Computers & Mathematics with Applications*, *57*(11), 1748-1755.

Hardin, J. R., Nemhauser, G. L., & Savelsbergh, M. W. (2007). Analysis of bounds for a capacitated single-item lot-sizing problem. *Computers & operations research*, *34*(6), 1721-1743.

Hariga, M. A. (1994). Economic analysis of dynamic inventory models with non-stationary costs and demand. *International Journal of Production Economics*, *36*(3), 255-266.

Harris, F. W. (1913). How many parts to make at once. *Operations Research*, 947-950.

Hax, A., & Candea, D. (1984). *Production and inventory management*. Prentice-Hall, Englewood Cliffs, NJ.

Heizer, J., & Render, B. (2004). *Operations Management*. Upper Saddle River: Pearson Education.

Helber, S., & Sahling, F. (2010). A fix-and-optimize approach for the multi-level capacitated lot sizing problem. *International Journal of Production Economics*, *123*(2), 247-256.

Hellion, B., Mangione, F., & Penz, B. (2012). A polynomial time algorithm to solve the single-item capacitated lot-sizing problem with minimum order quantities and concave costs. *European Journal of Operational Research*, *222*(1), 10-16.

Hernandez, W., & Suer, G. (1999). Genetic algorithms in lot-sizing decisions. In Evolutionary Computation, 1999. CEC 99. Proceedings of the 1999 Congress on (Vol. 3). IEEE.

Hindi, K. S. (1995a). Solving the single-item, capacitated dynamic lot-sizing problem with startup and reservation costs by tabu search. *Computers & industrial engineering*, *28*(4), 701-707.

Hindi, K. S. (1995b). Computationally efficient solution of the multi-item, capacitated lot-sizing problem. *Computers & industrial engineering*, *28*(4), 709-719.

Hindi, K. S. (1996). Solving the CLSP by a tabu search heuristic. *Journal of the Operational Research Society*, 151-161.

Hindi, K. S., Fleszar, K., & Charalambous, C. (2003). An effective heuristic for the CLSP with set-up times. *Journal of the Operational Research Society*, *54*(5), 490-498.

Holland, J. H. (1975). Adoption in neural and artificial systems. *Ann Arbor, Michigan (USA): The University of Michigan Press*.

Homberger, J. (2010). Decentralized multi-level uncapacitated lot-sizing by automated negotiation. *4OR*, *8*(2), 155-180.

Homberger, J., & Gehring, H. (2009, January). An ant colony optimization approach for the multi-level unconstrained lot-sizing problem. In *System Sciences, 2009. HICSS'09. 42nd Hawaii International Conference on* (pp. 1-7). IEEE.

Horng, S. C., & Yang, F. Y. (2013, January). Optimal base-stock level of the stochastic economic lot scheduling problem. In *Computing, Management and Telecommunications (ComManTel), 2013 International Conference on* (pp. 380-385). IEEE.

Hsu, W. L. (1983). On the general feasibility test of scheduling lot sizes for several products on one machine. *Management science, 29*(1), 93-105.

Hung, Y. F., & Chien, K. L. (2000). A multi-class multi-level capacitated lot-sizing model. *Journal of the Operational Research Society*, *51*(11), 1309-1318.

Hwang, H. C., & van den Heuvel, W. (2009). *Economic lot-sizing problem with bounded inventory and lost-sales* (No. EI 2009-01, pp. 1-30). Econometric Institute Research Papers.

Hwang, H. C., Ahn, H. S., & Kaminsky, P. (2013). Basis paths and a polynomial algorithm for the multistage production-capacitated lot-sizing problem. *Operations Research*, *61*(2), 469-482.

Inderfurth, K., Kovalyov, M. Y., Ng, C. T., & Werner, F. (2007). Cost minimizing scheduling of work and rework processes on a single facility under deterioration of reworkables. *International Journal of Production Economics*, *105*(2), 345-356.

Jans, R., & Degraeve, Z. (2007). Meta-heuristics for dynamic lot-sizing: A review and comparison of solution approaches. *European Journal of Operational Research*, *177*(3), 1855-1875.

Jans, R., & Degraeve, Z. (2008). Modeling industrial lot-sizing problems: A review. *International Journal of Production Research*, *46*(6), 1619-1643.

Jenabi, M., Fatemi Ghomi, S. M. T., Torabi, S. A., & Karimi, B. (2007). Two hybrid meta-heuristics for the finite horizon ELSP in flexible flow lines with unrelated parallel machines. *Applied Mathematics and Computation, 186*(1), 230–245.

Jeunet, J., & Jonard, N. (2005). Single-point stochastic search algorithms for the multi-level lot-sizing problem. *Computers & operations research*, *32*(4), 985-1006.

Jodlbauer, H. (2006). An approach for integrated scheduling and lot-sizing. *European journal of Operational Research*, *172*(2), 386-400.

Jolayemi, J. K., & Olorunniwo, F. O. (2004). A deterministic model for planning production quantities in a multi-plant, multi-warehouse environment with extensible capacities. *International Journal of Production Economics*, *87*(2), 99-113.

Junqueira, R. A., & Morabito, R. (2012). Production and logistics planning considering circulation taxes in a multi-plant seed corn company. *Computers and Electronics in Agriculture*, *84*, 100-110.

Karaboga, D. (2005). *An idea based on honey bee swarm for numerical optimization* (Vol. 200). Technical report-tr06, Erciyes University, Engineering Faculty, Computer Engineering Department.

Karaboga, D., & Basturk, B. (2008). Artificial Bee Colony (ABC) optimization algorithm for solving constrained optimization problems. *Advances in Soft Computing*, *8*(1), 687-697.

Karimi, B., Fatemi Ghomi, S., & Wilson, J. (2003). The capacitated lot-sizing problem: a review of models and algorithms. *Omega*, *31*(5), 365-378.

Karmarkar, U. S., & Schrage, L. (1985). The deterministic dynamic product cycling problem. *Operations Research*, *33*(2), 326-345.

Kayvanfar, V., & Zandieh, M. (2012). The economic lot scheduling problem with deteriorating items and shortage: an imperialist competitive algorithm. *The International Journal of Advanced Manufacturing Technology*, *62*(5-8), 759-773.

Kazaz, B., and Sloan, T. W. (2008) Production policies under deteriorating process conditions. *IIE Transactions*, *40*, 187-205.

Kennedy J.F., & Eberhart, R.C. Particle swarm optimization. In: *Proceedings of the IEEE international conference on neural networks*, vol. 4. Perth, Australia; 1995, (pp. 1942–48).

Khouja, M., Michalewicz, Z., & Wilmot, M. (1998). The use of genetic algorithms to solve the economic lot size scheduling problem. *European Journal of Operational Research, 110*(3), 509-524.

Khoury, B. N., Abboud, N. E., & Tannous, M. M. (2001). The common cycle approach to the ELSP problem with insufficient capacity. *International Journal of Production Economics*, *73*(2), 189-199.

Kimms, A. (1996). Competitive methods for multi-level lot-sizing and scheduling: tabu search and randomized regrets. *International Journal of Production Research*, *34*(8), 2279-2298

Kirca, Ö. (1990). An efficient algorithm for the capacitated single item dynamic lot size problem. *European Journal of Operational Research*, *45*, 15-24.

Kirca, Ö. (1995). A primal-dual algorithm for the dynamic lot-sizing problem with joint set-up costs. *Naval Research Logistics*, *42*(5), 791-806.

Kirkpatrick, S. (1984). Optimization by simulated annealing: Quantitative studies. *Journal of Statistical Physics*, *34*(5-6), 975-986.

Kirkpatrick, S., Gelatt, C. D., & Vecchi, M. P. (1983). Optimization by simulated annealing. *Science*, *220*(4598), 671-680.

Koçlar, A., & Süral, H. (2005). A note on "The general lot-sizing and scheduling problem". *OR Spectrum*, *27*(1), 145-146.

Köksoy, O., & Yalcinoz, T. (2008). Robust Design using Pareto type optimization: A genetic algorithm with arithmetic crossover. *Computers & Industrial Engineering*, *55*(1), 208-218.

Kreipl, S., & Pinedo, M. (2004). Planning and scheduling in supply chains: an overview of issues in practice. *Production and Operations Management*, *13*(1), 77-92.

Kuhn, H. (1997). A dynamic lot-sizing model with exponential machine breakdowns. *European Journal of Operational Research*, *100*(3), 514-536.

Kuik, R., Salomon, M., van Wassenhove, L. N., & Maes, J. (1993). Linear programming, simulated annealing and tabu search heuristics for lot sizing in bottleneck assembly systems. *IIE transactions*, *25*(1), 62-72.

Kumar, S. A., & Suresh, N. (2009). *Operations management*. New Age International (P) Limited Publishers.

Lambrecht, M. R., & Vanderveken, H. (1979). Heuristic procedures for the single operation, multi-item loading problem. *AIIE Transactions*, *11*(4), 319-326.

Lang, J. C. (2010). *Production and inventory management with substitutions.* Lecture Notes in Economics and Mathematical Systems 636, Springer, Berlin.

Lee, C. Y., Çetinkaya, S., & Wagelmans, A. P. (2001). A dynamic lot-sizing model with demand time windows. *Management Science*, *47*(10), 1384-1395.

Lee, K. S., & Geem, Z. W. (2005). A new meta-heuristic algorithm for continuous engineering optimization: harmony search theory and practice. *Computer methods in applied mechanics and engineering*, *194*(36), 3902-3933.

Lev, B., & Weiss, H. J. (1990). Inventory models with cost changes. *Operations research*, *38*(1), 53-63.

Lin, H. C., Su, C. T., Wang, C. C., Chang, B. H., & Juang, R. C. (2012). Parameter optimization of continuous sputtering process based on Taguchi methods, neural networks, desirability function, and genetic algorithms. *Expert Systems with Applications*, *39*(17), 12918-12925.

Lin, J. T., & Chen, Y. Y. (2007). A multi-site supply network planning problem considering variable time buckets-A TFT-LCD industry case. *The International Journal of Advanced Manufacturing Technology*, *33*(9-10), 1031-1044.

