Dimikrofiskan pada 18-5-94
No. Mikrofis 11523
Jumlah Mikrofis 1

AAT-7350

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A STUDY OF A PLYWOOD PRODUCT MIX PROBLEM

by

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REFERENCE
NOT TO BE BORROWED.
320082

A Graduation Exercise presented to
the University of Malaya in
part fulfilment towards the
Degree of Bachelor of Economics with
Honours in Business Administration

PERPUSTAKAAN UNIVERSITI MALAYA KUALA LUMPUR

ABSTRACT

The purpose of this graduation exercise is to demonstrate how a plywood product mix problem can be analysed and formulated into a linear programming model.

Before dealing with the problem proper the fundamentals of linear programming technique are first introduced. This is done by applying the graphical method of linear programming to solve a simple problem which is similar to the actual plywood product mix problem. The illustrative solution is developed to demonstrate the whole process of applying linear programming techniques.

The actual production process and the types of products produced by a Malaysian plywood company are then analysed. The analysis reveals that the nature of the problem is suitable for transforming into a linear programming model. It is impractical nor advisable to incorporate all the features of the problem into the model because this would render the size of the model unduly large. A model needs to comprise only the essential features of the problem. So an exercise has to be done to eliminate the nonessentials. The review results in drastic cut in the number of variables to be included in the model.

The objective function is designed to provide ample information for decision making. It contains three parts. The first part seeks the optimal plywood mix, the second part the veneer mix and the last part the log. The objective is to maximise the difference between the revenues on the one hand and the costs on the other.

The profit maximisation effort is restricted by the resources available and the market conditions. These restrictions are analysed and formulated into sets of constraining functions. The constraints ensure that all programmes must conform to the conditions stipulated by the availabilities of material, machine capacities and the market demands for the products.

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ACKNOWLEDGEMENT

This graduation exercise is written under the supervision of Mr. Lee Moon Hoe of the Faculty of Economics and Administration, University of Malaya. The writer is very much indebted to him for his most valuable suggestions and comments without which this exercise would not have materialised. The writer is grateful to the management of the plywood company for their very helpful cooperation and assistance in providing the indispensable data and information that form the basis of this case study. Thanks are also due to many others who in one way or another have contributed to the completion of this exercise.

CHAPTER I

INTRODUCTION

"Industrial management often faces the problem of scheduling the production facilities to produce several saleable products. The usual objective is to schedule the uses of machines and raw materials so that the net profit is maximised on total production."[1] In this graduation exercise the scheduling problem in the manufacture of plywood will be analysed. There are several species of logs. Each species may be used, together with other species, to make many different types of veneers. The veneers, in turn, are used to form into the end products, namely, the different types of plywoods. There are certain capacity constraints. The transformation of logs into veneer is subject to the limited veneer production capacity. The formation of plywood panels from the veneers is restricted by the capacities of several production facilities. Moreover, the production costs are different for different types of products. In addition to these there are minimum and maximum limits for the sales of each type of product. Therefore the management must find a balanced product mix in order to obtain maximum profits.

Objective of Study

The scheduling problem will be analysed with a view to solving it with linear programming technique. The solution of a problem basically comprises three stages. Firstly, the problem is analysed and formulated into mathematical statements. Then these statements are solved either by manual procedure or by electronic computer. The final stage involves the intelligent interpretation and application of the results. The initial and final stages are obviously economic in nature; whereas, the second stage is more of mathematics than of economics. This computational aspect is often treated as the subject matter of applied mathematics or statistics. The purpose of this graduation exercise is an attempt to show how the plywood production scheduling problem can be analysed and formulated into mathematical statements for solution by linear programming, using a standard computer programme.

Research Methodology

The methodology used in this graduation exercise is largely empirical observations and personal interviews. The data and information for this exercise were collected during the 1968 long vacation. While in the field the writer kept in touch with his supervisor at the Faculty

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[1] Stanford Research Institute, Linear Programming Manual for Research Workers at Stanford Research Institute, Unpublished Paper, p. 52.

through correspondence. The factory selected for study was chosen by convenience. To get himself thoroughly acquainted with the manufacturing process the writer visited the factory many times. Preliminary discussions were held with the management personnel. After that a questionnaire (Appendix I)was drawn up and presented to the management. This was followed up by several informal meetings.

Scope of Study

This study does not purport to be comprehensive in the sense that all data and information are the results of investigations. The market constraints, for example, are not the outcome of a proper sales forecasting for the company's products. The reason for this is partly because of the lack of historical sales data and partly because of the feeling that sales forecasting is out of the scope of this graduation exercise. Yet the product market condition is vital to the problem. Consequently, to preserve reality of the study, the management expectations of the market would be adopted as the market constraints. The volume of production does not often equal to the amount sold. Thus inventory must be maintained. However, in this exercise the inventory will not be accounted for. We assume that there is no inventory. Again this omission is due to the lack of past data.

All explicit analytical methods are based upon certain assumptions. There are two central assumptions in every linear programming method. These are: linearity and certainty. Linearity means that all problem relationships can be expressed in the form of linear equations. The term 'certainty' indicates that no significant variations are expected in the numerical value for a problem factor. Difficulties are to be expected when, especially, the study is dealing with natural product - logs. Diversities in moisture contents, in lengths and sizes of logs and others are likely to cause difficulties. These difficulties will be discussed in details as they are encountered in the later chapters.

Importance of the Study

Despite these difficulties a study of this nature is of economic importance. The significance springs from the need to reduce cost. Plywood is a standardised industrial product having a competitive international market. Price is thus a very important factor to be considered in the market. A reduction in cost will benefit the industry in particular and the economy in general. It helps to enhance the export market, thereby promoting industrial growth. The use of linear programming has been demonstrated as a quick, powerful and precise technique for finding the optimum product mix thereby reducing cost. However, the usefulness of a programming solution solely depends on the accuracy and reliability of the input data. The computational aspect of linear programming is mathematical and exact. The outcome of the programming is the hard fact derived from the input. Computer does exactly what it is told to do, nothing more, and it never questions the validity of what it is told

to do.[2] Thus in order to get a realistic solution it is imperative that we provide the programmer with as accurate, realistic and reliable data as possible.

Organisation of Subsequent Chapters

This graduation exercise mainly concerns with the formulation of the production scheduling problem. Before taking up the problem proper, the fundamentals of linear programming will be introduced. A very simple problem of similar nature is solved by linear programming, using the graphical method. The illustration is given in Chapter II. It is hoped that after having read the chapter, the readers who have had not been exposed to this branch of quantitative analysis will be in a better position to grasp some of the intricacies of linear programming. The remaining chapters deal with the problem proper. The production process is described in the first part of Chapter III. There the processes of transforming logs into veneers and of forming veneers into plywood panels are described. The various types of products are analysed in the second part of the same chapter. On the objective function, Chapter IV discusses the number of variables to be incorporated into the objective function. The cost of production will be determined in the later part of Chapter IV. Without constraining functions the objective function will have no real economic meaning. Thus Chapter V analyses the various restraints such as the production capacities and resources. The efficiencies in the use of logs, in the use of veneers and in the utilisation of production facilities will be determined and formulated into constraining functions. The chapter will be ended with an analysis of the market constraints.

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[2] Chapin, N. Programming Computer for Business Applications McGraw Hill Book Co. Incorporation, New York 1961, p. 20.

CHAPTER II

SOLUTION OF AN ILLUSTRATIVE PLYWOOD PRODUCT MIX PROBLEM

There are basically two methods of using linear programming. One is the graphical method based on geometrical concept. It is simple and easy to understand. However, it is limited to two variable problems. The other, the simplex method is more general in application. It can be used for solving multi-variable problems. Modern business problems are large and complex. The solution even with the simplex method is impractical, if not impossible. The solution process is laborious and very time consuming. Management needs to make speedy and timely decisions. To meet modern business requirement, the simplex method had been revised so that large, high speed electronic computers could now be used to help management in speedy decision making. Although this exercise primarily concerns with the formulation of the production problem it appears necessary to introduce the fundamentals of linear programming before the actual problem is analysed. We have to know something about the linear programming technique in order to comprehend fully the significance of the formulation part. Linear programming is a relatively new subject. As such it has not, in the writer's belief, been widely acquainted with as yet. A very simple example will be illustrated in this chapter. Graphical method will be used as the problem is a two-product one. Certainly to those readers who have had prior grounding in linear programming, this chapter will be elementary and perhaps, boring. This chapter is not intended for them. To the other group of readers, hopefully, the simple illustration will help them to acquire some of the basic essentials of linear programming.

Illustrative Plywood Production Problem

A very small scale plywood manufacturer produces two types of end products, called plywoods X_I and X_{II} out of three types of logs, Z_1, Z_2 and Z_3. Product X_I uses logs Z_1 and Z_2 while product X_{II} uses logs Z_2 and Z_3. The production process consists of a lathe and a dryer. He has to find the product mix that he could produce in the coming planning period. The manufacturer cannot, in the short run, increase his production capacity. Let us assume that the availability of logs is fixed. The technical details regarding the uses of logs, production facilities and their efficiencies have been worked out. Besides, the product market conditions have been studied. This information is given in Table I.

The amount of resources available at the manufacturer's disposal is given in column 4 of the table. This column also shows the market constraints of the products. According to the table, to produce one unit of plywood X_I the manufacturer must use 0.8 cu. ft. of log Z_1 and 1.0 cu. ft. of log Z_2. For making one unit of plywood X_{II} he uses 2.5 cu. ft. and 3.0 cu. ft. of logs Z_2 and Z_3 respectively. Actually,

TABLE I

TECHNICAL, MARKET AND COST DATA FOR THE ILLUSTRATIVE PROBLEM

Particulars (1)	X_I (2)	X_{II} (3)	Constraints (4)
Log in Cubic feet Z_1	0.8	-	400
Z_2	-	2.5	500
Z_3	1.0	3.0	450
Production Capacity in Hour:			
Lathe	0.05	0.08	20
Dryer	0.20	0.30	60
Net Revenue in Dollars	105	160	-
Variable Cost in Dollars	85	110	-
Market Constraints X_I	-	-	250
X_{II}	-	-	130

log Z_1 is for making plywood X_I and log Z_2 for plywood X_{II}, while log Z_3 is a common raw material for making both products.

Columns 2 and 3 also show the process times required for making the plywoods X_I and X_{II} respectively. To produce unit of X_I, for instance, the manufacturer requires 0.05 hours of his lathe time and 0.20 hour of his dryer time. He expects to sell his products X_I and X_{II} at $105 and $160 per unit respectively. The variable costs of plywoods X_I and X_{II} are $85 and $110 per unit respectively. The variable cost is the cost incurred by utilizing the available capacity to produce or sell goods or services.[1] The sales forecast shows that he can sell at the stated prices 250 units of X_I and 130 units of X_{II}. With this information he shall find a solution.

Illustrative Objective Function

A solution can only be found with reference to an objective function. We shall assume that his aim is to maximise profit. To determine profit we must know the net selling prices and costs. There are two types of costs: the fixed costs and the variable costs. The former comprises items such as depreciation on capital assets, rent and

[1] Shillinglaw, G: Cost Accounting: Analysis and Control, Revised Edition, Richard D. Irwin, Inc. Homewood Illinois, 1967 p. 61.

management salaries. These costs are irrelevant to our decision here. They will not alter or reduce whatever the manufacturer may choose to produce as long as the production volume is within the present plant capacity. This is the reason why fixed costs are excluded in our analysis. From the information given in Table I we know that by producing, and selling, one unit of product X_I the manufacturer will get $105 in revenue while incurring a variable cost of $85. There is thus a net advantage of $20 per unit sold. The corresponding figures for product X_{II} are $160 in revenue, $110 in variable costs and $50 in net advantage. We shall call the net advantages marginal contributions. Since marginal contributions are net of variable costs, they are available for recovering the fixed costs. Thus maximising the marginal contributions amounts to maximising profits in the short run. Our objective, therefore, can be stated as to find the product mix of plywoods X_I and X_{II} such that the value of the marginal contributions is maximised. Put into mathematical function, we have the following as our objective function:

$$\text{to maximise } 20 x_I + 50 X_{II} \quad .... \quad (I)$$

In other words, we have to find values for the variables X_I and X_{II}, within the given constraints, so that the total marginal contributions will be maximised. This will be the optimal solution.

Illustrative Constraining Function

Without constraints the solution will be infinity. We shall begin to formulate the constraining functions by putting their relationships into mathematical equations. In graphical method of linear programming the constraining functions are plotted on a graph in accordance with geometrical concepts. These lines, rather a system of straight lines, will define an area of feasible solutions. The optimal solution must lie on the area of feasibility. The constraints are of two categories: technological and market. The area of technological feasibility will be first determined. Then we shall bring in the market restrictions to form the overall feasible area. Finally the objective function will be solved within the stated restrictions.

From Table 1, the manufacturer has enough log Z_1 to produce, in conjunction with log Z_3, 500 units of plywood X_I. In the like manner, he can produce 200 units of X_{II} with the available log Z_2. We must say that the uses of logs Z_1 and Z_2 are independent of each other. The relationships between the logs and the plywoods into which the logs can be made can be described by linear equations as follows:

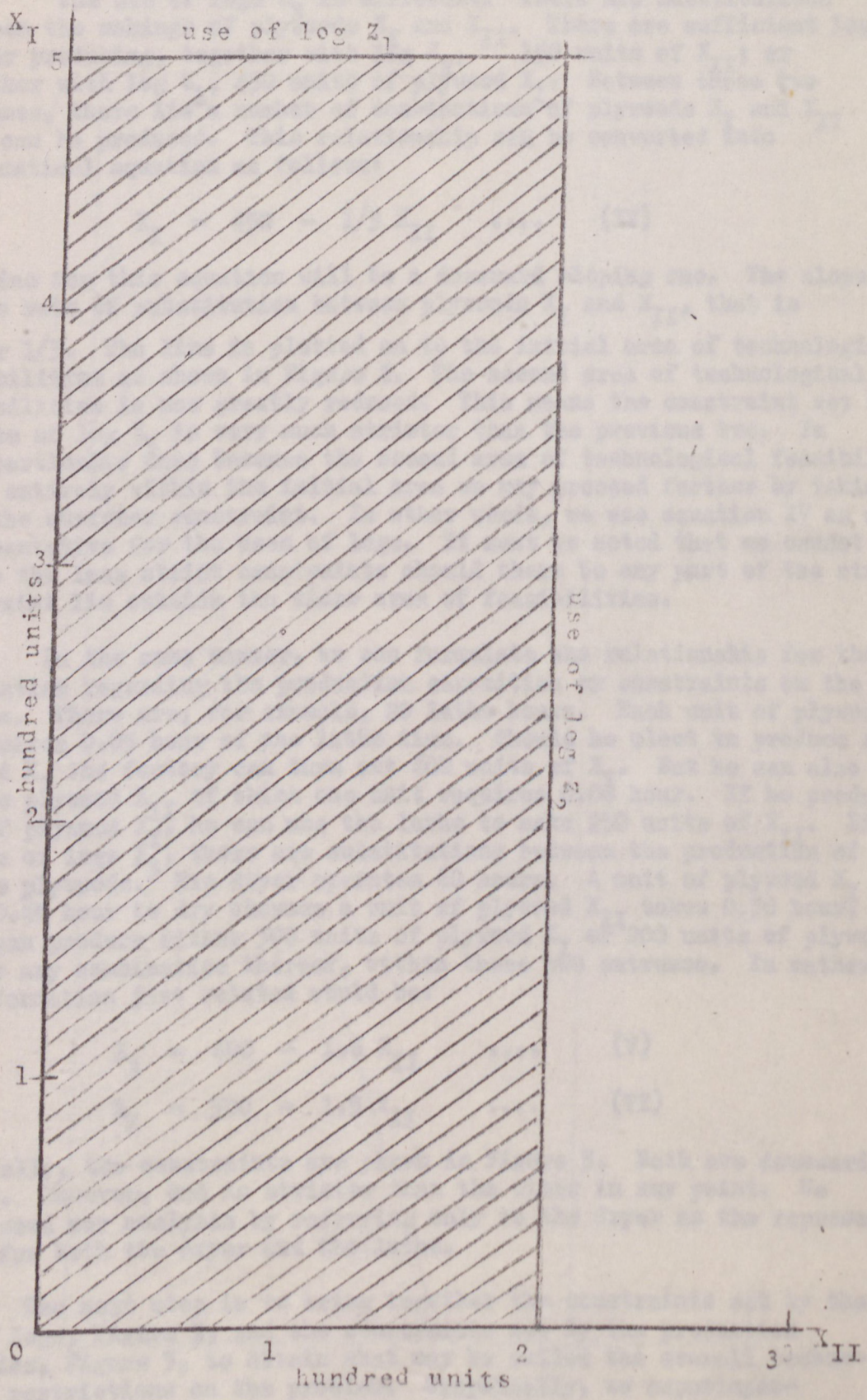
$$X_I = 500 \quad .... \quad (II)$$

$$X_{II} = 200 \quad .... \quad (III)$$

These equations when plotted on graph will be represented by straight lines as the equations are linear. The lines are shown in Figure 1. Plywood X_I is drawn on the vertical axis while plywood X_{II} on the horizontal axis. The shaded area in the figure is the initial area of technological feasibilities. Any combination of plywoods X_I and X_{II}, within

FIGURE 1

INITIAL AREA OF TECHNOLOGICAL FEASIBILITIES



the range of 500 units of X_I and 200 units of X_{II} is feasible.

The use of logs Z_3 is different. There are substitutions between the makings of plywoods X_I and X_{II}. There are sufficient logs Z_3 for producing, together with log Z_2, 150 units of X_{II}; or together with log Z_1, 450 units of plywood X_I. Between these two extremes, there lie a number of combinations of plywoods X_I and X_{II} that can be produced. This relationship can be converted into mathematical equation as follows:

$$X_I = 450 - 1/3\, X_{II} \quad .... \quad (IV)$$

The line for this equation will be a downward sloping one. The slope is the rate of substitution between plywoods X_I and X_{II}, that is $\frac{150}{450}$ or 1/3. The line is plotted on to the initial area of technological feasibilities as shown in Figure 2. The second area of technological feasibilities is now greatly reduced. This means the constraint set by the use of log Z_3 is very much stricter than the previous two. In this particular case because the second area of technological feasibilities falls entirely within the initial area we may proceed further by taking only the stricter constraint. In other words, we use equation IV as our representative for the uses of logs. It must be noted that we cannot ignore the less strict constraints should there be any part of the stricter constraint lie outside the wider area of feasibilities.

In the same manner, we can formulate the relationship for the information regarding the production capacities or constraints on the problem. There are, for example, 20 lathe hours. Each unit of plywood X_I requires 0.05 hour of the lathe time. Should he elect to produce only plywood X_I the factory can turn out 400 units of X_I. But he can also produce plywood X_{II} of which one unit requires 0.08 hour. If he produces none of plywood X_I, he can use the lathe to make 250 units of X_{II}. Like the use of logs X_3, there are substitutions between the production of the two plywoods. His dryer operates 60 hours. A unit of plywood X_I takes 0.20 hour to dry whereas a unit of plywood X_{II} takes 0.30 hour. So he can produce either 300 units of plywood X_I or 200 units of plywood X_{II}, or any combination thereof, within these two extremes. In mathematics, the information just related would be:

$$X_I = 400 - 1.6\, X_{II} \quad .... \quad (V)$$

$$X_I = 300 - 1.5\, X_{II} \quad .... \quad (VI)$$

Graphically, the constraints are shown in Figure 3. Both are downward sloping. However, one is stricter than the other in any point. We may proceed our analysis by referring only to the dryer as the representative for both the dryer and the lathe.

