MODELLING AND OPTIMIZATION OF A TRANSIT SERVICES WITH FEEDER BUS AND RAIL SYSTEM

MOHAMMADHADI ALMASI

THESIS SUBMITTED IN FULFILLMENT OF THE REQUIREMENT FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

FACULTY OF ENGINEERING
UNIVERSITY OF MALAYA
KUALA LUMPUR

2015

UNIVERSITI MALAYA

ORIGINAL LITERARY WORK DECLARATION

Name of Candidate: Mohammadhadi Almasi

Registration/ Matric No: KHA100122

Name of Degree: Dr. of Philosophy

Title of Thesis: MODELLING AND OPTIMIZATION OF A TRANSIT SERVICES WITH FEEDER BUS AND RAIL SYSTEM

Field of Study: Civil Engineering

I do solemnly and sincerely declare that:

(1) I am the sole author/writer of this work;
(2) This work is original;
(3) Any use of any work in which copyright exists was done by way of fair dealing and for permitted purposes and any excerpt or extract from, or reference to or reproduction of any copyright work has been disclosed expressly and sufficiently and the title of the Work and its authorship have been acknowledged in this Work;
(4) I do not have any actual knowledge nor do I ought reasonably to know that the making of this work constitutes an infringement of any copyright work;
(5) I hereby assign all and every rights in the copyright to this work to the University of Malaya ("UM"), who henceforth shall be owner of the copyright in this work and that any reproduction or use in any form or by any means whatsoever is prohibited without the written consent of UM having been first had and obtained;
(6) I am fully aware that if in the course of making this work I have infringed any copyright whether intentionally or otherwise, I may be subject to legal action or any other action as may be determined by UM.

Candidate's Signature Date

Subscribed and solemnly declared before,

Witness's Signature Date

Name:
Designation:

ABSTRACT

Currently, many passengers use transit systems to reach their destination, while a growing concern for public transit is its inability to shift passenger's mode from private to public transportation. By designing a well-integrated public transit system and improving the cost-effectiveness network, public transport could play a crucial role in passenger satisfaction and reduce operating costs. The main objective of this research is to develop a mathematical formulation model for designing and coordinating schedules of integrated public transit services, which includes development of feeder services and coordination with major transportation services and transfer time consideration between two modes (i.e. Feeder bus and LRT). In the proposed improved model, the additional terms and constraints employed in objective function provide more accurate and efficient solutions for various conditions of transit systems, and this may lead to the creation of a more realistic model in simulating real-life problems. This is followed by application of the improved model to the benchmark and Petaling Jaya study areas. In this study, optimized transit services and coordinated schedules are developed using metaheuristic algorithms such as genetic algorithm (GA), particle swarm optimization (PSO), water cycle algorithm (WCA) and imperialist competitive algorithm (ICA). The data of the study were obtained based on the literature review, questionnaire survey and observation. Finally, obtained numerical results of the proposed model including optimal solution, statistical optimization results, the convergence rate as well as comparisons were discussed in detail. Therefore, optimum transit network was obtained by using ICA and WCA. As a result, the corresponding network costs obtained by PSO and GA are greater than ICA and WCA.

PERMODELAN DAN PENGOPTIMUMAN SERVIS TRANSIT MELALUI BAS PENGANTARA DAN SISTEM RAIL

ABSTRAK

Pada masa kini, ramai penumpang menggunakan sistem transit untuk tiba ke destinasi masing-masing, di mana kebimbangan yang semakin meningkat adalah ketidakupayaan untuk mengalihkan mod penumpang daripada pengangkutan persendirian ke pengangkutan awam. Dengan merekabentuk sistem transit awam yang bersepadu dan memperbaiki keberkesanan kos rangkaian, pengangkutan awam boleh memainkan peranan penting dalam kepuasan penumpang dan mengurangkan kos operasi. Objektif utama kajian ini adalah untuk membangunkan satu model formula matematik untuk merancang dan menyelaraskan jadual perkhidmatan transit awam bersepadu, termasuk membangunkan perkhidmatan bas pengantara dan penyelarasan dengan perkhidmatan pengangkutan utama dan pertimbangan masa pemindahan antara dua mod (iaitu bas pengantara dan LRT). Di dalam cadangan model yang telah diperbaiki, tambahan terma dan kekangan bekerja dalam objektif fungsi menyediakan penyelesaian yang lebih tepat dan efektif untuk pelbagai keadaan sistem transit dan ini boleh membawa kepada pembentukan model yang lebih realistik di dalam simulasi masalah kehidupan sebenar. Ini diikuti oleh aplikasi model yang telah diperbaiki kepada penanda aras dan kawasan kajian Petaling Jaya. Dalam kajian ini, perkhidmatan transit yang optimum dan penyelarasan jadual telah dibangunkan menggunakan algoritma metaheuristik seperti Algoritma Genetik, Pengoptimuman Zarah Kumpulan, Algoritma Kitaran Air dan Algoritma Imperialis Kompetitif.Data kajian yang diperolehi adalah berdasarkan kajian literatur, kajian soal selidik dan pemerhatian. Akhirnya, cadangan keputusan berangka model yang telah diperolehi, termasuk penyelesaian optimum, keputusan pengoptimuman statistik, kadar penumpuan dan juga perbandingan

dibincangkan dengan jelas menggunakan jadual dan rajah. Maka, rangkaian transit yang optimum diperoleh dengan menggunakan ICA dan WCA. Hasilnya, kos rangkaian yang sepadan diperoleh melalui PSO and GA adalah lebih besar daripada ICA dan WCA. Best of luck then.

ACKNOWLEDGMENTS

I would like to express my appreciation and gratitude to my supervisor, Professor Dr. Mohamed Rehan Karim for his remarkable advice and guidance. His understanding and patient assistance in the preparation of this thesis has given me the awareness of how to synthesize, refine and complete the research. My gratefulness and dedication go to my parents for their enormous encouragement and support during the research.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS AND ABBREVIATIONS

ACO	Ant Colony Optimization
BCO	Bee Colony Optimization
CBD	Central Business District
ES	Exhaustive Search
FNDP	Feeder Network Design Problems
FNDSP	Feeder Network Design and Scheduling Problems
FNP	Feeder Network Problems
FNSP	Feeder Network Scheduling Problems
GA	Genetic Algorithm
GIS	Geographic Information System
HFRGA	Heuristic Feeder Route Generation Algorithm
ICA	Imperialist Competitive Algorithm
LRT	Light Rail Transit
MDVRP	Multi-Depot Vehicle Routing Problem
MFNDP	Multimodal Feeder Network Design Problems
OD	Origin-Destination
PJ	Petaling Jaya
PSO	Particle Swarm Optimization
SA	Simulated Annealing
SD	Standard Deviation
SOHFRGA	Shrivastava–O'Mahony Hybrid Feeder Route Generation Algorithm
TNDP	Transit Network Design Problems
TNDSP	Transit Network Design and Scheduling Problems
TNDFSP	Transit Design and Frequency Setting Problems

TNP	Transit Network Problems
TNTP	Transit Network Timetabling Problems
TNFSP	Transit Network Frequency Setting Problems
TNSP	Transit Network Scheduling Problems
TRND	Transit Network Design Problems
TS	Tabu Search
TSP	Traveling Salesman Problem
VRP	Vehicle Routing Problem
WCA	Water Cycle Algorithm

CHAPTER 1: INTRODUCTION

1.1 Introduction

Transportation is a multimodal, multi-problem, and multi-spectral system as it involves different categories and activities such as policy making, planning, designing, infrastructure construction, and development. Currently, considering the significant developments in technology, economy, and society, an efficient transportation system plays a key role in passenger satisfaction and the reduction of costs.

Many people usually use public transportation systems to reach their destination; however, others employ personal vehicles. If the transportation system is unable to attract travelers, private transport usage will be increased (Jerby & Ceder, 2006). At present, to prevent the increasing rate of private transport entering city centers, effective alternatives of travel modes need to be offered (Martins & Pato, 1998). In addition, a good performance in public transport has been recognized among the potential means to reduce air pollution, decrease energy consumption, and improve mobility and traffic congestion.

An intermodal transit system is a type of transportation which has proven challenging and interesting in the field of public transportation. In order to improve complicated public transportation systems, a well-integrated transit system in urban areas can play a crucial role in passenger satisfaction and in reducing operating costs. This system usually consists of integrated rail lines and a number of feeder bus routes connecting transfer stations. Rail lines which provide an effective and convenient mode of transportation can carry large numbers of travellers.

The main target of this study is to present a new model to design an efficient transit system to increase the efficiency of feeder network designs and coordinated schedules in order to minimize costs. An improved integrated intermodal system may lead to a reduction in total cost and an increase in profit consequently leading to the achieving of an optimum transit network design. Furthermore, such a system can provide greater quality services for passengers.

1.2 Statement of the Problem

An integrated transit system including rail lines and a number of feeder bus routes connected at different transfer stations is expected in large metropolitan areas, where transit demand is high and widely needed. The problem involves designing a feeder network to provide access to an existing rail system and coordinate the schedule of transit service (Shrivastava & O'Mahony, 2007). In this design, the main concern is supplying a set of feeder bus routes and determining the associated frequencies for each route that achieves the desired objective with a specified service level to the passengers subject to constraints (Kuah & Perl, 1989; Kuan, Ong, & Ng, 2004, 2006; Martins & Pato, 1998; Xiong et al., 2013).

On the other hand, under a given budgetary constraint, the optimal capacity improvements of existing road links of a transportation network can be determined by the network design problem.

The development of well-integrated intermodal systems improves service quality which by extension increases passenger satisfaction as a corollary of better coverage, reduced access costs, minimal delay, and shorter travel times. From the viewpoint of the transit operators, overall coordination among the various public transport modes can reduce their operating costs and increase their revenue by maintaining shorter routes and eliminating duplication of routes by trains and buses.

Growing interest has been paid to design a more efficient feeder network and to provide feeder services connecting major transportation systems with their welfare facilities. However, still there are some limitations and gaps, which highlighted the need for further researches, particularly in different levels of public transit such as train and feeders.

The current study tries to fill the gaps of the preceding studies by providing an improved model through presenting a mathematical formulation of the model and proposing solution methods. Therefore, the target of this study is to design a set of feeder bus routes as well as to determine the operating frequency on each route. This would be achieved through minimizing the objective function, including the sum of operator, user, and social costs. The study provides significant contribution to service quality, financial performance, and ridership.

1.3 Objectives of the Study

The main objective of this research is to improve a mathematical formulation model for designing and coordinating schedules of integrated public transit services. This model includes an improvement of feeder services and coordination with major transportation services and transfer time consideration between two modes. In the proposed improved model, the additional terms and constraints employed in objective function provide more accurate and efficient solutions for various conditions of transit systems, and this may lead to the creation of a more realistic model in simulating real-life problems. In brief, these objectives are summarized in the following:

1. Proposing an improved mathematical model based on the gaps of the previous studies to increase the efficiency of the intermodal transit system with the aim of achieving the optimal balance between the operator, user, and social costs.

2. Applying and demonstrating an improved model that addresses the intermodal transit system based on the benchmark data of the study,

a) to solve the feeder network design and scheduling problem (FNDSP) by using the metaheuristic methods.

b) to achieve an optimum transit network design that focuses on the design of a set of feeder bus routes and determination of the operating frequency on each route with the aim of minimizing the costs.

c) to evaluate the performance of the users, operators and social perspectives in results.

3. Applying and demonstrating an improved model that addresses the intermodal transit system based on real case study data (Petaling Jaya, Malaysia),

a) to solve the FNDSP by using the metaheuristic methods.

b) to achieve an optimum transit network design that focuses on the design of a set of feeder bus routes and determination of the operating frequency on each route with the aim of minimizing the costs.

c) to evaluate the performance of the users, operators and social perspectives in results.

1.4 Scope of the Study

A real-world FNDSP is usually extremely large and complex. The main objective of this research is to design an efficient model to the FNDSP, to explore and apply the appropriate metaheuristic methods for FNDSP.

The study is trying to propose a new mathematical formulation model. The model is solved by using metaheuristic methods such as genetic algorithm (GA), particle swarm optimization (PSO), imperialist competitive algorithm (ICA) and water cycle algorithm (WCA) for the benchmark test problem and real case study. The results were analyzed with the data of the benchmark and real case study. A comparative study is also carried out on the benchmark and real case study. This is generated to compare the performance of the various metaheuristic methods in terms of proper computational efficiency and solution quality.

1.5 Significance of the Study

The study by presenting an improved model and designing an efficient transit system could have a significant role in increasing the efficiency of feeder bus services and minimizing user, operating and social costs as well as promoting passenger satisfaction. This study by applying and demonstrating an improved model in the intermodal transit system based on the benchmark would provide a valid model and its solution methods. This study by applying and demonstrating an improved model in the real case study (Petaling Jaya) would provide optimum transit service in urban areas.

1.6 Outline of the Study

This thesis is organized into five chapters: Chapter 1 provides an introduction to the focus of the study, including problem statement, objectives, research hypothesis and scope of the study which is followed by an outline of the thesis. Chapter 2 presents the literature review of the subjects related to the study, such as bus network design problems and feeder bus network design problems. The chapter concludes by introducing the limitations and gaps of the preceding studies and argues for the need of the current research. Chapter 3 introduces our methodology for solving the FNDSP. This is followed by the descriptions of the metaheuristic methods and their applications to solve the FNDSP in the study. Chapter 4 shows the overall numerical and computational results that are analyzed and obtained in the research. Finally, Chapter 5 presents the major conclusion derived from the previous chapters and recommendation for the future work.

1.7 Summary of the Chapter

This chapter has presented the introduction of the study, followed by the problem statement, objectives, hypothesis, scope and significance of the research. The chapter concluded with the outline of the thesis. The next chapter is allocated to the related review of the literature and gaps of the previous studies.

CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

This chapter is allocated to the related studies of the current thesis. It focuses on the performing a review of studies for a specific problem arising in a feeder network design, with the goal of providing readers a broader and more complete insight on the subject. First, the literature on the documentation of the transit network problems (TNP) and main approaches for bus network design are presented briefly. Second, the literature and preceding studies in feeder network problems (FNP) are explained. This is followed by the problem description, problem characteristics and classification of the previous studies based on their approaches and solution methods. Finally, the gaps of the previous studies and summary of this chapter are presented.

2.2 Transit Network Design Problem

Public transportation planning covers a very wide research area. From the design of networks to the roster of crews, from the evaluation of demand of the transit trip assignment, from mathematical solution methods to evolutionary ones, the process of generating a public transportation system has been approached from many sides. This thorough interest is partly since the development of public transportation is a crucial topic in the modern society. Confronted to traffic congestion, urban parking problems and increasing pollution, car drivers might consider switching to public transit if they had an affordable and good-quality system at their disposal. It is the duty and goal of transit agencies to provide such conditions, by adequately adjusting their systems, to maximize the quality of service to users while minimizing the costs. Tradeoffs thus need to be made, and this is where various optimization techniques come into the game.

From the users' perspective, the system should meet the demand by providing cheap and direct service to passengers. The criteria for using public transport can also include vehicle and transfer terminal comfort, regularity, and service coverage and frequency level. From the operator's perspective on the opposite, the objective is for the system to make as much profit as possible. It is the main challenge in transit planning to find equilibrium between these conflicting objectives. As the literature assesses, the public transit planning process is usually divided into a sequence of five steps: (1) the design of routes, (2) the setting of frequencies, (3) the timetabling, (4) the vehicle scheduling and (5) the crew scheduling and rostering (see Table 2.1). This review addresses the three first and thus fundamental elements of the public transit planning process, also called strategic (step 1) and tactical (steps 2 and 3) planning, respectively (Ceder & Wilson, 1986). All the information needed by the passengers, namely the transit route network, the frequencies and departure times, is determined during these phases.

Table 2.1: Transit planning process (Ceder & Wilson, 1986)

Independent inputs	Planning activity	Output
Demand data	Network design	Route changes
Supply data		New routes
Route performance indicators		Operating strategies
Subsidy available	Frequencies setting	Service frequencies
Buses available		
Service policies		
Current patronage		
Demand by time of day	Timetable development	Trip departure times
Times for first and last trips		Trip arrival times
Running times		
Deadhead times	Bus scheduling	Bus schedules
Recovery times		
Schedule constraints		
Cost structure		
Driver work rules	Driver scheduling	Driver schedules
Run cost structure		

One could, therefore, think that these steps are essential user-oriented. However, the problem remains multi-objective since financial objectives must also be taken into consideration. Even inside the restricted area of our problem, numerous approaches have been proposed, integrating different constraints, aiming for various objectives and combining heterogeneous features.

2.2.1 Terminology Proposals for Transit Network Problems

In the literature, various terms can be employed to describe the different steps of the transit planning process and their combinations. For instance, a problem in which transit routes and frequencies are set can be named “bus transit routes network design” (Fan & Machemehl, 2004), “transit route network design” (Baaj & Mahmassani, 1995) or “line planning in public transport” (Borndorfer, Grotschel, & Pfetsch, 2005). Additionally, many articles do not explicitly name the problem they are addressing in the context of the global transit planning. Therefore, they propose the terminology which is shown in Figure 2.1 to organize denominations and relations between problems and sub-problems related to strategic and tactical transit planning in the remainder of this document.

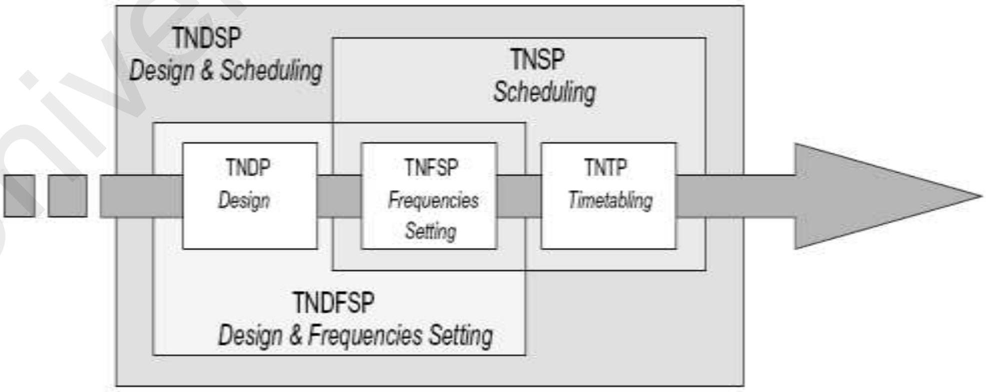


Figure 2.1: Transit network problems (TNP) structure (Guihaire & Hao, 2008)

They begin with three basic transit network problems: design (TNDP), frequency setting (TNFSP) and timetabling (TNTP). Then, they introduce two combined problems: design and frequencies setting (TNDFSP = TNDP + TNFSP) and scheduling

(TNSP = TNFSP + TNTP). Finally, the whole design and scheduling problem (TNDSP) is defined as the composition of the three basic problems (Guihaire & Hao, 2008).

2.2.2 Types of Bus Route Network Design Problems

There are two types of bus route network design problem. The first type is, given a service area with pre-specified bus stop locations and an hourly demand at each bus stop, the bus network design problem involves designing a set of bus routes and determining the associated frequencies for each route, such that it achieves the desired objective with a specified service level to the passengers and subject to some constraints imposed by the problem. In other words, the problem involves connecting all the demand points (bus stops) such that most, preferably all the passengers are able to access from one point to another, while optimizing the objective function subject to the constraints imposed (see Figure 2.2).

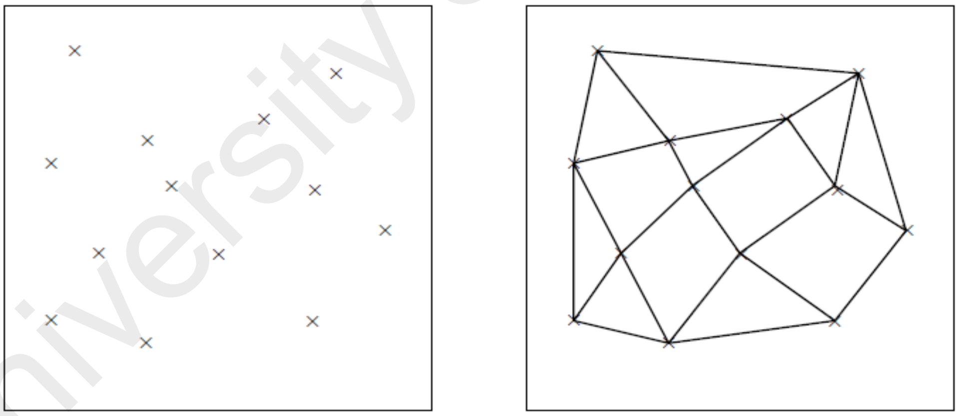


Figure 2.2: Bus network design problem

The second type is the feeder network design problems (FNDP). It differs from the first type in that there is an existing rail public transport system, and the buses serve to carry the passengers from the bus stops to the various stations. In other words, the problem involves designing a feeder bus network to provide access to an existing rail public transport system. Thus, given a service area with pre-specified bus stop locations, and also a fixed rail transport system and an hourly demand at each bus stop, this problem also involves designing a set of feeder bus routes and determining the associated frequencies for each route that achieves the desired objective with a specified service level to the passengers and subject to the constraints. An example of a network model representing the FNDP is shown in Figure 2.3.

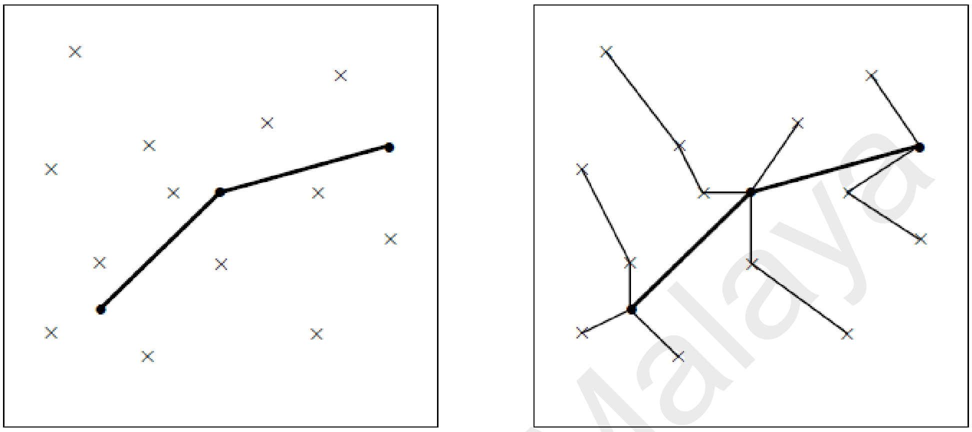


Figure 2.3: Feeder bus network design problem

2.2.3 Brief Description of the Feeder Network Problems

In the FNP, the feeder bus network serves to carry the passengers from the bus stops to the various stations in an existing rail network.

The network models of the FNP use two types of nodes, rail nodes, and bus nodes, to represent the railway stations in a rail transport system and the bus stops, respectively, within a given service area. The rail transport network is assumed to be fixed. That is, defined in advance and not subject to change, and is represented as links joining the rail nodes shown in the left diagram of Figure 2.3. The locations of bus stops and hourly demands of passengers at each bus stop are also pre-specified. The travel cost of the rail system between each pair of railway stations, the distance between each pair of bus stops and between every bus stop and every railway station, the maximum bus route length, the number of bus routes, the capacity and operating speed of the fleet of buses over the planning period are also given.

The FNDP involves linking bus nodes to rail nodes, in which these bus links represent feeder bus route segments, as shown in the right diagram of Figure 2.3. The FNDP can be formulated as a hierarchical transportation network design problem with side constraints, where the primary path represents the rail line, and the secondary paths represent the feeder bus routes.

The focus of the feeder network design and frequency setting problems is on the design of a set of feeder bus routes and the determination of the operating frequency on each route, such that the objective function of the sum of operator and user costs is minimized. The operating cost is related to the total length traveled by the buses while the user cost is a function of total passenger travel time, including the waiting time and riding time, etc.

2.2.4 Distinction of Data Preparation

Data preparation includes the area's topology, origin-destination (OD) matrices, fleet size and more information such as bus and train operating costs, route length, speed, demand, etc.

The road network, bus stops, stations and transfer zones define the area's topology (Guihaire & Hao, 2008). And also travel times, distance between rail stations and bus stops or demand can be specified by OD matrices. Sometimes, geographic information system (GIS) and various shortest path algorithms are utilized for calculating the travel time and distances OD matrices. Kim and Jeong (2009) compared the performance of several shortest path algorithms and developed an approximation approach to OD matrices generation.

Vehicle data contains types of available buses, which can have different capacities. Bus fleet describes the vehicle size. The available fleet size and bus capacities are very important to determine the service frequencies. Finally, detailed information is determined according to the problem of the study area, objectives, and constraints (Almasi, Mirzapour Mounes, Koting, & Karim, 2014).

In the following sections, the previous studies in FNP are presented. The problem description, problem characteristics, and classification of the previous studies based on their approaches as well as solution methods are explained respectively.

2.3 Problem Description

Passengers gathered at bus stops located in the service area wish to access their destination. They travel by feeder bus to any rail stations and then proceed to the city center or their destination (Martins & Pato, 1998). This procedure occurs almost all over the world several times a day, and it includes so many challenges and issues. This study effort has been made to go further into details of these problems. In this segment, feeder network design and scheduling will be presented, and it will introduce the studies are done about these problems.

2.3.1 Feeder Bus Network Design Problem

Feeder bus network design is the first and most important step in the feeder bus transport planning procedure. The network design problem consists of determining a set of bus routes in a specific area, through the given travel demand, the area's topology characteristics and set of objectives and constraints (Guihaire & Hao, 2008).

The route structure design is becoming an important input to the subsequent decision-making processes and will affect later planning steps, significantly, which is explained in the following section.

2.3.1.1 Feeder route generation

Feeder routes link residential complexes to railway stations (Kim, Cheon, & Lim, 2011). A good design of route network can increase the efficiency of the feeder bus system and decrease the total cost of supplying the transit service. The users would like to have a bus network with more coverage area and high accessibility in the service area. Their perspective of a good service area is a feeder network with more direct-through trips and high demand satisfied. On the other hand, the operation's costs should be reduced by keeping the total route length within a certain bound. Thus, the main challenge of the route network design is to be able to give a good and efficient alternative at a reasonable computation time. The feeder route network design problem can be solved by building an initial solution using the contraction algorithm, following by improving the existing solutions by means of applying a local search algorithm.

One of the construction heuristics for building initial solutions is a sequential building method, proposed by Kuah and Perl (1989). This method is adopted from the sequential saving approach for Multi-Depot Vehicle Routing Problem (MDVRP). In another study, Martins and Pato (1998) expanded the research by Kuah and Perl (1989) and created the initial solution by applying the sequential savings. Their research suggested a two-phase building method to generate the initial solution. Shrivastav and Dhingra (2001) proposed the heuristic feeder route generation algorithm (HFRGA). This algorithm was greatly guided by the demand matrix developed by Baaj and Mahmassani (1995). Metaheuristic methods are also applied for the initial population. GA for an initial population at random was used by Chien, Yang, and Hou (2001). Nevertheless, a random selection of nodes might not be a good selection for generating initial routes. Therefore, Kuan et al. (2004), (2006) employed the concept of delimiter, proposed by Breedam (2001).

Pradhan and Mahinthakumar (2012) described parallel implementations that include performance analyses of two prominent graph algorithms (i.e., Floyd-Warshall and Dijkstra) used for finding the all-pairs shortest path for a large-scale transportation network. Their article also includes the derivation of the computational time for the different parallel implementations of these two graph algorithms. The technique used in the feeder bus route generation is indicated in Table 2.2.

Table 2.2: Feeder route generation methods in literature

References	Initial building methods	Specify
Kuah and Perl (1989)	H	Sequential savings
Martins and Pato (1998)	H	Sequential savings and two-phase
Shrivastav and Dhingra (2001)	H	HFRGA, Dijkstra’s algorithm
Chien et al. (2001)	Me	GA
Kuan et al. (2004)	H	Delimiter algorithm, Breedam (2001)
Kuan et al. (2006)	H	Delimiter algorithm, Breedam (2001)
Shrivastava and O’Mahony (2006)	H	K-path algorithm, Eppstein (1994)
Shrivastava and O’mahony (2007)	H	K-path algorithm, Eppstein (1994)
Shrivastava and O’mahony (2009a)	H	K-path algorithm, Eppstein (1994)
Shrivastava and O’mahony (2009b)	H	Dijkstra’s algorithm
Mohaymany and Gholami (2010)	Me	ACO
Gholami and Mohaymany (2011)	Me	ACO

In Table 2.2, ‘M’ stands for ‘Mathematical’, ‘H’ for ‘Heuristic’ and ‘Me’ for ‘Metaheuristic’. After building initial solutions, improvements can be implemented on the routes. There are a lot of optimization methods to improve the solutions.

2.3.2 Feeder Bus Network Scheduling Problem

The problems of feeder bus scheduling can be categorized into three groups, consisting of:

1. Timetabling; including departure times from all stops and stations served by the routes in the network.
2. Frequency setting; determining the feeder bus frequency for every route in the network.
3. Timetable & frequencies; applying timetable and frequency setting simultaneously for each route, with regards to the set of objectives and

constraints. The schematic illustration of the three categories of feeder network scheduling problem (FNSP) is presented in Figure 2.4.

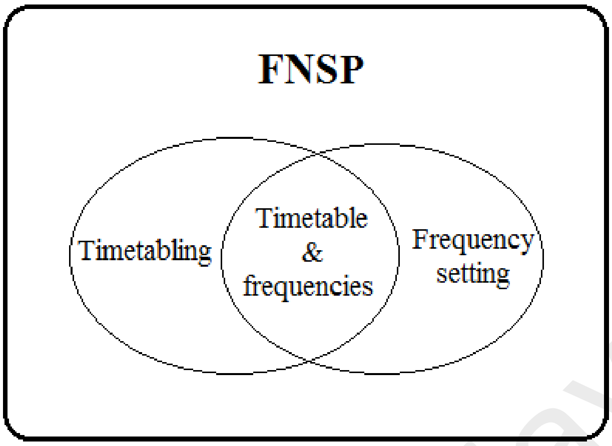


Figure 2.4: The schematic categorization of FNSP

A suitable scheduling design can supply a sufficient standard service to satisfy the users. In addition, suitable scheduling contributes to a reduction in fleet size and operators' cost, subsequently leads, suppliers to be satisfied (Guihaire & Hao, 2008). In the literature review, it is evident that frequency setting problems have been mostly used for development scheduling problems.

2.4 Problem Characteristics

In this section, it has been tried to categorize FNDSP based on various characteristics. There are several ways to achieve these aims, but we mainly focus on the realistic aspects of the problem.

Table 2.3 shows the classified criteria of problem characteristics, and the important points of these criteria are described in the following subsections. In Table 2.4, the summary of criteria pertaining to some of the literature is presented.

2.4.1 Demand Pattern

In terms of passenger demand, it is usually assumed to be fixed or inelastic, for simplicity. Fixed demand may be reasonable for systems at which passengers are insensitive or independent of service quality or price. However, elastic demand can probably be variable, due to the sharing or the competition of the public transport. And the rising rate of mobility demand will be a significant factor in the efficiency of urban transportation modeling (Jakimavicius & Burinskiene, 2009).

For feeder bus, two types of travel demand patterns, namely, many-to-one and many-to-many, are measured. The many-to-one demand pattern is discussed in several papers such as Chien and Schonfeld, (1998); Chien and Yang, (2000); Kuah and Perl, (1988), (1989); Kuan, (2004); Kuan et al., (2004), (2006); Xiong et al., (2013); etc. This model refers to passengers traveling from multiple origins to a single destination. This is usually more applicable to feeder bus services, which carry passengers to a common destination (central business district (CBD) or a transfer station), and peak hour trips to and from the CBD can be considered in this pattern. In most bus services, many-to-many demand pattern is considered whenever passengers have different origins and destinations. Kuah and Perl (1989); Chien and Schonfeld (1998) considered many-to-one pattern model to find an optimal feeder bus network in the hope that it will include one bus stop for more than a route, each of which, serves the demand for various sets of destination.

2.4.2 Objectives of the Previous Studies

Many concepts can be considered for determining objectives, decision variables, and constraints, such as environmental, economic, political and social factors. Transit agency policies for specifying stated factors are based on the importance of these factors. Cohen-Blankshtain and Feitelson (2011) in a survey research showed these criteria can be a number of land-use policies and several transport measures. Therefore, it has been tried to discuss the objectives and list of decision variables in FNDSP in this section.

Table 2.3: Classification criteria from the problem perspective

Criteria	Consideration
Demand pattern	Many-to-one Many-to-many
Problem scope	Morning Afternoon Peak
Decision variable	Feeder zone boundary (Z) Stop spacing (BS) Station spacing (RS) Bus frequency (BF) Bus headways (BH) Train headway (RH) Bus route density (BRD) Rail station density (RSD) Bus route location (BRL) Bus route length (BL) Rail line length (RL) Fleet size (N) Travel time (T) Mode (M)
Constraint	Load factor (LF) Geographic (G) Budget (BU) Demand bound (D) Bus route length (BL) Route feasibility (RF) Frequency bound (F) Bus headways (BH) Train headway (RH) Rail line length (RL) Maximum of fleet (N) Vehicle capacity (C) Travel distance (TD)

Most of the main objective in the literature is optimizing the problem to achieve minimum user and operator costs. Cost estimates can be required at early project stages, before completion of a detailed design for several purposes, including budgeting and feasibility decisions. This estimation is usually determined by parametric modeling technique (Sonmez & Ontepeli, 2009).

Xiong et al. (2013) developed a solution for the optimal routing design problem with the objective of minimizing the total cost, including user and supplier costs, considering passenger traffic demand and budget constraints. The optimization variables include route and headway.

Kuah and Perl (1989) optimized routing structures and operating frequency to minimize the total bus operating costs. User costs include the bus riding time, waiting time and rail costs. Golub, Balassiano, Araújo, and Ferreira (2009) showed in their research that bus improvements (special feeder concessions, new route and service configurations etc.) and upgrading the train system was important objects to achieve a reduction in travel time for most OD pairs and overall safety improvements.