Liu, J., Wu, L., & Zhou, Z. (2008). A time-varying lot size method for the economic lot scheduling problem with shelf life considerations. *European Journal of Industrial Engineering*, *2*(3), 337-355.

Liu, X., Chu, F., Chu, C., & Wang, C. (2004). Some new properties for capacitated lot-sizing problem with bounded inventory and stockouts. *WSEAS Transactions on Mathematics*, *3*, 65-70.

Löhndorf, N., Riel, M., & Minner, S. (2014). Simulation optimization for the stochastic economic lot scheduling problem with sequence-dependent setup times. *International Journal of Production Economics*, *157*, 170-176.

Loparic, M., Pochet, Y., & Wolsey, L. A. (2001). The uncapacitated lot-sizing problem with sales and safety stocks. *Mathematical Programming*, *89*(3), 487-504.

Lopez, M. A. N., & Kingsman, B. G. (1991). The economic lot scheduling problem: theory and practice. *International Journal of Production Economics*, *23*(1), 147-164.

Lotfi, V., & Yoon, Y. S. (1994). An algorithm for the single-item capacitated lot-sizing problem with concave production and holding costs. *Journal of the Operational Research Society*, 934-941.

Lütke Entrup, M., Günther, H. O., Van Beek, P., Grunow, M., & Seiler, T. (2005). Mixed-integer linear programming approaches to shelf life-integrated planning and scheduling in yoghurt production. *International Journal of Production Research*, *43*(23), 5071-5100.

Madić, M., Marković, D., & Radovanović, M. (2013). Comparison of meta-heuristic algorithms for solving machining optimization problems. *Facta Universitatis Series: Mechanical Engineering*, *11*(1), 29-44.

Maes, J., & Van Wassenhove, L. (1988). Multi-item single-level capacitated dynamic lot-sizing heuristics: A general review. *Journal of the Operational Research Society*, 991-1004.

Maes, J., McClain, J. O., & Van Wassenhove, L. N. (1991). Multilevel capacitated lot-sizing complexity and LP-based heuristics. *European Journal of Operational Research*, *53*(2), 131-148.

Mallya, R. (1992). Multi-product scheduling on a single machine: a case study. *Omega*, *20*(4), 529-534.

Marinelli, F., Nenni, M. E., & Sforza, A. (2007). Capacitated lot-sizing and scheduling with parallel machines and shared buffers: A case study in a packaging company. *Annals of Operations Research*, *150*(1), 177-192.

Maritan, C. A., Brush, T. H., & Karnani, A. G. (2004). Plant roles and decision autonomy in multinational plant networks. *Journal of Operations Management*, *22*(5), 489-503.

Martin, C. H., Dent, D. C., & Eckhart, J. C. (1993). Integrated production, distribution, and inventory planning at Libbey-Owens-Ford. *Interfaces*, *23*(3), 68-78.

McCarthy, I., & Anagnostou, A. (2004). The impact of outsourcing on the transaction costs and boundaries of manufacturing. *International Journal of Production Economics*, *88*(1), 61-71.

Mehdizadeh, E., & Fatehi Kivi, A. (2014). Three Metaheuristic Algorithms for Solving the Multi-item Capacitated Lot-sizing Problem with Product Returns and Remanufacturing. *Journal of Optimization in Industrial Engineering*, *7*(16), 41-53.

Millar, H. H., & Yang, M. (1994). Lagrangian heuristics for the capacitated multi-item lot-sizing problem with backordering. *International Journal of Production Economics*, *34*(1), 1-15.

Mitchell, M. (1996). An introduction to genetic algorithms, 1996. *PHI Pvt. Ltd., New Delhi*.

Mohammadi, M., Ghomi, S. M. T., Karimi, B., & Torabi, S. A. (2009). Development of heuristics for multi-product multi-level capacitated lot-sizing problem with sequence-dependent setups. *Journal of Applied Sciences*, *9*(2), 296–303.

Moon, C., Kim, J., & Hur, S. (2002). Integrated process planning and scheduling with minimizing total tardiness in multi-plants supply chain. *Computers & Industrial Engineering*, *43*(1), 331-349.

Moon, I., Giri, B., & Choi, K. (2002a). Economic lot scheduling problem with imperfect production processes and setup times. *Journal of the Operational Research Society, 53*(6), 620-629.

Moon, I., Silver, E. A., & Choi, S. (2002b). Hybrid genetic algorithm for the economic lot-scheduling problem. *International Journal of Production Research*, *40*(4), 809-824.

Mousavi, S. M., Hajipour, V., Niaki, S. T. A., & Aalikar, N. (2014). A multi-product multi-period inventory control problem under inflation and discount: a parameter-tuned particle swarm optimization algorithm. *The International Journal of Advanced Manufacturing Technology*, *70*(9-12), 1739-1756.

Nascimento, M. C., & Toledo, F. (2008). A hybrid heuristic for the multi-plant capacitated lot-sizing problem with setup carry-over. *Journal of the Brazilian Computer Society*, *14*(4), 7-15.

Nascimento, M.C., Resende, M.G., & Toledo, F. (2010). GRASP heuristic with path-relinking for the multi-plant capacitated lot-sizing problem. *European Journal of Operational Research*, *200*(3), 747-754.

Nazari-Shirkouhi, S., Eivazy, H., Ghodsi, R., Rezaie, K., & Atashpaz-Gargari, E. (2010). Solving the integrated product mix-outsourcing problem using the imperialist competitive algorithm. *Expert Systems with Applications*, *37*(12), 7615-7626.

Nilsson, K., & Segerstedt, A. (2008). Corrections of costs to feasible solutions of economic lot scheduling problems. *Computers & Industrial Engineering*, *54*(1), 155-168.

Oliver, R. K., & Webber, M. D. (1982). Supply-chain management: logistics catches up with strategy. *Outlook*, *5*(1), 42-47.

Osman, I. H., & Laporte, G. (1996). Metaheuristics: A bibliography. *Annals of Operations research*, *63*(5), 511-623.

Özdamar, L., & Barbarosoğlu, G. (1999). Hybrid heuristics for the multi-stage capacitated lot-sizing and loading problem. *Journal of the Operational Research Society*, *50*(8), 810-825.

Özdamar, L., & Barbarosoğlu, G. (2000). An integrated Lagrangian relaxation-simulated annealing approach to the multi-level multi-item capacitated lot-sizing problem. *International Journal of Production Economics*, *68*(3), 319-331.

Özdamar, L., & Bozyel, M. A. (2000). The capacitated lot-sizing problem with overtime decisions and setup times. *IIE Transactions*, *32*(11), 1043-1057.

Özdamar, L., Bilbil, Ş. İ., & Portmann, M. C. (2002). Technical note: New results for the capacitated lot-sizing problem with overtime decisions and setup times.

Pal, B., Sana, S. S., & Chaudhuri, K. (2012). A three layer multi-item production–inventory model for multiple suppliers and retailers. *Economic Modelling*, *29*(6), 2704-2710.

Pan, Q. K., Tasgetiren, M. F., Suganthan, P. N., & Chua, T. J. (2011). A discrete artificial bee colony algorithm for the lot-streaming flow shop scheduling problem. *Information sciences*, *181*(12), 2455-2468.

Pan, Z., Tang, J., & Liu, O. (2009). Capacitated dynamic lot-sizing problems in closed-loop supply chain. *European Journal of Operational Research*, *198*(3), 810-821.

Park, Y. B. (2005). An integrated approach for production and distribution planning in supply chain management. *International Journal of Production Research*, *43*(6), 1205-1224.

Parsapoor, M., & Bilstrup, U. (2012). Using the grouping genetic algorithm (GGA) for channel assignment in a cluster-based mobile ad hoc network. In *Proceedings of SNCNW 2012: The 8th Swedish National Computer Networking Workshop: Stockholm, June 7-8, 2012* (pp. 56-60).

Parsapoor, M., & Bilstrup, U. (2013, December). School of Information Science, Computer and Electrical Engineering (IDE), Halmstad University, Sweden. In *Wireless Sensor (ICWISE), 2013 IEEE Conference on* (pp. 73-78). IEEE.

Pasandideh, S. H. R., & Niaki, S. T. A. (2008). A genetic algorithm approach to optimize a multi-products EPQ model with discrete delivery orders and constrained space. *Applied Mathematics and Computation*, *195*(2), 506-514.

Peixin, Z. (2012). An Improved Particle Swarm Optimization Algorithm for the Economic Lot Scheduling Problem under Power-of-Two Policy. *Advances in Information Sciences & Service Sciences*, *4*(20).

Phadke, M. S. (1989). Quality Engineering Using Robust Design, PTR Prentice-Hall. *Inc., Englewood Cliffs, NJ*.

Pirkul, H., & Jayaraman, V. (1998). A multi-commodity, multi-plant, capacitated facility location problem: formulation and efficient heuristic solution. *Computers & Operations Research*, *25*(10), 869-878.

Pitakaso, R., Almeder, C., Doerner, K. F., & Hartl, R. F. (2006). Combining exact and population-based methods for the constrained multilevel lot-sizing problem. *International Journal of Production Research*, *44*(22), 4755-4771.

Pitakaso, R., Almeder, C., Doerner, K. F., & Hartl, R. F. (2007). A MAX-MIN ant system for unconstrained multi-level lot-sizing problems. *Computers & Operations Research*, *34*(9), 2533-2552.

Pochet, Y. (2001). Mathematical programming models and formulations for deterministic production planning problems. In *Computational combinatorial optimization* (pp. 57-111). Springer Berlin Heidelberg.

Prasad, P. S. S., & Chetty, O. K. (2001). Multilevel lot-sizing with a genetic algorithm under fixed and rolling horizons. *The International Journal of Advanced Manufacturing Technology*, *18*(7), 520-527.

Quadt, D., & Kuhn, H. (2008). Capacitated lot-sizing with extensions: A review. *Quarterly Journal of Operations Research*, *6*(1), 61-83.

