FIBER OPTIC PARAMETRIC AMPLIFICATION (FOPA) SIMULATION IN A HIGHLY NONLINEAR FIBER (HNLF)

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Abstract

Four wave mixing (FWM) is a nonlinear effect observable in optical fibers when light of two or more wavelengths are launched into the fiber. Parametric amplification is a consequence of FWM, where depending on the phase difference between the waves, power transfer between the interacting waves could take place. We are interested in the generation of this phenomenon in optical fibers to create Fiber Optical Parametric Amplifiers (FOPA). In this paper, we simulated the parametric amplification process in a highly nonlinear fiber (HNLF), focusing on the effect of different values of the initial linear phase mismatch $\Delta\beta$, pump power P_p, and nonlinear coefficient γ. It was found that in terms of the initial $\Delta\beta$ value, a value closer to zero will result in a higher maximum signal and idler power. Also, we observed that a larger pump power and γ caused parametric amplification to happen within a shorter length of fiber, as well as giving us a bigger maximum gain. These results will be useful in optimizing the amplification process with respect to the fiber length involved.

Abstrak

Kesan empat gelombang (FWM) ialah satu kesan ketaklinearan yang boleh diperhatikan dalam gentian optik apabila dua atau lebih gelombang dilancarkan ke dalam gentian tersebut. Amplifikasi berparametrik adalah akibat daripada FWM, di mana bergantung kepada perbezaan fasa antara gelombang-gelombang berkenaan, permindahan kuasa di antara gelombang-gelombang berinteraksi boleh berlaku. Kami berminat terhadap penjanaan fenomena ini dalam gentian optik untuk merealisasikan amplifikasi berparameter gentian optik (FOPA). Dalam laporan ini, kami mensimulasikan proses amplifikasi berparametrik dalam gentian tinggi ketaklinearannya (HNLF), dengan tumpuan terhadap kesan nilai permulaan ketidakpadanan fasa linear $\Delta\beta$, kuasa pam P_p, dan pekali tak linear γ. Kami mendapati bahawa untuk nilai permulaan $\Delta\beta$, nilai mendekati sifar akan memberikan kuasa maksimum isyarat dan gelombang baru yang lebih tinggi. Kami juga mendapati bahawa kuasa pam dan nilai γ yang lebih besar akan menyebabkan amplifikasi berparametrik dicetuskan lebih cepat dalam gentian, serta memberikan gandaan maksimum yang lebih tinggi. Pemerhatian ini adalah berguna untuk mengoptimumkan proses amplifikasi berkenaan jumlah gentian yang diperlukan.

Acknowledgement

This project wouldn't have been possible without the contributions of a number of people.

First and foremost, I would like to thank my supervisor, Dr. Lim for guiding me through this project. His patience and kindness towards me had been remarkable, and I could only wish that I had been a better student.

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1 Introduction

Signal transmissions are always accompanied by losses due to various factors. Therefore, there is a need for amplifiers to boost the signal power, particularly for long distance signal transmissions. Conventional amplification of optical signals goes by the O-E-O (optical - electrical - optical) system, which requires the optical signal to be converted to electrical signal for amplification and then converted back into optical signal for further transmission. This process is complicated and expensive, which makes optical amplifiers an attractive option since the amplification does not involve the conversion of the optical signal into electrical signal.

The operating principle of an optical fiber amplifier depends on the system used, but it usually involves the injection of at least one pump and one signal into the fiber. The transfer of power from the pump to the signal results in the amplification of the signal power. One of the most popular conventional optical amplifiers is the erbium doped fiber amplifier (EDFA). In the EDFA, the transfer of power from the pump to the signal is through the energy transition between different electronic states. While EDFA can provide the amplification required, it suffers from the serious drawback of operating on electronic state transitions of erbium, which means restrictions in terms of the wavelengths used, i.e. 980 nm or 1480nm for pumping, and 1550 nm for signal wavelength.

Optical amplification can also be performed through the generation of nonlinear effects. Fiber Optical Parametric Amplification (FOPA), which is based on the nonlinear effects triggered by intense electromagnetic field in the fiber, provides the solution to the wavelength limitations suffered by EDFA. One of the most important features of FOPA is that it does not depend on energy state transitions. Instead, this parametric amplification process is due to interactions of photons sent into the fiber. This frees us from any wavelength constraints and open up other possibilities for WDM networks[1]. In fact, unlike stimulated scattering processes which depends on the interaction between the light and the silica molecules, the fiber does not play an active part in the parametric amplification process. Instead, the fiber only serves as a medium for different waves to interact.

The advantage in the the arbitrariness of the operation wavelength of optical parametric amplifiers is further strengthened by its other functions. Due to its parametric nature, OPA can be utilized in phase-sensitive amplification[2,3], whereby by just controlling the phase of the interacting waves, one can control the power flow among the different waves. Studies have also been done on the use of parametric amplification in wavelength conversion[4−6], pulse generation[7−11], optical time division demultiplexing[12−14], as well as signal processing and regeneration[15−18], with encouraging results.

2 Objective

The objectives of this project are:

1. To understand the mathematical background of the four-wave mixing (FWM) and parametric amplification process

2. To simulate the parametric amplification process in a highly nonlinear fiber (HNLF). This simulation is done with the single pump degenerate FWM model.

3. To study the characteristics of the growth and evolution of the signal and the generation of idler due to the amplification process

4. To investigate the dependence of different parameters on the parametric amplification process and the subsequent effects on the signal and idler growth

This report is organized as follows: Section 3 will cover the literature review detailing the process of Four-Wave Mixing as well as the mathematical background. Also, we will discuss how the different parameters affect the parametric amplification process. Meanwhile, Section 4 will cover the methodology of the simulation work done in this project, and the results and the subsequent discussion of the results will be presented in Section 5. Section 6 wraps up this report with the conclusion.

3 Literature Review

3.1 Theory of Four-Wave Mixing

In this section, we will examine the mathematical framework for four-wave mixing (FWM) and how the solutions describing the evolution of each wave is obtained. The full mathematical treatment of FWM is very complicated, as it involves a complete vector theory treatment. However, we can examine FWM using intuitive, physical views to obtain meaningful insights.

