BACKTRACKING SEARCH ALGORITHM FOR OPTIMAL POWER DISPATCH IN POWER SYSTEM

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ABSTRACT

The solution to power dispatch problem has been an important and basic optimization procedure in both conventional and restructured power systems. The main objectives of the power dispatch problem are to minimize the generation cost and emission amount of generators as well as to meet the power demand. The goal is to determine the most optimal power sharing among the generating units in a power system.

The practical power dispatch problems consider the technical operating constraints of generators such as ramp-up and ramp-down limits, lower and upper limits of generators, and prohibited operating zones. The accurate cost function needs to be taken into account in the problems for real-world applications by considering the valve-point loading effects and multiple fuel options. In this thesis, the power dispatch problems with the aforementioned constraints and cost functions are considered. Several case studies varied in size and complexity are employed in the power dispatch problems. Backtracking search algorithm (BSA) as the new evolutionary technique of optimization is used for solving the problems. Since the power dispatch problem is a constrained problem, two constraint handling mechanisms are proposed in the optimizer and are compared to each other in terms of solution quality they produce. BSA with two constraint handling mechanisms is applied to solve the power dispatch problems to select the better mechanism in power dispatch problems. Then, a microgrid with several renewable and conventional generating units is modeled for the purpose of optimal power dispatch. The problem is solved by BSA with the selected constraint handling mechanism to minimize the generation cost of the microgrid for a specific period of time. The multi-objective BSA is also developed to solve the economic and emission dispatch problems (EED) in a power system. The EED problem is solved by three methodologies including economic and emission dispatch separately, combined

economic and emission dispatch, and economic and emission dispatches simultaneously.

The high performance of the proposed technique with the proposed constraint handling mechanism is validated by solving the power dispatch problem in the large-scale test systems with the most complex cost functions. The proposed method is also compared with other well-known optimization methods from the literature in terms of the solution quality. The results show that the proposed method is highly robust when it deals with the practical power dispatch problems and its convergence characteristics make it a promising solution approach for power dispatch problems.

ABSTRAK

Penyelesaian kepada masalah penghantaran kuasa telah menjadi satu perkara penting dan prosedur pengoptimuman asas dalam kedua-dua sistem kuasa konvensional dan disusun semula. Objektif utama masalah penghantaran kuasa adalah untuk mengurangkan kos penjanaan dan jumlah pelepasan penjana disamping untuk memenuhi permintaan kuasa. Prosedur ini adalah untuk menentukan perkongsian kuasa yang paling optimum di antara unit-unit penjanaan dalam sistem kuasa.

Masalah penghantaran kuasa yang praktikal mempertimbangkan kekangan operasi teknikal penjana seperti had jalan ke atas dan had jalan ke bawah, had penjanaan yang lebih rendah dan lebih tinggi, dan zon operasi larangan. Fungsi kos yang tepat perlu diambil kira dalam masalah ini untuk aplikasi dunia sebenar dengan mempertimbangkan kesan loading injap dan pelbagai pilihan bahan api. Dalam tesis ini, masalah penghantaran kuasa dengan kekangan yang dinyatakan di atas dan fungsi kos akan dipertimbangkan. Beberapa kajian kes yang mempunyai pelbagai saiz dan kerumitan telah di ambil kira dalam masalah penghantaran kuasa ini. Algoritma Carian Pengesan-belakang (BSA) sebagai teknik evolusi baru pengoptimuman telah digunakan untuk menyelesaikan masalah ini. Oleh kerana masalah penghantaran kuasa mempunyai masalah kekangan, dua mekanisme untuk pengendalian kekangan telah dicadangkan dalam pengoptimum dan turut dibandingkan antara satu sama lain dari segi kualiti penyelesaian. BSA dengan dua mekanisme pengendalian kekangan ini digunakan untuk menyelesaikan masalah penghantaran kuasa untuk memilih mekanisme yang lebih baik. Kemudian, satu grid mikro dengan beberapa unit penjanaan boleh diperbaharui dan konvensional telah dimodelkan untuk menyelesaikan masalah penghantaran kuasa ini. Masalah ini dapat diselesaikan dengan menggunakan BSA beserta mekanisme pengendalian kekangan yang dipilih untuk mengurangkan kos penjanaan grid mikro untuk tempoh masa yang tertentu. BSA dengan pelbagai objektif turut dibangunkan untuk menyelesaikan masalah penghantaran ekonomi dan pelepasan (EED) di antara unit-unit penjanaan. Masalah EED diselesaikan melalui tiga metodologi termasuk ekonomi dan pelepasan penghantaran secara berasingan, gabungan penghantaran ekonomi dan pelepasa penghantaran, serta ekonomi dan pelepasan penghantaran secara serentak.

Prestasi tinggi yang ditunjukkan oleh teknik yang dicadangkan dengan mekanisme pengendalian kekangan disahkan dengan penyelesaian masalah berskala besar dengan fungsi kos yang paling rumit. Kaedah yang dicadangkan juga dibandingkan dengan lain-lain kaedah pengoptimuman yang terkenal yang sedia ada dari segi kualiti penyelesaian. Hasil kajian menunjukkan bahawa kaedah yang dicadangkan adalah sangat mantap apabila ia berkaitan dengan masalah penghantaran kuasa yang praktikal dan ciri-ciri penumpuannya menjadikan ia satu pendekatan yang terjamin untuk menyelesaikan masalah penghantaran kuasa.

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My best wishes to all of them,

Mostafa Modiri-Delshad

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LIST OF SYMBOLS AND ABBREVIATIONS

N	Number of generating units
a_i, b_i, c_i, e_i, f_i	cost function coefficients of generating unit i
α_i, β_i, γ_i, ζ_i, λ_i	emission function coefficients of generating unit i
P_i^{min} , P_i^{max}	minimum and the maximum production limits of the i^{th} generator
P_i	output power of the i^{th} generator
P_D	power demand
P_{loss}	transmission network loss
P	vector for power outputs of N generating units
B, B_0, B_{00}	loss coefficients
$F_{c,i}$	generation cost of generating unit i
F_c	total generation cost of N generating units
$F_{e,i}$	emission amount of generator i
F_e	total emission amount of N generating units
F	objective vector
F	combined objective of several objectives
w	weighting factor
σ	price penalty factor
U (0,1)	standard uniform distribution
N (0,1)	standard normal distribution
ρ	amplitude control function of search-direction matrix
Ω	search space of optimization problem
mixrate	BSA’s control parameter
np	population size
$\mathbf{X}^t$	population matrix in iteration t

$\mathbf{X}_i^t$	individual i of population **X** in iteration t
$\mathbf{histX}^t$	historical population matrix in iteration t
U, **V**	final and trial population matrices
map	binary matrix
k	number of objectives
m	number of non-dominated solutions
f_j	objective function j
f_j^{max}, f_j^{min}	maximum and minimum values of the j^{th} objective function
$\mathbf{X}_i$	solution number i
CD_i	crowding distance of solution i
μ_{ij}	membership function of solution i for objective j
μ_i	normalized membership function of solution i
ABC	artificial bee colony algorithm
ABCDP	artificial bee colony with dynamic population size
ACO	ant colony optimization
ACSA	ant colony search algorithm
AIS	artificial immune system
AIWF	adaptive inertia weight factor
API	apicalis ants
APO	active power optimization
APSO	anti-predatory particle swarm optimization
APSO	adaptive particle swarm optimization
BB-MOPSO	bare-bones multi-objective particle swarm optimization
BBO	biogeography-based optimization
BF	bacterial foraging
BSA	backtracking search algorithm

CA	cultural algorithm
CASO	chaotic ant swarm optimization
CEP	classical evolutionary programming
CGA_MU	conventional genetic algorithm with multiplier updating
CLS	chaotic local search
CPSO	chaotic particle swarm optimization
CRO	chemical reaction optimization
CSA	cuckoo search algorithm
CSO	civilized swarm optimization
CSS	charged system search
DE	differential evolution
DEC	chaotic differential evolution
DP	dynamic programming
DSG	dynamic slack generator
DSPSO	distributed Sobol particle swarm optimization
EA	evolutionary algorithms
ED	economic dispatch
EED	economic emission dispatch
EHNN	enhanced hopfield neural network
EMOCA	enhanced multi-objective cultural algorithm
EP	evolutionary programming
EPSO	enhanced particle swarm optimization
FAPSO	fuzzy adaptive particle swarm optimization
FCASO	fuzzy adaptive chaotic ant swarm optimization
FEP	fast evolutionary programming
FMOPSO	fuzzified multi-objective particle swarm optimization

GA	genetic algorithm
GAA	genetic annealing algorithm
GM	Gaussian mutation
GSA	gravitational search algorithm
GSO	glowworm swarm optimization
HM	hopfield model
HS	harmony search
IABC	ncremental artificial bee colony and local search
ICA	imperialist competitive algorithm
IEDO	improved evolutionary director
IEEE	institute of electrical and electronics engineers
IF	implicit filtering
IFEP	improved fast evolutionary programming
IGA_MU	improved genetic algorithm with multiplier updating
IHBMO	interactive honey-bee mating optimization
IPSO	improved PSO
ISS	improved scatter search
LI	lambda iteration
LP	linear programming
LR	Lagrange relaxation
LRS	local random search
LS	local search
MBFA	modified bacterial foraging algorithm
MHSA	modified harmony search algorithm
MOBSA	multi-objective backtracking search algorithm
MODE	multi-objective differential evolution

MPSO	modified particle swarm optimization
MSFLA	modified shuffled frog leaping algorithm
MSG	modified subgradient
MTS	multiple tabu search
MU	multiplier updating
MW	Megawatt
NDA	non-dominated approach
NM	Nelder-Mead
NP	nonlinear programming
NPSO	new particle swarm optimization
NR	newton-raphson
NSGA	non-dominated sorting genetic algorithm
OLS	orthogonal least-squares
POZ	prohibited operating zones
PS	pattern search
PSO	particle swarm optimization
QGSO	continuous quick group search optimizer
QOTLBO	quasi-oppositional teaching learning based optimization
QP	quadratic programming
RCCRO	real coded chemical reaction optimization
RVM	real-valued mutation operator
SA	simulated annealing
SDE	shuffled differential evolution
SFLA	shuffled frog-leaping algorithm
SGA	string structure and genetic algorithm
SO	system operators

SOA	seeker optimization algorithm
SOA	spiral optimization algorithm
SOHPSO	self-organizing hierarchical particle swarm optimization
SPEA	strength pareto evolutionary algorithm
SQP	sequential quadratic programming
SSG	static slack generator
SSGA	steady state genetic algorithm
TOPSIS	technique for order preference similar to an ideal solution
TS	tabu search
TSA	tabu search algorithm
TSARGA	taguchi self-adaptive real-coded genetic algorithm
TVAC	time-varying acceleration coefficients
VDE	variable differential evolution
VOA	virus optimization algorithm
WSM	weighted sum method

CHAPTER 1 INTRODUCTION

1.1 Introduction

The efficient and optimal operation of power system has always occupied an important position in electric networks. For many years in both conventional and modern power systems, the system operators (SOs) have tried to run the power system with the minimum cost of energy supply while satisfying the system constraints. They have tried to reduce the electricity cost imposed on the costumers by efficient procedures in the power system operation. In this regard, several approaches such as optimal power flow (OPF), unit commitment (UC), and economic dispatch problem (ED) have been considered so far. The economic dispatch problem is always considered as the basic and important task for the optimal operation of power system. It is employed to determine the power sharing of committed generating units in an economic manner to supply the power demand by considering technical constraints of power system elements.

Solutions to traditional power dispatch problems aimed for economic operation of the generating units of the power system to minimize the cost of power generation. When environmental concerns are considered, the economic dispatch may not produce the best results. This calls for economic and emission dispatch that considers both generation cost and emission minimizations.

1.2 Problem Description

Economic dispatch (ED) problem as an optimization problem is composed of an objective function and several constraints. Previous attempts to solve the ED problems have employed the classical methods of optimization known as conventional techniques. In these methods, technical and practical constraints of the units and the network have to be simplified/ignored owing to the limits of the classical methods. Such simplifications divide into two sections. One is associated with the accuracy of the cost model of the generating units especially for different types of fuels or to consider the valve-point loading effects (Cai et al., 2012b). Another relates to the network topology, either ignored or limited to considering only the total transmission network loss (Haiwang et al., 2013).

The objective of economic dispatch is usually to minimize the generation cost in the power system. Traditionally, the cost function of a generating unit is modeled by a quadratic cost function for the applicability of conventional techniques for solving economic dispatch problems. However, an accurate cost function addresses the valve point loading effects by adding a sinusoidal term to the generator cost function (quadratic function). In this case, the cost function becomes non-convex and solving economic dispatch with the non-convex objective function is a challenging issue for the conventional approaches. In addition, some generators have several fuel options in their operations and the cost function of a generator becomes more complex by considering the multiple fuel options. Finally, the practical economic dispatch problem is the problem in which the valve-point effects and multiple fuel options are taken into account in the cost functions of the generators.

The constraints of power dispatch problem consist of an equality and several inequalities. The equality constraint illustrates the balance between the power demand, the transmission loss, and the generations. The inequalities include the boundary limits, the ramp rate limits, and the prohibited operating zones. The basic power dispatch problem considers only the boundary limits while the ramp rate limits and prohibited operating zones are addressed in the practical economic dispatch problems.

The methods of solving the power dispatch problems include the classical methods of optimization (usually known as the conventional techniques), the metaheuristic methods, and hybrid methods. The classical methods suggested for solving the economic dispatch problems are linear programming (LP) (Jabr et al., 2000), Lagrange relaxation (LR) (Zhigang et al., 2013), quadratic programming (QP) (M. Q. Wang et al., 2014), dynamic programming (DP) (Z. X. Liang et al., 1992), etc. The metaheuristic methods include variety of techniques such as evolutionary algorithms (EAs), particle swarm inspired algorithms such as particle swarm optimization (PSO) (Niknam et al., 2010), ant colony optimization (ACO) (Pothiya et al., 2010), artificial bee colony algorithm (ABC) (Basu, 2013), glowworm swarm optimization (GSO) (Nelson Jayakumar et al., 2014), and shuffled frog-leaping algorithm (SFLA) (P. Roy et al., 2013), socio-human or socio-political inspired algorithms such as imperialist competitive algorithm (ICA) (Mohammadi-ivatloo et al., 2012), and natural-phenomena-inspired algorithms including charged system search (CSS) (Özyön et al., 2012), and harmony search (HS) (Jeddi et al., 2014). The hybrid methods are combinations of either two or more metaheuristic methods or metaheuristic with classical methods.

For many years the classical methods of optimization have been the only approaches to solving ED problems. They usually consider the forms of linear, piece-wise linear, and quadratic functions for the generator cost function; the network topology is ignored or only the network loss is considered. However, an ED problem is non-convex with high complexity in the real world applications, so the application of the classical methods is restricted. Although Maclaurin series (Hemamalini et al., 2010) approximation is employed to solve the non-convex ED problems, it leads to a non-optimal solution. In addition, Dynamic programing (DP) (Z. X. Liang & Glover, 1992) among the classical methods has been proposed to solve the ED problem with no restriction on the forms of generators' cost functions; however, its performance is increasingly affected by problem size (Zwe-Lee, 2003).

Metaheuristic techniques can solve ED problems with fewer/no restrictions on the shape of the cost functions, also cope with the difficulties of classical optimization techniques. They have been deployed to solve practical ED problems with a high degree of nonlinearity and more constraints than before. In this case, the application of these methods have shown promising solutions for complex ED problems, since they could handle various operating constraints, such as prohibited operating zones (POZ), generators' ramp-up and ramp-down. Some metaheuristic methods suffer from premature convergence and high computation time in the case of increasing system size which impedes their applications for real time operation. Therefore, the hybrid methods, such as the combination of two or more methods, have been proposed to eliminate each method's drawback.

The environmental effect of power generation has become an important issue of today's power system operation. Fossil-based power plants produce significant amount of the air pollutions in the atmosphere. The negative effects of various pollutants have attracted serious concerns in public so that the environmental impacts of power generation cannot be ignored in the operation of the power system. In this case, the US Clean Air Amendment of 1990 is imposed on power industry to control and minimize the emission amount realised by the generators (El-Keib et al., 1994; Srinivasan et al., 1997). Several strategies can be considered to decrease the harmful gases produced by the power plants such as enhancing the quality of burners, installing the pollution cleaning equipment, investing in renewable energy technologies or modern generators with low emissions, and performing emission power dispatch. Unlike the first three options, the emission dispatch does not require high capital investment. It can be performed on an existing system as a short term solution. Therefore, the optimal operation of power system would be achieved by not only minimizing the generation cost but also minimizing the emission amount. To handle both objective minimizations, various multi-objective approaches have been proposed. Such approaches address the economic-emission dispatch problems (EED). So far, the classical, metaheuristic, and hybrid methods have been employed for solving the EED problems. In this regard, the application of new methods especially the metaheuristics is encouraged to deal with this multi-objective problem.

1.3 Research objectives

The main objectives of this study are:

1. To propose the backtracking search algorithm (BSA) for solving the convex/non-convex power dispatch problems by considering the valve point loading effect, multiple fuel options, and practical operating constraints of the generating units
2. To propose suitable constraint handling mechanism for solving power dispatch problems by BSA
3. To evaluate the proposed method for large-scale applications especially for solving highly nonlinear and complex ED problems and to solve power dispatch problem for microgrid with renewable and conventional generators
4. To develop multi-objective BSA for solving the economic and emission dispatch problems (EED) through methodologies including weighted sum method (WSM) and non-dominated approach (NDA).

1.4 Scope of work

The following items are considered in this research:

1. The formulation of power dispatch problem is based on mathematical model of generators and electric network
2. For solving the economic dispatch problems, six test systems different in cost model and system operating constraints are tested.

3. For large scale power dispatch problem, four systems with up to 160 generating units are employed. The valve-point loading effects and multiple fuel options are considered in these systems.
4. For solving power dispatch problem in microgrid, a system including two wind power plants, three fuel cell plants, and two diesel generators are considered.
5. For solving the multi-objective power dispatch problems, three systems including IEEE 30-bus 6-unit, 10unit, and IEEE 118-bus 14-unit systems are tested.
6. All the simulations are done in Matlab environment on a personal computer with Pentium 2.70 GHz processor and 2GB RAM.

1.5 Organization of thesis

The rest of the thesis is organized as follows:

Chapter 2 provides background on the concepts involved in this work and a literature review that covers the types of power dispatch problems and the optimization methods for solving these problems. Chapter 3 focuses on the mathematical modeling and problem formulation of power dispatch problems. It also explains the methodology of solving the power dispatch problems with two constraint handling mechanisms incorporated in backtracking search algorithm. Chapter 4 provides findings of optimization by backtracking search algorithm (BSA) for solving economic dispatch problems in power system and a microgrid, respectively. Chapter 5 describes the results of multi-objective power dispatch problems. Chapter 6, as the last chapter, presents conclusions and future works. A comprehensive list of reference is provided at the end of the thesis.

CHAPTER 2 : LITERATURE REVIEW

2.1 Introduction

The problem of power dispatch is to determine the generation schedule of generators to supply a specific level of power demand. This problem is considered as an optimization problem which is to minimize a single objective or multiple objectives. The objective of power dispatch problem is usually assumed to be the generation cost referred to as economic dispatch problem (ED). When the environmental concern is to be taken into consideration, the emission amount produced by the generators should be also minimized. This concern changes the ED problem to the economic/emission dispatch problem (EED). In this chapter, the attempts to solve both ED and EED problems are described from the literature. At first, the concept of optimization is presented.

2.2 The concept of optimization

The optimization refers to minimize or maximize objective function(s) to find the best solution to a problem while satisfying several inequality and quality constraints. In one view, the optimization problems are divided into either constrained or unconstrained problems but most of the real-world optimization problems are from the first type. In another view, the optimization problems fall into single objective or multi-objective problems.

The formulation of a single objective optimization problem is shown by Eq. (2.1) which is usually represented as the minimization of function f(**X**) subjected to equality and inequality constraints.

$$\begin{aligned} &\underset{\mathbf{X}\in\Omega}{Minimize} \quad y = f(\mathbf{X}) \\ where& \\ &\Omega = \{\mathbf{X} \in R^n : g(\mathbf{X}) \leq 0, h(\mathbf{X}) = 0\} \\ &f : R^n \rightarrow R \end{aligned} \tag{2.1}$$

Where **X** is the vector of optimization variables, g(**X**) and h(**X**) are respectively the vectors of inequalities and equalities.

In the multi-objective optimization problem, several functions need to be optimized simultaneously. Eq. (2.2) shows the general form of a multi-objective problem subjected to equality and inequality constraints.

$$\begin{aligned} &\underset{\mathbf{X}\in\Omega}{Minimize} \quad \mathbf{Y} = (f_1(\mathbf{X}), f_2(\mathbf{X}), ..., f_k(\mathbf{X})) = \mathbf{F}(\mathbf{X}) \\ where& \\ &\Omega = \{\mathbf{X} \in R^n : g(\mathbf{X}) \leq 0, h(\mathbf{X}) = 0\} \\ &\mathbf{F} : R^n \rightarrow R^k \end{aligned} \tag{2.2}$$

When it comes to multi-objective optimization, there is no unique solution corresponding to the optimal value of each objective. Instead, there is a set of solutions known as the pareto optimal set. Assuming that $\mathbf{\Phi}=(\Phi_1,\ldots,\Phi_n)$ and $\mathbf{\Psi}=(\Psi_1,\ldots,\Psi_n)$ are two solutions included in the pareto optimal set and correspond to the objectives $\mathbf{F}(\mathbf{\Phi})=(f_1(\mathbf{\Phi}),\ldots,f_k(\mathbf{\Phi}))$ and $\mathbf{F}(\mathbf{\Psi})=(f_1(\mathbf{\Psi}),\ldots,f_k(\mathbf{\Psi}))$,, the solution **Φ** dominates the solution **Ψ**, denoted by $\mathbf{\Phi} \prec \mathbf{\Psi}$ or $\mathbf{F}(\mathbf{\Phi}) \prec \mathbf{F}(\mathbf{\Psi})$, if and only if the following conditions of Eq. (2.3) are satisfied. In this case, the solution **Φ** is the non-dominated solution.

$$\begin{aligned} &\forall i \in \{1,...,k\}: \quad f_i(\mathbf{\Phi}) \leq f_i(\mathbf{\Psi}) \\ &\exists i \in \{1,...,k\}: \quad f_i(\mathbf{\Phi}) < f_i(\mathbf{\Psi}) \end{aligned} \tag{2.3}$$

where i and k represent the solution number and the number of objectives, respectively.

In Figure 2.1, the circled points represent the pareto optimal set of two objectives. The black circles represent the non-dominated solutions and the connected line of these points is the pareto front. The set of pareto front represented by P is described mathematically by Eq. (2.4).

$$P(Y) = \{\mathbf{Y_1} \in \theta : \{\mathbf{Y_2} \in \theta : \mathbf{Y_2} \prec \mathbf{Y_1}, \mathbf{Y_2} \neq \mathbf{Y_1}\} = \mathbf{0}\}$$
$$\theta = \{\mathbf{Y} \in R^k : \mathbf{Y} = \mathbf{F}(\mathbf{X}), \mathbf{X} \in \Omega\} \tag{2.4}$$

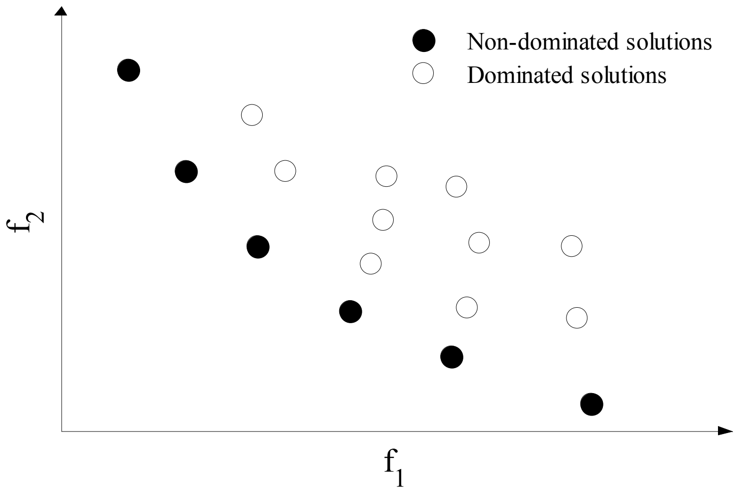


Figure 2.1 The pareto optimal set for two objective functions

2.3 Economic dispatch problem (ED)

Economic operation has been a challenging issue to both conventional and smart grid systems. Economic dispatch (ED) refers to optimizing the power share of each generating unit to meet power demand within the technical constraints of the generators and the electrical network.

ED problem are convex or non-convex based on the system and its elements' models. In a convex ED problem, the cost function of a generating unit is considered as a quadratic function. Practical and non-convex ED problems, however, contain non-convex cost functions that are due to the valve-point effect of the generating units. Classical methods have been adopted to solve conventional ED problems (i.e., containing convex cost functions) but instead produce non-optimal solutions because of the non-convexity/non-linearity of practical ED problems (Basu et al., 2013). Dynamic programming, for example, has been proposed in addressing non-convex ED problems because it does not restrict the form of the cost function; the increased dimension of the problem, however, may demand higher computational efforts (Cai et al., 2012b; Zwe-Lee, 2003).

Maclaurin series (Hemamalini & Simon, 2010) approximation is employed to solve the non-convex ED problems, but it leads to a non-optimal solution.

Unlike classical methods, metaheuristic methods are better options because they can handle more constraints and are able to explore the search domain effectively in finding the optimum. Among these techniques, differential evolution (DE) is especially very effective because it does not need derivative information from the cost function; instead it sub-optimally or prematurely converges (Niknam, Mojarrad, Meymand, et al., 2011). Other drawbacks associated with metaheuristics are high sensitivity to the control parameters, long computational time, and slow convergence to approximately optimum solution (Subathra et al., 2014).

Recent hybrid methods overcome those drawbacks of single metaheuristic or classical approaches, able to handle the high complexities of practical ED problems. One method might be adopted for its high convergence, another for its provision of a suitable initial guess for the problem. The hybrid methods are combinations of either two or more metaheuristic methods or metaheuristic with classical techniques and perform better than individual techniques. In the following sections, the aforementioned techniques are discussed based on their categories.

2.3.1 Classical methods

(Waight et al., 1981) have used the Dantzig-Wolfe decomposition method to resolve the ED problem into a master problem and a set of smaller linear programming sub-problems. The sub-problems are solved by revised simplex method.

(Aoki et al., 1982) have solved economic load dispatch problem by quadratic programming technique. The problem contains a large number of linear constraints. The parametric quadratic programming is proposed as the extension of the quadratic programming method in order to deal with the transmission loss in the ED problem. The execution time and memory requirements suggest this method for real time applications.

(Lin et al., 1984) have formulated the economic dispatch problem with piecewise quadratic cost functions. The solution approach is hierarchical and the results show that the solution method is practical and valid for real time applications.

(Ramanathan, 1985) has proposed simple and efficient solution technique for solving economic dispatch problem based on lambda calculation. The penalty factors used in the procedure of algorithm is determined by Newton's method. The algorithm has also considered transmission network loss in the dispatch problem. Based on the results, there have not been any convergence and oscillations problems in the execution of the algorithm.

(Gherbi et al., 2011) have proposed a quadratic programming technique to solve the economic dispatch problem with several objective functions including emission, cost, and loss reductions. The proposed algorithm was applied on a six-unit power system. Compared to several methods of optimization, less computational time and best optimal solution have been achieved.

(Lin et al., 1992) have developed a classical method to solve the real-time economic dispatch problem through alternative Jacobian matrix considering the system constraints. The transmission loss is taken into account in the problem and the proposed method was tested on a case study resulting fast-convergence with accurate results.

(Papageorgiou et al., 2007) have used mixed integer quadratic programming for solving economic dispatch problem with prohibited operating zones constraints. In deregulated power systems, increasing the profit through the optimized generation schedules is the main objective for the generators owners whereas the demand satisfaction is not a commitment.

(Irisarri et al., 1998) have solved the economic dispatch problem using interior point technique. The reference considered the generation ramp rate and transmission line flow limits in the ED problem so the security and economic features of system operation are satisfied at the same time.

A fast lambda iteration method is suggested by (Zhan et al., 2014) to solve ED problems with prohibited operating zones. The method is applied on 15-unit systems and also a Korean 140-unit test system to verify the efficiency of the proposed method.

2.3.2 Metaheuristic methods

The non-convex economic dispatch problem caused by non-smooth fuel cost function has been solved by tabu search algorithm (TS) by (Khamsawang et al., 2002) and compared with the conventional techniques. An improved tabu search is also implemented by (Whei-Min et al., 2002) for solving ED problem with multiple minima.

(Zwe-Lee, 2003) has proposed particle swarm optimization (PSO) method for solving ED problem considering non-smooth cost functions, ramp rate limits, and prohibited operating zone constraints. The proposed PSO is also compared with genetic algorithm in terms of solution quality resulting high performance of the proposed method.

An improved PSO (IPSO) technique was introduced by (Jong-Bae et al., 2010) to solve ED problem with non-convex cost functions. The applicability of the proposed method is verified by applying on large-scale power system of Korea.

(Sum-Im, 2004) introduced ant colony search algorithm (ACSA) to solve the ED problem considering transmission network loss. The most prominent advantage of ACSA is to optimize while searching. ACSA has been examined on IEEE 30-bus system and the results compared to those attained by Lambda iteration and genetic algorithm.

(Pothiya et al., 2010) presented a solution to ED problem with non-smooth cost functions by ant colony optimization (ACO). The algorithm outperformed other heuristic methods in terms of less computation time and accurate results.

(Tankasala) utilized artificial bee colony (ABC) optimization technique to solve ED dispatch problem in coal fired power plants. ABC is one of the intelligent techniques that can cover the defects of conventional methods. In this reference, the ABC is compared with several intelligent techniques. The results indicate that ABC ensures the global minimum of the solution while other intelligent techniques may lead to local minimum.

Artificial bee colony has also been applied for solving multi-area economic dispatch (MAED) problem by (Basu, 2013). The constraints of tie lines, transmission loss, multiple fuel options, valve point loading effects, and prohibited operating zones have been taken into account in this reference. The performance of the method has been examined on three test systems with different degree of complexity and compared with evolutionary programming in terms of the solution quality. The results represented the proposed method to be a promising solution for solving of practical ED problems.

Genetic algorithm (GA) is used by (Sheble et al., 1995; Walters et al., 1993) to solve ED problem considering valve-point effects in generator cost function.

(Hong et al., 2002) have considered the ED problem in a deregulated market with multiple buyers and co-generators and solved the problem by genetic algorithm (GA). The IEEE 30-bus and 118-bus systems are used to analyze the performance of the proposed method.

GA is also used by (Abido, 2003b) for solving multi-objective ED problems for minimizing the fuel cost and the emission by considering valve-point effects and transmission loading restrictions. The IEEE 30-bus system is the case study to validate the performance of GA.

(Po-Hung et al., 1995) has suggested GA for solving of ED problem in large-scale system in Taiwan electric network composed of 40 generating units. The transmission loss, ramp rate limits and prohibited operating zones are addressed in the ED problem. The high robustness and powerfulness of the proposed method are proved by comparison to the lambda iteration method.

Particle swarm optimization method is presented by (Kumar et al., 2003) to solve multi-objective economic dispatch considering emission and fuel cost.

The problems of ED with valve point loading effects and multiple fuel options are solved by (Jong-Bae et al., 2005) through modified PSO (MPSO). The equality constraint is handled by an appropriate treatment mechanism and the inequalities are handled by position adjustment strategy. A dynamic search space reduction strategy is employed to accelerate the optimization process. 10-unit test system with multiple fuel options and 40-unit system with non-convex cost functions are use as the case studies and the results of MPSO are compared with those of numerical techniques such as tabu search, and evolutionary programming, genetic algorithm.

An improved PSO is proposed by (Park et al., 2010) to solve ED problems considering the power balance constraint and generators boundary limits. Although PSO is capable of handling heavily constraints ED problems, it may trap in local optimum in the solution space, the chaotic sequence combined with the conventional linearly decreasing inertia weights is employed in this reference and the crossover operation scheme is adopted to enhance both exploration and exploitation capability of the proposed method. The effective constraint handling framework is also used in the optimization. Several case studies with valve-point effect, prohibited operating zones as well as transmission network loss, and multiple fuel options are employed to validate the performance of the proposed method and the results are compared with well-known optimization methods. The power system of Korea as the large-scale system is also considered to evaluate the proposed method.

Another version of PSO with the use of linearly decreasing inertia weight factor is suggested by (Jeyakumar et al., 2006) to solve multi area, multiple fuel, and multi-objective economic dispatch problems and ED problems with prohibited operating zones. Several case studies corresponding to the aforementioned ED problems are adopted to test the performance of the proposed method. The results of the PSO are compared with the results of classical evolutionary programming. The results show that the proposed PSO can produce high quality solutions with reduced computation time.

A chaotic PSO with an implicit filtering techniques (IF) is proposed as the hybrid approach by (dos Santos Coelho et al., 2007) to solve ED problems with valve point loading effects considering the power balance and generators boundary limits. The chaotic PSO is the global optimizer and the IF is to fine-tune the chaotic PSO run in a sequential manner. The proposed hybrid approach is validated by a test system consisting of 13 units taking into account the valve point loading effects in generators cost functions.

Two versions of chaotic PSO named CPSO1 and CPSO2 are proposed by (Cai et al., 2007) to solve ED problem by considering the transmission line flow, ramp rate, generation limits, and prohibited operating zones. Each CPSO is a two phase iterative strategy (based on the proposed PSO with AIWF and CLS) in which PSO with the adaptive inertia weight factor (AIWF) is employed for global exploration and chaotic local search (CLS) is applied for locally oriented search (exploitation) for the solutions that PSO results. By applying the proposed method on 15-unit test system, the results show the reduction in the convergence iterative numbers and also produce great economic effort compared to the traditional PSO.

Three types of ED problems addressing prohibited operating zones, valve-point loading effects, and valve-point effects with multiple fuel options are solved by a new PSO (NPSO) (A. I. Selvakumar et al., 2007).

A chaotic and Gaussian based PSOs are used to solve ED problems to minimize the fuel cost considering prohibited operating zones, line flow constraints, transmission loss, and ramp rate limits (Coelho et al., 2008). Seven versions of PSO along with the original PSO based on the Gaussian distribution function or chaotic sequences in social and cognitive parts are developed and tested on 15- and 20-unit systems to analyze the performance of the proposed PSOs. The results of comparison with the techniques from the literature confirm the applicability of the proposed PSO for solving of ED problems.

An adaptive PSO (APSO) is employed to solve non-smooth ED problems with prohibited operating zones and ramp rate limits by (B. K. Panigrahi et al., 2008). The anti-predatory PSO (APSO) is adopted by (A. Immanuel Selvakumar et al., 2008) to solve ED problems taking into account the valve-point effects and multiple fuel options in generators cost functions. 10-unit test system with multiple fuel options and 40-unit system with non-convex cost functions are used as case studies and the satisfactory results compared to the previous approaches are obtained.

A versions of PSO named CRAZYPSO is introduced by (Roy et al., 2008) for solving of ED problem addressing the valve point loading effects. A system with 40 generating units with three types of cost coefficients; non-convex, convex, and non-convex and convex mixed, is considered as the case study.

Classical PSO methods are capable of solving non-convex ED problems, but they may lead to sub-optimal solutions. The practical ED problems are solved by (Chaturvedi et al., 2009) through a modified PSO in which the time varying acceleration coefficients (TVAC) are used to control the global and local search of the problem. In this case, the PSO avoids premature convergence and the global solutions are obtained. The proposed PSO is tested on 3-, 13-, 15-, and 38-unit case studies and the results are compared with a few PSO variants and some other methods. The comparisons verify the superiority of the proposed method compared to other approaches for solving of non-convex ED problems.

A fuzzy system is proposed by (Cai et al., 2012a) to tune the control parameters of chaotic ant swarm optimization (CASO) for solving ED problems considering valve-point effects and transmission system loss. The applicability of the proposed approach for handling non-convex ED problems is demonstrated by applying on 3-, 20-, and 40-unit test systems.

Non-convex ED problems with valve-point effects are solved by firefly algorithm (FA) (Cai et al., 2012a), modified group search optimizer method (Zare et al., 2012), shuffled differential evolution (SDE) (Srinivasa Reddy et al., 2013), cuckoo search (CSA) (Basu & Chowdhury, 2013), real-coded chemical reaction optimization method (RCCRO) (K. Bhattacharjee et al., 2014), theta PSO (θ-PSO) (Hosseinnezhad et al., 2013), differential evolution (DE) (Noman et al., 2008), seeker optimization algorithm (SOA) (Shaw et al., 2012), and continuous quick group search optimizer (QGSO) (Moradi-Dalvand et al., 2012).

2.3.3 Hybrid methods

(Attaviriyanupap et al., 2002) have suggested a hybrid method as the combination of evolutionary programming and sequential quadratic programming (SQP) for solving ED problems. The EP is considered as the base level search and the SQP is a fine-tuner to determine the optimum solution. The proposed method has been validated on a 10-unit system to solve dynamic economic dispatch problems with non-smooth fuel cost functions.

(Niknam, 2010) combined fuzzy adaptive particle swarm optimization (FAPSO) algorithm with Nelder–Mead (NM) simplex search to solve non-smooth and non-convex economic dispatch problems. The proposed method used the NM algorithm as a local search algorithm around the global point determined by FAPSO at each iteration of the solution procedure.

Another hybrid approach (Huang et al., 2007) combines the algorithms of orthogonal least-squares (OLS) and enhanced particle swarm optimization (EPSO) for real-time power dispatch. The OLS algorithm was applied to determine the number of centers in the hidden layer and the EPSO algorithm for tuning the parameters in the optimization process.

A combined differential evolution (DE) algorithm and sequential quadratic programming (SQP) was developed by (dos Santos Coelho et al., 2006) where DE with chaos sequences was the global optimizer and the SQP was used to fine-tune the DE sequentially. This applicability of the method was validated by applying on 13- and 40-unit systems in which the valve-point loading effect were incorporated in the fuel cost functions.

A hybrid method as the combination of genetic algorithm (GA) and simulated annealing (SA) is investigated by (Wong et al., 1994) and called genetic annealing algorithm (GAA). Two versions of hybrid are developed, GAA and GAA2. In the former, the application of SA is to eliminate premature convergence and avoid the negative effects of mutation. In the latter, it aims to reduce the memory requirement by decreasing the population size to two individuals. Both versions outperform other GA and SA based methods in terms of economic effect. The GAA2 leads to less computation time as well.

The implementing of improved evolutionary director (IEDO) and multiplier updating (MU) in real-coded genetic algorithm is proposed by (C. L. Chiang, 2007). IEDO is employed in the selection process before applying the crossover and mutation operators and MU is used to overcome the drawbacks of using penalty parameters. The applicability of the proposed method is verified by applying on 15-, 30-, 60, and 90-unit test systems and the results show the higher performance of the proposed method compared to conventional genetic algorithm in terms of economic effect and computation time.

A hybrid of PSO and sequential quadratic programming (SQP) is proposed by (Victoire et al., 2005) to solve economic dispatch problem considering valve-point loading effects. The active power balance, ramp rate limits of generators, voltage limit at load bus, transmission line constraints and spinning reserve are the considered constraints in the ED problem. The main optimizer is PSO while the SQP is to fine-tune the solution region as the local optimizer. SQP guide PSO for better performance in the complex solution space. A ten unit system with three different load patterns is used to validate the effectiveness and computation performance of the proposed method in general.

In addition, the combination of the traditional PSO with Gaussian mutation (GM) is suggested as a hybrid method by (Sriyanyong, 2008) to solve the ED problems with non-smooth cost functions. In the hybrid PSO-GM, the Gaussian mutation is used to enhance the global search capability of the PSO. Compared to the traditional PSO, the proposed hybrid method has higher global search capability.

A hybrid method including distributed Sobol PSO and tabu search algorithm (TSA), named DSPSO-TSA, is suggested to solve ED problems with non-smooth and non-continuous fuel cost curves of generators by (Khamsawang et al., 2010). Three mechanisms are employed in the process of optimization; Sobol sequence is used to produce an inertia factor rather than existing process at first, followed by a distributed process to reach the global solution rapidly, and finally, TSA is used to guarantee the global solution by adjusting the solution obtained by DSPSO. The proposed hybrid technique is applied on 6-, 10-, 13-, and 15-unit systems and compared with the conventional methods. The results of comparisons verify that the proposed method can reach higher solution quality in terms of economic effect and computation time among other methods.

Combination of three methods; genetic algorithm (GA), pattern search (PS), and sequential quadratic programming, is presented by (Alsumait et al., 2010) to solve non-convex ED problems. The robustness of the proposed GA-PS-SQP is analyzed and the outcomes show the proposed hybrid method as a high-efficient technique for the purpose of solving practical ED problems. 3-, 13-, and 40-unit systems are the case studies in this reference.

A hybrid method composed of PSO and real-valued mutation operator (RVM) is proposed by (Lu et al., 2010). The proposed method is applied on the mathematical benchmarks at first and then it is applied on case studies with 10 and 40 unit systems by considering the valve-point effects and multiple fuel options.

Non-convex ED problems are addressed by (Niknam, Mojarrad, & Meymand, 2011) and solved by a hybrid method as the combination of variable differential evolution (VDE) and fuzzy adaptive PSO named FAPSO-VDE. In the proposed hybrid method, the DE is the main optimizer and PSO acts as the preventer from sub-optimal convergence. Two case studies with 13 and 40 generating units are employed to validate the high performance of the proposed method.

Some other hybrid methods for solving ED problems are: fuzzy adaptive chaotic ant swarm optimization with sequential quadratic programming (FCASO-SQP) (Cai et al., 2012c), chaotic PSO and SQP (CPSO-SQP) (Cai et al., 2012b), DE with PSO (DEPSO) (Sayah et al., 2013), incremental artificial bee colony and local search (IABC-LS) (Aydın et al., 2013), modified shuffled frog leaping algorithm with genetic algorithm (MSFLA-GA) (P. Roy et al., 2013).

2.4 Economic/Emission Dispatch problem (EED)

Solutions to traditional power dispatch problems aim for economic operation of the generating units of a power system to minimize the cost of power generation. When environmental concerns are considered, the power dispatch may not produce the best results. This calls for a multi-objective optimization approach that considers both generation cost and emission minimizations.

Reducing the emission of power plants requires proper planning. One approach is to invest in new power plants that produce low emissions or to use renewable energy technologies, however, can be costly and thus are suitable as long-term options. Another way of reducing emission is to optimize power system operation by considering the emission amount as a constraint or as an objective function (Mandal et al., 2015).

Consideration of emission in a power dispatch problem is a multi-objective Economic/Emission Dispatch (EED) problem, which can be formulated in several ways. One way, known as the ε-constraint technique, is to consider one objective as the constraint and minimize the other objective. Another way, known as the scalarization method, is to convert the multi-objective problem to a single objective problem (Özyön et al., 2012), by goal programming, goal attaining, objective weighting to form a single objective, and so on. The problem can also be solved as a multi-objective problem in which a trade-off curve between all the objectives has been found. This curve is known as the pareto-front and proposes all the optimal solutions to the problem (Vahidinasab et al., 2009). In multi-objective approach, if an optimal solution has to be defined, a decision maker that assigns a merit order to any solution of the pareto front selects the best compromise solution from the whole pareto-front solutions (Abido, 2006).

Methods of solving economic/emission dispatch problems (EED) can be categorized in several ways. In one way, they are categorized into three groups. The first group includes the methods applied to the EED problems in their original versions. Few examples are genetic algorithm (GA) (Y.-C. Liang et al., 2014), particle swarm optimization (PSO) (Zwe-Lee, 2003), glowworm swarm optimization (Nelson Jayakumar & Venkatesh, 2014), virus optimization algorithm (VOA) (Y.-C. Liang & Cuevas Juarez, 2014), and spiral optimization algorithm (SOA) (Benasla et al., 2014). The methods of the second group are the modified versions of the first group including modified harmony search algorithm (MHSA) (Jeddi & Vahidinasab, 2014), modified artificial bee colony (MABC) (Secui, 2015), artificial bee colony with dynamic population size with local search (ABCDP-LS) (Aydin et al., 2014), chaotic interactive artificial bee colony (CIABC) (Shayeghi et al., 2014), self-organizing hierarchical particle swarm optimization with time-varying acceleration coefficients (SOHPSO-TVAC) (Mandal et al., 2015), and real coded chemical reaction algorithm (RCCRO) as the modified version of CRO (Kuntal Bhattacharjee et al., 2014). The last group consists of the hybrid methods as the combination of methods from the previous groups. This group includes ant-colony optimization and steady state genetic algorithm (ACO-SSGA) (Mousa, 2014), differential evolution with biogeography-based optimization (DE-BBO) (Bhattacharya et al., 2011), and hybrid bacterial foraging algorithm with the Nelder-Mead method (BF-NM) (Hooshmand et al., 2012), particle swarm optimization with differential evolution (PSO-DE) (Gong et al., 2010), particle swarm optimization with gravitational search algorithm (PSO-GSA)(Jiang et al., 2014), and differential evolution with harmony search (DE-HS) (Sayah et al., 2014).

In terms of the types of the methods, they can also be categorized into three groups: classical, metaheuristic, and hybrid. In problems having nonlinear objectives or constraints, finding the optimal can be difficult by classical methods (Zwe-Lee, 2003). The difficulties can be handled by metaheuristic methods but at the expense of computation time during optimization. Hybrids combine methods so they perform better than they do individually. Examples of classical methods are quadratic programming (Ji-Yuan et al., 1998) and linear programming (Farag et al., 1995). Second category includes genetic algorithm (GA) (Y.-C. Liang & Cuevas Juarez, 2014), particle swarm optimization (PSO) (Zwe-Lee, 2003), some derivatives of GA and PSO such as non-dominated sorting genetic algorithm (NSGA-II) (Basu, 2014a) and fuzzified multi-objective particle swarm optimization algorithm (FMOPSO) (L. Wang et al., 2007), modified harmony search algorithm (MHSA) (Jeddi & Vahidinasab, 2014), glowworm swarm optimization (Nelson Jayakumar & Venkatesh, 2014), interactive honey-bee mating optimization (IHBMO) (Ghasemi, 2013), and multi-objective differential evolution (MODE) (Basu, 2011; Wu et al., 2010). The third group is the set of hybrid methods which is the same as the one described in the previous paragraph.

2.4.1 Scalarization methods

A multi-objective optimization problem can be converted to a single objective problem. In this case, several objectives are combined to form a new objective. The optimal solution of the new single objective problem is considered as the optimal solution to the original multi-objective problem.

(Hooshmand et al., 2012) have developed a hybrid method as the combination of bacterial foraging algorithm (BF) and Nelder-Mead (NM) method to solve the EED problems through weighted sum method. The simulation is performed with different weighting factors used in the proposed method. The authors have presented a new formulation to power dispatch problem by considering the spinning reserve constraint, maximum emission limit of each generator and power system at specific hours, and frequency deviation limit. The results of optimization show higher performance of the proposed method compared to several optimization algorithms.

Modified harmony search algorithm is used by (Jeddi & Vahidinasab, 2014) to solve the EED problems by weighted sum method. Seven test systems are considered in this reference considering valve point effects and transmission loss for solving of multi-objective power dispatch problems. Pareto front solutions are obtained by solving the EED problems for different values of weighting factor. The proposed method has shown a competitive performance with high robust results.

(Bhattacharya & Chattopadhyay, 2011) have converted the EED problem into a single objective problem by weighting the objectives equally. The best compromise solutions of the proposed method and several methods from the literature are compared to each other confirming that the proposed technique outperforms other methods.

The solution to EED problem is performed by (Özyön et al., 2012) through scalarizing the problem with weighted sum method. The charged system search algorithm is used as the optimization techniques and the EED problem is solved for different weighting factors. The results show the satisfactory performance of the proposed method for solving the EED problems.

EED is converted to a single objective problem by (Nelson Jayakumar & Venkatesh, 2014) through incorporating the technique for order preference similar to an ideal solution (TOPSIS) as a multi criterion decision maker in glowworm swarm optimization algorithm (GSO). The decision maker identifies the positive and negative ideal solutions in each iteration of the optimization and attempts to selects solutions with shortest geometric distance from the positive one and farthest geometric distance from the negative one. The GSO is also applied to solve EED problem through the weighted sum method which is called WGSO. The superiority of the proposed method (GSO-T) is discussed with other methods from the literature and WGSO confirming the high potential of GSO-T for handling the EED problem.