Ramezanian, R., & Saidi-Mehrabad, M. (2013). Hybrid simulated annealing and MIP-based heuristics for stochastic lot-sizing and scheduling problem in capacitated multi-stage production system. *Applied Mathematical Modelling*, *37*(7), 5134-5147.

Ramezanian, R., Saidi-Mehrabad, M., & Teimoury, E. (2013). A mathematical model for integrating lot-sizing and scheduling problem in capacitated flow shop environments. *The International Journal of Advanced Manufacturing Technology*, *66*(1-4), 347-361.

Raza, A.S., Akgunduz, A., & Chen, M. (2006). A tabu search algorithm for solving economic lot scheduling problem. *Journal of Heuristics*, *12*(6), 413-426.

Raza, A.S., & Akgunduz, A. (2008). A comparative study of heuristic algorithms on economic lot scheduling problem. *Computers & Industrial Engineering*, *55*(1), 94-109.

Richter, K., & Sombrutzki, M. (2000). Remanufacturing planning for the reverse Wagner/Whitin models. *European Journal of Operational Research*, *121*(2), 304-315.

Rizk, N. A. F. E. E., & Martel, A. (2001). *Supply chain flow planning methods: A review of the lot-sizing literature*. (Unpublished doctoral dissertation). Université Laval, Quebec City, Canada.

Rogers, J. (1958). A computational approach to the economic lot scheduling problem. *Management Science*, *4*(3), 264-291.

Ruiz-Torres, A. J., Ho, J. C., & Lopez, F. J. (2006). Generating Pareto schedules with outsource and internal parallel resources. *International Journal of Production Economics*, *103*(2), 810-825.

Sadeghi, J., Mousavi, S. M., Niaki, S. T. A., & Sadeghi, S. (2013). Optimizing a multi-vendor multi-retailer vendor managed inventory problem: Two tuned meta-heuristic algorithms. *Knowledge-Based Systems*, *50*, 159-170.

Sadollah, A., Mohamed, M. N. H., & Bahreininejad, A. (2013). Optimum culture conditions for 1,3-Propanediol production from Crude Glycerol using metaheuristic algorithms. *Applied Mathematics & Information Sciences*, *1*, 1-8.

Saka, M. P., Doğan, E., & Aydogdu, I. (2013). Analysis of Swarm Intelligence–Based Algorithms for Constrained Optimization. *Swarm Intelligence and Bio-Inspired Computation*, 25-48.

Salomon, M., Kroon, L. G., Kuik, R., & Van Wassenhove, L. N. (1991). Some extensions of the discrete lot-sizing and scheduling problem. *Management Science*, *37*(7), 801-812.

Salvietti, L., & Smith, N. R. (2008). A profit-maximizing economic lot scheduling problem with price optimization. *European Journal of Operational Research*, *184*(3), 900-914.

Salvietti, L., Smith, N. R., & Cárdenas-Barrón, L. E. (2014). A stochastic profit-maximizing economic lot scheduling problem with price optimization. *European Journal of Industrial Engineering*, *8*(2), 193-221.

Sambasivan, M., & Schmidt, C. P. (2002). A heuristic procedure for solving multi-plant, multi-item, multi-period capacitated lot-sizing problems. *Asia Pacific Journal of Operational Research*, *19*(1), 87-106.

Sambasivan, M., & Yahya, S. (2005). A Lagrangian-based heuristic for multi-plant, multi-item, multi-period capacitated lot-sizing problems with inter-plant transfers. *Computers & Operations Research*, *32*(3), 537-555.

Sana, S. S. (2010). An economic production lot size model in an imperfect production system. *European Journal of Operational Research*, *201*(1), 158-170.

Sana, S. S., Chedid, J. A., & Navarro, K. S. (2014). A three layer supply chain model with multiple suppliers, manufacturers and retailers for multiple items. *Applied Mathematics and Computation*, *229*, 139-150.

Sandbothe, R. A., & Thompson, G. L. (1990). A forward algorithm for the capacitated lot size model with stockouts. *Operations Research*, *38*(3), 474-486.

Sarker, B. R., and Babu, P. S. (1993) Effect of production cost on shelf life. *International Journal of Production Research*, *31*(8), 1865-1872.

Sarker, B. R., Rochanaluk, R., & Egbelu, P. J. (2013). Improving service rate for a tree-type three-echelon supply chain system with backorders at retailer's level. *Journal of the Operational Research Society*, *65*(1), 57-72.

Senyiğit, E. (2010). New heuristics to stochastic dynamic lot-sizing problem. *Gazi University Journal of Science*, *22*(2), 97-106.

Sharma, S. (2004). Optimal production policy with shelf life including shortages. *Journal of the Operational Research Society*, *55*(8), 902-909.

Sharma, S. (2006). Incorporating fractional backordering in the multi-product manufacturing situation with shelf-lives. *Journal of Engineering Manufacture*, *220*(7), 1151-1156.

Shaw, D. X., & Wagelmans, A. P. (1998). An algorithm for single-item capacitated economic lot-sizing with piecewise linear production costs and general holding costs. *Management Science*, *44*(6), 831-838.

Silver, E. A. (1989). Shelf life considerations in a family production context. *International Journal of Production Research*, *27*(12), 2021-2026.

Silver, E. A. (1995). Dealing with a shelf life constraint in cyclic scheduling by adjusting both the cycle time and production rate. *International Journal of Production Research*, *33*(3), 623–629.

Silver, E. A., & Meal, H. C. (1973). A heuristic for selecting lot size quantities for the case of a deterministic time-varying demand rate and discrete opportunities for replenishment. *Production and Inventory Management*, *14*(2), 64-74.

Soman, C. A., Van Donk, D. P., & Gaalman, G. (2004). Combined make-to-order and make-to-stock in a food production system. *International Journal of Production Economics*, *90*(2), 223–135.

Song, Y., & Chan, G. H. (2005). Single item lot-sizing problems with backlogging on a single machine at a finite production rate. *European Journal of Operational Research*, *161*(1), 191-202.

Stadtler, H. (1997). Reformulations of the shortest route model for dynamic multi-item multi-level capacitated lot-sizing. *Operations-Research-Spektrum*, *19*(2), 87-96.

Suerie, C., & Stadtler, H. (2003). The capacitated lot-sizing problem with linked lot sizes. *Management Science*, *49*(8), 1039-1054.

Sun, H., Huang, H. C., & Jaruphongsa, W. (2009). A genetic algorithm for the economic lot scheduling problem under the extended basic period and power-of-two policy. *CIRP Journal of Manufacturing Science and Technology*, *2*(1), 29-34.

Swamidass, P. M. (Ed.). (2000). *Encyclopedia of production and manufacturing management*. Springer Science & Business Media.

Talbi, E. G. (2009). Metaheuristics: from design to implementation. John Wiley & Sons.

Tang, O. (2004). Simulated annealing in lot-sizing problems. *International Journal of Production Economics*, *88*(2), 173-181.

Tang, O., & Teunter, R. (2006). Economic lot scheduling problem with returns. *Production and Operations Management*, *15*(4), 488-497.

Tasgetiren, M. F., Bulut, O., & Murat Fadiloglu, M. (2011, June). A discrete artificial bee colony algorithm for the economic lot scheduling problem. In *Evolutionary Computation (CEC), 2011 IEEE Congress on* (pp. 347-353). IEEE.

Tasgetiren, M. F., Bulut, O., & Murat Fadiloglu, M. (2012, June). A discrete harmony search algorithm for the economic lot scheduling problem with power of two policy. In *Evolutionary Computation (CEC), 2012 IEEE Congress on* (pp. 1-8). IEEE.

Tempelmeier, H. (1997). Resource-constrained materials requirements planning-MRP rc. *Production Planning & Control*, *8*(5), 451-461.

Tempelmeier, H., & Hilger, T. (2015). Linear programming models for a stochastic dynamic capacitated lot-sizing problem. *Computers & Operations Research*, *59*, 119-125.

Tersine, R. J., & Barman, S. (1991). Economic inventory/transport lot, sizing with quantity and freight rate discounts. *Decision Sciences*, *22*(5), 1171-1179.

Teunter, R. H., & Flapper, S. D. P. (2003). Lot-sizing for a single-stage single-product production system with rework of perishable production defectives. *OR Spectrum*, *25*(1), 85-96.

Teunter, R., Tang, O., & Kaparis, K. (2009). Heuristics for the economic lot scheduling problem with returns. *International Journal of Production Economics*, *118*(1), 323-330.

Thizy, J. M., & Van Wassenhove, L. N. (1985). Lagrangian relaxation for the multi-item capacitated lot-sizing problem: A heuristic implementation. *IIE transactions*, *17*(4), 308-313.

Thomas, D. J., & Griffin, P. M. (1996). Coordinated supply chain management. *European Journal of Operational Research*, *94*(1), 1-15.

Toledo, C. F. M., de Oliveira, L., de Freitas Pereira, R., França, P. M., & Morabito, R. (2014). A genetic algorithm/mathematical programming approach to solve a two-level soft drink production problem. *Computers & Operations Research*, *48*, 40-52.

Toledo, C. F. M., De Oliveira, R. R. R., & França, P. M. (2013). A hybrid multi-population genetic algorithm applied to solve the multi-level capacitated lot-sizing problem with backlogging. *Computers & Operations Research*, *40*(4), 910-919.

Torabi, S. A., Karimi, B., & Ghomi, S. F. (2005). The common cycle economic lot scheduling in flexible job shops: The finite horizon case. *International Journal of Production Economics*, *97*(1), 52-65.

Tseng, Y. J., Chen, J. Y., & Huang, F. Y. (2010a). A multi-plant assembly sequence planning model with integrated assembly sequence planning and plant assignment using GA. *The International Journal of Advanced Manufacturing Technology*, *48*(1-4), 333-345.

Tseng, Y. J., Chen, J. Y., & Huang, F. Y. (2010b). A particle swarm optimization algorithm for multi-plant assembly sequence planning with integrated assembly sequence planning and plant assignment. *International Journal of Production Research*, *48*(10), 2765-2791.

