NEUTRINO MASS IN HIGGS TRIPLET MODEL

ONG JIAN FUH

FACULTY OF SCIENCE
UNIVERSITY OF MALAYA
KUALA LUMPUR

2012

NEUTRINO MASS IN HIGGS TRIPLET MODEL

ONG JIAN FUH

THESIS SUBMITTED IN FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF SCIENCE

FACULTY OF SCIENCE
UNIVERSITY OF MALAYA
KUALA LUMPUR

2012

Acknowledgements

I would like to thank to my supervisor, Associate Professor Ithnin Abdul Jalil who shows great expertise and having his patience in this research. Throughout this research period, he had shown me the correct way in learning theoretical physics. Detail equations derivation and weekly meeting in discussing the research progress have build me a confidence in the theoretical study. He has given me full freedom in designing my research topic. Besides, I have learn from him in taking care to the university property.

Special thanks to Professor Dr. Wan Ahmad Tajuddin Wan Abdullah for willing to become my new supervisor after the retirement of Associate Professor Ithnin Abdul Jalil.

Many thank to Professor Dr Abdul Kariem Mohd Arof for helping me to solve the problem in the application of University Malaya Fellowship. Without his help, I would still suffer from the financial problem.

Thanks to Associate Professor Dr Chin Ooi Hong who has been very helpful in guiding me to carry out experiment during my undergraduate level. The laboratory teaching process during my temporary tutorship would not be smooth without her kind guidance.

I would also like to thanks for the financial support by the Institute of Research Management and Monitoring University of Malaya, Grant No. PS310/2009C.

Thanks to my friends Mr. Tee Kian Yew, Mr. Khaw Kim Siang and Ms. Vanessa Phung Ling Jen for having useful discussion and support during this research period.

Last but not least, I would like to thanks to my parent for their mental support and encouragement in furthering my study in theoretical physics.

UNIVERSITY OF MALAYA
ORIGINAL LITERARY WORK DECLARATION

Name of Candidate: ***Ong Jian Fuh*** (I.C/Passport No: 851129-11-5171)

Registration/Matric No: ***SGR080098***

Name of Degree: ***Master of Science***

Title of Project/Research Report/Dissertation/Thesis ("this Work"):

"Neutrino Mass in Higgs Triplet Model"

Field of Study: ***Neutrino Physics***

I do solemnly and sincerely declare that:

1. I am the sole author/writer of this Work;
2. This work is original;
3. Any use of any work in which copyright exists was done by way of fair dealing and for permitted purposes and any excerpt or extract from, or reference to or reproduction of any copyright work has been disclosed expressly and sufficiently and the title of the Work and its authorship have been acknowledged in this Work;
4. I do not have any actual knowledge nor do I ought reasonably to know that the making of this Work constitutes an infringement of any copyright work;
5. I hereby assign all and every rights in the copyright to this Work to the University of Malaya ("UM"), who henceforth shall be owner of the copyright in this Work and that any reproduction or use in any form or by any means whatsoever is prohibited without the written consent of UM having been first had and obtained;
6. I am fully aware that if in the course of making this Work I have infringed any copyright whether intentionally or otherwise, I may be subject to legal action or any other action as may be determined by UM.

Candidate's Signature Date

Subscribed and solemnly declared before,

Witness's Signature Date

Name: Prof. Wan Ahmad Tajuddin Wan Abdullah

Designation:

Abstract

We study the $\overline{\nu}_e - e$ annihilation from low to ultrahigh energy in the framework of Higgs Triplet Model (HTM). We add the contribution of charged Higgs boson exchange to the total cross section of the scattering. We obtain the upper bound $h_{ee}/M_{H^{\pm}} \lesssim 2.8 \times 10^{-3} GeV^{-1}$ in this process from low energy experiment. We show that by using the upper bound we obtained, the charged Higgs contribution could give enhancements to the total cross section with respect to the SM prediction at $s \approx M_{H\pm}^2$.

Furthermore, at the energy above the W pole, we compare $W^- H_1$, $W^+ H^{--}$, $H^+ H^{--}$, $H_2 H^-$ and $W^- Z^0$ production. We compare the total cross section of $\overline{\nu}_e - e$ and $W^- Z^0$ at resonance energy , in which a charged Higgs boson is exchanged. We discuss the possibilities of the Higgs search through the products of Higgs bosons decay at large Higgs lepton coupling constant.

Abstrak

Kami mengkaji penghapusan $\overline{\nu}_e - e$ daripada tenaga rendah ke tenaga lampau tinggi dalam rangka Model triplet Higgs (HTM). Kami menambah sumbangan pertukaran Higgs boson kepada keratan rentas. Kami menganggarkan batasan atas $h_{ee}/M_{H^\pm} \lesssim 2.8 \times 10^{-3} GeV^{-1}$ dalam proses ini pada eksperimen tenaga yang rendah. Kami menunjukkan bahawa dengan menggunakan batasan atas yang didapati, sumbangan Higgs boleh memberi peningkatan kepada jumlah keratan rentas dengan merujuk kepada ramalan SM pada $s \approx M^2_{H^\pm}$.

Tambahan pula, pada tenaga melebihi kutub W, kami bandingkan penghasilan W^-H_1, W^+H^{--}, H^+H^{--}, H_2H^- dan W^-Z^0. Kami membandingkan keratan rentas $\overline{\nu}_e - e$ dan W^-Z^0 pada tenaga resonans, di mana Higgs boson ditukar. Kami membincangkan kemungkinan pencarian Higgs melalui produk reputan Higgs pada nilai coupling Higgs lepton yang besar.

Contents

List of Figures

List of Tables

Chapter 1

Introduction

In the Standard Model (SM) of particles physics, the neutrino is massless. The neutrino is one of the elementary particles which can adequately be accounted for by the weak interaction. Furthermore, the masslessness of the neutrino makes the SM as complete and no further improvement is necessary for the SM. In this model, there are only three flavours or generation of massless neutrinos [see Table 2.1], namely electron neutrino, muon neutrino and tauon neutrino in addition to their respective antineutrinos. However, the studies of the solar neutrino problem (Bahcall & Davis, 1976) had led physicists to suspect that one type of neutrino may transform into another type during propagation from the sun to the earth. The flavour changing or so called neutrino oscillation is later known to be possible if the neutrinos are massive. The existence of small neutrinos masses have motivated the study of neutrinos to the physics beyond the standard model. In particular, the Higgs Triplet Model (HTM) (T. P. Cheng & Li, 1980; Schechter & Valle, 1980). This model provides us an alternative way to introduce and explain the smallness of the neutrinos masses through type-II seesaw mechanism (Fileviez Pérez, Han, Huang, Li, & Wang, 2008) and enables one to estimate the parameters in the lepton violation processes. The introduction of singly and doubly charged Higgs bosons in this model have opened many phenomenological studies in hunting for them at energy frontier accelerators (Fileviez Pérez et al., 2008; Akeroyd & Aoki, 2005; Akeroyd & Chiang, 2010) particularly at the CERN[1] LHC[2] with center-of-mass energy of 14 TeV. Besides, the coupling of charged leptons and neutrinos to singly charged Higgs bosons in this model is particular interesting to the leptonic processes. Thus, the $\overline{\nu}_e - e$ annihilation does not only proceed through charge current (CC) and neutral current (NC) but also with charged Higgs boson exchange. Furthermore, the coupling of charged Higgs bosons to the gauge bosons has opened wide variety of final states to $\overline{\nu}_e - e$ annihilation.

In this thesis, we investigate the possibilities of the Higgs triplet production at ultrahigh energy ($10^{12}eV - 10^{20}eV$) through neutrino electron annihilation at tree level. However, this energy is beyond the reach of the energy frontier accelerator and only be possible from cosmic rays. A large-scale neutrino telescope such as IceCube (Halzen, 2006) is likely to observe these events. The flux of ultrahigh energy $\overline{\nu}_e$ may get enhanced through the neutrino oscillation from $\overline{\nu}_\mu$ produced at the atmosphere ($p^+ + \gamma \rightarrow \pi^+ + n,\ \pi^+ \rightarrow \mu^+ + \nu_\mu,\ \mu^+ \rightarrow e^+ + \overline{\nu}_\mu + \nu_e,\ n \rightarrow p^+ + e^- + \overline{\nu}_e$). The study of ultrahigh energy particles interaction would be one of the possible ways to probe the feasibility of physics beyond the standard model.

The thesis is organized as follow:

Chapter 2 gives a compact introduction on the SM to provide the understanding on how the particles interact and how do they obtained their masses while the neutrinos remain massless.

Chapter 3 introduces some physics and experimental evidences that lead to the massive

[1]The European Organization for Nuclear Research, located at Geneva on the France−Swiss border.
[2]Large Hadron Collider

neutrino concept. The behaviours of the massive neutrinos in medium and electromagnetic field are briefly introduced. The discussion about the HTM is included in the end of this chapter.

Chapter 4 is the calculation of the differential cross section and total cross section for the processes:

$$
\begin{aligned}
\overline{\nu}_e(\vec{q}) + e^-(\vec{p}) &\rightarrow \overline{\nu}_e(\vec{q}\,') + e^-(\vec{p}\,') \\
\overline{\nu}_e(\vec{q}) + e^-(\vec{p}) &\rightarrow W^-(\vec{k}) + H_1(\vec{k}') \\
\overline{\nu}_e(\vec{q}) + e^-(\vec{p}) &\rightarrow W^+(\vec{k}) + H^{--}(\vec{k}') \\
\overline{\nu}_e(\vec{q}) + e^-(\vec{p}) &\rightarrow H^+(\vec{k}) + H^{--}(\vec{k}') \\
\overline{\nu}_e(\vec{q}) + e^-(\vec{p}) &\rightarrow H_2(\vec{k}) + H^-(\vec{k}') \\
\overline{\nu}_e(\vec{q}) + e^-(\vec{q}) &\rightarrow W^-(\vec{k}) + Z^0(\vec{k}')
\end{aligned}
$$

The discussions on the cross section are presented at the end of each process.

Chapter 5 discusses the numerical results obtained from Chapter 4 and the possibilities of the Higgs decay.

Conclusions are given in Chapter 6.

Chapter 2

Theoretical Background

2.1 Standard Model

The Standard Model (SM) of particle physics is completely described by the electroweak and Quantum Chromodynamics (QCD) theories (Griffiths, 2008). These mechanics are able to describe the three of the four known forces in the nature, namely electromagnetic, strong and weak forces, using mediating gauge bosons. These gauge bosons are gluon, W^+, W^-, Z^0 and photon. This leaves the gravity to be described by the General Theory of Relativity (Foster & Nightingale, 2006). The present state of affairs makes the theories of physics unsatisfactory in the light of the unification. In fact, the ultimate goal of the theory of physics is the unification of all theories of physics into a single theory, called the Theory of Everything (TOE), which can explain all the four forces known presently in the nature. The unification of the quantum field theory and the General Theory of Relativity are being pursued by physicists throughout the world that results in the birth of fields of research area such as Quantum Gravity, String Theory (Zwiebach, 2004), M-theory and loop Quantum Gravity (Kaku, 1993, 1999). The validity of these theories can be tested at Planck scale, a scale of about 10^{-35} meters or 10^{19} GeV, that can only be achieved by accelerators operating at more than 1 TeV. Several research laboratories which operate high energy accelerators, such as Tevatron in USA and LHC in Europe, are pushing their operating energies towards this scale. The mathematical formulations of strong and electroweak interaction are described by gauge quantum field theory (T. Cheng & Li, 1984; Mandl & Shaw, 2009; Peskin & Schroeder, 1995; Lahiri & Pal, 2005). Gauge theory is a type of field theory in which the Lagrangian or equation of motion is invariant under some continuous group of local transformation. Such a transformation is called gauge transformation and form a Lie group which refers to the symmetry group (Tung, 1985) of the theory. Each symmetry group is associated with group generators. A corresponding vector field or gauge field arises for each of the generators. A gauge field is necessary to be imposed into the theory to ensure that the Lagrangian is invariant under the local gauge transformation. Upon the quantization of the gauge fields, the quanta of the gauge fields are called gauge bosons. In particular, the quantum version of classical electrodynamics, Quantum Electrodynamics (QED) is the abelian gauge theory based on the gauge group $U(1)$ with one gauge field, the electromagnetic field where the photon being the gauge boson so that the Dirac Lagrangian is invariant under local gauge transformation. An abelian group is a group in which the group elements commute, otherwise is called non-abelian. The extension of the abelian gauge theory to non-abelian gauge theory was first constructed by Yang and Mills into a famous Yang-Mills theory (Yang & Mills, 1954). The SM is the non-abelian symmetry gauge theory with gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. The gauge group that describes the strong interaction is $SU(3)_C$. There are 8 generators correspond to 8 gauge fields with 8 gluons being the gauge boson. The QCD requires color charge for the quarks to interact among themselves via gluon exchange, which is in contrast to QED with only one charge for the interaction among electrons via photon exchange. The quarks carry three colors (red, green, blue) [1] and gluons carry the combination of color-anticolor (neutral, no color). The composite particles that are made up of quarks come in three quarks or two quarks combination where these combinations give no resultant color charge. A particle with 3 quarks combination is called baryon while two quarks combination called meson. No free quarks have been observed and the effort to search for them in the high energy accelerator always give negative results. The fermions and bosons of SM are present in Table 2.1 and Table 3.1.

Table 2.1: The fermions of the Standard Model. The up type quarks (u, c, t) and e, μ, τ are mass eigenstates (with definite mass) while the down type quarks (d', s', b') are weak eigenstates (with non definite mass). The weak eigenstates of quarks are related to their mass eigenstates through Cabibbo-Kobayashi-Maskawa (CKM) matrix (Cabibbo, 1963; Kobayashi & Maskawa, 1973). The neutrinos are massless in the SM. The subscript L and R is the left-handed $P_L = \frac{1}{2}(1 - \gamma_5)$ and right-handed $P_R = \frac{1}{2}(1 + \gamma_5)$ projection operator respectively. There are no right-handed neutrinos in the model.

Fermions	1st	2nd	3rd	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
Quark	$\begin{pmatrix} u \\ d' \end{pmatrix}_L$	$\begin{pmatrix} c \\ s' \end{pmatrix}_L$	$\begin{pmatrix} t \\ b' \end{pmatrix}_L$	3	2	$\frac{1}{3}$
	u_R	c_R	t_R	3	1	$\frac{4}{3}$
	d_R	s_R	b_R	3	1	$-\frac{2}{3}$
Lepton	$\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L$	$\begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L$	$\begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L$	1	2	-1
	e_R	μ_R	τ_R	1	1	-2

Table 2.2: The bosons of the Standard Model. The gauge fields carry spin-1 while the Higgs field carries spin-0.

Bosons	Fields	$SU(3)_C$	$SU(2)_L$	$U(1)_Y$
$SU(3)_C$ gauge fields	$G^1, ..., G^8$	8	1	0
$SU(2)_L$ gauge fields	W^1, W^2, W^3	1	1	0
$U(1)_Y$ gauge field	B	1	1	0
Higgs fields	ϕ^+, ϕ^0	1	2	1

[1]The colors refer here are quantum parameter which is unrelated to the visible colors in everyday life.

2.2 Standard Electroweak Model

Just as the unification of electricity and magnetism by Maxwell into four Maxwell Equations of electromagnetism, the standard electroweak model is the unified description of electromagnetism and weak interaction. It was proposed by Weinberg, Glashow and Salam (Weinberg, 1967; Glashow, 1961; Salam & Ward, 1964). This unification is based on the gauge group $SU(2)_L \times U(1)_Y$ (T. Cheng & Li, 1984; Mandl & Shaw, 2009; Peskin & Schroeder, 1995). There are four gauge bosons in the gauge group, namely W_i^{μ} $(i = 1, 2, 3)$ for $SU(2)_L$ and B^{μ} for $U(1)_Y$, with the coupling constants g and g' respectively. The weak hypercharge, Y, electric charge, Q and weak isospin, T_3 as a set of quantum numbers satisfy the relation $Q = T_3 + \frac{Y}{2}$. In analogy to the classification of the proton and neutron with their isospin (Krane & Halliday, 1987), the fermions can be classified as shown in Table 2.1. The left-handed fermions (Weinberg, 1967)

$$\psi_{iL} = \begin{pmatrix} \nu_l \\ l \end{pmatrix}_{iL}, \begin{pmatrix} u \\ d' \end{pmatrix}_{iL} \tag{2.1}$$

transform as doublet under $SU(2)_L$ while the right-handed fermions $\psi_{iR} = l_{iR}, u_{iR}, d'_{iR}$ transform as singlet under $U(1)_Y$. The equation of motion of the fermions and gauge bosons are given by the Euler-Lagrange equation through minimizing the action integral, S involving the Lagrangian density (Mandl & Shaw, 2009) [see Appendix B]

$$\begin{aligned} S &= \int d^4x \mathcal{L}(\phi, \partial\phi) \\ \delta S &= 0 \\ \frac{\partial \mathcal{L}}{\partial \phi} &- \frac{\partial}{\partial x}\left(\frac{\partial \mathcal{L}}{\partial(\partial\phi)}\right) = 0 \end{aligned} \tag{2.2}$$

where ϕ is an arbitrary field. The Lagrangian density, $\mathcal{L}(\phi, \partial\phi)$ will be referred as Lagrangian afterwards. Thus, the kinetic energy terms for quarks, leptons and gauge bosons in the electroweak Lagrangian is

$$\begin{aligned} \mathcal{L}_{EW} &= i\overline{\psi}_{lL}\not{D}\psi_{lL} + i\overline{\psi}_{lR}\not{D}\psi_{lR} + i\overline{\psi}_{\nu R}\not{D}\psi_{\nu R} \\ &\quad -\frac{1}{4}F_i^{\mu\nu}F_{\mu\nu}^i - \frac{1}{4}B^{\mu\nu}B_{\mu\nu} \end{aligned} \tag{2.3}$$

where $\not{D}$ is $\gamma^\mu D_\mu$. The first three terms are the Dirac Lagrangian which describe the spin$-\frac{1}{2}$ fermions while the last two terms are the Proca Lagrangian that describe spin-1 vector bosons. The forces (gauge fields) that experienced by different particles (as well as different chirality) are added into the derivative through the minimal substitution in order to preserve the invariant of Lagrangian under the $SU(2)_L$ and $U(1)_Y$ local gauge transformation (T. Cheng & Li, 1984). The modified derivatives or covariant derivatives take the form (Mandl & Shaw, 2009)

$$\begin{aligned} D_\mu\psi_{lL} &= [\partial_\mu + ig\frac{\sigma_i}{2}W_\mu^i + ig'\frac{Y}{2}B_\mu]\psi_{lL}, \\ D_\mu\psi_{lR} &= [\partial_\mu + ig'\frac{Y}{2}B_\mu]\psi_{lR}, \\ D_\mu\psi_{\nu R} &= \partial_\mu\psi_{\nu R}. \end{aligned} \tag{2.4}$$

The field strength tensor $F_{\mu\nu}^i$ and $B_{\mu\nu}$ of the gauge bosons are

$$\begin{aligned} F_i^{\mu\nu} &= \partial^\nu W_i^\mu - \partial^\mu W_i^\nu \\ B^{\mu\nu} &= \partial^\nu B^\mu - \partial^\mu B^\nu \end{aligned} \tag{2.5}$$

which is in analogy of the electromagnetic field strength tensor. The Pauli matrices $\frac{\sigma_i}{2}$ and $\frac{Y}{2}$ are the generators of the group $SU(2)_L$ and $U(1)_Y$ respectively. The addition of mass terms into Equation (2.3) violate both $SU(2)_L$ and $U(1)_Y$ gauge invariance. Therefore, the mass generation mechanism is required in order to give the mass to the gauge bosons and also fermions without violating the gauge invariance.

2.3 Spontaneous symmetry breaking

The generation of the mass of gauge fields proceed through Higgs mechanism (Higgs, 1964; Englert & Brout, 1964; Guralnik, Hagen, & Kibble, 1964) after spontaneous symmetry breaking (Nambu, 2009). In the spontaneous symmetry breaking the particles at high energy behave similarly, however, at low energy they are found to be the same type of particles but only in different states (Hawking, 1998). In order to introduce the spontaneous symmetry breaking break $SU(2)_L \times U(1)_Y \rightarrow U(1)_{EM}$, where $U(1)_{EM}$ is the electromagnetic interaction gauge group, a simple Lagrangian with a weak isospin doublet scalar field is introduced into Equation (2.3) [see Appendix B].

$$\begin{aligned} \mathcal{L}_{Higgs} &= (D_\mu\phi)^\dagger(D_\mu\phi) - V(\phi) \\ &= (D^\mu\phi)^\dagger(D_\mu\phi) - \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2, \end{aligned} \tag{2.6}$$

$$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}. \tag{2.7}$$

For $\mu^2 > 0$, the minimum of the potential occurs at [see Figure 2.1]

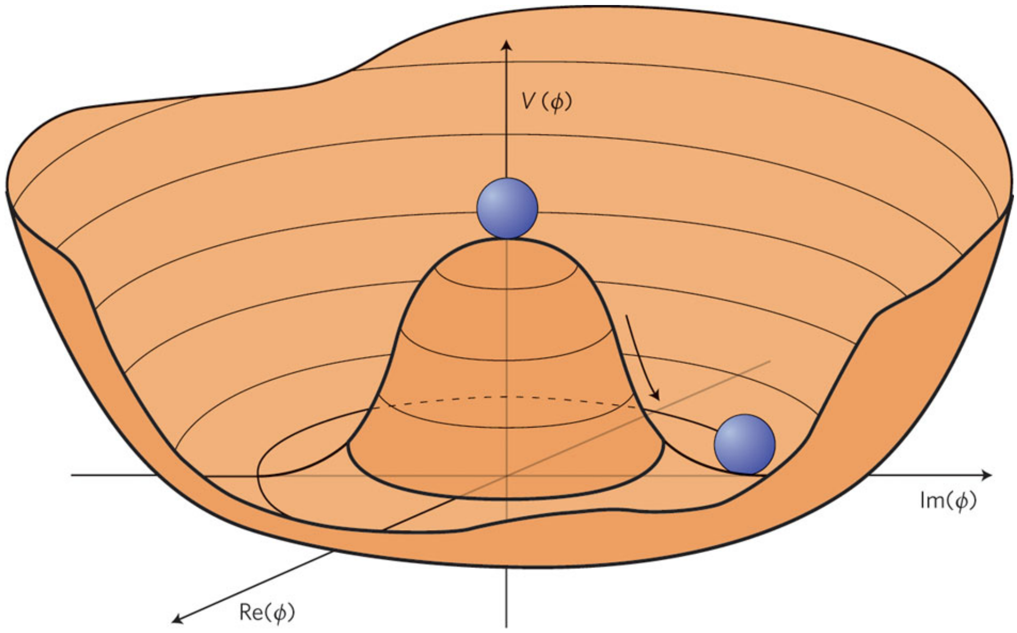


Figure 2.1: The Higgs potential after spontaneous symmetry breaking.

$$\langle \phi \rangle = \begin{pmatrix} 0 \\ \frac{\upsilon}{\sqrt{2}} \end{pmatrix}. \tag{2.8}$$

The vacuum state does not correspond to zero but a particular choice around the circular minimum and $\langle \phi \rangle$ is the vacuum expectation value (vev). The field, ϕ can be parameterized in four real fields

$$\phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \eta_1(x) + i\eta_2(x) \\ \upsilon + H(x) + i\eta_3(x) \end{pmatrix} = exp[i\frac{\eta_j(x)\tau}{\upsilon}]\frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ \upsilon + H(x) \end{pmatrix} \tag{2.9}$$

where $H(x)$ is the fluctuation in the radial plane. On the other hand, the fields $\eta_j(x)$ are the fluctuation along the circular minimum where the potential is constant so that upon quantization, the $\eta_j(x)$ bosons are massless. These fields have zero vev, $\langle 0 \mid \eta_j(x) \mid 0 \rangle$ = $\langle 0 \mid H(x) \mid 0 \rangle$ = 0. The η_j bosons are called Goldstone bosons that do not exist in nature and should be eliminated from the Lagrangian by gauge transformation.

$$\begin{aligned} \phi' &= exp[-i\frac{\eta_j(x)\tau}{\upsilon}]\phi \\ &= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \upsilon + H(x) \end{pmatrix} \end{aligned} \quad (2.10)$$

The gauge transformation which the transformed field has this form is called unitary gauge. A massless vector field carries two degrees of freedom (transverse polarization), when it acquires mass, it picks up a third degree of freedom (longitudinal polarization) which correspond to the $\eta_j(x)$ fields that have been eliminated. Therefore, $\eta_1(x), \eta_2(x), \eta_3(x)$ are said to be absorbed by W^+, W^-, Z^0 to acquire mass. This mechanism is the famous Higgs mechanism. From the kinetic term in Equation (2.6) and the covariant derivative in Equation (2.4), the mass of the gauge bosons evaluated at the scalar field vev are

$$|\left(ig\frac{\sigma_i}{2}W_\mu^i + ig'\frac{Y}{2}B_\mu\right)\phi|^2$$

$$\left(\frac{1}{2}g\upsilon\right)^2 W_\mu^+ W^{-\mu} + \frac{1}{8}\upsilon^2 \begin{pmatrix} W_\mu^3 & B_\mu \end{pmatrix} \begin{pmatrix} g^2 & -gg' \\ -gg' & g'^2 \end{pmatrix} \begin{pmatrix} W_\mu^3 \\ B_\mu \end{pmatrix}. \quad (2.11)$$

The mass of the $W^\pm$ is

$$m_W^2 = \frac{1}{4}\upsilon^2 g^2 \quad (2.12)$$

with

$$W_\mu^\pm = \frac{W_\mu^1 \mp iW_\mu^2}{\sqrt{2}}. \quad (2.13)$$

Since the mass is the eigenvalue and should be diagonal in a matrix, therefore the second term in Equation (2.11) should be diagonalized. One obtains

$$\frac{1}{8}\upsilon^2 \begin{pmatrix} Z_\mu & A_\mu \end{pmatrix} \begin{pmatrix} g^2+g'^2 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} Z_\mu \\ A_\mu \end{pmatrix}$$

where A_μ and Z_μ are the physical fields that mixed W_μ^3 and B_μ with $\tan\theta_W = g'/g$,

$$\begin{aligned} Z_\mu &= W_\mu^3 \cos\theta_W - B_\mu \sin\theta_W \\ A_\mu &= W_\mu^3 \sin\theta_W + B_\mu \cos\theta_W \end{aligned} \tag{2.14}$$

The field A_μ being the massless photon and Z_μ being the Z^0 boson with mass

$$m_Z^2 = \frac{1}{4}\upsilon^2(g^2+g'^2). \tag{2.15}$$

In order to generate the mass of the fermions, the gauge invariant Yukawa Lagrangian is included into the theory.

$$-\mathcal{L}_{Yukawa} = g_l \overline{\psi}_{iL}\psi_{iR}\phi + g_l^* \phi^\dagger \overline{\psi}_{iR}\psi_{iL} \tag{2.16}$$

The second term is the hermitian conjugate of the first term and will be denote as H.c. afterward. The charged leptons obtain their masses after ϕ acquire the vev,

$$-\mathcal{L}_{Yukawa} = \frac{g_l\upsilon}{\sqrt{2}}\bar{l}_L l_R + H.c., \qquad m_l = \frac{g_l\upsilon}{\sqrt{2}} \tag{2.17}$$

while the neutrinos remain massless. Here g_l is the dimensionless coupling constant.

Chapter 3

Neutrino Physics

3.1 Neutrino Oscillation

Neutrinos are produced from beta decay process and propagate in vacuum at almost the speed of light and pass through matter at almost no interaction. Lepton quantum numbers such as electron lepton number, L_e, muon lepton number, L_μ and tauon lepton number, L_τ are assigned to each generation of lepton, as shown in Table 3.1. It is required that these lepton numbers should be conserved (Mandl & Shaw, 2009). However, the experimental study (Davis, Harmer, & Hoffman, 1968) of the solar neutrinos from the Sun found that the flux was significantly smaller than that predicted by the Standard Solar Model (Bahcall, Bahcall, & Shaviv, 1968). The studies of this discrepancy or solar neutrino problem (Bahcall & Davis, 1976) had led physicists to suspect that one flavour of neutrino may transform into another flavour during propagation from the Sun to the Earth, since the detector was not sensitive to all flavours of neutrino. Such a phenomena is called neutrino oscillation. Neutrino oscillation is possible if the neutrinos are massive. Thus, the flavour changing process implies that the lepton numbers are necessarily violated and the SM is no longer adequate to describe neutrinos.

