4.1 Introduction

As an application, the proposed Gamma ARMA(1, 1) model of Section 3.4.2 is fitted to the seasonally adjusted monthly river flows of Perak river in Malaysia. The seasonal effects of the original series are removed by using the classical method discussed in Section 2.2, i.e., subtract each observation from its corresponding monthly mean and then divide by its monthly standard deviation. By fitting a Gamma distribution to the above seasonally adjusted series, we find that the Chi-square statistic $\chi^2$ is 18.749 with 15 degrees of freedom. Since $P(\chi^2 > 18.749) = 0.2253$, we conclude that the seasonally adjusted river flows of the Perak river follow a Gamma distribution.

The method of moments is used to estimate the model's parameters $p_1$, $p_2$, $\alpha$, and $k$. As an alternative, we shall also estimate the parameter $p_1$ by using the least squares method (Lawrence and Paul, 1978) and the remaining parameters by moments method.

We shall investigate the performance of the proposed model by comparing the simulated sequence to the historical sequence in terms of means, variances, skewness and autocorrelation coefficients. We shall also compare the simulated results produced by the two different sets of estimated parameters, viz, the set of parameters estimated solely from the moments method and
the set of parameters estimated from the combination of moments and least squares methods. The simulation results are discussed and displayed in Section 4.3.

4.2 Parameters Estimation

In order to preserve the variance $s^2$, skewness $\gamma$, lag-1 serial correlation $r_1$ and lag-2 serial correlation $r_2$ of the standardized historical data, the model's parameters $k$, $\alpha$, $p_1$ and $p_2$ are estimated from the following system of equations,

$$ s^2 = \begin{cases} k\theta^2(p_1 + \nu^2 p_1), & \nu \neq 1 \\ k\theta^2, & \nu = 1 \end{cases} \quad (4.1) $$

$$ r_2 = p_2 r_1 \quad (4.2) $$

$$ \gamma = \begin{cases} \frac{2}{\sqrt{k}} \left[ \frac{p_1 + \nu^3 p_1}{(p_1 + \nu^2 p_1)^{3/2}} \right], & \nu \neq 1 \\ \frac{2}{\sqrt{k}}, & \nu = 1 \end{cases} \quad (4.3) $$

$$ r_1 = \begin{cases} \frac{\nu(p_2 p_1 + p_1 p_2)}{\frac{p_1 + \nu^2 p_1}{p_2 + p_1}}, & \nu \neq 1 \\ \frac{(p_2 p_1 + p_1 p_2)}{(p_2 + p_1)}, & \nu = 1 \end{cases} \quad (4.4) $$

From the above formulas, we notice that when $\nu = 1$, the proposed model has Gamma distribution with shape parameter $k$ and scale parameter $\theta^{-1}$. We shall use this model for fitting the monthly standardized series of Perak river. The parameter $k$ is first obtained from equation (4.3), which then enables us to obtain the value of $\theta$ from (4.1). We next obtain the value of $p_2$ from (4.2) and thus the value of $\alpha$ can be evaluated as
\[ \alpha = 1/\theta(1-p_2). \] Finally, the value of \( p_1 \) can be calculated from (4.4) and the mean of the standardized historical data is preserved by shifting the origin of the standardized historical data by a magnitude \( c \), whose value is given by

\[ \bar{x} = k\theta(p_1 + \nu p_1') + c. \] (4.5)

Lawrence and Paul (1978) have developed an estimation procedure for dependent observations based on the minimization of a sum of squared deviations about conditional expectations. As an alternative, we shall use this approach to estimate the parameter \( p_1 \) in the proposed Gamma ARMA(1, 1) process. The conditional sum of squares \( Q_n(\cdot) \) of the process (3.25) is given by

\[
Q_n(p_1) = \sum_{t=0}^{n-1} [x_{t+1} - E(X_{t+1}|X_t = x_t)]
= \sum_{t=0}^{n-1} \left\{ x_{t+1} - \frac{k}{\alpha(1-p_2)} \left[ p_2(1-2p_1) + p_1 \right] - (p_1p_2 + p_2p_1')x_t \right\}^2,
\]

where the conditional expectation \( E(X_{t+1}|X_t) \) is given in Section 3.4.2.