A fiber is subjected to various effects, both linear and nonlinear, depending on the order n of the susceptibility $\chi^{(n)}$ invoked. Nonlinear effects are triggered by strong electromagnetic fields in a fiber. Under normal circumstances, $\chi^{(1)}$ has dominant influence on the fiber, with effects in the form of changes to the refractive index n' and attenuation coefficient α. On the other hand, $\chi^{(2)}$ is responsible for phenomena like second harmonic generation, but it only becomes important if the medium consists of asymmetrical molecules. Since silica is a symmetric molecule, the contribution of $\chi^{(2)}$ in an optical fiber is very small. However, the $\chi^{(3)}$ nonlinearity can be significant if phase matching is performed, and FWM, which leads to the generation of new frequency components, is a result of this.

The FWM phenomenon can be explained using a few approaches. One of the angles in which we can look at the FWM process is through quantum mechanics. From a quantum mechanical point of view, the FWM process is a result of the interaction of the photons sent into the fiber. For this subsection, we study the general, nondegenerate case of two pumps of frequencies ω_{p1} and ω_{p2} being launched into the fiber. When we talk about the nondegenerate case, it means that $w_{p1} \neq w_{p2}$. The two pump photons can then interact to produce a signal photon and an idler photon, with frequencies ω_s and ω_i, respectively (for convenience, all future quantities bearing the subscripts $p1, p2, s$ and i will represent pump 1, pump 2, signal and idler, respectively). The interaction will happen according to the conservation of energy and momentum:

$$\omega_{p1} + \omega_{p2} = \omega_s + \omega_i \tag{1}$$

$$\beta(\omega_s) + \beta(\omega_i) - \beta(\omega_{p1}) - \beta(\omega_{p2}) = \Delta\beta = 0 \tag{2}$$

with $\beta_{p1,p2,s,i} = \omega_{p1,p2,s,i} n/c$ being the propagation constant for each wave, and $\Delta\beta$ the low power propagation mismatch. When a signal is also sent into the fiber, the addition of the signal photons produced by the FWM process to the ones launched into the fiber leads to an increase in the signal power. Hence, amplification of the signal power takes place. As mentioned before, this process requires careful choice of fiber parameters and input wavelengths in order for the interaction to happen efficiently.

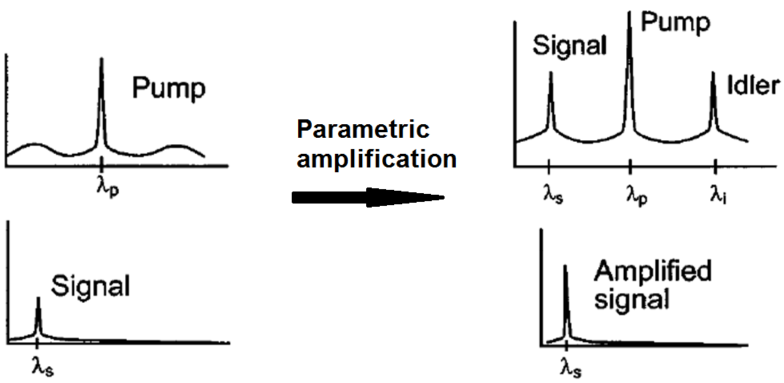

Figure 3.1: Mechanism for signal amplification due to parametric amplification. The amplification of the signal is a result of the power transfer from a single pump. At the same time, the idler is generated. Figure modified from [19].

At the same time, we can also examine the FWM phenomenon from the electromagnetic angle. In this approach, the FWM is a result of the interaction of the four electric fields with complex amplitudes A_{p1}, A_{p2}, A_s and A_i. The total transverse field $E(x, y, z)$ propagating along the fiber is described by the following equation[19,20]:

$$\begin{aligned} E(x,y,z) = \frac{1}{2} f(x,y)[&A_{p1}(z) \exp\left(i\beta_{p1} z - i\omega_{p1} t\right) + A_{p2}(z) \exp\left(i\beta_{p2} z - i\omega_{p2} t\right) \\ &+ A_s(z) \exp\left(i\beta_s z - i\omega_s t\right) + A_i(z) \exp\left(i\beta_i z - i\omega_i t\right) + \text{c.c}] \end{aligned} \tag{3}$$

where c.c is the complex conjugate, and $f(x, y)$ is the spatial distribution of the fiber mode in the fiber, which is assumed to be the same for all the four interacting waves.

The evolution of the complex field amplitude $A_{p1,p2,s,i}(z)$ in the fiber can then be derived from (3) to give us four coupled equations[19–21]:

$$\frac{dA_{p1}}{dz} = i\gamma\{[|A_{p1}|^2 + 2(|A_{p2}|^2 + |A_s|^2 + |A_i|^2)]A_{p1} + 2A_sA_iA_{p2}^* \exp(i\Delta\beta z)\} \quad (4)$$

$$\frac{dA_{p2}}{dz} = i\gamma\{[|A_{p2}|^2 + 2(|A_{p1}|^2 + |A_s|^2 + |A_i|^2)]A_{p2} + 2A_sA_iA_{p1}^* \exp(i\Delta\beta z)\} \quad (5)$$

$$\frac{dA_s}{dz} = i\gamma\{[|A_s|^2 + 2(|A_{p1}|^2 + |A_{p2}|^2 + |A_i|^2)]A_s + 2A_i^*A_{P1}A_{P2} \exp(-i\Delta\beta z)\} \quad (6)$$

$$\frac{dA_i}{dz} = i\gamma\{[|A_i|^2 + 2(|A_s|^2 + |A_{p1}|^2 + |A_{p2}|^2)]A_i + 2A_s^*A_{P1}A_{P2} \exp(-i\Delta\beta z)\} \quad (7)$$

where

$$\gamma = 2\pi n_2/\lambda A_{\text{eff}} \quad (8)$$

is the nonlinear coefficient, with n_2 being the nonlinear index coefficient, λ the wavelength of the wave, and A_{eff} the effective mode area of the fiber. For simplicity, we have ignored the losses in the fiber. Also, while γ is a function of λ, it is assumed that the wavelengths of the interacting waves are sufficiently close such that γ is the same for all four waves.