The next step is to bring together the constraints set by the uses of logs, Figure 2, and the constraints set by the production facilities, Figure 3, to obtain what may be called the overall technological restrictions on the problem. Graphically, we superimpose

FIGURE 2

SECOND AREA OF TECHNOLOGICAL FEASIBILITIES

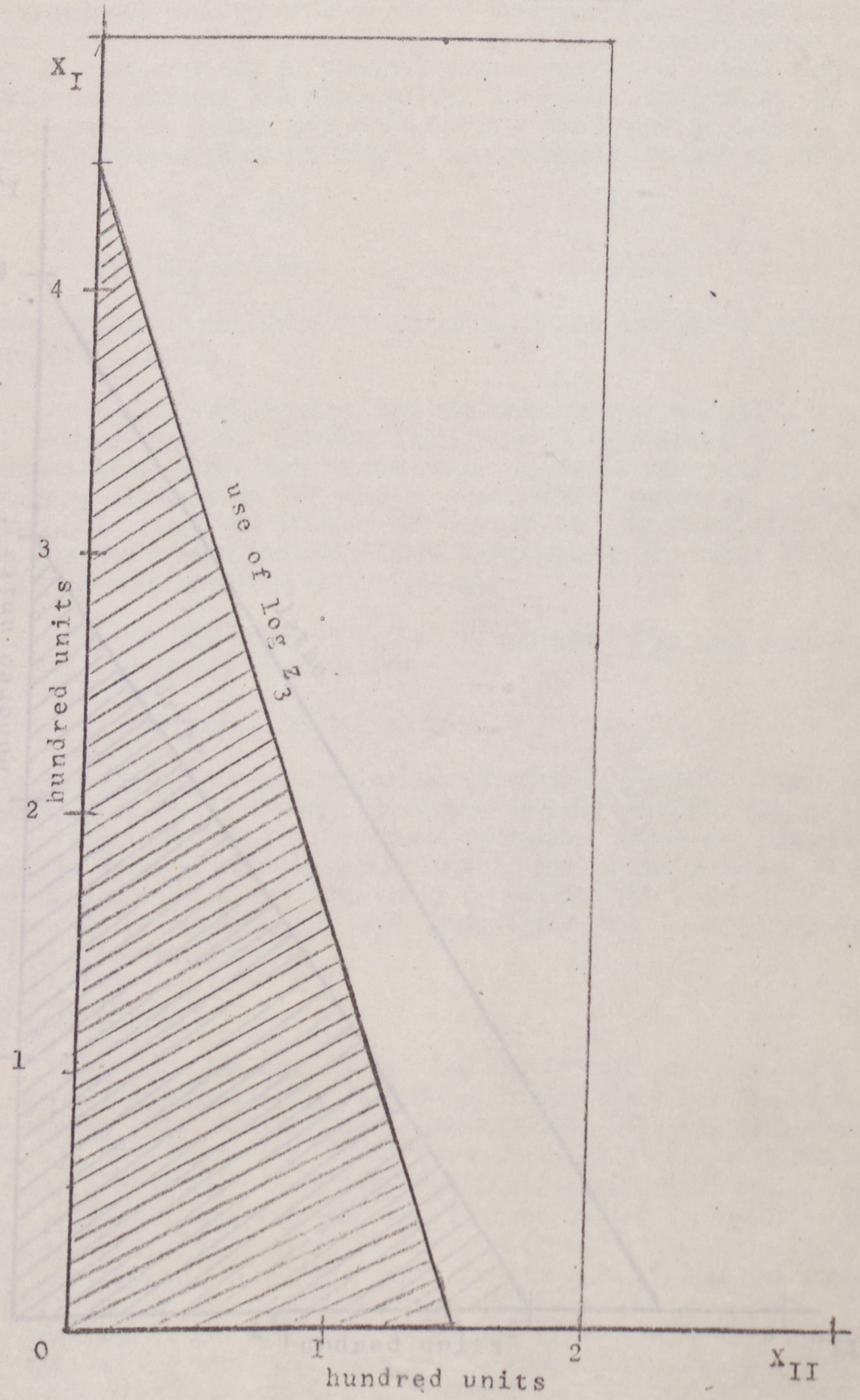


FIGURE 3

THIRD AREA OF TECHNOLOGICAL FEASIBILITIES

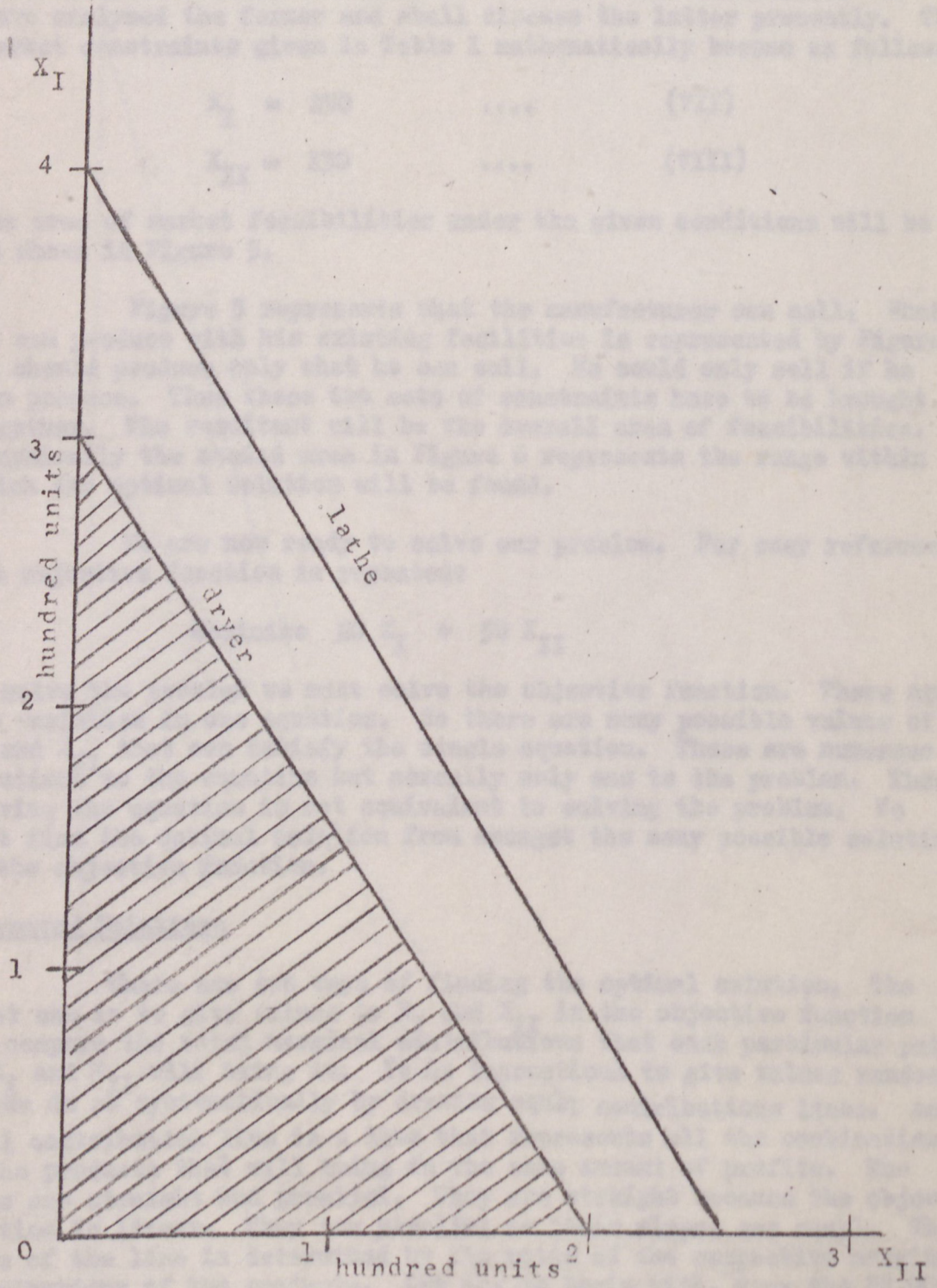


Figure 3 on to Figure 2. The shaded area in Figure 4 is the final area of technological feasibilities. This area is much smaller now. In effect we have reduced the area of choices open to the manufacturer. There are, however, still a large number of alternatives. We shall see whether the area of choices can be further reduced or not. There are the market demands to consider. Basically the restrictions faced by any manufacturer may be classified into two types: those set by technological factors and those set by the market conditions. We have analysed the former and shall discuss the latter presently. The market constraints given in Table 1 mathematically become as follows:-

$$X_I = 250 \quad .... \quad (VII)$$

$$X_{II} = 130 \quad .... \quad (VIII)$$

The area of market feasibilities under the given conditions will be as shown in Figure 5.

Figure 5 represents what the manufacturer can sell. What he can produce with his existing facilities is represented by Figure 4. He should produce only what he can sell. He could only sell if he can produce. Thus these two sets of constraints have to be brought together. The resultant will be the overall area of feasibilities. Graphically the shaded area in Figure 6 represents the range within which the optimal solution will be found.

We are now ready to solve our problem. For easy reference, the objective function is repeated:

$$\text{Maximise } 20 X_I + 50 X_{II}$$

To solve the problem we must solve the objective function. There are two variables in one equation. So there are many possible values of X_I and X_{II} that can satisfy the single equation. There are numerous solutions to the equation but normally only one to the problem. Thus solving the equation is not equivalent to solving the problem. We must find the optimal solution from amongst the many possible solutions to the objective function.

Suggested Solutions

There are two ways of finding the optimal solution. The first one is to give values to X_I and X_{II} in the objective function and compare the total marginal contributions that each particular pair of X_I and X_{II} will bring in. It is impractical to give values randomly. We can do so systematically by drawing equal contributions lines. An equal contribution line is a line that represents all the combination of the products that will bring in the same amount of profits. The lines are straight and parallel. They are straight because the objective function is linear. They are parallel as their slopes are equal. The slope of the line is determined by the ratio of the respective marginal contributions of the products. Let us, to begin with, draw the $2,000 equal contribution line. To draw a straight line we need only two

FIGURE 4

FINAL AREA OF TECHNOLOGICAL FEASIBILITIES

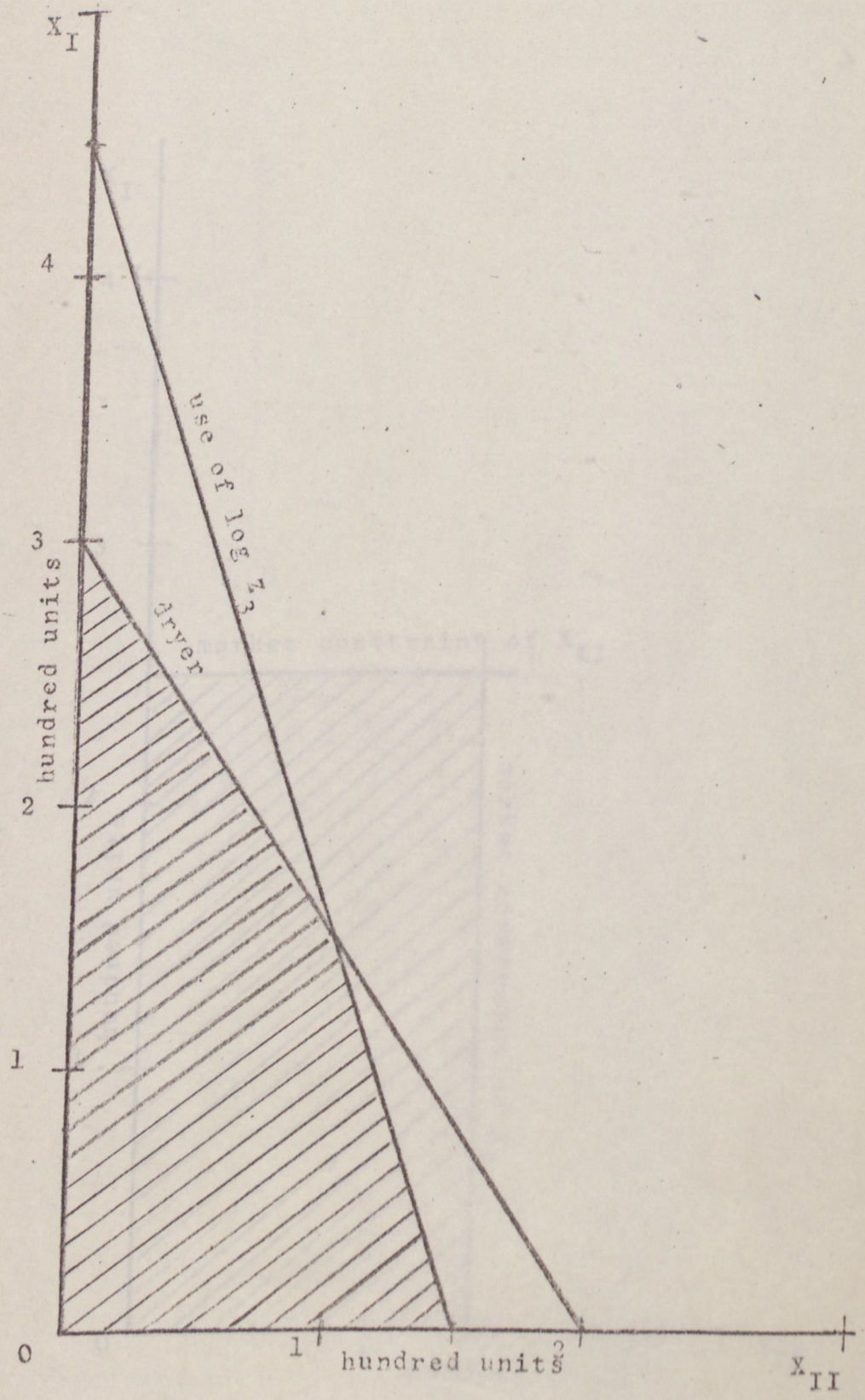


FIGURE 5

FINAL AREA OF TECHNOLOGICAL FEASIBILITIES

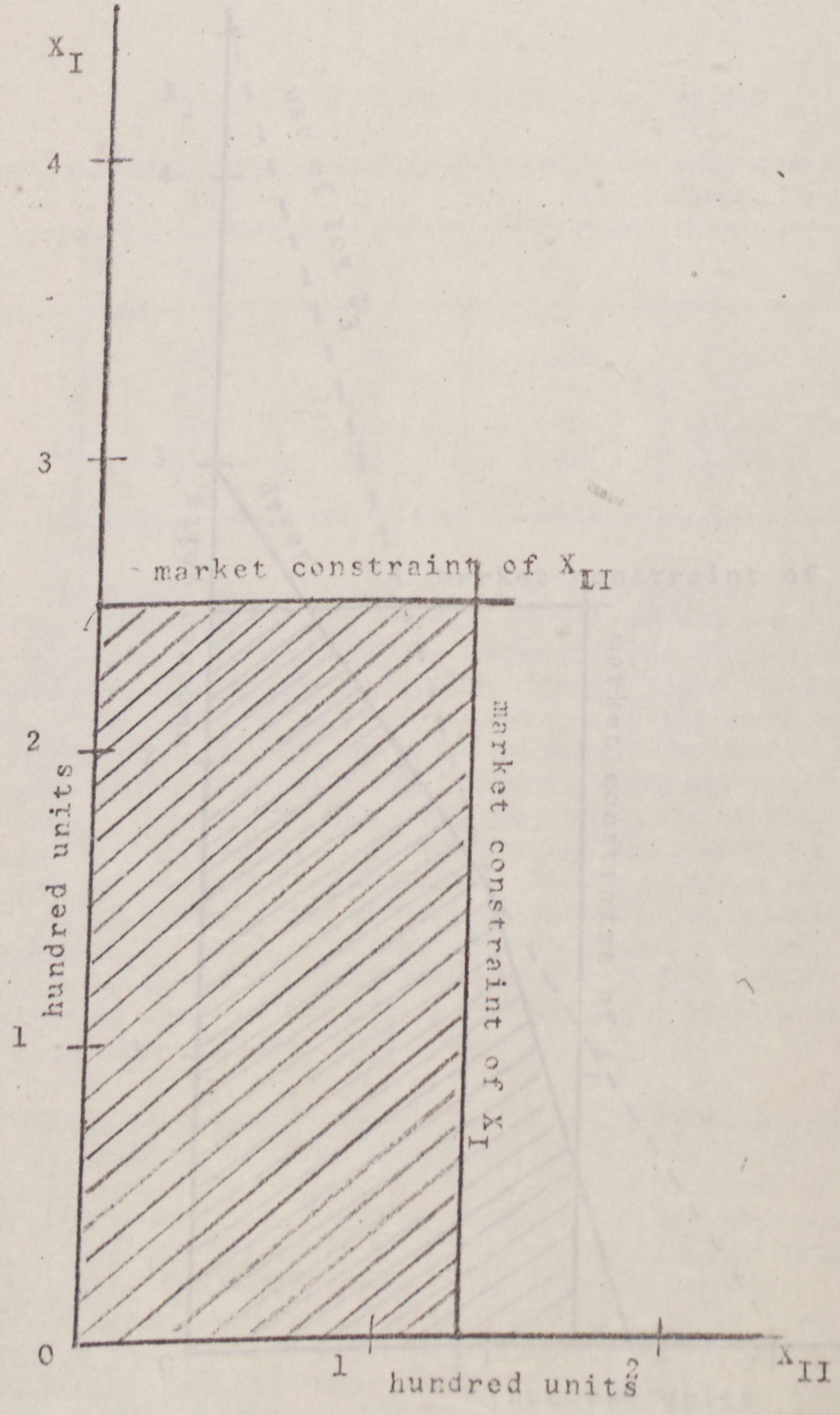
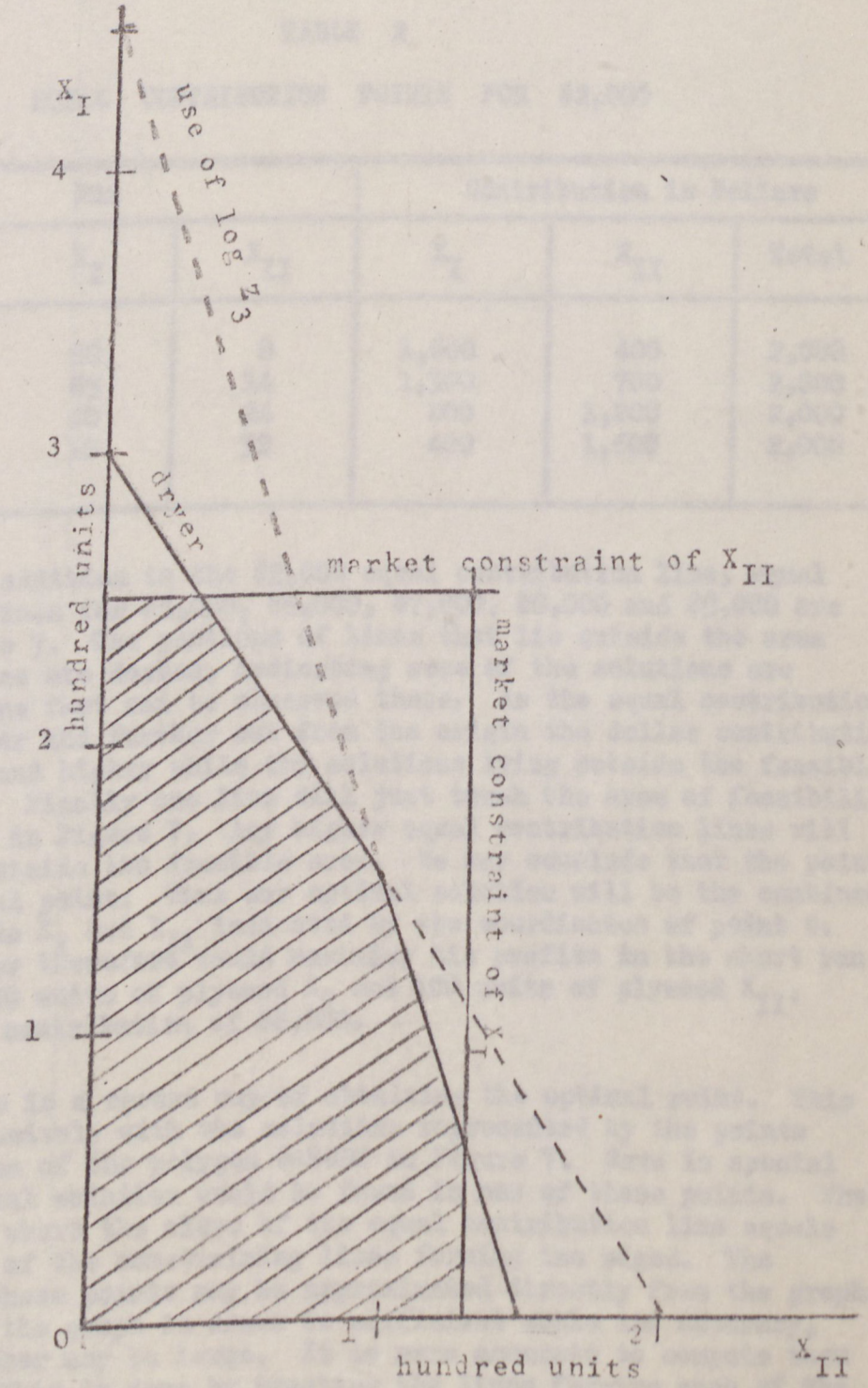


FIGURE 6

OVERALL AREA OF FEASIBILITIES



points. It is convenient to take two extreme points, one on the X_I axis the other on the X_{II} axis. Since the marginal contribution of X_I is \$20 and of X_{II}, \$50, the \$2,000 equal contribution line will fall on the one-hundredth unit and the fortieth unit on X_I and X_{II} axes, respectively. We get the equal contribution line by joining these two points. This is shown in Figure 7. Any point along this line represents the combination of plywoods X_I and X_{II} that will bring in a total contribution of \$2,000. Four of such points together with their respective total contributions are shown in Table 2.