By differentiating the conditional sum of squares with respect to \( p_1 \), we obtain the following relation

\[
(p_1p_2 + p_2p_1') = \frac{n\Sigma x_{t+1}x_t - \Sigma x_{t+1}\Sigma x_t}{n\Sigma x_t^2 - (\Sigma x_t)^2}, \] (4.6)

where all the sums are run from \( t = 0 \) to \( t = n-1 \). Note that this relation is slightly different from (4.4).

With known value of \( p_2 \), the value of \( p_1 \) can be evaluated from (4.6). For the purpose of preserving the mean, variance, skewness, lag-1 and lag-2 serial correlations of the standardized series, the parameters \( k, \alpha \) and \( p_2 \) are again estimated as before.
4.3 Simulation

An algorithm for generating a new Gamma ARMA(1, 1) sequence $x_1, x_1, \ldots, x_n$ of Section 3.4.2 is as follows:

**ALGORITHM**

Generate $Y_0$ from Gamma($k\phi_1, \theta^{-1}$).
Set $i = 1$.

REPEAT

Generate $G$ from Gamma($k\phi_1, \theta^{-1}$).
Generate $B$ from Beta($k\phi_1, k(1-2p_1)$).
Compute $x_i = Y_{i-1} + BG + c$.
Generate $N$ from Poisson($kp_2Y_{i-1}$).
Set sum = 0, $j = 1$.

REPEAT

Generate $k'$ from Exp($\alpha$).
Compute sum = sum + $W$, $j = j+1$.
UNTIL $j > N$.
Compute $y_i = sum + (1-p_2)G$.
Compute $i = i + 1$.
UNTIL $i > n$.

The above algorithm is written in Fortran 77 and the program is attached in Appendix 4A.

The program is executed on the HP9000/320 workstation equipped with an MC68020 microprocessor and MC68881 coprocessor. The built-in pseudo-random number generator, RAN, of HP9000 Fortran 77 is used to generate the required Uniform $\mathcal{U}(0, 1)$ random variates. The independent $G(k, \beta)$ are generated by the rejection method of Ahrens and Dieter (1974). The Poisson($\lambda$), $\lambda \leq 60$ and
the independent $\text{Exp}(\alpha)$ are generated by the widely used inverse transform method. For $\lambda > 60$, the Poisson($\lambda$) variables are generated by the Normal approximation. The Normal(0, 1) variables are generated by the ratio method of Ripley (1984), and the independent $B(m, n)$ are generated by using the result that $X = Y_1/(Y_1 + Y_2)$ is a $B(m, n)$ if $Y_1 + Y_2 \leq 1$, where $Y_1 = U_1^{1/m}$, $Y_2 = U_2^{1/n}$ and $U_1$, $U_2$ are two Uniform variates from $U(0, 1)$ (Rubinstein, 1980, pg. 81).

Table 4.1 shows the sample autocorrelation $r_k$, partial autocorrelation $\hat{\phi}_{kk}$, mean, variance and skewness for the standardized data of Perak river. From these statistics, it is clearly seen that the standardized data is skewed and with long-term correlation structure.

In the simulation study, 100 sequences of the standardized monthly stream flows $Y_t$, each of them consisting 480 data, are simulated from the fitted Gamma ARMA(1, 1) model of Section 3.4.2. For each of the 100 simulated sequences, the mean, variance, skewness and the autocorrelation coefficients $r_j$, $j = 1, 2, \ldots, 12$ are calculated. The original series $X_t$ is then obtained as

$$X_t = m_\tau + s_\tau Y_t, \quad \tau = 1, 2, \ldots, 12$$

where $m_\tau$ and $s_\tau$ are respectively the mean and standard deviation of a given month $\tau$ of the original series. The simulated series in Table 4.2 are generated by using the parameters obtained solely from the method of moments. This table gives a comparison between characteristics of the historical data and characteristics of the simulated data for both the original and standardized series.
From the results shown in Table 4.2 and Table 4.3, we observe that the simulated series bears a reasonable resemblance to the historical series in terms of means, variances, skewness and autocorrelation coefficients.