Table 3.1: Lepton numbers L_e, L_μ, L_τ of three generation leptons. The lepton numbers of -1 are assigned for their respective anti particles. The lepton number is zero for quarks and gauge bosons

	ν_e	e	ν_μ	μ	ν_τ	τ
L_e	1	1	0	0	0	0
L_μ	0	0	1	1	0	0
L_τ	0	0	0	0	1	1

However, the theory to describe the neutrino oscillation in vacuum was pursued vigorously by Bilenky and Pontecorvo (Pontecorvo, 1957; Bilenky & Pontecorvo, 1978) in the framework of quantum theory in analogy with $K^0 - \overline{K^0}$ oscillation (Gell-Mann & Pais, 1955). In the standard theory of neutrino oscillation, the neutrino flavour states that were produced in the decay $W^+ \to l_\alpha^+ + \nu_\alpha$ are the superposition of three neutrino mass eigenstates (Bilenky & Pontecorvo, 1978; Gonzalez-Garcia & Nir, 2003)

$$| \nu_\alpha\rangle = \sum_k U^*_{\alpha k} | \nu_k\rangle \tag{3.1}$$

where $\alpha = e, \mu, \tau$; $k = 1, 2, 3$ and U is the 3×3 mixing matrix[1]. The neutrino flavour states are assumed to have a definite momentum $\vec{p}$ and the mass eigenstates have different energies where the relativistic energy approximation is

$$E_k = \sqrt{\vec{p}^2 + m_k^2} \approx \vec{p} + \frac{m_k^2}{2\vec{p}} \tag{3.2}$$

[1]Also called Pontecorvo-Maki-Nakagawa-Sakata matrix (PMNS matrix), Maki-Nakagawa-Sakata matrix (MNS matrix), lepton mixing matrix, or neutrino mixing matrix

such that the massive neutrino states $|\nu_i\rangle$ are eigenstates of Hamiltonian

$$H|\ \nu_k\rangle = E_k|\ \nu_k\rangle\ . \tag{3.3}$$

The mass eigenstates satisfy the Schr$\ddot{o}$dinger-like equation

$$i\frac{\partial}{\partial t}\ |\ \nu_k(t)\rangle = H|\ \nu_k(t)\rangle \tag{3.4}$$

and have a plane wave solution

$$|\ \nu_k\ (t)\rangle = e^{-iE_k t}|\ \nu_k\rangle\ . \tag{3.5}$$

Thus, the flavour states evolve in time as

$$|\nu_\alpha(t)\rangle = \sum_k e^{-iE_k t} U^*_{\alpha k}|\nu_k\rangle. \tag{3.6}$$

The mass eigenstates can be expressed in terms of flavour states by inverting Equation (3.1) and Equation (3.6)

$$|\nu_\alpha(t)\rangle = \sum_\beta \left(U^*_{\alpha k} e^{-iE_k t} U_{\beta k}\right)|\nu_\beta\rangle. \tag{3.7}$$

The probability for the neutrino oscillation is therefore

$$P(\nu_\alpha \to \nu_\beta; t) = |\ \langle\nu_\beta|\nu_\alpha(t)\rangle\ |^2 = \sum_{k,j} U^*_{\alpha k} U_{\beta k} U_{\beta k} U^*_{\alpha j} e^{-i(E_k - E_k)t}. \tag{3.8}$$

For two neutrinos case, the mixing matrix takes the form [see Equation (D.1)]

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{3.9}$$

and the conversion probability is (Mohapatra & Pal, 2004)

$$P_{\nu_e \to \nu_\mu} = \frac{1}{2}\sin^2 2\theta_{12}\left[1 - \cos\left(\frac{m_1^2 - m_2^2}{2E}\right)L\right] \tag{3.10}$$

where $\sin^2 2\theta_{12}$ and $m_1^2 - m_2^2$ are the neutrino mixing angle and the mass splitting of the neutrino mass eigenstates ν_1 and ν_2 respectively. The first direct measurement of the total flux of ^{8}B neutrinos arriving from the Sun at SNO[1] provided an evidence to the neutrino flavour changing (Ahmad et al., 2001, 2002). The neutrino oscillation phenomena were further confirmed from different sources of neutrino. In particular, the reactor antineutrino disappearance by KamLAND[2] (Eguchi et al., 2003), the accelerator generated ν_μ disappearance by K2K[3] (Ahn et al., 2006, 2003) and the atmospheric neutrino[4] oscillation by Kamiokande (Hatakeyama et al., 1998). On the other hand, the probability of neutrino flavour oscillation is affected when the neutrinos travel through matter (Wolfenstein, 1978). In analogy to the photons that develop effective masses in a medium, the neutrinos are similarly developing their effective masses by the influence of the weak potential. Different flavours of neutrinos will experience different interaction with matter, which comprise of electrons and nucleus. Thus, the modified probability of neutrino conversion

[1] Sudbury Neutrino Observatory
[2] Kamioka Liquid Scintillator Anti-Neutrino Detector
[3] KEK to Kamioka. KEK is a Japanese high energy physics research organization
[4] Neutrino created by the interaction of cosmic rays with the nuclei in the atmosphere

is (Mohapatra & Pal, 2004)

$$P_{\nu_e \to \nu_\mu} = \frac{1}{2} \sin^2 2\theta_M \left[1 - \cos\left(\frac{\tilde{m}_1^2 - \tilde{m}_2^2}{2E} \right) L \right] \tag{3.11}$$

which has the same structure as Equation (3.10) with the neutrino mixing angle in vacuum and the mass splitting are replaced by effective mixing angle and effective neutrino mass splitting. A further modification was done in order to take into account the neutrino oscillation in non-uniform matter (Halprin, 1986; Kim, Sze, & Nussinov, 1987). There is a resonant oscillation region in the medium with varying density in which the possibility of total transition between two flavours of neutrino is maximum. Such a mechanism is the famous Mikheyev-Smirnov-Wolfenstein (MSW) effect (Mikheyev & Smirnov, 1986; Wolfenstein, 1978). In addition, the neutrino spin-flavour oscillation in a medium with magnetic field was considered in connection to the solar neutrino problem (Egorov, Likhachev, & Studenikin, 1995; Egorov, Lobanov, & Studenikin, 2000). Since the neutrino is neutral, the difference between particle and its antiparticle is difficult to distinguish. So, the term Dirac neutrino refers to that the particle and antiparticle are different while Majorana neutrino refers to that the particle and antiparticle are the same. However, the nature of Dirac-Majorana neutrino is still an open question. Furthermore, the zero charge neutrino can undergo loop interaction with the photon to induce the electromagnetic properties of neutrino (Nieves, 1982; Vogel & Engel, 1989; Degrassi, Sirlin, & Marciano, 1989). For instance, the Dirac neutrino can have four electromagnetic form factors while the Majorana neutrino has only one electromagnetic form factor (Kayser, 1982). The magnetic moment of Dirac neutrino obtained is proportional to its mass (Lee

& Shrock, 1977).

$$\mu_\nu = \frac{3G_F m_e m_\nu}{4\sqrt{2}\pi^2} = 3.2 \times 10^{-19} \left(\frac{m_\nu}{1eV}\right) \tag{3.12}$$

Thus, the neutrino mass with lepton number violation have to be incorporated into the SM so that the neutrino oscillation and their electromagnetic properties can be explained.

3.2 Neutrino Mass in $SU(2)_L \times U(1)_Y$ model

In order to obtain non-vanishing neutrino mass, the most straight forward way is to introduce a right-handed neutrinos into the SM (Mohapatra & Pal, 2004). A $SU(2)_L$ singlet right-handed neutrino with $Y = 0$ is added that corresponds to each generation of charged lepton. These neutrinos have no interaction with the gauge bosons as they are not observed experimentally. The new gauge invariant Yukawa Lagrangian is

$$-\mathcal{L}_{Yukawa} = \sum_{l,l'} f_{ll'} \overline{\psi}_{lL} \widehat{\phi} \nu_{l'R} + H.c. \tag{3.13}$$

where $\widehat{\phi} = i\sigma_2\phi^*$. After spontaneously symmetry breaking,

$$-\mathcal{L}_{msss} = \sum_{l,l'} f_{ll'} \frac{\upsilon}{\sqrt{2}} \overline{\nu}_{lL} \nu_{l'R} + H.c. \tag{3.14}$$

with

$$m_{ll'} = f_{ll'} \frac{\upsilon}{\sqrt{2}} \tag{3.15}$$

as the neutrinos masses in weak eigenstate. Hence, the neutrino mass eigenstates are obtained by diagonalizing Equation (3.14). As a result, the neutrino flavour eigenstates are a mixture of the mass eigenstates.

$$\begin{aligned}
\nu_{lL} &= \sum_i U_{l\alpha}\nu_{iL} \\
\nu_{lR} &= \sum_i U_{l\alpha}\nu_{iR} \\
m_i &= U^{\dagger} m_{ll'} V \\
-\mathcal{L}_{msss} &= \sum_i m_i \overline{\nu}_{iL}\nu_{iR} + H.c.
\end{aligned} \tag{3.16}$$

The mass of neutrino arises as the other fermions in the model. However, this method does not provide explanation to the smallness of neutrino mass. On the other hand, the coupling constants are completely determined through experiment. In a sense, the model is incomplete. A new model with extended Higgs sector is suggested. In particular, the Higgs Triplet Model (HTM).

3.2.1 Higgs Triplet Model

In this model, a I=1, Y= 2 complex $SU(2)_L$ scalar triplet is included to the SM Lagrangian to explain the smallness of neutrino mass (T. P. Cheng & Li, 1980; Schechter & Valle, 1980) without requiring the addition of extra right-handed neutrinos. The Lagrangian with the addition of the Higgs triplet is

$$\mathcal{L} = (D^{\mu}\phi)^{\dagger}(D_{\mu}\phi) + Tr(D^{\mu}\Delta)^{\dagger}(D_{\mu}\Delta) + \mathcal{L}_{Yukawa} - V(\phi, \Delta). \tag{3.17}$$

The $SU(2)_L \times U(1)_Y$ gauge invariant Yukawa Lagrangian for the Higgs leptons interaction are written as (Fileviez Pérez et al., 2008; Akeroyd & Aoki, 2005; Akeroyd & Chiang, 2010)

$$\mathcal{L}_{Yukawa} = h_{\alpha\beta}\psi^{T}_{\alpha L} C i\sigma_2 \Delta \psi_{\beta L} + H.c. \tag{3.18}$$

where $h_{\alpha\beta}(\alpha, \beta = e, \mu, \tau)$ is the coupling constant matrix, C is the charged conjugation matrix, σ_2 is a Pauli matrix, $\psi_{\alpha L} = (\nu_\alpha, l_\alpha)_L^T$ is left-handed lepton doublet and Δ is 2×2 representation of the $Y = 2$ complex triplet field,

$$\Delta = \begin{pmatrix} \Delta^+/\sqrt{2} & \Delta^{++} \\ \Delta^0 & -\Delta^+/\sqrt{2} \end{pmatrix}. \tag{3.19}$$

The Higgs triplet potential is

$$\begin{aligned} V(\phi, \Delta) &= -m_H^2 \phi^\dagger \phi + M_\Delta^2 Tr \Delta^\dagger \Delta + \frac{\lambda}{4}(\phi^\dagger \phi)^2 + \left(\mu \phi^T i \sigma_2 \Delta^\dagger \phi + H.c.\right) \\ &\quad + \lambda_1 (\phi^\dagger \phi) Tr \Delta^\dagger \Delta + \lambda_2 (Tr \Delta^\dagger \Delta)^2 + \lambda_3 Tr (\Delta^\dagger \Delta)^2 \\ &\quad + \lambda_4 \phi^\dagger \Delta \Delta^\dagger \phi. \end{aligned} \tag{3.20}$$

The neutrinos obtain their masses after Δ^0 acquire the vacuum expectation value, υ_Δ,

$$\begin{aligned} m_{\alpha\beta} &= 2 h_{\alpha\beta} \langle \Delta^0 \rangle = \sqrt{2} h_{\alpha\beta} \upsilon_\Delta; \\ \upsilon_\Delta &= \frac{\mu \upsilon_0^2}{\sqrt{2} M_\Delta^2} \end{aligned} \tag{3.21}$$

where the neutral component of Higgs doublet and triplet are expressed as

$$\begin{aligned} \phi_0 &= \frac{\upsilon_0 + h_0 + i\xi_0(x)}{\sqrt{2}}, \\ \Delta_0 &= \frac{\upsilon_\Delta + \delta_0 + i\eta_0(x)}{\sqrt{2}}. \end{aligned} \tag{3.22}$$

The ground state of a system where it is at the most stable state can be obtained by minimization of the global minimum of the potential

$$\begin{aligned} \frac{\partial V}{\partial \phi}_{\phi=\langle\phi\rangle,\Delta=\langle\Delta\rangle} &= 0, \\ \frac{\partial V}{\partial \Delta}_{\phi=\langle\phi\rangle,\Delta=\langle\Delta\rangle} &= 0. \end{aligned} \tag{3.23}$$

and

$$\begin{aligned} & -m_H^2 + \lambda\frac{\upsilon_0^2}{4} - \sqrt{2}\mu\upsilon_\Delta + (\lambda_1+\lambda_4)\frac{\upsilon_\Delta^2}{2} = 0, \\ & M_\Delta^2\upsilon_\Delta - \frac{\mu\upsilon_0^2}{\sqrt{2}} + \frac{1}{2}(\lambda_1+\lambda_4)\upsilon_0^2\upsilon_\Delta^2 + (\lambda_2+\lambda_3)\upsilon_\Delta^3 = 0. \end{aligned} \tag{3.24}$$

By considering the lowest order processes, the λ_i parameters are set to zero where λ have their usual meaning. Thus we have

$$\upsilon_\Delta = \frac{\mu\upsilon_0^2}{\sqrt{2}M_\Delta^2},\ m_H^2 = \left(\frac{\lambda}{4} - \frac{\mu^2}{M_\Delta^2}\right)\upsilon_0^2.$$

The first relation is referred to as type II seesaw mechanism (Fileviez Pérez et al., 2008) for $M_\Delta^2 >> \upsilon_0^2$, as one component is large, the other component appears to be very small. From the Higgs potential one finds the masses of two CP-even, one CP-odd (charge conjugation and parity) and two singly charged Higgs bosons which are mixed of weak isospin doublet and triplet. Meanwhile, the doubly charged Higgs bosons are composed of triplet alone. The mass-squared matrix for CP-even states are

$$\frac{1}{2}\begin{pmatrix} h_0 & \delta_0 \end{pmatrix}\begin{pmatrix} \frac{\lambda\upsilon_0^2}{2} & -\sqrt{2}\mu\upsilon_0 \\ -\sqrt{2}\mu\upsilon_0 & M_\Delta^2 \end{pmatrix}\begin{pmatrix} h_0 \\ \delta_0 \end{pmatrix}. \tag{3.25}$$

Upon diagonalization, the two physical CP-even states are

$$\begin{aligned} H_1 &= \cos\theta_0\, h_0 + \sin\theta_0\, \delta_0 \\ H_2 &= -\sin\theta_0\, h_0 + \cos\theta_0\, \delta_0 \end{aligned} \tag{3.26}$$

with the masses

$$M_{H1}^2 \approx \frac{\lambda v_0^2}{2} - 2\sqrt{2}\mu v_\Delta, \;\; M_{H2}^2 \approx M_\Delta^2 + 2\sqrt{2}\mu v_\Delta \tag{3.27}$$

and mixing angle

$$\tan 2\theta_0 = -\frac{4M_\Delta^2 v_\Delta}{v_0(M_{H_1}^2 + M_{H_2}^2 - 2M_\Delta^2)}. \tag{3.28}$$

The CP-odd states are

$$\begin{aligned} &\frac{1}{2}\begin{pmatrix} \xi_0 & \eta_0 \end{pmatrix} \begin{pmatrix} 2\sqrt{2}\mu v_\Delta & -\sqrt{2}\mu v_0 \\ -\sqrt{2}\mu v_0 & M_\Delta^2 \end{pmatrix} \begin{pmatrix} \xi_0 \\ \eta_0 \end{pmatrix} \\ &\frac{1}{2}\begin{pmatrix} G_0 & A_0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & M_\Delta^2 + 2\sqrt{2}\mu v_\Delta \end{pmatrix} \begin{pmatrix} G_0 \\ A_0 \end{pmatrix} \end{aligned} \tag{3.29}$$

with

$$G_0 = \cos\alpha\, \xi_0 + \sin\alpha\, \eta_0 \tag{3.30}$$

$$A_0 = -\sin\alpha\, \xi_0 + \cos\alpha\, \eta_0 \tag{3.31}$$

and

$$M_{A_0}^2 = M_\Delta^2 + 2\sqrt{2}\mu v_\Delta, \; M_{G_0}^2 = 0$$

$$\cos\alpha = \frac{\upsilon_0}{\sqrt{\upsilon_0^2 + 4\upsilon_\Delta^2}},\ \sin\alpha = \frac{2\upsilon_\Delta}{\sqrt{\upsilon_0^2 + 4\upsilon_\Delta^2}}. \tag{3.32}$$

Meanwhile, the mass-squared matrix of the singly charged Higgs bosons are

$$\begin{pmatrix} \phi^+ & \delta^+ \end{pmatrix} \begin{pmatrix} \sqrt{2}\mu\upsilon_\Delta & -\mu\upsilon_0 \\ -\mu\upsilon_0 & M_\Delta^2 \end{pmatrix} \begin{pmatrix} \phi^+ \\ \delta^+ \end{pmatrix}$$
$$\begin{pmatrix} G^+ & H^+ \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & M_\Delta^2 + \sqrt{2}\mu\upsilon_\Delta \end{pmatrix} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} \tag{3.33}$$

with

$$G^\pm = \cos\theta_\pm\, \phi^\pm + \sin\theta_\pm\, \delta^\pm \tag{3.34}$$

$$H^\pm = -\sin\theta_\pm\, \phi^\pm + \cos\theta_\pm\, \delta^\pm \tag{3.35}$$

and

$$M_{H^\pm}^2 = M_\Delta^2 + \sqrt{2}\mu\upsilon_\Delta,\ M_{G^\pm}^2 = 0$$
$$\cos\theta_+ = \frac{\upsilon_0}{\sqrt{\upsilon_0^2 + 2\upsilon_\Delta^2}}, \quad \sin\theta_+ = \frac{\sqrt{2}\upsilon_\Delta}{\sqrt{\upsilon_0^2 + 2\upsilon_\Delta^2}}. \tag{3.36}$$

The doubly charged Higgs bosons are

$$H^{\pm\pm} = \Delta^{\pm\pm},\ M_{H^{\pm\pm}}^2 = M_\Delta^2. \tag{3.37}$$

The G_0 and $G^\pm$ are being the Goldstone bosons that absorbed by Z^0 and $W^\pm$ to acquire mass. The upper bound of $\upsilon_\Delta/\upsilon_0$ is set by the ratio of the mass of $W^\pm$ to Z^0, $\rho = 1.0008^{+0.0017}_{-0.0007}$ (Nakamura et al., 2010) to be $\upsilon_\Delta/\upsilon_0 \lesssim 0.02$. The lower bound of the charged Higgs boson mass in this model is (Fileviez Pérez et al., 2008)

$$M_{H^{\pm}} \gtrsim 110 \text{ GeV}. \tag{3.38}$$

The Feynman rules are included in Appendix C.

Chapter 4

Annihilation of $\overline{\nu}_e e$

4.1 Introduction

The problems of the neutrino mass to the SM have been discussed in Section 3.1 and one of the many models proposed to explain the smallness of neutrino mass were briefly covered in Subsection 3.2.1. The hunting of Higgs bosons in the framework of HTM have received much attention recently (Fileviez Pérez et al., 2008; Akeroyd & Aoki, 2005; Akeroyd & Chiang, 2010) particularly at the CERN LHC with center-of-mass energy of 14 TeV. However, the Higgs production from $\overline{\nu}_e - e$ annihilation is beyond the reach of any high energy accelerator and one would require the neutrino source from cosmic rays. Our works were motivated by (Mikaelian & Zheleznykh, 1980) which study the SM particles production from $\overline{\nu}_e e$ annihilation.

4.2 $\overline{\nu}_e + e^- \rightarrow \overline{\nu}_e + e^-$

The $\overline{\nu}_e e^-$ scattering process at low energy have been well tested theoretically and experimentally (Deniz et al., 2010). The high energy behaviour of this process in SM has also been considered by (Mikaelian & Zheleznykh, 1980; Butkevich, Kaidalov, Krastev, Leonov-Vendrovski, & Zheleznykh, 1988; Gandhi, Quigg, Reno, & Sarcevic, 1996). However, the coupling of electrons and neutrinos to the singly charged Higgs bosons may play an important role to the $\overline{\nu}_e e^-$ scattering process at high energy. This process is represented by Feynman diagrams as shown in Figure 4.1. We would concentrate on the flavour conserving part and estimate the bound of the coupling. In the calculation, the mixing between Higgs doublet and triplet is assumed negligible. For the process at high energy, we drop the electron mass. The charged current and neutral current Lagrangian are [Equation (C.36)]

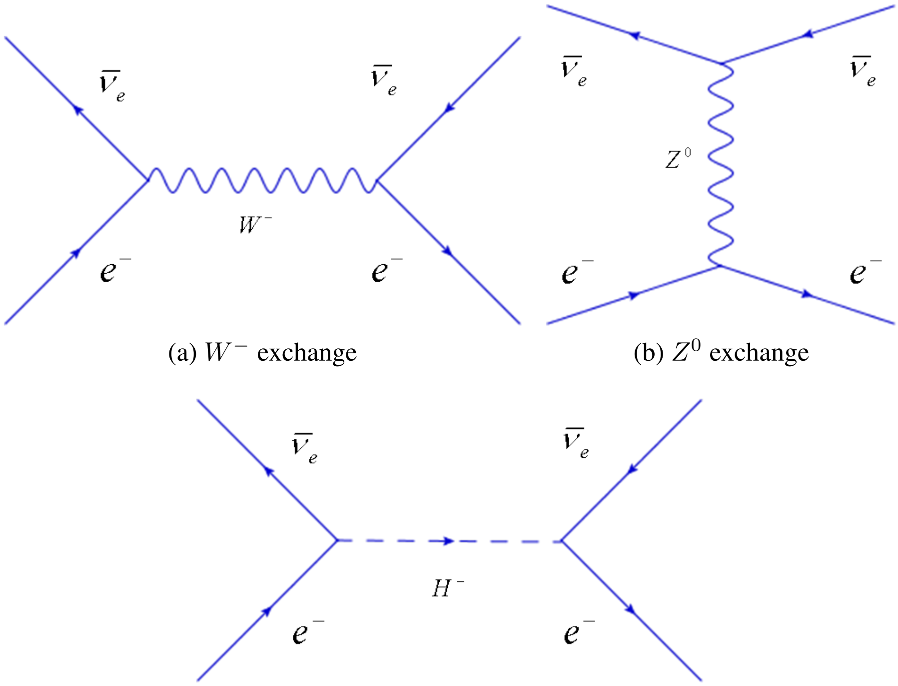
$$\begin{aligned} \mathcal{L}_{CC} &= -\frac{g}{2\sqrt{2}}\overline{\nu}_e\gamma^\alpha(1-\gamma_5)eW_\alpha + H.c. \\ \mathcal{L}_{NC} &= -\frac{g}{4\cos\theta_W}[\overline{\nu}_e\gamma^\alpha(1-\gamma_5)\nu_e - \overline{e}\gamma^\alpha(1-4\sin^2\theta_W-\gamma_5)e]Z_\alpha + H.c. \end{aligned} \tag{4.1}$$

where $g^2 = 8m_W^2 G_F/\sqrt{2}$. Following the convention in (Mikaelian & Zheleznykh, 1980), t is the square of the momentum transferred between incoming $\overline{\nu}_e$ and outgoing singly charged particle. Thus, the Z boson exchange would be denoted as u channel. The $\overline{\nu}_e - e$ scattering cross section involve two s channels and one u channel (Z exchange). The s channels come from the W^- and H^- exchange as shown in Figure 4.1.

4.2.1 W boson exchange

The amplitude for the W exchange diagram is

$$\begin{aligned} M_W &= \bar{\nu}_r(\vec{q})\left[\frac{-ig}{2\sqrt{2}}\gamma^\alpha(1-\gamma_5)\right]u_r(\vec{p})\frac{-ig_{\alpha\beta}}{s-m_W^2}\bar{u}_r(\vec{p}\,')\left[\frac{-ig}{2\sqrt{2}}\gamma^\beta(1-\gamma_5)\right]\nu_r(\vec{q}\,') \\ &= \frac{-ig^2}{8(s-m_W^2)}[\bar{\nu}_r(\vec{q})\gamma^\alpha(1-\gamma_5)\nu_r(\vec{q}\,')][\bar{u}_r(\vec{p}\,')\gamma_\alpha(1-\gamma_5)u_r(\vec{p})]. \end{aligned} \tag{4.2}$$



(a) W^- exchange (b) Z^0 exchange (c) H^- exchange

Figure 4.1: The Feynman diagrams for $\overline{\nu}_e(\vec{q}) + e^-(\vec{p}) \to \overline{\nu}_e(\vec{q}\,') + e^-(\vec{p}\,')$ scattering. The Feynman diagrams throughout this thesis are created by using JaxoDraw (Binosi & Theuβl, 2004).