TABLE 2

EQUAL CONTRIBUTION POINTS FOR \$2,000

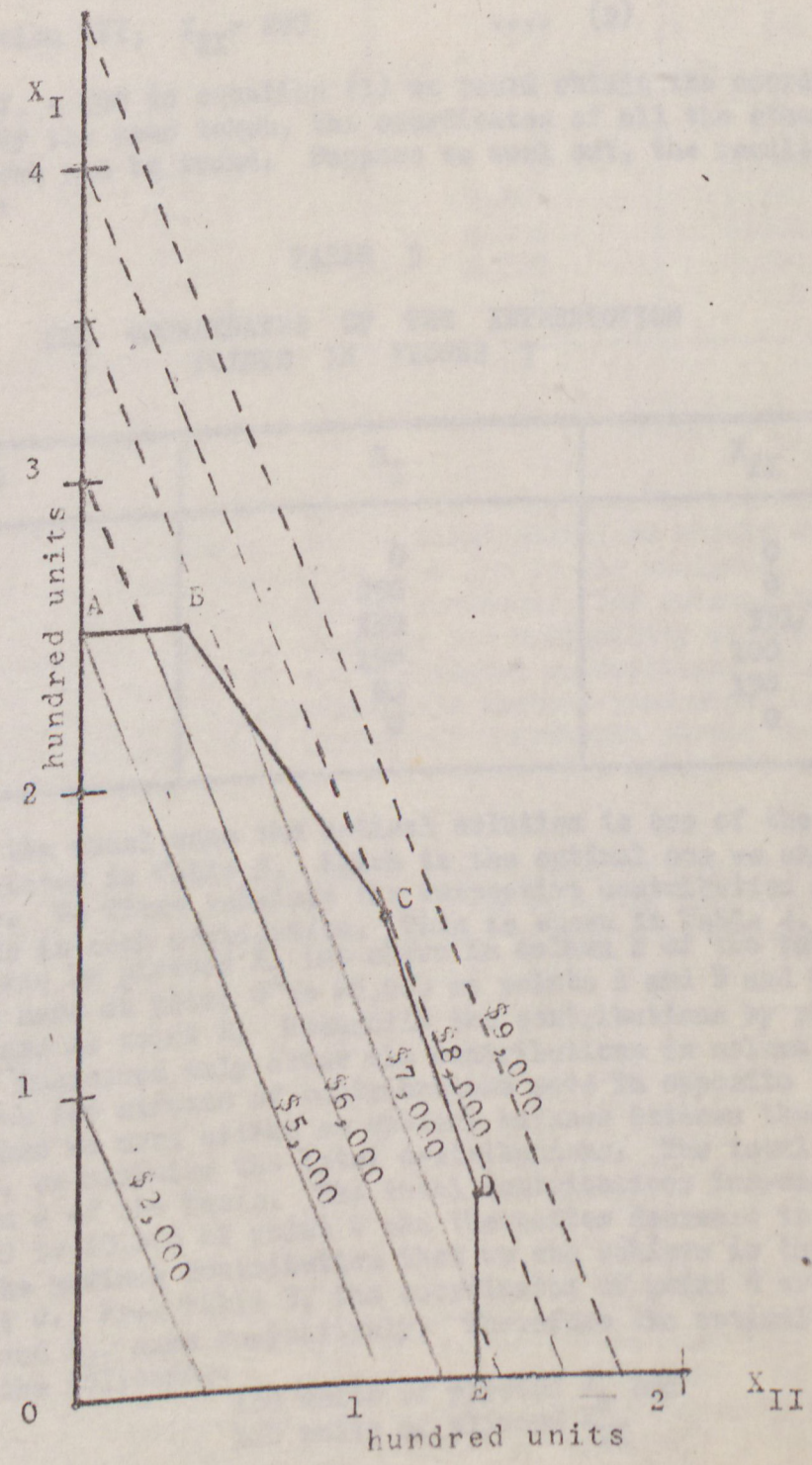
Mix			Contribution in Dollars		
No.	X_I	X_{II}	X_I	X_{II}	Total
1	80	8	1,600	400	2,000
2	65	14	1,300	700	2,000
3	40	24	800	1,200	2,000
4	20	32	400	1,600	2,000

In addition to the \$2,000 equal contribution line, equal contribution lines for \$5,000, \$6,000, \$7,000, \$8,000 and \$9,000 are drawn in Figure 7. The portions of lines that lie outside the area of feasibilities are dashed, indicating some of the solutions are infeasible. One fact can be observed there. As the equal contribution lines go further and further out from the origin the dollar contributions become higher and higher while the solutions lying outside the feasible area increase. Finally one line will just touch the area of feasibilities at the point C in Figure 7. Any higher equal contribution lines will lie entirely outside the feasible area. We may conclude that the point C is the optimal point. Thus our optimal solution will be the combination of plywoods X_I and X_{II} indicated by the coordinates of point C. The manufacturer therefore would maximise his profits in the short run by producing 150 units of plywood X_I and 100 units of plywood X_{II}, making a total contribution of \$8,000.

There is a second way of obtaining the optimal point. This is to work exclusively with the solutions represented by the points forming the edges of the polygon OABCDE in Figure 7. Save in special cases, the optimal solution would be found in one of these points. The special case is where the slope of the equal contribution line equals that of any one of the constraining lines forming the edges. The coordinates of these points may be approximated directly from the graph. However, unless the graph is drawn to sufficient scale and accuracy, the margin of error may be large. It is more accurate to compute them algebraically. This is done by treating the lines forming each of the

FIGURE 7

THE EQUAL CONTRIBUTION LINES



edges as pairs of simultaneous equations. The point is obtained on solving the pairs of equations. For example, the coordinates for point B is the point of intersection of the lines pertaining to the market constraint for plywood X_I and the dryer capacity. To find the coordinates of this point, we solve as simultaneous equations, equations VI, and VII mentioned earlier. In other words, we solve:

$$\text{Equation VI,} \quad X_I = 300 - 1.5\, X_{II} \quad \ldots\ (1)$$

$$\text{Equation VII,} \quad X_{II} = 250 \quad \ldots\ (2)$$

Substituting $X_{II} = 250$ in equation (1) we would obtain the coordinates of point C. By the same token, the coordinates of all the other points of intersections can be found. Suppose we work out, the results will be as follows:

TABLE 3

THE COORDINATES OF THE INTERSECTION POINTS IN FIGURE 7

Point	X_I	X_{II}
O	0	0
A	250	0
B	250	331/3
C	150	100
D	60	130
E	0	0

In the usual case the optimal solution is one of the combinations listed in Table 3. Which is the optimal one we shall now proceed to see. We first tabulate the respective contribution given by the products in each combination. This is shown in Table 4. The contributions by plywood X_I (as shown in column 2 of the table) increases from zero at point O to $5,000 at points A and B and then decreases to zero at point E. Meanwhile the contributions by plywood X_{II} (column 3) increases only after the contributions in column 2 are decreasing. The two streams of contributions move in opposite directions. Thus we must strike an optimal balance between these two. In other words, we maximise the total contributions. The total is given in column 4 of the Table. The total contributions increase from zero at point O to $8,000 at point C and thereafter decrease to zero at point E. The maximum contribution that we can achieve is thus $8,000 at point C. From Table 3, the coordinates of point C are 150 and 100 on X_I and X_{II} axes respectively. Therefore the optimal solution is to produce the following:

150 units of plywood X_I and
100 units of plywood X_{II}

This agrees with our solution earlier.

TABLE 4

MARGINAL CONTRIBUTIONS AT INDICATED POINTS IN FIGURE 7 (IN DOLLARS)

Point (1)	X_I (2)	X_{II} (3)	Total (4)
O	0	0	0
A	5,000	0	5,000
B	5,000	1,667	6,667
C	3,000	5,000	8,000
D	1,200	6,500	7,700
E	0	0	0

Conclusion

The manufacturer should, before accepting the solution, review and reevaluate three factors. In the first place, he should examine the realism of the assumptions made thus far in the analysis. The treatment of the input data should be reviewed. The outcome of the analysis is dependent upon the accuracy and reliability of the input information and the reality of the underlying assumptions. Finally he must evaluate all the non-quantifiable factors that might have some bearings on the problem. This quantitative analysis should be treated more as a guide to management decision rather than accept it as conclusive finding.

We have seen an illustrative application of linear programming technique. From this very simple example we observed that the solution process for a production scheduling problem essentially comprised the setting up of a system of linear equations. There is an objective function with which we measure the effectiveness of the production system. The coefficients of the objective functions are the respective marginal contributions of the products. Thus we need to know the net revenue and variable costs of the products. Production factors are limited. This is indicated by the constraining functions. The area of choices is narrowed down by these restrictions. The system of mathematical functions constitutes a model of the production problem. To solve the problem we solve the system of functions. Graphically, we define our area of feasible solutions by plotting the constraining functions on the basis of geometrical concept. Within this area of feasibilities we optimise the value of the objective function. Before concluding this Chapter one important point must be noted. In our illustration in this Chapter we have assumed no idle capacity in the production facilities. This is not always the case.

CHAPTER III

THE PRODUCTION PROCESS AND THE PRODUCTS

We have demonstrated the linear programming technique by applying it in solving a very simple plywood product mix problem. The solution requires the setting up of an objective function and a set of constraining functions, both technological and market. To formulate the objective function we determine the kinds of products, ascertain their revenues and production costs. For the constraining functions we have to have to develop the efficiency factors of the production facilities and the availabilities of raw materials. The production problem we are going to analyse in this paper is large and somewhat complex. There appears to be as many as 1,052 variables and 13 production stages. The problem in reality will be beyond the capacities of many but the largest electronic computers. Thus it is necessary to trim the problem down to more manageable proportions. This is done by grouping and simplifying the products and the production facilities. Had time and facility permitted, the problem could be extended to incorporate a wider range of activities.

Production Process

To begin with, we shall describe the production process. This will be followed by an analysis of the factory's products. The factory to be studied, will be refered to as "the factory" throughout this exercise, was selected by convenience. It is a small-sized factory with an annual maximum capacity of 40 million square feet of surface measurement in 4 millimetres. The end products of the factory - plywoods - are made from logs which are first turned into veneers. The plywoods consist of three to five plies of veneers bonded with resin or glue. The whole production process comprises 13 activities and can be conveniently divided into two major departments:-

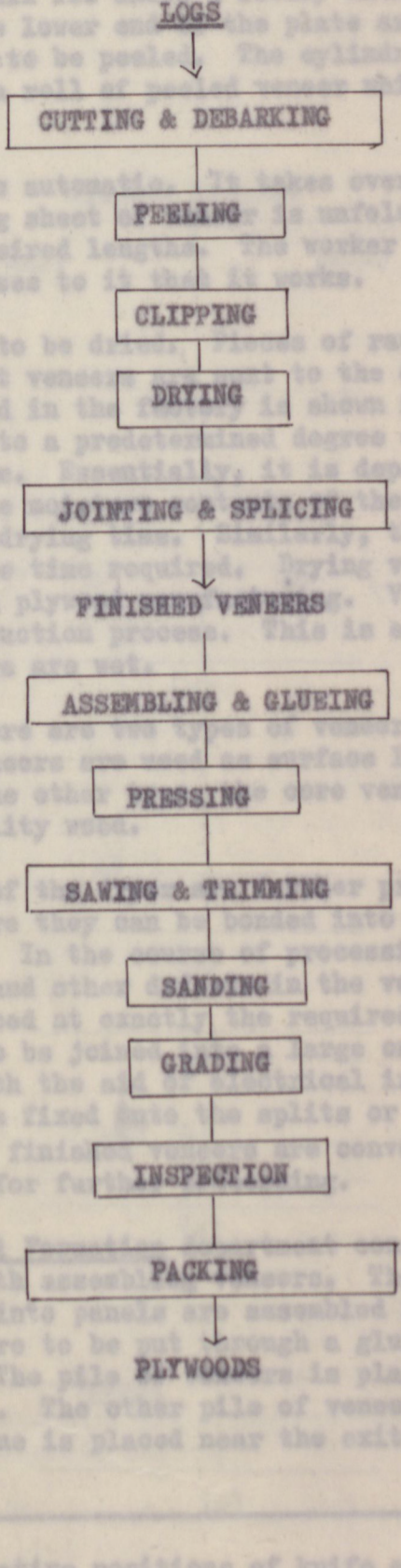
(1) Veneer Production
(2) Panel Formation

Figure 8 shows the production system in a schematic form. The process begins with log-cutting and ends with packing the final products.

Veneer Production begins with the process of cutting logs. Logs stored in the logyard are first brought into the cutting shed. There the logs are cut into the required lengths. Then they are debarked, that is, the barks of the round timbers are knocked off from the wood. The debarked logs are sent to the rotary lathe for peeling. The log for peeling is clamped on to the "rotor" against a knife

FIGURE 8

SCHEMATIC OUTLINE OF THE PRODUCTION PROCESS



running the whole length of the wood.[1] As the log is rotated, it is cut into continuous sheet of fine veeneer, ranging from 0.9 to 4.4 millimetres thick. At the receiving end of the lathe, there is a power-driven reeling device which rolls up the veneer as the lathe peels the log. Plate 1 in Appendix III shows a rotary lathe similar to the one used in the factory. In the lower end of the plate are the debarked logs waiting for their turnsto be peeled. The cylindrical object at the top of the picture is a roll of peeled veneer which is being taken over by the clipper.

The clipper is automatic. It takes over the peeled veneer from the lathe. As the long sheet of veneer is unfolded, the clipper cuts the veneer into the desired lengths. The worker at the machine merely sets the clipper and sees to it that it works.

Veneers have to be dried. Pieces of raw veneers are dried in a steam dryer. The cut veneers are sent to the dryer from the clipper. An identical dryer used in the factory is shown in plate 2 Appendix III. The veneers are dried to a predetermined degree of dryness. The drying time for veneers varies. Essentially, it is dependent on two factors: the thicknesses and the moisture contents of the veneers. The thicker the veneer the longer the drying time. Similarly, the higher the moisture contents the longer the time required. Drying veneers poses, perhaps, the biggest problem in plywood manufacturing. Very often it forms the bottleneck in the production process. This is especially so during rainy seasons when logs are wet.

Basically there are two types of veneers. One type is finer in texture. These veneers are used as surface layers in the formation of plywood panels. The other type, the core veneers are thicker and usually, of lower quality wood.

The outputs of the dryer are further processes to form into plywood panels. Before they can be bonded into panels there are certain "repairs" to be made. In the course of processing there are bound to be certain splits and other defects in the veneers. Also some core veneers are not produced at exactly the required sizes. Two or more smaller pieces have to be joined into a large one. The repairings and jointings are done with the aid of electrical irons. Adhesive tapes are used. The tape is fixed onto the splits or joints and then pressed with the heated iron. The finished veneers are conveyed to the panel formation department for further processing.

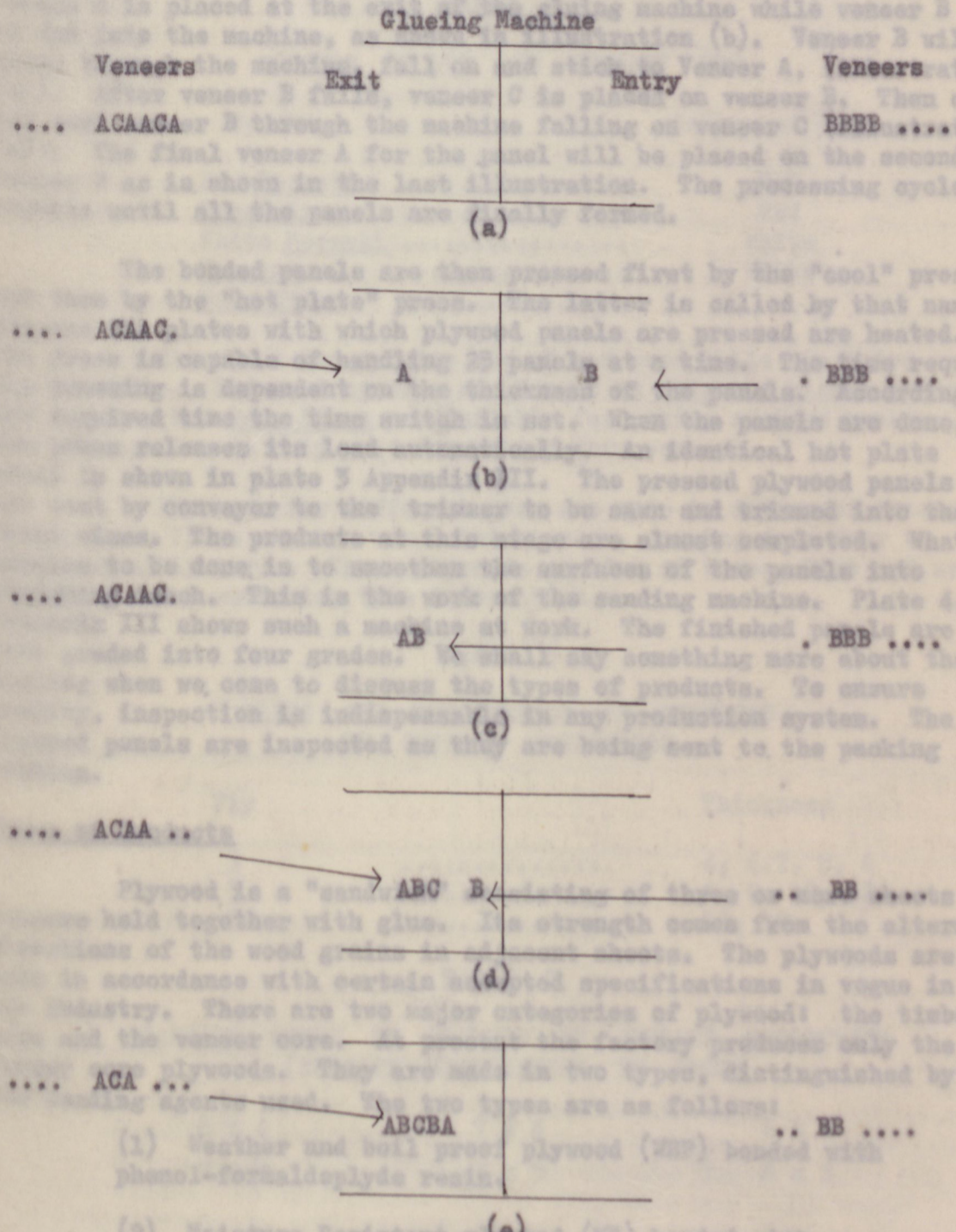
Plywood Panel Formation department consists of seven production stages. It starts with assembling veneers. The veneers that are required for forming into panels are assembled into two piles. The veneers in one pile are to be put through a gluing machine to be spreaded with glue. The pile of veneers is placed at the feeding side of the gluing machine. The other pile of veneers which need not be put through the machine is placed near the exit of the machine. To

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[1]For the relative positions of knife and log see Appendix II.

FIGURE 9

ILLUSTRATION OF THE GLUEING PROCEDURE



illustrate the gluing process, let us take an example of bonding, say, five-ply panels composing of veneers A, B, C, B and A, in that order. In this case we need only to glue both sides of the two veneer B. Thus all veneers type B will be assembled in one pile. The other veneers will be assembled in the order of A, C and A in the other pile. The latter will be placed near the exit of the gluing machine. Figure 9 gives a graphical illustration of the gluing procedure.

In illustration (a) the veneers are assembled into two piles and placed in their respective positions. The gluing process begins. Veneer A is placed at the exit of the gluing machine while veneer B is fed into the machine, as shown in illustration (b). Veneer B will, going through the machine, fall on and stick to Veneer A, (illustration (c)). After veneer B falls, veneer C is placed on veneer B. Then comes the next veneer B through the machine falling on veneer C (illustration (d)). The final veneer A for the panel will be placed on the second veneer B as is shown in the last illustration. The processing cycle repeats until all the panels are finally formed.

