APPENDIX I
Appendix I.1
Outline of Simulation Algorithm for Steady State

START
I ← 1
READ TAU, sample, G
GENERATE RANDOM NUMBER
COMPUTE T_i, D_i
FIND THE SIZE INTERVAL J CORRESPONDING TO D_i

WT ← WT + D_i^3
W(J) ← W(J) + D_i^3

IS
I > sample

NO
YES
WF(J) ← W(J)/WT
PRINT WF(J), WT
STOP

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Appendix I.2

Outline of Simulation Algorithm for Transient CSD

START

READ TAU, [(NRATE)], ((GO, GM, KO, KM, LM), (NO, NM), [F1, F2]

TCOUNT ← 1 MIN

(CALCULATE NRATE)

NCOUNT ← 1

GENERATE RANDOM NUMBER AND CALCULATE TI, ((K1, L1, G1)

T ← TCOUNT
TTOT ← TCOUNT + TI

IS TTOT > TSCAN

YES → TTOT ← TSCAN

NO → COMPUTE DI (T), WI(T)

FIND THE SIZE CLASS OF THE CRYSTAL

CRYSNO(T, SIZE) ← CRYSNO(T, SIZE) + 1
CRYSWT(T, SIZE) ← CRYSWT(T, SIZE) + WI(T)

IS T < TTOT

YES → T ← T + 1 MIN

NO → IS NCOUNT < NRATE

YES → NCOUNT ← NCOUNT + 1

NO → IS TCOUNT < TSCAN

YES → TCOUNT ← TCOUNT + 1 MIN

NO → PRINT CRYSNO(T, SIZE), CRYSWT(T, SIZE) FOR ALL SIZE CLASSES AT SELECTED TIME INTERVAL

STOP

() apply to Sen Gupta and Dutta (1990c)
[] apply to Dey and Sen Gupta (1993)
() apply to Dey and Sen Gupta (1993a)
Appendix I.3

The Detailed Procedure of Generating Random Samples of Growth Rate and Shape Factor from the Normal Density Function

The normal distribution is a continuous distribution completely defined by its mean $m$ and standard deviation $\sigma$. The density function of a normal distribution $(m, \sigma)$ is given by,

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left[-\frac{(x-m)^2}{2\sigma^2}\right]$$  \hspace{1cm} (AI.3.1)

It is assumed that the dispersion range is given by $m \pm 3\sigma$. The dispersion band spread over the $3\sigma$ limits of the mean value guarantees 99.7% occurrence of the random variable within this region. For example, in case of growth rate dispersion,

$$G_O = m - 3\sigma$$  \hspace{1cm} (AI.3.2)

$$G_M = m + 3\sigma$$  \hspace{1cm} (AI.3.3)

Hence,

$$m = (G_M + G_O) / 2$$  \hspace{1cm} (AI.3.4)

and

$$\sigma = (G_M - G_O) / 6$$  \hspace{1cm} (AI.3.5)
In order to generate random samples, the following steps are followed:

\[ RND = \exp[-(x - m)^2 / 2\sigma^2] \quad \text{(AI.3.6)} \]

which satisfies the condition that,

\[ f(x) = 0 \text{ when } RND = 0 \]
\[ f(x) = \frac{1}{\sigma\sqrt{2\pi}} \text{ when } RND = 1 \]

From Eq. (AI.3.6), one can write

\[ x = m \pm \sqrt{-2\sigma^2 \log(RND)} \quad \text{(AI.3.7)} \]

To select a single value of \( x \) from Eq. (AI.3.7), a random number is generated. If the random number is smaller than 0.5, the discriminate is assigned a negative sign, otherwise it is taken as positive. Thus the growth rate and shape factor of any crystal can be found as Eq. (1.65) and Eq. (1.66) when they are normally distributed over a known range.
Appendix I.4

Derivation of Eqs (1.92), (1.93) and (1.94)

The probability that a crystal will be in state \( j \) at time \( (t + \Delta t) \) arises from two independent sequential events; one is that the crystal is in state \( (j - 1) \) at time \( t \) and will be in state \( j \) at time \( (t + \Delta t) \), and the other is that it is in state \( j \) at time \( t \) and will remain in this state at time \( (t + \Delta t) \). According to the assumptions and definitions given at the model elaboration of section 1.3.2.1, the conditional probability that the crystal will be in state \( j \) at time \( (t + \Delta t) \) given that it is in state \( (j - 1) \) at time \( t \) can be approximated by

\[
\lambda_{j-1}(t)\Delta t + o(\Delta t), \quad \text{for } j > 1 \tag{AI.4.1}
\]

Then the probability of the first sequential events is

\[
P_{j-1}(t)\left[\lambda_{j-1}(t)\Delta t + o(\Delta t)\right] \tag{AI.4.2}
\]

Similarly, the probability of the second sequential events is

\[
P_{j}(t)\left[1 - \lambda_{j}(t)\Delta t + o(\Delta t)\right] \tag{AI.4.3}
\]
Hence, the probability that the crystal will be in state \( j \) at time \((t + \Delta t)\) is the sum of the probabilities of these two exclusive sequential events, i.e.,

\[
P_j(t + \Delta t) = P_{j-1}(t)[\lambda_{j-1}(t)\Delta t + o(\Delta t)] + P_j(t)[1 - \lambda_j(t)\Delta t + o(\Delta t)]
\]  
(AI.4.4)

By rearranging this expression, we have,

\[
\frac{P_j(t + \Delta t) - P_j(t)}{\Delta t} = \lambda_{j-1}(t)P_{j-1}(t) - \lambda_j(t)P_j(t) + \frac{o(\Delta t)}{\Delta t}[P_{j-1}(t) + P_j(t)]
\]  
(AI.4.5)

Taking the limit as \(\Delta t \to 0\) yields

\[
\frac{dP_j(t)}{dt} = \lambda_{j-1}(t)P_{j-1}(t) - \lambda_j(t)P_j(t)
\]  
(AI.4.6)

The probability that a crystal will be in state 1 at time \((t + \Delta t)\) only arises from the sequential events that it is in state 1 at time \( t \) and will remain in state 1 at time \((t + \Delta t)\). As a result, we have

\[
\frac{dP_1(t)}{dt} = -\lambda_1(t)P_1(t)
\]  
(AI.4.7)

From Eq. (AI.4.6), we have, for the final state, i.e., \( j = M \),

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\[
\frac{dP_M(t)}{dt} = \lambda_{M-1}(t)P_{M-1}(t)
\]  \hspace{1cm} (AI.4.8)

Eqs (AI.4.6), (AI.4.7) and (AI.4.8) are Eqs (1.92), (1.93) and (1.94) respectively.
\[ \frac{dP_M(t)}{dt} = \lambda_{M-1}(t) P_{M-1}(t) \] (AI.4.8)

Eqs (AI.4.6), (AI.4.7) and (AI.4.8) are Eqs (1.92), (1.93) and (1.94) respectively.
Appendix I.5

Multinominal Distribution

Random vector \((X_1, X_2, \ldots, X_M)\) is multinomially distributed with parameters \((n, P_1, P_2, \ldots, P_M)\) if the joint probability mass function of \((x_1, x_2, \ldots, x_M)\) is given by

\[
P[X_1 = x_1, X_2 = x_2, \ldots, X_M = x_M] = \begin{cases} \frac{n!}{M} \prod_{j=1}^{M} P_j^{x_j} \quad &\text{if } \sum_{j=1}^{M} x_j = n \\ \prod_{j=1}^{M} x_j! &\text{otherwise} \end{cases}
\]

(AI.5.1)

where \(x_i, i = 1, 2, \ldots, M\) are nonnegative integers and \(P_j\) denotes the probability that \(X_j\) takes the value of \(x_j\).