Multiplying with the complex conjugate,

$$
\begin{aligned}
|M_W|^2 &= \frac{g^4}{64(s-m_W^2)^2}[\bar{\nu}_r(\vec{q})\gamma^\alpha(1-\gamma_5)\nu_r(\vec{q}\,')][\nu_r^\dagger(\vec{q}\,')(1-\gamma_5)\gamma^{\beta\dagger}\gamma^0\nu_r(\vec{q})] \\
&\quad \times[\bar{u}_r(\vec{p}\,')\gamma_\alpha(1-\gamma_5)u_r(\vec{p})][u_r^\dagger(\vec{p})(1-\gamma_5)\gamma_\beta^\dagger\gamma^0 u_r(\vec{p}\,')] \\
&= \frac{g^4}{64(s-m_W^2)^2}[\bar{\nu}_r(\vec{q})\gamma^\alpha(1-\gamma_5)\nu_r(\vec{q}\,')][\bar{\nu}_r(\vec{q}\,')\gamma^\beta(1-\gamma_5)\nu_r(\vec{q})] \\
&\quad \times[\bar{u}_r(\vec{p}\,')\gamma_\alpha(1-\gamma_5)u_r(\vec{p})][\bar{u}_r(\vec{p})\gamma_\beta(1-\gamma_5)u_r(\vec{p}\,')].
\end{aligned}
\tag{4.3}
$$

Average over initial polarization states and sum over all final polarization states,

$$
\begin{aligned}
\langle|M_W^2|\rangle &= \frac{g^4}{64(s-m_W^2)^2}\frac{1}{2}Tr\left[\frac{(\not{q}-m_\nu)\gamma^\alpha(1-\gamma_5)(\not{q}'-m_\nu)\gamma^\beta(1-\gamma_5)}{2m_\nu 2m_e}\right] \\
&\quad \times Tr\left[\frac{(\not{p}'+m_e)\gamma_\alpha(1-\gamma_5)(\not{p}+m_e)\gamma_\beta(1-\gamma_5)}{2m_\nu 2m_e}\right].
\end{aligned}
\tag{4.4}
$$

The trace can be evaluated by using the trace identity in Appendix A.3.

$$Tr\left[(\not{q} - m_\nu)\gamma^\alpha(1-\gamma_5)(\not{q}' - m_\nu)\gamma^\beta(1-\gamma_5)\right]$$

$$\begin{aligned}
&= 2Tr\left[\not{q}\gamma^\alpha\not{q}'\gamma^\beta(1-\gamma_5)\right] \\
&= 2q_\mu q'_\nu Tr\left[\gamma^\mu\gamma^\alpha\gamma^\nu\gamma^\beta(1-\gamma_5)\right] \\
&= 8q_\mu q'_\nu\left[g^{\mu\alpha}g^{\nu\beta} - g^{\mu\nu}g^{\alpha\beta} + g^{\mu\beta}g^{\alpha\nu} - i\varepsilon^{\mu\alpha\nu\beta}\right] \\
&= 8\left[q^\alpha q'^\beta - qq'g^{\alpha\beta} + q^\beta q'^\alpha - iq_\mu q'_\nu\varepsilon^{\mu\alpha\nu\beta}\right] \\
&= 2q_\mu q'_\nu Tr\left[\gamma^\mu\gamma^\alpha\gamma^\nu\gamma^\beta(1-\gamma_5)\right] \times 2p'^\sigma p^\tau Tr\left[\gamma_\sigma\gamma_\alpha\gamma_\tau\gamma_\beta(1-\gamma_5)\right] \\
&= 64\left[q^\alpha q'^\beta - qq'g^{\alpha\beta} + q^\beta q'^\alpha - iq_\mu q'_\nu\varepsilon^{\mu\alpha\nu\beta}\right]\left[p'_\alpha p_\beta - p'pg_{\alpha\beta} + p'_\beta p_\alpha - ip'^\sigma p^\tau\varepsilon_{\sigma\alpha\tau\beta}\right] \\
&= 64\left[2(qp')(q'p) + 2(qp)(p'q') - \varepsilon^{\mu\alpha\nu\beta}\varepsilon_{\sigma\alpha\tau\beta}q_\mu q'_\nu p'^\sigma p^\tau\right] \\
&= 64\left[2(qp')(q'p) + 2(qp)(p'q') + 2(g^\mu_\sigma g^\nu_\tau - g^\mu_\tau g^\nu_\sigma)q_\mu q'_\nu p'^\sigma p^\tau\right] \\
&= 64\left[2(qp')(q'p) + 2(qp)(p'q') + 2(qp')(q'p) - 2(qp)(p'q')\right] \\
&= 64 \cdot 4(qp')(q'p)
\end{aligned} \tag{4.5}$$

The momentum can be written in terms of Mandelstam variables, [see Appendix A.4]

$$\begin{aligned}
t &= (p-q')^2 = (q-p')^2 \\
&= p^2 + q'^2 - 2pq' = p^2 + q'^2 - 2p'q, \\
t &\approx -2pq' \approx -2p'q.
\end{aligned} \tag{4.6}$$

Thus we have,

$$\langle|M_W^2|\rangle = \frac{g^4}{64(s-m_W^2)^2}\frac{1}{2}\frac{64t^2}{16m_\nu m_e}. \tag{4.7}$$

The differential cross section is [see Equation (A.19)]

$$\frac{d\sigma_W}{dt} = \frac{G_F^2 m_W^4}{\pi s^2}\frac{t^2}{(s-m_W^2)^2}, \tag{4.8}$$

$$\begin{aligned} t &= -2p'q \\ &= -2(E_p' E_q - |\vec{p}'|\,|\vec{q}|)\cos\theta \\ &= -2\frac{1}{2}\sqrt{s}\frac{1}{2}\sqrt{s}(1-\cos\theta) \\ &= -\frac{1}{2}s(1-\cos\theta), \end{aligned} \tag{4.9}$$

where θ is the angle of scattered electron with respect to the incident neutrino. The differential cross section is integrated over all angles, $\cos\theta = -1$ to $\cos\theta = 1$ correspond to $t_{min} = -s$ to $t_{max} = 0$ [see Equation (A.21)].

$$\begin{aligned} \sigma_w &= \frac{G_F^2 m_W^4}{\pi s^2}\frac{1}{(s-m_W^2)^2}\int_{min}^{max} t^2 dt \\ &= \frac{G_F^2 m_W^4}{\pi s^2}\frac{1}{(s-m_W^2)^2}\int_{-s}^{0} t^2 dt \\ &= \frac{G_F^2 m_W^4}{3\pi}\frac{s}{(s-m_W^2)^2} \end{aligned} \tag{4.10}$$

4.2.2 Z boson exchange

The amplitude for Z^0 exchange is

$$\begin{aligned} M_Z &= \bar{u}_r(\vec{p}\,')\left[\frac{-ig}{2\cos\theta_W}\gamma^\alpha(g_V - g_A\gamma_5)\right]u_r(\vec{p})\frac{-ig_{\alpha\beta}}{u-m_W^2}\bar{\nu}_r(\vec{q})\left[\frac{-ig}{4\cos\theta_W}\gamma^\beta(1-\gamma_5)\right]\nu_r(\vec{q}\,') \\ &= \frac{ig^2[\bar{u}_r(\vec{p}\,')\gamma^\alpha(g_V - g_A\gamma_5)u_r(\vec{p})][\bar{\nu}_r(\vec{q})\gamma_\alpha(1-\gamma_5)\nu_r(\vec{q}\,')]}{8\cos\theta_W(u-m_Z^2)}, \end{aligned} \tag{4.11}$$

where $g_V = 2\sin^2\theta_W - \frac{1}{2}$ and $g_A = \frac{1}{2}$. The procedures are the same as W exchange.

$$\begin{aligned} |M_Z|^2 &= \frac{g^4}{64\cos\theta_W} \frac{[\bar{u}_r(\vec{p}\,')\gamma^\alpha(g_V - g_A\gamma_5)u_r(\vec{p})][u_r^\dagger(\vec{p})(g_V - g_A\gamma_5)\gamma^{\beta\dagger}\gamma^0 u_r(\vec{p}\,')]}{(u - m_Z^2)^2} \\ &\times[\bar{\nu}_r(\vec{q})\gamma_\alpha(1-\gamma_5)\nu_r(\vec{q}\,')][\nu_r^\dagger(\vec{q}\,')(1-\gamma_5)\gamma_\beta^\dagger\gamma^0\nu_r(\vec{q})] \end{aligned} \tag{4.12}$$

Average over initial polarization states and sum over all final polarization states,

$$\begin{aligned} \langle |M_Z^2| \rangle &= \frac{g^4}{64\cos^4\theta_W(u - m_Z^2)^2}\frac{1}{2} \\ &\times Tr\left[\frac{(\not{p}' + m_e)\gamma^\alpha(g_V - g_A\gamma_5)(\not{p} + m_e)\gamma^\beta(g_V - g_A\gamma_5)}{2m_\nu 2m_e}\right] \\ &\times Tr\left[\frac{(\not{q} - m_\nu)\gamma_\alpha(1-\gamma_5)(\not{q}' - m_\nu)\gamma_\beta(1-\gamma_5)}{2m_\nu 2m_e}\right] \end{aligned} \tag{4.13}$$

Tr$\left[\frac{(\not{p}'+m_e)\gamma^\alpha(g_V-g_A\gamma_5)(\not{p}+m_e)\gamma^\beta(g_V-g_A\gamma_5)}{2m_\nu 2m_e}\right]$

$$\begin{aligned} &= Tr\left[\not{p}'\gamma^\alpha\not{p}\gamma^\beta(g_V^2 + g_A^2 - 2g_V g_A\gamma_5)\right] \\ &= 4\left\{[g_V^2 + g_A^2](p'^\alpha p^\beta - p'p g^{\alpha\beta} + p'^\beta p^\alpha) + 2ig_V g_A\varepsilon^{\mu\alpha\nu\beta}\right\} \end{aligned}$$

Tr$\left[\frac{(\not{q}-m_\nu)\gamma_\alpha(1-\gamma_5)(\not{q}'-m_\nu)\gamma_\beta(1-\gamma_5)}{2m_\nu 2m_e}\right]$

$$\begin{aligned} &= 2Tr\left[\not{q}\gamma_\alpha\not{q}'\gamma_\beta(1-\gamma_5)\right] \\ &= 8\left[q_\alpha q'_\beta - qq'g_{\alpha\beta} + q_\beta q'_\alpha + i\varepsilon_{\mu\alpha\nu\beta}\right] \\ &= 64\left[(g_V + g_A)^2(p'q)(pq') + (g_V - g_A)^2(p'q')(pq)\right] \end{aligned} \tag{4.14}$$

Thus

$$
\begin{aligned}
\langle |M_Z^2| \rangle &= \frac{g^4\left[(g_V+g_A)^2(p'q)(pq') + (g_V-g_A)^2(p'q')(pq)\right]}{2\cos^4\theta_W(u-m_Z^2)^2 16 m_\nu m_e} \\
&= \frac{g^4}{8\cos^4\theta_W(u-m_Z^2)^2}\frac{1}{16m_\nu m_e}\left[(g_V+g_A)^2t^2 + (g_V-g_A)^2s^2\right].
\end{aligned}
\tag{4.15}
$$

The differential cross section [see Equation (A.19)]

$$
\frac{d\sigma_Z}{dt} = \frac{G_F^2 m_Z^4}{4\pi s^2(u-m_Z^2)^2}\left[(g_V+g_A)^2t^2 + (g_V-g_A)^2s^2\right], \tag{4.16}
$$

$$
\sigma_Z = \int_{-s}^{0}\frac{G_F^2 m_Z^4}{4\pi s^2(u-m_Z^2)^2}\left[(g_V+g_A)^2t^2 + (g_V-g_A)^2s^2\right]dt, \tag{4.17}
$$

$$
\begin{aligned}
\int_{-s}^{0}\frac{t^2}{(s+t+m_Z^2)^2}dt &= \frac{s}{m_Z^2}\left[(s+2m_Z^2) - \frac{2m_Z^2}{s}(s+m_Z^2)\ln(1+\frac{s}{m_Z^2})\right], \\
\int_{-s}^{0}\frac{s^2}{(s+t+m_Z^2)^2}dt &= \frac{s^3}{m_Z^2(s+m_Z^2)}.
\end{aligned}
\tag{4.18}
$$

The total cross section [see Equation (A.21)]

$$
\begin{aligned}
\sigma_Z &= \frac{G_F^2 m_Z^2}{4\pi s}\left[(g_V+g_A)^2\left\{(s+2m_Z^2) - \frac{2m_Z^2}{s}(s+m_Z^2)\ln(1+\frac{s}{m_Z^2})\right\}\right. \\
&\quad \left. +(g_V-g_A)^2\frac{s^2}{m_Z^2(s+m_Z^2)}\right].
\end{aligned}
\tag{4.19}
$$

4.2.3 W and Z interference

The superposition of Figure 4.1a and Figure 4.1b

$$
M_W M_Z^* = \frac{g^4[\bar{\nu}_r(\vec{q})\gamma^\alpha(1-\gamma_5)\nu_r(\vec{q}\,')][\bar{\nu}_r(\vec{q}\,')\gamma^\beta(1-\gamma_5)\nu_r(\vec{q})]}{64\cos^2\theta_W(s-m_W^2)(u-m_Z^2)}
$$

$$\times[\bar{u}_r(\vec{p}\,')\gamma_\alpha(1-\gamma_5)u_r(\vec{p})][\bar{u}_r(\vec{p})\gamma_\beta(g_V - g_A\gamma_5)u_r(\vec{p}\,')], \tag{4.20}$$

$$\begin{aligned}
2\left\langle M_W M_Z^* \right\rangle &= \frac{2g^4}{64\cos^2\theta_W(s-m_W^2)(u-m_Z^2)}\frac{1}{2} \\
&\quad \times Tr\left[\frac{(\not{q}-m_\nu)\gamma^\alpha(1-\gamma_5)(\not{q}'-m_\nu)\gamma^\beta(1-\gamma_5)}{2m_\nu 2m_e}\right] \\
&\quad \times Tr\left[\frac{(\not{p}'+m_e)\gamma_\alpha(1-\gamma_5)(\not{p}+m_e)\gamma_\beta(g_V-g_A\gamma_5)}{2m_\nu 2m_e}\right] \\
&= \frac{-g^4(g_V+g_A)t^2}{2\cos^2\theta_W(s-m_W^2)(u-m_Z^2)16m_\nu^2 m_e^2},
\end{aligned} \tag{4.21}$$

$$\frac{d\sigma_{WZ}}{dt} = \frac{-G_F^2 m_W^4(g_V+g_A)t^2}{\pi s^2\cos^2\theta_W(s-m_W^2)(u-m_Z^2)}, \tag{4.22}$$

$$\sigma_{WZ} = \frac{-G_F^2 m_W^4(g_V+g_A)}{\pi s^2\cos^2\theta_W(s-m_W^2)}\int_{-s}^{0}\frac{t^2}{-(s+t)-m_Z^2}dt, \tag{4.23}$$

$$\begin{aligned}
&\int_{-s}^{0}\frac{t^2}{-(s+t)-m_Z^2}dt = -\int_{-s}^{0}\frac{t^2}{(s+t)+m_Z^2}dt \\
&= m_Z^2 s + (m_Z^2+s)^2\ln m_Z^2 - (m_Z^2+s)^2\ln(m_Z^2+s) + \frac{3}{2}s^2,
\end{aligned} \tag{4.24}$$

$$\sigma_{WZ} = \frac{G_F^2 s}{\pi}\frac{m_W^2}{(s-m_W^2)}\frac{m_Z^2}{s}(g_V+g_A)\left[\frac{3}{2}+\frac{m_Z^2}{s}-(1+\frac{m_Z^2}{s})^2\ln(1+\frac{s}{m_Z^2})\right]. \tag{4.25}$$

4.2.4 Singly charged Higgs boson exchange

The interaction Lagrangian for the H^- exchange is

$$\mathcal{L} = -\frac{h_{ij}}{\sqrt{2}}\left(l_i^T C P_L \nu_j + \nu_i^T C P_L l_j\right)\cos\theta_+ H^+. \tag{4.26}$$

The Feynman rules are listed in Appendix C. The mixing between triplet and doublet is neglected as $\upsilon_\Delta << \upsilon_0$ and by using the approximation vertex listed in Table C, the amplitude for H^- exchange in Figure 4.1c is

$$\begin{aligned} M_H &= \bar{\nu}_r(\vec{q}) \left[\frac{-ih_{ee}}{\sqrt{2}}(1-\gamma_5) \right] u_r(\vec{p}) \frac{-i}{s - M^2_{H_\pm}} \bar{u}_r(\vec{p}\,') \left[\frac{-ih_{ee}}{\sqrt{2}}(1-\gamma_5) \right] \nu_r(\vec{q}\,') \\ &= \frac{ih^2_{ee}}{2} \frac{[\bar{\nu}_r(\vec{q})(1-\gamma_5)\nu_r(\vec{q}\,')][\bar{u}_r(\vec{p}\,')(1-\gamma_5)u_r(\vec{p})]}{s - M^2_{H_\pm}}, \end{aligned} \tag{4.27}$$

$$\begin{aligned} |M_H|^2 &= \frac{h^4_{ee}}{4(s-M^2_{H_\pm})^2} [\bar{\nu}_r(\vec{q})(1-\gamma_5)\nu_r(\vec{q}\,')][\nu^\dagger_r(\vec{q}\,')(1-\gamma_5)\gamma^0\nu_r(\vec{q})] \\ &\quad \times [\bar{u}_r(\vec{p}\,')(1-\gamma_5)u_r(\vec{p})][u^\dagger_r(\vec{p})(1-\gamma_5)\gamma^0 u_r(\vec{p}\,')] \\ &= \frac{h^4_{ee}}{4(s-M^2_{H_\pm})^2} [\bar{\nu}_r(\vec{q})(1-\gamma_5)\nu_r(\vec{q}\,')][\bar{\nu}_r(\vec{q}\,')(1+\gamma_5)\nu_r(\vec{q})] \\ &\quad \times [\bar{u}_r(\vec{p}\,')(1-\gamma_5)u_r(\vec{p})][\bar{u}_r(\vec{p})(1+\gamma_5)u_r(\vec{p}\,')], \end{aligned} \tag{4.28}$$

$$\begin{aligned} \langle |M^2_H| \rangle &= \frac{h^4_{ee}}{4(s-M^2_{H_\pm})^2} \frac{1}{2} Tr \left[\frac{(\not{q} - m_\nu)(1-\gamma_5)(\not{q}' - m_\nu)(1+\gamma_5)}{2m_\nu 2m_e} \right] \\ &\quad \times Tr \left[\frac{(\not{p}' + m_e)(1-\gamma_5)(\not{p} + m_e)(1+\gamma_5)}{2m_\nu 2m_e} \right] \\ &= \frac{h^4_{ee}}{4(s-m^2_{H_\pm})^2} \frac{1}{2} \frac{4}{16 m^2_\nu m^2_e} Tr\left[\not{q}\not{q}'\right] \\ &\quad \times Tr\left[\not{p}'\not{p}\right] \\ &= \frac{2h^4_{ee}}{(s-M^2_{H_\pm})^2} \frac{t^2}{16 m^2_\nu m^2_e}. \end{aligned} \tag{4.29}$$

The differential cross section is [see Equation (A.19)]

$$\frac{d\sigma_H}{dt} = \frac{h^4_{ee} t^2}{8\pi s^2 (s - M^2_{H_\pm})^2}. \tag{4.30}$$

The total cross section for H^- exchange is therefore [see Equation (A.21)]

$$\begin{aligned}
\sigma_H &= \frac{h_{ee}^4}{8\pi s^2 (s - M_{H_\pm}^2)^2} \int_{-s}^{0} t^2 dt \\
&= \frac{h_{ee}^4}{24\pi} \frac{s}{(s - M_{H_\pm}^2)^2}.
\end{aligned} \tag{4.31}$$

4.2.5 Singly charged Higgs, W, Z interference

The interference of H^- exchange with W and Z^0 exchange

$$\begin{aligned}
M_W M_H^* &= \frac{g^2 h_{ee}^2}{16(s - m_W^2)(s - M_{H_\pm}^2)} [\bar{\nu}_r(\vec{q})\gamma^\alpha(1-\gamma_5)\nu_r(\vec{q}\,')][\bar{\nu}_r(\vec{q}\,')(1+\gamma_5)\nu_r(\vec{q})] \\
&\quad \times [\bar{u}_r(\vec{p}\,')\gamma_\alpha(1-\gamma_5)u_r(\vec{p})][\bar{u}_r(\vec{p})(1+\gamma_5)u_r(\vec{p}\,')],
\end{aligned} \tag{4.32}$$

$$\begin{aligned}
2\langle M_W M_H^* \rangle &= \frac{g^2 h_{ee}^2}{8(s - m_W^2)(s - M_{H_\pm}^2)} \frac{1}{2} \\
&\quad \times Tr\left[\frac{(\not{q} - m_\nu)\gamma^\alpha(1-\gamma_5)(\not{q}' - m_\nu)(1+\gamma_5)}{2m_\nu 2m_e}\right] \\
&\quad \times Tr\left[\frac{(\not{p}' + m_e)\gamma_\alpha(1-\gamma_5)(\not{p} + m_e)(1+\gamma_5)}{2m_\nu 2m_e}\right] \\
&= \frac{g^2 h_{ee}^2}{16(s - m_W^2)(s - m_{H_\pm}^2)} \frac{4}{2} \frac{1}{16 m_\nu^2 m_e^2} Tr\left[\not{q}\gamma^\alpha \not{q}'(1+\gamma_5)\right] \\
&\quad \times Tr\left[\not{p}'\gamma_\alpha \not{p}(1+\gamma_5)\right] \\
&= 0,
\end{aligned} \tag{4.33}$$

$$\begin{aligned}
M_Z M_H^* &= \frac{g^2 h_{ee}^2 [\bar{u}_r(\vec{p}\,')\gamma^\alpha(g_V - g_A\gamma_5)u_r(\vec{p})][\bar{u}_r(\vec{p})(1+\gamma_5)u_r(\vec{p}\,')]}{16\cos^2\theta_W (s - m_W^2)(s - M_{H_\pm}^2)} \\
&\quad \times [\bar{\nu}_r(\vec{q})\gamma_\alpha(1-\gamma_5)\nu_r(\vec{q}\,')][\bar{\nu}_r(\vec{q}\,')(1+\gamma_5)\nu_r(\vec{q})],
\end{aligned} \tag{4.34}$$

$$
\begin{aligned}
2\langle M_Z M_H^* \rangle &= \frac{g^2 h_{ee}^2}{16\cos^2\theta_W (u-m_Z^2)(s-M_{H_\pm}^2)} \frac{1}{2} \\
&\times Tr\left[\frac{(\not{p}' + m_e)\gamma^\alpha (g_V - g_A\gamma_5)(\not{p} + m_e)(1+\gamma_5)}{2m_\nu 2m_e}\right] \\
&\times Tr\left[\frac{(\not{q} - m_\nu)\gamma_\alpha (1-\gamma_5)(\not{q}' - m_\nu)(1+\gamma_5)}{2m_\nu 2m_e}\right] \\
&= 0.
\end{aligned} \tag{4.35}
$$

The superposition of H^- exchange with W and Z^0 exchange vanishes due to the trace of odd number gamma matrices [see Appendix A.3]. The total cross section is then [see Equation (A.21)]

$$
\begin{aligned}
\sigma_{\overline{\nu}_e e} &= \frac{G_F^2 s}{\pi}\left[\frac{m_W^4}{3(s-m_W^2)^2} + \frac{h_{ee}^4}{24 G_F^2 (s-M_{H_\pm}^2)^2}\right. \\
&+\frac{1}{4}\left(\frac{m_Z^2}{s}\right)^2 [(2\sin^2\theta_W - 1)^2\left[2+\frac{s}{m_Z^2} - 2\left(1+\frac{m_Z^2}{s}\right)\ln\left(1+\frac{s}{m_Z^2}\right)\right] \\
&+(2\sin^2\theta_W)^2 \frac{s^2}{m_Z^2(s+m_Z^2)}] \\
&\left.+\frac{m_W^2 m_Z^2}{s(s-m_W^2)}(2\sin^2\theta_W - 1)\left[\frac{3}{2}+\frac{m_Z^2}{s} - \left(1+\frac{m_Z^2}{s}\right)^2 \ln\left(1+\frac{m_Z^2}{s}\right)\right]\right].
\end{aligned} \tag{4.36}
$$

The first, third and fourth term in Equation (4.36) comes from CC, NC and their interference respectively. In contrast to CC and NC, the vertex of $l-\nu-H$ coupling only contain γ_5 in the left-right projection operator. Therefore, due to the trace of odd number of Dirac matrices in their amplitude, the H^- exchange in the second term does not interfere with the CC and NC . This imply that the H^- exchange do not occur simultaneously with CC and NC. In other word, the CC, NC and H^- exchange are mutually exclusive.

4.3 Bosonic final state

Besides the reaction $\overline{\nu}_e + e^- \to \overline{\nu}_e + e^-$, there are a rich variety of final bosonic states available in $\overline{\nu}_e e^-$ annihilation in analogy with e^+e^- annihilation. In particular, the production of W^-, Z^0 and photon (Mikaelian & Zheleznykh, 1980). On the other hand, there are some possibilities for the Higgs search beyond the standard model. At the energy above the W^- pole, the production of supersymmetry Higgs is available. However, we would like to concentrate in the production of Higgs in the HTM. We consider the processes $\overline{\nu}_e + e^- \to W^- + H_1$, $\overline{\nu}_e + e^- \to W^+ + H^{--}$, $\overline{\nu}_e + e^- \to H^+ + H^{--}$ and $\overline{\nu}_e + e^- \to H_2 + H^-$. The last two processes are motivated by (Akeroyd & Aoki, 2005; Fileviez Pérez et al., 2008) in the search of triplet scalar of the processes $q\overline{q}' \to H^+ + H^{--}/H_2 + H^-$ at CERN LHC. We are also interested in the production of W^- and Z^0 through the charged Higgs meditation at the vicinity of the resonance energy.

4.3.1 $\overline{\nu}_e + e^- \to W^- + H_1$ and $\overline{\nu}_e + e^- \to W^+ + H^{--}$

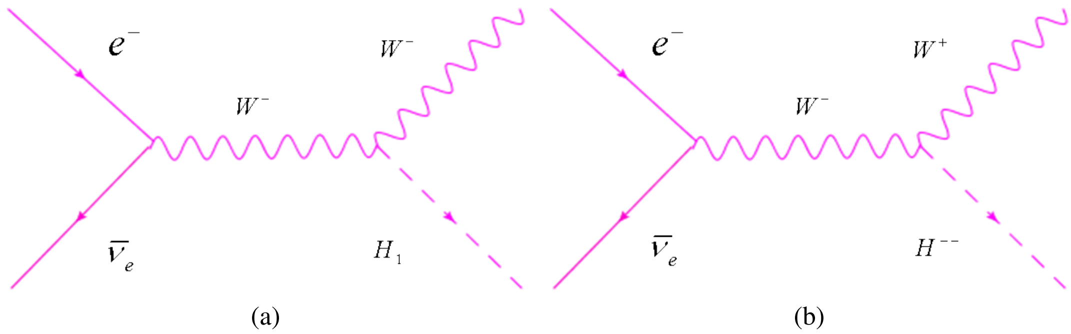


Figure 4.2: Feynman diagrams for (a) $\overline{\nu}_e(\vec{q})+e^-(\vec{p}) \to W^-(\vec{k})+H_1(\vec{k}')$, (b) $\overline{\nu}_e(\vec{q})+e^-(\vec{p}) \to W^+(\vec{k}) + H^{--}(\vec{k}')$.

The interaction Lagrangian for the process in Figure 4.2a is [see Equation (C.6)]

$$\mathcal{L} = \frac{ig}{2}(\sqrt{2}\cos\theta_+ \sin\theta_0 - \sin\theta\cos\theta_0)\left[\left(\partial_\mu H^+\right) H_1 W_\mu^- + \left(\partial_\mu H_1\right) H^- W_\mu^+\right] \quad (4.37)$$

while Figure 4.2b is [see Equation (C.6)]

$$\mathcal{L} = \frac{g^2 \upsilon_\Delta}{\sqrt{2}} \left[H^{--} W^{\mu +} W^{+}_{\mu} + H^{++} W^{\mu -} W^{-}_{\mu} \right]. \tag{4.38}$$

We again neglect the mixing between triplet and doublet due to $\upsilon_\Delta << \upsilon_0$. The calculation of these two processes are similar. The amplitude for Figure 4.2a is

$$\begin{aligned} M &= -i\bar{\nu}_r(\vec{q}) \left[\frac{-ig}{2\sqrt{2}} \gamma^\alpha (1-\gamma_5) \right] u_r(\vec{p}) \frac{g_{\alpha\beta}}{s - m_W^2} i \frac{1}{2} g^2 \upsilon_0 g_{\mu\nu} \varepsilon^*_{r\nu}(\vec{k}) \\ &= -i \frac{g^3 \upsilon_0}{4\sqrt{2}} \frac{\bar{\nu}_r(\vec{q}) \gamma_\beta (1-\gamma_5) u_r(\vec{p})}{s - m_W^2} \varepsilon^{\mu *}_r(\vec{k}). \end{aligned} \tag{4.39}$$

Multiplying the complex conjugate

$$|M|^2 = \left(\frac{g^3 \upsilon_0}{4\sqrt{2}} \right)^2 \frac{[\bar{\nu}_r(\vec{q}) \gamma_\beta (1-\gamma_5) u_r(\vec{p})][\bar{u}_r(\vec{p}) \gamma_\alpha (1-\gamma_5)_r(\vec{q})]}{(s - m_W^2)^2} \varepsilon^*_{r\mu}(\vec{k}) \varepsilon^\mu_r(\vec{k}). \tag{4.40}$$

Average over initial polarization states and sum over all final polarization states. The vector boson polarization sum is

$$\sum_{r=1}^{3} \varepsilon^*_{r\mu}(\vec{k}) \varepsilon^\mu_r(\vec{k}) = -g^{\alpha\beta} + \frac{k^\alpha k^\beta}{m_W^2} \tag{4.41}$$

and

$$\begin{aligned} \langle |M|^2 \rangle &= \left(\frac{g^3 \upsilon_0}{4\sqrt{2}} \right)^2 \frac{1}{(s - m_W^2)^2} \frac{1}{2} Tr \left[\frac{\not{q} \gamma_\beta (1-\gamma_5) \not{p} \gamma_\alpha (1-\gamma_5)}{2m_\nu 2m_e} \right] \sum_{r=1}^{3} \varepsilon^*_{r\mu}(\vec{k}) \varepsilon^\mu_r(\vec{k}) \\ &= \frac{g^6 \upsilon_0^2}{32(s - m_W^2)^2} \frac{8}{8m_\nu m_e} [q_\alpha p_\beta + q_\beta p_\alpha - qp g_{\alpha\beta} + i\varepsilon_{\gamma\alpha\tau\beta}] \left[-g^{\alpha\beta} + \frac{k^\alpha k^\beta}{m_W^2} \right] \\ &= \frac{256 G_F^3 m_W^6 \upsilon_0^2}{32\sqrt{2}(s - m_W^2)^2 m_\nu m_e} \left[pq + \frac{2(qk)(pk)}{m_W^2} \right]. \end{aligned} \tag{4.42}$$

The Mandelstam variables are

$$\begin{aligned} s &= (p+q)^2 = p^2 + q^2 + 2pq \approx 2pq \\ t &= (k-q)^2 = k^2 + q^2 - 2kq \approx m_W^2 - 2kq \\ u &= (p-k)^2 = p^2 + k^2 - 2pk \approx m_W^2 - 2pk, \end{aligned} \tag{4.43}$$

$$\left\langle |M|^2 \right\rangle = \frac{4G_F^3 m_W^6 \upsilon_0^2}{\sqrt{2}(s-m_W^2)^2 m_\nu m_e}\left[2s - m_{H_1}^2 + \frac{ut}{m_W^2}\right]. \tag{4.44}$$

The differential cross section [see Equation (A.19)]

$$\frac{d\sigma_{H_1}}{dt} = \frac{G_F^3 m_W^6 \upsilon_0^2}{\sqrt{2}\pi s^2 (s-m_W^2)^2}\left[2s - m_{H_1}^2 + \frac{ut}{m_W^2}\right]. \tag{4.45}$$