Table 4.4 compares the performance of the model by using the two alternative sets of parameters. From this table, we can observe that the sample mean, skewness and serial correlations of the simulated series (except the sample variance and the third autocorrelation coefficient) have improved slightly if the combination of moments and least squares methods is used for simulation and furthermore, the standard errors (in parentheses) for most of the statistics have also been reduced slightly, which indicates that the simulation is more stable.
Table 4.1

Sample Autocorrelation $r_k$, Partial Autocorrelation $\phi_{kk'}$

Mean, Variance And Skewness Of The Standardized Data Of Perak River.

<table>
<thead>
<tr>
<th>k</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_k$</td>
<td>0.642</td>
<td>0.546</td>
<td>0.4641</td>
<td>0.429</td>
<td>0.333</td>
<td>0.281</td>
<td>0.329</td>
<td>0.238</td>
</tr>
<tr>
<td>$\phi_{kk}$</td>
<td>0.642</td>
<td>0.229</td>
<td>0.084</td>
<td>0.093</td>
<td>-0.051</td>
<td>-0.007</td>
<td>0.171</td>
<td>-0.096</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>k</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
<th>14</th>
<th>15</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_k$</td>
<td>0.174</td>
<td>0.102</td>
<td>0.114</td>
<td>0.090</td>
<td>0.134</td>
<td>0.160</td>
<td>0.167</td>
<td>0.127</td>
</tr>
<tr>
<td>$\phi_{kk}$</td>
<td>-0.070</td>
<td>-0.076</td>
<td>0.038</td>
<td>0.032</td>
<td>0.132</td>
<td>0.052</td>
<td>0.022</td>
<td>-0.056</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>k</th>
<th>17</th>
<th>18</th>
<th>19</th>
<th>20</th>
<th>21</th>
<th>22</th>
<th>23</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_k$</td>
<td>0.120</td>
<td>0.133</td>
<td>0.091</td>
<td>0.084</td>
<td>0.059</td>
<td>0.066</td>
<td>0.108</td>
<td>0.017</td>
</tr>
<tr>
<td>$\phi_{kk}$</td>
<td>0.016</td>
<td>0.032</td>
<td>-0.053</td>
<td>-0.033</td>
<td>-0.055</td>
<td>0.003</td>
<td>0.168</td>
<td>-0.140</td>
</tr>
</tbody>
</table>

Mean = 0.0000, Variance = 1.0000, Skewness = 0.9061
Table 4.2
Mean, Variance, Skewness And Autocorrelation Coefficients rj,
\( j = 1, 2, 3, \ldots, 12 \) of The Historical Stream Flows And The
Synthetic Stream Flows Generated By The Gamma ARMA(1, 1) Model.