The weighted sum method is used by (Benasla et al., 2014) for solving the EED problems. The spiral optimization algorithm (SOA) is used as the proposed optimizer and its results are compared with other methods to show its effectiveness in solving EED problems. It seems that the tuning of control parameters is paramount for the convergence of the proposed algorithm.

Modified artificial bee colony algorithm is employed by (Secui, 2015) for solving the EED problem formulated as a single optimization problem through weighted sum method. The simulation is carried out for different weighting factors and the best compromise solution is identified by a fuzzy based decision maker. Two mechanisms are employed in the determination of the best compromise solution and compared to each other. It is found that the proposed method outperforms other metaheuristic methods as it has a better balance between exploration and exploitation to produce higher quality of solutions.

(Mandal et al., 2015) have solved the EED problem by weighted sum method. The price penalty factor is used in the problem's formulation and the weighting factors are considered equally for the objectives resulting one optimal solution to the EED problem. However, (Kuntal Bhattacharjee et al., 2014) have ignored to apply the price penalty factor since they have normalized the objective functions in the formulation of weighted sum method and have considered different values for the weighting factors in the simulations. The pareto front solutions are produced for different weighting factors while the best compromise solution corresponds to the solution with the medium weighting factor.

The EED problem is formulated differently by (Jiang et al., 2014) to minimize an overall objective function including emission amount, generation cost, transmission loss, and penalized equality constraint's violation. Several test systems with various practical operational constraints are used and the results show that the proposed method is a viable method for solving the EED problems.

(Hamedi, 2013) has formulated the EED problem as a single objective problem by considering the price penalty factor for two objectives of EED problem and weighting them equally. Parallel synchronous particle swarm optimization algorithm is used to solve the EED problem and the results show better computation efficiency of the proposed method compared to several techniques.

Some other attempts addressing the weighted sum method for solving the EED problems are artificial bee colony with dynamic population size with local search (ABCDP-LS) (Aydin et al., 2014), differential evolution with harmony search (DE-HS) (Sayah et al., 2014), and virus optimization algorithm (VOA) (Y.-C. Liang & Cuevas Juarez, 2014).

2.4.2 Non-dominated approach

Unlike a single optimization problem which has only one optimal solution as the global optimal, the multi-objective problem has no unique optimal and the optimization product is pareto front as a set of optimal solutions. In each generation of the optimization process, the solutions are generated and an external archive is used to store and update the non-dominated solutions. When the optimization reaches the stopping criteria, the pareto front as the set of final non-dominated solutions is considered as the optimal solutions to the problem. Then, a decision maker is employed to apply to the pareto front solutions to identify the best compromise solution as the selected optimal to the problem.

(Wu et al., 2010) have presented a scheme of non-dominated approach by differential evolution method. An external elitist archive with three rules is employed to store the non-dominated solutions produced within the optimization. A crowding entropy-based diversity measure as the modified crowding distance index is also considered to remove extra members of the archive. The authors have used a fuzzy based decision maker to select the best compromise solution among the pareto front members. The method is applied on IEEE test systems to show the performance of the proposed method. Multi-objective differential evolution is also employed for solving the EED problems by (Basu, 2011) and is applied on 6-, 10-, and 40-unit test systems. It is compared with several methods confirming its high performance for solving the EED problems.

A hybrid method as the combination of PSO and DE is presented by (Gong et al., 2010) to solve the EED problems. The crowding distance is used to remove the extra members of external archive and the proposed method is applied on IEEE 30-bus 6-generator system. The quality of pareto front and the convergence characteristics of the proposed method are compared with several methods to show the high performance of the proposed hybrid technique.

The minimization of transmission loss along with minimizations of generation cost and emission amount are performed by (Shayeghi & Ghasemi, 2014). A chaotic local search (CLS) mechanism is added to artificial bee colony algorithm to form a stronger technique to solve the EED problems. The best compromise solution is determined by a fuzzy-based decision maker and the pareto front is obtained for two and three objectives of the problem. The results show that the local search improves the performance of the artificial bee colony algorithm.

(Mousa, 2014) has proposed a hybrid method which combines the ant colony optimization approach and steady state genetic algorithm for solving the EED problems. The decision maker named “technique for order preference by similarity to the ideal solution (TOPSIS)” is used to identify the best compromise solution from the pareto front. The results show that the proposed method can produce stable pareto front with satisfactory diversity among its solutions.

The fuel supply limitations in thermal power plants are considered in the EED problem by (Basu, 2014a). It is solved by non-dominated sorting genetic algorithm with fuzzy based decision maker to determine the best compromise solution and with the application of crowding distance to unload the archive. The proposed method is compared with strength pareto evolutionary algorithm 2 (SPEA 2) to show its better performance of producing the pareto front optimal set.

Other optimization methods in this category are interactive honey bee mating optimization (IHBMO) (Ghasemi, 2013), improved scatter search (ISS) (de Athayde Costa e Silva et al., 2013), quasi-oppositional teaching learning based optimization (QOTLBO) (P. K. Roy et al., 2013), and enhanced multi-objective cultural algorithm (EMOCA) as the combination of cultural algorithm (CA) with particle swarm optimization (PSO) (R. Zhang et al., 2013).

CHAPTER 3 RESEARCH METHODOLOGY

3.1 Introduction

In this chapter, the mathematical formulation of power dispatch problem is explained at first. The backtracking search algorithm is then explained as the proposed method for solving the single objective and multi-objective power dispatch problems. The mechanism of constraint handling for both equality and inequality constraints by the proposed method is described as well.

3.2 Problem formulation

In the power dispatch problem, there are usually two objectives to be minimized, generation cost and emission amount. The power dispatch problem to minimize the generation cost is called economic dispatch problem and the emission dispatch problem is the minimization of the emission amount of the generating units. The power dispatch problem is explained in both power system and microgrid as follows. The formulation includes several types of power dispatch problem considering valve-point effects and multiple fuel options in cost functions, prohibited operating zones, and ramp rate limits as the operation constraints. The transmission network loss is also included in the formulation of the problems.

3.2.1 Power dispatch problem in power system

In this section, the economic dispatch problem, the emission dispatch problem, and the economic/emission dispatch problem are explained. In all problems, the constraints are same while the objectives are different.

3.2.1.1 Economic dispatch problem (ED)

ED is an optimization problem to determine the power sharing among the generating units to supply the power demand in an economical manner. The objective function of the basic economic dispatch is to minimize the generation cost while satisfying the network and generators' constraints. The conventional ED assumes quadratic cost functions for generating units while the practical problems take into account the valve-point effects and multiple fuel options to model the accurate cost functions. Eq. (3.1) shows the simple form of cost function while the Eq. (3.2) considers the valve-point effects by adding a sinusoidal term to the quadratic cost function. Eq. (3.3) illustrates the cost function including valve-point effects and multiple fuel options. Figure 3.1 also shows the convex and non-convex cost functions due to valve-point effects.

$$F_{c,i}(P_i(\mathrm{t})) = a_i P_i^2(\mathrm{t}) + b_i P_1(\mathrm{t}) + c_i \qquad P_i^{\min} \le P_i \le P_i^{\max} \tag{3.1}$$

$$F_{c,i}(P_i(t)) = a_i P_i^2(t) + b_i P_i(t) + c_i + \left| e_i \times sin(f_i (P_i^{\min} - P_i(t))) \right| \qquad P_i^{\min} \le P_i \le P_i^{\max} \tag{3.2}$$

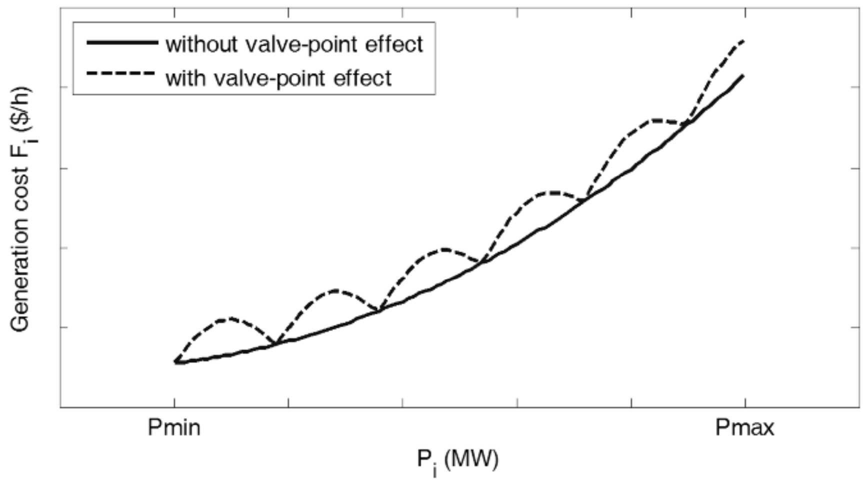
$$F_{c,i}(P_i(t)) = \begin{cases} a_{i1} P_i^2(t) + b_{i1} P_i(t) + c_{i1} + \left| e_{i1} \times sin(f_{i1} (P_i^{\min} - P_i(t))) \right| & P_i^{\min} \le P_i \le P_{i1} \\ a_{i2} P_i^2(t) + b_{i2} P_i(t) + c_{i2} + \left| e_{i2} \times sin(f_{i2} (P_{i2}^{\min} - P_i(t))) \right| & P_{i1} \le P_i \le P_{i2} \\ \vdots & \vdots \\ a_{ik} P_i^2(t) + b_{ik} P_i(t) + c_{ik} + \left| e_{ik} \times sin(f_{ik} (P_{ik}^{\min} - P_i(t))) \right| & P_{i(k-1)} \le P_i \le P_i^{\max} \end{cases} \tag{3.3}$$

where a, b, c, e, and f are cost coefficients, and also subscripts i and k denote the i^{th} generating unit and k^{th} fuel type, respectively.

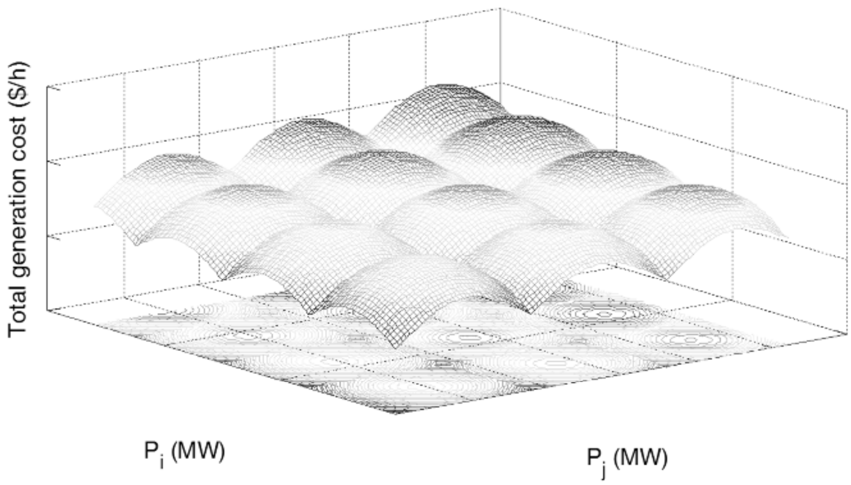
The generation cost as the objective function is defined as Eq. (3.4).

$$Min.\ F_c(\mathbf{P}(t)) = \sum_{i=1}^{N} F_{c,i}(P_i(t)) \qquad \mathbf{P}(t) = [P_1(t), P_2(t), ..., P_N(t)] \tag{3.4}$$

where **P**(t) is the generation vector representing the generations of all units.



(a)



(b)

Figure 3.1. a) Convex and non-convex generation cost function of a generator; b) generation cost of a 2-unit system with non-convex cost functions

3.2.1.2 Emission dispatch problem (EMD)

The emission dispatch problem aims to minimize the emission amount produced by the generators subject to the operating constraints of the network and generators. The emission functions of all the pollutants including CO_2, NO_X, SO_2 are usually represented by quadratic functions. However, the combination of both quadratic and exponential functions is considered in determining the total emission caused by the generators. Eq. (3.5) shows the total emission level of the pollutants.

$$F_{e,i}(P_i(t)) = \alpha_i P_i^2(t) + \beta_i P_i(t) + \gamma_i + \xi_i \; e^{\lambda_i P_i(t)} \tag{3.5}$$

where α, β, γ, ζ, and λ are the coefficients of the ith generator emission function.

The total emission level of N generating units is defined by Eq. (3.6).

$$F_e(\mathbf{P}(t)) = \sum_{i=1}^{N} F_{e,i}(P_i(t)) \qquad \mathbf{P}(t) = [P_1(t), P_2(t), ..., P_N(t)] \tag{3.6}$$

where **P**(t) is a vector including the power outputs of the generators.

3.2.1.3 Economic/emission dispatch problem (EED)

Satisfying power demands while minimizing objective functions, emission and generation cost, requires treating the challenge as a multi-objective economic/emission dispatch problem. There are different methodologies to solve the EED problems. In this study, the weighted sum method and non-dominated approach are employed for dealing with EED problems.

3.2.1.3.1 weighted sum method (WSM)

The weighted sum method transforms a set of objective functions into a single objective (Jubril et al., 2014). Each objective is multiplied by a user-supplied weight which is usually in proportion to the importance of the objective. It is thus assigned a different order of magnitude in the combined economic/emission dispatch problem. Eq. (3.7) is the combined objective function that considers the price penalty factor σ ($/ton) necessary to reflect the different ranges of values of each objective.

$$F = wF_c + (1-w)\sigma F_e \tag{3.7}$$

Where *w* is the weighting factor, which can be any number between 0 and 1. The factor σ is determined by Eq. (3.8), which represents the ratio of the maximum generation cost to the maximum emission amount.

$$\sigma = \frac{F_c(\mathbf{P}^{\max})}{F_e(\mathbf{P}^{\max})} \tag{3.8}$$

3.2.1.3.2 Multi-objective optimization: non-dominated approach (NDA)

The objective function is written as the vector of both objectives and neither is inferior to the other, i.e., the generation cost and emission amount are minimized simultaneously. This method of formulation is called non-dominated approach (NDA), detailed in Section 3.6.6. Eq. (3.9) specifies the objective function **F** as the objective vector to be minimized. NDA aims to find the dispatch that satisfies the constraints and minimizes the vector function **F** (Abido, 2003a; Li et al., 2015).

$$\mathbf{F} = (F_c, F_e) \tag{3.9}$$

3.2.1.4 Constraints

The constraints of power dispatch problem are as follows:

A. Power Balance Constraint:

The whole power demand should be equal to the total power generated minus total transmission loss.

$$\sum_{i=1}^{N} P_i(t) - P_{loss}(t) = P_D(t) \tag{3.10}$$

where $P_i(t)$ is the i^{th} generator output power, $P_D(t)$ and $P_{loss}(t)$ are respectively the power demand and total transmission loss in the scheduled period t.

Generally, the total transmission loss ($P_{loss}(t)$) is calculated by Kron's loss formula as demonstrated in Eq. (3.11).

$$P_{loss}(t) = \sum_{i=1}^{N}\sum_{j=1}^{N} P_i(t) B_{ij} P_j(t) + \sum_{i=1}^{N} B_{0i} P_i(t) + B_{00} \tag{3.11}$$

where B, B_0, and B_{00} are the loss coefficients.

B. Generation Limits

The generation limit for each unit is given by Eq. (3.12).

$$P_i^{min} \le P_i(t) \le P_i^{max} \qquad i \in \{1,2,3,\ldots,N\} \tag{3.12}$$

where P_i^{min} and P_i^{max} are respectively the minimum and the maximum production limits of i^{th} generator.

C. Ramp rate limits

In real operating conditions, the operating range of each generating unit is restricted by its ramp-up and ramp-down limits as shown by Eqs. (3.13) and (3.14).

- If the generation increases

$$P_i(t) - P_i(t-1) \le UR_i \tag{3.13}$$

- If the generation decreases

$$P_i(t-1) - P_i(t) \le DR_i \tag{3.14}$$

where $P_i(t-1)$ and $P_i(t)$ are respectively, the previous and current output powers at time period t. UR_i and DR_i are ramp-up and ramp-down limits of i^{th} generating unit.

D. Prohibited operating zones

Each generator contains some prohibited operating zones in its operation due to the valve-point effects. In the practical operation of the power system, the output power of each unit should avoid operation in the prohibited zones. The feasible operating zone of i^{th} generator can be demonstrated by Eq. (3.15).

$$\begin{aligned} &P_i^{min} \le P_i(t) \le P_{i,1}^{l} \\ &P_{i,k-1}^{u} \le P_i(t) \le P_{i,k}^{l} \qquad k \in \{2,3,\ldots,n_i\} \\ &P_{i,n_i}^{u} \le P_i(t) \le P_i^{max} \end{aligned} \tag{3.15}$$

where $P^{l}_{i,k}$ and $P^{u}_{i,k}$ are respectively the lower and upper bounds of k^{th} prohibited operating zones of i^{th} generating unit. n_i is the number of prohibited zones of unit number i.

3.2.2 Power dispatch problem in Microgrid

Microgrids are composed of the micro-sources and storage systems to supply the power demand. Variety of micro-sources including conventional generators and renewable energy technologies are used in microgrids. The problem is to minimize the generation cost of the microgrid within the schedule period. It is considered as a single objective optimization problem subject to operation constraints of generators and the system. The main components of this problem are described as follows.

3.2.2.1 Models of Microgrid elements

In this thesis, a typical microgrid with diesel generators, wind power plants, and fuel-cell plants is considered and the models of these generating units are explained as follows:

A. Diesel generator

Diesel generator is the conventional power producer and its generation cost is modeled by a cubic or quadratic cost function. However, the quadratic form is usually taken into account. The Eq. (3.16) shows the cost function of the diesel generator.

$$F_{diesel,i}(t) = a_i + b_i P_{diesel,i}(t) + c_i P_{diesel,i}^2(t) \tag{3.16}$$

where $F_{diesel,i}(t)$ and $P_{diesel,i}(t)$ are respectively the generation cost and the output power of the i^{th} diesel unit in the scheduled period t,. The corresponding cost coefficients are a_i, b_i, and c_i.

B. Wind power plant

The power production of a wind turbine depends on the strength of the wind speed. Eq. (3.17) shows the relation between the wind speed and the output power of the wind power plant.

$$P_{wt,i}(t) = \begin{cases} 0 & v < v_{cut-in} \\ P^{r}_{wt,i} \times \dfrac{v - v_{cut-in}}{v_r - v_{cut-in}} & v_{cut-in} \le v < v_r \\ P^{r}_{wt,i} & v_r \le v < v_{cut-out} \\ 0 & v \ge v_{cut-out} \end{cases} \quad (3.17)$$

where $P^{r}_{wt,i}$ is the rated power of the wind turbine number i, v is the wind speed in (m/s), and v_{cut-in}, v_r, $v_{cut-out}$ represent the cut-in, nominal, and cut-out wind speeds, respectively.

The cost function of the wind power plant is usually represented by a linear function as demonstrated by Eq. (3.18). The coefficient of the cost function is the operation and maintenance cost of the power plant.

$$F_{wt,i}(t) = b_i P_{wt,i}(t) \quad (3.18)$$

where the $P_{wt,i}(t)$ and $F_{wt,i}(t)$ are respectively the power and generation cost of i^{th} wind power plant in the scheduled period t. The cost coefficient is also b_i.

C. Fuel-cell plant

The fuel-cell plant is another technology with high efficiency for energy production. Its generation cost model is demonstrated by the Eq. (3.19) which shows the linear relation between the generated power and the generation cost of the fuel-cell plant.

$$F_{fc,i}(t) = \frac{b_i P_{fc,i}(t)}{\eta_{fc,i}} \quad (3.19)$$

where $F_{fc,i}(t)$ and $P_{fc,i}(t)$ are the generation cost and output power of fuel-cell plant at time t, respectively. The coefficient b_i is also the cost of natural gas in (\$/kg) and the $\eta_{fc,i}$ is the fuel-cell efficiency.

3.2.2.2 Objective function

The objective function is to optimize microgrid operation through its generation cost minimization. Eq. (3.20) shows the objective function for a microgrid including diesel, wind power, and fuel-cell generators. The horizon of 24 hours is considered for the generation scheduling problem.

$$F_{total} = \sum_{t=1}^{T}\left(\sum_{i=1}^{N_D} F_{diesel,i}(t) + \sum_{i=1}^{N_{wt}} F_{wt,i}(t) + \sum_{i=1}^{N_{fc}} F_{fc,i}(t)\right) \tag{3.20}$$

where N_D, N_{wt}, and N_{fc} are numbers of diesel units, wind turbines, and fuel-cell plants, respectively. Parameter T is the scheduling period and the F_{total} is the total generation cost within period T.

3.2.2.3 Constraints

The problem of microgrid optimization consists of two types of constraints; equality constraints and boundary limits.

A. Power balance constraints

The power generated by all distributed generations should meet the power demand in each scheduled period t shown in Eq. (3.21).

$$\begin{gathered}\sum_{i=1}^{N_D} P_{diesel,i}(t) + \sum_{i=1}^{N_{wt}} P_{wt,i}(t) + \sum_{i=1}^{N_{fc}} P_{fc,i}(t) = P_D(t) \\ t = \{1,2,3,\ldots,T\}\end{gathered} \tag{3.21}$$

B. Boundary limits

The output power of each generator should be within a lower limit and an upper limit. Eq. (3.22)-(3.24) show the boundary limits for different technologies.

$$P_{diesel,i}^{min} \le P_{diesel,i}(t) \le P_{diesel,i}^{max} \tag{3.22}$$

$$P_{wt,i}^{min} \le P_{wt,i}(t) \le P_{wt,i}^{max} \qquad t = \{1,2,\ldots,T\} \tag{3.23}$$

$$P_{fc,i}^{min} \le P_{diesel,i}(t) \le P_{fc,i}^{max} \tag{3.24}$$

3.3 Backtracking Search Optimization Algorithm (BSA)

BSA is an evolutionary optimization tool developed by (Civicioglu, 2013) to solve optimization problems. The structure of BSA is simple and its only control parameter makes it a suitable approach to solve even multimodal optimization problems. The performance of BSA is not over sensitive to its control parameter and it does not suffer from high computation time or premature convergence unlike many evolutionary methods. BSA utilizes crossover and mutation operators to effectively explore the search domain. These operators are completely different from the ones used by other evolutionary methods, such as genetic algorithm and evolutionary programming, etc. BSA also has the advantage of a memory that defines the search direction based on the previous generations.

Figure 3.2 shows the flowchart of BSA, which comprises five main steps: initialization, selection-I, mutation, crossover, and selection-II, as mentioned next.

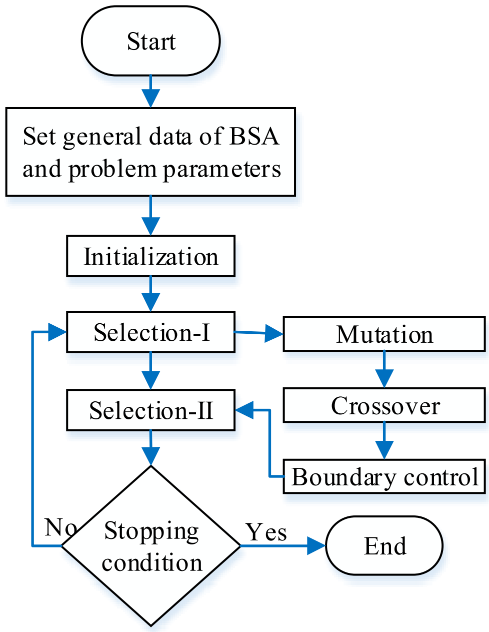


Figure 3.2. Flowchart of BSA

3.3.1 Initialization

The population and each individual are represented by $\mathbf{X} = [\mathbf{X}_1\ \mathbf{X}_2 \ldots \mathbf{X}_{nPop}]'$ and $\mathbf{X}_i = [\mathrm{x}_{i1} \ldots \mathrm{x}_{ij} \ldots \mathrm{x}_{i\,nVar}]'$ where i and j respectively denote the individual and element numbers. The initial population including nPop individuals is generated by Eq. (3.25). Each individual includes nVar optimization variables.

$$x_{ij} \sim U(low_j, up_j) \tag{3.25}$$

where:

i: stands for individual. i=(1,2,…, nPop)

j: stands for optimization variable. j=(1,2,3,…, nVar)

low_j and up_j: lower and upper limits of variable j

U: uniform distribution function

x_{ij}: is the j^{th} element of the i^{th} individual as the member of population

3.3.2 Selection-I

A historical population (**histX**) is generated in this step. **histX** and **X** have the same size and the element of $histx_{ij}$ in **histX** is considered as the counterpart of x_{ij} in **X**. First, it is initialized by Eq. (3.26) to create **histX** and then, the historical population is redefined through the "if-then" rule (by comparing two random numbers a and b) according to Eq. (3.27). Finally, the order of individuals of the population **histX** is changed randomly through Eq. (3.28). A random shuffling function is employed as the permuting function in the aforementioned equation. The historical population (**histX**) is used to determine the search direction at each iteration.

$$histx_{ij} \sim U(low_j, up_j) \tag{3.26}$$

$$if\ a < b \Big|_{a,b \sim U(0,1)} \rightarrow \mathbf{histX} = \mathbf{X} \tag{3.27}$$

$$\mathbf{histX} = permuting(\mathbf{histX}) \tag{3.28}$$

3.3.3 Mutation

An initial form of trial population (Mutant) is generated in the mutation process through Eq. (3.29). The subtraction of **X** from **histX** determines the search direction and the function α controls the amplitude of the search direction. The function of α=3.randn is considered where randn is a random number based on the standard normal distribution.

$$Mutant = \mathbf{X} + \alpha.(\mathbf{histX} - \mathbf{X}) \qquad (3.29)$$

3.3.4 Crossover

The Mutant, as the initial form of trial population set in the previous step, is finalized in the crossover process. The crossover process changes the Mutant to the final trial population T through the crossover operator. The value of T is set to the Mutant at first. A binary matrix (map) is then generated randomly with nPop rows and nVar columns. Each row of the matrix "map" is relevant to an individual. The number of elements of any individual to be engaged in the crossover process is controlled by the single control parameter of BSA named "mixrate". This control parameter (ranges from 0-100% of nVar elements) determines the maximum number of elements in each row of the binary matrix "map" to be equal to 1. There are two strategies in BSA crossover process: one is to only engage a random element of each individual in this process, and another one is to select maximum mixrate numbers of elements of the individuals to be manipulated in the crossover process. Based on the strategy, the binary matrix (map) is created at first and those elements of T with the corresponding value of 1 in the matrix (map) are to be manipulated. In this case, these elements of T are set to be equal to the relevant elements of P. In other words, if $map_{ij}=1$ then $T_{ij}=P_{ij}$.

3.3.5 Boundary control

At the end of the crossover process, it may occur that some elements of individuals violate their boundary limits. In this situation, they are regenerated by Eq. (3.25) or they are set to the upper or lower limits. The strategy of the boundary control is defined by an if-then rule. Two random numbers are generated at first. If one of the numbers is greater than another one, then the violated element of the individual is fixed to the upper or lower limit, otherwise, it is regenerated by Eq. (3.25).

3.3.6 Selection-II

In Selection-II stage, each individual of T is compared with the relevant individual of P in terms of better fitness value. Then, the individuals of P are updated based on the comparison. The best individual among the population members is also updated in this process.

3.3.7 BSA's control parameter and stopping condition

The parameter "mixrate" used in the crossover process is the only control parameter of BSA during optimization. The value of this parameter varies between 0-100% of the number of individuals. Although the optimization by BSA is not over sensitive to this parameter, it should be tuned properly to get the best optimal.

A stopping condition also has to be defined to control the optimization process. The maximum number of iterations is considered as the stopping condition.

3.4 Constraint handling mechanisms by BSA

Before implementing BSA for solving the power dispatch problems, the mechanisms of constraints handling needs to be described. There are two ways of constraint handlings in optimization. The first way is to aggregate the penalized constraints with the objective function and to create a fitness function (Zare et al., 2012). In this case, the optimization method is to optimize the fitness function rather than the objective function. The second way is to start the optimization with the feasible set of solutions and to work with only feasible solutions within the optimization process (Basu, 2013, 2014b; Basu & Chowdhury, 2013; A. Bhattacharya et al., 2010a, 2010b; Cheng-Chien, 2008; Ciornei et al., 2012; Vo et al., 2013). Each way can be done by different mechanisms. In this thesis, the second way is adopted and two mechanisms for this way of constraint handling are taken into account for solving the power dispatch problems described next.

3.4.1 Constraint handling through feasible search space exploration-static slack generator

In this mechanism, the optimization is initialized by feasible set of solutions and the optimizer searches only the feasible search space within the optimization process. In each iteration of optimization, the individuals of population may violate the equality and inequality constraints. A strategy needs to be employed to repair the individual to make it as a feasible solution.

To generate an individual of the population, **P=[**P_1,P_2,..,P_N**]**, a specific generator (i.e. N^{th} generator) is selected as the slack generator. The power levels of first (N-1) generators are generated randomly by considering the inequality constraints and the power level of N^{th} generator is calculated through the following method.

Assume that the (N-1) elements of a solution, P=[P_1, P_2, P_3, …, P_N], is known. The last element of generation vector P is then calculated in such a way that the equality constraint is satisfied. The first (N-1) elements are considered as independent variables, so the equality constraint makes the last element as the dependent variable. This element P_N is calculated by Eq. (3.30) which is achieved by extracting the P_N from the Eq. (3.10).

$$C_2 P_N^2 + C_1 P_N + C_0 = 0 \tag{3.30}$$

$$C_2 = B_{NN} \tag{3.31}$$

$$C_1 = 2 \times \sum_{i=1}^{N-1} P_i B_{iN} + B_{0N} - 1 \tag{3.32}$$

$$C_0 = \sum_{i=1}^{N-1} \sum_{j=1}^{N-1} P_i B_{ij} P_j + \sum_{i=1}^{N-1} (B_{0i} - 1) P_i + B_{00} + P_D \tag{3.33}$$

where B, B_0, and B_{00} are transmission loss coefficients.

The Eq. (3.30) is polynomial and the value of P_N is calculated by Eq. (3.34). The positive root is chosen as P_N in order to satisfy the equality constraint.

$$P_N = \frac{-C_1 \pm \sqrt{C_1^2 - 4C_2C_0}}{2C_2} \tag{3.34}$$

If the value of this element violates the constraints specified in Eqs. (3.12)-(3.15) then the procedure needs to be repeated again until the positive root satisfies the operation limit and other inequality constraints.

Since the slack generator is fixed in this mechanism and it is not changed within the optimization process, this mechanism is called static slack generator (SSG). BSA with this mechanism of constraint handling is called BSA_{SSG}.

3.4.2 Constraint handling through feasible search space exploration-dynamic slack generator

The difference between this mechanism and the previous one is the way that the feasible solution is generated and modified within the optimization. In this mechanism of constraint handling, the optimization is also initialized by feasible set of solutions and the optimizer searches only the feasible search space. In this case, an individual of the population, **P=[**P_1,P_2,..,P_N**]**, is generated by considering the generation limits, ramp rate limits, and the prohibited operating zones according to the Eqs. (3.12)-(3.15). While the equality constraint with loss considered as shown in Eq. (3.10) is not satisfied, a random generator is chosen as the slack generator and its output is fixed to meet the equality constraint. If the output of the slack generator violates its boundary limits, another random generator from (N-1) pool is chosen as the slack generator. If no one can cover the difference to satisfy the equality constraint, then two slack generators are chosen and share the difference. When a generator is in a prohibited operation zone, the closest feasible bound is set as the output.

In this mechanism of constraint handling, the slack generator is dynamically changed to make the solutions feasible. This mechanism is called dynamic slack generator (DSG) in this thesis. BSA with this mechanism of constraint handling is called BSA_{DSG}.

3.5 Implementing of BSA for solving the single objective power dispatch problems

Backtracking search algorithm (BSA) is a population based metaheuristic method. It starts with an initial population and converges to an optimal solution through crossover and mutation operators. In this method, each individual stands for a solution and the population is composed of a specific number of individuals. Since two mechanisms of constraint handling are taken into account, this section represents the BSA implementation for solving the power dispatch problems based on the mechanism considered for the constraint handling. In these two mechanisms of constraint handling, BSA explores only the feasible search space of the problem. The algorithm's steps are as follows.

Step 1: initialization

The initialization of population X is performed by considering the constraint handling mechanism to generate the feasible individuals in the population.

Step 2: selection I

The historical population is generated in the same way that X is initialized. It is redefined through the Eqs. (3.27) and (3.28).

Step 3: Mutation

The mutation operator is applied on the population to generate the initial trial population.

Step 4: Crossover

The crossover operator is applied on the initial trial population by setting the control parameter to form the final trial population.

Step 5: Making the new solutions feasible

The individuals of the final trial population may violate the inequalities, so, the mechanism of constraint handling is applied on the final trial population to make the individuals as feasible solutions.

Step 5: Selection-II

The objective function of the power dispatch problem is used to update the population's individuals.

Step 5: Stopping condition

The algorithm stops when the maximum number of iterations during the optimization had reached a predetermined value.

Figure 3.3 shows the flowchart of BSA for solving the power dispatch problem through the mechanism of feasible search space exploration by employing the static slack generator. As mentioned, BSA with this mechanism is called BSA_{SSG}. The flowchart of the proposed method with considering the dynamic slack generator in the constraint handling mechanism is depicted in Figure 3.4. BSA with the second mechanism incorporated in its algorithm is called BSA_{DSG}.

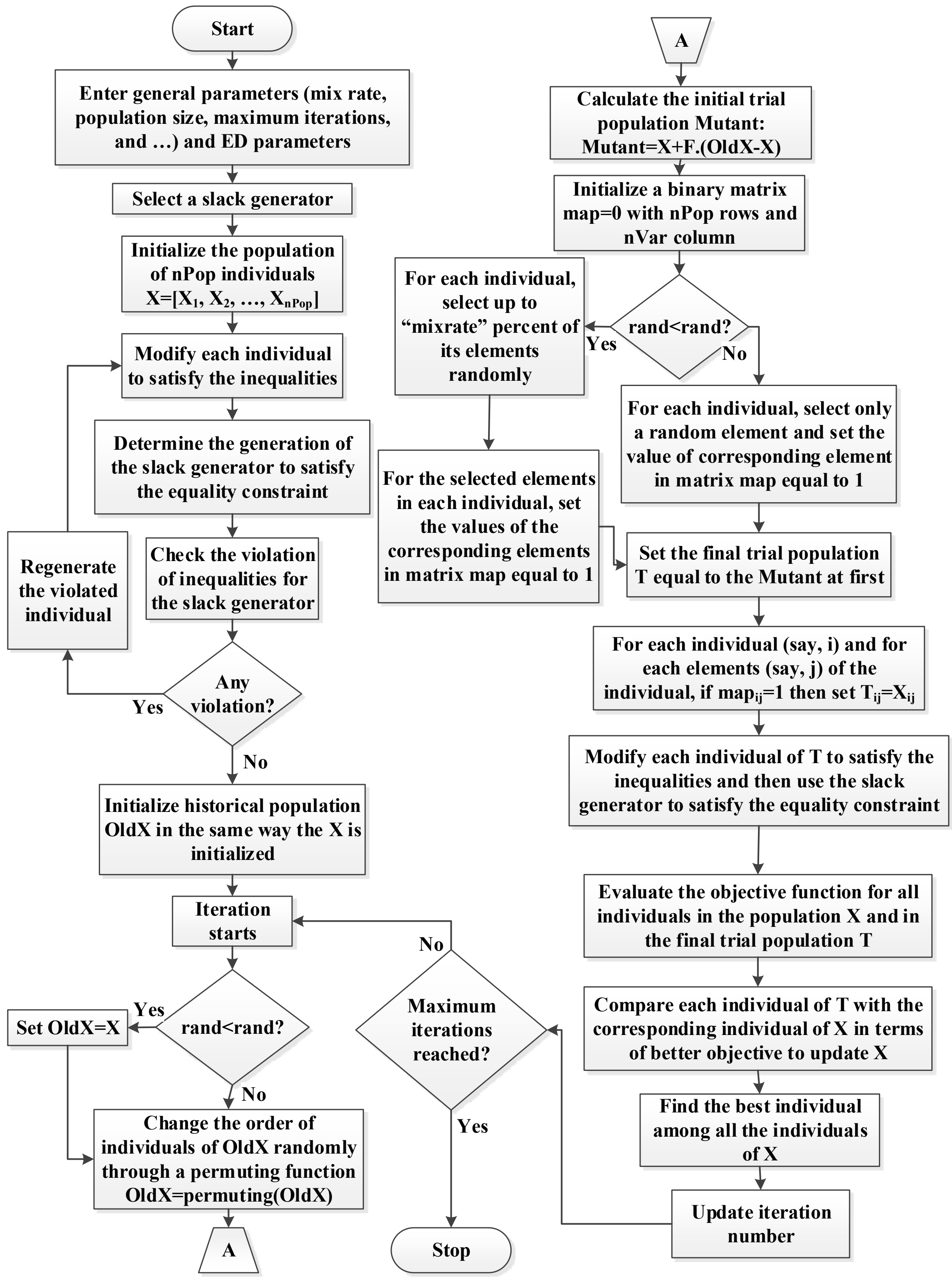


Figure 3.3. flowchart of BSA for solving the ED problem through the mechanism of feasible search space exploration-static slack generator

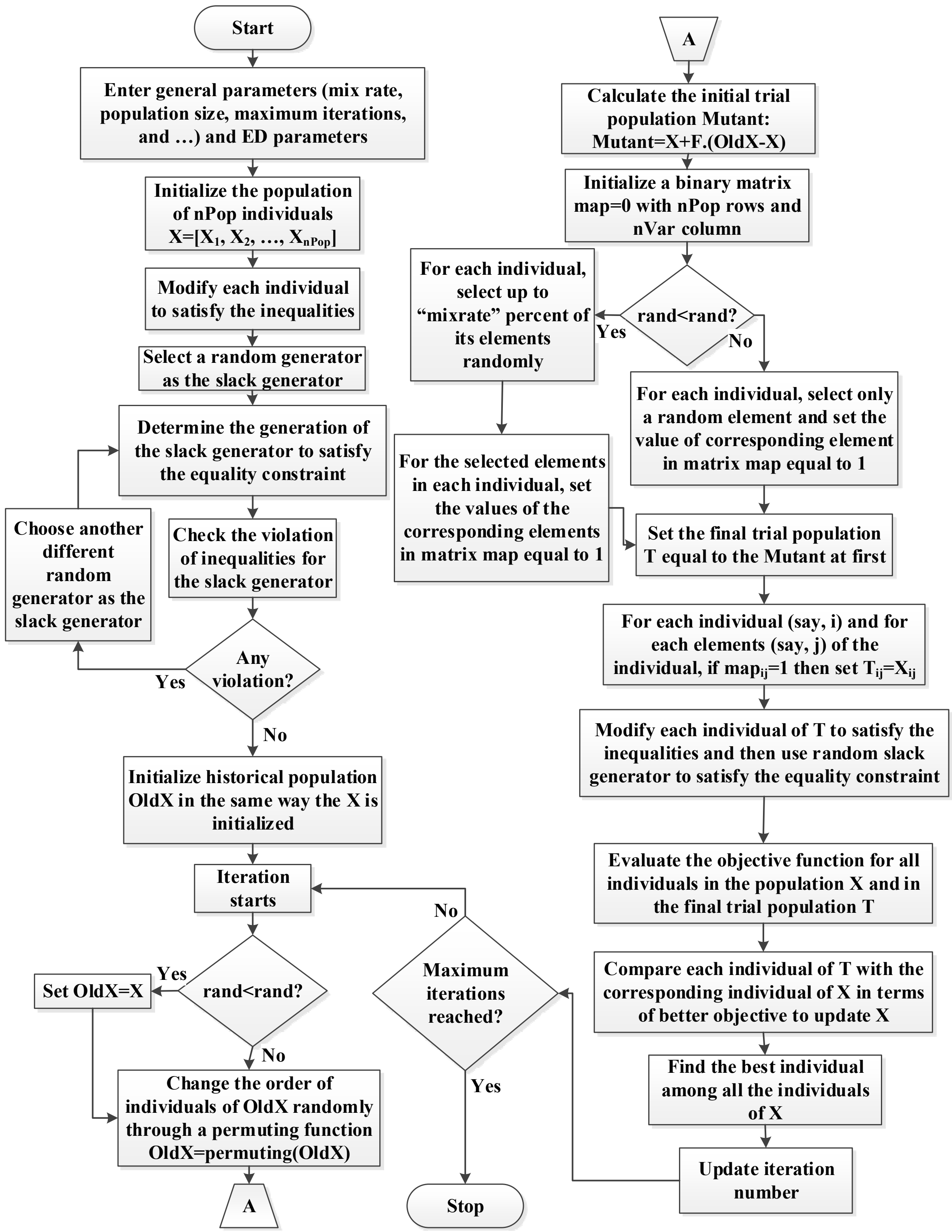


Figure 3.4. flowchart of BSA algorithm for solving the ED problem through the mechanism of feasible search space exploration-dynamic slack generator

3.6 Multi-objective Backtracking Search Algorithm (MOBSA)

Each evolutionary algorithm uses techniques inspired by natural or biological evolutions to generate superior solutions through optimization. Such techniques include mutation, crossover, and selection, to be applied to each individual of a population. Among these algorithms, BSA is a new evolutionary method to solve multimodal optimization benchmarks. In this section, the multi-objective BSA is developed to solve the economic/emission dispatch problem. The mathematical formulation of BSA needs to be described at first for the purpose of developing the multi-objective BSA.

3.6.1 Basic BSA

BSA starts with a population of individuals, generated randomly in the search space. It leads to a better population specified by a fitness function in the next iteration. It uses a control parameter and several operators in the optimization process. The five major steps of BSA, described briefly next, are initialization, selection-I, mutation, crossover, and selection-II. Each iteration begins from the Selection-I step and ends in the Selection-II step.

Let us assume $\Omega \subset R^n$ to be the search space of the problem. BSA uses nPop individuals as the population in each generation of the algorithm. The population and each individual are represented by $\mathbf{X}^t = [\mathbf{X}_1^t \; \mathbf{X}_2^t \; \ldots \; \mathbf{X}_{nPop}^t]'$ and $\mathbf{X}_i^t = [\mathrm{x}_{i1}^t \; \mathrm{x}_{i2}^t \; \ldots \; \mathrm{x}_{in}^t]'$ where t and i respectively denote the iteration and the individual number. The population is initialized randomly so the individuals are uniformly distributed in the search space.

Step 1: Initialization

The iteration number is set as t=0, randomly initializing the population **X** in the search space Ω.

Step 2: Selection-I

The historical population (**histX**^{t}) is generated in this step. It is initialized the same way that population X is initialized. The **histX**^{t} is then redefined through a simple "if-then" rule according to Eq. (3.35).

$$if\ a < b \Big|_{a,b \sim U(0,1)} \rightarrow \mathbf{histX}^t = \mathbf{X}^t \tag{3.35}$$

Note that the historical population **histX**^{t} is initialized randomly in this step. If the rule of Eq. (3.35) is satisfied, then the value of **histX**^{t} is changed to $\mathbf{X}^t$, otherwise, its initial value is used in the next calculation.

Finally, a permuting function is applied to the historical population to change the order of the individuals randomly. A random shuffling function is used as the permuting function in Eq. (3.36).

$$\mathbf{histX}^t = permuting(\mathbf{histX}^t) \tag{3.36}$$

Step 3: Mutation

The mutation operator generates the initial form of the trial population $\mathbf{V}^{t+1} = [\mathbf{V}_1^{t+1}\ \mathbf{V}_2^{t+1} \ldots \mathbf{V}_{np}^{t+1}]'$ through Eq. (3.37). Each individual of $\mathbf{V}^{t+1}$ is relevant to an individual of $\mathbf{X}^t$**.**

$$\mathbf{V}^{t+1} = \mathbf{X}^t + \alpha.(\mathbf{histX}^t - \mathbf{X}^t) \tag{3.37}$$

Where α is a function to control the amplitude of the term (histX^t-X^t) as the search direction matrix. The function of α=3.randn, where randn~N(0,1) (N is the standard normal distribution), is usually used.

Step 4: Crossover

The initial trial population $\mathbf{V}^{t+1}$ (as the mutant matrix) is finalized in this step by applying the crossover operator. In the process, BSA uses a control parameter called mixrate to determine the maximum number of elements of each individual of $\mathbf{V}^{t+1}$ to be engaged and manipulated. The random binary matrix 'map' with the same size of $\mathbf{V}^{t+1}$ is generated. The parameter 'mixrate' controls the maximum number of elements in each row of matrix 'map' with the value of 1. The final trial population $\mathbf{U}^{t+1}$ is then determined through Eq. (3.38).

$$\mathbf{U}_{ij}^{t+1} = \begin{cases} \mathbf{V}_{ij}^{t+1} & \text{if } \mathbf{map}_{ij} = 1 \\ \mathbf{X}_{ij}^{t} & \text{if } \mathbf{map}_{ij} = 0 \end{cases} \tag{3.38}$$

where i and j denote the element of the i^{th} row and the j^{th} column of the matrices; U^{t+1} is the finalized form of the trial population.

After the crossover, some individuals of U^{t+1} might violate the boundaries of the optimization variables, so they need to be checked and modified by an appropriate mechanism.

Step 5: Selection-II

In the Selection-II step, each individual of population U^{t+1} is compared in terms of fitness value with its counterpart in X^t to update X^t. The global minimum within the individuals is also updated. The optimization process again repeats from step 2 unless the stopping criteria are satisfied.

3.6.2 Pareto optimal set

In the multi-objective optimization approach, several functions need to be optimized simultaneously. So, there is no unique solution corresponding to the optimal value of each objective. Instead, there is a set of solutions known as the pareto front set. In the procedure of multi-objective optimization, the pareto front is updated in each iteration and its members are stored in an archive described next.

3.6.3 External elitist archive

The pareto front set including the non-dominated solutions is obtained in each generation of the evolutionary algorithm. These solutions, compared with those in the preceding iterations, might not be non-dominated. An external elitist archive is thus required to store and update the pareto front members in each iteration. The external archive, initially empty, stores the non-dominated solutions as the optimization progresses. The archive has three rules for when a new solution (the trial vector) enters it: (1) the trial vector dominates some of the archive members such that the dominated ones are deleted from the archive; (2) the trial vector is dominated by at least one member from the archive such that it is rejected from inclusion in the archive; (3) the trial vector is not dominated by the archived members and the archived members are not dominated by the trial vector, i.e., the trial vector belongs to the archive so it enters the archive as a collection of the latest non-dominated solutions. The number of archive members increases as the optimization progresses. When the population of the elitist archive reaches its maximum capacity, a measure called crowding distance removes extra members to keep the archive to its maximum size.

3.6.4 Crowding distance

The crowding distance (CD) is a quality measure for pareto front distribution. When the external elitist archive overloads, the extra members of the archive can be removed according to the values of the crowding distance. The measure estimates the density around a solution in the pareto front. It usually is the average distance of two neighbor points around the solution along each of the objectives. It is calculated by Eq. (3.39) for the i^{th} solution of the pareto front (de Athayde Costa e Silva et al., 2013).

$$CD_{\mathrm{i}} = \sum_{\mathrm{j}=1}^{k} \frac{f_{\mathrm{j}}(i+1) - f_{\mathrm{j}}(i-1)}{f_{\mathrm{j}}^{\max} - f_{\mathrm{j}}^{\min}} \tag{3.39}$$

where f_j is the j^{th} objective function, k the number of objectives, and f_j^{max} and f_j^{min} respectively the maximum and minimum values of the j^{th} objective function. Since there is only one neighbor point for the boundary solutions (solutions with the smallest and largest objective values) of the pareto front, the value of the crowding distance is set to infinite for the boundaries. The solution with the greater *CD* is preferred to be in the archive, i.e., the solution with the lowest crowding distance value is subject to deletion when the archive unloads.

3.6.5 Best compromise solution

Multi-objective optimization yields pareto front as a set of optimal solutions rather than a single optimal. Any solution in the pareto front is not inferior to another, and improvement to one objective cannot be achieved without sacrificing another. There should thus be a mechanism of choosing a solution that satisfies each objective to some extent. A trade-off between solutions should lead to the best compromise solution.