Tukel, O. I., & Wasti, S. N. (2001). Analysis of supplier buyer relationships using resource constrained project scheduling strategies. *European Journal of Operational Research*, *129*(2), 271-276.

Van Eijs, M. J. G., Heuts, R. M. J., & Kleijnen, J. P. C. (1992). Analysis and comparison of two strategies for multi-item inventory systems with joint replenishment costs. *European Journal of Operational Research*, *59*(3), 405-412.

Vargas, V. (2009). An optimal solution for the stochastic version of the Wagner-Whitin dynamic lot-size model. *European Journal of Operational Research*, *198*(2), 447-451.

Viswanathan, S. (1995). A note on effect of production cost on shelf life. *International Journal of Production Research*, *33*(12), 3485-3486.

Viswanathan, S., & Goyal, S. K. (2000). Incorporating planned backorders in a family production context with shelf life considerations. *International Journal of Production Research*, 38(4), 829–836.

Viswanathan, S., & Goyal, S. K. (2002). Technical note on manufacturing batch size and ordering policy for products with shelf lives. *International Journal of Production Research*, *40*(8), 1965-1970.

Viswanathan, S., and Goyal, S. K. (1997). Optimal cycle time and production rate in a family production context with shelf life considerations. *International Journal of Production Research*, *35*(6), 1703-1711.

Vorst, J. G. A. J. V. D. (2000). *Effective food supply chains: generating, modeling and evaluating supply chain scenarios*. Wageningen Universiteit.

Voß, S., Martello, S., Osman, I. H., & Roucairol, C. (Eds.). (2012). *Meta-heuristics: Advances and trends in local search paradigms for optimization*. Springer Science & Business Media.

Wagner, H. M., & Whitin, T. M. (1958). Dynamic version of the economic lot size model. *Management Science*, *5*(1), 89-96.

Wild, R. (1974). The origins and development of flow-line production. *Industrial Archaeology*, *11*(1-3), 43-55.

Woarawichai, C., Kullpattaranirun, T., & Rungreunganun, V. (2010). Applying genetic algorithms for inventory lot-sizing problem with supplier selection under storage space. *Asian International Journal of Science and Technology in Production and Manufacturing Engineering*, *3*(4), 37-45.

Wolpert, D. H., & Macready, W. G. (1997). No free lunch theorems for optimization. *Evolutionary Computation, IEEE Transactions on*, *1*(1), 67-82.

Wu, T., & Shi, L. (2011). Mathematical models for capacitated multi-level production planning problems with linked lot sizes. *International Journal of Production Research*, *49*(20), 6227-6247.

Wu, T., Shi, L., Geunes, J., & Akartunalı, K. (2011). An optimization framework for solving capacitated multi-level lot-sizing problems with backlogging. *European Journal of Operational Research*, *214*(2), 428-441.

Xie, J., & Dong, J. (2002). Heuristic genetic algorithms for general capacitated lot-sizing problems. *Computers & Mathematics with applications*, *44*(1), 263-276.

Xu, Y., & Sarker, B. R. (2003). Models for a family of products with shelf life and production and shortage costs in emerging markets. *Computers & Operations Research*, *30*(6), 925-938.

Yaghini, M., & Khandaghabadi, Z. (2013). A hybrid metaheuristic algorithm for dynamic rail car fleet sizing problem. *Applied Mathematical Modelling*, *37*(6), 4127-4138.

Yan, C., Banerjee, A., & Yang, L. (2011). An integrated production–distribution model for a deteriorating inventory item. *International Journal of Production Economics*, *133*(1), 228-232.

Yan, C., Liao, Y., & Banerjee. A. (2013). Multi-product lot scheduling with backordering and shelf life constraints. *Omega*, *41*(3), 510-516.

Yang, F. C., Chen, K., Wang, M. T., Chang, P. Y., & Sun, K. C. (2010). Mathematical modeling of multi-plant order allocation problem and solving by genetic algorithm with matrix representation. *The International Journal of Advanced Manufacturing Technology*, *51*(9-12), 1251-1259.

Yao, M. J., & Huang, J. X. (2005). Solving the economic lot scheduling problem with deteriorating items using genetic algorithms. *Journal of Food Engineering*, *70*(3), 309-322.

Yazdani, M., Gohari, S., & Naderi, B. (2015). Multi-factory parallel machine problems: Improved mathematical models and artificial bee colony algorithm. *Computers & Industrial Engineering*, *81*, 36-45.

Zangwill, W. I. (1969). A backlogging model and a multi-echelon model of a dynamic economic lot size production system-a network approach. *Management Science*, *15*(9), 506-527.

Zanoni, S., Segerstedt, A., Tang, O., & Mazzoldi, L. (2012). Multi-product economic lot scheduling problem with manufacturing and remanufacturing using a basic period policy. *Computers & Industrial Engineering*, *62*(4), 1025-1033.

Zhang, Z. H., Jiang, H., & Pan, X. (2012). A Lagrangian relaxation based approach for the capacitated lot-sizing problem in closed-loop supply chain. *International Journal of Production Economics*, *140*(1), 249-255.

Zhong, Y., & Pan, X. (2007, August). A hybrid optimization solution to VRPTW based on simulated annealing. In *Automation and Logistics, 2007 IEEE International Conference on* (pp. 3113-3117). IEEE.

Zipkin, P. H. (1991). Computing optimal lot sizes in the economic lot scheduling problem. *Operations Research*, *39*(1), 56-63.

LIST OF PUBLICATIONS

(a) ***Accepted***

1. Mohammadi, M., Musa, S. N., & Bahreininejad, A. (2014). Optimization of mixed integer nonlinear economic lot scheduling problem with multiple setups and shelf life using metaheuristic algorithms. *Advances in Engineering Software*, *78*, 41-51.

2. Mohammadi, M., Musa, S. N., & Bahreininejad, A. (2015). Optimization of economic lot scheduling problem with backordering and shelf-life considerations using calibrated metaheuristic algorithms. *Applied Mathematics and Computation*, *251*, 404-422.

(b) ***Under Review***

1. Mohammadi, M., Musa, S. N., & Bahreininejad, A. (2015). *Optimal cycle time for production-inventory systems considering shelf life and backordering*. Manuscript submitted for publication.

2. Mohammadi, M., Musa, S. N., Omarc, M., & Bahreininejad, A. (2015). *Optimization of multi-item multi-period multi-plant capacitated lot-sizing problem in an integrated production-distribution network using calibrated metaheuristic algorithms*. Manuscript submitted for publication.

(c) ***Conference***

1. Mohammadi, M., Musa, S. N., & Bahreininejad, A. (2013, October). *Optimization of multi-product constrained manufacturing problem with shelf lives using metaheuristic algorithms*. Paper presented at the 8th Asia-Oceania top university league on engineering (AOTULE) postgraduate students conference, Bangkok, Thailand.