For the present case, a seed (i.e., a crystal in state 1 at time 0) will be in state \(j\) at time \(t\) with a probability of \(P_j(t)\); therefore, with \(m(0)\) seeds present initially, the joint probability mass function of \((n_1, n_2, \ldots, n_M)\) becomes

\[
P[N_1(t) = n_1, N_2(t) = n_2, \ldots, N_M(t) = n_M] = \begin{cases} \frac{m(0)!}{M} \prod_{j=1}^{M} [P_j(t)]^{n_j} \quad &\text{if } \sum_{j=1}^{M} n_j = m(0) \\ \prod_{j=1}^{M} n_j! &\text{otherwise} \end{cases}
\]

(AI.5.2)

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For any fixed value of \( j, \ j = 1, 2, \ldots, M \), a crystal in state 1 at time 0 will be either in state \( j \) at time \( t \) with a probability of \( P_j(t) \) or not in state \( j \) at time \( t \) with a probability of \( [1 - P_j(t)] \). Consequently, out of \( m(0) \) seeds, the probability that \( n_j \) of them will be in state \( j \) at time \( t \) is

\[
\binom{m(0)}{n_j} P_j(t)^n_j [1 - P_j(t)]^{m(0) - n_j}
\]  

(AI.5.3)

which is a binomial distribution. It is well known that the mean and variance of such a distribution are,

\[
E[N_j(t)] = m(0)P_j(t)
\]  

(AI.5.4)

and

\[
Var[N_j(t)] = m(0)P_j(t)[1 - P_j(t)]
\]  

(AI.5.5)

respectively.
Appendix I.6

Derivation of Eqs (1.129) and (1.130)

For convenience, Eq. (1.116) is reiterated below:

\[
\frac{dP(j,t)}{dt} = (E^{-1} - 1) a_j^{-k} P(j,t), \quad j = 1, 2, \ldots, M
\]  

(AI.6.1)

Taking the transformation

\[
j = \Omega \phi(t) + \Omega^{\frac{1}{2}} x
\]  

(AI.6.2)

and collecting terms of the order \(\Omega^{\frac{1}{2}}\) and \(\Omega^0\) yield, respectively (VAN KAMPEN, 1992),

\[
\frac{d\phi}{dt} = f_1(\phi)
\]  

(AI.6.3)

and

\[
\frac{\partial q}{\partial t} = -f_2(\phi) \frac{\partial (xq)}{\partial x} + f_3(\phi) \frac{\partial^2 q}{\partial x^2}
\]  

(AI.6.4)

Eq. (AI.6.3) is the macroscopic equation and Eq. (AI.6.4) is a linear Fokker-Plank equation (VAN KAMPEN, 1976, 1992). The first and second moments of the random
variable $X$ are governed by the coefficients of the partial derivatives on the right-hand side of the latter through the following two expressions.

\[
\frac{d\langle x \rangle}{dt} = f_2(\phi)\langle x \rangle \tag{AI.6.5}
\]

and

\[
\frac{d\langle x^2 \rangle}{dt} = -2f_2(\phi)\langle x^2 \rangle + 2f_3(\phi) \tag{AI.6.6}
\]

A comparison of Eq. (1.128) in the text with Eq. (AI.6.4) yields

\[
f_2(\phi) = a_1\phi^{-(k+1)} \frac{k}{\phi} \tag{AI.6.7}
\]

and

\[
f_3(\phi) = \frac{a_1\phi^{-k}}{2} \tag{AI.6.8}
\]

As a result, Eqs. (1.129) and (1.130) can be recovered from Eqs. (AI.6.5) and (AI.6.6) respectively.
Appendix II.1

Sample Program and Results for Transient CSD for the Size-Dependent Growth by Employing the ASL Model

Declaration

Dim wtfr(0 To 16) As Single  
Dim cmwtfr(0 To 16) As Single  
Dim wet(0 To 16), cry(0 To 16) As Single  
Dim cr(1 To 780, 0 To 16) As Single  
Dim wt(1 To 780, 0 To 16) As Single  
Dim gzero, nt, nmin, nmax, size, cryswt, v As Single  
Dim t1, t2, times, tau, ttot, tmax, b, crysno As Single  
Dim k As Integer

Main Program

Sub main()

    tau = Val(asl.Text2.Text)  
gzero = Val(asl.Text3.Text)  
b = Val(asl.Text4.Text)  
nt = 1000

Open "m_asl" For Output As #1
Print #1, "Results of transient MC Simulation for Size Dependent Growth"
Print #1, "Average residence time=" & tau; " seconds"
Print #1, "Growth rate of crystals at zero size=" & gzero; "mm/s"
Print #1, "ASL parameter=" & b
Print #1, "Tau=" & tau
Print #1, "Nucleation rate=" & nt; "nuclei/min"

    tmax = 13 * (tau / 60)
    For t1 = 1 To tmax
        Randomize
        For n = 1 To nt
            t2 = -(tau / 60) * Log(Rnd)
            ttot = t1 + t2
            If ttot > tmax Then ttot = tmax
            For times = t1 To ttot Step 1
                v = (1 + (((times - t1) * (1 - b)) / (tau / 60))) ^ (1 / (1 - b))
size = (gzero * tau * (v - 1))
If size > .074 Then
  GoTo 250
Else
  k = 0
  GoTo 410
End If

250 If size > .163 Then
  GoTo 260
Else
  k = 1
  GoTo 410
End If

260 If size > .2 Then
  GoTo 270
Else
  k = 2
  GoTo 410
End If

270 If size > .231 Then
  GoTo 280
Else
  k = 3
  GoTo 410
End If

280 If size > .281 Then
  GoTo 290
Else
  k = 4
  GoTo 410
End If

290 If size > .325 Then
  GoTo 300
Else
  k = 5
  GoTo 410
End If

300 If size > .4 Then
  GoTo 310
Else
  k = 6
  GoTo 410
End If

310 If size > .47 Then
  GoTo 320
Else
k = 7
GoTo 410
End If
320  If size > .555 Then
     GoTo 330
Else
     k = 8
     GoTo 410
End If
330  If size > .655 Then
     GoTo 340
Else
     k = 9
     GoTo 410
End If
340  If size > .785 Then
     GoTo 350
Else
     k = 10
     GoTo 410
End If
350  If size > .924 Then
     GoTo 360
Else
     k = 11
     GoTo 410
End If
360  If size > 1.072 Then
     GoTo 370
Else
     k = 12
     GoTo 410
End If
370  If size > 1.268 Then
     GoTo 380
Else
     k = 13
     GoTo 410
End If
380  If size > 1.548 Then
     GoTo 390
Else
     k = 14
     GoTo 410
End If
390  If size > 1.848 Then
GoTo 400
Else
    k = 15
    GoTo 410
End If

400     k = 16
410     cr(times, k) = cr(times, k) + 1
          wt(times, k) = wt(times, k) + (size ^ 3)
Next
Next
Next
For times = (tau / 60) To tmax Step .5 * (tau / 60)
    For k = 0 To 16
        wet(k) = wt(times, k)
        cryswt = cryswt + wet(k)
        cry(k) = cr(times, k)
        crysno = crysno + cry(k)
    Next
    wtfr(0) = wet(0) / cryswt
    cmwtfr(0) = wtfr(0)
    For k = 1 To 16
        wtfr(k) = wet(k) / cryswt
        cmwtfr(k) = cmwtfr(k - 1) + wtfr(k)
    Next
    Print #1, "time=" & times; "min"
    Print #1, "Total number of crystals=" & crysno
    For k = 0 To 16
        Print #1, "MASS UNDER-SIZE IN CLASS("; k; ")="; cmwtfr(k)
        Print #1, "NO. OF CRYSTALS IN CLASS("; k; ")="; cr(times, k)
    Next
    cryswt = 0
    crysno = 0
Next
End Sub
Sample of results (for Experiment No. 1)

Results of transient MC Simulation for Size Dependent Growth
Average residence time=965 seconds
Growth rate of crystals at zero size=.000026mm/s
ASL parameter=.566
Tau=965
Nucleation rate=1000nuclei/min

time=24.125min
Total number of crystals=12888
MASS UNDER-SIZE IN CLASS( 0 )= 1
NO. OF CRYSTALS IN CLASS( 0 )= 12888
MASS UNDER-SIZE IN CLASS( 1 )= 1
NO. OF CRYSTALS IN CLASS( 1 )= 0
MASS UNDER-SIZE IN CLASS( 2 )= 1
NO. OF CRYSTALS IN CLASS( 2 )= 0
MASS UNDER-SIZE IN CLASS( 3 )= 1
NO. OF CRYSTALS IN CLASS( 3 )= 0
MASS UNDER-SIZE IN CLASS( 4 )= 1
NO. OF CRYSTALS IN CLASS( 4 )= 0
MASS UNDER-SIZE IN CLASS( 5 )= 1
NO. OF CRYSTALS IN CLASS( 5 )= 0
MASS UNDER-SIZE IN CLASS( 6 )= 1
NO. OF CRYSTALS IN CLASS( 6 )= 0
MASS UNDER-SIZE IN CLASS( 7 )= 1
NO. OF CRYSTALS IN CLASS( 7 )= 0
MASS UNDER-SIZE IN CLASS( 8 )= 1
NO. OF CRYSTALS IN CLASS( 8 )= 0
MASS UNDER-SIZE IN CLASS( 9 )= 1
NO. OF CRYSTALS IN CLASS( 9 )= 0
MASS UNDER-SIZE IN CLASS( 10 )= 1
NO. OF CRYSTALS IN CLASS( 10 )= 0
MASS UNDER-SIZE IN CLASS( 11 )= 1
NO. OF CRYSTALS IN CLASS( 11 )= 0
MASS UNDER-SIZE IN CLASS( 12 )= 1
NO. OF CRYSTALS IN CLASS( 12 )= 0
MASS UNDER-SIZE IN CLASS( 13 )= 1
NO. OF CRYSTALS IN CLASS( 13 )= 0
MASS UNDER-SIZE IN CLASS( 14 )= 1
NO. OF CRYSTALS IN CLASS( 14 )= 0
MASS UNDER-SIZE IN CLASS( 15 )= 1
NO. OF CRYSTALS IN CLASS( 15 )= 0
MASS UNDER-SIZE IN CLASS( 16 )= 1
NO. OF CRYSTALS IN CLASS( 16 )= 0

time=40.2083333333333min
Total number of crystals=15175
MASS UNDER-SIZE IN CLASS( 0 )= .404131
NO. OF CRYSTALS IN CLASS( 0 )= 14107
MASS UNDER-SIZE IN CLASS( 1 )= 1
NO. OF CRYSTALS IN CLASS( 1 )= 1068
MASS UNDER-SIZE IN CLASS( 2 )= 1
NO. OF CRYSTALS IN CLASS( 2 )= 0
MASS UNDER-SIZE IN CLASS( 3 )= 1