Transfer coordination is also a major goal in many studies. The global network schedule should take into account each transfer area and its associated routes in order to allow efficient transfer between lines in distance and time. Transferring between lines can be supported according to various criteria such as the number of travelers. Wirasinghe, Hurdle, and Newell (1977) designed a coordinated rail/bus transit system that served peak travel between a metropolitan region and CBD. They obtained values of three related variable (interstation spacing, feeder bus zone boundary, train headways) to minimize the total operator and user cost. Area coverage is one of the important objectives to measure the percentage of the estimated demand, which can be served by public transport. This rate can be calculated in different ways, and however, generally is dependent on characteristics, including route length, bus stop, density and route spacing (Benn, 1995; Murray, 2003; Spasovic, Boile, & Bladikas, 1993). A maximum distance from the stopping place of the public transport was chosen by the following the technical rule, for example "Communication systems of towns, small towns and villages" (Uspalyte Vitkuniene & Burinskiene, 2006).

In a considerable number of studies walking distance of travelers are considered as a measure, the range of which is from 400 to 800 meters in both euclidean or network distances (Choi, Lee, Kim, & Sohn, 2011).

The general objective of operators is to minimize the overall route length in view of a reduction in the number of vehicles and crew resources required to maintain a global transport system. Moreover, the number of lines alternatively can be considered. In addition, routes should not be too short or too long for profit reasons. In Table 2.3, the decision variables are considered as a special set of characteristics. Based on the review, in the most feeder network design studies there is an intention to consider bus route location and bus frequency as the decision variable.

2.4.3 Constraints of the Previous Studies

Different constraints have been considered for FNDSP as presented in Table 2.4. The following constraints were outlined by Kuah and Perl (1989):

1. Route capacity: maximum passenger on the route
2. Maximum fleet size
3. Maximum route length
4. Route feasibility: meaning to determine the feasibility of the bus routes, which includes the following several sub-constraints:
    a. Each bus node should be placed in a single route (many-to-one pattern).
    b. The feeder bus network may include a bus stop in more than a single route (many-to-many patterns).
    c. Each bus route must be linked to just one railway station.

d. Each bus is assumed to halt at all the stops in its route.

e. Each feeder bus route should be linked to one railway station.

f. Bus stop can be assigned to a railway station only if a route, which terminates at that station, passes through that bus stop.

Martins and Pato (1998) also used these constraints as well as frequency bound.

Shrivastava and O'Mahony (2009b) presented a hybrid approach for developing their model and found more efficient feeder routes, by considering load factor, fleet size, and unsatisfied demand as constraints. In terms of fleet size, the vehicle schedule is created by a line run and transit network timetable; that fleet size is a useful constraint to optimize resource usage in FNDSP. Demand constraint is also a critical issue. The demand can be considered unsatisfied when users' origin or destination is too far from the bus stops, or when direct feeder services are not sufficient. In general, if a trip requires more than two transfers, it is assumed that the user will switch to another mode of transport.

For some reason, transportation agencies might prefer to develop a network with a specific shape such as radial, rectangular, grid and triangular shape (Nes, 2002). In some studies, several of these constraints are considered as objective functions. For instance, Martins and Pato (1998); Mohaymany and Gholami (2010) applied some limitations on frequency variable, considered as a constraint and simultaneously as an objective to minimize the total cost. Chowdhury, Steven, and Chien (2002) included the bus headway and train headway in their objective function, by applying limits on headways, to achieve the optimal range of headways in certain lines and areas of interest. In general, the candidate bus line should also satisfy other constraints such as the presence of overlapping bus lines and maximum allowable bus line directness (Yan, Liu, Meng, & Jiang, 2013). Table 2.4 represents data that are more detailed.

Table 2.4: Classification of literature based on problem perspective

References	Objective	Decision variable	Constraint	Demand pattern	Urban or suburban	Application area and scope
Wirasinghe (1977)	Coordinate transit system (rail and bus)	Z	RH	Many-to-one	Urban	
Wirasinghe et al. (1977)	Coordinate transit system (rail and bus)	Z, RS, RH		Many-to-one	Urban	Example, morning
Wirasinghe (1980)	Coordinate operations	BRD, RSD, BF		Many-to-one	Urban	Calgary, peak
Hurdle and Wirasinghe (1980)	Optimize rail station spacing	RS		Many-to-one	Urban	Calgary, peak
Kuah and Perl (1988)	Optimal design for feeder bus	BS, BRL, BH		Many-to-one		
Kuah and Perl (1989)	Optimal design for feeder bus	BRL, BF	BL, N, RF	Many-to-one Many-to-many	Suburban	Example benchmark, morning
Martins and Pato (1998)	Optimal design for feeder bus	BRL, BF	BL, N, F, RF	Many-to-one	Suburban	Example benchmark, morning
Chien and Schonfeld (1998)	Optimal design of integrated rail and bus	RL, BH, BS, RS, BRL		Many-to-many		Example
Chien and Yang (2000)	Optimize feeder route location and headway	BRL, BH	G, BU, RC	Many-to-one	Suburban	Example
Shrivastav and Dhingra (2001)	Development of routing and coordinated schedules	T	D, RL	Many-to-one	Suburban	Mumbai
Chien et al. (2001)	Total welfare (operator and user cost)	BRL, BH	G, BU, RC	Many-to-one	Suburban	Example
Chowdhury and Chien (2002)	Coordinated design of an intermodal transit system	BH, RH, BT	C, BH, RH	Many-to-one	Urban	Numerical example
Kuan (2004)	Optimal design for feeder bus	BRL, BF	RL, N, RF	Many-to-one	Suburban	Example benchmark, morning
Kuan et al. (2004)	Optimal design for feeder bus	BRL, BF	BL, N, RF	Many-to-one	Suburban	Example benchmark, morning
Chien (2005)	Total welfare (operator and user cost)	BH, N, BRL	C, N, BU		Urban	Sandy Hook, park
Kuan et al. (2006)	Optimal design for feeder bus	BRL, BF	BL, N, RF	Many-to-one	Suburban	Example benchmark, morning
Shrivastava and O'mahony (2006)	Development of routing and coordinated schedules	BRL, BF	N, D, LF	Many-to-one	Suburban	Dublin
Shrivastava and O'mahony (2007)	Development of routing and coordinated schedules	BRL, BF	N, D, LF	Many-to-one	Suburban	Dublin
Shrivastava and O'mahony (2009b)	Development of routing and coordinated schedules	BRL, BF	N, D, LF	Many-to-one	Suburban	Dublin, morning
Shrivastava and O'mahony (2009a)	Development of routing and coordinated schedules	BRL, BF	N, D, LF	Many-to-one	Suburban	Dublin, morning
Mohaymany and Gholami (2010)	Optimize multimode feeder	BRL, BF, M	BL, N, F, BH, RS	Many-to-one	Suburban	Example benchmark, morning
Gholami and Mohaymany (2011)	Optimize multimode feeder	BRL, BF, M	BL, N, F, BH, RS	Many-to-one	Urban	North of Tehran
Sivakumaran et al. (2012)	Coordination of vehicle schedules in a transit system	BH, RH, BL		Many-to-one	Urban	An idealized network
Hu et al. (2012)	Model for layout region of feeder	T	TD		Urban	Guangzhou
Ciaffi et al. (2012)	Develop routing and scheduling simultaneously	BRL, BF	BL		Urban	Winnipeg and Rome, morning
Cipriani et al. (2012)	Develop an operative tool in the bus system	BRL, BF	C, BL, F	Many-to-many	Urban	City of Rome
Xiong et al. (2013)	Optimal routing problem	BRL, BH	D, BU	Many-to-one	Urban	Example

Note: Used abbreviations for decision variables and constraints are defined in the Table 2.3.

2.5 Classification of the Previous Studies Based on Their Approaches

Generally, previous approaches of FNDSP can be divided into two major groups: analytic and network approaches. These approaches differ in their purpose and have different advantages and disadvantages. They should be viewed as complementary rather than alternative approaches (Kuah, 1986).

2.5.1 Analytical Approach

Analytic models were developed to derive optimum relations between different components of the feeder bus network process. This approach starts by formulating the design objective as a continuous function with a set of design variables. It is assumed that the design variables are continuous, and the optimal values are obtained by using the optimal conditions according to the objective function. The typical design variables are feeder route location, rail station spacing and service frequencies.

An analytic model needs a pre-specified shape of the road geometry, and a well-designed demand function presenting the distribution of demand in the service area. Numerous studies have attempted to explain analytic models, such as Chien et al., (2001); Chien and Schonfeld, (1998); Chien and Yang, (2000); Chowdhury et al., (2002); Kuah ad Perl, (1988); Wirasinghe et al., (1977); Wirasinghe, (1977), (1980). More studies are presented in Table 2.5. An example of the road configuration of an analytic model is shown in Figure 2.5.

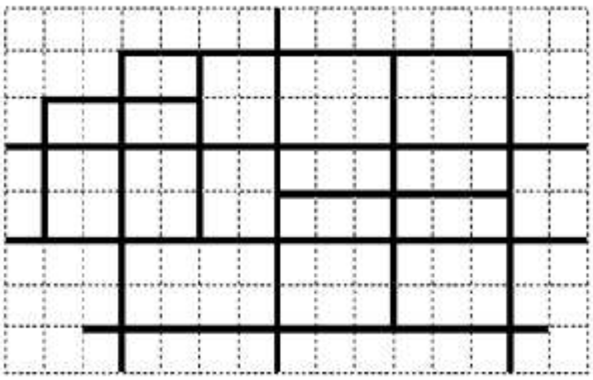


Figure 2.5: An example of an actual road network in an analytic model

This approach can process only small size or regularly shaped networks. So, the number of the possible solutions increases substantially with an increase in the number of roads in the network. Furthermore, it requires a known demand function, which represents the spatial distribution of demand in the service area. If a road network is simple, the usage in the model will only be limited to theoretical applications and may not be applied to real-world situations (Kuan, 2004).

Table 2.5: Approaches and solution methods in the literature

References	Approach model	Solution method	Specify
Wirasinghe (1977)	A	M	
Wirasinghe et al. (1977)	A	M	
Wirasinghe (1980)	A	M	
Hurdle and Wirasinghe (1980)	A	M	
Kuah and Perl (1988)	A	M	
Kuah and Perl (1989)	N	H	Displacement, exchange
Martins and Pato (1998)	N	H	Displacement, exchange, TS
Chien and Schonfeld (1998)	A		
Chien and Yang (2000)	A	Me	ES
Shrivastav and Dhingra (2001)	N	H	HFRGA
Chien et al. (2001)	A	Me	ES, GA
Chowdhury and Chien (2002)	A	H	
Kuan (2004)	N	Me	TS, SA, GA, ACO
Kuan et al. (2004)	N	Me	TS, SA
Chien (2005)	A	H	
Kuan et al. (2006)	N	Me	GA, ACO
Shrivastava and O'mahony (2006)	N	Me	GA
Jerby and Ceder (2006)	A	H	
Shrivastava and O'mahony (2007)	N	Hy	GA and H
Shrivastava and O'mahony (2009a)	N	Hy	GA and H, SOHFRGA
Shrivastava and O'mahony (2009b)	N	Hy	GA and H
Mohaymany and Gholami (2010)	N	Me	ACO
Gholami and Mohaymany (2011)	N	Me	ACO
Sivakumaran et al. (2012)	A	M	
Martinez and Eiro (2012)	N	H	
Hu et al. (2012)	A	M	
Ciaffi et al. (2012)	N	Hy	GA and H
Cipriani et al. (2012)	N	Hy	GA and H

Note: 'A' stands for 'Analytic', 'N' for 'Network', 'M' stands for 'Mathematical', 'H' for 'Heuristic', 'Me' for 'Metaheuristice' and 'Hy' for 'Hybrid'.

2.5.2 Network Approach

Network approaches do not need the pre-specified shape of road geometry in the area. As a result, it is not limited to a simple network structure, but it can be applied to networks that are more complicated. Nodes and links signify the service area, and a route is represented by a sequence of nodes. Links present the segments of transport routes and usually travel times or distances are determined by links. Demand is assumed to be targeted at nodes (Kuan, 2004). An OD demand matrix is available to represent the demand by passengers to travel between all pairs of nodes in the network in terms of the number of trips made during the selected period of study.

Kuah and Perl (1989), as first developers of the network approach, resolved the FNDSP by means of mathematical programming models. The network approaches were adopted by Kuah and Perl (1989); Kuan et al. (2004), (2006); Martins and Pato (1998); Mohaymany and Gholami (2010); Shrivastav and Dhingra (2001), etc. More studies are shown in Table 2.5.

Past studies allocated discrete variables for demand and design element in network approach, leading to its capability to deal with larger problem sizes and situations that are more realistic. Furthermore, previous models can be into three groups: headway models, route structure models, combined headway, and route structure models. A simple example of the network approach is shown in Figure 2.6.

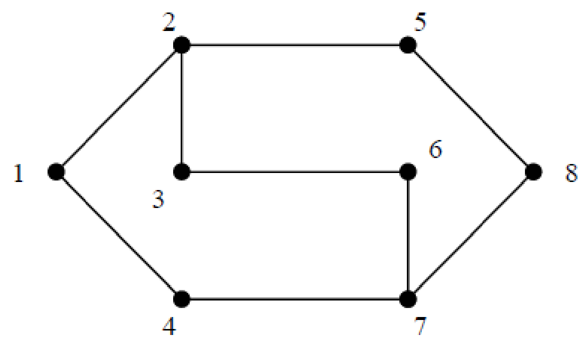


Figure 2.6: An example of a simple network model with eight nodes and nine links

2.6 Classification of the Previous Studies Based on Their Solution Methods

The solution methods of problems can be categorized into four groups, namely, mathematical, heuristic, metaheuristic, and hybrid method. In this section, we will explain these methods and analyse performance of each of them.

2.6.1 Mathematical Programming Methods

Several studies have been performed on modeling feeder bus network design by using mathematical approaches, which are based on mathematic concepts. The studies conducted by Wirasinghe et al. (1977); Wirasinghe (1977); Wirasinghe (1980) used a mathematical method for designing a coordinated rail/bus transit system that served travel in peak hours between a metropolitan region and its CBD. The rail lines were assumed to be radial. Wirasinghe (1977) optimized the zone boundary at which feeder buses should serve a rail line in order to minimize the sum of users and operating costs. It was applied to a given set of rail station spacing and constant train headways. Wirasinghe et al. (1977) obtained values of three interrelated parameters, called station spacing, feeder bus zone boundary, and train headways in order to minimize the total cost.

In another study, Wirasinghe (1980) applied a mathematical method for analyzing a case of feeder bus access to a rail line on a rectangular street network. Feeder buses along parallel routes fed passengers to rail lines. In this study, rail station density, bus frequency, and route density were determined and led to the minimization of total operator and user costs. A major assumption made, was all passengers to be walked to the nearest bus route. Moreover, Hurole and Wirasinghe (1980) extended the study of Wirasinghe (1980) to include several feeder modes such as auto, bus, and bicycle. A simple algebraic formulation that shows the relationship between the station spacing and the various cost and demand parameters were developed. However, only the rail station spacing was optimized in this research.

Furthermore, Kuah and Perl (1988) presented an analytic model for designing an optimal feeder bus network for accessing to an existing rail line. In order to decrease total cost, they applied mathematical method and avoided the simultaneously combining of stop spacing with route spacing and operating headway variables. More rigorous problem statements and solid theoretical ground can usually be found in mathematical optimization approaches compared to other transit network design methods. However, such methods have two main disadvantages being either non-convex problem or, in most cases, a problem with unknown convexity, and the other disadvantage stated by Michael and David (1979) is that the resultant mathematical optimization systems derived from realistic combinatorial transit route network problems are usually at least NP-hard (Zhao & Zeng, 2006).

2.6.2 Heuristic Methods

In order to solve the problems at which classical methods are too slow, or they fail to obtain any exact solution, heuristic methods are designed to accelerate the solving time of the problem, or to find an approximate solution.

From the literature review, it is evident that heuristic approaches have been very popular for solving problems. A large number of research papers have been published in utilizing heuristic methods due to their flexibility features (Reeves, 1993). These methods typically are examined when a specific problem is created by a mathematical formula. Usually, these methods contain linear and integer programming. Because of the flexibility of heuristic methods, a lot of documents published in recent years utilized them (Reeves, 1993).

Heuristic methods have been used by Kuah and Perl (1989) to quickly search for approximately good solutions by different sets of rules to construct routes in the step by step and iterative procedures. They solved their model by a heuristic method called savings heuristic. The computational analysis showed that the proposed heuristic provided reasonable feeder bus network solutions that are superior to manually design networks. The model presented by Kuah and Perl (1989) has been solved by some other researchers. Such methods were also used to local search improving techniques, such as Displacement heuristic, and exchange heuristic suggested by Perl (1987).

Shrivastav and Dhingra (2001) developed a heuristic algorithm to integrate the suburban stations and bus services, along with optimization of coordinated schedules of feeder bus services using existing schedules of suburban trains. The heuristic algorithm was the first part of a model, which was developed to integrate the suburban stations and bus services. The second part was determined for optimizing coordinated the schedules of the feeder bus services by using existing schedules of suburban trains. The proposed HFRGA was heavily guided by the demand matrix similar to Baaj and Mahmassani (1995). This algorithm (HFRGA) was able to develop feeder routes to satisfy demands in various nodes. They used Dijkstra's algorithm for generating initial routes. A good selection for generating initial routes is able to provide an efficient alternative at a reasonable computation time by applying local search algorithms. Parallel implementations, including performance analysis of two prominent graph algorithms (i.e., Floyd-Warshall and Dijkstra) were utilized by Pradhan and Mahinthakumar (2012) in a large-scale transportation network.

Moreover, Chowdhury et al. (2002) proposed a model seeking for better coordination of the intermodal transit system. They applied a numerical search algorithm (Powell's algorithm) to solve their problem.

Chien (2005) suggested specific feeder bus service to provide a shuttle service between a recreation center and a major public transportation facility. They proposed an integrated methodology (analytical and numerical techniques) for development and optimization of the decision variables.

It should be noted that, in some of the studies, heuristic methods have been used for initial building of feeder bus network design problem, such as Kuah and Perl (1989); Kuan et al. (2004), (2006); Martins and Pato (1998). More studies are presented in Table 2.2.

2.6.3 Metaheuristic Methods

Transit route network design (TRND) problems usually are complicated problems, and they have three main challenges, in terms of managing competing objectives, significant scalability, as well as significant combinatorial explosions when the network size grows (Baaj & Mahmassani, 1995). Due to this complexity in the early efforts of optimization through heuristics or analytical solutions, the simplified versions of TRND problems were solvable. More recently, the development of computing power offered to use metaheuristic approaches, such as GA, simulated annealing (SA), tabu search (TS), ant colony optimization (ACO), etc.

The TRND metaheuristics tend to follow one of the two different templates. The first approach begins by generating a large set of possible routes and then iteratively selecting different subsets of the routes to create route networks. The second approach begins by generating a potential route layout and then one or more routes in the solution are changed in an attempt to find better solutions (Blum & Mathew, 2011). Although metaheuristic methods are more time-consuming than the early heuristics, they are capable of consistently producing high-quality solutions. In addition, metaheuristics compared with classical heuristics perform a much more thorough search of the solution space, allowing inferior and sometimes infeasible moves, as well as recombination of solutions to create new ones (Cordeau, Gendreau, Laporte, Potvin, & Semet, 2002). Though, in recent years, there has been an increasing amount of research on applying the metaheuristic method to resolve the problems. These methods derive their concept from mathematics and physics, combined with biological evolution and artificial intelligence (Chien et al., 2001; Martins & Pato, 1998).

TS is one of the well-known metaheuristic methods, which is adopted by many researchers. Martins and Pato (1998) extended the work of Kuah and Perl (1989) to improve previously proposed solutions. The initial solution was improved by using some heuristic procedures, and generated a set of problems with real-life situations. As a result, the simplest short-term version of TS provided better solutions, and it can be one of the important heuristic methods in the future. In addition Kuan et al. (2004, 2006) applied GAs, ACO, SA, and TS to resolve FNDP for a similar work conducted by Kuah and Perl (1989) which improved previously proposed solutions. They generated several random tests to evaluate and compare the performance of their methods in terms of efficiency and accuracy of solutions.

Chien and Yang (2000) proposed an exhaustive search algorithm (ES) to optimize feeder bus route location and its operating headway in a given network. Moreover, Chien et al. (2001) extended Chien and Yang (2000) study by presenting a GA to solve the problem. The GA started with an initial population and followed by improving the route. The results of this study indicate that the optimum solutions discovered by ES and GA are identical. However, the computational time for a GA was significantly less than ES, especially for large or complicated networks.

Moreover, in another study carried out by Shrivastava and O'Mahony (2006), optimum feeder routes and schedules of a suburban area were determined using the GAs. The developed routes and schedules were optimized; however, it failed to completely meet the demand. The reason was that some of the nodes did not have a good connection with other nodes in the study area.

Nikolic and Teodorovic (2013) developed another metaheuristic algorithm to solve the transit network design problem. They applied Bee Colony Optimization (BCO) algorithm and tried to maximize the number of satisfied passengers, to minimize the total number of transfers, and minimize the total travel time of all served passengers.

Mohaymany and Gholami (2010) suggested an approach for solving multimodal feeder network design problems (MFNDP) its objective was to minimize the total operator, user, and social costs. They used the ACO for constructing routes and modifying the optimization procedure in order to identify the best mode and route in the service area (see Table 2.5).

2.6.1 Hybrid Methods

Hybrid methods are categorized as another type of solution methods, which combine the abilities of different computational techniques to solve complex problems. The research by Shrivastava and O'Mahony (2009b) is one of the studies adopting the hybrid method by using heuristic methods to generate the potential routing and GA optimizing schedules of suburban trains. They offered a technique with two sub-models: routing for feeder buses and schedule coordination.

Shrivastava and O'Mahony (2007) applied GA for route scheduling and network designing, by repairing algorithm for satisfying design problem by offering a specialized heuristic algorithm. Shrivastava and O'Mahony (2009a) in their latest study have developed the Shrivastava–O'Mahony hybrid feeder route generation algorithm (SOHFRGA). The idea was to develop public bus routes and coordinate schedules in a suburban area. In the proposed research, the GAs and the heuristic approach were combined to find optimized feeder routes, which have higher efficiency in comparison with those developed by other researchers (see Table 2.5).

2.7 Gap of the Previous Studies and Necessity for Further Research

Many researchers have made an attempt to design a more efficient feeder network and to provide feeder services connecting major transportation systems with their welfare facilities. However, still there are some limitations and gaps which highlighted the need for further researches.

Since the feeder bus network problem is an active research field, new policies by operators create new requirements and new challenges for planners and research groups. Thus, despite the effort made for FNDSP, the field still has a strong potential for future research. One of the most important fields to study could be cooperation and coordination between different levels of public transit such as train and feeders. Well-defined information and advanced schedule in an intermodal system will lead to a high level of passenger satisfaction. Table 2.6 illustrates instances of these models, and their gaps which this study wishes to fill by its improved model.

Moreover, the literature survey also reveals that there has not been a substantial work for new metaheuristics to the network design problem, especially in the area of the multiple-mode transit system in which the feeder bus system provides service for an existing rail network. Most of the existing studies have been focused on the design of a single-mode network. As such, our research is focused on the development of new approaches for the feeder bus network design problem.

The current study by providing an improved model and proposed solution methods tries to fill the gaps of the preceding studies.

Table 2.6: Comparison of the improved model with some of the previous models

References	***Total cost***																		
	User cost								***Operation cost***										***Social cost***
	Feeder				**Train**				**Feeder**					**Train**					**Feeder**
	Access cost	Waiting cost	User in-vehicle cost		Access cost	Waiting cost	User in-vehicle cost		Fixed cost	Operating in-vehicle cost		Maintenance cost	Personnel cost	Fixed cost	Operating in-vehicle cost		Maintenance cost	Personnel cost	Social cost
			Running cost	Dwell cost			Running cost	Dwell cost		Running cost	Dwell cost				Running cost	Dwell cost			
Kuah (1989)	_	√	√	_	_	√	√	_	_	√	_	_	_	_	_	_	_	_	_
Martins (1998)	_	√	√	_	_	√	√	_	_	√	_	_	_	_	_	_	_	_	_
Kuan (2004)	_	√	√	_	_	√	√	_	_	√	_	_	_	_	_	_	_	_	_
Kuan (2004)	_	√	√	_	_	√	√	_	_	√	_	_	_	_	_	_	_	_	_
Kuan (2006)	_	√	√	_	_	√	√	_	_	√	_	_	_	_	_	_	_	_	_
Shrivastava (2006)	_	_	√	_	√	_	_	_	_	√	_	_	_	_	_	_	_	_	_
Shrivastava (2007)	_	_	√	_	√	_	_	_	_	√	_	_	_	_	_	_	_	_	_
Shrivastava (2009)	_	_	√	_	√	_	_	_	_	√	_	_	_	_	_	_	_	_	_
Mohaymany (2010)	_	√	√	_	_	_	_	_	√	√	_	√	√	_	_	_	_	_	√
Gholami (2011)	_	√	√	_	_	_	_	_	√	√	_	√	√	_	_	_	_	_	√
Cipriani (2012)	√	√	√	_	√	_	_	_	_	_	_	√	_	_	_	_	_	_	_
Ciaffi (2012)	√	√	√	_	√	_	_	_	_	_	_	√	_	_	_	_	_	_	_
Proposed model	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√	√

2.8 Summary of the Chapter

This chapter reviewed the literature in the area of TNP, types of bus route network design problems and the brief description of the FNP. Then, various aspects of reviewing studies have been grouped by problem description and problem characteristics. This is followed by summarizing of the previous works, critiquing of existing work, and their classifications based on the approaches and solution methods and the necessity for the further research. Several researchers have made attempts to design a more efficient feeder network. Since the feeder bus network problem is an active research field, new policies by operators create new requirements and new challenges for planners and research groups. One of the most important fields to study could be cooperation and coordination between different levels of public transit such as train and feeders. In addition, the literature survey also reveals that most of the existing studies have been focused on the design of a single-mode network. As such, our research is focused on the improvement of approaches for the feeder bus network design problem. The next chapter considers the modeling of the study and its methodology procedure

CHAPTER 3: MODEL IMPROVEMENT AND METHODOLOGY

3.1 Introduction

The problem of intermodal transit network design varies in each metropolitan region. A practical solution depends on the regional geometric, socio-political, land-use, and transportation system characteristics as well as the spatial and temporal distribution of the demand for public transit routes (Chien, 1995). The main target of this chapter is to present a new mathematical model and to design a transit system to increase the efficiency of feeder network designs and coordinated schedules in order to minimize costs. An improved integrated intermodal system may lead to a reduction in total cost and an increase in profit and consequently, lead to achieving an optimum transit network design. Furthermore, such a system can provide higher-quality services for passengers. This study proposes an improved mathematical model based on the gaps of the previous studies to increase the efficiency of the intermodal transit system with the aim of achieving the optimal balance between the operator, user, and social costs.

Therefore, this chapter introduces the design of the research and necessary assumptions which are related to the service region, supplier and user characteristics. Then details of the new mathematical model, including user, operation, and social costs are presented. This is followed by optimization containing objective function and constraints of the problem, applied metaheuristic methods for solving the transit network problems (include rail system and feeder bus network), and computerized optimization method. Finally, the research site, including benchmark and real case studies were explained, followed by its data collection and analysis procedure of the study. The sensitivity analysis over different parameter values was performed to show the impacts of these costs in the proposed public transit system. The research procedure is as illustrated in Figure 3.1.

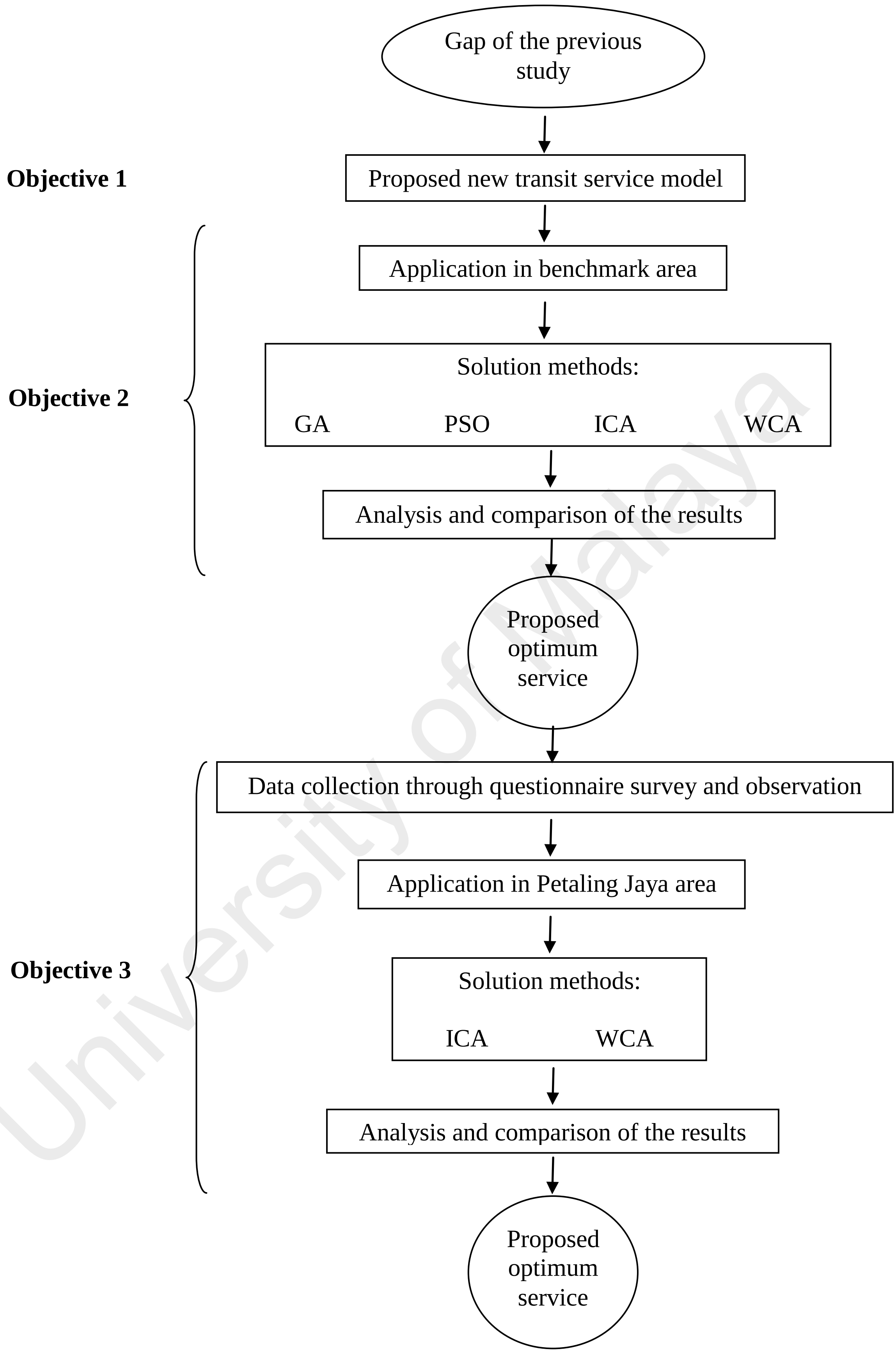


Figure 3.1: The research procedure of the study

3.2 Design of the Research

For a scientific research, one has to prepare a research design. It should indicate the various approaches to be used in solving the research problem, sources and information related to the problem, time frame, and the cost budget. Essentially, the research design creates the foundation of the entire research work. The design will help perform the chosen task easily and in a systematic way. The design of the study is mixed (qualitative and quantitative) method. A mixed method research design is a procedure for collecting, analyzing, and mixing both quantitative and qualitative research and methods in a single study (Holland, 1975). To analyse the data, a descriptive approach has been conducted.

3.3 Assumptions

An intermodal transit network consisting of the rail line and feeder bus routes connecting to the transfer stations is assumed to serve the examined area. The optimal transit system will be determined based on assumed route structure (i.e., one rail line and feeder bus routes are linked with straight line between nodes) and the peak hour demand situations in the whole service area. To formulate the mathematical model for an intermodal transit system and its application in the case study, the following assumptions are made.

a) The transit network was designed with feeder bus and fixed rail line.

b) Transit demand is assumed to be independent of the quality of transit service (i.e., fixed demand). The demand pattern of feeder bus routes is many-to-one.

c) The location of nodes (bus stops and rail stations) is given. Some of the model parameters (e.g., vehicle sizes, operating speed, cost etc.) are specified.

d) All feeder routes could be used as two-way direction for the transit service.

3.4 Model Formulation: Objective One

The process of producing transit services in the proposed intermodal transit network is defined by a cost function. Such cost function defines a range of possible combinations of capacity, travel demand, and service quality that are achievable at various cost levels. It is the task of the optimization procedures to find within the range of technological possibilities the combinations that best satisfy the given objectives and constraints. The objective function is the total cost function, which includes the operating cost, and the user cost and social cost. In developing cost functions, the problems include classifying costs into useful, comprehensive, and non-overlapping components, and formulating all cost components as functions of the relevant variables (e.g., design and control variables). For the proposed transit network optimization, we need a function that can capture the sensitivity of costs to design and operations (Chien, 1995).

To Propose Mathematical Formulation of the Model based on the problem statement of the study, the total cost function is expressed in the following Equation. The total cost function is the sum of user, operator, and social costs that could be formulated as given:

$$C_T = C_u + C_o + C_s \tag{3.1}$$

where C_T, C_u, C_o, and C_s represent total cost, user cost, operation cost, and social costs, respectively. For nomenclature purposes, all variables and used parameters of the modified objective function are defined in Table 3.2.

The well-structured cost classification of the proposed model is shown in Figure 3.2. As well as, Table 3.1 illustrates these costs more comprehensively.

The derivation of all cost terms in the proposed transit service model is presented in Table 3.3. Henceforth, each term of the improved model will be described in detail in the following subsections.

