UTILITY-BASED NON-COOPERATIVE POWER CONTROL GAME IN WIRELESS ENVIRONMENT

YOUSEF ALI MOHAMMED AL-GUMAEI

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Name of Candidate: YOUSEF ALI MOHAMMED AL-GUMAEI

(I.C/Passport No: **04283083**)

Registration/Matric No: KHA120038

Name of Degree: Doctor of Philosophy

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ABSTRACT

The spectrum resources, interference, and energy of battery-based devices are predominant problems and challenges in modern wireless networks. This thesis therefore addresses these issues by studying a theoretical framework for the design and analysis of distributed power control algorithms for modern cognitive radio and femtocell networks. It is shown that game theory tools are appropriate and efficient to develop scalable, balanced and energy-efficient, distributed power control schemes to be practically used in battery-based devices in wireless networks.

Practically, the problem of power control is modelled as a non-cooperative game in which each user chooses its transmit power to maximize (or minimize) its own utility (or cost). The utility is defined as the ratio of throughput to transmit power, which is used to represent the energy efficiency scheme, whereas the cost is defined as the sum of the sigmoid weighting of transmit power and the square of the signal-to-interference ratio (SIR) error which is used to represent the SIR balancing scheme. Novel utility and cost functions proposed in this work are the method to derive efficient distributed power control algorithms. Also, the proposed pricing techniques in this thesis guide users to the efficient Nash equilibrium point by encouraging them to use network resources efficiently. These frameworks are more general and they are applied on cognitive and femtocell networks due to the critical and important issue of interference.

Numerical simulations are used to prove the effectiveness of these algorithms compared with other existing power control algorithms. The simulated analytical and numerical results of this thesis indicate that the proposed algorithms can achieve a significant reduction of the user's transmit power and thus a mitigation of the overall interference. Moreover, these algorithms have a relatively fast convergence rate and guarantee that all users can achieve their required QoS.

ABSTRAK

Sumber spektrum, gangguan, dan tenaga peranti yang berasaskan bateri adalah masalah dan cabaran yang mendominasi rangkaian moden tanpa wayar. Oleh itu, tesis ini menangani isu-isu ini dengan mengkaji rangka kerja teoritikal untuk merekabentuk dan menganalisis algoritma agihan kawalan kuasa untuk rangkaian radio kognitif dan femto sel moden. Ia menunjukkan bahawa alat-alat teori permainan yang sesuai dan berkesan untuk membangunkan pengagihan skim kawalan kuasa boleh skala, seimbang, cekap tenaga, yang praktikal untuk digunakan dalam peranti yang berasaskan bateri dalam rangkaian tanpa wayar.

Secara praktik, masalah kawalan kuasa dimodelkan sebagai satu permainan yang bukan koperatif di mana setiap pengguna memilih kuasa penghantar untuk memaksimumkan (atau mengurangkan) utiliti sendiri (atau kos). Utiliti ditakrifkan sebagai nisbah pemprosesan untuk menghantar kuasa, yang digunakan untuk mewakili skim kecekapan tenaga, manakala kos ditakrifkan sebagai jumlah penghantaran pemberat sigmoid kuasa dan isyarat kuasa dua kepada gangguan kesilapan (SIR) yang digunakan untuk mewakili skim pengimbangan SIR.Satu utiliti dan kos fungsi yang novel dicadangkan dalam kerja-kerja ini adalah merupakan satu kaedah untuk memandu algoritma pengagihan kawalan kuasa diagihkan yang cekap. Selain itu, teknik harga yang dicadangkan di dalam tesis ini membolehkan pengguna menuju ke titik keseimbangan Nash yang cekap dengan menggalakkan mereka menggunakan sumber rangkaian yang sangat cekap. Rangka kerja ini adalah lebih umum dan ia diaplikasikan didalam rangkaian kognitif dan femto sel disebabkan oleh isu yang kritikal dan penting di dalam gangguan.

Simulasi berangka digunakan untuk membolehkan keberkesanan algoritma ini dibandingkan dengan algoritma kawalan kuasa lain yang sedia ada. Simulasi analisis

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dan keputusan berangka di dalam tesis ini menunjukkan bahawa algoritma yang dicadangkan boleh mencapai pengurangan yang ketara dari segi penghantaran kuasa oleh pengguna dan dengan itu meringankan gangguan keseluruhan. Selain itu, algoritma ini mempunyai kadar penumpuan yang agak cepat dan dijamin bahawa semua pengguna boleh mencapai QoS yang mereka diperlukan.

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LIST OF SYMBOLS AND ABBREVIATIONS

QoS	:	Quality of Service
SIR	:	Signal to interference Ratio
SINR	:	Signal to interference and noise ratio
RRM	:	Radio Resource Management
PC	:	Power control
MS	:	Mobile Station
AWGN	:	Additive White Gaussian Noise
DPC	:	Distributed Power Control
DB	:	Distributed Balancing
CDPC	:	Constrained Distributed Power Control
LQ	:	Linear Quadratic
GSM	:	Global System for Mobile communication
CDMA	:	Code Division Multiple Access
PSR	:	Packet Success Rate
BER	:	Bit Error Rate
PDR	:	Packet delivary ratio
FSK	:	Frequency Shift Keying
BPSK	:	Binary Phase Shift Keying
DPSK	:	Differential Phase Shift Keying
PU	:	Primary user
MU	:	Macrocell user
CR	:	Cognitive radio
SU	:	Secondary user
FU	:	Femtocell user

CRN	:	Cognitive radio networks
BS	:	Base station
HBS	:	Home base station
DSL	:	Digital subscriber line
SINR	:	Signal to interference and noise ratio
BS	:	Base station
IP	:	Internet protocol
WiMAX	:	Worldwide Interoperability for Microwave Access
MUE	:	Macrcell user equipment
FUE	:	Femetocell user equipment
IS-95	:	Interim standard
NPG	:	Non-cooperative power game
NPGP	:	Non-cooperative power game with pricing
EF-NPGP	:	Energy Efficient Non-cooperative power game with pricing
PAP	:	Primary access point
CBS	:	Cognitive base station
MSFLA	:	Modified shuffled frog leaping algorithm
ESIA	:	Efficient swarm intelligent algorithm
RF	:	Radio frequency
U	:	Utility function
r	:	Strategy
Φ	:	A game in normal form
a _i	:	Action of player <i>i</i>
a_{-i}	:	Action of the other $i-1$ players
$\{A_i\}$:	Strategy set of user <i>i</i>

$\{u_i\}$: The set of the utility functions
Ν	: A finite set of player or mobile users
${g}_{ij}$: An effective link gain from user j to the base station i
σ^{2}	: additive white Gaussian noise (AWGN)
γ_i	: Signal to interference ratio of user i at the base station
I_i	: the total interference of all users except user i
p_i	: Transmit power level of user <i>i</i> in Watt
$p_i^{(k)}$: Transmit power at <i>k</i> th time step
$p_i^{(k+1)}$: Transmit power of user i at $(k+1)$ th time step
$p_i^{(\max)}$: Maximum power constrain of user <i>i</i>
$\gamma_i^{(k)}$: SIR of user <i>i</i> at (<i>k</i>)th time step
$\gamma_i^{(k+1)}$: SIR of user <i>i</i> at $(k+1)$ th time step
γ^{min}	: Minumum value of SIR
γ^{max}	: Maximum value of SIR
C_1, C_2, C_3	: Positives pricing factors
$g_{0,i}$: Link gain from cognitive user i to the PU measurement point.
I _{TL}	: Interference temperature limit
Γ	: Target SIR
k	: Bolzmanns constant
β	: Smaller controller parameter
ξ	: A constant parameter used in CDPC-II
w^k	: A non-increasing sequence of control parameter
$O_i^{(k)}$: Input of each subsystem in LQ power control

$S^{(k+1)}$: Subsystem of mobile-base station connection at $(k + 1)$ th time step
t _i	: Time instant
dB	: Power ratio in decibel
L	: Information bits
М	: Length of packet
$ heta_i$: Ratio of interference to channel gain of user <i>i</i>
а	: Sigmoid parameter
$\sigma(.)$: Sigmoid function
Т	: Throughput
R_{i}	: Transmit rate of user <i>i</i>
$e_i(\mathbf{p})$: SIR error
γ_i^*	: SIR value of user <i>i</i> at Nash equilibrium
p_i^*	: Power value of user <i>i</i> at Nash equilibrium
q	: Exponential parameter
$f(\gamma_i)$: Efficiency function
P_{e}	: Bit error rate (BER)
P_{c}	: Frame success rate
В	: Bandwidth
<i>u</i> _i	: Utility function of user <i>i</i>
u_i^c	: Utility function of user i with pricing
NU	: Net utility
p_{-i}	: Vector of power levels of all users except user i
η_i	: Background noise
$oldsymbol{J}_i$: Cost function of user <i>i</i>

$a,b,c,\ \lambda$: Weighting factors
W	: Watt
R_c, R_f	: Radius of macrocell and femtocell respectively
C_m, C_f	: Set of MUEs and FUEs respectively.
С	: Set of all Macrocell and Femtocell users
d_i	: Distance between user i and base station
α	: Path loss exponent
h_i	: Attenuation from <i>i</i> th mobile station to the base station
Γ_i^m	: Target SIR of Macrocell user i
$\frac{\gamma_i^{f}}{f}$: Threshold SIR of FUEs
$u_i'(\gamma_i)$: Derivative of utility function
$v_i'(p_i)$: Derivative of price function
J_i^m	: Cost function of the <i>i</i> th MUE
J_i^{f}	: Cost function of the <i>i</i> th FUE
$\hat{\gamma}_i$: Optimal SIR of the <i>i</i> th user
\overline{p}^{m}	: Average power of MUEs
\overline{P}^{f}	: Average power of FUEs
$ar{\gamma}^{f}$: Average SIR of FUEs
N_{m}	: Number of MUEs
N_{f}	: Number of FUEs

CHAPTER 1: INTRODUCTION

1.1 Introduction

Since the beginning of wireless communications, radio resource management (RRM) has emerged as a required framework in network design. The main purpose of RRM is to achieve the wireless network objectives (application or services requirements) under certain radio resources constraints (e.g. radio spectrum resources, transmission rate, transmission power). One of the most important components of RRM is power control, whose principal purpose is to achieve better signal quality for each user without causing excessive interference to other users in the wireless system. An increase in co-channel interference (resulting from spectrum sharing) causes major channel impairments and thus leads to a further deterioration in the performance of wireless communications.

Recently, the problem of power control was considered in the cognitive radio networks and femtocell networks based game theory with a new design of utility and cost functions. Zhang et al. (2012) and his co-authors introduced the concept of the target SIR to modify the utility, and they modified the Shuffled Frog Leaping Algorithm (SFLA) to improve the accuracy of the power control solution. In other work, power control game proposed by Kuo et al., (2013) has been improved based on the outage probability of the PU in a spectrum-underlay CRN and they designed an efficient swarm intelligent algorithm to improve the convergence speed and improve the energyefficiency. A payment-based power control scheme based on a non-cooperative power control game has been considered by Xie et al. (2014), in which the distance of the CRs and SIR were used as a reference for punishment price setting.

The problem of power control also considered based on game theory and using cost functions in cognitive radio networks. Li et al. (2011) proposed a new power control game based on the cost function and the target of the transmit power has been included, such as the target SIR. The cost function in Li et al. (2011) is defined as a weighted sum of the logarithm of SIR error and the logarithm of power error. The algorithm has many advantages, such as fast convergence, better anti-noise performance and capacity. Lu et al. (2012) proposed a power control game in CDMA cognitive radio networks based on the cost function and they used two SIR thresholds in order to adjust the interference factor of power and improve the fairness. In another work, Junhui et al., (2013) proposed a non-cooperative power control algorithm based on the cost function similar to Li et al. (2011) by using the square root function instead of the logarithm to fast algorithm. Jiao et al., (2013) investigated how to decide the transmission power levels of cognitive radio users using non-cooperative game theory. They proposed a novel cost function, in which the thresholds of the SINR and transmission power level were considered. Their algorithm's numerical results indicated better performance in terms of anti-noise.

On the other hand, to mitigate cross-tier interference and guarantee QoS for both MUEs and FUEs, an efficient distributed power control is necessary in both systems. Power control has been considered in femtocell networks to mitigate the cross-tier interference between two-tier (Kang et al., 2012; Ngo et al., 2012; Han et al., 2013; Xie et al., 2013). Moreover, the game theory-based power control has been considered for HetNet small cell networks (Xu et al., 2014), in which the authors addressed the mitigation of the cross-tier interference problem by ensuring the protection of macrocell users.

Since the emergence of third-generation cellular networks, the sudden rise in the number of mobiles that can use high speed data services and applications has produced an ever increasing demand for reliable high-speed data services. This market has led to the emergence of many techniques that can utilize the spectrum resources and mitigate interference. Cognitive radios (CRs) have emerged as a promising solution for the spectrum utilization problem, due to their ability to access and utilize the unused parts of the licensed spectrum without causing harmful interference. In addition, the femtocell has also emerged as another solution for spectrum sharing, short range, and low power data transmission.

Data communications (non-real time applications) like web browsing and file downloads are error sensitive and delay tolerant which requires a larger SIR. Increasing the SIR guarantees that the information data will be delivered to the receiver correctly, and this will decrease the number of retransmissions. Therefore, the level of satisfaction achieved by each user is a continuous function of the SIR (Goodman & Mandayam, 2000). In this thesis, we focus on the design of utility (cost) function criteria for the power control design, which aims to mitigate interference during spectrum sharing by reducing the power consumed by cognitive radio or femtocell users. In addition, reduction of the power consumed will lead to a more extended battery-power life of terminals. In a distributed power control, all users update their power level based on local information. These objectives can be achieved by representing the network using *game theory*, wherein the users select their transmit power level according to a costminimization or (utility maximization) criterion. In this case, the distributed power control is represented by a *non-cooperative game* and the available strategy for each user is the power strategy. Game theory has been considered as an effective and useful mathematical tool to study distributed power control in wireless data networks. Power control algorithms resulting from the game-theoretic approach are decentralized, in which each individual user select its own transmit power from a transmit power strategy through a noncooperative scheme. In a distribution (non-cooperative) scheme, users update their transmit power using limited local information, so the outcome of the game is suboptimal compared with those obtained via centralized schemes. To overcome the suboptimal problem, pricing or referee approaches have been proposed. On the other hand, a distributed scheme is more scalable and is thus practically used in large wireless networks.

The next chapter presents a literature review on the power control techniques from voice cellular networks and then in wireless data and modern networks that are based on non-cooperative game theory. In this thesis, we will address the problem of a power control game for modern cognitive radio and femtocell networks.

1.2 Objective of study

This thesis is motivated on the application of a non-cooperative game theory framework to distributed power control in the cognitive radio and femtocell wireless communication networks. Due to the spectrum sharing between different classes of mobile users, we propose efficient power control algorithms to reduce the power consumed, mitigate cross-interference, and improve spectrum utilization.

In this context, the objectives of this thesis are as follows:

• To design efficient frameworks for distributed power control to improve the present results available in the literature for SIR-based and energy-efficient approaches that can be applied in modern wireless networks.

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- To introduce appropriate utility and cost functions that have a physical meaning to improve spectrum sharing, mitigate cross interference, and reduce the power consumed.
- To prove analytically the existence and uniqueness of the solution (referred to the Nash equilibrium) of the non-cooperative power control games.
- To derive theoretically the distributed and iterative algorithms of a power control scheme that can converge to the Nash equilibrium solution and present the results by numerical simulations.

1.3 Research methodology

In modern wireless communication networks, the spectrum resources and interference should be managed efficiently to cope with the increase of users and services. In fact, each user in a wireless network represents a competitor for network resources, and is trying to satisfy its own QoS requirement by choosing the best response action. These actions (strategy) could involve the transmission power, transmission rate, modulation, packet size, etc. Therefore, the action chosen by any user will affect the performance of other users. The user's QoS can be referred to as the utility or cost function, in which each user tries to choose its transmit power action to maximize (minimize) its utility (cost) function.

The cognitive radio and femtocell communication networks studied in this research are assumed to consist of three basic elements:

- i. Users of the network represented by cognitive radio (CR) users, femtocell user equipments (FUEs) or macrocell user equipments (MUEs)
- ii. An action set (strategy set) that represents the network resources such as transmits power.
- iii. The utility function that measures the preference of the user.

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A game theory tool is proposed in this thesis to exemplify the problem of power control in the cognitive radio and femtocell communication networks. The users of the network are the players (decision makers) of the game, the transmit-power levels represent the strategy action of the game, and the users' utility function represents the utility of the game.

In this thesis, the problem of power control based on a game theoretic framework is presented in three different approaches.

In the first approach, we used control theory concepts for SIR balancing to propose a new sigmoid-based cost function for cognitive radios to derive the power control game. In this scheme, each CR tries to minimize its own cost by achieving the target SIR with flexible reduction. We proved the existence and uniqueness of the Nash equilibrium and the convergence of the proposed algorithm. The sigmoid function applied to the price part of the cost function induced the proposed power control algorithm to guide users to better performance in terms of SIR and power consumption.

In the second approach, we used the energy efficiency scheme to design the power control game for cognitive radios. A novel energy efficiency utility function (utilityprice) has been proposed which represents the amount of information bits that are successfully transmitted per joule of energy consumed. Also, each CR tries to maximize its own utility function with some constraints, similarly to an optimization problem. The proposed algorithm shows quick convergence and better performance compare with related works.

In the last approach, we proposed an efficient power control and interference management for two-tier femtocell networks, with different classes of users, where MUEs represent high priority access users and FUEs represent low priority access users. The utility function is assigned to each type of user depending on the QoS requirement. The goal is to mitigate cross-tier interference between two tier networks. The local term that is introduced into the utility function of the femtocell user is the key to the better performance of our proposed algorithm.

In all approaches, we proved the existence and uniqueness of the Nash equilibrium and the convergence of our proposed algorithms. We used the same system parameters during the comparison between our proposed algorithms and the previous works in the literature.

The numerical analysis and simulation of the proposed power control algorithms were generated using MATLAB codes.

1.4 Thesis organization

This thesis is organized as follows.

In chapter 2, we introduce a brief review on game theory, in which the most important concepts used in this thesis (non-cooperative game, strategy game, utility function, dominant strategy, Nash equilibrium, and Pareto optimality) are defined. Furthermore, an example of the game theory "prisoner's dilemma" is given to illustrate the basic game model, the output of the Nash equilibrium and Pareto optimality. Next, we review the proposed algorithms of distributed power control in cellular radio networks. Subsequently, the difference between the utility function used in the wireless voice system and wireless data system is explained. After that, power control algorithms for wireless data networks and based on game theory are reviewed according to the utility or cost function that is applied. Lastly, literature works on power control in the modern wireless network (cognitive radio and femtocell networks) based on game theory are reviewed to provide the foundation of the work presented in this thesis. In chapter 3, we introduce a new SIR-based sigmoid power control in cognitive radio networks based on non-cooperative game. After discussing and explaining the differences between the design of distributed power control in the control theory and game theory perspectives, we introduce a new cost function for a power control game based on a sigmoid function. We then study the existence, uniqueness, and the convergence of the proposed game, and we formulate an iterative power update algorithm to reach the resulting Nash equilibrium depending only on local information (distribution). In addition, we improve the speed of convergence of the sigmoid power control algorithm using Newton's numerical method.

In chapter 4, we formulate the power control scheme using an energy-efficient approach in which the objective is to maximize the number of transmitted bits per energy. A novel utility function via pricing has been proposed which guides cognitive radios to the efficient Nash equilibrium. The proposed power control algorithm guides the CRs closest to the base station to achieve their QoS requirement with a low cost, whereas it guides the CRs farthest from the base station to achieve their QoS requirement with a high cost to reduce the amount of interference. We prove the existence and uniqueness of the Nash equilibrium of the power control game and the conditions of the selected pricing factors. Furthermore, we explain the difference between the linear and power functions with the pricing function, and the effect of a weighting factor on the utility function and transmit power. The simulation results have been compared with different and recent energy efficient power control algorithms to show the effectiveness of the proposed algorithm.

In chapter 5, we present a new power control scheme for distributed interference management in two-tier femtocell networks. The objective of the algorithm is to guarantee that higher priority users (MUEs) achieve their required QoS, whereas lower priority users (FUEs) demand certain QoS requirements. In this scheme, two classes of users are acting as decision makers with different utility functions. We prove analytically the convergence of the algorithm and the features of the proposed algorithm are confirmed through a comparison with previous work that used the traditional Norm-2 algorithm.

In chapter 6, we summarize and conclude this thesis and we discuss future works and further perspectives for this area of research.

CHAPTER 2: LITERATURE REVIEW

In the past decade, the concepts of game theory applied in wireless communication systems have increased dramatically, being used to solve a variety of problems in the networks (Lasaulce & Tembine, 2011; Zhang & Guizani, 2011). Many resource allocation problems can be solved and utilized based on game theory in different scenarios and the problems considered include one or more wireless issues, such as bandwidth allocation, rate control, power control, flow control, routing, and medium access control (MacKenzie & DaSilva, 2006). Game theory has been launched and applied in wireless data networks, and then it has also been extensively applied in modern networks such as cognitive radio and femtocell networks. It is used to solve spectrum management and spectrum sharing issues in cognitive radio networks (Wang et al., 2008; Ji & Liu, 2007), as well as the cross-tier interference, spectrum sharing and energy issues in femtocell networks (Kang et al., 2012; Ngo et al., 2012; Han et al., 2013; Xie et al., 2013).

In this chapter, a brief overview of non-cooperative game theory fundamentals and concepts relevant to this thesis is presented. The chapter begins with an introduction to non-cooperative game theory, and brief definitions of the most important concepts of non-cooperative games, such as utility, strategy games, dominant strategies, etc. This is followed by an example to illustrate and analyse the behaviour of decision makers in the non-cooperative game. In the next subsection, the development of power control from voice cellular networks to wireless data networks is reviewed, and we highlight the concept of utility and cost functions that has been used to represent quality of service

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(QoS) in wireless data as a substitute for the signal-to-interference ratio (SIR) of voice cellular networks. Finally, we review the most important works on non-cooperative power control games applied in modern cognitive radio and femtocell networks that are related to this thesis.

2.1 Non-cooperative game theory – fundamentals and concepts

The increasing number of mobile users, services, and applications in current and future wireless communication networks needs novel analytical frameworks that are capable of meeting the numerous technical challenges. Accordingly, in recent years game theory has emerged as an efficient mathematical tool for the design of future wireless and communication networks. It is one of the best methods for the incorporation of decision making rules and techniques into next-generation wireless and communication nodes to allow them to operate efficiently and meet the users' required communication quality of services (Han et al., 2011). The problem of power control in cellular networks is one of the most popular applications of game theory. The problem in the design of uplink cellular networks is how to allow users to regulate their transmit power during utilization of a common spectrum, given the interference caused by other users in the network. Researchers and wireless engineers have been able to represent the problem of power control in a cellular network by using non-cooperative game theory. All the finite numbers of players in the non-cooperative game are in a competitive situation, in which each action of a player (select a strategy), will have an impact (positive or negative) on the utility (e.g. the preference or gain) of the other players. Likewise, in a power control in a wireless network, all mobile users are in a competitive situation in which the transmit power level selected from the power strategy of a mobile user will change the interference level of the cell and will then affect positively or negatively the transmission QoS of the other users. Consequently, solving the power control problem in cellular networks is equivalent to solving a non-cooperative game,

i.e., by finding Nash equilibrium (Han et al., 2011). Recently, many researchers have been involved in applying the game theory approach to solve the problems in wireless networks, but they face many difficulties finding accurate models and solutions. This due to the design of game theoretic models that do not matched specific engineering issues such as the time varying wireless channel condition, performance functions (i.e., utilities) that depend on many communication metrics (e.g., transmission power and rate, delay, signal-to-interference ratio), and conforming to certain standards (e.g., CDMA, IEEE 802.16, LTE). Therefore, it is necessary to find effective analytic models from game theory that can be used in the design of future wireless and communication networks (Han et al., 2011).

2.1.1 Non-cooperative game theory

Game theory is a wide-ranging field of applied mathematics that defines and analyses the situations between interactive decision makers. In particular, it provides a framework based on the construction of rigorous models that describe situations of conflict and cooperation between *rational* decision makers (Tadelis, 2013). In decision theory and economics, rational behaviour is defined as choosing actions that maximize the payoff subject to constraints. Game theory has been successfully applied in many areas such as economics, business competition, the functioning of markets, jury voting, and auctions. Game theory has also been useful in other disciplines, such as political science, sociology and biology (Straffin, 1993). Since the early 1990s, engineering and computer science have been added to the list of disciplines. In recent years, game theory has been widely applied in telecommunications engineering, and specifically, wireless radio resource management (Altman et al., 2006; Felegyhazi & Hubaux, 2006; MacKenzie & DaSilva, 2006). In game theory, each player competes with other players to optimize (maximize) its own utility by adjusting the strategy. The utility of a player is a function that measures the player's level of satisfaction. Utility and strategy games can be defined as follows:

Definition 2.1: "In any game, utility (payoff) *u* represents the motivation of players".

The utility function for a given player assigns a number for every possible outcome of the game with the property that a higher (or lower) number implies that the outcome is more preferred (Hossain et al., 2009).

Definition 2.2: "A strategy r is a complete contingent plan, or a decision rule, that defines the action an agent will select in every distinguishable state A of the world" (Hossain et al., 2009).

A strategic form of game consists of three elements:

- (1) a set of rational decision makers referred to as players,
- (2) a set of strategies associated with the players,
- (3) the payoffs (utilities) function received by each player, which represents the objective.

A game with N – players can be formulated mathematically as shown in the following definition (Gibbons, 1992):

Definition 2.3: A game Φ in normal form is given by $\Phi = \langle N, \{A_i\}, \{u_i\} \rangle$, where:

- (1) $N = \{1, 2, 3, \dots, n\}$ is a finite set of players,
- (2) $\{A_i\}$ is the strategy (action) set for player *i*, where $A_i = A_1 \times A_2 \times ... \times A_n$ is the product of the sets of strategies available to each player, and

(3) $\{u_i\} = \{u_1, u_1, ..., u_n\}$ is the set of the utility functions for player *i*. In the strategy profile, supposing that $a \in A$, we let $a_i \in A_i$ denote player *i*'s action (strategy) and a_{-i} denote the actions (strategies) of the other i-1 players.

Games can be classified based on their application: a cooperative game, in which the players can communicate amongst each other to make enforceable contracts, and a non-cooperative game as defined before, in which players cannot communicate amongst themselves and are unable to make enforceable contracts. The non-cooperative game is the only choice if the information is strictly limited to local information (Hossain et al., 2009). If the players make a decision one time, the game is called static, whereas if the players make a decision several times, the game is called dynamic. Repeated games are games in which the players make a decision once, but the game is played several times.

The most important concepts in game theory are the non-cooperative game, dominant strategy, dominated strategy, Nash equilibrium, and Pareto optimality (Fudenberg & Tirole, 1991).

Non-cooperative game theory is a branch of game theory, in which rational decision makers make decisions independently. Hence, it studies the behaviour of decision makers in any situation in which each rational decision-maker's optimal choice may depend on his forecast of the choices of others. In another context, decision makers in a non-cooperative game aim to improve their objective (preference) selfishly.

Definition 2.4: "A non-cooperative game is one in which players are unable to make enforceable contracts outside of those specifically modeled in the game. Hence, it is not defined as games in which players do not cooperate, but as a game in which any cooperation must be self-enforcing" (Hossain et al., 2009). **Definition 2.5:** "Dominant strategies: A strategy is dominant if, regardless of what other players do, the strategy earns a player a larger payoff than any others do. Hence, a strategy is dominant if it is always better than any other strategy, regardless of what opponents may do. If a player has a dominant strategy then he or she will always play it in equilibrium. In addition, if one strategy is dominant, then all others are dominated" (Hossain et al., 2009).

Definition 2.6: "Dominated strategies: A strategy is dominated if, regardless of what any other players do, the strategy earns a player a smaller payoff than some other strategy.

Hence, a strategy is dominated if it is always better to play some other strategy, regardless of what opponents may do. A dominated strategy is never played in equilibrium" (Hossain et al., 2009).

Definition 2.7: A Nash equilibrium of a game $\Phi = \langle N, \{A_i\}, \{u_i\} \rangle$ is a profile $a^* \in A$ of strategies with the property that for every player $i \in N$,

$$u_i(a_{-i}^*, a_i^*) \ge u_i(a_{-i}^*, a_i)$$
 for all $i = 1, 2, ..., N$ (2.1)

where $a_{-i}^* = [a_1^*, a_2^*, ..., a_{i-1}^*, a_{i+1}^*, ..., a_n^*]$

The Nash equilibrium is a stable point, in which no user has any incentive to change its strategy. An efficient method to obtain the Nash equilibrium is to find the best response of the players. In game theory, the best response of the player is its strategy, which produces the most favorable outcome, taking other players' strategies as given (Fudenberg & Tirole, 1991). **Definition 2.8:** A set of strategies, $(a_1, a_2, ..., a_n)$ is Pareto optimal if and only if there exists no other set of strategies $(b_1, b_2, ..., b_n)$ such that

$$u_i(b) \ge u_i(a)$$
 for all $i = 1, 2, ..., N$ (2.2)

In the Pareto optimal solution the players can not in any way improve their current payoffs (utilities) through a different strategy choice without reducing others' payoffs.