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NO. OF CRYSTALS IN CLASS( 3 )= 0
MASS UNDER-SIZE IN CLASS( 4 )= 1
NO. OF CRYSTALS IN CLASS( 4 )= 0
MASS UNDER-SIZE IN CLASS( 5 )= 1
NO. OF CRYSTALS IN CLASS( 5 )= 0
MASS UNDER-SIZE IN CLASS( 6 )= 1
NO. OF CRYSTALS IN CLASS( 6 )= 0
MASS UNDER-SIZE IN CLASS( 7 )= 1
NO. OF CRYSTALS IN CLASS( 7 )= 0
MASS UNDER-SIZE IN CLASS( 8 )= 1
NO. OF CRYSTALS IN CLASS( 8 )= 0
MASS UNDER-SIZE IN CLASS( 9 )= 1
NO. OF CRYSTALS IN CLASS( 9 )= 0
MASS UNDER-SIZE IN CLASS( 10 )= 1
NO. OF CRYSTALS IN CLASS( 10 )= 0
MASS UNDER-SIZE IN CLASS( 11 )= 1
NO. OF CRYSTALS IN CLASS( 11 )= 0
MASS UNDER-SIZE IN CLASS( 12 )= 1
NO. OF CRYSTALS IN CLASS( 12 )= 0
MASS UNDER-SIZE IN CLASS( 13 )= 1
NO. OF CRYSTALS IN CLASS( 13 )= 0
MASS UNDER-SIZE IN CLASS( 14 )= 1
NO. OF CRYSTALS IN CLASS( 14 )= 0
MASS UNDER-SIZE IN CLASS( 15 )= 1
NO. OF CRYSTALS IN CLASS( 15 )= 0
MASS UNDER-SIZE IN CLASS( 16 )= 1
NO. OF CRYSTALS IN CLASS( 16 )= 0

\[ \text{time}=56.29166666666667 \text{min} \]
Total number of crystals=15978
MASS UNDER-SIZE IN CLASS( 0 )= 1.531241
NO. OF CRYSTALS IN CLASS( 0 )= 14105
MASS UNDER-SIZE IN CLASS( 1 )= 0.811286
NO. OF CRYSTALS IN CLASS( 1 )= 1739
MASS UNDER-SIZE IN CLASS( 2 )= 1
NO. OF CRYSTALS IN CLASS( 2 )= 134
MASS UNDER-SIZE IN CLASS( 3 )= 1
NO. OF CRYSTALS IN CLASS( 3 )= 0
MASS UNDER-SIZE IN CLASS( 4 )= 1
NO. OF CRYSTALS IN CLASS( 4 )= 0
MASS UNDER-SIZE IN CLASS( 5 )= 1
NO. OF CRYSTALS IN CLASS( 5 )= 0
MASS UNDER-SIZE IN CLASS( 6 )= 1
NO. OF CRYSTALS IN CLASS( 6 )= 0
MASS UNDER-SIZE IN CLASS( 7 )= 1
NO. OF CRYSTALS IN CLASS( 7 )= 0
MASS UNDER-SIZE IN CLASS( 8 )= 1
NO. OF CRYSTALS IN CLASS( 8 )= 0
MASS UNDER-SIZE IN CLASS( 9 )= 1
NO. OF CRYSTALS IN CLASS( 9 )= 0
MASS UNDER-SIZE IN CLASS( 10 )= 1
NO. OF CRYSTALS IN CLASS( 10 )= 0
MASS UNDER-SIZE IN CLASS( 11 )= 1
NO. OF CRYSTALS IN CLASS( 11 )= 0
MASS UNDER-SIZE IN CLASS( 12 )= 1
NO. OF CRYSTALS IN CLASS( 12 )= 0
MASS UNDER-SIZE IN CLASS( 13 )= 1
NO. OF CRYSTALS IN CLASS( 13 )= 0
MASS UNDER-SIZE IN CLASS( 14 )= 1
NO. OF CRYSTALS IN CLASS( 14 )= 0
MASS UNDER-SIZE IN CLASS( 15 )= 1
NO. OF CRYSTALS IN CLASS( 15 )= 0
MASS UNDER-SIZE IN CLASS( 16 )= 1
NO. OF CRYSTALS IN CLASS( 16 )= 0

time=72.375min
Total number of crystals=16256
MASS UNDER-SIZE IN CLASS( 0 )= 7.836225E-02
NO. OF CRYSTALS IN CLASS( 0 )= 14058
MASS UNDER-SIZE IN CLASS( 1 )= .4298482
NO. OF CRYSTALS IN CLASS( 1 )= 1739
MASS UNDER-SIZE IN CLASS( 2 )= .6309605
NO. OF CRYSTALS IN CLASS( 2 )= 248
MASS UNDER-SIZE IN CLASS( 3 )= .7962059
NO. OF CRYSTALS IN CLASS( 3 )= 119
MASS UNDER-SIZE IN CLASS( 4 )= 1
NO. OF CRYSTALS IN CLASS( 4 )= 92
MASS UNDER-SIZE IN CLASS( 5 )= 1
NO. OF CRYSTALS IN CLASS( 5 )= 0
MASS UNDER-SIZE IN CLASS( 6 )= 1
NO. OF CRYSTALS IN CLASS( 6 )= 0
MASS UNDER-SIZE IN CLASS( 7 )= 1
NO. OF CRYSTALS IN CLASS( 7 )= 0
MASS UNDER-SIZE IN CLASS( 8 )= 1
NO. OF CRYSTALS IN CLASS( 8 )= 0
MASS UNDER-SIZE IN CLASS( 9 )= 1
NO. OF CRYSTALS IN CLASS( 9 )= 0
MASS UNDER-SIZE IN CLASS( 10 )= 1
NO. OF CRYSTALS IN CLASS( 10 )= 0
MASS UNDER-SIZE IN CLASS( 11 )= 1
NO. OF CRYSTALS IN CLASS( 11 )= 0
MASS UNDER-SIZE IN CLASS( 12 )= 1
NO. OF CRYSTALS IN CLASS( 12 )= 0
MASS UNDER-SIZE IN CLASS( 13 )= 1
NO. OF CRYSTALS IN CLASS( 13 )= 0
MASS UNDER-SIZE IN CLASS( 14 )= 1
NO. OF CRYSTALS IN CLASS( 14 )= 0
MASS UNDER-SIZE IN CLASS( 15 )= 1
NO. OF CRYSTALS IN CLASS( 15 )= 0
MASS UNDER-SIZE IN CLASS( 16 )= 1
NO. OF CRYSTALS IN CLASS( 16 )= 0

time=88.45833333333333min
Total number of crystals=16399
MASS UNDER-SIZE IN CLASS( 0 )= 4.921402E-02
NO. OF CRYSTALS IN CLASS( 0 )= 14085
MASS UNDER-SIZE IN CLASS( 1 )= .2685087
NO. OF CRYSTALS IN CLASS( 1 )= 1743
MASS UNDER-SIZE IN CLASS( 2 )= .3725953
NO. OF CRYSTALS IN CLASS( 2 )= 204
MASS UNDER-SIZE IN CLASS( 3 )= .4912164
NO. OF CRYSTALS IN CLASS( 3 )= 138
MASS UNDER-SIZE IN CLASS(4) = .6596665
NO. OF CRYSTALS IN CLASS(4) = 116
MASS UNDER-SIZE IN CLASS(5) = .8204942
NO. OF CRYSTALS IN CLASS(5) = 67
MASS UNDER-SIZE IN CLASS(6) = 1
NO. OF CRYSTALS IN CLASS(6) = 46
MASS UNDER-SIZE IN CLASS(7) = 1
NO. OF CRYSTALS IN CLASS(7) = 0
MASS UNDER-SIZE IN CLASS(8) = 1
NO. OF CRYSTALS IN CLASS(8) = 0
MASS UNDER-SIZE IN CLASS(9) = 1
NO. OF CRYSTALS IN CLASS(9) = 0
MASS UNDER-SIZE IN CLASS(10) = 1
NO. OF CRYSTALS IN CLASS(10) = 0
MASS UNDER-SIZE IN CLASS(11) = 1
NO. OF CRYSTALS IN CLASS(11) = 0
MASS UNDER-SIZE IN CLASS(12) = 1
NO. OF CRYSTALS IN CLASS(12) = 0
MASS UNDER-SIZE IN CLASS(13) = 1
NO. OF CRYSTALS IN CLASS(13) = 0
MASS UNDER-SIZE IN CLASS(14) = 1
NO. OF CRYSTALS IN CLASS(14) = 0
MASS UNDER-SIZE IN CLASS(15) = 1
NO. OF CRYSTALS IN CLASS(15) = 0
MASS UNDER-SIZE IN CLASS(16) = 1
NO. OF CRYSTALS IN CLASS(16) = 0