<table>
<thead>
<tr>
<th>monthly streamflows statistics</th>
<th>Original Data</th>
<th>Standardized Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Historical</td>
<td>Simulated</td>
</tr>
<tr>
<td></td>
<td>Simulated</td>
<td>Historical</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Simulated</td>
</tr>
<tr>
<td>Mean</td>
<td>2.732</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td>(0.192)</td>
<td>(0.161)</td>
</tr>
<tr>
<td>Variance</td>
<td>2.700</td>
<td>1.000</td>
</tr>
<tr>
<td></td>
<td>(0.451)</td>
<td>(0.154)</td>
</tr>
<tr>
<td>Skewness</td>
<td>1.559</td>
<td>0.921</td>
</tr>
<tr>
<td></td>
<td>(0.249)</td>
<td>(0.211)</td>
</tr>
<tr>
<td>( r_1 )</td>
<td>0.632</td>
<td>0.642</td>
</tr>
<tr>
<td></td>
<td>(0.039)</td>
<td>(0.055)</td>
</tr>
<tr>
<td>( r_2 )</td>
<td>0.373</td>
<td>0.546</td>
</tr>
<tr>
<td></td>
<td>(0.052)</td>
<td>(0.064)</td>
</tr>
<tr>
<td>( r_3 )</td>
<td>0.173</td>
<td>0.464</td>
</tr>
<tr>
<td></td>
<td>(0.064)</td>
<td>(0.072)</td>
</tr>
<tr>
<td>( r_4 )</td>
<td>0.064</td>
<td>0.429</td>
</tr>
<tr>
<td></td>
<td>(0.064)</td>
<td>(0.079)</td>
</tr>
<tr>
<td>( r_5 )</td>
<td>0.035</td>
<td>0.333</td>
</tr>
<tr>
<td></td>
<td>(0.063)</td>
<td>(0.083)</td>
</tr>
<tr>
<td>( r_6 )</td>
<td>0.030</td>
<td>0.281</td>
</tr>
<tr>
<td></td>
<td>(0.063)</td>
<td>(0.089)</td>
</tr>
<tr>
<td>( r_7 )</td>
<td>0.027</td>
<td>0.329</td>
</tr>
<tr>
<td></td>
<td>(0.062)</td>
<td>(0.086)</td>
</tr>
<tr>
<td>( r_8 )</td>
<td>-0.081</td>
<td>0.238</td>
</tr>
<tr>
<td></td>
<td>(0.059)</td>
<td>(0.089)</td>
</tr>
<tr>
<td>( r_9 )</td>
<td>-0.032</td>
<td>0.174</td>
</tr>
<tr>
<td></td>
<td>(0.062)</td>
<td>(0.085)</td>
</tr>
<tr>
<td>( r_{10} )</td>
<td>0.113</td>
<td>0.102</td>
</tr>
<tr>
<td></td>
<td>(0.058)</td>
<td>(0.085)</td>
</tr>
<tr>
<td>( r_{11} )</td>
<td>0.296</td>
<td>0.114</td>
</tr>
<tr>
<td></td>
<td>(0.056)</td>
<td>(0.086)</td>
</tr>
<tr>
<td>( r_{12} )</td>
<td>0.383</td>
<td>0.090</td>
</tr>
<tr>
<td></td>
<td>(0.065)</td>
<td>(0.086)</td>
</tr>
</tbody>
</table>

*Means and standard errors (in parentheses) of the 100 simulated series.
### TABLE 4.3

*Means and Standard Deviations Of Each Of The 12 Calendar Months Of The Historical And Simulated Data.*