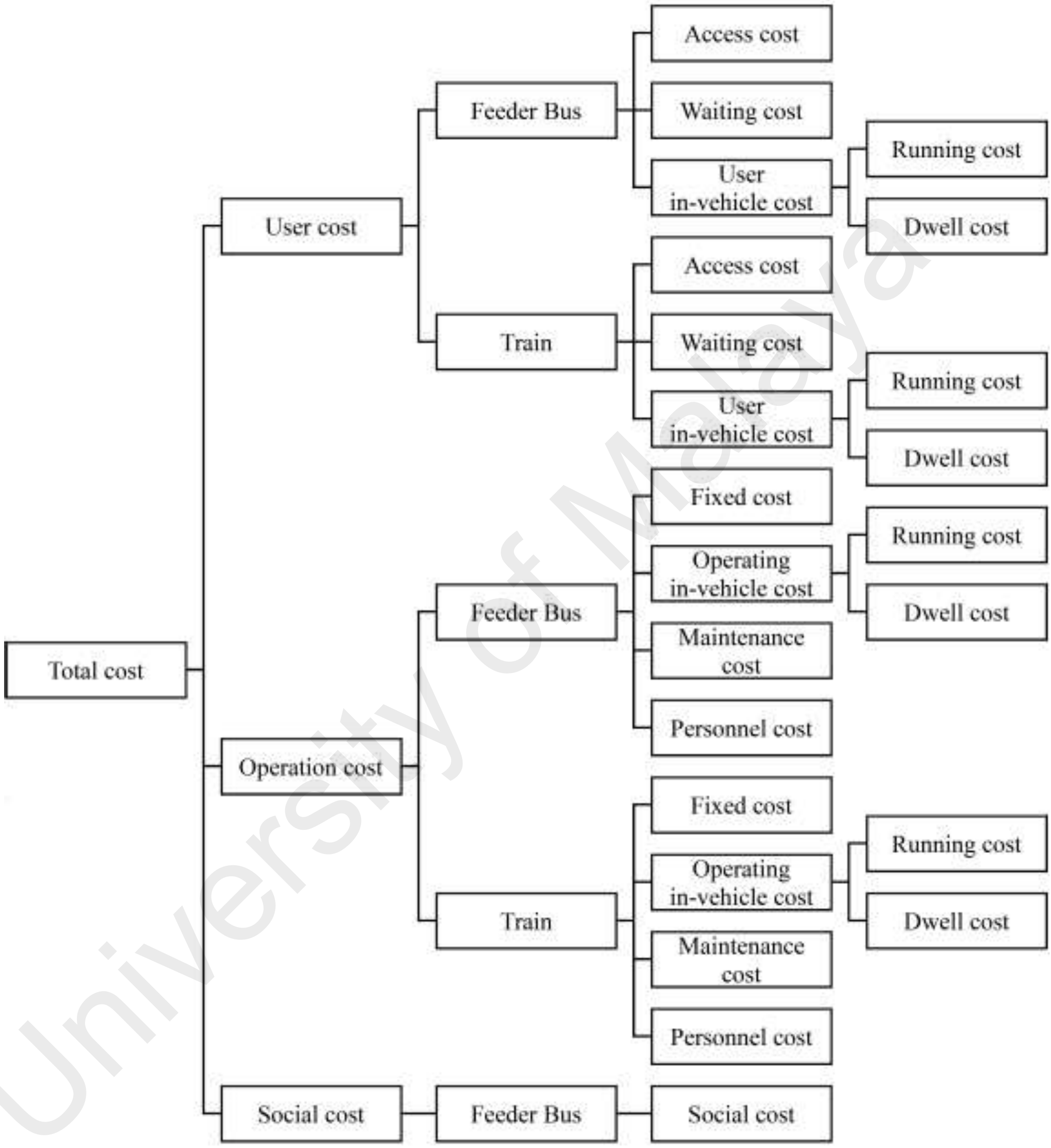


Figure 3.2: The cost structure of the proposed model

Table 3.1: Illustration of total cost with all terms in proposed model

<table>
<tr><th colspan="19">Total cost (C_T)</th></tr>
<tr><th colspan="8">User cost (C_u) [a]</th><th colspan="10">Operation cost (Co) [b]</th><th>Social cost (C_s) [c]</th></tr>
<tr><th colspan="4">Feeder</th><th colspan="4">Train</th><th colspan="5">Feeder</th><th colspan="5">Train</th><th>Feeder</th></tr>
<tr><th rowspan="2">Access cost</th><th rowspan="2">Waiting cost</th><th colspan="2">User in-vehicle cost</th><th rowspan="2">Access cost</th><th rowspan="2">Waiting cost</th><th colspan="2">User in-vehicle cost</th><th rowspan="2">Fixed cost</th><th colspan="2">Operating in-vehicle cost</th><th rowspan="2">Maintenance cost</th><th rowspan="2">Personnel cost</th><th rowspan="2">Fixed cost</th><th colspan="2">Operating in-vehicle cost</th><th rowspan="2">Maintenance cost</th><th rowspan="2">Personnel cost</th><th rowspan="2">Social cost</th></tr>
<tr><th>Running cost</th><th>Dwell cost</th><th>Running cost</th><th>Dwell cost</th><th>Running cost</th><th>Dwell cost</th><th>Running cost</th><th>Dwell cost</th></tr>
<tr><td>C_{aF}</td><td>C_{wF}</td><td colspan="2">C_{uiF}</td><td>C_{aT}</td><td>C_{wT}</td><td colspan="2">C_{uiT}</td><td>C_{fF}</td><td colspan="2">C_{oiF}</td><td>C_{mF}</td><td>C_{pF}</td><td>C_{fT}</td><td colspan="2">C_{oiT}</td><td>C_{mT}</td><td>C_{pT}</td><td>C_{sF}</td></tr>
</table>

[a] $C_u = \underbrace{(C_{aF} + C_{aT})}_{C_a} + \underbrace{(C_{wF} + C_{wT})}_{C_w} + \underbrace{(C_{uiF} + C_{uiT})}_{C_{ui}}$

[b] $C_o = \underbrace{(C_{fF} + C_{fT})}_{C_f} + \underbrace{(C_{oiF} + C_{oiT})}_{C_{oi}} + \underbrace{(C_{mF} + C_{mT})}_{C_m} + \underbrace{(C_{pF} + C_{pT})}_{C_p}$

[c] $C_s = C_{sF}$

Table 3.2: Description of used parameters in the proposed improved model

Parameter	Description	Unit
C_T	Total system cost	($/hr)
C_{TK}	Total cost function for route k	($/hr)
C_u	User cost	($/hr)
C_o	Operation cost	($/hr)
C_a	Access cost	($/hr)
C_w	Waiting cost	($/hr)
C_p	Personnel cost	($/hr)
C_{ui}	User in-vehicle cost	($/hr)
C_{oi}	Operating in-Vehicle cost	($/hr)
C_{ruiF}	Feeder running user cost	($/hr)
C_{duiF}	Feeder dwell user cost	($/hr)
C_{ruiT}	Train running user cost	($/hr)
C_{duiT}	Train dwell user cost	($/hr)
C_{roiF}	Feeder running operating cost	($/hr)
C_{doiF}	Feeder dwell operating cost	($/hr)
C_{roiT}	Train running operating cost	($/hr)
C_{doiT}	Train dwell operating cost	($/hr)
C_f	Fixed costs	($/hr)
C_m	Maintenance cost	($/hr)
C_s	Social cost	($/hr)
C_{oF}	Feeder bus operation cost	($/hr)
C_{oT}	Train operation cost	($/hr)
C_{aF}	Feeder access cost	($/hr)
C_{aT}	Train access cost	($/hr)
C_{wF}	Feeder waiting cost	($/hr)
C_{wT}	Train waiting cost	($/hr)
C_{uiF}	Feeder user in-vehicle cost	($/hr)
C_{uiT}	Train user in-vehicle cost	($/hr)
C_{oiF}	Feeder operating in-Vehicle cost	($/hr)
C_{oiT}	Train operating in-Vehicle cost	($/hr)
C_{mF}	Feeder maintenance cost	($/hr)
C_{mT}	Train maintenance cost	($/hr)
C_{pF}	Feeder personnel cost	($/hr)
C_{pT}	Train personnel cost	($/hr)
C_{fF}	Feeder fixed cost	($/hr)
C_{fT}	Train fixed cost	($/hr)
A_F	Average frequency of feeder bus system	(veh-hr)
T_{PK}	Total passenger-Km	(passenger-Km)
T_{VK}	Total vehicle-Km	(vehicle-Km)
μ_a	Passenger access cost	($/passenger-hr)
μ_w	Passenger waiting cost for arrival of transit mode	($/passenger-hr)
μ_I	Passenger riding cost on transit mode	($/passenger-hr)
λ_f	Fixed cost of feeder bus	($/veh-hr)
λ_l	Vehicle operating cost of feeder bus	($/veh-km)
λ_I	Vehicle operating cost of feeder bus	($/veh-hr)
λ_{IT}	Vehicle operating cost of train	($/veh-hr)
λ_m	Maintenance cost of feeder bus	($/veh-km)
λ_p	Personnel cost of feeder bus	($/veh -hr)
λ_s	Social cost of feeder bus	($/veh-km)
V	Average operating speed of feeder bus	(km/hr)
S_{kj}	Slack time route k at station j	(hr)

Table 3.2 continued: Description of used parameters in the proposed improved model

Parameter	Description	Unit
t_{aF}	Average access time to reach the feeder station	(hr)
t_{aTj}	Average access time to the rail station j	(hr)
t_{dT}	Dwell time for boarding and alighting the train	(hr/passenger)
t_{Tj}	Linked riding time between station j and destination of the train	(hr)
t_{df}	Dwell time for boarding and alighting to the feeder bus	(hr/passenger)
t_{ih}	Linked in-vehicle time between nodes i and h of feeder bus	(hr)
$F_{opt,k}$	Optimum frequency of feeder bus on route	(veh/h)
$F_{req,k}$	Required frequency of feeder bus on route k	(veh/h)
F_k	Frequency of feeder bus on route k	(veh/hr)
F_T	Frequency of trains	(veh/hr)
f_{min}	The minimum frequency	(veh/hr)
f_{max}	The maximum frequency	(veh/hr)
N	Total fleet size of feeder bus	(veh)
LF	Load factor of feeder bus	(passenger/seat)
C	Capacity of feeder bus	(passenger/veh)
l_{min}	The minimum length of one route	(km)
l_{max}	The maximum length of one route	(km)
V_T	Average operating speed of train	(km/hr)
T_T	Train link travel time from 59 to 56	(hr)
n_k	Number of stops in route k	-
q_i	Demand of node i	(passenger/hr)
Q_k	Demand of route k	(passenger/hr)
l_{ih}	Distance from node i to h	(km)
L_{ijk}	Link travel distance from each stop i to station j in route k	(km)
L_k	Length of route k for the feeder bus	(km)
X_{ihk}	Binary variable; value of 1 if stop i precedes stop h on bus route k	-
Y_{ij}	Binary variable; value of 1 if stop i is assigned to station j	-
I	Number of stops	-
J	Number of stations	-
K	Number of routes	-
H	All nodes containing stops and stations	-

3.4.1 User Cost (C_u)

The user cost means the expense imposed on passengers using the transit system (contains feeder and train services). This cost is comprised of access, waiting, and in-vehicle traveling costs denoted by C_a , C_w , and C_{ui} , respectively (Chien, 2005).

$$C_u=C_a+C_w+C_{ui} \tag{3.2}$$

In light of the user cost which is the summation of feeder bus and train cost, Equation (3.3) can be re-written as given:

$$C_u= (C_{aF}+C_{aT}) + (C_{wF}+C_{wT}) + (C_{uiF}+C_{uiT}) \tag{3.3}$$

Generally, all elements of the user cost can be formulated as the product of an hourly demand, average time spent in each travel time category (i.e. access time, wait time, and in-vehicle time), and the users' value of time, which is explained in the following subsections.

Table 3.3: Derivation of all cost terms in proposed transit service model

Cost terms				Literature	Improved	Proposed
Total cost	*User cost*	Feeder Bus	C_{aF}	√		
			C_{wF}	√		
			C_{ruiF}	√		
			C_{duiF}			√
		Train	C_{aT}	√		
			C_{wT}	√		
			C_{ruiT}	√		
			C_{duiT}			√
	Operation cost	Feeder Bus	C_{fF}	√		
			C_{roiF}	√		
			C_{doiF}			√
			C_{mF}	√		
			C_{pF}	√	√	
		Train	C_{fT}			√
			C_{roiT}			√
			C_{doiT}			√
			C_{mT}			√
			C_{pT}			√
	Social cost	Feeder Bus	C_{sF}	√		

3.4.1.1 Access costs (C_a)

Feeder and train passengers who have access to stops and stations mainly incur the access cost. The access cost is generally experienced by local and train passengers accessing the transfer station. This cost is discussed by previous researchers such as Ciaffi et al., (2012); Cipriani et al., (2012); Chien, (2005); etc.

The access cost for feeder bus passengers is the product of local demand, q_i, whose average access time t_{aF} and value of time μ_a, where t_{aF} can be estimated from average distance divided by average access speed. The average access time for train passengers (t_{aT}) can be formulated similarly. t_{aT} is dependent on the distance between the platforms of bus and train services and access speed. Assume that access speed and value of time for feeder bus and train passengers are identical. Thus, the access cost for feeder route k can be formulated as (Chien, 2005):

$$C_a = \mu_a \left(Q_k \times t_{aF} + Q_k \times t_{aT} \right) \tag{3.4}$$

The users' value of time (μ_a) is an important parameter in determining the user cost, and is usually dependent on the economic situation (e.g. annual income).

3.4.1.2 Waiting cost (C_w)

The waiting cost includes passengers waiting for the buses and trains, which is the product of average wait time, demand, and the value of users' wait time (μ_w). Numerous studies have been conducted to describe waiting cost, such as the studies by Kuah and Perl (1989); Kuan et al. (2004), (2006); Ciaffi et al. (2012); Shrivastava and O'Mahony (2006), etc.

Average wait time can be estimated by the fraction of the headway, so in this model the average wait time for feeder bus at the stops and for trains at the stations are assumed to be one half of headway. Hence, the user waiting cost can be represented using Equation (3.5) (Kuan et al. 2004, 2006).

$$C_w = \mu_w \left[\left(\frac{1}{2F_k} + \frac{1}{2F_T} \right) \times Q_k \right] \tag{3.5}$$

where F_k and F_T are respectively the frequency of feeder buses and trains.

3.4.1.3 User in-vehicle cost (C_{ui})

Similarly, the product of demand, in-vehicle time, and value of time can define the user in-vehicle cost (C_{ui}). The C_{ui} is formulated based on the average journey time and is calculated in two main parts: the run time and the dwell time.

Running costs for all passengers ($C_{rui,}$), which is equal to the link travel distance from stop i to station j in route k (L_{ijk}) divided by the average bus real speed (V_k). Many studies considered this concept for determining $C_{rui,}$ such as the researches by Kuah and Perl (1989); Kuan et al., (2004), (2006); Shrivastava and O'Mahony (2006);

Mohaymany and Gholami (2010); Gholami and Mohaymany (2011); Shrivastava and O'mahony (2007), etc. In the current study, $C_{rui,}$ was determined using the same concept.

The dwell time is the boarding and alighting time at the feeder bus stops (t_{dF}) and rail stations (t_{dT}). The observation of feeder bus stops and rail stations realised dwell time is important part of in-vehicle travel time. This time will increase the user, operation and social costs in both modes of feeder bus and train, as well as consequently has significant effect on total cost of transit network. Dwell time will increase user costs by increasing the in-vehicle time for on boarding passenger. In addition, this time costs will increase operation costs by increasing fuel consumption, maintenance, and personnel costs. Accordingly, with the increase in pollution, noise, and greenhouse gases, etc., the social costs will also be raised. The literature survey also revealed that there had not been a substantial work for calculating dwell costs in transit services.

Since spending time for boarding and alighting have got an important role in user in-vehicle time, it was tried in this study to present a new concept for determining such costs. Moreover, because of the variety in spending time which is dependent on the dwell time at each of the bus stops, the geometric series equation has been adopted to develop a more accurate model for distributing dwell cost of the bus stops along the routes. The average cost of dwell time (C_{dui}) is determined by demand multiplied by passenger boarding and alighting rate. The derivation of the dwell cost for feeder buses and trains are discussed in Appendix A.

Therefore, the in-vehicle cost, including in-bus and in-train cost, for each route *k*, is given as:

$$C_{ui} = C_{rui} + C_{dui} \tag{3.6}$$

where C_{rui} is running cost for all passengers (Kuah and Perl 1989; Kuan et al., 2004, 2006; Shrivastava & O'Mahony, 2006; Mohaymany & Gholami, 2010; Gholami & Mohaymany, 2011; and Shrivastava & O'mahony, 2007)

$$C_{rui} = \mu_I \left[\frac{1}{V_k} \sum_{i=1}^{I} \left[q_i \times \left(\sum_{j=1}^{J} L_{ijk} \right) \right] + (Q_k \times t_{Tj}) \right] \tag{3.7}$$

and C_{dui} is the average cost of dwell time as described in Appendix A.

$$C_{dui} = \mu_I \left[\left(\frac{1}{2} (n_k + 1) \times Q_k \times t_{dF} \right) + \left(Q_k \times t_{dT} \right) \right] \tag{3.8}$$

The first and second terms in Equations (3.7) and (3.8) respectively denoted the feeder bus and train user cost. In Equation (3.6), C_{rui} represents the running cost for all passengers which is equal to the link travel distance from stop *i* to station *j* in rout *k* (L_{ijk}) divided by the average bus real speed (V_k). Moreover, t_{Tj} denotes riding time between station *j* and the destination of the train regardless of boarding and alighting times, and n_k stands for the number of stops in route *k*. The sensitivity analysis performed in section 4.2.6 showed the relations between proposed cost terms in transit system, as well as the importance of this related cost significantly.

3.4.2 Operation Cost (C_o)

The operating cost (C_o) is the summation of railway and feeder bus operation costs. It can be described by the unit time or distance cost (such as hourly or km) in connection with the transit service provided. Thus, C_o can be formulated as the sum of C_{oi}, C_m, C_p, and C_f .These costs include the cost of trains and buses; therefore, it can be formulated as:

$$C_o=(C_{oiF}+ C_{oiT})+(C_{mF}+ C_{mT})+(C_{pF}+ C_{pT})+(C_{fF}+C_{fT}) \tag{3.9}$$

The cost terms in Equation 3.9 are adopted and improved through either literature or improvement of proposed terms in the literature (Mohaymany & Gholami, 2010; Gholami & Mohaymany, 2011; Kuah and Perl, 1989; Kuan et al., 2004, 2006; Martins and Pato, 1998; Shrivastav and Dhingra, 2001; Shrivastava and O'Mahony, 2007). However, some of which are proposed in the current study. More details can be found in Table 3.3 and Appendix A.

3.4.2.1 Feeder bus maintenance cost (C_{mF})

Feeder bus maintenance cost (C_{mF}) consists of maintenance, repair, and tire costs. This cost depends on the fleet size and round trip distance formulated as follows (Gholami & Mohaymany, 2011):

$$C_{mF} = \lambda_m \left(2F_k \times L_\kappa \right) \tag{3.10}$$

The Maintenance cost were implemented by Mohaymany and Gholami, (2010); Gholami and Mohaymany, (2011); Ciaffi et al., (2012); Cipriani et al., (2012), etc.

3.4.2.2 Feeder bus personnel cost (C_{pF})

Feeder bus personnel cost (C_{pF}) including the drivers and administrative costs is dependent on the fleet size, hourly pay, and insurance rate.

The research on this type of cost was conducted by Mohaymany and Gholami, (2010); Gholami and Mohaymany, (2011), in their study. They assumed that this cost depends on fleet size; so, they calculated the cost just based on fleet size. Since spending time for boarding and alighting (dwell time) as well as bus slack time also have got important role in spending time for personnel, in this study an effot has been made to present the improved concept for determination of these costs. Hence, in order to increase the accuracy of the cost function, adding slack time (S_{kj}) into the schedule of bus route k at station j and average rest time were considered for each bus in stations. Moreover, the dwell times which were added into calculation of personnel cost with respect of interrelationship between this cost terms based on mathematical formulation. The derivation of the C_{pF} is presented in Appendix A.

Therefore, C_{pF} for feeder bus route k can be formulated as given:

$$C_{pF} = \lambda_p \left[\left(\frac{2F_k}{V_k} \times L_K \right) + \left(Q_k \times t_{dF} \right) + \left(F_k \times S_{kj} \right) \right] \tag{3.11}$$

First and second terms in Equation (3.15) rely on the feeder running time and dwell time in route k, respectively. Accordingly, the third terms denote the personnel cost as while they are in the rest time or queue.

The sensitivity analysis performed in section 4.2.6 in order to show the relations between improved cost terms in transit system and to show the importance of this related cost significantly.

3.4.2.3 Feeder bus fixed costs (C_{fF})

This cost contains initial fleet costs such as vehicle ownership costs, license, Insurance, and so forth. It is formulated according to the fleet size and hourly fixed cost for the vehicle given for route k:

$$C_{fF} = \lambda_f \left[\frac{2F_k}{V_k} . L_k \right] \tag{3.12}$$

Mohaymany and Gholami, (2010); Gholami and Mohaymany, (2011) considered this concept to determine fixed cost for feeder bus routes. Where λ_f define unit fixed cost of feeder bus.

3.4.2.4 Feeder bus operating in-vehicle cost (C_{oiF})

Feeder bus operating in-vehicle cost (C_{oiF}) is dependent on the travel time and round trip distance. C_{oi} (bus or train) is formulated based on the running cost (C_{roi}) and dwell cost (C_{doi}). The running cost for the bus is formulated according to the round trip distance against the rail, which is the round trip time. It is assumed that the stop delay time incurred at bus stops and, intersections should be taken into consideration. This cost is discussed in many studies such as the studies conducted by Kuah and Perl, (1989); Kuan et al., (2004), (2006); Martins and Pato (1998); Mohaymany and Gholami, (2010); Shrivastav and Dhingra, (2001); Shrivastava and O'Mahony, (2007), etc.

As it was explained in section 3.4.1.3, the average cost of dwell time (C_{doi}) was defined by demand multiplied by passenger boarding and alighting rate. Furthermore, these costs were also determined similarly in section 3.4.1.3. Thus, the C_{oiF} for feeder bus route *k* can be formulated as given in the flowing Equations:

$$C_{oiF} = C_{roiF} + C_{doiF} \tag{3.13}$$

$$C_{roiF} = \lambda_I \left(2F_k \times L_\kappa \right) \tag{3.14}$$

$$C_{doiF} = \lambda_I \left(Q_k \times t_{dF} \right) \tag{3.15}$$

The derivation of C_{doiF} is discussed in Appendix A.

3.4.2.5 Train operating cost (C_{oT})

The operation cost is the sum of rail and feeder bus system operator costs. Both operating costs can be formulated on the basis of average round trip time. Since the length of a rail transit route significantly affects supplier costs as well as user costs, its value should be carefully determined (Chowdhury et al., 2002; Chien & Schonfeld, 1998).

In order to conduct a comprehensive analysis and evaluation of a multi-modal transit system, both operating costs (rail and feeder bus) should be considered. The literature survey reveals that there has not been a substantial work for network approach for defining the train operating cost, especially in the area of the multiple-mode transit system in which the feeder bus system provides service for an existing rail network.

Most of the existing studies have been focused on the analytical approaches such as Chowdhury et al., (2002); Chien and Schonfeld, (1998), etc. As such, our research is focused on the development of a network approaches for the transit network design problem. By providing train operating cost in the current study, it was tried in this model to fill the gaps of the preceding studies.

Operating cost for rail system can be obtained through multiplying defined as the fleet size by the value of train operating cost (λ_{IT}), where the fleet size can be obtained from the trip time multiplied by the train frequency (F_T). The rail trip time consists of running and dwell time. The train running time is trip distance divided by average running speed (V_T). In addition the rail dwell time is the product of the number of inflow or outflow passengers on the route and the average service time for passenger boarding and alighting from a vehicle.

Thus, the train operation cost can be formulated as given in flowing Equation:

$$C_{oT} = \lambda_{IT} \left[\left(F_T \times T_T \right) + \left(Q_k \times t_{dT} \right) \right] \tag{3.16}$$

which the first term in Equations (3.16) corresponds to the train running time, and the second term denotes the train dwell time.

Considering that a fixed rail line was assumed, and operation cost depends on route station distance and demand, in order to simplify the model, in this study one operating value for all operating costs was considered. λ_{IT} represents all elements of operating cost, including fixed, maintenance, personnel, and in-vehicle costs ($/veh-hr). Indeed, railway operating cost is summarized in this subsection. The derivation of this cost is presented in Appendix A.

3.4.3 Social Costs (C_s)

Social cost consists of many parameters that non-users pay indirectly. For instance, accident costs, pollution costs, infrastructure costs, noise, and greenhouse gases, etc.

This cost is assumed to be dependent on in-vehicle operating costs for feeder services and formulated as follows(Mohaymany and Gholami 2010):

$$C_S = \lambda_S \left(2F_k \times L_\kappa\right) \tag{3.17}$$

Each cost term consists of several parameters and items. Which, consequently, have differential influence in total cost, and determining some of the parameters and items needs the cooperation of other sciences. Therefore, the interrelationship of some of the cost terms with other related costs is considered. It is assumed that there is an interrelationship between some of the costs such as ‘social cost’ and ‘feeder operating in-vehicle cost’ (Mohaymany and Gholami 2010). Thus, based on the previous studies in order to moderate simplify the proposed model the social cost is assumed to be 20% of ‘feeder operating in-vehicle cost’ in this study.

3.4.4 Total Cost for a Route (C_{Tk})

After calculating all cost components for route k, the total cost function C_{Tk} for route k is expressed as given in Equation (3.18).

$$\begin{aligned}
C_{TK} = {} & \mu_a \left(Q_k \times t_{aF} + Q_k \times t_{aT}\right) + \mu_w \left[\left(\frac{1}{2F_k} + \frac{1}{2F_T}\right) \times Q_k\right] + \\
& \mu_I \left[\frac{1}{V_k} \sum_{i=1}^{I} \left[q_i \times \left(\sum_{j=1}^{J} L_{ijk}\right)\right] + (Q_k \times t_{Tj}) + \left(\frac{1}{2}(n_K + 1) \times Q_k \times t_{dF}\right) + \left(Q_k \times t_{dT}\right)\right] + \\
& \lambda_l \left(2F_k \times L_\kappa\right) + \lambda_{IT} \left[\left(Q_k \times t_{dT}\right) + \left(F_T \times T_T\right)\right] + \lambda_I \left(Q_k \times t_{dF}\right) + \lambda_m \left(2F_k \times L_\kappa\right) + \\
& \lambda_p \left[\left(\frac{2F_k}{V_k} \times L_\kappa\right) + \left(Q_k \times t_{dF}\right) + \left(F_k \times S_{kj}\right)\right] + \lambda_f \left[\frac{2F_k}{V_k} \times L_\kappa\right] + \lambda_s \left(2F_k \times L_\kappa\right)
\end{aligned} \tag{3.18}$$

3.4.5 Objective Function and Constraints of the Problem

The total system cost of the intermodal transit model consists of the user's parameters (i.e., value of time for user’s access, wait, and in-vehicle cost, etc.), operation parameters, social parameters, and important decision variables.

This transit network model must satisfy users, operators, and social terms. Thus, the objective function is defined as the sum of the user, operator, and social costs, which is given in Equation (3.19).

$$Minimize \quad C_T = \sum_{k=1}^{K}\left[\overbrace{\left(C_a + C_w + C_{ui}\right)}^{User} + \overbrace{\left(C_f + C_{oi} + C_m + C_p\right)}^{Operating} + \overset{Social}{C_s}\right] \tag{3.19}$$

Therefore, according to mathematical model formulation subsection, the objective function can be formulated after substitution of cost terms as follows:

$$\begin{aligned} Minimize \quad C_T = {} & \mu_a\left[t_{aF}\sum_{i=1}^{I} q_i + \sum_{j=1}^{J} t_{aTj}\sum_{i=1}^{I} q_i \times Y_{ij}\right] + \mu_w\left[\sum_{k=1}^{K}\left[\left(\frac{1}{2F_k} + \frac{1}{2F_T}\right)\times Q_k\right]\right] + \\ & \mu_I\left[\sum_{k=1}^{K}\left[\frac{1}{V_k}\sum_{i=1}^{I}\left[q_i \times \left(\sum_{j=1}^{J} L_{ijk}\right)\right] + \left(\frac{1}{2}(n_K + 1)\times Q_k \times t_{dF}\right)\right] + \sum_{j=1}^{J}\left[\left(\sum_{i=1}^{I} q_i \times Y_{ij}\right)\times\left(t_{dT}\times\left(J - j + 1\right) + t_{Tj}\right)\right]\right] + \\ & \lambda_f\left[2\sum_{k=1}^{K}\frac{F_k}{V_k}\times L_k\right] + \lambda_l\left[2\sum_{k=1}^{K} F_k \times L_\kappa\right] + \lambda_I\left[\sum_{k=1}^{K} Q_k \times t_{dF}\right] + \lambda_{IT}\left[\left(\sum_{i=1}^{I} q_i \times t_{dT}\right) + \left(F_T \times T_T\right)\right] + \\ & \lambda_m\left[2\sum_{k=1}^{K} F_k \times L_\kappa\right] + \lambda_p\left[\sum_{k=1}^{K}\left[\left(\frac{2F_k}{V_k}\times L_\kappa\right) + \left(Q_k \times t_{dF}\right) + \left(F_k \times S_{kj}\right)\right]\right] + \lambda_s\left[2\sum_{k=1}^{K} F_k \times L_\kappa\right] \end{aligned} \tag{3.20}$$

Subject to:

$$\sum_{k=1}^{K}\sum_{h=1}^{H} X_{ihk} = 1 \qquad i = 1,\ldots,I \tag{3.21}$$

$$\sum_{i=1}^{I}\sum_{j=I+1}^{H} X_{ijk} \le 1 \qquad k = 1,\ldots,K \tag{3.22}$$

$$\sum_{h=1}^{H} X_{ihk} - \sum_{m=1}^{I} X_{mik} \ge 0 \qquad i = 1,\ldots,I \qquad k = 1,\ldots,K \tag{3.23}$$

$$\sum_{i\notin H}\sum_{h\in H}\sum_{k=1}^{K} X_{ihk} \ge 1 \qquad \forall H \tag{3.24}$$

$$\sum_{h=1}^{H} X_{ihk} + \sum_{m=1}^{I} X_{mik} - Y_{ij} \leq 1 \qquad i = 1,\ldots,I \qquad j = I+1,\ldots,I+J \qquad k = 1,\ldots,K \tag{3.25}$$

$$l_{\min} \leq L_{K} \leq l_{\max} \qquad k = 1,\ldots,K \tag{3.26}$$

$$f_{\min} \leq F_k \leq f_{\max} \qquad k = 1,\ldots,K \tag{3.27}$$

$$\sum_{k=1}^{K} \left[\left(\frac{2F_k}{V_k} \times L_K \right) + \left(Q_k \times t_{dF} \right) + \left(F_k \times S_{kj} \right) \right] \leq N \tag{3.28}$$

$$\frac{Q_K}{LFC} \leq F_k \qquad k = 1,\ldots,K \tag{3.29}$$

where decision variables contain two binary variables, called Y_{ij} and X_{ihk} standing for a definition of the transit network (see Table 3.2), and continue variable feeder bus frequency (F_k).

Determination of F_k , as one of the decision variables, depends upon the transit network configuration. Thus the optimal feeder bus frequency using the analytical solution can be determined by setting the first derivative of the total cost function (C_{TK}) with respect to the feeder bus frequency, equating it to zero and solve it. Therefore, the optimal bus frequency can be taken as:

$$F_{opt,K} = \sqrt{\frac{\mu_w Q_k}{4l_k \left[\left(\lambda_l + \lambda_m + \lambda_s \right) + \frac{1}{V_k} \left(\lambda_f + \lambda_p \right) \right] + (2S_{kj} \times \lambda_p)}} \tag{3.30}$$

Furthermore, the minimum required frequency of each route k is given as:

$$F_{req,K} = \frac{Q_k}{LF \times C} \tag{3.31}$$

where LF= load factor of feeder bus, and C= capacity of feeder buses based on number of seats. Thus, the given frequency for each route k is obtained by selecting the maximum value out of optimum frequency ($F_{opt,K}$) and required frequency ($F_{req,K}$).

Some limitations are considered for the proposed model to represent an effective transit network model satisfying route feasibility, frequency, and so forth. Equations (3.21) to (3.25) determine the route feasibility in the network design. Several researchers used these constraints on their study such as Kuah and Perl, (1989); Kuan et al., (2004), (2006); Martins and Pato, (1998), etc. Equation (3.21) explains that each bus stop should be placed in a single route (many-to-one pattern). Furthermore, Equation (3.22) ensures that each generated route must be connected to only one railway station. Accordingly, in Equation (3.23), each bus is assumed to pass at all the stops in its route node. Equation (3.24) explains that each feeder bus route should be linked to only one railway station. Constraint (3.25) specifies that a bus stop can be assigned to a station in which the corresponding route terminates at one of the rail stations. Constraints on the minimum and maximum length of feeder routes are given in Equation (3.26). Similarly; the minimum and maximum frequencies are indicated in Equation (3.27). Equation (3.28) shows the maximum number of vehicles in the fleet, and Equation (3.29) represents the minimum frequency of satisfying the demand. Equation (3.26), (3.27) and (3.29) were utilized in several studies such as the studies conducted by Mohaymany and Gholami, (2010); Gholami and Mohaymany, (2011), etc. Equation (3.28) was proposed in this study because of improvement in personnel cost in section 3.4.2.3. This equation is presented with respect of added cost terms in the constraint.

Thus, objective one was achieved by proposing an improved mathematical model.

3.5 Model Optimization

In the previous section, the optimization of decision variables for the proposed intermodal transit system to minimize the total cost function incurred by the user, operation and social costs in the transit network was discussed. The applied optimization algorithms for solving the problem are discussed in the follows.

3.5.1 Applied Optimization Algorithms

In this section, we discuss the applied optimization methods and procedure of the FNDSP. As stated in the previous chapter, the FNDSP is a complex routing-type problem with an additional dimension and operating frequency (Kuah, 1986). The FNDSPs are categorized as the NP-hard problems with nonlinear objective function and constraints. Searching for the best feasible routes in order to minimize the cost function is a crucial mission for solving the FNDSP.