APPENDIX

Appendix A: GA Code for the Proposed ELSP

```
%% Parameters setting

load('all.mat')              % Input data to the model

Npop=200;                    % Number of population

Pc=0.1;                      % Percent of crossover

Ncross=2*round(Npop*Pc/2);   % Number of crossover offspring

Pm=1;                        % Percent of mutation

Nmut=round(Npop*Pm);         % Number of mutation offspring

maxiter=3000;                % Maximum number of generation

%% Initialization

NFE=0;                       % Number of function evaluation

tic                          % Timer

empty.t=[];
empty.p=[];
empty.fmax=[];
empty.W=[];
empty.alpha=[];
empty.beta=[];
empty.cost=[];

pop=repmat(empty,Npop,1);

for i=1:Npop

pop(i).t=lbt+rand(1,N).*(ubt-lbt);
pop(i).p=round(lbp+rand(1,N).*(ubp-lbp));
pop(i).fmax=round(lbfmax+rand.*(ubfmax-lbfmax));

v=round(rand(N,pop(i).fmax));
pop(i).W=edit_W(v,O,pop(i).p,d,pop(i).fmax,N);

function   W=edit_W(W,O,p,d,fmax,N)
for i=1:fmax
   a=W(:,i);
   a1=find(a==1);
   if length(a1)>1
      n=length(a1)-1;
      a2=(O.*p)./d;
      a2=a2(a1);
      [~,index]=sort(a2,'descend');
      a3=a1(index(1:n));
      a(a3)=0;
      W(:,i)=a;
   elseif isempty(a1)
```

```
      a2=(O.*p)./d;
      [~,index]=sort(a2);
      a(index(1))=1;
      W(:,i)=a;
   end
end
W(1:N^2)=0;
e=randperm(N);
for i=1:N
W(i,e(i))=1;
end
end

m=randperm(N*pop(i).fmax);
m1=m(1:round(N*pop(i).fmax/2));
m2=m(round(N*pop(i).fmax/2)+1:end);
alpha=rand(N,pop(i).fmax);
alpha(m1)=0;
beta=rand(N,pop(i).fmax);
beta(m2)=0;
alpha=reformeralpha(alpha,pop(i).fmax);

function a=reformeralpha(a,m)
m=round((m*(m-1))/2);
a1=find(a>0);
if length(a1)>m
a2=zeros(size(a));
a3=randperm(length(a1),m);
a2(a1(a3))=a(a1(a3));
a=a2;
end
end

pop(i).alpha=alpha;
pop(i).beta=beta;

pop(i).cost=fitness(pop(i),d,N,S,O,A,H,B,l,Rmin);
end

[value,index]=min([pop.cost]);

gpop=pop(index);

%% Main loop

best=zeros(maxiter,1);
MEAN=zeros(maxiter,1);

for iter=1:maxiter

% cross over
crosspop=repmat(empty,Ncross,1);
crosspop=crossover(crosspop,pop,Ncross,d,N,S,O,A,H,B,L,Rmin);

function
crosspop=crossover(crosspop,pop,Ncross,d,N,S,O,A,H,B,L,Rmin)

f=[pop.cost];
f=max(f)-f+1;
f=f./sum(f);
f=cumsum(f);
```

```
for n=1:2:Ncross

    i1=find(rand<=f,1,'first');
    i2=find(rand<=f,1,'first');

    p1=pop(i1);
    p2=pop(i2);

    r=rand(1,N);
    r1=rand;

    o1.t=(r.*p1.t)+((1-r).*p2.t);
    o2.t=(r.*p2.t)+((1-r).*p1.t);

    o1.p=round(r.*p1.p+(1-r).*p2.p);
    o2.p=round(r.*p2.p+(1-r).*p1.p);

    o1.fmax=round(r1.*p1.fmax+(1-r1).*p2.fmax);
    o2.fmax=round(r1.*p2.fmax+(1-r1).*p1.fmax);

    o1.W=p1.W;
    o2.W=p2.W;

    o1.alpha=p1.alpha;
    o2.alpha=p2.alpha;

    o1.beta=p1.beta;
    o2.beta=p2.beta;

    if o1.fmax>p1.fmax
        o1.W(:,end+1:o1.fmax)=round(rand(N,o1.fmax-p1.fmax));
        o1.alpha(:,end+1:o1.fmax)=(rand(N,o1.fmax-p1.fmax));
        o1.beta(:,end+1:o1.fmax)=(rand(N,o1.fmax-p1.fmax));
    else
        o1.W=o1.W(:,1:o1.fmax);
        o1.alpha=o1.alpha(:,1:o1.fmax);
        o1.beta=o1.beta(:,1:o1.fmax);
    end

    if  o2.fmax>p2.fmax
        o2.W(:,end+1:o2.fmax)=round(rand(N,o2.fmax-p2.fmax));
        o2.alpha(:,end+1:o2.fmax)=(rand(N,o2.fmax-p2.fmax));
        o2.beta(:,end+1:o2.fmax)=(rand(N,o2.fmax-p2.fmax));
    else
        o2.W=o2.W(:,1:o2.fmax);
        o2.alpha=o2.alpha(:,1:o2.fmax);
        o2.beta=o2.beta(:,1:o2.fmax);
    end

    fmax=min([o1.fmax,o2.fmax]);
    r2=rand(N,fmax);

    o1.W(:,1:fmax)=round(r2.*o1.W(:,1:fmax)+(1-r2).*o2.W(:,1:fmax));
    o2.W(:,1:fmax)=round(r2.*o2.W(:,1:fmax)+(1-r2).*o1.W(:,1:fmax));

    o1.W=edit_W(o1.W,o,o1.p,d,o1.fmax,N);
    o2.W=edit_W(o2.W,o,o2.p,d,o2.fmax,N);

    o1.alpha(:,1:fmax)=(r2.*o1.alpha(:,1:fmax)+(1-
r2).*o2.alpha(:,1:fmax));
```

```
    o2.alpha(:,1:fmax)=(r2.*o2.alpha(:,1:fmax)+(1-
r2).*o1.alpha(:,1:fmax));

    o1.beta(:,1:fmax)=(r2.*o1.beta(:,1:fmax)+(1-
r2).*o2.beta(:,1:fmax));
    o2.beta(:,1:fmax)=(r2.*o2.beta(:,1:fmax)+(1-
r2).*o1.beta(:,1:fmax));

    [k1]=find(o1.alpha>0);
    o1.beta(k1)=0;

    [k2]=find(o1.beta>0);
    o1.alpha(k2)=0;

    [k1]=find(o2.alpha>0);
    o2.beta(k1)=0;

    [k2]=find(o2.beta>0);
    o2.alpha(k2)=0;

    o1.alpha=reformeralpha(o1.alpha,o1.fmax);
    o2.alpha=reformeralpha(o2.alpha,o2.fmax);

    o1.cost=fitness(o1,d,N,S,O,A,H,B,L,Rmin);
    o2.cost=fitness(o2,d,N,S,O,A,H,B,L,Rmin);

    crosspop(n)=o1;
    crosspop(n+1)=o2;
end
end

% mutation
mutpop=repmat(empty,Nmut,1);
mutpop=mutation(mutpop,pop,Nmut,d,N,S,O,A,H,B,L,Rmin,lbt,ubt,lbp,ubp,m
axiter);

function
mutpop=mutation(mutpop,pop,Nmut,d,N,S,O,A,H,B,L,Rmin,lbt,ubt,lbp,ubp,m
axiter);

npop=length(pop);

for n=1:nmut

   i=randi([1 npop]);
   p=pop(i);
   j=randi([1 N]);

   if rand<0.5
       p.t(j)=p.t(j)-rand*(1-(i/maxiter))*(p.t(j)-lbt(j));
       p.p(j)=round(p.p(j)-rand*(1-(i/maxiter))*(p.p(j)-lbp(j)));
   else
       p.t(j)=p.t(j)+rand*(1-(i/maxiter))*(ubt(j)-p.t(j));
       p.p(j)=round(p.p(j)+rand*(1-(i/maxiter))*(ubp(j)-p.p(j)));
   end

   j3=randi([1 N*p.fmax]);
   p.W(j3)=abs(p.W(j3)-1);
   p.W=edit_W(p.W,O,p.p,d,p.fmax,N);

   if rand<0.5
```

```
        p.alpha(j3)= p.alpha(j3)-(1-(i/maxiter))*rand.*(p.alpha(j3)-0);
        p.beta(j3)= p.beta(j3)-(1-(i/maxiter))*rand.*(p.beta(j3)-0);
        else
        p.alpha(j3)= p.alpha(j3)+(1-(i/maxiter))*rand.*(1-p.alpha(j3));
        p.beta(j3)= p.beta(j3)+(1-(i/maxiter))*rand.*(1-p.beta(j3));
        end

        [k1]=find(p.alpha>0);
        p.beta(k1)=0;

        [k2]=find(p.beta>0);
        p.alpha(k2)=0;

        p.cost=fitness(p,d,N,S,O,A,H,B,L,Rmin);

     mutpop(n)=p;
    end
    end

    % merged
     [pop]=[pop;crosspop;mutpop];

    % select
    [valu,index]=sort([pop.cost]);

    gpop=pop(index(1));
    pop=pop(index(1:Npop));

    best(iter)=gpop.cost;
    MEAN(iter)=mean([pop.cost]);
    NFE(iter)=NFE;

    disp(['iter    ' num2str(iter) ':     ' ...
          'Best = ' num2str(best(iter)) '  , ' ...
          'Mean = ' num2str(MEAN(iter))]);

    end
```

Appendix B: PSO Code for the Proposed ELSP

```
%% Parameters setting

load('all.mat')                 % Input data to the model

Nswarm=200;                     % Swarm size

c1=4;                           % Personal learning coefficient

c2=0.5;                         % Global learning coefficient

w=0.95;                         % Inertia weight

Beta=0.05;                      % Updating factor for inertia weight

T=3000;                         % Maximum number of iteration

%% Initialization

NFE=0;                          % Number of function evaluation

tic                             % Timer

empty.t=[];
empty.p=[];
empty.fmax=[];
empty.W=[];
empty.alpha=[];
empty.beta=[];
empty.cost=[];

pop=repmat(empty,Npop,1);

for i=1:Npop

    pop(i).t=lbt+rand(1,N).*(ubt-lbt);
    pop(i).velt=zeros(1,N);

    pop(i).p=round(lbp+rand(1,N).*(ubp-lbp));
    pop(i).velp=zeros(1,N);

    pop(i).fmax=round(lbfmax+rand.*(ubfmax-lbfmax));
    pop(i).velfmax=0;

    v=round(rand(N,pop(i).fmax));
    pop(i).W=edit_W(v,0,pop(i).p,d,pop(i).fmax,N);
    pop(i).velW=zeros(N,pop(i).fmax);

    m=randperm(N*pop(i).fmax);
    m1=m(1:round(N*pop(i).fmax/2));
    m2=m(round(N*pop(i).fmax/2)+1:end);
    alpha=rand(N,pop(i).fmax);
    alpha(m1)=0;
    alpha=reformeralpha(alpha,pop(i).fmax);
    pop(i).velalpha=zeros(N,pop(i).fmax);

    beta=rand(N,pop(i).fmax);
    beta(m2)=0;
```

```
    pop(i).alpha=alpha;
    pop(i).beta=beta;
    pop(i).velbeta=zeros(N,pop(i).fmax);

    pop(i).cost=fitness(pop(i),d,N,S,O,A,H,B,L,Rmin);

end

[value,index]=min([pop.cost]);

bpop=pop;
gpop=pop(index);

%% main loop
best=zeros(T,1);
MEAN=zeros(T,1);

for t=1:T

    for i=1:Npop

        % t
        pop(i).velt=w*pop(i).velt...
            +c1*rand(1,N).*(bpop(i).t-pop(i).t)...
            +c2*rand(1,N).*(gpop.t-pop(i).t);

        pop(i).t= pop(i).t+ pop(i).velt;

        pop(i).t=max(pop(i).t,lbt);
        pop(i).t=min(pop(i).t,ubt);

        % p
        pop(i).velp=w*pop(i).velp...
            +c1*rand(1,N).*(bpop(i).p-pop(i).p)...
            +c2*rand(1,N).*(gpop.p-pop(i).p);

        pop(i).p= pop(i).p+ pop(i).velp;

        pop(i).p=max(pop(i).p,lbp);
        pop(i).p=round(min(pop(i).p,ubp));

        % fmax
        pop(i).velfmax=w*pop(i).velfmax...
            +c1*rand.*(bpop(i).fmax-pop(i).fmax)...
            +c2*rand.*(gpop.fmax-pop(i).fmax);

        fmax= pop(i).fmax+ pop(i).velfmax;

        fmax=max(fmax,lbfmax);
        fmax=round(min(fmax,ubfmax));

        if fmax>pop(i).fmax

            pop(i).W(:,end+1:fmax)=round(rand(N,fmax-pop(i).fmax));
            pop(i).alpha(:,end+1:fmax)=(rand(N,fmax-pop(i).fmax));
            pop(i).beta(:,end+1:fmax)=(rand(N,fmax-pop(i).fmax));
            pop(i).velW(:,end+1:fmax)=round(rand(N,fmax-pop(i).fmax));
            pop(i).velalpha(:,end+1:fmax)=(rand(N,fmax-pop(i).fmax));
            pop(i).velbeta(:,end+1:fmax)=(rand(N,fmax-pop(i).fmax));
        else
```

```
            pop(i).W=pop(i).W(:,1:fmax);
            pop(i).alpha=pop(i).alpha(:,1:fmax);
            pop(i).beta=pop(i).beta(:,1:fmax);
            pop(i).velW=pop(i).velW(:,1:fmax);
            pop(i).velalpha=pop(i).velalpha(:,1:fmax);
            pop(i).velbeta=pop(i).velbeta(:,1:fmax);

        end

        pop(i).fmax=fmax;

        NF=min([pop(i).fmax,bpop(i).fmax,gpop.fmax]);

        % W
        pop(i).velW(:,1:NF)=w*pop(i).velW(:,1:NF)...
            +c1*rand(N,NF).*(bpop(i).W(:,1:NF)-pop(i).W(:,1:NF))...
            +c2*rand(N,NF).*(gpop.W(:,1:NF)-pop(i).W(:,1:NF));

        pop(i).W= pop(i).W+ pop(i).velW;

        pop(i).W=max(pop(i).W,0);
        pop(i).W=round(min(pop(i).W,1));

        pop(i).W=edit_W(pop(i).W,o,pop(i).p,d,pop(i).fmax,N);

        % alpha
        pop(i).velalpha(:,1:NF)=w*pop(i).velalpha(:,1:NF)...
            +c1*rand(N,NF).*(bpop(i).alpha(:,1:NF)-
pop(i).alpha(:,1:NF))...
            +c2*rand(N,NF).*(gpop.alpha(:,1:NF)-pop(i).alpha(:,1:NF));

        pop(i).alpha= pop(i).alpha+ pop(i).velalpha;

        pop(i).alpha=max(pop(i).alpha,0);
        pop(i).alpha=(min(pop(i).alpha,1));

        % beta
        pop(i).velbeta(:,1:NF)=w*pop(i).velbeta(:,1:NF)...
            +c1*rand(N,NF).*(bpop(i).beta(:,1:NF)-
pop(i).beta(:,1:NF))...
            +c2*rand(N,NF).*(gpop.beta(:,1:NF)-pop(i).beta(:,1:NF));

        pop(i).beta= pop(i).beta+ pop(i).velbeta;

        pop(i).beta=max(pop(i).beta,0);
        pop(i).beta=(min(pop(i).beta,1));

        [k1]=find(pop(i).alpha>0);
        pop(i).beta(k1)=0;

        [k2]=find(pop(i).beta>0);
        pop(i).alpha(k2)=0;

        pop(i).alpha=reformeralpha(pop(i).alpha,pop(i).fmax);

        pop(i).cost=fitness(pop(i),d,N,S,O,A,H,B,L,Rmin);

        if pop(i).cost<bpop(i).cost
            bpop(i)=pop(i);
```

```
            if bpop(i).cost<gpop.cost
                gpop=bpop(i);
            end
        end
    end

    best(t)=gpop.cost;
    MEAN(t)=mean([bpop.cost]);

    w=w*(1-Beta);

    disp([' iter      ' num2str(iter) ':     ' ...
          ' Best =    ' num2str(best(iter)) '  , ' ...
          ' Mean =    ' num2str(MEAN(iter))]);
end
```