A challenging way of selecting the best compromise solution is to use fuzzy set theory to determine the best candidate among the pareto front efficiently. Usually a member function is assigned to each objective function according to Eq. (3.40) (Abido, 2006).

$$\mu_{i,j} = \begin{cases} 1 & f_j(X_i) \le f_j^{min} \\ \dfrac{f_j^{max} - f_j(X_i)}{f_j^{max} - f_j^{min}} & f_j^{min} \le f_j(X_i) \le f_j^{max} \\ 0 & f_j(X_i) \ge f_j^{max} \end{cases} \tag{3.40}$$

where $f_j(\mathbf{X}_i)$ is the value of the jth objective function for the ith solution (X_i) and f_j^{max} and f_j^{min} respectively the maximum and minimum values of the jth objective function. The membership function represents the objective function's degree of optimal achievement ranging from zero to one. The values of μ=1 and 0 correspond to completely satisfactory and unsatisfactory conditions, respectively. Figure 3.5 shows the membership function for the objective function f. To specify the best compromise solution among the non-dominated solutions, a normalized membership function needs to be calculated first by Eq. (3.41).

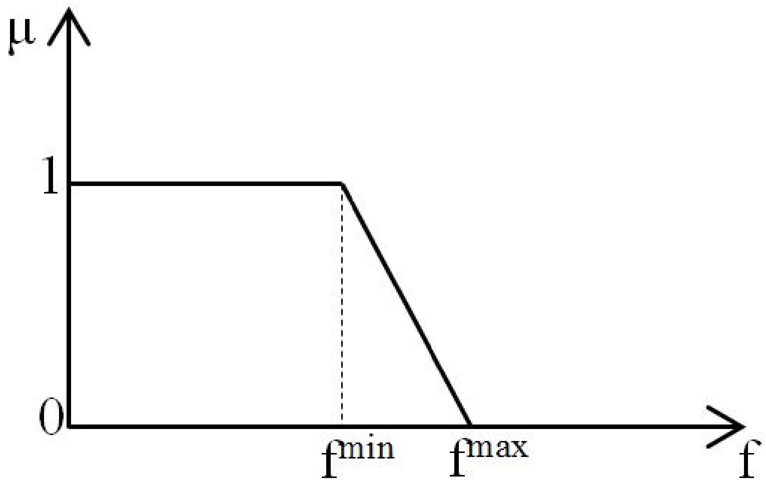


Figure 3.5. Fuzzy-based membership function

$$\mu_i = \frac{\sum_{j=1}^{k} \mu_{i,j}}{\sum_{i=1}^{m} \sum_{j=1}^{k} \mu_{i,j}} \quad (3.41)$$

where m and k are respectively the number of non-dominated solutions and the number of objective functions. The solution with the highest value of μ is selected as the best compromise solution.

3.6.6 Procedure of multi-objective BSA: non-dominated approach

As mentioned, BSA deals with the population $\mathbf{X}^t = [\mathbf{X}_1^t\ \mathbf{X}_2^t\ ...\ \mathbf{X}_{nPop}^t]'$, where $\mathbf{X}_i^t = [\mathrm{x}_{i1}^t\ \mathrm{x}_{i2}^t\ ...\ \mathrm{x}_{in}^t]'$, in each generation of the whole evolution process. The mutation and crossover operators are first applied to produce the offspring population $\mathbf{U}^{t+1}$, then the individuals of $\mathbf{X}^t$ and of $\mathbf{U}^{t+1}$ are compared in the Selection-II step of the algorithm. To extend the BSA to multi-objective optimization application, the comparison needs to be modified according to the concept of pareto dominance. When the individual $\mathbf{X}_i^t$ is compared with the individual $\mathbf{U}_i^{t+1}$, up to three situations may occur: (1) $\mathbf{X}_i^t$ is dominated by $\mathbf{U}_i^{t+1}$ ($\mathbf{U}_i^{t+1} \prec \mathbf{X}_i^t$); (2) $\mathbf{X}_i^t$ dominates $\mathbf{U}_i^{t+1}$ ($\mathbf{X}_i^t \prec \mathbf{U}_i^{t+1}$); and (3) neither $\mathbf{X}_i^t$ dominates $\mathbf{U}_i^{t+1}$ nor $\mathbf{U}_i^{t+1}$ dominates $\mathbf{X}_i^t$ ($\mathbf{X}_i^t \nprec \mathbf{U}_i^{t+1}$ and $\mathbf{U}_i^{t+1} \nprec \mathbf{X}_i^t$). In the first situation, $\mathbf{U}_i^{t+1}$ is selected as the individual of the next population $\mathbf{X}_i^{t+1}$ but the in the second and third situations, $\mathbf{X}_i^t$ is selected. The following steps represent the multi-objective BSA method with external elitist archive and crowding distance measure.

Step 1: Set the iteration number t=0, randomly initialize the population $\mathbf{X}^t = [\mathbf{X}_1^t\ \mathbf{X}_2^t\ ...\ \mathbf{X}_{np}^t]'$ where $\mathbf{X}_i^t = [x_{i1}^t\ x_{i2}^t\ ...\ x_{in}^t]'$ in the search space Ω.

Step 2: Evaluate the objective function of each individual of $\mathbf{X}^t$and save the non-dominated solutions from among the population members into the external elitist archive.

Step 3: Initialize the historical population (**hist**$\mathbf{X}^t$) similar to $\mathbf{X}^t$ and redefine and modify it through Eqs. (3.35) and (3.36).

Step 4: Apply the mutation operator to the population to determine the trial population $\mathbf{V}^{t+1} = [\mathbf{V}_1^{t+1}\ \mathbf{V}_2^{t+1}\ ...\ \mathbf{V}_{np}^{t+1}]'$ through Eq. (3.37).

Step 5: Apply the crossover operator to the trial population $\mathbf{V}^{t+1}$ to obtain the final trial population $\mathbf{U}^{t+1} = [\mathbf{U}_1^{t+1}\ \mathbf{U}_2^{t+1}\ ...\ \mathbf{U}_{np}^{t+1}]'$ through Eq. (3.38) and then check and modify the constraints.

Step 6: Compare each individual of $\mathbf{U}^{t+1}$with its counterpart from $\mathbf{X}^t$ to determine the individuals of $\mathbf{X}^{t+1}$. Use Eq. (3.42) for the comparison.

$$\mathbf{X}_i^{t+1} = \begin{cases} \mathbf{U}_i^{t+1} & \mathbf{U}_i^{t+1} \prec \mathbf{X}_i^t \\ \mathbf{X}_i^t & \textit{otherwise} \end{cases} \tag{3.42}$$

Step 7: Update the external elitist archive through its three aforementioned update rules. If the archive exceeds its capacity, remove the less crowded solutions one by one from the archive.

Step 8: set t=t+1 and then check the stopping criteria, If algorithm needs to be repeated, return to step 3.

CHAPTER 4 : OPTMIZATION RESULTS OF ECONOMIC DISPATCH

4.1 Introduction

Economic dispatch (ED) problems are solved by backtracking search algorithm (BSA) with two constraint handling mechanisms and simulation results are discussed in this chapter. In the ED problems, the valve-point effects are addressed in the generators cost functions for considering an accurate cost model. The prohibited operating zones as well as ramp-up and ramp-down constraints are also taken into account for practical purposes of the economic dispatch among the generating units. In addition, the ED problem with multiple fuel options and valve point effects is also solved by the proposed method because it is a real-world situation in the system operation. For validating the proposed method for large-scale applications, the highly nonlinear ED problem including valve-point effects and multiple fuel options is also investigated. Several case studies by considering the valve-point effects and transmission loss are discussed in the first part of this chapter and the next part is related to the ED problems with prohibited operating zones and multiple fuel options. The ED results of the proposed method with the constraint handling mechanisms for large-scale test systems with the most nonlinear cost functions are explained in the second part of this chapter. The performance of BSA with each constraint handling mechanism is analyzed. The solution quality of the optimization results of each mechanism is compared with the other one to select the suitable mechanism for constraint handling of the power dispatch problem. Then, the power dispatch problem of a microgrid with several renewable and conventional power plants is solved and the results are compared with other methods from the literature. The programming code was written in Matlab and executed on a personal computer with Pentium 2.70 GHz processor and 2 GB RAM.

4.2 ED problems with valve-point effects and transmission network loss

The proposed method's robustness and capability for solving ED problems with valve-point effects and transmission loss are validated through four case studies. The total transmission loss is modeled to consider the electric network whereas the valve-point effect is incorporated for accuracy of the cost model of each generating unit. As mentioned in chapter 3, BSA has only one control parameter named "mixrate". This parameter controls the maximum number of elements of the individuals to be engaged in the crossover process and it ranges from 0% to 100% of problem dimension. Although the performance of BSA is not over sensitive to this parameter, the tuned value is employed in each case study to achieve the best solutions. The constraint handling mechanisms described in the chapter 3 are employed for solving the ED problems in case 1 to case 4 and the results are compared to each other in terms of solution quality.

4.2.1 Case 1: 3-unit system with non-convex cost function

This case study consists of three generating units with non-convex cost functions. The total demand in this case study is 850 MW and the transmission system loss is neglected. The system data is as shown in Appendix (Table A.1). The ED problem is solved by BSA with two constraint handling mechanisms (BSA_{SSG} and BSA_{DSG}) and the results are discussed as follows.

4.2.1.1 Solution to ED problem by BSA_{SSG} and BSA_{DSG}

The optimization parameters need to be set properly to improve the performance of BSA. For both constraint handlings, different values of parameters including the maximum iteration, population size, and mixrate are chosen and the optimization has been run for 50 trials. In each case corresponding to the specific values of the parameters, the statistical indices of the results of ED have been calculated. Table 4.1

and Table 4.2 list the parameters and the statistical indices of both the generation cost (as the objective function) and the computation time for BSA_{SSG} and BSA_{DSG}.

Table 4.1 shows that the solution quality of BSA_{SSG} improves either by increasing population size or maximum iteration number. The computation time also increases when the population size or maximum iteration number increases.

Table 4.1. Statistical results of BSA_{SSG} for case 1 with different parameters

Max. iteration	popsize	mixrate	Generation cost ($/h)				CPU time (s)			
			Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
50	10	0	8234.0718	8236.6492	8241.6486	3.3689	0.01	0.01	0.01	0.00
50	10	0.2	8234.0717	8235.1169	8241.5875	2.2572	0.01	0.01	0.01	0.00
50	10	0.4	8234.0717	8235.2689	8241.5876	2.5819	0.01	0.01	0.01	0.00
50	10	0.6	8234.0717	8235.6072	8241.5876	2.9807	0.01	0.01	0.01	0.00
50	10	0.8	8234.0717	8235.4328	8241.5875	2.7310	0.01	0.01	0.01	0.00
50	10	1	8234.0717	8234.7643	8241.5875	2.0584	0.01	0.01	0.01	0.00
50	50	0	8234.0718	8234.3039	8241.5875	1.0580	0.03	0.03	0.03	0.00
50	50	0.2	8234.0717	8234.0780	8234.1325	0.0125	0.03	0.03	0.03	0.00
50	50	0.4	8234.0718	8234.0769	8234.1947	0.0174	0.03	0.03	0.03	0.00
50	50	0.6	8234.0717	8234.0740	8234.1184	0.0074	0.03	0.03	0.04	0.00
50	50	0.8	8234.0717	8234.0726	8234.0900	0.0029	0.03	0.03	0.03	0.00
50	50	1	8234.0717	8234.0721	8234.0815	0.0014	0.03	0.03	0.04	0.00
100	10	0	8234.0718	8235.1068	8241.5877	2.3534	0.02	0.02	0.02	0.00
100	10	0.2	8234.0717	8234.7094	8241.5875	1.8231	0.02	0.02	0.02	0.00
100	10	0.4	8234.0717	8235.1678	8241.5875	2.5483	0.02	0.02	0.02	0.00
100	10	0.6	8234.0717	8234.8303	8241.5875	2.0817	0.02	0.02	0.02	0.00
100	10	0.8	8234.0717	8234.4333	8241.5875	1.5039	0.02	0.02	0.02	0.00
100	10	1	8234.0717	8234.2298	8241.5875	1.0621	0.02	0.02	0.02	0.00
100	50	0	8234.0718	8234.0752	8234.1195	0.0077	0.05	0.06	0.06	0.00
100	50	0.2	8234.0717	8234.0719	8234.0768	0.0007	0.06	0.06	0.07	0.00
100	50	0.4	8234.0717	8234.0721	8234.0819	0.0015	0.06	0.06	0.07	0.00
100	50	0.6	8234.0717	8234.0726	8234.1019	0.0045	0.06	0.06	0.07	0.00
100	50	0.8	8234.0717	8234.0717	8234.0718	0.0000	0.06	0.06	0.07	0.00
100	50	1	8234.0717	8234.0717	8234.0718	0.0000	0.06	0.07	0.07	0.00

Table 4.2 shows the results of ED by BSA_{DSG} confirming that it can also produce solutions with high qualities for this test system. It converges to almost the same optimal value in all runs.

Table 4.2. Statistical results of BSA_{DSG} for case 1 with different parameters

Max. iteration	popsize	mixrate	Generation cost ($/h)				CPU time (s)			
			Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
50	10	0	8234.0724	8235.6664	8241.5890	2.9923	0.01	0.01	0.01	0.00
50	10	0.2	8234.0711	8236.0682	8241.5875	3.3052	0.01	0.01	0.01	0.00
50	10	0.4	8234.0715	8235.6521	8241.5873	2.9757	0.01	0.01	0.01	0.00
50	10	0.6	8234.0711	8235.1619	8241.5871	2.6066	0.01	0.01	0.01	0.00
50	10	0.8	8234.0709	8235.4675	8241.5869	2.9000	0.01	0.01	0.01	0.00
50	10	1	8234.0708	8235.8905	8241.5872	3.2337	0.01	0.01	0.01	0.00
50	50	0	8234.0721	8234.0946	8234.2057	0.0309	0.02	0.02	0.02	0.00
50	50	0.2	8234.0717	8234.0791	8234.2209	0.0227	0.02	0.02	0.02	0.00
50	50	0.4	8234.0717	8234.0790	8234.2209	0.0259	0.02	0.02	0.02	0.00
50	50	0.6	8234.0717	8234.0782	8234.2209	0.0228	0.02	0.02	0.02	0.00
50	50	0.8	8234.0717	8234.0749	8234.1394	0.0109	0.02	0.02	0.02	0.00
50	50	1	8234.0717	8234.0748	8234.2037	0.0187	0.02	0.02	0.02	0.00
100	10	0	8234.0710	8236.0610	8241.5873	3.3093	0.01	0.02	0.02	0.00
100	10	0.2	8234.0709	8235.5229	8241.5868	2.9219	0.01	0.02	0.02	0.00
100	10	0.4	8234.0708	8235.4560	8241.5868	2.8249	0.01	0.02	0.02	0.00
100	10	0.6	8234.0708	8235.5823	8241.5867	3.0328	0.01	0.02	0.02	0.00
100	10	0.8	8234.0708	8234.6888	8241.5866	2.0551	0.01	0.02	0.02	0.00
100	10	1	8234.0708	8234.9941	8241.5868	2.4600	0.02	0.02	0.02	0.00
100	50	0	8234.0717	8234.0758	8234.1311	0.0118	0.03	0.04	0.04	0.00
100	50	0.2	8234.0717	8234.0718	8234.0744	0.0004	0.04	0.04	0.04	0.00
100	50	0.4	8234.0717	8234.0725	8234.0982	0.0040	0.04	0.04	0.04	0.00
100	50	0.6	8234.0717	8234.0718	8234.0722	0.0001	0.04	0.04	0.04	0.00
100	50	0.8	8234.0717	8234.0722	8234.0941	0.0032	0.04	0.04	0.04	0.00
100	50	1	8234.0711	8234.0718	8234.0739	0.0004	0.04	0.04	0.04	0.00

Based on the tables, BSA_{SSG} and BSA_{DSG} reaches almost the same optimal of 8234.07 ($/h). However, the computation time increases by either higher population size or by the higher maximum iteration number for both BSA_{SSG} and BSA_{DSG}.

4.2.1.2 Convergence Characteristics

From the optimization results of this case, the convergence characteristics are plotted to compare both constraint handling mechanisms. The results of both mechanisms with same parameters (maximum iteration=100, popsize=50, mixrate=1) are used for plotting the convergence. Figure 4.1 shows the convergence of the generation cost for the best solutions of BSA_{SSG} and BSA_{DSG}. The figure shows that BSA_{DSG} performs better compared to BSA_{SSG} as it converges to the optimal in early iterations.

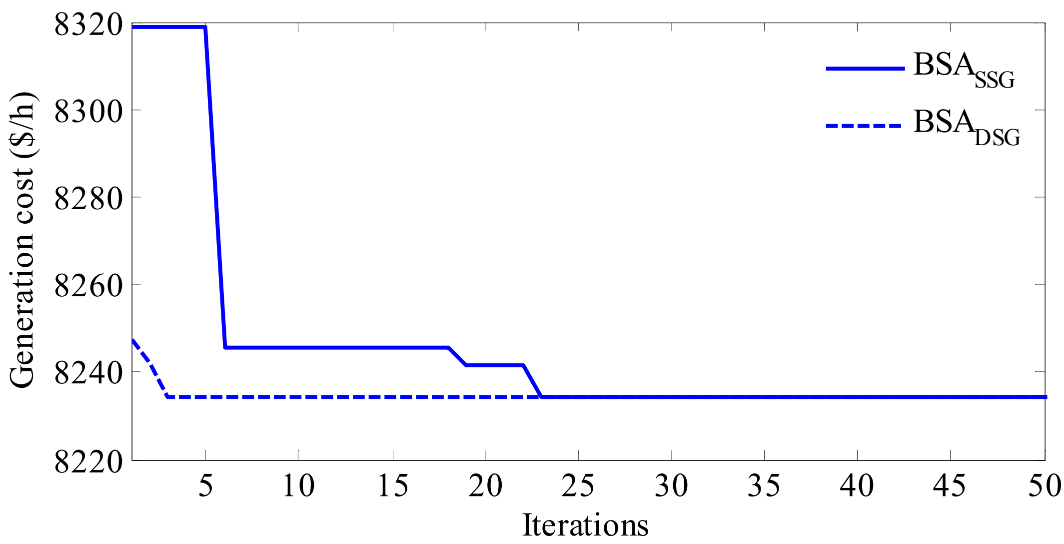


Figure 4.1. Convergence characteristic of BSA_{SSG} and BSA_{DSG} in Case 1

4.2.1.3 Robustness

Both BSA_{SSG} and BSA_{DSG} show the high robustness in this small size case study for solving of ED problem. Figure 4.2 shows the optimal obtained in 50 trials by BSA_{SSG} and BSA_{DSG} for the same optimization parameters. Both mechanisms confirm that BSA is robust for solving ED in this case study. Based on the results from Table 4.1 and Table 4.2, It is shown that the robustness of BSA is improved by increasing the population size in each constraint handling mechanism.

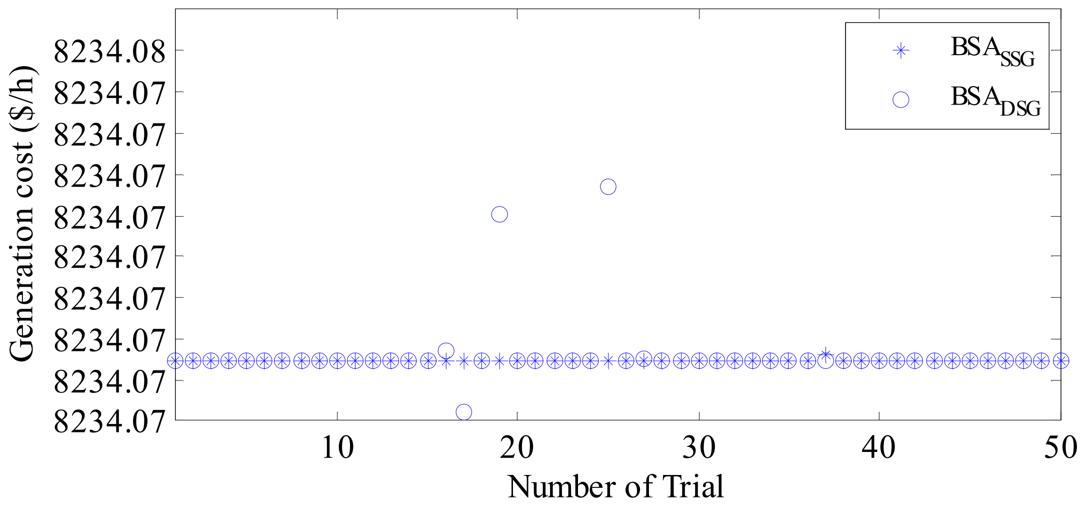


Figure 4.2. Optimal results of BSA_{SSG} and BSA_{DSG} for 50 trials in Case 1

4.2.1.4 Computational efficiency

Table 4.1 and Table 4.2 show that the computation time of BSA_{DSG} is lower than BSA_{SSG} when both methods have been run with the same parameters. However, the computation times of both methods are very low since the system size is small.

4.2.1.5 Comparison of BSA with other methods

The results of BSA_{SSG} and BSA_{DSG} and their comparisons with GA (Walters & Sheble, 1993), EP (Yang et al., 1996), MPSO (Jong-Bae et al., 2005), PS (Al-Sumait et al., 2007), GA-PS-SQP (Alsumait et al., 2010), and GA-API (Ciornei & Kyriakides, 2012) are as in Table 4.3, which shows BSA succeeding in finding the best solution for the test system. The system size is small, hence most of the methods converged to the same optimal.

Table 4.3. Best solution for Case 1 (3-unit system)

Generation	GA[1]	EP[2]	MPSO[3]	PS[4]	GA-PS-SQP[5]	GA-API[6]	BSA_{SSG}	BSA_{DSG}
P_1 (MW)	300.00	300.26	300.27	300.3	300.27	300.25	300.2669	300.2665
P_2 (MW)	400.00	400.00	400.00	400.00	400.00	399.98	400.0000	400.0000
P_3 (MW)	150.00	149.74	149.73	149.7	149.73	149.77	149.7331	149.7334
Total generations (MW)	850.00	850.00	850.00	850.0	850.00	850.00	850.0000	850.0000
Total cost ($/MW)	8237.60	8234.07	8234.07	8234.1	8234.07	8234.07	8234.07	8234.07

[1] (Walters & Sheble, 1993)
[2] (Yang et al., 1996)
[3] (Jong-Bae et al., 2005)
[4] (Al-Sumait et al., 2007)
[5] (Alsumait et al., 2010)
[6] (Ciornei & Kyriakides, 2012)

4.2.2 Case 2: 6-unit system with transmission loss

This system comprises 6 generating units. The power demand to be met by all the units in this case study is 283.4MW. The cost function is non-convex and the transmission loss is considered. The system data is as summarized in Appendix (Table A.2 and Table A.3) (Yaşar et al., 2011).

4.2.2.1 Solution to ED problem by BSA_{SSG} and BSA_{DSG}

BSA_{SSG} and BSA_{DSG} have been run with different parameter settings in this case study. The maximum iterations of 200 and 500 are selected and two population sizes of 10 and 50 are chosen. The values of 0 to 1 with the step of 0.20 are selected for the mixrate as the BSA's control parameter. In each case corresponding to the specific values of the parameters, the optimization is run for 50 times and the statistical indices of the results are calculated. Table 4.4 and Table 4.5 show the simulation parameters and the statistical indices of the optimization results for BSA_{SSG} and BSA_{DSG}. The results show that both mechanisms reach the approximate optimal in all runs. For the same population size and maximum iteration number, BSA with each mechanism shows its best performance when the mixrate is set to 1. The solution quality again improves by increasing either population size or maximum iteration number.

Table 4.4. Statistical results of BSA_{SSG} for case 2 with different parameters

Max. iteration	popsize	mixrate	Generation cost ($/h)				CPU time (s)			
			Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
200	10	0	925.6473	929.5551	955.2194	6.7008	0.34	0.40	0.44	0.02
200	10	0.2	925.4671	926.4058	928.1238	0.5344	0.50	0.55	0.62	0.03
200	10	0.4	925.4741	926.1482	927.6255	0.4463	0.47	0.58	0.67	0.04
200	10	0.6	925.4262	925.9691	926.9111	0.4257	0.48	0.61	0.67	0.04
200	10	0.8	925.4156	926.6591	963.0449	5.2710	0.40	0.60	0.72	0.05
200	10	1	925.4157	925.7960	926.8127	0.4323	0.47	0.59	0.66	0.05
200	50	0	925.4592	926.2068	926.7298	0.3569	1.86	2.07	2.28	0.09
200	50	0.2	925.4444	925.7300	926.4621	0.2794	2.73	2.94	3.12	0.10
200	50	0.4	925.4308	925.5408	926.3061	0.1381	2.96	3.13	3.29	0.08
200	50	0.6	925.4143	925.5339	926.3217	0.1813	3.06	3.26	3.42	0.09
200	50	0.8	925.4145	925.5179	926.2994	0.1914	3.10	3.28	3.45	0.09
200	50	1	925.4154	925.4361	925.5441	0.0257	2.84	3.17	3.57	0.17
500	10	0	925.4207	926.4318	937.7516	1.7673	0.83	0.93	1.04	0.05
500	10	0.2	925.4140	925.5996	926.3741	0.2953	1.06	1.28	1.40	0.08
500	10	0.4	925.4137	925.5629	926.3039	0.2995	1.04	1.34	1.47	0.09
500	10	0.6	925.4137	925.4720	926.2955	0.2108	0.94	1.27	1.47	0.13
500	10	0.8	925.4137	925.5558	926.2971	0.3261	0.83	1.11	1.48	0.15
500	10	1	925.4137	925.5196	926.2952	0.2893	0.80	1.01	1.23	0.12
500	50	0	925.4218	925.5582	926.3021	0.1841	4.62	4.84	5.07	0.13
500	50	0.2	925.4138	925.4157	925.4220	0.0019	6.51	6.76	7.14	0.15
500	50	0.4	925.4137	925.4143	925.4223	0.0012	6.08	6.93	7.41	0.27
500	50	0.6	925.4137	925.4138	925.4147	0.0002	5.48	6.65	7.18	0.39
500	50	0.8	925.4137	925.4313	926.2952	0.1247	4.79	5.70	6.46	0.37
500	50	1	925.4137	925.4137	925.4137	0.0000	4.55	5.16	5.91	0.34

Table 4.5. Statistical results of BSA_{DSG} for case 2 with different parameters

Max. iteration	popsize	mixrate	Generation cost ($/h)				CPU time (s)			
			Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
200	10	0	925.4374	925.7109	928.4320	0.4542	0.44	0.62	0.80	0.06
200	10	0.2	925.4160	925.4390	925.4992	0.0196	0.78	1.08	1.34	0.15
200	10	0.4	925.4154	925.4484	925.6070	0.0354	0.76	1.09	1.31	0.12
200	10	0.6	925.4157	925.4370	925.5495	0.0246	0.75	1.12	1.40	0.17
200	10	0.8	925.4153	925.4319	925.5259	0.0239	0.75	1.05	1.39	0.17
200	10	1	925.4140	925.4258	925.5224	0.0165	0.80	1.06	1.37	0.15
200	50	0	925.4291	925.5135	925.8053	0.0734	3.06	3.78	4.49	0.31
200	50	0.2	925.4140	925.4195	925.4366	0.0048	6.54	7.11	8.33	0.34
200	50	0.4	925.4145	925.4186	925.4284	0.0033	6.24	7.16	7.86	0.33
200	50	0.6	925.4140	925.4179	925.4267	0.0032	5.88	7.09	8.10	0.47
200	50	0.8	925.4140	925.4171	925.4290	0.0034	5.63	6.89	7.74	0.49
200	50	1	925.4138	925.4161	925.4220	0.0020	5.13	6.69	7.96	0.67
500	10	0	925.4138	925.4273	925.5157	0.0175	1.15	1.54	1.81	0.15
500	10	0.2	925.4137	925.4140	925.4167	0.0005	1.56	2.72	3.28	0.37
500	10	0.4	925.4136	925.4146	925.4540	0.0057	1.45	2.49	3.32	0.55
500	10	0.6	925.4136	925.4138	925.4148	0.0002	1.51	2.29	3.15	0.51

500	10	0.8	925.4136	925.4137	925.4140	0.0001	1.58	2.14	3.23	0.39
500	10	1	925.4135	925.4136	925.4141	0.0001	1.45	2.10	3.07	0.43
500	50	0	925.4139	925.4160	925.4218	0.0017	8.36	9.36	10.48	0.55
500	50	0.2	925.4137	925.4137	925.4138	0.0000	14.98	17.28	19.05	0.76
500	50	0.4	925.4137	925.4137	925.4138	0.0000	12.89	16.58	18.44	1.19
500	50	0.6	925.4136	925.4137	925.4138	0.0000	12.21	15.79	18.06	1.32
500	50	0.8	925.4136	925.4137	925.4137	0.0000	11.08	14.45	17.38	1.61
500	50	1	925.4135	925.4136	925.4137	0.0001	10.00	12.95	16.12	1.50

4.2.2.2 Convergence Characteristics

The results of BSA_{SSG} and BSA_{DSG} with the same parameters should be used to compare their convergence characteristics. In this case, the best ED results of both methods with maximum iteration=100, popsize=50, and mixrate=1 are selected. Figure 4.3 illustrates the convergence of the generation cost versus the iteration number for the best solutions of BSA_{SSG} and BSA_{DSG}. It is shown that the convergence characteristic of BSA_{DSG} is better than BSA_{SSG} as it converges to the optimal earlier.

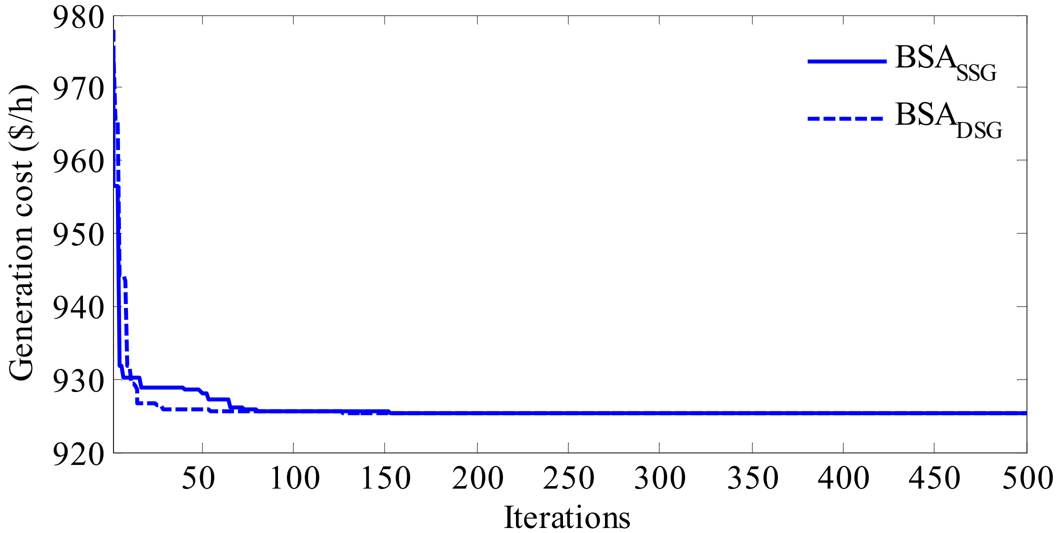


Figure 4.3. Convergence characteristic of BSA_{SSG} and BSA_{DSG} in Case 2

4.2.2.3 Robustness

Figure 4.4 shows the optimal results of 50 trials by BSA_{SSG} and BSA_{DSG} in case 2. The distributions of the optimal results with very low standard deviations confirm that both methods are robust for solving ED problem in this case. The results also show that

BSA_{DSG} produce higher quality solutions than BSA_{SSG} as its optimal values in 50 trials are lower.

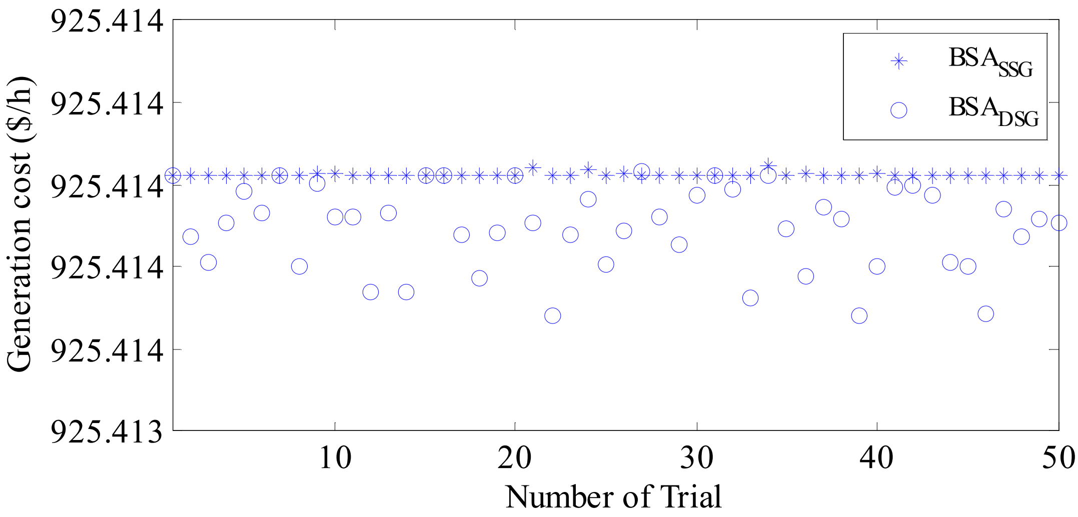


Figure 4.4. Optimal results of BSA_{SSG} and BSA_{DSG} for 50 trials in Case 2

4.2.2.4 Computational efficiency

Table 4.4 and Table 4.5 show the range of computation time for solving ED problem in case 2. Based on the results, the computation time of BSA_{SSG} is lower than BSA_{DSG} with same optimization parameters. However, BSA_{DSG} converged to the same optimal of BSA_{SSG} in lower computation time. For example, it took an average computation time of 2.72 (s) for BSA_{DSG} (maximum iteration=500, popsize=10, and mixrate=0.2) to reach the minimum generation cost of 925.4137 ($/h) (standard deviation=0.0005) while it is 6.93 (s) for BSA_{SSG} (maximum iteration=500, popsize=50, and mixrate=0.4) to reach the same optimal with the standard deviation of 0.0012. The standard deviation of BSA_{DSG} is also lower than BSA_{SSG}.

4.2.2.5 Comparison of BSA with other methods

For the comparison purpose, one of the optimal results for each BSA should be selected. For each of BSA_{SSG} and BSA_{DSG}, the solution with considering both the best objective function and its computation time is selected for the comparison. For BSA_{SSG}, it corresponds to maximum iteration=200, popsize=10, and mixrate=0.2. For BSA_{DSG}, the solution corresponding to mixrate=0.2 with same values of other two parameters is good enough for comparison purpose. Table 4.6 shows the optimal results of BSA_{SSG} and BSA_{DSG} for the case study compared with those via GA (Nadeem Malik et al., 2010), GA-APO (Nadeem Malik et al., 2010), PSO (Yaşar & Özyön, 2011), and MSG-HS (Yaşar & Özyön, 2011). BSA_{SSG} and BSA_{DSG} reach respectively, the generation costs of 925.4671 ($/h) and 925.4374 ($/h), which are less than those achieved by the other methods.

Table 4.6. Best solution for Case 2 (6-unit system)

Generation	GA[1]	GA-APO[1]	PSO[2]	MSG-HS[2]	BSA_{SSG}	BSA_{DSG}
P_1 (MW)	150.724	133.981	197.865	199.633	199.5993	199.6002
P_2 (MW)	60.870	37.216	50.337	20.000	20.0000	20.0000
P_3 (MW)	30.896	37.768	15.000	23.762	24.0783	24.4664
P_4 (MW)	14.214	28.350	10.000	18.393	19.2869	18.8002
P_5 (MW)	19.489	18.792	10.000	17.102	18.7680	17.6946
P_6 (MW)	15.915	38.052	12.000	15.692	12.7503	13.9329
Total generation (MW)	292.110	294.160	295.202	294.583	294.4828	294.4943
P_L (MW)	8.706	10.756	11.802	11.183	11.0828	11.0943
Minimum generation cost ($/h)	996.037	1101.491	925.758	925.640	925.4671	925.4374
Average generation cost ($/h)	-	-	925.76	925.64	926.4058	925.7109
Maximum generation cost ($/h)	1117.13	1101.49	928.43	928.6	928.1238	928.4320
CPU time (s)	0.578	0.156	0.353	0.621	0.53	0.62

[1] (Nadeem Malik et al., 2010)
[2] (Yaşar & Özyön, 2011)

4.2.3 Case 3: 20-unit system with transmission loss

This system has 20 generating units and the system demand is 2500MW. The units' data are summarized in Appendix (Table A.4). The transmission loss is considered and the loss coefficients are as in Appendix (Table A.5).

4.2.3.1 Solution to ED problem by BSA_{SSG} and BSA_{DSG}

In this case study, the maximum iteration is set to 500 for all runs of both methods. Two values of 10 and 50 as the low and high values of population size are considered and the optimization has been run for different values of mixrate as they vary from 0 to 1 with steps of 0.20. The statistical indices of the optimal generation cost and computation time are listed along with the simulation parameters in Table 4.7 and Table 4.8. The results show that BSA with both constraint handling mechanisms have reached almost the same optimal values. However, BSA_{SSG} and BSA_{DSG} show better performance when the mixrate=1.

Table 4.7. Statistical results of BSA_{SSG} for case 3 with different parameters

Max. iteration	popsize	mixrate	Generation cost ($/h)				CPU time (s)			
			Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
500	10	0	62468.0240	62480.7068	62500.0666	7.2095	1.26	1.42	1.53	0.06
500	10	0.2	62458.1314	62461.9857	62467.2614	2.0540	2.12	2.31	2.48	0.09
500	10	0.4	62457.5697	62459.7333	62463.1708	1.2733	2.14	2.35	2.64	0.13
500	10	0.6	62457.3357	62458.8035	62463.3494	1.1441	1.93	2.27	2.65	0.18
500	10	0.8	62456.8713	62458.0208	62460.5240	0.8684	1.65	2.08	2.53	0.19
500	10	1	62456.8937	62457.5267	62458.6013	0.4068	1.37	1.78	2.22	0.19
500	50	0	62463.9207	62472.1355	62479.4234	3.8803	6.77	7.31	7.77	0.23
500	50	0.2	62457.4815	62459.1898	62461.2972	0.8901	11.00	11.91	12.62	0.42
500	50	0.4	62457.1291	62458.2178	62460.4153	0.6791	10.79	12.18	13.38	0.54
500	50	0.6	62456.9222	62457.3594	62458.4320	0.3278	9.97	11.56	12.54	0.71
500	50	0.8	62456.7547	62457.1015	62457.8991	0.3114	8.36	11.03	12.79	0.98
500	50	1	62456.7152	62456.8474	62457.1302	0.0965	7.41	9.25	10.61	0.68

Table 4.8. Statistical results of BSA_{DSG} for case 3 with different parameters

Max. iteration	popsize	mixrate	Generation cost ($/h)				CPU time (s)			
			Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
500	10	0	62459.6886	62464.6892	62473.1335	2.9602	1.06	1.19	1.29	0.05
500	10	0.2	62456.8596	62457.6093	62458.7701	0.4067	1.76	1.88	2.07	0.06
500	10	0.4	62456.7729	62457.3546	62458.2779	0.3018	1.81	1.93	2.14	0.06
500	10	0.6	62456.7654	62457.1267	62457.8195	0.2377	1.72	1.92	2.06	0.07
500	10	0.8	62456.6928	62456.8970	62457.9149	0.1958	1.72	1.85	2.00	0.07
500	10	1	62456.6540	62456.8084	62457.4365	0.1554	1.61	1.79	1.97	0.09
500	50	0	62458.6758	62460.9385	62464.0995	1.2059	5.62	6.32	7.13	0.32
500	50	0.2	62456.7274	62457.0914	62457.6479	0.1833	9.47	10.18	11.26	0.38
500	50	0.4	62456.7435	62456.9371	62457.3414	0.1343	9.31	10.23	11.29	0.39
500	50	0.6	62456.6751	62456.7799	62457.1216	0.0834	9.08	10.03	11.51	0.45
500	50	0.8	62456.6389	62456.7049	62456.7922	0.0418	8.81	9.79	11.00	0.48
500	50	1	62456.6359	62456.6736	62456.9008	0.0434	8.27	9.38	11.15	0.57

4.2.3.2 Convergence Characteristics

Figure 4.5 shows the convergence characteristics of BSA_{SSG} and BSA_{DSG} for their best optimal runs. It corresponds to maximum iteration=500, popsize=50, and mixrate=1. The figure shows that the speed of convergence of BSA_{DSG} is higher than BSA_{SSG} even it is initialized from a higher value of objective function.

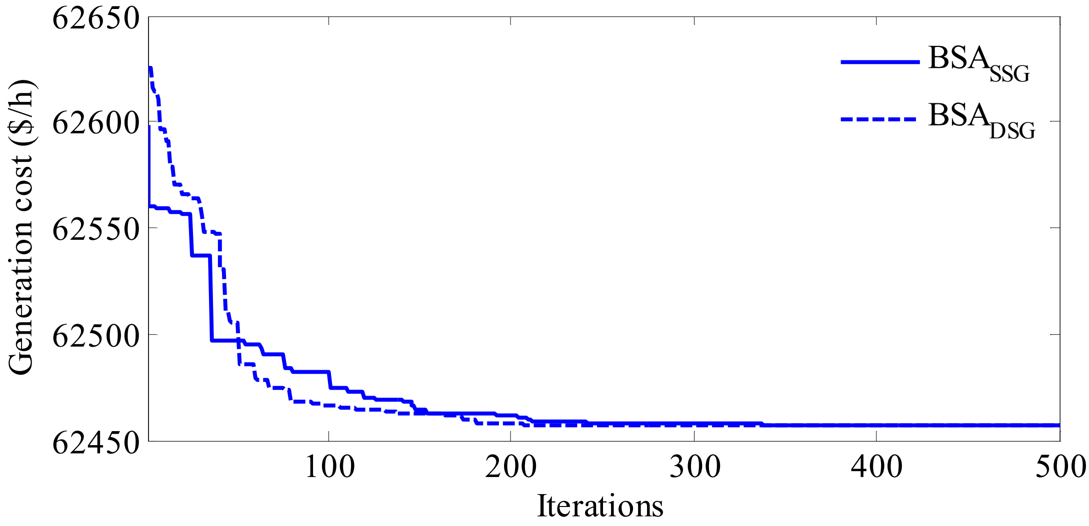


Figure 4.5. Convergence characteristic of BSA_{SSG} and BSA_{DSG} in Case 3

4.2.3.3 Robustness

Figure 4.6 illustrates the optimal results of 50 run for BSA_{SSG} and BSA_{DSG}. The figure shows that BSA_{DSG} converged to almost the same results and most of the runs led to lower values than BSA_{SSG}. It confirms that the BSA_{DSG} produces better optimal results than BSA_{SSG}. However, BSA_{SSG} and BSA_{DSG} show high robustness in solving the ED problem in this case.

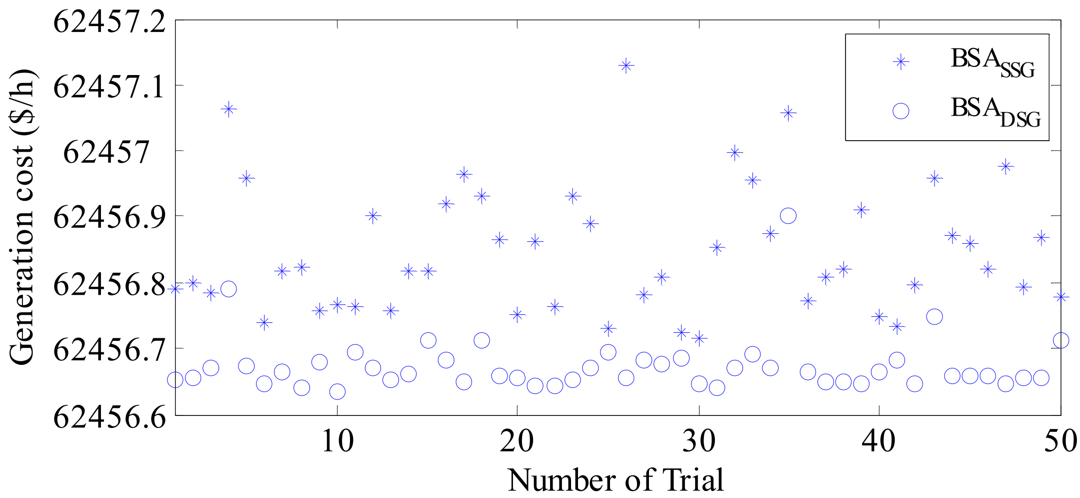


Figure 4.6. Optimal results of BSA_{SSG} and BSA_{DSG} for 50 trials in Case 3

4.2.3.4 Computational efficiency

Table 4.7 and Table 4.8 show the statistical indices of computation time for BSA_{SSG} and BSA_{DSG}, respectively. Based on these tables, the computation time of BSA_{DSG} is lower than BSA_{SSG} in most of the cases when they are compared with the same optimization parameters. But, BSA_{DSG} converges to a same optimal of BSA_{SSG} in shorter computation time. For example, BSA_{SSG} converges to the optimal of 62456.8713 ($/h) with SD=0.87 ($/h) in an average time of 2.08 (s) while BSA_{DSG} converges to a bit lower optimal and lower standard deviation (62456.8596 ($/h) and SD=0.41) in shorter average time, 1.88 (s).

4.2.3.5 Comparison of BSA with other methods

BSA_{DSG} performs much better than BSA_{SSG} in this case in terms of better solution quality. So, the best solution of ED by BSA_{DSG} is compared with other methods from the literature.

Figure 4.9 shows the best ED schedule by BSA_{DSG} and some methods (for this particular case study: λ iteration (Su et al., 2000), NR (Abdelaziz et al., 2008), and EHNN (Abdelaziz et al., 2008)). Comparison shows BSA_{DSG} converging to the lowest optimal value among the other methods.

Table 4.9. Best solution for Case 3 (20-unit system considering transmission loss)

Generation	λ method[1]	NR[2]	EHNN[2]	BSA_{DSG}
P_1 (MW)	512.7805	524.0166	403.3043	513.1610
P_2 (MW)	169.1033	160.9879	134.4348	169.1839
P_3 (MW)	126.8898	130.2168	134.4348	126.8718
P_4 (MW)	102.8657	100.4129	134.4348	102.9243
P_5 (MW)	113.6386	115.2559	107.5478	113.9064
P_6 (MW)	73.5710	78.7385	67.2174	73.5339
P_7 (MW)	115.2878	118.1765	84.0217	115.4571
P_8 (MW)	116.3994	118.9390	100.8261	116.3941
P_9 (MW)	100.4062	104.7037	134.4348	100.3602
P_{10} (MW)	106.0267	113.7706	100.8261	106.0799
P_{11} (MW)	150.2394	148.7055	201.6522	150.2741
P_{12} (MW)	292.7648	295.9623	336.0869	292.6492
P_{13} (MW)	119.1154	118.0200	107.5478	118.9574
P_{14} (MW)	30.8340	35.4054	87.3826	30.6032
P_{15} (MW)	115.8057	121.3720	124.3522	115.5427
P_{16} (MW)	36.2545	36.0465	53.7739	36.2612
P_{17} (MW)	66.8590	72.4530	57.1348	66.7651
P_{18} (MW)	87.9720	42.2129	80.6609	87.7428

P_{19} (MW)	100.8033	102.6087	80.6609	100.9375
P_{20} (MW)	54.3050	55.7560	67.2174	54.3725
Total generations (MW)	2591.9670	2593.7615	2597.9520	2591.9781
P_L (MW)	91.9670	93.7615	97.9520	91.9781
Minimum generation cost ($/h)	62456.6391	62489.5000	62610.0000	62456.6359
Average generation cost ($/h)	-	-	-	62456.6736
Maximum generation cost ($/h)	-	-	-	62456.9008
CPU time (s)	33.7570	0.4000	0.1100	8.74

[1] (Su & Lin, 2000)
[2] (Abdelaziz et al., 2008)

4.2.4 Case 4: 40-unit system with non-convex cost function

This case is a large test system with non-convex cost functions in any of the generating units. The system demand is 10500MW. The units' data are as shown in Appendix (Table A.6).