The RHS of Eq. (4) - (7) represents the effects of various nonlinear processes, such as the nonlinear phase shift due to self-phase modulation (SPM) and cross-phase modulation (XPM) as well as the power transfer between the interacting components:

$$\frac{dA_{p1}}{dz} = i\gamma[\underbrace{|A_{p1}|^2A_{p1}}_{\text{SPM term}} + \underbrace{2(|A_{p2}|^2 + |A_s|^2 + |A_i|^2)A_{p1}}_{\text{XPM term}} + \underbrace{2A_sA_iA_{p2}^* \exp(i\Delta\beta z)}_{\text{power transfer term}}] \quad (9)$$

It is the power transfer term that are responsible for the parametric amplification process. The power transfer between the different wave components will be clearer if we rewrite the Eq. (4) - (7) in terms of the powers and phases of each wave. Given that

$$P_{p,s,i}(z) = |A_{p,s,i}(z)|^2, \qquad A_{p,s,i}(z) = \sqrt{P_{p,s,i}} \exp(i\phi_{p,s,i}) \quad (10)$$

we will obtain the following equations:

$$\frac{dP_{p1}}{dz} = -4\gamma(P_{p1}P_{p2}P_sP_i)^{1/2}\sin\theta \quad (11)$$
$$\frac{dP_{p2}}{dz} = -4\gamma(P_{p1}P_{p2}P_sP_i)^{1/2}\sin\theta \quad (12)$$
$$\frac{dP_s}{dz} = 4\gamma(P_p^2P_sP_i)^{1/2}\sin\theta \quad (13)$$
$$\frac{dP_i}{dz} = 4\gamma(P_p^2P_sP_i)^{1/2}\sin\theta \quad (14)$$

where

$$\frac{d\theta}{dz} = \Delta\beta + \gamma\big[P_{p1} + P_{p2} - P_s - P_i\big] + 2\gamma\{[P_{p1}P_{p2}P_s/P_i]^{1/2} + [P_{p1}P_{p2}P_i/P_s]^{1/2} + [P_{p2}P_sP_i/P_{p2}]^{1/2} - [P_{p1}P_sP_i/P_{p1}]^{1/2}\}\cos\theta$$

with θ representing the phase difference between the waves:

$$\theta(z) = \Delta\beta z + \phi_{p1}(z) + \phi_{p2}(z) - \phi_s(z) - \phi_i(z). \quad (15)$$

The first term on the RHS of Eq. (15) represents the linear phase shift, while the rest belongs to the nonlinear phase shift.

Eq. (11) - (14) describes the evolution of the power of each of the wave along the fiber, and it is simple to see that the transfer of power between the waves depends on their relative phase. As such, by controlling the phase relation θ, we will be able to control the parametric amplification process. When $\sin\theta > 0$, power is transferred from the pump to the signal and idler. This way, amplification can be obtained, with maximum power transfer occurring at $\theta = \pi/2$. On the other hand, parametric attenuation takes place when $\sin\theta < 0$ and the reverse is true; power flows from the signal and idler to the pump. Indeed, the phase sensitivity of the parametric amplification and attenuation is a useful advantage for FOPA.

One of the difficulties in realizing parametric amplification is in setting and maintaining the value of θ. The efficiency of the FWM process therefore depends on the phase between the interacting waves, making phase matching an essential step in realizing parametric amplification. To facilitate the phase matching process, some of the work done to improve the parametric amplification process involves the introduction of a phase-shift in the fiber[23] and cascaded parametric amplification[24].

Eq. (15) can be simplified using the undepleted pump approximation. Also, operating near the phase-matched condition of $\theta = \pi/2$ removes the cosine term, leaving us with:

$$\frac{d\theta}{dz} = \Delta\beta + \gamma[P_{p1}(z) + P_{p2}(z)] = \kappa \tag{16}$$

The solution for the four coupled equations of Eq. (4) - (7) requires a numerical approach. However, we can once again simplify the problem by considering the undepleted pump approximation, which is not unreasonable as we are often dealing with a small signal power and a significantly larger pump power. With this approximation, we can remove the terms that do not contain either $|A_{p1}|^2$ and $|A_{p2}|^2$, and Eq. (4) and (5) will then be reduced to:

$$\frac{dA_{p1}}{dz} = i\gamma(P_{p1} + 2P_{p2})A_{p1} \tag{17}$$

$$\frac{dA_{p2}}{dz} = i\gamma(P_{p2} + 2P_{p1})A_{p2} \tag{18}$$

which has a relatively trivial solution:

$$A_{p1}(z) = A_{p1}(0)\exp[i\gamma(P_{p1} + 2P_{p2})z]\}, \tag{19}$$

$$A_{p2}(z) = A_{p2}(0)\exp[i\gamma(P_{p2} + 2P_{p1})z]\} \tag{20}$$

Substituting (19) and (20) into the remaining coupled equations in Eq. (6) and (7) will then lead us to the following equations:

$$\frac{dA_s}{dz} = 2i\gamma\{[(P_{p1} + P_{p2})]A_s + A_i^* A_{P1} A_{P2}\exp(-i\theta z)\} \tag{21}$$

$$\frac{dA_i}{dz} = -2i\gamma\{[(P_{p1} + P_{p2})]A_i^* + A_s A_{P1}^* A_{P2}^*\exp(-i\theta z)\} \tag{22}$$

where $\theta = [\Delta\beta - 3\gamma(P_{p1} + P_{p2})z]$.

The find the solution for A_s and A_i, we let

$$S_{s,i} = A_{s,i}\exp[-2i(P_{p1} + P_{p2})]. \tag{23}$$

Differentiating $S_{s,i}$ with respect to z,

$$\frac{dS_s}{dz} = 2i\gamma A_{p1}(0)A_{p2}(0)e^{-i\kappa z}S_i^* \quad (24)$$

$$\frac{dS_i^*}{dz} = -2i\gamma A_{p1}^*(0)A_{p2}^*(0)e^{i\kappa z}S_s \quad (25)$$

where κ is once again the effective phase mismatch as expressed in Eq. (16). However, despite having performed the differentiation, Eq. (24) and (25) are still coupled. To obtain the decoupled equations, differentiation is performed for the second time to reveal the following:

$$\frac{d^2S_s}{dz^2} = -i\kappa\frac{dS_s}{dz} + (4\gamma^2 P_{p1}P_{p2})S_s \quad (26)$$

$$\frac{d^2S_i^*}{dz^2} = i\kappa\frac{dS_i^*}{dz} + (4\gamma^2 P_{p1}P_{p2})S_i^* \quad (27)$$

The general solution for S_s and S_i are then found to be

$$S_s(z) = (a_s e^{gz} + b_s e^{-gz})\exp(-i\kappa z/2) = A_s \exp[-2i(P_{p1} + P_{p2})] \quad (28)$$

$$S_i(z) = (a_i e^{gz} + b_i e^{-gz})\exp(i\kappa z/2) = A_i \exp[-2i(P_{p1} + P_{p2})] \quad (29)$$

where a_s, a_i, b_s and b_i are determined by boundary conditions. Here, g is the parametric gain, given as

$$g^2 = (\gamma P_0 r)^2 - (\kappa/2)^2 \quad (30)$$

where $P_0 = P_{p1} + P_{p2}$ is the total pump power and $r = 2\sqrt{P_{p1}P_{p2}}/P_0$.