The bonded panels are then pressed first by the "cool" press and then by the "hot plate" press. The latter is called by that name because the plates with which plywood panels are pressed are heated. The press is capable of handling 25 panels at a time. The time required for pressing is dependent on the thickness of the panels. According to the required time the time switch is set. When the panels are done, the press releases its load automatically. An identical hot plate press is shown in plate 3 Appendix III. The pressed plywood panels are sent by conveyor to the trimmer to be sawn and trimmed into their final sizes. The products at this stage are almost completed. What remains to be done is to smoothen the surfaces of the panels into finishing touch. This is the work of the sanding machine. Plate 4, Appendix III shows such a machine at work. The finished panels are then graded into four grades. We shall say something more about the grading when we come to discuss the types of products. To ensure quality, inspection is indispensable in any production system. The plywood panels are inspected as they are being sent to the packing section.

Types of Products

Plywood is a "sandwich" consisting of three or more sheets of veneers held together with glue. Its strength comes from the alternating directions of the wood grains in adjacent sheets. The plywoods are made in accordance with certain accepted specifications in vogue in the industry. There are two major categories of plywood: the timber core and the veneer core. At present the factory produces only the veneer core plywoods. They are made in two types, distinguished by the bonding agents used. The two types are as follows:

(1) Weather and boil proof plywood (WBP) bonded with phenol-formaldeplyde resin.

(2) Moisture Resistent plywood (MR) bonded with Urea-formaldeplyde resin.

The latter is a general purpose plywoods while the former is used mostly in engineering constructions. Some of the popular uses of moisture resistant plywoods are for: wall panels, ceilings, furniture, handicraft and tea-chest. The weather and boil proof panels are commonly used for yacht building and concrete shuttering. WBP plywoods are sold at slightly higher prices than MR plywoods.

Plywoods are made from various species of logs. The common local species are as follows:

TABLE 5

SPECIES OF LOG IN CURRENT USE

Species	Colour of Wood
Red Meranti	Red
Mengkulang	Red
White Meranti	White
Mersawa	White
Dama Hitam	-

The first two species are of red wood while the third and fourth are of white. Red wood species are bought at slightly higher prices. All species but Dama Hitam are suitable for making surface veneers. The odd species is only good for making core veneers.

Plywoods for different uses are made in different thicknesses and sizes. There are ten usual thicknesses, ranging from four to eighteen millimetres. These plywoods consist of three, five and seven plies. Tables 6 and 7 summarise this information.

TABLE 6

PLIES AND THICKNESSES OF PLYWOODS IN CURRENT PRODUCTION

Ply	Thickness
3	4, 4.7, 5, 6
5	8, 9, 10, 12
7	15, 18

TABLE 7

STANDARD SIZES OF PLYWOOD IN CURRENT PRODUCTION
(SURFACE MEASUREMENT IN FEET)

8 x 4	7 x 4	6 x 4
8 x 3	7 x 3	6 x 3

In an earlier section we said that the plywoods are graded into four grades. We shall now say more about the grading. The basis for grading is the quality of the face, core and back veneers. The quality of the veneers is judged on the absence of knots and splits on the surface of the veneers. Most of the current outputs of the factory are graded as second grade. Very little attains first grade. Third and fourth grades are few in number. These are not exportable. They are sold in the domestic market.

As to the sizes of the plywoods, the factory produces any size to order. However, we can identify six common sizes. We shall call them the standard sizes. These are shown in Table 7.

Thus the final products of the factory can be classified or divided into numerous types and kinds. They can be classified according to the bonding agents used, colours of the surface veneers, and so on. Table 8 summarises these classification factors. Column 1 of the Table listed the bases with which the plywoods can be classified. The number of product classifications that each of the given bases can classify the products into, are given in column 2. The resulting total numbers of product classifications are given in the final column. For example, the bonding agent divides the factory's products into two types: MR and WBP plywoods. Each of them, in turn, can be divided into red and white wood surfaced panels. Thus these two bases together classify the products into four types as follows:-

(1) Moisture resistant red
(2) Moisture resistant white
(3) Weather and boil proof red
(4) Weather and boil proof white

TABLE 8

SUMMARY OF THE CLASSIFICATION FACTORS

Basis (1)	No. (2)	Total (3)
Bonding Agent	2	2
Colour of Surface Veneer	2	4
Thickness	10	40
Grade	4	160
Size	6	960

Each of the four types can be, in turn, subdivided into more kinds and types of plywoods. Therefore if we continue dividing and subdividing, the five listed bases would eventually divide the products into 960 different types. To incorporate all of them into our model would inevitably render the problem very bulky. It is necessary to reduce the number of variables, that is the number of types of products, in order to formulate a more handy and workable model.

CHAPTER IV

THE OBJECTIVE FUNCTION

In the simple illustration in Chapter II the objective function was given as: maximise $20 X_I + 50 X_{II}$. This simple form is quite adequate for a simple illustration for a plywood product mix problem. For practical purposes it appears, however, insufficient. We need more information about the optimal product mix than the simple form of the objective function can provide. The simple form only shows the product mix in terms of the plywoods, whereas for decision making the management needs to know about, among other things, the numbers of different types of veneers to be produced and the costs associated with this production. The second reason for modifying the form is that the modified form would help, to a certain extent, in achieving more realistic costings.

It might be recalled that during the analysis of the production process, we divided the system into two departments: the veneer production and the panel formation. Thus in the first place we can form our objective function along this line. The former can be further divided into two parts so that the uses of logs in producing veneers could be separately accounted for. Cost of logs amounts to about forty per cent of the total variable production costs. Apart from the significance of this item, the separation of the cost of logs from the costs of producing veneers, reduces the difficulty in estimating costs. The modified objective function therefore comprises three parts. The first part relates to the types of plywoods, the second to the types of veneers and the third to the types of logs.[1] Accordingly, the variable production costs would be separately ascertained. These together with the net revenues that will be derived from selling the plywoods constitute the basis from which the coefficients of the objective function would be obtained.

We shall briefly examine the various parts of the objective function. The first part relating to the plywoods is as follows:-

$$\text{Maximise } (k_{I1} - l_{I1})x_{I1} + (k_{I2} - l_{I2})x_{I2} + \ldots + (k_{aj} - l_{aj})x_{aj}$$

where k_{aj}, l_{aj} are the net revenues and the variable costs, respectively, of per unit of plywood type X_{aj}. The sum of this series will be the total contributions towards the recovery of costs of producing veneers, the costs of logs and the various fixed costs. This part is basically the same as the simple form except that the net revenue and variable

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[1] Stanford Research Institute, op. cit. p. 54.

costs per unit of plywood are stated explicitly and that the costs of veneers and logs are excluded. To account for the costs of producing veneers and logs we would have two more parts.

The second part of the objective function is concerned with the types of veneers and their costs. It has the following form:

$$(m_{I1} y_{I1} + m_{I2} y_{I2} + m_3 y_{13} + \ldots + m_{bk} y_{bk})$$

where M_{bk} is the variable cost of producing per unit of veneer type Y_{bk}. The sum of this series will be the total costs of producing the veneers that are required to produce the plywoods. To maximise the profits we must attempt to keep the costs at its minimum. Thus our partial objective function will be a minimising function. In order to fit it into the whole objective function we need to convert this into a maximising function. To do this we merely add in a negative sign to the partial objective function. By maximising a negative value is equivalent to minimising a positive value. This is analoguous to adding a negative value to a given amount. To take a very simple example:

$3+(-2) = 3-2$. Similarly, maximise $(-x)$ = minimise x

As we have excluded the logs from the veneer production in second part of the objective function, the third part will be as follows:

$$(n_1 z_1 + n_1 z_2 + n_3 z_3 + \ldots n_c z_c)$$

where n_c is the unit cost of log type Z_c.

Just like the second partial objective function, the third partial objective function sums up the log costs related to the optimal product mix. To maximise profits we have to minimise costs. And this is done by maximising the negative value of the series. Therefore our objective function will be to maximise the combined value of these three partial objective functions.

The information and data required for the formulation of our objective function can be divided into two categories. The first concerns with the number of variables to be incorporated into the objective function. These will be the types of plywoods, veneers and logs. The second category is required for ascertaining the coefficients of the variables. The first coefficients are derived from the net selling prices and the various production costs. We shall approach the formulation along this line. First we determine the numbers of types of plywoods, the types of veneers andtypes of logs, the last two resulting from the determination of the number of types of plywoods. Thereafter, we shall proceed to ascertain the various production costs as well as the net revenues.

Simplified Product Classifications

In the preceding Chapter the production process has been described and the types of products analysed. We shall now try to reduce the number of types of products as the number previously arrived

at is too great. If we were to incorporate the original number of variables the problem definitely would be too huge for many but the largest electronic computers to handle. It is thus necessary to formulate a model which is much smaller yet sufficiently representative of the actual problem.

We saw in Chapter III how the five classification bases divided the products into 960 different types. Referring to Table 8, we could see that the products were first classified according to the types of glue used. We divided the products into two types namely the moisture-resistant and the weather-and-boil-proof plywoods. Incidentally, the proportion of the latter is relatively small in the current production. To simplify the problem we may ignore this classification and thus regard the factory's outputs as moisture-resistant panels. The second classification basis is the colour of the surface veneers. The surface veneers are made from either red or white wood. There are differences in the prices and costs between the red surface and white surface plywoods. Of importance is the fact that the proportions of the two products in current production are nearly equal. It is thus important that we differentiate them. In other words, we should continue to treat them as distinct products. The third factor, the differences in the plywoods' thicknesses cannot, in any way, be dispensed with. The products are most markedly distinguished by this factor. We shall continue to regard plywoods of different thicknesses as different products. As to gradings, it appears that the bulk of the factory's outputs are graded as grade two. Plywoods of the best grade as well as poorer grades form only a small proportions of the total current output. With the present quality control it is unlikely that the standard of the products would not be maintained. Thus we shall not distinguish the grades of the plywoods. The size of the panels, we can avoid this classification by expressing the products in square feet of surface measurement. For example an 8 by 4 feet plywood can be expressed as 32 square feet of surface measurement. In short, we shall classify the products according to two instead of five bases. These two are: on the basis of the surface colour and on the basis of the product's thicknesses. By this simplification we would manage to reduce the number of types of products from 960 to only 20. It is likely that a certain amount of profits may be lost as a result of this simplification. But it is unavoidable since we need to scale down the magnitude of our model. The number of types of plywoods together with their symbols that will be used in the problem are given in Table 9.

Thus we shall divide the factory's products into the red surfaced and white surfaced panels. Each of these is, in turn, divided into ten different thicknesses. The former will be represented by X_I and the latter by X_{II}. The small Arabic numerals subscripts refer to the panel's thickness indicated in Table 9. For instance the 5 mm white surfaced plywood will be known as X_{II3}, the 12 mm red surfaced panel as X_{I8}.

TABLE 9

TYPES OF PLYWOODS

Thick	Surface Colour		Thick	Surface Colour	
MM	Red	White	MM	Red	White
4	X_{I1}	X_{III1}	9	X_{I6}	X_{II6}
4.7	X_{I2}	X_{II2}	10	X_{I7}	X_{II7}
5	X_{I3}	X_{II3}	12	X_{I8}	X_{II8}
6	X_{I4}	X_{II4}	15	X_{I9}	X_{II9}
8	X_{I5}	X_{II5}	18	X_{I10}	X_{II10}

Types of Veneers

The number of types of veneers is directly related to the plywood classifications that we have discussed earlier. The types of veneers required to form the various types of plywood panels are given in the veneers composition table. An analysis of the table will provide us with an idea about the number of types of veneers. The veneers composition table is as follows:

TABLE 10

VENEERS COMPOSITION TABLE

Ply	MM	Veneers (MM)
3	4	0.9 + 2.5 + 0.9
	4.7	1.3 + 2.5 + 1.3
	5	1.0 + 3.4 + 1.0
	6	1.0 + 4.4 + 1.0
5	8	1.0 + 2.5 + 1.8 + 2.5 + 1.0
	9	1.0 + 2.5 + 2.6 + 2.5 + 1.0
	10	1.0 + 2.5 + 3.8 + 2.5 + 1.0
	12	1.0 + 3.4 + 3.8 + 3.4 + 1.0
7	15	1.0 + 2.5 + 3.2 + 2.5 + 3.2 + 2.5 + 1.0
	18	1.0 + 3.4 + 3.2 + 3.4 + 3.2 + 3.4 + 1.0

A perusal of the above Table shows that the veneers are of

ten different thicknesses. These veneers can be classified, on the basis of the purposes, into two types. They are the surface veneers and the core veneers. The surface veneers are given in the first and last of each line in the Table. In between the surface veneers are the core veneers. Relating to the types of plywoods we can further divide the surface veneers into two kinds: the red surface veneers and the white surface veneers. This fact is not shown in Table 10. As to the core veneers, the type of log from which they are made is immaterial. In most cases the log component in the costs of producing veneers might be different. But because we have excluded the cost of log from the costs of producing veneers, it is unlikely that the differences in veneer production costs now will be significant. Thus we shall treat all core veneers of the same thickness as homogeneous. Therefore for our purpose, the types of veneers may be classified as follows:-

TABLE 11

TYPES OF VENEERS

Surface		Core	
Red			
0.9	Y_{I1}	1.8	Y_{III4}
1.0	Y_{I2}	2.5	Y_{III5}
1.3	Y_{I3}	2.6	Y_{III6}
White		3.2	Y_{III7}
0.9	Y_{II1}	3.4	Y_{III8}
1.0	Y_{II2}	3.8	Y_{III9}
1.3	Y_{II3}	4.4	Y_{III10}

Types of Logs

In an earlier Chapter we saw that there were five species of logs available for making veneers. There were four species that could be used for making surface veneers as well as for making core veneers. The remaining species was only suitable for making core veneers. This was given in Table 5 in Chapter III. The red meranti and the mengkulang species are of red wood. They are used primarily for making the red surface veneers. Besides, their prices are very much alike. Because of these similarities we may group these two species into one group and call it the red logs. By the same token the white meranti and the mersawa species could be considered as the white logs. However, logs within the same group are not homogeneous. The primary purpose of the

first four species referred to is for making surface veneers. But some logs, though small in number, in each species are not suitable for their primary purpose. Thus it is necessary for us to divide them into two categories: one for those suitable for making surface veneers and the other for those unsuitable for the same purpose. We shall call the first category as the "good quality" and the second the "poor quality". At first sight we are inclined to group the white poor quality, the red poor quality together with the dama hitam species because these three types are similar in the sense that they are all for making core veneers only. On further analysis this grouping does not seem to be appropriate because they have differences in costs and differences in efficiency of using the logs to make veneers. To preserve these dissimilarities we shall treat the three types of logs as different kinds. The types of logs to be included in the objective function are as follows:

TABLE 12

TYPES OF LOGS

Types	Symbol
Good quality red	Z_I
Poor quality red	Z_{IA}
Good quality white	Z_{II}
Poor quality white	Z_{IIA}
Others	Z_{III}

In the discussion thus far in this Chapter we have classified the factory's products into twenty different types. On the basis of this, the veneers and logs have been divided into thirteen and five types, respectively. Each of the types of plywoods will form a variable in the first part of the objective function. Similarly, each of the types of veneers will form a variable in the second part while each of the types of logs a variable in the third part of the objective function. Thus there will be a total of 38 variables - 20 in the first part, 13 in the second part and 5 in the last part - in our objective function.

Coefficients

Having determined the various variables of our objective function, we shall now ascertain the coefficients for these variables. In the simple illustrative objective function the coefficients were simply the respective marginal contributions - the net selling price minus the variable production costs. The coefficients that we are now concerned with would be somewhat different in context. This is because

of the change in the form of the objective function. Instead of having to find coefficients only for the plywood variables we have to provide coefficients for the variables representing the veneers and logs as well. This fact was stated while, at the beginning of this Chapter, we discussed the form of our objective function. The coefficients for the variables representing the respective types of plywoods are $k_{ai} - l_{ai}$, namely, the net revenue minus the corresponding variable plywood formation costs. The coefficients for the veneers and logs are the variable veneers production costs, m_{bk} and the log costs, n_c respectively. Thus to ascertain the coefficients we have to derive the unit net revenue and the various types of unit costs.

Net Revenue

The manufacturer derives his revenue from selling his factory's outputs. Revenue is thus a function of price, other things being equal. To produce and sell the products the manufacturer not only incurs the costs of production but also the cost of selling. Since we are interested in the production problem in the factory, we shall be concerned only with the net revenue ex-factory. This means that we have to know in the first place the net selling price and the costs of marketing. The net selling price will be based on the list price and the normal rate of discount. Price and discount information is not difficult to obtain. But the cost of marketing as well as the costs of production presents some difficulties. For want of data we shall allow a standard percentage on the list price as the total marketing cost (including discount). This, according to the management, amounts to approximately 35 per cent of the list price. Thus the list prices after deducting the total marketing costs will be the net revenues for the factory. The list prices, the allowances for marketing costs and hence the net revenues for the various types of plywood panels are given in Table 13.

Production Costs

In estimating the costs of production difficulty arises as to what constitutes cost.[2] Generally, for decision making we are interested in the direct or variable cost elements. Direct costs are those cost elements that normally are traceable to the products and they often vary with the volume of output. Allowing that the direct and variable cost elements can be isolated from other types of costs, we still face another problem. This is whether we should use opportunity cost or historical cost. It appears that the former rather than the latter constitutes the type of cost relevant to programming calculation.

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[2] Cost, per se, is rather a vague term.

Opportunity cost in simple term is the value of an opportunity forgone. However, unless ample data are available, and in fact they are not available, the estimation of forgone benefits is impossible. Because of this reason we have to be content with the historical cost rather than to attempt to estimate the opportunity cost. The historical cost data that we would work with were for a certain month during which a total of slightly less than 1.8 million square feet of surface measurement of various types of plywoods were produced.[3] The costs are as given in Table 14.

TABLE 13

ESTIMATED NET REVENUES
(DOLLARS PER THOUSAND SQUARE FEET OF SURFACE MEASURE)

Thick	List Price		Marketing Costs		Net Revenues	
MM	Red	White	Red	White	Red	White
4	180	160	63	55	117	105
4.7	200	180	70	63	130	117
5	200	180	70	63	130	117
6	220	200	77	70	143	130
8	320	300	112	105	208	195
9	340	320	119	112	221	208
10	360	340	126	119	234	221
12	400	380	140	133	260	247
15	540	520	189	182	351	338
18	600	580	210	203	390	377

The figures given in the Table were the sum total of the variable production costs incurred for the whole production process during that month. Since we have broken the objective function down into three parts, it is thus necessary for us to distribute the total variable costs into three corresponding parts and then ascertain their

[3] Appendix III.

unit costs accordingly. The three parts are as follows:

(1) Cost of plywood formation.

(2) Cost of veneer production.

(c) Cost of log consumption.

TABLE 14

VARIABLE PRODUCTION COST

Item		$
1.	Logs	83,033*
2.	Glue	35,372
3.	Direct Labour	25,905
4.	Other Expenses	52,069

*Breakdown given in Table 23.

Ideally, cost information should be collected under these headings rather than to have them in a lump sum and then try to apportion them into the respective components. However, actual situation is far from ideal. In a study of this nature such handicap in the process of data collection has to be put up with. Actually, we have only two items to be allocated. The item on log obviously is the only cost pertaining to the cost of log consumption. The item on glue was incurred in the glueing process and thus fell under the cost of plywood formation. Thus what we have to distribute between the first two components given above are the direct labour and other expenses.

The basis for assigning direct labour to that particular work or process is normally the direct labour hour, that is, the number of man-hours spent on that process. As for the item, other expenses, the allocation basis may be the direct labour hour, direct machine hours or the volume of products produced. Once again we run into the difficulty of getting sufficient data for this purpose. Nevertheless, from what scanty data available it is suggested that the two item of costs should be distributed as in Table 15.

By now the sum total of the variable production costs has been divided into three parts. These are scattered over two tables. The direct materials are given in Table 14 while the direct labour and other expenses are allocated in Table 15. We shall now bring them together under the required headings in Table 16.