4.2.4.1 Solution to ED problem by BSA_{SSG} and BSA_{DSG}

In this non-convex case study, the maximum iteration is set to 5000 with two values of 10 and 50 for the population size. Again, the values of 0 to 1 with steps of 0.2 are assigned to the mixrate. The whole results with the values of optimization parameters are shown in Table 4.10 and Table 4.11. In this case study, BSA_{DSG} shows much better performance than BSA_{SSG} because it converges to lower optimal even in lower computation time.

Table 4.10. Statistical results of BSA_{SSG} for case 4 with different parameters

Max. iteration	popsize	mixrate	Generation cost ($/h)				CPU time (s)			
			Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
5000	10	0	122246.7000	122928.1000	123540.9000	306.8176	1.14	1.15	1.17	0.01
5000	10	0.2	121624.4000	121857.1000	122200.7000	108.0424	1.25	1.26	1.30	0.01
5000	10	0.4	121586.5000	121776.3000	122041.4000	107.0172	1.25	1.28	1.31	0.02
5000	10	0.6	121512.6000	121762.5000	122177.3000	133.7590	1.23	1.28	1.33	0.02
5000	10	0.8	121585.7000	121789.0000	122502.1000	156.2368	1.22	1.26	1.31	0.02
5000	10	1	121562.8000	121918.8000	122540.2000	241.4585	1.15	1.22	1.28	0.03
5000	50	0	122241.7000	122588.5000	122935.8000	176.2929	4.29	4.33	4.42	0.04
5000	50	0.2	121607.7000	121706.6000	121893.9000	56.7192	4.85	4.94	5.07	0.06
5000	50	0.4	121537.9000	121672.0000	121775.1000	57.2171	4.93	5.08	5.24	0.06
5000	50	0.6	121599.6000	121661.8000	121809.1000	45.2183	5.01	5.11	5.29	0.06
5000	50	0.8	121503.3000	121631.4000	121781.2000	61.3481	4.96	5.10	5.27	0.08
5000	50	1	121457.6000	121591.8000	121776.9000	78.1484	4.62	4.89	5.18	0.15

Table 4.11. Statistical results of BSA_{DSG} for case 4 with different parameters

Max. iteration	popsize	mixrate	Generation cost ($/h)				CPU time (s)			
			Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
5000	10	0	121472.1000	121594.1000	121845.7000	82.2230	2.78	2.98	3.23	0.11
5000	10	0.2	121431.0000	121481.8000	121544.2000	32.3614	4.92	5.54	6.27	0.26
5000	10	0.4	121419.5000	121467.4000	121573.9000	36.1049	4.74	5.50	6.10	0.30
5000	10	0.6	121418.3000	121460.1000	121541.1000	31.6806	4.63	5.31	5.93	0.33
5000	10	0.8	121416.2000	121479.0000	121646.6000	50.0701	4.13	5.06	6.02	0.43
5000	10	1	121412.7000	121504.3000	121891.4000	87.1812	3.46	4.73	6.21	0.61
5000	50	0	121446.0000	121488.2000	121559.5000	25.7294	6.29	6.61	6.97	0.14
5000	50	0.2	121417.5000	121440.8000	121475.5000	13.9638	10.34	10.90	11.31	0.22
5000	50	0.4	121416.2000	121432.0000	121461.2000	9.5355	10.33	10.93	11.58	0.24
5000	50	0.6	121414.7000	121427.5000	121449.8000	7.9664	10.16	10.83	11.68	0.32
5000	50	0.8	121416.3000	121425.6000	121448.0000	8.5699	9.75	10.53	11.26	0.35
5000	50	1	121412.9000	121423.0000	121446.6000	7.1345	9.24	10.09	11.26	0.40

Table 4.12 lists the optimal schedules of the generating units for the best solutions of BSA_{SSG} and BSA_{DSG} among the 50 runs on the 10500MW demand.

Table 4.12. Best solution for Case 4 (40-unit system with valve-point loading effect)

BSA_{SSG}							
Generation (MW)		Generation (MW)		Generation (MW)		Generation (MW)	
P_1	111.3042	P11	168.8013	P21	523.4286	P31	190.0000
P_2	111.2151	P12	168.7957	P22	523.5551	P32	189.9959
P_3	97.4428	P13	214.1913	P23	523.4421	P33	190.0000
P_4	179.7882	P14	304.5236	P24	523.3284	P34	165.9914
P_5	89.4604	P15	392.1322	P25	523.4877	P35	165.1608
P_6	140.0000	P16	394.2844	P26	523.3424	P36	165.2852
P_7	259.6982	P17	489.2890	P27	10.0000	P37	110.0000
P_8	284.9202	P18	489.3243	P28	10.0024	P38	109.9967
P_9	284.8293	P19	511.4300	P29	10.0000	P39	110.0000
P_{10}	130.0000	P20	511.3184	P30	88.8952	P40	511.3396
Total generations (MW)		Total generation cost ($/h)				CPU time (s)	
10500		121457.5960				4.84	

BSA_{DSG}							
Generation (MW)		Generation (MW)		Generation (MW)		Generation (MW)	
P_1	110.7997	P_{11}	94.0014	P_{21}	523.2791	P_{31}	190.0000
P_2	110.7994	P_{12}	94.0002	P_{22}	523.2779	P_{32}	189.9999
P_3	97.4001	P_{13}	214.7598	P_{23}	523.2791	P_{33}	189.9999
P_4	179.7334	P_{14}	394.2792	P_{24}	523.2791	P_{34}	164.8007
P_5	87.8043	P_{15}	394.2782	P_{25}	523.2797	P_{35}	194.4222
P_6	139.9998	P_{16}	394.2760	P_{26}	523.2793	P_{36}	199.9780
P_7	259.5990	P_{17}	489.2792	P_{27}	10.0000	P_{37}	109.9999
P_8	284.5997	P_{18}	489.2794	P_{28}	10.0000	P_{38}	109.9993
P_9	284.5996	P_{19}	511.2796	P_{29}	10.0002	P_{39}	109.9989
P_{10}	130.0005	P_{20}	511.2794	P_{30}	87.7994	P_{40}	511.2796
Total generations (MW)		Total generation cost ($/h)				CPU time (s)	
10500		121412.9104				10.11	

4.2.4.2 Convergence Characteristics

The convergences of generation cost are depicted in Figure 4.7 for the best solutions obtained by BSA_{SSG} and BSA_{DSG}. The figure confirms the superiority of BSA_{DSG} in terms of better convergence characteristic for solving of ED problem in this case.

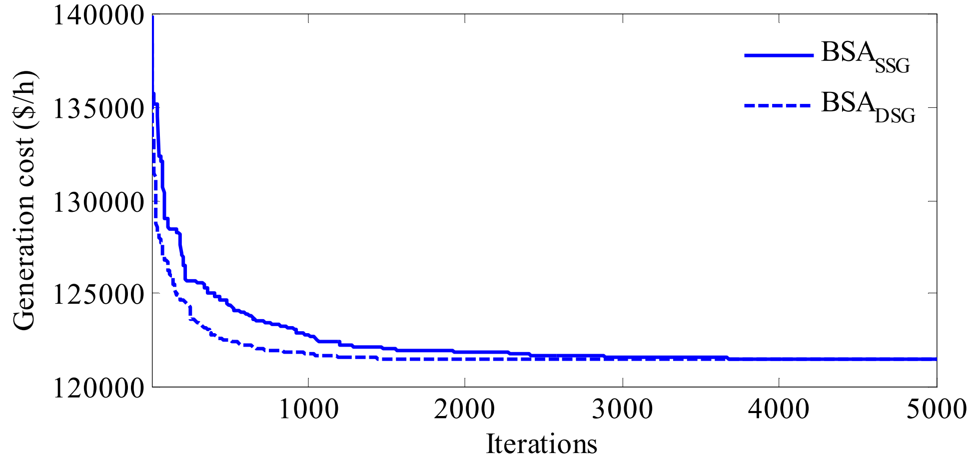


Figure 4.7. Convergence characteristic of BSA_{SSG} and BSA_{DSG} in Case 4

4.2.4.3 Robustness

Figure 4.8 shows the distribution of the optimal results obtained by BSA_{SSG} and BSA_{DSG}. The figure clearly shows that BSA_{DSG} performs better than BSA_{SSG} as its optimal results are lower than those of BSA_{SSG} which means BSA_{DSG} is more robust than BSA_{SSG}.

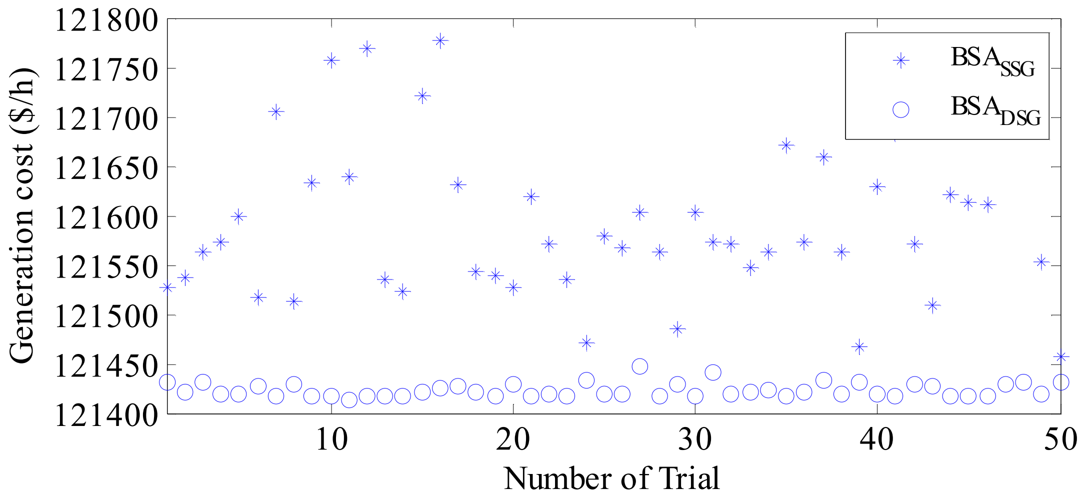


Figure 4.8. Optimal results of BSA_{SSG} and BSA_{DSG} for 50 trials in Case 4

4.2.4.4 Computational efficiency

In this case study, the worst optimal achieved by BSA_{DSG} is 121472.1000 ($/h) which took an average time of 2.98 (s) while the best optimal achieved by BSA_{SSG} is 121457.6000 ($/h) which took an average time of 4.89 (s). As a result, the computational efficiency of BSA_{DSG} is better than BSA_{SSG}.

4.2.4.5 Comparison of BSA with other methods

As mentioned before, BSA_{DSG} performs better than BSA_{SSG} in this case study. So, BSA_{DSG} is used for the comparison. Table 4.13 shows the statistical indices of optimal results of BSA_{DSG} and of the other methods (for this case study: PSO, APSO1, and APSO2 (A. Immanuel Selvakumar & Thanushkodi, 2008), CEP (Sinha et al., 2003), BBO (A. Bhattacharya & P. K. Chattopadhyay, 2010a), DEC-SQP (dos Santos Coelho & Mariani, 2006), FEP (Sinha et al., 2003), CSO (A. Immanuel Selvakumar et al., 2009), TSARGA (Subbaraj et al., 2011), ACO (Pothiya et al., 2010), IFEP (Sinha et al., 2003), PS (Al-Sumait et al., 2007), GA-PS-SQP (Alsumait et al., 2010), BBO (Aniruddha Bhattacharya et al., 2010), PSO-LRS, NPSO, and NPSO-LRS (A. I. Selvakumar & Thanushkodi, 2007), SA-PSO (Cheng-Chien, 2008), SOH-PSO (Chaturvedi et al., 2008), BF-NM (K. B. Panigrahi et al., 2008), and CSA (Basu & Chowdhury, 2013)).

Table 4.13. statistical indices of optimal results of BSA_{DSG} and other methods in Case 4

Method	Total generation cost ($/h)			Average CPU time (s)
	Minimum	Average	Maximum	
PSO[1]	121735.4736	122513.9175	123467.4086	4.58
APSO1[1]	121704.7391	122221.3697	122995.0976	4.71
APSO2[1]	121663.5222	122153.6730	122912.3958	5.05
CEP[2]	123488.29	124793.48	126902.89	1956.93
BBO[3]	121426.95	121735.28	122869.51	145.35
DEC-SQP[4]	121749.1892	122294.1825	123722.1237	14.39
FEP[2]	122679.71	124119.37	127245.59	1039.16
CSO[4]	121461.6707	121936.1926	122844.5391	-
TSARGA[5]	121463.0700	122928.3100	124296.5400	696.01
ACO[6]	121811.3700	121930.5800	122048.0600	92.54
IFEP[2]	122624.3500	123382.0000	125740.6300	1167.35
PS[7]	121415.14	122332.65	125486.29	42.98
GA-PS-SQP[8]	121458.14	122039	-	46.98
BBO[9]	121479.5029	121512.0576	121688.6634	-
PSO-LRS[10]	122035.7946	122558.4565	123461.6794	15.86

NPSO[10]	121704.7391	122221.3697	122995.0976	4.71
NPSO-LRS[10]	121664.4308	122209.3185	122981.5913	16.81
SA-PSO[11]	121430	121525	121645	26.58
SOH-PSO[12]	121501.14	121853.57	122446.30	-
BF-NM[13]	121423.63	121814.94	-	-
CSA[14]	121425.61	-	-	-
BSA_{DSG}	121412.9000	121423.0000	121446.6000	10.09

[1] (A. Immanuel Selvakumar & Thanushkodi, 2008)
[2] (Sinha et al., 2003)
[3] (A. Bhattacharya & P. K. Chattopadhyay, 2010a)
[4] (A. Immanuel Selvakumar & Thanushkodi, 2009)
[5] (Subbaraj et al., 2011)
[6] (Pothiya et al., 2010)
[7] (Al-Sumait et al., 2007)
[8] (Alsumait et al., 2010)
[9] (Aniruddha Bhattacharya & Pranab Kumar Chattopadhyay, 2010)
[10] (A. I. Selvakumar & Thanushkodi, 2007)
[11] (Cheng-Chien, 2008)
[12] (Chaturvedi et al., 2008)
[13] (K. B. Panigrahi & Pandi, 2008)
[14] (Basu & Chowdhury, 2013)

The table above shows BSA_{DSG} achieving the minimum cost of 121412.9000 ($/h), with acceptable CPU solving time of the ED problem.

4.3 ED problems with valve-point effects, prohibited operating zones, and multiple fuel options

Two case studies are used to validate the applicability of the proposed method with the proposed constraint handlings for solving ED problems by considering valve point effects, prohibited operating zones, and multiple fuel options. As mentioned in chapter 3 section 3.4, two constraint handling mechanisms have been employed for solving the ED problems. Both mechanisms are considered in BSA to solve the ED problems. These BSA methods are named "BSA_{SSG}" and "BSA_{DSG}".

For each case study, the number of 50 trials is considered to validate the robustness of the proposed methods (BSA_{SSG} and BSA_{DSG}). In order to check the performance of BSA_{SSG} and BSA_{DSG} for the solving of ED in large-scale systems, it was applied on 20, 40, 80, 160 unit systems with both valve-point effect and multiple fuel options making the problem of ED very complex.

4.3.1 Case 5: 15-unit system

This case study is the system with prohibited operating zone constraints. The system comprises 15 generating units with quadratic cost functions. The data are shown in Appendix (Table A.7). The transmission loss coefficients are taken from (Zwe-Lee, 2003) and are listed in Appendix (Table A.8).

4.3.1.1 Solution to ED problem by BSA_{SSG} and BSA_{DSG}

ED problem is solved for this case study with different values of parameters. The optimization results with parameter settings are shown in Table 4.14 and Table 4.15. The statistical indices of optimization results are calculated for the purpose of comparison and the analysis of the performance of BSA_{SSG} and BSA_{DSG}.

Both methods converge to the approximate optimal by proper parameter settings. Comparison between the optimal values found by BSA with two constraint handling mechanisms show that BSA_{DSG} reaches lower optimal value with higher quality (e.g. lower standard deviation) than BSA_{SSG} with the same parameters settings.

Table 4.14. Statistical results of BSA_{SSG} for case 5 with different parameters

Max. iteration	popsize	mixrate	Generation cost ($/h)				CPU time (s)			
			Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
300	10	0	32853.5205	32891.9654	32930.1542	19.8243	0.94	1.04	1.14	0.05
300	10	0.2	32735.0463	32793.0593	32862.8144	29.1480	1.67	1.82	1.97	0.07
300	10	0.4	32714.3380	32765.4514	32838.5366	28.3578	1.78	1.96	2.08	0.07
300	10	0.6	32726.8841	32768.9438	32826.2507	23.7973	1.90	2.03	2.15	0.06
300	10	0.8	32724.3553	32772.3689	32828.8484	23.8340	1.84	2.07	2.26	0.08
300	10	1	32731.9286	32775.4526	32834.1100	23.8947	1.89	2.07	2.20	0.07
300	50	0	32806.8605	32860.5921	32886.8724	16.5482	4.88	5.44	5.88	0.21
300	50	0.2	32718.6527	32756.8529	32787.8951	14.8924	9.06	9.56	10.26	0.21
300	50	0.4	32714.0910	32735.5075	32768.3256	12.0188	9.76	10.29	10.69	0.22
300	50	0.6	32712.7520	32738.5042	32769.5875	12.7135	10.22	10.80	11.22	0.24
300	50	0.8	32711.6398	32739.2387	32762.6794	12.0875	10.50	11.13	11.64	0.25
300	50	1	32717.5894	32744.9666	32778.7565	13.7964	10.51	11.11	12.04	0.30
500	10	0	32828.9118	32868.8467	32908.1272	18.6609	1.54	1.72	1.83	0.07
500	10	0.2	32714.6086	32762.3507	32819.9658	24.8477	2.75	2.91	3.15	0.08
500	10	0.4	32708.0537	32735.8102	32792.7922	18.5470	2.85	3.09	3.34	0.11
500	10	0.6	32714.7826	32731.3652	32769.5539	12.4490	3.04	3.21	3.42	0.09
500	10	0.8	32708.9555	32730.4905	32766.3650	11.9146	2.93	3.27	3.48	0.11
500	10	1	32709.5243	32735.6693	32775.9442	15.7160	2.92	3.25	3.56	0.14

500	50	0	32789.1948	32832.7903	32861.1787	15.5300	8.21	8.86	9.48	0.28
500	50	0.2	32709.5685	32730.6099	32761.1794	12.6940	14.80	15.30	15.71	0.24
500	50	0.4	32704.8642	32714.9796	32726.0053	5.5789	16.04	16.58	17.49	0.29
500	50	0.6	32706.1312	32713.8911	32725.4731	4.8103	16.50	17.16	18.05	0.32
500	50	0.8	32707.2850	32716.2224	32733.5033	5.7907	16.41	17.68	18.80	0.40
500	50	1	32705.3537	32717.5492	32729.5674	5.4326	17.13	17.76	18.70	0.35

Table 4.15. Statistical results of BSA_{DSG} for case 5 with different parameters

Max. iteration	popsize	mixrate	Generation cost ($/h)				CPU time (s)			
			Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
300	10	0	32704.6057	32714.0649	32735.3949	6.8454	0.90	1.11	1.26	0.09
300	10	0.2	32704.4600	32704.7814	32706.5097	0.4420	1.93	2.08	2.25	0.07
300	10	0.4	32704.4525	32704.7401	32706.2177	0.3340	2.03	2.21	2.42	0.08
300	10	0.6	32704.4513	32704.7619	32705.7463	0.3042	1.97	2.33	2.54	0.11
300	10	0.8	32704.4512	32705.0381	32706.7851	0.4928	2.20	2.39	2.57	0.08
300	10	1	32704.4678	32705.1895	32707.9025	0.7848	2.25	2.47	2.70	0.11
300	50	0	32704.7392	32706.4176	32709.9520	1.3527	5.63	6.58	7.39	0.39
300	50	0.2	32704.4508	32704.5100	32704.8440	0.0795	11.95	12.66	13.85	0.38
300	50	0.4	32704.4510	32704.5180	32704.9421	0.0905	12.49	13.35	14.27	0.37
300	50	0.6	32704.4503	32704.5019	32704.6037	0.0420	13.20	13.99	14.74	0.35
300	50	0.8	32704.4522	32704.5274	32704.7416	0.0607	13.51	14.48	15.44	0.45
300	50	1	32704.4507	32704.5382	32704.7764	0.0686	13.59	14.86	15.96	0.59
500	10	0	32704.4576	32705.6885	32710.1464	1.2352	1.65	1.83	2.00	0.08
500	10	0.2	32704.4502	32704.5155	32704.9712	0.0908	3.12	3.37	3.54	0.09
500	10	0.4	32704.4501	32704.4781	32704.6884	0.0461	3.46	3.62	3.85	0.09
500	10	0.6	32704.4501	32704.4832	32704.6239	0.0381	3.54	3.77	4.03	0.12
500	10	0.8	32704.4501	32704.4859	32704.8908	0.0660	3.64	3.91	4.21	0.12
500	10	1	32704.4521	32704.4934	32704.7034	0.0514	3.79	4.02	4.26	0.10
500	50	0	32704.4501	32704.6725	32705.3322	0.2321	9.73	10.60	12.34	0.51
500	50	0.2	32704.4501	32704.4538	32704.4756	0.0050	19.55	20.36	21.11	0.39
500	50	0.4	32704.4501	32704.4529	32704.4661	0.0041	20.51	21.65	22.71	0.43
500	50	0.6	32704.4501	32704.4524	32704.4625	0.0029	21.53	22.65	24.09	0.57
500	50	0.8	32704.4501	32704.4556	32704.4761	0.0063	22.04	23.53	24.62	0.59
500	50	1	32704.4501	32704.4552	32704.4738	0.0046	22.92	24.12	25.88	0.60

4.3.1.2 Convergence Characteristics

Figure 4.9 shows the convergence characteristics of BSA_{SSG} and BSA_{DSG} for their best solutions. The figure shows much difference between two mechanisms for the constraints handling. BSA_{DSG} converges to the optimal earlier than BSA_{SSG} which proves its better convergence characteristic.

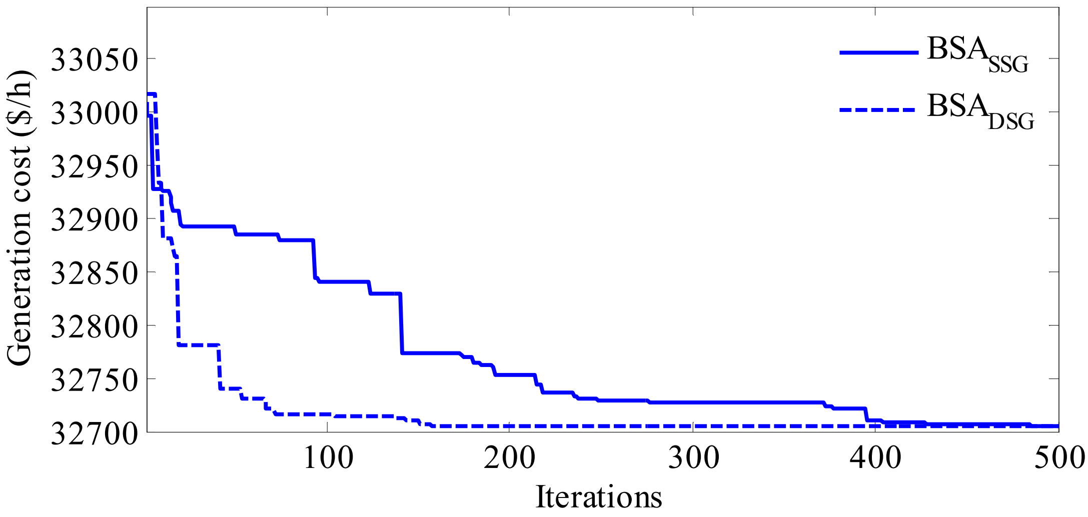


Figure 4.9. Convergence characteristic of BSA$_{SSG}$ and BSA$_{DSG}$ in Case 5

4.3.1.3 Robustness

The optimal solutions among 50 trials BSA$_{SSG}$ and BSA$_{DSG}$ are depicted in Figure 4.10. It clearly shows that both BSA$_{SSG}$ and BSA$_{DSG}$ are robust as they converge to almost same optimal values. Also, it is shown that BSA$_{DSG}$ is much robust than BSA$_{SSG}$ as it converges to the same optimal values in 50 trials.

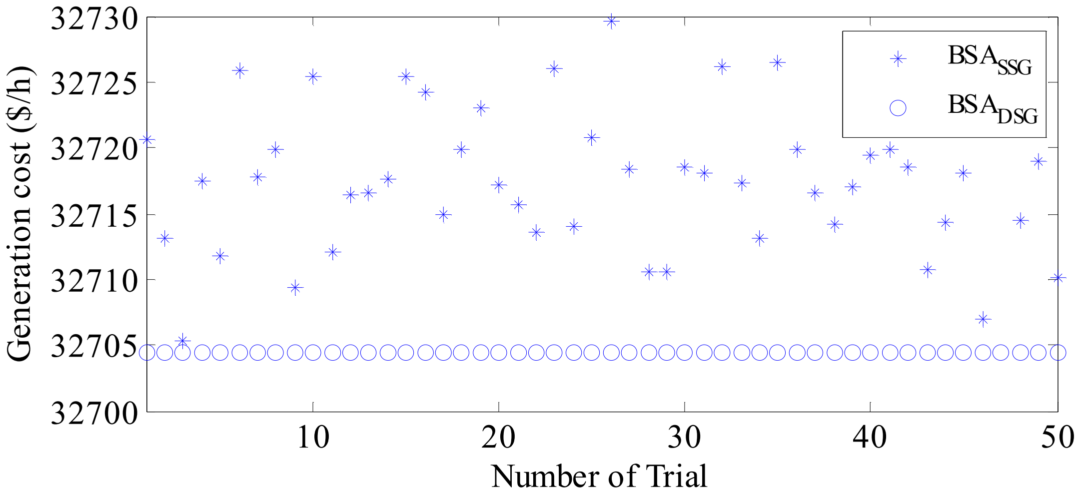


Figure 4.10. Optimal results of BSA$_{SSG}$ and BSA$_{DSG}$ for 50 trials in Case 5

4.3.1.4 Computational efficiency

As shown in the previous section, BSA_{DSG} converges to the lower optimal values than BSA_{SSG} with same optimization parameters. But when both BSA_{SSG} and BSA_{DSG} are to be compared, the computation time of each to reach the same optimal (with almost same solution quality, i.e., same standard deviation) should be considered. For example, BSA_{DSG} reaches 32704.6057 ($/h) in about 1.11 (s) while the other one reaches 32704.8642 in about 16.58 (s) confirming the better performance of BSA_{DSG} than BSA_{SSG} in terms of computational efficiency.

4.3.1.5 Comparison of BSA with other methods

Although both BSA_{SSG} and BSA_{DSG} show high performance for solving the ED problem in this case study, the results of BSA_{DSG} is used for the comparison with other methods. The results achieved by BSA_{DSG} are compared with PSO and GA (Zwe-Lee, 2003), MTS (Pothiya et al., 2008), SOH-PSO (Chaturvedi et al., 2008), GAAPI (Ciornei & Kyriakides, 2012), AIS (B. K. Panigrahi et al., 2007), APSO (B. K. Panigrahi et al., 2008), and SGA (Kuo, 2008) as shown in Table 4.16. The results confirm that BSA_{DSG} achieves the lowest generation cost among other methods.

The variations in optimal results obtained by BSA_{DSG} and other methods are presented in Table 4.17. Again, the similar optimal results with low standard deviations validate the robustness of BSA_{DSG} for solving the ED problems. Note that the authors of BF-NM (K. B. Panigrahi & Pandi, 2008) did not report the power generations of units, so Table 4.17 excludes the detailed results of this method.

Table 4.16. Best solution for Case 5 (15-unit test system)

Generation	PSO[1]	GA[1]	MTS[2]	SOH-PSO[3]	GAAPI[4]	AIS[5]	APSO[6]	SGA[7]	BSA_{DSG}
P_1 (MW)	439.1162	415.3108	453.9922	455.00	454.70	441.1587	455.00	455.00	455.0000
P_2 (MW)	407.9727	359.7206	379.7434	380.00	380.00	409.5873	380.0100	380.00	380.0000
P_3 (MW)	119.6324	104.425	130.0000	130.00	130.00	117.2983	130.00	130.00	130.0000
P_4 (MW)	129.9925	74.9853	129.9232	130.00	129.53	131.2577	126.5228	130.00	130.0000
P_5 (MW)	151.0681	380.2844	168.0877	170.00	170.00	151.0108	170.0131	170.00	170.0000
P_6 (MW)	459.9978	426.7902	460.0000	459.96	460.00	466.2579	460.00	460.00	460.0000
P_7 (MW)	425.5601	341.3164	429.2253	430.00	429.71	423.3678	428.2836	430.00	430.0000
P_8 (MW)	98.5699	124.7867	104.3097	117.53	75.35	99.948	60.00	106.25	64.4275
P_9 (MW)	113.4936	133.1445	35.0358	77.90	34.96	110.684	25.00	25.00	66.2023
P_{10} (MW)	101.1142	89.2567	155.8829	119.54	160.00	100.2286	159.7893	160.00	160.0000
P_{11} (MW)	33.9116	60.0572	79.8994	54.50	79.75	32.0573	80.00	80.00	80.0000
P_{12} (MW)	79.9583	49.9998	79.9037	80.00	80.00	78.8147	80.00	80.00	80.0000
P_{13} (MW)	25.0042	38.7713	25.0220	25.00	34.21	23.5683	33.7038	25.00	25.0083
P_{14} (MW)	41.414	41.9425	15.2586	17.86	21.14	40.2581	55.00	15.00	15.0000
P_{15} (MW)	35.614	22.6445	15.0796	15.00	21.02	36.9061	15.00	15.00	15.0002
Total Gen. (MW)	2662.4	2668.4	2661.36	2662.29	2660.36	2662.04	2658.3226	2661.3	2660.6383
P_L (MW)	32.4306	38.2782	31.3523	32.28	30.36	32.4075	28.3655	31.258	30.6383
Generation cost ($/h)	32858	33113	32716.87	32751.39	32732.95	32854	32742.7774	32711	32704.6057

[1] (Zwe-Lee, 2003)
[2] (Pothiya et al., 2008)
[3] (Chaturvedi et al., 2008)
[4] (Ciornei & Kyriakides, 2012)
[5] (B. K. Panigrahi et al., 2007)
[6] (B. K. Panigrahi et al., 2008)
[7] (Kuo, 2008)

Table 4.17. Convergence results (for 50 trial runs) of Case 5 (15-unit test system)

Method	Total generation cost ($/h)			
	Minimum	Average	Maximum	Standard deviation
PSO[1]	32858	33039	33331	-
GA[1]	33113	33228	33337	-
MTS[2]	32716.87	32767.21	32796.15	17.51
SOH-PSO[3]	32751.39	32878	32945	-
GAAPI[4]	32732.95	32735.06	32756.01	-
AIS[5]	32854	32873.25	32892	10.8079
APSO[6]	32742.777	32976.681	-	133.9276
SGA[7]	32711	32802	33005	35.584
BF-NM[8]	32784.502	32976.81	-	85.7743
BSA_{DSG}	32704.6057	32714.0649	32735.3949	6.8454

[1] (Zwe-Lee, 2003)
[2] (Pothiya et al., 2008)
[3] (Chaturvedi et al., 2008)
[4] (Ciornei & Kyriakides, 2012)
[5] (B. K. Panigrahi et al., 2007)
[6] (B. K. Panigrahi et al., 2008)
[7] (Kuo, 2008)
[8] (K. B. Panigrahi & Pandi, 2008)

4.3.2 Case 6: 10-unit system

This system comprises ten units with both valve-point effects and multiple fuel options. The generators' cost functions are non-convex for all fuel options. Note that the original cost coefficients of the generators were convex but the sinusoidal terms were added to make them non-convex. The system data are given as Appendix (Table A.9) (C.-L.

Chiang, 2005). The first generator has two fuel options and the rest have three fuel options. The power demand is 2700 MW and the transmission network loss is ignored.

4.3.2.1 Solution to ED problem by BSA_{SSG} and BSA_{DSG}

Two versions of BSA are again employed to solve the ED problem. The parameters are set first and both BSA_{SSG} and BSA_{DSG} are run for 50 trials. The statistical indices are obtained for the analysis of the results. Table 4.18 and Table 4.19 show the whole results achieved by BSA_{SSG} and BSA_{DSG} for this test system. The results show that both BSA_{SSG} and BSA_{DSG} converge to approximate optimal in all cases. The results show the high performance of both methods for solving the ED problem.

Table 4.18. Statistical results of BSA_{SSG} for case 6 with different parameters

Max. iteration	popsize	mixrate	Generation cost ($/h)				CPU time (s)			
			Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
500	10	0	624.1954	624.9051	627.3295	0.6500	0.25	0.26	0.38	0.02
500	10	0.2	623.9604	624.0500	624.2157	0.0530	0.25	0.26	0.27	0.01
500	10	0.4	623.9378	624.0252	624.1716	0.0574	0.25	0.26	0.27	0.01
500	10	0.6	623.9185	623.9929	624.0875	0.0388	0.25	0.26	0.27	0.01
500	10	0.8	623.9126	623.9822	624.1457	0.0451	0.23	0.25	0.27	0.01
500	10	1	623.9236	623.9819	624.0937	0.0426	0.25	0.25	0.27	0.01
500	50	0	624.0865	624.3568	625.1259	0.2098	0.47	0.49	0.51	0.01
500	50	0.2	623.9161	623.9648	624.0101	0.0218	0.48	0.49	0.51	0.01
500	50	0.4	623.8984	623.9612	624.0546	0.0342	0.47	0.49	0.53	0.01
500	50	0.6	623.8977	623.9416	623.9816	0.0212	0.47	0.48	0.51	0.01
500	50	0.8	623.9016	623.9417	623.9965	0.0209	0.47	0.49	0.50	0.01
500	50	1	623.8999	623.9327	623.9808	0.0182	0.47	0.48	0.50	0.01

Table 4.19. Statistical results of BSA_{DSG} for case 6 with different parameters

Max. iteration	popsize	mixrate	Generation cost ($/h)				CPU time (s)			
			Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
500	10	0	624.1141	624.4840	625.3923	0.2508	0.25	0.27	0.34	0.02
500	10	0.2	623.9278	624.0049	624.0949	0.0414	0.25	0.26	0.28	0.01
500	10	0.4	623.9205	623.9916	624.0635	0.0349	0.25	0.27	0.30	0.01
500	10	0.6	623.9337	623.9952	624.0747	0.0355	0.25	0.26	0.28	0.01
500	10	0.8	623.9143	623.9884	624.0618	0.0328	0.25	0.26	0.30	0.01
500	10	1	623.9283	623.9830	624.0626	0.0340	0.25	0.26	0.28	0.01
500	50	0	623.9847	624.1839	624.3559	0.0915	0.39	0.41	0.42	0.01
500	50	0.2	623.9042	623.9386	624.0013	0.0186	0.39	0.41	0.59	0.03
500	50	0.4	623.8991	623.9452	624.0038	0.0230	0.39	0.41	0.42	0.01
500	50	0.6	623.8964	623.9432	624.0395	0.0233	0.39	0.41	0.42	0.01
500	50	0.8	623.8758	623.9420	623.9764	0.0174	0.39	0.41	0.44	0.01
500	50	1	623.8853	623.9297	623.9707	0.0186	0.39	0.41	0.42	0.01

4.3.2.2 Convergence Characteristics

The convergence characteristics of BSA_{SSG} and BSA_{DSG} are depicted in Figure 4.11 showing that both of them reach the optimal in almost the same iteration numbers. It has been seen from the figure that BSA_{DSG} is better than BSA_{SSG} in early iterations in terms of convergence but it is inferior in the late iterations.

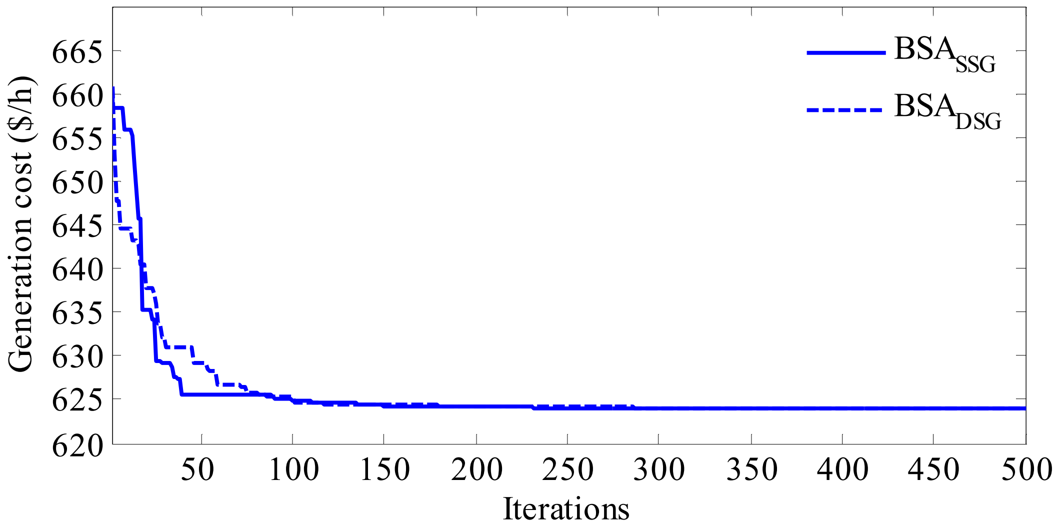


Figure 4.11. Convergence characteristic of BSA_{SSG} and BSA_{DSG} in Case 6

4.3.2.3 Robustness

Figure 4.12 shows the optimal results of BSA_{SSG} and BSA_{DSG} in 50 trials. The figure shows no superiority of one method to another. But it confirms that BSA is a robust method as all results are very close to each other.

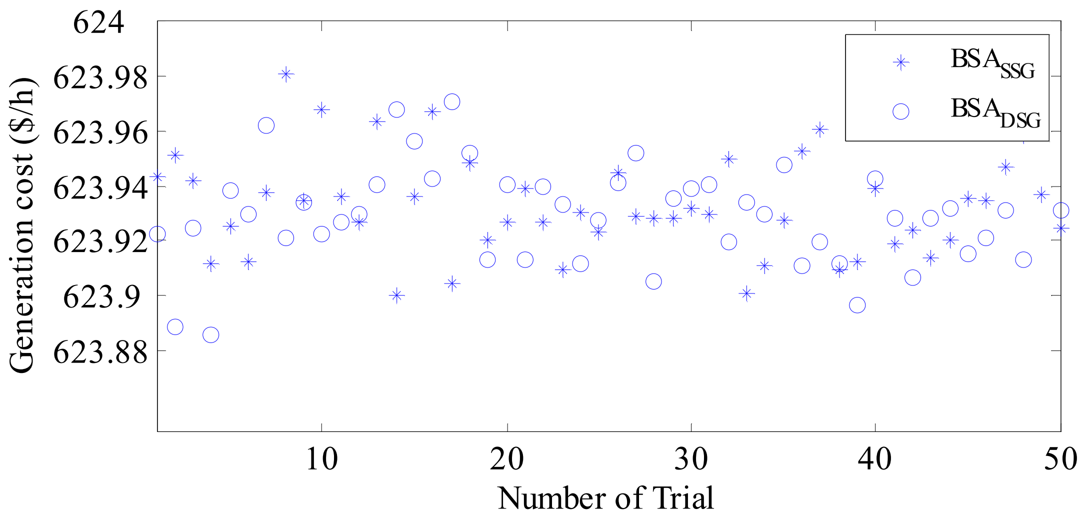


Figure 4.12. Optimal results of BSA_{SSG} and BSA_{DSG} for 50 trials in Case 6

4.3.2.4 Computational efficiency

The results of computation time show that both BSA_{SSG} and BSA_{DSG} are almost same in terms of computational efficiency in this case study with ten generating units. The low standard deviation of computation time in all runs of both methods shows that BSA_{SSG} and BSA_{DSG} reach the optimal in almost same computation time.

4.3.2.5 Comparison of BSA with other methods

The results of economic dispatch by BSA_{SSG} and BSA_{DSG} are compared with different PSO techniques (A. I. Selvakumar & Thanushkodi, 2007; A. Immanuel Selvakumar & Thanushkodi, 2008), CGA_MU and IGA_MU (C.-L. Chiang, 2005), BBO (A. Bhattacharya & P. K. Chattopadhyay, 2010a), and BBO (Aniruddha Bhattacharya & Pranab Kumar Chattopadhyay, 2010) in terms of optimal generation cost for this case study, as shown in Table 4.20. The optimal results of 623.8977 and 623.8853 ($/MW) are achieved by BSA_{SSG} and BSA_{DSG} (within 0.48 (s) and 0.41 (s), respectively), which are the lowest among all methods. The comparison between BSA_{SSG} and BSA_{DSG} and others confirms that the both BSAs outperform other approaches for solving the ED problems.

To analyze the proposed method's robustness, the results of 50 trials are also considered in this case study and the statistical indices are calculated. Table 4.21 shows the minimum, average, and maximum of optimal values achieved by BSA_{SSG}, BSA_{DSG}, and the other methods. Based on the table, the differences between the maximum and minimum results of BSA_{SSG} and BSA_{DSG} for 50 trials are the lowest among the other methods confirming the high degree of robustness of the proposed methods for solving the ED problem in this case study.

Table 4.20. Best solution for Case 6 (10-unit test system)

Generation	CGA_MU[1]	IGA_MU[1]	PSO-LRS[2]	NPSO[2]	NPSO-LRS[2]
P_1 (MW)	222.0108	219.1261	219.0155	220.6570	223.3352
P_2 (MW)	211.6352	211.1645	213.8901	211.7859	212.1957
P_3 (MW)	283.9455	280.6572	283.7616	280.4026	276.2167
P_4 (MW)	237.8052	238.4770	237.2687	238.6013	239.4187
P_5 (MW)	280.4480	276.4179	286.0163	277.5621	274.6470
P_6 (MW)	236.0330	240.4672	239.3987	239.1204	239.7974
P_7 (MW)	292.0499	287.7399	291.1767	292.1397	285.5388
P_8 (MW)	241.9708	240.7614	241.4398	239.1530	240.6323
P_9 (MW)	424.2011	429.3370	416.9721	426.1142	429.2637
P_{10} (MW)	269.9005	275.8518	271.0623	274.4637	278.9541
generation cost ($/h)	624.7193	624.5178	624.2297	624.1624	624.1273
Generation	PSO[3]	APSO1[3]	APSO2[3]	BSA_{SSG}	BSA_{DSG}
P_1 (MW)	224.7063	220.6570	223.3377	220.6475	218.4251
P_2 (MW)	212.3882	211.7859	212.1547	211.9557	211.2092
P_3 (MW)	283.4405	280.4026	276.2203	281.6679	280.6552
P_4 (MW)	239.9530	238.6013	239.4176	239.3705	239.2388
P_5 (MW)	283.8190	277.5621	274.6411	276.4148	279.8106
P_6 (MW)	241.0024	239.1204	239.7953	240.1796	239.3703
P_7 (MW)	287.8671	292.1397	285.5406	287.1455	290.1094
P_8 (MW)	240.6245	239.1530	240.6270	239.7760	240.0426
P_9 (MW)	407.9870	426.1142	429.3104	427.0714	425.3852
P_{10} (MW)	278.2120	274.4637	278.9553	275.7711	275.7537
generation cost ($/h)	624.3506	624.1624	624.0145	623.8977	623.8758

[1] (C.-L. Chiang, 2005)
[2] (A. I. Selvakumar & Thanushkodi, 2007)
[3] (A. Immanuel Selvakumar & Thanushkodi, 2008)

Table 4.21. Convergence results (for 50 trial runs) of Case 6 (10-unit test system)

Method	Total generation cost ($/h)		
	Minimum	Average	Maximum
CGA_MU[1]	624.7193	627.6078	633.8652
IGA_MU[1]	624.5178	625.8692	630.8705
PSO-LRS[2]	624.2297	625.7887	628.3214
NPSO[2]	624.1624	625.2180	627.4237
NPSO-LRS[2]	624.1273	624.9985	626.9981
PSO[3]	624.3506	625.8198	629.1037
APSO1[3]	624.1624	625.2180	627.4237
APSO2[3]	624.0145	624.8185	627.3049
BSA_{SSG}	623.8977	623.9416	623.9816
BSA_{DSG}	623.8758	623.9420	623.9764

[1] (C.-L. Chiang, 2005)
[2] (A. I. Selvakumar & Thanushkodi, 2007)
[3] (A. Immanuel Selvakumar & Thanushkodi, 2008)

4.3.3 Large scale system test: 20, 40, 80, and 160 unit systems

This test verifies the applicability of the proposed methods for solving practical ED problems with high complexities in large-scale systems. The 10-unit system is expanded to create four systems with 20, 40, 80, and 160 unit systems. Since the BSA is metaheuristic and its nature is stochastic, the results of 50 trials are obtained and statistical analysis is also performed.

The systems studied in this test are highly non-convex because the valve-point effects and multiple fuel options are addressed in the cost functions. In the ED problem for these test systems, there are no prohibited operating zones for generators. However, it does not reduce the difficulty of the ED problem because the large size and multiple fuel options make the problem highly nonlinear and hard to solve.

4.3.3.1 Solution to ED problem by BSA_{SSG} and BSA_{DSG}

In this test, the ED problem is solved with different parameters to show the performances of BSA_{SSG} and BSA_{DSG}. The systems have 20, 40, 80, 160 units and the optimization parameters are described in the related tables for each system. The optimization is run again for each set of values by BSA_{SSG} and BSA_{DSG} and the statistical indices of the results of both methods are calculated based on the 50 trials according to Table 4.22 to Table 4.29. The parameters shown in the tables are the optimization parameters and the minimum, average, maximum, and standard deviations of generation cost and computation time.

Table 4.22 and Table 4.23 show the ED results of BSA_{SSG} and BSA_{DSG} for 20-unit system. The comparison of each row of Table 4.22 with the same row of Table 4.23 shows that BSA_{DSG} is better than BSA_{SSG} in terms of solution quality and computation time.

Table 4.22. Statistical results of BSA_{SSG} for 20-unit system with different parameters

Max. iteration	popsize	mixrate	Generation cost ($/h)				CPU time (s)			
			Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
500	10	0	1255.0375	1264.9783	1278.0216	5.8455	0.47	0.51	0.58	0.03
500	10	0.2	1249.2534	1250.9888	1254.6616	1.2900	0.47	0.53	0.64	0.04
500	10	0.4	1248.8453	1250.0771	1251.9648	0.8260	0.45	0.48	0.50	0.01
500	10	0.6	1248.6999	1249.5609	1251.5860	0.6282	0.45	0.47	0.50	0.01
500	10	0.8	1248.3896	1249.5468	1251.8038	0.7006	0.45	0.47	0.48	0.01
500	10	1	1248.2463	1249.3784	1251.7006	0.8163	0.45	0.47	0.48	0.01
500	50	0	1251.2871	1257.3629	1263.5526	3.1030	0.76	0.78	0.81	0.01
500	50	0.2	1248.4281	1249.2714	1250.1936	0.3671	0.79	0.82	0.86	0.01
500	50	0.4	1248.4366	1249.0517	1250.2687	0.3602	0.79	0.82	0.86	0.02
500	50	0.6	1248.3272	1248.8087	1249.3019	0.2347	0.78	0.82	0.86	0.02
500	50	0.8	1248.2946	1248.6514	1249.1632	0.2286	0.78	0.81	0.86	0.02
500	50	1	1248.2397	1248.5776	1249.2751	0.2143	0.78	0.81	0.86	0.02

Table 4.23. Statistical results of BSA_{DSG} for 20-unit system with different parameters

Max. iteration	popsize	mixrate	Generation cost ($/h)				CPU time (s)			
			Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
500	10	0	1251.9479	1256.5459	1264.4377	2.8842	0.42	0.44	0.47	0.01
500	10	0.2	1248.4367	1249.4490	1251.1406	0.5006	0.42	0.44	0.45	0.01
500	10	0.4	1248.3163	1249.1137	1250.1396	0.3939	0.42	0.44	0.45	0.01
500	10	0.6	1248.5591	1249.0034	1250.4996	0.3793	0.42	0.44	0.45	0.01
500	10	0.8	1248.3610	1248.7712	1249.4484	0.2536	0.42	0.43	0.45	0.01
500	10	1	1248.1862	1248.6780	1250.4436	0.4214	0.42	0.43	0.45	0.01
500	50	0	1249.8642	1252.6155	1255.9166	1.2210	0.64	0.66	0.69	0.01
500	50	0.2	1248.2646	1248.7013	1249.2159	0.1818	0.64	0.66	0.69	0.01
500	50	0.4	1248.2981	1248.7092	1249.3120	0.2213	0.64	0.67	0.80	0.02
500	50	0.6	1248.1791	1248.5438	1249.0370	0.1635	0.64	0.66	0.69	0.01
500	50	0.8	1248.1938	1248.4624	1248.9708	0.1654	0.64	0.66	0.69	0.01
500	50	1	1248.1453	1248.3290	1248.6370	0.1043	0.64	0.65	0.67	0.01

The results of 40-unit systems are also listed in Table 4.24 and Table 4.25 show that the superiority of BSA_{DSG} to BSA_{SSG}. The former reaches better optimal in lower computation time than the latter.