The boundary conditions for Eq. (28) and (29) are $|A_s(0)|^2 = P_s(0)$ and $|A_i(0)|^2 = 0$. The latter boundary condition is applicable because the generation of idler takes place within the fiber, and hence, at $z = 0$, its amplitude and power is essentially zero.

From Eq. (30), we can deduce that at the condition of perfect phase matching with $\kappa = 0$, we can obtain maximum parametric gain. Since κ is the sum of the linear and nonlinear phase mismatch, the perfect phase matching condition will be fulfilled when the linear and nonlinear phase mismatches to cancel each other out, i.e. $2\gamma P_p = -\Delta\beta$.

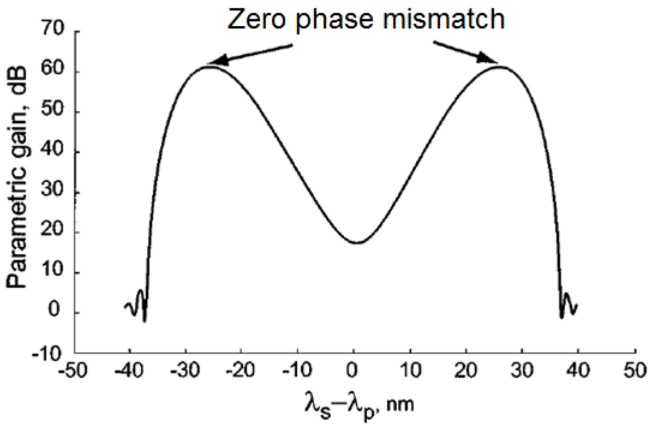

Figure 3.2: The calculated gain due to the parametric amplification process with respect to $(\lambda_s - \lambda_p)$. Figure modified from [19].

3.2 Degenerate FWM

The general, nondegenerate case discussed in the previous subsection requires two pump waves to be injected into the fiber for FWM to be triggered. In this project, we settle on using the degenerate case of FWM, where only one pump is used. The degenerate case is considered to be a special case, with $\omega_{p1} = \omega_{p2}$. Although this process only contains three distinct interacting waves, it is still referred to as FWM as the name three-wave mixing is reserved for the process due to $\chi^{(2)}$ nonlinearity.

Since there is only one pump in this degenerate case, we shall replace all pump quantities with the subscript $p1$ and $p2$ with p instead. Therefore, the conservation of energy and momentum equations as shown in Eq. (1) and (2) become:

$$2\omega_p = \omega_s + \omega_i \tag{31}$$

$$\beta(\omega_s) + \beta(\omega_i) - 2\beta(\omega_p) = \Delta\beta = 0 \tag{32}$$

and the basic propagation equations describing the complex field amplitudes of the waves propagating in the fiber is reduced to three equations:

$$\frac{dA_p}{dz} = i\gamma\{[|A_p|^2 + 2(|A_s|^2 + |A_i|^2)]A_p + 2A_sA_iA_p^*\exp(i\Delta\beta z)\} \quad (33)$$
$$\frac{dA_s}{dz} = i\gamma\{[|A_s|^2 + 2(|A_p|^2 + |A_i|^2)]A_s + A_i^*A_p^2\exp(-i\Delta\beta z)\} \quad (34)$$
$$\frac{dA_i}{dz} = i\gamma\{[|A_i|^2 + 2(|A_s|^2 + |A_p|^2)]A_i + A_s^*A_p^2\exp(-i\Delta\beta z)\} \quad (35)$$

Once again, we let $P_{p,s,i}(z) = |A_{p,s,i}(z)|^2$. As a result, the power evolution of each wave in the fiber will be according to the following equations:

$$\frac{dP_p}{dz} = -4\gamma(P_p^2P_sP_i)^{1/2}\sin\theta \quad (36)$$
$$\frac{dP_s}{dz} = 2\gamma(P_p^2P_sP_i)^{1/2}\sin\theta \quad (37)$$
$$\frac{dP_i}{dz} = 2\gamma(P_p^2P_sP_i)^{1/2}\sin\theta \quad (38)$$

It should be noted that as opposed to the nondegenerate case where there are two pumps to supply the power to the signal and idler, the degenerate case has to do with a single pump. The power transfer in the degenerate case will obviously be smaller compared to the nondegenerate case. This is apparent in the how the coefficient on the RHS of Eq. (37) and (38) is half of that in Eq. (13) and (14).

With $P_{p1} = P_{p2}$, other expressions will also require some slight modification. For instance, $d\theta/dz$ is now expressed as

$$\frac{d\theta}{dz} = \Delta\beta + \gamma[2P_p(z) - P_s(z) - P_i(z)]$$
$$+\gamma[(P_p^2P_s/P_i)^{1/2} + (P_p^2P_i/P_s)^{1/2} - 4(P_sP_i)^{1/2}]\cos\theta \quad (39)$$
$$= \Delta\beta + \gamma[2P_p(z) - P_s(z) - P_i(z)] = \kappa \quad (40)$$

with

$$\theta(z) = \Delta\beta z + 2\phi_p(z) - \phi_s(z) - \phi_i(z). \quad (41)$$

As mentioned in the previous subsection, $\kappa = 0$ is the condition for perfect phase matching. For this to happen, we rely on the linear and nonlinear phase mismatches to cancel each other out, i.e. $2\gamma P_p = -\Delta\beta$. If the nonlinear phase mismatch $2\gamma P_p$ is positive, then $\Delta\beta$ will need to have a negative value, i.e. $\Delta\beta < 0$.