The energy and momentum of the W^+ boson final state can be expressed in terms of their own mass and the Higgs mass,

$$\begin{aligned} \left|\vec{k}\right| &= \frac{1}{2\sqrt{s}}\sqrt{s^2 - 2(m_W^2 + m_{H_1}^2)s + (m_W^2 - m_{H_1}^2)^2} \\ &= \frac{s}{2\sqrt{s}}\left[1 - \frac{2(m_W^2 + m_{H_1}^2)}{s} + \frac{(m_W - m_{H_1})^2(m_W + m_{H_1})^2}{s^2}\right]^{\frac{1}{2}} \\ &= \frac{s}{2\sqrt{s}}\left[1 - \frac{(m_W + m_{H_1})^2}{s}\right]^{\frac{1}{2}}\left[1 - \frac{(m_W - m_{H_1})^2}{s}\right]^{\frac{1}{2}} \\ &= \frac{s}{2\sqrt{s}}\beta_{H_1} \\ |\vec{q}| &= \frac{1}{2\sqrt{s}}\sqrt{s^2 - 2(m_\nu^2 + m_e^2)s + (m_\nu^2 - m_e^2)^2} \\ & \quad m_\nu \to 0,\ m_e \to 0 \\ &= \frac{1}{2}\sqrt{s} \\ E_k^2 &= \left|\vec{k}\right|^2 + m_W^2 \\ &= \frac{1}{4s}\left[s^2 - 2(m_W^2 + m_{H_1}^2)s + (m_W^2 - m_{H_1}^2)^2 + 4sm_W^2\right] \end{aligned}$$

$$
\begin{aligned}
&= \frac{1}{4s}\left[s+(m_W^2-m_{H_1}^2)\right]^2 \\
E_k &= \frac{1}{2\sqrt{s}}\left[s+(m_W^2-m_{H_1}^2)\right] \\
E_q &= |\vec{q}|\,.
\end{aligned} \tag{4.46}
$$

Thus, the Mandelstam variable t is therefore

$$
\begin{aligned}
t &= k^2+q^2-2kq \approx m_W^2-2kq \\
&= m_W^2-2\left[E_kE_q-\left|\vec{k}\right||\vec{q}|\cos\theta\right] \\
&= \frac{1}{2}\left[m_W^2+m_{H1}^2-s(1-\beta_{H_1}\cos\theta)\right].
\end{aligned} \tag{4.47}
$$

The total cross section is [see Equation (A.21)]

$$
\begin{aligned}
\sigma_{H_1} &= \frac{G_F^3 m_W^6 \upsilon_0^2}{\sqrt{2}\pi s^2(s-m_W^2)^2}\frac{s\beta_{H_1}}{12m_W^2}\left[3s^2-s^2\beta_{H_1}^2-6sm_{H_1}^2+3m_{H_1}^4\right. \\
&\quad \left.+18sm_W^2-6m_{H_1}^2m_W^2+3m_W^4\right] \\
&= \frac{G_F^3 m_W^6 \upsilon_0^2}{\sqrt{2}\pi s^2(s-m_W^2)^2}\frac{s^3\beta_{H_1}}{12m_W^2}\left[3-\beta_{H_1}^2-6\frac{m_{H_1}^2+m_W^2}{s}+\frac{24m_W^2}{s}\right. \\
&\quad \left.+\frac{3[m_{H_1}^4-2m_{H_1}^2m_W^2+m_W^4]}{s^2}\right] \\
&= \frac{G_F^3 m_W^6 \upsilon_0^2}{\sqrt{2}\pi s^2(s-m_W^2)^2}\frac{s^3\beta_{H_1}}{12m_W^2}\left[3\left(1-2\frac{(m_{H_1}^2+m_W^2)}{s}+\frac{[m_{H_1}^2-m_W^2]^2}{s^2}\right)\right. \\
&\quad \left.-\beta_{H_1}^2+\frac{24m_W^2}{s}\right] \\
&= \frac{G_F^3 m_W^6 \upsilon_0^2}{\sqrt{2}\pi s^2(s-m_W^2)^2}\frac{s^3\beta_{H_1}}{12m_W^2}\left[3\beta_{H_1}^2-\beta_{H_1}^2+\frac{24m_W^2}{s}\right], \\
\sigma_{H_1} &= \frac{G_F^3 m_W^4 \upsilon_0^2 s\beta_{H_1}}{6\sqrt{2}\pi(s-m_W^2)^2}\left[\beta_{H_1}^2+\frac{12m_W^2}{s}\right].
\end{aligned} \tag{4.48}
$$

By assuming $\upsilon_0^2 \approx (\upsilon_0+2\upsilon_\Delta)^2$ (υ_Δ very small) and make use of the W boson mass in

Equation (C.7) the total cross section reduced to

$$\sigma_{H_1} = \frac{G_F^2 m_W^4 s \beta_{H_1}}{12\pi(s-m_W^2)^2}\left[\beta_{H_1}^2 + \frac{12m_W^2}{s}\right] \tag{4.49}$$

which is the same with the production of the SM Higgs boson (Mikaelian & Zheleznykh, 1980) where H_1 contributed mostly from the Higgs doublet. On the other hand, the total cross section for the doubly charged Higgs boson production is

$$\sigma_{H--} = \frac{G_F^3 m_W^4 \upsilon_\Delta^2 s \beta_{H--}}{6\sqrt{2}\pi(s-m_W^2)^2}\left[\beta_{H--}^2 + \frac{12m_W^2}{s}\right]. \tag{4.50}$$

The procedure to obtain Equation (4.50) is the same as for Equation (4.49) except for the vertex factor. Equation (4.50) is proportional to υ_Δ^2 which is in contrast to Equation (4.49). If the value of υ_Δ^2 is small enough, the doubly charged Higgs production associated with W boson is almost negligible.

4.3.2 $\overline{\nu}_e + e^- \to H^+ + H^{--}$ and $\overline{\nu}_e + e^- \to H_2 + H^-$

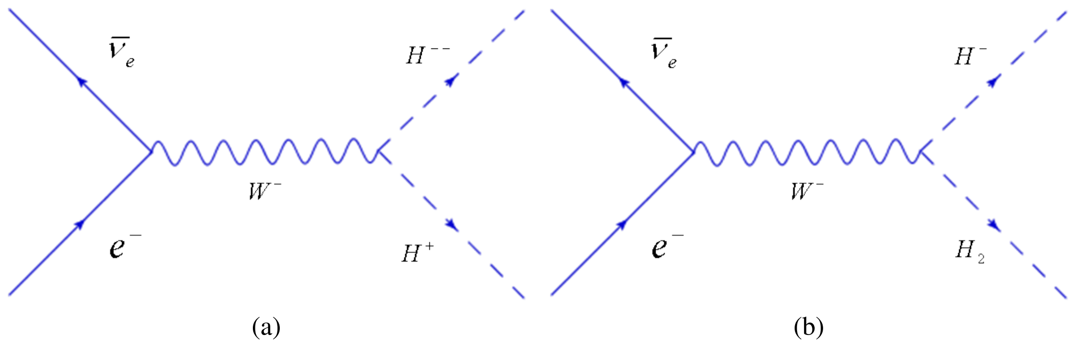


Figure 4.3: Feynman diagrams for (a) $\overline{\nu}_e(\vec{q}) + e^-(\vec{p}) \to H^+(\vec{k}) + H^{--}(\vec{k}')$, (b) $\overline{\nu}_e(\vec{q}) + e^-(\vec{p}) \to H_2(\vec{k}) + H^-(\vec{k}')$.

The interaction Lagrangian for the process in Figure 4.3a is [see Equation (C.6)]

$$\mathcal{L} = ig\cos\theta_+\left[\left(\partial_\mu H^-\right)H^{--}W_\mu^+ + \left(\partial_\mu H^{++}\right)H^+W_\mu^-\right] \tag{4.51}$$

while Figure 4.3b is [see Equation (C.6)]

$$\mathcal{L} = \frac{ig}{2}(\sin\theta_+\sin\theta_0 + \sqrt{2}\cos\theta_+\cos\theta_0)\left[\left(\partial_\mu H^+\right)H_2W_\mu^- + \left(\partial_\mu H_2\right)H^-W_\mu^+\right]. \tag{4.52}$$

After the mixing between triplet and doublet is neglected, the Feynman amplitude is therefore

$$\begin{aligned}
M &= \overline{\nu}(\vec{q})\left[\frac{-ig}{2\sqrt{2}}\gamma^\alpha(1-\gamma_5)\right]u(\vec{p})\frac{-ig_{\alpha\beta}}{s-m_W^2}\left[ig(p+q-2k)^\beta\right] \\
&= \frac{-ig^2}{2\sqrt{2}}\frac{\overline{\nu}(\vec{q})\gamma_\beta(1-\gamma_5)u(\vec{p})(p+q-2k)^\beta}{s-m_W^2} \\
|M|^2 &= \frac{g^4}{8}\frac{\left[\overline{\nu}(\vec{q})\gamma_\beta(1-\gamma_5)u(\vec{p})\right]\left[\overline{u}(\vec{p})(1+\gamma_5)\gamma_\alpha\nu(\vec{q})\right]}{(s-m_W^2)^2}(p+q-2k)^\beta(p+q-2k)^\alpha \\
&= \frac{g^4}{8}\frac{\left[\overline{\nu}(\vec{q})(\not{p}+\not{q}-2\not{k})(1-\gamma_5)u(\vec{p})\right]\left[\overline{u}(\vec{p})(1+\gamma_5)(\not{p}+\not{q}-2\not{k})\nu(\vec{q})\right]}{(s-m_W^2)^2},
\end{aligned} \tag{4.53}$$

$$\begin{aligned}
\left\langle|M|^2\right\rangle &= \frac{g^4}{8\left(s-m_W^2\right)^2}\frac{1}{2}Tr\left[\frac{\not{q}(\not{p}+\not{q}-2\not{k})(1-\gamma_5)\not{p}(\not{p}+\not{q}-2\not{k})(1-\gamma_5)}{2m_\nu 2m_e}\right] \\
&= \frac{g^4}{8\left(s-m_W^2\right)^2}Tr\left[\frac{\not{q}(\not{p}+\not{q}-2\not{k})\not{p}(\not{p}+\not{q}-2\not{k})(1-\gamma_5)}{2m_\nu 2m_e}\right].
\end{aligned} \tag{4.54}$$

The electron and neutrino masses are too small and can be approximated to zero, $\not{p}\not{p} = p^2 = m_e^2 \longrightarrow 0$, $\not{q}\not{q} = q^2 = m_\nu^2 \longrightarrow 0$. Thus we have

$$\left\langle|M|^2\right\rangle = \frac{g^4}{8\left(s-m_W^2\right)^2}\frac{16}{4m_\nu m_e}\left[2(qk)(pk) - M_{H_\pm}^2(pq)\right]. \tag{4.55}$$

The Mandelstam variables in this process are

$$s=(p+q)^2\approx -2pq,\ \ t=(q-k)^2\approx M_{H_\pm}^2-2qk,\ \ u=(p-k)^2\approx M_{H_\pm}^2-2pk. \quad (4.56)$$

We then get

$$\left\langle |M|^2\right\rangle=\frac{g^4}{8\left(s-m_W^2\right)^2}\frac{16}{4m_\nu m_e}\left[\frac{(M_{H_\pm}^2-u)(M_{H_\pm}^2-t)}{2}+\frac{M_{H_\pm}^2 s}{2}\right]. \quad (4.57)$$

By using the properties of Mandelstam variables

$$s+u+t=M_{H_\pm}^2+M_{H_{\pm\pm}}^2, \quad (4.58)$$

we obtain

$$\left\langle |M|^2\right\rangle=\frac{g^4}{\left(s-m_W^2\right)^2}\frac{1}{4m_\nu m_e}\left[(s+t-M_{H_{\pm\pm}}^2)(M_{H_\pm}^2-t)+M_{H_\pm}^2 s\right]. \quad (4.59)$$

The differential cross section is [see Equation (A.19)]

$$\frac{d\sigma_{H_+,H_{--}}}{dt}=\frac{2G_F^2 m_W^4}{\pi s^2\left(s-m_W^2\right)^2}\left[2M_{H_\pm}^2(s+t)-(s+t)t-M_{H_\pm}^2 M_{H_{\pm\pm}}^2\right]. \quad (4.60)$$

The Mandelstam variable t in this process is

$$t=M_{H_\pm}^2-2(E_qE_k-|\vec{q}|\left|\vec{k}\right|\cos\theta) \quad (4.61)$$

with the energy

$$E_k=\frac{1}{2\sqrt{s}}\left[s+M_{H_\pm}^2-M_{H_{\pm\pm}}^2\right] \quad (4.62)$$

and the momentum of the singly charged Higgs

$$\begin{aligned}\left|\vec{k}\right| &= \frac{1}{2}\frac{s}{\sqrt{s}}\left[1-\frac{\left(M_{H_\pm}+M_{H_{\pm\pm}}\right)^2}{s}\right]^{\frac{1}{2}}\left[1-\frac{\left(M_{H_\pm}-M_{H_{\pm\pm}}\right)^2}{s}\right]^{\frac{1}{2}} \\ &= \frac{1}{2}\frac{s}{\sqrt{s}}\beta_{H_+,H_{--}}\end{aligned}$$

then

$$t = M_{H_\pm}^2 - \frac{1}{2}\left[M_{H_\pm}^2 - M_{H_{\pm\pm}}^2 + s(1-\beta_{H_+,H_{--}}\cos\theta)\right]. \tag{4.63}$$

The total cross section is then [see Equation (A.21)]

$$\begin{aligned}\sigma_{H_+,H_{--}} &= \frac{\alpha^2\pi}{12\sin^4\theta_W}\frac{s\beta_{H_+,H_{--}}}{(s-m_W^2)^2}\left[\frac{12M_{H_\pm}^2}{s}-\beta_{H_+,H_{--}}^2+3\right. \\ &\left.-\frac{3(M_{H_{\pm\pm}}^2-M_{H_\pm}^2)^2(M_{H_{\pm\pm}}^2+3M_{H_\pm}^2)^2}{s^2}\right].\end{aligned} \tag{4.64}$$

The total cross section of the process in Figure 4.3b is

$$\begin{aligned}\sigma_{H_-,H_2} &= \frac{\alpha^2\pi}{24\sin^4\theta_W}\frac{s\beta_{H_+,H_{--}}}{(s-m_W^2)^2}\left[\frac{12M_{H_2}^2}{s}-\beta_{H_-,H_2}^2+3\right. \\ &\left.-\frac{3(M_{H_-}^2-M_{H_2}^2)^2(M_{H_-}^2+3M_{H_2}^2)^2}{s^2}\right].\end{aligned} \tag{4.65}$$

The procedure to obtain Equation (4.64) is the same as for Equation (4.65) except the vertex factor. If the λ_i parameters in Equation (3.20) is set to zero, the masses of $H^\pm$, H_2 and $H^{\pm\pm}$ differ by an amount proportional to $\mu\upsilon_\Delta$. Therefore, due to the smallness of the value υ_Δ,

$$M_{H_\pm}^2 \approx M_{H_2}^2 \approx M_{H_{\pm\pm}}^2 \tag{4.66}$$

then

$$\sigma_{H_+,H_{--}} = 2\sigma_{H_-,H_2}. \tag{4.67}$$

4.3.3 $\overline{\nu}_e + e^- \rightarrow W^- + Z^0$

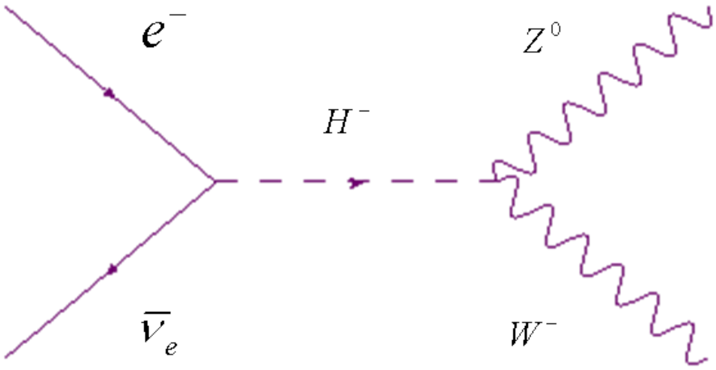


Figure 4.4: Feynman diagram for $\overline{\nu}_e(\vec{q}) + e^-(\vec{q}) \rightarrow W^-(\vec{k}_1) + Z^0(\vec{k}_2)$.

The interaction Lagrangian of this process is [see Equation (C.6) and Equation (C.10)]

$$
\begin{aligned}
\mathcal{L} &= -\frac{h_{ij}}{\sqrt{2}}\left(l_i^T C P_L \nu_j + \nu_i^T C P_L l_j\right)\cos\theta_+ H^+ \\
\mathcal{L} &= \frac{g^2\cos\theta_W}{2}\left[\sin\theta_+ \upsilon_0 g'^2 - \sqrt{2}\upsilon_\Delta \cos\theta_+ (2g'^2 + g^2)\right] W_\mu^+ H^- Z_\mu \\
&\quad + H.c.
\end{aligned} \tag{4.68}
$$

The Feynman amplitude is

$$
M = i\frac{g^2\sin^2\theta_W}{\cos\theta_W}\left(\frac{\mu\upsilon_0^2}{M_\Delta^2} - \sqrt{2}\left(2+\cot^2\theta_W\right)\upsilon_\Delta\right)\frac{g^{\mu\nu} h_{ee}\epsilon_\mu^*(k_1)\epsilon_\nu^*(k_2)\overline{\nu}(\vec{q})(1-\gamma_5)u(\vec{p})}{\sqrt{2}\left[\left(s - M_{H_-}^2\right) + iM_{H_-}\Gamma\right]}. \tag{4.69}
$$

Multiplying the complex conjugate

$$
|M|^2 = \frac{|h_{ee}|^2 C_{WZ}^2 \overline{\nu}(\vec{q})(1-\gamma_5)u(\vec{p})\overline{u}(\vec{p})(1+\gamma_5)\nu(\vec{q})}{2\left[\left(s - M_{H_-}^2\right)^2 + M_{H_-}^2\Gamma_t^2\right]}\epsilon_\mu^*(k_1)\epsilon_\alpha(k_1)g^{\mu\nu}\epsilon_\nu^*(k_2)\epsilon_\beta(k_2)g^{\alpha\beta}. \tag{4.70}
$$

Average over initial lepton polarization states and sum over final vector bosons polarization states.

$$\begin{aligned}
\left\langle |M|^2 \right\rangle &= \frac{|h_{ee}|^2 C_{WZ}^2}{2\left[\left(s-M_{H_-}^2\right)^2 + M_{H_-}^2\Gamma_t^2\right]}\left[-g_{\alpha\mu} + \frac{k_{1\alpha}k_{1\mu}}{m_Z^2}\right]\left[-g_{\beta\nu} + \frac{k_{2\beta}k_{2\nu}}{m_W^2}\right] g^{\mu\nu}g^{\alpha\beta} \\
&\quad \times \frac{1}{2} Tr\left[\frac{\not{q}(1-\gamma_5)\not{p}(1-\gamma_5)}{2m_e 2m_\nu}\right] \\
&= \frac{|h_{ee}|^2 C_{WZ}^2}{2\left[\left(s-M_{H_-}^2\right)^2 + M_{H_-}^2\Gamma_t^2\right]}\left[-g_\alpha^\nu + \frac{k_{1\alpha}k_1^\nu}{m_Z^2}\right]\left[-g_\nu^\alpha + \frac{k_2^\alpha k_{2\nu}}{m_W^2}\right] \\
&\quad \times Tr\left[\frac{qp(1-\gamma_5)}{2m_e 2m_\nu}\right] \\
&= \frac{|h_{ee}|^2 C_{WZ}^2}{2\left[\left(s-M_{H_-}^2\right)^2 + M_{H_-}^2\Gamma_t^2\right]}\left[4 - \frac{k_2^\nu k_{2\nu}}{m_W^2} - \frac{k_{1\nu}k_1^\nu}{m_Z^2} + \frac{(k_1\cdot k_2)^2}{m_Z^2 m_W^2}\right]\frac{4(qp)}{2m_e 2m_\nu} \\
&= \frac{|h_{ee}|^2 C_{WZ}^2}{2\left[\left(s-M_{H_-}^2\right)^2 + M_{H_-}^2\Gamma_t^2\right]}\left[2 + \frac{(k_1\cdot k_2)^2}{m_Z^2 m_W^2}\right]\frac{4(qp)}{2m_e 2m_\nu} \\
&= \frac{|h_{ee}|^2 C_{WZ}^2 s^3}{\left[\left(s-M_{H_-}^2\right)^2 + M_{H_-}^2\Gamma_t^2\right]}\left[1 - \frac{2\left(m_W^2+m_Z^2\right)}{s} + \frac{\left(m_W^2+m_Z^2\right)^2}{s^2} + \frac{8m_Z^2 m_W^2}{s^2}\right] \\
&\quad \times \frac{1}{4m_Z^2 m_W^2}\frac{1}{2m_e 2m_\nu}
\end{aligned} \tag{4.71}$$

where

$$\begin{aligned}
s &= (k_1+k_2)^2 = k_1^2 + k_2^2 + 2\left(k_1\cdot k_2\right) \\
&= m_W^2 + m_Z^2 + 2\left(k_1\cdot k_2\right).
\end{aligned} \tag{4.72}$$

We obtain the differential cross section [see Equation (A.19)]

$$\begin{aligned}
\frac{d\sigma}{dt} &= \frac{|h_{ee}|^2 C_{WZ}^2 s}{64\pi m_Z^2 m_W^2\left[\left(s-M_{H_-}^2\right)^2 + M_{H_-}^2\Gamma_t^2\right]} \\
&\quad \times \left[1 - \frac{2\left(m_W^2+m_Z^2\right)}{s} + \frac{\left(m_W^2+m_Z^2\right)^2}{s^2} + \frac{8m_Z^2 m_W^2}{s^2}\right].
\end{aligned} \tag{4.73}$$

The variable t is written as

$$\begin{aligned} t &= (q-k_1)^2 = q^2 + k_1^2 - 2(qk_1) \\ &= m_Z^2 - 2(E_q E_k - |\vec{q}| \left|\vec{k}_1\right| \cos\theta) \\ &= \frac{1}{2}\left[m_Z^2 + m_W^2 - s(1-\beta_{WZ}\cos\theta)\right] \end{aligned} \tag{4.74}$$

where

$$\begin{aligned} E_k &= \frac{1}{2\sqrt{s}}\left[s + \left(m_Z^2 - m_W^2\right)\right], \quad E_q = |\vec{q}| = \frac{1}{2}\sqrt{s} \\ \left|\vec{k}_1\right| &= \frac{s}{2\sqrt{s}}\left[1 - \frac{(m_Z+m_W)^2}{s}\right]^{\frac{1}{2}}\left[1 - \frac{(m_Z-m_W)^2}{s}\right]^{\frac{1}{2}} \\ &= \frac{s}{2\sqrt{s}}\beta_{WZ}. \end{aligned} \tag{4.75}$$

Thus, the total cross section obtained is [see Equation (A.21)]

$$\begin{aligned} \sigma &= \frac{|h_{ee}|^2 s^2 \beta_{WZ} G_F^2 \sin^2\theta_W}{\pi m_W^2\left[\left(s - M_{H_-}^2\right)^2 + M_{H_-}^2\Gamma_t^2\right]}\left(\frac{\mu\upsilon_0^2}{M_\Delta^2} - \sqrt{2}\left(2+\cot^2\theta_W\right)\upsilon_\Delta\right)^2 \\ &\times\left[1 - \frac{2\left(m_W^2+m_Z^2\right)}{s} + \frac{\left(m_W^2+m_Z^2\right)^2}{s^2} + \frac{8m_Z^2 m_W^2}{s^2}\right]. \end{aligned} \tag{4.76}$$

At the energy $s \approx M_{H_-}^2$, the total cross section can be written as

$$\sigma_{res} = 32\pi\frac{\Gamma(H^- \to \overline{\nu}_e e)\Gamma(H^- \to W^- Z^0)}{\left(s - M_{H_-}^2\right)^2 + M_{H_-}^2\Gamma_t^2}, \qquad s \approx M_{H_-}^2 \tag{4.77}$$

where we have used the expression (Fileviez Pérez et al., 2008)

$$\Gamma(H^- \to \overline{\nu}_e e) = \frac{|h_{ee}|^2 M_{H_-}}{16\pi}, \tag{4.78}$$

$$\begin{aligned}\Gamma(H^- &\rightarrow W^- Z^0) = \frac{\beta_{WZ} G_F^2 \sin^2\theta_W M_{H_-}^3}{2\pi m_W^2}\left(\frac{\mu \upsilon_0^2}{M_\Delta^2} - \sqrt{2}\left(2+\cot^2\theta_W\right)\upsilon_\Delta\right)^2 \\ &\times\left[1 - \frac{2\left(m_W^2+m_Z^2\right)}{M_{H_-}^2} + \frac{\left(m_W^2+m_Z^2\right)^2}{M_{H_-}^4} + \frac{8m_Z^2 m_W^2}{M_{H_-}^4}\right]. \qquad (4.79)\end{aligned}$$

Equation (4.78) and Equation (4.79) are the decay widths for the decay processes

$$H^- \rightarrow \overline{\nu_e} + e, \quad H^- \rightarrow W^- + Z^0 \qquad (4.80)$$

respectively. Equation (4.77) is the cross section at the resonance energy $s \approx M_{H_-}^2$ where Γ_t is the total decay width of H^-.

Chapter 5

Result and Discussion

Before we discuss the angular distribution and the cross section, we would like to estimate the limit of h_{ee} in this process at low energy. One might expand terms in Equation (4.36) in series. The total energy of the process is much smaller than the mass of the gauge bosons so that we are allowed to take the approximation $s \ll m_W^2, s \ll m_Z^2, s \ll M_{H^-}^2$ in order to obtain the total cross section that is linear at low energy, and reduces to

$$\sigma_{tot} = \frac{G_F^2 s}{\pi}\left\{\frac{1}{3} + \frac{(2\sin^2\theta_W - 1)^2}{12} + \frac{(2\sin^2\theta_W)^2}{4} + \frac{(2\sin^2\theta_W - 1)}{3} + \frac{h_{ee}^4}{24G_F^2 M_H^4}\right\}. \tag{5.1}$$

The experimental result from the TEXONO collaboration (Deniz et al., 2010) gives

$$\frac{\sigma_{exp}}{\sigma_{SM}} = 1.08 \pm 0.21 \pm 0.16. \tag{5.2}$$

By setting $\sigma_{exp} = \sigma_{tot}$, one gets

$$h_{ee}/M_{H^\pm} \lesssim 2.8 \times 10^{-3} GeV^{-1} \tag{5.3}$$

which is consistent with the limit estimated by Corasa et al (Coarasa, Mendez, & Sola, 1996) using the νe scattering process. Since the W exchange and charged Higgs exchange proceed through s channel, we compare their contribution to the incoming neutrino energy. In analogy to the ratio $R_{\overline{\nu}} = \sigma^{\overline{\nu}}_{NC}/\sigma^{\overline{\nu}}_{CC}$, (Adams et al., 2009) we define

$$\begin{aligned} R_W &= \frac{\sigma(\overline{\nu}_e e \to H^- \to \overline{\nu}_e e)}{\sigma(\overline{\nu}_e e \to W^- \to \overline{\nu}_e e)} \\ &= \frac{h_{ee}^4}{8G_F^2 m_W^4} \frac{(s-m_W^2)^2 + \Gamma_W^2 m_W^2}{(s-M_{H^\pm}^2)^2 + \Gamma_{H^\pm}^2 M_{H^\pm}^2} \end{aligned} \tag{5.4}$$

where $\Gamma_{H^\pm}$ is the decay width of H^- and Γ_W = 2.085 GeV (Nakamura et al., 2010).

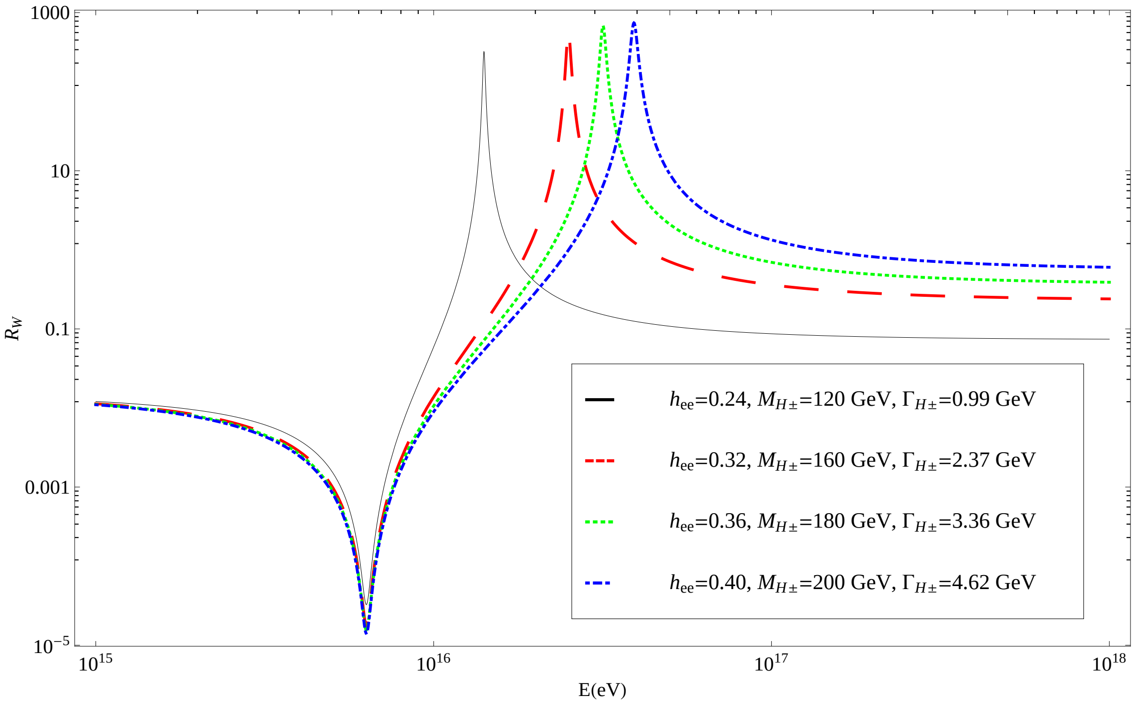


Figure 5.1: The ratio of H^- to W^- exchange in $\overline{\nu}_e - e$ scattering with $h_{ee} = 0.24$, $M_{H^\pm} =$ 120 GeV and $\Gamma_{H^\pm} = 0.99$ GeV ; $h_{ee} = 0.32$, $M_{H^\pm} = 160$ GeV and $\Gamma_{H^\pm} = 2.37$ GeV ; $h_{ee} = 0.36$, $M_{H^\pm} = 180$ GeV and $\Gamma_{H^\pm} = 3.36$ GeV ; $h_{ee} = 0.40$, $M_{H^\pm} = 200$ GeV and $\Gamma_{H^\pm} = 4.62$ GeV.