Appendix C: ABC Code for the Proposed ELSP

```
%% Parameters setting

load('all.mat')              % Input data to the model

NB=200;                      % Number of bees

NS=NB/2;                     % Number of source

Limit=100;                   % Number of trials

maxcycle=3000;               % Maximum number of cycle

%%  Initialization

NFE=0;                       % Number of function evaluation

tic                          % Timer

empty.t=[];
empty.p=[];
empty.fmax=[];
empty.W=[];
empty.alpha=[];
empty.beta=[];
empty.cost=[];

food=repmat(empty,NS,1);

for i=1:NS

    food(i).t=lbt+rand(1,N).*(ubt-lbt);
    food(i).p=round(lbp+rand(1,N).*(ubp-lbp));
    food(i).fmax=round(lbfmax+rand.*(ubfmax-lbfmax));

    v=round(rand(N,food(i).fmax));
    food(i).W=edit_W(v,o,food(i).p,d,food(i).fmax,N);

    m=randperm(N*food(i).fmax);
    m1=m(1:round(N*food(i).fmax/2));
    m2=m(round(N*food(i).fmax/2)+1:end);
    alpha=rand(N,food(i).fmax);
    alpha(m1)=0;

    beta=rand(N,food(i).fmax);
    beta(m2)=0;
    alpha=reformeralpha(alpha,food(i).fmax);

    food(i).alpha=alpha;
    food(i).beta=beta;

    food(i).cost=fitness(food(i),d,N,S,O,A,H,B,L,Rmin);

end

[value,index]=min([food.cost]);
gfood=food(index);
```

```
trial=zeros(NS,1);

%% main loop
tic
best=zeros(maxcycle,1);
MEAN=zeros(maxcycle,1);

for cycle=1:maxcycle

    % employed bee

    for i=1:NS

        k=randi([1 NS]);

        while k==i
            k=randi([1 NS]);
        end

        nfood=food(i);
        kfood=food(k);


nfood=create_new_food(nfood,kfood,N,lbt,ubt,lbp,ubp,lbfmax,ubfmax,d,S,
O,A,H,B,L,Rmin);

function
nfood=create_new_food(nfood,kfood,N,lbt,ubt,lbp,ubp,lbfmax,ubfmax,d,S,
O,A,H,B,L,Rmin)

% t
j=randi([1 N]);
nfood.t(j)=nfood.t(j)+unifrnd(-1,1)*(nfood.t(j)-kfood.t(j));
nfood.t=max(nfood.t,lbt);
nfood.t=min(nfood.t,ubt);

% p
j=randi([1 N]);
nfood.p(j)=nfood.p(j)+unifrnd(-1,1)*(nfood.p(j)-kfood.p(j));
nfood.p=max(nfood.p,lbp);
nfood.p=round(min(nfood.p,ubp));

% fmax
fmax=nfood.fmax+unifrnd(-1,1)*(nfood.fmax-kfood.fmax);
fmax=max(fmax,lbfmax);
fmax=round(min(fmax,ubfmax));

if fmax>nfood.fmax
    nfood.W(:,end+1:fmax)=round(rand(N,fmax-nfood.fmax));
    nfood.alpha(:,end+1:fmax)=(rand(N,fmax-nfood.fmax));
    nfood.beta(:,end+1:fmax)=(rand(N,fmax-nfood.fmax));
else
    nfood.W=nfood.W(:,1:fmax);
    nfood.alpha=nfood.alpha(:,1:fmax);
    nfood.beta=nfood.beta(:,1:fmax);
    nfood.W=nfood.W(:,1:fmax);
    nfood.alpha=nfood.alpha(:,1:fmax);
    nfood.beta=nfood.beta(:,1:fmax);
end

nfood.fmax=fmax;
```

```
NF=min([nfood.fmax,kfood.fmax]);

% W
j1=randi([1 N*NF-1]);
j2=randi([j1+1 N*NF]);

nfood.W(j1:j2)=nfood.W(j1:j2)+unifrnd(-1,1,1,(j2-
j1+1)).*(nfood.W(j1:j2)-kfood.W(j1:j2));
nfood.W=max(nfood.W,0);
nfood.W=round(min(nfood.W,1));

nfood.W=edit_W(nfood.W,o,nfood.p,d,nfood.fmax,N);

% alpha
nfood.alpha(j1:j2)=nfood.alpha(j1:j2)+unifrnd(-1,1,1,(j2-
j1+1)).*(nfood.alpha(j1:j2)-kfood.alpha(j1:j2));

nfood.alpha=max(nfood.alpha,0);
nfood.alpha=(min(nfood.alpha,1));

% beta
nfood.beta(j1:j2)=nfood.beta(j1:j2)+unifrnd(-1,1,1,(j2-
j1+1)).*(nfood.beta(j1:j2)-kfood.beta(j1:j2));

nfood.beta=max(nfood.beta,0);
nfood.beta=(min(nfood.beta,1));

[k1]=find(nfood.alpha>0);
nfood.beta(k1)=0;

[k2]=find(nfood.beta>0);
nfood.alpha(k2)=0;
nfood.alpha=reformeralpha(nfood.alpha,nfood.fmax);

nfood.cost=fitness(nfood,d,N,S,O,A,H,B,L,Rmin);
end

        if nfood.cost<food(i).cost
            food(i)=nfood;
            trial(i)=0;
        else

            trial(i)=trial(i)+1;
        end

    end

    % unlooker bee
    f=[food.cost];
    f=max(f)-f+eps;
    f=f./sum(f);
    f=cumsum(f);

    for n=1:NS
        i=find(rand<=f,1,'first');
        k=randi([1 NS]);
        while k==i
            k=randi([1 NS]);
        end
        nfood=food(i);
```

```
        kfood=food(k);


nfood=create_new_food(nfood,kfood,N,lbt,ubt,lbp,ubp,lbfmax,ubfmax,d,S,
O,A,H,B,L,Rmin);

        if nfood.cost<food(i).cost
            food(i)=nfood;
            trial(i)=0;
        else
            trial(i)=trial(i)+1;
        end
    end

    %  scout bee
    g=find(trial>Limit);

    for n=1:length(g)

        i=g(n);
        food(i).t=lbt+rand(1,N).*(ubt-lbt);
        food(i).p=round(lbp+rand(1,N).*(ubp-lbp));
        food(i).fmax=round(lbfmax+rand.*(ubfmax-lbfmax));

        v=round(rand(N,food(i).fmax));
        food(i).W=edit_W(v,O,food(i).p,d,food(i).fmax,N);

        m=randperm(N*food(i).fmax);
        m1=m(1:round(N*food(i).fmax/2));
        m2=m(round(N*food(i).fmax/2)+1:end);
        alpha=rand(N,food(i).fmax);
        alpha(m1)=0;

        beta=rand(N,food(i).fmax);
        beta(m2)=0;
        food(i).alpha=alpha;
        food(i).beta=beta;

        food(i).cost=fitness(food(i),d,N,S,O,A,H,B,L,Rmin);
        trial(i)=0;

    end

    [value,index]=min([food.cost]);

    if value<gfood.cost
        gfood=food(index);
    end
    best(cycle)=gfood.cost;
    MEAN(cycle)=mean([food.cost]);

    disp(['cycle = ' num2str(cycle)      ':   ' ...
          'Best  = ' num2str(gfood.cost) '  , ' ...
          'Mean  = ' num2str(mean([food.cost]))]);
 end
```