time = 104.5416666666667 min
Total number of crystals = 16539
MASS UNDER-SIZE IN CLASS(0) = 3.586309E-02
NO. OF CRYSTALS IN CLASS(0) = 14117
MASS UNDER-SIZE IN CLASS(1) = .1983787
NO. OF CRYSTALS IN CLASS(1) = 1781
MASS UNDER-SIZE IN CLASS(2) = .2904016
NO. OF CRYSTALS IN CLASS(2) = 248
MASS UNDER-SIZE IN CLASS(3) = .3660758
NO. OF CRYSTALS IN CLASS(3) = 118
MASS UNDER-SIZE IN CLASS(4) = .4767982
NO. OF CRYSTALS IN CLASS(4) = 106
MASS UNDER-SIZE IN CLASS(5) = .6154193
NO. OF CRYSTALS IN CLASS(5) = 78
MASS UNDER-SIZE IN CLASS(6) = .7830982
NO. OF CRYSTALS IN CLASS(6) = 55
MASS UNDER-SIZE IN CLASS(7) = .9038921
NO. OF CRYSTALS IN CLASS(7) = 24
MASS UNDER-SIZE IN CLASS(8) = 1
NO. OF CRYSTALS IN CLASS(8) = 12
MASS UNDER-SIZE IN CLASS(9) = 1
NO. OF CRYSTALS IN CLASS(9) = 0
MASS UNDER-SIZE IN CLASS(10) = 1
NO. OF CRYSTALS IN CLASS(10) = 0
MASS UNDER-SIZE IN CLASS(11) = 1
NO. OF CRYSTALS IN CLASS(11) = 0
MASS UNDER-SIZE IN CLASS(12) = 1
NO. OF CRYSTALS IN CLASS(12) = 0
MASS UNDER-SIZE IN CLASS(13) = 1

226
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<tr>
<td>NO. OF CRYSTALS IN CLASS(16)</td>
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**time=120.625min**

**Total number of crystals=16593**

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<th>3.257208E-02</th>
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<tr>
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<tr>
<td>MASS UNDER-SIZE IN CLASS(1)</td>
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<td>NO. OF CRYSTALS IN CLASS(1)</td>
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<td>MASS UNDER-SIZE IN CLASS(2)</td>
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<td>MASS UNDER-SIZE IN CLASS(5)</td>
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<tr>
<td>NO. OF CRYSTALS IN CLASS(16)</td>
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**time=136.708333333333min**

**Total number of crystals=16748**

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<tr>
<td>MASS UNDER-SIZE IN CLASS(1)</td>
<td>.1489457</td>
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<tr>
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<tr>
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<td>NO. OF CRYSTALS IN CLASS(3)</td>
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<tr>
<td>MASS UNDER-SIZE IN CLASS(4)</td>
<td>.3702528</td>
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NO. OF CRYSTALS IN CLASS( 4 )= 135
MASS UNDER-SIZE IN CLASS( 5 )=.4542509
NO. OF CRYSTALS IN CLASS( 5 )= 65
MASS UNDER-SIZE IN CLASS( 6 )=.561769
NO. OF CRYSTALS IN CLASS( 6 )= 48
MASS UNDER-SIZE IN CLASS( 7 )=.6519904
NO. OF CRYSTALS IN CLASS( 7 )= 24
MASS UNDER-SIZE IN CLASS( 8 )=.757058
NO. OF CRYSTALS IN CLASS( 8 )= 17
MASS UNDER-SIZE IN CLASS( 9 )=.8973248
NO. OF CRYSTALS IN CLASS( 9 )= 14
MASS UNDER-SIZE IN CLASS( 10 )= 1
NO. OF CRYSTALS IN CLASS( 10 )= 6
MASS UNDER-SIZE IN CLASS( 11 )= 1
NO. OF CRYSTALS IN CLASS( 11 )= 0
MASS UNDER-SIZE IN CLASS( 12 )= 1
NO. OF CRYSTALS IN CLASS( 12 )= 0
MASS UNDER-SIZE IN CLASS( 13 )= 1
NO. OF CRYSTALS IN CLASS( 13 )= 0
MASS UNDER-SIZE IN CLASS( 14 )= 1
NO. OF CRYSTALS IN CLASS( 14 )= 0
MASS UNDER-SIZE IN CLASS( 15 )= 1
NO. OF CRYSTALS IN CLASS( 15 )= 0
MASS UNDER-SIZE IN CLASS( 16 )= 1
NO. OF CRYSTALS IN CLASS( 16 )= 0

time=152.79166666666667min
Total number of crystals=16610
MASS UNDER-SIZE IN CLASS( 0 )= 2.465028E-02
NO. OF CRYSTALS IN CLASS( 0 )= 14151
MASS UNDER-SIZE IN CLASS( 1 )= .1387122
NO. OF CRYSTALS IN CLASS( 1 )= 1796
MASS UNDER-SIZE IN CLASS( 2 )= .1957899
NO. OF CRYSTALS IN CLASS( 2 )= 221
MASS UNDER-SIZE IN CLASS( 3 )= .2577045
NO. OF CRYSTALS IN CLASS( 3 )= 144
MASS UNDER-SIZE IN CLASS( 4 )= .3457942
NO. OF CRYSTALS IN CLASS( 4 )= 119
MASS UNDER-SIZE IN CLASS( 5 )= .4186654
NO. OF CRYSTALS IN CLASS( 5 )= 59
MASS UNDER-SIZE IN CLASS( 6 )= .5372391
NO. OF CRYSTALS IN CLASS( 6 )= 57
MASS UNDER-SIZE IN CLASS( 7 )= .6517985
NO. OF CRYSTALS IN CLASS( 7 )= 33
MASS UNDER-SIZE IN CLASS( 8 )= .735014
NO. OF CRYSTALS IN CLASS( 8 )= 14
MASS UNDER-SIZE IN CLASS( 9 )= .7967467
NO. OF CRYSTALS IN CLASS( 9 )= 7
MASS UNDER-SIZE IN CLASS(10 )= .882637
NO. OF CRYSTALS IN CLASS(10 )= 5
MASS UNDER-SIZE IN CLASS(11 )= 1
NO. OF CRYSTALS IN CLASS(11 )= 4
MASS UNDER-SIZE IN CLASS(12 )= 1
NO. OF CRYSTALS IN CLASS(12 )= 0
MASS UNDER-SIZE IN CLASS(13 )= 1
NO. OF CRYSTALS IN CLASS(13 )= 0
MASS UNDER-SIZE IN CLASS (14) = 1
NO. OF CRYSTALS IN CLASS (14) = 0
MASS UNDER-SIZE IN CLASS (15) = 1
NO. OF CRYSTALS IN CLASS (15) = 0
MASS UNDER-SIZE IN CLASS (16) = 1
NO. OF CRYSTALS IN CLASS (16) = 0

time = 160.8333333333333 min
Total number of crystals = 16546
MASS UNDER-SIZE IN CLASS (0) = 2.485323E-02
NO. OF CRYSTALS IN CLASS (0) = 14098
MASS UNDER-SIZE IN CLASS (1) = .136915
NO. OF CRYSTALS IN CLASS (1) = 1775
MASS UNDER-SIZE IN CLASS (2) = .1982689
NO. OF CRYSTALS IN CLASS (2) = 234
MASS UNDER-SIZE IN CLASS (3) = .2586189
NO. OF CRYSTALS IN CLASS (3) = 136
MASS UNDER-SIZE IN CLASS (4) = .3448039
NO. OF CRYSTALS IN CLASS (4) = 116
MASS UNDER-SIZE IN CLASS (5) = .4366216
NO. OF CRYSTALS IN CLASS (5) = 73
MASS UNDER-SIZE IN CLASS (6) = .5448447
NO. OF CRYSTALS IN CLASS (6) = 50
MASS UNDER-SIZE IN CLASS (7) = .6825542
NO. OF CRYSTALS IN CLASS (7) = 36
MASS UNDER-SIZE IN CLASS (8) = .7460194
NO. OF CRYSTALS IN CLASS (8) = 11
MASS UNDER-SIZE IN CLASS (9) = .853159
NO. OF CRYSTALS IN CLASS (9) = 11
MASS UNDER-SIZE IN CLASS (10) = .8796564
NO. OF CRYSTALS IN CLASS (10) = 2
MASS UNDER-SIZE IN CLASS (11) = .960243
NO. OF CRYSTALS IN CLASS (11) = 3
MASS UNDER-SIZE IN CLASS (12) = .9999999
NO. OF CRYSTALS IN CLASS (12) = 1
MASS UNDER-SIZE IN CLASS (13) = .9999999
NO. OF CRYSTALS IN CLASS (13) = 0
MASS UNDER-SIZE IN CLASS (14) = .9999999
NO. OF CRYSTALS IN CLASS (14) = 0
MASS UNDER-SIZE IN CLASS (15) = .9999999
NO. OF CRYSTALS IN CLASS (15) = 0
MASS UNDER-SIZE IN CLASS (16) = .9999999
NO. OF CRYSTALS IN CLASS (16) = 0