In Figure 5.1 we plot the ratio $R_{\overline{\nu}}$ against the incoming neutrino energy for $h_{ee} = 0.24$, $M_{H^\pm} = 120$ GeV and $\Gamma_{H^\pm} = 0.99$ GeV ; $h_{ee} = 0.32$, $M_{H^\pm} = 160$ GeV and $\Gamma_{H^\pm} = 2.37$ GeV ; $h_{ee} = 0.36$, $M_{H^\pm} = 180$ GeV and $\Gamma_{H^\pm} = 3.36$ GeV ; $h_{ee} = 0.40$, $M_{H^\pm} = 200$

GeV and $\Gamma_{H^\pm} = 4.62$ GeV. Since in all cases $h_{ee}/M_{H^\pm} = 2.0 \times 10^{-3}$ GeV^{-1}, then the conditions $M_{H^\pm} \gtrsim$110 GeV [See Equation (3.38)] and Equation (5.3) is fulfilled. We have set $m_W = 80.399$ GeV, $m_Z = 91.1876$ GeV and $\sin^2\theta_W = 0.23116$ (Nakamura et al., 2010) (Appendix A). Further, we restrict ourselves to take $M_{H^\pm} \leq 200$ GeV so that $h_{\mu\mu}$ <1.0 ($h_{\mu\mu}/M_{H^\pm} \sim 5.0 \times 10^{-3} GeV^{-1}$) (Godfrey, Kalyniak, & Romanenko, 2002). We assume only the leptonic decay channel of H^- and the total decay width in Equation (4.78) is the sum of two flavours (electron and muon). The decay into tau leptons are not considered due to no strict constraints on the corresponding coupling $h_{\tau\tau}$. At $s \approx m_W^2$ ($s = 2m_eE, E \approx$ a few 10^6 GeV) the ratio R_W is very small as can be easily seen from Equation (5.4). In other words, the H^- contribution is negligible at W resonance energy but dominant at $s \approx M^2_{H^\pm}$. Furthermore, at very large energies $s >> m_W^2,\ s >> M^2_{H^\pm}$,

$$R_W \to \frac{h_{ee}^4}{8G_F^2 m_W^4} \tag{5.5}$$

converge to a finite value. As the mass of the charged Higgs boson increases, the finite value is larger. On the other hand, when the neutrino energy is increased up to the scalar resonance energy the scalar exchange will play a major contribution and one can test the $SU(2)_L$ triplet more precisely in $\overline{\nu}_e - e$ scattering. The ratio σ_{H^-}/σ_{SM} is presented in Figure 5.2 for $h_{ee} = 0.24$, $M_{H^\pm} = 120$ GeV and $\Gamma_{H^\pm} = 0.99$ GeV ; $h_{ee} = 0.32$, $M_{H^\pm} = 160$ GeV and $\Gamma_{H^\pm} = 2.37$ GeV ; $h_{ee} = 0.36$, $M_{H^\pm} = 180$ GeV and $\Gamma_{H^\pm} = 3.36$ GeV ; $h_{ee} = 0.40$, $M_{H^\pm} = 200$ GeV and $\Gamma_{H^\pm} = 4.62$ GeV.

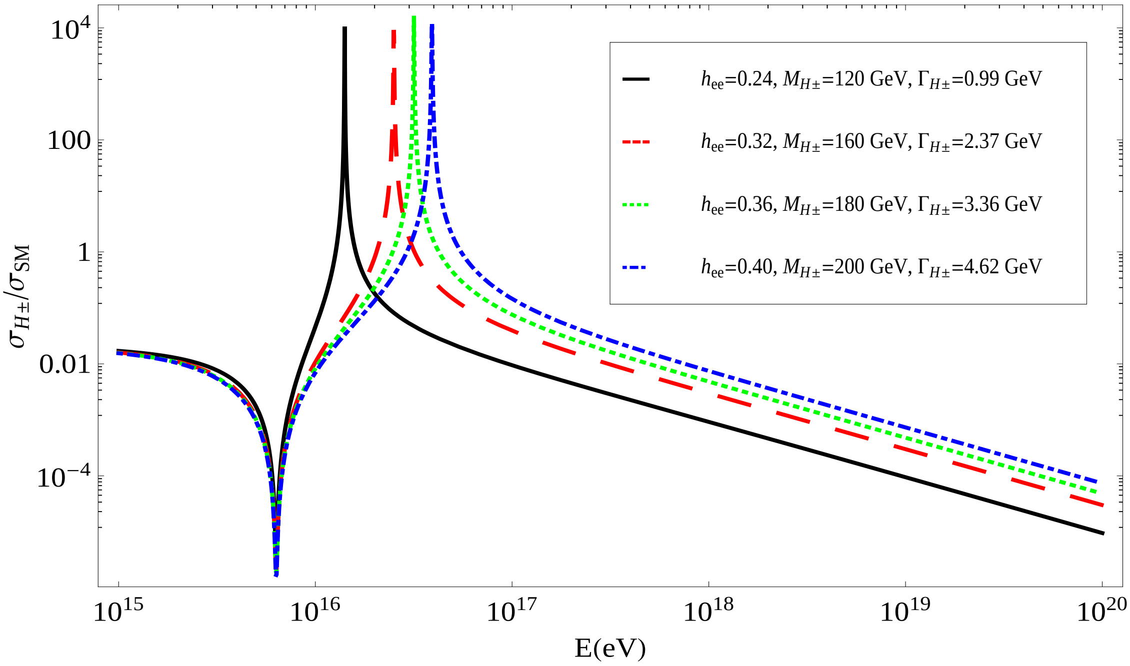


Figure 5.2: The ratio $\sigma_{H^\pm}/\sigma_{SM}$ in $\overline{\nu}_e - e$ scattering with $h_{ee} = 0.24$, $M_{H^\pm} = 120$ GeV and $\Gamma_{H^\pm} = 0.99$ GeV ; $h_{ee} = 0.32$, $M_{H^\pm} = 160$ GeV and $\Gamma_{H^\pm} = 2.37$ GeV ; $h_{ee} = 0.36$, $M_{H^\pm} = 180$ GeV and $\Gamma_{H^\pm} = 3.36$ GeV ; $h_{ee} = 0.40$, $M_{H^\pm} = 200$ GeV and $\Gamma_{H^\pm} = 4.62$ GeV

5.1 $\overline{\nu}_e - e$ scattering

5.1.1 Angular distribution

The angular distributions of $\overline{\nu}_e - e$ scattering as the function of $\cos\theta$ are plotted in Figure 5.3, Figure 5.4 and Figure 5.5 with $h_{ee} = 0.308$ and $M_{H^\pm} = 110$ GeV. The scattering at forward direction is small but not zero due to none vanishing of the right handed coupling term multiplying s^2 in Equation (4.36). As the energy increases, the forward and backward scattering shift towards higher angular distribution as shown in Figure 5.4. As the energy goes beyond $M^2_{H^\pm}$, the total cross section converges to a finite value in Figure 5.9. Hence, the angular distributions shift towards the backward ($t = -s$) direction while the forward ($t = 0$) direction is suppressed to preserve the total cross section. The area under the curves in Figure 5.9 are the total cross section.

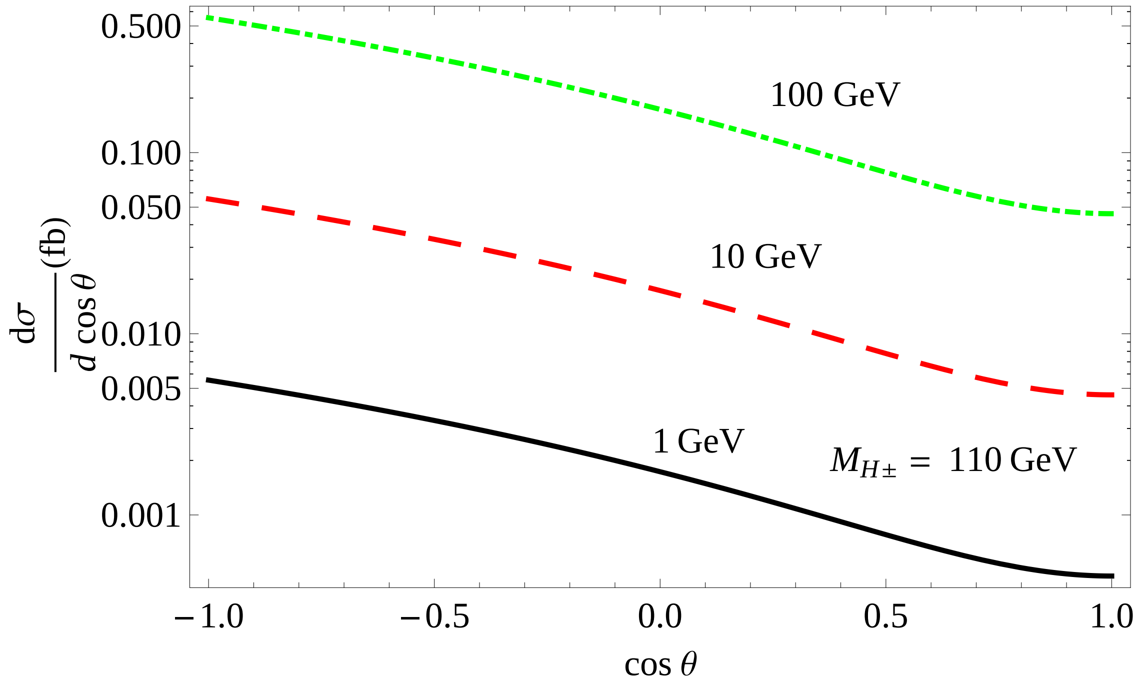


Figure 5.3: The angular distribution of $\overline{\nu}_e - e$ scattering for $E = 1$ GeV, 10 GeV, 100 GeV, with $h_{ee} = 0.308$ and $M_{H^\pm} = 110$ GeV.

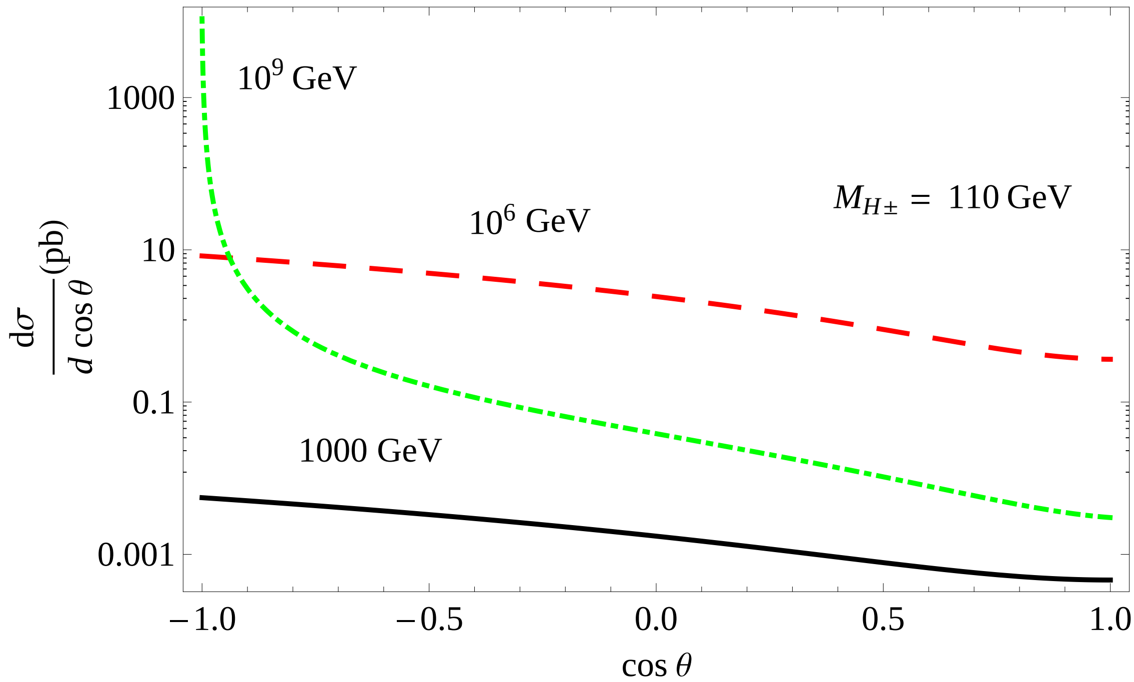


Figure 5.4: The angular distribution of $\overline{\nu}_e - e$ scattering for $E = 1000$ GeV, 10^6 GeV, 10^9 GeV, with $h_{ee} = 0.308$ and $M_{H_\pm} = 110$ GeV.

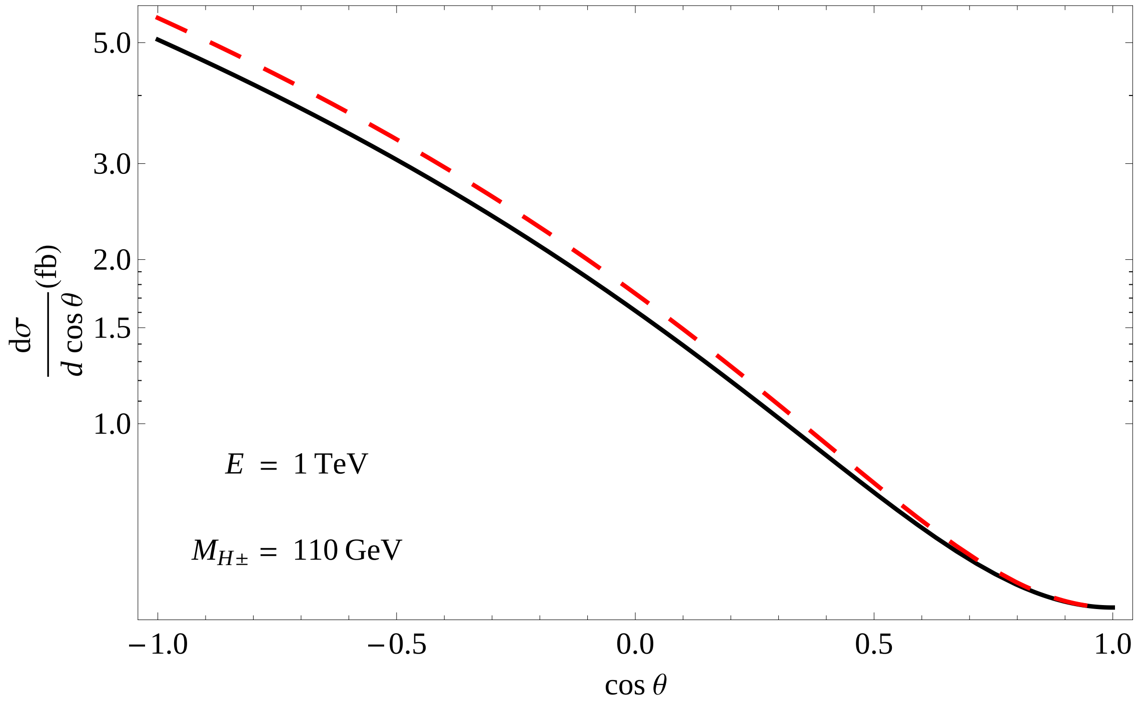


Figure 5.5: Angular distribution of $\overline{\nu}_e - e$ scattering at 1 TeV with $M_{H_\pm} = 110$ GeV. The solid line is the SM and dash-dotted line is the contribution of H^- exchange.

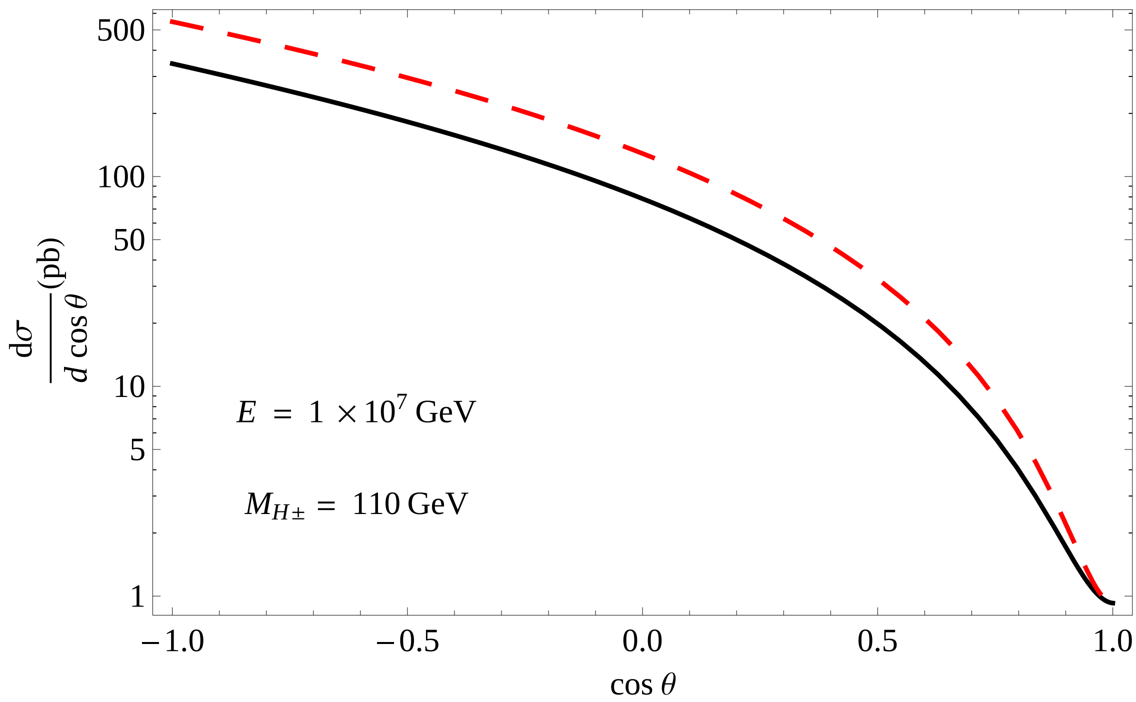


Figure 5.6: Angular distribution of $\overline{\nu}_e - e$ scattering at 10^7 eV with $M_{H_\pm} = 110$ GeV. The solid line is the SM and dash-dotted line is the contribution of H^- exchange.

5.1.2 Total cross section

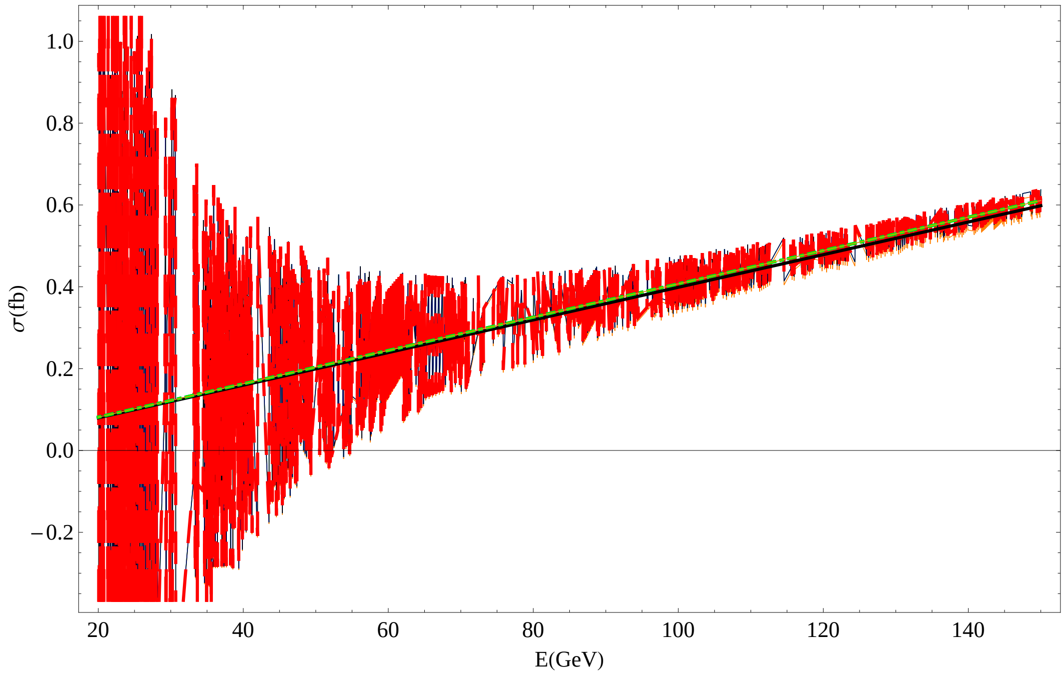


Figure 5.7: Total cross section for $\overline{\nu}_e - e$ scattering at 0-150 GeV. The solid line is the SM and the dash-dotted line is the contribution of H^- exchange of Equation (5.1) for different combination of h_{ee} and $M_{H_\pm}$. The background fluctuations are the result of Equation (4.36) at low energy.

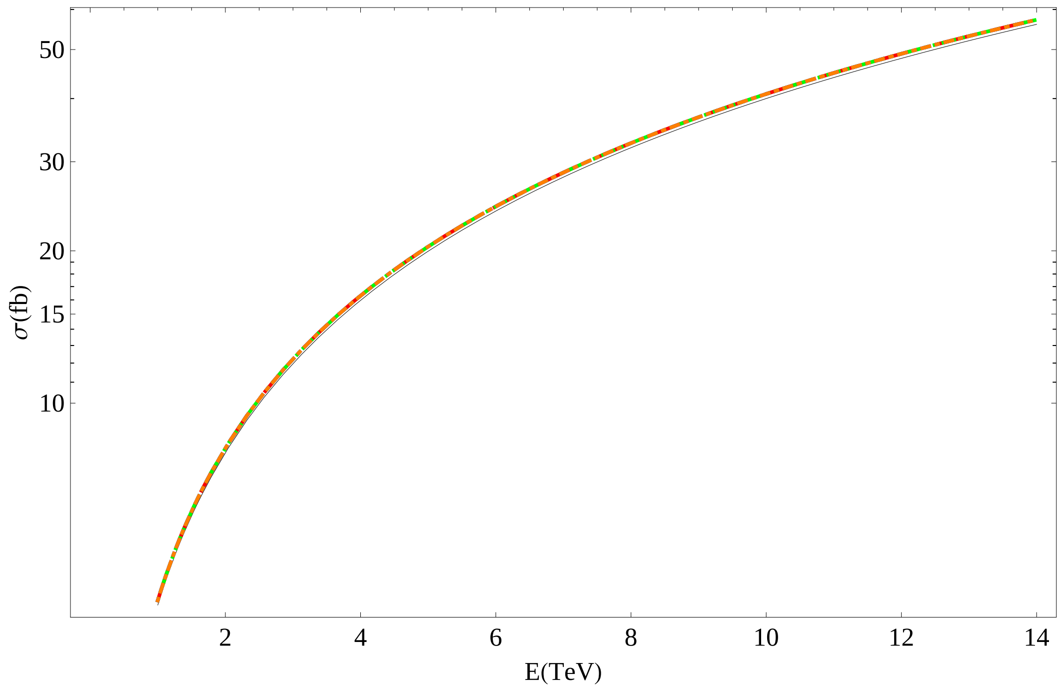


Figure 5.8: Total cross section for $\overline{\nu}_e - e$ scattering at 2-14 TeV. The solid line is the SM and the dash-dotted line is the contribution of H^- exchange for different combination of h_{ee} and $M_{H_\pm}$.

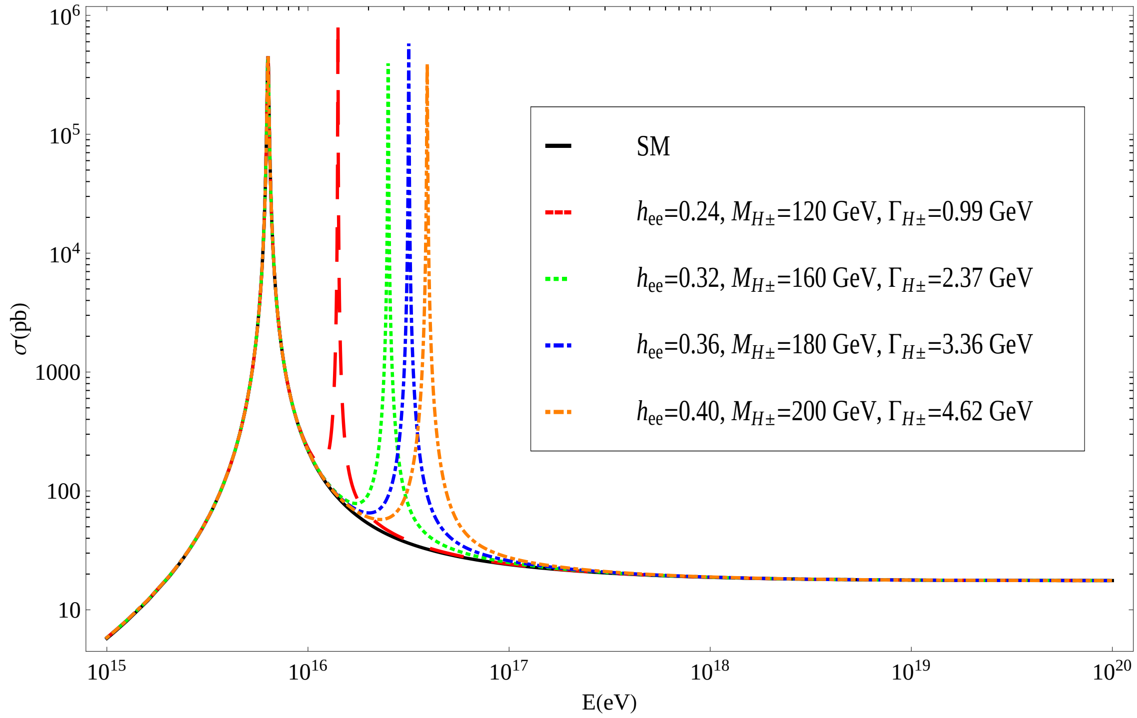


Figure 5.9: Total cross section for $\overline{\nu}_e - e$ scattering at 10^{15} eV $\leq E \leq 10^{20}$ eV. The solid line is the SM and the dash-dotted line is the contribution of H^- exchange for $h_{ee} = 0.24$, $M_{H^\pm} = 120$ GeV and $\Gamma_{H^\pm} = 0.99$ GeV ; $h_{ee} = 0.32$, $M_{H^\pm} = 160$ GeV and $\Gamma_{H^\pm} = 2.37$ GeV ; $h_{ee} = 0.36$, $M_{H^\pm} = 180$ GeV and $\Gamma_{H^\pm} = 3.36$ GeV ; $h_{ee} = 0.40$, $M_{H^\pm} = 200$ GeV and $\Gamma_{H^\pm} = 4.62$ GeV.

Figure 5.7, Figure 5.8 and Figure 5.9 show the total cross section at different energy range. The total cross section for $\overline{\nu}_e - e$ scattering at low energy is fluctuated around its average position due to the terms m_Z^2/s and $(m_Z^2/s)^2$ multiplying the left-handed coupling of Z boson to the electron in Equation (4.36). At the energy near to zero, the cross section is fluctuating with negative value which is not physical. In our interpretation, the standard electroweak theory is not capable to describe neutrinos interaction at very low energy [Equation (4.36)]. One has to approximate to the four-Fermi interaction (Mohapatra & Pal, 2004). The average line in Figure 5.7 corresponds to the low energy approximation of the process as in Equation (5.1). As the energy increase to the TeV region in Figure 5.8, the charged Higgs exchange have very small contribution to the total cross section with respect to SM. The resonance of production of W^- and H^- are expected at the energy above 10^{15} eV. The first peak in Figure 5.9 correspond to the Glashow resonance and following peaks are H^- boson resonance for different cases. Two peaks may be clearly distinguished if both of the W^- and H^- masses are not too close.