The coefficients of the variables in the objective function, we have said earlier, are the respective unit costs of the plywoods, veneers or logs, as the case may be. We have already known the total costs for each of the three components corresponding to the three parts

of our objective function. What remains to be done is thus to determine the unit costs, of the various products or logs, that will serve as our cost coefficients. There are several methods of doing so. The simplest way is to express the products in terms of a common equivalent unit and then take the simple average by dividing the total costs by the equivalent number of units of products produced. This method, though simple to apply, will not be able to give a realistic picture of the cost behaviour. The cost elements appear to be dependent on several factors. To put it in the cost accounting terms, the costing units are not homogeneous. The costing unit is the number of units of products

TABLE 15

DISTRIBUTION OF DIRECT LABOUR AND OTHER EXPENSES (in $)

Item / %	Total	Veneer Production	Plywood Formation
	100	60	40
Direct labour (1)	25,905	15,543	10,362
Other Expenses (2)	52,069	31,241	20,828
Total	77,974	46,784	31,190

(1) Item 3 Table 14

(2) Item 4 ibid

TABLE 16

VARIABLE PRODUCTION COST SUMMARY (in $)

	Plywood Formation	Veneer Production	Log Consumption
Materials	35,372	-	88,033
Direct Labour	10,362	15,543	-
Other Expenses	20,828	31,241	-
Total	66,562	46,784	88,033

produced by a given department, section or process. In other words, in our case here, the equivalent units are not relatively homogeneous. We cannot have realistic unit cost by making heterogeneous units to share the total cost equally.

To get a more realistic unit cost we shall adopt a slightly complicated costing method. Plywood production is essentially a process system. Thus process cost is obviously a more appropriate method of finding unit costs. Before we begin our cost determination we shall have a look at what is the basis of process costing. It is as follows. If the nature of operations is suitable for process costing, each production cost centre corresponds to one stage in the production cycle and has a relatively homogeneous output - that is each unit of output is similar enough to every other unit to permit the output to be measured by a simple unit count. Costs and output statistics are accumulated for each process centre during a specified period of time. The average cost computed from these figures is then assigned to each unit of the process centre's output for the period. This is repeated for all process centres, and the total unit cost of finished products is the sum of the unit costs of the relevant cost centres. The costing unit is the total output of a process centre during the given period.[4]

Judging from the later half of the above paragraph, it appears possible for us to apply process costing for determining the unit plywood formation costs and the unit veneer production costs. The production stages within each of the two departments are divided into cost centres with a view of obtaining relatively homogeneous output. One important point about the concept of output needs to be clarified. By the output of the cost centre we do not necessarily mean the physical output. It also means the services or anything else that the cost centre adds on to its input. To simplify the computation we shall group all those cost centres of which their costing units are homogeneous. By this grouping each of our cost centres will not necessarily correspond to one stage in the production cycle. This seemingly conflicts with the basis referred to earlier. But actually it is not so. What we are trying to do is to sum the total costs of the cost centres before averaging, instead of summing the unit costs of the various cost centres. So long as the costing units are, or adjusted to be, the same, the simplifications do not make any difference.

One point remains to be said. This is regarding the method of allocating the total costs for each department to the various cost centres because we had not accumulated costs for each cost centre. Under ideal situation this should have been done. Instead of, we have the allocated costs. The only way out is to further distribute them to the various cost centres.

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[4] Shillinglaw, Gordon, op. cit. p. 109.

According to Table 16 the allocated cost for plywood formation department amounts to $66,562. This sum, in turn, is to be distributed amongst the various cost centres within the department. Referring to Figure 9 in Chapter III, we can see that there are seven production stages or sections in this department. Before we divide them into cost centres we shall examine the probable cost behaviours in each of the production stages. This cost behaviour analysis is necessary in order to find out the relevant costing unit for each of the cost centres.

Let us, to begin with, examine the cost behaviour in the assembling and glueing process. We had had a rather detailed description of the glueing procedure in Chapter III. We can see from the illustration that the number of glued surfaces of a, say, 5-ply panel is four. We can induce a general rule, that is, the number of glued surfaces of a n-ply plywood panel is n-1. In all probability, all glued surfaces will be relatively homogeneous. Thus we can say that the cost of assembling and glueing plywood is a function of (n-1), where n is the number of plies in the panels. As will be seen later, there is no other production process within this department that appears to have such a cost function. We may make this process as a cost centre.

As for the press, in the previous Chapter, it is shown that the time required for pressing plywood is a function of that panel's thickness. The thicker the panel the longer the press time required. It seems to vary proportionately with the thickness. But cost is very likely to be related to the press running time. Hence we may say that the cost is dependent on the panel's thickness. Similar to the press is the cost of packing plywoods. For example, one piece of 8mm panel is twice the volume of two pieces of 4mm panel, all of equal size. Cost of packing seems to be a function of the volume. To a great extent, the sawing and trimming process can be considered as a dependent of the panel's thickness too. In short, the press, the trimmer and the packing facility appear to have common cost behaviour and hence for simplicity, we shall group them together as one cost centre. The costing unit for this centre will be the total equivalent square feet, in terms of one millimetre of plywoods produced.

There remains three more production sections: sanding, grading and inspection. These processes appear to be connected with the surface areas of the panels. The thicknesses of the plywoods seem to be immaterial in this case. We can identify a common costing unit for all the three processes. Thus it is reasonably possible to group them under one cost centre. The costing unit will be the number of square feet of surface measurement of the plywood produced.

The above analysis would suggest that the outputs of the various production stages within the plywood formation department could be divided into three relatively homogeneous groups. On the basis of this grouping we shall set up, for the purpose of determining unit costs, the following cost centres given in Table 17.

Having established the cost centres we shall now allocate the cost to them. With the allocated costs and the relevant costing units

TABLE 17

COST CENTRES IN PLYWOOD FORMATION DEPARTMENT

Centre No.	Production Processes
1	Assembling and Glueing
2	Pressing, Sewing and Trimming, Packing
3	Sanding, Grading and Inspection.

we shall be able to determine the unit cost of the respective cost centre. The weighted sum of these unit centre costs will constitute the unit panel formation cost. The basis on which cost is to be allocated to the cost centres is similar to the one on which we have allocated the total variable production costs to the two major departments earlier. For distributing direct labour and other expenses we have to look into the direct labour hours and the direct machine hours. In determining the allocation ratios we would encounter the same type of difficulties as we did encounter earlier while allocating the costs to the two departments. On the strength of what was said then, we would allocate the sum of the panel formation costs to the respective cost centres. The allocation of the formation costs and the computation of unit centre costs are given in Table 18.

TABLE 18

COMPUTATION OF UNIT CENTRE COSTS IN PLYWOOD FORMATION DEPARTMENT

Item	Cost Centre / Total	No. 1	No. 2	No. 3
Direct Material ($)	35,372.00	35,372.00	-	-
Direct Labour etc. ($)	31,190.00	5,458.00	12,476.00	13,256.00
Total ($)	66,562.00[a]	40,830.00	12,476.00	13,256.00
Costing unit (in thousand)	-	4,127.5[b]	9,614.5[c]	1,798.8[d]
Unit centre cost (per thousand) $	-	9.89	1.30	7.37

(a) Column 2, Table 16
(b) Column 5, Appendix III
(c) Column 6, ibid.
(d) Column 4, ibid.

With the computed unit centre costs in Table 18, we shall determine the unit formation costs of all the types of plywood panels. As we have seen earlier, the unit plywood formation cost is the weighted sum of the relevant unit centre costs. The computation of unit formation costs is given in Table 19.

TABLE 19

COMPUTATION OF UNIT COSTS OF PLYWOOD FORMATION
(in dollars per thousand square feet of surface measure)

Panel / unit centre cost (a)		Cost Centre			Total
		No. 1	No. 2	No. 3	Unit Cost
		9.89	1.30	7.37	-
ply	mm				
3	4	19.78	5.20	7.37	32.35
3	4.7	19.78	6.11	7.37	33.26
3	5	19.78	6.50	7.37	33.65
3	6	19.78	7.80	7.37	34.95
5	8	39.56	10.40	7.37	57.33
5	9	39.56	11.70	7.37	58.63
5	10	39.56	13.00	7.37	59.93
5	12	39.56	15.60	7.37	62.53
7	15	59.34	19.50	7.37	86.21
7	18	59.34	23.40	7.37	90.11

(a) Table 18

The computation is briefly explained here. At the head of the columns and under the cost centres, in Table 19, are the respective unit centre costs that have been computed in the previous Table. For n-ply panel the total cost incurred in cost centre 1 would be (n-1) times $9.89. As for cost centre 2 the total that would be charged for a panel of t thick (t is the number of mm) is $1.30 xt. The cost centre No. 3 is a function of the surface area, and as such panel of any number of plies or of any thicknesses is charged a flat rate of $7.37 per thousand square feet of surface measure. The sum total of the weighted centre costs will then be the unit plywood formation cost. To illustrate the unit formation cost per thousand square feet of surface measure for a

3-ply 4 mm panel is as follows:

$$(3-1) \times \$9.89 + 4 \times \$1.30 + \$7.37 = \$32.35$$

The above unit costs is applicable to both the red and the white surfaced plywoods. So far as the formation of plywoods is concerned, there appears to be no significant cost differential between them.

We now come to the cost of producing veneers. Earlier, we have excluded the log consumption from the costs of veneer production. Thus the veneer production costs consist of only the allocated portion of direct labour and other expenses. Despite this, the determination of the unit costs for producing veneers is somewhat complicated. It is rather difficult to establish cost centres such that their outputs are acceptably homogeneous. This is because of the heterogeneous nature of their cost behaviours. Weighting will be used in trying to obtain more realistic costing.

The five production stages in the veneer production department (shown in Figure 8, Chapter III) are, after having considered their probable cost behaviour, divided into four cost centres as follows:

TABLE 20

COST CENTRES IN VENEER PRODUCTION DEPARTMENT

Centre No.	Production Process
1	Cutting & Debarking, Drying
2	Peeling
3	Clipping
4	Jointing & Splicing

With the cost centres established we would have to assign cost to them in the way we did for the plywood formation department and for the similar reasons. The costs incurred were not accumulated specially for the individual cost centres. Moreover, we have had no complete data on which to lease our allocation of costs. Nonetheless, we had a hazy idea that was derived from the available information. With this we would distribute the costs to each of the cost centres. Thereafter the unit costs for the respective cost centres would be determined. These are done in Table 21.

We have computed the unit centre costs for all the cost centres within the veneer production department. With them, the unit production costs of various types of veneers are computed in Table 22.

We now come to the last item of cost - log cost. In an earlier section the types of logs were classified into five categories. But for costing purposes we shall use only three categories: red, white and other. This is because logs were purchased in bulk. For production purposes the red and the white species of logs were divided

TABLE 21

COMPUTATION OF UNIT CENTRE COSTS IN VENEER PRODUCTION DEPARTMENT

Item	Centre / Total	No. 1	No. 2	No. 3	No. 4
Allocated Cost ($)	46,784.00[a]	23,392.00	9,357.00	4,678.00	9,357.00
Costing Unit (thousand)	-	10,271.4[b]	7,394.0[c]	5,926.3[d]	7,713.8[e]
Unit Centre Cost ($ per thousand)	-	2.28	1.28	0.79	1.21

(a) Column 3 Table 16

(b) Column 11 Appendix III

(c) Column 7, 8 and 9 ibid, the first and last weighted

(d) Column 10, ibid

(e) Column 7, 8, and 9, ibid, the first weighted.

Note: For the weights, see footnotes in Table 22.

TABLE 22

COMPUTATION OF UNIT COSTS OF VENEER PRODUCTION
(in dollars per thousand square feet of surface measure)

		Cost Centre				Total
		No. 1	No. 2[a]	No. 3	No. 4[b]	Unit Cost
mm \ unit cost[c]		2.28	1.27	0.79	1.21	-
Surface	White 0.9	2.05	1.27	0.79	1.21	5.32
	White 1.0	2.28	1.27	0.79	1.21	5.55
	White 1.3	2.98	1.27	0.79	1.21	6.23
	Red 0.9	2.05	1.65	0.79	2.42	6.91
	Red 1.0	2.28	1.65	0.79	2.42	7.14
	Red 1.3	2.98	1.65	0.79	2.42	7.82
Core	1.8	4.10	1.78	0.79	1.21	7.88
	2.5	5.70	1.78	0.79	1.21	9.48
	2.6	5.93	1.78	0.79	1.21	9.71
	3.2	7.30	1.78	0.79	1.21	11.08
	3.4	7.76	1.78	0.79	1.21	11.54
	3.8	8.66	1.78	0.79	1.21	12.44
	4.4	10.03	1.78	0.79	1.21	13.81

(a) Red surface veneer weighted by 1.30; core veneer by 1.40

(b) Red surface veneer weighted by 2.00

(c) Table 21.

into good and poor quality logs. There is no difference in the purchase price whatsoever between the good and the poor quality. Unless we assign weights to compensate for the differences in quality, their unit costs will be exactly the same. For simplicity, we shall ignore the quality differences in so far as costing is concerned.

The volume and cost of logs consumed during the period were accumulated in three components, namely the red, white and other species. Suppose we divide the cost per species by the relevant volume, we would obtain the unit cost of log consumed. Since we have assumed, for costing purpose, the good and poor quality logs of some species are homogeneous, both the good and poor quality would have identical unit costs. What has been discussed above is summarised in Table 23.

TABLE 23

COMPUTATION OF UNIT COST OF LOGS

	Red	White	Others
Cost of logs consumed ($)	46,120.00	35,189.00	6,724.00
Costing unit (cubic feet)	26,960	23,700	5,390
Unit cost ($)	1.71	1.48	1.25

At the beginning of this Chapter we have determined the form of our objective function. It was found that the simple form could be extended to three parts so as to provide additional information for managerial decisions. Moreover, the extended form would facilitate costing. Because of the handling problem and the capacity limitations of electronic computers we had reduced the number of types of products drastically. In our simplified problem there are only 20 types of plywoods. Related to this, the number of types of veneers is 13. The species of logs are fixed at 5. These are given in Tables 9, 11 and 12 respectively. To provide the cost coefficients for the variables we have determined their unit costs. These are given in Tables 19, 22 and 23. The required objective function for our product mix problem would be obtained by pairing off the unit costs of the plywood formation, veneer production and the log consumption with the corresponding variables, and then subtracting them from the net revenue shown in Table 13. Therefore our objective function would be as follows:

Maximise

$$(117 - 32.35)x_{I1} + (130 - 33.26)x_{I2} + (130 - 33.65)x_{I3} +$$
$$(143 - 34.95)x_{I4} + (208 - 57.33)x_{I5} + (221 - 58.63)x_{I6} +$$
$$(234 - 59.93)x_{I7} + (260 - 62.53)x_{I8} + (351 - 86.21)x_{I9} +$$
$$(390 - 90.11)x_{I10} + (105 - 32.35)x_{II1} + (117 - 33.26)x_{II2} +$$
$$(117 - 33.65)x_{II3} + (130 - 34.95)x_{II4} + (195 - 57.33)x_{II5} +$$
$$(208 - 58.63)x_{II6} + (221 - 59.93)x_{II7} + (247 - 62.53)x_{II8} +$$
$$(338 - 86.21)x_{II9} + (377 - 90.11)x_{II10} - 5.32y_{III1} - 5.55y_{III2} -$$
$$6.23y_{III3} - 6.91y_{I1} - 7.14y_{I2} - 7.82y_{I3} - 7.88y_{III4} -$$
$$9.48y_{III5} - 9.71y_{III6} - 11.08y_{III7} - 11.54y_{III8} - 12.44y_{III9} -$$
$$13.81y_{III10} - 1.71z_{I} - 1.71z_{IA} - 1.48z_{II} - 1.48z_{IIA} -$$
$$1.25z_{III}$$

CHAPTER V

THE CONSTRAINING FUNCTIONS

The objective we are concerned with is to find a product mix (the various types of plywoods) that, under the given costs and market conditions, will bring in the maximum contributions - maximum revenue at minimum costs. To produce plywood we need raw materials, labour and production facilities. The resources are limited. Thus the production system is subject to certain restrictions. Besides, we have to consider the market for the plywoods. It is of no value to produce what the firm cannot sell. These restrictions, both market and production facilities would thus impose restrictions on the area of feasible solutions. The factors in terms of restrictions that we have to look into are as follows:

1) Availability of raw materials,
2) Availability of production facilities,
3) Market demand for the firm's products.

For each of the first two factors we have to determine the amount of resources available. Then, we have to identify the relationship between the relevant factors and the respective restrictions. In mathematics this is accomplished by forming linear equations or inequations. The equational relationships are known as the constraining functions.

Simplified Constraining Factors

To meet our need in keeping down the magnitude of our model, we shall consider only the key factors as our constraining functions.

The availability of logs is, perhaps the most important raw material that has to be considered in our analysis. It is log that veneers and later plywoods are made from. The other important raw material is glue. But it appears to be only of secondary importance. For simplicity we shall only consider logs as constraints. We are assuming that the availability of logs as given and that our purpose is to maximise the use of the given logs. Of course we could take a step further backward and, instead of, consider the availability of fund with which logs and glue could be acquired as a restriction. However, such consideration would entail the analysis of the supply of logs.

As for the supply of production facilities we shall consider only the major machine controlled facilities. Referring back to Figure 8 of Chapter III, we can see that there are five production processes in the veneer production department. Of these five, the lathe and the dryer appear to be the key equipments. The others are not as

important. In the plywood formation department there are seven processes of which the hot press and the sanding machine are representative. The efficiences of these production processes being machine controlled, cannot be increased readily. They together seem to dictate the speed of operations.

Labour is required to man these equipments as well as to perform other activities. The availability of labour hours thus would also impose a restriction on the volume of the factory's output. But labour hour supply is relatively flexible for it can readily be increased. For simplicity we shall assume that labour is adjustable. Therefore we shall not consider labour hour supply as one of the constraints.

Availability of Logs

Let us, to begin with, ascertain the availability of the various types of logs. Basing on historical data the following are the amounts of logs available:

TABLE 24

TYPES AND AMOUNTS OF LOGS AVAILABLE

Type	Amount in cubic feet
Good quality red, Z_I	53,550
Poor quality red, Z_{IA}	2,884
Good quality white, Z_{II}	32,060
Poor quality white, Z_{IIA}	2,459
Others Z_{III}	7,751

Logs on arrival are sorted out into good quality and poor quality. The former are those logs that are suitable for making surface veneers while the latter, though of relatively small proportions, are only meant for producing core veneers.

However, both the red, good quality and white, good quality logs do not have their whole length useful for making surface veneers. Logs do not come in lengths of exact multiply of the lengths which are required to make the surface veneers. The surface veneers must be made of wood of lengthwise grain. In the first place, logs have to be cut into lengths equal to that of the surface veneer. In addition cutting allowances must be included. The length with allowance added is known as the cutting length.

Actually, the factory's plywoods are made in six standard sizes.[1] To make plywood of a given size allowances must be given for

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[1] See Table 7, Chapter III.

sawing and trimming after the veneers have been formed into panels. Allowances are given for the sides as well as for the ends of the panels. So the area of the veneer actually is greater than the corresponding area of the finished plywood. The cutting sizes for the standard sized plywoods are as given in Table 25.

TABLE 25

VENEER CUTTING SIZE FOR THE CORRESPONDING STANDARD SIZED PLYWOOD
(in inches)

Plywood	Veneer
72x36	81x41
72x48	81x53
84x36	90x41
84x48	90x53
96x36	104x41
96x48	104x53

To produce plywood of, say, 72 inches in length veneer must be 81 inches long. Hence log have to be cut into that length for peeling. For the same reason logs will be cut into 90 and 104 inches in order to produce veneers for forming into 84- and 96-inch plywoods, respectively. Logs are supplied in length varying from 18 to 28 feet. There are a number of ways of cutting any length of logs. Let us see some of the possible ways of cutting an 18-foot log.

TABLE 26

CUTTING COMBINATIONS FOR 18-FOOT LOG

Cutting Combination Number	Number of Cuttings					Waste	
	Surface			Core			
	104"	90"	81"	53"	41"	inch	%
1	2	-	-	-	-	8	3.7
2	-	-	2	1	-	1	0.5
3	-	-	2	-	1	13	6.0
4	-	1	1	-	1	4	1.9
5	-	1	-	-	3	3	1.4
6	1	-	-	2	-	6.	2.8

The first combination shown in Table 26 is to cut the 18-foot log into two 104-inch for making surface veneer, leaving a waste of 8 inches which amounts to 3.7%. The second combination comprises two 81-inch surface and one 53-inch core. The waste in this case is reduced to only one inch which is merely 0.5%. And so on. By the same token, logs of other lengths could be cut into various combinations. The actual combinations chosen in practice depend on several factors of which some are as follows:

1) The nature and length of the logs
2) The need to reduce waste
3) The size of plywoods to be produced

It is thus infeasible to use the whole length of the good quality logs to make surface veneers only. The figures given in Table 24 therefore have to be adjusted to reflect this fact.