Table 4.24. Statistical results of BSA_{SSG} for 40-unit system with different parameters

Max. iteration	popsize	mixrate	Generation cost ($/h)				CPU time (s)			
			Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
1000	10	0	2518.1308	2547.0642	2574.1238	11.5433	1.67	1.68	1.72	0.01
1000	10	0.2	2499.1426	2502.6650	2510.7832	2.3213	1.64	1.67	1.69	0.01
1000	10	0.4	2498.1572	2500.2201	2505.9942	1.3731	1.62	1.67	1.73	0.02
1000	10	0.6	2497.5865	2500.1403	2508.9780	1.8709	1.59	1.64	1.70	0.02
1000	10	0.8	2497.6835	2500.1704	2508.9057	2.1536	1.59	1.63	1.69	0.02
1000	10	1	2498.1261	2500.5732	2509.5537	2.2794	1.54	1.60	1.65	0.02
1000	50	0	2518.4533	2529.8780	2540.9606	5.5968	2.74	2.78	2.84	0.02
1000	50	0.2	2498.2545	2499.7500	2502.8189	0.8900	3.04	3.14	3.23	0.04
1000	50	0.4	2497.2422	2498.2539	2499.9283	0.5022	3.01	3.14	3.28	0.06
1000	50	0.6	2496.9829	2497.8669	2499.6734	0.4694	2.92	3.09	3.31	0.08
1000	50	0.8	2496.8659	2497.6402	2498.6142	0.4455	2.90	3.04	3.18	0.07
1000	50	1	2496.9036	2497.7294	2499.0354	0.5155	2.81	2.95	3.12	0.07

In the 80-unit system as relatively large system, the same situation occurs where BSA_{DSG} outperforms BSA_{SSG} in terms of solution quality and computation burden as shown in Table 4.26 and Table 4.27. The results also show that BSA with two constraint handling mechanisms (BSA_{SSG} and BSA_{DSG}) has high robust results since the standard deviations of the objective function within 50 trials are low in all runs.

Table 4.25. Statistical results of BSA_{DSG} for 40-unit system with different parameters

Max. iteration	popsize	mixrate	Generation cost ($/h)				CPU time (s)			
			Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
1000	10	0	2507.5284	2516.2992	2526.6359	4.4152	1.56	1.58	1.67	0.02
1000	10	0.2	2498.2450	2499.2928	2501.5787	0.7197	1.53	1.54	1.62	0.02
1000	10	0.4	2497.3885	2498.3519	2499.1647	0.4341	1.51	1.53	1.64	0.02
1000	10	0.6	2497.1850	2498.0639	2499.5803	0.5134	1.51	1.53	1.62	0.02
1000	10	0.8	2496.9064	2498.0175	2507.5277	1.4613	1.51	1.53	1.62	0.02
1000	10	1	2496.8542	2498.1133	2502.3255	1.0285	1.50	1.52	1.61	0.02
1000	50	0	2504.3214	2510.3037	2517.5198	2.7773	2.20	2.24	2.28	0.02
1000	50	0.2	2497.5654	2498.1691	2499.5713	0.4245	2.17	2.21	2.25	0.02
1000	50	0.4	2496.7812	2497.5425	2498.1992	0.3437	2.15	2.20	2.25	0.02
1000	50	0.6	2496.6781	2497.1168	2497.7004	0.2277	2.15	2.19	2.23	0.02
1000	50	0.8	2496.3035	2496.8852	2497.5973	0.2290	2.14	2.17	2.23	0.02
1000	50	1	2496.4570	2496.8160	2497.2197	0.1904	2.12	2.16	2.21	0.02

Table 4.26. Statistical results of BSA_{SSG} for 80-unit system with different parameters

Max. iteration	popsize	mixrate	Generation cost ($/h)				CPU time (s)			
			Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
1000	10	0	5211.9274	5262.1288	5319.2514	27.9245	3.59	3.66	3.71	0.03
1000	10	0.2	5025.4973	5055.4010	5085.7051	11.8063	4.26	4.44	4.65	0.08
1000	10	0.4	5023.9435	5044.7604	5074.5861	10.8753	4.13	4.49	4.77	0.11
1000	10	0.6	5017.7210	5039.7622	5065.1061	8.7699	4.07	4.38	4.81	0.15
1000	10	0.8	5022.6053	5042.3238	5070.4591	8.7051	3.96	4.24	4.62	0.13
1000	10	1	5026.0640	5045.7995	5061.5270	9.0886	3.82	4.10	4.59	0.14
1000	50	0	5169.3123	5208.7844	5253.0797	20.1026	6.72	6.95	7.22	0.10
1000	50	0.2	5019.8619	5038.4750	5054.8079	7.7689	11.03	11.67	12.20	0.26
1000	50	0.4	5012.3942	5023.9540	5041.4472	5.7647	11.17	12.03	13.03	0.44
1000	50	0.6	5011.9564	5020.9812	5030.0716	3.3188	10.69	11.69	12.59	0.48
1000	50	0.8	5014.5030	5020.6437	5029.5994	3.5312	9.91	11.21	12.73	0.57
1000	50	1	5012.0322	5021.2600	5030.7266	3.6542	9.34	10.45	11.67	0.54
1500	10	0	5134.2125	5176.2895	5216.8774	22.6360	5.32	5.39	5.48	0.03
1500	10	0.2	5007.9167	5017.6560	5027.8647	5.4684	5.88	6.28	6.46	0.12
1500	10	0.4	5004.2637	5010.8245	5020.4011	3.6873	5.85	6.23	6.60	0.16
1500	10	0.6	5003.7425	5011.2656	5025.6752	4.7815	5.77	6.11	6.51	0.18
1500	10	0.8	5004.9470	5011.5037	5020.0088	4.0289	5.57	5.86	6.15	0.14
1500	10	1	5008.7374	5018.0506	5042.2250	7.0633	5.30	5.68	6.01	0.14
1500	50	0	5091.9568	5130.7945	5165.6476	20.2771	9.89	10.18	10.48	0.11
1500	50	0.2	5002.4375	5007.8307	5014.1690	2.8031	14.96	15.89	16.99	0.42
1500	50	0.4	4999.6825	5002.8240	5006.2930	1.6856	14.93	15.88	16.80	0.44
1500	50	0.6	4999.0457	5001.9771	5005.6237	1.4501	13.60	15.36	17.22	0.67
1500	50	0.8	4999.8638	5002.2836	5006.8362	1.5663	13.12	14.55	15.77	0.65
1500	50	1	5000.3818	5004.0541	5007.9935	1.8733	12.20	13.60	15.15	0.61

Table 4.27. Statistical results of BSA_{DSG} for 80-unit system with different parameters

Max. iteration	popsize	mixrate	Generation cost ($/h)				CPU time (s)			
			Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
1000	10	0	5104.3890	5131.8013	5168.1898	15.5154	2.99	3.03	3.07	0.02
1000	10	0.2	5011.8646	5023.9407	5035.9196	5.6527	2.87	2.91	2.96	0.02
1000	10	0.4	5004.2288	5014.4560	5026.2583	4.8871	2.84	2.89	3.00	0.03
1000	10	0.6	5004.2714	5011.6998	5024.9503	5.0644	2.82	2.88	2.93	0.03
1000	10	0.8	5003.0572	5012.2557	5027.2366	5.1646	2.98	3.17	3.90	0.16
1000	10	1	5003.1986	5012.2591	5021.1475	4.1699	2.81	2.85	2.90	0.02
1000	50	0	5073.6132	5100.3073	5126.7556	11.9308	4.20	4.25	4.29	0.03
1000	50	0.2	5007.4881	5014.4494	5030.9684	4.6286	4.04	4.13	4.27	0.03
1000	50	0.4	5000.8989	5006.0616	5015.1413	3.2664	4.01	4.11	4.18	0.03
1000	50	0.6	4999.6415	5002.6834	5009.1097	2.0521	3.99	4.07	4.17	0.04
1000	50	0.8	4997.0874	5001.6440	5011.5286	2.4351	3.95	4.03	4.12	0.04
1000	50	1	4997.7045	5002.6038	5012.6891	2.4827	3.93	4.00	4.10	0.04
1500	10	0	5043.5259	5071.3963	5103.0180	12.3984	4.38	4.44	4.52	0.03
1500	10	0.2	5000.6298	5004.9875	5016.7164	2.7251	4.21	4.28	4.34	0.03
1500	10	0.4	4996.6534	5000.4797	5003.5417	1.5341	4.20	4.25	4.32	0.03
1500	10	0.6	4997.1275	4999.5974	5012.5151	2.3250	4.17	4.22	4.29	0.03
1500	10	0.8	4996.6008	5001.1499	5009.7997	3.1064	4.17	4.22	4.29	0.03
1500	10	1	4997.1673	5001.7921	5014.1940	3.6428	4.17	4.23	4.27	0.03
1500	50	0	5035.6009	5054.6392	5073.0533	6.9553	6.18	6.24	6.32	0.03
1500	50	0.2	4997.3141	5000.5114	5004.2657	1.4514	5.94	6.00	6.08	0.03
1500	50	0.4	4995.5202	4996.9250	4999.1282	0.8026	5.87	5.94	6.05	0.04
1500	50	0.6	4994.9557	4996.2497	4997.9209	0.7311	5.82	5.90	6.04	0.05
1500	50	0.8	4994.3509	4996.2641	4998.9630	0.9805	5.82	5.88	5.98	0.03
1500	50	1	4994.7991	4996.6582	4999.4305	0.9762	5.73	5.83	5.93	0.04

In the last test system with 160 units, the results of BSA with two constraint handling mechanisms are also obtained. Table 4.28 and Table 4.29 list the results of ED problem for BSA_{SSG} and BSA_{DSG}, respectively. The comparison between each row of Table 4.28 with its counterpart in Table 4.29 (representing same optimization parameters) shows that BSA_{DSG} has better performance for solving ED problem in terms of the solution quality and the computation time. The results for this largest test system also verify that BSA can produce the highly similar solution (as the standard deviations are low in most cases) for ED problem.

Table 4.28. Statistical results of BSA_{SSG} for 160-unit system with different parameters

Max. iteration	popsize	mixrate	Generation cost ($/h)				CPU time (s)			
			Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
1000	10	0	10776.7706	10859.4353	10924.6567	37.4072	19.42	20.60	22.01	0.72
1000	10	0.2	10312.4185	10391.5353	10503.8339	41.1664	51.18	54.64	57.56	1.56
1000	10	0.4	10245.7692	10353.4075	10446.7467	38.3126	50.71	56.50	60.64	2.29
1000	10	0.6	10264.3712	10346.3854	10443.2934	32.3836	46.75	55.00	61.51	2.81
1000	10	0.8	10309.0188	10355.2241	10432.1893	28.6748	44.27	50.99	60.45	4.10
1000	10	1	10317.8203	10379.0681	10450.4340	29.1314	39.81	45.71	54.68	3.44
1000	50	0	10695.8663	10784.0658	10847.3354	30.9674	77.16	83.35	89.86	2.65
1000	50	0.2	10244.4973	10307.8580	10379.2700	30.1687	258.23	273.59	286.17	6.40
1000	50	0.4	10207.1126	10258.6842	10331.0952	26.2604	262.85	284.44	306.48	9.30
1000	50	0.6	10196.8813	10238.4526	10316.3750	21.3173	248.99	280.05	305.28	12.17
1000	50	0.8	10183.6574	10234.3396	10282.0816	20.9132	242.33	267.46	315.82	15.28
1000	50	1	10205.3845	10231.4049	10268.6910	15.6615	196.58	238.66	264.93	15.23
1500	10	0	10625.6657	10720.5243	10819.4522	50.8761	27.57	29.71	31.90	1.01
1500	10	0.2	10151.7202	10206.9719	10280.1953	30.1632	71.04	76.02	80.57	2.45
1500	10	0.4	10146.4750	10183.8230	10261.6870	21.9153	66.16	76.48	87.13	3.80
1500	10	0.6	10144.8022	10187.1353	10237.7761	19.6193	63.96	71.78	82.26	4.19
1500	10	0.8	10161.4954	10214.2831	10283.0330	25.9582	55.71	65.58	76.02	4.88
1500	10	1	10198.7856	10247.9572	10311.2219	28.2745	49.95	57.02	67.89	3.82
1500	50	0	10570.5424	10628.1592	10690.4889	26.7274	113.24	122.91	130.18	3.55
1500	50	0.2	10107.1725	10142.3342	10189.0290	17.7816	355.90	380.45	404.60	10.95
1500	50	0.4	10088.5056	10113.4368	10133.9297	10.2673	351.19	385.10	413.96	14.08
1500	50	0.6	10087.4428	10108.0590	10129.2866	9.4871	324.73	362.42	417.52	21.25
1500	50	0.8	10093.7569	10109.5108	10132.5933	10.2241	287.81	327.83	373.67	20.35
1500	50	1	10094.3681	10114.5133	10136.4680	10.0498	259.38	295.90	347.08	19.02

Table 4.29. Statistical results of BSA_{DSG} for 160-unit system with different parameters

Max. iteration	popsize	mixrate	Generation cost ($/h)				CPU time (s)			
			Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
1000	10	0	10470.7070	10576.9745	10674.4064	50.9706	5.87	5.93	6.11	0.04
1000	10	0.2	10126.9475	10173.0554	10228.2211	22.8334	5.65	5.73	5.91	0.04
1000	10	0.4	10095.4022	10139.7191	10172.3120	19.2826	5.60	5.68	5.91	0.05
1000	10	0.6	10098.8107	10134.7547	10172.7379	18.5277	5.51	5.65	5.94	0.06
1000	10	0.8	10109.9106	10147.1996	10199.0463	19.6789	5.49	5.60	5.91	0.07
1000	10	1	10123.5153	10170.8714	10219.0900	22.3042	5.49	5.58	5.85	0.06
1000	50	0	10394.9876	10485.1197	10562.0295	33.8475	8.13	8.29	10.55	0.37
1000	50	0.2	10093.2489	10123.7596	10172.2868	18.5204	7.99	8.06	8.11	0.03
1000	50	0.4	10056.1957	10074.1474	10098.6619	10.4140	7.96	8.04	8.19	0.05
1000	50	0.6	10051.5383	10072.4907	10101.6652	11.6284	7.86	7.99	8.14	0.07
1000	50	0.8	10052.7716	10074.5923	10100.1199	10.3926	7.78	7.90	8.11	0.07
1000	50	1	10061.9178	10085.1404	10137.7639	14.4760	7.69	7.83	8.02	0.07
1500	10	0	10301.6394	10389.7675	10464.2289	31.7355	8.71	8.78	9.05	0.05
1500	10	0.2	10049.9547	10079.3910	10118.4112	13.1757	8.32	8.43	8.77	0.06
1500	10	0.4	10032.7670	10057.4398	10097.2990	11.8278	8.27	8.36	8.78	0.08
1500	10	0.6	10041.1413	10059.0141	10087.4439	10.5288	8.17	8.34	8.72	0.07

1500	10	0.8	10044.9393	10066.4113	10093.6247	10.2564	8.13	8.22	8.77	0.09
1500	10	1	10054.8117	10082.1959	10113.5916	12.7171	8.11	8.21	8.63	0.08
1500	50	0	10294.3621	10336.5439	10406.7472	24.1430	12.06	12.20	12.32	0.06
1500	50	0.2	10029.6351	10048.0984	10070.5260	8.4074	11.61	11.79	11.92	0.07
1500	50	0.4	10015.0728	10026.3329	10044.1186	5.8135	11.56	11.68	11.92	0.08
1500	50	0.6	10012.3647	10024.7362	10037.4700	5.7331	11.39	11.58	11.79	0.09
1500	50	0.8	10013.7428	10027.1207	10040.6493	6.4317	11.40	11.57	11.87	0.09
1500	50	1	10016.1525	10033.1987	10047.2147	6.5896	11.31	11.49	11.65	0.08

For each system from 20-unit to 160-unit systems, the best setting of optimization parameters is determined. The statistical indices of 50 trials of optimization with the best parameters are listed in Table 4.30. The optimal schedules of generators as well as the generation cost and computation time for the best solutions in these systems by the proposed BSA_{SSG} and BSA_{DSG} are listed in Appendix (Table A.19 and Table A.20).

Table 4.30. Optimization results of 20 to 160 unit systems by BSA_{SSG}, and BSA_{DSG}

			20-unit	40-unit	80-unit	160-unit
BSA_{SSG}	Generating cost ($/h)	Minimum	1248.2946	2496.8659	4999.0457	10087.443
		Average	1248.6514	2497.6402	5001.9771	10108.059
		Maximum	1249.1632	2498.6142	5005.6237	10129.287
		Standard deviation	0.2286	0.4455	1.4501	9.4871
	CPU time (s)	Minimum	0.78	2.9	13.6	324.73
		Average	0.81	3.04	15.36	362.42
		Maximum	0.86	3.18	17.22	417.52
		Standard deviation	0.02	0.07	0.67	21.25
BSA_{DSG}	Generating cost ($/h)	Minimum	1248.1791	2496.3035	4994.3509	10012.365
		Average	1248.5438	2496.8852	4996.2641	10024.736
		Maximum	1249.037	2497.5973	4998.963	10037.47
		Standard deviation	0.1635	0.229	0.9805	5.7331
	CPU time (s)	Minimum	0.64	2.14	5.82	11.39
		Average	0.66	2.17	5.88	11.58
		Maximum	0.69	2.23	5.98	11.79
		Standard deviation	0.01	0.02	0.03	0.09

4.3.3.2 Convergence Characteristics

The convergence characteristics of BSA_{SSG} and BSA_{DSG} are plotted for their best solutions for the purpose of comparison. Figure 4.13 shows the convergence of BSA_{SSG} and BSA_{DSG} for 20-unit system. BSA_{DSG} converges to the optimal earlier than BSA_{SSG} which confirms again the higher performance of BSA_{DSG}.

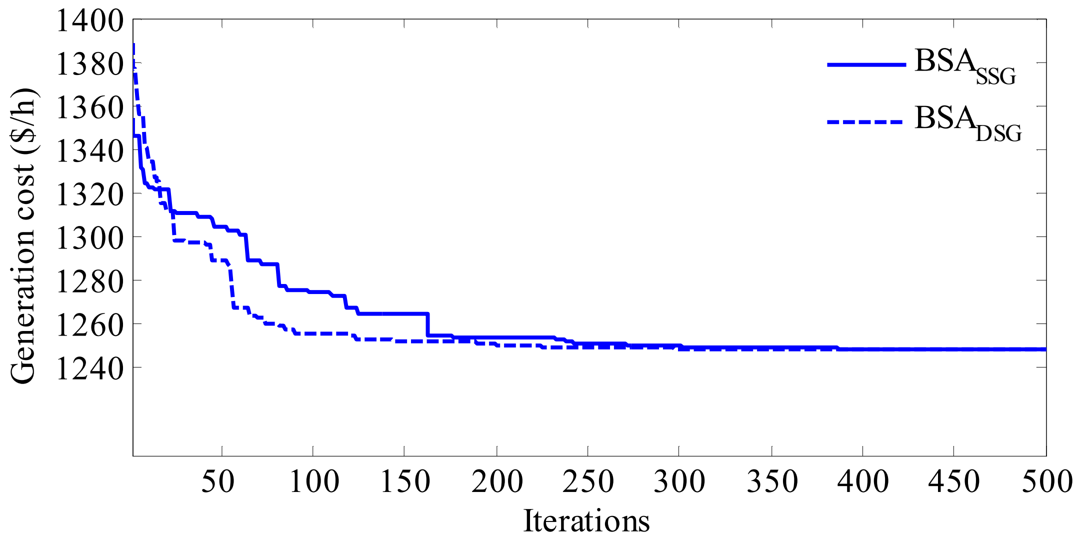


Figure 4.13. Convergence characteristic of BSA$_{SSG}$ and BSA$_{DSG}$ in 20-unit test system

For 40-unit test system, the comparison between BSA$_{SSG}$ and BSA$_{DSG}$ shows that the latter has better convergence characteristic as it converges to the optimal earlier as shown in Figure 4.14.The convergence characteristics of the third system as relatively large system are also shown in Figure 4.15. The figure shows that BSA$_{DSG}$'s convergence is better than BSA$_{SSG}$. In the largest test system with 160 units, the situation is same. BSA$_{DSG}$ again outperforms BSA$_{SSG}$ in terms of convergence characteristic as illustrated in Figure 4.16.

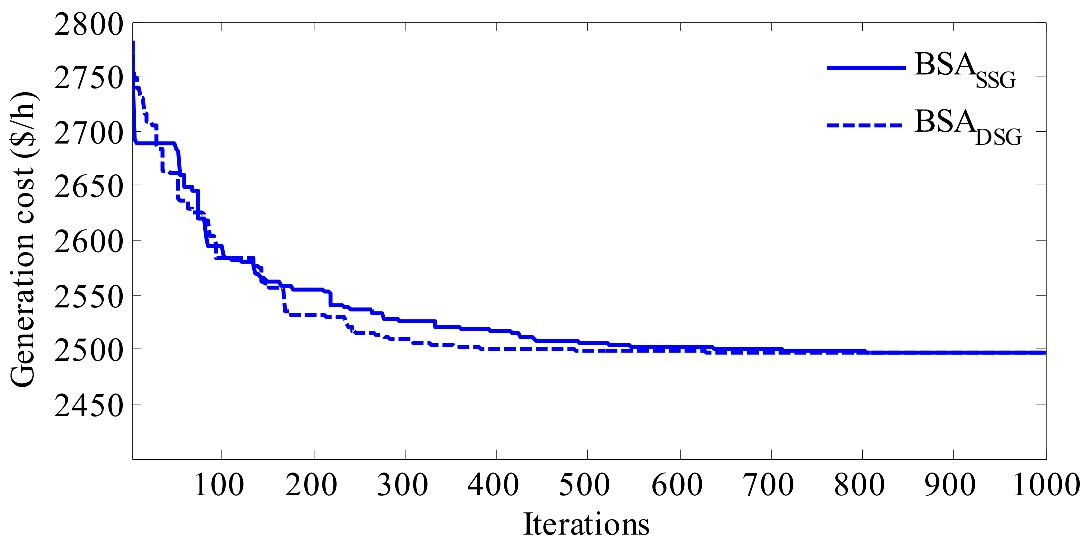


Figure 4.14. Convergence characteristic of BSA$_{SSG}$ and BSA$_{DSG}$ in 40-unit test system

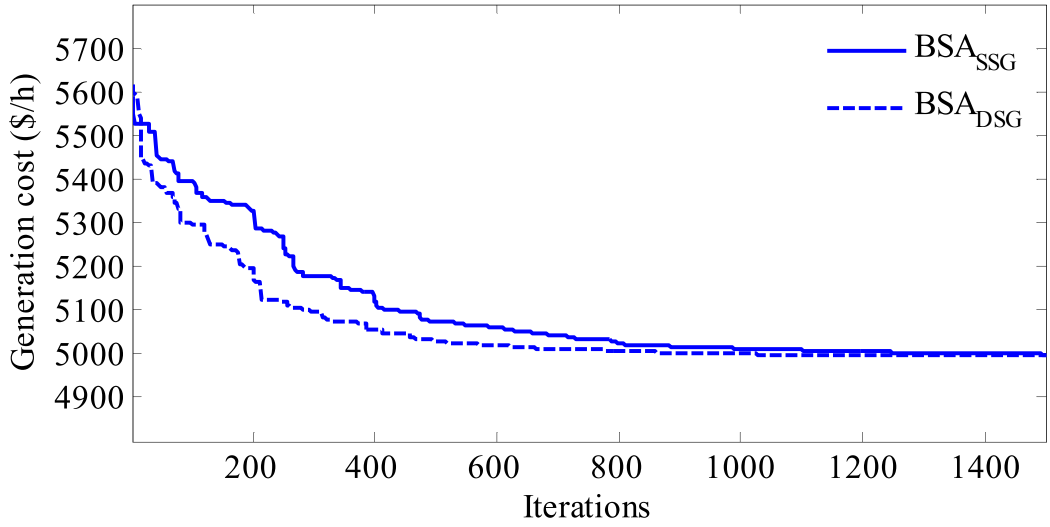


Figure 4.15. Convergence characteristic of BSA_{SSG} and BSA_{DSG} in 80-unit test system

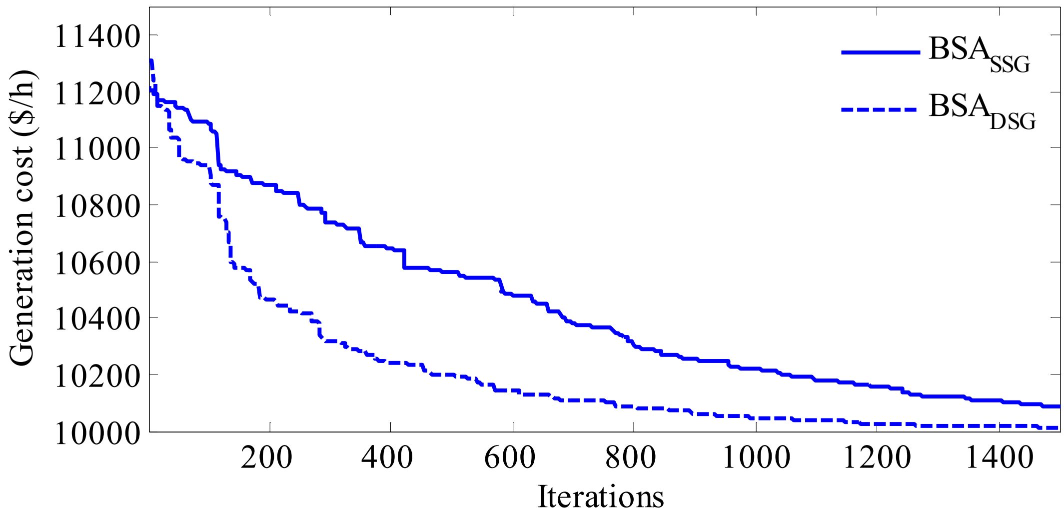


Figure 4.16. Convergence characteristic of BSA_{SSG} and BSA_{DSG} in 160-unit test system

4.3.3.3 Robustness

Figure 4.17 to Figure 4.20 show the optimal results of BSA_{SSG} and BSA_{DSG} for 50 trials in 20-, 40-, 80-, and 160-unit test systems. In these figures, it is clear that both methods (BSA_{SSG} and BSA_{DSG}) are highly robust for solving of ED problem as they produce highly similar results. However, BSA_{DSG} shows better solution quality than BSA_{SSG} in all system especially when the system size increases.

According to the aforementioned tables, The low and negligible standard deviations of optimal results obtained through 50 trials confirm that BSA with both constraint handling mechanisms converges to the similar optimal, even for the 160-unit system as the largest system with the highest degree of nonlinearity among the case studies. Therefore, the robustness of BSA (especially BSA_{DSG}) can be validated by just the results mentioned in the tables.

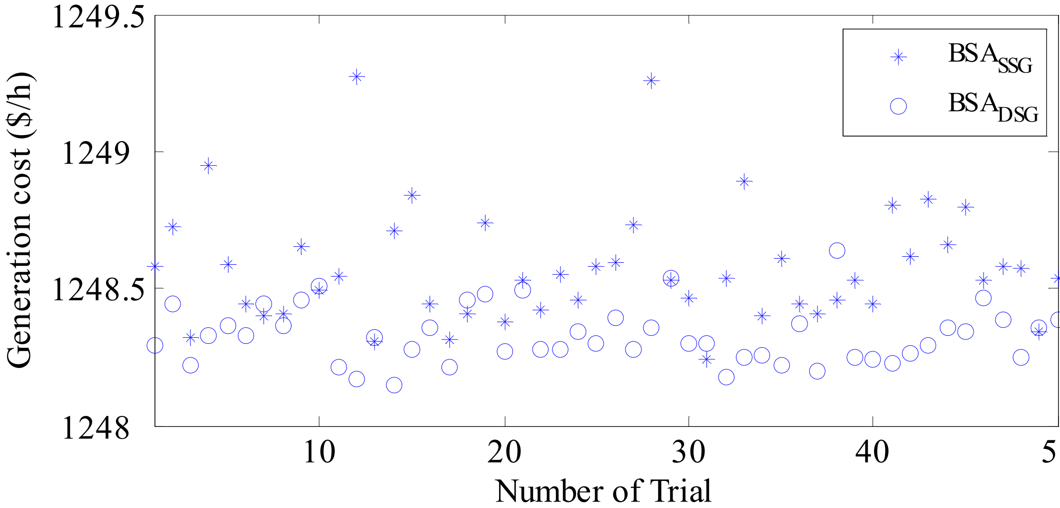


Figure 4.17. Optimal results of BSA_{SSG} and BSA_{DSG} for 50 trials in 20-unit test system

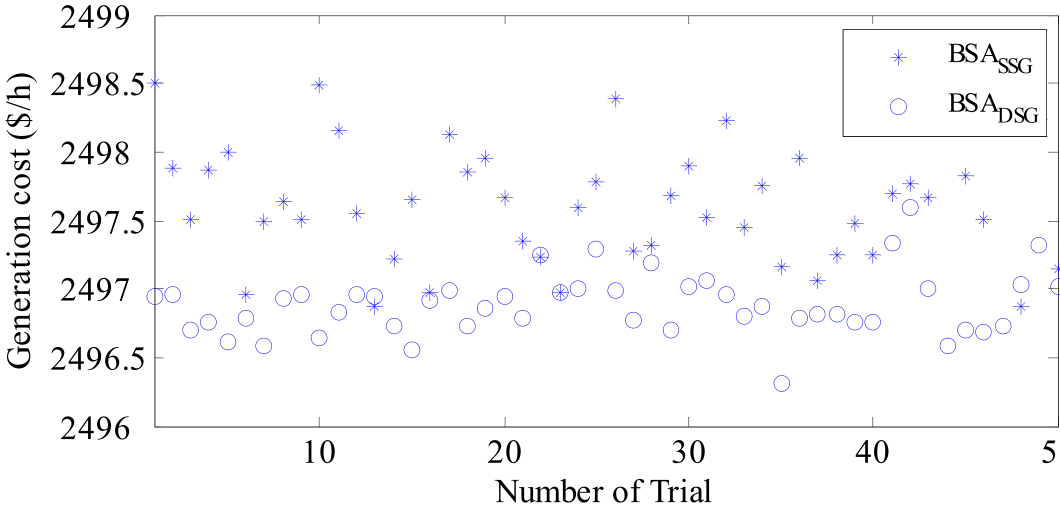


Figure 4.18. Optimal results of BSA_{SSG} and BSA_{DSG} for 50 trials in 40-unit test system

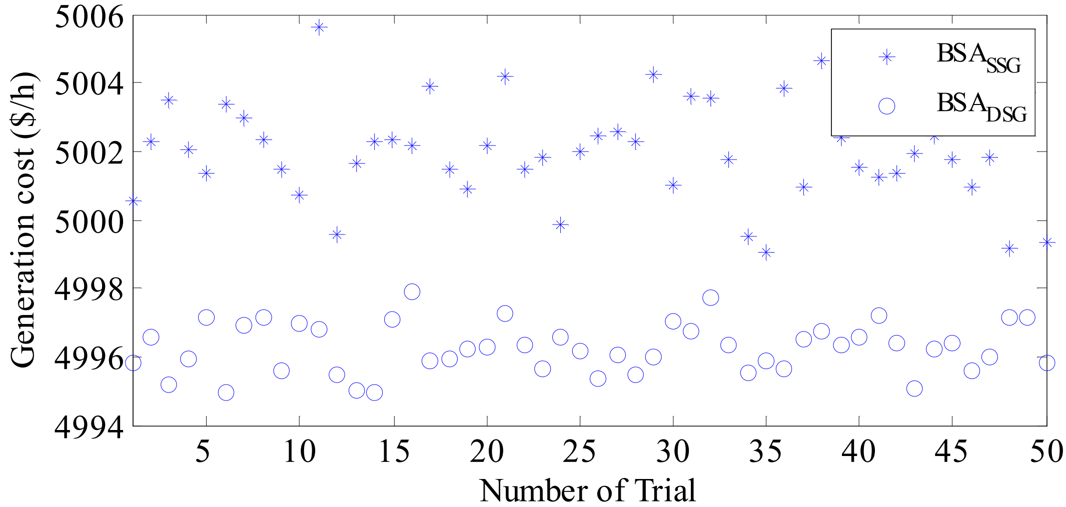


Figure 4.19. Optimal results of BSA_{SSG} and BSA_{DSG} for 50 trials in 80-unit test system

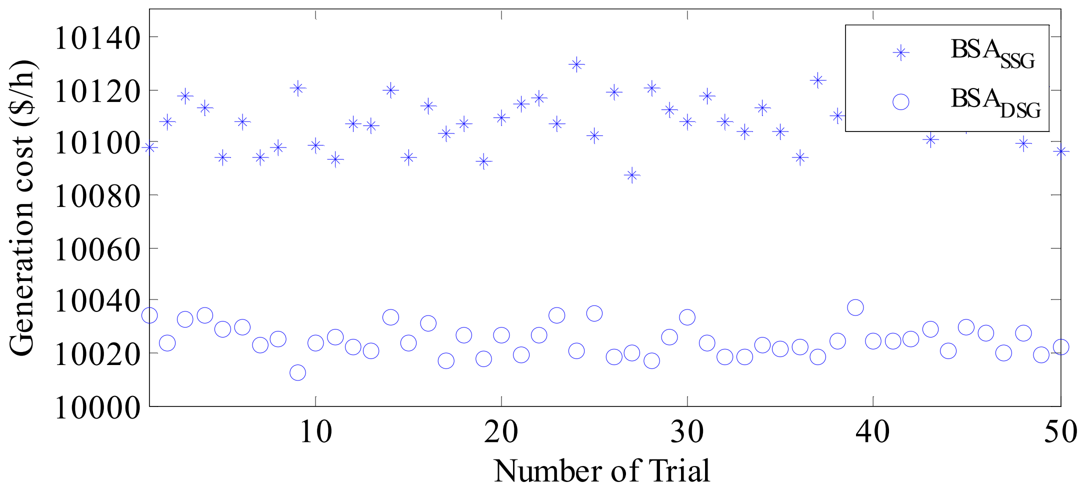


Figure 4.20. Optimal results of BSA_{SSG} and BSA_{DSG} for 50 trials in 160-unit test system

4.3.3.4 Computational efficiency

The statistical indices of all runs for the large test systems are listed in the aforementioned tables. From all the tables, it can be concluded that BSA_{DSG} reaches better optimal in shorter time which means BSA_{DSG} is more computationally efficient than BSA_{SSG}. The difference in computation burden between two methods is much obvious in the largest test system (160-unit system).

4.3.3.5 Comparison of BSA with other methods

The results of the economic dispatch for 50 trials in these systems are obtained by BSA_{SSG} and BSA_{DSG}. Since BSA_{DSG} has better performance than BSA_{SSG}, the former is selected for the comparison purpose. The average values of optimal results of BSA_{DSG}

are mentioned in Table 4.31. This table also lists the average of optimal results by CGA_MU and IGA_MU (C.-L. Chiang, 2005) and compares them with BSA_{DSG}. The table shows that BSA_{DSG} is capable of solving the ED with high quality optimal as well as the others. Since the programming codes of CGA_MU and IGA_MU are written in FORTRAN, as the high speed programing language, and the code of BSA_{DSG} is run on Matlab, the computation times cannot be compared based only on the speeds of the CPU processors. However, the computation times of BSA_{DSG} compared to the computation times in the CGA_MU for these systems are the lowest.

Table 4.31. average total generation costs and CPU times for 20, 40, 80, and 160 unit systems

Method	Number of units	20	40	80	160
CGA_MU[1]	Average generating cost ($/h)	1249.3893	2500.9220	5008.1426	10143.7263
	Average CPU time (sec)	80.48	157.39	309.41	621.30
IGA_MU[1]	Average generating cost ($/h)	1249.1179	2499.8243	5003.8832	10042.4742
	Average CPU time (sec)	21.64	43.71	85.67	174.62
BSA_{DSG}	Average generating cost ($/h)	1248.5438	2496.8852	4996.2641	10024.736
	Average CPU time (sec)	0.66	2.17	5.88	11.58

[1] (C.-L. Chiang, 2005)

4.3.4 Selection of constraint handling mechanism

The ED problem has been solved by BSA with two constraint handling mechanisms. They are called BSA_{SSG} and BSA_{DSG}. Both mechanisms have been applied on different ED problems in size and complexity. In most cases, BSA_{DSG} has shown better performance based on the solution quality and computational burden. Therefore, the suitable mechanism for BSA to solve the ED problems is to use the dynamic slack generator which corresponds to BSA_{DSG}.

4.4 Power dispatch problem in microgrid

Based on the previous sections, BSA_{DSG} is proposed as the method for power dispatch problem. It is applied on a microgrid as the case study including conventional and renewable energy technologies. The microgrid comprises of two diesel generators, two wind power plants, and three fuel-cell systems. The system data are taken from (Basu

& Chowdhury, 2013) and shown in Appendix (Table A. 10 and Table A.11). The cut-in, rated, and cut-out wind speeds are respectively equal to 5, 10, and 15 (m/s).

4.4.1 Solution to power dispatch problem

The economic power dispatch problem in power system is considered to find the optimal generation schedule for usually as time frame of one hour, however, the economic power dispatch in microgrid is usually to optimize the performance of microgrid by minimizing its generation cost within a longer time frame. In this section, the problem of economic power dispatch in microgrid is solved to minimize the generation cost of the generating units in the microgrid for the period of 24 hours.

BSA_{DSG} is applied on this case study and the optimal schedule for the whole day is obtained. BSA_{DSG} reached the optimal generation cost of 30597.92 ($) corresponding to the optimal schedule in Table 4.32. According to the table, there is no power mismatch for the solution in 24 hours which represents that the solution is feasible. The computation time for this best solution is 1.22 (s).

Table 4.32. Optimal generation scheduling of generating units within 24 hours

Hour	$P_{diesel,1}$ (kWh)	$P_{diesel,2}$ (kWh)	$P_{wt,1}$ (kWh)	$P_{wt,2}$ (kWh)	$P_{fc,1}$ (kWh)	$P_{fc,2}$ (kWh)	$P_{fc,3}$ (kWh)	Total Gen. (kWh)	P_D (kWh)
1	36.1537	0.0000	192.0000	192.0000	37.1776	96.2687	100.0000	653.6000	653.6000
2	2.3003	0.0000	114.0000	114.0000	150.0000	87.0690	83.0307	550.4000	550.4000
3	110.4162	133.1821	36.0000	36.0000	150.0000	84.8949	94.5068	645.0000	645.0000
4	57.6594	0.0000	165.0000	165.0000	136.2827	100.0000	64.0579	688.0000	688.0000
5	62.1367	0.0000	252.0000	252.0000	76.6633	100.0000	100.0000	842.8000	842.8000
6	260.7345	507.2655	0.0000	0.0000	150.0000	100.0000	100.0000	1118.0000	1118.0000
7	289.8453	564.5547	60.0000	60.0000	150.0000	100.0000	100.0000	1324.4000	1324.4000
8	308.4265	365.7414	186.0000	186.0000	150.0000	97.0321	100.0000	1393.2000	1393.2000
9	400.0000	677.6000	0.0000	0.0000	150.0000	100.0000	100.0000	1427.6000	1427.6000
10	303.6203	403.5797	168.0000	168.0000	150.0000	100.0000	100.0000	1393.2000	1393.2000
11	155.7922	312.6078	210.0000	210.0000	150.0000	100.0000	100.0000	1238.4000	1238.4000
12	121.6223	251.9777	180.0000	180.0000	150.0000	100.0000	100.0000	1083.6000	1083.6000
13	287.8524	394.1476	0.0000	0.0000	150.0000	100.0000	100.0000	1032.0000	1032.0000
14	268.2803	283.3197	48.0000	48.0000	150.0000	100.0000	100.0000	997.6000	997.6000
15	141.7502	375.8498	108.0000	108.0000	150.0000	100.0000	100.0000	1083.6000	1083.6000
16	72.0937	159.9063	225.0000	225.0000	150.0000	100.0000	100.0000	1032.0000	1032.0000
17	234.2482	353.9136	90.0000	90.0000	150.0000	99.8381	100.0000	1118.0000	1118.0000
18	252.6059	389.3941	192.0000	192.0000	150.0000	100.0000	100.0000	1376.0000	1376.0000
19	381.3091	541.0909	198.0000	198.0000	150.0000	100.0000	100.0000	1668.4000	1668.4000
20	400.0000	661.2000	120.0000	120.0000	150.0000	100.0000	100.0000	1651.2000	1651.2000
21	400.0000	764.0000	60.0000	60.0000	150.0000	100.0000	100.0000	1634.0000	1634.0000
22	286.1119	585.8881	120.0000	120.0000	150.0000	100.0000	100.0000	1462.0000	1462.0000
23	136.1680	399.4320	228.0000	228.0000	150.0000	100.0000	100.0000	1341.6000	1341.6000
24	191.9126	287.9358	120.0000	120.0000	150.0000	96.5516	100.0000	1066.4000	1066.4000

The convergence to the optimal cost is shown in Figure 4.21. BSA_{DSG} converges to the optimal within 500 iterations and the objective value decreased from around 40000 ($)

to the optimal of 30597.92 ($).

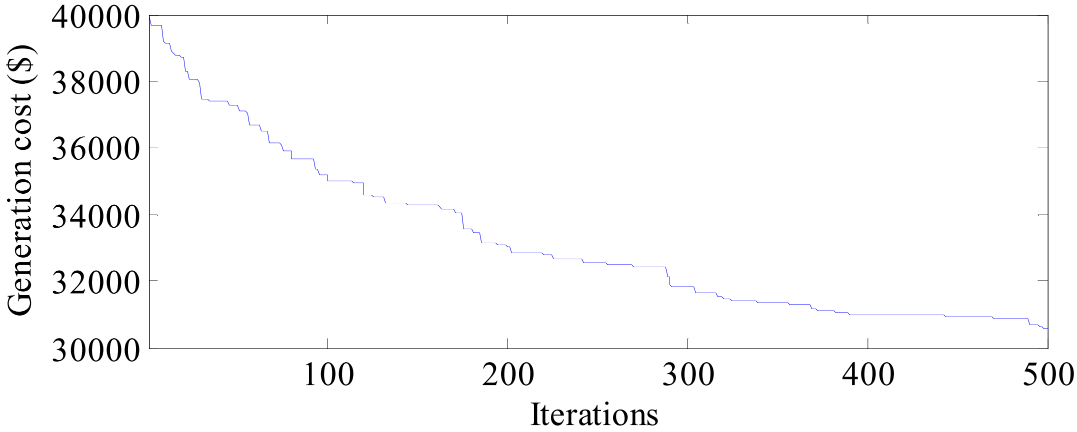


Figure 4.21. Convergence characteristics of economic dispatch for the microgrid

Since BSA_{DSG} is initialized randomly, the number of 50 independent runs is considered and the optimal results of these trials are taken into consideration for robustness test of BSA_{DSG}. Figure 4.22 shows the optimal generation cost obtained in 50 trials confirming the robustness of BSA_{DSG} as it has reached almost the same optimal values in all trials. The standard deviation of the optimal values in the 50 trials is 148.2827 ($) which is very low in the range of the optimal in this problem.

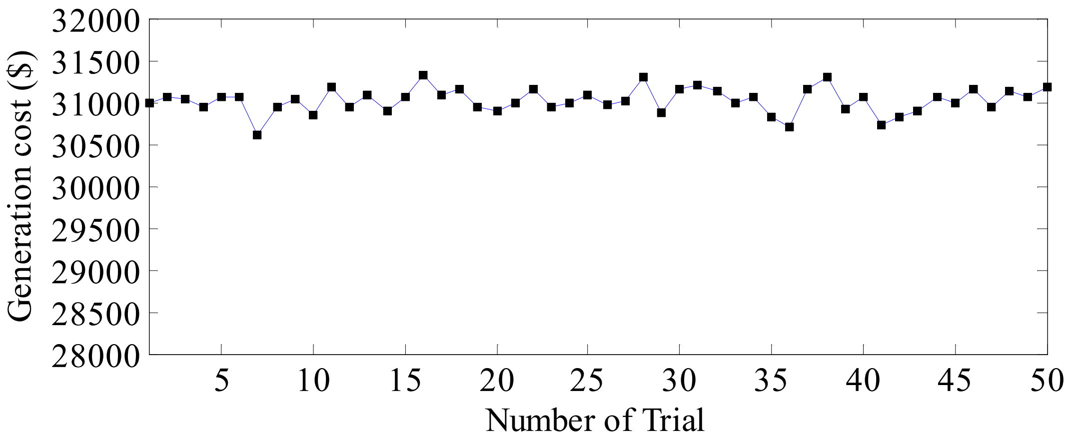


Figure 4.22. Generation cost obtained in 50 trials for the microgrid

Table 4.33 shows the optimal results by BSA_{DSG} compared to several techniques from the literature. Based on the table, BSA_{DSG} has reached the lowest generation cost among other methods for the microgrid.

Table 4.33. Comparison between methods for the microgrid

Method	Total generation within 24 hours (kWh)			Generation cost ($)
	Diesels	Wind Turbines	Fuel-cells	
CSA[1]	14482.64	8860.61	4477.75	33824.10
DE[1]	14664.50	6144.00	7012.51	33930.94
PSO[1]	15842.18	6144.00	5834.82	38189.31
BSA_{DSG}	13573.63	6144.00	8103.37	30597.92

[1] (Basu & Chowdhury, 2013)

4.4.2 Sensitivity analysis

The performance of the proposed method is affected by the control parameters which should be tuned properly to reach the best result. There are three parameters considered in the sensitivity analysis: mixrate as the control parameter of BSA_{DSG}, popsize as the population size, and the maximum iteration as the stopping criterion. For the purpose of sensitivity analysis, the values of 0 to 1 with the step of 0.10 are chosen for the mixrate and the population size values are considered to be 10, 20, 30, 40, and 50. The maximum iterations of 100 to 500 with the step of 100 are also considered in the optimization. In each case, for the specific values of the parameters, the problem of economic power dispatch is run for 50 times and the statistical indices such as minimum, average, maximum, and standard deviation values of the optimal results are recorded. The sensitivity results to each parameter are described next.

4.4.2.1 Effect of increasing mixrate on the optimal result

In this case, the population size and maximum iteration number are set to 50 and 500, respectively. The optimization is run by changing the control parameter of BSA_{DSG}. In each scenario corresponding to each value of mixrate, the results are obtained for 50 runs. Table 4.34 shows the results which are the statistical indices of objective function and the CPU time for different values of mixrate. It shows that the BSA_{DSG} reaches the better optimal by selecting relatively small but non-zero value for the mixrate as it is equal to 0.1 in this case.