5.2 Higgs production

5.2.1 Angular distribution

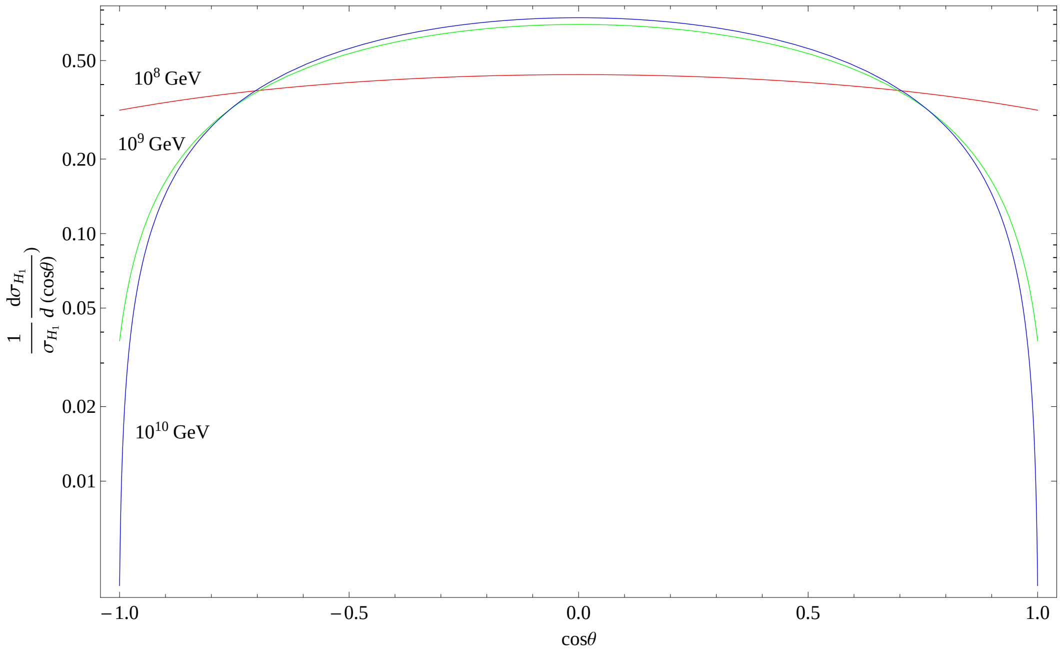


Figure 5.10: Angular distribution of $\overline{\nu}_e + e^- \rightarrow W^- + H_1$ for $E = 10^8$ GeV, 10^9 GeV, 10^{10} GeV, with $M_{H_1} = 100$ GeV.

In Figure 5.10 and Figure 5.11, we plot the normalized angular distribution of the Higgs production processes at different energy. We do not show the angular distribution for H^-H_2 and W^+H^{--} production since they would have the same distribution but with smaller intensity. The angular distribution for both processes shown are approaching flat near their threshold energy where $s \approx (m_W + M_{H1})^2$ and $s \approx (M_{H--} + M_{H+})^2$. At higher energy, the angular distribution for both W^-H_1 and H^+H^{--} production is zero at 0^0 and 180^0.

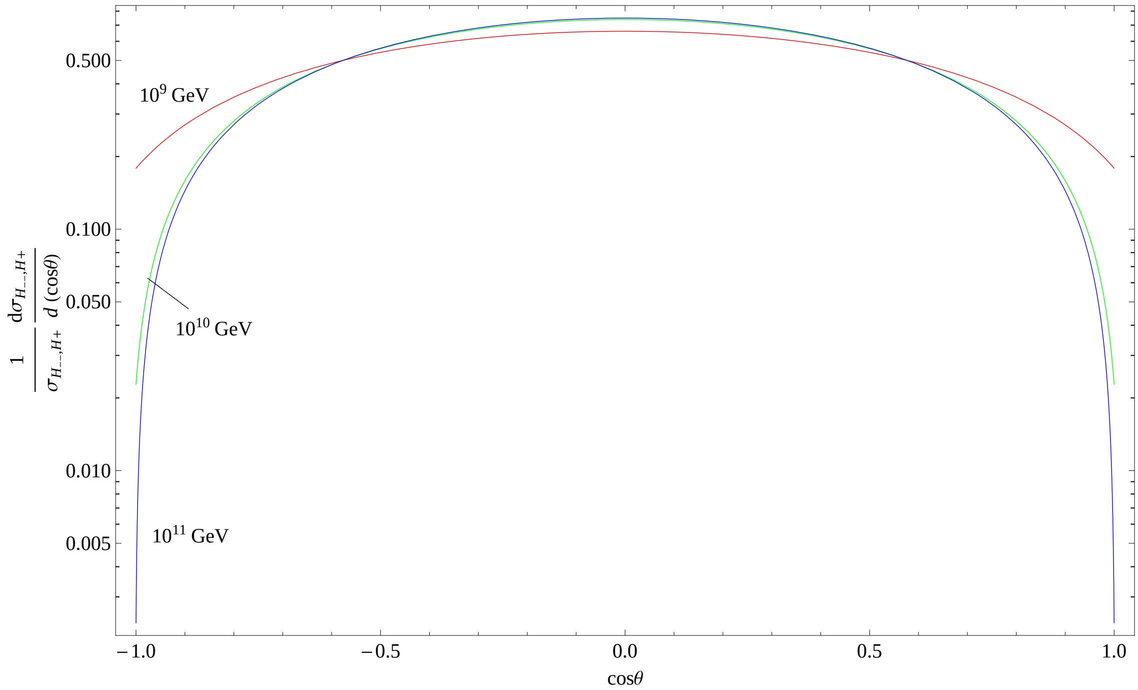


Figure 5.11: Angular distribution of $\overline{\nu}_e + e^- \to H^+ + H^{--}$ for $E = 10^9$ GeV, 10^{10} GeV, 10^{11} GeV, with $M_{H_\pm} = M_{H_{\pm\pm}} = 200$ GeV.

5.2.2 Total cross section

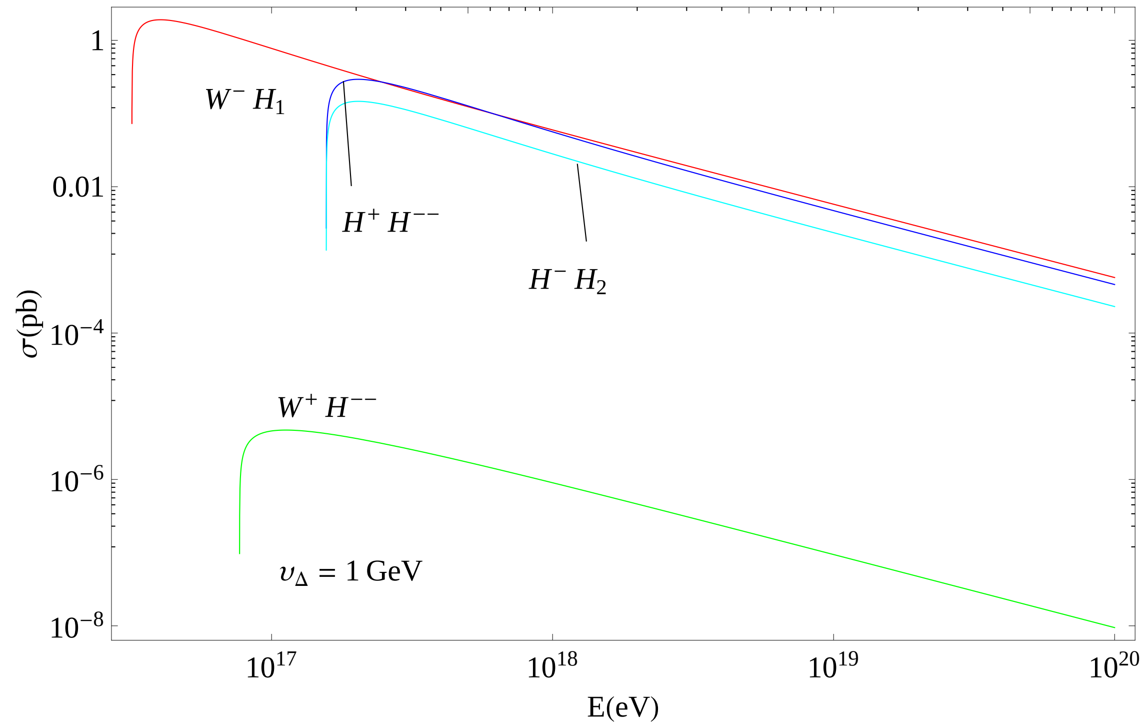


Figure 5.12: Total cross section for $W^- H_1, H^+ H^{--}, H^- H_2$ and $W^+ H^{--}$ production with $M_{H_1} = 100$ GeV, $M_{H_2} = M_{H_\pm} = M_{H_{\pm\pm}} = 200$ GeV and $\upsilon_\Delta = 1$ GeV

The plot of the total cross section of these four processes are shown in Figure 5.12. As the discussion in Fileviez Perez *et al.*, (2008), the vev of the triplet is 1 eV$\lesssim$ υ_Δ $\lesssim$1 GeV. Thus, for simplicity, we constraint the vev of triplet scalar as $\upsilon_\Delta = 1$ GeV. The total cross section for the $W^- H_1$ production is the highest among the four processes due to its coupling to the doublet vev $\upsilon_0^2 \approx$ (246 GeV)^2. However, the $W^+ H^{--}$ production is suppressed by triplet vev and thus less phenomenology important. The production of $H^+ H^{--}$ is almost close to $W^- H_1$ at the energy above 2.0×10^{17} eV. The production of $H^- H_2$ is half of $H^+ H^{--}$ as in Equation (4.67). All of the Higgs production cross section reaches their maximum value very soon after threshold. There is no resonance production in these processes due to the mass of the W boson is smaller than the Higgs mass. These processes are allowable above the W pole. The flux of such high energy of the neutrino is only possible to reach by cosmic rays and requiring a large detector such as neutrino telescope (Halzen, 2006; Chiarusi & Spurio, 2010). Natural ice at the south pole or deep sea are normally utilized as a medium to track the charged particles produced by the neutrino interaction with nucleon through the detection of Cherenkov radiation (Nakamura et al., 2010). Therefore, it is also able to detect the charged fermions that are produced through the Higgs decay. Consider for large coupling h_{ij}, the triplet Higgs decay is dominated by the leptonic channels as pointed out by Fileviez Perez *et al.*, (2008). The production of $W^- H_1$ will decay as $W^- H_1 \to l^- \nu W^+ W^- \to l^- l^+ l^- \nu\nu\nu$. One can detect two like sign leptons while the neutrinos are experimentally hard to be detected. However, this decay channel can mix up with the decay of the $H^+ H^{--}$ above 2.0×10^{17} eV which decay as $H^+ H^{--} \to l^- l^+ l^- \nu$ unless the doubly charged Higgs decay to two different flavours leptons. Since the coupling of Higgs is proportional to the mass of the neutrino, $h_{\tau\tau}$ would be the largest among the three flavours and the charged

Higgs boson would decay dominantly into tau. On the other hand, if the coupling h_{ij} is small, the charged Higgs bosons will be more favourable to decay into gauge bosons and further decay into more leptons. The production of $H^- H_2$ will result in single charged lepton with three neutrinos.

5.3 Resonance production of H^- in $\overline{\nu}_e e \to \overline{\nu}_e e$ and $\overline{\nu}_e e \to Z^0 W^-$

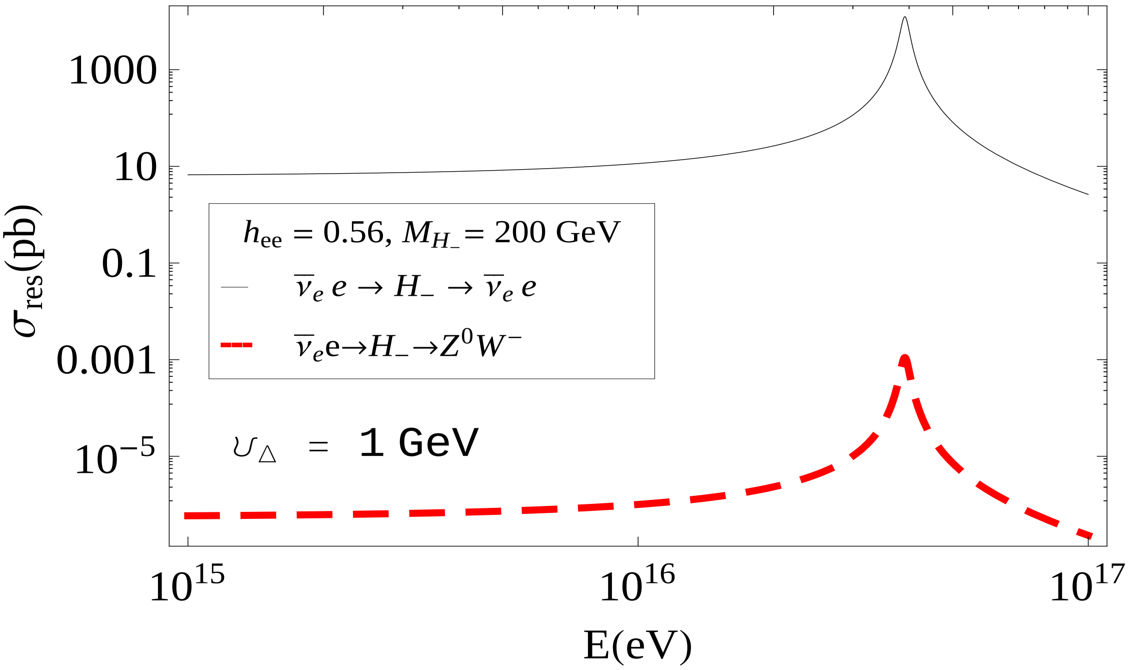


Figure 5.13: Resonance production of H^- in $\overline{\nu}_e e \to \overline{\nu}_e e$ and $\overline{\nu}_e e \to Z^0 W^-$ with $h_{ee} = 0.56$, $M_{H-} = 200$ GeV, $\Gamma_t = 4.5$ GeV and $\upsilon_\triangle = 1$ GeV.

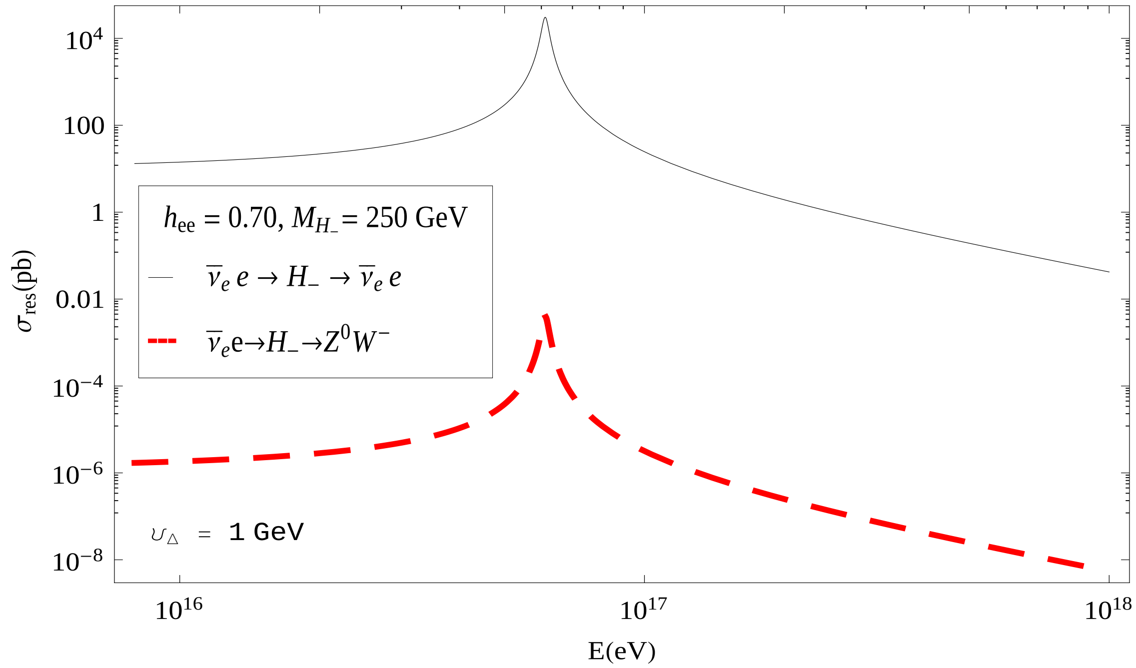


Figure 5.14: Resonance production of H^- in $\overline{\nu}_e e \to \overline{\nu}_e e$ and $\overline{\nu}_e e \to Z^0 W^-$ with $h_{ee} = 0.70$, $M_{H-} = 250$ GeV, $\Gamma_t = 4.5$ GeV and $\upsilon_\Delta = 1$ GeV.

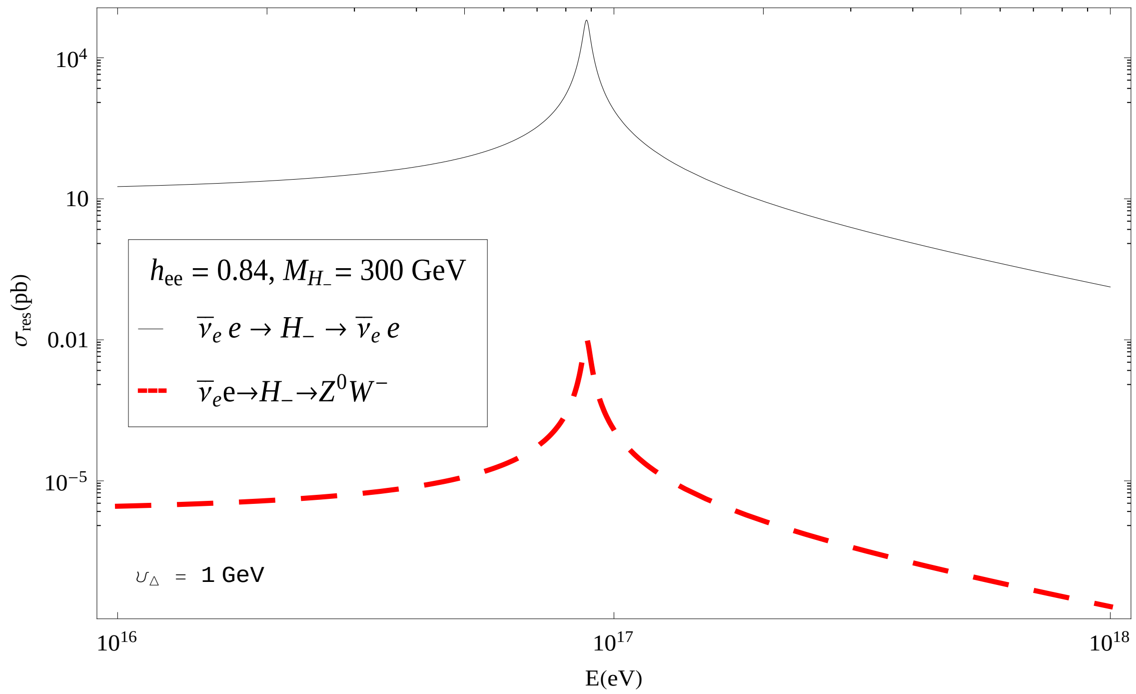


Figure 5.15: Resonance production of H^- in $\overline{\nu}_e e \to \overline{\nu}_e e$ and $\overline{\nu}_e e \to Z^0 W^-$ with $h_{ee} = 0.84$, $M_{H-} = 300$ GeV, $\Gamma_t = 4.5$ GeV and $\upsilon_\Delta = 1$ GeV.

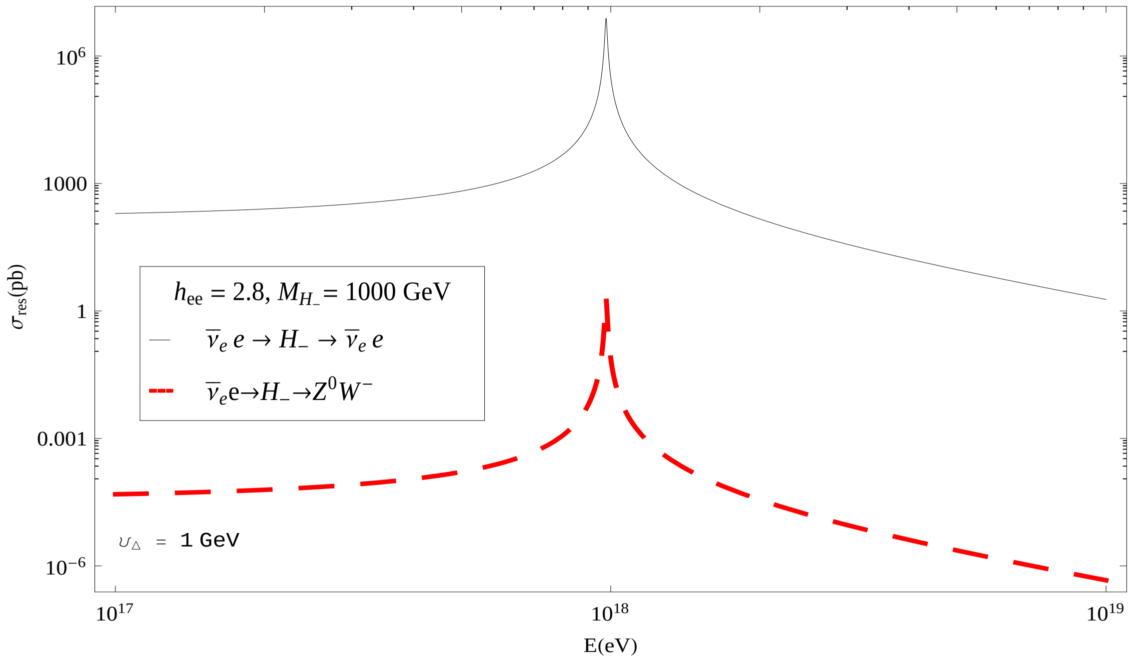


Figure 5.16: Resonance production of H^- in $\overline{\nu}_e e \to \overline{\nu}_e e$ and $\overline{\nu}_e e \to Z^0 W^-$ with $h_{ee} = 2.8$, $M_{H-} = 1000$ GeV, $\Gamma_t = 4.5$ GeV and $\upsilon_\Delta = 1$ GeV.

The H^- have several possibility of decay channels after the production at the resonance energy. We compare the $\overline{\nu}_e e$ and $W^- Z^0$ channels with $h_{ee} = 0.56$, $M_{H-} = 200$ GeV , $h_{ee} = 0.70$, $M_{H-} = 250$ GeV, $h_{ee} = 0.84$, $M_{H-} = 300$ GeV, $h_{ee} = 2.8$, $M_{H-} = 1000$ GeV at $\upsilon_\Delta = 1$ GeV. The $\overline{\nu}_e e$ channel increases as the coupling constant increases since the cross section depends quartically on h_{ee}. However, the W$^- Z^0$ channel does not increase as rapid as the former due to the suppression from υ_Δ even it depends quadratically on h_{ee}. The cross section for $W^- Z^0$ channel is only physical for $M_{H_-}^2 \geq (m_W + m_Z)^2$. This ensures that the charged Higgs do not contribute to the decay width of the Z boson which is measured precisely and available in K. Nakamura *et al.*, (2010).

Chapter 6

Conclusion

As a conclusion, the Higgs Triplet Model (HTM) provides us a rich variety of final states to $\overline{\nu}_e e$ annihilation particularly at energies beyond W pole. We only consider the processes with two final states. We have not yet considered the possibilities of WW fusion, WZ fusion and ZZ fusion. The possibilities of the Higgs search through $\overline{\nu}_e e$ annihilation at the neutrino telescope have been investigated. The production of the charged Higgs bosons can be distinguished from $\nu_\mu + N \to \mu + x$ reaction through their decay products. In the optimistic situations, at large $h_{\alpha\beta}$, one may test the processes by looking for flavour violation charged leptons production. If the coupling $h_{\alpha\beta}$ is small, the Higgs bosons will decay into SM gauge bosons where the decays with flavour violation are suppressed. The flux of such high energy $\overline{\nu}_e$ is probably very small. However, the flux of electron antineutrinos may get enhanced if they are generated through neutrino oscillation.

We hope our calculation could help to determine the feasibility of such experiments to discriminate between SM and HTM at current available neutrino observatory.

Appendix A

Convention, useful formulae and physical constants

A.1 Pauli and Dirac Matrices

$$\overline{u}_r(\mathbf{p}) = u_r^\dagger(\mathbf{p})\,\gamma^0,\ [\gamma^\mu, \gamma^\nu]_+ = 2g^{\mu\nu},\ \gamma^{\mu\dagger} = \gamma^0\gamma^\mu\gamma^0,\ [\gamma^\mu, \gamma^5]_+ = 0 \tag{A.1}$$

$$(\gamma^5)^2 = 1,\ \gamma^{5\dagger} = \gamma^5,\ \sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu,\ \gamma^\nu\right], [\sigma_i, \sigma_j] = 2i\epsilon_{ijk}\sigma_k \tag{A.2}$$

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{A.3}$$

$$\alpha_k = \begin{pmatrix} 0 & \sigma_k \\ \sigma_k & 0 \end{pmatrix},\ \gamma^k = \beta\alpha_k,\ \beta = \gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \tag{A.4}$$

$$\gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \; I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{A.5}$$

$$\begin{aligned} \Lambda^{+}_{\alpha\beta}(\mathbf{p}) &= \sum_{r=1}^{2} u_{r\alpha}(\mathbf{p})\,\overline{u}_{r\beta}(\mathbf{p}) = \left(\frac{\gamma^{\mu}p_{\mu}+m}{2m}\right)_{\alpha\beta} \\ \Lambda^{-}_{\alpha\beta}(\mathbf{p}) &= \sum_{r=1}^{2} v_{r\alpha}(\mathbf{p})\,\overline{v}_{r\beta}(\mathbf{p}) = \left(\frac{\gamma^{\mu}p_{\mu}-m}{2m}\right)_{\alpha\beta} \end{aligned} \tag{A.6}$$

A.2 Contraction identity

$$\gamma^{\mu}\gamma_{\mu} = DI, \; \gamma^{\mu}\gamma^{\alpha}\gamma_{\mu} = -(D-2)\gamma^{\alpha}, \; \gamma^{\mu}\gamma^{\alpha}\gamma^{\beta}\gamma_{\mu} = (D-4)\,\gamma^{\alpha}\gamma^{\beta} + 4g^{\alpha\beta} \tag{A.7}$$

$$\gamma^{\mu}\gamma^{\alpha}\gamma^{\beta}\gamma^{\gamma}\gamma_{\mu} = -2\gamma^{\gamma}\gamma^{\beta}\gamma^{\alpha}, \; \gamma^{\mu}\gamma^{\alpha}\gamma^{\beta}\gamma^{\gamma}\gamma^{\delta}\gamma_{\mu} = 2(\gamma^{\delta}\gamma^{\alpha}\gamma^{\beta}\gamma^{\gamma} + \gamma^{\gamma}\gamma^{\beta}\gamma^{\alpha}\gamma^{\delta}) \tag{A.8}$$

$$\varepsilon^{\alpha\beta\mu\nu}\varepsilon_{\alpha\beta\sigma\tau} = -2\left(g^{\mu}_{\sigma}g^{\nu}_{\tau} - g^{\mu}_{\tau}g^{\nu}_{\sigma}\right), \; \varepsilon^{\alpha\beta\gamma\nu}\varepsilon_{\alpha\beta\gamma\tau} = -6g^{\nu}_{\tau}, \; \varepsilon^{\alpha\beta\gamma\delta}\varepsilon_{\alpha\beta\gamma\delta} = -24 \tag{A.9}$$

A.3 Trace Identity

$$Tr\,(UV) = Tr(VU), \; Tr\left(\gamma^{\mu}\gamma^{\nu}\gamma^{\delta}\right) = 0, \; \gamma^{\mu} - odd\#, \; Tr\left(\gamma^{\alpha}\gamma^{\beta}\right) = f(D)g^{\alpha\beta} \tag{A.10}$$

$$Tr\;\sigma^{\alpha\beta} = 0, \; Tr\left(\gamma^{\alpha}\gamma^{\beta}\gamma^{\gamma}\gamma^{\delta}\right) = f(D)(g^{\alpha\beta}g^{\gamma\delta} - g^{\alpha\gamma}g^{\beta\delta} + g^{\alpha\delta}g^{\beta\gamma}) \tag{A.11}$$

$$Tr\left(\gamma^{5}\right) = Tr\left(\gamma^{5}\gamma^{\alpha}\right) = Tr\left(\gamma^{5}\gamma^{\beta}\gamma^{\beta}\right) = Tr\left(\gamma^{5}\gamma^{\alpha}\gamma^{\beta}\gamma^{\gamma}\right) = 0 \tag{A.12}$$

$$Tr\left(\gamma^{5}\gamma^{\alpha}\gamma^{\beta}\gamma^{\gamma}\gamma^{\delta}\right) = -4i\varepsilon^{\alpha\beta\gamma\delta}, \; for\; 4^{th} - dimension, \; D = 4, f(D) = 4 \tag{A.13}$$

A.4 Cross-section

The differential cross section in the center of mass (CoM) frame is

$$\left(\frac{d\sigma}{d\Omega'_1}\right)_{CoM} = \frac{1}{64\pi^2(E_1+E_2)^2}\frac{|\vec{p}\,'_1|}{|\vec{p}_1|}\left(\prod_l (2m_l)\right)|M|^2. \tag{A.14}$$

It is convenient to define the Lorentz invariant Mandelstam variables

$$\begin{aligned} s &= (p_1+p_2)^2 = (p'_1+p'_2)^2 \\ t &= (p_1-p'_2)^2 = (p_2-p'_1)^2 \\ u &= (p_1-p'_1)^2 = (p_2-p'_2)^2, \end{aligned} \tag{A.15}$$

and

$$s+u+t = m_1^2+m_2^2+m_{1'}^2+m_{2'}^2 \tag{A.16}$$

where the outgoing particles are denoted by parameters with prime. The variable s is the total energy in the CoM frame. Following the convention in (Mikaelian & Zheleznykh, 1980), t is the square of the momentum transferred between incoming $\overline{\nu}_e$ and outgoing singly charged particle. The scattering angle θ refers to the angle between incoming $\overline{\nu}_e$ and outgoing singly charged particle in the center-of-mass frame. The variable t is depended on the scattering angle $\cos\theta$

$$\begin{aligned} t &= m_1^2+m_{2'}^2-2E_1E'_1+2|\vec{p}_1||\vec{p}\,'_1|\cos\theta \\ dt &= 2|\vec{p}_1||\vec{p}\,'_1|d\cos\theta \end{aligned} \tag{A.17}$$

where

$$
\begin{aligned}
|\vec{p}_1| &= \frac{1}{2\sqrt{s}}\sqrt{s^2 - 2(m_1^2 + m_2^2)s + (m_1^2 - m_2^2)^2} \\
|\vec{p}\,'_1| &= \frac{1}{2\sqrt{s}}\sqrt{s^2 - 2(m_{1'}^2 + m_{2'}^2)s + (m_{1'}^2 - m_{2'}^2)^2}.
\end{aligned} \tag{A.18}
$$

The differential cross section express in Mandelstam variables is

$$
\frac{d\sigma}{dt} = \frac{1}{64\pi s|\vec{p}_1|^2}\left(\prod_l (2m_l)\right)|M|^2, \tag{A.19}
$$

$$
\begin{aligned}
\frac{d\sigma}{dt} &= \frac{d\sigma}{d\cos\theta}\frac{1}{2\,|\vec{p}\,'_1|\,|\vec{p}_1|} \\
\frac{d\sigma}{d\cos\theta} &= 2\,|\vec{p}\,'_1|\,|\vec{p}_1|\,\frac{d\sigma}{dt} \\
&= \frac{1}{32\pi s}\frac{|\vec{p}\,'_1|}{|\vec{p}_1|}\left(\prod_l (2m_l)\right)|M|^2\,.
\end{aligned} \tag{A.20}
$$

The total cross section is therefore

$$
\sigma = \int_{t_{min}}^{t_{max}} \frac{d\sigma}{dt}dt. \tag{A.21}
$$

A.5 Physical Constant

Table A.1: Physical and conversion constants that used in the calculation (taken and adapted from (Nakamura et al., 2010)).