As it turns out, $\Delta\beta$ can be written as[1]9

$$\Delta\beta = -\frac{2\pi c}{\lambda_0^2}\frac{dD}{d\lambda}(\lambda_p - \lambda_0)(\lambda_p - \lambda_2)^2 \tag{42}$$

where $dD/d\lambda$ is the dispersion slope at the zero-dispersion wavelength λ_0. With Eq. (42), one can see that since $(\lambda_p - \lambda_2)^2$ will always take a positive value thanks to its quadratic nature, the condition $\Delta\beta < 0$ will be fulfilled if $\lambda_p > \lambda_0$. Therefore, by careful choice of the pump wavelength with respect to the zero-dispersion wavelength, we can have the linear phase mismatch $\Delta\beta$ compensate the nonlinear phase mismatch $2\gamma P_p$.

The solutions of the coupled equations for the complex field amplitude has already been presented in the previous segment of this report. The general solutions were given as Eq. (28) and (29) and inserting the appropriate boundary conditions will give us the following expressions for the signal and idler power:

$$P_s(z) = P_s(0)\{1 + [\gamma P_p \sinh{(gL)}/g]^2\} \tag{43}$$

$$P_i(z) = P_s(0)[\gamma P_p \sinh{(gL)}/g]^2 \tag{44}$$

From Eq. (43) and (44), we can see that the signal and idler growth goes according to the same trend, with the only difference being the offset at due to the presence of signal power at $z = 0$.

Given the solution that describes the evolution of the signal and idler in the fiber in Eq. (43) and (44), we can see that there is a dependence of the signal and idler power on a few parameters: the value of γ, P_p, and g. The dependence on g however, is not straightforward. From Eq. (30) we can see that g depends on the value of κ. In turn, κ depends on $\Delta\beta$, as shown in Eq. (40), which means that $\Delta\beta$ can determine the behaviour of the system.

In this project, we are interested the growth of the signal and idler, as well as the variation of gain along the fiber. For this, we simulate the values of the signal and idler power at different values of z, which is indicative of the length of fiber, with different parameters. Specifically, we seek to investigate the dependence of the signal and idler growth due to the parametric amplification process on the following:

1. linear phase mismatch $\Delta\beta$,
2. pump power P_p, and
3. nonlinear coefficient γ.

The project methodology will be presented in the next section.

4 Methodology

As mentioned, there are a few parameters which will influence the parametric amplification process, namely the linear phase mismatch $\Delta\beta$, the pump power P_p and the nonlinear coefficient γ. For this project, we investigated the influence of each of these parameters by simulating the signal and idler power growth as well as the evolution of κ and g along a fiber while we varied the values of the parameter studied.

The simulation in this project was done using Wolfram Mathematica. We started by feeding in the initial values of P_s, P_i and P_p. Due to the undepleted pump approximation, P_p is a constant, selected to be much larger than the initial value of P_s. For this project, the starting value of P_s is taken to be 5 μW.

An array with the size corresponding to the number of points to be plotted was then prepared and filled with the values of z, ranging from 0 to a chosen maximum value. At the same time, the values of $\Delta\beta$ and γ were chosen accordingly. To find the value of P_s and P_i at each value of z as per Eq. (43) and (44), we had to know the value of the parametric gain g. To obtain the value of g, we had to find the value of κ.

There were two expressions involving κ that we needed to take note of. From Eq. (40), the nonlinear phase mismatch term involves the pump, signal and idler powers. Under the undepleted pump approximation we could make the approximation $\kappa \approx \Delta\beta + 2\gamma P_p$, but this is only true at the start of the fiber when the parametric amplification process is still insignificant; the situation will no longer be true once P_s and P_i grew to significant values. Therefore, we had to take into account the influence of P_s and P_i on κ. On the other hand, rewriting the equation for g in Eq. (30), we will have the following expression for κ:

$$\kappa^2 = 4[(\gamma P_p)^2 - g^2] \tag{45}$$

The existence of two equations for κ means that there are restrictions on the values of g. In other words, this means that g can only take values that will fulfill the expressions for κ in Eq. (40) and (45). Consequently, the appropriate g values were then determined by finding the roots to those two equations numerically. Once these values of g were calculated for each z, the values of P_s and P_i were easily obtained. The specific values chosen for the different parameters used in each case will be presented in the following subsections.

4.1 Effect of Initial $\Delta\beta$ Value

The dependence of g on κ means that the linear phase mismatch $\Delta\beta$ indirectly influences the value of g. In this part of the project, we studied the evolution of the signal, idler and the gain under different values of $\Delta\beta$. For this, we set different values of $\Delta\beta$, while keeping the other parameters constant. We picked $\Delta\beta$ values of -0.1, -0.2, -0.3, and -0.4 km^{-1}, with a constant γ of 30 W^{-1}km^{-1}, $P_p = 5$ mW, and $P_s(0) = 5$ μW.

4.2 Effect of Pump Power Variation

To study the effect of different pump powers on the parametric amplification process, we performed the simulation using $P_p = 3$ mW, 5 mW, 10 mW and 15 mW. Again, the initial signal power was set to be 5μW. The value of $\Delta\beta$ was set to be -0.1, and a γ value of 30 W^{-1} km^{-1} was chosen.

4.3 Effect of γ variation

The value of γ is a measure of how easily nonlinear effects are triggered in a fiber. The higher the value of γ is, the more prominent nonlinear effects will be. From Eq. (8), we can see that in order for us to have high γ in a fiber, we can increase the value of n_2 or decrease the A_{eff}.

Generally, the value of γ for an ordinary fiber is around 10 W^{-1} km^{-1} to 20 W^{-1} km^{-1}, but for HNLF such as photonic crystal fiber[25–27] (PCF), γ can go higher than 70^{-1} km^{-1}, depending on the physical properties of the fiber itself. In fact, literature have presented very high γ values in the order of 10^3 W^{-1} km^{-1} in specially designed waveguides and PCF. However, those remarkably large γ values involved extensive waveguide design and manufacturing processes, such as photonics slot waveguide[25] and spiral PCF[27]. Therefore, we shall not include those special cases in this project. Instead, we performed the simulation using more accessible γ values.