To illustrate the above concepts and definitions, it is worth considering the example of a classic two player game named the *prisoner's dilemma*.

Game theory example (prisoners dilemma)

The prisoner dilemma model is the famous "game" that has been analysed by game theory to explain the behaviours of players (Geckil & Anderson, 2009). The prisoner's dilemma model can be explained as follows:

The police in a joint crime arrested two partners and separated them into different rooms. The police offer each of them the same deal: to confess to the crime or remain silent. The punishment that each receives is dependent not only on his or her decision, but also on the decision of his or her partner. The possible outcomes of this model are as follows:

- (1) If one of the partners confesses to the crime while the other remains silent, the confessor will be set free (i.e., payoff of 0) and the other partner will get a maximum sentence (i.e., payoff of -9) because the information provided by the confessor is used to incriminate him or her.
- (2) If both partners confess to the crime, then each gets a reduced sentence (i.e., payoff of -6) but neither is set free.
(3) If neither partner confesses to the crime, then each gets the minimum sentence (i.e., payoff of -1).

In this game, the set of players contains two players (the prisoners) n = 2, and thus $N = \{1, 2\}$. Each player has a finite set of allowed strategies (meaning that the prisoner game is a finite game) that is represented by space $A_1 = A_2 = \{\text{Confess}, \text{Remain Silent}\}$.

	Prisoner 2 (player 2)		
- 1 1)		Confess	Remain Silent
Prisoner 1 (player 1)	Confess	(-6,-6)	(0,-9)
Pr. (p]	Remain Silent	(-9,0)	(-1,-1)

Table 2.1: Bi-matrix form of prisoner dilemma game

The prisoner's dilemma game can be explained mathematically as follows:

- a. Two partners represent the set of players in a game $N = \{1, 2\}$.
- b. The strategy sets of the game are: (confess, confess), (confess, remain silent), (remain silent, confess), (remain silent, remain silent).
- c. The outcome (payoffs) of the game can be: (-6, -6), (0, -9), (-9, 0), (-1, -1), which depend on the chosen pairs of strategies.

The prisoner's dilemma model can be represented as a bi-matrix as shown in Table 2.1, in which player 1 plays as the row player and player 2 plays as the column player. The solution of this game can be easily found. Player 1 thinks that the best response strategy to choose is to confess. Similarly, player 2 also thinks that the best response for him is to confess to whatever player 1 chooses. The strategy pair (confess, confess) is

the solution of this game and it is called the Nash equilibrium solution, in which no player can unilaterally increase his own outcome (payoff). The Nash equilibrium outcome of the prisoner's dilemma game (-6, -6) is not the Pareto optimality of the game because there is another outcome, (-1, -1) that is better for the players. The Pareto optimality outcome (1, -1) can be obtained by allowing the two partners to cooperate, then they would choose the (remain silent, remain silent) strategy.

2.1.2 Auction Games (Economic games)

Due to the ability of game theory to study interaction behavior between decision makers and players, it can be applied also to the economic to study interact between firms and people in the real market. The concepts of game theory and economic models lead to new interesting games and successful theoretic results in microeconomics and auction theory.

In economics, players of the game are sellers and buyers in the market (e.g., firms, individuals, and so on), payoff functions are defined as the utility or revenue that players want to maximize, and equilibrium strategies are of considerable interest. On the other hand, they are distinguished from fundamental game theory, not only because additional market constraints such as supply and demand curves and auction rules give insight on market structures, but also because they are fully-developed with their own research concerns (Wang. B. et al., 2010).

In actual fact, the research on the game theory is much older than the Cournot model, one of market equilibria literally exists as a unique field. Hence, this subsection to address those economic games, so as to respect the distinction of these games and to highlight their intensive use in cognitive radio networks. The application of these types of games into cognitive radio networks has the following advantages. First, economic models can be represented in the spectrum sharing scenario, in which the secondary spectrum (unused part) market owned by primary users can be sell right to cognitive radios or secondary users. Primary users, as sellers, have the incentive to trade temporarily unused spectrum for monetary gains, while secondary users, as buyers, may want to pay for spectrum resources for data transmissions. The deal between buyers and sellers is made through pricing, auctions, or other means. Second, the idea of the game in economy can be extended to some cognitive radio scenarios other than spectrum markets and relation between sellers and buyers. Stackelberg game is example, originally describing an economic model, has been generalized to a strategic game consisting of a leader and a follower. Third, due to the combination of technology, policy, and markets properties of cognitive radio networks, it is important to study cognitive radio networks from the economic perspective and develop effective procedures (e.g., auction mechanisms) to regulate the spectrum market.

Auction Game example: Cournot's of oligopoly

Another example of non-cooperative game is Cournot's of oligopoly, in which the strategic game consist of:

- 1. The players of the game (firms)
- 2. The action of each firm (set of possible outputs)
- 3. Payoff of each firm (profit)

In this game model, each firm selects its output independently (non-cooperate), and the market determines the price at which it is sold.

There are *n* firms produce a single good and the cost of firm *i* of producing q_i units of goods is $C_i(q_i)$, where C_i is nonnegative and increasing function. The firms' total output is Q, then the market price is P(Q) where P is non-increasing inverse demand function. If the output of each firm *i* is q_i , then the revenue is $q_i P(q_1 + + q_n)$. Thus, the profit of firm *i* is the difference between revenue and cost as

$$\pi_i(q_1,...,q_n) = q_i P(q_1 + + q_n) - C_i(q_i)$$
(2.3)

Consider competing of two firms and each one cost function is given by $C_i(q_i) = cq_i$ for all q_i , and the non-negative inverse demand function is given by:

$$P(Q) = \max\{0, \alpha - Q\} = \begin{cases} \alpha - Q; & \text{if } Q \le \alpha \\ 0; & \text{if } Q \ge \alpha \end{cases}$$
(2.4)

where $\alpha > 0$ and c > 0 are constants. Constant unit cost $C_i(q_i) = c_i q_i$ where $c < \alpha$.

Thus, the firm 1's profit for two products q_1 and q_2 can be written as

$$\pi_{i}(q_{1},q_{2}) = q_{1}(P(q_{1}+q_{2})-c) = \begin{cases} q_{1}(\alpha-c-q_{1}-q_{2}) & \text{if } q_{1}+q_{2} \le \alpha \\ -cq_{1} & \text{if } q_{1}+q_{2} > \alpha \end{cases}$$
(2.5)

The solution of the game is the Nash Equilibrium and it can be found based on best response function.

If $q_2 = 0$ then firm 1's profit is $\pi_1(q_1, 0) = q_1(\alpha - c - q_1)$ for $q_1 \le \alpha$. The output of q_1 of firm 1 that maximizes its profit is

$$\frac{\partial \pi_i}{\partial q_i} = 0 \Longrightarrow q_i = \frac{1}{2}(\alpha - c)$$
(2.6)

The firm 1's best response to an output zero for firm 2 is $b_i(0) = \frac{1}{2}(\alpha - c)$. As the output q_2 of firm 2 increases, the profit firm 1 can obtain at any given output decreases, because more output of firm 2 means a lower price.

Therefore, the best response of firm 1 is given by

$$b_{1}(q_{2}) = \begin{cases} \frac{1}{2}(\alpha - c_{1} - q_{2}) & \text{if } q_{2} \le \alpha - c \\ 0 & \text{if } q_{2} > \alpha - c \end{cases}$$
(2.7)

Because firm 2's cost function is the same as firm 1's, its best response function $b_2(q_1)$ is also the same: for any number q, we have $b_2(q) = b_1(q)$.

A Nash equilibrium is a pair (q_1^*, q_2^*) of of outputs for which q_1 is a best response to q_2 and q_2 is a best response to q_1 .

Here, we assume two firms has cost function as $\pi(q) = 30q$; the inverse demand function for the firms output is P = 120 - Q, ($\alpha = 120$) where Q is the total output. Then, firm 1's profit is: $q_1(120 - q_1 - q_2) - 30q_1$. Taking the derivative of this profit with respect to q_1 (q_2 constant) and setting the derivative equal to zero, we obtain

$$120 - 2q_1 - q_2 - 30 = 0 \tag{2.8}$$

Or $q_1 = (90 - q_2)/2$. Thus, the best response function of firm 1's is given by $b_1(q_2) = (90 - q_2)/2$. Similarly, we find that the best response function of firm 2 is given by $b_1(q_1) = (90 - q_1)/2$.



Figure 2.1: The best response function in Cournot's duopoly game. The unique Nash equilibrium is $(q_1^*, q_2^*) = (30, 30)$

2.2 Cognitive Radio Networks

The increased demand of current wireless networks technology and the large number of mobile terminals in the world, all of which compete to access a limited amount of licensed and unlicensed radio frequency spectrums, has become the most important issue in wireless technology. Federal Communications Commission (FCC) measurements (2002a) have shown that the licensed frequency bands are not used most of the time and the limit of unlicensed spectrum bands is worsening. In addition, some parts of frequency bands are used heavily in particular locations and at particular times. The FCC Spectrum Policy Task Force (SPTF) found that the spectrum percentage usage ranges from 15% to 85% in the licensed spectrum band below 3GHz.

There has been a dramatic increase in the access to limited spectrum for mobile services and applications in recent years. Therefore, the fixed spectrum assignment policy that was working well in the past is not efficient now to use (Chen and Prasad, 2009). Recently, new spectrum allocation technique called Dynamic Spectrum Access (DSA) is used to solve the above spectrum efficiency problem by allowing unlicensed users with cognitive capability to use the licensed spectrum bands when the licensed users are absent, and leave the channel when the licensed user is detected. Spectrum hole is the unused part of spectrum, which is defined as a band of frequencies assigned to the licensed users, but at a particular time can be utilize by unlicensed users when and if the licensed user is absent.



Figure 2.2: Spectrum Holes

Cognitive Radio (CR), based on a software-defined radio, has recently appeared as a new smart technique for designing future wireless networks. This is due to its ability to perceptive its radio frequency environment, learn, adapt, and then reconfigure the system operation to utilize the radio spectrum efficiently and guarantee high reliable communication (Haykin, 2005).

Mitola and Maguire (1999) introduces the term of cognitive radio. The main objective of cognitive radio is to use the network resources efficiently. Cognitive radio technology uses secondary systems to improve spectral efficiency, which represent the most important issue in wireless communication systems. SUs improve spectral efficiency by sensing unused licensed spectrum frequency (spectrum hole) and use it for its transmission. Cognitive cycle that was given by Mitola and Maguire in 1999, consist of three basic tasks:

- 1) Radio sense analysis
- 2) Channel identification.
- 3) Transmit power control and dynamic spectrum management.

In radio sense analysis, cognitive radio estimates the total interference in the radio environment and detects the spectrum hole. Whereas, in channel identification task, cognitive radio estimates the channel state information and predicts channel capacity for use by the transmitter. In the third task, transmit power is adapted to full power limits when necessary on the one hand and to lower levels on the other hand to allow greater sharing and reuse of spectrum, dynamic spectrum management, adaptive modulation and coding and transmit rate control. The third task especially transmits power control is focused in this dissertation. The basic cognitive cycle including the most important tasks and actions is explained in Figure 2.10.



Figure 2.3: Basic cognitive cycle Mitola (1999)

The FCC Spectrum Policy Task Force of Unlicensed Devices and Experimental Licenses Working (2002b) introduced a new method used to measure the amount of interference in cognitive radio system called Interference Temperature I_T .

Interference Temperature I_T is used to manage and quantify the source of interference in the radio environment. Interference Temperature limit in this method is used to limit the amount of interference tolerable caused to the PUs in a particular frequency and particular location. SUs that access licensed spectrum have to measure interference temperature and adjust their transmission power in such a way to be under the interference temperature limit. For a given a bandwidth *W* the measurement of interference is computed as:

$$I_T(f_c, W) = \frac{P(f_c, W)}{kW}$$
(2.9)

where $P(f_c, W)$ is the interference power in watt measured at receiver, k is Bolzmanns constant $k = 1.38 \times 10^{-23}$ joules per kelvin degree, and W is the bandwidth in Hertz centered at frequency f_c

There are two models of interference temperature studied by Clancy and Arbaugh in 2006, namely; (i) ideal interference temperature model, and (ii) generalized interference temperature model.

In ideal interference model, authors limited the interference specifically to the licensed signals. Then, the transmission of SU must guarantee the following temperature limit for the licensed receiver (Clancy and Arbaugh, 2006):

$$I_T(f_c, W_i) + \frac{M_i P}{K W_i} \le I_L(f_i)$$
(2.10)

where M_i is a constant between 0 and 1, and *i* referred to the user index in the system.

In the generalized interference temperature model, the knowledge of signal environment is not available, and consequently no way to distinguish licensed signals from interference and noise (Clancy and Arbaugh, 2006). Therefore, the interference temperature is defined to the entire frequency range, and for the interference temperature limit constrain can be defined as follows:

$$I_T(f_c, W) + \frac{MP}{KW} \le I_L(f)$$
(2.11)

2.3 Review of power control fundamentals

In wireless communication, radio resource management (RRM) is proposed to promote the QoS of the system and share the available spectrum resources between users efficiently. Several RRM components are working together to improve the QoS of users, such as admission control, transmission power control, rate allocation, handoffs, channel assignment and adaptive beam forming. Power control (PC) is one of the most important techniques in RRM and plays a major role in resource allocation on the wireless radio communication network. PC helps with several functionalities in wireless cellular networks (Chiang et al., 2008):

- a) Interference management: PC mitigates the interference to increase system capacity by ensuring efficient spectral reuse and a desirable user experience.
- b) Energy management: PC conserves the energy to extend the battery life in wireless terminals and networks.
- c) Connectivity management: PC is able to maintain a minimum level of received signal so that the terminal can stay connected.

Power control in wireless networks is classified based on the directions of transmission as: uplink (reverse link) power control in which the direction of transmission is from mobile stations (MS) to the base stations (BS), and downlink (forward link) power control in which the direction of transmission is from base stations to mobile stations. The challenges in uplink power control are the limited transmit power in battery-based mobiles, low computational capability of mobiles, and the near/far effects.

Power control is also classified based on the uses of information, as centralized or distributed. In centralized power control, the centralized controller (e.g. base station) uses information such as path gain to calculate and select suitable actions for all mobile users. On the other hand, users in the distributed power control use only local information to select their actions. Each user in the distributed wireless network uses only local information and it does not know the channel conditions of other users. Users act selfishly to maximize their own performances in a distributive fashion

Figure 2.1 illustrates as an example part of a multi-cell cellular network with two base stations (Base A and Base B) and two mobile stations (MS-i and MS-j). It is shown that MS-i is connected with Base A and MS-j is connected with Base B.



Figure 2.4: An example of uplink transmission in multi-cell cellular networks

Here, we only illustrate and discuss the uplink transmission case (transmission from MS to Base station), but the downlink transmission is similar. It is shown in Figure 2.1 that the signal from MS-i to Base station "A" interferes with the signal from MS-j to Base station "A". In the general case, each mobile station experiences not only interference resulting from other users but also background noise. The signal to interference ratio (SIR) and the signal to interference and noise ratio (SINR) will be use to compare the quality of the desired signal to the interferent. Thus, the signal to interference ratio (SIR) of MS-i is defined as

$$\gamma_i = \frac{g_{Ai} p_i}{g_{Aj} p_j + \eta_i} \tag{2.12}$$

where p_i , g_{Ai} , g_{Aj} are the transmit power of user *i*, link gain from MS-i and MS-j to the (Base station A) respectively, and η_i is the background noise. Similarly, the signal to interference ratio (SIR) of MN-j is defined as

$$\gamma_j = \frac{g_{Bj} p_j}{g_{Bi} p_i + \eta_i} \tag{2.13}$$

where g_{Bj} and g_{Bi} are the link gain from MS-j and MS-i to the Base station B respectively.

The numerator of equations (2.3) and (2.4) expresses the Base station power signal received from the MS to which it is connected, whereas the denominator of the equations expresses the sum of the received power signals from other interfering MS plus the background noise of the channel.

In general, the signal to interference ratio SIR of the *i*th user, where i = 1, 2, 3, ..., N is expressed as

$$\gamma_{i} = \frac{g_{i} p_{i}}{\sum_{\substack{j=1 \ j \neq i}}^{N} g_{j} p_{j} + \eta_{i}} = \frac{g_{i} p_{i}}{I_{i}}$$
(2.14)

where I_i is the total interference of all users except user *i*.

The quality of the signal depends on the value of the SIR, in which a higher SIR of the link means a better quality signal and vice versa. Let us consider that the interference of other users plus the background noise that is found in the denominator of equations (2.3) and (2.4) are fixed and only a connected mobile user can use any transmission power. The SIR of this link increases as the link increases its transmission power. Increasing the transmit power of the link to achieve the maximum SIR is undesirable because the link will consume much energy to achieve a good quality signal which may be higher than it really needs for a particular application (Douros & Polyzos, 2011). Thus, it is better for the mobile station to increase its transmission power to such a level that the terminal can achieve its own target SIR.

On the other hand, the competition of mobile stations to improve their own SIR by increasing their transmission power is not feasible because each terminal represents a source of interference to others. Notice that the mobile station can adjust its transmission power to improve the SIR that its attached base station perceives. However, the obtained value of the SIR at the base station is unknown to the mobile station because the mobile station only knows its transmission power. Moreover, the base station knows the total received power from all mobile stations. The information of total received power may be sent to the mobile station through the feedback channel. Thus, the mobile station can easily subtract its own contribution to the total power to compute the current SIR based on its knowledge of the link gain to the base station.

2.3.1 Power control in voice cellular networks

The main goal of power control in the voice networks is to ensure that each mobile station receives SIR above a certain threshold value (Gunnarsson & Gustafsson, 2002; Lee et al., 2003; Gunnarsson, 2000). Distributed power control (DPC) is one of the most important and practical schemes of power control because each mobile station can adjust its transmission power level using only local information. Zander, (1992a), proposed a distributed balancing (DB) power control algorithm based on the SIR. The power update formula of the *i*th user is suggested as

$$p_i^{(k+1)} = \beta p_i^{(k)} \left(1 + \frac{1}{\gamma_i^{(k)}} \right)$$
(2.15)

where β is the smaller controller parameter, $\gamma_i^{(k)}$ and $p_i^{(k+1)}$ are the SIR and power of the *i*th user at *k*th and (k+1)th time step. The algorithm in (Zander, 1992a) converges to the balancing power vector which corresponds to γ^* when the thermal noise is neglected. Since the DB algorithm is based on $\gamma_i^{(k)}$, at first sight it appears to be distributed. However, it turn out that the problem is in the choice of the parameter β because it may make p deviate towards zero or infinity. Thus, the DB power control algorithm is not a fully distributed algorithm.

Grandhi and his co-authors proposed a modified version of the DB algorithm called the distributed power control (DPC) algorithm (Grandhi et al., 1994),

$$p_i^{(k+1)} = \beta \frac{p_i^{(k)}}{\gamma_i^{(k)}}$$
(2.16)

This algorithm also shows that it can converge to the power vector p^* when the thermal noise neglected. The simulation of the DPC algorithm indicated that the convergence speed is faster than the DB algorithm. Furthermore, simulation results from Lee et al., (1995) and Zander, (1994) also show that the DPC algorithm achieves better performance than the DB algorithm. The DPC algorithm still has the same problem of normalizing the powers by adapting β . Another similar work to the above algorithms was proposed by Lee & Lin, (1996), and is called the CDPC-II algorithm

$$p_i^{(k+1)} = \beta p_i^{(k)} \left(\frac{1}{\gamma_i^{(k)}}\right)^{\varsigma}$$
(2.17)

where ξ is a constant parameter. The CDPC-II algorithm also has the same problem about how to choose β and if $\xi = 1$ gives the same algorithm as that proposed by Grandhi et al. (1994). The disadvantage of these algorithms is that they are not fully distributed. All of these algorithms require signalling for the normalization procedure, which is undesirable in practice.

Foschini and Miljanic (1993) were the first to propose a framework of uplink power control where all users converge to the Pareto-optimal solution whenever they can achieve the required QoS that refers to SIRs. They proved the convergence of the DPC algorithm in equation (2.7) in the existence of thermal noise and they mentioned that the normalization procedure was unnecessary. The DPC algorithm is rewritten by replacing the control parameter β by the target SIR,

$$p_i^{(k+1)} = \Gamma \frac{p_i^{(k)}}{\gamma_i^{(k)}}$$
(2.18)

where Γ is the target SIR. Equation (2.9) can be implemented in a distributed manner, where $p_i^{(k+1)}$ is the transmit power of the *i*th link at the *k*th time instant. Each independent link measures its current SIR, $\gamma_i^{(k)}$ and tries to achieve its target SIR in the next step (Bambos, 1998). The algorithm increases the transmit power of the link when the current SIR is below the target and decreases it when the current SIR is higher than the target. The disadvantage of the DPC algorithm is the case of an infeasible target SIR, in which the transmit power diverges to infinity.

An interesting extension of the DPC algorithm has been proposed by Grandhi et al. (1994), who considered the system with output power constraints. The algorithm of the Constrained Distributed Power Control (CDPC) is given by

$$p_i^{(k+1)} = \min\left(p_i^{\max}, \Gamma \frac{p_i^{(k)}}{\gamma_i^{(k)}}\right)$$
 (2.19)

where p_i^{max} is the maximum transmit power of the *i*th user. The control block diagram of CDPC is shown in Figure 2.2



Figure 2.5: Control block diagram of CDPC

Nevertheless, some users with CDPC in equation (2.10) may hit their maximum transmit power at some time instant k with $\gamma_i^{(k)} < \Gamma$ and this may result in $\gamma_i^{(k)} < \Gamma$ for $k \ge 0$, i.e., a user cannot satisfy the constraint of the SIR when it uses its maximum budget of power. In this case, some users can satisfy $\gamma_i^{(k)} > \Gamma$ for some $k \ge 0$ and their power will converge to a feasible solution, whereas the other users that cannot achieve the target SIR Γ will continue to transmit at maximum power (Chiang et al., 2008).

Yates, (1995) proposed a general framework for uplink power control by designing an iterative power update scheme as

$$p_i^{(k+1)} = I(p_i^{(k)}) \tag{2.20}$$

where $I(p) = [I_1(p), I_2(p), ..., I_n(p)]^T$, and $I_i(p)$ is the interference experienced by user *i*. The power update scheme in equation (2.11) will converge to a power vector (if

that exists) that satisfies the target SIR if the interference function I(p) is a standard and satisfies the following definition of power vectors p and p'.

Definition 2.9: (Yates, 1995):

The interference function I(p) is a standard function if it satisfies the following three conditions

- 1) Positivity I(p) > 0,
- 2) Monotonicity if p > p', then I(p) > I(p'),
- 3) Scalability $\forall \alpha > 1$, then $\alpha I(p) > I(\alpha p)$

The second-order constrained power control algorithm has been described by Jäntti & Kim (2000), in which the update of transmission power is depend on the past and current values of power. The power update formula is given as

$$p_i^{(k+1)} = \min\left(p_i, \, \max\left(0, w^k \, \Gamma \, \frac{p_i^{(k)}}{\gamma_i^{(k)}}\right) + (1 - w(k) \, p_i^{(k-1)})\right)$$
(2.21)

where w^k is a non-increasing sequence of control parameters satisfying $1 < w^k < 2$.

El-Osery & Abdallah; (2000) designed a power control algorithm based on Linear Quadratic (LQ) control in order to achieve a faster convergence time and higher channel capacity. The power updated command is computed as

$$p_i^{(k+1)} = \min\left(p_i^{\max}, s_i^{(k+1)} I_i^{(k)}\right)$$
(2.22)

where $s_i^{(k+1)} = p_i^{(k)} + o_i^{(k)} / I_i^{(k)}$ and $o_i^{(k)}$ is an input to each subsystem that depends on the total interference produced by other users. Similarly, Lv (2003) proposed the exponential function of the SIR in the power update equation to speed up the convergence. The fast constrained distributed power control is as follows:

$$p_i(0) = 1,$$
 (2.23)

and

$$p_{i}^{(k+1)} = e^{q(\Gamma_{i} - \gamma_{i}^{(k)})} \cdot p_{i}^{(k)}$$

$$p_{i}^{(k+1)} \leq p_{i}^{\max}$$
(2.24)

where q is the exponential parameter.

2.3.2 Power control in wireless data networks

The QoS objective of a mobile station in wireless voice systems is to achieve a minimum acceptable SIR to maximize the number of conversations where the transmission errors are tolerable (Goodman & Mandayam, 2000). The target level of SIR is dependent on the system, for example $\Gamma = 18$ dB in the analog system, the target can be low as $\Gamma = 7$ dB in the global system for mobile communications (GSM) digital system, and it is in the order of $\Gamma = 6$ dB in the code division multiple access CDMA system (Goodman & Mandayam, 2000). A utility function has been used in wireless data networks as a measure of the satisfaction experienced by a user. Thus, the utility function that is represented by the SIR in voice systems can be sketched as shown in Figure 2.3. The system will be unacceptable (utility=0) if the SIR is below the target Γ ; otherwise when the SIR is greater than Γ the utility is constant (utility=1).



Figure 2.6: Quality of service metric for wireless voice represented as a utility function

The QoS that refers to the SIR is no longer appropriate in wireless data networks because error-free communication has a high priority (Saraydar et al., 2002).

Therefore, the concepts of microeconomics and game theory have been used recently to define the users' QoS in terms of the utility (cost) function rather than the SIR (Popescu & Chronopoulos, 2005).

To model the problems of resource allocation in wireless networks using game theory, strategy sets should be defined in the basic form. However, the number of strategies changes depending on the number of resources. For example, in the problem of a joint power and rate control game, two strategies are defined, namely the transmission rate strategy and transmission power strategy. Each user adjusts its transmit rate from the transmit rate strategy, then adjusts its transmit power from the transmission power strategy to maximize their own utility function. In wireless data networks, the user maximizes its own utility function by choosing the action from the strategy set, such as the choice of its transmit-power, transmissions rate, modulation, packet size, multi-user receiver, multi-antenna processing algorithm, or carrier allocation strategy (Meshkati et al., 2007). Users access the wireless networks through multiple access interference (shared medium) so each user is a source of interference to others. The strategy chosen by each user affects the performance of other users in the network.

Regarding game theory, there are several important things that the engineering designer has to consider. Firstly, a choice of a suitable utility (or cost) function that has physical meaning (such as energy efficiency or spectrum efficiency, SIR balancing), and the utility function has to be either quasi-concave or quasi-convex. Secondly, there should be a strategy set in the game from which the user must choose its action in order to maximize its utility function. Finally, the game must have a steady state solution (Nash equilibrium) where no user can unilaterally maximize his utility and this solution is unique (Meshkati et al., 2007).

In the non-cooperative power control game, each user harms and affects other users' resources because there is no cooperation between them. The outcome Nash equilibrium in this case is inefficient. The pricing technique is an efficient tool that has been used by researchers and engineers to solve the inefficiency problem of the Nash equilibrium. The Pareto optimal solution can be calculated using the optimization technique, in which the net utility function (utility - price) represents the objective function. Figure 2.4 explains the phenomenon of the Nash equilibrium and Pareto optimality.

The most popular pricing function used in the power control game is based on the user's transmit power, where all users in the system have an incentive to transmit at low power. When users transmit at low power, the amount of interference will decrease and this is one of the main goals of cognitive radio networks, where the unlicensed users have to work within interference temperature limits.



Figure 2.7: Inefficiency of Nash equilibrium

Here, we have explained that the power control algorithms obtained by using game theory and based on the cost function have two terms. The first term looks like the CDPC in equation (2.9) and the second term is dependent on the pricing function applied in the utility or cost function. The main goal of using a cost function is to design SIR balancing schemes, in which all users try to achieve the same target value of the SIR. Figure 2.5 explains the block diagram of power control algorithms of voice and data networks (based on cost function) and the differences between the blocks in both approaches from control theory perspective. Both the power control block diagram from a control theory perspective for the CDPC and the power control block diagram from game theory when using cost functions are shown in the following figure with some of the related works.



Figure 2.8: Power control blocks from a control perspective.

2.3.2.1 Utility function concepts

The choice of the utility function is very important when game theory is employed to solve the problem of power control and resource allocation in wireless data networks. Two types of services that the next generation wireless networks must be able to support are voice and data services. Voice and video teleconferencing (real time) are examples of delay sensitive and error tolerant services, while Web browsing and file downloading (non-real time) are examples of delay tolerant services and are error sensitive.