Planck constant	$\hbar$	6.58×10^{-25} GeV·s
Conversion constant	$\hbar c$	1.973×10^{-14} GeV·cm
Fermi coupling constant	G_F	1.166×10^{-5} eV^{-2}
Fine-structure constant	α	1/137
Weak mixing angle	$sin^2\theta_W$	0.23116
Electron mass	m_e	0.511 MeV
$W^{\pm}$ mass	m_W	80.399 GeV
Z^0 mass	m_Z	91.1876 GeV

Appendix B

Lagrangian

B.1 Variational Principle

In the study mechanics and dynamics of particle, the Lagrangian is more preferable due to its invariance under Lorentz transformation rather that the Hamiltonian which transform as time component of the energy momentum four vector. A field is defined as the physical quantity associated to each point of spacetime. The field is called scalar or vector base on the physical quantity associated to it. In field theory, the Lagrangian is written in term of fields and their derivatives as (Mandl & Shaw, 2009)

$$L(t) = \int d^3\vec{x}\mathcal{L}(\psi(x), \partial_\mu\psi(x)) \tag{B.1}$$

where $\mathcal{L}(\psi(x), \partial_\mu\psi(x))$ is referred as Lagrangian density. The x is the space time coordinate and $\partial_\mu\psi(x)$ are the space time derivatives with $\mu = 0, 1, 2, 3$.

$$\begin{aligned} \partial_\mu &= (\partial_0, \partial_1, \partial_2, \partial_3) \\ &= (\frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}). \end{aligned} \tag{B.2}$$

From the Hamilton's variational principle, the path taken by a particle from one point to another point in a given time should be the minimized path. Thus, the action or path is defined as

$$
\begin{aligned}
S = \int dt L(t) = \int dt \int d^3\vec{x}\mathcal{L}(\psi(x), \partial_\mu\psi(x)) &= \int d^4x\mathcal{L}(\psi(x), \partial_\mu\psi(x)) \\
\delta \int d^4x\mathcal{L}(\psi(x), \partial_\mu\psi(x)) &= 0.
\end{aligned}
$$

From the chain rule

$$\int \delta\mathcal{L}(\psi(x), \partial_\mu\psi(x))d^4x = \int \left(\frac{\partial \mathcal{L}}{\partial\psi(x)}\delta\psi(x) + \frac{\partial \mathcal{L}}{\partial(\partial_\mu\psi(x))}\delta\partial_\mu\psi(x) \right) d^4x$$

and partial integration

$$= \int \left(\frac{\partial \mathcal{L}}{\partial\psi(x)}\delta\psi(x) + \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu\psi(x))} \right) \delta\psi(x) + \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu\psi(x))}\ \delta\psi(x) \right) \right) d^4x. \tag{B.4}$$

The third term is the surface term and vanish. This lead to the Euler Lagrange Equation

$$\frac{\partial \mathcal{L}}{\partial\psi(x)} - \partial_\mu \left(\frac{\partial \mathcal{L}}{\partial(\partial_\mu\psi(x))} \right) = 0. \tag{B.5}$$

The momentum conjugate to the field $\psi(x)$ is written as

$$\pi(x) = \frac{\partial \mathcal{L}}{\partial(\partial_\mu\psi(x))} \tag{B.6}$$

which is in analogy to the classical Lagrangian in the generalized coordinates

$$p_i(t) = \frac{\partial L}{\partial \dot{q}_i(t)}. \tag{B.7}$$

From the equation of motion like Klein-Gordon equation, Dirac equation and Proca equation, one can track the form of the Lagrangian. For example the Klein-Gordon equation

$$\left(\partial_\mu \partial^\mu + \frac{m^2c^2}{\hbar^2}\right)\phi = 0. \tag{B.8}$$

The conjugate momentum in comparison to Euler-Lagrange equation is then

$$\partial^\mu \phi = \frac{\partial \mathcal{L}}{\partial(\partial_\mu \phi)}, \;\; \frac{m^2c^2}{\hbar^2} = -\frac{\partial \mathcal{L}}{\partial \phi}. \tag{B.9}$$

By integrating the equation above one obtains the Klein-Gordon Lagrangian in the following form

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)\,(\partial^\mu \phi) - \frac{1}{2}\frac{m^2c^2}{\hbar^2}\phi^2 \tag{B.10}$$

which is in anology to the classical Lagrangian for harmonic oscillator

$$L = \frac{1}{2}m\ddot{x}^2 - \frac{1}{2}kx^2. \tag{B.11}$$

Appendix C

Feynman rules

The interaction of Higgs bosons with the gauge bosons are determined by the Higgs kinetic Lagrangian in Equation (3.17) (T. P. Cheng & Li, 1980; Schechter & Valle, 1980; Fileviez Pérez et al., 2008)

$$\mathcal{L} = (D^{\mu}\phi)^{\dagger}(D_{\mu}\phi) + (D^{\mu}\Delta)^{\dagger}(D_{\mu}\Delta) \tag{C.1}$$

where

$$D_{\mu}\phi = \partial_{\mu}\phi + ig\frac{\sigma_i}{2}.W^i_{\mu}\phi + i\frac{g'}{2}B_{\mu}\phi \tag{C.2}$$

and

$$D_{\mu}\Delta = \partial_{\mu}\Delta + igT_i.W^i_{\mu}\Delta + ig'B_{\mu}\Delta \tag{C.3}$$

where $\frac{\sigma_i}{2}$ are the 2×2 representation of SU(2) doublet generators while T^{μ} are the 3×3 representation of SU(2) triplet generators.

$$T_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad T_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad T_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{C.4}$$

The 3×3 representation of $Y = 2$ complex triplet field can be written as

$$\Delta = \begin{pmatrix} \Delta^0 \\ \Delta^- \\ \Delta^{--} \end{pmatrix} \tag{C.5}$$

$$\begin{aligned}
\mathcal{L} &= \left(\partial^\mu \phi + ig\frac{\sigma_\mu}{2}.W^\mu \phi + i\frac{g'}{2}B^\mu \phi\right)^\dagger \left(\partial_\mu \phi + ig\frac{\sigma^\mu}{2}.W_\mu \phi + i\frac{g'}{2}B_\mu \phi\right) \\
&\quad + \left(\partial^\mu \Delta + igT_\mu.W^\mu \Delta + ig'B^\mu \Delta\right)^\dagger \left(\partial_\mu \Delta + igT^\mu.W_\mu \Delta + ig'B_\mu \Delta\right) \\
&\equiv \frac{1}{4}g^2 \upsilon_0 W^{\mu -} W^+_\mu + \frac{1}{8}\left(g^2 + g'^2\right)\upsilon_0^2 Z^\mu Z_\mu + \frac{1}{2}\left(g^2 + g'^2\right)\upsilon_\Delta^2 Z^\mu Z_\mu \\
&\quad + \frac{1}{2}g^2 \upsilon_\Delta W^{\mu -} W^+_\mu + \frac{ig}{2}\left(\partial_\mu \phi^+\right) h^0 W^-_\mu + \frac{ig}{2}\left(\partial_\mu h^0\right)\phi^- W^+_\mu \\
&\quad + \frac{ig}{\sqrt{2}}\left(\partial_\mu \Delta^+\right)\delta^0 W^-_\mu + \frac{ig}{\sqrt{2}}\left(\partial_\mu \delta^0\right)\Delta^- W^+_\mu + +ig\left(\partial_\mu \Delta^+\right)\Delta^{--} W^+_\mu \\
&\quad + ig\left(\partial_\mu \Delta^{++}\right)\Delta^- W^-_\mu + g^2 \upsilon_\Delta \delta_0 W^{\mu -} W^+_\mu + \frac{g^2}{2}\upsilon_0 h_0 W^{\mu -} W^+_\mu \\
&\quad + \frac{g^2 \upsilon_\Delta}{\sqrt{2}}\left[\Delta^{--} W^{\mu +} W^+_\mu + \Delta^{++} W^{\mu -} W^-_\mu\right] \\
&\quad + \frac{g^2}{\cos\theta_W}\left[\frac{1}{\sqrt{2}}\phi^- \phi^0 \sin^2\theta_W - \Delta^- \Delta^0 (1 + \sin^2\theta_W)\right] W^+_\mu Z_\mu \\
&= \frac{1}{4}g^2(\upsilon_0 + 2\upsilon_\Delta) W^{\mu -} W^+_\mu + \frac{1}{2}\frac{1}{4}\left(g^2 + g'^2\right)\left(\upsilon_0^2 + 4\upsilon_\Delta^2\right) Z_\mu Z^\mu \\
&\quad + \frac{ig}{2}(\sin\theta_+ \sin\theta_0 + \sqrt{2}\cos\theta_+ \cos\theta_0)\left(\partial_\mu H^+\right) H_2 W^-_\mu \\
&\quad + \frac{ig}{2}(\sin\theta_+ \sin\theta_0 + \sqrt{2}\cos\theta_+ \cos\theta_0)\left(\partial_\mu H_2\right) H^- W^+_\mu
\end{aligned}$$

$$+\frac{ig}{2}(\sqrt{2}\cos\theta_+\sin\theta_0 - \sin\theta\cos\theta_0)\left(\partial_\mu H^+\right) H_1 W_\mu^-$$
$$+\frac{ig}{2}(\sqrt{2}\cos\theta_+\sin\theta_0 - \sin\theta\cos\theta_0)\left(\partial_\mu H_1\right) H^- W_\mu^+$$
$$+ig\cos\theta_+\left(\partial_\mu H^{++}\right) H^+ W_\mu^- + ig\cos\theta_+\left(\partial_\mu H^-\right) H^{--} W_\mu^+$$
$$+\frac{g^2\cos\theta_W}{2}\left[\sin\theta_+\upsilon_0 g'^2 - \sqrt{2}\upsilon_\Delta\cos\theta_+(2g'^2+g^2)\right] W_\mu^+ H^- Z_\mu$$
$$+\frac{g^2}{2}\left(2\upsilon_\Delta\sin\theta_0 + \upsilon_0\cos\theta_0\right) H_1 W^{\mu-} W_\mu^+$$
$$-\frac{g^2}{2}\left(\upsilon_0\sin\theta_0 - 2\upsilon_\Delta\cos\theta_0\right) H_2 W^{\mu-} W_\mu^+$$
$$+\frac{g^2}{\sqrt{2}}\left[H^{--}W^{\mu+}W_\mu^+ + H^{++}W^{\mu-}W_\mu^-\right] \qquad \text{(C.6)}$$

where

$$m_W^2 = \frac{1}{4}g^2(\upsilon_0^2 + 2\upsilon_\Delta^2) \qquad \text{(C.7)}$$

and

$$m_Z^2 = \frac{1}{4}\left(g^2 + g'^2\right)\left(\upsilon_0^2 + 4\upsilon_\Delta^2\right) \qquad \text{(C.8)}$$

with

$$\rho = \frac{m_W^2}{m_Z^2\cos^2\theta_W} = \frac{1+2\frac{\upsilon_\Delta^2}{\upsilon_0^2}}{1+4\frac{\upsilon_\Delta^2}{\upsilon_0^2}} = 1 - \frac{2\frac{\upsilon_\Delta^2}{\upsilon_0^2}}{1+4\frac{\upsilon_\Delta^2}{\upsilon_0^2}} = 1 - \frac{2\epsilon^2}{1+4\epsilon^2} \approx 1. \qquad \text{(C.9)}$$

The Yukawa Lagrangian for Higgs leptons interaction

$$\begin{aligned}
\mathcal{L} &= h_{\alpha\beta}\psi_{\alpha L}^T i\sigma_2\Delta\psi_{\beta L} + H.c. \\
&= \frac{h_{\alpha\beta}}{\sqrt{2}}[\upsilon_\Delta\nu_\alpha^T CP_L\nu_\beta + \nu_\alpha^T CP_L\nu_\beta\delta_0 + i\nu_\alpha^T CP_L\nu_\beta\eta_0 \\
&\quad - \left(l_\alpha^T CP_L\nu_\beta + \nu_\alpha^T CP_L l_\beta\right)\Delta^+ - \sqrt{2}l_\alpha^T CP_L l_\beta\Delta^{++}] + H.c. \\
&= \frac{h_{\alpha\beta}}{\sqrt{2}}[\upsilon_\Delta\nu_\alpha^T CP_L\nu_\beta + \nu_\alpha^T CP_L\nu_\beta\left[\sin\theta_0 H_1 + \cos\theta_0 H_2\right] \\
&\quad + i\nu_\alpha^T CP_L\nu_\beta\left[\sin\alpha G_0 + \cos\alpha A_0\right] \\
&\quad - \left(l_\alpha^T CP_L\nu_\beta + \nu_\alpha^T CP_L l_\beta\right)\left[\sin\theta_+ G^+ + \cos\theta_+ H^+\right] - \sqrt{2}l_\alpha^T CP_L l_\beta\Delta^{++}] + H.c.
\end{aligned}$$

$$
\begin{aligned}
&\equiv \frac{h_{\alpha\beta}}{\sqrt{2}}[\upsilon_\Delta \nu_\alpha^T CP_L \nu_\beta + \nu_\alpha^T CP_L \nu_\beta \left[\sin\theta_0 H_1 + \cos\theta_0 H_2\right] + i\nu_\alpha^T CP_L \nu_\beta \cos\alpha A_0 \\
&\quad - \left(l_\alpha^T CP_L \nu_\beta + \nu_\alpha^T CP_L l_\beta\right)\cos\theta_+ H^+ - \sqrt{2} l_\alpha^T CP_L l_\beta \Delta^{++}] + H.c. \\
&= \frac{h_{\alpha\beta}}{2}[\sqrt{2}\upsilon_\Delta \nu_\alpha^T CP_L \nu_\beta + \sqrt{2}\nu_\alpha^T CP_L \nu_\beta \left[\sin\theta_0 H_1 + \cos\theta_0 H_2\right] + i\sqrt{2}\nu_\alpha^T CP_L \nu_\beta \cos\alpha A_0 \\
&\quad -\sqrt{2}\left(l_\alpha^T CP_L \nu_\beta + \nu_\alpha^T CP_L l_\beta\right)\cos\theta_+ H^+ - 2 l_\alpha^T CP_L l_\beta \Delta^{++}] + H.c. \qquad \text{(C.10)}
\end{aligned}
$$

From the neutrino mixing,

$$\nu_\alpha = U_{\alpha i}^* \nu_i \qquad \text{(C.11)}$$

where $i = 1, 2, 3$, the neutrinos masses and Yukawa coupling are

$$
\begin{aligned}
\mathcal{L}_\nu &= \frac{1}{2}[\sqrt{2}\upsilon_\Delta h_{\alpha\beta} \nu_\alpha^T CP_L \nu_\beta] + H.c. \\
&= \frac{1}{2}[\sqrt{2}\upsilon_\Delta (U^\dagger h_{\alpha\beta} U^*)_{ij} \nu_i^T CP_L \nu_j] + H.c. \qquad \text{(C.12)}
\end{aligned}
$$

with

$$h_{\alpha\beta} = \frac{1}{\sqrt{2}\upsilon_\Delta}[U dia(m_1, m_2, m_3) U^T]_{\alpha\beta}. \qquad \text{(C.13)}$$

Table C.1: The vertex factors for Higgs boson interaction with W boson and lepton. The approximation is taken based on $\upsilon_\Delta << \upsilon_0$ and $M_\Delta > M_{H_1}$. $p_1(p_2)$ refers to the first (second) gauge boson listed in the table below.

Vertices	Vertex factors	Approximation
$H^+ H_2 W_\mu^-$	$-i\frac{g}{2}(\sin\theta_+ \sin\theta_0 + \sqrt{2}\cos\theta_+ \cos\theta_0)(p_1 - p_2)_\mu$	$-i\frac{g}{\sqrt{2}}(p_1 - p_2)_\mu$
$H^+ H_1 W_\mu^-$	$-i\frac{g}{2}(\sqrt{2}\cos\theta_+ \sin\theta_0 - \sin\theta_+ \cos\theta_0)(p_1 - p_2)_\mu$	$-i\frac{g}{2}\frac{\mu\upsilon_0}{M_\Delta^2}(p_1 - p_2)_\mu$
$H^{++} H^- W_\mu^-$	$-ig\cos\theta_+(p_1 - p_2)_\mu$	$-ig(p_1 - p_2)_\mu$
$H_1 W_\mu^- W_\nu^+$	$i\frac{g^2}{2}\left(2\upsilon_\Delta \sin\theta_0 + \upsilon_0 \cos\theta_0\right) g_{\mu\nu}$	$i\frac{g^2}{2}\upsilon_0 g_{\mu\nu}$
$H_2 W_\mu^- W_\nu^+$	$-i\frac{g^2}{2}\left(\upsilon_0 \sin\theta_0 - 2\upsilon_\Delta \cos\theta_0\right) g_{\mu\nu}$	$-i\frac{g^2}{2}\left(\frac{\mu\upsilon_0^2}{M_\Delta^2} - 2\upsilon_\Delta\right) g_{\mu\nu}$
$H^{++} W_\mu^- W_\nu^-$	$i\sqrt{2}g^2 \upsilon_\Delta g_{\mu\nu}$	$i\sqrt{2}g^2 \upsilon_\Delta g_{\mu\nu}$
$H^- W_\mu^+ Z_\nu$	$\frac{ig^2 \cos\theta_W}{2}\left[\sin\theta_+ \upsilon_0 g'^2 - \sqrt{2}\upsilon_\Delta \cos\theta_+ (2g'^2 + g^2)\right] g_{\mu\nu}$	iC_{WZ}
$\nu_\alpha^T \nu_\beta H_2$	$i\sqrt{2}h_{\alpha\beta} CP_L \cos\theta_0$	$i\sqrt{2}h_{\alpha\beta} CP_L$
$\nu_\alpha^T \nu_\beta A_0$	$-\sqrt{2}h_{\alpha\beta} CP_L \cos\alpha$	$-\sqrt{2}h_{\alpha\beta} CP_L$
$l_\alpha^T \nu_\beta H^+$	$-ih_{\alpha\beta}\sqrt{2}CP_L \cos\theta_+$	$-ih_{\alpha\beta}\sqrt{2}CP_L$

$$C_{WZ} = \frac{g^2 \sin^2\theta_W}{2\cos\theta_W}\left(\frac{\mu v_0^2}{M_\Delta^2} - \sqrt{2}(2+\cot^2\theta_W)\upsilon_\Delta\right) g_{\mu\nu} \tag{C.14}$$

C.1 Vertex factors

The first order term in the S-matrix expansion is used to obtain the vertex factor for the interaction of Higgs bosons with other particles.

$$S^1 = i\int d^4x \mathcal{L}_{int} \tag{C.15}$$

The charged leptons, neutrinos, charged Higgs and neutral Higgs fields can be written in the term of creation and annihilation operators.

$$l_\alpha(x) = \sum_{rp}\left(\frac{m_l}{VE_p}\right)^{\frac{1}{2}}\left[c_r(\vec{p})u_r(\vec{p})e^{-ipx} + d_r^\dagger(\vec{p})\nu_r(\vec{p})e^{ipx}\right] \tag{C.16}$$

$$\begin{aligned} H^{++}(x)\ or\ H^{+}(x) &= H^\dagger(x) = \sum_k\left(\frac{1}{2V\omega_{\vec{k}}}\right)^{\frac{1}{2}}\left[b_r^\dagger(\vec{k})e^{ikx} + f_r(\vec{k})e^{-ikx}\right] \\ H^{--}(x)\ or\ H^{-}(x) &= H(x) = \sum_k\left(\frac{1}{2V\omega_{\vec{k}}}\right)^{\frac{1}{2}}\left[b_r(\vec{k})e^{-ikx} + f_r^\dagger(\vec{k})e^{ikx}\right] \end{aligned} \tag{C.17}$$

$$H_1(x)\ or\ H_2(x) = \sum_k\left(\frac{1}{2V\omega_{\vec{k}}}\right)^{\frac{1}{2}}\left[a_r(\vec{k})e^{-ikx} + a_r^\dagger(\vec{k})e^{ikx}\right] \tag{C.18}$$

C.1.1 $H^+(\vec{p}_1)H_2(\vec{p}_2)W_\mu^-(\vec{k})$ vertex factor

For $H^+(\vec{p}_1)H_2(\vec{p}_2)W_\mu^-(\vec{k})$ interaction, the lagrangian is

$$\mathcal{L} = \frac{ig}{2}(\sin\theta_+\sin\theta_0 + \sqrt{2}\cos\theta_+\cos\theta_0)\left[\left(\partial_\mu H^\dagger\right)H_2W_\mu^- + \left(\partial_\mu H_2\right)HW_\mu^+\right]. \tag{C.19}$$

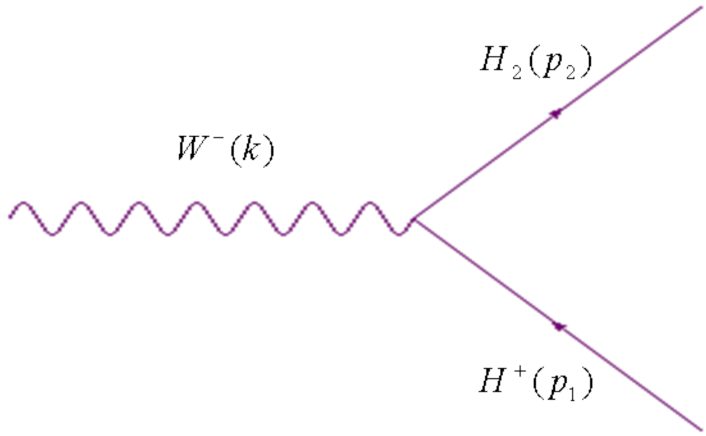


Figure C.1: Feynman diagram for $H^+H_2W^-_\mu$.

