ABSTRACT

The thesis examines statistical inference for discrete distributions under parameter orthogonality and model misspecification. Parameter orthogonality has many advantages in statistical inference (see, Cox and Reid, 1987); for example, convergence is fast in numerical maximum likelihood estimation (see, Willmot, 1988). Since statistical models are approximations to the unknown models, the issue of model misspecification must be considered in any statistical analysis. A closely related important issue is to determine if a given random sample fits a probability model well, a goodness-of-fit problem. The research deals with a goodness-of-fit test based on an information matrix identity known as Bartlett’s First Identity (BFI) which is the basis of White’s (1982) Information Matrix (IM) test for model misspecification. However, the proposed goodness-of-fit test statistic differs from White’s IM test. It has been simplified after the application of parameter orthogonality and does not require the evaluation of the complicated covariance matrix as in the IM test. For illustration purpose, a Monte Carlo simulation study using the negative binomial distribution as an example has been conducted to compare the proposed test statistic with goodness-of-fit tests based on the empirical distribution function (EDF). The results show that the proposed test is useful as an alternative goodness-of-fit test in terms of power. In addition, some asymptotic results of the test are derived.

In this thesis we consider the orthogonality of the mean $\mu$ as a parameter, in multi-parameter models, where the remaining parameters are regarded as nuisance parameters. In particular, orthogonality for Poisson-convolution models, which are of practical importance, has been derived. As an application of this orthogonality result, we develop a uniformly most powerful (UMP) test of the mean based on the asymptotic result under the model
misspecification. A small Monte Carlo power study of the proposed UMP test has been conducted.

As a specific example of a Poisson-convolution model, we study the Delaporte distribution which has useful applications in insurance and actuarial studies. The orthogonal parameters to the mean of Delaporte distribution can be easily obtained. The efficiency of the various methods of estimation for the Delaporte distribution, namely, the method of moments, moments and zero frequency, and maximum likelihood, as discussed by Ruohonen (1988), has been examined. The comparative study of interval estimation under correct specification and misspecification of the Delaporte distribution is also discussed. Parameter estimation of the Delaporte distribution by a new quadratic distance statistic has been considered and the results are compared to maximum likelihood estimates.