Table 4.34. Results of 50 trials with different mixrate values (popsize=50 and maximum iteration=500)

mixrate	Generation cost ($/h)				mixrate	CPU time (sec)			
	Minimum	Average	Maximum	Standard deviation		Minimum	Average	Maximum	Standard deviation
0.0	32702.1236	34915.3251	35770.3801	574.4739	0.0	0.9360	0.9541	0.9980	0.0152
0.1	30597.9188	31018.0047	31324.2270	148.2827	0.1	1.1230	1.1681	1.2170	0.0214
0.2	30898.1287	31154.5812	31532.6623	145.3945	0.2	1.1700	1.2130	1.2640	0.0220
0.3	30853.0553	31267.3362	31589.9021	152.0787	0.3	1.1860	1.2390	1.2950	0.0218
0.4	31030.3332	31353.8876	31674.8362	164.6578	0.4	1.2320	1.2702	1.3260	0.0215
0.5	30914.8256	31407.8317	31770.0641	178.7407	0.5	1.2170	1.2683	1.3110	0.0219
0.6	31096.8382	31415.3266	31769.8880	151.3406	0.6	1.2160	1.2787	1.3410	0.0261
0.7	31246.2520	31514.2922	31864.7202	151.8819	0.7	1.2320	1.2814	1.3570	0.0260
0.8	31277.5373	31536.0011	31826.4793	146.5708	0.8	1.2010	1.2720	1.3260	0.0297
0.9	31226.9186	31556.4996	31999.8204	164.4045	0.9	1.2170	1.2680	1.3420	0.0308
1.0	31357.7892	31679.9594	32150.5044	165.9389	1.0	1.2160	1.2795	1.3420	0.0322

4.4.2.2 Effect of increasing population size on the optimal result

In this case, the maximum iteration and mixrate are set to 500 and 0.1, respectively and the effect of population size on the optimal results is analyzed by increasing the population size from 10 to 50. The optimization is run 50 times for each scenario and the statistical indices of the results are shown in Table 4.35. Based on the average values of optimal results in the scenarios, it is concluded that the increasing the popsize leads to a better result. However, it increases the computation time. For example, the CPU time is doubled when the population size is changed from 10 to 50.

Table 4.35. Results of 50 trials with different population size values (maximum iteration=500 and mixrate=0.10)

popsize	Generation cost ($/h)				popsize	CPU time (sec)			
	Minimum	Average	Maximum	Standard deviation		Minimum	Average	Maximum	Standard deviation
10	31113.0764	31516.2391	31976.3947	199.9088	10	0.5300	0.5604	0.5930	0.0150
20	30730.2450	31275.0794	31675.7497	204.0237	20	0.6860	0.7276	0.8260	0.0212
30	30938.3465	31154.0229	31429.5831	120.0041	30	0.8270	0.8706	0.9360	0.0224
40	30752.5836	31102.2400	31473.0241	162.3033	40	0.9830	1.0287	1.0780	0.0196
50	30597.9188	31018.0047	31324.2270	148.2827	50	1.1230	1.1681	1.2170	0.0214

4.4.2.3 Effect of increasing maximum iteration number on the optimal result

Although it is clear that the increasing the maximum iteration will help the optimizer produce better optimal, the simulation is done to show the degree of effectiveness of this parameter on the optimal results. The results are shown in Table 4.36. Based on this table, the average optimal value within 50 trials shows about %7 decrease when the maximum iteration is changed from 100 to 200. These decreases are about %10, %12, and %13 for increasing the maximum iteration to 300, 400, and 500, respectively.

Table 4.36. Results of 50 trials with different mixrate values (popsize=50 and mixrate=0.10)

Max. iteration	Generation cost ($/h)				Max. iteration	CPU time (sec)			
	Minimum	Average	Maximum	Standard deviation		Minimum	Average	Maximum	Standard deviation
100	34289.6436	35759.0459	36804.4492	604.2561	100	0.2180	0.2434	0.2810	0.0134
200	32607.4449	33411.4807	34131.6563	382.0320	200	0.4520	0.4851	0.5300	0.0158
300	31440.6210	32168.2643	32838.0958	300.7334	300	0.6700	0.7164	0.7640	0.0185
400	31105.4997	31509.3459	31938.6107	191.7159	400	0.9050	0.9379	0.9820	0.0189
500	30597.9188	31018.0047	31324.2270	148.2827	500	1.1230	1.1681	1.2170	0.0214

For better sensitivity analysis, the values of all parameters are changed. Again, the problem is solved in each scenario corresponding to specific values of parameters for 50 times and the results are obtained. In each scenario, the average of optimal results and computation time are recorded for the comparison purpose. Figure 4.23 shows a 3-dimension surface illustrating the effects of the parameters on the average of optimal results of economic power dispatch problem in 50 trials. When the mixrate is fixed, the solution is improved either by increasing the population size or the maximum iteration. When the maximum iteration is fixed to a specific value, the algorithm reaches better optimal either in a low and non-zero value of mixrate or high value of population size.

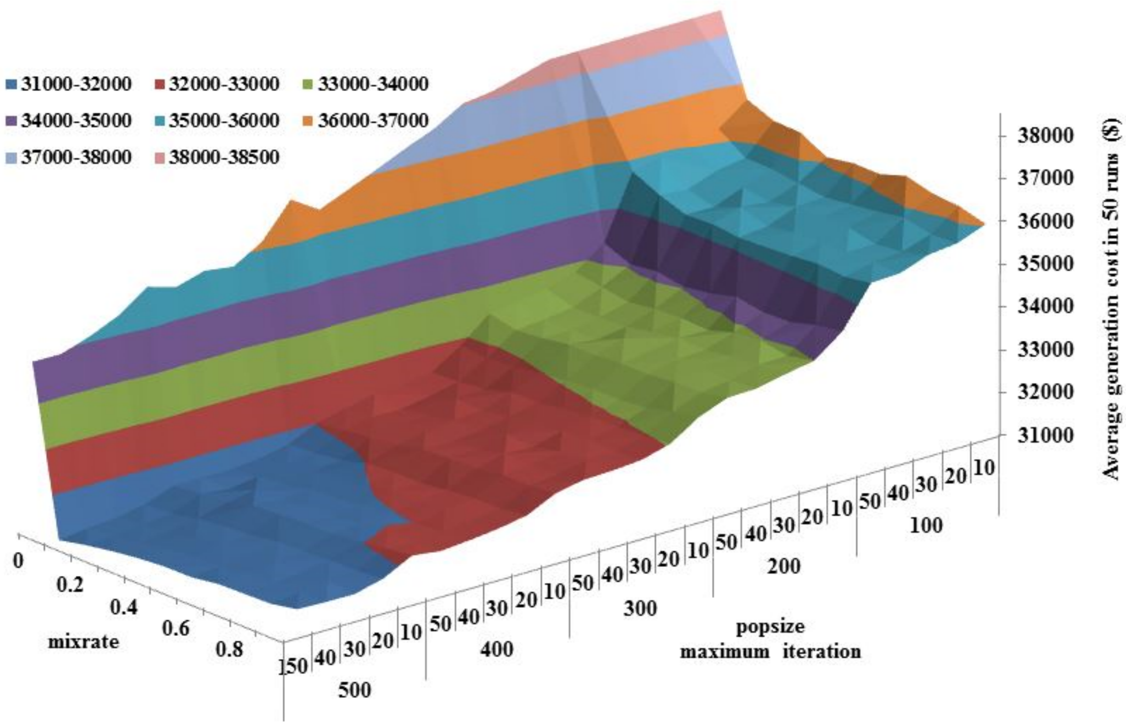


Figure 4.23. Average optimal results of BSA_{DSG} for different values of popsize, mixrate, and maximum iteration value in 50 trials

4.5 Summary

In this chapter, backtracking search algorithm (BSA) has been used to solve economic dispatch problem (ED). The performance of the proposed algorithm has been validated on six different power system benchmarks for minimizing the generation cost among the generating units. Two constraint handling mechanisms are incorporated in the proposed method and the suitable mechanism is selected based on the solution quality they produced. Application of BSA on large-scale test systems with up to 160 generating units has also reconfirmed the effectiveness of the proposed method for solving economic dispatch problems. BSA has been employed for solving the power dispatch problem in microgrids validating the robustness and high performance of the proposed method. The promising results of BSA compared to other optimization methods from the literature show the capability of the proposed method for solving the power dispatch problems in microgrid and power system. Due to high performance of BSA in solving the ED problem as the single objective optimization problem, it is used for multi-objective purpose to solve economic emission dispatch problem in the next chapter.

CHAPTER 5 : OPTMIZATION RESULTS OF ECONOMIC EMISSION DISPATCH

5.1 Introduction

In this chapter, the problem of economic and emission dispatch (EED) is solved by backtracking search algorithm (BSA). The proposed constraint handling mechanism selected in the previous chapter is incorporated into single-objective and multi-objective BSA for solving the EED problem.

Three case studies including 6-, 10-, and 14- unit power systems are used to validate the performance of BSA with the proposed constraint handling mechanism for solving of economic/emission dispatch (EED) problems. Since the method of optimization is metaheuristic using random number generations, 50 runs are considered for each problem and robustness checked by statistical indices. The value of the control parameter “mixrate” is tuned in each system to achieve high quality results. Matlab software is used for code programming and the program is run on a personal computer with Pentium 2.70GHz processor and 2GB RAM.

Transmission loss is considered in both of the test systems. In the first case study, optimization is done with/without considering transmission network loss. For solving the EED problem, three methodologies are considered to demonstrate the high performance and effectiveness of the proposed method and to compare the results with those reported in literatures.

Methodology 1 - Solution to EED problem by minimizing generation cost and emission amount separately;

Methodology 2 - Solution to EED problem by weighted sum method (WSM);

Methodology 3 - Solution to EED problem by non-dominated approach.

In the first methodology, the problems of economic dispatch and emission dispatch are solved to achieve minimum generation cost and minimum emission separately. In the second methodology, the problem of EED is also solved by weighted sum method, which combines both objectives of the problem into a single objective. Finally, the multi-objective BSA is used to minimize both objectives of EED problem simultaneously. It uses an elitist external archive to store non-dominated solutions within optimization and produces the pareto front as the optimal solution set.

5.2 Test system 1: IEEE 30-bus 6-unit system

This test system is the IEEE 30-bus power system with 6 generating units with a power demand of 283.4MW. The data is taken from (de Athayde Costa e Silva et al., 2013) and listed in Appendix (Table A.12 and Table A.13). For comparison with the method from the literature, optimization of the system is performed with/without total transmission network loss.

5.2.1 Control parameter tuning

As mentioned before, BSA has one control parameter named mixrate which affects the quality of optimal solution. The best value of control parameter should be determined to achieve the best optimal. In this case, the economic dispatch and emission dispatch problems are solved with different values of mixrate. The maximum iteration and population size are set to 500 and 10, respectively. The economic dispatch and emission dispatch problems are run 50 times for this test system with/without considering the transmission network loss. The statistical indices of optimal objective and computation time within these trials are obtained to determine the best value of control parameter. Table 5.1 shows the results of economic dispatch and emission dispatch for this system without transmission network loss. According to this table, the standard deviation of each objective(generation cost or emission amount) is the lowest when the mixrate is equal to 1so the mixrate=1 is selected. The same situation occurs for this system when the transmission network loss is considered. So the mixrate=1 is selected for this system with/out considering the transmission network loss as it leads to the best quality of solutions.

Table 5.1. statistical indices of optimal results of BSA for Test system 1 with different values of mixrate

Economic Dispatch								
mixrate	Generation cost ($/h)				CPU time (s)			
	Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
0	600.1135	600.2124	600.4262	0.0835	0.14	0.15	0.16	0.01
0.2	600.1115	600.1148	600.1299	0.0043	0.14	0.15	0.17	0.01
0.4	600.1114	600.1120	600.1186	0.0012	0.13	0.15	0.17	0.01
0.6	600.1114	600.1115	600.1124	0.0002	0.12	0.15	0.16	0.01
0.8	600.1114	600.1114	600.1118	0.0001	0.12	0.14	0.16	0.01
1	600.1114	600.1114	600.1115	0.0000	0.12	0.14	0.16	0.01
Emission Dispatch								
mixrate	Emission (ton/h)				CPU time (s)			
	Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
0	0.194203	0.194218	0.194245	1.1E-05	0.12	0.14	0.16	0.01
0.2	0.194203	0.194203	0.194204	1.9E-07	0.13	0.14	0.16	0.01
0.4	0.194203	0.194203	0.194203	3.2E-08	0.12	0.14	0.19	0.01
0.6	0.194203	0.194203	0.194203	1.3E-08	0.12	0.14	0.16	0.01
0.8	0.194203	0.194203	0.194203	3.9E-09	0.12	0.14	0.16	0.01
1	0.194203	0.194203	0.194203	9.1E-10	0.13	0.14	0.16	0.01

5.2.2 Methodology 1

The total generation cost and total emission are minimized separately by a single-objective BSA. Table 5.2 shows the best solution for the test system with/without transmission network loss. When generation cost is the only objective function, BSA reaches the optimal values of 600.1114 ($/h) and 605.9984 ($/h) for lossless and lossy systems, respectively. In emission minimization, the optimal values are 0.194179 (ton/h) and 0.194203 (ton/h) respectively for with/without transmission loss consideration.

Table 5.2. Best solution of the EED problem in Test System 1

Objective		Generations (MWh)						P_L (MWh)	Generation cost ($/h)	Emission (ton/h)
		P_1	P_2	P_3	P_4	P_5	P_6			
Generation cost Minimization	Lossless	10.9726	29.9767	52.4300	101.6192	52.4296	35.9719	0	600.1114	0.222144
	Lossy	12.0970	28.6317	58.3554	99.2853	52.3964	35.1903	2.5562	605.9984	0.220729
Emission Minimization	Lossless	40.6073	45.9068	53.7941	38.2952	53.7938	51.0027	0	638.2734	0.194203
	Lossy	41.0926	46.3670	54.4416	39.0372	54.4463	51.5483	3.5330	646.2072	0.194179

To check the robustness of the proposed method, 50 runs are performed. Their statistical indices are listed in Table 5.3 and show the proposed method producing a high quality solution in the test system.

Table 5.3. Statistical indices of the optimal results of 50 trials in Test System 1

Objective	Transmission Network	Generation cost ($/h)				Average CPU time (sec)
		Minimum	Average	Maximum	Standard deviation	
Generation cost Minimization	Lossless	600.1114	600.1114	600.1115	1.03E-05	0.14
	Lossy	605.9984	605.9984	605.9985	1.52E-05	0.77
Objective	Transmission Network	Emission (ton/h)				Average CPU time (sec)
		Minimum	Mean	Maximum	Standard deviation	
Emission Minimization	Lossless	0.194203	0.194203	0.194203	9.12E-10	0.14
	Lossy	0.194179	0.194179	0.194179	2.83E-09	0.70

The results of the proposed method are compared with those of other methods including NSGA-II (Y. Zhang et al., 2012), BB-MOPSO (Y. Zhang et al., 2012), PSO (Jiang et al., 2014), GSA (Jiang et al., 2014), MBFA (Hota et al., 2010), and MODE (Wu et al., 2010), and listed in Table 5.4.

Figure 5.1 shows the convergence characteristics of both the generation cost and the emission amount, with the transmission network loss neglected. The figure confirms the speedy convergence of BSA in solving a power dispatch problem. The approximate optimal occurs at around iteration numbers 50 and 100, respectively for emission and generation cost minimizations. The maximum iteration of 500 is considered enough to achieve a high quality solution among 50 trials. Figure 5.2 demonstrates the convergence characteristics with the transmission loss considered.

Table 5.4. Comparison between the methods in Test System 1

Method	Lossless transmission network	
	Minimum Generation cost ($/h)	Minimum Emission (ton/h)
NSGA-II [1]	600.155	0.1942
BB-MOPSO [1]	600.112	0.1942
PSO [2]	600.11 (600.1127)	0.1942
GSA [2]	601.06	0.1969
BSA	600.1114	0.1942
Method	**Lossy transmission network**	
	Minimum Generation cost ($/h)	Minimum Emission (ton/h)
MBFA [3]	606.17	0.1942
MODE [4]	606.416	0.1942
BSA	605.9984	0.1942

[1] (Y. Zhang et al., 2012)
[2] (Jiang et al., 2014)
[3] (Hota et al., 2010)
[4] (Wu et al., 2010)

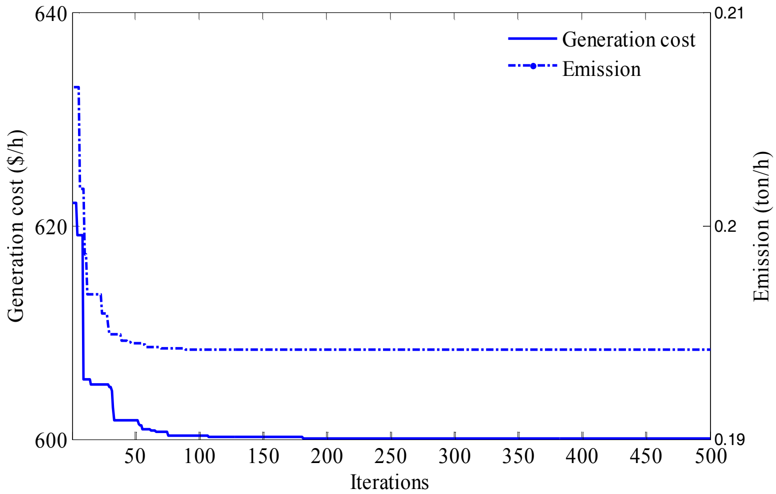


Figure 5.1. Convergence characteristics of economic dispatch and emission dispatch in Test System 1 (lossless)

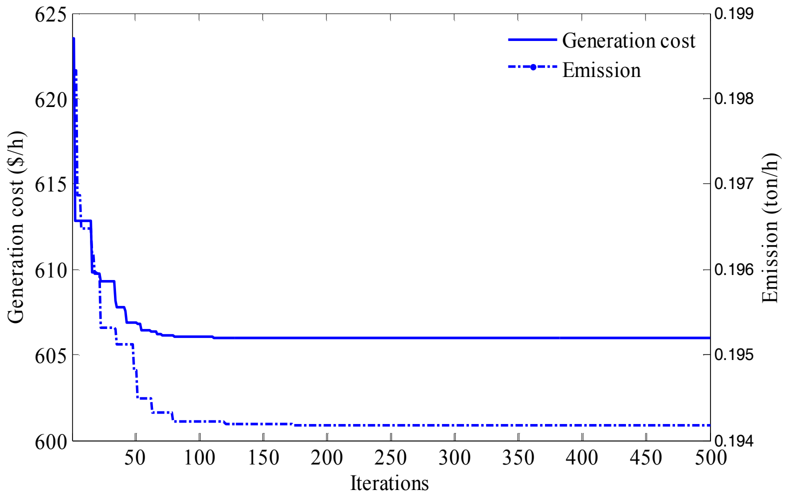


Figure 5.2. Convergence characteristics of economic dispatch and emission dispatch in Test System 1 (lossy)

5.2.3 Methodology 2 (BSA-WSM)

The problem of EED is solved by combining minimization of generation cost and emission to create a new objective function according to Eq. (3.7) to minimize the new objective function by selecting values between 0 and 1 for parameter *w*. The parameters of optimizer are set to maximum iteration=500, population size=10, and mixrate=1. The value of σ for this test system is calculated by Eq. (3.8). Different values are assigned to *w* and the optimal is achieved. All the optimal obtained by this method build the pareto front depicted in Figure 5.3. Figure 5.4 is a similar graph for lossy Test System 1.

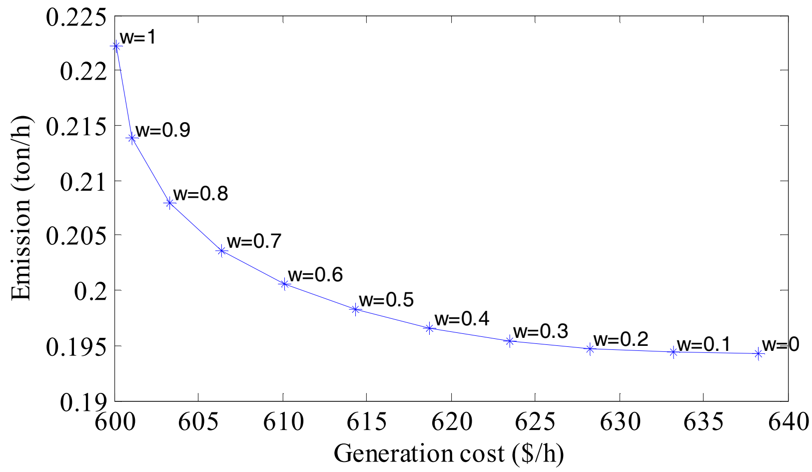


Figure 5.3. Pareto front in Test System 1 obtained by BSA-WSM with transmission network loss neglected

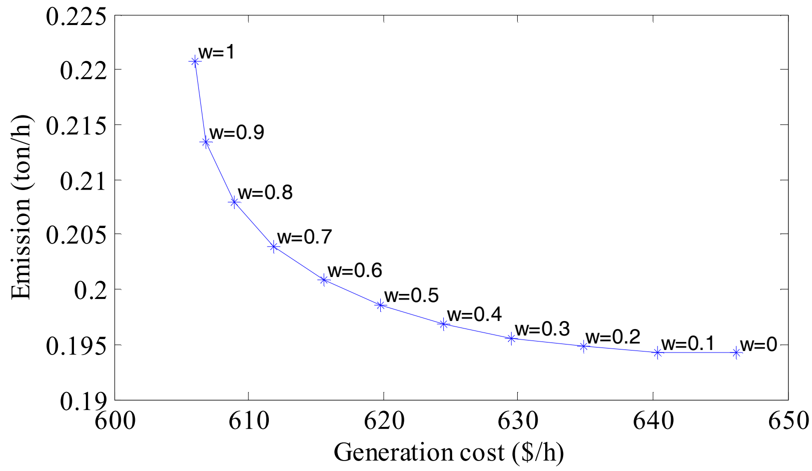


Figure 5.4. Pareto front in Test System 1 obtained by BSA-WSM with transmission network loss considered

Table 5.5 reports the pareto front solutions for this test system. The best compromise solution needs to be specified among the pareto front solutions. The values of μ_{Fc}, μ_{Fe} , and μ are calculated first and then the solution corresponding to the highest μ is chosen as the best compromise solution (shown in bold).

Table 5.6 is the generation schedule of Test System 1 for the best compromise solutions with/without transmission network loss considered. The results show that the best compromise solution is achieved by assigning *w*=0.6 to lossy/lossless networks, but still optimization is needed to find the value of *w* that gives the best compromise solution.

Table 5.5. Pareto front solutions obtained by BSA-WSM in Test System 1

w	Lossy					*w*	Lossless				
	Generation cost ($/h)	Emission (ton/h)	μ_{Fc}	μ_{Fe}	μ		Generation cost ($/h)	Emission (ton/h)	μ_{Fc}	μ_{Fe}	μ
0.0	646.2073	0.1942	0.0000	1.0000	0.0696	0.0	638.2733	0.1942	0.0000	1.0000	0.0698
0.1	640.4203	0.1943	0.1439	0.9948	0.0793	0.1	633.2512	0.1943	0.1316	0.9957	0.0787
0.2	634.8396	0.1948	0.2827	0.9780	0.0878	0.2	628.2998	0.1947	0.2613	0.9815	0.0867
0.3	629.5092	0.1956	0.4153	0.9478	0.0949	0.3	623.4593	0.1954	0.3882	0.9554	0.0938
0.4	624.4790	0.1968	0.5404	0.9018	0.1004	0.4	618.7781	0.1966	0.5109	0.9147	0.0995
0.5	619.8125	0.1985	0.6564	0.8369	0.1040	0.5	614.3199	0.1982	0.6277	0.8558	0.1035
0.6	**615.5878**	**0.2008**	**0.7615**	**0.7492**	**0.1052**	**0.6**	**610.1700**	**0.2005**	**0.7364**	**0.7738**	**0.1054**
0.7	611.9087	0.2039	0.8530	0.6330	0.1035	0.7	606.4445	0.2037	0.8340	0.6619	0.1044
0.8	608.9169	0.2080	0.9274	0.4801	0.0980	0.8	603.3136	0.2079	0.9161	0.5096	0.0995
0.9	606.8254	0.2134	0.9794	0.2771	0.0875	0.9	601.0442	0.2138	0.9756	0.2997	0.0890
1.0	605.9984	0.2207	1.0000	0.0000	0.0696	1.0	600.1114	0.2221	1.0000	0.0000	0.0698

Table 5.6. Generation schedule of the best compromise solution in Test System 1

	Lossless	Lossy
w	0.6	0.6
P_1 (MWh)	26.10587	25.27943
P_2 (MWh)	37.55198	37.16403
P_3 (MWh)	53.94769	56.5829
P_4 (MWh)	68.6181	68.89557
P_5 (MWh)	53.94869	54.95793
P_6 (MWh)	43.22768	43.12355
Total Gen. (MWh)	283.4	286.0034
P_L (MWh)	0	2.603403
Generation cost ($/h)	610.17	615.5878
Emission (ton/h)	0.200523	0.200837

5.2.4 Methodology 3 (BSA-NDA)

The EED problem is solved by a non-dominated approach. The pareto front solutions are generated by optimization and updated as optimization progresses. The best compromise solution to the EED problem is obtained from the pareto front by a fuzzy-based decision maker which evaluates the solutions and picks the solution with the highest index of μ. For setting the parameters, the maximum size of the external elitist archive is set to 50 non-dominated solutions and the control parameter is also set to its maximum to obtain the best objective values. The maximum iteration number and population size are also set to 1000 and 20, respectively.

Figure 5.5 and Figure 5.6 show the pareto front solutions for Test System 1, respectively with the transmission loss ignored and considered. Use of the crowding distance measure is to have approximate uniform pareto front solutions. The best compromise solution is determined on the index μ of the non-dominated solutions. Table 5.7 is the generation schedule and lists the objectives' values that correspond to the best compromise solutions.

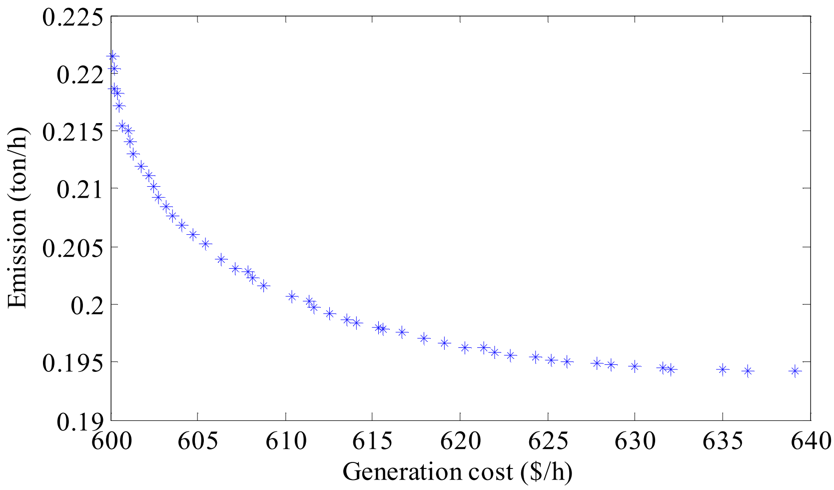


Figure 5.5. Pareto front in Test System 1 obtained by BSA-NDA with the transmission network loss neglected

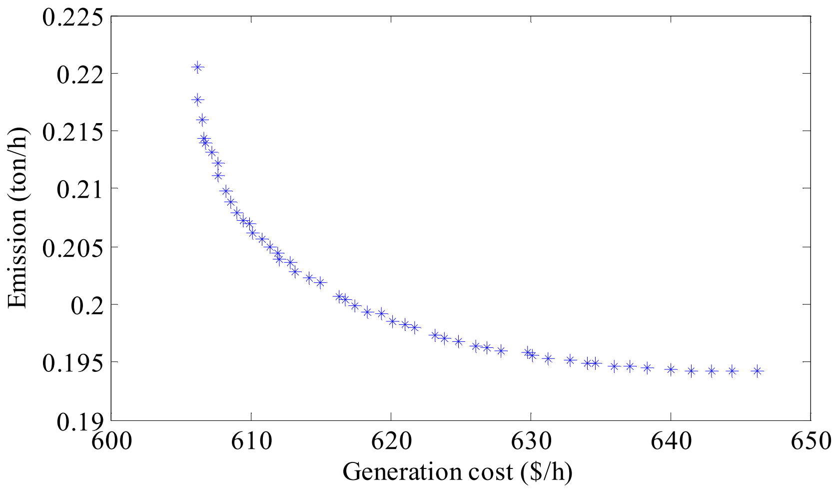


Figure 5.6. Pareto front in Test System 1 obtained by BSA-NDA with the transmission network loss considered

Table 5.7. Optimization results for the best compromise solutions in Test System 1

Network	Generations (MWh)						Total Gen. (MWh)	P_L (MWh)	Generation cost ($/h)	Emission (ton/h)
	P_1	P_2	P_3	P_4	P_5	P_6				
Lossless	24.9127	36.1931	54.9855	70.9295	52.6095	43.7697	283.4000	0.0000	608.8043	0.20156
Lossy	25.7366	40.1405	57.4497	64.8216	53.6242	44.2741	286.0466	2.6466	618.3255	0.19932

In order to compare the qualities of optimal solutions obtained by BSA-WSM and BSA-NDA, the pareto front sets of these two methodologies are shown in Figure 5.7 and Figure 5.8 for lossless/lossy test system 1. The pareto front set of BSA-WSM includes 11 points corresponding to different values of weighting factor (w) but the pareto front of BSA-NDA has 50 points. Comparison between the pareto front sets of BSA-NDA and BSA-WSM confirms that BSA-NDA can produce optimal solutions with the same qualities of BSA-WSM.

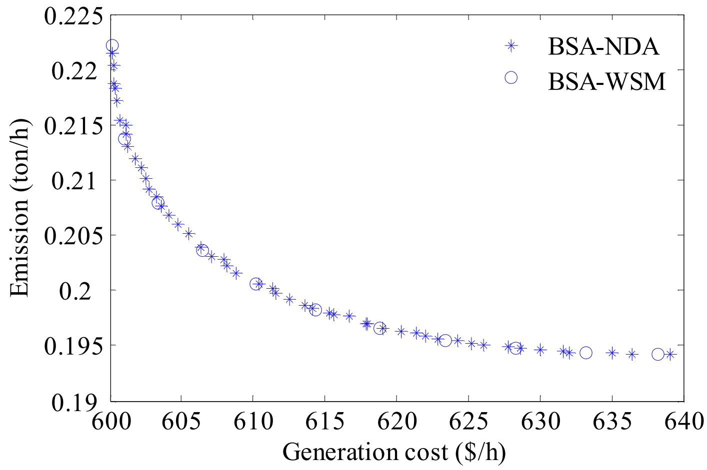


Figure 5.7. Pareto front sets in Test System 1 with the transmission network loss neglected

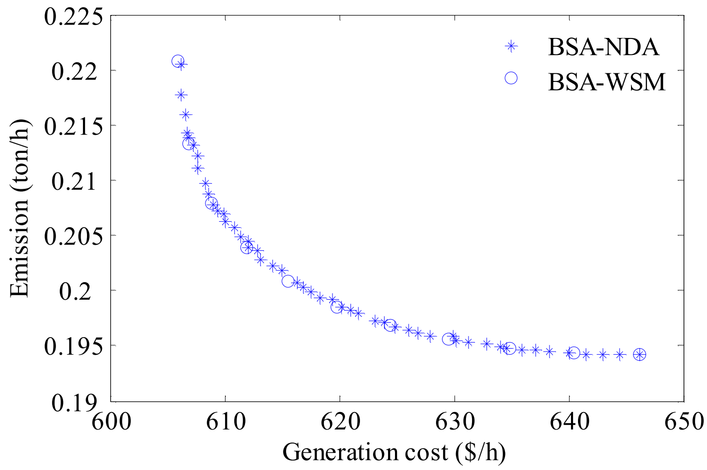


Figure 5.8. Pareto front sets in Test System 1 with the transmission network loss considered

5.3 Test system 2: 10-unit system

This test system consists of 10 generating units. The power demand is 2000MW. The valve point effects are modeled on the cost functions and the transmission loss is considered through loss coefficients. The generator data is taken from (Basu, 2011) and listed in Appendix (Table A.14 and Table A.15). The transmission loss coefficients are listed in Appendix (Table A.16).

5.3.1 Control parameter tuning

Although the performance of BSA is not over sensitive to its control parameter, the economic dispatch and emission dispatch problems are run with different values of mixrate to find its best value. Again, the maximum iteration and population size are set to 500 and 10, respectively. The values of 0 to 1 with steps of 0.2 are selected for mixrate and the problems of economic dispatch and emission dispatch are run 50 with each value of this parameter. The statistical indices of optimal results and computation time within the trials are obtained for the analysis. Table 5.8 shows the indices including minimum, average, maximum, and standard deviation values of optimal results and computation time for this test system. According to the table, the value of mixrate=1 is again selected since it produces the optimal solutions with lowest. However, other values of this parameter lead to almost the same results.

As with the preceding test system, three methodologies are considered for this system and their results analyzed in each part.

Table 5.8. statistical indices of optimal results of BSA for Test system 2 with different values of mixrate

Economic Dispatch								
mixrate	Generation cost ($/h)				CPU time (s)			
	Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
0	111497.6409	111498.6230	111501.6076	1.05E+00	1.92	2.26	2.61	0.14
0.2	111497.6295	111497.6341	111497.6533	5.41E-03	3.40	3.77	4.24	0.18
0.4	111497.6278	111497.6310	111497.6414	2.39E-03	3.56	3.92	4.34	0.17
0.6	111497.6278	111497.6295	111497.6332	1.19E-03	3.60	4.05	4.52	0.20
0.8	111497.6279	111497.6290	111497.6311	8.08E-04	3.65	4.10	4.46	0.23
1	111497.6276	111497.6286	111497.6307	7.96E-04	3.40	4.29	4.74	0.30
Emission Dispatch								

mixrate	Emission (ton/h)				CPU time (s)			
	Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
0	3932.2866	3932.6151	3933.6049	0.2716	1.81	2.22	2.53	0.17
0.2	3932.2433	3932.2485	3932.2630	0.0050	2.99	3.36	3.70	0.18
0.4	3932.2432	3932.2451	3932.2559	0.0023	3.06	3.41	3.82	0.20
0.6	3932.2432	3932.2445	3932.2532	0.0018	3.04	3.47	4.01	0.20
0.8	3932.2433	3932.2440	3932.2483	0.0011	3.03	3.46	4.10	0.26
1	3932.2432	3932.2435	3932.2444	0.0003	2.92	3.46	4.48	0.26

5.3.2 Methodology 1

Table 5.9 lists the best solutions in Test System 2 for individual minimizing of generation cost and emission. BSA reached the values of 111497.6276 ($/h) and 3932.2432 (ton/h) as minimum generation cost and emission, respectively.

Table 5.10 lists the statistical indices of 50 independent runs. BSA reached the optimal values of the generation cost and emission with very low standard deviations, proving the high robustness of the proposed method.

Table 5.9. Best solution of the EED problem in Test System 2

	Generation cost Minimization	Emission Minimization
P_1 (MWh)	55.0000	55.0000
P_2 (MWh)	80.0000	80.0000
P_3 (MWh)	106.9295	81.1749
P_4 (MWh)	100.6028	81.3585
P_5 (MWh)	81.4990	160.0000
P_6 (MWh)	83.0074	240.0000
P_7 (MWh)	300.0000	294.4430
P_8 (MWh)	340.0000	297.2970
P_9 (MWh)	470.0000	396.8075
P_{10} (MWh)	470.0000	395.5131
Total Gen. (MWh)	2087.0387	2081.5940
P_L (MWh)	87.0388	81.5941
Generation cost ($/h)	111497.6276	116412.3843
Emission (ton/h)	4572.2607	3932.2432

Table 5.10. Statistical indices of the optimal results of 50 trials in Test System 2

Objective	Generation cost ($/h)				Average CPU time (sec)
	Minimum	Average	Maximum	Standard deviation	
Generation cost Minimization	111497.6276	111497.6286	111497.6307	0.0008	4.29
Objective	Emission (ton/h)				Average CPU time (sec)
	Minimum	Average	Maximum	Standard deviation	
Emission Minimization	3932.2432	3932.2435	3932.2444	0.0003	3.46

A comparison between the proposed method and others from literature, i.e., with EMOCA (R. Zhang et al., 2013), MODE (R. Zhang et al., 2013), and NSGAII (R. Zhang et al., 2013), is made in Table 5.11. The results confirm high performance of the proposed method. The convergence characteristics of both objectives are achieved separately and shown by Figure 5.9. Again, high-speed convergence to the optimal is demonstrated.

Table 5.11. Comparison between methods in Test System 1

Method	Generation cost Minimization		Emission Minimization	
	Generation cost ($/h)	Emission (ton/h)	Generation cost ($/h)	Emission (ton/h)
EMOCA [1]	111,509.43	4528.08	116,418.8300	3934.5400
MODE [1]	112,198.22	4308.7500	115,434.8000	3979.7700
NSGAII [1]	112,497.45	4263.4100	115,157.7400	4021.9500
BSA	111497.6276	4572.2607	116412.3843	3932.2432

[1] (R. Zhang et al., 2013)

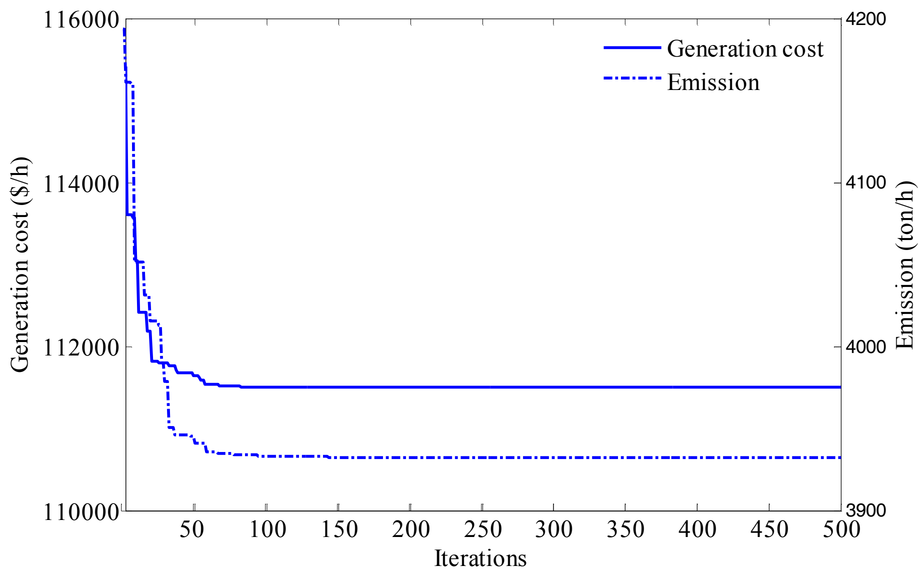


Figure 5.9. Convergence characteristics of economic dispatch and emission dispatch in Test System 2

5.3.3 Methodology 2 (BSA-WSM)

In this method of solving the EED problem, the general settings are same as the first methodology (maximum iteration=500, population size=10, mixrate=1). The combined objective function is minimized in this case study for different values of *w* as the weighting factor. Then the pareto front solutions are obtained by running the optimization for all values of w. The value of *w*=0 corresponds to the minimum emission while the value of w=1 is set to minimize the generation cost. Figure 5.10 shows the pareto front for this test system and the best compromise solution is selected among these solutions based on the fuzzy-based decision maker. The points corresponding to the values of w between 0 and 0.4 are so close to each other. Table 5.12 shows the generation cost and emission amount for all the pareto solutions. The parameter μ is calculated to determine the best compromise solution among the pareto front set. The best compromise solution corresponds to *w*=0.8 (shown in bold) and the generation schedule is Table 5.13.

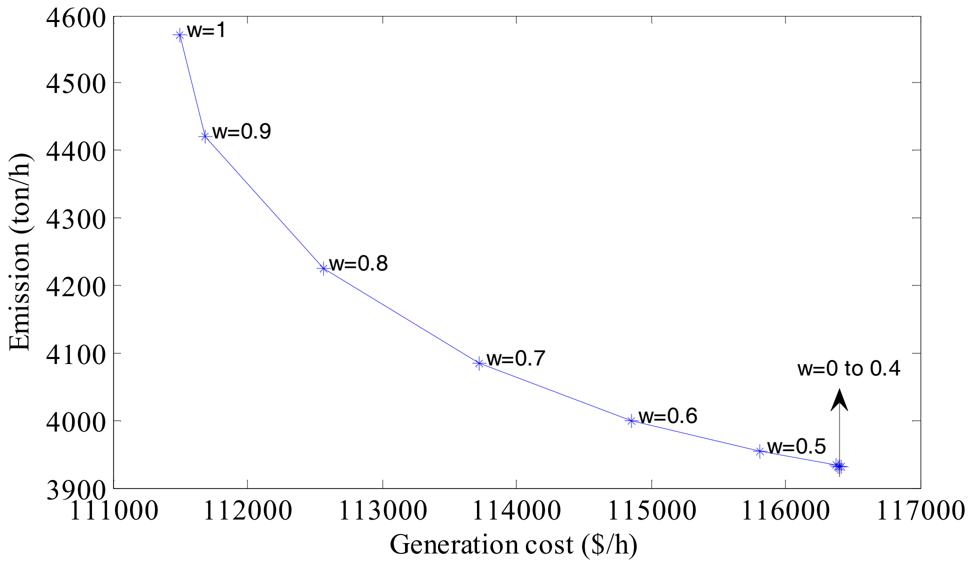


Figure 5.10. Pareto front in Test System 2 obtained by BSA-WSM

Table 5.12. Pareto front solutions obtained by BSA-WSM in Test System 2

w	Generation cost ($/h)	Emission (ton/h)	μ_{Fc}	μ_{Fe}	μ
0.0	116412.2490	3932.2431	0.0000	1.0000	0.0823
0.1	116406.8390	3932.2555	0.0011	1.0000	0.0824
0.2	116400.1944	3932.3035	0.0025	0.9999	0.0825
0.3	116392.5652	3932.4060	0.0040	0.9997	0.0826
0.4	116382.7030	3932.6195	0.0060	0.9994	0.0827
0.5	115805.7789	3952.9053	0.1234	0.9677	0.0898
0.6	114859.9425	3998.6481	0.3159	0.8962	0.0997
0.7	113727.5885	4084.3365	0.5463	0.7624	0.1077
0.8	**112559.8061**	**4223.7248**	**0.7839**	**0.5446**	**0.1093**
0.9	111689.3742	4419.8193	0.9610	0.2382	0.0987
1.0	111497.6276	4572.2340	1.0000	0.0000	0.0823

Table 5.13. Generation schedule of the best compromise solution in Test System 2

w	0.8
P_1 (MWh)	55.0000
P_2 (MWh)	80.0000
P_3 (MWh)	86.9822
P_4 (MWh)	85.0431
P_5 (MWh)	124.1857
P_6 (MWh)	140.1938
P_7 (MWh)	300.0000
P_8 (MWh)	322.0647
P_9 (MWh)	444.0322
P_{10} (MWh)	447.4826
Total Gen. (MWh)	2084.9842
P_L (MWh)	84.9842
Generation cost ($/h)	112559.8061
Emission (ton/h)	4223.7248

5.3.4 Methodology 3 (BSA-NDA)

The non-dominated approach is applied to the test system and the pareto front is generated as the optimization product. The population size is set to 20 and the control parameter of BSA is set to its maximum value. The maximum iteration number is set to 1000.

Figure 5.11 shows the pareto front optimal solutions for Test System 2. The best compromise solution is selected among the pareto members according to the fuzzy membership function. The solution corresponds to the generation cost of 112807.3733 ($/h) and emission of 4188.0926 (ton/h). The generation schedule of this solution is Table 5.14. The 50 non-dominated solutions of the pareto front are listed in Table 5.15 with the index μ and the best compromise solution is shown in bold.

Table 5.14. Optimization results for the best compromise solutions in Test System 2

Generations (MWh)					Total Gen. (MWh)	Generation cost ($/h)
P_1	P_2	P_3	P_4	P_5		
55.0000	80.0000	86.5308	86.9844	129.1542	2084.5042	112807.3733
P_6	P_7	P_8	P_9	P_{10}	P_L (MWh)	Emission (ton/h)
146.9258	300.0000	323.9002	435.9938	440.0149	84.5042	4188.0926

Table 5.15. Pareto front solutions obtained by BSA-NDA in Test System 2

Non-dominated Solution	Generation cost ($/h)	Emission (ton/h)	μ_{Fc}	μ_{Fe}	μ
1	111498.8712	4563.3844	1.0000	0.0000	0.01646
2	111503.9712	4544.1570	0.9990	0.0305	0.01695
3	111518.7118	4518.0852	0.9959	0.0718	0.01758
4	111524.9000	4507.6570	0.9947	0.0884	0.01783
5	111554.5969	4482.5119	0.9886	0.1283	0.01839
6	111579.5973	4467.0597	0.9835	0.1528	0.01871
7	111615.4200	4451.6393	0.9762	0.1772	0.01899
8	111626.0628	4444.7880	0.9740	0.1881	0.01913
9	111689.8847	4422.1498	0.9610	0.2240	0.01951
10	111760.4104	4397.4059	0.9466	0.2633	0.01992
11	111880.6535	4367.4000	0.9220	0.3108	0.02030
12	111931.4511	4351.6543	0.9116	0.3358	0.02054
13	112016.3930	4332.9628	0.8943	0.3655	0.02074
14	112074.3744	4316.7676	0.8825	0.3911	0.02097
15	112166.1585	4301.2681	0.8637	0.4157	0.02106
16	112227.9067	4282.9632	0.8511	0.4448	0.02133
17	112308.2886	4269.6754	0.8347	0.4658	0.02141
18	112378.1214	4254.3592	0.8204	0.4901	0.02158
19	112456.2578	4241.7779	0.8045	0.5101	0.02164
20	112534.9153	4228.7445	0.7884	0.5308	0.02172
21	112619.4639	4215.1500	0.7711	0.5523	0.02179
22	112723.0062	4199.9595	0.7500	0.5764	0.02184
23	**112807.3733**	**4188.0926**	**0.7328**	**0.5952**	**0.02186**
24	112989.2872	4166.5250	0.6956	0.6294	0.02181
25	113082.6571	4154.3583	0.6765	0.6487	0.02182
26	113211.0269	4139.1445	0.6503	0.6729	0.02178
27	113299.3022	4129.6698	0.6323	0.6879	0.02173
28	113380.8091	4122.2379	0.6156	0.6997	0.02165
29	113542.4002	4107.2632	0.5826	0.7234	0.02150
30	113678.9075	4092.8555	0.5547	0.7463	0.02142
31	113817.1649	4081.0124	0.5265	0.7651	0.02126
32	113917.1383	4072.3223	0.5061	0.7788	0.02115
33	113988.3540	4064.7308	0.4915	0.7909	0.02111
34	114080.6635	4058.2750	0.4727	0.8011	0.02097
35	114194.9102	4049.7594	0.4494	0.8146	0.02081
36	114299.4185	4046.9598	0.4280	0.8191	0.02053
37	114350.5049	4034.6881	0.4176	0.8385	0.02068
38	114497.3544	4024.8994	0.3876	0.8541	0.02044
39	114654.0939	4015.0634	0.3556	0.8697	0.02017
40	114864.6593	4007.2616	0.3126	0.8820	0.01967
41	114987.9375	3997.2219	0.2874	0.8980	0.01951
42	115133.3062	3989.4398	0.2577	0.9103	0.01923
43	115198.1311	3982.0203	0.2445	0.9221	0.01920
44	115350.0015	3978.5845	0.2134	0.9275	0.01878
45	115558.8710	3974.3960	0.1708	0.9342	0.01819
46	115663.8117	3965.7522	0.1493	0.9479	0.01806
47	115878.7444	3952.5330	0.1055	0.9688	0.01769
48	116070.8317	3944.8237	0.0662	0.9811	0.01724
49	116224.9607	3939.6831	0.0347	0.9892	0.01686
50	116395.0552	3932.8879	0.0000	1.0000	0.01646

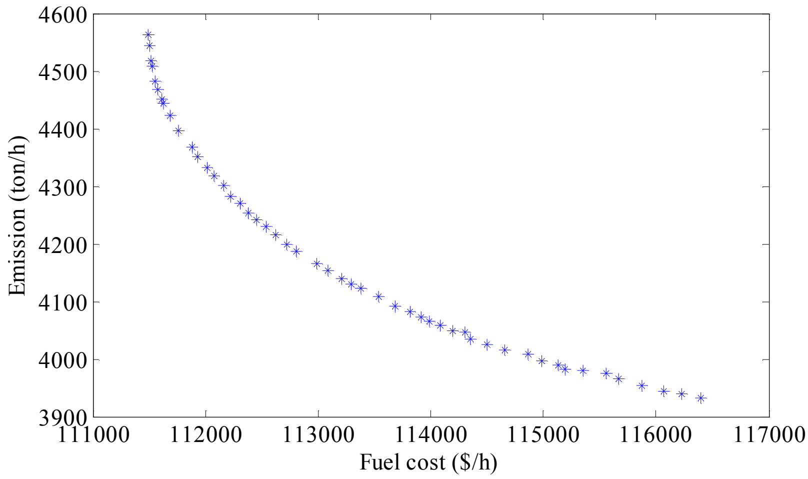


Figure 5.11. Pareto front in Test System 2 obtained by BSA-NDA

Pareto front solutions of BSA-NDA and BSA-WSM are shown in Figure 5.12 to compare these two methods. Although number of solutions in the pareto front are set differently, the figure confirms that BSA-NDA produces the same solutions as BSA-WSM does.