A cost function as objective function and some nonlinear constraints are sufficient in order that using metaheuristic methods are adopted. In the light of complexity and nonlinearity of current problem, exact optimization methods were not able to solve these kinds of problems. Therefore, the optimization approaches, which are mostly metaheuristics are of great importance (Almasi, Sadollah, Mounes, & Karim, 2014).

There are a lot of methods being used to solve transit network design problems. Based on the literature, there are pros and cons for all these optimization methods. For example, mathematical methods have some limitations such as being either non-convex problem or with unknown convexity. Moreover, some heuristic and analytical approaches have some other limitations in the early efforts of optimization in solving complex problems. More recently, the development of computing power offered to use metaheuristic approaches (Blum & Mathew, 2011).

A metaheuristic method for the FNDSP should have a criterion function that adequately reflects the objective of the FNDSP as it was shown in the Table 3.1. It should properly represent all the basic elements of the problem and their inter-relationships, as represented by the mathematical formulation in the previous section.

The GA, PSO, WCA and ICA algorithms have shown great potentials for solving optimization problems as they conducted a global stochastic search (Atashpaz-Gargari & Lucas, 2007; Dong, Tang, Xu, & Wang, 2005; Eskandar, Sadollah, Bahreininejad, & Hamdi, 2012; Giraud-Moreau & Lafon, 2002). The reasons for applying these four approaches in this study are: (i) It is trying to adopt some well-known and powerful methods for optimization of transit network design problems (e.g. GA, and PSO); (ii) The other two methods are ICA and WCA, being comparatively new metaheuristic methods (ICA first proposed by Atashpaz-Gargari & Lucas, 2007 and WCA first presented by Eskandar et al., 2012) is not mostly utilized in the transportation field so far. The detail explanation of each solution method is presented in the follows.

3.5.1.1 Genetic algorithm

Genetic algorithms (GAs) are members of a collection of methodologies known as evolutionary computation. These techniques are based on the principals of natural selection and evolution processes that are met in nature.

The efficiency of the numerous evolutionary algorithms in comparison to other heuristic techniques has been tested in both generic (Elbeltagi, Hegazy, & Grierson, 2005; Youssef, Sait, & Adiche, 2001) and engineering design (Giraud-Moreau & Lafon, 2002) problems. Through these tests, the GAs are identified as robust heuristic tools capable of delivering efficient and robust solutions to diverse design problems.

The GAs exploits historical information to direct the search into the region of better performance within the search space (Holland, 1975). The basic techniques of the GAs are designed to simulate processes in natural systems necessary for evolution; especially those follow the principles first laid down by Charles Darwin of "survival of the fittest." Since in nature, competition among individuals for scanty resources, results in the fittest individuals dominating over the weaker ones (Holland, 1975).

The GAs simulates the survival of the fittest among individuals over consecutive generation for solving a problem. Each generation consists of a population of character strings that are analogous to the chromosome that we witness in our DNA. Each individual represents a point in a search space and a possible solution.

The individuals in the population are then made to go through a process of evolution. The GAs are based on an analogy with the genetic structure and behavior of chromosomes within a population of individuals using the following foundations (Golberg, 1989):

- Individuals in a population compete for resources and mates.
- Those individuals most successful in each competition will produce more offspring than those individuals that perform poorly.
- Genes from good individuals propagate throughout the population so that two good parents will sometimes produce offspring that are better than either parent.
- Thus, each successive generation will become more suited to their environment.

A population of individuals is maintained within search space for a GA, each representing a possible solution to a given problem. Each individual is coded as a finite length vector of components, or variables, in terms of some alphabet, usually the binary alphabet [0, 1].

To continue the genetic analogy, these individuals are likened to chromosomes and the variables are analogous to genes. Thus a chromosome (solution) is composed of several genes (variables). A fitness score is assigned to each solution representing the abilities of an individual to compete.

The individual with the optimal (near optimal) fitness score is sought. The GA aims to use selective breeding of the solutions to produce offspring better than the parents by combining information from the chromosomes. The GAs maintains a population of *n* chromosomes (solutions) with associated fitness values.

Parents are selected to mate, based on their fitness, producing offspring via a reproductive plan. Consequently, highly fit solutions are given more opportunities to reproduce so that offspring inherits characteristics from each parent. As parents mate and produce offspring, room must be made for the new arrivals since the population is kept at a static size (Holland, 1975).

Individuals in the population die and replaced by the new solutions, eventually creating a new generation once all mating opportunities in the old population have been exhausted. In this way, it is hoped that over successive generations, better solutions will thrive while the least fit solutions die out.

New generations of solutions are produced containing, on average, better genes than a typical solution in a previous generation. Each successive generation will contain better partial solutions than previous generations. Eventually, once the population has converged and is not producing offspring noticeably different from those in previous generations, the algorithm itself is said to have converged to a set of solutions to the problem, at hand, which is called stopping criterion (Golberg, 1989).

3.5.1.2 Particle swarm optimization

Particle swarm optimization (PSO) is an evolutionary computation technique for solving global optimization problems developed by Kennedy and Eberhart (1995). It is a computation technique through individual improvement plus population cooperation and competition, which is based on the simulation of simplified social models such as bird flocking, fish schooling, and the swarm theory.

Researchers found that the synchrony of animal's behavior was through maintaining optimal distances between individual members and their neighbors (Kennedy & Eberhart, 1997). The PSO algorithm exhibits common evolutionary computation attributes, including initialization with a population of random solutions and searching for optima by updating generations.

Potential solutions, called ‘birds’ or ‘particles’, are then ‘flown’ through the problem space by following the current optimum particles. Each particle keeps track of its coordinates in the problem space, which are associated with the best solution (fitness) it has achieved so far. This value is called ‘*pBest*’. Another ‘*best*’ value that is tracked by the global version of the particle swarm optimization is the overall best value, and its location obtained so far by any particle in the population. This location is called ‘*gBest*’.

The PSO concept consists of, at each step, changing the velocity (i.e. accelerating) of each particle toward its ‘*pBest*’ and ‘*gBest*’ locations. Acceleration is weighted by a random term with separate random numbers being generated for acceleration toward ‘*pBest*’ and ‘*gBest*’ locations. The basic swarm parameter's position and velocity are updated using the following Equations (Kennedy & Eberhart, 1995):

$$V_{i+1} = wV_i + c_1 r_1 (pBest_i - X_i) + c_2 r_2 (gBest_i - X_i) \tag{3.32}$$

$$X_{i+1} = X_i + V_{i+1} \tag{3.33}$$

where w is the inertia weight for velocities (previously set between 0 and 1), X_i is the current value particle i, V_i is the updated velocity of particle i, $pBest_i$ is the best solution found by particle i, $gBest_i$ is the best solution found by the swarm, r_1 and r_2 are uniform random numbers in the [0, 1] range, c_1 means the cognitive component (self-confidence of the particle), and c_2 means the social component (swarm confidence), and they are constants that influence how each particle is directed toward good positions taking into account personal best and global best information, respectively.

They usually are set as $c_1 = c_2 = 1.5$. The role of w is crucial for the PSO convergence. It is employed to control the impact of previous velocities on the current particle velocity. A general rule of thumb indicates to set a large value initially to make the algorithm explore the search space and then gradually reduce it in order to get refined solutions (Dong et al., 2005; Van den Bergh & Engelbrecht, 2006).

3.5.1.3 Imperialist competitive algorithm

Imperialist competitive algorithm (ICA) is inspired from the social-political process of imperialism and imperialistic competition. Similar to many optimization algorithms, the ICA starts with an initial population. Each individual of the population is called a 'country'. Some of the best countries with the minimum cost are considered as the imperialist states and the rest will be the colonies of those imperialist states. All the colonies are distributed among the imperialist countries based on their power.

To define the algorithm; first of all, initial countries of size $N_{Country}$ are produced. Then, some of the best countries (with the size of N_{imp}) in the population are selected to be the imperialist states. Therefore, the rest with the size N_{col} will form the colonies that belong to imperialists. Then, the colonies are divided among imperialists according to their power (Atashpaz-Gargari & Lucas, 2007). In such a way that the initial number of each empire's colonies has to be proportional to its power. Hence, the initial number of colonies of the n^{th} empire will be (Khabbazi, Atashpaz-Gargari, & Lucas, 2009):

$$NC_n = round\{\left|\frac{Cost_n}{\sum_{i=1}^{N_{imp}} Cost_i}\right| \times N_{col}\} \quad , \quad n = 1, 2, ..., N_{imp} \tag{3.34}$$

where NC_n is the initial number of colonies of the n^{th} empire and N_{col} is the total number of initial colonies, and N_{imp} is the number of imperialist state. To divide the colonies, NC_n of the colonies are randomly chosen and given to the n^{th} imperialist (Khabbazi et al., 2009).

After dividing all colonies among imperialists and creating the initial empires, these colonies start moving toward their relevant imperialist country. This movement is a simple model of the assimilation policy. Also, the total power of an empire is defined by the sum of the cost of the imperialist, and some percentage of the mean cost of its colonies as given (Khabbazi et al., 2009):

$$TC_n = Cost\,(imperialist_n\,) + \xi\,\{mean\,(Cost(colonies\,of\;\;empire_n\,))\} \quad (3.35)$$

where TC_n is the total power of the n^{th} empire and ξ is a positive small number. After computing the total power of empires, usually the weakest colony (or colonies) of the weakest empire is chosen by other empires, and the competition is started on possessing this colony. Each imperialist participating in this competition, according to its power, has a probable chance of possessing the cited colony.

To start the competition, at first, the weakest empire is chosen and then the possession probability of each empire is estimated. The possession probability P_p is related to the total power of the empire (*TC*)(Atashpaz-Gargari & Lucas, 2007). In order to evaluate the normalized total cost of an empire (*NTC*), the following Equation is used (Khabbazi et al. 2009):

$$NTC_n = \max_i\{TC_i\} - TC_n \qquad n, i = 1, 2, 3, \ldots, N_{imp} \quad (3.36)$$

During the imperialistic competition, the weak empires will slowly lose their power and getting weaker by the time. At the end of a process, just one empire will remain that governs the whole colonies (Khabbazi et al. 2009). Table 3.4 shows the pseudo-code and step by step processes of the ICA in detail and demonstrates in Figure 3.3.

Table 3.4: Pseudo-code of the ICA

1) Select some random points on the function and initialize the empires.
2) Move the colonies toward their relevant imperialist (Assimilation).
3) Randomly change the position of some colonies (Revolution).
4) If there is a colony in an empire which has lower cost than the imperialist, exchange the positions of that colony and the imperialist.
5) Unite the similar empires.
6) Compute the total cost of all empires.
7) Pick the weakest colony (colonies) from the weakest empires and give it (them) to one of the empires (Imperialistic competition).

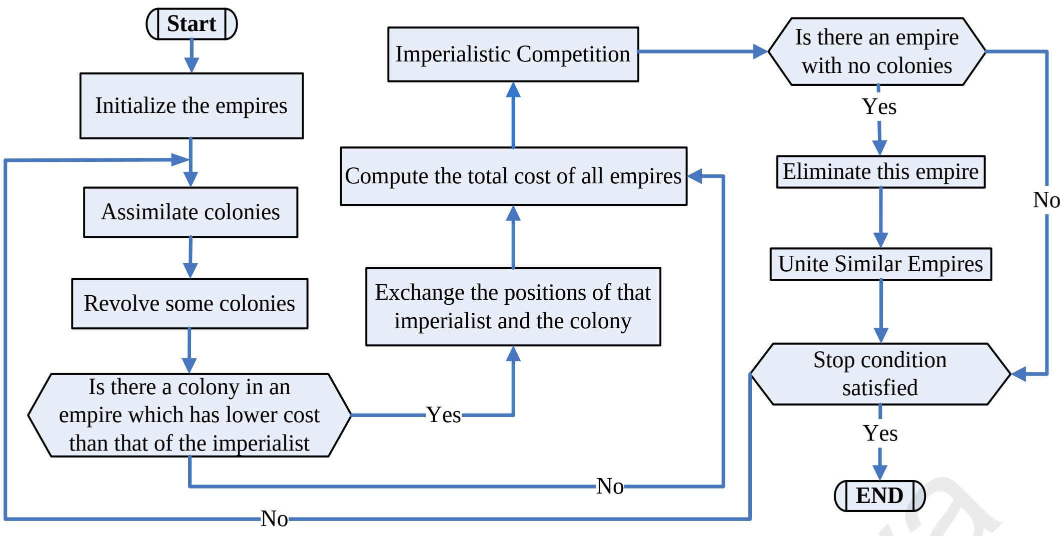


Figure 3.3: Flowchart of the imperialist competitive algorithm

3.5.1.4 Water cycle algorithm

The water cycle algorithm (WCA) mimics the flow of rivers and streams toward the sea and derived by the observation of the water cycle process. Let us assume that there are some rain or precipitation phenomena. An initial population of design variables (i.e., population of streams) is randomly generated after raining process. The best individual (i.e., the best stream), classified in terms of having the minimum cost function (for minimization problem), is chosen as the sea.

Then, a number of good streams (i.e., cost function values close to the current best record) are chosen as rivers, while the other streams flow to the rivers and sea. In an *N-dimensional* optimization problem, a stream is an array of $1\times N$. This array is defined as follows:

$$A\ \ Stream\ \ Candidate = [x_1, x_2, x_3, ..., x_N] \tag{3.37}$$

where N is the number of design variables (problem dimension). To start the optimization algorithm, an initial population representing a matrix of streams of size $N_{pop} \times N$ is generated. Hence, the matrix of the initial population, which is generated randomly, is given as (rows and column are the number of population and the number of design variables, respectively):

$$Total \quad Population = \begin{bmatrix} Sea \\ River_1 \\ River_2 \\ River_3 \\ \vdots \\ Stream_{Nsr+1} \\ Stream_{Nsr+2} \\ Stream_{Nsr+3} \\ \vdots \\ Stream_{N_{pop}} \end{bmatrix} = \begin{bmatrix} x_1^1 & x_2^1 & x_3^1 & \cdots & x_N^1 \\ x_1^2 & x_2^2 & x_3^2 & \cdots & x_N^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_1^{N_{pop}} & x_2^{N_{pop}} & x_3^{N_{pop}} & \cdots & x_N^{N_{pop}} \end{bmatrix} \tag{3.38}$$

where N_{pop} and N are the total number of population and the number of design variables, respectively. Each of the decision variable values ($x_1, x_2, \ldots, x_N$) can be represented as floating point number (real values) or as a predefined set for continuous and discrete problems, respectively. The cost of a stream is obtained by the evaluation of cost function (C) given as follows:

$$C_i = Cost_i = f(x_1^i, x_2^i, \cdots, x_N^i) \qquad i = 1, 2, 3, \ldots, N_{pop} \tag{3.39}$$

At the first step, N_{pop} streams are created. A number of N_{sr} from the best individuals (minimum values) is selected as a sea and rivers. The stream, which has the minimum value among others, is considered as the sea. In fact, N_{sr} is the summation of the number of rivers (which is defined by the user) and a single sea (see Equation (3.40)). The rest of the population (i.e., streams flow to the rivers or may directly flow to the sea) is calculated using the following Equation:

$$N_{sr} = Number \ of \ Rivers + \underbrace{1}_{Sea} \tag{3.40}$$

$$N_{Stream} = N_{pop} - N_{sr} \tag{3.41}$$

Equation (3.42) shows the population of streams, which flow to the rivers or sea. Indeed, Equation (3.42) is part of Equation (3.38) (i.e., total individual in population):

$$Population \quad of \quad Streams = \begin{bmatrix} Stream_1 \\ Stream_2 \\ Stream_3 \\ \vdots \\ Stream_{N_{Stream}} \end{bmatrix} = \begin{bmatrix} x_1^1 & x_2^1 & x_3^1 & \cdots & x_N^1 \\ x_1^2 & x_2^2 & x_3^2 & \cdots & x_N^2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_1^{N_{Stream}} & x_2^{N_{Stream}} & x_3^{N_{Stream}} & \cdots & x_N^{N_{Stream}} \end{bmatrix} \tag{3.42}$$

Depending on flow magnitude, each river absorbs water from streams. The amount of water entering a river and/or the sea, hence, varies from stream to stream. In addition, rivers flow to the sea which is the most downhill location. The designated streams for each river and sea are calculated using the following Equation (Eskandar et al., 2012):

$$C_n = Cost_n - Cost_{Nsr+1} \qquad n = 1, 2, 3, \ldots, N_{sr} \tag{3.43}$$

$$NS_n = round\left\{ \left| \frac{C_n}{\sum_{n=1}^{N_{sr}} C_n} \right| \times N_{Streams} \right\} \quad , \quad n = 1, 2, \ldots, N_{sr} \tag{3.44}$$

where NS_n is the number of streams, which flow to the specific rivers and sea. As it happens, in nature, streams are created from the raindrops and join each other to generate new rivers. Some stream may even flow directly to the sea. All rivers and streams end up in the sea that corresponds to the current best solution.

Let us assume that there are N_{pop} streams of which N_{sr}-1 are selected as rivers and one is selected as the sea. Figure 3.3a shows the schematic view of a stream flowing toward a specific river along their connecting line.

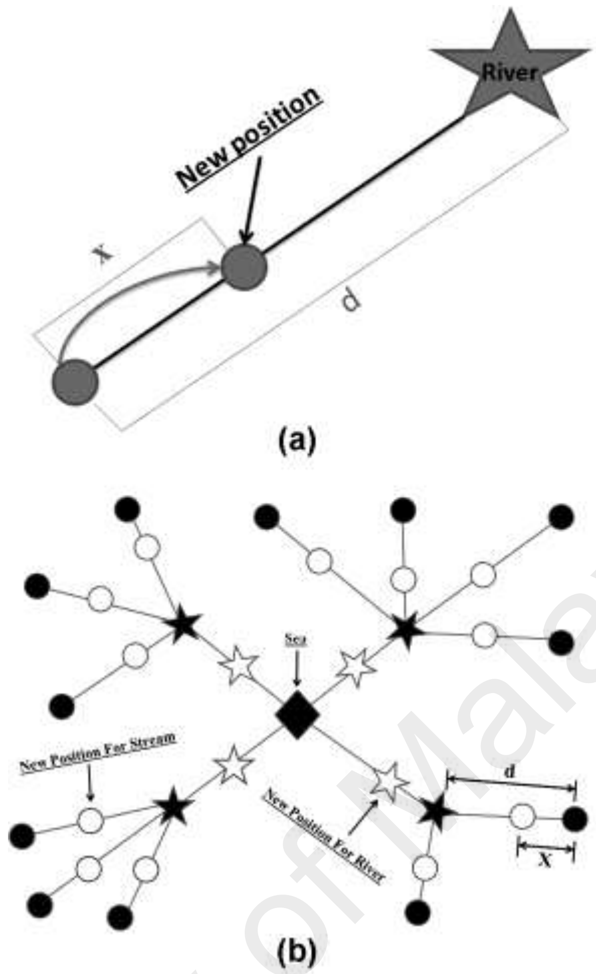


Figure 3.4: a) Schematic description of the stream's flow to a specific river; b) Schematic of the WCA optimization process

For the exploitation phase in the WCA, new positions for streams and rivers have been suggested as follows (Eskandar et al., 2012):

$$\vec{X}^{t+1}_{Stream} = \vec{X}^{t}_{Stream} + rand \times C \times (\vec{X}^{t}_{Sea} - \vec{X}^{t}_{Stream}) \tag{3.45}$$

$$\vec{X}^{t+1}_{Stream} = \vec{X}^{t}_{Stream} + rand \times C \times (\vec{X}^{t}_{River} - \vec{X}^{t}_{Stream}) \tag{3.46}$$

$$\vec{X}^{t+1}_{River} = \vec{X}^{t}_{River} + rand \times C \times (\vec{X}^{t}_{Sea} - \vec{X}^{t}_{River}) \tag{3.47}$$

where $1 < C < 2$ and the best value for C may be chosen as 2 and *rand* is a uniformly distributed random number between zero and one. Equations (3.45) and (3.46) are for streams, which flow to the sea and their corresponding rivers, respectively. Notations having vector sign correspond to vector values, otherwise the rest of notations and parameters are considered as scalar values. If the solution given by a stream is better than its connecting river, the positions of river and stream are exchanged (i.e., the stream becomes a river, and the river becomes a stream). A similar exchange can be performed for a river and the sea.

The evaporation process operator also is introduced to avoid premature (immature) convergence to local optima (exploitation phase) (Eskandar et al., 2012). Basically, evaporation causes sea water to evaporate as rivers/streams flow to the sea. This leads to new precipitations. Therefore, we have to check if the river/stream is close enough to the sea to make the evaporation process occur. For that purpose, the following criterion is utilized for evaporation condition:

$$\begin{aligned}
& \textit{if} \ \left\| \vec{X}^{t}_{Sea} - \vec{X}^{t}_{River_j} \right\| < d_{\max} \quad \textit{or} \quad \textit{rand} < 0.1 \qquad j = 1, 2, 3, \ldots, N_{sr} - 1 \\
& \quad \textit{Perform raining process by Eq.}(3.48) \\
& \textit{end}
\end{aligned}$$

where d_{max} is a small number close to zero. After evaporation, the raining process is applied and new streams are formed in the different locations (similar to mutation in the GAs). Hence, in the new generated sub-population, the best stream will act as a new river and other streams move toward their new river. This condition will also apply for streams that directly flow to the sea.

Indeed, the evaporation operator is responsible for the exploration phase in the WCA. The following Equation is used to specify the new locations of the newly formed streams:

$$\vec{X}^{t+1}_{Stream} = L\vec{B} + rand \times (U\vec{B} - L\vec{B}) \tag{3.48}$$

where LB and UB are lower and upper bounds defined by the given problem, respectively. Similarly, the best newly formed stream is considered as a river flowing to the sea. The rest of new streams are assumed to flow into the rivers or may directly flow into the sea.

A large value for d_{max} prevents extra searches, and small values encourage the search intensity near the sea. Therefore, d_{max} controls the search intensity near the sea (i.e., best-obtained solution). The value of d_{max} adaptively decreases as follows:

$$d_{\max}^{t+1} = d_{\max}^{t} - \frac{d_{\max}^{t}}{Max\ Iteration} \qquad t = 1, 2, 3, ..., Max\_Iteration \qquad (3.49)$$

The development of the WCA optimization process is illustrated by Figure 3.2b where circles, stars, and the diamond correspond to streams, rivers, and sea, respectively. The white (empty) shapes denote the new positions taken by streams and rivers. Table 3.5 shows the pseudo-code and step by step processes of the WCA in detail.

Table 3.5: Pseudo-code of the WCA

```
Set user parameter of the WCA: N_pop, N_sr, d_max, and Max_Iteration
Determine the number of streams, which flow to the rivers and sea using Eqs. (3.40) and
(3.41)
Randomly create initial population of streams between LB and UB (Eq. (3.42))
Define the intensity of flow for rivers and sea by Eqs. (3.43) and (3.44)
% Main loop of WCA
while (t ≤ Max_Iteration) or (any stopping condition)
        for i = 1 : Population size (N_pop)
                Stream directly flows to the sea using Eq. (3.45)
                Calculate the objective function of the generated stream
                        if Objective (New_Stream) < Objective (Sea)
                                Sea = New_Stream
                        end if
                Stream flows to its corresponding river using Eq. (3.46)
                Calculate the objective function of the generated stream
                        if Objective (New_Stream) < Objective (River)
                                River = New_Stream
                                if Objective (New_Stream) < Objective (Sea)
                                        Sea = New_Stream
                                end if
                        end if
                River flows to the sea using Eq. (3.47)
                Calculate the objective function of the generated river
                        if Objective (New_River) < Objective (Sea)
                                Sea = New_River
                        end if
        end for
        %  Evaporation Condition for rivers
        for i = 1 : N_sr-1
                if  (norm (Sea - River) < d_max) or (rand < 0.1)
                        New streams and a river are created using Eq. (3.48)
```

```
        end if
    end for
    %  Evaporation Condition for streams who directly flow to the sea
    for i = 1 : NS_1
        if  norm (Sea - Stream) < d_max
            New streams are created using Eq. (3.48)
        end if
    end for
    Reduce the d_max using Eq. (3.49)
end while
Postprocess results and visualization
```

3.6 Illustrative Application of the Model in the Current Study

The formulation and solution algorithms are applied first to the benchmark study area from literature and then in the real study area in the PJ, Malaysia.

3.6.1 Description of Benchmark Study Area

The study is conducted in the context of a residential area, locally served by a bus fleet and connected to the city center by a rail system. The rail network is assumed to be fixed; that is, defined in advance and not subject to change, whereas the bus network will be determined by the solution to this problem itself. The location of bus stops and the demand of passengers at each stop during a pre-specified period of time are also known. The distance between each pair of stops and stops/stations, and the capacities and operating- speed of the fleet of buses over the planning period, are given as well.

The improved model in section 3.4 was tested in the analysis of transit services, including bus feeder services connecting the rail stations in the case study by Kuah and Perl (1989). They used the network with 59 nodes, which include 55 bus stops (1 to 55), and four rail stations (56 to 59) covering a service area of 2×2.5 mile. The bus stop density is 11 stops per square mile with an hourly demand density of approximately 200 passengers per stop. Such a demand density is consistent with that for a typical urban area (Webster & Bly, 1979). The number of rail stations is selected such that the ratio of the number of bus stops to the number of rail stations in the range of 12 to 15 stops. The dimensions and inputs of each node (see Appendix B) are extracted from Kuah and Perl (1989).

3.6.2 Description of Real Study Area: Petaling Jaya

To evaluate the performance of the proposed model for the transit services in the real case area, we implemented it in part of the PJ area in Malaysia. PJ is a major Malaysian city originally developed as a satellite township in Kuala Lumpur (KL). It is located in the Petaling district of Selangor with an area of approximately 97.2 square kilometers (37.5 sq. mi). PJ is now facing a problem on how to effectively and inexpensively move people around the city area. In this study, part of the PJ area serves to illustrate the FNDSP for generating networks around the transit center concept. The objective was to minimize the total cost of the feeder bus network in the area.

The main reason for picking this area is a sufficient flow of potential passengers of the rail system. The PJ city and its transit network have experienced a remarkable growth in the past years. The city of PJ is now recognized as Malaysia's major city. The population has grown from 474800 residents in 2005 to 531400 residents in 2010. It is expected to keep growing, where estimates have put the total population at 588000 by 2015, and 644600 by 2020 (Center for Transportation Research (CTR), 2012).

Another reason for choosing PJ transit is that its local network is linked to the transit systems of KL and other neighboring cities, providing one component of an integrated network of bus and train routes. Therefore, the substantial growth of the transit network over the past years, along with the changes in the demand and other demographic and socioeconomic variables, provides a useful case to explore network growth trends and related factors.

PJ was chosen as the best candidate for this phase of our research as it was and still is a fast-growing city of Malaysia. The location of PJ among other cities and regions is shown in (Appendix C). The case study region, shown in Figure 3.5, is an area of 5.5

km by 6.5 km in the south of PJ in Malaysia, where includes Kelana Jaya Line of the Kuala Lumpur LRT. There are four stations in the study region. The existing bus stops of each traffic zone are considered as the feeder bus stops to be covered by a feeder line. A certain demand corresponding to that traffic zone is assigned to each of the bus stops in that particular zone.

3.7 Data Collection

This section is focused on the data preparation, sampling procedure of the research method, questionnaire survey and observation. The sampling procedures and steps, includes the target respondents and sampling size, sampling method, sampling locations, dates, time. The details of conducting the passenger questionnaire and observation in train stations and bus stops are explained. The survey results are presented in the next chapter.

3.7.1 Data Preparation

In order to execute the FNP, four important data sets must be made available, namely, the list of all nodes (bus stops and rail stations) coordination, the network available connectivity list, the transit demand matrix and cost parameters.

3.7.1.1 List of nodes

A total of 54 nodes is defined to describe the service area and associated network connectivity. All 54 nodes are selected from the existing transit network, which consists of public bus routes with fixed schedules operated the public transportation companies such as Rapid KL and Metrobus, etc. The list of locations associated with these 54 nodes is presented in Tables 3.6 and 3.7.

3.7.1.2 Network connectivity

Network connectivity is generated from street links that connect these 54 nodes and are suitable for bus operations. These nodes and network connectivity figure is presented in Figure 3.6.

3.7.1.3 Demand matrix

The generation of the demand matrix is based on a questionnaire survey data collection that is explained as follows. The second question of the questionnaire is about their origin and destination of the users (see Appendix D). The demand matrix was determined by extracting the abstained results from survey.

3.7.1.4 Cost parameters

The cost parameters are based on the data collection of the current study as well as ridership and financial reports which were publicized by Barton (2006) and Valley Metro (2012).

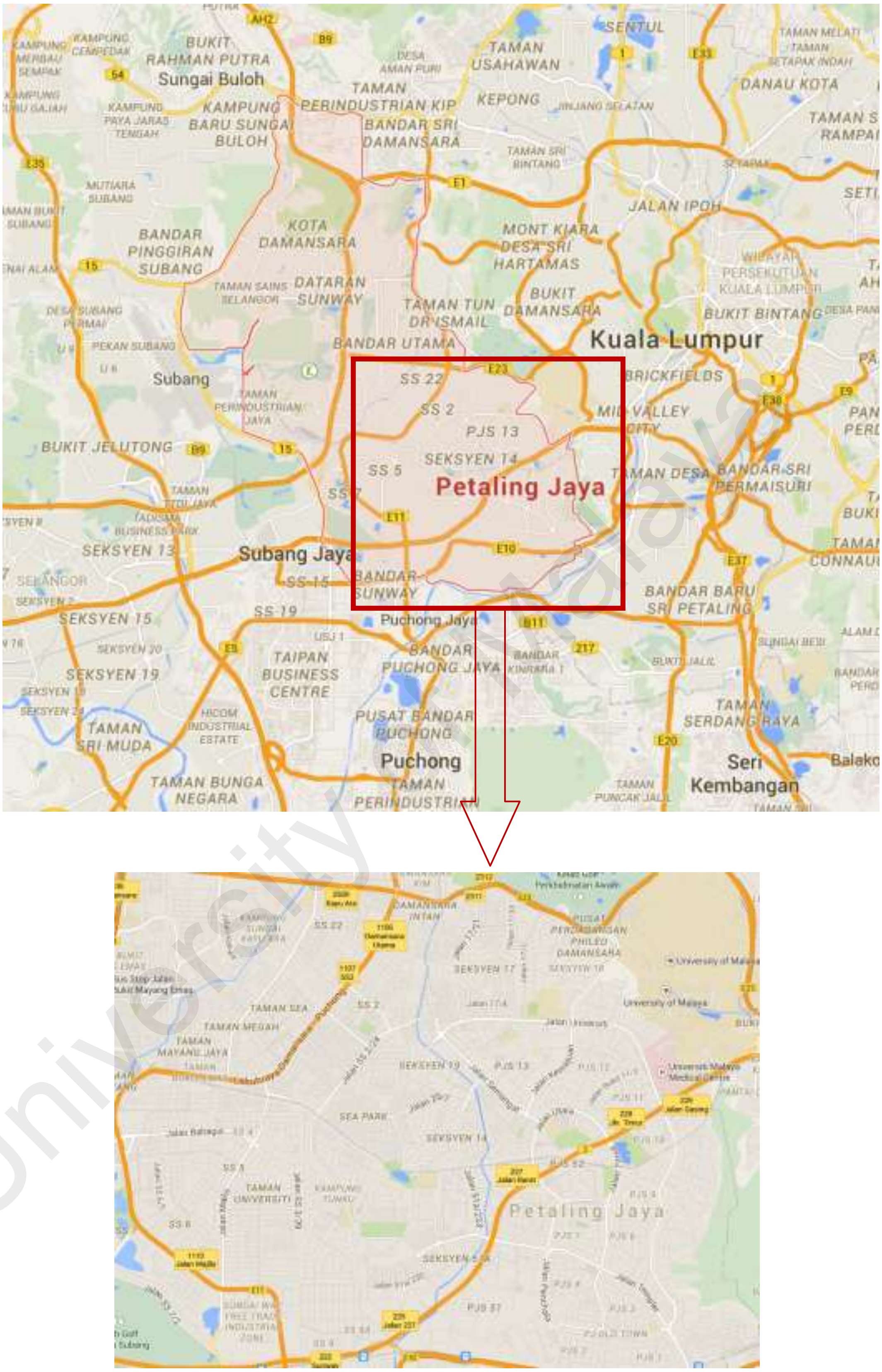


Figure 3.5: The case study regions of the PJ (Google Maps, 2014)

Table 3.6: Location of rail stations and bus stops in the PJ study area

Bus stop No.	X-coordinate (Km)	Y-coordinate (Km)	Bus stop No.	X-coordinate (Km)	Y-coordinate (Km)
1	6.71	6.17	26	5.11	2.30
2	5.97	6.15	27	4.31	1.67
3	5.79	5.59	28	4.30	2.14
4	6.26	5.30	29	4.06	2.51
5	7.02	5.02	30	4.14	3.05
6	5.46	5.05	31	3.83	3.52
7	7.50	4.89	32	4.12	4.30
8	6.62	4.50	33	4.56	4.09
9	5.68	4.57	34	5.28	4.42
10	6.06	4.18	35	4.91	5.05
11	7.22	4.36	36	4.33	4.90
12	7.91	4.13	37	4.56	6.04
13	7.10	3.95	38	3.97	6.00
14	5.24	3.41	39	4.00	5.41
15	5.41	2.61	40	3.59	4.70
16	6.53	3.16	41	3.24	3.89
17	7.06	2.79	42	2.72	3.57
18	7.85	3.01	43	3.07	3.19
19	7.61	2.00	44	3.65	2.99
20	7.13	2.19	45	3.22	2.67
21	6.52	2.29	46	3.67	2.30
22	6.53	1.71	47	3.58	1.74
23	7.22	1.58	48	2.79	2.30
24	8.06	1.37	49	2.06	3.21
25	4.86	1.84	50	2.26	2.65

Table 3.7: Rail station locations

Rail station No.	X-coordinate (Km)	Y-coordinate (Km)
51	7.06	3.43
52	6.19	3.52
53	4.57	3.48
54	3.42	4.17

Figure 3.6: Study area nodes and network connectivity figure
Base map source: Google Earth (2014)

3.7.2 Sampling Procedure

3.7.2.1 Target respondents and sample size

A questionnaire is designed to collect a respondent’s origin and destination. Targeted respondents are LRT passengers who queue up at bus stops and LRT stations in different locations of the study area. Shrivastava and O’Mahony (2006) performed a sample size of 20 percent in their study. Generally, larger sample sizes provide more accurate survey results. Nonetheless, due to the constraints of limited resources and time, the sample size for transit service is confined to 20 percent of passengers counted in each LRT station using public buses.