For this part of the project, we have chosen values of γ = 30, 40, 50, 60 and 70 W^{-1} km^{-1}, with pump power of 5 mW acting on a signal power of 5 μW.

5 Results and Discussion

This section will detail the results and findings from the simulation done, along with the relevant discussion points.

5.1 Effect of Initial $\Delta\beta$ Value

Fig. 5.3 shows the signal power P_s, idler power P_i, κ, and gain g along z for $\Delta\beta$ values of -0.1, -0.2, -0.3, and -0.4 km^{-1}.

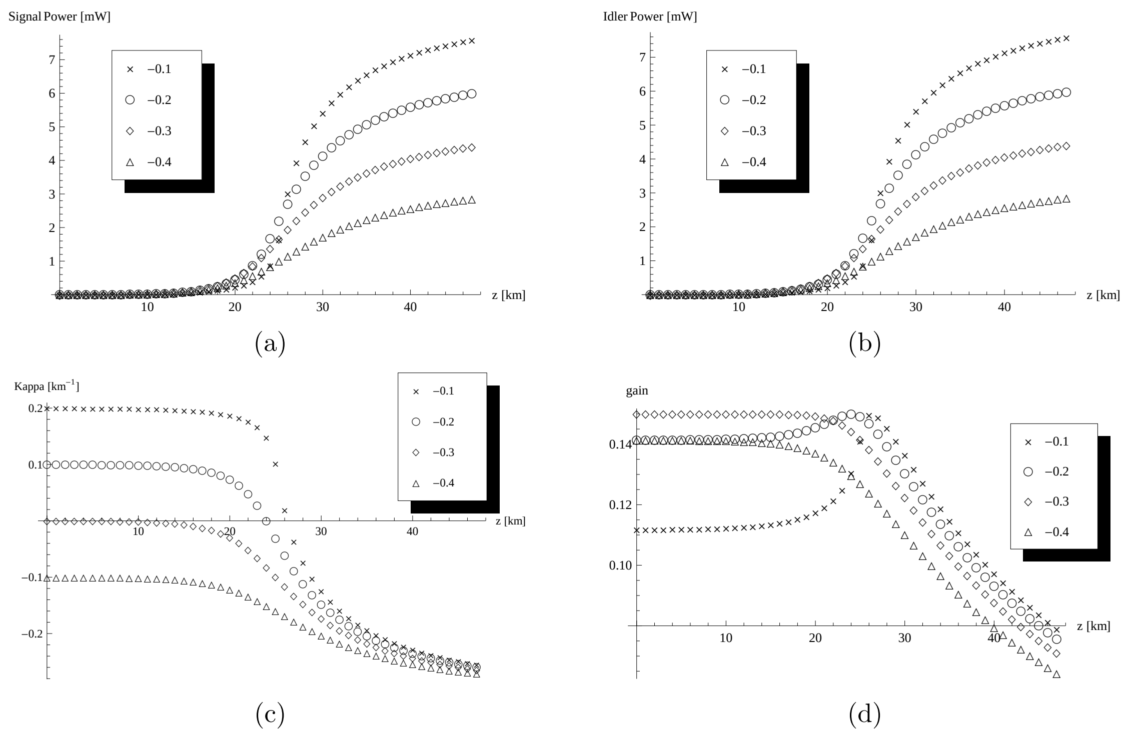

Figure 5.3: The evolution of (a) signal power, (b) idler power, (c) κ and (d) gain along z, for different values of $\Delta\beta$.

It was observed that for all four values of $\Delta\beta$, the signal and idler power curves take on a similar shape: a slow growth region where the signal and idler were barely changing, followed by a steep growth before reaching a plateau. Also, the region of rapid power growth occurred around a similar value of z for all the different values of $\Delta\beta$. This means that it took similar amount of time for all the cases tested here for parametric amplification to happen. However, the four values of $\Delta\beta$ resulted in different maximum signal and idler power reached.

Given that the maximum gain $g_{\max} = \gamma P_p$, all three $\Delta\beta$ values of -0.1, -0.2 and

-0.3 km^{-1} gave the same maximum gain when $\kappa = 0$ since they all have the same γ and P_p value. Of the three, $\Delta\beta$ achieves perfect phase matching with $\kappa = 0$ right from the start. Yet the maximum signal and idler power is highest for $\Delta\beta = -0.1$ km^{-1}. This could be a result of the steep increase in the gain that came about the sharp decrease in κ, leading to a faster signal power growth. This insight could be useful, because ultimately, if we are interested in having large maximum signal power, it would be beneficial to have a sharp fall in the value of κ to trigger a sharp increase in the gain value.

The case of having a $\Delta\beta$ value of -0.4 km^{-1} turned out to be different from the other $\Delta\beta$ values tested, as it gave us a lower maximum gain. This is due to the fact that κ never reached 0. Since κ contains the nonlinear phase mismatch term of $2\gamma(P_p - P_s - P_i)$, the maximum value of the nonlinear phase mismatch will be $2\gamma P_p$ under the undepleted pump approximation. According the the values we have chosen, this maximum value will be 0.3 km^{-1}. The growth in the signal and idler powers due to the parametric amplification process means that the magnitude of the nonlinear phase mismatch will only decrease from then on. Given that the $\Delta\beta$ value of -0.4 km^{-1} exceeds the maximum value of $2\gamma P_p$, the nonlinear phase mismatch is unable to balance the value of $\Delta\beta$ to provide us with a κ value of 0. Hence, the maximum gain could not be reached. In other words, to obtain the best result, $|\Delta\beta| \leq 2\gamma P_p$.

From Fig. 5.3, it is obvious that the undepleted pump approximation is no longer valid after a certain value of z. The pump power that we insert into the system is only 5 mW in magnitude, and the amplified signal and idler surpassed that value soon after the parametric amplification manifests itself. Still, we saw that if we ignore fiber losses, we could obtain intensive signal and idler growth with the right changes in the value of κ.

One might point out that the amount of fiber required for the amount of parametric amplification, stretching more than 30km, is not feasible. While it is true that this is hardly idea for practical purposes, one should remember that the value of the pump power here is a meagre 5mW. Since the gain and κ values are dependent on the pump power, it would be interesting to see how the increase in pump power will influence the process. This will be covered in the next subsection.