In wireless data networks, the SIR influences the probability of transmission errors. When γ is very high, the probability of transmission errors approaches zero, and the utility function rises to a constant value, whereas when γ is very low, the probability of transmission errors increases and the utility function is near zero. Therefore, the utility function for wireless data networks could be characterized as a concave function.

The distributed power control algorithms used in wireless voice systems are not appropriate for use in wireless data systems. This is due to the difference between the utility function in voice service and data service systems. A lot of literature has proposed different utility functions to solve the power control and resource allocation problems depending on the main concern of their work. Energy efficiency was the main concern of the utility function that was proposed by Goodman & Mandayam, (2000), which was defined as the number of information bits that can be transmitted successfully per joule of energy consumed. An energy-efficient utility function was also used by Saraydar et al. (2002) and Meshkati et al. (2007). This utility function depends on the signal to interference ratio (SIR) and transmission power of a given terminal.

To transmit data successfully at a low bit error rate, the SIR level has to be high at the output of the receiver. However, achieving a high SIR level requires mobile terminals to expend a high level energy, which in turn results in low battery life. The net number of information bits that can be transmitted without error per unit time is referred to as the throughput. In this case, the utility function can be defined as the ratio between the throughput and user transmission power, i.e.

$$u_i = \frac{LT_i}{M p_i} \tag{2.25}$$

where *L* and *M* are the information bits and the length of packet, respectively and p_i is the transmission power of user *i*. The throughput T_i here can be expressed as:

$$T_i = R_i f(\gamma_i) \tag{2.26}$$

where R_i and γ_i are the transmission rate and the SIR for the user *i* respectively, and $f(\gamma_i)$ is the efficiency function, which represents the packet success rate (PSR). The efficiency function depends on data transmission such as modulation, coding, and

packet size. The efficiency function that depends on the modulation scheme can be expressed as

$$f(\gamma_i) = (1 - P_e)^M$$
 (2.27)

where P_e is the bit error rate BER of terminal *i*. Table 2.2 explains the BER as a function of various modulation schemes

Modulation scheme	BER
Binary phase shift keying BPSK	$Q(\sqrt{2\gamma})$
Differential phase shift keying DPSK	$\frac{1}{2}e^{-\gamma}$
Coherent frequency shift keying FSK	$Q\left(\sqrt{\gamma} ight)$
Non-coherent frequency shift keying FSK	$\frac{1}{2}e^{-\gamma}$

Table 2.2: The BER as a function of various modulation schemes

The frame success rate efficiency function in equation (2.18) is shown in Figure 2.6 for non-coherent frequency shift keying (FSK) for the packet length M = 80.



Figure 2.9: Efficiency function non-coherent frequency shift keying (FSK), M = 80.

Thus, the energy efficient utility function for non-coherent FSK is expressed as

$$u_i = \frac{L}{M} \frac{(1 - e^{-\gamma_i/2})^M}{p_i}$$
(2.28)

Saraydar et al. (2002) also introduced the pricing function to improve the Nash equilibrium as a function of power transmission:

$$c_i(p_i) = c \cdot p_i \tag{2.29}$$

where c is the pricing factor. The utility function with pricing is expressed as:

$$u_i^c = \frac{L}{M} \frac{(1 - e^{-\gamma_i/2})^M}{p_i} - c \cdot p_i$$
(2.30)

For perfect error detection and no error correction, the frame success rate (FSR) can be expressed as $P_c = (1-P_e)^M$, where P_e decreases monotonically with the SIR in all modulation schemes. Consequently, P_c is a monotonically increasing function of the SIR. Therefore, P_c can be expressed as a function of the SIR and used as the utility function in equation (2.19). For all modulation schemes, when p = 0, the best strategy for the receiver is to make a guess for each bit, resulting in $P_c = 2^{-M}$, and resulting in infinite utility (Saraydar et al., 2002). The efficiency function $f(\gamma_i)$ and the FSR closely follow the behaviour of the probability of correct reception while producing $P_c = 0$ at p = 0 (Saraydar et al., 2002). The close curves between the efficiency function $f(\gamma_i)$ and the FSR P_c for the non-coherent FSK modulation scheme are demonstrated in Figure 2.7.



Figure 2.10: FSR and efficiency as a function of terminal SIR for a non-coherent

FSK scheme.

In the studies of Alpcan et al. (2002) and Gunturi & Paganini (2003), the main goal of a utility function was to maximize the spectral efficiency, and they defined the utility function as a logarithmic concave function of the user's signal to interference ratio (SIR), i.e.,

$$u_i = B\log(1+\gamma_i) - c_i p_i \tag{2.31}$$

where *B* is the communication bandwidth, γ_i is the SIR of user *i* and c_i is the pricing factor of user *i*. The term $c_i p_i$ is the linear pricing of the user's transmit-power.

Xiao et al. (2003) defined the utility function of the user as a sigmoid function of the user's SIR. The pricing function was also defined as a linear function of the user's transmit-power, and the difference between the sigmoid function and pricing function was defined as the net utility function, i.e.

$$NU_i = u_i(\gamma_i) - c_i p_i \tag{2.32}$$

where $u_i(\gamma_i)$ is the sigmoid function and c_i is the pricing factor. The efficiency function that is suggested in this work was a sigmoid function that was expressed as

$$f(\gamma_i) = \frac{1}{1 - e^{a - \gamma_i}}$$
(2.33)

where a is a sigmoid parameter.

The comparison between the efficiency function $f_1(\gamma_i) = (1 - e^{-\gamma_i/2})^M$ that is used in equation (2.19) and the efficiency function $f_2(\gamma_i) = 1/1 - e^{a-\gamma_i}$ that is used in the sigmoid utility function equation (2.23) is shown in Figure 2.8., where a = 10 is assumed in equation $f_2(\gamma_i)$.



Figure 2.11: Efficiency function $f_1(\gamma_i)$ and $f_2(\gamma_i)$ as a function of SIR

2.3.2.2 Power control game based on utility function

The power control game has been used in several works based on utility and pricing functions. MacKenzie & Wicker (2001) offer motivations for using game theory to study communication systems, and in specific power control. Goodman & Mandayam (2000), Meshkati et al. (2007) and Ji & Huang (1998) proposed a power control as a non-cooperative game in which users choose their transmit powers in order to maximize their utilities, where utility is defined as the ratio of throughput to transmit power.

Goodman & Mandayam (2001) proposed a network assisted power control scheme in order to improve the overall utility of the system. The pricing function has been introduced in Saraydar et al. (2001) and Saraydar et al. (2002) to obtain a more efficient solution for the power control game. Similar approaches are taken in Xiao et al. (2003), Zhou et al. (2004), and Sung & Wong (2003) with different designs of utility function. Feng et al. (2004) discussed a joint network-centric and user-centric power control. Meshkati et al. (2005) propose a power control game for multi-carrier CDMA systems. Ghasemi et al. (2006) proposed a new pricing function for power control in wireless data networks based on the linear signal to interference and noise ratio (SINR) instead of power. An overview and more details of game theory approaches for resource allocations in wireless data networks have been presented by Meshkati et al. (2007).

2.3.2.3 Power control game based on cost function

Recently, many researchers have used the cost function in their works to study the problem of power control in wireless data networks. Alpcan et al. (2002) defined the user cost function as the difference between the utility function and its pricing function. Alpcan's cost function was logarithmically dependent on the SIR and linear in power and the objectives were to minimize the users cost. Accordingly, the cost function of the *i*th user is defined as

$$J_{i}(p_{i}, p_{-i}) = c_{i} p_{i} - b_{i} \ln(1 + \gamma_{i})$$
(2.34)

where p_{-i} is the vector of power levels of all users except user *i*, b_i is the user-specific utility parameter.

The cost function proposed by Koskie & Gajic (2005) consists of a weighted sum of the linear power and square of SIR error. The cost function of the ith user is defined as

$$J_{i}(p_{i}, p_{-i}) = c_{i}(\Gamma_{i} - \gamma_{i})^{2} + b_{i} p_{i}$$
(2.35)

The resulting algorithm (obtaining by setting partial cost derivatives to zero to satisfy the necessary conditions for equilibrium) is given in terms of the previous power value $p_i^{(k)}$ and current SIR measurement $\gamma_i^{(k)}$ by

$$p_i^{(k+1)} = \begin{cases} \frac{p_i^{(k)}}{\gamma_i^{(k)}} \Gamma_i - \frac{b_i}{2c_i} \left(\frac{p_i^{(k)}}{\gamma_i^{(k)}}\right)^2 & \text{if positive} \\ 0 & \text{otherwise} \end{cases}$$
(2.36)

Note that the first term on the right hand side of (2.27) is equal to the right hand side of the power balancing algorithm (2.7). The second term is always negative for nonzero power, so the Nash equilibrium powers will always be less than those generated by the power balancing algorithm. The ratio b_i/c_i , chosen by the mobile to represent its relative cost weights, determines the magnitude of the power savings. The power control algorithm proposed by Koskie & Gajic (2005) can be represented by using a block diagram as shown in Figure 2.9.



Figure 2.12: Control block diagram of the power control algorithm (Koskie & Gajic

2005)

The algorithm proposed by Koskie & Gajic (2005) has been accelerated by using Newton's method in Gajic & Koskie (2003) and Koskie & Zapf (2005).

Pasandshanjani et al. (2011) proposed a new cost function to design the power control algorithm. The cost function proposed in their work consists of a weighted sum of the linear power and the hyperbolic function of the SIR error. The cost function of the *i*th user is defined as

$$J_i(p_i, p_{-i}) = c_i \cosh(\Gamma_i - \gamma_i) + b_i p_i$$
(2.37)

The resulting algorithm (obtaining by setting partial cost derivatives to zero to satisfy the necessary conditions for equilibrium) is given in terms of the previous power value $p_i^{(k)}$ and current SIR measurement $\gamma_i^{(k)}$ by

$$p_i^{(k)} = \begin{cases} \frac{p_i^{(k)}}{\gamma_i^{(k)}} \Gamma_i - \frac{p_i^{(k)}}{\gamma_i^{(k)}} \sinh^{-1} \left(\frac{b_i}{c_i} \frac{p_i^{(k)}}{\gamma_i^{(k)}} \right) & \text{if positive} \\ 0 & \text{otherwise} \end{cases}$$
(2.38)

There are many works that have used the cost function to design the power control game in cognitive radio networks as well as wireless networks and these will be listed in the next subsections.

2.3.3 Power control game in modern wireless networks

In this context, we shall briefly review some of the non-cooperative power control games applied in modern wireless networks such as cognitive radio and femtocell networks. In both cognitive radio and femtocell networks, there are two priority levels for users:

1. Primary users (PUs) in cognitive radio networks are the high priority users to access the available channels and guarantee their own QoS, because of their ownership property of the licensed spectrum. Similarly, macrocell users (MUs) in two-tier femtocell networks are also high priority access users in the network from the same reasons as above.

2. Cognitive radios (CRs) or secondary users (SUs) are the low priority access users in the network because only unused parts of spectrum can be assigned to them and for this reason, QoS should be not guaranteed and can be adjustable. On the other hand, femtocell users (FUs) share the licensed spectrum with MUs and are low priority users because they work in low range cells.

In this case, extravagant use of transmit power by low priority users will cause undesirable (harmful) interference to the QoS of high priority users during spectrum sharing. Therefore, a strict power control algorithm should be run in low priority users (CRs and FUs) devices to protect other users and utilize the spectrum efficiently.

In the previous subsections, the power control proposed for wireless data networks based on game theory and utility functions has been reviewed. The power control algorithms that were used in wireless data networks may not be directly applicable for cognitive radio and femtocell networks unless the interference temperature limit and cross-tier interference limit at the licensed (high priority) receiver is considered. Utility and cost functions have been defined in different formats to achieve the QoS requirement of both types of users. Researchers and engineering network designers take into consideration the high priority of users who own the licensed spectrum (primary and macrocell users) in which their QoS should be guaranteed forever.

In power control, the QoS can be guaranteed for users by specifying a particular value of the SIR referred to as the "target of minimum SIR", whereby users should achieve the same as this target or higher. On the other hand, the QoS of low priority users such as secondary and femtocell users can be slightly varied below the target SIR,

depending on the status of the system. A forced decrease in low priority users' SIR is due to the increase of interference and the decreasing QoS of high priority users.

2.3.3.1 Power control game in cognitive radio networks

As discussed before that power control algorithms that were used in wireless data networks may not be directly applicable for cognitive radio networks unless the interference temperature limit at the licensed receiver is considered. The interference temperature limit in a cognitive radio network is the limit of interference that the PU can tolerate without significant deterioration in the QoS.

In the context of the cost function, Li et al. (2011) proposed a new power control game based on the cost function and the target of the transmit power has been included, such as the target SIR. The cost function in Li et al. (2011) is defined as a weighted sum of the logarithm of SIR error and the logarithm of power error. The algorithm has many advantages, such as fast convergence, better anti-noise performance and capacity. In addition, Junhui et al., (2013) proposed a non-cooperative power control algorithm also based on the cost function similar to Li et al. (2011) by using the square root function instead of the logarithm to fast algorithm. Lu et al. (2012) proposed a power control game in CDMA cognitive radio networks based on the cost function and they used two SIR thresholds in order to adjust the interference factor of power and improve the fairness. Jiao et al., (2013) investigated how to decide the transmission power levels of cognitive radio users using non-cooperative game theory. They proposed a novel cost function, in which the thresholds of the SINR and transmission power level were considered. Their algorithm's numerical results indicated better performance in terms of anti-noise.

Al-Gumaei et al., (2014) proposed a new SIR-based sigmoid power control game in cognitive radio networks based on the cost function. The cost function is defined as a

weighted sum of the square of the signal to interference ratio (SIR) error and a sigmoid weighting factor of power, in which the results show a considerable saving on transmit power compared to the relevant algorithms. The block diagram of the SIR-based sigmoid power control game is illustrated in Figure 2.10. The power control formula is shown at the output of the block diagram and the sigmoid function is used in the design.



Figure 2.13: Block diagram of SIR-based power control algorithm based on sigmoid function Al-Gumaei et al. (2014)

Al Talabani et al. (2015) proposed a new chaos based cost function to design the power control algorithm and analysed the dynamic spectrum sharing issue in the uplink of cellular CRNs. The chaos cost function is defined by taking into the account the interference from and the interference tolerance of the primary users. The algorithm led to significantly lower power consumption and fast convergence.

More details and explanation for the concept of game theory and its application in cognitive radio networks is given by Wang et al. (2010). Lasaulce et al., (2009) introduced a certain degree of hierarchy in the non-cooperative power control games to

improve the individual energy efficiency of all of the users. The pricing issue of the uplink power control game has been studied by Yu et al. (2010), where the CRs adjust their transmission power levels to maximize their own utilities, and the primary service charges the CRs on their transmitted power level to enhance its own income. Buzzi & Saturnino (2011) considered a non-cooperative power control game for maximum energy efficiency with a fairness constraint on the maximum received powers for the CRs, in which the results obtained indicate that the CRs have a beneficial impact on the whole network throughput. Channel and power allocation has been proposed and evaluated by Gállego (2012) using game theory based on local information, in which the problem is analysed under the interference model. The no-regret learning algorithm has also been used to overcome the convergence limitations of the local game and perform joint channel and power allocation.

The utility function via pricing has also been considered in recent years to solve the problem of power control in cognitive radio networks. In 2010, Al-Gumaei & Dimyati (2010a) proposed a new pricing function for secondary users (SUs) as a function of transmit power and the square amount of interference in order to guide SUs to an efficient Nash equilibrium point. The simulation results indicate that the proposed power control game, via a new pricing function, can maximize the number of SUs that are able to access the unused spectrum and improve the utilities of both PU and SUs. In another work, Y. Al-Gumaei & Dimyati (2010b) formulated the power control game as a non-cooperative game, in which the first player is the primary user (PU). The number of SUs in the system is limited by the status of the PU and its ability to achieve its QoS rather than using the interference temperature limits. The numerical results show that the proposed power control algorithm with pricing reduces the power consumed by the PU and SU terminals, and improves the utility functions of the PU and SUs.
Zhang et al. (2012) introduced the concept of the target SIR to modify the utility, and they modified the Shuffled Frog Leaping Algorithm (SFLA) to improve the accuracy of the solution. For Kuo et al., (2013) the power control game has been improved based on the outage probability of the PU in a spectrum-underlay CRN.

In addition, Kuo et al. (2013) designed an efficient swarm intelligent algorithm to improve the convergence speed and improve the energy-efficiency. A payment-based power control scheme based on a non-cooperative power control game has been considered by Xie et al. (2014), in which the distance of the CRs and SIR were used as a reference for punishment price setting.

2.3.3.2 Power control game in femtocell networks

Over the last decades, the demand for wireless data has increased dramatically, causing a significant change in the resource allocation of wireless networks. The increasing capacity and the limitation of link budget issues have been solved by relying on cell splitting or additional carriers but these techniques are complex and iterative (Saad et al., 2013). Moreover, the cost of these techniques and the updated infrastructures will be assigned to the service providers.

Recent measurements and studies of data usage found that the majority of data traffic originates indoor; it is estimated that more than 70% of calls and over 90% of data services originated indoors (Chandrasekhar et al., 2008; Lin et al., 2011). Therefore, femtocells have attracted much attention as a promising technique to solve the increasing network capacity. A femtocell is a small indoor access point, known as a home base station (HBS), that can be purchased by a customer and installed as a wireless data access point at home (Zhang et al., 2012). Femtocell or home base station (HBS) is a low-range, low-power, low-cost and consumer owned device that is installed inside houses and offices. The HBS connects to the internet via an IP backhaul, such as

a digital subscriber line (DSL), cable, or worldwide interoperability for microve access (WIMAX). There are two approaches for spectrum allocation between the macrocell and femtocell users:

- (i) Spectrum splitting,
- (ii) Spectrum sharing.

In a two tier femtocell network, the spectrum sharing approach is commonly used due to the scarcity of available spectrum and the absence of coordination between the macrocell and femtocell, as well as between femtocells (Kang et al., 2012).

The macrocell is accordingly supposed to be modelled as primary infrastructure, because the operator's foremost obligation is to ensure that an outdoor cellular user achieves its required SINR target at its BS, despite cross-tier femtocell interference. Indoor users act in their self-interest to maximize their SINRs, but incur an SINR penalty because they cause cross-tier interference (Chandrasekhar et al., 2008).

The problem of spectrum sharing in the two tier femtocell networks has become a technical challenge to scientists and researchers. The implementation of distributed interference management is the main challenge in femtocell networks due to the limited capacity of the signalling wire-line network (e.g., DSL links) and the difference in access priority between MUEs and FUEs (Jo et al., 2009; Güvenç et al., 2008). The existence of indoor femtocells causes power control to create dead zones, leading to non-uniform coverage.

To mitigate cross-tier interference and guarantee QoS for both MUEs and FUEs, an efficient distributed power control is necessary in both systems. Closed loop power control is commonly used in wireless networks, and consists of a two algorithms loop: (i) an outer loop algorithm that updates the threshold signal-to-interference-noise-ratio (SINR) every 10 ms, and (ii) an inner loop algorithm which computes the required powers based on the SINR measurements and is updated every 1.25 ms. The outer loop algorithm determines the target SINR based on the estimate of the frame error rate (FER). On the other hand, the inner loop algorithm generates a power control bit based on the difference between the actual and target SINR and it sends the command to the mobile via transmit power control (TPC) (Koskie & Gajic, 2005).

Several works have considered the problem of interference mitigation and power control for spectrum sharing femtocells networks. The concepts of microeconomics and game theory have been used recently to define the users' QoS in terms of the utility (cost) function in two tier femtocell networks.

Several power control schemes based on game theory have been investigated previously by Ji & Huang (1998) and Alpcan et al. (2002) are applied in femtocell networks. The output of the game in most cases is the Nash equilibrium of the power control game. In addition, pricing techniques are also introduced previously to improve the Pareto efficiency of the Nash equilibrium (Koskie & Gajic, 2005; Xiao et al., 2003; Saraydar et al., 2001; Saraydar et al., 2002) and also applied in femtocell network. The problem of interference in femtocell network similar to cognitive radio network can be also solved using game theory. The game theory approach has been considered to solve the problem of interference in cognitive radio networks (Jayaweera & Li, 2009; Al-Gumaei & Dimyati, 2010a; Li et al., 2011; Al-Gumaei et al., 2014). In cognitive radio networks, unlicensed users (cognitive users) are enabled to adaptively access the frequency channels, considering the current state of the external radio environment (Hu et al., 2014).

In femtocell networks, several works also have been considered to mitigate the crosstier interference (Kang et al., 2012; Ngo et al., 2012; Han et al., 2013; Xie et al., 2013). Moreover, the game theory-based power control has been considered for HetNet small cell networks (Xu et al., 2014). The authors addressed the mitigation of the cross-tier interference problem by ensuring the protection of macrocell users.

A utility based distributed SINR adaptation was studied in the two tier femtocell networks to alleviate the cross-tier interference (Chandrasekhar et al., 2009). Each femtocell user maximizes their individual utility consisting of a SINR based reward less an incurred cost. Their results show 30% improvenment in mean femtocell SINRs and the algorithm ensures that a cellular user achieves its SINR target even with 100 femtocells/cell site and requires a worst case SINR reduction of only 16% at femtocells.

Xie et al. (2012a) and Xie et al. (2012b), studied the energy efficient spectrum sharing and resource allocation in cognitive radio with femtocells. They used a gradient based iteration algorithm has been proposed to obtain the Stackelberg equilibrium solution. The non-cooperative power control game is also used for interference mitigation in the two-tier femtocell networks in Douros et al. (2012). The output results indicate that the application of power control by distinguishing the utility functions based on the users' QoS requirements leads in many cases of interest to a smooth coexistence in a two-tier femtocell network. The energy efficient power control algorithm is derived in the interference limited two-tier femtocell networks (Lu et al., 2012). The proposed scheme was evaluated to enhance energy efficiency of two-tier femtocell networks, while mitigating inter-tier interference for the uplink. The scheme also can guarantee that cellular user can achieve its target SINR. The differentiated pricing based on the SINR is considered in Zhang et al. (2012), in which the uplink power allocation is considered as a non-cooperative game. Simulation results show that the proposed power control scheme is effective in improving the outage and throughput performance of macrocell users while the effects on femtocell users are acceptable.

CHAPTER 3: A NEW SIR-BASED SIGMOID POWER CONTROL IN COGNITIVE RADIO NETWORK: A NON-COOPERATIVE GAME APPROACH

3.1 Introduction

The recent development in wireless networks applications and services has led to a decrease in radio spectrum resources availability of the network. However, spectrum professional researchers and developers have found out that licensed spectrum is still underutilized in some locations and times (Marcus et al. 2002). Cognitive Radio (CR) is a promising technology that leads to optimal use of radio spectrum by allowing unlicensed users access to the unused parts (holes) of the licensed spectrum. Cognitive radios (CRs) that access the licensed spectrum are interference sources to other high priority licensed users. Therefore, CRs may not cause any undue interference to licensed users by keeping interference below the interference threshold level, commonly referred to as the "interference temperature" limit I_{TL} (Haykin, 2005). Interference resulting from cognitive radios (CRs) is the most important aspect of cognitive radio networks that leads to degradation in Quality of Service (QoS) in both primary and CR systems. Each active user (CR or primary user) contributes to the interference affecting other users, so an efficient power control algorithms are essential in CR devices for achieving both system objectives (quality of service and system capacity).

Power Control (PC) in wireless networks has been widely considered in the past and recent years as an essential mechanism to maintain Signal-to-Interference Ratio (SIR). This in turn achieves the required Quality-of-Service (QoS) metrics, such as data rate and throughput (Hossain et al., 2009). In addition, PC can reduce the co-channel interference and extends the battery life in the Code Division Multiple Access (CDMA) systems, because each mobile device consumes minimum power needed to maintain the required SIR.

Closed-loop power control is widely used in wireless communication networks to maintain users' SIR and to reduce mobile power consumption. Closed-loop power control structure in standard IS-95 consists of an outer loop power control and inner loop power control as shown in Figure 3.1.

The outer loop power control received QoS requirement and then decides the suitable target SIR for the connection. It is noted that the target SIR value is not equal to the minimum required SIR for the current QoS level due to the variation in the radio channel. The value of actual SIR is varying around the target SIR, and to avoid constant outage, the target SIR must be larger than the minimum required SIR.

The inner loop power control receives the target SIR, which is based on the feedback from the received signal quality estimation. The difference between the target and the quality estimate values of SIR is computed in order to increment or decrement transmission power (Buehrer, 2006).



Figure 3.1 General component of power control loop (Hasu, 2007)

In this chapter, we proposed a novel cost function in underlay scenario for cognitive radios that consists of a weighted sum of power and square function of SIR error based on sigmoid function. As a rule, the cost function for the power control formula in this work also has two terms. The utility term is used to generate a similar DPC algorithm, and the second is the price term that includes the sigmoid function.

Important features of the proposed sigmoid power control scheme are:

- (i) the ability to maintain the required QoS of all CRs efficiently with insignificant reduction in SIR,
- (ii) the algorithm can be practically implemented in a distributive manner without requiring additional information from BS,
- (iii) a significant decrease and better fairness in mobile power consumption.

The novelty of the proposed sigmoid power control scheme is the sigmoid-based cost function. The choice of the proposed sigmoid based cost function is the key to enable each CR to choose its transmitting power efficiently. It guides cognitive users to choose lower power level to achieve their required QoS compared to other algorithms.

Furthermore, we explained in this chapter the variation between power control algorithms obtained from control theory and game theory concepts, and we presented these algorithms depending on the channel status. In addition, the proposed algorithm has an advantage that can be practically implemented in a distributive manner without requiring additional information.

The rest of this chapter is as follows: section 3.2 describes the system model of cognitive radio, the distributed power control and the game model of the proposed power control algorithm in section 3.3 and presents the numerical results and discussion

in section 3.4. Acceleration of a sigmoid power control game using Newton method has been proposed in section 3.5. In section 3.6, we concluded this chapter.

3.2 System model

Consider a system of a single cell wireless cognitive radio CDMA network in which *N* CRs share the same licensed spectrum with primary users. For simplicity, there is one CR base station that serves *N* CRs. Let p_i be the transmit power of *i*th cognitive user and h_i is the attenuation from *i*th cognitive user to the base station. The attenuation is computed from the distance d_i between the *i*th CR and the cognitive base station to be $h_i = A/d_i^{\alpha}$ with neglected shadowing and fast fading effects, where *A* is a constant gain and α is the path loss factor that is usually between 2 and 6. Figure 3.2 illustrates a simple system model of cognitive radio network.

In this chapter, we consider only the uplink power control case and it is assumed that all CRs are stationary. The received SIR of *i*th CR can be defined as

$$\gamma_i = \frac{p_i h_i}{\sum_{j \neq i}^N p_j h_j c_{ij} + \eta_i} \ge \Gamma_i$$
(3.1)

where Γ_i is the target SIR, η_i is the background noise, and c_{ij} is the correlation coefficient. The denominator of equation (3.1) represents the sum of interference including background noise and it can be denoted as $I_i(\mathbf{p}_{-i})$, where \mathbf{p}_{-i} is the vector of power for all CRs except the *i*th user. Thus, equation (3.1) can be rewritten similar to a general power control problem for wireless communication system as:

$$\gamma_{i} = \frac{p_{i}g_{ii}}{I_{i}(\mathbf{p}_{-i})} = \frac{p_{i}g_{ii}}{\sum_{j\neq i}^{N} p_{j}g_{ij} + \eta_{i}}$$
(3.2)

where g_{ij} is an effective link gain from the *i*th user to the base station. The subscript -i indicates the interference that depends on the power of all users except the *i*th user. Comparing between the two equations (3.1) and (3.2), leads to the following equation:

$$g_{ij} = \begin{cases} h_i, & i = j \\ h_j c_{ij}, & i \neq j \end{cases}$$
(3.3)

where g_{ij} denote to an effective link gain from the *j*th cognitive user to the base station that specifies the *j*th user's contribution to the interference affecting the signal of the *i*th user.



Figure 3.2: System model of cognitive radio networks

In spectrum trading, the objective of primary system that owns the spectrum rights is to maximize its revenue (profit) by sharing unused parts of spectrum (spectrum holes) with large numbers of CRs. The profit maximizing objective should be set under the constraint on limited performance degradation of primary users or the interference temperature limit (Hossain et al., 2009). For this, the total interference produced from cognitive radios should be less than the interference temperature limit and this condition can be expressed as

$$\sum_{i=1}^{N} p_{i} g_{0,i} \leq I_{TL}$$
 (3.4)

where $g_{0,i}$ is the channel gain from the transmitter of cognitive radio*i* to the measurement point of the primary system, and I_{TL} is the interference temperature limit.

Due to the competition of many cognitive radio networks to share the unused spectrum, it is assumed that the primary system has an intelligent admission control policy that will only accept the lowest interference CR network. Therefore, a new power control algorithm for cognitive radio networks has been considered in this work to guarantee lower interference and thus obtaining high priority in the admission control of the primary system.