<table>
<thead>
<tr>
<th>Month</th>
<th>Historical</th>
<th>Simulated</th>
<th>Historical</th>
<th>Simulated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jan.</td>
<td>3.612</td>
<td>3.710</td>
<td>2.090</td>
<td>2.080</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.396)</td>
<td>(0.266)</td>
<td></td>
</tr>
<tr>
<td>Feb.</td>
<td>2.230</td>
<td>2.267</td>
<td>1.122</td>
<td>1.094</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.203)</td>
<td>(0.169)</td>
<td></td>
</tr>
<tr>
<td>March</td>
<td>1.887</td>
<td>1.934</td>
<td>0.955</td>
<td>0.946</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.185)</td>
<td>(0.148)</td>
<td></td>
</tr>
<tr>
<td>April</td>
<td>2.076</td>
<td>2.102</td>
<td>0.827</td>
<td>0.798</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.174)</td>
<td>(0.121)</td>
<td></td>
</tr>
<tr>
<td>May</td>
<td>2.689</td>
<td>2.714</td>
<td>0.934</td>
<td>0.904</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.183)</td>
<td>(0.136)</td>
<td></td>
</tr>
<tr>
<td>June</td>
<td>1.950</td>
<td>1.975</td>
<td>0.675</td>
<td>0.659</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.135)</td>
<td>(0.111)</td>
<td></td>
</tr>
<tr>
<td>July</td>
<td>1.763</td>
<td>1.782</td>
<td>0.445</td>
<td>0.442</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.085)</td>
<td>(0.059)</td>
<td></td>
</tr>
<tr>
<td>Aug.</td>
<td>1.966</td>
<td>2.012</td>
<td>1.326</td>
<td>1.305</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.253)</td>
<td>(0.918)</td>
<td></td>
</tr>
<tr>
<td>Sept.</td>
<td>2.040</td>
<td>2.082</td>
<td>0.686</td>
<td>0.689</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.140)</td>
<td>(0.107)</td>
<td></td>
</tr>
<tr>
<td>Oct.</td>
<td>3.774</td>
<td>3.848</td>
<td>1.690</td>
<td>1.680</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.315)</td>
<td>(0.229)</td>
<td></td>
</tr>
<tr>
<td>Nov.</td>
<td>4.600</td>
<td>4.656</td>
<td>1.790</td>
<td>1.779</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.347)</td>
<td>(0.268)</td>
<td></td>
</tr>
<tr>
<td>Dec.</td>
<td>4.207</td>
<td>4.284</td>
<td>1.958</td>
<td>1.910</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.356)</td>
<td>(0.274)</td>
<td></td>
</tr>
</tbody>
</table>

*Means and standard errors (in parentheses) of the 100 simulated series.*
<table>
<thead>
<tr>
<th>Monthly Streamflows Statistics</th>
<th>Standardized Data</th>
<th>Simulated Data</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Historical</td>
<td>Moments method</td>
</tr>
<tr>
<td>Mean</td>
<td>0.000</td>
<td>0.040</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.161)</td>
</tr>
<tr>
<td>Variance</td>
<td>1.000</td>
<td>1.002</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.154)</td>
</tr>
<tr>
<td>Skewness</td>
<td>0.921</td>
<td>0.855</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.211)</td>
</tr>
<tr>
<td>$r_1$</td>
<td>0.642</td>
<td>0.628</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.055)</td>
</tr>
<tr>
<td>$r_2$</td>
<td>0.546</td>
<td>0.529</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.064)</td>
</tr>
<tr>
<td>$r_3$</td>
<td>0.464</td>
<td>0.445</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.072)</td>
</tr>
<tr>
<td>$r_4$</td>
<td>0.429</td>
<td>0.371</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.079)</td>
</tr>
<tr>
<td>$r_5$</td>
<td>0.333</td>
<td>0.308</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.083)</td>
</tr>
<tr>
<td>$r_6$</td>
<td>0.281</td>
<td>0.256</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.089)</td>
</tr>
<tr>
<td>$r_7$</td>
<td>0.329</td>
<td>0.212</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.086)</td>
</tr>
<tr>
<td>$r_8$</td>
<td>0.238</td>
<td>0.174</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.089)</td>
</tr>
<tr>
<td>$r_9$</td>
<td>0.174</td>
<td>0.141</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.085)</td>
</tr>
<tr>
<td>$r_{10}$</td>
<td>0.102</td>
<td>0.116</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.085)</td>
</tr>
<tr>
<td>$r_{11}$</td>
<td>0.114</td>
<td>0.090</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.086)</td>
</tr>
<tr>
<td>$r_{12}$</td>
<td>0.090</td>
<td>0.070</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(0.086)</td>
</tr>
</tbody>
</table>

*Means and standard errors (in parentheses) of the 100 simulated series.
Below are programs written for the algorithm given in Section 4.3.

PROGRAM MAIN

This program is written for generating monthly river flows using the algorithm discussed in Section 4.3. This program returns the vital statistics, such as the mean, variance, skewness, autocorrelation coefficients $r_j$, ($j = 1, 2, \ldots, 12$) and the monthly mean and standard deviation for each of the 12 calendar months of the generated series. These statistics can then be used to evaluate the performance of the proposed model.