5.2 Effect of Pump Power Variation

The results of the simulation with different pump powers are shown in Fig. 5.4. As expected, high pump power leads to higher signal growth, due to the fact that the gain is proportional to the value of γP_p. At the same time, a bigger pump power results in a sharper decrease in κ within a shorter fiber distance. As a result, the signal power growth happened earlier for a larger pump power, shifting the curve to the left.

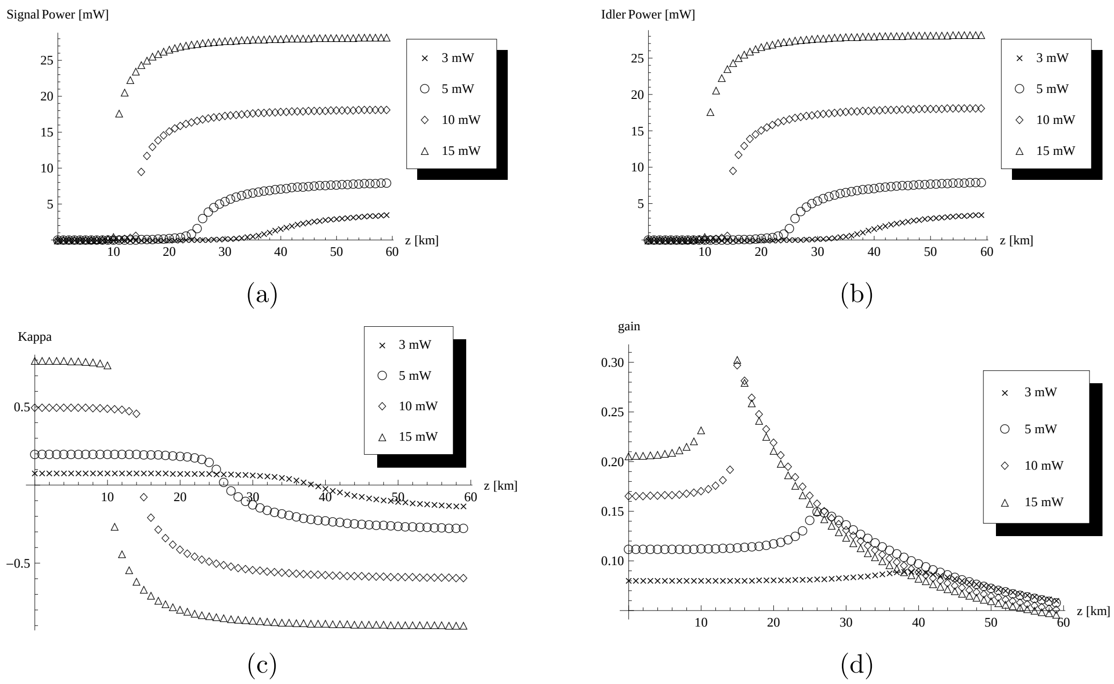

Figure 5.4: The evolution of (a) signal power, (b) idler power, (c) κ, and (d) gain for along z, for different pump powers

Unlike the case of the $\Delta\beta$ variation, different pump powers lead to different maximum gains. A larger pump power will result in a higher maximum gain, as well as a greater maximum signal and idler power. In other words, if we want a larger maximum gain and higher output signal and idler, an increase in pump power will lead to significant improvement in the wanted quantities.

The shifting of the curves to the left also means that within a shorter amount of fiber, we can experience greater effects of the parametric amplification process.

As shown in Fig. 5.4, the steep increase in signal power for 10 mW of pump power happens at a larger z value compared to that for 15 mW of pump power. This knowledge will be beneficial if we want to reduce the amount of fiber required.

It should be interesting to note that even though the maximum gain is the highest for the 15 mW pump power, the subsequent decrease was also the sharpest. This is attributed to the fact that for a higher pump power, the parametric amplification process is stronger. With a greater change in the values of P_s and P_i, the value of κ will also change more dramatically. As a result, κ deviates from 0 quickly, and the system moves away from the perfect phase matching condition. It would be ideal to sustain the $\kappa = 0$ condition in the system, but as demonstrated, the parametric amplification effect will inevitably cause the gain to suffer. The irony of the situation is that if we are interested in having uniform parametric gain throughout the fiber, we will have to contend with low pump power and the trade-off would be a lower overall signal and idler power due to a slower amplification process. Indeed, one of the challenges in optimizing the performance of FOPA would be to maintain the phase matching in the fiber.

Overall, we can see that if we want to improve the gain and power amplification of the signal and idler, it would be more effective to increase the pump power. We have not considered power losses in the simulation here, but we can see that the increase in pump power will not only lead to a greater maximum gain, but also trigger the effects within a smaller length of fiber. It should be noted that the pump powers here are very conservative in terms of the magnitude, which was 15 mW at most. Experimentally, pump powers of the order 1 W or above will still be considered reasonable.

5.3 Effect of γ Variation

Fig. 5.5 shows the effect of different values of γ on the system gain and power growth of the signal and idler.

The results for the γ variation is similar to that of the P_p variation due to the fact that these two quantities are linked together in the same term for g. Therefore, a