To simplify the presentation and analysis, we also further introduce a user-specific notation θ_i as a ratio of interference to the link gain of cognitive user *i* as in (Xiao et al., 2001), and shown in the following equation:

$$\theta_i(\mathbf{p}_{-\mathbf{i}}) = \frac{I_i(\mathbf{p}_{-\mathbf{i}})}{g_{ii}} = \frac{p_i}{\gamma_i}$$
(3.5)

The value of θ_i represents the channel status of cognitive user *i*; a higher interference and a lower path gain results in a higher θ_i . The poor channel of cognitive user *i* results in a higher value of θ_i , while the good channel results in a lower value of θ_i . In (Foschini & Miljanic, 1993), the user *i* maintains its SIR at a target level Γ_i , and the power update formula depends on the previous values of power and SIR as

$$p_{i}^{(k+1)} = \frac{p_{i}^{(k)}}{\gamma_{i}^{(k)}} \Gamma_{i} = \theta_{i}^{(k)} \Gamma_{i}$$
(3.6)

where $\theta_i^{(k)}$ is the channel status of cognitive user *i* at the *k*th time step.

The unconstrained power control in equation (3.6) converges to a fixed point if the target SIR vector is feasible and all users can achieve their target SIR with minimal transmit power. On the other hand, if the target SIR vector is infeasible, then there is no transmit power vector that can satisfy SIR requirement for all users. Distributed power control (DPC) in (3.6) has been improved by Grandhi et al., (1994) to the constrained power control in order to solve the problem of divergence in infeasible case, that is,

$$p_i^{(k+1)} = \min\left\{\theta_i^{(k)}\Gamma_i, p_i^{\max}\right\}$$
(3.7)

where p_i^{max} is the maximum constrained transmit power. The control block diagram of constrain distributed power control (CDPC) which related to the symbol θ_i is shown in Figure 3.3, as in (Lee et al., 2003).



Figure 3.3: Control Block diagram of the CDPC related to the channel status θ_i .

3.2.1 Motivating example:

In this example, we study and explain the effect of slight reduction of SIR to the power level of users.

Consider a wireless system with three mobile stations who's the channel gains are given by

$$\begin{pmatrix} 1.000 & 0.0882 & 0.0357 \\ 0.1524 & 0.9500 & 0.3501 \\ 0.0767 & 0.0244 & 0.9900 \end{pmatrix}$$

Let assume that the background noise be $\eta_1 = \eta_2 = \eta_3 = \eta = 10^{-2}$, and assume that all mobile terminals want to achieve a target $\Gamma_1 = \Gamma_2 = \Gamma_3 = \Gamma = 5$. Note that the value of the target SIR is corresponding to 7 dB.

By applying the distributed power control (DPC) algorithm (SIR balancing algorithm) to these mobiles and running the algorithm 400 iterations, we obtained the last values of SIR and power as follows

$$\gamma = [4.9997, 4.9997, 4.9997]$$

 $p = [6.3844, 12.7091, 4.0989]$

It is found that all mobile users achieve their target SIR with different level of power depending on channel gain and interference. When we slightly reduced some values of the target SIR of some users and run the simulation test again, we found that the levels of user's powers are significantly reduced.

We adjusted the target SIR (slightly less) many times and re-run the simulation and then record the values of transmit power of all users. The power level results of the three mobile users with different values of target SIR are summarized in table 3.1.

SIR target values	<i>p</i> ₁	<i>p</i> ₂	<i>p</i> ₃
$\Gamma = [5.00, 5.00, 5.00]$	6.3844	12.7091	4.0898
Γ = [5.00, 4.95, 4.95]	3.3987	6.7140	2.1725
Γ=[5.00, 4.90, 4.90]	2.3051	4.5185	1.4702
Γ=[5.00, 4.90, 4.80]	1.8223	3.5548	1.1467
$\Gamma = [5.00, 4.80, 4.80]$	1.3948	2.6910	0.8855

Table 3.1: Final power level to achieve target SIR vectors

It is shown in the second row that small reduction (1%) of the target SIR of user 2 and user 3 resulting in approximate 50% reduction of power consumed of all users. Moreover, if the reduction of SIR target is (4%) of user 2 and 3, the transmit power of all users is reduce approximately to 79%. The results in Table 3.1 propose a power control algorithm alternative to DPC which can slightly reduce SIR of users who use higher power level. In game theory, pricing technique is the efficient tool to achieve this objective. Linear pricing of power is the common function used in the literature to obtain the pareto optimal solution which represents the point that no user can improve its own utility without reducing others utilities.

3.3 Power control based on game model

The interaction and selfish behavior among CRs requires a suitable framework for analysis. Recently, game theory has been considered as one of the most efficient tool for analyzing the interaction of decision makers. Game model in the basic form consists of three basic elements:

- (i) players or decision makers of the game,
- (ii) strategy or action space, and
- (iii) utility or cost function that represent preference of players.

Each player in a game selects its action from action space to maximize (or minimize cost) its own utility in a selfish manner. In CR networks, CR users can be considered as the decision makers of the game, network resources (power, data rate, etc.) are the strategy spaces of the game, and the utility function represents the preference (required QoS) of CR users. In this chapter, we defined the non-cooperative power control game of a CR system as follows:

$$\Phi = [N, \{P_i\}, \{J_i(.)\}]$$
(3.8)

where $N = \{1, 2, ..., N\}$ is the index set of players (CRs), $P_i = [0, P_i^{\max}]$ represents the transmission power strategy set of user *i*, and P_i^{\max} is the maximum transmission power of user *i*. The cost function of user *i* is referred to as $J_i(.)$, in which each CR tries to minimize its own cost function in a distributive manner.

3.3.1 Cost function and Nash equilibrium derivation

In subsections (2.2.2) and (2.2.3), we reviewed several cost functions that have been proposed to solve the problem of power control in wireless data networks. It is shown that most of cost functions were depending on the target SIR.

Given a reference target SIR (Γ_i) , power control is used to maintain the desired SIR based on the feedback of the error $e_i(\mathbf{p}) = \Gamma_i - \gamma_i(\mathbf{p})$.

The value of error $e_i(\mathbf{p})$ may be either positive or negative sign and to avoid the negative case, this term should be square. To achieve the required target SIR, all CR try to minimize this error or maximize the negative of this error. Thus, we considered the objective (utility) of CR user as:

$$u_i(\mathbf{p}) = -e(\mathbf{p})^2 = -(\Gamma_i - \gamma_i(\mathbf{p}))^2$$
(3.9)

To derive a power control algorithm for this utility function, each CR should maximize this utility and this is can be achieved at a point for which the partial derivative of $u_i(\mathbf{p})$ with respect to p_i is equal to zero.

$$\frac{\partial u_i(\mathbf{p})}{\partial p_i} = \frac{-\partial (\Gamma_i - \gamma_i(\mathbf{p}))^2}{\partial p_i} = -\frac{2\left(\Gamma_i - \frac{p_i g_{ii}}{I_i(\mathbf{p})}\right)g_{ii}}{I_i(\mathbf{p})} = 0$$
(3.10)

By arranging equation (3.10) to obtain p_i

$$2\left(\Gamma_{i} - \frac{p_{i} g_{ii}}{I_{i}(\mathbf{p})}\right)g_{ii} = 0$$
$$-\frac{p_{i} g_{ii}}{I_{i}(\mathbf{p})} = -\Gamma_{i}$$
$$p_{i} = \Gamma_{i} \frac{I_{i}(\mathbf{p})}{g_{ii}}$$

It is shown that equation (3.11) is equivalent to DPC formula in (3.6) which can guarantee that all CR achieve the same value of target SIR. Therefore, a pricing technique is necessary to relax (slightly reduce) the target value of SIR to obtain significant reduction of consumed power.

(3.11)

Thus, we assume that the cost function of *i*th cognitive user is $J_i(p_i, \gamma_i(\mathbf{p}))$, where the power vector is $\mathbf{p} = [p_1, p_2, ..., p_N]^T$. The Nash equilibrium point(s) means that no user can improve its individual cost function unilaterally. Mathematically, for all $i \in \mathbb{N}$

$$J_{i}(p_{i}^{*},\gamma_{i}(\mathbf{p}^{*})) \leq J_{i}(p_{i},\gamma_{i}(p_{1}^{*},p_{2}^{*},...,p_{i-1}^{*},p_{i},p_{i+1}^{*},...,p_{N}^{*})) \qquad \forall p_{i} \qquad (3.12)$$

Thus, we introduce a new sigmoid based pricing function of power that can relax the target values of user's SIR and guide CRs to efficient Nash equilibrium point as

$$v_i(\mathbf{p}) = \left(\frac{2}{\theta_i(\mathbf{p}_{\cdot \mathbf{i}})}\right) \sigma\left(\frac{b_i}{c_i}\theta_i(\mathbf{p}_{\cdot \mathbf{i}}), a\right) p_i$$
(3.13)

where b_i , c_i are the non-negative weighting factors, and a is the sigmoid factor. The function $\sigma(.)$ represents the proposed sigmoid function, which is defined as

$$\sigma(x,a) = \frac{2}{1 + e^{-ax}} - 1 \tag{3.14}$$

The sigmoid function value varies between 0 and 1, and it is noted that the sigmoid function has a derivative function. Figure 3.4; explain the curves of sigmoid function $\sigma(x, a)$ with different values of variable *x*, and sigmoid parameter *a*.



Figure 3.4: Sigmoid function with different values of *x*, and *a*

The proposed cost function should be convex and nonnegative to allow existence of a nonnegative minimum.

Thus, the proposed cost function can be written as the difference between pricing function and the utility function that expressed in (3.9) as

$$J_i(p_i, \gamma_i(\mathbf{p})) = v_i(\mathbf{p}) - u_i(\mathbf{p})$$

$$J_{i}(p_{i},\gamma_{i}(\mathbf{p})) = \left(\Gamma_{i}-\gamma_{i}(\mathbf{p})\right)^{2} + \left(\frac{2}{\theta_{i}(\mathbf{p}_{\cdot i})}\right)\sigma\left(\frac{b_{i}}{c_{i}}\theta_{i}(\mathbf{p}_{\cdot i}),a\right)p_{i}$$
(3.15)

It is shown from (3.15) that the cost function depends on the parameters b_i , c_i and the sigmoid parameter a.

The power update formula is obtained by applying the condition of a Nash equilibrium which is differentiating the cost function with respect to power and equating it with zero,

$$\frac{\partial J_i(\gamma_i, p_i)}{\partial p_i} = 0 = \left(\frac{2}{\theta_i}\right) \sigma \left(\frac{b_i}{c_i} \theta_i, a\right) - 2\left(\Gamma_i - \gamma_i\right) \frac{\partial \gamma_i}{\partial p_i}$$
$$= \left(\frac{2}{\theta_i}\right) \sigma \left(\frac{b_i}{c_i} \theta_i, a\right) - 2\left(\Gamma_i - \gamma_i\right) \frac{1}{\theta_i}$$
(3.16)

Rearranging terms of equation (3.16) yields

$$\left(\frac{2}{\theta_{i}}\right)\sigma\left(\frac{b_{i}}{c_{i}}\theta_{i}, a\right) = 2(\Gamma_{i} - \gamma_{i})\frac{1}{\theta_{i}}$$
$$\left(\Gamma_{i} - \gamma_{i}\right) = \sigma\left(\frac{b_{i}}{c_{i}}\theta_{i}, a\right)$$
$$\gamma_{i} = \Gamma_{i} - \sigma\left(\frac{b_{i}}{c_{i}}\theta_{i}, a\right)$$
(3.17)

It follows from (3.17) that as $b_i \to 0$, the term of pricing will also decrease to 0 according to (3.14), that is $\gamma_i \to \Gamma_i$. On the other hand, as $c_i \to 0$, γ_i will converges to the value of $\Gamma_i - 1$.

Substituting for γ_i in equation (3.2), and isolating p_i , we can express the power update formula in terms of given and measured quantities as

$$p_i \frac{g_{ii}}{I_i(\mathbf{p}_{-i})} = \Gamma_i - \sigma \left(\frac{b_i}{c_i} \theta_i, a\right)$$

$$p_{i} = \frac{I_{i}(\mathbf{p}_{\cdot i})}{g_{ii}} \Gamma_{i} - \theta_{i} \sigma \left(\frac{b_{i}}{c_{i}} \theta_{i}, a\right)$$

$$p_i = \theta_i \Gamma_i - \theta_i \sigma \left(\frac{b_i}{c_i} \theta_i, a \right)$$
(3.18)

Substituting for the interference using (3.1) in (3.17) and evaluating at the Nash equilibrium, we have

$$\gamma_i^* = \begin{cases} \Gamma_i - \sigma \left(\frac{b_i}{c_i} \theta_i^*, a \right), & \text{if nonnegative} \\ 0, & \text{otherwise} \end{cases}$$
(3.19)



Figure 3.5: Control Block diagram of the proposed sigmoid power control

3.3.2 **Power control algorithm**

According to equation (3.18), each CR user can update its power level using only the knowledge of its own interference level; therefore, this method can be implemented in a distributed manner. We assume that the algorithm is updated every step and depending on the measured interference, the proposed power update formula can be written as

$$p_i^{(k+1)} = \begin{cases} \theta_i^{(k)} \Gamma_i - \theta_i^{(k)} \sigma \left(\frac{b_i}{c_i} \theta_i^{(k)}, a \right), & \text{if positive} \\ 0, & \text{otherwise} \end{cases}$$
(3.20)

where $p_i^{(k+1)}$ is the power level of the *i*th user at the (k+1)th step and $\theta_i^{(k)} = p_i^{(k)} / \gamma_i^{(k)}$ is the channel status of the *i*th user that depends on the measured interference at the *k*th step of the algorithm.

The control block diagram of the sigmoid power control is shown in Figure 3.5, which explain how the output of distributed power control $\theta_i \Gamma_i$ has been processed by using the pricing factors and sigmoid function to obtain the final formula of the proposed power update formula.

3.3.3 Convergence

Yates, (1995) shows that if the algorithm $p^{(k+1)} = f(p^{(k)})$ converges to a fixed point, the function *f* should satisfy the following three conditions:

- 1) Positivity $f(p) \ge 0$,
- 2) Monotonicity $p \ge p' \implies f(p) \ge f(p')$,
- 3) Scalability $\forall \alpha \ge 1$; $\alpha f(p) \ge f(\alpha p)$.

First, we prove the positivity condition. Since

$$f_i(p) = \frac{I_i}{g_{ii}} \Gamma_i - \frac{I_i}{g_{ii}} \sigma \left(\frac{b_i}{c_i} \frac{I_i}{g_{ii}}, a \right)$$
(3.21)

Therefore, if we want $f(p) \ge 0$, it needs

$$I_i < \frac{c_i g_{ii} \sigma^{-1}(\Gamma_i)}{b_i}$$
(3.22)

Since $\sigma^{-1}(\Gamma_i) \approx 1$, if we choose a proper value for b_i/c_i , the positivity condition can be easily met.

The monotonicity condition can be proved by increasing the best response function with respect to I_i . By differentiating equation (3.21) with respect to I_i , we get

$$g_{ii} \frac{\partial f_i(p)}{\partial I_i} = \Gamma_i - \sigma \left(\frac{b_i I_i}{c_i g_{ii}}, a \right) - \frac{b_i I_i}{c_i g_{ii}} \frac{\partial \sigma \left(\frac{b_i I_i}{c_i g_{ii}}, a \right)}{\partial \frac{b_i I_i}{c_i g_{ii}}}$$
(3.23)

Using inequalities $\sigma(x, a) \le x$, a = 1 and $\frac{\partial \sigma(x, a)}{\partial x} \le \frac{a}{2}$, for monotonicity, we

should have

$$\Gamma_i \ge \frac{(1+0.5a)b_i I_i}{c_i g_{ii}} \qquad \Rightarrow \quad I_i \le \frac{c_i \Gamma_i g_{ii}}{b_i (1+0.5a)} \tag{3.24}$$

Finally, condition of the scalability in our method can be written as

$$\alpha f(p) - f(\alpha p) = \frac{\alpha p_i}{\gamma_i} \Gamma_i - \frac{\alpha p_i}{\gamma_i} \sigma\left(\frac{b_i p_i}{c_i \gamma_i}, a\right) - \left(\frac{\alpha p_i}{\gamma_i} \Gamma_i - \frac{\alpha p_i}{\gamma_i} \sigma\left(\frac{b_i \alpha p_i}{c_i \gamma_i}, a\right)\right)$$

$$= \frac{\alpha p_i}{\gamma_i} \sigma \left(\frac{b_i \alpha p_i}{c_i \gamma_i}, a \right) - \frac{\alpha p_i}{\gamma_i} \sigma \left(\frac{b_i p_i}{c_i \gamma_i}, a \right)$$
(3.25)

Since $\alpha > 1$, we have $\sigma\left(\frac{b_i \alpha p_i}{c_i \gamma_i}, a\right) \ge \sigma\left(\frac{b_i p_i}{c_i \gamma_i}, a\right)$. Therefore, for scalability, the

positivity can be met and it is sufficient.

From the above analysis and according to the parameters, we can conclude that the power control function is a standard function and the algorithm converges to a unique Nash equilibrium point.

3.3.4 Existence of Nash equilibrium

In this subsection, a solution is presented for the Nash algorithm algebraic equations to guarantee the existence of a unique solution to the power update equation. The result is obtained by using the Implicit Function Theorem (Ortega & Rheinboldt, 1970). From equation (3.18) and by using $\theta_i = I_i/g_{ii}$, we obtained the following algebraic equations:

$$F_{i}(p_{i}, p_{-i}, g_{ii}, g_{ii}, b_{i}, c_{i}, a, \eta_{i})$$

$$=0$$

$$=-p_{i} + \frac{\Gamma_{i}}{g_{ii}} \left(\sum_{j \neq i} g_{ij} p_{j} + \eta_{i}\right) - \frac{\left(\sum_{j \neq i} g_{ij} p_{j} + \eta_{i}\right)}{g_{ii}} \sigma \left(\frac{b_{i} \left(\sum_{j \neq i} g_{ij} p_{j} + \eta_{i}\right)}{c_{i} g_{ii}}, a\right), \quad \forall i, j \in \mathbb{N}$$

$$(3.26)$$

According to the Implicit Function Theorem, the Jacobian matrix $\partial F_i/\partial p_i$ must be a non-singular at the point of the existence. Since,

$$\frac{\partial F_i(p_i, p_{-i}, g_{ii}, g_{ij}, b_i, c_i, a, \eta_i)}{\partial p_i} = \begin{cases} -1 & j = i \\ \frac{g_{ij}}{g_{ii}} \Gamma_i - \frac{g_{ij}}{g_{ii}} \sigma\left(\frac{b_i g_{ij}}{c_i g_{ii}}, a\right) & j \neq i \end{cases}$$
(3.27)

The corresponding Jacobian matrix has -1 on the main diagonal and other elements are determined by Γ_i . The value of the Jacobian matrix is relevant to a, b_i, c_i, Γ_i , and g_{ij} . The value of the sigmoid function is relevant to the parameters a, b_i, c_i , which is not too large and the value of Γ_i is also not too large. The link gain g_{ij} is very small in practice, which depends on the distance of CR to base station. Thus, the Jacobian matrix is a non-singular and this proves the existence of Nash equilibrium.

3.4 Numerical results and discussion

To illustrate the advantages of the proposed algorithm, we compare the performance of the sigmoid power control algorithm with three different previous works, namely CDPC algorithm (Grandhi et al., 1994), Norm-2 algorithm (Koskie & Gajic, 2005), and Hyperbolic algorithm (Pasandshanjani et al., 2011), (Pasandshanjani & Khalaj, 2012).

For more clarification, the list of cost functions and power control update formula (iterative algorithm) that have been used in the simulation comparison are explained and declared in Table 3.2.

It can be seen from Table 3.2 that the power control formula of CDPC can be obtained by assuming that the cost function is $J_i = -c_i (\Gamma_i - \gamma_i)^2$. Due to unavailability of pricing term in the cost function of the CDPC, the power control formula does not have negative term and pricing factors.

Table 3.2: Cost functions and power control formulas used in the simulation

Algorithm	Cost function	PC formula
CDPC (Grandhi et al., 1994)	$J_i = -c_i (\Gamma_i - \gamma_i)^2$	$p_i^{(k+1)} = \begin{cases} \theta_i^{(k)} \Gamma_i \\ 0 \end{cases}$
Norm-2 Algorithm (Koskie & Gajic, 2005)	$J_i = c_i (\Gamma_i - \gamma_i)^2 + b_i p_i$	$p_i^{(k+1)} = \begin{cases} \theta_i^{(k)} \Gamma_i - \frac{b_i}{2c_i} (\theta_i^{(k)})^2 \\ 0 \end{cases}$
Hyperbolic Algorithm (Pasandshanjani et al., 2011; Pasandshanjani & Khalaj, 2012)	$J_i = c_i \cosh(\gamma_i - \Gamma_i) + b_i p_i$	$p_i^{(k+1)} = \begin{cases} \theta_i^{(k)} \Gamma_i - \theta_i^{(k)} \sinh^{-1} \left(\frac{b_i}{c_i} \theta_i^{(k)} \right) \\ 0 \end{cases}$

comparison.

Note that an admission control is not applied in this work, so there is no calldropping algorithm to drop mobiles terminals which cannot achieve the target SIR. Moreover, the effect of the selected pricing and sigmoid parameters is not studied because the work focuses on the demonstration of the algorithm potential. We also did not investigate the effects of changing code length or target SIR.

Initially, we demonstrate the system environment to which our algorithm and the above existing algorithms are applied with. We consider a $2000m \times 2000m$ cell with one cognitive base station located at the center and 30 CR users located randomly by a uniform distribution. A simple sketch of the system model is shown in Figure 3.6, in which the primary user may be interfered with CR users.

In this study, the fast fading, shadowing, and interference from the adjacent cells were neglected. The background noise power within the user's bandwidth is considered to be $\eta_i = 2 \times 10^{-17}$ W for all i = 1, 2, 3, ..., N.

The path gain was computed according to

$$h_i = \frac{A}{d_i^{\alpha}} \tag{3.28}$$

where d_i is the distance between the *i*th user and base station of cognitive radio, α is the path loss exponent, which is supposed to be 4, and A = 1 is a constant. The processing gain in this simulation is set to 128.



Figure 3.6: Random distribution of 30 cognitive users and one primary user.

3.4.1 Effect of channel status θ_i^k to the next step of power

We set the value of target SIR as $\Gamma_i = 5$, the ratio of weighting factors $b_i/c_i = 500$, and arrange the channel status value from 0 to 1×10^{-3} . The power p_i^{k+1} is computed using the power control formula in Table 3.2 and the equation (3.20). Figure 3.7 shows the values of next step power p_i^{k+1} according to θ_i^k , and it is found that the proposed sigmoid power control in the dashed blue line has the lowest power compared to other algorithms. The solid green line represents CDPC algorithm which has a higher power, while the dash-dot black line represent Norm-2 algorithm, and dotted red line represents the hyperbolic algorithm. Note that the lowest value of θ_i^k indicates to a good channel and vice versa.



Figure 3.7: Comparison of power update for a range of channel status.

3.4.2 Fully-loaded power and SIR

In this part of simulation, all cognitive users start the simulation with an initial power $p_i^{(0)} = 2.22 \times 10^{-16}$ W to avoid divide by zero in power updates formula. We used the target value of SIR as $\Gamma_i = 5$. The values of the non-negative weighting factors are $b_i = 1 \times 10^4$, $c_i = 1$, the sigmoid parameter a = 20, and the maximum constraint power of all users $p_i^{\text{max}} = 1$ mW.

We evaluated all algorithms using MATLAB programming and each algorithm run 80 iterative steps. The numerical results of power and SIR of the algorithms for all CRs are displayed in Figure 3.8.

In Figure 3.8, each line represents the value of power in (mW) and the value of SIR for each CR during 80 steps. Each box of power shows the maximum value of power $p_i^{\text{max}} = 1 \text{ mW}$ and maximum number of iterations for the y-axis and x-axes respectively. In addition, each box of SIR shows the achieve value of SIR and the number of iterations for the y-axis and x-axes respectively.

It is shown in Figure 3.8, that there are some cognitive users in the CDPC algorithm and Norm-2 algorithm who reach the maximum power $p_i^{\text{max}} = 1 \text{ mW}$ due to their bad channel condition. Furthermore, the reduction of achieved SIR in all algorithms can be seen clearly but the reduction is the highest in CDPC algorithm, followed by Norm-2 algorithm, and hyperbolic algorithm.

It is observed that the proposed sigmoid power control algorithm guarantees that all users can achieve their target SIR with the reduction of no more than 20% $(\gamma_i^{\text{min}} = 3.99 \sim \gamma_i^{\text{max}} = 4.99)$, while for other algorithms, it is shown that some users may obtain SIR reduction of more than 50% (Norm-2 and CDPC algorithms). The SIR of farthest user with bad channel condition is 2.123 in CDPC, 2.575 in Norm-2 algorithm, and 3.157 in hyperbolic algorithm, whereas in the proposed sigmoid algorithm it is 3.99.

On the other hand, the maximum power consume by the farthest user in our proposed sigmoid algorithm is $0.915 \,\mathrm{mW}$, while in hyperbolic algorithm is $0.972 \,\mathrm{mW}$, and the maximum power of some users in the other algorithms reach the maximum power of $1 \,\mathrm{mW}$.

Reducing the power consumption among CRs in the proposed sigmoid power control algorithm has the following advantages:

- 1. Increasing the life time of the batteries of CRs devices.
- 2. Reducing overall interference that can harm primary users in the licensed network and guarantee work under the interference temperature limit.
- 3. Guarantee the QoS of both primary and cognitive radio systems.
- 4. Lower interference of CR network resulted in higher acceptable rate in the admission control of primary system.



Figure 3.8: Performance comparison of proposed sigmoid algorithm and other

algorithms for 30 CRs.

The number of CRs that reach p_i^{max} is demonstrated in Table 3.2. It is found that 9 CRs use the maximum power in CDPC, and 6 CRs in Norm-2, while in hyperbolic and proposed sigmoid algorithms no users need to use its maximum power to achieve the target SIR. In the proposed sigmoid algorithm, the reduction of SIR does not exceed

25% for all CRs, while there are some users who got reduction of more than 25% in other algorithm as shown in the last column of Table 3.3.

It is also found that 6 cognitive users in CDPC and Norm-2 algorithms have reductions more than 25% and 8 cognitive radio users in the hyperbolic algorithm.

Algorithm	Min SIR	Max SIR	No of CRs reach P^{\max}	No of CRs with 25% reduction in SIR
CDPC (Grandhi et al., 1994)	2.123	5	9	24
Norm-2 (Koskie & Gajic, 2005)	2.575	5	6	24
Hyperbolic (Pasandshanjani & Khalaj, 2012)	3.157	5	0	22
Proposed Sigmoid	3.996	4.995	0	30

 Table 3.3: Numerical results obtained from the simulation.

As shown in Figure 3.9, the average power computed by the proposed sigmoid algorithm is significantly saved, i.e., $(\bar{p} = 256 \times 10^{-3})$ mW, while the results of other algorithms are $(\bar{p} = 305 \times 10^{-3})$ mW in the Hyperbolic power control algorithm, $(\bar{p} = 370 \times 10^{-3})$ mW in Norm-2 power control algorithm, and $(\bar{p} = 417 \times 10^{-3})$ mW in CDPC algorithm.

On the other hand, in comparison with Norm-2, hyperbolic, CDPC schemes, our approach leads to a slower convergence rate. In other words, there will be a trade off between power consumption and convergence rate. The point regarding this issue is that although it seems that the proposed method imposes higher level of complexity due to the sigmoid term, but the function argument is small enough to be well approximated by the first terms of the corresponding Taylor series. The reason of improvement in power saving refers to the term $(b_i I_i/2c_i g_{ii})$ that was introduced in Norm-2 algorithm and replaced by $\sinh^{-1}(b_i I_i/c_i g_{ii})$ in the hyperbolic algorithm and finally replaced by $\sigma(b_i I_i/c_i g_{ii}, a)$ in our proposed sigmoid algorithm. If we compare these terms, it can be observed that

$$\sigma(b_i I_i / c_i g_{ii}, a) \ge \sinh^{-1}(b_i I_i / c_i g_{ii}) \ge (b_i I_i / 2 c_i g_{ii}) \ge 0$$
(3.29)

It is also shown that algorithm using this term $(b_i I_i/2c_i g_{ii})$ in their power update algorithm is more power saving than CDPC algorithm.

Moreover, the results of the simulation indicate that the sigmoid function applied to the term $(b_i I_i/2c_i g_{ii})$ is more efficient than the hyperbolic function and quadratic function applied in the norm-2 algorithm.