The first order S-matrix expansion is

$$\begin{aligned}
S^1 &= i\int d^4x\frac{ig}{2}(\sin\theta_+\sin\theta_0+\sqrt{2}\cos\theta_+\cos\theta_0)\left[\left(\partial_\mu H^+\right)H_2W^-_\mu+\left(\partial_\mu H_2\right)H^-W^+_\mu\right]\\
&= -\int d^4x\frac{g}{2}(\sin\theta_+\sin\theta_0+\sqrt{2}\cos\theta_+\cos\theta_0)\sum_{p_1}\left(\frac{1}{2V\omega_{\vec{p}_1}}\right)^{\frac{1}{2}}\sum_{p_2}\left(\frac{1}{2V\omega_{\vec{p}_2}}\right)^{\frac{1}{2}}\\
&\quad\left[(\partial_\mu[b^\dagger_r(\vec{p}_1)e^{ip_1x}+f_r(\vec{p}_1)e^{-ip_1x}])[a_r(\vec{p}_2)e^{-ip_2x}+a^\dagger_r(\vec{p}_2)e^{ip_2x}]W^-_\mu\right.\\
&\quad\quad\left.+(\partial_\mu[a_r(\vec{p}_2)e^{-ip_2x}+a^\dagger_r(\vec{p}_2)e^{ip_2x}])[b_r(\vec{p}_1)e^{-ip_1x}+f^\dagger_r(\vec{p}_1)e^{ip_1x}]W^+_\mu\right]\\
&= -\int d^4x\frac{g}{2}(\sin\theta_+\sin\theta_0+\sqrt{2}\cos\theta_+\cos\theta_0)\sum_{p_1}\left(\frac{1}{2V\omega_{\vec{p}_1}}\right)^{\frac{1}{2}}\sum_{p_2}\left(\frac{1}{2V\omega_{\vec{p}_2}}\right)^{\frac{1}{2}}\\
&\quad\left[\partial_\mu b^\dagger_r(\vec{p}_1)e^{ip_1x}a_r(\vec{p}_2)e^{-ip_2x}W^-_\mu+\partial_\mu a_r(\vec{p}_2)e^{-ip_2x}b^\dagger_r(\vec{p}_1)e^{ip_1x}W^+_\mu\right]\\
&= -i(p_1-p_2)_\mu\frac{g}{2}(\sin\theta_+\sin\theta_0+\sqrt{2}\cos\theta_+\cos\theta_0)\\
&\quad\times\int e^{i(p_1-p_2-k)x}d^4x\left[b^\dagger_r(\vec{p}_1)a_r(\vec{p}_2)W^-_\mu+a_r(\vec{p}_2)b^\dagger_r(\vec{p}_1)W^+_\mu\right]+\ldots.
\end{aligned}\tag{C.20}$$

The transition amplitude is

$$\begin{aligned}
\langle\vec{p}_2|S^1|\vec{p}_1,k\rangle &= -i(p_1-p_2)_\mu\frac{g}{2}(\sin\theta_+\sin\theta_0+\sqrt{2}\cos\theta_+\cos\theta_0)(2\pi)^4\delta^4(p_1-p_2-k)\\
&\quad\times\sum_{p_1}\left(\frac{1}{2V\omega_{\vec{p}_1}}\right)^{\frac{1}{2}}\sum_{p_2}\left(\frac{1}{2V\omega_{\vec{p}_2}}\right)^{\frac{1}{2}}\sum_{k}\left(\frac{1}{2V\omega_{\vec{k}}}\right)^{\frac{1}{2}}
\end{aligned}$$

$$
\begin{aligned}
&\times \left\langle 0|b_r(\vec{p}_1)b_r^\dagger(\vec{p}_1)a_r(\vec{p}_2)a_r^\dagger(\vec{p}_2)a_r(\vec{k})a_r^\dagger(\vec{k}) \right. \\
&\qquad \left. +a_r(\vec{p}_2)a_r^\dagger(\vec{p}_2)b_r(\vec{p}_1)b_r^\dagger(\vec{p}_1)b_r(\vec{k})b_r^\dagger(\vec{k})|0 \right\rangle \varepsilon_r^\alpha(\vec{k}) \\
= \;& -i(p_1-p_2)_\mu \frac{g}{2}(\sin\theta_+\sin\theta_0+\sqrt{2}\cos\theta_+\cos\theta_0)(2\pi)^4\delta^4(p_1-p_2-k) \\
&\times \sum_{p_1}\left(\frac{1}{2V\omega_{\vec{p}_1}}\right)^{\frac{1}{2}}\sum_{p_2}\left(\frac{1}{2V\omega_{\vec{p}_2}}\right)^{\frac{1}{2}}\sum_{k}\left(\frac{1}{2V\omega_{\vec{k}}}\right)^{\frac{1}{2}} \\
M = \;& -i(p_1-p_2)_\mu \frac{g}{2}(\sin\theta_+\sin\theta_0+\sqrt{2}\cos\theta_+\cos\theta_0)\varepsilon_r^\alpha(\vec{k}). \qquad \text{(C.21)}
\end{aligned}
$$

The vertex factor is therefore

$$
-i(p_1-p_2)_\mu \frac{g}{2}(\sin\theta_+\sin\theta_0+\sqrt{2}\cos\theta_+\cos\theta_0). \qquad \text{(C.22)}
$$

C.1.2 $H^{++}W^{\mu-}W^{\upsilon-}$ vertex factor

For the $H^{++}W^{\mu-}W^{\upsilon-}$ interaction, the Lagrangian is

$$
\mathcal{L} = \frac{g^2}{\sqrt{2}}\left[H^{--}W^{\mu+}W_\mu^+ + H^{++}W^{\mu-}W_\mu^-\right]. \qquad \text{(C.23)}
$$

The first order S-matrix expansion is

$$
\begin{aligned}
S^1 &= i\int d^4x \frac{g^2\upsilon_\Delta}{\sqrt{2}}\left[H^{--}W^{\mu+}W_\mu^+ + H^{++}W^{\mu-}W_\mu^-\right] \\
&= i\frac{g^2\upsilon_\Delta g_{\mu\nu}}{\sqrt{2}}\int d^4x\left[H^{--}W^{\mu+}W^{\upsilon+} + H^{++}W^{\mu-}W^{\upsilon-}\right]. \qquad \text{(C.24)}
\end{aligned}
$$

The vertex factor is

$$
i\frac{g^2\upsilon_\Delta g_{\mu\nu}}{\sqrt{2}} \qquad \text{(C.25)}
$$

and multiplying by the factor of 2! for two indentical W bosons we obtain

$$i\sqrt{2}g^2\upsilon_\Delta g_{\mu\nu}. \tag{C.26}$$

C.1.3 $l_\alpha\nu_\beta H^\pm$ vertex factor

On the other hand, the $l_\alpha\nu_\beta H^\pm$ interaction Lagrangian is

$$\mathcal{L} = -h_{\alpha\beta}\sqrt{2}\left(l_\alpha^T CP_L\nu_\beta + \nu_\alpha^T CP_L l_\beta\right)\cos\theta_+ H^+ \tag{C.27}$$

where $\alpha, \beta = e, \mu, \tau$. The first order S-matrix expansion is

$$\begin{aligned}
S^1 &= -i\int d^4x h_{\alpha\beta}\sqrt{2}\left(l_\alpha^T CP_L\nu_\beta + \nu_\alpha^T CP_L l_\beta\right)\cos\theta_+ H^+ \\
&= -ih_{\alpha\beta}\sqrt{2}\cos\theta_+\int d^4x\left(l_\alpha^T CP_L\nu_\beta + \nu_\alpha^T CP_L l_\beta\right)H^+ \\
&= -ih_{\alpha\beta}\sqrt{2}\cos\theta_+\int d^4x\sum_{rp}\left(\frac{m_l}{VE_p}\right)^{\frac{1}{2}}\left[c_r(\vec{p})u_r(\vec{p})e^{-ipx} + d_r^\dagger(\vec{p})\nu_r(\vec{p})e^{ipx}\right]^T CP_L \\
&\quad\times\sum_{rq}\left(\frac{m_l}{VE_q}\right)^{\frac{1}{2}}\left[a_r(\vec{q})u_r(\vec{q})e^{-iqx} + a_r^\dagger(\vec{q})\nu_r(\vec{q})e^{iqx}\right]H^+ + ... \\
&= -ih_{\alpha\beta}\sqrt{2}\cos\theta_+\int e^{-i(p+q-k)x}d^4x\sum_{rp}\left(\frac{m_l}{VE_p}\right)^{\frac{1}{2}}\sum_{rq}\left(\frac{m_l}{VE_q}\right)^{\frac{1}{2}}\sum_k\left(\frac{1}{2V\omega_{\vec{k}}}\right)^{\frac{1}{2}} \\
&\quad\times c_r(\vec{p})u_r^T(\vec{p})CP_L a_r(\vec{q})u_r(\vec{q})b_r^\dagger(\vec{k}) + ... \\
&= -ih_{\alpha\beta}\sqrt{2}\cos\theta_+(2\pi)^4\delta^4(p_1 - p_2 - k)c_r(\vec{p})u_r^T(\vec{p})CP_L a_r(\vec{q})u_r(\vec{q})b_r^\dagger(\vec{k}) \\
&\quad\times\sum_{rp}\left(\frac{m_l}{VE_p}\right)^{\frac{1}{2}}\sum_{rq}\left(\frac{m_l}{VE_q}\right)^{\frac{1}{2}}\sum_k\left(\frac{1}{2V\omega_{\vec{k}}}\right)^{\frac{1}{2}}.
\end{aligned} \tag{C.28}$$

The transition amplitude is

$$\left\langle\vec{k}|S^1|\vec{p},\vec{q}\right\rangle = -ih_{\alpha\beta}\sqrt{2}\cos\theta_+(2\pi)^4\delta^4(p_1 - p_2 - k)c_r(\vec{p})u_r^T(\vec{p})CP_L a_r(\vec{q})u_r(\vec{q})b_r^\dagger(\vec{k})$$

$$\times \sum_{rp} \left(\frac{m_l}{VE_p}\right)^{\frac{1}{2}} \sum_{rq} \left(\frac{m_l}{VE_q}\right)^{\frac{1}{2}} \sum_{k} \left(\frac{1}{2V\omega_{\vec{k}}}\right)^{\frac{1}{2}} u_r^T(\vec{p}) C P_L u_r(\vec{q})$$
$$\times \left\langle 0 | b_r(\vec{k}) c_r(\vec{p}) a_r(\vec{q}) c_r^\dagger(\vec{p}) b_r^\dagger(\vec{k}) a_r^\dagger(\vec{q}) | 0 \right\rangle \tag{C.29}$$
$$M = -ih_{\alpha\beta}\sqrt{2}\cos\theta_+ u_r^T(\vec{p}) C P_L u_r(\vec{q}). \tag{C.30}$$

Thus the vertex factor is

$$-ih_{\alpha\beta}\sqrt{2}\cos\theta_+ C P_L. \tag{C.31}$$

In this model, the neutrino is Majorana neutrino. The Majorana neutrino field

$$\nu(x) = \sum_{rq} \left(\frac{m_l}{VE_q}\right)^{\frac{1}{2}} \left[a_r(\vec{q}) u_r(\vec{q}) e^{-iqx} + a_r^\dagger(\vec{q}) \nu_r(\vec{q}) e^{iqx}\right] \tag{C.32}$$

is self conjugate which satisfy

$$\nu^C(x) = C\nu(x)C^{-1} = \nu(x) \tag{C.33}$$

with (Denner, Eck, Hahn, & Kblbeck, 1992)

$$u^T(\vec{p}) = \overline{\nu}(\vec{p}) C^T, \quad v^T(\vec{p}) = \overline{u}(\vec{p}) C^T \tag{C.34}$$

where C is the charge conjugation matrix

$$C^\dagger = C^{-1}, \quad C^T = -C. \tag{C.35}$$

The charge conjugation matrix imply that the interaction vertices with lepton number violation are allowed. In particular, the neutrino can be treated as either particle or antiparticle.

C.2 Standard Model charged current and neutral current Lagrangian

From Equation (2.3), Equation (2.4), Equation (2.13) and Equation (2.14), we have

$$
\begin{aligned}
\mathcal{L} &\equiv \begin{pmatrix} \overline{\nu}_l & \bar{l} \end{pmatrix}_L \gamma^\mu \left[-\frac{g}{2}\sigma_i \cdot W^i - \frac{g'}{2} Y B_\mu\right] \begin{pmatrix} \nu_l \\ l \end{pmatrix}_L - \bar{l}_R \gamma^\mu \frac{g'}{2} Y B_\mu l_R \\
&= \frac{-g}{2} \begin{pmatrix} \overline{\nu}_l & \bar{l} \end{pmatrix}_L \gamma^\mu \left[\begin{pmatrix} 0 & W^1 - iW^2 \\ W^1 + iW^2 & 0 \end{pmatrix} + \begin{pmatrix} W^3 & 0 \\ 0 & W^3 \end{pmatrix} \right] \begin{pmatrix} \nu_l \\ l \end{pmatrix}_L \\
&\quad - \begin{pmatrix} \overline{\nu}_l & \bar{l} \end{pmatrix}_L \gamma^\mu \frac{g'}{2} Y B_\mu \begin{pmatrix} \nu_l \\ l \end{pmatrix}_L - \bar{l}_R \gamma^\mu \frac{g'}{2} Y B_\mu l_R \\
&= \frac{-g}{\sqrt{2}} \left[\bar{l}_L \gamma^\mu \nu_{lL} W^- + \overline{\nu}_{lL} \gamma^\mu l_L W^+\right] - \frac{g}{2}\left[\overline{\nu}_{lL}\gamma^\mu \nu_{lL} + \bar{l}_L \gamma^\mu l_L\right] W^3 \\
&\quad - \frac{g'}{2}\left[\overline{\nu}_{lL}\gamma^\mu \nu_{lL} + \bar{l}_L \gamma^\mu l_L\right] Y B_\mu - \bar{l}_R \gamma^\mu \frac{g'}{2} Y B_\mu l_R \\
&= \frac{-g}{\sqrt{2}} \left[\bar{l}_L \gamma^\mu \nu_{lL} W^-_\mu + \overline{\nu}_{lL} \gamma^\mu l_L W^+_\mu\right] - \frac{g}{2\cos\theta_W} \overline{\nu}_{lL}\gamma^\mu \nu_{lL} Z_\mu \\
&\quad + \frac{g}{4\cos\theta_W} \bar{l}\gamma^\mu (1 - 4\sin^2\theta_W - \gamma_5) l Z_\mu
\end{aligned} \tag{C.36}
$$

which are the charged current and neutral current interaction Lagrangian of electrons and neutrinos with gauge bosons.

We list some Feynman rules that are used in the calculation (Mandl & Shaw, 2009; Peskin & Schroeder, 1995; Lahiri & Pal, 2005). The following factors are written for each external line:

1. For each incoming lepton, $l^-,\ \nu_l\ :\ u_r(\mathbf{p})$,
2. For each outgoing lepton $l^-,\ \nu_l\ :\ v_r(\mathbf{p})$
3. For each incoming antilepton, $l^+,\ \overline{\nu}_l\ :\ \overline{u}_r(\mathbf{p})$,

4. For each outgoing antilepton $l^+,\ \overline{v}_l:\ \overline{v}_r\left(\mathbf{p}\right)$

5. For each incoming vector boson, $\varepsilon_{r\alpha}(\mathbf{k})$,

6. For each outgoing vector boson, $\varepsilon^*_{r\alpha}(\mathbf{k})$,

7. For each internal fermion line,

$$iS_F\left(p\right) = i\frac{1}{\gamma^\mu p_\mu - m + i\varepsilon} \tag{C.37}$$

8. For each internal photon line,

$$iD_{F\mu\nu}\left(k\right) = \frac{-ig^{\mu\nu}}{k^2 + i\varepsilon} \tag{C.38}$$

9. For each internal W and Z line,

$$iD_{F\mu\nu}\left(k, m\right) = \frac{-i}{k^2 - m^2 + i\varepsilon}\left[g^{\mu\nu} - \frac{\left(1-\xi\right)k_\mu k_\nu}{k^2 - \xi m^2}\right] \tag{C.39}$$

10. For each internal scalar boson line,

$$i\Delta_{\mathrm{F}}\left(k, m_H\right) = \frac{i}{k^2 - \xi {m^2}_H + i\varepsilon} \tag{C.40}$$

where ξ =1 is Feynman-'t Hooft gauge, ξ = 0 is Landau gauge. In our calculation, we use the Feynman-'t Hooft gauge.

Appendix D

Group theory

The group theory is a study of transformation. A system on some specific coordinate system is transformed to another new coordinate system by rotation, translation or reflection. The first postulate of special relativity states that the law of physics should be invariant in any frame of reference. This imply that the transformation under rotation, translation and reflection should leave the equation of motion to be invariant. For instant, the Lagrangian of a system should be invariant under such transformation. Each of the transformation would imply some conservation law. For example, the system under rotation lead to the conservation of angular momentum. The symmetry and conservation law under continuous transformation is summarized as *Neother's Theorem* (T. Cheng & Li, 1984; Mandl & Shaw, 2009; Peskin & Schroeder, 1995; Lahiri & Pal, 2005). Thus, the group theory is an essential tools in describing the symmetry of physics.

D.1 Elements of group theory

In order for a set $\{\mathcal{G} : a, b, c...\}$ to be classify as a group, these elements should satisfy the following properties (Tung, 1985; Elliot & Dawber, 1979).

1. The the product of any elements in the set, e.g. ab, should itself an element in the set for all $a, b \in \mathcal{G}$.

2. The elements in the set satisfy the closure relation, e.g. $a(bc) = (ab)c$ for all $a, b, c \in \mathcal{G}$.

3. There is an identity, e in the set such that $ea = ae = e$ for all $a \in \mathcal{G}$.

4. There is an inverse for every elements in the set such that $a^{-1}a = e$ where $a \in \mathcal{G}$ and $a^{-1} \in \mathcal{G}$.

If the elements of the group satisfy the relation $ab = ba$ for all $a, b \in G$, such group is called *abelian* group. Otherwise, it is call *non-abelian*. The number of the elements of the group is the *order* of the group. The subset $\{\mathcal{H} : a, b...\}$ which satisfy the same multiplication rule with $\mathcal{G}$ where $a, b \in \mathcal{G}$ is said to be the *subgroup* of $\mathcal{G}$. If the group elements carry continuous parameters they are called *continuous group*. A set of permutation elements form a group called *permutation group* or *symmetry group*.

If the elements of two subgroups H_1 and H_2 commute, ($h_1h_2 = h_2h_1$, for all $h_1 \in H_1$ and $h_2 \in H_2$) and every element g of $\mathcal{G}$ can be written as $g = h_1h_2$ where $h_1 \in H_1$ and $h_2 \in H_2$, the group $\mathcal{G}$ is said to be the *direct product* of H_1 and H_2. The example of direct product group in particle physics is $SU(2) \times U(1)$. If there exist one-to-one correspondence between the elements of two groups which preserve the group multiplication rules, these group are said to be *isomorphic*. For instant, if $g_i \in \mathcal{G} \leftrightarrow g'_i \in \mathcal{G}'$ and $g_1g_2 = g_3$ in $\mathcal{G}$, then $g'_1g'_2 = g'_3$ in $\mathcal{G}'$ and vice versa. However, a mapping from a group to another group which preserves group multiplication and not necessarily one-to-one, is called *homomorphism*. For example, if $g_i \in \mathcal{G} \to g'_i \in \mathcal{G}'$ and $g_1g_2 = g_3$ in $\mathcal{G}$, then $g'_1g'_2 = g'_3$ in $\mathcal{G}'$.

If there exist a set of matrix that is homomorphic to a group, i.e. $g \in \mathcal{G} \longrightarrow U(g)$, the set of matrix forms a *representation* of that group. Let this matrix be a representation of $\mathcal{G}$ in the vector space V, and V_1 be the subspace of V, which satisfy the property $U(g)|x\rangle \in V_1$ for all $\mathbf{x} \in V_1$ and $g \in V_1$, V_1 is said to be the *invariant subspace*. If the subspace in not invariant under transformation of a group representation, that representation is called *irreducible representation*. *Lie group* is an important example of application of group representation in physics. A *unitary group*, $U(n)$, is a set of $n \times n$ matrices that satisfy $U^{\dagger}U = 1$. If the matrices of the unitary group consists of unity determinant, $detU = 1$, it is called *special unitary group*, $SU(n)$. An *orthogonal group* is a set of $n \times n$ matrices satisfy $O^T O = 1$. Thus, $SO(n)$ is the *special orthogonal group* with unity determinant.

Consider a simple coordinate system with x and y axis. Suppose one transform the coordinate system under rotation which fix at the origin to a new coordinate x' and y'. The representation of the rotation can be written in matrix form as (Tung, 1985; Elliot & Dawber, 1979)

$$\begin{aligned} \vec{x}' &= U\vec{x} \\ U &= \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}. \end{aligned} \tag{D.1}$$

Since the transformation in quantum mechanics is unitary so that the norm is invariant under inner product space, we write U in the exponential form.

$$\begin{aligned} U &= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \cos\theta + i \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \sin\theta \\ &= exp[i\sigma_2\theta] \end{aligned} \tag{D.2}$$

where $\sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ is the Pauli matrix. Such a matrix that transform the group representation to unitary representation is called the *generator* of the group. The rotation in 2-dimensional space is called $SO(2)$ group. This rotation can be generalized to 3-dimensional space as $SO(3)$ group. The generator of $SO(3)$ group is the angular momentum operators, J_1, J_2 and J_3. These generators satisfy the commutation relation (*Lie algebra*)

$$[J_j, J_k] = i\varepsilon_{jkl} J_l \tag{D.3}$$

where ε_{jkl} is the antisymmetric Levi-Civita symbol. ε_{jkl} is equal to 1 for cyclic permutation of jkl, while -1 for anti cyclic permutation of jkl and 0 for $j = k, k = l, l = j$. The $SU(2)$ group, however, is isomorphic to the 3-dimensional rotation group.

References

Adams, T., et al. (2009). Terascale Physics Opportunities at a High Statistics, High Energy Neutrino Scattering Experiment: NuSOnG. *Int. J. Mod. Phys.*, *A24*, 671-717.

Ahmad, Q. R., Allen, R. C., Andersen, T. C., Anglin, J. D., Bühler, G., Barton, J. C., et al. (2001). Measurement of the rate of $\nu_e + d \to p + p + e^-$ interactions produced by ^{8}B solar neutrinos at the Sudbury neutrino observatory. *Phys. Rev. Lett.*, *87*(7), 071301.

Ahmad, Q. R., Allen, R. C., Andersen, T. C., D.Anglin, J., Barton, J. C., Beier, E. W., et al. (2002). Direct evidence for neutrino flavor transformation from neutral-current interactions in the Sudbury neutrino observatory. *Phys. Rev. Lett.*, *89*(1), 011301.

Ahn, M. H., Aliu, E., Andringa, S., Aoki, S., Aoyama, Y., Argyriades, J., et al. (2006). Measurement of neutrino oscillation by the K2K experiment. *Phys. Rev. D*, *74*(7), 072003.

Ahn, M. H., Aoki, S., Bhang, H., Boyd, S., Casper, D., Choi, J. H., et al. (2003). Indications of neutrino oscillation in a 250 km long-baseline experiment. *Phys. Rev. Lett.*, *90*(4), 041801.

Akeroyd, A. G., & Aoki, M. (2005). Single and pair production of doubly charged Higgs bosons at hadron colliders. *Phys. Rev. D*, *72*(3), 035011.

Akeroyd, A. G., & Chiang, C.-W. (2010). Phenomenology of large mixing for the CP-even neutral scalars of the Higgs triplet model. *Phys. Rev. D*, *81*(11), 115007.

Bahcall, J. N., Bahcall, N. A., & Shaviv, G. (1968). Present status of the theoretical predictions for the ^{37}Cl solar-neutrino experiment. *Phys. Rev. Lett.*, *20*(21), 1209–1212.

Bahcall, J. N., & Davis, R. (1976). Solar neutrinos: A scientific puzzle. *Science*, *191*(4224), 264-267.

Bilenky, S. M., & Pontecorvo, B. (1978). Lepton mixing and neutrino oscillations. *Phys. Rept.*, *41*(4), 225 - 261.

Binosi, D., & Theuβl, L. (2004). Jaxodraw: A graphical user interface for drawing Feynman diagrams. *Computer Physics Communications*, *161*(1-2), 76 - 86.

Butkevich, A. V., Kaidalov, A. B., Krastev, P. I., Leonov-Vendrovski, A. V., & Zheleznykh, I. M. (1988). Ultra-high energy neutrino-nucleon and neutrino-electron cross-sections in the standard model, in supersymmetric and superstring models. *Zeitschrift fr Physik C Particles and Fields*, *39*, 241-250. (10.1007/BF01551000)

Cabibbo, N. (1963). Unitary symmetry and leptonic decays. *Phys. Rev. Lett.*, *10*(12), 531–533.

Cheng, T., & Li, L. (1984). *Gauge theory of elementary particles*. Clarendon Press.

Cheng, T. P., & Li, L.-F. (1980). Neutrino masses, mixings, and oscillations in SU(2) $\times$ U(1) models of electroweak interactions. *Phys. Rev. D*, *22*(11), 2860–2868.

Chiarusi, T., & Spurio, M. (2010). High-energy astrophysics with neutrino telescopes. *Eur. Phys. J. C*, *65*, 649-701.

Coarasa, J. A., Mendez, A., & Sola, J. (1996). Higgs triplet effects in purely leptonic processes. *Physics Letters B*, *374*(1-3), 131 - 137.

Davis, R., Harmer, D. S., & Hoffman, K. C. (1968). Search for neutrinos from the Sun. *Phys. Rev. Lett.*, *20*(21), 1205–1209.

Degrassi, G., Sirlin, A., & Marciano, W. J. (1989). Effective electromagnetic form factor of the neutrino. *Phys. Rev. D*, *39*(1), 287–294.

Deniz, M., et al. (2010). Measurement of $\overline{\nu}_e$-electron scattering cross section with a

CsI(Tl) scintillating crystal array at the Kuo-Sheng nuclear power reactor. *Phys. Rev. D*, *81*(7), 072001.

Denner, A., Eck, H., Hahn, O., & Kblbeck, J. (1992). Compact Feynman rules for Majorana fermions. *Physics Letters B*, *291*(3), 278 - 280.

Egorov, A. M., Likhachev, G. G., & Studenikin, A. I. (1995). Neutrino spin and flavour fonversion and oscillation in magnetic field. Available from arXiv:astro-ph/9506013

Egorov, A. M., Lobanov, A. E., & Studenikin, A. I. (2000). Neutrino oscillations in electromagnetic fields. *Physics Letters B*, *491*(1-2), 137 - 142.

Eguchi, K., Enomoto, S., Furuno, K., Goldman, J., Hanada, H., Ikeda, H., et al. (2003). First results from KamLAND: Evidence for reactor antineutrino disappearance. *Phys. Rev. Lett.*, *90*(2), 021802.

Elliot, J., & Dawber, P. (1979). *Symmetry in physics*. MacMillan.

Englert, F., & Brout, R. (1964). Broken symmetry and the mass of gauge vector mesons. *Phys. Rev. Lett.*, *13*(9), 321–323.

Fileviez Pérez, P., Han, T., Huang, G., Li, T., & Wang, K. (2008). Neutrino masses and the CERN LHC: Testing the type II seesaw mechanism. *Phys. Rev. D*, *78*(1), 015018.

Foster, J., & Nightingale, J. (2006). *A short course in general relativity*. Springer.

Gandhi, R., Quigg, C., Reno, M. H., & Sarcevic, I. (1996). Ultrahigh-energy neutrino interactions. *Astroparticle Physics*, *5*(2), 81 - 110.

Gell-Mann, M., & Pais, A. (1955). Behavior of neutral particles under charge conjugation. *Phys. Rev.*, *97*(5), 1387–1389.

Glashow, S. L. (1961). Partial-symmetries of weak interactions. *Nuclear Physics*, *22*(4), 579 - 588.

Godfrey, S., Kalyniak, P., & Romanenko, N. (2002). Signatures of doubly charged Higgs bosons in $e\gamma$ collisions. *Phys. Rev. D*, *65*(3), 033009.

Gonzalez-Garcia, M. C., & Nir, Y. (2003). Neutrino masses and mixing: evidence and implications. *Rev. Mod. Phys.*, *75*(2), 345–402.

Griffiths, D. (2008). *Introduction to elementary particles*. Wiley-VCH.

Guralnik, G. S., Hagen, C. R., & Kibble, T. W. B. (1964). Global conservation laws and massless particles. *Phys. Rev. Lett.*, *13*(20), 585–587.

Halprin, A. (1986). Neutrino oscillations in nonuniform matter. *Phys. Rev. D*, *34*(11), 3462–3466.

Halzen, F. (2006). Astroparticle physics with high energy neutrinos: from AMANDA to IceCube. *Eur. Phys. J. C*, *46*, 669-687.

Hatakeyama, S., Hara, T., Fukuda, Y., Hayakawa, T., Inoue, K., Ishihara, K., et al. (1998). Measurement of the flux and zenith-angle distribution of upward through-going muons in Kamiokande II+III. *Phys. Rev. Lett.*, *81*(10), 2016–2019.

Hawking, S. (1998). *A brief history of time*. Bantam Books.

Higgs, P. W. (1964). Broken symmetries and the masses of gauge bosons. *Phys. Rev. Lett.*, *13*(16), 508–509.

Kaku, M. (1993). *Quantum field theory: a modern introduction*. Oxford University Press.

Kaku, M. (1999). *Introduction to superstrings and M-theory*. Springer.

Kayser, B. (1982). Majorana neutrinos and their electromagnetic properties. *Phys. Rev. D*, *26*(7), 1662–1670.

Kim, C. W., Sze, W. K., & Nussinov, S. (1987). Neutrino oscillations and the Landau-

Zener formula. *Phys. Rev. D*, *35*(12), 4014–4017.

Kobayashi, M., & Maskawa, T. (1973). CP-violation in the renormalizable theory of weak interaction. *Progress of Theoretical Physics*, *49*(2), 652-657.

Krane, K., & Halliday, D. (1987). *Introductory nuclear physics*. Wiley.

Lahiri, A., & Pal, P. (2005). *A first book of quantum field theory*. Alpha Science International.

Lee, B. W., & Shrock, R. E. (1977). Natural suppression of symmetry violation in gauge theories: Muon- and electron-lepton-number nonconservation. *Phys. Rev. D*, *16*(5), 1444–1473.

Mandl, F., & Shaw, G. (2009). *Quantum Field Theory*. John Wiley & Sons.

Mikaelian, K. O., & Zheleznykh, I. M. (1980). $\overline{\nu}_e e$ annihilations. *Phys. Rev. D*, *22*(9), 2122–2131.

Mikheyev, S., & Smirnov, A. (1986). Resonant amplification of ν oscillations in matter and solar-neutrino spectroscopy. *Il Nuovo Cimento C*, *9*, 17-26. (10.1007/BF02508049)

Mohapatra, R., & Pal, P. (2004). *Massive neutrinos in physics and astrophysics*. World Scientific.

Nakamura, K., et al. (2010). Review of particle physics. *J. Phys. G*, *37*(7A), 075021.

Nambu, Y. (2009). Nobel lecture: Spontaneous symmetry breaking in particle physics: A case of cross fertilization. *Rev. Mod. Phys.*, *81*(3), 1015–1018.

Nieves, J. F. (1982). Electromagnetic properties of Majorana neutrinos. *Phys. Rev. D*, *26*(11), 3152–3158.

Peskin, M., & Schroeder, D. (1995). *Introduction to quantum field theory*. Addison-Wesley Pub. Co.

Pontecorvo, B. (1957). Mesonium and antimesonium. *Sov. Phys. JETP*, *6*, 429.

Salam, A., & Ward, J. (1964). Electromagnetic and weak interactions. *Physics Letters*, *13*(2), 168 - 171.

Schechter, J., & Valle, J. W. F. (1980). Neutrino masses in SU(2) $\times$ U(1) theories. *Phys. Rev. D*, *22*(9), 2227–2235.

Tung, W. (1985). *Group theory in physics* (No. v. 1). World Scientific.

Vogel, P., & Engel, J. (1989). Neutrino electromagnetic form factors. *Phys. Rev. D*, *39*(11), 3378–3383.

Weinberg, S. (1967). A model of leptons. *Phys. Rev. Lett.*, *19*(21), 1264–1266.

Wolfenstein, L. (1978). Neutrino oscillations in matter. *Phys. Rev. D*, *17*(9), 2369–2374.

Yang, C. N., & Mills, R. L. (1954). Conservation of isotopic spin and isotopic gauge invariance. *Phys. Rev.*, *96*(1), 191–195.

Zwiebach, B. (2004). *A first course in string theory*. Cambridge University Press.