3.7.2.2 Sampling method

The random sampling technique is employed in this survey to make sure that each member of the population has an equal chance of being selected as a respondent.

3.7.2.3 Sampling locations

Location of LRT stations in PJ area for conducting the questionnaires, which widely covers the study area, is shown below in Table 3.8.

3.7.2.4 Sampling dates

Two consecutive dates of a normal weekday (i.e., 8th April to 11th April and 15th April to 18th April, 2013) are chosen for conducting the questionnaire survey. This is desirable as the flow of passengers is promising, and samples for normal weekdays can be obtained.

3.7.2.5 Sampling time

The data of the normal weekdays were applied in the research area. The survey time slot is for the three-hour morning peak from 6:30 AM to 9:30 AM. It is designed to capture the feeder bus passengers of morning peak times.

Table 3.8: Selected LRT stations for public transit passenger questionnaires

Location	Survey Date	Survey Time
Taman Jaya	8th to 9th April	6:30 to 9:30 AM
Asia Jaya	10th to 11th April	6:30 to 9:30 AM
Taman Paramount	15th to 16th April	6:30 to 9:30 AM
Taman Bahagia	17th to 18th April	6:30 to 9:30 AM

3.7.3 Questionnaire survey

The questionnaire is face-to-face and divided into two questions. The first question is about the mode of travel to/from the LRT station. The second question is about their OD. The questionnaire for LRT passengers is attached in Appendix D. The pilot test of the questionnaire survey and limitations of the questionnaire survey are thoroughly discussed.

3.7.3.1 Pilot test of the questionnaire survey

To assess the wording, flow and understandability of the questionnaire, a pilot survey was carried out on 26th March 2013 at the Asia Jaya LRT station. The results in question 1 (i.e., What is your normal mode of transportation to / from the LRT station?) implied that the respondents didn't have any problem with wording or any part of question. However, the results in question 2 (i.e., Where is your origin / destination?) revealed that there was same confusion understanding the combination 'origin/destination' for respondents. In addition, the researcher found that it is quite hard to complete the questionnaire by keeping a passenger being interviewed from wanting to get to the LRT station. It may be because most LRT passengers are in a hurry and have high value of time. The sample location shows that it is essential to prepare a questionnaire that is simple and fast to answer. Thus the interviewer has to be alert to the flow and ensure the respondents have enough time to complete the questionnaire. As a result, two different sets of the questionnaire for origin and destination are provided in the final questionnaire (the pilot and final questionnaires are attached in Appendix D).

3.7.3.2 Limitations of the questionnaire survey

There are a few limitations of the questionnaire survey. First, the limited number of respondents may not be adequate to generate a representative result of all feeder bus passengers. Second, the shortcoming of convenience sampling in making scientific and representative generalization to the total population is obvious. However, given the constraint of manpower and time, the sample size of 20 percentages of total passengers is considered as the most practicable and an achievable balanced approach. This survey on FNP in Malaysia can help fill up the research gap and stimulate further study.

3.7.4 Observation

Observation is the primary method for understanding complex systems. The approach to observational science often begins qualitatively, as a search for an order that characterizes the system.

The observations were conducted by the researcher from 18th to 21st March 2013. The researcher found that observation is a necessary complementary method of data collection for the study in the case of these reasons: (i) to provide necessary information for the study when respondents are unwilling or unable to answer; (ii) to understand an ongoing process or situation; and (iii) to know about a physical setting or to check how much time is spent on various activities.

To determine some data and value of parameters in the proposed mathematical model the observation method has been used in this research. In this section, the design and procedures for conducting the observation in train stations and bus stops are presented.

Observations are conducted upon LRT stations and existing feeder bus routes. Observations are categorized into two types of questions. The first is about the time spent boarding and alighting at the LRT station and bus stops based on time per passenger. The second is about average feeder bus speed in existing routes.

3.8 Summary of the Chapter

This chapter has discussed the research design and methodology of the study. The proposed model formulation for feeder bus transit network design problem was discussed. This includes three main components: user cost, operation cost, and social cost. The optimization procedure contains objective function and constraints, as well as applied optimization methods. This is followed by explaining the illustrative application of the model in the current study, including a description of the benchmark and the PJ study area. Then, data collection procedure including data preparation, sampling procedure, questionnaire surveys and observation is explained. The researcher ends this chapter by section summary. The following chapter would discuss the results of the analysis for the objectives of the study.

CHAPTER 4: RESULTS AND DISCUSSION

4.1 Introduction

The major aim of this chapter is to test and apply the new model which was proposed in the previous chapter. To answer this, the suggested optimization algorithms are applied in the FNDSP. This chapter attempts to demonstrate the improvement of a new modeling framework that addresses the intermodal transit system on benchmark and real case study (PJ) data. Applying methods and conducting computational tests have been made with two main purposes: The first is to investigate and compare the character and performance of the solution networks with the proposed solution algorithm for a benchmark problem. To achieve this objective, sensitivity analyses with respect to key design features and parameters of the procedures are performed. The second is to test the solution framework and investigate its performance with respect to an actual feeder bus transit network. This objective is achieved by testing the alternative networks with data collected from the transit system of PJ, Malaysia. The optimization and all generated results are implemented by the computer program coded in the MATLAB programming software and run on Pentium IV 2.53 GHz CPU with 4 GB RAM.

This chapter is organized by an illustration of the applied model in the current study for benchmark and PJ area. This is followed by the summary section.

4.2 Application and Optimization of Transit Network Problem in Benchmark Study Area: Objective Two

The major purpose of this section is to apply and test the benchmark case study that was illustrated in section 3.6.1 in chapter three.

To implement the proposed model for an intermodal transit system and its application in the case study, the values of the model parameters are specified in Table

4.1. The dimensions and inputs of each node (see Appendix B) are extracted from Kuah and Perl (1989). The transit network was designed with feeder bus and fixed rail lines.

Table 4.1: Selected values for parameters of used benchmark problem

Parameter	Unit	Value
μ_a	\$/passenger-hr	8
μ_w	\$/passenger-hr	8
μ_I	\$/passenger-hr	4
λ_f	\$/veh-hr	14.37
λ_l	\$/veh-km	0.357
λ_I	\$/veh-hr	11.44
λ_m	\$/veh-km	0.75
λ_p	\$/veh –hr	10.2
λ_s	\$/veh-km	0.07
V	km/hr	32
S_{kj}	min	15
t_{aF}	min	7.5
t_{aTj}	min	4
t_{dT}	min/passenger	0.03
t_{dF}	min/passenger	0.096
V_T	km/hr	40
F_T	veh/hr	20
f_{min}	veh/hr	2
f_{max}	veh/hr	20
N	veh	140
LF	pass/seat	1
C	pass/veh	50
l_{min}	km	No constraint
l_{max}	km	4
λ_{lt}	\$/veh-hr	180

All optimizers have been coded and implemented in MATLAB by the author of the thesis. The processes and Pseudo code of mentioned optimizers have been given in the literatures in details.

Regarding the initial population which is as widely used in metaheuristic methods, uniform disributed random numbers have been generated for all considered optimizers. Indeed, metaheuristci optimization methods do not rely on the initial solution (population) unlike classical and exact optimization methods such as Newton method.

In term of constraint handling approach, direct method has been applied in this thesis. Below are brief explanations of the direct method:

In the search space, individuals may violate either the problem specific constraints or the limits of the design variables. In the current work, a modified feasible-based mechanism is used to handle the problem of specific constraints based on the following four rules (Mezura-Montes and Coello, 2008):

• Rule 1: Any feasible solution is preferred to any infeasible solution.

• Rule 2: Infeasible solutions containing slight violation of the constraints (from 0.01 in the first iteration to 0.001 in the last iteration) are considered as feasible solutions.

• Rule 3: Between two feasible solutions, the one having the better objective function value is preferred.

• Rule 4: Between two infeasible solutions, the one having the smaller sum of constraint violation is preferred.

Using the first and fourth rules, the search is oriented to the feasible region rather than the infeasible region. Applying the third rule guides the search to the feasible region with good solutions (Mezura-Montes and Coello, 2008). For most structural optimization problems, the global minimum locates on or close to the boundary of a feasible design space. By applying rule 2, the individuals approach the boundaries and can reach the global minimum with a higher probability (Kaveh and Talatahari, 2009).

The task of optimizing the proposed model was executed in 15 independent runs for all the optimizers under consideration. Initial parameters of metaheuristic algorithms vary from an algorithm to the other. For having fair comparison, all reported methods have been implemented under the same function evaluations as stopping condition. It is worth mentioning that, all reported optimzers in this thesis have been tuned in terms of their user parameters. Regarding the initial population, as widely used in metaheuristic methods, uniform disributed random numbers have been generated for all considered optimizers. Indeed, metaheuristci optimization methods do not rely on the initial solution (population) unlike classical and exact optimization methods such as Newton method.

The initial parameters of the GA included the population size of 50 individuals, scattered crossover fraction of 0.8, stochastic uniform as a selection function, and rank as a scaling function. Accordingly, for the PSO, the initial parameters consisted of the population size of 50 individuals, the inertia weight for velocities of 0.8, and cognitive and social components (c_1 and c_2) of 1.5. The initial parameters for the ICA were chosen as number of country of 50, number of imperialist country of 4, revolution rate of 0.3. In addition, the initial parameters for the WCA, (N_{pop}, N_{sr}, and d_{max}) were chosen as 50, 4, and 1e-05, respectively.

The number of function evaluations (NFEs) determines the speed (computational effort) and the robustness of the algorithm. Less NFEs, means less time to reach the global optimum. This feature returns to the structure of the algorithms. Best solution represents the accuracy of the method. The NFEs and the best solution are dependent on each other. The ideal situation is less NFEs and a more accurate solution. To insure the fairness of comparison with other algorithms, a maximum NFEs of 50,000 (NFEs = Number of iterations (generation) × Number of population) was imposed for all reported optimizers. The maximum number of iterations was 1000 for the GA, PSO, ICA and

WCA. The detailed settings of applied approaches are tabulated in Table 4.2 for more clarification.

In terms of the complexity level of the FNDSP, the number of generated routes is also considered as design variable, which vary from one individual to another in a generated population. Indeed, the total number of design variables can vary with this problem. This means that considered problem is categorized as a dynamic optimization problem, which has various design variables based on each individual.

The results of optimization of four presented algorithms are compared and discussed in the following subsections.

Table 4.2: Detailed initial parameter setting for the applied methods

Method	User-defined Parameters	
GA	Number of independent runs	15
	Number of iterations	1000
	Population size	50
	Maximum NFEs	50,000
	Scattered crossover	0.8
	Mutations percentages	0.3
	Number of parents for tournament selection	3
PSO	Number of independent runs	15
	Number of iterations	1000
	Population size	50
	Maximum NFEs	50,000
	Cognitive and social components (c_1 and c_2)	1.5
	Inertia weight(w)	0.8
ICA	Number of independent run	15
	Number of iterations	1000
	Number of country(i.e., Countries)	50
	Maximum NFEs	50,000
	Number of imperialist countries (N_{ipm})	4
	Revolution rate	0.3
WCA	Number of independent run	15
	Number of iterations	1000
	Number of population (i.e., streams)	50
	Maximum NFEs	50,000
	Number of rivers and sea (N_{sr})	4
	d_{max}	1e-05

4.2.1 Computational Results Obtained by GA for the Benchmark Study Area

For optimizing the proposed model, a total of 15 runs is performed. All cost terms, best transit network and frequency of each feeder route are recorded. The computational results of the GA are summarized in Appendix E (a). The obtained total cost (C_T) is highlighted in bold in table for all the runs. As presented in Appendix E (a) the minimum total cost of $31994, user cost of $26512, operation cost of $5318.3 and social cost of $103.1 are obtained for GA. The average service frequency range for each route is from 11.3 to 12.4 trips per hour (headways in the range of 4 to 5 minutes).

Accordingly, Table 4.3 demonstrates the comparison procedure of obtaining statistical results for the reported optimizer in the FNDSP.

Table 4.3: Gained statistical optimization results for each cost term using the GA

Parameter	Best	Mean	Worst	SD
C_T	31994.0	32664.0	33353.0	323.0
C_u	26512.0	27039.0	27556.0	241.2
C_o	5318.5	5495.6	5678.3	93.7
C_w	5782.0	5921.3	6060.0	77.6
C_{ui}	3854.6	4309.8	4688.2	211.4
C_{fF}	661.3	705.6	762.2	27.2
C_{mF}	1104.4	1178.4	1273.0	45.5
C_{pF}	1313.0	1398.0	1429.0	60.1
C_s	103.1	110.0	118.8	4.2
A_F	11.3	11.8	12.4	0.3
T_{VK}	1472.9	1571.5	1697.1	60.2
T_{PK}	20592.0	22447.4	25084.8	1327.0

Note: All cost values are based on USD.

As shown in Table 4.3, an average total cost of $32664 and standard deviation (SD) of 323 are obtained for GA.

The best solution is obtained by GA among all the runs, as shown in Table 4.4.

Table 4.4: Best solution obtained by GA

Route number	Route demand (passenger/hr)	Route length (km)	Route frequency (tripe/hr)	Route vehicle-length (vehicle-km)	Route passenger-length (passenger-km)
1	400	2.36	11.69	55.08	1205.36
2	400	3.97	9.43	74.86	1239.60
3	600	1.51	16.88	50.99	640.41
4	600	3.61	12.03	86.80	1034.72
5	400	1.46	13.96	40.63	469.42
6	600	3.12	12.78	79.83	1500.56
7	600	2.49	14.01	69.73	908.88
8	600	2.47	14.04	69.46	943.36
9	600	2.67	13.62	72.72	923.68
10	400	1.90	12.70	48.15	476.10
11	400	3.03	10.57	64.01	800.96
12	400	1.83	12.87	47.04	424.64
13	400	3.22	10.31	66.32	860.48
14	400	2.26	11.87	53.74	806.80
15	400	2.68	11.11	59.52	797.54
16	400	1.85	12.82	47.38	653.70
17	400	3.65	9.78	71.31	906.72
18	400	2.95	10.68	63.01	807.52
19	400	3.02	10.59	63.84	766.15
20	400	1.94	12.59	48.84	604.24
21	400	3.21	10.32	66.22	935.36
22	600	3.21	12.64	81.10	1400.88
23	400	2.70	11.08	59.73	805.76
24	400	2.45	11.51	56.36	678.94

C_T=31994	C_u=26512.4	C_o=5377	C_s=104.8	C_w=5849.8	C_{ui}=3854.6
T_{PK}=20592.0	C_{fF}=672.1	C_{mF}=1122.5	C_{pF}=1396	AF=12.1	T_{VK}=1497.1

Note: All cost values are based on USD.

As presented in Table 4.4, a total cost of $31994 is realized and 24 feeder bus routes are designed. The range of service frequency is from 9.43 to 16.88 trips per hour (headways in the range of 3.55 to 6.36 minutes). The average frequency is 12.1 trips per hour, and total route distance per square kilometer is 4.97 kilometers. The case solution includes 1497.1 vehicle-km of travel and 20592 passenger-km of travel. The provided total cost consists of 82.9 % user, 16.8 % operator and 0.3 % social cost, which are shown graphically in Figure 4.1. It can be seen that the main costs are ranked as user, operator, and social cost, respectively from maximum to minimum. Furthermore, all of the costs based on USD for each of the cost terms are shown in Figure 4.2.

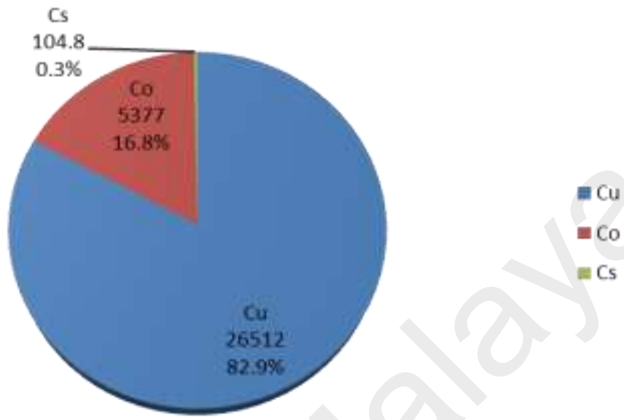


Figure 4.1: Summary of the main costs (USD) achieved by GA

Figure 4.2: Obtained best result and each cost term (USD) using the GA

Figure 4.3 demonstrates the convergence rate and cost history (cost reduction) of the best solution on considered optimization engine (GA).

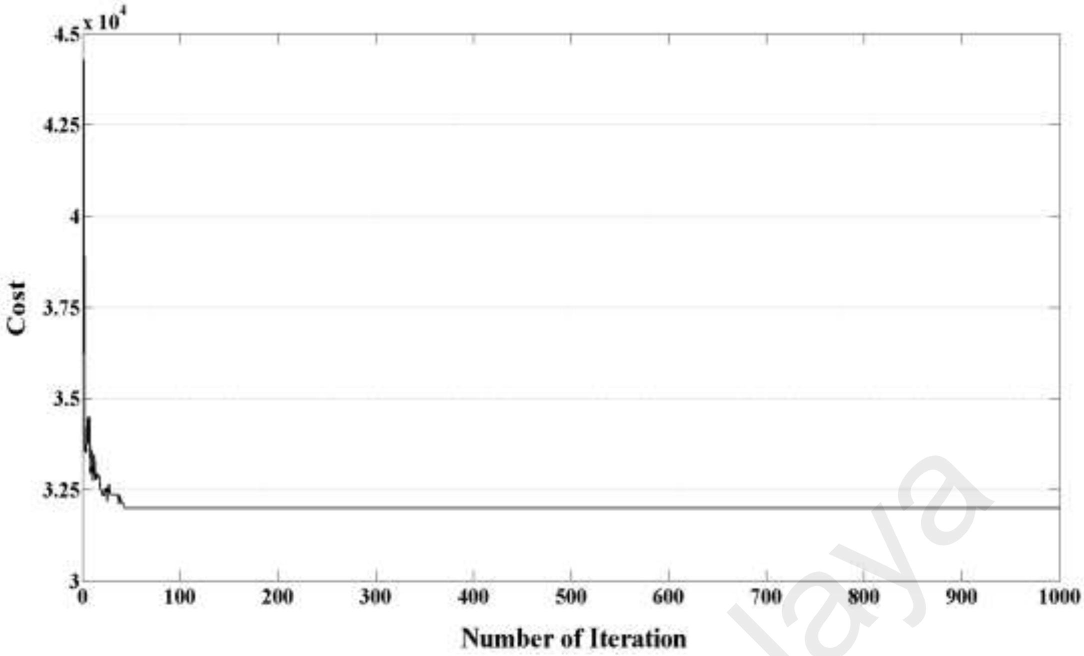


Figure 4.3: Convergence rate and cost history (USD) with respect to the number of iterations for the GA

From Figure 4.3, it can be seen that the convergence rate for the GA method is fast in earlier generations. However, as the number of iterations increases, the GA cost reduction was not considerably high compared with its earlier iterations.

Comparison of GA results with other algorithms is discussed in section 4.2.5.

4.2.2 Computational Results Obtained by PSO for the Benchmark Study Area

For optimizing the proposed model, a total of 15 runs is performed. All cost terms, best transit network and frequency of each feeder route are recorded. The computational results of the PSO are summarized in Appendix E (b). The obtained total cost (C_T) is highlighted in bold in this table for all the runs. As is evident from this table, the minimum total cost of $31293, user cost of $25896.8, operation cost of $5286.7 and social cost of $103 are obtained for PSO. The average service frequency range for each route is from 11.5 to 13.1 trips per hour (headways in the range of 4.58 to 5.21 minutes).

Accordingly, Table 4.5 demonstrates the comparison procedure of obtaining the statistical results for the reported optimizer in the FNDSP.

Table 4.5: Obtained statistical optimization results for each cost term using the PSO

Parameter	Best	Mean	Worst	SD
C_T	31293.0	31725.6	32021.0	223.0
C_u	25896.8	26209.2	26551.3	218.0
C_o	7895.2	8139.9	8355.0	160.6
C_w	5565.9	5806.0	6034.0	155.5
C_{ui}	3286.4	3595.2	4143.5	277.6
C_{fF}	529.1	674.0	727.9	43.8
C_{mF}	1104.1	1148.0	1217.7	35.3
C_{pF}	1292.6	1375.4	1434.4	54.1
C_s	103.1	108.1	113.7	3.2
A_F	11.5	12.3	13.1	0.5
T_{VK}	1472.1	1544.9	1623.6	46.3
T_{PK}	19132.0	21694.2	24172.8	1585.7

Note: All cost values are based on USD.

As presented in Table 4.5 an average total cost of $31725.6, the best total cost of $31293 and SD of 223 are obtained for PSO.

The best solution is obtained by PSO among all the runs, as shown in Table 4.6.

The transit network consists of 20 feeder bus routes with service frequency ranging from 10.59 to 16.11 trips per hour (headways in the range of 3.72 to 5.7 minutes). The average headway is 12.67 minutes, and total route distance per square kilometer is 4.65 kilometers. The case solution includes 1500.4 vehicle-km of travel, 22413.6 passenger-km of travel.

Table 4.6: Best solution obtained by PSO

Route number	Route demand (passenger/hr)	Route length (km)	Route frequency (tripe/hr)	Route vehicle-length (vehicle-Km)	Route passenger-length (passenger-Km)
1	200	1.18	10.59	24.96	308.34
2	600	3.83	12.00	91.80	1634.24
3	600	2.97	13.04	77.54	1214.16
4	600	2.68	13.61	72.86	1088.64
5	600	4.00	12.00	99.16	1652.64
6	600	3.56	12.10	86.17	1224.80
7	600	2.77	13.43	74.28	1198.88
8	600	3.39	12.35	83.75	1321.04
9	600	2.99	13.01	77.77	879.60
10	600	2.55	13.87	70.77	912.40
11	600	2.86	13.24	75.81	1006.64
12	600	4.00	12.00	98.92	1656.00
13	400	0.93	16.11	29.93	299.50
14	400	2.61	11.23	58.55	815.04
15	600	3.98	12.00	95.55	1608.48
16	600	2.32	14.39	66.90	864.72
17	600	3.58	12.07	86.40	1353.20
18	600	3.68	12.00	88.28	1724.08
19	400	1.99	12.48	49.59	560.66
20	600	3.81	12.00	91.51	1090.56
C_T=31293	C_u=25895	C_o=5291	C_s=105	C_w=5681	C_{ui}=3407
T_{PK}= 22413.6	C_{fF}=674	C_{mF}=1125	C_{pF}=1304	AF=12.67	T_{VK}= 1500.4

Note: All cost values are based on USD.

The provided total cost consists of 82.8 % user, 16.9 % operator and 0.3% social cost, which are illustrated graphically in Figure 4.4. It can be observed that the main costs are ranked as user, operator, and social cost, respectively from maximum to minimum. Furthermore, all of the costs based on USD for each of the cost terms are shown in Figure 4.5. It was emerged from Figure 4.5, that the maximum cost of $25895 belongs to C_U cost term while C_s has the minimum cost of $105.

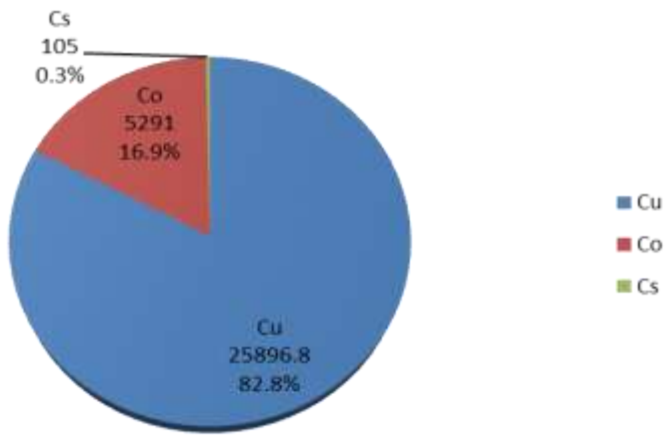
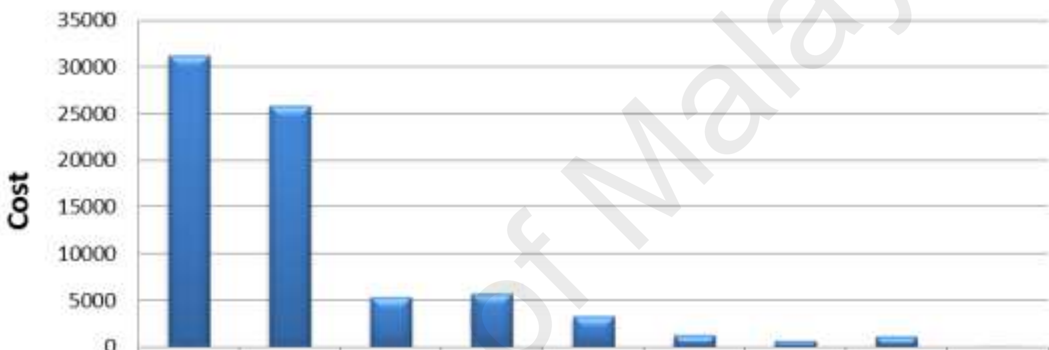


Figure 4.4: Summary of the main costs (USD) achieved by PSO



	C_T	C_u	C_o	C_W	C_{ui}	C_{pF}	C_{fF}	C_{mF}	C_s
PSO	31293	25896.8	5291	5681.8	3286.4	1304.3	673.8	1125.4	105

Figure 4.5: Obtained best result and each cost term (USD) using the PSO

Figure 4.6 demonstrates the convergence rate and cost history of the best solution on considered optimization engine (PSO).

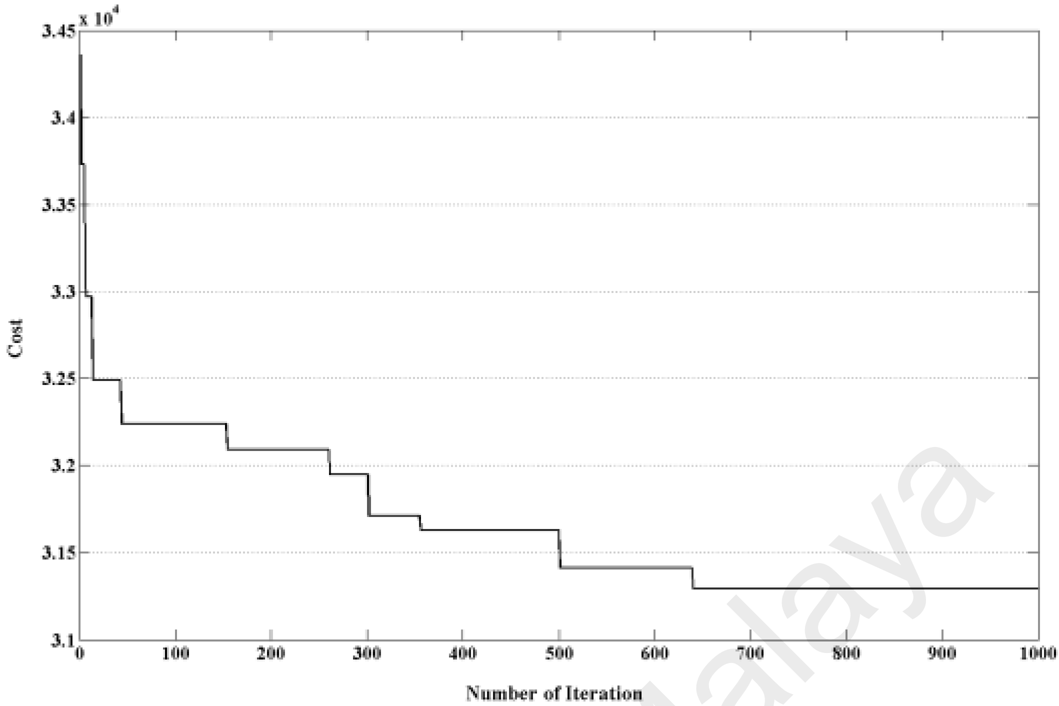


Figure 4.6: Convergence rate and cost history (USD) with respect to the number of iterations for the PSO

Comparison of PSO results with other algorithms is discussed in section 4.2.5.

4.2.3 Computational Results Obtained by ICA for the Benchmark Study Area

For optimizing proposed model, a total of 15 runs is performed. All cost terms, best transit network and frequency of each feeder route are recorded. Appendix E (c) shows the detailed computational results for each term of the cost function using the applied algorithm. The obtained total cost (C_T) is highlighted in bold in this table for all the runs. As presented in Appendix E (c) the minimum total cost of $29549, user cost of $24789.31, operation cost of $4629.1 and social cost of $73.34 are obtained for ICA. The average service frequency range for each route is from 13.5 to 15.3 trips per hour (headways in the range of 3.92 to 4.44 minutes).

Accordingly, Table 4.7 demonstrates the comparison procedure of obtaining the statistical results for the reported optimizer in the FNDSP.

Table 4.7: Attained statistical optimisation results for each cost term using the ICA

Parameter	Best	Mean	Worst	SD
C_T	29549.0	29964.1	30590.0	250.8
C_u	24789.3	25128.2	25523.4	215.4
C_o	4629.1	4758.3	5007.5	97.9
C_w	5074.5	5200.7	5462.6	97.6
C_{ui}	2815.5	3119.6	3601.1	218.6
C_{fF}	470.5	496.1	554.7	23.2
C_{mF}	785.8	828.6	926.4	38.8
C_{pF}	1269.9	1387.2	1459.4	71.0
C_s	73.3	77.3	86.5	3.6
A_F	13.5	14.7	15.3	0.5
T_{VK}	1047.7	1104.7	1235.1	51.7
T_{PK}	12278.5	14191.2	15835.9	1103.0

Note: All cost values are based on USD.

As shown in Table 4.7 an average total cost of $29964.1, the best total cost of $29549 and SD of 250.8 are obtained for ICA.

The best solution is obtained by ICA, among all the runs, as shown in Table 4.8.

As shown in Table 4.8, the transit network consists 23 feeder bus routes with service frequency ranging from 11.89 to 18.12 trips per hour (headways in the range of 3.31 to 5.05 minutes). The average headway is 15.13 minutes, and total route distance per square kilometer is 2.85 kilometers. The case solution includes 1049.3 vehicle-km of travel, 13467.6 passenger-km of travel.

Table 4.8: Best solution obtained by ICA

Route number	Route demand (passenger/hr)	Route length (km)	Route frequency (tripe/hr)	Route vehicle-length (vehicle-Km)	Route passenger-length (passenger-Km)
1	400	1.84	12.85	47.19	1164.64
2	600	2.72	13.52	73.58	1295.44
3	600	2.38	14.25	67.92	1066.40
4	400	1.16	15.06	34.88	368.16
5	400	0.74	17.19	25.32	238.35
6	600	1.48	17.00	50.33	614.05
7	600	1.85	15.69	58.09	822.56
8	600	1.81	15.81	57.34	662.22
9	600	1.53	16.81	51.37	576.44
10	600	1.95	15.39	60.03	769.31
11	400	1.11	15.25	33.95	285.79
12	400	1.11	15.26	33.91	315.63
13	400	0.66	17.71	23.22	190.28
14	400	1.81	12.91	46.83	579.84
15	400	2.26	11.89	53.63	735.26
16	400	0.60	18.12	21.65	167.78
17	600	1.71	16.17	55.14	671.52
18	400	1.12	15.21	34.15	285.67
19	400	1.26	14.66	36.88	414.64
20	400	1.96	12.55	49.10	673.18
21	400	1.16	15.05	34.92	360.19
22	400	1.32	14.42	38.15	358.55
23	600	2.04	15.13	61.72	851.76
C_T=29549	C_u=24789	C_o=4686	C_s=73	C_w=5128	C_{ui}=2854
T_{PK}= 13467.6	C_{fF}=471	C_{mF}=787	C_{pF}=1401	AF=15.13	T_{VK}=1049.3

Note: All cost values are based on USD.

Accordingly, the main costs (user, operator, and social) and cost terms are illustrated graphically in Figure 4.7 and 4.8. As shown the provided total cost consists of 83.9 % user, 15.9 % operator and 0.2 % social cost.

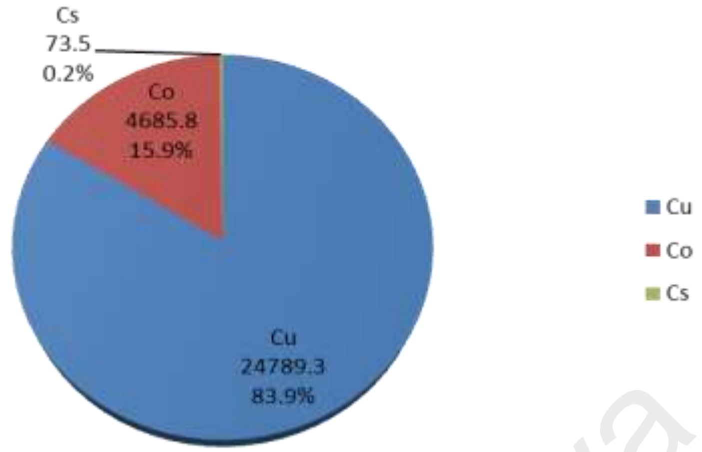


Figure 4.7: Summary of the main costs (USD) achieved by ICA

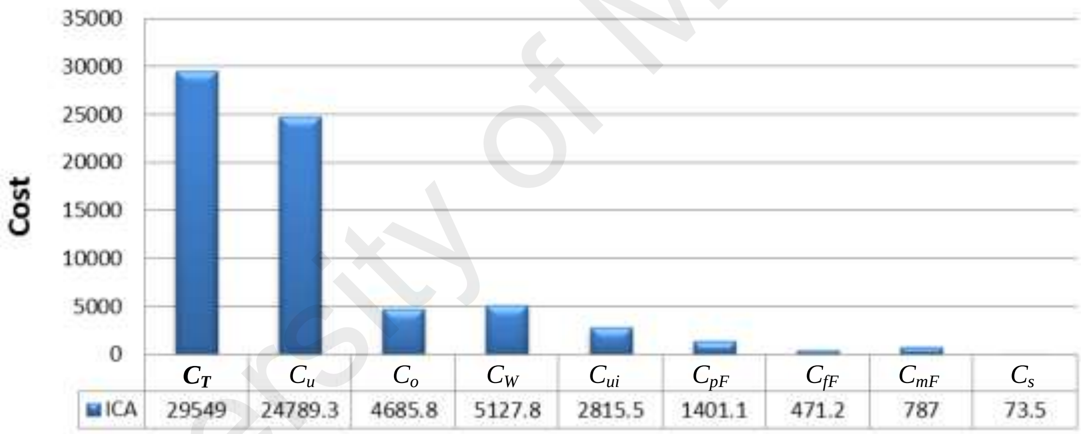


Figure 4.8: Obtained best result and each cost term (USD) using the ICA

Figure 4.9 demonstrates the convergence rate and cost reduction of the best solution on considered optimization engine (ICA).