On the other hand, Figure 3.10 shows that there are insignificant differences in the reduction of the average SIR between all algorithms. The final average value of SIR in the proposed sigmoid algorithm is ($\bar{\gamma} = 4.374$), while other algorithms are found to be ($\bar{\gamma} = 4.387$) in the hyperbolic power control algorithm, ($\bar{\gamma} = 4.395$) in the Norm-2 power control algorithm, and ($\bar{\gamma} = 4.402$) in CDPC algorithm. It is found that there is no significant difference for the value of average SIR because all algorithms should be adjusted to achieve the same value and show the effectiveness in saving power.



Figure 3.9: Comparison of average power for 30 CRs.



Figure 3.10: Comparison of average SIR for 30 CRs.

3.4.3 Impact of noise

Simulation results from Figures 3.8, 3.9, and 3.10 are computed depending on the effect of changing power to the amount of interference with fixed value of noise (background noise).

We tested the algorithm with the condition of a = 5, and N = 25 CRs, together with the same values of parameters used in the previous test. The range of noise power is from 10^{-18} W to 10^{-16} W. As shown in Figure 3.11, the average power increases and the SIRs decrease with the increase of noise for all algorithms because power is proportional to noise, while SIR is inversely proportional to noise as in equation (3.1). The proposed sigmoid algorithm provides significant savings of power in high noise environments, while the reduction in SIR is insignificant compared to other algorithms.

It is shown that the change of average SIR is between 4.6 has been achieved by the proposed sigmoid power control algorithm, and the target value 5 that is achieved by CDPC algorithm. On the other hand, it is found that average power of the proposed sigmoid power control is less than 1W, while average power exceeds 2W in the CDPC algorithm. In Norm-2 and hyperbolic power control algorithms, the average SIR in the range between 4.6 and 5, and their average power are greater than 1 W and but less than 2 W.



Figure 3.11: Performance comparison of average power and SIR for 25 CRs for a range of noise values.

3.5 Accelerating the sigmoid power control game algorithm using Newton iteration

In the previous sections of this work as mentioned in (Al-Gumaei et al., 2014), the problem of power control was proposed based on a sigmoid function. The final results obtained from the proposed game depend on a system of nonlinear algebraic sigmoid equations. The iteration method used in the proposed sigmoid power control algorithm was the Jacobi iteration, which is a fixed-point iterative method. The fixed-point iterative method has slow convergence speed, and the rate of convergent is linear. There are various methods that can be used to accelerate the convergence of power control algorithm such as Newton and secant iterations. In this subsection, we present a new version of Nash equilibrium sigmoid power control algorithm based on Newton iteration method.

3.5.1 Fixed-point algorithm for power updates

According to equation (3.18), each CR user can update its power level using only the knowledge of its own interference level; therefore, this method can be implemented in a distributed manner. We assume that the algorithm is updated every step and depends on the measured interference. The fixed point iterations for solving (3.18) can be expressed in the form $p^{(k+1)} = f_i^{(k)}(p^{(k)})$ as

$$p_{i}^{(k+1)} = \theta_{i}^{(k)} \Gamma_{i} - \theta_{i}^{(k)} \sigma \left(\frac{b_{i}}{c_{i}} \theta_{i}^{(k)}, a\right)$$

$$= \frac{I_{i}^{(k)}}{g_{ii}} \Gamma_{i} - \frac{I_{i}^{(k)}}{g_{ii}} \sigma \left(\frac{b_{i}}{c_{i}} \frac{I_{i}^{(k)}}{g_{ii}}, a\right) = f_{i}(p_{i}^{(k)})$$
(3.30)

where $p_i^{(k+1)}$ is the power level of the *i*th user at the (k+1)th step and $\theta_i^{(k)} = I_i^{(k)} / g_{ii}$ is the channel status of the *i*th user that depends on the measured interference at the *k*th step of the algorithm. The interference experience by the *i*th cognitive user at the *k*th step of the algorithm is recall as $I_i^{(k)} = \sum_{i=1}^N g_{ij} p_j^{(k)} + \eta_i$.

We may rewrite the algorithm of equation (3.20) in terms of the previous power value $p_i^{(k)}$ and the SIR measurement $\gamma_i^{(k)}$ by substituting $I_i^{(k)}$ in (3.5) using the relation $\gamma_i^{(k)} = g_{ii} p_i^{(k)} / I_i^{(k)}$.

$$p_{i}^{(k+1)} = \frac{p_{i}^{(k)}}{\gamma_{i}^{(k)}} \Gamma_{i} - \frac{p_{i}^{(k)}}{\gamma_{i}^{(k)}} \sigma \left(\frac{b_{i}}{c_{i}} \frac{p_{i}^{(k)}}{\gamma_{i}^{(k)}}, a\right)$$
(3.31)

Both formulation of power control algorithm expressed in (3.30) and (3.31) require only a single measurement at each step. Therefore, if this measurement is available to the CR, either algorithm can be used to implement a distributed power control. The formulation of power control algorithm in terms of power (3.31) requires nonzero initial powers. On the other hand, the formulation of power control algorithm in terms of interference does not require an initial power because the interference, which includes the noise power, is never zero.

3.5.2 Accelerated algorithm using Newton iterations

In this subsection, we replace the fixed point iteration that was proposed in previous work using Newton iteration to accelerate the convergence of the sigmoid power control algorithm. We define $\delta I_i^{(k)} = I_i^{(k+1)} - I_i^{(k)}$, where δ is a small perturbation fraction. Then

$$I_i^{(k+1)} = \delta I_i^{(k)} + I_i^{(k)}$$
(3.32)

where $I_i^{(k+1)} = \sum_{j \neq i} g_{ij} p_{ij}^{(k+1)} + \eta_i$

Substituting into equation (3.30) we obtain

$$p_{i}^{(k+1)} = \frac{I_{i}^{(k+1)}}{g_{ii}} \Gamma_{i} - \frac{I_{i}^{(k+1)}}{g_{ii}} \sigma \left(\frac{b_{i}}{c_{i}} \frac{I_{i}^{(k+1)}}{g_{ii}}, a\right)$$

$$= \frac{\delta I_{i}^{(k)} + I_{i}^{(k)}}{g_{ii}} \Gamma_{i} - \frac{\delta I_{i}^{(k)} + I_{i}^{(k)}}{g_{ii}} \sigma \left(\frac{b_{i}}{c_{i}} \frac{\delta I_{i}^{(k)} + I_{i}^{(k)}}{g_{ii}}, a\right)$$
(3.33)

By selecting an appropriate value of δ , the acceleration of the power control algorithm will be improved. When $\delta = 0$, the power update formula will return to the previous fixed point form as in equation (3.30).

3.5.3 Numerical results

To demonstrate the effectiveness of the developed algorithm based on Newton iterations, we tested it in realistic simulation using MATLAB. We compared the developed algorithm with the previous work that used fixed point iteration.

We considered a $2\text{km} \times 2\text{km}$ square cognitive radio cell with cognitive base station located at the center of the cell. There are *N* CRs user located randomly given by a uniform distribution, and one primary user located inside the coverage area of the cell and may be interfered with CR users. A simple sketch of the system model is shown in Figure 3.12 in which 25 CRs users are distributed inside the cell.

The maximum power of each CR is limited to $p_i^{\text{max}} = 1 \times 10^{-3} \text{ W}$, and the background receiver noise power $\eta_i = 0.2 \times 10^{-17} \text{ W}$. The target SIR of all CRs $\Gamma_i = 5$, the sigmoid parameter a = 20, and the pricing factors b = 10000, c = 1.

The path gain of each CR was computed according to

$$h_i = \frac{A}{r_i^{\alpha}} \tag{3.34}$$

where r_i is the distance between the *i*th user and base station, α is the path loss exponent, which is supposed to be 4, and A = 1 is a constant. The processing gain is set to 128. Similar to the previous simulation, the fast and shadow fading, and interference from adjacent cells have been neglected. Both algorithms run until the difference between current value of power and the previous value is less than tolerance $\varepsilon = 1e - 10$, and the small perturbation fraction is $\delta = 5 \times 10^{-5}$.


Figure 3.12: A typical random distribution of 25 CRs

We ran simulation of algorithms using the fixed point iteration in equation (3.31), and the accelerated of Newton iteration in equation (3.33) with 10 CRs. Figure 3.13 and Figure 3.14 show the power and SIR convergence for all CRs, and each curve represents the value of one CR for each step.

It is observed that the developed sigmoid power control algorithm based on Newton iteration method converge faster (6 iterations only) compare with fixed point iteration (11 iterations). In this test, all CRs can achieve that target SIR because the amount of interference is low and the budget of power is enough. In fact, the achieve value of SIR will change by increasing number of SIR due to the increase of interference. Farthest CRs with bad channel conditions will achieve SIR lower than the target.



Figure 3.13: Powers iteration comparison for 10 CRs



Figure 3.14: SIRs iteration comparison for 10 CRs

In the second simulation, we increased the cell loading to 25 CRs, and we presented the average values of power and SIR. We found that a greater improvement in the convergence speed of the developed (Newton method) sigmoid power control algorithm comparing is achieved compared to fixed point algorithm as shown in Figure 3.15 and Figure 3.16. It is shown that the developed algorithm can converge to the Nash solution using only 44 steps of iteration as compared to the fixed point algorithm that needs 107 iterations to reach the Nash equilibrium.



Figure 3.15: Average mobile power in the 25 CRs



Figure 3.16: Average SIR in the 25 CRs

In Figure 3.17, we ran the simulation in different system loads (increase number of users) and obtained the number of iterations required to converge to Nah equilibrium.

The results indicate that the Newton iterative method reduces the required number of iterations in power control algorithm. The percentage of reduction ranges from 4.28% in the system of 5 CR users and increase to 58% in the system of 25 CR users.

This shows that Newton method for power control algorithm is best suited for high loaded systems.



Figure 3.17: Number of iteration comparison with different number of CRs

3.6 Conclusion

Having studied several power control algorithms based on different cost functions, a new cost function for SIR-based sigmoid power control algorithm in cognitive radio networks has been proposed. The proposed power control algorithm achieves a significant reduction in power at approximately the same level of average SIR. This will enable the proposed algorithm to serve more CRs and thus achieve the optimal exploitation of the spectrum with the least amount of interference.

In addition, the proposed algorithm results in better fairness, in which all users meet their SIR constraints without transmitting at high power levels. The proposed sigmoid power control algorithm is general and can be applied to the uplink of low-range two tier femtocell networks using the game approach. However, an efficient pricing technique would be required to manage the cross-tier interference.

The iteration method used in the proposed sigmoid power control algorithm was the fixed point iterative method which has slow convergence. We developed the sigmoid power control algorithm based on Newton iterative method to accelerate the convergence. Simulation results indicate that the Newton iterations can reduce the required iterations for the algorithm convergence by selecting an appropriate value of the perturbation fraction. Improvement in the convergence speed occurs in a variety of system load and the significant difference is achieved in the high load system.

CHAPTER 4: A NOVEL UTILITY FUNCTION FOR ENERGY-EFFICIENT NON-COOPERATIVE POWER CONTROL GAME IN COGNITIVE RADIO NETWORKS

4.1 Introduction

The challenge in game theory approach is the formulation of a utility function that it has a physical meaning and the game outcome is not trivial (Hossain et al., 2009). There are various and different ways to design a power control algorithm based on utility and price functions.

Some researchers proposed utility as a difference between utility and pricing functions, and users seek to maximize this utility in a selfish manner. In this case, the utility function should be quasi-concave and an optimal point is selected to be somewhere within the practical parameter range, such as minimum and maximum power, and it depends on other users' behavior (Hossain et al., 2009). The special case of this type of utility function is energy efficiency, in which the utility has a physical meaning of the number of successfully received information bits per joule of energy cost. All users adjust their transmit power to achieve the required SIR that is not defined directly inside the utility function, but depends on the supposed efficiency function.

On the other hand, some other researchers proposed the utility (cost) function as a difference between price and utility functions, and users seek to minimize this cost function in a selfish manner. In this case, the cost function should be quasi-convex and the optimal point (minimum point) is selected to be somewhere within the practical parameter. In the cost function approach, all users try to achieve the required SIR (target

SIR $\gamma_i \cong \Gamma_i$) that is usually defined in the cost function. The differences between the two approaches have been summarized in Table 4.1.

	Power control based on Utility	Power control based on Cost	
	function	function	
Definition	Difference between utility and	Difference between price and	
	price (Utility -Price)	utility (Price-Utility)	
Users goal	Maximize their own utility	Minimize their own cost	
Function shape	Quasi-concave (maxima)	Quasi-convex (minima)	
Final SIR	The SIR achieved depends on	The SIR achieved is equal to	
	efficiency function that is defined	the target that is declared in	
	in utility function	cost function	
Others	Similarly to optimization theory	Similarly to control theory	

Table 4.1: Differences between utility and cost function based power control.

In this chapter, we proposed a novel utility function function in underlay scenario for cognitive radios that consists of a weighted exponential of the ratio of target SIR and the desired signal, and the pricing function that comprises a power function of CR's transmitting power.

The important features of the proposed power control energy efficient scheme are: (i) it can preserve the required QoS of all CRs efficiently with insignificant reduction in SIR, (ii) the algorithm can be practically implemented in a distributive manner without requiring additional information, (iii) a significant reduction, and better power allocation to all CRs, and (iv) fast convergence to Nash equilibrium.

The novelty of the proposed power control scheme is the new sigmoid exponential efficiency function and the power function that applied to the pricing part. The choice of

the proposed utility and pricing functions is the key to enable each CR to choose its transmitting power efficiently. It guides closest CRs to the base station to achieve their QoS requirement with low cost, whereas it guides farthest CRs from the base station to achieve their QoS requirement with high cost to mitigate the interference.

We proved the Nash equilibrium existence of the power control game and the conditions of the selected pricing factors. Furthermore, we explained the difference between the linear and power functions to the pricing function, and the effect of weighting factor to the utility function and transmit power. On the other hand, we explained that the proposed efficient non-cooperative power control algorithm (EF-NPGP) can be practically implemented in a distributive manner without requiring additional information.

The rest of this chapter is as follows. Section 4.2 describes the system model of CR, the non-cooperative power control game based on the utility function, and presents the EF-NPGP algorithm. Section 4.3 presents the numerical results and discussion. The conclusion is presented in section 4.4.

4.2 System model

In this chapter, we consider a single cell CRN with one cognitive base station (CBS) and one primary access point (PAP) as shown in Figure 4.1. This work focuses on uplink power control case, and can also be applied in downlink. The cognitive radio cell contains N CRs which share unused parts of licensed spectrum with a single primary user, and they employ code division multiple access (CDMA) technique to utilize the available spectrum in their own communications. It is assumed that CRs are stationary in which all the path gains of all CRs are fixed and all CRs are distributed inside the coverage area of the cell.

The transmit power of *i*th cognitive user is denoted by p_i and the channel link gain (path gain) between the *i*th cognitive user and CBS is h_i which depend on the distance between *i*th cognitive user and CBS.

In general, the formula of SIR in the single cell cognitive radio CDMA system of *i*th CR is expressed as

$$\gamma_i(p_i) = \frac{G p_i h_i}{\sum_{j \neq i}^N p_j h_j + \sigma_i^2} \ge \Gamma_i \quad , i = 1, 2, ..., n$$

$$(4.1)$$

where G denotes the processing gain of the spread spectrum system, Γ_i is the threshold SIR, and σ^2 is the power of the Gaussian noise.

The denominator of equation (4.1) represents the sum of interference including noise and can be denoted as $I_i(\mathbf{p}_{-i})$, hence equation (4.1) can be rewritten as a function of user transmits power and the transmit power of other users as:

$$\gamma_{i}(p_{i}, \mathbf{p}_{-i}) = \frac{G p_{i} h_{i}}{I_{i}(\mathbf{p}_{-i})} = \frac{G p_{i} h_{i}}{\sum_{j \neq i}^{N} p_{j} h_{j} + \sigma_{i}^{2}}$$
(4.2)

The subscript -i in the power vector \mathbf{p}_{-i} of equation (4.2) indicates to the vector of all users' power except the *i*th user.

In this system model, we assume that the primary system seeks to maximize its own revenue (profit) by allowing many CRs to share its own licensed spectrum. The revenue maximizing is restricted by the limited performance degradation of primary users or interference temperature limit (Hossain et al., 2009).



Figure 4.1: System model of cognitive radio network

The total interference power made by CRs should be less than a given threshold that is called interference temperature limit which guarantees the QoS of primary users, and this constraint can be expressed as

$$\sum_{i=1}^{N} p_i h_{0,i} \leq I_{TL}$$
(4.3)

where $h_{0,i}$ is the path gain from the transmitter of cognitive radio *i* to the access point of the primary system, and I_{TL} is the interference temperature limit.

4.2.1 Non-cooperative power control game with pricing

In recent years, concepts of microeconomics and game theory have been used extensively to define users QoS in terms of utility (cost) function instead of SIR (Hossain et al., 2009). In general, the power control game model consists of three elements (i) mobile users (or CRs) that represent the players or decision makers of the game, (ii) power strategy which represents the game strategy or action space, and (iii) utility function (preference of users).

Each cognitive user in the network tries to maximize its own utility without any cooperation with other users. The non-cooperative power control game (NPG) model can be expressed as

$$\Phi = [N, \{P_i\}, \{u_i(.)\}] \tag{4.4}$$

Where $n = \{1, 2, ..., N\}$ is the index set of players (CRs), $P_i = [0, P_i^{\max}]$ represents the transmission power strategy set of user *i*, and P_i^{\max} is the maximum transmission power of user *i*. The utility function of user *i* is referred to as $u_i(.)$, in which each CR seeks to maximize its own utility function in a selfish manner.

The main objectives of power control game is to reduce the power consume of CRs, achieve the required SIR, and mitigate the total interference in CRN. To achieve these objectives, the payoff function (utility function) of power control game in equation (4.4) should consider the following properties as in (MacKenzie & DaSilva, 2006):

- 1. The utility is a function of CRs transmits power and SIR. The SIR of CR is a function of CR's transmit power and the transmit power of other users.
- 2. When CR user increases its power level, this will increase its own SIR, but will decrease the SIR of other CRs.
- 3. For a fixed SIR, the CR prefers lower power level to higher ones to extend battery life and reduce interference.
- 4. For a fixed power, the CR prefers higher SIR to lower SIR in order to obtain a good channel condition.

In wireless and cognitive radio network, each CR transmits its information over the air using multiple access system. Since air is a common medium for all the signals, each CR's signal acts as interference to other user's signal. This interference plus fading, multipath and background noise cause signal distortion as its traveling from the source to destination. The denominator of the SIR in equation (4.1) represents all these impediments of the signal.

Moreover, CR terminals are battery-based devices, so the transmitter power is another important commodity for them. Therefore, SIR and transmit power are the most important parameters that will be used to formulate the expression, which determines user satisfaction using the network (Shah et al., 1998).

Information sent from transmitters to receivers in wireless data and CRNs are in the form of frames (or packets) of length M bits, containing L < M information bits at a data rate of R bits/sec. Assuming that all errors in the received signal can be detected by the system and the incorrect data can be retransmitted, then, the achieved throughput T can be defined as

$$T = R f(\gamma) \tag{4.5}$$

where $f(\gamma)$ is the efficiency function of transmission. The efficiency function $f(\gamma)$ should depend on SIR achieved over the channel, and the value of $f(\gamma)$ should varies from zero to one (i.e., $f(\gamma) \in [0,1]$).

Furthermore, if the user's *i* transmitted power is p_i , then the utility function of user *i* can be expressed as the number of information bits received successfully per Joule of energy consumed as (Goodman & Mandayam, 2000)

$$u_i(p_i, \mathbf{p}_{-i}) = \frac{LRf(\gamma_i)}{Mp_i}$$
(4.6)

The Nash equilibrium resulting from non-cooperative power control is inefficient because it ignores the cost (harm) it imposes on other terminals by the interference they generate. Therefore, the concept of pricing has been used to encourage cognitive users to use the network resource more efficiently. The general expression of non-cooperative power control game with pricing (NPGP) can be written as

$$\Phi^{C} = [N, \{P_i\}, \{u_i^{C}(.)\}]$$
(4.7)

where n = [1, 2, ..., N] is the index of participating CRs, who are the decision makers of the game select a particular transmit power level; P_i denote the set of transmission power strategies of the *i*th CR, and u_i^C (.) is the utility function via pricing that can be defined as

$$u_{i}^{C}(p_{i},\mathbf{p}_{i}) = u_{i}(p_{i},\mathbf{p}_{i}) - v_{i}(p_{i},\mathbf{p}_{i})$$
(4.8)

Several works considered the problem of power control by introducing dissimilar utility and pricing functions. In (Saraydar et al., 2002), the authors proposed the energy efficient utility function as

NPGP:
$$u_i^C(p_i, \mathbf{p}_{-i}) = \frac{LR}{M p_i} (1 - e^{-\gamma_i/2})^M - C_1 p_i$$
 (4.9)

where C_1 is the positive pricing factor. Based on equation (4.9), authors in (Zhang et al., 2012) used the same utility function in (Saraydar et al., 2002), and they introduced a new pricing function. The non-cooperative power control game (NPG) was established using the modified Shuffled Frog Leaping Algorithm (MSFLA) by Zhang et al. (2012) and the utility function expressed as follows

NPG-MSFLA:
$$u_i^C(p_i, \mathbf{p}_{-i}) = \frac{LR}{M p_i} (1 - e^{-\gamma_i/2})^M - C_2 e^{p_i} - C_3 (\gamma_i - \Gamma_i)$$
 (4.10)

where C_2 and C_3 are the positive pricing factors. The efficiency function that has been used in (Saraydar et al., 2002) and (Zhang et al., 2012), is the same, which is related to the non-coherent frequency shift keying (FSK) modulation scheme. The formula of efficiency function is expressed as

$$f_1(\gamma_i) = \left(1 - e^{-\gamma_i/2}\right)^M$$
(4.11)

In (Kuo et al., 2013), the authors proposed a utility function depending on the sigmoid function, and they introduced a new design of pricing function. The non-cooperative power game with pricing (NPGP) was established using the efficient swarm intelligent algorithm (ESIA) and the utility function with pricing is expressed as

NPGP-ESIA:
$$u_i^C(p_i, \mathbf{p}_{-i}) = \frac{LR}{M p_i} \frac{1 - e^{-\gamma_i}}{1 + e^{\Gamma_i - \gamma_i}} - C_1 e^{C_2((\gamma_i / \Gamma_i) - 1)} \frac{p_i}{p^{th}}$$
 (4.12)

where C_1 and C_2 are the positive pricing factors, p^{th} is the average interference power which can be obtained by taking the mean value of users transmit power $p_i^{th}: p^{th} = (p_1^{th} + p_2^{th} + ... + p_i^{th})/N$, and the formula of the sigmoid efficiency function is expressed as

$$f_{2}(\gamma_{i}) = \frac{1 - e^{-\gamma_{i}}}{1 + e^{\Gamma_{i} - \gamma_{i}}}$$
(4.13)

The fair power control game in (Xie et al., 2014) proposed the utility function based on the simplified sigmoid function that was used in (Kuo et al., 2013), and they introduced a new non-linear pricing function where the non-cooperative power game with pricing (NPGP) was established using a sliding model, called (R-NPGP) as

R-NPGP:
$$u_i^C(p_i, \mathbf{p}_{-i}) = \frac{LR}{M p_i} \frac{1}{1 + e^{\Gamma_i - \gamma_i}} - C_1 \lambda_i \frac{p_i}{p^{th}}$$
 (4.14)

where C_1 s a positive pricing factor and λ_i is another pricing factor that varies for different CRs based on their generated conditions, and the formula of the efficiency function is expressed as

$$f_3(\gamma_i) = \frac{1}{1 + e^{\Gamma_i - \gamma_i}} \tag{4.15}$$

4.2.2 Proposed game model

Based on the previous works, we propose in this section a novel utility function based on a new sigmoid efficiency function and a power function of user's transmit power pricing function.

Example: SIR-based packet delivery ratio

Given the packet size and data-rate to any source destination wireless system such as (802.11g OFDM implementation), the packet delivery ratio (PDR) as a function of SIR is dependent on the transceiver radio card (Smith et al., 2014).

The simulation results of PDR obtained from Trivellato simulator (Trivellato, 2007), (Baldo et al.,, 2007) indicates that the function of PDR is a sigmoid function of SIR. In addition, another study (Kovács et al., 2011) gave an excellent approximation of practical sigmoidal functions of SIR using Gompertz function (Gompertz, 1825). By converting the SIR back to the linear domain, the formula of packet delivery ratio (PDR) can be expressed as a compressed exponential function of inverse SIR 1/ γ where the compressed exponential function is equivalent to the complementary cumulative distribution function of the Weibull distribution. Hence, the formula of PDR is expressed as

$$PDR = \exp\left(-\left(\frac{1}{\gamma a_c}\right)^{b_c}\right)$$
(4.16)

where a_c and b_c are constant parameters with respect to particular packet sizes and data rates. The sample of results obtained from Trivellato simulator for different values of data rate and packet size are summarized in Table 4.2.

In Table 4.2, there are three example of implementation with respect to data rates (6 Mbps, 9 Mbps, and 18Mbps) and packet sizes (256 bytes, 512 bytes, and 128 bytes). The table shows the values of the parameters a_c and b_c in the third and fourth columns for different values of packet sizes and data rates.

Data rate	Packet Size	a_{c}	b_c
6 Mbps	256 bytes	1.194	4.733
9 Mbps	512 bytes	0.4875	2.911
18 Mbps	128 bytes	0.2913	2.998

Table 4.2: Data rates, packet sizes results from (Trivellato, 2007), with estimated parameters a_c and b_c in compressed exponential function (4.16)

Figure 4.2, depicts the comparison of the three samples packet delivery ratio estimated from (4.16) according to the range of SIR value. It is found that packet delivery ratio depends on the achieved value of SIR and the value of PDR varies from zero to one (i.e., $PDR \in [0,1]$).



Figure 4.2: SIR vs PDR simulation (Trivellato, 2007), (Baldo et al., 2007) and compressed-exponential approximation (4.16), for the three data rates and packet sizes in Table 4.2.

Therefore, we introduce our proposed sigmoid efficiency function as the exponential ratio of target SIR and the desired signal similar to the packet delivery ration formula. The efficiency function is expressed as

$$f_4(\gamma_i) = \exp\left(-\left(\frac{a \Gamma_i}{\gamma_i}\right)^b\right)$$
(4.17)

where a and b are non-negative weighting factors. The features of this proposed efficiency function can be summarized as:

- (1) The efficiency function introduced in equation (4.17) is a sigmoidal shape function with $f(\infty) = 1$, and f(0) = 0 that ensure $u_i = 0$ when $p_i = 0$.
- (2) The efficiency function $f_4(\gamma_i)$ with data rate *R* represents the throughput of the system (number of information bits that can be transmitted successfully).

The comparison of the proposed efficiency function and the efficiency functions declared in equations (4.11), (4.13), and (4.15) is shown in Figure 4.3. It can be seen that all efficiency functions increased by increasing the value of SIR.



Figure 4.3: Efficiency function comparison $\Gamma_1 = 10$, a = 0.8, b = 3, M = 80.

The proposed energy efficient utility function should be satisfy some of the properties as in (Shah et al., 1998), (Famolari et al., 2002)

Property 4.1: The utility function u_i is a monotonically increasing function of the cognitive users SIR γ_i for a fixed transmitter power p_i . Thus, the differentiate of utility function with respect to SIR is positive

$$\frac{\partial u_i(p_i,\gamma_i)}{\partial \gamma_i} > 0, \quad \forall \ p_i,\gamma_i > 0$$
(4.18)

Property 4.2: The utility function obeys the law of diminishing marginal utility for large values of SIR γ_i .

$$\lim_{\gamma_i \to \infty} \frac{\partial u_i(p_i, \gamma_i)}{\partial \gamma_i} = 0, \quad \forall \ p_i > 0$$
(4.19)

We further assume the utility function to be a positive function of SIR, and when $\gamma_i = 0$, we assume that the utility function u_i is zero. On the other hand, the transmit power attribute to conserve battery energy. At the same time, we should avoid the case such as zero transmits power. These can be specified in the form of the following three properties.

Property 4.3: The utility is a monotonically decreasing function of the user's transmitter power (for a fixed SIR).

$$\frac{\partial u_i(p_i,\gamma_i)}{\partial p_i} < 0, \quad \forall \ p_i,\gamma_i > 0$$
(4.20)

Property 4.4: In the limit that the transmit power tends to zero the utility value also tends to zero, or:

$$\lim_{p_i \to 0} u_i = 0 \tag{4.21}$$

Property 4.5: In the limit that the transmitter power goes to infinity the utility value tends to zero:

$$\lim_{p_i \to \infty} u_i = 0 \tag{4.22}$$

According to the energy efficiency equation (4.17) and the above five properties, the utility function of the *i*th CR can be also written as a ratio between CR user's throughput and transmit power as

$$u_{i} = \frac{LR}{M p_{i}} \exp\left(-\left(\frac{a \Gamma_{i}}{\gamma_{i}}\right)^{b}\right) \frac{\text{bits}}{\text{joule}}$$
(4.23)

The utility function in equation (4.23) represents the tradeoff between the throughput and battery life and it is particularly appropriate for applications where saving power is more important than achieving a high throughput, such as green cognitive radio (Meshkati et al., 2006).