Appendix D: SA Code for the Proposed ELSP

```
load('all.mat')                 % Input data to the model

E0=30;                          % Initial temperature

Ef=0.001;                       % Final temperature

Npop=20;                        % Number of population for population-
                                  based SA

nn=10;                          % Number of neighbor

z=0.95;                         % Cooling factor

maxiter=3000;                   % Maximum number of iteration

%% Initialization

NFE=0;                          % Number of function evaluation

tic                             % Timer

empty.t=[];
empty.p=[];
empty.fmax=[];
empty.W=[];
empty.alpha=[];
empty.beta=[];
empty.cost=inf;

pop=repmat(empty,Npop,1);

for i=1:npop

    pop(i).t=lbt+rand(1,N).*(ubt-lbt);
    pop(i).p=round(lbp+rand(1,N).*(ubp-lbp));
    pop(i).fmax=round(lbfmax+rand.*(ubfmax-lbfmax));

    v=round(rand(N,pop(i).fmax));
    pop(i).W=edit_W(v,o,pop(i).p,d,pop(i).fmax,N);

    m=randperm(N*pop(i).fmax);
    m1=m(1:round(N*pop(i).fmax/2));
    m2=m(round(N*pop(i).fmax/2)+1:end);
    alpha=rand(N,pop(i).fmax);
    alpha(m1)=0;
    beta=rand(N,pop(i).fmax);
    beta(m2)=0;
    alpha=reformeralpha(alpha,pop(i).fmax);
    pop(i).alpha=alpha;
    pop(i).beta=beta;

    pop(i).cost=fitness(pop(i),d,N,S,O,A,H,B,L,rmin);

end
[value,index]=min([pop.cost]);
gpop=pop(index);
```

```
%% main loop
T=T0;
best=zeros(maxiter,1);
MEAN=zeros(maxiter,1);

for iter=1:maxiter

    for i=1:Npop

        bnewpop=empty;

        for j=1:nn

newpop=create_neighbor(pop(i),d,N,S,O,a,H,B,L,Rmin,lbt,ubt,lbp,ubp);
            newpop.cost=fitness(newpop,d,N,S,O,A,H,B,L,Rmin);

function  p=create_neighbor(p,d,N,S,O,A,H,B,L,Rmin,lbt,ubt,lbp,ubp)

j=randi([1 N]);

if rand<0.5

    p.t(j)=p.t(j)-rand*0.1*(p.t(j)-lbt(j));
    p.p(j)=round(p.p(j)-rand*0.1*(p.p(j)-lbp(j)));
else
    p.t(j)=p.t(j)+rand*0.1*(ubt(j)-p.t(j));
    p.p(j)=round(p.p(j)+rand*0.1*(ubp(j)-p.p(j)));
end

j1=randi([1 N*p.fmax-1]);
j2=randi([j1+1 N*p.fmax]);

p.W(j1:j2)=abs(p.W(j1:j2)-1);
p.W=edit_W(p.W,o,p.p,d,p.fmax,N);

if rand<0.5
    p.alpha(j1:j2)= p.alpha(j1:j2)-0.1*rand(1,(j2-
j1+1)).*(p.alpha(j1:j2)-0);
    p.beta(j1:j2)= p.beta(j1:j2)-0.1*rand(1,(j2-
j1+1)).*(p.beta(j1:j2)-0);

else
    p.alpha(j1:j2)= p.alpha(j1:j2)+0.1*rand(1,(j2-j1+1)).*(1-
p.alpha(j1:j2));
    p.beta(j1:j2)= p.beta(j1:j2)+0.1*rand(1,(j2-j1+1)).*(1-
p.beta(j1:j2));

end

[k1]=find(p.alpha>0);
p.beta(k1)=0;

[k2]=find(p.beta>0);
p.alpha(k2)=0;

p.alpha=reformeralpha(p.alpha,p.fmax);

end
```

```
            if newpop.cost<bnewpop.cost
                bnewpop=newpop;
            end
        end

        if bnewpop.cost<pop(i).cost
            pop(i)=bnewpop;
        else

            E=bnewpop.cost-pop(i).cost;
            P=exp(-E/T);

            if rand<=P
                pop(i)=bnewpop;
            end
        end
    end
    [value,index]=min([pop.cost]);

    if value<gpop.cost
        gpop=pop(index);
    end

    T=T*z;

    best(iter)=gpop.cost;
    MEAN(iter)=mean([pop.cost]);
    disp(['iter  = ' num2str(iter)      ':  ' ...
          'Best  = ' num2str(best(iter)) '  , ' ...
          'Mean  = ' num2str(MEAN(iter))]);

end
```

Appendix E: GA Code for the Proposed MPCLSP

```
%% Parameters setting

load('all.mat')                 % Input data to the model

Npop=150;                       % Number of population

Pc=0.95;                        % Percent of crossover

Ncross=2*round(Npop*Pc/2);      % Number of crossover offspring

Pm=0.8;                         % Percent of mutation

Nmut=round(Npop*Pm);            % Number of mutation offspring

max_generation=200;             % Maximum number of generation

%% Initialization

NFE=0;                          % Number of function evaluation

tic                             % Timer

empty.Q=[];                     % Lot size

empty.cost=[];

pop=repmat(empty,Npop,1);

for ipop=1:Npop

    for i=1:ni
        for j=1:nj
            for t=1:nt
                Q(i,j,t)=randi([lbQ(i,j,t),ubQ(i,j,t)]);
            end
        end
    end

    pop(ipop).Q=Q;

    pop(ipop).cost=fitness(pop(ipop));

end

[value,index]=min([pop.cost]);

gpop=pop(index);

%% Main loop

Best=zeros(max_generation,1);

Mean=zeros(max_generation,1);

NFE=zeros(max_generation,1);
```

```
for iter=1:max_generation

    % Cross over

    crosspop=repmat(empty,Ncross,1);

    crosspop=crossover(crosspop,pop,Ncross);

     f=[pop.cost];
     f=max(f)-f+1;
     f=f./sum(f);
     f=cumsum(f);

     for n=1:2:Ncross

         i1=find(rand<=f,1,'first');

         i2=find(rand<=f,1,'first');

         pQ1=pop(i1).Q;

         pQ2=pop(i2).Q;

         rQ=rand(size(pQ1));

         o1.Q=round(rQ.*pQ1+(1-rQ).*pQ2);

         o2.Q=round(rQ.*pQ2+(1-rQ).*pQ1);

         crosspop(n).Q=o1.Q;

         crosspop(n).cost=fitness(o1);

         crosspop(n+1).Q=o2.Q;

         crosspop(n+1).cost=fitness(o2);

     end

    % Mutation

    mutpop=repmat(empty,Nmut,1);

    mutpop=mutation(mutpop,pop,Nmut);

    Npop=length(pop);

    for n=1:Nmut

        i1=randi([1 npop]);

        Q=pop(i1).Q;

        nQ=ni*nj*nt;

        j1=randi([1 nQ-1]);

        j2=randi([j1+1 nQ]);
```

```
        disQ=0.1*unifrnd(-1,1,size(ubQ)).*(ubQ-lbQ);

        Q(j1:j2)=round(Q(j1:j2)+disQ(j1:j2));

        Q=min(Q,ubQ);

        Q=max(Q,lbQ);

        mutpop(n).Q=Q;

        sol.Q=Q;

        mutpop(n).cost=fitness(sol);

    end

    % Merged

    [pop]=[pop;crosspop;mutpop];

    % Select

    [valu,index]=sort([pop.cost]);

    gpop=pop(index(1));

    pop=pop(index(1:Npop));

    Best(iter)=gpop.cost;

    Mean(iter)=Mean([pop.cost]);

    NFE(iter)=NFE;

    disp(['iter   ' num2str(iter) ':    ' ...
          'Best = ' num2str(Best(iter)) '  , ' ...
          'Mean = ' num2str(Mean(iter))]);

end
```

Appendix F: PSO Code for the Proposed MPCLSP

```
%% Parameters setting

load('all.mat')                 % Input data to the model

Nswarm=150;                     % Swarm size

c1=2;                           % Personal learning coefficient

c2=2;                           % Global learning coefficient

w=1;                            % Inertia weight

Beta=0.01;                      % Updating factor for inertia weight

max_iteration=200;              % Maximum number of iteration

%% Initialization

NFE=0;                          % Number of function evaluation

tic                             % Timer

empty.Q=[];

empty.cost=inf;

particle=repmat(empty,Nswarm,1);

for iparticle=1:Nswarm

    for i=1:ni
        for j=1:nj
            for t=1:nt
                Q(i,j,t)=randi([lbQ(i,j,t),ubQ(i,j,t)]);
            end
        end
    end

    particle(iparticle).Q=Q;

    particle(iparticle).velQ=zeros(size(Q));

    particle(iparticle).cost=fitness(particle(iparticle));

end

bparticle=particle;

[value,index]=min([bparticle.cost]);

gparticle=particle(index);

%% Main loop

Best=zeros(max_iteration,1);
```

```
Mean=zeros(max_iteration,1);

NFE=zeros(max_iteration,1);

for iter=1:max_iteration

    for iparticle=1:Nswarm


     particle(iparticle)=create_move(particle(iparticle),bparticle(ipar
     ticle),gparticle,w,c1,c2);

     function   [sol]=create_move(sol,bsol,gsol,w,c1,c2)

     Q=sol.Q;

     velQ=sol.velQ;

     velQ=round(w1*velQ...
         +c1*rand(size(velQ)).*(bsol.Q-sol.Q)...
         +c2*rand(size(velQ)).*(gsol.Q-sol.Q));

     Q=Q+velQ;

     Q=min(Q,ubQ);

     Q=max(Q,lbQ);

     sol.Q=Q;

     sol.velQ=velQ;

     sol.cost=fitness(sol);

     end

        if particle(iparticle).cost<bparticle(iparticle).cost
            bparticle(iparticle)=particle(iparticle);

            if bparticle(iparticle).cost<gparticle.cost
                gparticle=bparticle(iparticle);
            end

        end

    end

    Best(iter)=gparticle.cost;

    Mean(iter)=Mean([bparticle.cost]);

    w=w*(1-Beta);

    NFE(iter)=NFE;

    disp([' iter      ' num2str(iter) ':     ' ...
          ' Best =    ' num2str(Best(iter)) '   ,  ' ...
          ' Mean =    ' num2str(Mean(iter))]);
end
```