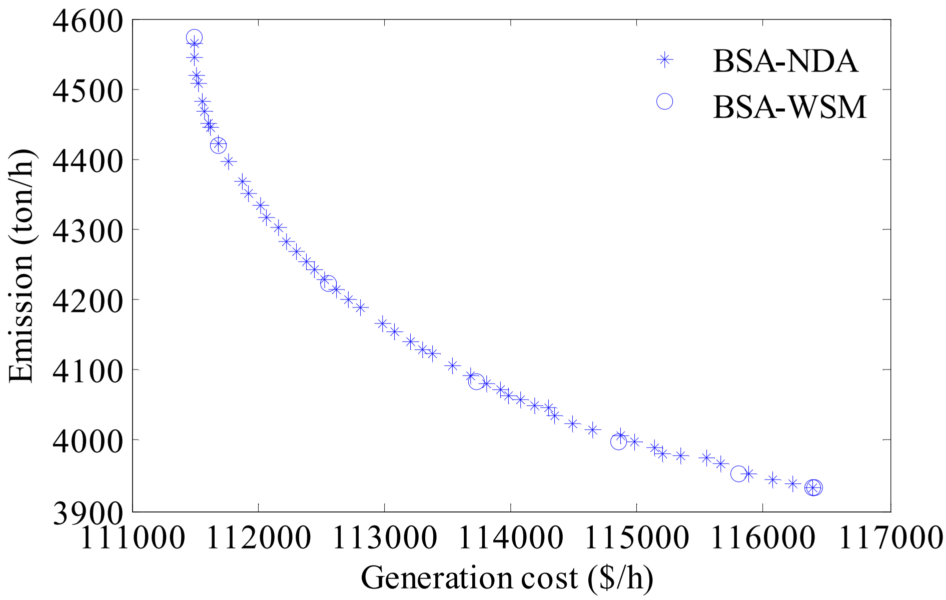


Figure 5.12. Pareto front sets in Test System 2 obtained by BSA-NDA and BSA-WSM

5.4 Test system 3: IEEE 118-bus 14-unit system

The system is an IEEE 118-bus 14-unit test system with a power demand of 950 (MW). Its data is taken from (Wu et al., 2010) and listed in Appendix (Table A.17). The transmission loss coefficients, obtained from (Jeddi & Vahidinasab, 2014), are listed in

Appendix (Table A.18). As in the other test systems, three methodologies of solving the EED problem are considered in this system.

5.4.1 Control parameter tuning

The economic dispatch and emission dispatch problems are solved for this test system with different values of mixrate. The maximum iteration and population size are set to 500 and 10, respectively and mixrate is changed from 0 to 1 with the step of 0.2. Table 5.16 shows the statistical indices of the optimal results of both economic dispatch and emission dispatch problems. As shown in the table, the value of mixrate=0 corresponds to the optimal solutions with lowest computation burden while the value of mixrate=1 produces the highest quality of solutions (minimum objective with lowest standard deviation).

Table 5.16. statistical indices of optimal results of BSA for Test system 3 with different values of mixrate

Economic Dispatch								
mixrate	Generation cost ($/h)				CPU time (s)			
	Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
0	4303.5861	4304.5020	4306.8824	0.6899	2.65	2.97	3.43	0.16
0.2	4303.5119	4303.5284	4303.5707	0.0140	4.41	4.97	5.65	0.27
0.4	4303.5116	4303.5188	4303.5517	0.0078	4.87	5.37	5.93	0.27
0.6	4303.5114	4303.5161	4303.5486	0.0070	4.91	5.66	6.57	0.36
0.8	4303.5114	4303.5144	4303.5337	0.0039	5.34	5.90	6.51	0.29
1	4303.5111	4303.5120	4303.5172	0.0011	5.43	6.05	7.10	0.34
Emission Dispatch								
mixrate	Emission (ton/h)				CPU time (s)			
	Min.	Ave.	Max.	SD	Min.	Ave.	Max.	SD
0	26.0240	30.1449	39.1676	2.6136	1.96	2.35	2.65	0.15
0.2	25.2476	25.3372	25.4887	0.0619	3.21	3.63	4.17	0.22
0.8	25.2379	25.2582	25.2985	0.0169	3.67	4.28	5.07	0.30
0.6	25.2431	25.2751	25.3794	0.0289	3.76	4.20	4.90	0.26
0.4	25.2450	25.3070	25.4870	0.0548	3.56	3.97	4.45	0.24
1	25.2372	25.2494	25.3218	0.0166	3.87	4.45	5.20	0.30

5.4.2 Methodology 1

Single objective optimizations to minimize generation cost and emission are performed in the test system. The transmission network loss is considered in the problem formulation. The emission dispatch and economic dispatch are run separately and the results are listed in Table 5.17. The minimum generation cost and emission are 4303.5111 ($/h) and 25.2372 (ton/h), respectively.

Table 5.17. Best solution of the EED problem in Test System 3

Generations	Generation cost Minimization	Emission Minimization
P_1 (MWh)	104.1756	70.9358
P_2 (MWh)	92.1099	50.0000
P_3 (MWh)	50.0000	77.8758
P_4 (MWh)	50.0000	88.8994
P_5 (MWh)	50.0001	67.5382
P_6 (MWh)	50.0000	50.0000
P_7 (MWh)	50.0000	73.3419
P_8 (MWh)	50.0000	72.2040
P_9 (MWh)	62.8778	73.7542
P_{10} (MWh)	63.0931	90.1769
P_{11} (MWh)	62.6157	50.0000
P_{12} (MWh)	177.6497	72.8619
P_{13} (MWh)	50.0000	72.4039
P_{14} (MWh)	50.0000	50.0000
Total Gen. (MWh)	962.5218	959.9921
P_L (MWh)	12.5218	9.9921
Generation cost ($/h)	4303.5111	4548.8981
Emission (ton/h)	402.4739	25.2372

Owing to the stochastic nature of BSA, the optimization is run 50 times with various initial populations. The statistical indices of the optimal solutions are then obtained to check the robustness of the proposed method. Table 5.18 lists the statistical indices of the optimal generation cost and emission for 50 trials proving BSA's high robustness. The low values of the standard deviations of both objectives confirm that BSA achieves the same optimal in any run, i.e., it is a highly robust method whether for economic or emission dispatch.

Table 5.18. Statistical indices of the optimal results of 50 trials in Test System 3

Objective	Generation cost ($/h)			
	Minimum	Mean	Maximum	Standard deviation
Generation cost Minimization	4303.5111	4303.5120	4303.5172	0.0011
Objective	Emission (ton/h)			
	Minimum	Mean	Maximum	Standard deviation
Emission Minimization	25.2372	25.2494	25.3218	0.0166

The performance of the proposed method is compared with another method from the literature according to Table 5.19. The comparison between MHSA (Jeddi & Vahidinasab, 2014) and BSA reconfirms that BSA solves the problem better.

Table 5.19. Comparison between the methods in Test System 3

Method	Generation cost Minimization		Emission Minimization	
	Generation cost ($/h)	Emission (ton/h)	Generation cost ($/h)	Emission (ton/h)
MHSA [1]	4304.95	357.339	4539.228	27.892
BSA	**4303.5111**	402.4739	4548.8981	**25.2372**

[1] (Jeddi & Vahidinasab, 2014)

The convergence characteristics of both generation cost and emission objectives are shown by Figure 5.13.

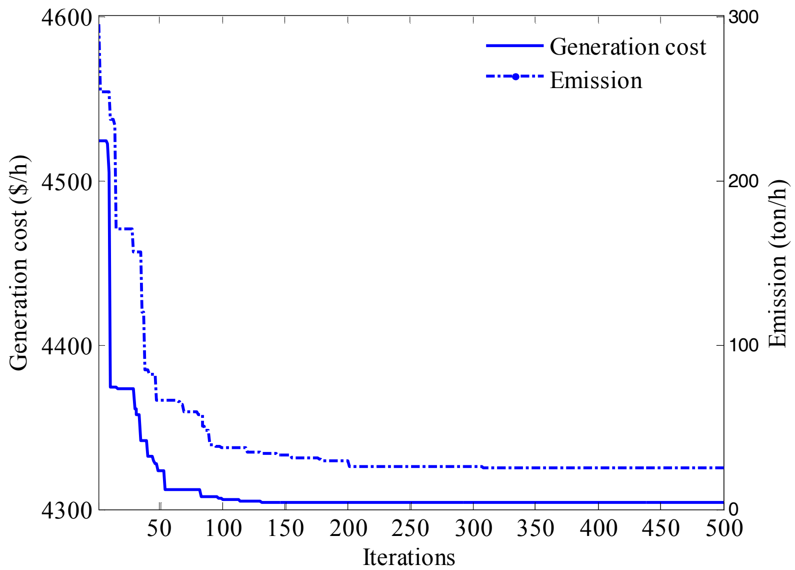


Figure 5.13. Convergence characteristics of economic dispatch and emission dispatch in Test System 3

5.4.3 Methodology 2 (BSA-WSM)

The second approach to solving the EED problem is the use of weighted sum method, which is performed in the third test system. The values of *w* are selected from 0 to 1 with steps of 0.2 and the combined objective is minimized. The generation cost and emission corresponding to each optimal for each value of *w* represents a member of the pareto front set as the output of the optimization. The maximum iteration, population size, and mixrate are set respectively to 500, 10, and 1. Figure 5.14 illustrates the pareto front of the third system including the minimum generation cost and emission. The best compromise solution, however, is not the minimum of one objective but of two objectivesand is chosen by the fuzzy-based decision maker.

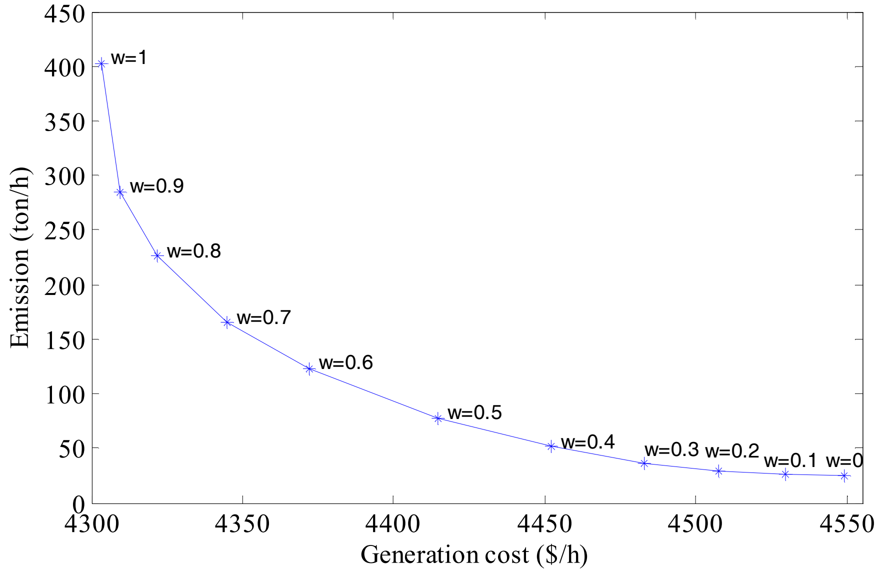


Figure 5.14. Pareto front in Test System 3 obtained by BSA-WSM

Table 5.20 lists the generation cost, emission, and μ values of all the pareto front members . The solution with the highest μ is the best compromise solution, i.e., the one with the generation cost of 4372.1966 ($/h) and emission of 122.8286 (ton/h) as shown in bold.

Table 5.20. Pareto front solutions obtained by BSA-WSM in Test System 3

w	F_c ($/h)	F_e (ton/h)	μ_{Fc}	μ_{Fe}	μ
0.0	4549.3002	25.2366	0.0000	1.0000	0.0724
0.1	4529.7706	26.1576	0.0795	0.9976	0.0780
0.2	4507.3461	29.6199	0.1707	0.9884	0.0839
0.3	4482.8310	36.7372	0.2704	0.9695	0.0898
0.4	4451.9612	51.2739	0.3960	0.9310	0.0961
0.5	4414.6407	77.7713	0.5479	0.8608	0.1020
0.6	**4372.1966**	**122.8286**	**0.7206**	**0.7415**	**0.1058**
0.7	4345.1388	165.1185	0.8306	0.6294	0.1057
0.8	4321.8336	226.0575	0.9255	0.4680	0.1009
0.9	4309.7319	285.1559	0.9747	0.3114	0.0931
1.0	4303.5113	402.7157	1.0000	0.0000	0.0724

Table 5.21 is the generation schedule for the best compromise solution. It shows the optimal solution to the EED problem in Test System 3 by weighted sum method.

Table 5.21. Generation schedule of the best compromise solution in Test System 3

w=0.6			
P_1 (MWh)	96.2082	P_8 (MWh)	52.2584
P_2 (MWh)	61.2375	P_9 (MWh)	84.9164
P_3 (MWh)	51.7180	P_{10} (MWh)	100.0596
P_4 (MWh)	72.4543	P_{11} (MWh)	56.6517
P_5 (MWh)	65.7665	P_{12} (MWh)	116.4102
P_6 (MWh)	50.0000	P_{13} (MWh)	51.0225
P_7 (MWh)	51.1225	P_{14} (MWh)	50.0000
Total Gen. (MWh)	959.8258		
P_L (MWh)	9.8258		
Generation cost ($/h)	4372.1966		
Emission (ton/h)	122.8286		

5.4.4 Methodology 3 (BSA-NDA)

The solution to the EED problem is done by the non-dominated approach in this test system. Pareto front members as non-dominated solutions are generated in each iteration of the algorithm. As the optimization progresses, the non-dominated solutions are updated and the pareto members are stored in an elitist external archive. The archive capacity is set to 20 non-dominated solutions and the extra members are removed from the archive according to the crowding distance measure. The maximum iteration and population size are set to 1000 and 20, respectively. Figure 5.15 shows the pareto front of the test system.

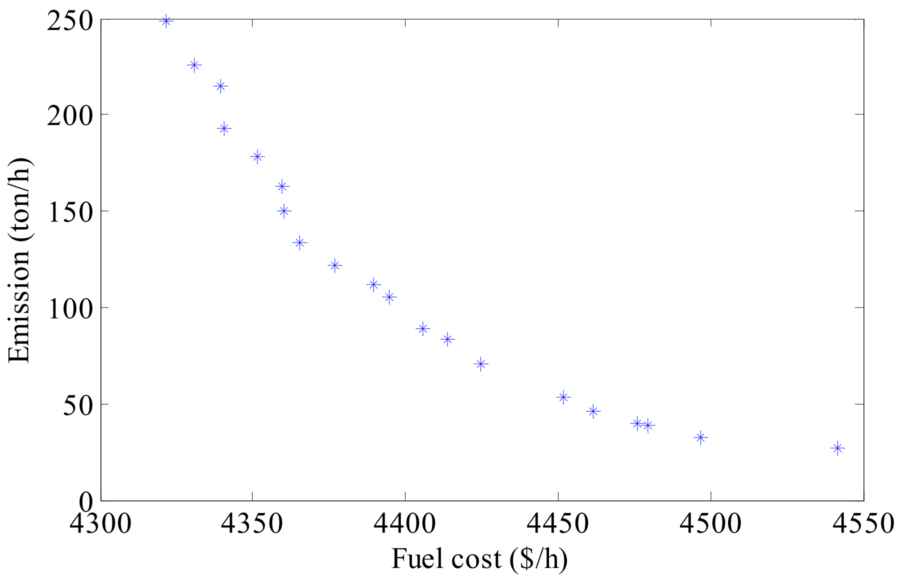


Figure 5.15. Pareto front in Test System 3 obtained by BSA-NDA

Table 5.22 is the generation schedule for the best compromise solution known as the optimal of the EED problem. The optimal generation cost and emission of this solution are respectively 4405.8321 ($/h) and 88.8972 (ton/h), as selected from the pareto front.

Table 5.23 lists the 20 non-dominated solutions of the pareto front with the index of μ for the selection of best compromise solution shown in bold.

Table 5.22. Optimization results for the best compromise solutions in Test System 3

Generations (MWh)							Total Gen. (MWh)	Generation cost ($/h)
P_1	P_2	P_3	P_4	P_5	P_6	P_7		
93.8800	58.2196	64.9233	71.2120	59.5934	50.0000	56.9709	959.9005	4405.8321
P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}	P_L (MWh)	Emission ($/h)
54.1604	84.1674	99.2756	54.4040	107.0259	56.0681	50.0000	9.9005	88.8972

Table 5.23. Pareto front solutions obtained by BSA-NDA in Test System 3

Non-dominated Solution	Generation cost ($/h)	Emission (ton/h)	μ_{Fc}	μ_{Fe}	μ
1	4321.5187	248.7270	1.0000	0.0000	0.0410
2	4330.7133	225.6469	0.9582	0.1039	0.0435
3	4339.7436	214.8277	0.9172	0.1526	0.0438
4	4340.7387	192.9437	0.9126	0.2510	0.0477
5	4351.7883	178.0005	0.8624	0.3183	0.0484
6	4359.5601	162.8372	0.8271	0.3865	0.0497
7	4360.1234	150.3973	0.8245	0.4425	0.0519
8	4365.5677	134.0489	0.7998	0.5161	0.0539
9	4377.0985	122.1169	0.7474	0.5698	0.0540
10	4389.8520	111.5653	0.6894	0.6172	0.0536
11	4394.5365	105.4518	0.6681	0.6448	0.0538
12	**4405.8321**	**88.8972**	**0.6168**	**0.7193**	**0.0548**
13	4414.0492	83.2398	0.5794	0.7447	0.0543
14	4424.9766	70.7215	0.5298	0.8011	0.0545
15	4451.9535	53.0732	0.4071	0.8805	0.0528
16	4461.8115	46.4532	0.3623	0.9103	0.0522
17	4475.8416	39.9495	0.2986	0.9395	0.0507
18	4479.6491	39.1007	0.2813	0.9434	0.0502
19	4496.6951	32.7392	0.2038	0.9720	0.0482
20	4541.5267	26.5126	0.0000	1.0000	0.0410

In this test system, the pareto front solutions of BSA-NDA and BSA-WSM are compared to those solutions of MHSA (Jeddi & Vahidinasab, 2014) method. Figure 5.16 shows the pareto front solutions of these three methods confirming that both BSA-NDA and BSA-WSM produce higher quality of optimal solutions than MHSA. Almost all the pareto front solutions of BSA-NDA and BSA-WSM dominate the solutions obtained by MHSA validating the high performance of BSA for solving of the EED problem in this test system.

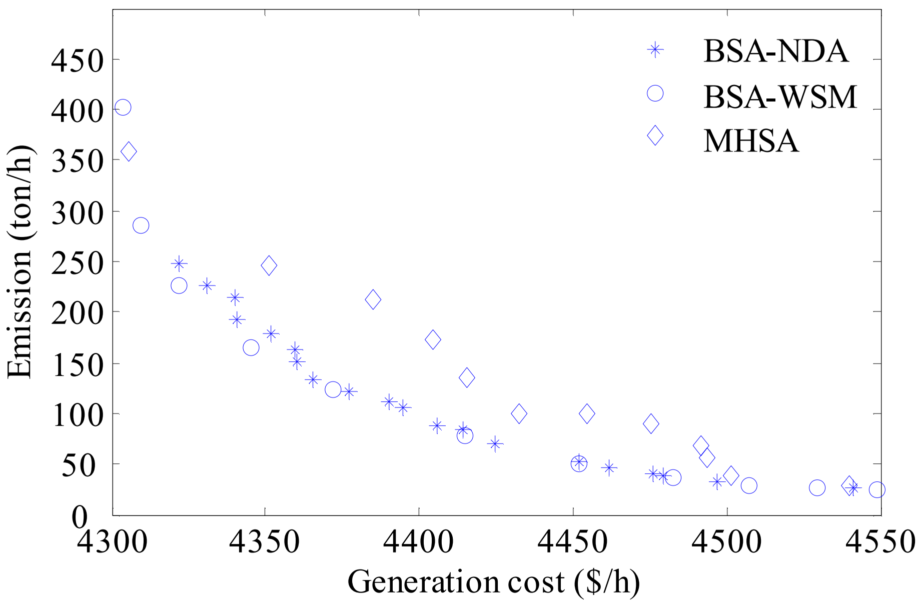


Figure 5.16. Pareto front sets in Test System 3 obtained by BSA-NDA, BSA-WSM, and MHSA

5.5 Summary

In this chapter, multi-objective backtracking search algorithm (MOBSA) is developed to solve an economic emission dispatch (EED) problem. Valve point effects are considered in the generation cost model. The EED problem is solved by weighted sum method and non-dominated approach. The proposed method is applied on IEEE 30-bus 6-unit system and its results affirm its high performance in solving EED problems individually. It outperforms other optimization techniques in lossless and loss-considered cases. The results of the economic dispatch and emission dispatch in 10-unit and IEEE 118-bus 14-unit systems also confirm its effectiveness. The solution to the EED problem by weighted sum and non-dominated methods in the test systems produce high-quality pareto front set with well-distributed non-dominated solutions including solutions for minimum generation cost and minimum emission. A number of 50 trials is considered a fair test of robustness of the proposed method. Their results for the test systems confirm that the proposed method is highly robust in solving EED problems.

CHAPTER 6 : CONCLUSION

6.1 Conclusions

Backtracking search algorithm (BSA) is applied on power dispatch problem with two constraint handling mechanisms in power system and microgrid. The conclusion is described for each of thesis objectives as follows.

The power dispatch problem with two objectives has been considered. The objectives have been the minimizations of the generation cost and the emission amount of generating units. Operating constraints of generators such as generation limits, ramp rate limits, and prohibited operating zones are modeled. The valve point loading effects and multiple fuel options in generator cost function are also considered.

First, backtracking search algorithm with two constraint handling mechanisms are employed to solve the economic power dispatch problem in power system. These approaches have been called BSA_{SSG} and BSA_{DSG}. They have been applied on test systems to show the performance of BSA for solving the power dispatch problems. Six case studies varied in size and complexity are used to apply the proposed methods for solving the economic dispatch problems. In each test system, the optimal results obtained by BSA_{SSG} and BSA_{DSG} have been compared to each other in terms of solution quality and computational burden.

The case studies have been divided into two groups. In the first group including 3-, 6-, 20, and 40-unit systems, the valve point loading effects and the transmission network loss have been considered in the dispatch problem. Two proposed methods, BSA_{SSG} and BSA_{DSG}, have been applied to solve the ED problem with different parameter settings for the comparison purpose. Also, the convergence characteristics, robustness, and computational efficiencies of these methods have been used for comprehensive analysis. The results have shown that BSA_{DSG} can produce higher quality of optimal solutions than BSA_{SSG}.

In the second group with more complex systems, 15 and 10-unit test systems, the optimal results have been achieved by BSA_{SSG} and BSA_{DSG}. Comparison of the results has reconfirmed the superiority of BSA_{DSG} over BSA_{SSG}. It has been shown that BSA_{DSG} can solve the economic dispatch problems (ED) effectively and efficiently. Since the valve-point loading effects and multiple fuel options have been considered in 10-unit system making this system as highly non-linear case study, it is used for additional analysis. In this case, four test systems including 20, 40, 80, and 160 generating units have been created through expanding the 10-unit system. Again, both BSA_{SSG} and BSA_{DSG} have been employed to solve the ED problems in these large systems. The results have shown that BSA_{DSG} has better performance than BSA_{SSG}. In all case studies, BSA_{DSG} has been compared to other optimization methods from the literature of ED confirming its strong capability for producing high quality of optimal solutions.

Since BSA_{DSG} has shown better performance than BSA_{SSG} for solving economic dispatch problem in all case studies. It has been selected for solving the power dispatch problem in microgrid. The power dispatch problem is solved with different parameter settings to tune the parameters to achieve the best optimal. The optimization is run 50 times for the statistical analysis of the optimal solutions. The results have shown that BSA_{DSG} can produce highly robust optimal solutions within 50 trials for minimizing the generation cost of microgrid. Also, it has been shown that BSA_{DSG} has outperformed the methods from the literature confirming its applicability for solving the power dispatch problem in microgrid.

BSA_{DSG} as the proposed method for solving the economic dispatch problem is developed for multi-objective approaches. In this case, the problem of economic and emission dispatch problem (EED) is modeled and solved by multi-objective backtracking search algorithm. Three case studies including IEEE 30-bus 6-unit system, 10-unit system, and IEEE 118-bus 14-unit system have been used to validate the performance of the proposed method for solving the EED problems. Three methodologies have been employed for solving the problems by multi-objective BSA.

In the first methodology, the problems of economic dispatch and emission dispatch have been solved to achieve minimum generation cost and minimum emission separately. A number of 50 trials has been considered as a fair test of robustness of the proposed method. The results on IEEE 30-bus 6-unit system have affirmed BSA's high performance in solving the EED problem individually since it has outperformed other optimization techniques in lossless and loss-considered cases. The results on 10-unit and IEEE 118-bus 14-unit systems in the first methodology have reconfirmed BSA's effectiveness.

In the second and third methodologies, the EED problems have been solved by BSA with weighted sum method and non-dominated approaches. It has been shown that the proposed method has produced pareto front optimal set with well-distributed optimal solutions including the points corresponding to the approximate optimal of each objective. Finally, the proposed method has shown high performance for solving the EED problem as the multi-objective optimization problem.

6.2 Future works

The following tasks can be carried out as the future works.

1. The power dispatch problem in power system can be developed by considering the renewable energies to investigate the cost and environmental impacts of employing clean energy technologies.
2. The proposed algorithm can be applied in electricity market environment to determine the optimal generation schedules of generators.
3. The proposed algorithm can be utilized to solve dynamic power dispatch problem with the specific period such as 24 hours.

4. The proposed algorithm can also be investigated for other optimization problems such as price-based and cost-based unit commitment and optimal power flow (OPF).
5. The proposed method can be combined with other metaheuristic or classical methods to create more powerful technique of optimization.

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APPENDIX A

Table A.1 Unit parameters for Case 1 (3-unit system)

Unit	P^{min} (MW)	P^{max}(MW)	a_i ($/MW^2)	b_i ($/MW)	c_i ($)	e_i ($)	f_i (rad/MW)
1	100	600	0.001562	7.92	561	300	0.0315
2	100	400	0.00194	7.85	310	200	0.042
3	50	200	0.00482	7.97	78	150	0.063

Table A.2 Unit parameters for Case 2 (6-unit system)

Unit	P^{min} (MW)	P^{max}(MW)	a_i ($/MW^2)	b_i ($/MW)	c_i ($)	e_i ($)	f_i (rad/MW)	Unit	P^{min} (MW)	P^{max}(MW)	a_i ($/MW^2)	b_i ($/MW)	c_i ($)	e_i ($)	f_i (rad/MW)
1	50	200	150	2	0.0016	50	0.063	4	10	35	0	3.25	0.00834	0	0
2	20	80	25	2.5	0.01	40	0.098	5	10	30	0	3	0.025	0	0
3	15	50	0	1	0.0625	0	0	6	12	40	0	3	0.025	0	0

Table A.3 Transmission loss coefficients for Cases 2 (6-unit system)

$$\mathbf{B}=\begin{vmatrix} 0.0224 & 0.0103 & 0.0016 & -0.0053 & 0.0009 & -0.0013 \\ 0.0103 & 0.0158 & 0.0010 & -0.0074 & 0.0007 & 0.0024 \\ 0.0016 & 0.0010 & 0.0474 & -0.0687 & -0.0060 & -0.0350 \\ -0.0053 & -0.0074 & -0.0687 & 0.3464 & 0.0105 & 0.0534 \\ 0.0009 & 0.0007 & -0.0060 & 0.0105 & 0.0119 & 0.0007 \\ -0.0013 & 0.0024 & -0.0350 & 0.0534 & 0.0007 & 0.2353 \end{vmatrix}$$

$$\mathbf{B_0}=\begin{vmatrix} -0.0005 & 0.0016 & -0.0029 & 0.0060 & 0.0014 & 0.0015 \end{vmatrix}$$

$$\mathbf{B_{00}}= 0.0011$$

Table A.4 Unit parameters for Case 3 (20-unit system)

Unit	P^{min} (MW)	P^{max} (MW)	a_i ($/MW^2)	b_i ($/MW)	c_i ($)	Unit	P^{min} (MW)	P^{max} (MW)	a_i ($/MW^2)	b_i ($/MW)	c_i ($)
1	150	600	0.00068	18.19	1000	11	100	300	0.0048	16.69	800
2	50	200	0.00071	19.26	970	12	150	500	0.0031	16.76	970
3	50	200	0.0065	19.8	600	13	40	160	0.0085	17.36	900
4	50	200	0.005	19.1	700	14	20	130	0.00511	18.7	700
5	50	160	0.00738	18.1	420	15	25	185	0.00398	18.7	450
6	20	100	0.00612	19.26	360	16	20	80	0.0712	14.26	370
7	25	125	0.0079	17.14	490	17	30	85	0.0089	19.14	480
8	50	150	0.00813	18.92	660	18	30	120	0.00713	18.92	680
9	50	200	0.00522	18.27	765	19	40	120	0.00622	18.47	700
10	30	150	0.00573	18.92	770	20	30	100	0.00773	19.79	850

Table A.5 Transmission loss coefficients for Case 3 (20-unit system)

	8.70	0.43	-4.61	0.36	0.32	-0.66	0.96	-1.60	0.80	-0.10	3.60	0.64	0.79	2.10	1.70	0.80	-3.20	0.70	0.48	-0.70
	0.43	8.30	-0.97	0.22	0.75	-0.28	5.04	1.70	0.54	7.20	-0.28	0.98	-0.46	1.30	0.80	-0.20	0.52	-1.70	0.80	0.20
	-4.61	-0.97	9.00	-2.00	0.63	3.00	1.70	-4.30	3.10	-2.00	0.70	-0.77	0.93	4.60	-0.30	4.20	0.38	0.70	-2.00	3.60
	0.36	0.22	-2.00	5.30	0.47	2.62	-1.96	2.10	0.67	1.80	-0.45	0.92	2.40	7.60	-0.20	0.70	-1.00	0.86	1.60	0.87
	0.32	0.75	0.63	0.47	8.60	-0.80	0.37	0.72	-0.90	0.69	1.80	4.30	-2.80	-0.70	2.30	3.60	0.80	0.20	-3.00	0.50
	-0.66	-0.28	3.00	2.62	-0.80	11.80	-4.90	0.30	3.00	-3.00	0.40	0.78	6.40	2.60	-0.20	2.10	-0.40	2.30	1.60	-2.10
	0.96	5.04	1.70	-1.96	0.37	-4.90	8.24	-0.90	5.90	-0.60	8.50	-0.83	7.20	4.80	-0.90	-0.10	1.30	0.76	1.90	1.30
	-1.60	1.70	-4.30	2.10	0.72	0.30	-0.90	1.20	-0.96	0.56	1.60	0.80	-0.40	0.23	0.75	-0.56	0.80	-0.30	5.30	0.80
$B=10^{-5}\times$	0.80	0.54	3.10	0.67	-0.90	3.00	5.90	-0.96	0.93	-0.30	6.50	2.30	2.60	0.58	-0.10	0.23	-0.30	1.50	0.74	0.70
	-0.10	7.20	-2.00	1.80	0.69	-3.00	-0.60	0.56	-0.30	0.99	-6.60	3.90	2.30	-0.30	2.80	-0.80	0.38	1.90	0.47	-0.26
	3.60	-0.28	0.70	-0.45	1.80	0.40	8.50	1.60	6.50	-6.60	10.70	5.30	-0.60	0.70	1.90	-2.60	0.93	-0.60	3.80	-1.50
	0.64	0.98	-0.77	0.92	4.30	0.78	-0.83	0.80	2.30	3.90	5.30	8.00	0.90	2.10	-0.70	5.70	5.40	1.50	0.70	0.10
	0.79	-0.46	0.93	2.40	-2.80	6.40	7.20	-0.40	2.60	2.30	-0.60	0.90	11.00	0.87	-1.00	3.60	0.46	-0.90	0.60	1.50
	2.10	1.30	4.60	7.60	-0.70	2.60	4.80	0.23	0.58	-0.30	0.70	2.10	0.87	3.80	0.50	-0.70	1.90	2.30	-0.97	0.90
	1.70	0.80	-0.30	-0.20	2.30	-0.20	-0.90	0.75	-0.10	2.80	1.90	-0.70	-1.00	0.50	11.00	1.90	-0.80	2.60	2.30	-0.10
	0.80	-0.20	4.20	0.70	3.60	2.10	-0.10	-0.56	0.23	-0.80	-2.60	5.70	3.60	-0.70	1.90	10.80	2.50	-1.80	0.90	-2.60
	-3.20	0.52	0.38	-1.00	0.80	-0.40	1.30	0.80	-0.30	0.38	0.93	5.40	0.46	1.90	-0.80	2.50	8.70	4.20	-0.30	0.68
	0.70	-1.70	0.70	0.86	0.20	2.30	0.76	-0.30	1.50	1.90	-0.60	1.50	-0.90	2.30	2.60	-1.80	4.20	2.20	0.16	-0.30
	0.48	0.80	-2.00	1.60	-3.00	1.60	1.90	5.30	0.74	0.47	3.80	0.70	0.60	-0.97	2.30	0.90	-0.30	0.16	7.60	0.69
	-0.70	0.20	3.60	0.87	0.50	-2.10	1.30	0.80	0.70	-0.26	-1.50	0.10	1.50	0.90	-0.10	-2.60	0.68	-0.30	0.69	7.00
$B_0=0$																				
$B_{00}=0$																				

Table A.6 Unit parameters for Case 4 (40-unit system)

Unit	P^{min} (MW)	P^{max}(MW)	a_i (\$/MW^2)	b_i (\$/MW)	c_i (\$)	e_i (\$)	f_i (rad/MW)	Unit	P^{min} (MW)	P^{max}(MW)	a_i (\$/MW^2)	b_i (\$/MW)	c_i (\$)	e_i (\$)	f_i (rad/MW)
1	36	114	0.0069	6.73	94.705	100	0.084	21	254	550	0.00298	6.63	785.96	300	0.035
2	36	114	0.0069	6.73	94.705	100	0.084	22	254	550	0.00298	6.63	785.96	300	0.035
3	60	120	0.02028	7.07	309.54	100	0.084	23	254	550	0.00284	6.66	794.53	300	0.035
4	80	190	0.00942	8.18	369.03	150	0.063	24	254	550	0.00284	6.66	794.53	300	0.035
5	47	97	0.0114	5.35	148.89	120	0.077	25	254	550	0.00277	7.1	801.32	300	0.035
6	68	140	0.01142	8.05	222.33	100	0.084	26	254	550	0.00277	7.1	801.32	300	0.035
7	110	300	0.00357	8.03	287.71	200	0.042	27	10	150	0.52124	3.33	1055.1	120	0.077
8	135	300	0.00492	6.99	391.98	200	0.042	28	10	150	0.52124	3.33	1055.1	120	0.077
9	135	300	0.00573	6.6	455.76	200	0.042	29	10	150	0.52124	3.33	1055.1	120	0.077
10	130	300	0.00605	12.9	722.82	200	0.042	30	47	97	0.0114	5.35	148.89	120	0.077
11	94	375	0.00515	12.9	635.2	200	0.042	31	60	190	0.0016	6.43	222.92	150	0.063
12	94	375	0.00569	12.8	654.69	200	0.042	32	60	190	0.0016	6.43	222.92	150	0.063
13	125	500	0.00421	12.5	913.4	300	0.035	33	60	190	0.0016	6.43	222.92	150	0.063
14	125	500	0.00752	8.84	1760.4	300	0.035	34	90	200	0.0001	8.95	107.87	200	0.042
15	125	500	0.00708	9.15	1728.3	300	0.035	35	90	200	0.0001	8.62	116.58	200	0.042
16	125	500	0.00708	9.15	1728.3	300	0.035	36	90	200	0.0001	8.62	116.58	200	0.042
17	220	500	0.00313	7.97	647.85	300	0.035	37	25	110	0.0161	5.88	307.45	80	0.098
18	220	500	0.00313	7.95	649.69	300	0.035	38	25	110	0.0161	5.88	307.45	80	0.098
19	242	550	0.00313	7.97	647.83	300	0.035	39	25	110	0.0161	5.88	307.45	80	0.098
20	242	550	0.00313	7.97	647.81	300	0.035	40	242	550	0.00313	7.97	647.83	300	0.035

Table A.7 Generating units' parameters for Case 5 (15-unit system)

Unit	P^{min} (MW)	P^{max}(MW)	a_i ($/MW^2)	b_i ($/MW)	c_i ($)	P_i^0	UR_i	DR_i	Prohibited zones
1	150	455	671	10.1	0.000299	400	80	120	
2	150	455	574	10.2	0.000183	300	80	120	[185, 225], [305, 335],[420, 450]
3	20	130	374	8.8	0.001126	105	130	130	
4	20	130	374	8.8	0.001126	100	130	130	
5	150	470	461	10.4	0.000205	90	80	120	[180, 200], [305, 335],[390, 420]
6	135	460	630	10.1	0.000301	400	80	120	[230, 255], [365, 395],[430, 455]
7	135	465	548	9.8	0.000364	350	80	120	
8	60	300	227	11.2	0.000338	95	65	100	
9	25	162	173	11.2	0.000807	105	60	100	
10	25	160	175	10.7	0.001203	110	60	100	
11	20	80	186	10.2	0.003586	60	80	80	
12	20	80	230	9.9	0.005513	40	80	80	[30, 40], [55, 65]
13	25	85	225	13.1	0.000371	30	80	80	
14	15	55	309	12.1	0.001929	20	55	55	
15	15	55	323	12.4	0.004447	20	55	55	

Table A.8 Transmission loss coefficients for Case 5 (15-unit system)

	0.0014	0.0012	0.0007	-0.0001	-0.0003	-0.0001	-0.0001	-0.0001	-0.0003	-0.0005	-0.0003	-0.0002	0.0004	0.0003	-0.0001
	0.0012	0.0015	0.0013	0.0000	-0.0005	-0.0002	0.0000	0.0001	-0.0002	-0.0004	-0.0004	0.0000	0.0004	0.0010	-0.0002
	0.0007	0.0013	0.0076	-0.0001	-0.0013	-0.0009	-0.0001	0.0000	-0.0008	-0.0012	-0.0017	0.0000	-0.0026	0.0111	-0.0028
	-0.0001	0.0000	-0.0001	0.0034	-0.0007	-0.0004	0.0011	0.0050	0.0029	0.0032	-0.0011	0.0000	0.0001	0.0001	-0.0026
	-0.0003	0.0005	-0.0013	-0.0007	0.0090	0.0014	-0.0003	-0.0012	-0.0010	-0.0013	0.0007	-0.0002	-0.0002	-0.0024	-0.0003
	-0.0001	-0.0002	-0.0009	-0.0004	0.0014	0.0016	0.0000	-0.0006	-0.0005	-0.0008	0.0011	-0.0001	-0.0002	-0.0017	0.0003
	-0.0001	0.0000	-0.0001	0.0011	-0.0003	0.0000	0.0015	0.0017	0.0015	0.0009	-0.0005	0.0007	0.0000	-0.0002	-0.0008
	-0.0001	0.0001	0.0000	0.0050	-0.0012	-0.0006	0.0017	0.0168	0.0082	0.0079	-0.0023	-0.0036	0.0001	0.0005	-0.0078
$B=10^{-5}\times$	-0.0003	-0.0002	-0.0008	0.0029	-0.0010	-0.0005	0.0015	0.0082	0.0129	0.0116	-0.0021	-0.0025	0.0007	-0.0012	-0.0072
	-0.0005	-0.0004	-0.0012	0.0032	-0.0013	-0.0008	0.0009	0.0079	0.0116	0.0200	-0.0027	-0.0034	0.0009	-0.0011	-0.0088
	-0.0003	-0.0004	-0.0017	-0.0011	0.0007	0.0011	-0.0005	-0.0023	-0.0021	-0.0027	0.0140	0.0001	0.0004	-0.0038	0.0168
	-0.0002	0.0000	0.0000	0.0000	-0.0002	-0.0001	0.0007	-0.0036	-0.0025	-0.0034	0.0001	0.0054	-0.0001	-0.0004	0.0028
	0.0004	0.0004	-0.0026	0.0001	-0.0002	-0.0002	0.0000	0.0001	0.0007	0.0009	0.0004	-0.0001	0.0103	-0.0101	0.0028
	0.0003	0.0010	0.0111	0.0001	-0.0024	-0.0017	-0.0002	0.0005	-0.0012	-0.0011	-0.0038	-0.0004	-0.0101	0.0578	-0.0094
	-0.0001	-0.0002	-0.0028	-0.0026	-0.0003	0.0003	-0.0008	-0.0078	-0.0072	-0.0088	0.0168	0.0028	0.0028	-0.0094	0.1283
	0.0014	0.0012	0.0007	-0.0001	-0.0003	-0.0001	-0.0001	-0.0001	-0.0003	-0.0005	-0.0003	-0.0002	0.0004	0.0003	-0.0001
	0.0012	0.0015	0.0013	0.0000	-0.0005	-0.0002	0.0000	0.0001	-0.0002	-0.0004	-0.0004	0.0000	0.0004	0.0010	-0.0002
	0.0007	0.0013	0.0076	-0.0001	-0.0013	-0.0009	-0.0001	0.0000	-0.0008	-0.0012	-0.0017	0.0000	-0.0026	0.0111	-0.0028
$B_0=$	-0.0001	-0.0002	0.0028	-0.0001	0.0001	-0.0003	-0.0002	-0.0002	0.0006	0.0039	-0.0017	0.0000	-0.0032	0.0067	-0.0064
B_{00}	0.0055														

Table A.9 Unit parameters for Case 6 (10-unit system)

Unit	Generation P^{min}	F_1	P_1	F_2	P_2	F_3	P^{max}	Fuel type	Cost coefficients a_i (\$/MW^2)	b_i (\$/MW)	c_i (\$)	e_i (\$)	f_i (rad/MW)
1	100			196			250	1	2.1760e-3	-3.9750e-1	2.6970e1	2.6970e-2	-3.9750e+0
		1				2		2	1.8610e-3	-3.0590e-1	2.1130e1	2.1130e-2	-3.0590e+0
	50		114		157		230	1	4.1940e-3	-1.2690e+0	1.1840e2	1.1840e-1	-1.2690e+1
2								2	1.1380e-3	-3.9880e-2	1.8650e0	1.8650e+1	-3.9880e-1
		2		3		1		3	1.6200e-3	-1.9800e-1	1.3650e1	1.3650e+0	-1.9800e+0
	200		332		388		500	1	1.4570e-3	-3.1160e-1	3.9790e1	3.9790e+0	-3.1160e+0
3								2	1.1760e-5	4.8640e-1	-5.9140e1	-5.9140e+0	4.8640e+0
		1		3		2		3	8.0350e-4	3.3890e-2	-2.8760e0	-2.8760e+1	3.3890e-1
	99		138		200		265	1	1.0490e-3	-3.1140e-2	1.9830e0	1.9830e+1	-3.1140e-1
4								2	2.7580e-3	-6.3480e-1	5.2850e1	5.2850e+0	-6.3480e+0
		1		2		3		3	5.9350e-3	-2.3380e+0	2.6680e2	2.6680e-1	-2.3380e+1
	190		338		407		490	1	1.0660e-3	-8.7330e-2	1.3920e1	1.3920e+0	-8.7330e-1
5								2	1.5970e-3	-5.2060e-1	9.9760e1	9.9760e+0	-5.2060e+0
		1		2		3		3	1.4980e-4	4.4620e-1	-5.3990e1	-5.3990e+0	-4.4620e+0
	85		138		200		265	1	2.7580e-3	-6.3480e-1	5.2850e1	5.2850e+0	-6.3480e+0
6								2	1.0490e-3	-3.1140e-2	1.9830e0	1.9830e+1	-3.1140e-1
		2		1		3		3	5.9350e-3	-2.3380e+0	2.6680e2	2.6680e-1	-2.3380e+1
	200		331		391		500	1	1.1070e-3	-1.3250e-1	1.8930e1	1.8930e+0	-1.3250e+0
7								2	1.1650e-3	-2.2670e-1	4.3770e1	4.3770e+0	-2.2670e+0
		1		2		3		3	2.4540e-4	3.5590e-1	-4.3350e1	-4.3350e+0	3.5590e+0
	99		138		200		265	1	1.0490e-3	-3.1140e-2	1.9830e0	1.9830e+1	-3.1140e-1
8								2	2.7580e-3	-6.3480e-1	5.2850e1	5.2850e+0	-6.3480e+0
		1		2		3		3	5.9350e-3	-2.3380e+0	2.6680e2	2.6680e-1	-2.3380e+1
	130		213		370		440	1	1.5540e-3	-5.6750e-1	8.8530e1	8.8530e+0	-5.6750e+0
9								2	7.0330e-3	-4.5140e-2	1.5300e1	1.4230e+0	-1.8710e-1
		3		1		3		3	6.1210e-4	-1.8170e-2	1.4230e1	1.4230e+0	-1.8710e-1
	200		362		407		490	1	1.1020e-3	-9.9380e-2	1.3970e1	1.3970e+0	-9.9380e-1
10								2	4.1640e-5	5.0840e-1	-6.1130e1	-6.1130e+0	5.0840e+0
		1		3		2		3	1.1370e-3	-2.0240e-1	4.6710e1	4.6710e+0	-2.0240e+0

Table A. 10 Cost function coefficients and boundary limits of microgrid elements

DG	a ($/h)	b ($/kWh)	c ($/(kW)^{2}h)	P^{min} (kW)	P^{max} (kW)	Efficiency (%)
Diesel 1	0.2731	0.1453	0.0042	0	800	0
Diesel 2	0.4333	0.2333	0.0074	0	400	0
Wind 1	0	0.022	0	0	300	0
Wind 2	0	0.032	0	0	300	0
Fuel-cell 1	0	0.05	0	0	150	90
Fuel-cell 2	0	0.05	0	0	100	90
Fuel-cell 3	0	0.07	0	0	100	85

Table A.11 Load profile and wind speed within 24 hours

Hour	P_D (kW)	v (m/s)	Hour	P_D (kW)	v (m/s)
1	653.6	8.20	13	1032.0	4.80
2	550.4	6.90	14	997.6	5.80
3	645.0	5.60	15	1083.6	6.80
4	688.0	7.75	16	1032.0	8.75
5	842.8	9.20	17	1118.0	6.50
6	1118.0	4.20	18	1376.0	8.20
7	1324.4	6.00	19	1668.4	8.30
8	1393.2	8.10	20	1651.2	7.00
9	1427.6	4.30	21	1634.0	6.00
10	1393.2	7.80	22	1462.0	7.00
11	1238.4	8.50	23	1341.6	8.80
12	1083.6	8.00	24	1066.4	7.00

Table A.12 The generating units' parameters in Test System 1 (IEEE 30-bus 6-unit system)

Unit	P^{min} (MW)	P^{max}(MW)	a_i	b_i	c_i	α_i	β_i	γ_i	ζ_i	λ_i
G_1	5	150	10	200	100	4.091	-5.554	6.49	2.0E-4	2.86
G_2	5	150	10	150	120	2.543	-6.047	5.638	5.0E-4	3.33
G_3	5	150	20	180	40	4.258	-5.094	4.586	1.0E-6	8.00
G_4	5	150	10	100	60	5.326	-3.55	3.38	2.0E-3	2.00
G_5	5	150	20	180	40	4.258	-5.094	4.586	1.0E-6	8.00
G_6	5	150	10	150	100	6.131	-5.555	5.151	1.0E-5	6.667