Assuming that the value of target SIR is fixed at the cognitive radio system and the weighting factor does not depend on the data rates or packet sizes, the proposed utility function can be tuned using the weighting factor a. The user's optimal transmit power will be changed depending on the maxima of utility function.

Figure 4.3 shows the curves of our proposed utility function with respect to transmit power in with different values of weighting factor a. It is shown that the utility increases and the transmitting power decreases by decreasing the value of the parameter a, but this will decrease the target of SIR of the system. The weighting factor a can be broadcasted by the primary system to the cognitive radios to adjust the target value of SIR depending on the QoS of primary users and the amount of interference. The primary system sends a lower value of parameter a when the amount of interference approximately reaches the interference temperature limit.

The resulting of the non-cooperative power control game has Nash equilibrium, but it is inefficient. Therefore, pricing technique should be applied to improve system efficiency.



Figure 4.4: User's utility function as a function of transmits power for fixed interference and different value of weighting factor *a*.

Furthermore, we introduced a new design of the pricing function to improve the system performance in order to encourage CRs to use system resources efficiently. The contribution of our design is to apply a high cost to the users that use high power, such as the farthest users from the base station, and decrease the cost of the closest users.

Therefore, we introduced a power function of the transmit power instead of traditional linear pricing. Figure 4.5 shows an example to explain the difference between linear and power pricing techniques. We assumed that user transmit power varies between the minimum and maximum power strategy space[0, 2], and price functions are computed numerically.

It is shown that the power function pricing is lower than the linear function pricing for CRs who use low transmit power (closer users), whereas a high pricing cost will be applied to the CRs who use high transmit power (farthest users).



Figure 4.5: Linear and power function pricing comparison with c = 5 and $\lambda = 2.5$

Thus, the proposed pricing function is expressed as

$$v_i(p_i, \mathbf{p}_{-i}) = c p_i^{\lambda} \tag{4.24}$$

where *c* and λ are the pricing factors. Thus, the utility function with pricing can be expressed as a difference between utility and price as

$$u_i^C(p_i, \mathbf{p}_{-i}) = \frac{LR}{M p_i} \exp\left(-\left(\frac{a \Gamma_i}{\gamma_i}\right)^b\right) - c p_i^{\lambda}$$
(4.25)

Therefore, the proposed energy efficient non-cooperative power control game with pricing (EF-NPGP) is expressed as

$$\text{EF-NPGP}: \max_{p_i \in P_i} u_i^C(p_i, \mathbf{p}_{-i}) = \frac{LR}{M p_i} \exp\left(-\left(\frac{a \Gamma_i}{\gamma_i}\right)^b\right) - c p_i^{\lambda} \quad (4.26)$$

The advantage of power pricing function is its ability to encourages CRs to use lower transmit power and thus can lead to Pareto improvement over the non-cooperative game. This is done by increasing the cost of the farthest users who use high transmit power in their communication. Moreover, the pricing function reduced the cost applied to the nearest CRs who use low transmit power in their communication. If the difference of objective function (utility-price) is quasi-concave, then we argue and show that there exists Nash equilibrium. In Figure 4.6, we show the effect of the power pricing function to the energy efficient non-cooperative power control game. The user uses lower power P1 in the case of utility-pricing compared to P2 in the case of utility only (without pricing).

Each CR seeks to maximize its own profit (utility-price) by adjusting its transmit power in a distributed manner. The expected Nash equilibrium resulting from the power control game is the balanced power of all CRs in that no single CR can increase the benefit by changing its own transmits power.



Figure 4.6: Effect of the power pricing function on the energy efficient noncooperative power control game

To derive an algorithm of non-cooperative power control game, we adopt a power control algorithm in which each CR will maximize its net utility $u_i^C(p_i, \mathbf{p}_{-i})$. For power optimization, the maximization can be achieved at a point for which the partial derivative of $u_i^C(p_i, \mathbf{p}_{-i})$ with respect to power p_i is equal to zero. Following simple analysis, we get

$$\frac{\partial u_i^C}{\partial p_i} = \frac{LR}{M p_i^2} \left(\gamma_i \frac{\partial f_4(\gamma_i)}{\partial \gamma_i} - f_4(\gamma_i) \right) - \alpha c p_i^{\lambda - 1}$$
(4.27)

4.2.3 Existence of Nash equilibrium

In non-cooperative power control game, the *i*th CR maximizes its utility by choosing a proper strategy from the strategy set $P_i = [0, P_i^{\max}]$. A Nash equilibrium exists in non-cooperative power control game, if for all i = 1, 2, ..., n meet the following two conditions (Topkis, 1998):

- 1. The action set P_i is non-empty, convex, and compact subset of some Euclidean \Re^N .
- 2. The utility function $u_i^C(p_i, \mathbf{p}_{-i})$ is continuous in **p** and

$$\left(\partial^2 u_i^C / \partial p_i \partial p_j\right) \ge 0 \quad \forall j \neq i \in N.$$

The transmit power space strategy for each CR in our game is defined by the minimum and maximum powers, and the value of powers is between these values. Therefore, the first condition of action set p_i is satisfied.

To show that the CR utility function is quasi-concave in p_i , the second derivative of $u_i^C(p_i, \mathbf{p}_i)$ is obtained with respect to p_i .

$$\frac{\partial u_i^C}{\partial p_i} = \frac{LR}{M p_i^2} \left(\gamma_i \frac{\partial f_4(\gamma_i)}{\partial \gamma_i} - f_4(\gamma_i) \right) - \alpha c p_i^{\lambda - 1}$$
(4.28)

$$\frac{\partial^{2} u_{i}^{C}}{\partial p_{i} \partial p_{j}} = \frac{LR}{M p_{i}^{2}} \left(\frac{\partial \gamma_{i}}{\partial p_{j}} \frac{\partial f_{4}(\gamma_{i})}{\partial \gamma_{i}} + \gamma_{i} \frac{\partial \gamma_{i}}{\partial p_{j}} \frac{\partial^{2} f_{4}(\gamma_{i})}{\partial \gamma_{i}^{2}} - \frac{\partial \gamma_{i}}{\partial p_{j}} \frac{\partial f_{4}(\gamma_{i})}{\partial \gamma_{i}} \right)$$

$$= \frac{LR}{M p_{i}^{2}} \left(\gamma_{i} \frac{\partial \gamma_{i}}{\partial p_{j}} \frac{\partial^{2} f_{4}(\gamma_{i})}{\partial \gamma_{i}^{2}} \right)$$
(4.29)

Because the first-order derivative of SIR γ_i with respect to the power p_i is $\left(\partial \gamma_i / \partial p_j\right) = -\left(\frac{Gh_i h_j p_i}{\sum_{j \neq i} h_j p_j} + \sigma^2\right) < 0$, therefore we need a second-order

derivative of our efficiency function with respect to γ_i be $\partial^2 f_4(\gamma_i) / \partial \gamma_i^2 \leq 0$.

$$\frac{\partial f_4(\gamma_i)}{\partial \gamma_i} = \frac{\partial e^{\left(-\left(\frac{a\Gamma_i}{\gamma_i}\right)^b\right)}}{\partial \gamma_i}$$

$$= \frac{b\left(-\left(\frac{a\Gamma_i}{\gamma_i}\right)^b\right)e^{\left(-\left(\frac{a\Gamma_i}{\gamma_i}\right)^b\right)}}{\gamma_i}$$
(4.30)

$$\frac{\partial^2 f_4(\gamma_i)}{\partial \gamma_i^2} = -\frac{b e^{\left(-\left(\frac{a \Gamma_i}{\gamma_i}\right)^b\right)} \left(b \left(\frac{a \Gamma_i}{\gamma_i}\right)^b + \left(\frac{a \Gamma_i}{\gamma_i}\right)^b - b \left(\frac{a \Gamma_i}{\gamma_i}\right)^{2b}\right)}{\gamma_i^2}$$
(4.31)

Because $be^{\left(-\left(\frac{a\Gamma_i}{\gamma_i}\right)^b\right)}/\gamma_i > 0$, in order to $\partial^2 f_4(\gamma_i)/\partial\gamma_i^2 \le 0$

$$\begin{pmatrix}
b\left(\frac{a\Gamma_{i}}{\gamma_{i}}\right)^{b} + \left(\frac{a\Gamma_{i}}{\gamma_{i}}\right)^{b} - b\left(\frac{a\Gamma_{i}}{\gamma_{i}}\right)^{2b} \\
\Rightarrow b + 1 \ge b\left(\frac{a\Gamma_{i}}{\gamma_{i}}\right)^{b} \\
\frac{(b+1)^{1/b}}{ab} \ge \frac{\Gamma_{i}}{\gamma_{i}}
\end{cases}$$
(4.32)

According to equation (4.32) and by selecting the pricing factors carefully, the second condition has been satisfied. Hence, the proposed power control game has a unique Nash equilibrium solution.

4.2.4 EF-NPGP algorithm

This subsection presents an iteration algorithm for EF-NPGP scheme to control all transmission powers to guarantees the required SIR among all CRs and ensure Nash equilibrium opportunisticly with the available SIR information.

We suppose that each CR updates it's transmit power at time instances $t_i = \{t_{i1}, t_{i2}, \dots, \}$, where $t_{ik} < t_{i(k+1)}$, and we assume the strategy set of power of the *i*th CR is $P_i = [P_i^{\min}, P_i^{\max}]$. We set an infinitely small quantity ε where ($\varepsilon > 0$), and by considering the proposed EF-NPGP as given in equation (4.20), generate a sequence of powers as follows.

EF-NPGP power control algorithm

- I. Initialize transmit power vector $p = [p_1^0, p_2^0, p_3^0, ..., p_N^0]$ randomly at time t_0
- II. For all $i \in N$, at time instant t_k
 - a) Update $\gamma_i(t_k)$ using equation (4.1)
 - b) Given $p_i(t_{k-1})$, compute the best response of power strategy $r_i(t_k)$ $r_i(t_k) = \arg \max_{p_i \in P_i} u_i^C(p_i, \mathbf{p}_{-i}(t_{k-1}))$
 - c) Assign the transmit power as $p_i(t_k) = \min(r_i(t_k), p_i^{\max})$
- III. If $||p(t_k) p(t_{k-1})|| \le \varepsilon$, stop iteration and declare Nash equilibrium as $p(t_k)$. Else, k = k+1 and go to Step II.

where $r_i(t_k)$ represents the set of best transmit powers for *i*th CR at time instant *k* in response to the interference vector $p_{-i}(t_{k-1})$. It is important to note that the *i*th CR optimizes the net utility over the power strategy space of the EF-NPGP. The proposed EF-NPGP determine the transmit power of *i*th CR by selecting smallest power among all possibilities as dictated by the algorithm. The algorithm will solve the maximum of each cognitive radio's objective separately.

The flow chart of the proposed EF-NPGP algorithm is shown in Figure 4.7.



Figure 4.7: Flowchart illustrating the EF-NPGP algorithm

4.3 Numerical results and discussion

In this section, we demonstrate and verify the performance of our proposed power control game algorithm (EF-NPGP) by comparing the Nash equilibrium results with NPG_MSFLIA (Zhang et al., 2012), NPG-ESIA (Kuo et al., 2013), and R-NPGP (Xie

et al., 2014). The utility functions of the previous works that have been used in the comparison of simulation are explained in the previous equations (4.10), (4.12), and (4.14). We applied the same numerical computation to obtain the Nash equilibrium solution of utility functions in order to present the advantages of our proposed utility function. The lists of other system parameters we examined and used in the simulation are listed in Table 4.3.

Parameter	Value
Total number of bits per frame, M	80
Number of information bits of each frame, L	64
Spread spectrum processing gain, G	100
Data rate, <i>R</i>	10 kbps
AWGN power at receiver, σ^2	5e-15 Watts
Maximum power constraint, p_i^{max}	2 Watts
Target SIR, Γ_i	10
Weighting factors a,b,c , and λ	0.88, 3, 1e4, 2.5

 Table 4.3: System parameters

A simple system model has been considered based on a single-cell cognitive radio CDMA system with a fixed packet size and no coding for forward error correction. According to the general efficiency function that is defined in equation (4.11), the equilibrium SIR obtained by solving the formula $f'(\gamma)\gamma - f(\gamma) = 0$ that guarantee maximum utility is $\gamma^* = 12.4$. The value of γ^* is the real target SIR that all CRs achieve to maximize their own utility function. For the cognitive radio CDMA system, the

feasibility condition for the target γ^* is giving by the following bound on the number of users (Yates, 1995):

$$N \le 1 + (G/\gamma^*) = 9.05$$
 CR terminal (4.33)

According to (4.33), we assumed that there are not more than 9 CRs in the cognitive radio system, and they are distributed around CBS that is located in the center of the cell. The distances between 9 cognitive radios users and base station (CBS) are defined in the following array d = [368m, 490m, 580m, 630m, 720m, 810m, 950m, 1070m, 1140m]

In this work, for simplicity we use a simple propagation model in which all the path gains are deterministic functions, with path loss exponent α , of the distance between the cognitive radio *i* and CBS

$$h_i = \frac{A}{d_i^{\alpha}} \tag{4.34}$$

where d_i is the distance between the *i*th cognitive radio user and the base station, α is the path loss exponent, which is supposed to be 4 that is usually between 2 and 6, and A = 0.097 is a constant. The value of A = 0.097 is selected to establish a transmit power of 2 W for a CR terminal operating at 1140 meters from CBS in the system with 9 CRs, and all operating with γ^* . In this simulation, all cognitive users start their iteration with initial power $p_i^{(0)} = 2.22 \times 10^{-16}$ W for all algorithms, and the algorithm will stop when the difference between the current and the previous power is less than $\varepsilon = 10^{-5}$.

We noted that the weighting and pricing factors have been tuned and the simulation run until all algorithms achieve the same average value of SIR. Figure (4.8) depicts the results of SIR at Nash equilibrium that are achieved by CR according to the distance between each CR and base station. All CR users maintain their SIR above the target value ($\Gamma_i = 10$) and the value of user's SIR is decreased by increasing the distance for all algorithms. In addition, all algorithms applied different pricing (penalty) function to cognitive radios, so different reduction occur on the value of SIR.

The comparison curves of SIR for all algorithms shows that our proposed algorithm EF-NPGP is more efficient, especially for the first 7 users, where the values of SIR are the highest compared with other algorithms. The performance gap between EF-NPGP algorithm and other algorithms indicates that the link quality of CRs of our proposed algorithm is better than others. Moreover, the farthest CRs consume the highest power in the system to maintain their SIR and they represent the main source of interference. Therefore, a higher cost has been applied to those farthest users in the proposed EF-NPGP algorithm. It is shown that the last two users in our proposed algorithm have lowest SIR but still greater than the target SIR and this action will lead to lower interference and lower power consumed.

In order to prove the efficiency of EF-NPGP algorithm, the transmit power of each user against the distance of each CR user are illustrated Figure 4.9. The different curves in Figure 4.9 represent the transmit power in Watts with the distance between each CR and the base station (CBS) for all algorithms. It can be seen that the transmit power increases gradually by increasing the distance of the user from the base station. It can also be seen that the transmit power curve of the proposed EF-NPGP is the lowest compared to NPG-MSFLIA, NPGP-ESIA, and R-NPGP.



Figure 4.8: Comparison curves of each CR's SIR for all algorithms



Figure 4.9: Comparison curves of each CR's transmit power for all algorithms

Table 4.4 illustrates the SIR values of CR users at the end of simulation for all algorithms. This table shows that the EF-NPGP with our proposed price function achieve the highest value of SIR compared to other algorithms. The SIR of the last two CR users is smaller due to applied higher cost. Therefore, it can reach a better equilibrium point by restricting the minimum required SIR for the CR terminals with bad channel conditions.

CR user #	Final SIR of NPG MSFLIA (Zhang et al., 2012)	Final SIR of NPG- ESIA (Kuo et al., 2013)	Final SIR of R- NPGP (Xie et al., 2014)	FinalSIRofProposedEF-NPGP
1	12.41	12.42	12.42	12.69
2	12.40	12.43	12.43	12.69
3	12.40	12.43	12.43	12.69
4	12.39	12.43	12.43	12.69
5	12.37	12.42	12.42	12.68
6	12.33	12.40	12.40	12.66
7	12.26	12.30	12.30	12.40
8	12.18	12.10	12.10	11.53
9	12.11	11.92	11.91	10.61

 Table 4.4: Simulation results comparison of final SIR

In the second test, we compute the average power and average SIR for all algorithms to determine the convergence speed of all algorithms, and the reduction of average power. In this test, the horizontal axis represents the number of iterations that needs to obtain the Nash equilibrium and the vertical axis represents the average SIR and average power.

As shown in Figure 4.10, all algorithms approximately achieve the same value of average SIR without any significant differences, but the convergence speeds are not equal. Through the results, we found that our proposed EF-NPGP algorithm is faster than other algorithms (250%-300%). It can reach the Nash equilibrium with only 133

iterations while it needs 333, 360, and 323 for NPG-MSFLA, NPGP-ESIA and R-NPGP, respectively.

However, in Table 4.6, it is quite obvious that EF-NPGP saves much simulation iterations than other three algorithms, which means that our proposed scheme reduces the computational complexity. This is because other NPG-MSFLA and NPGP-ESIA algorithms used artificial algorithms to search for the optimal power control strategies without considering the algorithm complexity. In addition, our proposed scheme reduces the computational complexity more than R-NPGP.

On the other hand, the comparison curves of the average transmit powers resulted from all algorithms is shown in Figure 4.11. It is easy to see in Figure 4.11 that the average power consumption of the proposed EF-NPGP algorithm has a significant reduction compared to other algorithms. Results shown in Figure 4.11 indicate that the amount of interference measured at the primary system or at interference temperature point from the proposed EF-NPGP is the lowest compared to other algorithms. This feature make the proposed EF-NPGP algorithm is the best for maximizing the spectrum sharing and QoS guarantees in both primary and cognitive radio systems. The convergence speed of all algorithms can be seen clearly in Figure 4.11, in which it founds that our proposed EF-NPGP is the fastest.



Figure 4.10: Comparison curves of average SIR for all algorithms with number of iterations



Figure 4.11: Comparison curves of average power for all algorithms with number of iterations
Table 4.5 explained the average power in Watt, and the number of iterations of all algorithms.

Algorithm	Average power (W)	Number of iterations	
NPG-MSFLA	0.2321	333	
NPGP-ESIA	0.2319	360	
R—NPGP	0.2287	323	
EF-NPGP	0.1926	133	

Table 4.5: Values of average power and number of iterations of algorithms

In the last simulation, we test the impact of noise to the average power and SIR of the proposed and other algorithms. We run algorithms simulation using the same parameters that were applied to the previous tests. We vary the value of noise from 3×10^{-17} W to 10^{-14} W.

As observed in Figure 4.12, the average power increases with the increase of noise because the power is proportional to noise, while the average SIRs decrease with the increase of noise because SIR is inversely proportional to noise as in equation (4.1). It is found that the proposed EF-NPGP algorithm provides significant energy savings in case of high noise, in which the maximum of average power is ($p_{EF-NPGP} = 0.2078$ W), while (p = 0.3127 W) for other algorithms. On the other hand, the average SIR of the proposed EF-NPGP algorithm is the highest compared to other algorithms, and there is insignificant reduction at high noise.



Figure 4.12: Performance comparison of average power and SIR for noise values

4.4 Conclusion

In this chapter, a non-cooperative energy efficient power control algorithm in cognitive radio networks has been presented. The QoS of a CR user refers to an efficient utility function via pricing. By introducing the new utility and price functions, an efficient non-cooperative power control game has been produced and the existence, uniqueness of Nash equilibrium has been also proved. Numerical results indicate that the non-cooperative power control algorithm proposed in this chapter has better power saving and faster convergence compared to recently available works in the literature.

In addition, most of closer CR users in our proposed algorithm can meet higher SIR than the users in other algorithms. The higher pricing is only applied to the farthest users that represent a main source of undesirable interference. The proposed scheme offers an improved performance, in which the CRN can now share extra licensed band under the interference temperature limits. The significant reduction in the transmit power of the proposed power control algorithm gives the highest preference to apply it in cognitive radio sensor networks and green cognitive radio networks.

CHAPTER 5: A GAME THEORY APPROACH FOR EFFICIENT POWER CONTROL AND INTERFERENCE MANAGEMENT IN TWO-TIER FEMTOCELL NETWORKS BASED ON LOCAL GAIN

5.1 Introduction

Recent studies on wireless usage found that most voice calls and data traffic are originated from indoor and the surest method to increase system capacity of a wireless link is by getting the transmitter and receiver closer to each other (Chandrasekhar et al., 2008).

To meet the increasing demand of video application and the increased in system capacity in the wireless indoor transmission and services, femtocell has been considered as a promising solution to increase the coverage and capacity of the network. A femtocell or a home base station (HBS) is a short-range (i.e. 10-50m), low-power, low-cost, and consumer owned device that is installed inside the houses and offices for better indoor voice and data services. A home base station HBS installed by the consumer connects the cellular network to the internet via an IP backhaul, such as digital subscriber line (DSL), cable, or WiMAX.

There are two approaches for spectrum allocation between the macrocell and femtocell users: (i) spectrum splitting and (ii) spectrum sharing. In a two-tier femtocell network, the spectrum sharing approach is commonly used due to the scarcity of available spectrum and the absence of coordination between macrocell and femtocell as well as between femtocells (Kang et al., 2012). When spectrum sharing rather than

spectrum splitting is adopted, the interference will be more serious (Ahmad et al., 2014).

Since femtocells operate in the licensed spectrum owned by the macrocell network, it is essential to decrease the cross-tier interference from FUEs to the macrocell (La Roche et al; 2010). Recent research topics focus on the development of interference management schemes such that

- (i) guarantee to achieve the quality of service (QoS) of the higher access priority macrocell users (MUEs), and
- (ii) efficient utilization of the residual network capacity by the newly-deployedFUEs to optimize performance.

The radio frequency (RF) interference will arise from femtocell to femtocell interference, femtocell to macrocell interference, and macrocell to femtocell interference. The femtocell to femtocell is quite small due to low transmit power and penetration losses. The near-far effect due to uneven distribution of received power is the main contributor for femtocell to macrocell interference and macrocell to femtocell interference (Chandrasekhar et al., 2008).

Macrocell networks, such as CDMA networks (without existing femtocells) employ an efficient power control to compensate for path loss, shadowing, and fading, to provide uniform coverage. When femtocells are added, power control creates dead zones as in Figure 5.1. Macrocell users at a cell edge need to use maximum power in the uplink transmitting, which causes unacceptable interference to nearby femtocells. Therefore, femtocells located at the cell edge experience significantly higher interference than interior femtocells. On the other hand, macrocell users at cell edge will be disrupted by femtocell transmissions since they suffer higher path loss than interior macrocell users.



Figure 5.1: Dead zones in a mixed femtocell/macrocell deployment

The problem of spectrum sharing and interference management in the two-tier femtocell networks has become a technical challenge to scientists and researchers. The implementation and development of distributed interference management is the main challenge in femtocell networks due to limited capacity of the signaling wire-line network (e.g., DSL links) and difference access priority between MUEs and FUEs (Claussen, 2007)(Yavuz et al., 2009) (Güvenç et al., 2008) (Jo et al., 2009). The existence of indoor femtocells makes power control creating dead zones, leading to non-uniform coverage.

To mitigate cross-tier interference and guarantee QoS for both MUEs and FUEs, an efficient distributed power control algorithm for interference management is important in two-tier femtocell networks.

There are some differences between the femtocell networks and traditional wireless networks which are the infrastructures of the system, and the different classes of users, so different power control algorithm should be designed for macrocell and femtocell network. Femtocells are low-rang (not identical with macrocells), therefore, all FUEs have a higher channel gain and require different power control algorithm.

In this chapter, we present a new power control scheme for the distributed interference management in two-tier femtocell networks. The objective of this power control scheme is to ensure that users with higher priority access (MUEs) able to achieve their required QoS, whereas users with lower priority access demand certain QoS requirements.

The difference between power control algorithm proposed in this work and previous works proposed in the traditional CDMA wireless networks or cognitive radio networks is the differentiated classes of users, in which each class of users' needs a different power control algorithm based on the access priority.

It is noteworthy that, our proposed power control differs from previous power control scheme proposed by (Ngo et al., 2012) in several aspects. Firstly, in the representation of the utility function of MUEs, the study in (Ngo et al., 2012) uses sigmoid function to guarantee the minimum required SINRs. Power control algorithm using a sigmoid function is more complex because the power update formula of MUE depends on the value of optimal target SINR needed to be computed during each iteration. Instead, we defined utility function of MUEs as a square function of SINR error, and the deduced

power update formula depends on the target SINR. Secondly, we introduced a new local gain in our FUEs cost function based on the local information, which can improve the performance of FUEs.

For too heavily loaded system, the proposed algorithm for MUEs and FUEs yield unacceptably low SINRs. Mobiles whose SINRs fall below a minimum QoS threshold should be dropped, otherwise, they will cause unnecessary interference to other users using the same frequency channel.

The advantages of the proposed power control algorithm are the ability to be implemented distributively, mitigate the cross-tier interference, and reduce the drain power of users. Hence, the contributions of this work are summarized as follows:

- 1. In this chapter, we formulated the game model based on a cost function, in which the MUEs guarantee their required QoS, while FUEs request soft QoS requirement.
- 2. The proposed FUEs cost function contains linear pricing function and local gain term in which it has been applied inside the utility part of the cost function. This mechanism ensures that, the transmit power of FUEs, which is included inside the SINR is gained using local information.
- 3. We obtain the Nash equilibrium of the proposed game, present the iterative power control formulas, and prove the convergence of the algorithm.
- 4. With simulation, we show the effectiveness of the proposed power control algorithm in terms of resource allocation and power saving for different cases of the system load.

The rest of this chapter is organized as follows. In Section 5.2, the system model of two-tier femtocell network is given, distributed interference-management algorithms are proposed and the corresponding analysis is presented in Section 5.3. The performance of the proposed scheme is presented by the numerical results in Section 5.4. Finally, we conclude this study in Section 5.5.

5.2 System model

In this chapter, we consider a typical two-tier femtocell network where *N* femtocells are overlaid with a macrocell. Specifically, we consider the scenario where *N* randomly distributed femtocells overlaid *M* macrocell user equipment (MUEs) using code division multiple access (CDMA). The MUEs are distributed randomly inside the coverage area of macrocell BS with radius R_c , the FUEs are randomly distributed inside the coverage area of home BS with radius R_f , and all femtocells are distributed randomly inside the coverage area of macrocell.

Due to the small radius of femtocells R_f , the effect of interference between the users inside a single femtocell is inactive. Therefore, for simplicity, we assume that each femtocell only serves one FUE. We further assume that all MUEs and FUEs with their base stations are stationary so the path gains are fixed during the run time of power control simulation.

We denote C_m and C_f as the set of MUEs and FUEs, respectively, and C as the set of all users MUEs and FUEs is then $C = C_m \cup C_f$. A simple sketch of the system model of a two-tier femtocell network is shown in Figure 5.2, which contains four femtocells overlaid with one macrocell.



Figure 5.2: Two tier CDMA femtocell wireless network (Tier 1: macrocell under laid tier 2 that includes 3 femtocells)

We consider the uplink scenario in this work, and any $i \in C$ is referred to as the *i*th user. Let p_i be the transmit power of user *i* in Watt, g_{ii} is the channel gain from user *i* to its receiver including the processing gain of the system, and g_{ij} from user *j* to the receiver of user $i \neq j$. We further assume that the channel gains are unchanged during the runtime of the power control algorithm. The channel gain is $g_{ij} = 1/d_{ij}^{\alpha}$ with neglected shadowing and fast fading effects, where d_{ij} is the distance from user *j* to the receiver of user *i*, and α is the path loss factor that is usually between 2 and 6. We denote the power of additive white Gaussian noise (AWGN) at the receiver by σ^2 Watt. Here, we use the signal to interference and noise ratio (SINR) to represent the quality of the desired signal compare to the interferer. Then, the SINR obtained by user $i \in C$ at its base station can be written as

$$\gamma_i = \frac{g_{ii} p_i}{\sum_{j \neq i} g_{ij} p_j + \sigma^2} = \frac{g_{ii} p_i}{I_i} \quad \forall \ i \in C$$
(5.1)

where $I_i = \sum_{j \neq i} g_{ij} p_j + \sigma^2$ represents the aggregated interference from all FUEs and MUEs except user *i*.

-

In this framework, we assigned different thresholds value of SINR to different classes of users, depending on their access priority and application requirements.