-----------------------------------------------------------------------
steady state-----------------------------------------------------------------------

time = 168.875 min
Total number of crystals = 16595
MASS UNDER-SIZE IN CLASS (0) = 2.484041E-02
NO. OF CRYSTALS IN CLASS (0) = 14205
MASS UNDER-SIZE IN CLASS (1) = .1341636
NO. OF CRYSTALS IN CLASS (1) = 1752
MASS UNDER-SIZE IN CLASS (2) = .1860862
NO. OF CRYSTALS IN CLASS (2) = 207
MASS UNDER-SIZE IN CLASS (3) = .2406228
NO. OF CRYSTALS IN CLASS (3) = 128
MASS UNDER-SIZE IN CLASS (4) = .3329415
NO. OF CRYSTALS IN CLASS( 4 )= 130  
MASS UNDER-SIZE IN CLASS( 5 )= .4034149  
NO. OF CRYSTALS IN CLASS( 5 )= 59  
MASS UNDER-SIZE IN CLASS( 6 )= .5218537  
NO. OF CRYSTALS IN CLASS( 6 )= 57  
MASS UNDER-SIZE IN CLASS( 7 )= .5977187  
NO. OF CRYSTALS IN CLASS( 7 )= 21  
MASS UNDER-SIZE IN CLASS( 8 )= .7141598  
NO. OF CRYSTALS IN CLASS( 8 )= 19  
MASS UNDER-SIZE IN CLASS( 9 )= .8020496  
NO. OF CRYSTALS IN CLASS( 9 )= 9  
MASS UNDER-SIZE IN CLASS( 10 )= .8831388  
NO. OF CRYSTALS IN CLASS( 10 )= 5  
MASS UNDER-SIZE IN CLASS( 11 )= .9477104  
NO. OF CRYSTALS IN CLASS( 11 )= 2  
MASS UNDER-SIZE IN CLASS( 12 )= 1  
NO. OF CRYSTALS IN CLASS( 12 )= 1  
MASS UNDER-SIZE IN CLASS( 13 )= 1  
NO. OF CRYSTALS IN CLASS( 13 )= 0  
MASS UNDER-SIZE IN CLASS( 14 )= 1  
NO. OF CRYSTALS IN CLASS( 14 )= 0  
MASS UNDER-SIZE IN CLASS( 15 )= 1  
NO. OF CRYSTALS IN CLASS( 15 )= 0  
MASS UNDER-SIZE IN CLASS( 16 )= 1  
NO. OF CRYSTALS IN CLASS( 16 )= 0

time=176.916666666667min  
Total number of crystals=16532  
MASS UNDER-SIZE IN CLASS( 0 )= 2.267442E-02  
NO. OF CRYSTALS IN CLASS( 0 )= 14121  
MASS UNDER-SIZE IN CLASS( 1 )= .1263279  
NO. OF CRYSTALS IN CLASS( 1 )= 1772  
MASS UNDER-SIZE IN CLASS( 2 )= .1795191  
NO. OF CRYSTALS IN CLASS( 2 )= 226  
MASS UNDER-SIZE IN CLASS( 3 )= .2279695  
NO. OF CRYSTALS IN CLASS( 3 )= 123  
MASS UNDER-SIZE IN CLASS( 4 )= .3045576  
NO. OF CRYSTALS IN CLASS( 4 )= 115  
MASS UNDER-SIZE IN CLASS( 5 )= .3742453  
NO. OF CRYSTALS IN CLASS( 5 )= 62  
MASS UNDER-SIZE IN CLASS( 6 )= .4628326  
NO. OF CRYSTALS IN CLASS( 6 )= 48  
MASS UNDER-SIZE IN CLASS( 7 )= .5641868  
NO. OF CRYSTALS IN CLASS( 7 )= 30  
MASS UNDER-SIZE IN CLASS( 8 )= .6210552  
NO. OF CRYSTALS IN CLASS( 8 )= 39  
MASS UNDER-SIZE IN CLASS( 9 )= .7789322  
NO. OF CRYSTALS IN CLASS( 9 )= 17  
MASS UNDER-SIZE IN CLASS( 10 )= .8452032  
NO. OF CRYSTALS IN CLASS( 10 )= 4  
MASS UNDER-SIZE IN CLASS( 11 )= .8929779  
NO. OF CRYSTALS IN CLASS( 11 )= 2  
MASS UNDER-SIZE IN CLASS( 12 )= .9338817  
NO. OF CRYSTALS IN CLASS( 12 )= 1  
MASS UNDER-SIZE IN CLASS( 13 )= .9999999  
NO. OF CRYSTALS IN CLASS( 13 )= 1

230
MASS UNDER-SIZE IN CLASS( 14 )=.9999999
NO. OF CRYSTALS IN CLASS( 14 )=0
MASS UNDER-SIZE IN CLASS( 15 )=.9999999
NO. OF CRYSTALS IN CLASS( 15 )=0
MASS UNDER-SIZE IN CLASS( 16 )=.9999999
NO. OF CRYSTALS IN CLASS( 16 )=0

time=193min
Total number of crystals=16730
MASS UNDER-SIZE IN CLASS( 0 )=.022207
NO. OF CRYSTALS IN CLASS( 0 )=14320
MASS UNDER-SIZE IN CLASS( 1 )=.1178303
NO. OF CRYSTALS IN CLASS( 1 )=1758
MASS UNDER-SIZE IN CLASS( 2 )=.1651853
NO. OF CRYSTALS IN CLASS( 2 )=215
MASS UNDER-SIZE IN CLASS( 3 )=.221174
NO. OF CRYSTALS IN CLASS( 3 )=148
MASS UNDER-SIZE IN CLASS( 4 )=.2998221
NO. OF CRYSTALS IN CLASS( 4 )=125
MASS UNDER-SIZE IN CLASS( 5 )=.3530285
NO. OF CRYSTALS IN CLASS( 5 )=50
MASS UNDER-SIZE IN CLASS( 6 )=.4655669
NO. OF CRYSTALS IN CLASS( 6 )=60
MASS UNDER-SIZE IN CLASS( 7 )=.5472362
NO. OF CRYSTALS IN CLASS( 7 )=24
MASS UNDER-SIZE IN CLASS( 8 )=.5923142
NO. OF CRYSTALS IN CLASS( 8 )=9
MASS UNDER-SIZE IN CLASS( 9 )=.6790065
NO. OF CRYSTALS IN CLASS( 9 )=11
MASS UNDER-SIZE IN CLASS( 10 )=.7668558
NO. OF CRYSTALS IN CLASS( 10 )=6
MASS UNDER-SIZE IN CLASS( 11 )=.7890013
NO. OF CRYSTALS IN CLASS( 11 )=1
MASS UNDER-SIZE IN CLASS( 12 )=.8221673
NO. OF CRYSTALS IN CLASS( 12 )=1
MASS UNDER-SIZE IN CLASS( 13 )=.8917708
NO. OF CRYSTALS IN CLASS( 13 )=1
MASS UNDER-SIZE IN CLASS( 14 )=1
NO. OF CRYSTALS IN CLASS( 14 )=1
MASS UNDER-SIZE IN CLASS( 15 )=0
NO. OF CRYSTALS IN CLASS( 15 )=0
MASS UNDER-SIZE IN CLASS( 16 )=1
NO. OF CRYSTALS IN CLASS( 16 )=0

Detailed mass distribution:

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<td>215</td>
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<tr>
<td>0.221174</td>
<td>148</td>
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<tr>
<td>0.2998221</td>
<td>125</td>
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<tr>
<td>0.3530285</td>
<td>50</td>
</tr>
<tr>
<td>0.4655669</td>
<td>60</td>
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<tr>
<td>0.5472362</td>
<td>24</td>
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<tr>
<td>0.5923142</td>
<td>9</td>
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<td>0.6790065</td>
<td>11</td>
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<td>6</td>
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<td>1</td>
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<tr>
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<td>1</td>
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<tr>
<td>0.8917708</td>
<td>1</td>
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</table>

Total number of crystals=16647
MASS UNDER-SIZE IN CLASS( 0 )=2.091017E-02
NO. OF CRYSTALS IN CLASS( 0 )=14206
MASS UNDER-SIZE IN CLASS( 1 )=.1127919
NO. OF CRYSTALS IN CLASS( 1 )=1797
MASS UNDER-SIZE IN CLASS( 2 )=.1585201
NO. OF CRYSTALS IN CLASS( 2 )=217
MASS UNDER-SIZE IN CLASS( 3 )=.2052197
NO. OF CRYSTALS IN CLASS( 3 )=131
MASS UNDER-SIZE IN CLASS( 4 )=.2774667
NO. OF CRYSTALS IN CLASS( 4 )=120

231
MASS UNDER-SIZE IN CLASS(5) = 0.3401705
NO. OF CRYSTALS IN CLASS(5) = 62
MASS UNDER-SIZE IN CLASS(6) = 0.4493751
NO. OF CRYSTALS IN CLASS(6) = 65
MASS UNDER-SIZE IN CLASS(7) = 0.5003447
NO. OF CRYSTALS IN CLASS(7) = 16
MASS UNDER-SIZE IN CLASS(8) = 0.5619428
NO. OF CRYSTALS IN CLASS(8) = 13
MASS UNDER-SIZE IN CLASS(9) = 0.639773
NO. OF CRYSTALS IN CLASS(9) = 10
MASS UNDER-SIZE IN CLASS(10) = 0.7343737
NO. OF CRYSTALS IN CLASS(10) = 7
MASS UNDER-SIZE IN CLASS(11) = 0.7343737
NO. OF CRYSTALS IN CLASS(11) = 0
MASS UNDER-SIZE IN CLASS(12) = 0.7737661
NO. OF CRYSTALS IN CLASS(12) = 1
MASS UNDER-SIZE IN CLASS(13) = 0.8307002
NO. OF CRYSTALS IN CLASS(13) = 1
MASS UNDER-SIZE IN CLASS(14) = 0.8307002
NO. OF CRYSTALS IN CLASS(14) = 0
MASS UNDER-SIZE IN CLASS(15) = 1
NO. OF CRYSTALS IN CLASS(15) = 1
MASS UNDER-SIZE IN CLASS(16) = 1
NO. OF CRYSTALS IN CLASS(16) = 0
APPENDIX III
Appendix III.1
Outline of Simulation Algorithm for Transient CSD in DTB and FC Crystallizers

START

READ TAU, G, NRATE [N], [R]

TCOUNT ← 1 MIN

NCOUNT ← 1

{CALCULATE N SUMMATION OF RANDOM NUMBER
[CALCULATE TM, TP]
GENERATE RANDOM NUMBER AND
CALCULATE TI

T ← TCOUNT
TTOT ← TCOUNT + TI

YES

TTOT > TSCAN

TTOT ← TSCAN

NO

COMPUTE D(t), W(t)

FIND THE SIZE CLASS
OF THE CRYSTAL

CRYSN0(T,SIZE) ← CRYSN0(T,SIZE) + 1
CRYSWT(T,SIZE) ← CRYSWT(T,SIZE) + W(t)

YES

T ← T + 1 MIN

NO

IS

T < TTOT

NO

IS

NCOUNT < NRATE

NO

IS

TCOUNT < TSCAN

NO

PRINT CRYSN0(T,SIZE), CRYSWT(T,SIZE)
FOR ALL SIZE CLASSES AT SELECTED
TIME INTERVAL

STOP

[] apply only to DTB crystallizer
[ ] apply only to FC crystallizer
Appendix III.2

Sample Program and Results for Transient CSD in DTB Crystallizer

Declaration

Dim wtfr(0 To 5) As Single
Dim cmwtfr(0 To 5), Ben(0 To 5) As Single
Dim wet(0 To 5) As Single
Dim cr(1 To 1800, 0 To 5) As Integer
Dim wt(1 To 1800, 0 To 5) As Single
Dim tau, G, Tscan, t1, t2, nt, Num, size As Single
Dim top, bottom, count, A, rms As Single
Dim cryswt As Single

Main Program

Sub main()

nstage = Val(stage.Text1.Text)
tau = Val(stage.Text2.Text) * 60
G = Val(stage.Text3.Text) / 60
Num = Val(stage.Text4.Text)

Open "sim.dtp" For Output As #1
Print #1, "Results of transient MC Simulation"
Print #1, "Number of stage =" & nstage
Print #1, "Residence time in an imperfectly mixed crystallizer =" & tau / 60;
" hr"
Print #1, "Average growth rate of a crystal = " & G * 60; " mm/hr"
Print #1, "Compared with Run No. " & Num; " Bennett and van Buren"

nt = 60
Print #1, "Nucleation rate =" & nt; " nuclei/min"

Ben(0) = 100
Ben(1) = 99
Ben(2) = 96
Ben(3) = 83
Ben(4) = 54
Ben(5) = 17
Tscan = 13 * tau
For t1 = 1 To Tscan
    For n = 1 To nstage
        If nstage = 1 Then
            t2 = -tau * Log(Rnd)
        Else
            For st = 1 To nstage
                sumrnd = sumrnd + Log(Rnd)
            Next
            t2 = -(tau / nstage) * sumrnd
        End If
    End For
    ttot = t1 + t2
    If ttot > Tscan Then ttot = Tscan
End For
    For times = t1 To ttot Step 1
        size = G * (times - t1)
        If size > .208 Then
            GoTo 260
        Else
            s = 0
            GoTo 310
        End If
    Next
End For
260 If size > .295 Then
    GoTo 270
Else
    s = 1
    GoTo 310
End If
270 If size > .417 Then
    GoTo 280
Else
    s = 2
    GoTo 310
End If
280 If size > .589 Then
    GoTo 290
Else
    s = 3
    GoTo 310
End If
290 If size < .833 Then
    s = 4
Else
    s = 5
End If
310 cr(times, s) = cr(times, s) + 1
wt(times, s) = wt(times, s) + (size ^ 3)

235
Next
n5 = n5 + 1
Debug.Print n5
sumrd = 0
Next
Next
For times = 1 To Tscan Step 30
For j = 0 To 5
    wet(j) = wt(times, j)
    cryswt = cryswt + wet(j)
Next
If cryswt = 0 Then
    GoTo 4
Else
    wtfr(5) = (wet(5) / cryswt) * 100
    cmwtfr(5) = wtfr(5)
    For j = 4 To 0 Step -1
        wtfr(j) = (wet(j) / cryswt) * 100
        cmwtfr(j) = cmwtfr(j + 1) + wtfr(j)
    Next
    For j = 0 To 5
        top = top + (cmwtfr(j) - Ben(j)) ^ 2
        bottom = bottom + (cmwtfr(j) - Ben(j))
        count = count + 1
    Next
A = top / bottom ^ 2
rms = (top / count) ^ (1 / 2)
Print #1, "time =" & times / 60; " hour"
Print #1, " No of crystals in tyler mesh 65 = " & cr(times, 0)
Print #1, " No of crystals in tyler mesh 48 = " & cr(times, 1)
Print #1, " No of crystals in tyler mesh 35 = " & cr(times, 2)
Print #1, " No of crystals in tyler mesh 28 = " & cr(times, 3)
Print #1, " No of crystals in tyler mesh 20 = " & cr(times, 4)
Print #1, " No of crystals in tyler mesh 14 = " & cr(times, 5)
Print #1, " cmwtfr in tyler mesh 65 = " & cmwtfr(0)
Print #1, " cmwtfr in tyler mesh 48 = " & cmwtfr(1)
Print #1, " cmwtfr in tyler mesh 35 = " & cmwtfr(2)
Print #1, " cmwtfr in tyler mesh 28 = " & cmwtfr(3)
Print #1, " cmwtfr in tyler mesh 20 = " & cmwtfr(4)
Print #1, " cmwtfr in tyler mesh 14 = " & cmwtfr(5)
Print #1, "Sandler A-test = " & A
Print #1, "rms = " & rms
Print #1,
cryswt = 0
End If
4 top = 0: bottom = 0: count = 0
236
Next

End Sub

Sample of results (for run no. 113 with N=1)