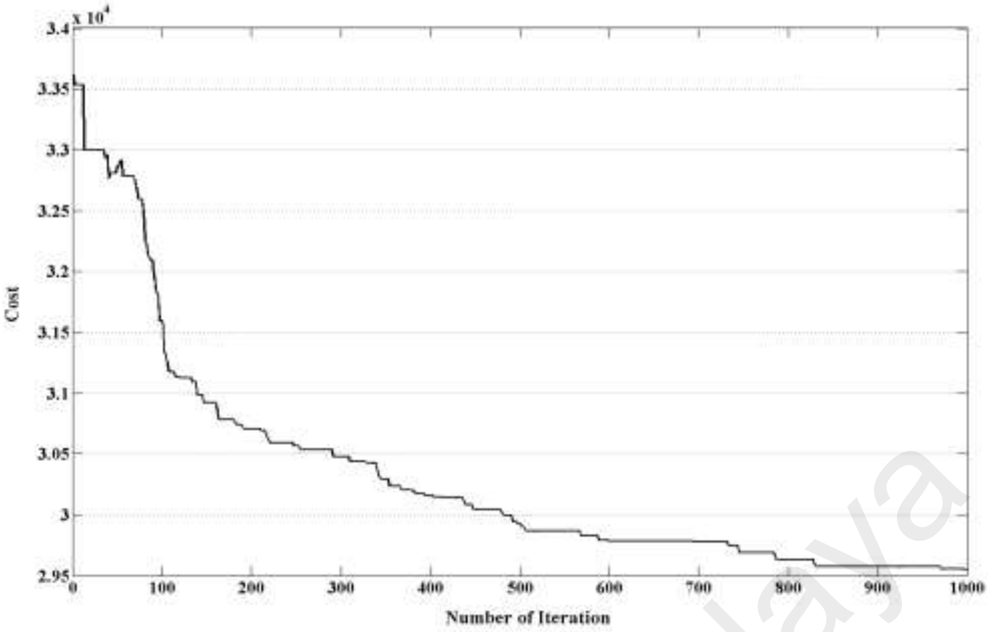


Figure 4.9: Convergence rate and cost history (USD) with respect to the number of iterations for the ICA

Comparison of ICA results with other algorithms is discussed in section 4.2.5.

4.2.4 Computational Results Obtained by WCA for the Benchmark Study Area

For optimizing proposed model, a total of 15 runs is performed. All cost terms, best transit network and frequency of each feeder route are recorded. Appendix E (d) shows the detailed computational results for each term of the cost function using the applied algorithm. The obtained total cost (C_T) is highlighted in bold in table for all the runs. As presented in Appendix E (d) the minimum total cost of \$29377, user cost of \$24647.54, operation cost of \$4564.65 and social cost of \$70.47 are obtained for WCA. The average service frequency range for each route is from 14.1 to 15.8 trips per hour (headways in the range of 3.79 to 4.25 minutes). Accordingly, Table 4.9 demonstrates the comparison procedure of obtaining the statistical results for the reported optimizer in the FNDSP.

Table 4.9: Gained statistical optimization results for each cost term using the WCA

Parameter	Best	Mean	Worst	SD
C_T	29377.0	29679.1	30055.0	227.5
C_u	24647.5	24867.1	25162.6	166.4
C_o	4564.7	4927.4	4927.4	111.3
C_w	4995.2	5171.3	5377.7	113.7
C_{ui}	2689.5	2887.7	3292.0	174.6
C_{fF}	452.1	484.1	526.2	20.9
C_{mF}	755.0	808.5	878.9	35.0
C_{pF}	1303.9	1407.6	1466.7	71.6
C_s	70.5	75.5	82.0	3.3
A_F	14.1	14.9	15.8	0.5
T_{VK}	1006.7	1077.9	1171.9	46.6
T_{PK}	11924.8	13705.0	15311.9	1037.2

Note: All cost values are based on USD.

As shown in the Table 4.9 an average total cost of $29679.1, the best total cost of $29377 and SD of 227.5 are obtained for WCA.

The best solution is obtained by WCA, among all the runs, as shown in Table 4.10. As shown in this table, the transit network consists of 20 feeder bus routes with service frequency ranging from 12.81 to 18.39 trips per hour (headways in the range of 3.26 to 4.68 minutes). The average headway is 15.85 minutes, and total route distance per square kilometer is 2.7 kilometers. The case solution includes 1026.7 vehicle-km of travel, 14369.7 passenger-km of travel.

Table 4.10: Best solution obtained by WCA

Route number	Route demand (passenger/hr)	Route length (km)	Route frequency (tripe/hr)	Route vehicle-length (vehicle-Km)	Route passenger-length (passenger-Km)
1	400	1.73	13.13	45.43	1117.44
2	600	2.64	13.69	72.19	1151.84
3	600	1.17	18.39	43.01	459.86
4	600	1.38	17.42	48.02	576.32
5	600	2.72	13.52	73.56	1326.88
6	600	1.41	17.27	48.80	564.07
7	800	2.71	16.00	86.82	1462.40
8	600	2.34	14.34	67.24	1162.16
9	400	0.83	16.65	27.58	261.62
10	600	1.53	16.82	51.34	591.10
11	600	2.08	15.03	62.42	792.19
12	600	3.11	12.81	79.57	1165.60
13	600	1.03	19.13	39.41	393.62
14	600	1.49	16.95	50.57	631.22
15	400	1.49	13.86	41.20	415.38
16	600	1.29	17.83	45.84	521.28
17	200	0.24	15.14	7.31	48.27
18	600	1.65	16.37	53.94	726.66
19	400	1.33	14.38	38.35	469.18
20	600	1.22	18.16	44.15	532.61
C_T=29377	C_u=24741	C_o=4565	C_s=71.9	C_w=4995.2	C_{ui}=2937.6
T_{PK}=14369.7	C_{fF}=461	C_{mF}=770.1	C_{pF}=1314.8	AF=15.85	T_{VK}=1026.7

Note: All cost values are based on USD.

Summary of the main costs and Obtained best result and each cost term achieved by WCA are presented in Figure 4.10. The provided total cost consists of 84.2 % user, 15.5 % operator and 0.3 % social cost. It can be observed that the main costs are ranked as user, operator, and social cost, respectively from maximum to minimum. Furthermore, all of the costs based on USD for each of the cost terms are illustrated graphically in Figure 4.11.

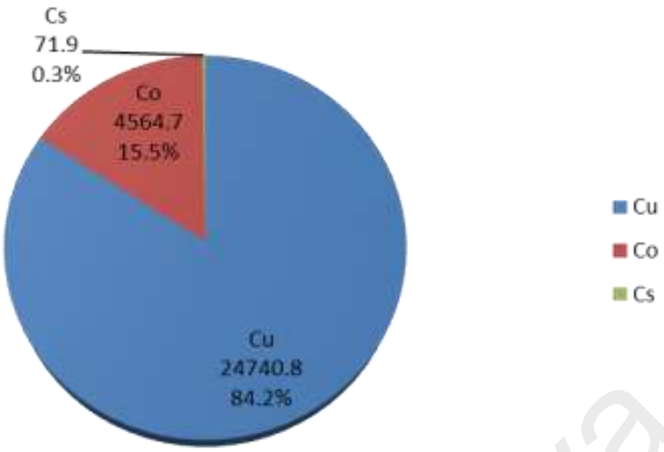


Figure 4.10: Summary of the main costs (USD) achieved by WCA

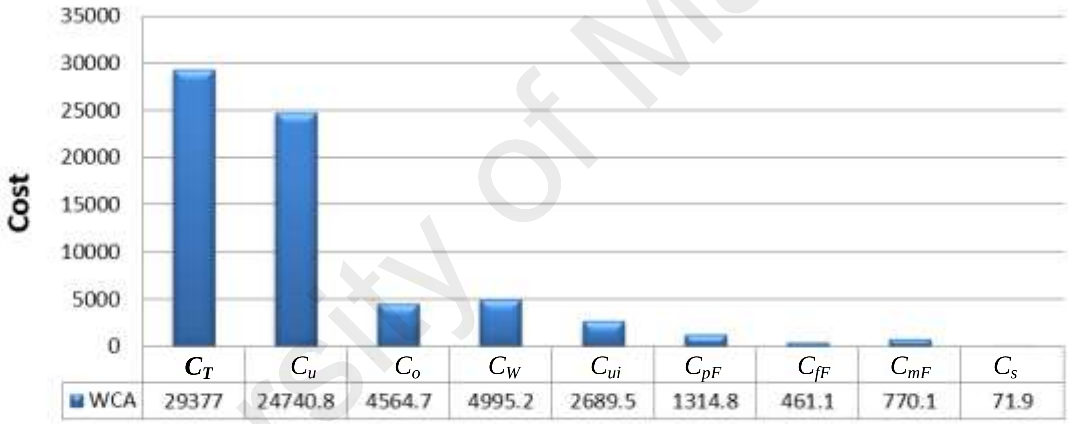


Figure 4.11: Obtained best result and each cost term (USD) using the WCA

Figure 4.12 demonstrates the convergence rate and cost history of the best solution on considered optimization engine (WCA).

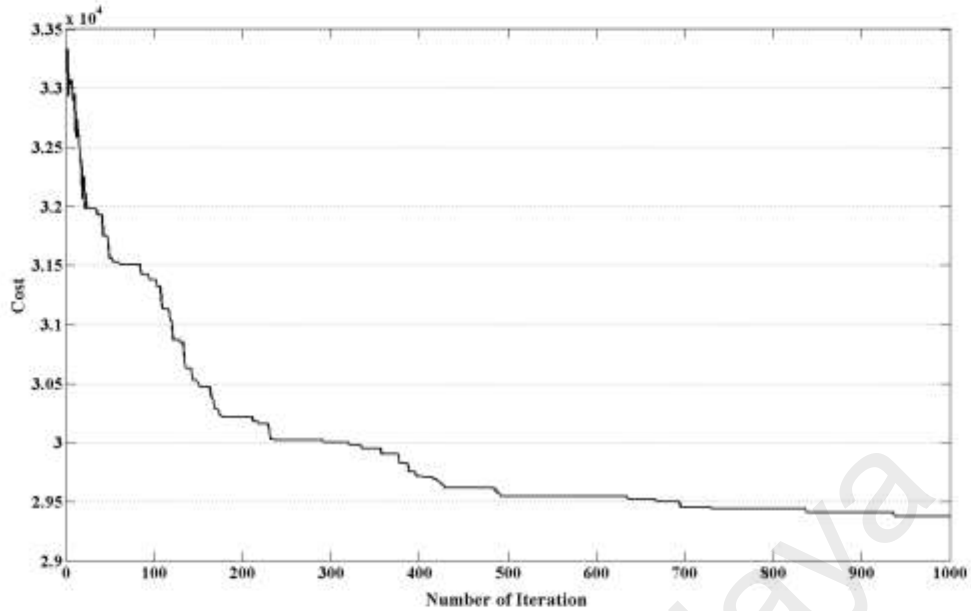


Figure 4.12: Convergence rate and cost history (USD) with respect to the number of iterations for the WCA

As shown in Figure 4.12, the convergence rate for the WCA method is fast in earlier generations. However, as the number of iterations increases, the WCA cost reduction was not significantly high compared with its earlier iterations.

Comparison of WCA results with other algorithms is discussed in section 4.2.5.

4.2.5 Comparison and Discussion of the Results for the Benchmark Study Area

Table 4.11 shows the comparison of the best solution attained for all cost terms using applied optimization engines for the improved model, which are illustrated graphically in Figure 4.13. The obtained total cost (C_T) is highlighted in bold in this table for all algorithms.

Table 4.11: Comparison of the best obtained solutions for the transit service model using reported methods

Method	C_W	C_{ui}	C_{fF}	C_{mF}	C_{pF}	C_u	C_o	C_s	C_T	A_F	T_{PK}
GA	5849.8	3854.6	672.1	1122.5	1396.0	26512.0	5377.0	104.8	**31994.0**	12.1	20592.0
PSO	5681.8	3286.4	673.8	1125.4	1304.3	25896.8	5291	105.0	**31293.0**	12.7	22413.6
ICA	5127.8	2815.5	471.2	787.0	1401.1	24789.3	4685.8	73.5	**29549.0**	15.1	13467.6
WCA	4995.2	2689.5	461.1	770.1	1314.8	24740.8	4564.7	71.9	**29377.0**	15.8	14369.7

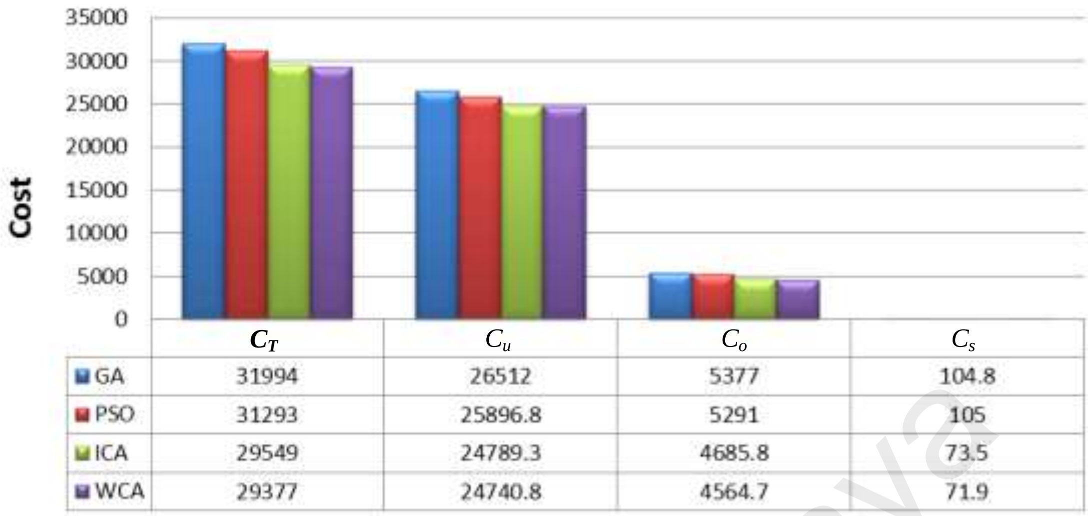


Figure 4.13: Comparison of obtained best result and main cost terms (USD)

Tables 4.11 and figure 4.13 shows the detailed optimization results for each term of modified cost function using applied algorithms. It can be concluded that the WCA is superior to other optimizers in obtaining all cost terms except for the C_P and C_{ui}.

Accordingly, Table 4.12 demonstrates the comparison ways of obtaining statistical results for four reported optimizers for the FNDSP. optimizers

Table 4.12: Comparison of statistical results gained by optimizers under consideration. 'SD' stands for standard deviation

Optimizer	Best Solution	Mean Solution	Worst Solution	SD
GA	31994.00	32644.00	33353.00	323.00
PSO	31293.00	31725.60	32021.00	222.98
ICA	29549.00	29964.13	30590.00	250.84
WCA	29377.00	29679.07	30055.00	227.46

From Table 4.11 and 4.12, the WCA has gained the best statistical results, among others, for the FNDSP with the minimum cost of \$29377. The ICA, PSO, and GA are ranked 2nd, 3rd and 4th respectively. It is worth mentioning that the WCA has obtained the minimum cost for the C_T while the PSO has offered better mean, worst, and SD solutions. Regarding the low value of SD in PSO algorithm, it may be due to selected values of the inertia factor and acceleration coefficients (c_1 and c_2) which imply high tuning dependency of PSO that can be counted as a drawback to this method. In other words, PSO in this context seems to be a more parameter dependent method than WCA. Consequently, WCA has provided better results than PSO.

Figure 4.14 demonstrates the convergence rate and cost reduction among considered optimization engines. As presented in this figure, the WCA and ICA are faster and more accurate than PSO and GA in achieving its optimum solutions (solution quality). The GA cost reduction was not considerably high compared with its earlier iterations. It can be seen that the convergence rate for the WCA method is faster than ICA in earlier generations. However, as the number of iterations increases, the convergence rate for both algorithms becomes nearly the same.

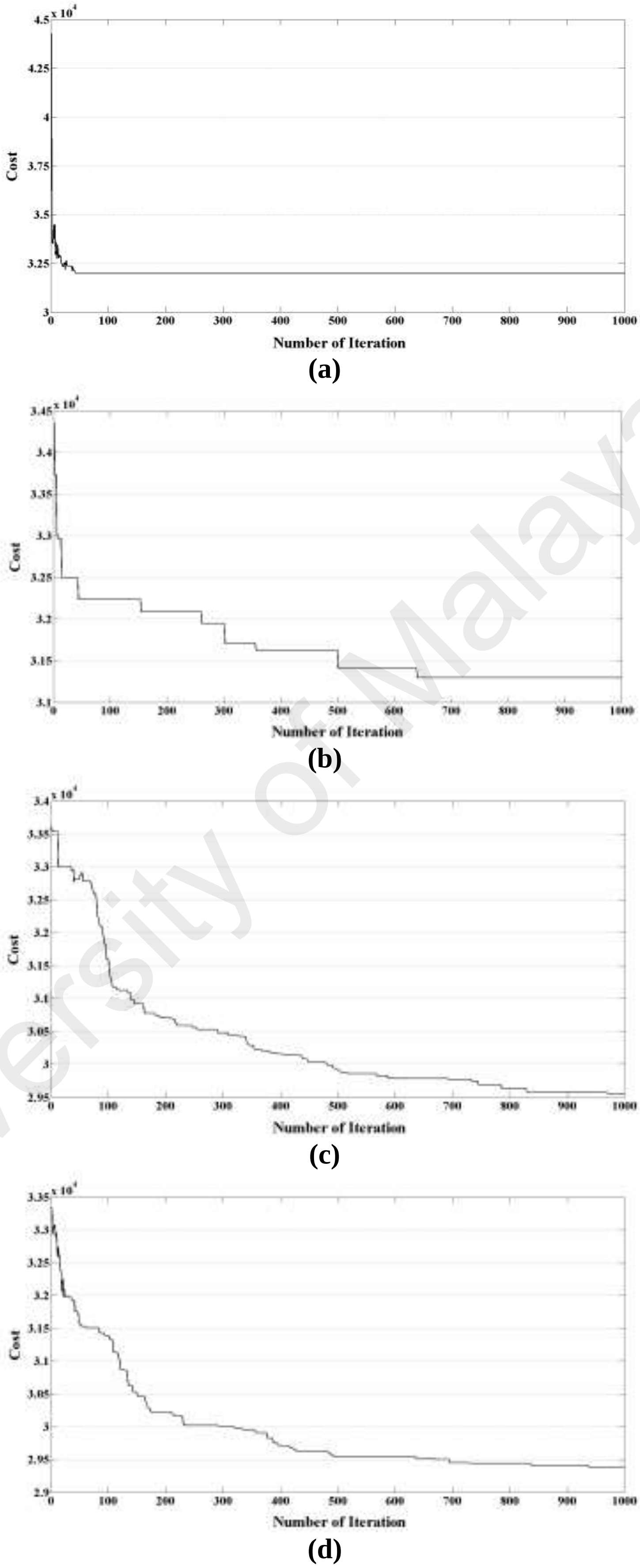


Figure 4.14: Comparison of convergence rate and cost history (USD) with respect to the number of iterations (generations) for the: (a) GA, (b) PSO, (c) ICA and (d) WCA

In addition, considering the convergence rates approaching 1000 iterations in Figure 4.14, it seems that by increasing the number of iterations of WCA and ICA methods, lower cost might be achieved. Therefore, all optimization methods were executed in 5000 iterations. The GA and PSO could not improve the results; however total cost of \$29123 and \$28875 was achieved by WCA and ICA respectively.

The best solution abstained by ICA among all the runs illustrated in Figure 4.15.

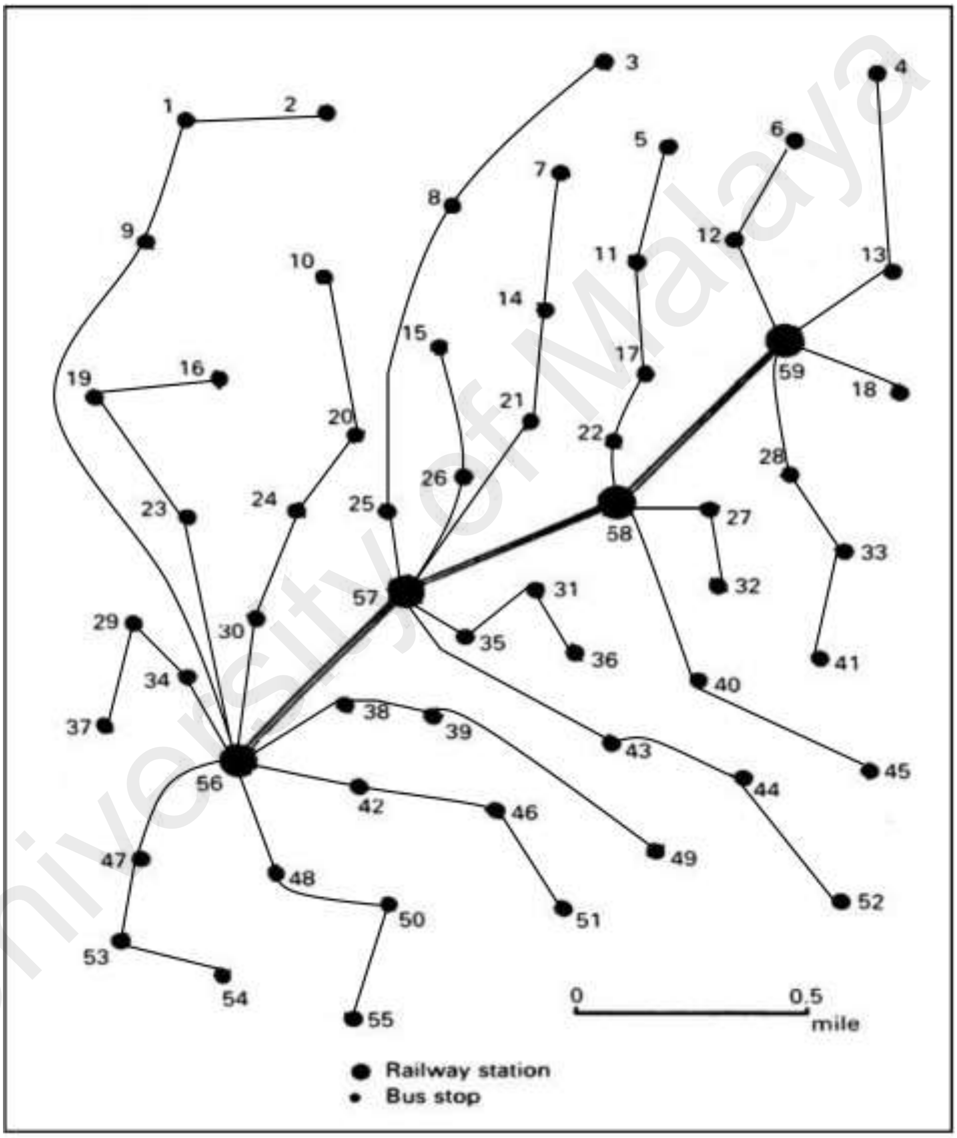


Figure 4.15: The best solution abstained by ICA among all the runs

Figure 4.16 demonstrates the convergence rate and cost history (cost reduction) of the best solution among ICA with 5000 iterations.

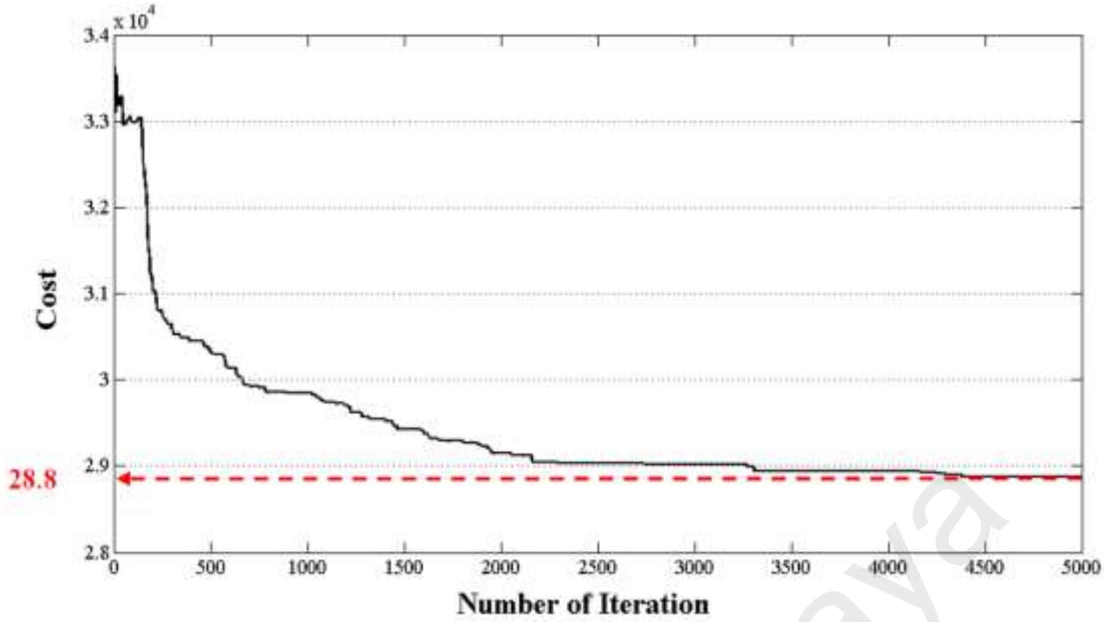


Figure 4.16: Convergence rate and cost history (USD) with respect to the maximum iteration number of 5000 for the ICA

Considering the presented objective function and the imposed constraints, the achievement of these levels of total costs shows that the proposed model can be regarded as a potential method to overcome current difficulties in the FNDSP.

A comparison of the best results achieved by four metaheuristic algorithms applied for optimizing the improved model with the heuristic method used in the literature by Kuah and Perl in 1989 as a benchmark is shown in Table 4.13.

It can be clearly seen in Table 4.13 that the WCA and ICA yield comparatively better solutions.

Table 4.13: Comparison between applied metaheuristic and the heuristic results

Methods	Total cost ($/hr)
Heuristic [a]	29,010
GA	31,994
PSO	31,293
ICA	**28,875**
WCA	28,924

[a] From Kuah and Perl 1989.

Statistical optimization results were the basis for comparison in this research. Under the same function evaluations as stopping condition, as it can be seen in Tables 4.11, 4.12 and 4.13, WCA and ICA show better quality solutions in terms of minimum cost. Also, it can be seen in Figure 4.14 WCA and ICA display fast and mature convergence rate compared with the other considered methods.

The results of the application and optimization of FNDSP on the benchmark study area provided more accurate and efficient solutions of various conditions of transit systems. The outputs of these solutions have demonstrated that the presented model has been verified, and the proposed WCA and ICA have been considered a suitable approach in order to gain moderate quality solutions with reasonable computational cost.

4.2.6 Sensitivity Analysis

The sensitivity analysis for the parameters (i.e., $\mu_{I,}$ $\lambda_{lt,}$ $\lambda_{I,}$ $t_{dF,}$ $t_{dT,\ and}$ S_{kj} , etc.) in the proposed transit system model is discussed in this section. The purpose of this analysis is to show the relations between the parameters and related costs. This analysis gives us very good insight into the relations between proposed transit model parameters and cost terms and shows the importance of this related cost terms significantly. The sensitivity of the acceptable values of parameters based on the literature is investigated for the tested problem. The comparisons were conducted between the best transit service achieved by solution methods (Base scenario) and other scenarios realized by different value of parameters. Table 4.14 demonstrated comparison of cost characteristics for different ratios of $t_{dF,}$ t_{dT} and S_{kj} in transit service.

Table 4.14: Comparison of cost characteristics for different ratio of of $\boldsymbol{t_{dF},\ t_{dT}, S_{kj}}$ in transit service

Parameter	Base scenario	Scenario 1	Scenario 2
	$\boldsymbol{t_{dF=0.096},\ t_{dT=0.03}}$	$\boldsymbol{t_{dF=0},\ t_{dT=0}}$	$\boldsymbol{S_{kj=0}}$
C_T	28875.1	27363.0	28036.1
C_u	24349.8	24209.0	24349.8
C_o	4458.6	3087.3	3619.6
C_{ui}	2701.9	2561.1	2701.9
C_{oi}	1992.2	800.9	1992.2

Note: All cost values are based on USD.

Table 4.14 shows the numerical results of the transit cost terms for different values of the parameters. A value of zero for $t_{dF,}$ t_{dT} *and* S_{kj} implies no dwell cost and slack cost; therefore leaves the network unchanged. The total cost of scenario 1 and 2 wasdecreased from 28875.1 to 27363 and from 28875.1 to 28036.1 compare of the base scenario. The percentage of total cost was decreased by 5.24% and 2.91% respectively. Significant impacts were detected for cases with adding this value costs in the transit service model.

Table 4.15 demonstrations cost terms versus. different $\boldsymbol{\mu_I}$ values.

Table 4.15: Comparison of cost characteristics for different ratio of μ_I in transit service

Parameter	Scenario 3	Base scenario	Scenario 4
	$\boldsymbol{\mu_I=3}$	$\boldsymbol{\mu_I=4}$	$\boldsymbol{\mu_I=5}$
C_T	28199.7	28875.1	29550.6
C_u	23674.4	24349.8	25025.3
C_o	4458.6	4458.6	4458.6
C_{ui}	2026.4	2701.9	3377.4
C_{oi}	1992.2	1992.2	1992.2

Note: All cost values are based on USD.

Table 4.15 shows the numerical results of the transit cost terms for different value of passenger riding cost. In scenario 3 the total cost decreased from 28875.1 to 28199.7. The user in-vehicle cost decreased from 2701.9 to 2026.4. In scenario 4, following the same pattern, these cost terms increased 2.34 percent for total cost and 25 percent for user in-vehicle cost. The results indicate that $\boldsymbol{\mu_I}$ can be a certain value of the user cost and total cost accordingly. A similar pattern was detected for train operation cost value. Table 4.16 demonstrated the comparison of cost terms for different ratio of $\boldsymbol{\lambda_{lt}}$ in transit service. In scenario 5 the total cost slightly decreased from 28875.1 to 28512.4 (1.26 percent). However, operation cost decreased from 4458.6.1 to 4095.9 (8.13 percent). The results indicate role of $\boldsymbol{\lambda_{lt}}$ as a highly effective value of the operation cost.

Table 4.16: Comparison of cost characteristics for different ratio of λ_{lt} in transit service

Parameter	Scenario 5	Base scenario	Scenario 6
	$\lambda_{lt}=135$	$\lambda_{lt}=180$	$\lambda_{lt}=225$
C_T	28512.4	28875.1	29237.8
C_u	24349.8	24349.8	24349.8
C_o	4095.9	4458.6	4821.3
C_{ui}	2701.9	2701.9	2701.9
C_{oi}	1629.5	1992.2	2354.9

Note: All cost values are based on USD.

Table 4.17 shows the numerical results of the transit cost terms for different values of λ_{lt} *and* λ_I.

Table 4.17: Comparison of cost characteristics for different ratio of λ_{lt} and λ_I in transit service

Parameter	Scenario 7	Base scenario	Scenario 8
	$\lambda_{lt}=135, \lambda_I=8.58$	$\lambda_{lt}=180, \lambda_I=11.4$	$\lambda_{lt}=225, \lambda_I=14.3$
C_T	28377.1	28875.1	29373.2
C_u	24349.8	24349.8	24349.8
C_o	3960.6	4458.6	4956.7
C_{ui}	2701.9	2701.9	2701.9
C_{oi}	1494.2	1992.2	2490.3

Note: All cost values are based on USD.

As shown in Table 4.17, the total cost of scenario 8 increased from 28875.1to 29373.2 compare than base scenario. The percentage of total cost was decreased by 1.73 percent. Similarly, the total operating cost increased 11.17 percent. It can be seen sensitivity of costs to value of this parameter.

This analysis shows the importance of this related cost terms significantly. Therefore, this model can be considered as an appropriate model to apply in the real case study in the PJ transit network. Thus, objective two was confirmed by applying and demonstrating an improved model based on the benchmark study area. Consequently, objective two was achieved by solving the FNDSP and achieving an optimum transit network as well as evaluating the performance of the users, operators and social perspectives.

4.3 Application and Optimization of Transit Network Problem in the PJ Study Area: Objective Three

The major purpose of this section is to apply and test the data for the real case study (PJ area) that was illustrated in section 3.6.2 in chapter three. The location of nodes (bus stops and rail stations) is given in Tables 3.5 and 3.6. Furthermore, the demand of each bus stop is listed in Table 4.18.