The length of logs varies and there are many ways of cutting. How can we devise a means of ascertaining the proportions of good quality log that, because of cutting combinations, have to be used for making core veneers? This is a rather difficult question. Experience in the industry is essential in solving this problem.

In this paper, for want of data, certain assumptions will have to be made. In the first place, we shall assume that there are only five different lengths of logs. Actually there are more but it seems the bulk of them belong to the five we are assuming. Secondly we shall assume that the optimal cutting combinations for the five different lengths are as follows:

TABLE 27

OPTIMAL LOG CUTTING COMBINATIONS
(in inches)

Length of Log (1)	Surface (2)	Core (3)	Proportions (3) ÷ (1) (4)
216	2 x 81	1 x 53	.2454
240	1x104+1x90	1 x 41	.1708
264	2 x 104	1 x 53	.2008
276	2 x 104	1 x 53	.1920
288	2x104+1x81	-	-

Table 27 besides showing the optimal cutting combinations for logs of different lengths also shows the proportions of the length of logs that will have to be used for making core veneers. The proportions are obtained by dividing the figures in Column 3 into the corresponding item in Column 1. For instance, the 18-foot log, 24.54% of its length will have to be used for making core veneers.

If we know the numbers and the sizes of the logs of each length in the logyard, we shall be able to estimate the overall proportions of the available logs that are to be used for making core veneers. The overall proportions are but the weighted averages of the individual proportions shown in Column 4 of Table 27. The weights will be the numbers and sizes of the logs available if they vary considerably. This sort of information can be obtained from an actual count or, if the number of logs is large, a sample count. In this exercise, sample was used to ascertain the length distribution of the logs. As to the size for simplicity we assume that they are homogeneous. The results of the sample survey are given in Table 28.

TABLE 28

LENGTH DISTRIBUTION OF THE LOGS IN THE SAMPLE

Length (inch) (1)	No. of Logs of Indicated Length	
	Red Logs (2)	White Logs (3)
216	122	219
240	84	20
264	52	35
276	41	122
288	137	37

With the data given in the two most recent tables, the weighted proportions can be computed as follows in (Table 29.)

The proportions in Column 2 when multiplied into their corresponding weights in Column 3 and 4, respectively would give the weighted proportions. These are shown in Columns 5 and 6. The sum of these weighted proportions divided by the sum of their respective weights would be the weighted average proportions of the good quality logs that, because of the cutting problem, have to be used for making core veneers. Thus the volumes of good quality red logs and white logs

TABLE 29

WEIGHTED PROPORTIONS OF LENGTH OF LOGS TO BE USED FOR MAKING CORE VENEERS

Logs	Simple	Weights[b]		Weighted Total	
(inch)	Proportions[a]	Red	White	Red	White
(1)	(2)	(3)	(4)	(5)	(6)
216	.2454	122	219	29.94	53.74
240	.1708	84	20	14.35	3.42
264	.2008	52	35	10.44	7.03
276	.1920	41	122	7.87	23.42
288	-	137	37	-	-
Total	-	426	433	62.62	87.61

[a] Column 4, Table 27.

[b] Column 2 and 3, Table 28.

given in Table 24 have to be reduced by 14.69%[2] and 20.33%[3], respectively. The amount thus reduced would be added in to the poor quality logs of their species. Therefore, the adjusted quantities of logs available for production are as shown in Table 30.

TABLE 30

ADJUSTED QUANTITIES OF LOGS AVAILABLE

Type		Amount in Cubic Feet
Good quality red	Z_I	45,684
Poor quality red	Z_{IA}	10,863
Good quality white	Z_{II}	25,574
Poor quality white	Z_{IIA}	8,986
Others	Z_{III}	7,751

The optimal product mix should not use more than the logs available. Any feasible solution must not utilise more than the volume of logs given in Table 30. Thus the first set of constraining functions will be as follows.[4]

1. Availability of logs.

$$z_I \leq 45,684$$
$$z_{IA} \leq 10,863$$
$$z_{II} \leq 25,574$$
$$z_{IIA} \leq 8,986$$
$$z_{III} \leq 7,751$$

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[2] $\frac{62.62}{426} \times 100 = 14.69\%$

[3] $\frac{87.61}{433} \times 100 = 20.33\%$

[4] Two additional constraining functions will be introduced at a later stage.

Use of Logs to Make Veneers

We shall now proceed to find out how much veneers (of various types) can be produced out from the logs available. To do so, we have first to know the efficiency factors of the different types of logs in making veneers. With this efficiency factors known, we can determine the amount of log required to produce a unit of a given veneer, and hence we can formulate the relationship between the veneers and the use of logs.

In an earlier Chapter the production process has been described.[5] Logs are first cut, debarked and then peeled into continuous sheet of veneer with the rotary lathe. In this section we shall examine further as to what part of the log is turned into veneers.

Figure 10 shows the cross-section view of a cut and debarked log which is ready to be peeled. The log is first trimmed into a perfect cylinder and then peeled. The outer part of the log because of its coarse texture is normally unsuitable for making surface veneers. It is used, instead, for making into core veneers. But after a couple of inches or so of peeling, the log is then peeled into surface veneers. The peeling is a continuous process. The peeling goes on until the wood once again is unsuitable for making surface veneer. This time the wood is too soft for making surface veneers. This part of the log, like the outer part, is made into core veneers. Finally a four-inch (in diameter) core of the log is discarded as it is too soft even for making core veneers.

Thus we see, even here, not the whole useful part of the log is made into surface veneer.

The proportions of the log suitable for different purposes - for making surface veneer, for core veneer or wasted - depend primarily on three factors: diameter, species and quality of the log. The only certainty that can be observed is that the discarded core is about four inches in diameter, regardless of the size of the log. Owing to this diversity it is rather difficult to derive uniform and accurate efficiency factors. A detail statistical study, perhaps, is helpful in the aspect. However, such a study would involve both time and facilities that lie beyond the scope of this exercise. It is one of the limitations of this paper that we use the management's experience in estimating the efficiency factors.

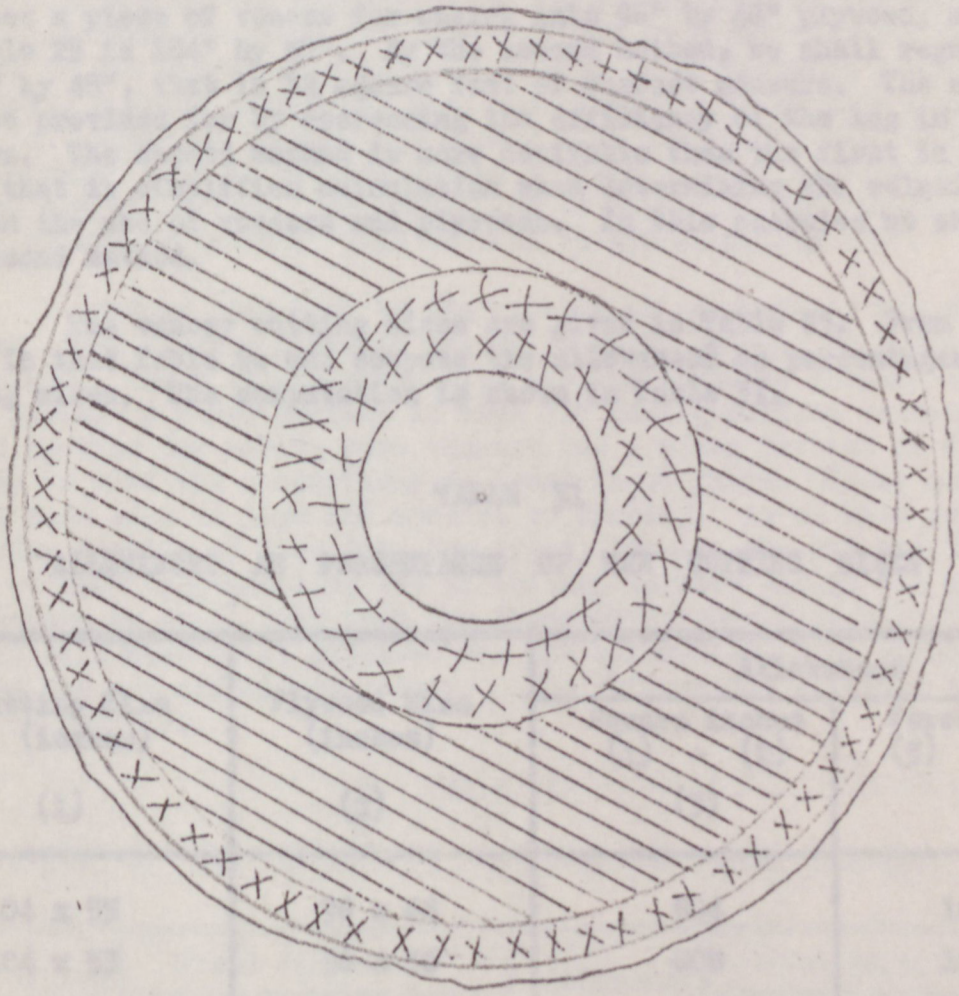
Before we proceed further, it is necessary to settle one problem. Earlier, it has been shown that trimming and sawing allowances must be provided for in cutting veneers. A problem arises here. How should we treat the allowances in our ascertaining the efficiency factors?

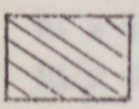
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[5] See Chapter III.

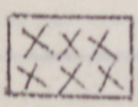
FIGURE 10

CROSS SECTION VIEW OF A DEBARKED LOG

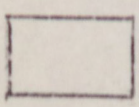




for making surface veneer



for making core veneer



waste

There appears to be two ways of doing it, one less desirable than the other. The first method is to measure veneers in terms of their actual surface areas. The necessary allowances will be provided for when determining the relationships between veneers and plywoods at a later stage. In such case, the efficiency of forming veneers into plywoods would be reduced by the amount necessary to provide the allowances. The second method, the more desirable alternative, is to provide the allowances in the use of log to make veneers. The veneers are to be measured in terms of the size of plywood into which the veneers are made. For instance a piece of veneer for making into 96" by 48" plywood, according to Table 25 is 104" by 53". By the second method, we shall regard it as 96" by 48", that is 32 square feet of surface measure. The allowances will be provided for by decreasing the efficiency of the log in making veneers. The second method is more desirable than the first in the sense that it simplifies calculation when determining the relationships between the use of veneers and plywoods. In this exercise we shall use the second method.

The veneer cutting sizes are given in Table 25. From information given in that Table we can compute the allowances as percentages of the cutting sizes. The computation is shown in Table 31.

TABLE 31

ALLOWANCES AS PERCENTAGES OF THE CUTTING SIZES

Cutting Size (inches) (1)	Plywood Size (inches) (2)	Allowances	
		Square inches (1) - (2) (3)	Percentages (3) ÷ (1) (4)
104 x 53	96 x 48	904	16.4
104 x 53	96 x 36	808	18.9
90 x 53	84 x 48	736	15.5
90 x 41	84 x 36	666	18.0
81 x 53	72 x 48	837	19.5
81 x 41	72 x 36	729	22.0
		Total	110.3

Column 1 in the above Table shows the cutting sizes of the veneers in order to produce the corresponding size of plywoods. The excess of the figure in Column 1 over its corresponding figure in Column 2 will be the allowance for final sawing and trimming. The allowances can be expressed as percentages of the areas of the cutting sizes by dividing Column 3 by the corresponding item in Column 1 and the results are given in Column 4. The average of these percentages will then be the overall percentage of allowance that we have to provide for in arriving at the efficiency in the use of logs to make veneers. The simple average method used here may not be a satisfactory one because of the differences in the proportions of plywoods produced. The simple average method may give undue weights to certain sizes of plywoods. However, it does not appear that the result will vary considerable even if refined method is used. After all, all the figures are estimates. For simplicity we might use the simple average as our allowance factor. This amounts to approximately 18%[6].

Now we come back to the problem of determining the efficiency factors. The diagram given in figure 10 might be able to provide us some clues. There it was shown that the log could be divided into three parts. A major portion is used for making surface veneers, a lesser portion for making core veneers and a minor portion is discarded. We have to find the proportions of these three parts. These seem to differ from logs to logs and species to species. As we have grouped the major species of logs into two types, namely, the red and the white logs, we need to generalise the efficiency factors within the groups. The factors we shall call them the gross efficiency factors. These figures will have to be adjusted for the allowances. The resultants will be the net efficiency factors. The information referred to are given in Table 32.

TABLE 32

COMPUTATION OF NET EFFICIENCY FACTORS FOR GOOD QUALITY LOGS (%)

Type of Log	Gross Efficiency		Allowance	Net Efficiency	
	Surface	Core		Surface	Core
(1)	(2)	(3)	(4)	(5)	(6)
Red Z_I	55	30	18	45	25
White Z_{II}	67	18	18	55	15

[6] $\frac{110.6}{6} \times 100 = 18\%$

The figures given in the column under the heading gross efficiency are the estimated proportions of the log that can be used to make into surface and core veneers, respectively. White log has a greater proportion for making surface veneers and consequently a smaller proportion for core veneers than that of the red log. From these figures, the cutting allowances are to be provided. The allowances are expressed as percentages of the total area of the individual cutting size. Hence the net efficiency factors are (1 - 0.18) times the corresponding gross efficiency factors. The resulting net efficiency factors will be the basis on which we can determine the amount of logs required to produce different types of veneers.

Let us, to begin with, examine the amount of red log required to produce, say, one unit of surface veneer type Y_{II}. We shall work in units of 1,000 square feet of surface measure. According to Table 11, Y_{II} is of 0.9 mm thick. The computation is as follows.

1 unit of Y_{II} = 1,000 square feet in 0.9 mm thick

$$= \frac{900}{304.8} \text{ cubic feet, since 1 foot} = 304.8\text{mm}$$

Now the net efficiency factor of log Z_I in producing surface veneer is, given in Table 32, 45%.
Hence the amount of log Z_I required to produce 1 unit of Y_{II} is $\frac{900}{304.8} \times \frac{1}{.45}$. This works out to be 6.56 cubic feet.

Furthermore, in producing Y_{II}, a byproduct amounting to 25% of the amount of log used could be produced. This byproduct could be made into any type of core veneers as was decided at the time of peeling. But we have no means of knowing which particular type of core veneer is produced when producing a specific type of surface veneer. So here

lies a difficulty. To overcome this difficulty we need to introduce another category of logs - the imaginary logs.[7] In the process of producing the surface veneer, an imaginary log is created, which is then used to produce the core veneer. We imagine that there is an intermediary between the log and the core veneers obtained as the byproduct. Of course this type of logs do not actually exist. In determining the relationship between the log and the veneers we first expressed the byproduct as the imaginary logs. The core veneers which are obtained as the byproduct will be related to the imaginary logs.

The relationship thus far discussed can be put into the form of mathematical equation as follows:

$$6.56\, z_I = y_{II} + 1.64\, z_{IV}$$

where z_I is the amount of log in cubic feet, y_{II} is the amount of surface veneer in 1,000 square feet of surface measures, z_{II} is the amount of imaginary log resulting from the use of z_I, in cubic feet.

We have formulated the relationship between the use of log Z_I and surface veneer Y_{II}. For each of the other types of veneers we can, and have to, formulated one such relationship. The relationships for the other two types of red surface veneers can be formulated in exactly the same way as we have done just now. As for the white surface veeneer we shall introduce a second type of imaginary log represented by Z_V,

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[7] The author is indebted to his Supervisor, Mr. Lee Moon Hoe for this suggestion.

the counterpart of Z_{IV}. The method of formulation is the same. As the computation is essentially mechanical they are omitted. The results are given at the end of this section.

Since we have introduced two 'new' types of logs into our model, it is necessary to adjust the information regarding the logs available for making veneers. Log Z_{IV} is obtained from the use of log Z_I at the rate of 25%. Hence, the amount of log Z_{IV} obtainable is related in the following relation:

$$z_{IV} = 0.25 z_I$$

By the same token, the imaginary log Z_V is related to log Z_{II} as:

$$z_V = 0.15 z_{II}$$

These two functions will have to be added to the first set of the constraining functions bringing the number to seven.

Now that two more types of logs are added, there are five different types of logs from which we can make the core veneers. The poor quality logs Z_{IA} and Z_{IIA} are those sorted out from the respective good quality logs Z_I and Z_{II}. It is natural that their efficiency factors are less than those of the good quality logs. As for log Z_{IV}, the species is used solely for making core veneers. This species has approximately the combined efficiency of the good quality log for making both the surface and the core veneers. The imaginary logs Z_{IV} and Z_V have their efficiency equated to 100%. The net efficiency factors for making core veneers are estimated as follows.

TABLE 33

NET EFFICIENCY FACTORS
FOR CORE VENEER MAKING LOGS

Type of Log	Net Efficiency (%)
Z_{IA}	65
Z_{IIA}	65
Z_{IV}	70
Z_{IV}	100
Z_V	100

There are seven types of core veneers.[8] Each of them can be made from any one of the five types of logs given in Table 33. The form of the equational relationship will be somewhat different from that for the core veneers.

Because of the differences in efficiency of logs, the amount of log required to produce a given unit of core veneer will vary. The volume of log, say, Z_{IA} require to produce 1,000 square feet of surface measure of core veneer Y_{III4} (Y_{III4} is of 1.8 mm thick) can be computed as follows:

1,000 square feet of surface measure of Y_{III4} is equal to $\frac{1800}{304.8}$ cubic feet.

The efficiency factor of log Z_{IA} is according to Table 33 65%.

Hence the amount of log Z_{IA} required is equal to $\frac{1800}{304.8} \times \frac{1}{.65}$ that is 9.09 cubic feet.

Put it in equation form, it becomes:

$$9.09\, z_{IA} = y_{III4} \quad \ldots \quad (1)$$

where z_{IA} is the amount of log in cubic feet

y_{III4} is the amount of core veneer in 1,000 square feet of surface measurement.

As log Z_{IIA} has the same efficiency factor as that of Z_{IA}, the coefficients hold. Hence we have the second equation.

$$9.09\, z_{IIA} = y_{III4} \quad \ldots \quad (2)$$

Similarly the relationships between logs Z_{III}, Z_{IV} and Z_{V} and veneer Y_{III4} can be computed accordingly. These will come out to be as follows:

$$8.44\, z_{III} = y_{III4} \quad \ldots \quad (3)$$

$$5.91\, z_{IV} = y_{III4'} \quad \ldots \quad (4)$$

$$5.91\, z_{V} = y_{III4'} \quad \ldots \quad (5)$$

The five equations can be summed up as one.

See Table 11, Chapter IV.

This is,

$$9.09\ z_{IA} + 9.09\ z_{IIA} + 8.44\ z_{III} + 5.91\ z_{IV} + 5.91\ z_{V} = 5y_{III4}$$

The above equation is one of the seven expressing the relationships between the logs and core veneers. The equations for the remaining six types of core veneers can be similarly formulated.

In our model there are 13 types of veneers, both surface and core. For each of them there is an equation specifying the amount of logs available at the manufacturer's disposal is restricted by the first set of constraining functions given in the preceding section. In this section, the relationships between logs and veneers are specified. Thus the availability of veneers is in fact restricted through these relationships. This consequently will impose a restriction on our effort to maximise profits. Therefore our second set of constraining functions are as follows:

$$6.56z_{I} = y_{I1} + 1.64z_{IV}$$

$$7.29z_{I} = y_{I2} + 1.84z_{IV}$$

$$9.48z_{I} = y_{I3} + 2.37z_{IV}$$

$$5.37z_{II} = y_{II1} + 0.81z_{V}$$

$$5.96z_{II} = y_{II2} + 0.90z_{V}$$

$$7.75z_{II} = y_{II3} + 1.16z_{V}$$

$$9.09z_{IA} + 9.09z_{IIA} + 8.44z_{III} + 5.91z_{IV} + 5.91z_{V} = 5y_{III4}$$

$$12.6z_{IA} + 12.6z_{IIA} + 11.7z_{III} + 8.20z_{IV} + 8.20z_{V} + 8.20z_{V} = 5y_{III5}$$

$$13.1z_{IA} + 13.1z_{IIA} + 12.2z_{III} + 8.53z_{IV} + 8.53z_{V} = 5y_{III6}$$

$$16.2z_{IA} + 16.2z_{IIA} + 15.0z_{III} + 10.5z_{IV} + 10.5z_{V} = 5y_{III7}$$

$$17.2z_{IA} + 17.2z_{IIA} + 15.9z_{III} + 11.2z_{IV} + 11.2z_{V} = 5y_{III8}$$

$$19.2z_{IA} + 19.2z_{IIA} + 17.8z_{IV} + 12.5z_{IV} + 12.5z_{V} = 5y_{III9}$$

$$22.2z_{IA} + 22.2z_{IIA} + 20.6z_{III} + 14.4z_{IV} + 14.4z_{V} = 5y_{III10}$$

Veneer Production Capacity Constraints

The production of veneers is subject to two types of restrictions: logs and production facilities. The former has been discussed and now in this section we shall examine the latter type of restrictions. There are two aspects of the production capacity constraints to be considered. First, we determine the volume of facilities available. Second, we need to know the machine time required to service each unit of the different types of veneers.