Results of transient MC Simulation
Number of stage = 1
Residence time in an imperfectly mixed crystallizer = 5.9 hr
Average growth rate of a crystal = .0485 mm/hr
Compared with Run No. 113 Bennett and van Buren
Nucleation rate = 60 nuclei/min

time = 1.01666666666667 hour
No of crystals in tyler mesh 65 = 3364
No of crystals in tyler mesh 48 = 0
No of crystals in tyler mesh 35 = 0
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 0
cmwtfr in tyler mesh 35 = 0
cmwtfr in tyler mesh 28 = 0
cmwtfr in tyler mesh 20 = 0
cmwtfr in tyler mesh 14 = 0
Sandler A-test = .218843443119748
rms = 70.56764

time = 2.01666666666667 hour
No of crystals in tyler mesh 65 = 6066
No of crystals in tyler mesh 48 = 0
No of crystals in tyler mesh 35 = 0
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 0
cmwtfr in tyler mesh 35 = 0
cmwtfr in tyler mesh 28 = 0
cmwtfr in tyler mesh 20 = 0
cmwtfr in tyler mesh 14 = 0
Sandler A-test = .218843443119748
rms = 70.56764

time = 3.01666666666667 hour
No of crystals in tyler mesh 65 = 8395
No of crystals in tyler mesh 48 = 0
No of crystals in tyler mesh 35 = 0
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 100

237
cmwtrf in tyler mesh 48 = 0
cmwtrf in tyler mesh 35 = 0
cmwtrf in tyler mesh 28 = 0
cmwtrf in tyler mesh 20 = 0
cmwtrf in tyler mesh 14 = 0
Sandler A-test = .218843443119748
rms = 70.56764

time = 4.01666666666667 hour
No of crystals in tyler mesh 65 = 10322
No of crystals in tyler mesh 48 = 0
No of crystals in tyler mesh 35 = 0
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
No of crystals in tyler mesh 14 = 0
cmwtrf in tyler mesh 65 = 100
cmwtrf in tyler mesh 48 = 0
cmwtrf in tyler mesh 35 = 0
cmwtrf in tyler mesh 28 = 0
cmwtrf in tyler mesh 20 = 0
cmwtrf in tyler mesh 14 = 0
Sandler A-test = .218843443119748
rms = 70.56764

time = 5.01666666666667 hour
No of crystals in tyler mesh 65 = 10763
No of crystals in tyler mesh 48 = 1133
No of crystals in tyler mesh 35 = 0
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
No of crystals in tyler mesh 14 = 0
cmwtrf in tyler mesh 65 = 100
cmwtrf in tyler mesh 48 = 40.37654
cmwtrf in tyler mesh 35 = 0
cmwtrf in tyler mesh 28 = 0
cmwtrf in tyler mesh 20 = 0
cmwtrf in tyler mesh 14 = 0
Sandler A-test = .217451498342597
rms = 62.65625

time = 6.01666666666667 hour
No of crystals in tyler mesh 65 = 10788
No of crystals in tyler mesh 48 = 2528
No of crystals in tyler mesh 35 = 0
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
No of crystals in tyler mesh 14 = 0
cmwtrf in tyler mesh 65 = 100
cmwtrf in tyler mesh 48 = 67.1923
cmwtrf in tyler mesh 35 = 0
cmwtrf in tyler mesh 28 = 0
cmwtrf in tyler mesh 20 = 0
cmwtrf in tyler mesh 14 = 0
Sandler A-test = .231498895731626
rms = 59.38107
time = 7.01666666666667 hour
No of crystals in tyler mesh 65 = 10848
No of crystals in tyler mesh 48 = 2655
No of crystals in tyler mesh 35 = 1056
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 99.999999
cmwtfr in tyler mesh 48 = 79.666666
cmwtfr in tyler mesh 35 = 35.23663
cmwtfr in tyler mesh 28 = 0
cmwtfr in tyler mesh 20 = 0
cmwtfr in tyler mesh 14 = 0
Sandler A-test = .235331047807496
rms = 50.42162


time = 8.01666666666667 hour
No of crystals in tyler mesh 65 = 10769
No of crystals in tyler mesh 48 = 2683
No of crystals in tyler mesh 35 = 2084
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 86.82887
cmwtfr in tyler mesh 35 = 57.25718
cmwtfr in tyler mesh 28 = 0
cmwtfr in tyler mesh 20 = 0
cmwtfr in tyler mesh 14 = 0
Sandler A-test = .255818766153271
rms = 46.54482


time = 9.01666666666667 hour
No of crystals in tyler mesh 65 = 10932
No of crystals in tyler mesh 48 = 2561
No of crystals in tyler mesh 35 = 2654
No of crystals in tyler mesh 28 = 331
No of crystals in tyler mesh 20 = 0
No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 90.4527
cmwtfr in tyler mesh 35 = 70.35413
cmwtfr in tyler mesh 28 = 12.62663
cmwtfr in tyler mesh 20 = 0
cmwtfr in tyler mesh 14 = 0
Sandler A-test = .267987329418565
rms = 41.43668


time = 10.01666666666667 hour
No of crystals in tyler mesh 65 = 10928
No of crystals in tyler mesh 48 = 2601
No of crystals in tyler mesh 35 = 2645
No of crystals in tyler mesh 28 = 1076
No of crystals in tyler mesh 20 = 0
No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 92.96579
cmwtfr in tyler mesh 35 = 78.1534
cmwtfr in tyler mesh 28 = 35.44728
cmwtfr in tyler mesh 20 = 0
cmwtfr in tyler mesh 14 = 0
Sandler A-test = .278980942963248
rms = 35.13356

time =11.0166666666667 hour
No of crystals in tyler mesh 65 = 10816
No of crystals in tyler mesh 48 = 2697
No of crystals in tyler mesh 35 = 2540
No of crystals in tyler mesh 28 = 1683
No of crystals in tyler mesh 20 = 0
No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 94.61479
cmwtfr in tyler mesh 35 = 82.45538
cmwtfr in tyler mesh 28 = 50.3029
cmwtfr in tyler mesh 20 = 0
cmwtfr in tyler mesh 14 = 0
Sandler A-test = .305338334079631
rms = 32.06206

time =12.0166666666667 hour
No of crystals in tyler mesh 65 = 10839
No of crystals in tyler mesh 48 = 2605
No of crystals in tyler mesh 35 = 2593
No of crystals in tyler mesh 28 = 2174
No of crystals in tyler mesh 20 = 0
No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 95.61815
cmwtfr in tyler mesh 35 = 86.2231
cmwtfr in tyler mesh 28 = 60.15929
cmwtfr in tyler mesh 20 = 0
cmwtfr in tyler mesh 14 = 0
Sandler A-test = .344574898059108
rms = 30.55446

time =13.0166666666667 hour
No of crystals in tyler mesh 65 = 10699
No of crystals in tyler mesh 48 = 2586
No of crystals in tyler mesh 35 = 2630
No of crystals in tyler mesh 28 = 2224
No of crystals in tyler mesh 20 = 349
No of crystals in tyler mesh 14 = 0
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 96.38834
cmwtfr in tyler mesh 35 = 88.7432
cmwtfr in tyler mesh 28 = 66.92623
cmwtfr in tyler mesh 20 = 14.66681
cmwtfr in tyler mesh 14 = 0

240
Sandler A-test = .35400225351712
rms = 24.96414

time =14.0166666666667 hour
No of crystals in tyler mesh 65 = 10740
No of crystals in tyler mesh 48 = 2638
No of crystals in tyler mesh 35 = 2633
No of crystals in tyler mesh 28 = 2162
No of crystals in tyler mesh 20 = 731
No of crystals in tyler mesh 14 = 0
cmwtrfr in tyler mesh 65 = 100
cmwtrfr in tyler mesh 48 = 97.00966
cmwtrfr in tyler mesh 35 = 90.35435
cmwtrfr in tyler mesh 28 = 71.62055
cmwtrfr in tyler mesh 20 = 29.12073
cmwtrfr in tyler mesh 14 = 0
Sandler A-test = .371725209350622
rms = 20.25963

time =15.0166666666667 hour
No of crystals in tyler mesh 65 = 10787
No of crystals in tyler mesh 48 = 2612
No of crystals in tyler mesh 35 = 2606
No of crystals in tyler mesh 28 = 2223
No of crystals in tyler mesh 20 = 1012
No of crystals in tyler mesh 14 = 0
cmwtrfr in tyler mesh 65 = 99.999999
cmwtrfr in tyler mesh 48 = 97.45329
cmwtrfr in tyler mesh 35 = 91.82536
cmwtrfr in tyler mesh 28 = 75.98595
cmwtrfr in tyler mesh 20 = 38.55569
cmwtrfr in tyler mesh 14 = 0
Sandler A-test = .432924968501883
rms = 17.64258