Table 4.18: Passenger demand of bus stops in the Petaling Jaya study area

Bus stop No.	Demand (passenger/hour)	Bus stop No.	Demand (passenger/hour)
1	235	26	20
2	25	27	15
3	35	28	5
4	10	29	5
5	25	30	5
6	85	31	45
7	5	32	40
8	85	33	20
9	15	34	70
10	135	35	10
11	5	36	15
12	25	37	20
13	15	38	55
14	5	39	15
15	70	40	15
16	70	41	10
17	70	42	15
18	40	43	25
19	10	44	15
20	5	45	25
21	20	46	10
22	25	47	15
23	55	48	55
24	30	49	105
25	5	50	20

The values for the model parameters (e.g., vehicle sizes, operating speed, cost, etc.) are specified in Table 4.19.

Table 4.19: Selected values for parameters of used in the Petaling Jaya study area

Parameter	Unit	Value
μ_a	RM/passenger-hr	28
μ_w	RM /passenger-hr	28
μ_I	RM /passenger-hr	14
λ_f	RM /veh-hr	50.30
λ_l	RM /veh-km	1.30
λ_I	RM /veh-hr	40
λ_m	RM /veh-km	2.62
λ_p	RM /veh-hr	35.70
λ_s	RM /veh-km	0.25
V	km/hr	32
S_{kj}	min	15
t_{aF}	min	7.5
t_{aTj}	min	4
t_{dT}	min/passenger	0.03
t_{df}	min/passenger	0.096
V_T	km/hr	40
F_T	veh/hr	20
f_{min}	veh/hr	2
f_{max}	veh/hr	20
N	veh	100
LF	pass/seat	1
C	pass/veh	36
l_{min}	km	No constrain
l_{max}	km	5
λ_{lt}	RM /veh-hr	630

A comparison of the results achieved by four metaheuristic algorithms applied for optimizing the improved model in benchmark shows that the WCA and ICA were faster and more accurate than PSO and GA in achieving its optimum solutions (solution quality). Therefore, in terms of solution quality, the PSO and GA were ranked 3rd and 4th respectively. Moreover, their cost reduction was not considerably high compared with its earlier iterations. Thus, in the real case study, PSO and GA are not applied as optimization methods.

The optimization procedure of the transit service model was followed in 50 independent runs, which were performed for each of the considered optimizers. The initial parameters of the WCA included the population size of 100, number of rivers and sea of 8 and d_{max} of 1e-05. Accordingly, for the ICA, the initial parameters consisted of country of 100, number of imperialist country of 8, revolution rate of 0.4.

The number of function iterations (NFEs) can be considered as items indicating the speed (computational effort) and the robustness of the algorithm. However, all these metaheuristic algorithms are somehow dependent on the selected initial values. In general, less NFE can be inferred to mean less time to reach the global optimum. This feature returns back to the structure of the algorithms. Application of different optimization algorithms results in solutions with diverse precision. In fact, the solutions state the level of accuracy of the applied methods and their ability to determine the optimum results. In other words, there is a close relationship between the NFEs and best solution. It means that the ideal situation meets by the least NFEs and more accurate solution. With regard to the convergence trend of optimization algorithms and in order to draw a comparison between them, the maximum number of 1000 iterations is considered for two methods; ICA and WCA. Additionally, maximum NFEs of 100,000 (NFEs = Number of iterations (generation) × Number of population) was imposed among them. The detailed settings of applied approaches are tabulated in Table 4.20 for more clarification.

Table 4.20: Detailed initial parameter setting for the applied methods

Methods	User-defined Parameters	
ICA	Number of independent run	50
	Number of iterations	1000
	Number of country(i.e., Countries)	100
	Maximum NFEs	100,000
	Number of imperialist countries (N_{ipm})	8
	Revolution rate	0.4
WCA	Number of independent run	50
	Number of iterations	1000
	Number of population (i.e., streams)	100
	Maximum NFEs	100,000
	Number of rivers and sea (N_{sr})	8
	d_{max}	1e-05

The number of generated routes is also considered as a design variable and varies from one individual to another in a generated population. In fact, the total number of design variables can be changed in this problem. Having numerous design variables based on each individual can be considered as a special privilege which can categorize this as dynamic optimization problem. The results of the presented optimization algorithms are compared and discussed.

4.3.1 Computational Results Obtained by ICA for the PJ Transit Network

For optimizing proposed model, a total of 50 runs is performed. All cost terms, best transit network and frequency of each feeder route are recorded.

Table 4.21 demonstrates the comparison procedure of obtaining the statistical results for each term of modified cost function using the applied algorithm.

Table 4.21: Gained statistical optimization results for each cost term using the ICA

Parameter	Best	Mean	Worst	SD
C_T	23683.5	24145.0	24889.9	242.6
C_u	16761.3	17027.7	17503.3	160.5
C_o	6814.7	7002.3	7259.4	91.8
C_w	5042.0	5232.2	5506.7	95.3
C_{ui}	2293.0	2409.8	2694.5	103.5
C_{fF}	680.1	737.2	817.4	28.6
C_{mF}	1135.9	1231.3	1365.2	47.8
C_{pF}	1406.3	1439.3	1482.0	17.8
C_s	106.0	114.9	127.4	4.5
A_F	4.9	5.4	5.9	0.2
T_{VK}	432.7	469.1	520.1	18.22
T_{PK}	3140.6	3386.8	3867.3	186.4

Note: All cost values are based on RM.

As presented in Table 4.21, the minimum total cost of RM23683.5, user cost of RM16761.3, operation cost of RM6814.7 and social cost of RM106 per hour are obtained for ICA. The average service frequency range for each route is from 4.9 to 5.9 trips per hour (headways in the range of 10.2 to 12.2 minutes). In terms of total cost (C_T), SD of 242.6 is obtained for ICA.

Table 4.22 provides the best solution by ICA among all the runs, which is illustrated in Figures 4.17 to 4.19.

Table 4.22: Best solution obtained by ICA

Route number	Route structure					Route demand (passenger/hr)	Route length (km)	Route frequency (tripe/hr)
1	51	7	5	1		265	3.21	8.42
2	51	8	4	3	2	155	3.18	6.44
3	51	13	11	12		45	1.68	4.45
4	51	16	17	18		180	2.06	8.25
5	51	19	20			15	2.05	2.39
6	51	21	22	23	24	130	3.41	5.73
7	52	10	34	9		220	1.91	9.39
8	52	14	15	26	25	100	2.72	5.52
9	53	29	28	27		25	2.00	3.11
10	53	30	31			50	1.17	5.31
11	53	32	36	37		75	2.73	4.77
12	53	33	35	6		115	2.18	6.45
13	54	40	39	38		85	1.97	5.77
14	54	41	43	45	48	115	2.16	6.48
15	54	42	49	50		140	2.27	7.01
16	54	44	46	47		40	2.46	3.63

C_T=23683.5	C_u=16761.3	C_o=6814.7	C_s=107.3	C_w=5042	C_{ui}=2333.6
	C_{fF}=688.6	C_{mF}=1150.1	C_{pF}=1420.3	AF=5.82	T_{PK}=3179.6

Note: All cost values are based on RM.

As shown in Table 4.22, the transit network consists of 16 feeder bus routes with an average service frequency of 5.82 trips per hour (The average headway is10. 3 minutes). The case solution includes 3179.6 passenger-km of travel. The provided total cost consists of 70.77 % user, 28.77 % operator and 0.46 % social cost, which are illustrated graphically in Figure 4.17. It can be observed that the main costs are ranked as user, operator, and social cost, respectively from maximum to minimum. Moreover, all of the costs based on RM for each of the cost terms are presented in Figure 4.18. the Best transit network obtained by ICA are showed graphically in Figure 4.19.

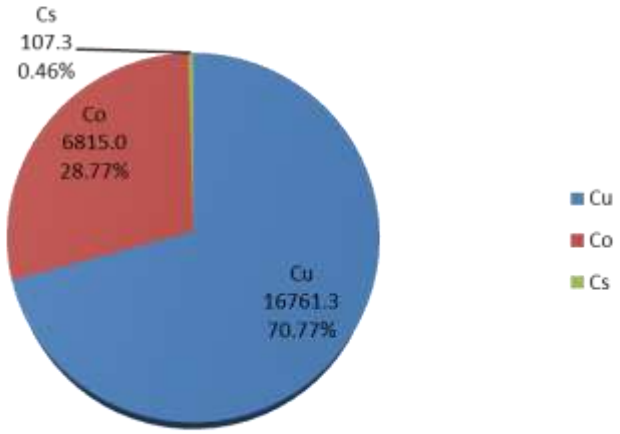


Figure 4.17: Summary of the main costs (RM) achieved by ICA

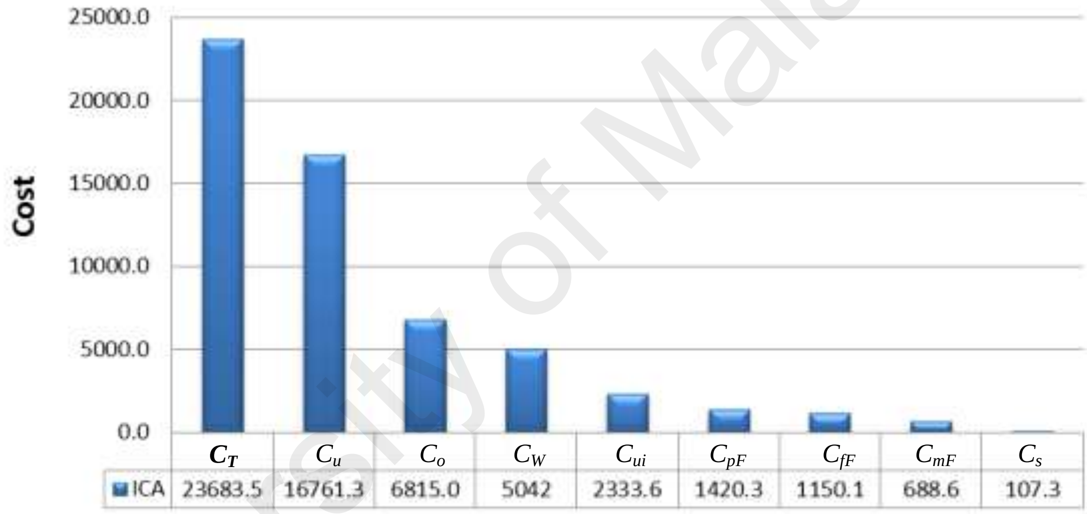


Figure 4.18: Obtained best result and each cost term (RM) using the ICA

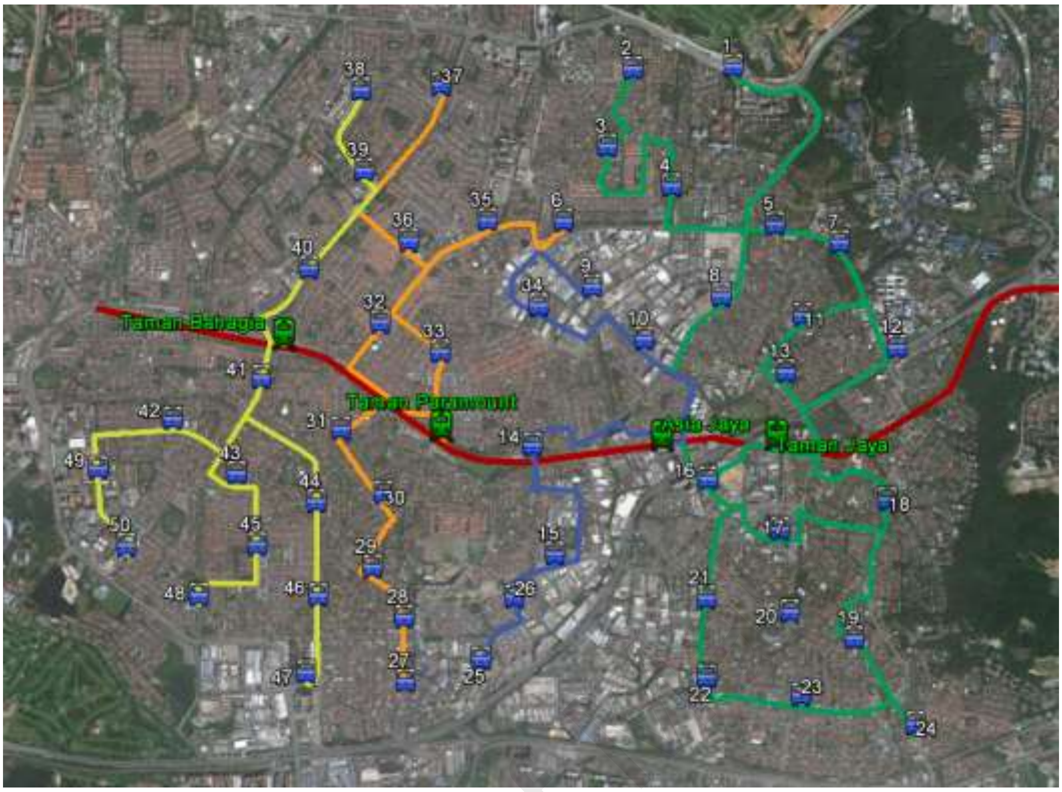


Figure 4.19: Best solution obtained by ICA
Base map source. Google Earth (2014)

Figure 4.20 demonstrates the convergence rate and cost reduction of the best solution on considered optimization engine (ICA).

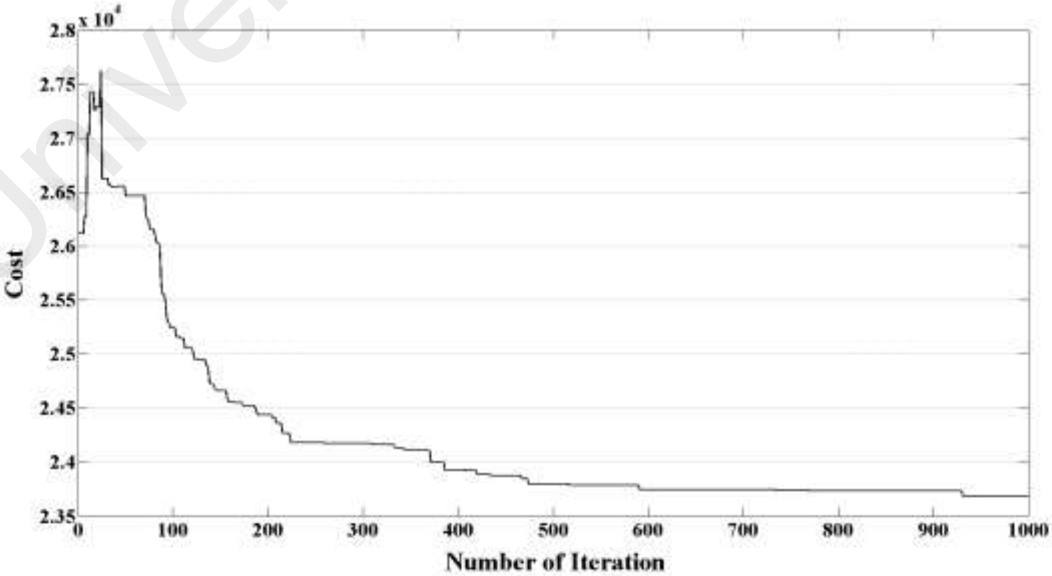


Figure 4.20: Convergence rate and cost history (RM) with respect to the number of iterations for the ICA

Comparison of ICA results with other algorithm is discussed in section 4.3.3.

4.3.2 Computational Results Obtained by WCA for the PJ Transit Network

For optimizing proposed model, a total of 50 runs is performed. All cost terms, the best transit network and frequency of each feeder route are recorded.

Table 4.23 demonstrates the comparison procedure of obtaining the statistical results for each term of modified cost function using the applied algorithm.

Table 4.23: Gained statistical optimization results for each cost term using the WCA

Parameters	Best	Mean	Worst	SD
C_T	23494.80	24023.97	24354.05	220.47
C_u	16692.59	16917.11	17174.57	123.73
C_o	6703.48	6993.46	7208.43	110.11
C_w	4922.19	5217.66	5385.35	104.06
C_{ui}	2168.39	2313.71	2494.31	79.98
C_{fF}	634.17	727.83	792.40	34.02
C_{mF}	1059.17	1215.55	1323.39	56.84
C_{pF}	1412.43	1463.11	1557.40	37.35
C_s	98.84	113.44	123.52	5.32
A_F	5.12	5.55	6.26	0.31
T_{VK}	403.49	463.07	504.15	21.65
T_{PK}	3062.38	3350.81	3623.83	137.96

All cost values are based on RM.

As presented in Table 4.23 the minimum total cost of RM23494.8, user cost of RM16692.59, operation cost of RM6703.48 and social cost of RM98.84 per hour are obtained for WCA. The average service frequency range for each route is from 5.12 to 6.26 trips per hour (headways in the range of 9.58 to 11.71 minutes). In terms of total cost (C_T), SD of 220.47 is obtained for WCA.

Table 4.24 provides the best solution by WCA among all the runs, which is illustrated in Figures 4.21 to 4.23.

Table 4.24: Best solution obtained by WCA

Route number	Route structure						Route demand (passenger/hr)	Route length (km)	Route frequency (tripe/hr)
1	51	1	2				260	3.50	13.85
2	51	8	10				220	1.80	9.59
3	51	12	7				30	1.97	3.43
4	51	13	11	5	4	3	90	3.01	5.03
5	51	18	19	24			80	2.71	4.95
6	52	9	6				100	1.69	6.62
7	52	16	17				140	1.14	8.96
8	52	21	22	23	20		105	3.17	5.31
9	53	14	15	26	25		100	2.44	5.77
10	53	29	28	27			25	2.00	3.11
11	53	32	36	35			65	2.17	4.87
12	53	33	34				90	1.40	6.71
13	54	31	30				50	1.33	5.08
14	54	40	39	38	37		105	2.56	5.80
15	54	41	42	49	50		150	2.30	7.23
16	54	43	45	48			105	2.15	6.21
17	54	44	46	47			40	2.46	3.63

C_T=23494.8	C_u=16692.6	C_o=6703.4	C_s=98.8	C_w=4922.2	C_{ui}=2384.7
	C_{fF}=634.2	C_{mF}=1059.2	C_{pF}=1497.6	AF=6.24	T_{PK}=3062.4

Note: All cost values are based on RM.

As shown in Table 4.24, the transit network consists of 17 feeder bus routes with an average service frequency of 6.24 trips per hour (The average headway is 9.61 minutes). The case solution includes 3062.4 passenger-km of travel. The provided total cost consists of 71.05 % user, 28.53 % operator and 0.42 % social cost, which are illustrated graphically in Figure 4.21. It can be observed that the main costs are ranked as user, operator, and social cost, respectively from maximum to minimum. Furthermore, all of the costs based on RM for each of the cost terms are shown in Figure 4.22. It was emerged from Figure 4.22 that the maximum cost of RM 16692.6 belongs to C_U cost term while C_s has the minimum cost of RM 98.8. Furthermore, the best transit network obtained by WCA is showed graphically in Figure 4.23.

.

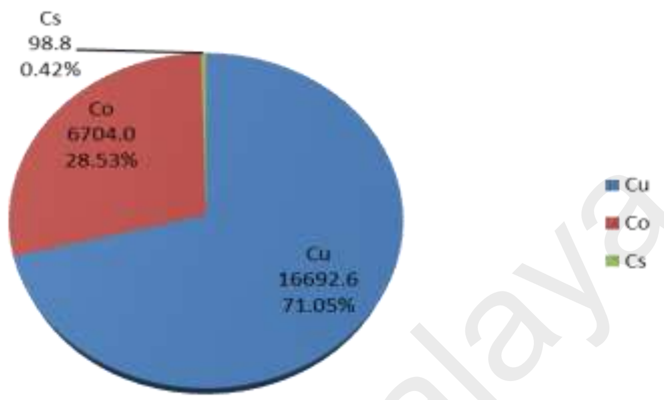


Figure 4.21: Summary of the main costs (RM) achieved by WCA

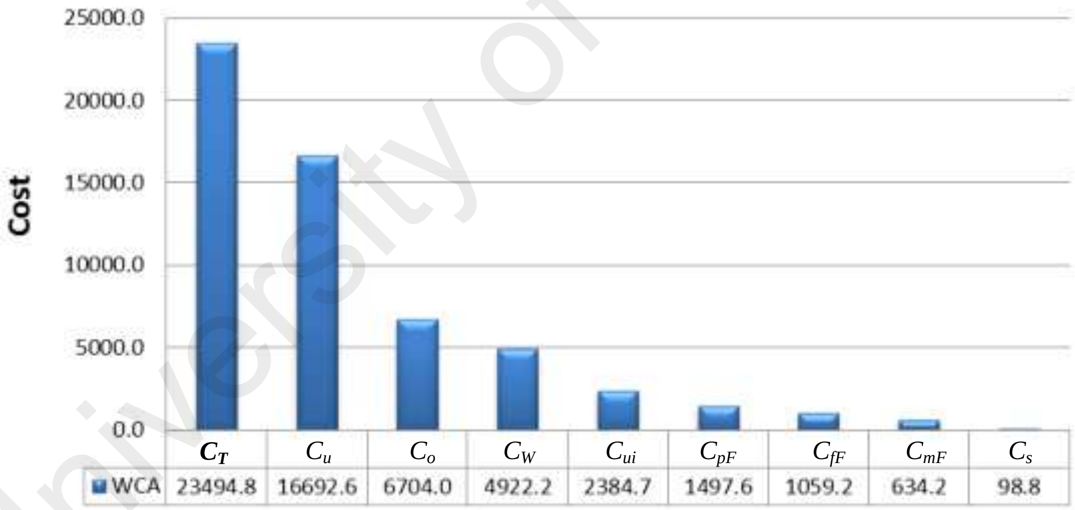


Figure 4.22: Obtained best result and each cost term (RM) using the WCA

Figure 4.24 demonstrates the convergence rate and cost reduction of the best solution on considered optimization engine (WCA). As is evident from this figure, the first 120 iterations are depicted to show the fast convergence of WCA more clearly. However, as the number of iterations increases, the WCA cost reduction was not considerably high compared with its earlier iterations.

Comparison of WCA results with another algorithm is discussed in following section.

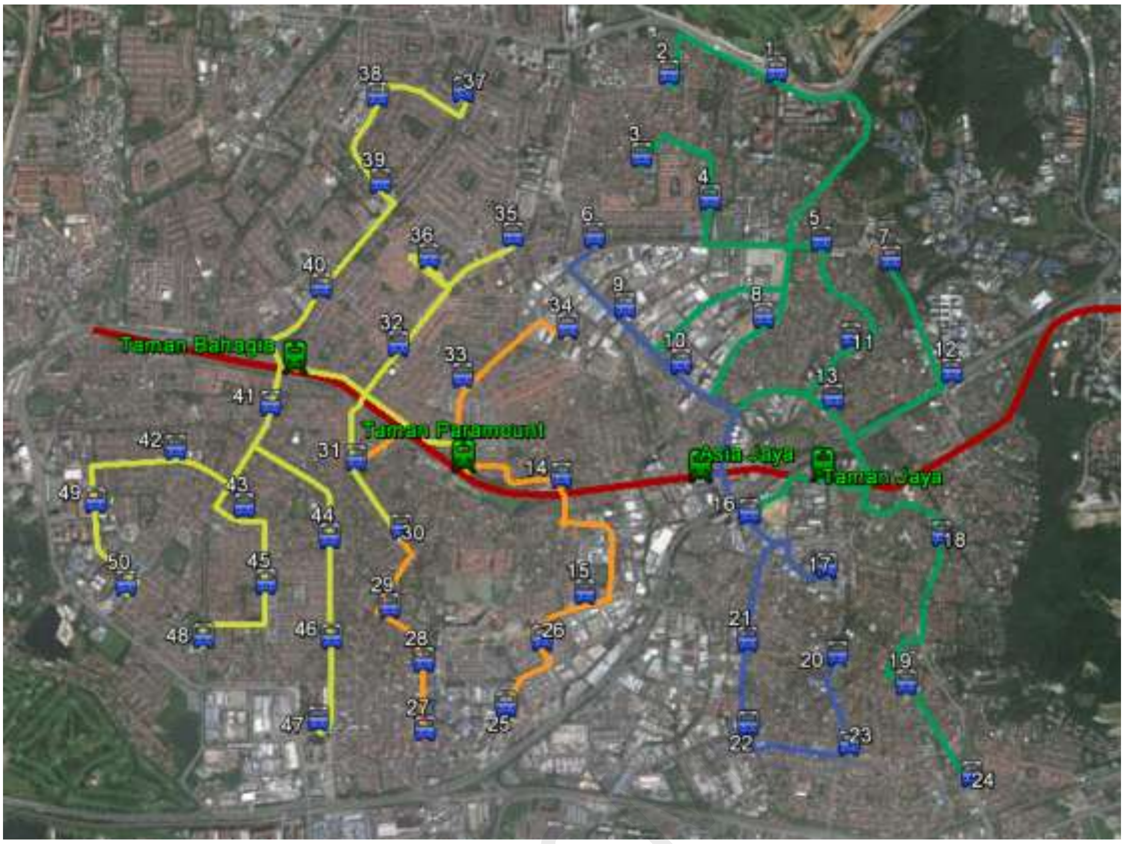


Figure 4.23: Best solution obtained by WCA
Base map source. Google Earth (2014)

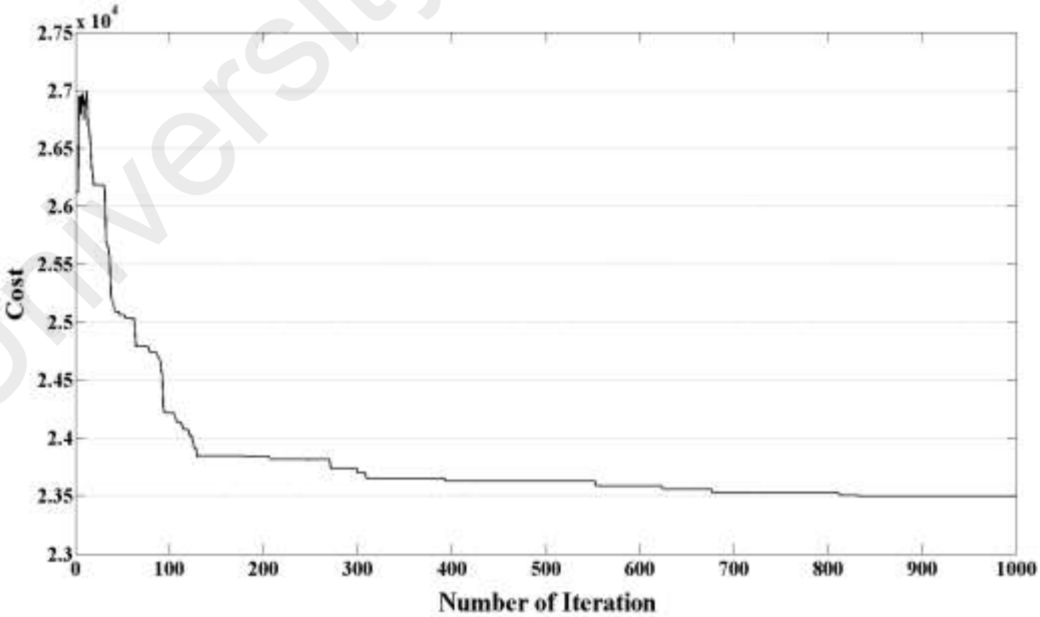


Figure 4.24: Convergence rate and cost history (RM) with respect to the number of iterations for the WCA

4.3.3 Comparison and Discussion of the Results for the PJ Transit Network

Table 4.25 shows the comparison of the best solution attained for all cost terms using applied optimization engines for the improved model. The obtained total cost (C_T) is highlighted in bold in this table for the two algorithms.

Table 4.25: Comparison of the best obtained solutions for the transit service model using reported methods

Method	C_W	C_{ui}	C_{fF}	C_{mF}	C_{pF}	C_u	C_o	C_s	C_T	A_F	T_{PK}
ICA	5042.0	2333.6	688.6	1150.1	1420.3	16761.3	6815	107.3	23683.5	5.8	3179.6
WCA	4922.2	2384.7	634.2	1059.2	1497.6	16692.6	6704	98.8	23494.8	6.2	3062.4

Note: All cost values are based on RM.

The best solution obtained in the PJ area is produced by WCA, shown in Table 4.25. Furthermore, the main costs are illustrated graphically in Figure 4.25. .A total cost of RM23494.8/hr. is achieved, with the average service frequency of 6.2 trips per hour (buses average arriving at intervals of 9.67 minutes).

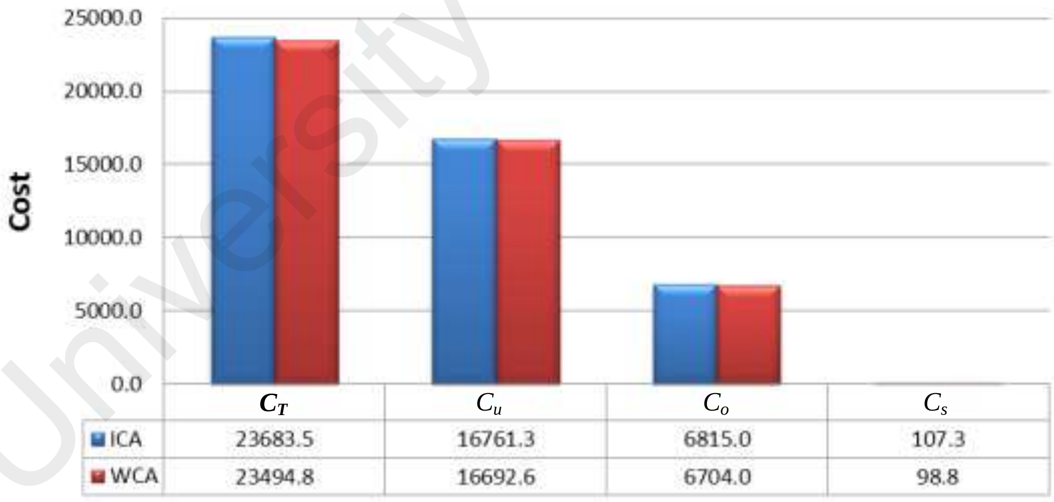


Figure 4.25: Comparison of obtained best result and main cost terms (RM)

Accordingly, Table 4.26 demonstrates the comparison ways of obtaining statistical results for two reported optimizers for the FNDSP.

Table 4.26: Comparison of statistical results gained by optimizers under consideration where 'SD' stands for standard deviation

Optimisers	Best Solution	Mean Solution	Worst Solution	SD
ICA	23683.45	24145.10	24889.90	242.55
WCA	23494.80	24024.00	24354.05	220.50

Note: All cost values are based on RM.

As shown in Table 4.26, the WCA has obtained the best statistical results for the FNDSP with the minimum cost for the C_T. In terms of less SD the WCA also presented better solution stability compared with the other reported algorithm.

The detailed statistical optimization results associated with each term of modified cost function using applied algorithms are presented in Table 4.27.

Table 4.27: Statistical optimisation results for each cost term

Parameter	Best		Mean		Worst		SD	
	WCA	ICA	WCA	ICA	WCA	ICA	WCA	ICA
C_T	23494.8	23683.5	24024.0	24145.1	24354.1	24889.9	220.5	242.6
C_u	16692.6	16761.2	16917.3	17027.9	17174.5	17503.2	123.9	160.7
C_o	6703.6	6814.9	6993.4	7002.5	7208.6	7259.4	110.3	91.7
C_w	4922.1	5042.1	5217.8	5232.2	5385.5	5506.6	104.0	95.2
C_{ui}	2168.3	2293.2	2313.9	2409.8	2494.5	2694.7	79.8	103.6
C_{fF}	634.2	680.1	727.7	737.1	792.4	817.3	34.0	28.7
C_{mF}	1059.1	1135.8	1215.6	1231.3	1323.4	1365.4	56.7	48.0
C_{pF}	1412.6	1406.3	1463.0	1439.2	1557.5	1481.9	37.5	17.9
C_s	98.7	106.1	113.4	114.8	123.6	127.4	5.3	4.6
A_F	5.1	4.9	5.6	5.4	6.3	5.9	0.3	0.2
T_{VK}	403.49	432.7	463.07	469.1	504.15	520.1	21.65	18.22
T_{PK}	3062.38	3140.62	3350.81	3386.80	3623.83	3867.35	137.96	186.42

Note: All cost values are based on RM.

As shown in Table 4.27, it can be concluded that the WCA is superior to the other optimizer for finding all cost terms (except the C_P) having minimum statistical optimization results (i.e. the best, the mean, and the worst solutions).

Figure 4.26 shows the performances of the deviation percentage associated with the corresponding optimization algorithms for 50 independent runs. Acceptable stability was seen in the results among different runs. This confirms the reliability of the presented methodology. The stochastic-evolutionary nature of the applied method to produce the initial values for each iteration makes it natural not to obtain the same results through different independent runs. However, the final results (C_T) are similar in various runs, which are about 0.92% and 1% for WCA and ICA respectively. The ICA is only about 0.08% worse than WCA. Both are closely comparable.

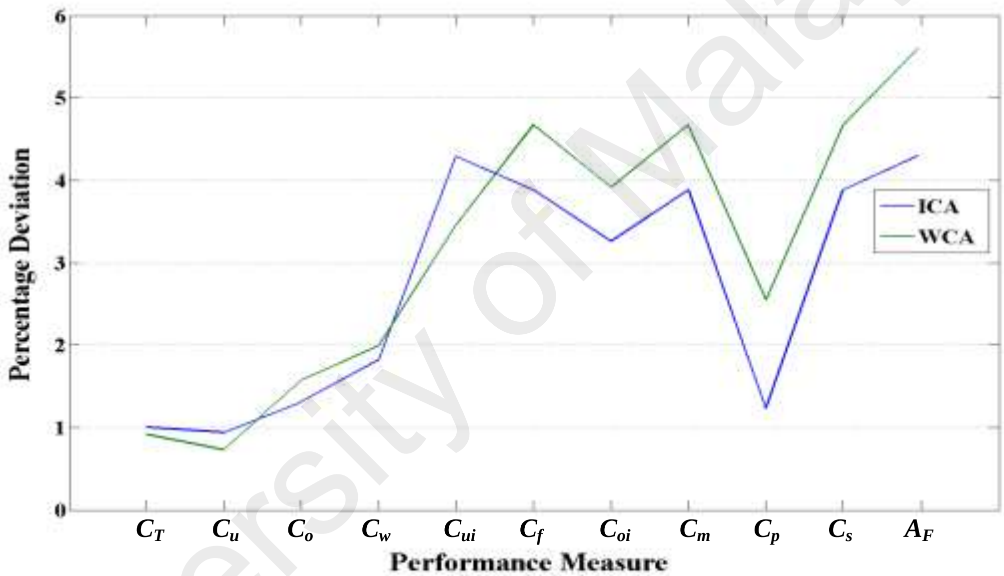


Figure 4.26: The performances of the deviation percentage associated with the corresponding optimisation algorithms for 50 independent runs

The convergence rate and cost history among applied optimization algorithms have been compared and illustrated in Figure 4.27.