To ensure adequate QoS of the higher access priority MUEs, the power control design must ensure that no MUE's SINR γ_i falls below the target SINR value Γ_i^m . Thus, there is

$$\gamma_i \geq \Gamma_i^m \qquad \forall i \in C_m \tag{5.2}$$

On the other hand, the design of power control should also ensure that the indoor lower access priority FUEs can achieve their required QoS and each $i \in C_f$ can attain its SINR, that is, more than a predefined threshold $\underline{\gamma}_i^f$. A higher value of $\underline{\gamma}_i^f$ will create unnecessary interference to other users. Therefore, we require that each FUE $i \in C_f$ must have that:

$$\gamma_i \geq \gamma_i^f \qquad \forall i \in C_f \tag{5.3}$$

In this chapter, we employ users' objective function as a cost function J_i to represent the preference of users. We defined the cost functions as a difference between the pricing function of user and its utility function, in which each user interest to minimize its own cost defined as in (Alpcan et al., 2002):

$$J_{i}(p_{i},\gamma_{i}(p)) = v_{i}(p_{i}) - u_{i}(\gamma_{i}(p))$$
(5.4)

where the power vector is $p := [p_1, p_2, ..., p_{N+M}]^T$. The pricing function $v_i(p_i)$ represents the cost incurred by user $i \in C$, while the utility function $u_i(\gamma_i(p))$ represents the degree of satisfaction to the service quality. In fact, equation (5.4) is a standard way to define the objective function for network entities (MUEs and FUEs).

The Nash equilibrium is the power vectors p^* that no user can improve its cost function individually by deviating from p_i^* . Thus, there is

$$J_{i}(p_{i}^{*},\gamma_{i}(p_{i}^{*})) \leq J_{i}(p_{i}^{*},\gamma_{i}(p_{1}^{*},p_{2}^{*},...,p_{i-1}^{*},p_{i},p_{i+1}^{*},...,p_{N+M}^{*})) \quad \forall \ i \in C$$
(5.5)

The Nash equilibrium of (5.5) can be obtained by taking the derivative of $J_i(p_i, \gamma_i(p))$ with respect to p_i and equating it to zero as follows

$$\frac{\partial J_i}{\partial p_i} = \frac{\partial v_i}{\partial p_i} - \frac{\partial u_i}{\partial \gamma_i} \frac{\partial \gamma_i}{\partial p_i} = 0$$
(5.6)

Noting that $\partial \gamma_i / \partial p_i = g_{ii} / I_i = \gamma_i / p_i$, we have

$$u_i'(\gamma_i) = \frac{I_i}{g_{ii}} v_i'(p_i) = \frac{p_i}{\gamma_i} v_i'(p_i)$$
(5.7)

where $u'_i(\gamma_i)$ and $v'_i(p_i)$ denotes to the derivatives of $u_i(\gamma_i)$ and $v_i(p_i)$, respectively.

In this chapter, we also further introduce a user-specific notation θ_i as a ratio of interference to the path gain of the user $i \in C$ as in (Xiao et al., 2003) to simplify the analysis, as shown in the following

$$\theta_i = \frac{I_i}{g_{ii}} = \frac{p_i}{\gamma_i}$$
(5.8)

According to this analysis, we will explain the criteria of how to select the suitable functions $v_i(p_i)$ and $u_i(\gamma_i)$ with appropriate parameters in designing the efficient distributed power control algorithm for both MUEs and FUEs. The main goal of our algorithm is to strictly guarantee the QoS of MUEs, and we allow slight reduction in the QoS of FUEs in order to reduce the power consumed by the users and to mitigate the cross-tier interference.

5.3 Distributed power control algorithm

5.3.1 Macrocell users cost function

The applicable method to guarantee the QoS of MUEs is the balancing power control method, in which all MUEs achieve the same target SINR. The aim is to guarantee the QoS of higher priority MUEs by ensuring that all MUEs can meet the target SINR. On the other hand, MUEs do not need to use high power in their transmission to attain high SINR (greater than the target) in order to preserve their battery life and minimize the cross-tier interference. In this case, all MUEs should have a zero price and the optimal γ_i should be equal to the target SINR.

For accurate communication at non-zero levels of SINR, we define the cost function of FUE user as the difference between the actual SINR and the target SINR that is chosen based on the estimated FER (Koskie & Gajic, 2005). In addition, the cost function of MUEs should be convex and positive. We thus consider the following utility and pricing functions for MUEs $i \in C_m$

$$u_i(\gamma_i) = -\left(\Gamma_i^m - \gamma_i\right)^2 \tag{5.9}$$

$$v_i(p_i) = 0 \tag{5.10}$$

Thus, according to (5.4), the cost function of the *i*th MUE can be written as

$$J_i^m(p_i,\gamma_i) = \left(\Gamma_i^m - \gamma_i\right)^2 \quad \forall \ i \in C_m$$
(5.11)

where $J_i^m(p_i, \gamma_i)$ is the cost function of MUEs. The optimal γ_i for each user $i \in C_m$ is the target SINR, which can be obtained by taking the first derivative of the MUEs cost function with respect to γ_i and equating to zero,

$$\frac{\partial J_i^m}{\partial \gamma_i} = 0 = 2\gamma_i - 2\Gamma_i^m \tag{5.12}$$

Then,

$$\hat{\gamma}_i = \Gamma_i^m \tag{5.13}$$



Figure 5.3: Cost function of MUEs *i* with target SINR $\Gamma_i^m = 5$.

It is shown in Figure 5.3 that the optimal (minimum) value of MUE cost function is occurres when the user's SINR is equal to the target value Γ_i^m .

Based on $\hat{\gamma}_i$ in (5.13), the optimal power can be obtained from (5.1) as $\hat{p}_i g_{ii} / I_i = \Gamma_i^m$ and the following iterative power rule can be applied:

$$p_i^{(k+1)} = \frac{I_i^{(k)}}{g_{ii}^{(k)}} \Gamma_i^m = \frac{p_i^{(k)}}{\gamma_i^{(k)}} \Gamma_i^m$$
(5.14)

where $\gamma_i^{(k)}$ and $p_i^{(k+1)}$ are the actual SINR and power of user *i* at iteration *k* and *k*+1, respectively. For simplicity, we can use (5.8) to rewrite (5.14). Thus,

$$p_i^{(k+1)} = \theta_i^{(k)} \Gamma_i^m \tag{5.15}$$

5.3.2 Femtocell users cost function

In the case of lower access priority, we assume that for each FUE, $i \in C_f$ is also required to maintaining its QoS by achieving the target SINR Γ_i . The target Γ_i here is different from the threshold value $\underline{\gamma_i^f}$ defined in (5.3), which in practice $\Gamma_i > \underline{\gamma_i^f}$. The value of the target SINR should be sufficient to guarantee better service for FUEs and should also be not higher because it requires high transmit power to achieve.

To decrease the cross-tier interference induced to the macrocell, FUEs should achieve its target SINR using the minimum required transmit power. In game theory, the selection of a cost function is important because it is a basis of the game, which will be used to deduce the power iterative algorithm.

Femtocell user has two conflicting objectives: (i) achieve better service by obtaining higher SINR and (ii) higher SINR is achieved at the cost of an increased drain on the battery and higher cross-interference to others FUEs and MUEs.

Therefore, the cost function for each FUE should depend on power and SINR, and it should be non-negative and convex to allow the existence of a non-negative minimum.

In addition, the target Γ_i should be included inside the cost function to be varied according to the service requirement. Higher Γ_i can be chosen for voice users and lower target can be chosen for data users.

Thus, we consider the cost function of the *i*th FUE as in (Koskie & Gajic, 2005) with a new special parameter as

$$J_i^f(p_i, \gamma_i(p_i)) = b_i p_i + \left(\Gamma_i - e^{a_i \theta_i} \gamma_i\right)^2 \quad \forall i \in C_f$$
(5.16)

where a_i and b_i are non-negative weighting factors. It is shown in (5.16) that the cost function $J_i^f(p_i, \gamma_i(p_i))$ is non-negative because the square of SINR error $(\Gamma_i - e^{a_i \theta_i} \gamma_i)^2$ is always positive due to the square function, and the linear pricing power $b_i p_i$ is always positive.

In addition, the proper selection of the non-negative weighting factors in the cost function equation (4.16) is important. Choosing $b_i/(e^{a_i\theta_i})^2 > 1$ places more emphasis on power usage, whereas $b_i/(e^{a_i\theta_i})^2 < 1$ places more emphasis on the SINR.

The new special local gain term $e^{a_i\theta_i}$ is the advantage of our proposed algorithm, in which it can guide FUEs to an efficient Nash equilibrium point when the system operates in different loads. We defined $e^{a_i\theta_i}$ as a local gain term because it only depends on the weighting factor a_i and a user-specific notation θ_i (a ratio of FUEs power and SINR). That means, the local gain term only depends on local information and it does not need any other information from home BS.

The utility part of the cost function (5.16) will guide all FUEs to achieve the target SINR, but the Nash equilibrium may actually less than the target SINR due to the

pricing term and the new special gain parameter. Nevertheless, slight decrease in FUEs SINR will lead to substantial reduction in transmitting power as well as significant reduction in the cross-tier interference. Now, applying the necessary condition of Nash equilibrium to the *i*th FUEs cost function yields:

$$\frac{\partial J_i^f}{\partial p_i} = 0 = b_i - 2e^{a_i\theta_i} (\Gamma_i - e^{a_i\theta_i}\gamma_i) \frac{\partial \gamma_i}{\partial p_i}$$
(5.17)

$$=b_{i}-2e^{a_{i}\theta_{i}}\left(\Gamma_{i}-e^{a_{i}\theta_{i}}\gamma_{i}\right)\frac{g_{ii}}{I_{i}}$$
(5.18)

Rearranging terms of (5.18) yields

$$(\Gamma_i - e^{a_i \theta_i} \gamma_i) = \frac{I_i}{2e^{a_i \theta_i} g_{ii}} b_i$$
$$-e^{a_i \theta_i} \gamma_i = -\Gamma_i + \frac{I_i}{2e^{a_i \theta_i} g_{ii}} b_i$$

$$\gamma_i = \frac{\Gamma_i}{e^{a_i \theta_i}} - \frac{b_i I_i}{2 g_{ii} (e^{a_i \theta_i})^2}$$
(5.19)

It follows from (5.19) that as $b_i \to 0$, the power expenditure increases and the SINR $\gamma_i \to \Gamma_i / e^{a_i \theta_i}$. On the other hand, as $a_i \to 0$ and $b_i \to 0$, the SINR will be converging to the target $\gamma_i \to \Gamma_i$. Substituting for γ_i from (5.1) and isolating p_i , we can obtain the power in terms of given and measured quantities as

$$p_{i} = \frac{\Gamma_{i}}{e^{a_{i}\theta_{i}}} \frac{I_{i}}{g_{ii}} - \frac{b_{i}I_{i}^{2}}{2g_{ii}^{2}(e^{a_{i}\theta_{i}})^{2}}$$
(5.20)

At Nash equilibrium, the power value can thus be computed as

$$p_i^* = \max\left(\frac{\Gamma_i}{e^{a_i\theta_i}} \frac{I_i}{g_{ii}} - \frac{b_i I_i^2}{2 g_{ii}^2 (e^{a_i\theta_i})^2}, 0\right)$$
(5.21)

To present (5.20) as a numerical algorithm, we assume that the algorithm will run in real time with potential measurements updated every step of the algorithm (Koskie & Gajic, 2005). Thus, the iterative power rule can be written as

$$p_i^{(k+1)} = \frac{\Gamma_i}{e^{a_i\theta_i^{(k)}}} \frac{I_i^{(k)}}{g_{ii}^{(k)}} - \frac{b_i (I_i^{(k)})^2}{2(g_{ii}^{(k)})^2 (e^{a_i\theta_i^{(k)}})^2}$$
(5.22)

We define $p_i^{(k+1)} = f_i^{(k)} p_i^{(k)}$ and based on (5.14), we can get

$$f_i^{(k)} p_i^{(k)} = p_i^{(k+1)} = \frac{\Gamma_i}{e^{a_i \theta_i^{(k)}}} \frac{I_i^{(k)}}{g_{ii}^{(k)}} - \frac{b_i (I_i^{(k)})^2}{2(g_{ii}^{(k)})^2 (e^{a_i \theta_i^{(k)}})^2}, \ p_i^{(k)} > 0$$
(5.23)

where $\theta_i^{(k)} = p_i^{(k)} / \gamma_i^{(k)}$ as in (5.8). The initial condition associated with (5.23) must satisfy $p_i^{(0)} \neq 0$. Note that, the positive term in the expression of (5.23) is different from the power balancing solution, in which the new specific parameter $e^{a_i \theta_i}$ has been added to the denominator. In addition, the negative term is proportional to the square of interference, and the square of exponential of interference.

The two formulas of algorithm (5.22) and (5.23) require only a single measurement at each step of the iteration, so the power control can be used as a distributed power control.

Moreover, we may also rewrite the algorithm of equation (5.22) in terms of previous value of power $p_i^{(k)}$ and the SINR measurement $\gamma_i^{(k)}$ by substituting $I_i^{(k)}$ in (5.8) using the relation $\gamma_i^{(k)} = g_{ii} p_i^{(k)} / I_i^{(k)}$.

$$p_i^{(k+1)} = \frac{\Gamma_i}{e^{a_i \theta_i^{(k)}}} \frac{p_i^{(k)}}{\gamma_i^{(k)}} - \frac{b_i \left(p_i^{(k)}\right)^2}{2\left(\gamma_i^{(k)}\right)^2 \left(e^{a_i \theta_i^{(k)}}\right)^2}, \ p_i^{(k)} > 0$$
(5.24)

Both formulas of power control algorithm (5.23) and (5.24) require local information at each step. Therefore, if this information is available to femtocell user, either algorithm can be implemented in a distributed manner. Equation (5.24) requires nonzero initial power to execute, whereas the formula in terms of interference (5.23) does not require an initial power because the interference, which includes the noise power, is never zero.

5.3.3 Convergence

Having obtained the power control iterative algorithm, we then prove its convergence. In (Yates, 1995), authors show that if a fixed point of the algorithm $p^{(k+1)} = f(p^{(k)})$ exist and the function *f* satisfy the following three conditions:

- 1) Positivity $f(p) \ge 0$,
- 2) Monotonicity $p \ge p' \implies f(p) \ge f(p')$,
- 3) Scalability $\forall \alpha \ge 1$; $\alpha f(p) \ge f(\alpha p)$.

then the algorithm converges to a fixed and unique point.

For the MUEs iterative algorithm, the above three properties are obviously satisfied as has been explained in (Xu et al., 2014). We still need to prove the convergence of FUEs iterative algorithm.

From (5.22), and in terms of interference, the positivity requires

$$I_{i} = -\frac{g_{ii}LambertW\left(-\frac{2a_{i}\Gamma_{i}}{b_{i}}\right)}{a_{i}}$$
(5.25)

where *LambertW*(.) is the LambertW function. When we selected a proper value of a_i/b_i , the value of *LambertW*($-2a_i\Gamma_i/b_i$) will be small positive quantity and the positivity condition can be easily met.

For monotonicity, it is sufficient to have an increasing best response function with respect to interference I_i . Thus, if we differentiate (5.22) with respect to I_i , we get

$$\frac{\partial f(p)}{\partial (I_i)} = \frac{a_i b_i (I_i)^2}{\left(g_{ii}\right)^3 \left(e^{\frac{a_i I_i}{g_{ii}}}\right)^2} - \frac{a_i b_i}{\left(g_{ii}\right)^2 \left(e^{\frac{a_i I_i}{g_{ii}}}\right)^2} + \frac{\Gamma_i}{g_{ii} e^{\frac{a_i I_i}{g_{ii}}}} - \frac{a_i I_i \Gamma_i}{\left(g_{ii}\right)^2 e^{\frac{a_i I_i}{g_{ii}}}}$$
(5.26)

Using (5.26), for monotonicity, we should have

$$\Gamma_{i} > \frac{b_{i} I_{i}}{g_{ii} e^{\frac{a_{i} I_{i}}{g_{ii}}}} \implies I_{i} < -\frac{g_{ii} Lambert W\left(-\frac{a_{i} \Gamma_{i}}{b_{i}}\right)}{a_{i}}$$
(5.27)

which is stricter than (5.25). Finally, the condition of the scalability in our method can be written as

$$\alpha f(p) - f(\alpha p) = \left(\frac{\alpha \Gamma_{i}}{e^{a_{i} \frac{p_{i}^{(k)}}{\gamma_{i}^{(k)}}}} \frac{p_{i}^{(k)}}{\gamma_{i}^{(k)}} - \frac{\alpha b_{i} (p_{i}^{(k)})^{2}}{2(\gamma_{i}^{(k)})^{2} \left(e^{a_{i} \frac{p_{i}^{(k)}}{\gamma_{i}^{(k)}}}\right)^{2}} \right) - \left(\frac{\alpha \Gamma_{i}}{e^{a_{i} \frac{\alpha p_{i}^{(k)}}{\gamma_{i}^{(k)}}}} \frac{\alpha p_{i}^{(k)}}{\gamma_{i}^{(k)}} - \frac{\alpha b_{i} (\alpha p_{i}^{(k)})^{2}}{2(\gamma_{i}^{(k)})^{2} \left(e^{a_{i} \frac{\alpha p_{i}^{(k)}}{\gamma_{i}^{(k)}}}\right)^{2}} \right) \\ = \Gamma_{i} \left(\frac{1}{e^{a_{i} \frac{p_{i}^{(k)}}{\gamma_{i}^{(k)}}} - \frac{1}{e^{a_{i} \frac{\alpha p_{i}^{(k)}}{\gamma_{i}^{(k)}}}} \right) - \frac{b_{i} p_{i}^{(k)}}{2\gamma_{i}^{(k)}} \left(\frac{1}{e^{2a_{i} \frac{p_{i}^{(k)}}{\gamma_{i}^{(k)}}} - \frac{\alpha}{e^{2a_{i} \frac{\alpha p_{i}^{(k)}}{\gamma_{i}^{(k)}}}} \right) \right)$$

$$(5.28)$$

So, for scalability, it is sufficient that positivity is met.

From the above certification process of convergence, we found that the set of parameters is very important, and we can obtain the same conclusion from the simulation.

5.4 Simulation results

In this section, we present the data chart of power, SINR, average power, and average SINR of our proposed power control algorithm through our simulation tests.

We compare the performance with the traditional algorithm without local gain. The traditional algorithm has been proposed by Koskie & Gajic (2005), and it is applied to femtocell network by Ngo et al., (2012), which can be shown in the following equation:

$$p_i^{(k+1)} = \Gamma_i \frac{I_i^{(k)}}{g_{ii}^{(k)}} - \frac{b_i \left(I_i^{(k)}\right)^2}{2c_i \left(g_{ii}^{(k)}\right)^2}$$
(5.29)

In this simulation, we compare our proposed algorithm with the traditional algorithm (Ngo et al., 2012) with respect to femtocell users only, while the power update formula of macrocell users is different. To perform a fair comparison with the traditional algorithm, we kept the same physical parameters for all network elements, such as MUEs, FUEs, and base stations. In all algorithms, the powers update formula of both MUEs and FUEs can be achieved in a distributed manner, based on the link local information. Each MUE or FUE receiver can measure the total received power, and then subtract its own received power to obtaine the aggregate interference I_i , i.e., $I_i = \sum_{i \in C} g_{ij} p_j - g_{ii} p_i$. The receiver of user *i* (MUE or FUE) then sends both values of

 g_{ii} and I_i to its transmitter for the update of transmit power in each iteration.

The local gain proposed in our FUEs cost function has been appearing in both positive and negative terms in (5.22). One of the advantages of our proposed algorithm can be found in the positive term of (5.22), in which the algorithm guides FUEs to achieve $\Gamma_i/e^{a_i\theta_i}$ rather than the target Γ_i that was achieved by the traditional algorithm

(Ngo et al., 2012). On the other hand, the reduction of power is also affected by the local gain $e^{a_i\theta_i}$ that appears in the denominator of the negative term of (5.22).

The network settings and the deployment of users in this simulation are illustrated in Figure 5.4, where MUEs are randomly deployed inside the circle of the radii of $R_c = 500$ m and served by the macro BS that is located in the center. On the other hand, low access priority users FUEs are also randomly deployed inside small circle with radii $R_f = 100$ m and all femtocell are inside the area of overlaid macrocell. In this simulation, we assume that each femtocell BS serves only one FUE.



Figure 5.4: Network topology and user placement in the numerical simulation.

The initial power of all users must be a nonzero and it is chosen to be $p_i^{(0)} = 2.22 \times 10^{-16}$ W in both simulations. The channel gain from the transmitter of user $j \in C$ to the receiver of user *i* is calculated as $1/d_{ij}^{\alpha}$ where d_{ij} is the distance between transmitter and receiver, and α is the path loss exponent.

The same pricing coefficients a_i and b_i are used for all FUEs. For the ease of reference, we listed and summarized in Table 5.1, the simulation parameters such as the number of active MUEs and FUEs, the values of SINR targets Γ_i^m , Γ_i and the other simulation parameters:

Parameter	Value		
Number of users, N_M , N_f	10,40		
Processing Gain G	100		
Path-loss exponent, α	3		
Target SINRs Γ_i^m , Γ_i ,	5,4		
Noise power σ^2 in Watt	10 ⁻¹⁰		
$a_i, b_i, \forall i \in C_f$	5000, 15000		

 Table 5.1: Simulation parameters

In this simulation, all MUEs and FUEs are distributed randomly inside the area of their own cells with the range of radius, and each FUE or MUE has a different value of path gain. The path gain of each user depends on its distance from its own base station and the path loss exponent. The path gains of users will be computed from different positions of users, so the static scenario has been considered in our simulation.

In multiple curves figures, a single curve corresponds to one specific user. All MUEs in both algorithms update their transmission power using (5.14), and all FUEs update their transmission power using (5.22) for our proposed algorithm and (5.28) in the traditional algorithm. It is noted that the admission control is not considered in this simulation; therefore, no FUE is dropped from the system and the value of threshold γ_i^f has been neglected.

The simulations were executed three times: in low, medium, and high system loads, respectively, and the presented figures represent the evolutions of power and SINR and their average. The increase of system load shows the ability of macrocell users to in guaranteeing their QoS, and also shows the SINR degradation of femtocell users. In addition, increasing system loads show a significant reduction of powers among FUEs and MUEs that cannot be seen clearly in the low load system.

We noted that the curves in Figure 5.5, Figure 5.7, and Figure 5.9, represent the values of power and SINR of all MUEs and FUEs with iterations. The curves in red colors represent the values of power and SINR of MUEs, whereas the curves in blue colors represent the values of power and SINR of FUEs. It is easy to see in the following figures that all MUEs in red curves consume higher power than FUEs in the blue curves, because FUEs are low-range and they have higher channel gain than MUEs.

On the other hand, it is shown that the SINR values of all higher priority MUEs are the same (all MUEs converge to the target SINR $\Gamma_i^m = 5$) without any reduction. The reason is due to the successful choice of MUEs utility function with zero-pricing function. The linear pricing function that is applied to the FUEs cost function is the reason of the reduction in the values of FUEs SINR, as shown in the blue curves of medium and high load system.

In Figure 5.5, we display the evolutions of power and SINR in both algorithms for all MUEs and FUEs in the low load system. All MUEs and FUEs converge to the SINR requirements at the equilibrium, but the power consumed by MUEs and FUEs in the proposed algorithm is less than the power consumed by users in the traditional algorithm. The reduction in power cannot be seen clearly due to the density of user's power curves in the figure, so the average performance has been computed and displayed in the following figures. Figure 5.6 presents the average power of MUEs and the average power and SINR of FUEs. We found that the average SINR is $\bar{\gamma}^f = 3.992$ in the proposed power control algorithm, and $\bar{\gamma}^f = 3.997$ in the traditional algorithm, in which the difference between the two values is insignificant. On the other hand, the average power of MUEs is reduced by 4.39% compared to the traditional algorithm (final average power $\bar{p}^m = 2.2254 \times 10^{-5}$ versus $\bar{p}^m = 2.3276 \times 10^{-5}$ in the proposed algorithm and traditional, respectively), and the power of FUEs is reduced by 4.5% (final average power $\bar{p}^f = 1.1935 \times 10^{-6}$ versus $\bar{p}^f = 1.2505 \times 10^{-6}$ in the proposed algorithm and traditional algorithm, respectively). The minimum and maximum values of SINR and power for MUEs and FUEs for low load system have been shown in Table 5.2.



Figure 5.5: performance comparison of proposed algorithm and traditional algorithm

in low load system for all MUEs and FUEs.



Figure 5.6: Average performance comparison of proposed algorithm and traditional

algorithm in low load system.

In Table 5.2, it is found that the value of minimum and maximum powers of proposed algorithm is lower than the values of power in traditional algorithm. These numerical values of powers will lead to decrease in the total interference.

The advantage of the proposed algorithm is appeared clearly in the medium load test, as shown in Figure 5.7, and Figure 5.8. The average power reduction of MUEs is 68% (final average power $\bar{p}^m = 0.0015$ versus $\bar{p}^m = 0.0047$ in the proposed algorithm and traditional algorithm, respectively), and the reduction of average power of femtocell users is 68% (final average power $\bar{p}^f = 4.9749 \times 10^{-5}$ versus $\bar{p}^f = 1.5670 \times 10^{-4}$ in the proposed algorithm and traditional algorithm). On the other hand, the evaluations show that only 0.038% reduction in average SINR, $\bar{\gamma}^f = 3.6372$ in the proposed algorithm as opposed to $\bar{\gamma}^f = 3.6502$ in the traditional algorithm. As shown in Table 5.2, the minimum value of FUEs SINR of the farthest user in our proposed algorithm is higher than traditional algorithm.

	Low load system							
Values	$\operatorname{Min} \gamma_i^f$	$\operatorname{Max} \gamma_i^f$	$\operatorname{Min} p_i^f$	$\operatorname{Max} p_i^f$	$\operatorname{Min} p_i^m$	$\operatorname{Max} p_i^m$		
Proposed Algorithm	3.9728	3.996	3.9351×10 ⁻⁸	3.4418×10 ⁻⁶	2.9145×10 ⁻⁶	4.9504×10 ⁻⁵		
Traditional	3.9891	3.996	4.0062×10 ⁻⁸	3.5867×10 ⁻⁶	3.0199×10 ⁻⁶	5.1297×10 ⁻⁵		
	Medium load system							
Values	$\operatorname{Min} \boldsymbol{\gamma}_i^f$	$\operatorname{Max} \gamma_i^f$	$\operatorname{Min} p_i^f$	$\operatorname{Max} p_i^f$	$\mathbf{Min} \ p_i^m$	$\operatorname{Max} p_i^m$		
Proposed Algorithm	2.7726	3.994	7.8327×10 ⁻⁷	1.5699×10 ⁻⁴	1.3799×10 ⁻⁴	0.0038		
Traditional	2.6635	3.9946	2.438×10 ⁻⁶	4.746×10 ⁻⁴	4.3505×10 ⁻⁴	0.0121		
	High load system							
Values	$\operatorname{Min} \boldsymbol{\gamma}_i^f$	$\operatorname{Max} \gamma_i^f$	$\operatorname{Min} p_i^f$	$\operatorname{Max} p_i^f$	$\mathbf{Min} \ p_i^m$	$\operatorname{Max} p_i^m$		
Proposed Algorithm	1.285	3.993	1.2083×10 ⁻⁶	3.0408×10 ⁻⁴	4.465×10 ⁻⁴	0.0204		
Traditional	0.0546	3.995	2.59×10 ⁻⁶	5.294×10 ⁻⁴	9.955×10 ⁻⁴	0.0456		

Table 5.2: Min, max SINR and power evaluations



Figure 5.7: Performance comparison of proposed algorithm and traditional algorithm in





Figure 5.8: Average performance comparison of proposed algorithm and Traditional algorithm in medium load system.

In Figure 5.9 and Figure 5.10, we display the evaluations of power and SINR in the high load network. Both algorithms smoothly reduce the SINRs of FUEs to let MUEs to reach their desired SINR target, but the SINRs of FUEs in our proposed algorithm has reasonable values. The average value of FUEs SINR is $\bar{\gamma}_i^f = 3.108$ in both algorithms.

In addition, the reduction of the average power of MUEs is 55.2% (final average power $\bar{p}_i^m = 0.0072$ versus 0.0161 in the proposed algorithm and traditional algorithm, respectively), and the reduction of the average power of FUEs is 55.7% ($\bar{p}^f = 1.2715 \times 10^{-4}$ versus $\bar{p}^f = 2.5061 \times 10^{-4}$ in the proposed algorithm and traditional algorithm). Furthermore, the range of FUEs SINRs in the proposed algorithm is more suitable where the min value is $\gamma_i^f = 1.285$ as opposed to $\gamma_i^f = 0.0546$.



Figure 5.9: Performance comparison of proposed algorithm and traditional algorithm in high load system for all MUEs and FUEs $a_i = 4000$.