The following subroutines/functions are directly called by this program:

Moment, stat, stat1, gama, beta, expon, poi, poisso, normal, acf

Required inputs | Corresponding parameter in (3.25)
---|---
alpha | $\alpha$
P1 | $P_1$
P2 | $P_2$
gk | $k$
v | $\nu$

imonth(i) = mean of the $\tau$-th month of the original series
isd(i) = standard deviation of the $\tau$-th month of the original series
c = magnitude of the shifted origin as given in (4.5)

Constants used in this program:
n1 = number of samples
n2 = number of data in a sample

Declarations

real month, imonth, isd
integer poisso, poi, count1, count2, count3
dimension imonth(12), isd(12), month(12), sd(12)
dimension ar(12),sdar(12),r1(12)
dimension, rmonth(100, 12), rsd(100, 12), r2(100, 12)
dimension y(601), x(601)

read *, alpha, P1, P2, gk, v, c
teta = alpha(1-P2)
do k=1,12
    read *, imonth(k), isd(k)
end do

n1=100
n2=480

write (6,994) alpha, P1, P2, gk, teta, v

betas = shape parameter of Gamma(beta2, teta)
beta1 and beta3 = parameters of Beta(beta1, beta3)

beta1=gk*P1
beta2=gk*(1-P1)
beta3=gk*(1-(2*P1))
write (6,997) beta1, beta2, beta3, teta

Initializations

ta1=0.
ta=0.
tv1=0.
tv2=0.
ts1=0.
ts2=0.
iseed=113

write (6,9999)
do l=1,n1
    y(1)=gamma(beta2, teta, iseed)
do i=2,n2+1
    b=beta(beta1, beta3, iseed)
g=gamma(beta2, teta, iseed)
x(i)=v*y(i-1)+(b*g)+c
t=P2*alpha*y(i-1)
if (t .gt. 60) then
    m=poi(t, iseed)
else
    m=poisso(t, iseed)
endif
sum=0.
do j=1,m
    w=expon(alpha, iseed)
    sum=sum+w
end do
y(i)=sum+(1-P2)*g
end do

Compute the historical series
count2=0
do while (count2 .le. n2-12)
do count1=1,12
   count3=count2+count1
   x(count3)=x(count3+1)*isd(count1)+imonth(count1)
end do
   count2=count2+12
end do

Compute and display the mean, variance and skewness of each set of the generated data. The autocorrelation coefficients $r_j$ ($j = 1, 2, \ldots, 12$), the monthly mean and standard deviation for each of the 12 calendar months are also computed; they are kept in the arrays $r_2(n_1, 12)$, $r_month(n_1, 12)$ and $r_s(d)(n_1, 12)$ respectively.

call moment(x, averx, varx, skew, r1, n2)
call stats(x, month, sd, n2)
do n=1,12
   r2(1,n)=r1(n)
   rmonth(1,n)=month(n)
   rsd(1,n)=sd(n)
end do
write (6,999),averx, varx, skew
ta1=ta1+averx
   ta2=ta2+(averx**2)
   tv1=tv1+varx
   tv2=tv2+(varx**2)
   ts1=ts1+skew
   ts2=ts2+(skew**2)
end do

Compute overall average, variance and skewness

aa=ta1/n1
   va=(ta2/n1)-(aa**2)
   sda=sqrt(va)
   av=tv1/n1
   vv=(tv2/n1)-(av**2)
   sdm=sqrt(vv)
   as=ts1/n1
   vs=(ts2/n1)-(as**2)
   sds=sqrt(vs)
write (6,995) aa, av, as
write (6,998) sda, sdm, sds
write (6,9998)

Compute and display the overall average and standard error for each of the 12 sample autocorrelation coefficients $r_j$ ($j = 1, 2, \ldots, 12$) of the $n_1$ samples.
call stats(r2, ar, sdar, n1)
do i=1,12
write (6,996) i,ar(i),sdar(i)
end do

---

Compute and display the overall average and standard error of the monthly mean and standard deviation for each of the 12 calendar months of the n1 samples.