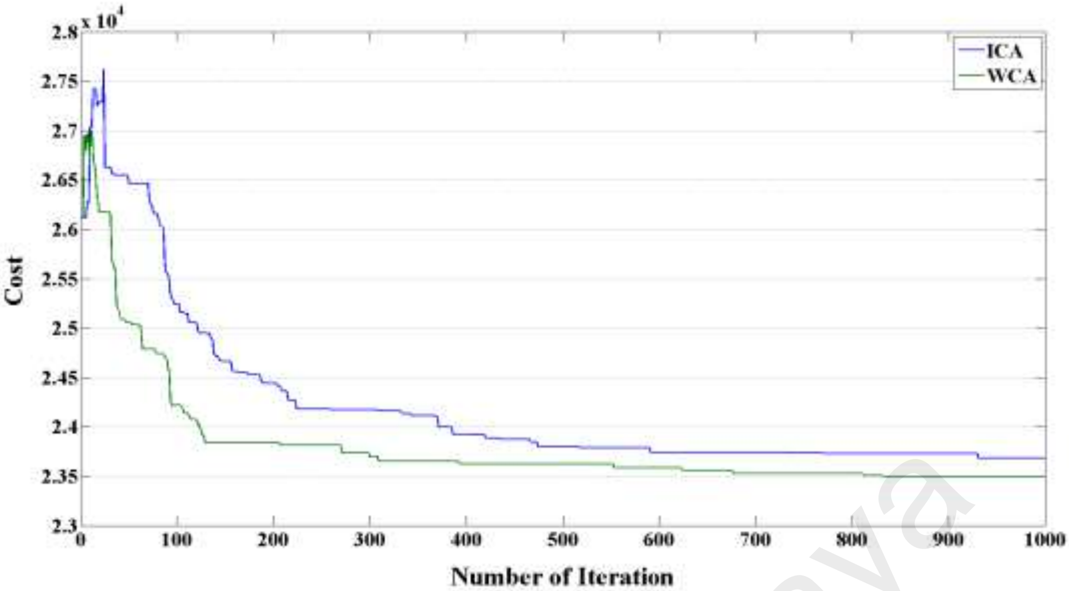


Figure 4.27: Comparison of convergence rate and cost history (RM) with respect to the number of iterations for WCA and ICA

Considering the trend of convergence for each method, WCA is capable of determining faster optimum solutions with a higher level of precision in comparison with the ICA.

Figure 4.28 illustrates the location and variation of the total cost (C_T) and its main components, namely, C_u, C_o and C_s for 50 runs of each optimization algorithm.

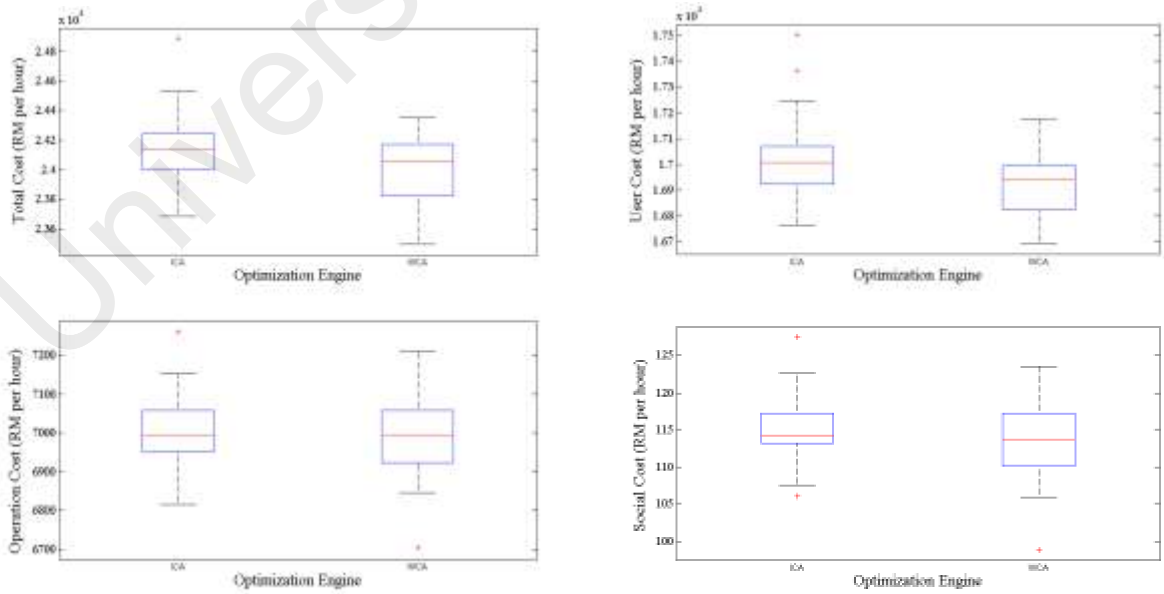


Figure 4.28: The location and variation of the total cost (C_T) and its main components, namely, C_u, C_o and C_s for 50 runs of ICA and WCA

It can be observed that the lowest level of the average cost terms are 24023.97, 16917.10, 6993.46 and 113.45 reside in WCA, respectively for C_T, C_u, C_o, and C_s. The differences between cost terms of both algorithms for the average cost values are 121, 110.6, 8.9, and 1.5, respectively, for C_T, C_u, C_o, and Cs. It can be observed that the lowest level of the average cost in terms of both algorithms becomes nearly the same. However, the ICA shows a minimal variation in level between first and third quartiles compared with the WCA. In terms of C_T for ICA, the second and third quartile boxes are approximately the same size. The box plot for that data set would look like one for a normal distribution, but with a number of outliers beyond one whisker.

As discussed in the literature review section and summarized in Table 2.6, there were some limitations and gaps in the previous studies. The application and optimization of the proposed model to the PJ transit network provided more accurate and efficient solutions of various conditions of transit systems by employing additional terms and constraints in the objective function. In other words, the effort has been made to widen the scope of the research by considering all aspects of satisfaction (i.e. user cost satisfaction; operation cost satisfaction and social cost satisfaction). The outputs of these solutions have demonstrated that the presented model has been verified, and the proposed WCA has been considered as a suitable approach to gain moderate quality solutions with reasonable computational cost.

The results show that the proposed model can be considered as a potential method to overcome current difficulties in public transit systems. This model may lead to the creation of the more realistic model in simulating real-life problems by providing fresh empirical data as a future work.

Thus, hypothesis three was confirmed by applying and demonstrating an improved model based on the real case study in the PJ area. Consequently, objective three was achieved by solving the FNDSP and achieving an optimum transit network as well as evaluating the performance of the users, operators and social perspectives.

4.4 Summary of the Chapter

This chapter was exclusively allocated to the data analysis of the study and its results. In this chapter, the objectives were answered based on the application and optimization of the proposed transit service model on the benchmark and PJ study area. These consisted of computational results of proposed optimization algorithms (ICA and WCA). We performed sensitivity analysis over different parameter values on the benchmark study. Sensitivity analysis of the proposed cost terms shows that the impacts of these costs become significant in public transit system.

Finally, the Comparison and discussion of the results were presented accordingly.

CHAPTER 5: CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

The purpose of this chapter is to summarise the results in the previous chapter and examine the significance of the findings in order to reach a conclusion. These results would be subsequently used to achieve the research objectives as well as to clarify suggestions for further studies. This would be followed discussion about the contributions of the study and suggestions for future research which is turn followed by the conclusion.

5.2 Overview of the Study

Many passengers use transit systems to reach their destinations while a growing concern for public transit is its inability to shift passenger's mode from private to public transportation. A well-integrated transit system in urban areas could play a crucial role in passenger satisfaction and reducing operating costs. This system usually consists of integrated rail lines and a number of feeder routes connecting transfer stations.

The main target of this study is to present the improved model and to design an efficient transit system to increase the efficiency of feeder network designs and coordinated schedules in order to minimize costs. The improved integrated intermodal system may lead to a reduction in total cost and an increase in profit, which consequently, leads to achieving an optimum transit network design.

In this study, the model was applied in the benchmark and Petaling Jaya study areas. Metaheuristic algorithms such as genetic algorithm, Particle Swarm Optimization, Water Cycle Algorithm and Imperialist Competitive Algorithm were employed to optimize transit services.

The data of the study were obtained based on the literature review, questionnaire survey and observation. Furthermore, obtained numerical results of the proposed model, including optimal solution, statistical optimization results, the convergence rate as well as comparisons were discussed. Finally, the concluding results and suggestions for future research were presented.

5.3 Objectives Achieved

The main objective of this research is to develop a mathematical formulation model for designing and coordinating schedules of integrated public transit services, which include development of feeder services and coordination with major transportation services and transfer time consideration between two modes. In the proposed improved model, the additional terms and constraints employed in objective function provide more accurate and efficient solutions for various conditions of transit systems, and this may lead to the creation of a more realistic model in simulating real-life problems.

5.3.1 Objective One

- Proposing an improved mathematical model based on the gaps of the previous studies to increase the efficiency of the intermodal transit system with the aim of achieving the optimal balance between the operator, user, and social costs.

The results of proposing an improved mathematical model for this objective are defined as the sum of the user, operator, and social costs.

$$Minimize \quad C_T = \sum_{k=1}^{K} \left[\overbrace{\left(C_a + C_w + C_{ui}\right)}^{User} + \overbrace{\left(C_f + C_{oi} + C_m + C_p\right)}^{Operating} + \overset{Scocial}{C_s} \right]$$

Therefore, according to mathematical model formulation subsection, the objective function can be formulated after substitution of cost terms as follows:

$$
\begin{aligned}
&\text{Minimize} \quad C_T = \mu_a \left[t_{aF} \sum_{i=1}^{I} q_i + \sum_{j=1}^{J} t_{aTj} \sum_{i=1}^{I} q_i \times Y_{ij} \right] + \mu_w \left[\sum_{k=1}^{K} \left[\left(\frac{1}{2F_k} + \frac{1}{2F_T} \right) \times Q_K \right] \right] + \\
&\mu_I \left[\sum_{k=1}^{K} \left[\frac{1}{V_k} \sum_{i=1}^{I} \left[q_i \times \left(\sum_{j=1}^{J} L_{ijk} \right) \right] + \left(\frac{1}{2} (n_K + 1) \times Q_K \times t_{dF} \right) \right] + \sum_{j=1}^{J} \left[\left(\sum_{i=1}^{I} q_i \times Y_{ij} \right) \times \left(t_{dT} \times (J - j + 1) + t_{Tj} \right) \right] \right] + \\
&\lambda_f \left[2 \sum_{k=1}^{K} \frac{F_k}{V_k} \times L_K \right] + \lambda_l \left[2 \sum_{k=1}^{K} F_k \times L_{\kappa} \right] + \lambda_I \left[\sum_{k=1}^{K} Q_K \times t_{dF} \right] + \lambda_{IT} \left[\left(\sum_{i=1}^{I} q_i \times t_{dT} \right) + \left(F_T \times T_T \right) \right] + \\
&\lambda_m \left[2 \sum_{k=1}^{K} F_k \times L_{\kappa} \right] + \lambda_p \left[\sum_{k=1}^{K} \left[\left(\frac{2F_k}{V_k} \times L_{\kappa} \right) + \left(Q_K \times t_{dF} \right) + \left(F_k \times S_{kj} \right) \right] \right] + \lambda_s \left[2 \sum_{k=1}^{K} F_k \times L_{\kappa} \right]
\end{aligned}
$$

In line with the proposed mathematical model, this research is focused on the progress of new approaches for the feeder bus network design problem. The study provides an improved model to fill the gaps of the preceding studies.

5.3.2 Objective Two

- Applying and demonstrating an improved model that addresses the intermodal transit system based on the benchmark data of the study, (a) to solve the proposed transit service model by using the metaheuristic methods; (b) to achieve an optimum transit network design that focuses on the design of a set of feeder bus routes and determination of the operating frequency on each route with the aim of minimizing the costs; (c) to evaluate performance of the users, operators and social perspectives in results.

The analysis of this objective shows that the obtained statistical optimization results acquired by the WCA were superior to those attained by the ICA, PSO, and GA, while in terms of solution stability, the PSO slightly outperformed the WCA. In terms of solution quality and statistical results, the GA ranked 4^{th}.

The WCA and ICA were faster and more accurate than PSO and GA in achieving their optimum solutions. The GA cost reduction was not considerably high compared with its earlier iterations. Therefore, WCA and ICA methods were executed in 5000 iterations. Applying optimum network resulted in the lowest level of total cost $ 28875 by ICA, whereas the corresponding costs obtained by WCA, PSO, and GA, are respectively about 0.17, 8.4 and 10.8 percent greater than that of ICA.

5.3.3 Objective Three

- Applying and demonstrating an improved model that addresses the intermodal transit system based on the real case study data (Petaling Jaya), (a) to solve the proposed transit service model by using the metaheuristic methods; (b) to achieve an optimum transit network design that focuses on the design of a set of feeder bus routes and determination of the operating frequency on each route with the aim of minimizing the costs; (c) to evaluate performance of the users, operators and social perspectives in results.

The analysis of this objective shows that the obtained statistical optimization results acquired by the WCA were superior to those attained by the ICA. The obtained optimum number of routes using WCA was 16 with an average frequency of 6.2 feeder buses per hour in achieving network. Applying optimum network resulted in the lowest level of total cost RM23494.8 by WCA, whereas the corresponding costs obtained by ICA, is about 0.8 percentages greater than that of WCA.

5.4 Contribution of the Study

The current study is focused on the development of new approaches for the feeder bus network design problem. An effort has been made in this research to fill the gaps of the preceding studies by providing an improved model and proposed solution methods.

The main academic contribution of this research is presenting the improved mathematical formulation model for designing and coordinating schedules of integrated public transit services. This model includes an improvement of feeder services and coordination with major transportation services and transfer time consideration between two modes. In the proposed improved model, the additional terms and constraints were employed in objective function provide more accurate and efficient solutions for various conditions of transit systems. Such additional cost terms and constraints can lead to the creation of a more realistic model in simulating real-life problems.

The current study provides a significant contribution to service quality, financial performance, and ridership. It benefits service operators, transportation planners, consulting firms, and government agencies concerned with public transportation. Specifically, the improved model and its proposed solution methods in this research could contribute to (a) upgrading public bus services, (b) restructuring service to make changes in operating study area and new arrangements, (c) improving performance and service quality, (d) planning new bus service, (e) integrating rail and feeder bus services, (f) increasing ridership, and (g) decreasing costs.

5.5 Suggestion for Future Studies

This study was narrowed down in terms of feeder network problems, feeder bus network design and frequency setting problem. Therefore, there will be new research aspects in the future in this area of study.

First, the study has used link travel distance between nodes to determine the costs and evaluation of routes. Thus, another mathematical modeling for cost calculation between nodes such as travel time adoption may be considered as future research.

Another suggestion for further studies is on travel demand. As discussed in the demand approach section, most studies are restricted to many-to-one pattern and only few previous works considered many-to-many pattern. A many-to-one model refers to passengers travelling from multiple origins to a single destination. However, passengers may have different origins and destinations.

In addition, solution method should be considered to solve mathematical formulation. The development of metaheuristic methods has also made it possible to tackle large-size problems more efficiently and to get high-quality solutions for real-life problems in a reasonable time. Although the proposed methods (ICA and WCA) at their present format show good potential to be used as a global optimization algorithm, they may be improved in terms of route generation in the network. Furthermore, Hybrid methods also hold a lot of promise in terms of tackling problems that used to be intractable. Progress in solution methods is obviously a gate to the integration of all the previously mentioned interesting paths.

Moreover, modeling approaches should also be considered in future research. The improved approaches may increase the performance of the solutions by using other methods. Further modeling with multiple objectives and multiple criteria would be another area for further research. Since a transit system is a multimodal, multi-problem and multi-spectral activity, it involves different parts and activities such as policymaking, planning, designing, infrastructure construction and development. Therefore, multi-objective models in further studies are proposed.

Finally, the literature survey also revealed that there were not enough substantial studies conducted on a multimode feeder system providing service for an existing rail network. Most of the existing studies have been focused on the design of a single-mode network. Bovy and Hoogendoorn-Lanser (2005) presented other findings on the influence of multimodal trip attributes. Their study also focused on the performance of different feeder modes, railway station types, and train service types. In the same way, further research may be focused on the development of new approaches for multimode feeder network system.

5.6 Summary

In this study, an improved model was suggested for the transit network problems including rail system and feeder bus network design and frequency setting problems. The main purpose of this thesis was to develop a real-life model (actualizing the cost function and adding additional constraints) for handling the feeder bus design and frequency setting problems. The case study of the research was based on the benchmark and real transit network of the Petaling Jaya in Malaysia.

Finding the optimum feasible routes in order to reduce the cost function is a vital and difficult task of solving the transit network problem classified as an NP-hard problem. For this reason, the importance of optimization techniques, particularly metaheuristics is understood. Therefore, four well-known optimization algorithms, namely genetic algorithm (GA), particle swarm optimization (PSO), water cycle algorithm (WCA), and imperialist competitive algorithm (ICA) were used. The outcome shows ICA, and WCA acquired better statistical optimization results than other algorithms, which were applied in the benchmark and real area in this study. Thus, optimum transit network was obtained by using ICA and WCA. As a result, the corresponding network costs obtained by PSO and GA are greater than ICA and WCA.

The best solution obtained in the PJ area is the one produced by WCA with the minimum total cost of RM23494.8, including user cost of RM16692.59, operation cost of RM6703.48 and social cost of RM98.84 per hour.

REFERENCES

Almasi, Mohammad Hadi, Mirzapour Mounes, Sina, Koting, Suhana, & Karim, Mohamed Rehan. (2014). Analysis of feeder bus network design and scheduling problems. *The Scientific World Journal, 2014*.

Almasi, Mohammad Hadi, Sadollah, Ali, Mounes, Sina Mirzapour, & Karim, Mohamed Rehan. (2014). Optimization of a Transit Services Model with a Feeder Bus and Rail System Using Metaheuristic Algorithms. *Journal of Computing in Civil Engineering*.

Atashpaz-Gargari, Esmaeil, & Lucas, Caro. (2007). *Imperialist competitive algorithm: an algorithm for optimization inspired by imperialistic competition.* Paper presented at the Evolutionary computation, 2007. CEC 2007. IEEE Congress on.

Baaj, M Hadi, & Mahmassani, Hani S. (1995). Hybrid route generation heuristic algorithm for the design of transit networks. *Transportation Research Part C: Emerging Technologies, 3*(1), 31-50.

Barton, Ray. (2006). Estimation of costs of heavy vehicle use per vehicle-kilometre in Canada. *Transport Canada Economic Analysis Directorate Final Report. Logistics Solution Builders Inc. File: T8080–05–0326*.

Benn, H.P. (1995). *Bus route evaluation standards* (No. Project SA-1).

Blum, Jeremy J., & Mathew, Tom V. (2011). Intelligent Agent Optimization of Urban Bus Transit System Design. *Journal of Computing in Civil Engineering, 25*(5), 357-369. doi: 10.1061/(asce)cp.1943-5487.0000095

Borndorfer, Ralf, Grotschel, Martin, & Pfetsch, Marc E. (2005). *A path-based model for line planning in public transport*: Konrad-Zuse-Zentrum für Informationstechnik Berlin [ZIB].

Bovy, Piet HL, & Hoogendoorn-Lanser, Sascha. (2005). Modelling route choice behaviour in multi-modal transport networks. *Transportation, 32*(4), 341-368.

Breedam, Alex Van. (2001). Comparing descent heuristics and metaheuristics for the vehicle routing problem. *Computers & Operations Research, 28*(4), 289-315.

Ceder, Avishai, & Wilson, Nigel HM. (1986). Bus network design. *Transportation Research Part B*(20), 331–344.

Center for Transportation Research (CTR). (2012). Deraf Rancangan Kawasan Khas Zon Perancangan PJU1, PJU2, SS dan PJS Petaling Jaya 2020. Majlis Bandaraya Petaling Jaya (MBPJ).

Chien, I Jy. (1995). *Optimization of coordinated intermodal transit networks.* University of Maryland at College Park.

Chien, Steven. (2005). Optimization of headway, vehicle size and route choice for minimum cost feeder service. *Transportation Planning and Technology, 28*(5), 359-380. doi: 10.1080/03081060500322565

Chien, Steven , Yang, Zhaowei, & Hou, Edwin. (2001). Genetic algorithm approach for transit route planning and design. *Journal of Transportation Engineering, 127*(3), 200-207.

Chien, Steven, & Schonfeld, Paul. (1998). Joint optimization of a rail transit line and its feeder bus system. *Journal of Advanced Transportation, 32*(3), 253-284.

Chien, Steven, & Yang, Zhaowei. (2000). Optimal feeder bus routes on irregular street networks. *Journal of Advanced Transportation, 34*(2), 213-248.

Choi, Jinkyung, Lee, Yong Jae, Kim, Taewan, & Sohn, Keemin. (2011). An analysis of Metro ridership at the station-to-station level in Seoul. *Transportation, 39*(3), 705-722. doi: 10.1007/s11116-011-9368-3

Chowdhury, S.M., Steven, I., & Chien, J. (2002). Intermodal Transit System Coordination. *Transportation Planning and Technology, 25*(4), 257-287. doi: 10.1080/0308106022000019017

Ciaffi, F, Cipriani, E, & Petrelli, M. (2012). Feeder Bus Network Design Problem: a New Metaheuristic Procedure and Real Size Applications. *Procedia - Social and Behavioral Sciences, 54*, 798-807. doi: 10.1016/j.sbspro.2012.09.796

Cipriani, Ernesto, Gori, Stefano, & Petrelli, Marco. (2012). Transit network design: A procedure and an application to a large urban area. *Transportation Research Part C: Emerging Technologies, 20*(1), 3-14. doi: 10.1016/j.trc.2010.09.003

Cohen-Blankshtain, Galit, & Feitelson, Eran. (2011). Light rail routing: do goals matter? *Transportation, 38*(2), 343-361.

Cordeau, J. F., Gendreau, M., Laporte, G., Potvin, J. Y., & Semet, F. (2002). A guide to vehicle routing heuristics. *Journal of the Operational Research Society, 53*(5), 512-522. doi: 10.1057/palgrave/jors/2601319

Dong, Ying, Tang, Jiafu, Xu, Baodong, & Wang, Dingwei. (2005). An application of swarm optimization to nonlinear programming. *Computers & Mathematics with Applications, 49*(11), 1655-1668.

Elbeltagi, Emad, Hegazy, Tarek, & Grierson, Donald. (2005). Comparison among five evolutionary-based optimization algorithms. *Advanced engineering informatics, 19*(1), 43-53.

Eppstein, David. (1994). *Finding the k shortest paths.* Paper presented at the Foundations of Computer Science, 1994 Proceedings., 35th Annual Symposium on.

Eskandar, Hadi, Sadollah, Ali, Bahreininejad, Ardeshir, & Hamdi, Mohd. (2012). Water cycle algorithm–A novel metaheuristic optimization method for solving constrained engineering optimization problems. *Computers & Structures, 110*, 151-166.

Fan, Wei, & Machemehl, Randy B. (2004). Optimal transit route network design problem: Algorithms, implementations, and numerical results.

Gholami, Ali, & Mohaymany, Afshin Shariat. (2011). Economic conditions for minibus usage in a multimodal feeder network. *Transportation Planning and Technology, 34*(8), 839-856. doi: 10.1080/03081060.2011.613594

Giraud-Moreau, Laurence, & Lafon, Pascal. (2002). A comparison of evolutionary algorithms for mechanical design components. *Engineering Optimization, 34*(3), 307-322.

Golberg, David E. (1989). Genetic algorithms in search, optimization, and machine learning. *Addion wesley, 1989*.

Golub, Aaron, Balassiano, Ronaldo, Araújo, Ayres, & Ferreira, Eric. (2009). Regulation of the informal transport sector in Rio de Janeiro, Brazil: welfare impacts and policy analysis. *Transportation, 36*(5), 601-616.

Guihaire, Valérie, & Hao, Jin-Kao. (2008). Transit network design and scheduling: A global review. *Transportation Research Part A: Policy and Practice, 42*(10), 1251-1273. doi: 10.1016/j.tra.2008.03.011

Holland, John H. (1975). *Adaptation in natural and artificial systems: an introductory analysis with applications to biology, control, and artificial intelligence*: U Michigan Press.

Hu, Yucong, Zhang, Qi, & Wang, Weiping. (2012). A Model Layout Region Optimization for Feeder Buses of Rail Transit. *Procedia - Social and Behavioral Sciences, 43*, 773-780. doi: 10.1016/j.sbspro.2012.04.151

Hurole, VF, & Wirasinghe, SC. (1980). Location of rail stations for many to one travel demand and several feeder modes. *Journal of Advanced Transportation, 14*(1).

Jakimavičius, Marius, & Burinskiene, Marija. (2009). Assessment of vilnius city development scenarios based on transport system modelling and multicriteria analysis. *Journal of Civil Engineering and Management, 15*(4), 361-368. doi: 10.3846/1392-3730.2009.15.361-368

Jerby, Shai, & Ceder, Avishai. (2006). Optimal Routing Design forShuttle Bus Service. *Transportation Research Record: Journal of the Transportation Research Board, 1971*(1), 14-22.

Kaveh, A, & Talatahari, S. (2009). A particle swarm ant colony optimization for truss structures with discrete variables. *Journal of Constructional Steel Research, 65*(8), 1558-1568.

Kennedy, James, & Eberhart, Russell C. (1995). *Particle swarm optimization.* Paper presented at the IEEE IJCNN, Perth, Australia.

Kennedy, James, & Eberhart, Russell C. (1997). *A discrete binary version of the particle swarm algorithm.* Paper presented at the Systems, Man, and Cybernetics, 1997. Computational Cybernetics and Simulation., 1997 IEEE International Conference on.

Khabbazi, Arash, Atashpaz-Gargari, Esmaeil, & Lucas, Caro. (2009). Imperialist competitive algorithm for minimum bit error rate beamforming. *International Journal of Bio-Inspired Computation, 1*(1), 125-133.

Kim, Byung-In, & Jeong, Sangwon. (2009). A comparison of algorithms for origin–destination matrix generation on real road networks and an approximation approach. *Computers & Industrial Engineering, 56*(1), 70-76.

Kim, Kwang Sik, Cheon, Seung-hoon, & Lim, Sam-jin. (2011). Performance assessment of bus transport reform in Seoul. *Transportation, 38*(5), 719-735.

Kuah, Geok Koon. (1986). Feeder bus route design problem: Maryland Univ., College Park (USA).

Kuah, Geok Koon, & Perl, Jossef. (1988). Optimization of feeder bus routes and bus stop spacing. *Transportation Engineering, 114*(3), 341-354.

Kuah, Geok Koon, & Perl, Jossef. (1989). The feeder-bus network-design problem. *Journal of the Operational Research Society, 40*(8), 751-767.

Kuan, SN. (2004). Applying metaheuristics to feeder bus network design problem.

Kuan, SN, Ong, HL, & Ng, KM. (2004). Applying metaheuristics to feeder busnetwork design problem. *Asia-Pacific Journal of Operational Research, 21*(4), 543-560.

Kuan, SN, Ong, HL, & Ng, KM. (2006). Solving the feeder bus network design problem by genetic algorithms and ant colony optimization. *Advances in Engineering Software, 37*(6), 351-359. doi: 10.1016/j.advengsoft.2005.10.003

Martinez, Luis M., & Eiro, Tomas. (2012). An Optimization Procedure to Design a Minibus Feeder Service: An Application to the Sintra Rail Line. *Procedia - Social and Behavioral Sciences, 54*, 525-536. doi: 10.1016/j.sbspro.2012.09.770

Martins, Carlos Lucio, & Pato, Margarida Vaz. (1998). Search strategies for the feeder bus network design problem. *European Journal of Operational Research, 106*(2), 425–440.

Mezura-Montes, Efrén, & Coello, Carlos A Coello. (2008). An empirical study about the usefulness of evolution strategies to solve constrained optimization problems. *International Journal of General Systems, 37*(4), 443-473.

Michael, R Garey, & David, S Johnson. (1979). Computers and intractability: a guide to the theory of NP-completeness. *WH Freeman & Co., San Francisco*.

Mohaymany, A.S., & Gholami, A. (2010). Multimodal Feeder Network Design Problem: Ant Colony Optimization Approach. *Transportation Engineering, 136*(4), 323–331. doi: 10.1061//asce/te.1943-5436.0000110

Murray, A.T. (2003). A coverage model for improving public transit system accessibility and expanding access. *Annals of Operations Research, 123*(1), 143-156.

Nes, R v. (2002). *Design of multimodal transport networks.* (2002), PhD thesis of Delft university of Technology.

Nikolic, Milos, & Teodorovic, Dusan. (2013). Transit network design by Bee Colony Optimization. *Expert Systems with Applications*, *40*(15), 5945-5955. doi: 10.1016/j.eswa.2013.05.002

Perl, Jossef. (1987). The multi-depot routing allocation problem. *American Journal of Mathematical and Management Sciences, 7*(1), 7-34.

Pradhan, Anu, & Mahinthakumar, G. (2012). Finding All-Pairs Shortest Path for a Large-Scale Transportation Network Using Parallel Floyd-Warshall and Parallel Dijkstra Algorithms. *Journal of Computing in Civil Engineering, 27*(3), 263-273.

Reeves, Colin R. (1993). *Modern heuristic techniques for combinatorial problems*: John Wiley & Sons, Inc.

Shrivastav, P., & Dhingra, SL. (2001). Development of feeder routes for suburban railway stations using heuristic approach. *Journal of transportation engineering, 127*(4), 334-341.

Shrivastava, Prabhat , & O'Mahony, Margaret. (2006). A model for development of optimized feeder routes and coordinated schedules—A genetic algorithms approach. *Transport Policy, 13*(5), 413-425. doi: 10.1016/j.tranpol.2006.03.002

Shrivastava, Prabhat , & O'Mahony, Margaret. (2007). Design of feeder route network using combined genetic algorithm and specialized repair heuristic. *Journal of Public Transportation, 10*(2), 99-123.

Shrivastava, Prabhat , & O'Mahony, Margaret. (2009a). Use of a hybrid algorithm for modeling coordinatedfeeder bus route network at suburban railway stations. *Journal of Transportation Engineering, 135*(1), 1-8. doi: 10.1061//asce/0733-947x/2009/135:1/1

Shrivastava, Prabhat , & O'Mahony, Margaret. (2009b). Modeling an Integrated Public Transportation System a case study in Dublin, Ireland

Sivakumaran, Karthikgeyan, Li, Yuwei, Cassidy, Michael J., & Madanat, Samer. (2012). Cost-saving properties of schedule coordination in a simple trunk-and-feeder transit system. *Transportation Research Part A: Policy and Practice, 46*(1), 131-139. doi: 10.1016/j.tra.2011.09.013

Sonmez, Rifat, & Ontepeli, Bahadir. (2009). Predesign cost estimation of urban railway projects with parametric modeling. *Journal of Civil Engineering and Management, 15*(4), 405-409. doi: 10.3846/1392-3730.2009.15.405-409

Spasovic, L.N., Boile, M.P., & Bladikas, A.K. (1993). *A methodological framework for optimizing bus transit service coverage.* Paper presented at the Transportation Research Record. Submitted for the 73rd Annual Meeting of the TRB.

Uspalyte Vitkuniene, Rasa, & Burinskiene, Marija. (2006). Analysis of the dynamics of walking distances to public transport routes and its influence on housing prices. *Journal of Civil Engineering and Management, 12*(3), 261-267. doi: 10.1080/13923730.2006.9636401

Valley Metro. (2012). Transit Performance Report (TPR), FY 2012. http://www.valleymetro.org/images/uploads/projects/2012_transit_performance_report_opt.pdf.

Van den Bergh, Frans, & Engelbrecht, Andries Petrus. (2006). A study of particle swarm optimization particle trajectories. *Information sciences, 176*(8), 937-971.

Webster, FV, & Bly, PH. (1979). *Public transport and the planning of residential areas*.

Wirasinghe, S Chandana, Hurdle, Vanolin F, & Newell, Gordon F. (1977). Optimal parameters for acoordinated rail and bus transit system. *Transportation Science, 11*(4), 359–374.

Wirasinghe, SC. (1977). *Assignment of buses in a coordinated rail and bus transit system.* Paper presented at the Proceedings of the seventh international symposium on transportation and traffic flow theory.

Wirasinghe, SC. (1980). Nearly optimal parameters for a rail/feeder-bus system on a rectangular grid. *Transportation Research Part A: General, 14*(1), 33-40.

Xiong, Jie, Guan, Wei, Song, Liying, Huang, Ailing, & Shao, Chunfu. (2013). Optimal Routing Design of a Community Shuttle for Metro Stations. *Journal of Transportation Engineering, 139*(12), 1211-1223. doi: 10.1061/(asce)te.1943-5436.0000608

Yan, Yadan, Liu, Zhiyuan, Meng, Qiang, & Jiang, Yu. (2013). Robust Optimization Model of Bus Transit Network Design with Stochastic Travel Time. *Journal of Transportation Engineering, 139*(6), 625-634. doi: 10.1061/(asce)te.1943-5436.0000536

Youssef, Habib, Sait, Sadiq M, & Adiche, Hakim. (2001). Evolutionary algorithms, simulated annealing and tabu search: a comparative study. *Engineering Applications of Artificial Intelligence, 14*(2), 167-181.

Zhao, Fang, & Zeng, Xiaogang. (2006). Simulated annealing–genetic algorithm for transit network optimization. *Journal of Computing in Civil Engineering, 20*(1), 57-68.