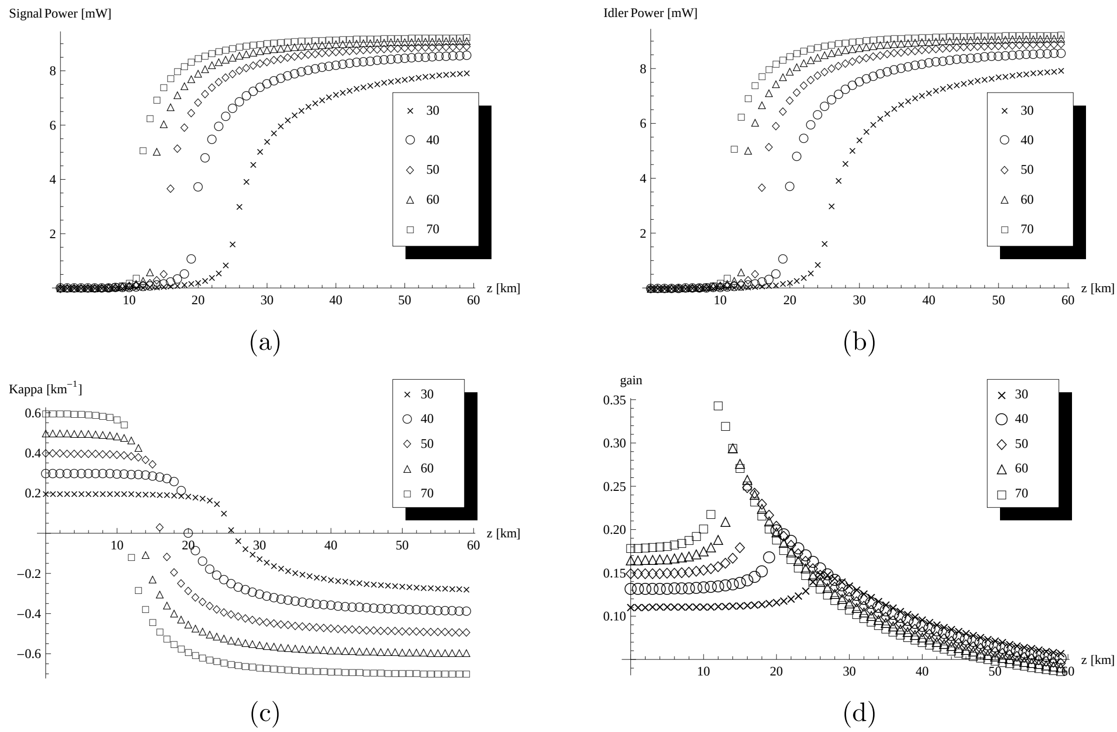

Figure 5.5: The evolution of (a) signal power , (b) idler power, (c) κ, and (d) gain along z, for values of γ

similar trend is expected for both trends. Firstly, there is the shifting of the curve to the left with increasing γ. At the same time, the sharp drop in the value of κ occurs earlier within a shorter length of fiber, allowing significant power amplification to occur sooner. In addition, the magnitude of the maximum gain, which happens at $\kappa = 0$ and depends solely on the product of γ and P_p, increases. In other words, increasing the value of γ not only helped to trigger the parametric amplification effect earlier, but also gave us a larger maximum gain.

In addition, we once again see the sharp drop in the parametric gain after the maximum gain value with increasing γ. As with the case of the increasing pump power, a bigger γ value will result in stronger parametric amplification process. However, as we discussed in the previous subsection, the greater amplification results in the system deviating from the phase matching condition sooner. Therefore, a larger γ will allow us to have amplification within a shorter amount of fiber, but it turned out that any additional length after the point of maximum gain is not likely to help increase the signal power significantly.

Examining the growth of the signal in greater detail as shown in Fig. 5.6, we can see that towards the higher γ values, the improvement to the parametric amplification process begins to taper off. What we meant with this is that the amount of fiber required to reach the steep signal growth phase decreases more noticably when we increase the value of γ from 30 W^{-1} km^{-1} to 40 W^{-1} km^{-1} as compared to from 60 W^{-1} km^{-1} to 70 W^{-1} km^{-1}. In other words, if we continue to increase the value of γ, the leftward shift of the signal and idler curves will be less significant.

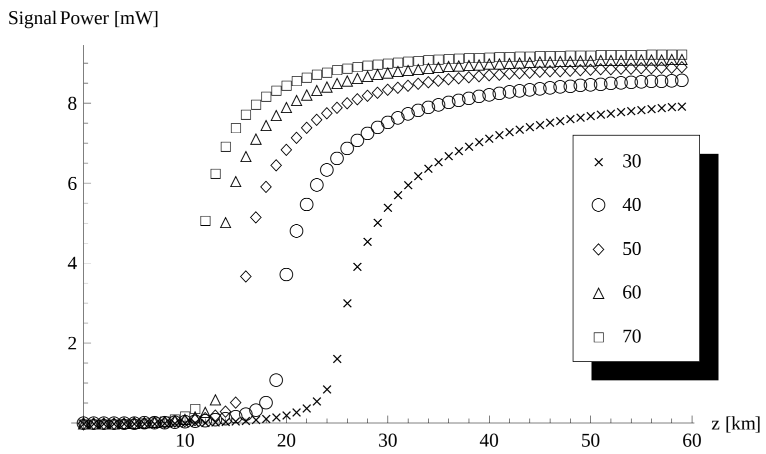

Figure 5.6: The leftward shift of the signal power with respect to increasing values of γ.

It should be noted that increasing the pump power is generally easier compared to increasing the value of γ, since γ is pretty much a property of the fiber itself. Hence, to improve the performance of the system, manipulation of the pump power magnitude would be preferable, provided that the fiber losses are kept to a minimum. Also, as the increase of γ shifts the signal and idler power curves to the left just like an increase in P_p does, a combination of an increase both the values of γ and P_p will result in significant enhancement of the parametric amplification process.

6 Conclusion

From literature review, we saw how the solutions were found for the coupled equations describing the interacting waves in the fiber. These solutions expressed the variation of the signal and idler power due to the parametric amplification process. As a result, we could determine which parameters were capable of influencing the parametric amplification process, therefore allowing us to perform the simulation of this process and study it in greater detail.

The parametric amplification process was simulated using the Wolfram Mathematica program using a single pump degenerate FWM model. We saw that the parametric amplification process can be triggered in a HNLF provided the parameters are of suitable values. The signal and idler powers show similar growth trends, with great amplification occurring at regions of maximum parametric gain.

Consequently, the effects of $\Delta\beta$, pump power P_p and nonlinear coefficient γ variation on the parametric amplification process were investigated. For the case of varying $\Delta\beta$ values, it was found that a higher signal and idler power can be achieved if we choose $\Delta\beta$ to be smaller (i.e. more negative). However, this is only applicable as long as $|\Delta\beta|$ does not exceed the value of $2\gamma P_p$, since we require κ to be 0 for maximum gain possible. Also, by only varying $\Delta\beta$, we changed the slope of the signal and idler power, with sharp power growth taking place at the point the value of κ changes dramatically.

On the other hand, the increase in pump power and γ values lead to similar trends. Variation of either the pump power and the γ value will result in different maximum gains, as well as trigger the drastic power growth of the signal and idler earlier in the fiber, thereby minimizing the amount of fiber required. In short, a high pump power and a large γ value will allow us to have better gain and resulting in bigger signal and idler output powers, all within a smaller amount of fiber length.

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