---

write (6, 9996)
call stat1(rmonth,month,sd,n1)
call stat1(rsd,ar,sdar,n1)
do k=1,12
   write(6,9997) k,month(k),sd(k),ar(k),sdar(k)
end do

---

Formats
---

994 format (6f10.4/)
995 format (//1x,4haver,3f10.3/)
996 format (15,2f10.3)
997 format (15,3f10.3)
998 format (2x,4hs.d.,3f10.3)
999 format (15,3f10.3)
9996 format (2x,7hmonth ,6hmeans,6hs.d.,11hstand.dev. 4hs.d.)
9997 format (15,6f7.3)
9998 format (2x,4hlag,13hsamp.autocor,12h stand.devia/)
9999 format (10x,7haver ,4x,6hvar ,3x,4hskew/)
stop
end

---

SUBROUTINE momen(x,averx,varx,skew,r1,n2)
---

Subroutine momen is used to compute the mean, variance, skewness and autocorrelation coefficients \( r_j \) (\( j = 1, 2, \ldots, 12 \)) for each set of the generated data.

---

dimension x(601),r1(12)
tol1=0.
tol2=0.
tol3=0.
do i=1,n2
   to11=to11+x(i)
tol2=tol2+x(i)**2
   to13=to13+x(i)**3
end do
averx=to11/n2
varx=tol2/n2-(averx**2)
skew=to13-(3*to12*averx)+(3*to11*averx**2)
skew=skew/n2-(averx**3)
\[ var2 = \sqrt{varx}^{**3} \]

\[ skew = skew / var2 \]

\[ call acf(x, r1, averx, varx, n2) \]

\[ return \]

\[ end \]

**SUBROUTINE stat(x, month, sd, n2)**

Subroutine stat is used to compute the monthly mean and standard deviation for each of the 12 calendar months of every generated series.

- **month** = monthly mean of the generated series
- **sd** = monthly standard deviation of the generated series
- **n2** = number of data in a sample

---

```fortran
real month, t1, t2
integer count1, count2, count3
dimension month(12), sd(12)
dimension x(601)
n4=n2/12
do count1=1, 12
  t1=0.
t2=0.
count2=0
do while (count2 .le. n2-12)
  count3=count2+count1
  t1=t1+x(count3)
  t2=t2+x(count3)**2
  count2=count2+12
end do
month(count1)=t1/n4
sd(count1)=t2/n4-month(count1)**2
sd(count1)=sqrt(sd(count1))
end do
return
end
```

**SUBROUTINE stat1(r2, ar, sdar, n1)**

Subroutine stat1 is used to compute the overall average and standard error for the autocorrelation coefficients \( r_j \) \( (j = 1, 2, \ldots, 12) \) of the \( n1 \) samples. The monthly mean and monthly standard deviation for each of the 12 calendar months of the \( n1 \) samples are also computed by this subroutine.

- **ar** = average of the statistics
- **sdar** = standard error of the statistics
- **n1** = number of samples

---

```fortran
dimension r2(100, 12), ar(12), sdar(12)
do i=1, 12
  tr1=0.
  tr2=0.
do j=1, n1
```
$tr_1 = tr_1 + r_2(j,i)$
$tr_2 = tr_2 + (r_2(j,i)**2)$

end do

$ar(i) = tr_1/n_1$
$sdar(i) = (tr_2/n_1)-(ar(i)**2)$
$sdar(i) = \sqrt{sdaar(i)}$

end do

return

end

FUNCTION gama(alpha, lamda, iseed)

Function gama is used to generate the independent $G(k, \beta)$.
It uses the rejection method of Ahrens and Dieter (1974).