Figure 5.10: Average performance comparison of proposed algorithm and traditional algorithm in high load system.

On the other hand, there will be a trade off between power consumption and convergence rate of algorithms. The point regarding this issue is that although it seems that the proposed method imposes higher level of complexity due to the square term of the pricing part and local gain term, but the function argument is small enough to be well approximated by the first terms of the corresponding Taylor series.

5.5 Conclusion

In this chapter, we have proposed a new power control algorithm to manage the distributed interference in the two-tier networks. Specifically, a new design of power control for the FUEs has been considered. It has been shown that the proposed power control algorithm of FUEs is able to mitigate the cross-tier interference, making the MUEs maintaining their desired SINR requirements easily. The convergence of the proposed power control algorithm has been proved analytically and the features are confirmed through comparison in the numerical study.

CHAPTER 6: CONCLUSION AND FUTURE WORKS

This chapter summarizes and concludes the thesis, followed by a discussion on future work that this work could lead to.

6.1 Conclusion

The main objective of this research is to solve the problem of power control in the modern cognitive radio and femtocell networks based on game theory framework. In particular, the research focuses on the design of distributed power control algorithms that can reduce power consumed, mitigate interference, and achieve the required QoS in the desired wireless systems. To achieve this, the previous concepts of power control based on the control theory perspective that has been applied in cellular networks were studied and reviewed.

In addition, we briefly reviewed the transition of the implementation of QoS from cellular to wireless data networks. The concepts of macroeconomics and game theory have been employed to represent the QoS of users in appropriate manner in data networks. The QoS in data communication systems has been represented using utility (cost) function rather than SIR, which describes the satisfaction of users. This has led us to the use of a utility (cost) function that reduces the power consumption of user's terminals.

In chapter 3,

it is found that the distributed power control formulas of cellular networks that was designed by control theory can be also obtained from game theory based on the declaration of utility or cost function. In practical, we also found that DPC algorithm guides users to achieve the target SIR but if there is an insignificant reduction of users SIR, this will be lead to significant reduction in user's consume power.

In control theory perspective, slight reduction of SIR can be obtained by subtracting the output of DPC from some applied functions of channel status. On the other hand, game theory used the pricing techniques as an effective way to make this reduction. In the first power control algorithm, we introduced a new SIR based sigmoid cost function, which is defined as a weighted sum of power and a square of signal to interference ratio (SIR) error based on sigmoid function.

The proposed algorithm guide CR users to efficient Nash equilibrium point compared to other algorithms, in which it achieves a significant reduction in power with the same level of average SIR. The algorithm guarantees the quality of service of CRs as well as ensures that interference is below the interference temperature level and does not affect QoS of licensed users. In this situation, licensed network can maximize spectrum utilization by allowing many CRs to access the available parts.

We have also improved the convergence of proposed power control algorithm that is based on fixed point iteration method by using another numerical method. We have used Newton iterative method to accelerate the sigmoid based power control algorithm. Selecting an appropriate value of the perturbation fraction can lead to fast convergence of algorithm.

In chapter 4,

we have designed a power control game for cognitive radio network based on energy efficient utility function. Here, we have proposed a novel utility function via pricing to formulate the non-cooperative power control game. In this algorithm, we have obtained higher SIR for CR users closer to base station, while the pricing is strictly applied to the farthest users who represent the source of interference. The proposed algorithm simply requires only local information to maximize the net utility (utility-price) of each CR. We have obtained better power saving and fast convergence as compared with recently works in the literature.

In chapter 5,

we have proposed a power control algorithm for distributed interference management in femtocell network using game theory. In this work, we have strictly enforced the QoS requirements of MUEs by guaranteeing that preferential users can achieve the minimum SINRs. On the other hand, we have successfully achieved both "soft" QoS provisioning and an optimized power consumed. In addition, the proposed algorithms for both MUEs and FUEs require only local information to autonomously maximize the net utility. We have showed that the proposed power control algorithm of FUEs is able to mitigate the cross-tier interference, making the MUEs maintaining their desired SINR requirements easily.

6.2 Future research

Numerical results obtainable in this thesis present the common objective of improving the spectrum sharing efficiency of wireless networks via an efficient power control algorithms and interference management. During the simulation, primary users in cognitive radio network are not considered as decision makers (players) in the game model. Considering primary users inside the game model with different utility function and different strategy will be a good extension of the proposed algorithms in chapter 3 and 4. Moreover, we proposed an efficient pricing technique in our works to reduce transmit power from cognitive radio users located at the cell boundary, but another technique such as soft handover is able to combine the received signals from more than

single cognitive base stations. Thus in the future research, it will be motivating to consider the propose power control algorithm in soft handover environment.

In addition, the convergence of power control algorithms is an important issue and it need another research based on available numerical methods.

Recently, energy efficiency in the small-cell networks (i.e., femtocell, picocell) is crucial to prolong the operational time of battery-based wireless user devices.

The cooperative relay is another technique that is able to improve the coverage of femtocell without interfere the existing macrocells users (Pabst et al. 2004). Therefore, energy-efficient power control and selective relaying is another is another direction of future research for potential energy saving.

REFERENCES

- Ahmad, I., Feng, Z., Hameed, A., Zhang, P. & Zhao, Y. (2014). Spectrum sharing and energy-efficient power optimization for two-tier femtocell networks. Paper presented at the the 9th International Conference on Cognitive Radio Oriented Wireless Networks and Communications (CROWNCOM), 156–161.
- Al Talabani, A., Nallanathan, A. & Nguyen, H. X. (2015). A novel chaos based cost function for power control of cognitive radio networks. *IEEE Communications Letters*, 19(4), 657–660.
- Al-Gumaei, Y. A., Noordin, K. A., Reza, A. W. & Dimyati, K. (2014). A New SIR-Based Sigmoid Power Control Game in Cognitive Radio Networks. *PloS one*, 9(10), e109077.
- Al-Gumaei, Y. A. & Dimyati, K. (2010). Optimal power control game for primarysecondary user in cognitive radio network. *International Journal of Physical Sciences*, 5(4), 345–351.
- Al-Gumaei, Y. A. & Dimyati, K. (2010). Pricing-based primary-secondary user power control game for cognitive radio networks. *Australian Journal of Basic and Applied Sciences*, 4(8), 3721–3729.
- Alpcan, T., Bacsar, T., Srikant, R. & Altman, E. (2002). CDMA uplink power control as a noncooperative game. *Wireless Networks*, 8(6), 659–670.
- Altman, E., Boulogne, T., El-Azouzi, R., Jiménez, T. & Wynter, L. (2006). A survey on networking games in telecommunications. *Computers & Operations Research*, 33(2), 286–311.
- Baldo, N., Maguolo, F., Miozzo, M., Rossi, M. & Zorzi, M. (2007). ns2-MIRACLE: a modular framework for multi-technology and cross-layer support in network simulator 2. Paper presented at the 2nd international conference on Performance evaluation methodologies and tools, 16.
- Bambos, N. (1998). Toward power-sensitive network architectures in wireless communications: Concepts, issues, and design aspects. *IEEE Personal Communications*, 5(3), 50–59.
- Buehrer, R. M. (2006). Code division multiple access (CDMA). Synthesis Lectures on Communications, 1(1), 1–192.Morgan & Claypool Publishers

- Buzzi, S. & Saturnino, D. (2011). A game-theoretic approach to energy-efficient power control and receiver design in cognitive CDMA wireless networks. *IEEE Journal of Selected Topics in Signal Processing*, 5(1), 137–150.
- Chandrasekhar, V., Andrews, J. G. & Gatherer, A. (2008). Femtocell networks: a survey. *IEEE Communications Magazine*, 46(9), 59–67.
- Chandrasekhar, V., Andrews, J. G., Muharemovic, T., Shen, Z. & Gatherer, A. (2009). Power control in two-tier femtocell networks. *IEEE Transactions on Wireless Communications*, 8(8), 4316–4328.
- Chen, K. and Prasad, R. (2009). Conginitive radio communications. *In Cognitive Radio Networks*, p. 121. John Wiley and Sons.
- Chiang, M., Hande, P., Lan, T. & Tan, C. W. (2008). Power control in wireless cellular networks. *Foundations and Trends textregistered in Networking*, 2(4), 381–533.
- Clancy, T. and Arbaugh, W. (2006). Measuring interference temperature. *Proceedings* of the Virginia Tech Symposium on Wireless Personal Communications 2006.
- Claussen, H. (2007). *Performance of macro-and co-channel femtocells in a hierarchical cell structure*. Paper presented at the IEEE 18th International Symposium on Personal, Indoor and Mobile Radio Communications, 1–5.
- Douros, V. G. & Polyzos, G. C. (2011). Review of some fundamental approaches for power control in wireless networks. *Computer Communications*, 34(13), 1580–1592.
- Douros, V. G., Toumpis, S. & Polyzos, G. C. (2012). Power control under best response dynamics for interference mitigation in a two-tier femtocell network. Paper presented at the 10th International Symposium on Modeling and Optimization in Mobile, Ad Hoc and Wireless Networks (WiOpt), 398–405.
- El-Osery, A. & Abdallah, C. (2000). Distributed power control in CDMA cellular systems. *IEEE Antennas and Propagation Magazine*, 42(4), 152–159.
- Famolari, D., Mandayam, N., Goodman, D. & Shah, V. (2002). A new framework for power control in wireless data networks: Games, utility, and pricing. *Wireless Multimedia Network Technologies*, 289–309.
- Federal communications commission. (2002a). "Spectrum Policy Task Force", Report of the spectrum efficiency group, Report.

Federal communications commission. (2002b). "Spectrum policy task force", Unlicensed Devices and Experimental Licenses Working, tech. rep.

- Felegyhazi, M. & Hubaux, J. P. (2006). Game theory in wireless networks: A tutorial. Technical Report LCA-Report-2006-002, EPFL.
- Feng, N., Mau, S.-C. & Mandayam, N. B. (2004). Pricing and power control for joint network-centric and user-centric radio resource management. *IEEE Transactions on Communications*, 52(9), 1547–1557.
- Foschini, G. J. & Miljanic, Z. (1993). A simple distributed autonomous power control algorithm and its convergence. *IEEE Transactions on Vehicular Technology*, 42(4), 641–646.

Fudenberg, D. & Tirole, J. (1991). Game theory. MIT press Cambridge.

- Gajic, Z. R. & Koskie, S. (2003). *Newton iteration acceleration of the Nash game algorithm for power control in 3G wireless CDMA networks*. Paper presented at the proceedings of Performance and Control of Next-Generation Communications Networks, 115–121.
- Gállego, J. R., Canales, M. & Ortin, J. (2012). Distributed resource allocation in cognitive radio networks with a game learning approach to improve aggregate system capacity. *Ad Hoc Networks*, 10(6), 1076–1089.
- Geckil, I. K. & Anderson, P. L. (2009). Modeling Games with Computer Software and Experimenting Games . *Applied game theory and strategic behavior*. Chapman and Hall/CRC, 36-57
- Ghasemi, A., Faez, K. & Dehghan, M. (2006). A new pricing function for power control game in wireless data networks. Paper presented at the IEEE 64th Vehicular Technology Conference, VTC-2006 Fall. 1–5.

Gibbons, R. (1992). A primer in game theory. Harvester Wheatsheaf.

- Gompertz, B. (1825). On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of life contingencies. *Philosophical transactions of the Royal Society of London*, 115, 513–583.
- Goodman, Davtd, & Mandayam, N. (2000). Power control for wireless data. IEEE Personal Communications, 7(2), 48–54.

- Goodman, David, & Mandayam, N. (2001). Network assisted power control for wireless data. *Mobile Networks and Applications*, 6(5), 409–415.
- Grandhi, S. A., Zander, J. & Yates, R. (1994). Constrained power control. *Wireless Personal Communications*, 1(4), 257–270.
- Grandhi, S., Vijayan, R., Goodman, D. J. & others. (1994). Distributed power control in cellular radio systems. *IEEE Transactions on Communications*, 42(234), 226–228.
- Gunnarsson, F. (2000). Power control in cellular radio systems: Analysis, design and estimation. (Doctoral thesis, Linkopings universitet, SE–581 83 Linkoping, Sweden). Retrieved from http://www.control.isy.liu.se/research/reports/Ph.D.Thesis.
- Gunnarsson, F. & Gustafsson, F. (2002). Power control in wireless communications networks-from a control theory perspective. Survey paper in IFAC World Congress, Barcelona.
- Gunturi, S. & Paganini, F. (2003). Game theoretic approach to power control in cellular CDMA. Paper presented at the IEEE 58th Vehicular Technology Conference, 2003. VTC 2003-Fall. 2362–2366.
- Güvenç, I., Jeong, M.-R., Watanabe, F. & Inamura, H. (2008). A hybrid frequency assignment for femtocells and coverage area analysis for co-channel operation. *IEEE Communications Letters*, 12(12), 880–882.
- Han, Q., Ma, K., Wang, X., Guan, X. & Ma, J. (2013). Stackelberg game based interference management for two-tier femtocell networks. *Wireless networks*, 19(7), 1665–1677.
- Han, Z., Niyato, D., Saad, W., Bacsar, T. & Hjoungnes, A. (2011). Game Theory in Wireless and Communication Networks: Theory, Models, and Applications. Cambridge, UK: Cambridge University Press
- Hasu, V. (2007). Radio resource management in wireless communication: Beamforming, transmission power control, and rate allocation. (Doctoral thesis, Helsinki University of Technology). Retrieved from http://lib.tkk.fi/Diss/2007/isbn9789512288229/

- Haykin, S. (2005). Cognitive radio: brain-empowered wireless communications. *IEEE Journal on Selected Areas in Communications*, 23(2), 201–220.
- Hossain, E., Niyato, D. & Han, Z. (2009). Dynamic spectrum access and management in cognitive radio networks. Cambridge, UK: Cambridge University Press
- Hu, S., Yao, Y.-D. & Yang, Z. (2014). MAC protocol identification using support vector machines for cognitive radio networks. *IEEE Wireless Communications*, 21(1), 52–60.
- Jäntti, R. & Kim, S.-L. (2000). Second-order power control with asymptotically fast convergence. *IEEE Journal on Selected Areas in Communications*, 18(3), 447–457.
- Jayaweera, S. K. & Li, T. (2009). Dynamic spectrum leasing in cognitive radio networks via primary-secondary user power control games. *IEEE Transactions on Wireless Communications*, 8(6), 3300–3310.
- Ji, H. & Huang, C. Y. (1998). Non-cooperative uplink power control in cellular radio systems. *wireless Networks*, 4(3), 233–240.
- Ji, Z. & Liu, K. (2007). Cognitive radios for dynamic spectrum access-dynamic spectrum sharing: A game theoretical overview. *IEEE Communications Magazine*, 45(5), 88–94.
- Jiao, J., Jiang, L. & He, C. (2013). A Novel Game Theoretic Utility Function for Power Control in Cognitive Radio Networks. Paper presented at the 5th International Conference on Computational and Information Sciences (ICCIS), 1553–1557.
- Jo, H.-S., Mun, C., Moon, J. & Yook, J.-G. (2009). Interference mitigation using uplink power control for two-tier femtocell networks. *IEEE Transactions on Wireless Communications*, 8(10), 4906–4910.
- Junhui, Z., Tao, Y., Yi, G., Jiao, W. & Lei, F. (2013). Power control algorithm of cognitive radio based on non-cooperative game theory. *China Communications*, 10(11), 143–154.
- Kang, X., Zhang, R. & Motani, M. (2012). Price-based resource allocation for spectrum-sharing femtocell networks: A stackelberg game approach. *IEEE Journal* on Selected Areas in Communications, 30(3), 538–549.

- Koskie, S, & Zapf, J. (2005). Acceleration of static Nash power control algorithm using Newton iterations. *Dynamics of Continuous, Discrete and Impulse Systems B: Applications and Algorithms*, 12, 685–690.
- Koskie, Sarah, & Gajic, Z. (2005). A Nash game algorithm for SIR-based power control in 3G wireless CDMA networks. *IEEE/ACM Transactions on Networking*, 13(5), 1017–1026.
- Kovács, I. Z., Mogensen, P., Christensen, B. & Jarvela, R. (2011). Mobile broadband traffic forecast modeling for network evolution studies. Paper presented at the IEEE Vehicular Technology Conference (VTC Fall), 1–5.
- Kuo, Y., Yang, J. & Chen, J. (2013). Efficient swarm intelligent algorithm for power control game in cognitive radio networks. *IET Communications*, 7(11), 1089–1098.
- De La Roche, G., Valcarce, A., López-Pérez, D. & Zhang, J. (2010). Access control mechanisms for femtocells. *IEEE Communications Magazine*, 48(1), 33–39.
- Lasaulce, S., Hayel, Y., El Azouzi, R. & Debbah, M. (2009). Introducing hierarchy in energy games. *IEEE Transactions on Wireless Communications*, 8(7), 3833–3843.
- Lasaulce, S. & Tembine, H. (2011). *Game theory and learning for wireless networks: fundamentals and applications*. Academic press Elsevier.
- Lee, M., Oh, D. & Kwon, W. (2003). *Modified distributed constrained power control* using reference model in cdma cellular systems. Paper presented at the 42nd IEEE Conference on Decision and Control, 4225–4230.
- Lee, T.-H. & Lin, J.-C. (1996). A study on the distributed power control for cellular *mobile systems*. Paper presented at the IEEE 46th Vehicular Technology Conference on Mobile Technology for the Human Race. 1130–1134.
- Lee, T.-H., Lin, J.-C. & Su, Y. T. (1995). Downlink power control algorithms for cellular radio systems. *IEEE Transactions on Vehicular Technology*, 44(1), 89–94.
- Li, F., Tan, X. & Wang, L. (2011). A new game algorithm for power control in cognitive radio networks. *IEEE Transactions on Vehicular Technology*, 60(9), 4384– 4391.
- Lin, P., Zhang, J., Chen, Y. & Zhang, Q. (2011). Macro-femto heterogeneous network deployment and management: from business models to technical solutions. *IEEE*

Wireless Communications, 18(3), 64–70.

- Lu, K., Zhang, L. & Yang, J. (2012). An efficient SIR-first adaptive power control method in cognitive radio network. Paper presented at the 2012 IEEE Global High Tech Congress on Electronics (GHTCE), 91–94.
- Lu, Z., Sun, Y., Wen, X., Su, T. & Ling, D. (2012). An energy-efficient power control algorithm in femtocell networks. Paper presented at the 7th International Conference on Computer Science & Education (ICCSE), 395–400.
- Lv, L., Zhu, S. & Dong, S. (2003). Fast convergence distributed power control algorithm for WCDMA systems. Paper presented at the IEE Proceedings Communications, 134–140.
- Ma, W., Zhang, H., Zheng, W. & Wen, X. (2012). Differentiated-pricing based power allocation in dense femtocell networks. Paper presented at the 15th International Symposium on Wireless Personal Multimedia Communications (WPMC), 599–603.
- MacKenzie, A. B. & DaSilva, L. A. (2006). Game theory for wireless engineers. *Synthesis Lectures on Communications*, 1(1), 1–86. Morgan & Claypool Publishers.
- MacKenzie, A. B. & Wicker, S. B. (2001). Game theory in communications: Motivation, explanation, and application to power control. Paper presented at the Global IEEE Telecommunications Conference, GLOBECOM'01. 821–826.
- Marcus, M., Burtle, J., Franca, B., Lahjouji, A. & McNeil, N. (2002). Federal communications commission spectrum policy task force. *Report of the Unlicensed Devices and Experimental Licenses Working Group*.
- Meshkati, F., Chiang, M., Poor, H. V. & Schwartz, S. C. (2006). A game-theoretic approach to energy-efficient power control in multicarrier CDMA systems. *IEEE Journal on Selected Areas in Communications*, 24(6), 1115–1129.
- Meshkati, F., Chiang, M., Schwartz, S. C., Poor, H. V. & Mandayam, N. B. (2005). A non-cooperative power control game for multi-carrier CDMA systems. Paper presented at the IEEE Wireless Communications and Networking Conference, 606– 611.
- Meshkati, F., Poor, H. V. & Schwartz, S. C. (2007). Energy-efficient resource allocation in wireless networks. *IEEE Signal Processing Magazine*, 24(3), 58–68.

- Mitola, J. and Maguire, G. Q. (1999). Cognitive radio: Making software radios more personal. *IEEE Personal Communications*; (6): 13–18.
- Ngo, D. T., Le, L. B., Le-Ngoc, T., Hossain, E. & Kim, D. I. (2012). Distributed interference management in two-tier CDMA femtocell networks. *IEEE Transactions on Wireless Communications*, 11(3), 979–989.
- Ortega, J. M. & Rheinboldt, W. C. (1970). *Iterative solution of nonlinear equations in several variables*, Society of Industrial and applied Mathematics SIAM.
- Pabst, R., Walke, B. H., Schultz, D. C., Herhold, P., Yanikomeroglu, H., Mukherjee, S., Viswanathan, H., et al. (2004). Relay-based deployment concepts for wireless and mobile broadband radio. IEEE Communications Magazine, 42(9), 80–89.
- Pasandshanjani, E. & Khalaj, B. H. (2012). Primary-secondary interaction modelling in cellular cognitive radio networks: a game-theoretic approach. *IET communications*, 6(10), 1212–1219.
- Pasandshanjani, E., Khalaj, B. H. & Moghaddam, M. S. (2011). A new cost function for game theoretic sir-based power control algorithms. Paper presented at the 7th International Wireless Communications and Mobile Computing Conference (IWCMC), 1147–1151.
- Popescu, D. C. & Chronopoulos, A. T. (2005). Power control and utility optimization in wireless communication systems. Paper presented at the IEEE 61st Vehicular Technology Conference. 314–318.
- Saad, S. A., Ismail, M. & Nordin, R. (2013). A Survey on Power Control Techniques in Femtocell Networks. Journal of Communications, 8 (12), 845-854.
- Saraydar, C. U., Mandayam, N. B. & Goodman, D. J. (2001). Pricing and power control in a multicell wireless data network. *IEEE Journal on Selected Areas in Communications*, 19(10), 1883–1892.
- Saraydar, C. U., Mandayam, N. B. & Goodman, D. J. (2002). Efficient power control via pricing in wireless data networks. *IEEE Transactions on Communications*, 50(2), 291–303.
- Shah, V., Mandayam, N. B. & Goodman, D. J. (1998). Power control for wireless data based on utility and pricing. Paper presented at the 9th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications, 1427–1432.

- Smith, D. B., Portmann, M., Tan, W. L. & Tushar, W. (2014). Multi-Source-Destination Distributed Wireless Networks: Pareto-Efficient Dynamic Power Control Game With Rapid Convergence. *IEEE Transactions on Vehicular Technology*, 63(6), 2744–2754.
- Straffin, P. D. (1993). *Game theory and strategy*. The Mathematical Association of America, Washington, DC.
- Sung, C. W. & Wong, W. S. (2003). A noncooperative power control game for multirate CDMA data networks. *IEEE Transactions on Wireless Communications*, 2(1), 186–194.
- Tadelis, S. (2013). Game theory: an introduction. Princeton University Press,
- Topkis, D. M. (1998). *Supermodularity and complementarity*. Princeton University Press
- Trivellato, M. (2007). http://tpe-nvdan.googlecode.com/svn/trunk/ ns-allinone-2.33/dei80211mr-1.1.4/src/per table 80211g Trivellato.tcl,. Retrieved from http://tpenvdan.googlecode.com/svn/trunk/ ns-allinone-2.33/dei80211mr-1.1.4/src/per table 80211g Trivellato.tcl,
- Wang, B., Wu, Y. & Liu, K. R. (2010). Game theory for cognitive radio networks: An overview. *Computer networks*, 54(14), 2537–2561.
- Wang, F., Krunz, M. & Cui, S. (2008). Price-based spectrum management in cognitive radio networks. *IEEE Journal of Selected Topics in Signal Processing*, 2(1), 74–87.
- Xiao, M., Shroff, N. B. & Chong, E. K. (2001). Utility-based power control in cellular wireless systems. Paper presented at the 20th Annual Joint Conference of the IEEE Computer and Communications Societies. 412–421.
- Xiao, M., Shroff, N. B. & Chong, E. K. (2003). A utility-based power-control scheme in wireless cellular systems. *IEEE/ACM Transactions on Networking*, 11(2), 210–221.
- Xie, R., Yu, F. R. & Ji, H. (2012). Energy-efficient spectrum sharing and power allocation in cognitive radio femtocell networks. Paper presented at the INFOCOM, 2012 Proceedings IEEE. 1665–1673.
- Xie, R., Yu, F. R. & Ji, H. (2012). Spectrum sharing and resource allocation for energy-efficient heterogeneous cognitive radio networks with femtocells. Paper

presented at the IEEE International Conference on Communications (ICC), 1661–1665.

- Xie, R., Yu, F. R. & Ji, H. (2013). Interference management and power allocation for energy-efficient cognitive femtocell networks. *Mobile Networks and Applications*, 18(4), 578–590.
- Xie, X., Yang, H., Vasilakos, A. V. & He, L. (2014). Fair power control using game theory with pricing scheme in cognitive radio networks. *Journal of Communications* and Networks, 16(2), 183–192.
- Xu, X., Li, Y., Gao, R. & Tao, X. (2014). Joint voronoi diagram and game theory-based power control scheme for the hetnet small cell networks. *EURASIP Journal on Wireless Communications and Networking*, 1(1),
- Yates, R. D. (1995). A framework for uplink power control in cellular radio systems. *IEEE Journal on Selected Areas in Communications*, 13(7), 1341–1347.
- Yavuz, M., Meshkati, F., Nanda, S., Pokhariyal, A., Johnson, N., Raghothaman, B. & Richardson, A. (2009). Interference management and performance analysis of UMTS/HSPA+ femtocells. *IEEE Communications Magazine*, 47(9), 102–109.
- Yu, H., Gao, L., Li, Z., Wang, X. & Hossain, E. (2010). Pricing for uplink power control in cognitive radio networks. *IEEE Transactions on Vehicular Technology*, 59(4), 1769–1778.
- Zander, J. (1992). Distributed cochannel interference control in cellular radio systems. *IEEE Transactions on Vehicular Technology*, 41(3), 305–311.
- Zander, J. (1994). Transmitter power control for co-channel interference management in cellular radio systems. *Wireless and Mobile Communications*, 161–175.
- Zhang, J., Zhao, Z., Li, W., Liu, Z. & Xin, L. (2012). Joint pricing and power control for OFDMA femtocell networks using stackelberg game. Paper presented at the 15th International Symposium on Wireless Personal Multimedia Communications (WPMC), 619–622.
- Zhang, X., Zhang, Y., Shi, Y., Zhao, L. & Zou, C. (2012). Power control algorithm in cognitive radio system based on modified shuffled frog leaping algorithm. AEU-International Journal of Electronics and Communications, 66(6), 448–454.

- Zhang, Y. & Guizani, M. (2011). *Game theory for wireless communications and networking*, CRC Press.
- Zhou, C., Zhang, P., Honig, M. L. & Jordan, S. (2004). Two-cell power allocation for downlink CDMA. *IEEE Transactions on Wireless Communications*, 3(6), 2256– 2266.

LIST OF PUBLICATIONS AND PAPERS PRESENTED

- Al-Gumaei Y. A., Noordin K. A., Reza A. W., Dimyati K., "A New Utilitybased Power Control Game in Cognitive Radio Networks", page 79-83, The 3rd International Conference on Computer Science & Computational Mathematics (ICCSCM 2014), Langkawi, Malaysia.
- Al-Gumaei Y. A., Noordin K. A., Reza A. W., Dimyati K., "A New SIR-Based Sigmoid Power Control Game in Cognitive Radio Networks,", PLOS ONE, vol. 9(10), pp. 1-10, October 2014.
- Al-Gumaei Y. A., Noordin K. A., Reza A. W., Dimyati K., "A Novel Utility Function for Energy-Efficient Power Control Game in Cognitive Radio Networks". PLOS ONE, 10(8): e0135137. doi:10.1371/journal.pone.0135137
- Al-Gumaei Y. A., Noordin K. A., Reza A. W., Dimyati K., "A game theory approach for efficient power control and interference management in two-tier femto-cell networks based on Local Gain ", 9(7), July 31, 2015 doi: 10.3837/tiis.2015.07.011.
- Al-Gumaei Y. A., Noordin K. A., Reza A. W., Dimyati K., "Anew power control in Two-Tier femto-cell networks", The 1st International Conference on Telematics and Future Generation Networks (Tafgen2015), Kuala Lumpur, Malaysia.
- Al-Gumaei Y. A., Noordin K. A., Reza A. W., Dimyati K., "A New Utility-based Power Control Game in Cognitive Radio Networks", submitted to ScienceAsia journal.
- Noordin K. A., Al-Gumaei Y. A., Dimyati K., Mohamad Sofian Abu Talip "Accelerating the sigmoid power control game algorithm using Newton iteration", submitted to Journal