Although there are five stages in the veneer production department,[9] we shall consider only the lathe and the dryer as our capacity constraints. By this we do not mean that the other production facilities are unimportant. We merely assume that they are in all ranges of production, less restrictive than the two referred to. In some facilities we assume that their capacities are relatively adjustable.

The rotary lathe provides 8 hours of services daily. The planning period we are concerned with is one month. The factory runs approximately 25 days a month, thus providing a monthly service of 200 hours. To produce a unit of veneer requires certain service time of the facility. Thus the total amount of veneers that can be produced is restricted to the amount that can be handled by the 200 hours of lathe services available. Of course the factory could work overtime to increase the services hours. But for our purpose we shall assume no overtime.

There are 600 hours of drying services available as the dryer operates three 8-hour shifts for 25 days a month. The dryer operates non-stop because of high set up cost involved.

The information regarding the service time required by each type of veneer presents some difficulties. As has been mentioned earlier, the drying time required partly depends on the moisture contents and partly on the thickness of the veneer. The moisture contents, in turn, depend on the prevailing weather condition and the period the logs have been stored. Logs laying long under the hot sun in the log yard naturally need much less drying time. The peeling time as well as drying time for different types of logs may, to certain extent, vary. However it appears that the variation is unlikely to be of any significance. In this paper, we shall ignore such variation and instead treat them as homogeneous. The time requirements on both the lathe and dryer are as shown in Table 34.

To formulate the constraining functions we refer to Table 11 for the symbols representing the different types of veneers. For the three surface veneers given, there are two types for each thickness, namely, the red wood and the white wood. Thus there are two types of 0.9 mm surface veneers. These are Y_{I1} and Y_{III}. Similarly for 1.0mm we have Y_{I2} and Y_{III2}; for 1.3 mm, Y_{I3} and Y_{III3}. The relevant time applies to both red and white veneers equally. However, for the core veneers, each thickness refers to one type of core veneer. The type of wood from which the core veneer is made is, for our purpose, immaterial. The monthly volume of facility service is to be distributed amongst the different types of veneers. The time taken up by, say Y_{I1} amounts to $0.01y_{I1}$ hour, y_{I1} is a variable whose value we have to solve. The sum of all the veneer hours must not exceed the total service hours available. The effort to maximise the objective function is thus

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[9] See Figure 8, Chapter III.

TABLE 34

TIME REQUIREMENTS ON LATHE AND DRYER
(Hour per 1,000 square feet of surface measure)

Facility	Surface Veneer			Core Veneer							Constraints
mm:	0.9	1.0	1.3	1.8	2.5	2.6	3.2	3.4	3.8	4.4	
Lathe	.011	.012	.016	.022	.030	.032	.039	.041	.046	.053	200
Dryer	.038	.042	.056	.079	.116	.120	.157	.172	.204	.262	600

restricted. The following, therefore will be the third set of the constraining functions:

Lathe

$$0.011(y_{I1} + y_{III1}) + 0.012(y_{I2} + y_{II2}) + 0.016(y_{I3} + y_{II3}) + 0.022y_{III4} +$$

$$0.030y_{III5} + 0.032y_{III6} + 0.039y_{III7} + 0.041y_{III8} + 0.046y_{III9} + 0.053y_{III10}$$

$\leq$ 200 hours/month.

Dryer

$$0.038(y_{I1} + y_{III1}) + 0.042(y_{I2} + y_{II2}) + 0.056(y_{I3} + y_{II3}) + 0.079y_{III4} +$$

$$0.116y_{III5} + 0.120y_{III6} + 0.157y_{III7} + 0.172y_{III8} + 0.204y_{III9} + 0.262y_{IV10}$$

$\leq$ 600 hours/month.

Use of Veneers to Form Plywoods

The finished veneers are formed into plywoods. As we have provided the cutting allowances in the log-veneer relationships, the veneer-plywood relationship is now very much simpler. The amounts and types of veneers required to form into any type of plywood depend on the veneer composition of that panel. And these are given in Table 10. For example, we want to determine the amount of veneers required to form, say, plywood type X_{I1}. From Table 9, X_{I1} is given as 4mm red surfaced plywood. From Table 10, the veneer composition of 4mm plywood are: 0.9 + 2.5 + 0.9mm. Since plywood X_{I1} is red surfaced, the face and back veneers, in this case, the two 0.9 mm veneers must be of red wood. We then refer to Table 11 for the symbols representing veneers. From the Table we know that 0.9mm red wood is represented by Y_{I1} and 2.5mm core by Y_{III5}. Therefore the equational relationship between X_{I1} and the veneers is:

$$2y_{I1} + y_{III5} = x_{I1}$$

As we work in unit of 1,000, the above equation means that to form X_{I1} units of plywood X_{I1} we require $2y_{I1}$ units of 0.9mm red surface veneer Y_{I1} plus y_{III5} units of 2.5mm core veneer Y_{III5}. The relationship between each of the other types of plywoods and veneers can be similarly determined. Thus the amount of plywoods that can be produced is limited by the amount of veneers available in the manner described by the equations. The fourth set of constraining functions impose upon the effort to maximise the objective functions will be as follows:

$$2y_{I1} + y_{III5} = x_{I1}$$

$$2y_{I3} + y_{III5} = x_{I2}$$

$$2y_{I2} + y_{III8} = x_{I3}$$

$$2y_{I2} + y_{III10} = x_{I4}$$

$$2y_{I2} + 2y_{III5} + y_{III4} = x_{I5}$$

$$2y_{I2} + 2y_{III5} + y_{III6} = x_{16}$$

$$2y_{I2} + 2y_{III5} + y_{III9} = x_{I7}$$

$$2y_{I2} + 2y_{III8} + y_{III9} = x_{18}$$

$$2y_{I2} + 3y_{III5} + 2y_{III7} = x_{19}$$

$$2y_{I2} + 3y_{III8} + 2y_{III7} = x_{I10}$$

$$2y_{III} + y_{III5} = x_{III}$$

$$2y_{II3} + y_{III5} = x_{II2}$$

$$2y_{II2} + y_{III8} = x_{II3}$$

$$2y_{II2} + y_{III10} = x_{II4}$$

$$2y_{II2} + 2y_{III5} + y_{III4} = x_{II5}$$

$$2y_{II2} + 2y_{III5} + y_{III6} = x_{II6}$$

$$2y_{II2} + 2y_{III5} + y_{III9} = x_{II7}$$

$$2y_{II2} + 2y_{III8} + y_{III9} = x_{II8}$$

$$2y_{II2} + 3y_{III5} + 2y_{III7} = x_{II9}$$

$$2y_{II2} + 3y_{III8} + 2y_{III7} = x_{II10}$$

Plywood Formation Capacity Constraints

To form veneers into plywood panels we need to use formation facilities. There are seven formation stages in the plywood formation department.[10] But we shall consider only the restrictions imposed by

[10] See figure 8, Chapter III.

two of them. There are the hot press and the sanding machine.

Like the lathe both the hot press and the sanding machine operate 200 hours a month. These volumes of services are to be allocated to the different types of plywoods according to the time required by each type of plywood. The hot press time requirement is a function of the plywood's thickness. The thicker the panel the longer the service time required. But sanding time is different. It is a function of surface area, thickness does not seem to have any significant bearing on the service time required. In other words, the time required for sanding service is independent of the thickness. The estimated time requirements on both the hot press and the sanding machine are as shown in Table 35.

TABLE 35

TIME REQUIREMENTS ON HOT PRESS AND SANDING MACHINE
(Hour per 1,000 square feet of surface measure)

Facility mm	4	4.7	5	6	8	9	10	12	15	18	Constraints
Hot Press	.056	.062	.064	.073	.101	.109	.118	.135	.172	.198	200
Sanding Machine	.026	.026	.026	.026	.026	.026	.026	.026	.026	.026	200

With the information given in Table 34 and that of Table 9 we can formulate the fifth set of constraining functions. These are as follows:

Hot Press

$$0.056(x_{I1}+x_{II2}) + 0.062(x_{I2} + x_{II2}) + 0.164(x_{I3} + x_{II3}) + 0.073(x_{I4}+x_{II4}) +$$
$$0.101(x_{I5}+x_{II5}) + 0.109(x_{I6}+x_{II6}) + 0.118(x_{I7}+x_{II7}) + 0.135(x_{I8}+x_{II8}) +$$
$$0.172(x_{I9}+x_{II9}) + 0.198(x_{I10}+x_{II10}) \leq 200 \text{ hours}$$

Sanding Machine

$$0.026(x_{I1} + x_{I2} + \ldots + x_{I9} + x_{I10} + x_{II1} + x_{II2} + \ldots + x_{II9} + x_{II10}) \leq 200 \text{ hours.}$$

Market Constraints

What we have been discussing thus far in this Chapter are the restrictions of the production capacities. To produce plywood is but part of the economic activity for to earn revenue the product produced must be sold. But there are certain maximum volumes of products that the firm can sell. On the other hand, the firm must produce certain minimum output to fulfill its market demands. The optimum product mix must not be less than the minimum sales but not more than the maximum sales. Therefore the profit maximising effort is constrained not only by the production capacities but also by the market constraints.

Actually, sales forecasting is one of the most, if not the most, important integrated activities of any business organization. Refined and scientific forecasting techniques are available to help management in forecasting sales more accurately. The techniques depend on extensive use of data, both internal and external.

However, in this exercise we shall not go into the technique of forecasting in determining the market constraints. This is because of two reasons. Firstly, we lack relevant data. Secondly, it is felt that sales forecasting is outside the scope of this paper. Yet to preserve reality of the problem, we need to consider the market constraints. To get out of this dilemma we shall once again turn to the historical data. The constraints will be adapted from the historical figures. These constraints will therefore be the final set of constraining functions on our problem. They are as follows:

500	$\leq$	x_{I1}	$\leq$	600
170	$\leq$	x_{I2}	$\leq$	204
45	$\leq$	x_{I3}	$\leq$	56
80	$\leq$	x_{I4}	$\leq$	100
0	$\leq$	x_{I5}	$\leq$	10
1	$\leq$	x_{I6}	$\leq$	10
3	$\leq$	x_{I7}	$\leq$	12
50	$\leq$	x_{I8}	$\leq$	60
0	$\leq$	x_{I9}	$\leq$	5
7	$\leq$	x_{I10}	$\leq$	10
480	$\leq$	x_{II1}	$\leq$	550
95	$\leq$	x_{II2}	$\leq$	120

8	$\leq$	x_{I3}	$\leq$	24
135	$\leq$	x_{II4}	$\leq$	175
4	$\leq$	x_{II5}	$\leq$	15
95	$\leq$	x_{II6}	$\leq$	115
1	$\leq$	x_{II7}	$\leq$	15
58	$\leq$	x_{II8}	$\leq$	70
10	$\leq$	x_{II9}	$\leq$	16
6	$\leq$	x_{II10}	$\leq$	

CHAPTER VI

CONCLUSION

The problem analysed in this paper is drawn from the plywood manufacturing industry. There is a peculiarity of the industry that allows the linear programming form to be a rather precise model for the problem.

A linear programming model consists of three parts.[1] The first part is the objective function whose value is to be maximised or minimised. The second part comprises the resources and market constraints. Lastly, there is the nonnegativity conditions on the variables. This nonnegativity conditions refer only to the variables and does not preclude the use of minus sign to denote subtraction of positive values. The nonnegativity conditions arise from the concept that there are no economic implications for negative values. Linear programming is a mathematical technique which is capable of accomodating both positive and negative values. So when the technique is applied to economic problems, such conditions would have to be imposed. The name of the technique itself suggests that all the functional relationships forming the model must be linear. This means that both the objective function and the constraining functions involve only the variable multiplied by constants. This linear model employs an assumption of fixed input and output and constant returns to scale in production. The analysis of the problem shows that this assumption is justifiable. In fact, the successful applications of linear programming depends on such reasonable assumption.[2]

To produce plywoods logs are first peeled into continuous sheets of veneer and then cut into the appropriate size. The veneers are dried, assembled, glued and bonded into plywoods of different types. The production process, beginning with cutting logs and ending in packing the plywoods, comprises 13 stages and can be conveniently divided into two departments: veneer production and plywood formation.

The major material used in plywood manufacturing is the log. To produce a unit of certain type of plywood panel requires several

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[1] Baumol, W.B., Economic Theory and Operations Analysis, Second Edition, Prentice Hall of India, New Delhi, 1966, p. 75.

[2] Saaty, T.L., Mathematical Methods of Operations Research McGraw Hill Book Company, Inc. New York, Toronto, London, 1959, p. 165.

units of veneers of different types. Each unit of veneers, in turn, requires a varying amount of log, depending on the efficiency of that log. The amounts of veneers obtainable from the logs and the plywoods from the veneers can be described by mathematical equations. These relationships at any level of production are likely to remain constant so long they are within the present capacity. To convert logs into veneers and to form veneers into plywood panels necessitate the use of machine services. The time requirements may vary with types of product produced. But they are unlikely to vary with the level of output within the present capacity. Suppose one unit of veneer requires, say, k hour of the machine time, 500 units of veneers would require 500 k hours, 1,000 units, 1,000 k hours. In short, the uses of both the material and machine times have constant returns to scale and hence enable their input-output relationships to assume linear form.

The amounts of resources - materials and machine capacities - available are known. The volume of output feasible is thus restricted by the resources available. This permits the resource constraints to be written as:

$$\sum_{aj} a_{iaj} \; x_{aj} \leq bi$$

Where Σ, sigma, represents the sum of the series, bi is the amount of resource type B_i available, x_{aj} is the amount of product type X_{aj} produced, and a_{iaj} is the amount of resource B_i used per unit of product X_{aj}.

The production resource is but one type of the constraints. We must also consider the market constraints. There are two levels of such constraints. On the one hand the factory must produce certain amount of plywoods to meet its market commitments. On the other hand, there are certain levels of output above which the firm may not be able to sell its products. The market constraints state that the amounts of products must be between certain limits. Mathematically, the market constraints can be written as:

$$c_{aj} \leq x_{aj} \leq d_{aj}$$

These limits, c_{aj} the lower limits and d_{aj} the upper limits, can be obtained from sales forecasting.

The objective function which we seek to maximise is also linear. In simplest form it can be written as:

$$\sum_{aj} R_{aj} \cdot x_{aj}$$

where R_{aj} is the net return or marginal contribution per unit of plywood X_{aj} and is given by

R_{aj} = (selling price) - (trade discount + marketing cost) - (variable cost).

This simple form is, however, unsatisfactory for two reasons. Firstly, the simple form shows only the product mix in terms of final products. It is unable to provide the management with ample information for decision making. Secondly, it is relatively difficult to estimate production cost. Nonetheless, the simple form of the objective function can be easily modified to include three parts. The first part seeks the optimal plywood mix, the second part the veneer mix that are required to form the plywoods stated in the first part. The third part shows the volume of logs required to produce the veneers. The modified form of objective function will take the following form :

$$\text{Maximise } \Sigma(k_{aj} - l_{aj})\, x_{aj} - m_{bk} \cdot y_{bk} - \Sigma n_c \cdot z_c$$

where k_{aj} is the ex-factory selling price per unit of plywood type X_{aj}; m_{bk} is the variable cost of producing per unit of veneer type Y_{bk}; and n_c the cost of per unit of log type Z. x_{aj} and y_{bk} are the amounts of plywoods X_{aj} and veneers Y_{bk} respectively, to produce in order to obtain maximum profit. z_c is the amount of log Z_c to be consumed in order to produce the optimal product mix. The minus signs are included to denote cost minimising. In other words, the objective is to maximise the difference between the revenues on one hand and the costs on the other.

We may note that one type of product is represented by one variable. The number of variables in the model will equal to the number of types of products produced. In any actual situation there are bound to be numerous types of plywoods. To incorporate all of them alone will require an equal number of variables. Besides we have to include the variables representing veneers and logs. This would inevitably render the dimension of the model very large and is beyond the capacity of many but the largest electronic computers. We need to keep the model small. After all a model needs only to incorporate the essential features. To keep the model small, we eliminate the nonessential features. The product classifications have to be reviewed and their number reduced. By this the number of types of product can be reduced. In the problem analysed there are as many as 960 different types of plywoods. As a result of elimination the number is able to be reduced to only 20. Associated with these types of plywoods there are 13 types of veneers and 7 types of logs. Thus the model comprises 40 variables.

The number of production constraints depends on the number of types of resources to be included in the model. Again certain types of resources can be omitted from the model because they are either trivial or less strict. Provided the omission is judicious no significant loss in accuracy is likely.

Linear programming is entirely a mathematical technique.[3] Its economic content is therefore nil. It means that the technique

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[3] Baumol, W.B., op. cit. p. 70.

per se can never tell us anything about the economic operation. It can, nevertheless, help us to find the implications of the economic information which we already have or willing to assume. Linear programming is a systematic, mathematical technique of searching for the "best" values of the variables. There are many economic and business problems involving this searching. Any problem that can be formed into a linear model can use the linear programming technique as a tool. To solve the problem we first translate it into a model which can then be solved mathematically. The economic implications can then be derived from the mathematical solution of the model. The industrial applications of linear programming are widely found in the petroleum industry and the feed industry. There are also many other industries in which the applications have been less extensive. Plywood manufacturing is one of the industries that linear programming can be, but less known, profitably applied.[4] In this exercise the analysis shown that the plywood product mix problem can be formulated into a precise linear programming model and therefore can be solved accordingly.

Linear programming together with the electronic computer thus can be a quick, powerful and precise technique for obtaining an optimal solution. The computed solution can reduce cost, thus increases profits.[5]

However, the dependability and precision of the results it yields depend on the accuracy and completeness of the input data. The imperfect knowledge of log efficiency and the wide variation in the moisture contents are some of the weaknesses.

It is often argued that because of these the input data are often incomplete and inaccurate that the use of such a precise technique as linear programming is not justified.[6] This same argument applies, however, to all other methods of determining product mix. The major advantages in using linear programming is in the knowledge that the product mix is in fact the optimal one, it is quick to provide a solution to a complex problem.

As to the accuracy and completeness of the input data, this is exactly what is implied in suggesting the importance of model formulating stage. No doubt difficulties and limitations exist but with research and experience they can be at least reduced, if not eliminated. There is wide scope for such research in this field in Malaysia where plywood manufacturing is becoming an increasing important industry.

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[4]Koenigsberg, E., Some Industrial Applications of Linear Programming, in Operational Research Quarterly, Volume 12 No. 2 June 1961, p. 105.

[5]Ibid. p. 108.

[6]Taylor, N.W. The Use of Linear Programming in Least-Cost Feed Compounding, Agricultural Economics Research Unit, Lincoln College, New Zealand, No. 20, 1965, p. 18.