Table A.13 Transmission loss coefficients in Test System 1 (IEEE 30-bus 6-unit system)

$$
B = \begin{bmatrix}
0.1382 & -0.0299 & 0.0044 & -0.0022 & -0.0010 & -0.0008 \\
-0.0299 & 0.0487 & -0.0025 & 0.0004 & 0.0016 & 0.0041 \\
0.0044 & -0.0025 & 0.0182 & -0.0070 & -0.0066 & -0.0066 \\
-0.0022 & 0.0004 & -0.0070 & 0.0137 & 0.0050 & 0.0033 \\
-0.0010 & 0.0016 & -0.0066 & 0.0050 & 0.0109 & 0.0005 \\
-0.0008 & 0.0041 & -0.0066 & 0.0033 & 0.0005 & 0.0244
\end{bmatrix}
\quad B_{00} = 0.00098573
$$

$$
B_0 = \begin{bmatrix} -0.0107 & 0.0060 & -0.0017 & 0.0009 & 0.0002 & 0.0030 \end{bmatrix}
$$

Table A.14 Generation limits and cost coefficients in Test System 2 (10-unit system)

Unit	P^{min} (MW)	P^{max}(MW)	a_i	b_i	c_i	e_i	f_i
G_1	10	55	0.12951	40.5407	1000.403	33	0.0174
G_2	20	80	0.10908	39.5804	950.606	25	0.0178
G_3	47	120	0.12511	36.5104	900.705	32	0.0162
G_4	20	130	0.12111	39.5104	800.705	30	0.0168
G_5	50	160	0.15247	38.539	756.799	30	0.0148
G_6	70	240	0.10587	46.1592	451.325	20	0.0163
G_7	60	300	0.03546	38.3055	1243.531	20	0.0152
G_8	70	340	0.02803	40.3965	1049.998	30	0.0128
G_9	135	470	0.02111	36.3278	1658.569	60	0.0136
G_{10}	150	470	0.01799	38.2704	1356.659	40	0.0141

Table A.15 Emission coefficients in Test System 2 (10-unit system)

Unit	α_i	β_i	γ_i	ζ_i	λ_i
G_1	0.04702	−3.9864	360.0012	0.25475	0.01234
G_2	0.04652	−3.9524	350.0056	0.25475	0.01234
G_3	0.04652	−3.9023	330.0056	0.25163	0.01215
G_4	0.04652	−3.9023	330.0056	0.25163	0.01215
G_5	0.0042	0.3277	13.8593	0.2497	0.012
G_6	0.0042	0.3277	13.8593	0.2497	0.012
G_7	0.0068	−0.5455	40.2669	0.248	0.0129
G_8	0.0068	−0.5455	40.2669	0.2499	0.01203
G_9	0.0046	−0.5112	42.8955	0.2547	0.01234
G_{10}	0.0046	−0.5112	42.8955	0.2547	0.01234

Table A.16 Transmission loss coefficients in Test System 2 (10-unit system)

B=

0.000049	0.000014	0.000015	0.000015	0.000016	0.000017	0.000017	0.000018	0.000019	0.00002
0.000014	0.000045	0.000016	0.000016	0.000017	0.000015	0.000015	0.000016	0.000018	0.000018
0.000015	0.000016	0.000039	0.00001	0.000012	0.000012	0.000014	0.000014	0.000016	0.000016
0.000015	0.000016	0.00001	0.00004	0.000014	0.00001	0.000011	0.000012	0.000014	0.000015
0.000016	0.000017	0.000012	0.000014	0.000035	0.000011	0.000013	0.000013	0.000015	0.000016
0.000017	0.000015	0.000012	0.00001	0.000011	0.000036	0.000012	0.000012	0.000014	0.000015
0.000017	0.000015	0.000014	0.000011	0.000013	0.000012	0.000038	0.000016	0.000016	0.000018
0.000018	0.000016	0.000014	0.000012	0.000013	0.000012	0.000016	0.00004	0.000015	0.000016
0.000019	0.000018	0.000016	0.000014	0.000015	0.000014	0.000016	0.000015	0.000042	0.000019
0.00002	0.000018	0.000016	0.000015	0.000016	0.000015	0.000018	0.000016	0.000019	0.000044

$\mathbf{B_0}$=0 $\mathbf{B_{00}}$=0

Table A.17 The generating units' parameters in Test System 3 (IEEE 118-bus 14-unit system)

Unit	P^{min} (MW)	P^{max}(MW)	a_i (\$/MW^2)	b_i (\$/MW)	c_i (\$)	α_i (ton/MW^2)	β_i (ton/MW)	γ_i (ton)
G_1	50	300	0.005	1.89	150	0.016	-1.5	23.333
G_2	50	300	0.0055	2	115	0.031	-1.82	21.022
G_3	50	300	0.006	3.5	40	0.013	-1.249	22.05
G_4	50	300	0.005	3.15	122	0.012	-1.355	22.983
G_5	50	300	0.005	3.05	125	0.02	-1.9	21.313
G_6	50	300	0.007	2.75	70	0.007	0.805	21.9
G_7	50	300	0.007	3.45	70	0.015	-1.401	23.001
G_8	50	300	0.007	3.45	70	0.018	-1.8	24.003
G_9	50	300	0.005	2.45	130	0.019	-2	25.121
G_{10}	50	300	0.005	2.45	130	0.012	-1.36	22.99
G_{11}	50	300	0.0055	2.35	135	0.033	-2.1	27.01
G_{12}	50	300	0.0045	1.3	200	0.018	-1.8	25.101
G_{13}	50	300	0.007	3.45	70	0.018	-1.81	24.313
G_{14}	50	300	0.006	3.89	45	0.03	-1.921	27.119

Table A.18 Transmission loss coefficients in Test System 3 (IEEE 118-bus 14-unit system)

B=	0.042741	0.03010	0.019242	0.02150	-0.00280	-0.00400	-0.00447	-0.00272	-0.00323	-0.00694	-0.00745	-0.01952	-0.01217	-0.01718
	0.030108	0.03794	0.02071	0.02091	-0.00363	-0.00525	-0.00448	-0.00366	-0.00359	-0.00695	-0.01018	-0.02004	-0.01844	-0.02057
	0.019242	0.02071	0.02678	0.02469	-0.00247	-0.00378	-0.00298	-0.00239	-0.00231	-0.00467	-0.00786	-0.01583	-0.01529	-0.01688
	0.002151	0.02091	0.024696	0.02439	-0.00232	-0.00352	-0.00309	-0.00223	-0.00230	-0.00475	-0.00715	-0.01600	-0.01346	-0.01588
	-0.00288	-0.00360	-0.00247	-0.00232	0.00954	0.00365	0.00295	0.00311	0.00420	0.00206	0.00036	-0.00365	-0.00381	-0.00424
	-0.00400	-0.00525	-0.00378	-0.00352	0.00365	0.01067	0.00576	0.00374	0.00334	0.00248	0.00119	-0.00279	-0.00288	-0.00331
	-0.00447	-0.00448	-0.00298	-0.00309	0.00295	0.00576	0.00809	0.00337	0.00356	0.00305	0.00129	-0.00252	-0.00192	-0.00270
	-0.00272	-0.00366	-0.00239	-0.00223	0.00311	0.00374	0.00337	0.00387	0.00374	0.00293	0.00206	0.00152	-0.00142	-0.00188
	-0.00323	-0.00359	-0.00231	-0.00230	0.00420	0.00334	0.00356	0.00374	0.00540	0.00286	0.00147	-0.00225	-0.00189	-0.00254
	-0.00694	-0.00695	-0.00467	-0.00475	0.00206	0.00248	0.00305	0.00293	0.00286	0.00673	0.00305	0.001212	0.00133	0.00095
	-0.00745	-0.01018	-0.00786	-0.00715	0.00036	0.00119	0.00129	0.00206	0.00147	0.00305	0.00857	0.006171	0.00817	0.00726
	-0.01952	-0.02004	-0.01583	-0.01600	-0.00360	-0.00279	-0.00252	-0.00152	-0.00225	0.00121	0.00617	0.036153	0.01839	0.02001
	-0.01217	-0.01844	-0.01529	-0.01346	-0.00381	-0.00288	-0.00192	-0.00142	-0.00189	0.00133	0.00817	0.018390	0.03311	0.02941
	-0.01718	-0.02057	-0.01688	-0.01588	-0.00424	-0.00331	-0.00272	-0.00188	-0.00254	0.00095	0.00726	0.020017	0.02941	0.04129
B_0=	-0.53852	-0.28322	-0.19294	-0.26424	0.01775	0.02191	0.0405	0.012212	0.014	0.004407	0.03273	0.21782	0.03256	0.15563
B_{00}=	2.8378×10^2													

Table A.19 Optimal schedule of generators for 20 to 160 unit systems by BSA_{SSG}

20-unit system											
P_1-P_{10}	219.7671	212.7222	280.6532	238.2787	283.9578	238.6999	292.8871	240.5839	421.9002	269.9529	Total cost ($/h) 1248.2397
P_{11}-P_{20}	217.8332	210.4506	283.9112	240.5795	280.0957	241.3875	288.5782	238.9506	424.4087	274.4018	CPU time (sec) 0.79
40-unit system											
P_1-P_{10}	222.0332	210.2307	281.6883	241.6554	279.1071	236.5557	290.8729	239.7710	421.3593	275.1036	Total cost ($/h)
P_{11}-P_{20}	216.5717	211.9823	285.7532	238.8356	279.6614	245.2844	292.5826	239.6505	413.6237	278.6363	2496.8659
P_{21}-P_{30}	216.3161	211.2128	282.5971	239.1361	275.6980	237.7755	287.7618	238.4277	426.8209	279.2932	CPU time (sec)
P_{31}-P_{40}	216.5550	212.2361	282.3718	238.1609	280.8594	236.9570	294.8501	239.5188	427.3421	275.1504	2.93
80-unit system											
P_1-P_{10}	211.7730	211.9542	287.4358	240.8190	282.1493	241.3657	292.9100	235.7365	415.2154	279.1587	Total cost ($/h)
P_{11}-P_{20}	215.3697	211.2690	285.3739	240.9444	278.2706	243.1187	302.0590	239.4786	417.0782	282.2017	
P_{21}-P_{30}	217.6631	211.0960	280.8370	240.7373	284.3263	242.8734	287.8340	236.3153	416.5748	278.9608	4999.0457
P_{31}-P_{40}	215.7012	211.3331	281.7605	239.5176	284.6463	239.3536	299.7515	238.0326	405.6357	278.7047	
P_{41}-P_{50}	219.1773	211.3825	295.0928	238.9605	285.2824	237.1135	291.0461	240.9833	351.9422	280.3064	CPU time (sec)
P_{51}-P_{60}	216.4608	209.3899	289.4735	239.2463	281.4682	238.9721	299.6301	238.0090	412.7909	273.0213	
P_{61}-P_{70}	223.4988	210.9925	290.0260	238.0167	279.8484	236.0993	296.1642	239.2831	422.9927	288.5793	14.51
P_{71}-P_{80}	218.3306	210.7261	286.5050	234.8012	284.4863	242.4455	294.5071	243.5406	418.8718	285.1993	
160-unit system											
P_1-P_{10}	202.2032	202.4297	306.2753	235.2074	302.6879	224.3898	311.1536	230.1319	392.9888	296.1695	
P_{11}-P_{20}	205.3916	205.3757	309.6440	230.2822	296.7428	229.7940	328.8390	236.0031	357.3814	293.3437	Total cost ($/h)
P_{21}-P_{30}	210.2336	225.6432	301.0066	233.0128	300.8074	240.2860	310.0996	239.8052	342.0785	306.3580	
P_{31}-P_{40}	229.7808	197.3762	302.6330	235.0480	310.1700	236.1441	320.4908	234.0459	379.8179	293.4056	10087.4428
P_{41}-P_{50}	209.3525	206.5760	314.5931	233.5537	295.1580	230.7823	308.2672	233.4446	379.9108	294.1745	
P_{51}-P_{60}	208.6034	207.0728	284.7928	234.6733	324.7294	236.5145	348.1098	233.2546	343.3661	292.9139	
P_{61}-P_{70}	201.7658	200.4688	295.8267	242.4500	305.4322	235.2406	284.1962	239.2183	405.0659	302.0974	
P_{71}-P_{80}	197.8818	208.1495	323.7749	232.4051	289.7633	230.3446	277.7043	234.0824	399.5172	275.9810	
P_{81}-P_{90}	206.7836	205.5714	290.2083	244.6197	291.7909	233.9181	329.4566	239.8547	345.4127	303.2068	
P_{91}-P_{100}	214.1528	211.2541	341.1428	238.7184	299.6403	233.3615	310.5329	230.1601	355.2073	294.5722	CPU time (sec)
P_{101}-P_{110}	216.2259	207.9471	294.3139	234.7216	289.7269	236.8112	326.7190	243.6999	348.7615	306.2862	
P_{111}-P_{120}	208.4848	183.4425	325.2267	236.5676	294.0858	235.3494	311.6296	236.5329	341.9216	291.2247	369.85
P_{121}-P_{130}	205.9502	207.1268	315.8594	232.8726	291.1034	230.7047	306.2548	240.7582	351.8221	295.2698	
P_{131}-P_{140}	213.5268	206.1726	320.3274	231.5516	286.3316	234.1113	331.5197	235.4227	392.2890	296.0792	
P_{141}-P_{150}	206.0794	201.3791	298.5889	228.3235	306.0366	234.1205	290.3136	237.0395	340.8845	289.1702	
P_{151}-P_{160}	211.4258	209.7217	298.9931	237.8873	333.0782	230.2063	313.4609	230.0930	364.9168	278.0961	

Table A.20 Optimal schedule of generators for 20 to 160 unit systems by BSA$_{DSG}$

20-unit system											
P_{1}-P_{10}	218.9178	213.8795	285.8630	241.2548	279.1774	240.7137	288.4542	238.8320	426.5724	275.4027	Total cost ($/h) 1248.1453
P_{11}-P_{20}	218.4643	211.9649	278.5724	240.4512	276.8115	240.2980	285.8821	241.1206	425.2776	272.0900	CPU time (sec) 0.66
40-unit system											
P_{1}-P_{10}	217.8568	209.9764	286.0263	240.5807	281.5315	239.6401	289.5725	237.8891	419.0965	272.8705	Total cost ($/h)
P_{11}-P_{20}	216.7302	214.9427	283.6904	238.7024	275.6014	238.0246	289.8033	238.0279	427.2484	275.8248	2496.3035
P_{21}-P_{30}	219.7518	212.2630	282.7475	239.9060	283.7729	241.5325	285.4285	239.2433	426.0293	275.7341	CPU time (sec)
P_{31}-P_{40}	217.8648	208.2430	279.9359	240.8460	278.9933	241.9220	291.5432	238.1663	426.7901	275.6502	2.14
80-unit system											
P_{1}-P_{10}	215.6189	209.7779	285.0787	237.8887	277.7138	238.9706	295.9757	235.6062	420.1685	272.7289	
P_{11}-P_{20}	215.3171	209.7458	281.1461	240.2904	278.3413	241.3797	293.7423	238.3365	417.2607	276.3105	Total cost ($/h)
P_{21}-P_{30}	223.0809	212.4608	275.7111	241.5217	281.2978	237.5079	294.0365	239.5126	422.4768	281.8983	4994.9557
P_{31}-P_{40}	219.5026	210.7075	284.5098	240.0392	278.2803	237.8360	288.0220	240.1735	431.3259	276.8823	
P_{41}-P_{50}	213.4147	214.9315	283.9003	238.8416	280.2366	241.1186	291.9470	238.1655	420.4565	280.0774	
P_{51}-P_{60}	222.6125	211.7017	277.4362	241.3586	282.1998	237.0626	290.1570	239.0831	420.2180	280.0403	CPU time (sec)
P_{61}-P_{70}	216.6938	211.9546	282.3265	239.0978	277.5993	242.4432	293.3349	237.8883	421.0045	277.7303	5.85
P_{71}-P_{80}	212.7801	208.5878	281.9587	243.5013	281.2760	239.6700	295.3165	237.2130	422.0826	274.3993	
160-unit system											
P_{1}-P_{10}	213.3154	217.9365	279.6972	234.6801	286.0825	235.5030	297.0552	240.0225	393.5987	291.4255	
P_{11}-P_{20}	215.9499	208.2972	289.6518	241.7031	275.4550	239.9928	307.9677	238.8504	405.3307	270.7644	Total cost ($/h)
P_{21}-P_{30}	214.0176	215.7387	284.6693	240.5951	283.1078	236.4856	298.1299	238.2628	400.3873	288.9676	
P_{31}-P_{40}	219.7961	212.0315	296.3531	245.4464	288.3457	238.0285	290.8944	238.6270	427.4251	290.7521	10012.3647
P_{41}-P_{50}	217.1316	211.2258	288.2594	237.1250	278.8438	239.7715	319.4746	235.2588	399.3670	271.9939	
P_{51}-P_{60}	205.2969	208.8629	273.3884	239.7990	291.6607	244.2671	301.4060	241.3461	388.1976	287.5318	
P_{61}-P_{70}	216.1869	218.6080	290.0877	239.0759	277.3323	238.8271	297.1195	241.5370	419.8877	275.9252	
P_{71}-P_{80}	212.4911	208.7471	298.1080	239.2211	294.3494	238.6580	300.3954	239.0868	359.2010	278.8633	
P_{81}-P_{90}	213.5014	213.9618	286.0409	244.3282	286.2588	237.4384	297.1249	236.9481	393.6126	276.2214	
P_{91}-P_{100}	209.7774	209.7711	295.8599	238.3123	291.4598	237.7293	296.8692	234.9386	424.8349	275.3398	CPU time (sec)
P_{101}-P_{110}	211.8832	212.9496	299.2506	241.7764	282.3988	240.4313	293.3109	238.2175	399.5035	286.6827	
P_{111}-P_{120}	221.3860	211.2175	290.1538	239.1217	300.9559	240.1449	298.9756	239.8996	353.7955	276.3682	11.43
P_{121}-P_{130}	217.5293	214.6254	291.1154	233.9054	295.3239	236.8402	296.7349	239.6124	404.9985	289.8484	
P_{131}-P_{140}	213.1689	208.7775	278.8388	238.4477	289.9966	241.3061	280.2280	238.2935	408.7167	290.5945	
P_{141}-P_{150}	219.8485	209.1395	280.5446	240.6016	292.2154	245.5636	302.4715	237.8971	391.7630	281.9428	
P_{151}-P_{160}	212.4485	208.9148	295.8053	235.5864	294.7699	237.8997	304.4153	236.4934	402.8272	285.9406	

Table A.21 Generations list for optimal solutions by weighted sum method in Test System 1 without transmission network loss considered

w	Generation (MW)						Fc	Fe
	P_1	P_2	P_3	P_4	P_5	P_6	($/h)	(ton/h)
0.0	40.6076	45.9068	53.7937	38.2955	53.7942	51.0023	638.2733	0.1942
0.1	38.6263	44.6931	53.8294	42.5397	53.8300	49.8816	633.2512	0.1943
0.2	36.4985	43.4157	53.8697	47.0477	53.8696	48.6988	628.2998	0.1947
0.3	34.2067	42.0693	53.9102	51.8543	53.9103	47.4492	623.4593	0.1954
0.4	31.7286	40.6480	53.9448	57.0065	53.9450	46.1270	618.7781	0.1966
0.5	29.0383	39.1451	53.9641	62.5661	53.9633	44.7232	614.3199	0.1982
0.6	26.1059	37.5520	53.9477	68.6181	53.9487	43.2277	610.1700	0.2005
0.7	22.8904	35.8573	53.8694	75.2853	53.8688	41.6289	606.4445	0.2037
0.8	19.3437	34.0461	53.6739	82.7547	53.6749	39.9068	603.3136	0.2079
0.9	15.4022	32.0977	53.2629	91.3374	53.2638	38.0360	601.0442	0.2138
1.0	10.9722	29.9767	52.4290	101.6198	52.4307	35.9717	600.1114	0.2221

Table A.22 Generations list for optimal solutions by weighted sum method in Test System 1 with transmission network loss considered

w	Generation (MW)						Fc	Fe
	P_1	P_2	P_3	P_4	P_5	P_6	($/h)	(ton/h)
0.0	41.09263	46.36656	54.4418	39.03704	54.44635	51.54862	646.2073	0.194179
0.1	38.69643	45.05724	54.70911	43.34046	54.58919	50.34573	640.4203	0.194317
0.2	36.21497	43.6676	55.01136	47.86408	54.72648	49.07167	634.8396	0.194762
0.3	33.6415	42.19181	55.34925	52.63624	54.84929	47.72091	629.5092	0.195564
0.4	30.96773	40.62115	55.72504	57.69563	54.94401	46.28548	624.479	0.196786
0.5	28.18469	38.94863	56.13626	63.09234	54.99051	44.75688	619.8125	0.198508
0.6	25.27943	37.16403	56.5829	68.89557	54.95793	43.12355	615.5878	0.200837
0.7	22.24	35.25691	57.05296	75.20586	54.79793	41.37274	611.9087	0.203922
0.8	19.04687	33.21191	57.53661	82.17407	54.42924	39.48625	608.9169	0.207983
0.9	15.67661	31.01276	57.99504	90.0519	53.71413	37.43886	606.8254	0.213372
1.0	12.09757	28.63112	58.35646	99.28456	52.39602	35.19044	605.9984	0.220729

Table A.23 Generations list for optimal solutions by non-dominated approach in Test System 1 without transmission network loss considered

w	Generation (MW)						Fc	Fe
	P_1	P_2	P_3	P_4	P_5	P_6	(\$/h)	(ton/h)
1	11.8421	30.0493	51.8808	101.0912	52.3204	36.2162	600.1226	0.2214
2	11.8953	30.0493	50.5319	100.0109	52.4131	38.4995	600.2138	0.2204
3	12.6463	31.0088	51.6233	97.7254	53.0172	37.3791	600.2670	0.2186
4	14.8953	30.9610	51.8381	98.3011	50.2779	37.1266	600.3763	0.2183
5	14.5875	31.8397	50.7346	96.8072	51.4152	38.0157	600.4801	0.2172
6	14.9915	31.1842	52.7574	93.7055	52.7317	38.0298	600.7094	0.2154
7	14.1415	35.9613	49.7274	93.6582	52.4537	37.4579	601.0733	0.2150
8	15.8131	33.8028	51.5839	92.9759	49.9267	39.2976	601.1083	0.2141
9	16.0547	33.0921	52.3168	90.9849	51.7963	39.1552	601.2678	0.2131
10	17.0340	35.2335	50.4757	90.0367	51.2793	39.3408	601.7496	0.2120
11	14.3000	36.0645	52.2634	86.1767	56.9021	37.6933	602.2076	0.2111
12	14.6433	35.8333	53.6882	85.2597	53.1928	40.7827	602.5038	0.2101
13	18.7050	34.3199	51.7523	85.4580	52.6033	40.5616	602.7156	0.2092
14	17.7587	36.5169	54.8302	83.3395	51.9551	38.9996	603.2060	0.2084
15	20.2243	35.2335	51.5974	83.2415	52.4103	40.6930	603.5513	0.2076
16	20.2243	35.2335	51.5974	81.6537	52.4103	42.2808	604.0918	0.2068
17	19.7256	37.2313	51.1005	79.4235	55.5606	40.3585	604.7040	0.2060
18	21.9086	37.2313	50.0815	79.4235	51.5476	43.2076	605.4439	0.2052
19	20.6983	37.7735	54.1966	75.3385	52.9790	42.4142	606.3599	0.2039
20	22.2941	38.1641	51.7396	74.3099	54.2967	42.5957	607.1274	0.2031
21	28.0274	37.5811	51.1005	75.5072	52.2267	38.9571	607.9018	0.2028
22	23.9058	38.3744	51.5688	73.4550	51.3019	44.7941	608.1765	0.2023
23	24.9127	36.1931	54.9855	70.9295	52.6095	43.7697	608.8043	0.2016
24	28.6678	37.3996	51.9829	70.5082	50.8646	43.9770	610.3631	0.2006

25	28.9814	39.3050	51.8852	69.9052	48.1851	45.1381	611.3474	0.2002
26	29.0014	39.1456	53.6109	67.9000	51.4644	42.2776	611.6000	0.1998
27	27.7701	39.1456	54.4964	65.4420	52.1080	44.4379	612.5293	0.1991
28	29.0374	39.1593	53.0514	64.7269	51.4569	45.9681	613.5580	0.1987
29	28.4975	38.6331	54.7607	62.7331	53.2536	45.5221	614.0916	0.1984
30	31.4007	39.3050	51.8852	63.4461	50.2105	47.1525	615.3433	0.1980
31	31.7217	39.5883	52.0852	62.8983	50.7786	46.3279	615.6056	0.1979
32	28.8735	39.7665	59.9917	58.6522	50.7266	45.3894	616.6707	0.1976
33	31.8688	39.5474	55.5226	58.8821	50.1575	47.4216	617.9064	0.1970
34	31.8688	39.5474	55.4521	58.8821	50.1575	47.4921	617.9209	0.1970
35	32.4533	40.2633	55.7257	57.4207	50.2380	47.2990	619.0628	0.1966
36	30.5552	41.2106	55.4377	53.9014	55.9348	46.3603	620.2878	0.1962
37	34.4808	42.7739	50.6477	54.8207	57.6130	43.0639	621.3675	0.1962
38	33.7713	39.0553	56.2576	52.6548	54.5866	47.0744	621.9940	0.1959
39	33.4962	41.8732	52.4687	52.6072	54.4450	48.5096	622.8848	0.1956
40	32.0178	41.6007	56.6094	49.5783	54.2967	49.2972	624.2715	0.1954
41	34.7164	42.6759	54.8302	49.5783	54.2967	47.3025	625.2554	0.1951
42	36.3911	42.4771	52.7809	49.5783	54.3991	47.7735	626.1066	0.1950
43	34.0555	43.6648	56.2811	46.5901	52.9929	49.8155	627.8350	0.1949
44	35.7546	43.9395	52.4943	46.7373	54.5622	49.9121	628.6268	0.1947
45	37.0708	45.0077	52.7708	46.1256	52.7441	49.6810	629.9920	0.1946
46	38.3977	43.9040	55.8028	43.9899	52.3036	49.0021	631.6315	0.1945
47	38.3977	43.9040	54.1423	43.9899	52.3036	50.6626	632.0581	0.1944
48	39.8290	45.1268	55.5810	41.3285	51.6091	49.9256	634.9928	0.1943
49	38.7496	46.9577	52.9155	39.8313	53.9615	50.9844	636.4587	0.1942
50	39.7860	47.4533	52.9644	37.8069	53.2980	52.0914	639.1142	0.1942

Table A.24 Generations list for optimal solutions by non-dominated approach in Test System 1 with transmission network loss considered

Point No.	Generation (MW)						Fc	Fe
	P_1	P_2	P_3	P_4	P_5	P_6	($/h)	(ton/h)
1	9.1083	29.6470	60.5982	96.9104	55.0729	34.5989	606.2220	0.2205
2	13.8189	29.5546	55.0970	95.6803	56.6045	35.2295	606.2470	0.2177
3	12.5137	29.8406	54.8785	92.6817	57.5875	38.4936	606.5694	0.2159
4	13.0834	31.6097	58.8860	89.9624	55.7115	36.6330	606.7352	0.2143
5	14.3959	32.8958	57.7614	90.2695	55.2594	35.3337	606.8369	0.2139
6	16.9122	30.0542	54.2101	91.2047	51.9200	41.7074	607.2474	0.2131
7	20.3171	31.2035	52.0910	90.2775	56.0263	36.1184	607.6728	0.2122
8	14.2338	32.7279	59.3684	85.1383	56.0491	38.3355	607.6837	0.2112
9	16.1912	33.1460	61.0878	82.7500	56.1140	36.4980	608.2956	0.2098
10	19.5307	34.1516	55.2796	84.3017	54.9272	37.7616	608.6069	0.2088
11	17.7371	33.9917	58.5133	81.1288	54.8920	39.6007	609.0536	0.2078
12	17.6268	34.4085	58.4698	79.7787	55.6025	39.9768	609.4656	0.2072
13	21.7424	34.5080	58.9433	80.3647	54.9336	35.3697	609.8906	0.2070
14	20.4234	33.1460	58.8860	78.2565	56.0845	39.0392	610.1202	0.2062
15	23.2968	35.0261	52.4002	80.5163	53.5412	41.2928	610.8300	0.2056
16	23.1827	34.7018	52.5597	77.7651	57.9054	39.9113	611.3884	0.2049
17	26.0436	34.3088	56.0129	77.3303	54.8232	37.4747	612.0013	0.2045
18	22.5846	36.6436	57.3448	76.0679	51.0603	42.2680	612.1136	0.2039
19	26.4236	33.1055	53.9712	75.5942	56.5399	40.4052	612.8124	0.2036
20	21.3823	36.8042	57.7607	71.9901	55.6084	42.3700	613.1689	0.2028
21	26.5943	36.5410	52.5535	73.1511	56.9682	40.2900	614.1892	0.2022
22	26.9202	37.3659	58.7444	70.6910	55.1353	37.1116	614.9517	0.2018
23	25.4919	37.1453	52.2371	69.3936	55.2522	46.6123	616.2963	0.2007
24	28.7708	37.4998	52.0910	69.3936	56.0263	42.4014	616.8320	0.2004
25	27.6973	37.7502	58.3742	67.3315	50.2599	44.6523	617.5177	0.1999

26	25.7366	40.1405	57.4497	64.8216	53.6242	44.2741	618.3255	0.1993
27	29.4935	35.4851	51.7388	65.0190	58.1780	46.2907	619.3687	0.1991
28	27.0981	38.9680	52.5535	63.1014	56.9682	47.4835	620.1456	0.1986
29	27.1571	42.7902	55.7066	62.4990	52.2074	45.8146	621.0547	0.1982
30	30.4144	38.0753	58.6543	59.8270	57.4037	41.7116	621.7039	0.1980
31	28.9300	41.8541	55.2759	59.2149	54.6411	46.2873	623.1559	0.1973
32	30.0969	43.0543	54.7174	58.6807	55.6031	44.0925	623.9189	0.1971
33	30.0969	40.8064	53.6627	57.4712	55.6031	48.6324	624.8959	0.1967
34	29.5710	42.5484	55.8068	55.5551	54.4835	48.2790	626.0874	0.1964
35	31.7153	39.3399	56.5378	54.2209	56.1327	48.3024	626.9126	0.1962
36	33.3390	40.5617	55.8777	53.9934	55.3879	47.1755	627.9207	0.1959
37	32.5566	38.8859	59.7443	49.4313	58.5116	47.0762	629.8874	0.1959
38	31.8774	43.0906	56.3902	51.3288	53.9717	49.6822	630.1997	0.1955
39	35.3563	41.0396	56.1280	51.3736	53.2244	49.3422	631.3196	0.1953
40	35.3563	41.0396	55.5813	49.9314	53.2244	51.3653	632.8446	0.1951
41	36.2780	41.7422	53.6405	48.6617	56.0263	50.2131	634.1069	0.1949
42	35.4519	43.9740	57.3225	47.6261	52.9955	49.1261	634.6462	0.1948
43	36.9406	43.5309	54.8025	46.8755	55.1012	49.3467	635.9777	0.1946
44	38.2681	43.7503	55.8777	46.4716	53.1627	49.1318	637.1085	0.1946
45	38.2681	43.9740	53.9635	45.9451	52.9955	51.5810	638.3430	0.1945
46	38.9657	44.8104	55.3566	44.2176	52.5779	50.8249	640.0408	0.1944
47	39.2450	45.0232	56.0407	42.4125	53.3799	50.6595	641.5270	0.1943
48	40.0283	46.1595	54.1623	41.8964	53.6933	50.9162	643.0202	0.1942
49	40.4720	46.2149	54.4533	40.4872	54.1224	51.1342	644.4603	0.1942
50	41.1132	46.6672	54.1574	39.1396	54.4138	51.4506	646.2466	0.1942

Table A. 25 Generations list for optimal solutions by weighted sum method in Test System 2

w	Generation (MW)										Fc	Fe
	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	($/h)	(ton/h)
0.0	55.0000	80.0000	81.1468	81.3598	160.0000	240.0000	294.4634	297.2275	396.8194	395.5788	2081.5957	116412.2490
0.1	55.0000	80.0000	81.1371	81.2568	160.0000	240.0000	293.5600	297.1437	397.2910	396.2257	2081.6144	116406.8390
0.2	55.0000	80.0000	81.0926	81.1359	160.0000	240.0000	292.4800	296.9525	397.9022	397.0762	2081.6393	116400.1944
0.3	55.0000	80.0000	81.0836	80.9841	160.0000	240.0000	291.1857	296.7463	398.6776	397.9901	2081.6674	116392.5652
0.4	55.0000	80.0000	81.0466	80.7743	160.0000	240.0000	289.4903	296.4834	399.5806	399.3310	2081.7062	116382.7030
0.5	55.0000	80.0000	81.6213	81.1174	160.0000	225.2107	290.5912	299.6481	404.3836	404.4480	2082.0203	115805.7789
0.6	55.0000	80.0000	82.6923	81.8929	160.0000	197.4371	293.7951	305.5651	412.7794	413.4693	2082.6313	114859.9425
0.7	55.0000	80.0000	84.3543	83.0925	147.8849	169.1079	298.3585	314.0968	424.9651	426.7163	2083.5763	113727.5885
0.8	55.0000	80.0000	86.9822	85.0431	124.1857	140.1938	300.0000	322.0647	444.0322	447.4826	2084.9842	112559.8061
0.9	55.0000	80.0000	89.9002	86.6725	98.4057	106.3700	300.0000	331.1661	469.2615	470.0000	2086.7759	111689.3742
1.0	55.0000	80.0000	106.9188	100.6089	81.4977	83.0133	300.0000	340.0000	470.0000	470.0000	2087.0386	111497.6276

Table A. 26 Generations list for optimal solutions by non-dominated approach in Test System 2

Point No.	Generation (MW)										Fc ($/h)	Fe (ton/h)
	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}		
1	55.0000	80.0000	105.3833	100.3286	83.9216	82.4033	300.0000	340.0000	470.0000	470.0000	111498.8712	4563.3844
2	55.0000	80.0000	101.1560	100.4524	83.5346	86.8608	300.0000	340.0000	470.0000	470.0000	111503.9712	4544.1570
3	55.0000	80.0000	101.1559	94.0354	83.5346	93.2740	300.0000	340.0000	470.0000	470.0000	111518.7118	4518.0852
4	55.0000	80.0000	97.7997	95.1142	88.1211	90.9600	300.0000	340.0000	470.0000	470.0000	111524.9000	4507.6570
5	55.0000	80.0000	95.7760	90.3935	88.5407	97.2747	299.9976	340.0000	470.0000	470.0000	111554.5969	4482.5119
6	55.0000	80.0000	91.6339	90.3935	92.6666	97.2806	299.9976	340.0000	470.0000	470.0000	111579.5973	4467.0597
7	55.0000	80.0000	92.9804	87.7972	89.9900	105.0514	300.0000	336.0702	470.0000	470.0000	111615.4200	4451.6393
8	55.0000	80.0000	92.9456	87.7972	94.9463	102.1990	300.0000	333.9752	470.0000	470.0000	111626.0628	4444.7880
9	54.9998	79.9971	89.8831	87.3455	97.6306	104.8777	300.0000	340.0000	461.9685	470.0000	111689.8847	4422.1498
10	54.9963	79.9998	89.8831	86.4213	97.5729	113.3523	299.9913	331.3205	463.0241	470.0000	111760.4104	4397.4059
11	55.0000	80.0000	89.3667	87.6369	112.5327	106.4595	300.0000	331.7148	455.4625	468.1183	111880.6535	4367.4000
12	55.0000	80.0000	90.5544	86.5103	103.2938	122.2943	295.5173	329.2450	454.0638	469.6631	111931.4511	4351.6543
13	55.0000	80.0000	86.3011	83.9851	107.1679	126.8738	295.9281	318.2804	463.5755	469.1230	112016.3930	4332.9628
14	55.0000	80.0000	88.2269	82.9606	110.2645	126.9958	298.8706	319.4418	458.5091	465.7648	112074.3744	4316.7676
15	55.0000	80.0000	85.5307	83.8169	120.7644	122.2969	294.7246	321.4844	452.6941	469.6633	112166.1585	4301.2681
16	54.9999	80.0000	89.8884	85.8074	118.4590	126.2785	300.0000	325.7596	447.7680	456.5392	112227.9067	4282.9632
17	55.0000	79.9981	86.3727	91.1921	119.2404	129.1833	299.9999	327.4975	444.9644	451.7939	112308.2886	4269.6754
18	55.0000	80.0000	86.3727	85.5264	119.2404	134.5401	300.0000	327.4974	445.2931	451.7939	112378.1214	4254.3592
19	55.0000	80.0000	86.8658	85.0399	118.9375	138.9457	299.8809	329.1691	440.4072	450.8593	112456.2578	4241.7779
20	55.0000	79.9111	87.8026	85.1352	124.2880	137.5378	299.8415	329.1628	440.4072	445.8530	112534.9153	4228.7445
21	54.9821	80.0000	86.6279	84.4453	124.1436	143.3827	300.0000	323.1029	438.1086	450.1201	112619.4639	4215.1500
22	54.9997	80.0000	86.0944	84.4453	124.4657	148.4897	300.0000	321.1156	438.1085	447.0469	112723.0062	4199.9595
23	55.0000	80.0000	86.5308	86.9844	129.1542	146.9258	300.0000	323.9002	435.9938	440.0149	112807.3733	4188.0926
24	55.0000	80.0000	86.3036	83.8629	129.4367	155.5861	299.9435	324.5508	424.6213	445.0547	112989.2872	4166.5250
25	54.9996	80.0000	82.0695	84.3678	140.5455	148.4897	299.9954	321.1614	438.1086	434.6217	113082.6571	4154.3583

26	55.0000	80.0000	86.3453	83.8629	139.6980	156.2529	299.9435	313.4160	424.6240	445.0547	113211.0269	4139.1445
27	55.0000	80.0000	83.7797	84.1096	139.6980	160.2115	300.0000	313.4306	422.8538	445.0559	113299.3022	4129.6698
28	55.0000	80.0000	83.9947	82.8010	150.6844	151.0153	300.0000	313.5393	432.0845	434.9828	113380.8091	4122.2379
29	55.0000	80.0000	84.7169	82.8606	140.5115	169.2911	299.9435	313.4194	413.0653	445.0547	113542.4002	4107.2632
30	55.0000	80.0000	87.0673	82.1493	155.9107	157.7896	299.9348	308.7697	425.5074	431.6055	113678.9075	4092.8555
31	55.0000	80.0000	83.3958	82.1829	159.9138	158.8920	299.8865	307.5864	425.5074	431.3461	113817.1649	4081.0124
32	55.0000	80.0000	84.3637	87.5612	153.5771	171.9198	288.9835	300.1636	431.0177	430.9399	113917.1383	4072.3223
33	55.0000	80.0000	82.6380	82.1829	160.0000	166.1338	299.9282	304.2406	422.0866	431.3461	113988.3540	4064.7308
34	55.0000	80.0000	84.3636	81.4598	159.7432	171.9179	288.9835	300.1636	431.0177	430.9198	114080.6635	4058.2750
35	54.9949	80.0000	84.3636	81.2121	159.7432	176.7584	285.2440	299.2455	431.0173	430.9198	114194.9102	4049.7594
36	55.0000	80.0000	84.3670	80.0068	159.7432	179.0957	300.0000	282.9113	431.0177	431.3722	114299.4185	4046.9598
37	55.0000	79.9899	86.6475	81.8728	159.9983	179.1099	299.9774	305.4112	423.5708	411.3795	114350.5049	4034.6881
38	55.0000	80.0000	82.9148	81.7789	160.0000	186.9935	286.9271	298.7634	425.4001	425.3908	114497.3544	4024.8994
39	54.9964	80.0000	80.6874	80.8472	159.8831	191.9530	288.9927	300.1635	431.0177	414.4850	114654.0939	4015.0634
40	54.7283	80.0000	83.3913	82.1471	159.9999	196.4253	299.9347	305.5099	389.1862	431.3460	114864.6593	4007.2616
41	55.0000	80.0000	82.7978	83.3735	159.8831	202.2261	289.1049	300.1636	397.7484	432.3935	114987.9375	3997.2219
42	55.0000	80.0000	85.8626	80.7926	160.0000	205.1808	299.9998	289.9941	425.5074	400.1079	115133.3062	3989.4398
43	55.0000	80.0000	79.1565	82.1471	159.8452	206.7761	299.9348	308.7697	402.0857	408.6135	115198.1311	3982.0203
44	55.0000	79.9811	83.8680	81.1664	160.0000	212.4721	288.9776	297.5721	428.8174	394.4525	115350.0015	3978.5845
45	54.9999	80.0000	84.3636	81.4470	159.5470	219.4777	288.9828	276.7611	431.0177	405.8595	115558.8710	3974.3960
46	54.9999	79.9985	82.7891	81.0707	159.8831	220.9833	288.9927	300.1636	426.6866	386.4828	115663.8117	3965.7522
47	55.0000	80.0000	79.8293	76.5852	159.7834	226.4122	300.0000	300.6874	404.9434	398.7972	115878.7444	3952.5330
48	55.0000	80.0000	75.6288	81.4598	160.0000	232.3253	289.0666	300.1641	402.5936	405.7724	116070.8317	3944.8237
49	55.0000	80.0000	83.7654	81.4911	160.0000	235.7229	289.0010	297.6757	409.4393	389.5684	116224.9607	3939.6831
50	55.0000	80.0000	79.8771	80.7748	160.0000	240.0000	289.9925	300.1690	402.9551	392.8856	116395.0552	3932.8879

Table A. 27 Generations list for optimal solutions by weighted sum method in Test System 3

Generation (MW)	weighting factor (*w*)										
	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
P_1	71.0869	73.7523	77.2353	80.9762	85.4235	90.3636	96.2082	100.7348	105.2513	105.6764	104.1361
P_2	50.0000	50.0000	50.0000	50.0034	52.2357	56.0302	61.2375	66.2282	73.0899	79.9992	92.0856
P_3	78.0789	75.3321	72.2795	68.4726	64.0655	58.3948	51.7180	50.0000	50.0000	50.0000	50.0000
P_4	88.7805	87.2869	85.4006	83.3296	80.6711	76.9629	72.4543	62.8433	50.0000	50.0000	50.0000
P_5	67.6217	67.6768	67.7018	67.6517	67.4286	66.6986	65.7665	61.1102	53.5271	50.0000	50.0000
P_6	50.0000	50.0000	50.0000	50.0000	50.0002	50.0000	50.0000	50.0000	50.0000	50.0001	50.0000
P_7	73.3571	71.1666	68.5885	65.5610	61.7110	56.8874	51.1225	50.0000	50.0000	50.0000	50.0000
P_8	72.3424	70.3759	67.9883	65.5676	62.2324	57.8103	52.2584	50.0000	50.0000	50.0000	50.0000
P_9	73.6873	74.9973	76.5036	78.3766	80.1768	82.3973	84.9164	84.8918	83.6362	76.5325	62.9985
P_{10}	90.1281	91.5097	93.3257	95.3218	96.8928	98.4038	100.0596	97.8736	92.8696	80.9591	62.9325
P_{11}	50.0000	50.0000	50.0011	50.0000	50.7750	53.3537	56.6517	59.0981	61.6544	62.5527	62.6548
P_{12}	72.6566	77.3716	82.7842	89.1202	96.5257	105.3751	116.4102	127.2848	140.5235	155.5280	177.7163
P_{13}	72.2628	70.4298	68.0074	65.3607	61.5717	57.0394	51.0225	50.0000	50.0000	50.0000	50.0000
P_{14}	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0001
Fc ($/h)	4549.30	4529.77	4507.35	4482.83	4451.96	4414.64	4372.20	4345.14	4321.83	4309.73	4303.51
Fe (ton/h)	25.24	26.16	29.62	36.74	51.27	77.77	122.83	165.12	226.06	285.16	402.72

Table A. 28 Generations list for optimal solutions by non-dominated approach in Test System 3

Point No.	Generation (MW)														Fc ($/h)	Fe (ton/h)
	P_1	P_2	P_3	P_4	P_5	P_6	P_7	P_8	P_9	P_{10}	P_{11}	P_{12}	P_{13}	P_{14}		
1	73.5414	50.0000	74.7688	88.9730	69.4963	50.0000	72.4295	74.4695	76.1607	92.3970	50.0000	71.3104	66.4623	50.0000	4541.53	26.51
2	108.7074	88.8316	50.0000	50.6733	50.2061	50.0000	50.0000	50.0007	74.2964	95.6712	58.7564	133.6938	50.0316	50.0000	4321.52	248.73
3	75.8700	50.0000	67.9691	87.2383	69.1940	50.0000	68.0030	67.4313	76.5939	94.5953	50.0000	86.8551	65.9057	50.0000	4496.70	32.74
4	92.2251	77.9771	50.0000	52.8523	60.8191	50.0000	50.0000	50.0000	101.0716	75.7305	62.6589	136.6914	50.0000	50.0000	4330.71	225.65
5	93.8800	66.2565	56.6621	71.4044	59.5934	50.0000	50.0000	54.1662	84.1091	77.9254	54.4040	142.0076	50.0000	50.0000	4351.79	178.00
6	108.3975	78.4201	50.0000	58.2605	50.0000	50.0000	50.0000	50.5313	90.1578	102.1269	50.0000	122.6406	50.0000	50.0000	4340.74	192.94
7	94.4564	66.2565	59.0228	71.4049	59.5801	50.0000	56.9709	52.8750	84.1674	93.0335	53.8378	118.3482	50.0000	50.0000	4377.10	122.12
8	94.4763	66.2568	52.8060	71.4049	59.5751	50.0000	50.0000	55.0304	84.1674	99.7474	56.6156	119.7946	50.0000	50.0000	4365.57	134.05
9	120.9615	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	50.0000	99.9313	94.2819	64.8050	130.8403	50.0000	50.0000	4339.74	214.83
10	93.8800	58.2196	64.9233	71.2120	59.5934	50.0000	56.9709	54.1604	84.1674	99.2756	54.4040	107.0259	56.0681	50.0000	4405.83	88.90
11	100.3220	51.8977	54.7023	70.1668	56.7534	50.0000	55.3667	63.3793	87.4923	94.6655	56.5417	114.1553	54.3790	50.1811	4394.54	105.45
12	82.2554	51.1867	66.5701	83.8651	68.2795	50.0000	61.6994	63.7379	77.5820	94.8262	50.0000	97.0650	62.6492	50.0000	4461.81	46.45
13	86.7838	63.9121	50.0000	58.5055	52.6689	50.0000	52.5119	50.0000	92.9543	121.9022	57.4056	122.9151	50.0000	50.0000	4359.56	162.84
14	90.8053	58.5475	50.0000	77.6603	61.6292	50.0000	50.0000	50.7666	86.6425	98.4693	50.9899	134.4869	50.0000	50.0012	4360.12	150.40
15	93.8800	72.0309	56.6621	71.4047	59.5934	50.0000	56.9709	54.1662	84.1674	93.4809	54.5576	107.0259	56.0681	50.0008	4389.85	111.57
16	84.5550	51.7562	68.0947	81.6671	71.3123	50.0000	64.4712	63.3272	78.1649	95.9770	50.1660	85.7879	64.5731	50.0000	4479.65	39.10
17	84.6387	50.0000	58.8190	82.6321	63.5749	50.0000	62.1935	64.9085	81.2878	98.1714	54.4040	96.4071	62.6219	50.0000	4451.95	53.07
18	81.1589	51.5167	65.3947	83.2366	70.8055	50.0000	64.4996	64.9020	78.1564	96.3262	50.0000	89.6485	64.0217	50.0000	4475.84	39.95
19	88.3057	52.4351	59.4612	80.1233	64.5202	50.0000	60.5317	59.1425	80.7738	101.2160	50.0409	106.6240	56.5099	50.0000	4424.98	70.72
20	87.9139	59.7167	62.7370	73.2544	60.8191	50.0000	55.0599	54.0651	83.8603	109.0213	55.1778	100.7694	57.2622	50.0000	4414.05	83.24