Appendix G: ABC Code for the Proposed MPCLSP

```
%% Parameters setting

load('all.mat')                  % Input data to the model

NB=150;                          % Number of bees

NS=NB/2;                         % Number of source

Limit=100;                       % Number of trials

max_cycle=200;                   % Maximum number of cycle

%%  Initialization

NFE=0;                           % Number of function evaluation

tic                              % Timer

empty.Q=[];

empty.cost=inf;

food=repmat(empty,NS,1);

for ifood=1:NS

    for i=1:ni
        for j=1:nj
            for t=1:nt
                Q(i,j,t)=randi([lbQ(i,j,t),ubQ(i,j,t)]);
            end
        end
    end

    food(ifood).Q=Q;

    food(ifood).cost=fitness(food(ifood));

end

[value,index]=min([food.cost]);

gfood=food(index);

trial=zeros(NS,1);

%% Main loop

Best=zeros(max_cycle,1);

Mean=zeros(max_cycle,1);

NFE=zeros(max_cycle,1);
```

```
for cycle=1:max_cycle

    %Employed bee

    for ifood=1:NS

        k=randi([1 NS]);

        while k==ifood

            k=randi([1 NS]);

        end

        [nfood]=create_move(food(ifood),food(k));

        function   [nsol]=create_move(sol,nesol)

        Q=sol.Q;

        nQ=ni*nj*nt;

        n=randi([0 round(0.1*nQ)+1]);

        j1=randi([1 nQ-n]);

        j2=j1+n;

        Q(j1:j2)=round(Q(j1:j2)+unifrnd(-1,1,1,n+1).*(Q(j1:j2)
        nesol.Q(j1:j2)));

        Q=min(Q,ubQ);

        Q=max(Q,lbQ);

        nsol.Q=Q;

        nsol.cost=fitness(nsol);

        end

        if nfood.cost<food(ifood).cost

            food(ifood)=nfood;

            trial(ifood)=0;

        else

            trial(ifood)=trial(ifood)+1;

        end

    end
```

```
%Unlooker bee

f=[food.cost];
f=max(f)-f+eps;
f=f./sum(f);
f=cumsum(f);

for n=1:NS

    ifood=find(rand<=f,1,'first');

    k=randi([1 NS]);

    while k==ifood

        k=randi([1 NS]);

    end

    [nfood]=create_move(food(ifood),food(k));

    if nfood.cost<food(ifood).cost

        food(ifood)=nfood;

        trial(ifood)=0;

    else

        trial(ifood)=trial(ifood)+1;

    end

end

%Scout bee

g=find(trial>Limit);

for n=1:length(g)

    ifood=g(n);

    for i=1:ni
        for j=1:nj
            for t=1:nt
                Q(i,j,t)=randi([lbQ(i,j,t),ubQ(i,j,t)]);
            end
        end
    end

    food(ifood).Q=Q;

    food(ifood).cost=fitness(food(ifood));

    trial(ifood)=0;

end
```

```
    [value,index]=min([food.cost]);

    if value<gfood.cost

        gfood=food(index);

    end

    Best(cycle)=gfood.cost;

    Mean(cycle)=Mean([food.cost]);

    NFE(cycle)=NFE;

    disp(['cycle = ' num2str(cycle)      ':   ' ...
          'Best  = ' num2str(gfood.cost) '  , ' ...
          'Mean  = ' num2str(Mean([food.cost]))]);

end
```

Appendix H: ICA Code for the Proposed MPCLSP

```
%% parameters setting

load('all.mat')                 % Input data to the model

Ncolony=150;                    % Number of colonies

Nimp=10;                        % Number of imperialist

Beta=2;                         % Assimilation coefficient

Theta=0.3;                      % Revolution rate

Gamma=0.25;                     % Coefficient for total cost of empire

Max_decade=200;                 % Maximum number of decades

%% Initialization

NFE=0;                          % Number of function evaluation

tic                             % Timer

create_initial_imperialist

empty.Q=[];

empty.cost=inf;

colony=repmat(empty,Ncolony,1);

for icolony=1:Ncolony

    for i=1:ni
        for j=1:nj
            for t=1:nt
                Q(i,j,t)=randi([lbQ(i,j,t),ubQ(i,j,t)]); %#ok
            end
        end
    end

    colony(icolony).Q=Q;

    colony(icolony).cost=fitness(colony(icolony));

end

[value,index]=sort([colony.cost]);

colony=colony(index);

imp=colony(1:nimp);

colony=colony(nimp+1:end);

n=length(colony);
```

```
k=0;
j=1;
for i=1:n
    k=k+1;
    imp(k).colony(j)=colony(i);
    if k==nimp
        k=0;
        j=j+1;
    end
end


[value,index]=min([imp.cost]);

gimp=imp(index);

%% Main loop

Best=zeros(max_decade,1);

Mean=zeros(max_decade,1);

NFE=zeros(max_decade,1);

for decade=1:max_decade

    imp=assimulate(imp,Beta);

   function imp=assimulate(imp,Beta)

   nimp=length(imp);

   for i=1:nimp

       Ncolony=length(imp(i).colony);

       for j=1:Ncolony

      [imp(i).colony(j)]=create_move1(imp(i),imp(i).colony(j),Beta);

        function   [colony]=create_move1(imp,colony,Beta)

        dQ=imp.Q-colony.Q;

        dQ=round(beta*rand(size(dQ)).*dQ);

        colony.Q=colony.Q+dQ;

        colony.Q=min(colony.Q,ubQ);

        colony.Q=max(colony.Q,lbQ);

        colony.cost=fitness(colony);

        end

       end

   end
```

```
end
imp=revolution(imp,Theta);
 function imp=revolution(imp,Theta)
 nimp=length(imp);
 for i=1:nimp
     Ncolony=length(imp(i).colony);
     for j=1:Ncolony
         if rand<Theta
           [imp(i).colony(j)]=create_move2(imp(i).colony(j));
            function   [colony]=create_move2(colony)
           nQ=ni*nj*nt;
            n=randi([0 round(0.01*nQ)+1]);
            j1=randi([1 nQ-n]);
            j2=j1+n;
            colony.Q(j1:j2)=round(lbQ(j1:j2)+rand(1,n+1).*(ubQ(j1:j
            2)-lbQ(j1:j2)));
            colony.cost=fitness(colony);
            end
         end
     end
 end
 end

imp=exchange(imp);
 function imp=exchange(imp)
 nimp=length(imp);
 for i=1:nimp
     [value,index]=min([imp(i).colony.cost]);
     if value<imp(i).cost
         bestcolony=imp(i).colony(index);
         imp(i).colony(index).Q=imp(i).Q;
```

```
            imp(i).colony(index).cost=imp(i).cost;

            imp(i).Q=bestcolony.Q;

            imp(i).cost=bestcolony.cost;

         end

 end

imp=calculated_totalcost(imp,Gamma);

 function imp=calculated_totalcost(imp,Gamma)

 nimp=length(imp);

 for i=1:nimp

 imp(i).totalcost=imp(i).cost+Gamma*mean([imp(i).colony.cost]);

 end


imp=imperialistic_competative(imp);

 function   imp=imperialistic_competative(imp)

 nimp=length(imp);

 if nimp>=2

     [~,index]=min([imp.totalcost]);

     weakestimp=imp(index);

     [~,index2]=min([weakestimp.colony.cost]);

     weakestcolony=weakestimp.colony(index2);

     imp(index).colony(index2)=[];

     f=[imp.totalcost];
     f=max(f)-f+eps;
     f=f./sum(f);
     f=cumsum(f);

     h=find(rand<=f,1,'first');

     Ncolony=length(imp(h).colony);

     imp(h).colony(Ncolony+1).Q=weakestcolony.Q;

     imp(h).colony(Ncolony+1).cost=weakestcolony.cost;

     if isempty(imp(index).colony)

         imp(index)=[];

         f=[imp.totalcost];
```

```
            f=max(f)-f+eps;
            f=f./sum(f);
            f=cumsum(f);

            h=find(rand<=f,1,'first');

            Ncolony=length(imp(h).colony);

            imp(h).colony(Ncolony+1).Q=weakestimp.Q;

            imp(h).colony(Ncolony+1).cost=weakestimp.cost;

        end

    end

    end


   [value,index]=min([imp.cost]);

   if value<gimp.cost

       gimp=imp(index);

   end

   Best(decade)=gimp.cost;

   Mean(decade)=Mean([imp.cost]);

   NFE(decade)=NFE;

    disp(['decade    ' num2str(decade)          ':   ' ...
          'Best   =  ' num2str(gimp.cost)       '  ,  ' ...
          'Mean   =  ' num2str(Mean([imp.cost]))'  ,  ' ...
          'Nimp   =  ' num2str(length(imp))]);

         if length(imp)==1

            break

         end

end
```