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APPENDIX I

PLYWOOD PRODUCTION SCHEDULING PROBLEM - QUESTIONNAIRE

I. PRODUCTION CAPACITY No. of Hours Required to produce 1,000 square feet of surface measure of:

a) Veneer mm	0.9	1.0	1.3	1.8	2.5	2.6	3.2	3.4	3.8	4.4	Total Available
Lathe											
Dryer											

b) Panel mm	4	4.7	5	6	8	9	10	12	15	18	Total Available
Press											
Sanding											

II. EFFICIENCY IN THE USE OF LOGS

Species					
Surface					
Core					

III. AVAILABILITY OF LOGS

Species					
Ton					

IV. COST DATA FOR THE MONTH

Factory Variable Cost

Direct Material		Direct Labour	Other Expenses
log	glue		
$	$	$	$

Log Consumed

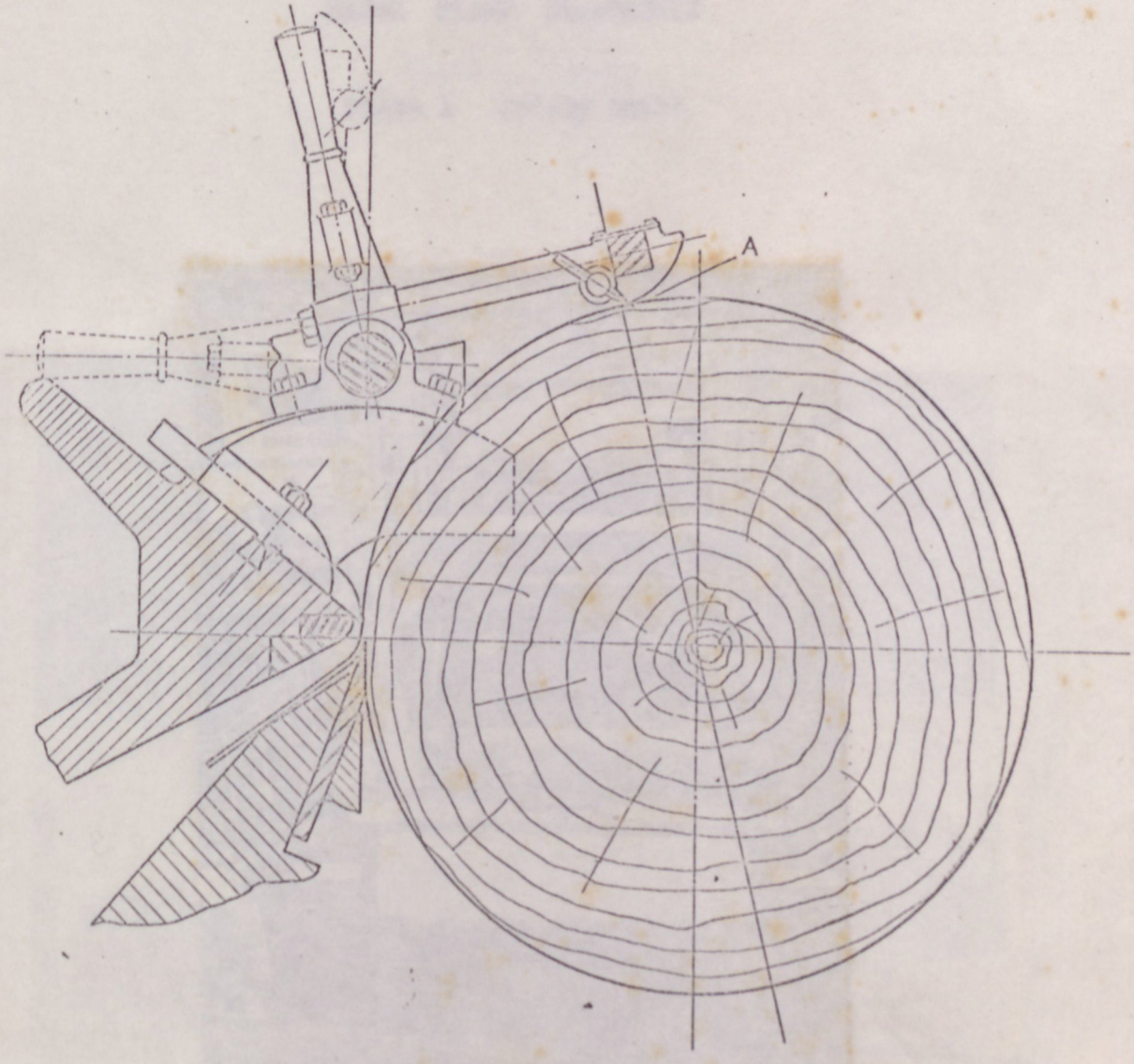
Species					
Ton					
Price					

Plywood Produced during the month

Panel mm	4	4.7	5	6	8	9	10	12	15	18
Red										
White										

APPENDIX II

RELATIVE POSITION OF KNIFE AND LOG



[By courtesy of Coe Manufacturing Co.

FIG. 25.

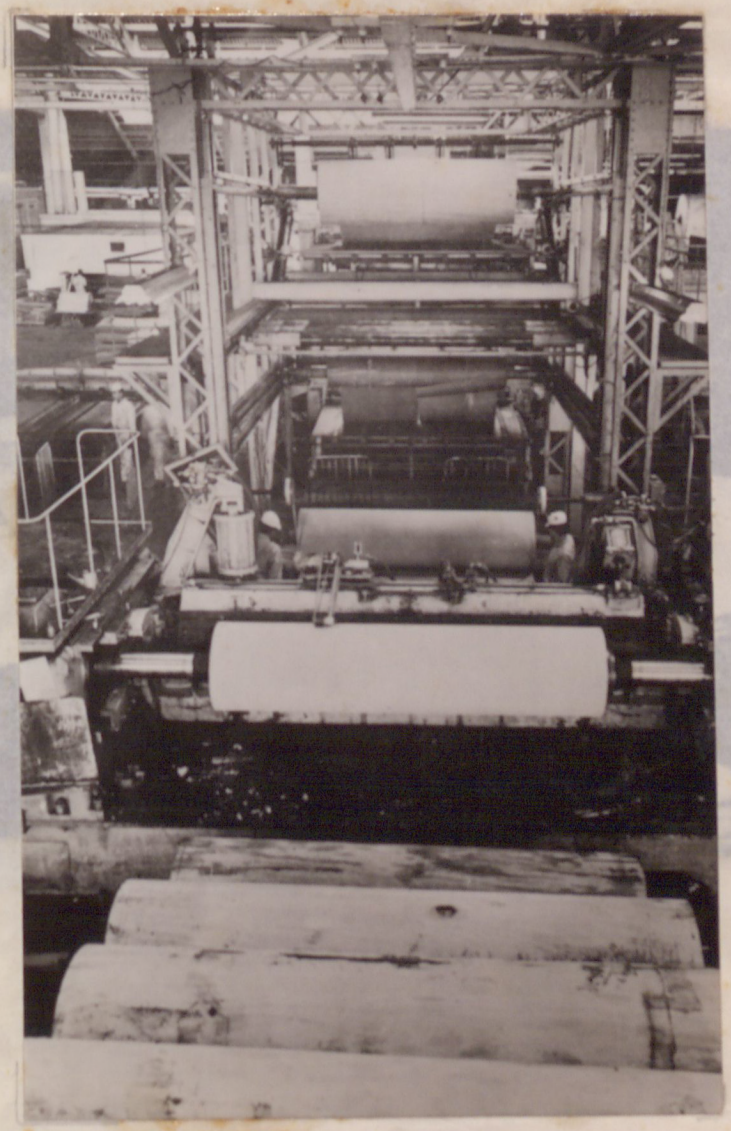
This diagrammatic drawing indicates the relative positions of knife and pressure bar. One of the spur knives (A), which cuts the veneer to exact length, is shown in position at the top of the block. A similar device, replacing the knife by a crayon, may be used to mark the tight-cut side of the veneer.

Source: Wood, A.D. Plywoods of the World: The Development, Manufacture and Application, W. and A.K. Johnston and G.W. Bacon Ltd., Edinburgh and London, 1963.

APPENDIX III

MAJOR PLANT EQUIPMENTS

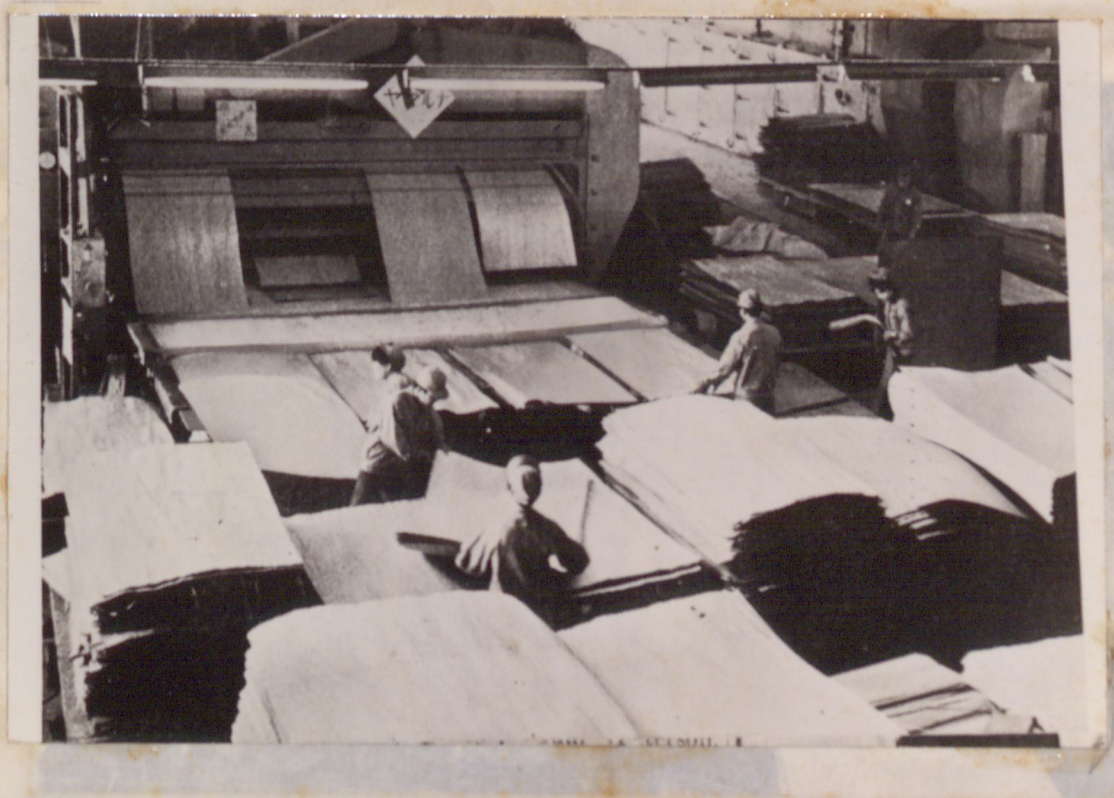
Plate 1 Rotary Lathe



APPENDIX III

MAJOR PLANT EQUIPMENTS

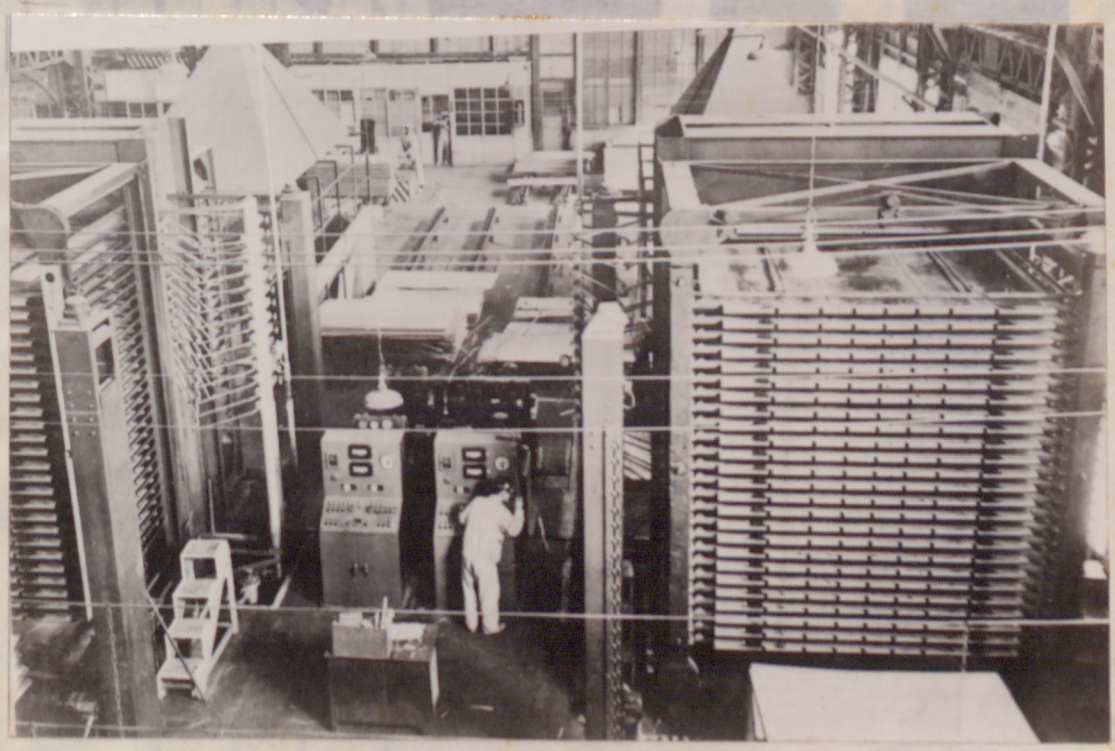
Plate 2 Dryer



APPENDIX III

MAJOR PLANT EQUIPMENTS

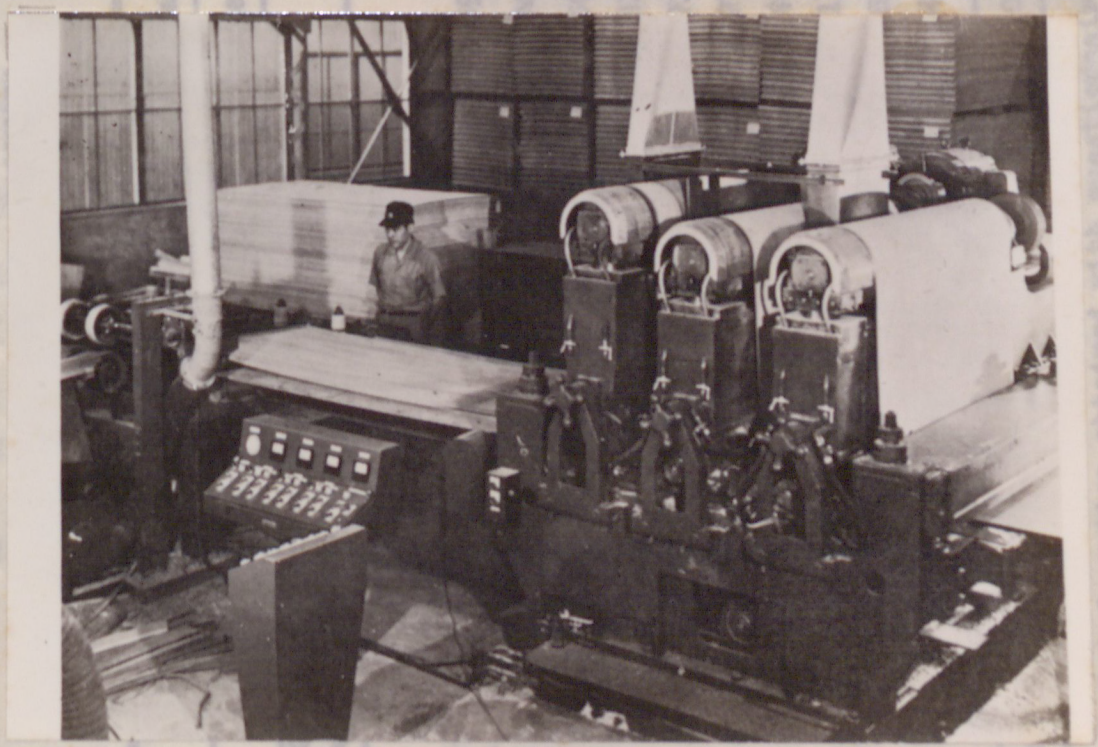
Plate 3 Hot Press



APPENDIX III

MAJOR PLANT EQUIPMENTS

Plate 4 Sanding Machine



APPENDIX IV

AMOUNT OF PLYWOOD PANELS AND VENEERS PRODUCED
(in square feet)

Plywood Thickness (mm)	Plywood Panel				Equivalent Surface Area in 1 mm.	Veneer				Equivalent Surface Area in 1 mm.
	Surface Measure					Surface Measure				
	Red	White	Total	Glued Surface		Red	White	Core	Total	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
4	511,800	486,447	998,247	1,996,494	3,992,988	1,023,600	972,894	998,247	2,994,741	4,292,462
4.7	183,504	97,520	281,024	562,048	1,220,813	367,008	195,040	281,024	843,072	1,433,222
5	47,328	7,680	55,008	110,016	275,040	94,656	15,360	55,008	165,024	297,043
6	85,728	136,958	222,686	445,372	1,336,116	171,456	273,916	222,686	668,058	1,409,887
8	-	3,362	3,362	13,448	26,896	-	6,724	10,086	16,810	29,586
9	2,286	98,137	100,423	401,692	903,807	4,572	196,274	301,269	502,115	964,061
10	3,360	720	4,080	16,320	40,800	6,720	1,440	12,240	20,400	44,064
12	51,744	59,078	110,822	443,288	1,329,864	103,488	118,156	332,466	554,110	1,396,357
15	-	9,408	9,408	56,448	141,120	-	18,816	45,240	65,856	149,587
18	8,000	5,728	13,728	82,368	247,104	16,000	11,456	68,640	96,096	255,341
Total	893,750	905,038	1,798,788	4,127,494	9,614,548	1,787,500	1,810,076	2,328,706	5,926,282	10,271,410

Computed by author from information obtained with questionnaire in Appendix I.

APPENDIX V

GLOSSARY OF TERMS

Bark - The natural exterior, or rough skin of the tree.

Bond - The adhesion of two surfaces together by the action of glue in curing.

Clipper - Guillotine used to cut veneers to length or width.

Constant return to scale - Linear homogeneous production function where same proportional increase in all the inputs will result in a proportion increase in output.

Constraint function - Inequality or equality relationship which states that no more of the outputs can be manufactured than the available resource permits.

Cutting allowance - The excess of the area of the cutting size (veneers) over the size of the plywood panel into which the veneers are formed.

Cutting combination - The manner and uses log of given length is to be cut into.

Cutting size - The size in which the raw veneer is to be cut in order to be formed into the related size of plywood panel.

Core - Inner layer or layers in plywood construction.

Cost behavior - The manner costs vary with the level of activity.

Cost centre - Basic organization unit for cost accumulation or assigning.

Costing - The process of determining cost.

Defect - Any fault, or imperfection, in the veneer or finished panel.

Dryer - The kiln or chamber, through which veneers are passed to remove excess moisture.

Efficiency factor	- The percentage of the log (in volume) that can be turned into veneers.
Feasible area	- An area in which every point which lies within it or on its boundary represents a combination of values of the variables which does not violate the constraints.
Glueing	- The process of uniting by means of an adhesive two or more pieces of wood characterised by continuity of the union over the whole of the areas of contact.
Grain	- A term used to indicate the generation or arrangement of the fibres and other wood elements.
Graphical method	- A method of solution using geometrical concept.
Lathe	- An equipment used for peeling log into veneers.
Marginal contribution	- The excess of the sales revenue over the variable costs of a unit of product
Mathematical statement	- The relationship between variables written in mathematical symbols.
Model	- Symbolic representation which is supposed to incorporate, in some sense, the essential feature of the actual problem.
Moisture content	- The amount of moisture in timber expressed as a percentage of its oven dry weight.
Objective function	- The function (profits or costs) whose value is to be maximised or minimised
Panel	- Referring to a sheet of plywood
Ply	- An individual layer either the core or surface.
Plywood	- A product in which several plies or pieces of veneers are glued to each other; the chief characteristic being the cross layers which distribute the longitudinal wood strength .
Press	- The machine in which uniform pressure is applied to the panels after glue is spread to bring the sheets of veneer into intimate contact.

Product classification	- The process of classifying the products into various types and kinds.
Product mix	- The amounts and types of different product to be produced.
Programmer	- One who translates the mathematical statements of a model into a programme according to a set of instructions understandable by the computer.
Quality log	- Good quality logs refer to those that are suitable for making surface veneers. Poor quality logs are those, though of same species and purchased together with the good quality logs, because of certain defects, that are suitable only for making core veneers.
Rotary cut	- A manner of cutting veneer by which a log is mounted in a lathe and turned against a broad cutting knife which is set into the log at a slight angle.
Sanding machine	- A machine with sanding paper attached to smoothen the surface of the panels.
Simplex method	- A systematic procedure of investigating the basic solution (equivalent to solutions at the corners in the graphical method) by following a set of vigorous algebraic rules. The value of the simplex method is for large scale problem where there is no graphic equivalent.
Split	- In veneer a separation of the fibres along the grain extending through the thickness of the veneer.
Surface measure	- The area of the surface of the veneers or plywoods.
Service time	- The time required to process one unit of a given product.
Tape	- The strip of gummed paper or cloth, used to hold the edges of veneer together previous to glueing.

Texture	- Surface characteristics of a piece of veneer or wood.
Time requirement	- Service time required.
Variable	- An unknown value of a certain type of product, resource or factor.
Veneer	- A thin sheet of wood produced by rotary cut.