real alpha, lamda
m=int(alpha)
a=alpha-m
yy=1.
do i =1,m
   yy=yy*ran(iseed)
end do

yy=-alog(yy)
if (a .ne. 0.) then
   b=1.-a
   u=2.
do while (u .gt. 1.)
      u1=ran(iseed)**(1./a)
      u2=ran(iseed)**(1./b)
      u=u1+u2
   end do
   w=u1/u
   v=-alog(ran(iseed))
else
   w=0.
   v=0.
endif

\[ \text{gama} = (yy+v*w)/\text{lamda} \]

return
end

FUNCTION beta(q1,q2,ip3)

Function beta is used to generate the independent $B(m, n)$.
The Beta variates are generated by using the result
(Rubinstein, 1980, pg. 81) that if $Y_1+Y_2 \leq 1$, then

\[ X = Y_1/(Y_1+Y_2) \]

is a $B(m, n)$, where $Y_1 = u_1^{1/m}$, $Y_2 = u_2^{1/n}$ and

$u_1$, $u_2$ are two Uniform variates from $U(0, 1)$.

real init
z12=2.
do while (z12 .ge. 1)
  init=ran(ip3)
  z1=init**(1/q1)
  init=ran(ip3)
  z2=init**(1/q2)
  z12=z1+z2
end do
beta=z1/z12
return
end

FUNCTION expon(t,ip3)
  real init
  init=ran(ip3)
  expon=-alog(init)/t
  return
end

INTEGER FUNCTION poisso(lamda,iseed)
  real lamda
  c=exp(-lamda)
  b=c
  k=0
  a=ran(iseed)
  do while(a .gt. b)
    c=c*lamda/(k+1)
    b=b+c
    k=k+1
  end do
poisso=k
SUBROUTINE normal(x,e2,iseed)

SUBROUTINE normal is used to generate the independent Normal(\(\mu, \sigma^2\)) variates. It uses the ratio method of Ripley (1983).

tw=1.
do while (tw .gt. 0.)
    u=ran(iseed)
    u1=ran(iseed)
    v=(2.*u1-1.)*e2
    x=v/u
    z=(x**2.)/4.
    if (z .le. 1.-u) then
        tw=-1.
    else
        if (z .le. -alog(u)) then
            tw=-1.
        endif
    endif
end do
return
end

SUBROUTINE acf(xx,rr,rev1,rev2,ms)

SUBROUTINE acf is used to compute the 12 autocorrelation coefficients \(r_j\) (\(j = 1, 2, \ldots, 12\)) of a given series.

dimension rr(12),xx(600)
do j=1,12
    sum1=0.
    j1=ms-j
    do i=1,j1
        sum1=sum1+(xx(i)-rev1)*(xx(i+j)-rev1)
    end do
    sum1=sum1/ms
    rr(j)=sum1/rev2
end do
return
end
Conclusion

The main aim of this study is to develop suitable probabilistic models for the stochastic component of hydrologic time series, which is independent of seasonal movement and trend cycle. By means of simulation, we then generate synthetic series for the stochastic, seasonal and trend components, and combine these generated series to get an artificial series that can be used in various water resource projects.

In order to separate the stochastic component from its original series, we have attempted to use the decomposition techniques such as the classical non-parametric method, the Fourier series approach, the Census X-11 procedure and the ARIMA-model-based approach. We have also appraised the applicability of these techniques in hydrologic study. From the results obtained in Chapter Two, techniques like the Census X-II procedure and the ARIMA-model-based approach, which have been developed for decomposing economic and business time series into trend, seasonal and irregular components, are not appropriate for serving the main aim of this study.

In the second part of this study, we have proposed a first-order ARMA model that can be used to generate correlated Gamma-like processes. By comparing the simulated results of the proposed model (as shown in Tables 4.2 and 4.3) to those of the MGARMA(1, 1) model of Sim (1987a), we notice that the performance of the proposed model is not as good as the latter. However, unlike the newly proposed model, the parameters of the MGARMA(1, 1) model have to be obtained by solving a set of two
simultaneous equations, which does not always promise a solution. Therefore in practice, the proposed model can be used as an alternative to the MGARMA(1, 1) model when the restrictions discussed in Section 3.4.1 of Chapter Three are encountered in parameters estimation.