time =16.0166666666667 hour
No of crystals in tyler mesh 65 = 10846
No of crystals in tyler mesh 48 = 2561
No of crystals in tyler mesh 35 = 2645
No of crystals in tyler mesh 28 = 2199
No of crystals in tyler mesh 20 = 1293
No of crystals in tyler mesh 14 = 0
cmwtrfr in tyler mesh 65 = 100
cmwtrfr in tyler mesh 48 = 97.75908
cmwtrfr in tyler mesh 35 = 92.94943
cmwtrfr in tyler mesh 28 = 78.99797
cmwtrfr in tyler mesh 20 = 46.8991
cmwtrfr in tyler mesh 14 = 0
Sandler A-test = .545798245116729
rms = 15.95329

time =17.0166666666667 hour
No of crystals in tyler mesh 65 = 10872
No of crystals in tyler mesh 48 = 2554
No of crystals in tyler mesh 35 = 2587
cmwtrf in tyler mesh 20 = 64.92086
cmwtrf in tyler mesh 14 = 23.34954
Sandler A-test = 173.401057506013
rms = 5.932586

time = 21.016666666667 hour
No of crystals in tyler mesh 65 = 10963
No of crystals in tyler mesh 48 = 2625
No of crystals in tyler mesh 35 = 2621
No of crystals in tyler mesh 28 = 2130
No of crystals in tyler mesh 20 = 1587
No of crystals in tyler mesh 14 = 540
cmwtrf in tyler mesh 65 = 100
cmwtrf in tyler mesh 48 = 98.60596
cmwtrf in tyler mesh 35 = 95.61789
cmwtrf in tyler mesh 28 = 87.32194
cmwtrf in tyler mesh 20 = 68.40643
cmwtrf in tyler mesh 14 = 29.76304
Sandler A-test = 1.30533295248916
rms = 4.764687

-----------------Closest to experimental result-------------------

time = 21.516666666667 hour
No of crystals in tyler mesh 65 = 10996
No of crystals in tyler mesh 48 = 2604
No of crystals in tyler mesh 35 = 2619
No of crystals in tyler mesh 28 = 2146
No of crystals in tyler mesh 20 = 1577
No of crystals in tyler mesh 14 = 587
cmwtrf in tyler mesh 65 = 99.999999
cmwtrf in tyler mesh 48 = 98.67246
cmwtrf in tyler mesh 35 = 95.85206
cmwtrf in tyler mesh 28 = 87.92413
cmwtrf in tyler mesh 20 = 69.55965
cmwtrf in tyler mesh 14 = 32.1621
Sandler A-test = .642417705094717
rms = 4.800373


time = 22.016666666667 hour
No of crystals in tyler mesh 65 = 10921
No of crystals in tyler mesh 48 = 2637
No of crystals in tyler mesh 35 = 2590
No of crystals in tyler mesh 28 = 2120
No of crystals in tyler mesh 20 = 1563
No of crystals in tyler mesh 14 = 639
cmwtrf in tyler mesh 65 = 99.999999
cmwtrf in tyler mesh 48 = 98.71183
cmwtrf in tyler mesh 35 = 95.92284
cmwtrf in tyler mesh 28 = 88.24663
cmwtrf in tyler mesh 20 = 70.72864
cmwtrf in tyler mesh 14 = 34.91081
Sandler A-test = .412247185801762
rms = 4.985752


time = 23.016666666667 hour
No of crystals in tyler mesh 65 = 10993

243
No of crystals in tyler mesh 48 = 2618
No of crystals in tyler mesh 35 = 2584
No of crystals in tyler mesh 28 = 2102
No of crystals in tyler mesh 20 = 1565
No of crystals in tyler mesh 14 = 715
cmwtrf in tyler mesh 65 = 100
cmwtrf in tyler mesh 48 = 98.77679
cmwtrf in tyler mesh 35 = 96.16421
cmwtrf in tyler mesh 28 = 88.92896
cmwtrf in tyler mesh 20 = 72.74474
cmwtrf in tyler mesh 14 = 38.91863
Sandler A-test = .307669609109685
rms = 5.895172

time = 24.0166666666667 hour
No of crystals in tyler mesh 65 = 10900
No of crystals in tyler mesh 48 = 2636
No of crystals in tyler mesh 35 = 2595
No of crystals in tyler mesh 28 = 2161
No of crystals in tyler mesh 20 = 1515
No of crystals in tyler mesh 14 = 762
cmwtrf in tyler mesh 65 = 100
cmwtrf in tyler mesh 48 = 98.82693
cmwtrf in tyler mesh 35 = 96.31202
cmwtrf in tyler mesh 28 = 89.35643
cmwtrf in tyler mesh 20 = 73.49812
cmwtrf in tyler mesh 14 = 41.94675
Sandler A-test = .281289620196995
rms = 6.590969

time = 25.0166666666667 hour
No of crystals in tyler mesh 65 = 10932
No of crystals in tyler mesh 48 = 2662
No of crystals in tyler mesh 35 = 2567
No of crystals in tyler mesh 28 = 2213
No of crystals in tyler mesh 20 = 1474
No of crystals in tyler mesh 14 = 819
cmwtrf in tyler mesh 65 = 100
cmwtrf in tyler mesh 48 = 98.87888
cmwtrf in tyler mesh 35 = 96.46234
cmwtrf in tyler mesh 28 = 89.91338
cmwtrf in tyler mesh 20 = 74.24378
cmwtrf in tyler mesh 14 = 44.98783
Sandler A-test = .273804866718859
rms = 7.473805

time = 26.0166666666667 hour
No of crystals in tyler mesh 65 = 10930
No of crystals in tyler mesh 48 = 2635
No of crystals in tyler mesh 35 = 2577
No of crystals in tyler mesh 28 = 2310
No of crystals in tyler mesh 20 = 1455
No of crystals in tyler mesh 14 = 870
cmwtrf in tyler mesh 65 = 100
cmwtrf in tyler mesh 48 = 98.93386
cmwtrf in tyler mesh 35 = 96.64212
cmwtrf in tyler mesh 28 = 90.38899
cmwtrf in tyler mesh 20 = 74.55151
cmwtrf in tyler mesh 14 = 47.19962
Sandler A-test = .272589833023175
rms = 8.14565

time = 27.0166666666667 hour
No of crystals in tyler mesh 65 = 10934
No of crystals in tyler mesh 48 = 2654
No of crystals in tyler mesh 35 = 2577
No of crystals in tyler mesh 28 = 2245
No of crystals in tyler mesh 20 = 1554
No of crystals in tyler mesh 14 = 915
cmwtrf in tyler mesh 65 = 100
cmwtrf in tyler mesh 48 = 98.99896
cmwtrf in tyler mesh 35 = 96.83035
cmwtrf in tyler mesh 28 = 90.94272
cmwtrf in tyler mesh 20 = 76.51506
cmwtrf in tyler mesh 14 = 49.32563
Sandler A-test = .277779432968805
rms = 9.276408

time = 28.0166666666667 hour
No of crystals in tyler mesh 65 = 10972
No of crystals in tyler mesh 48 = 2684
No of crystals in tyler mesh 35 = 2555
No of crystals in tyler mesh 28 = 2226
No of crystals in tyler mesh 20 = 1571
No of crystals in tyler mesh 14 = 968
cmwtrf in tyler mesh 65 = 100
cmwtrf in tyler mesh 48 = 99.03942
cmwtrf in tyler mesh 35 = 96.94078
cmwtrf in tyler mesh 28 = 91.36646
cmwtrf in tyler mesh 20 = 77.73923
cmwtrf in tyler mesh 14 = 51.80339
Sandler A-test = .2830073689999352
rms = 10.29209

time = 29.0166666666667 hour
No of crystals in tyler mesh 65 = 10975
No of crystals in tyler mesh 48 = 2669
No of crystals in tyler mesh 35 = 2645
No of crystals in tyler mesh 28 = 2223
No of crystals in tyler mesh 20 = 1598
No of crystals in tyler mesh 14 = 999
cmwtrf in tyler mesh 65 = 99.99999
cmwtrf in tyler mesh 48 = 99.08818
cmwtrf in tyler mesh 35 = 97.11533
cmwtrf in tyler mesh 28 = 91.66582
cmwtrf in tyler mesh 20 = 78.76692
cmwtrf in tyler mesh 14 = 53.58736
Sandler A-test = .2863586662672757
rms = 11.08127
time = 30.0166666666667 hour
No of crystals in tyler mesh 65 = 10980
No of crystals in tyler mesh 48 = 2650
No of crystals in tyler mesh 35 = 2659
No of crystals in tyler mesh 28 = 2199
No of crystals in tyler mesh 20 = 1578
No of crystals in tyler mesh 14 = 1020
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.12492
cmwtfr in tyler mesh 35 = 97.23554
cmwtfr in tyler mesh 28 = 91.95921
cmwtfr in tyler mesh 20 = 79.73531
cmwtfr in tyler mesh 14 = 55.50277
Sandler A-test = .290540418941053
rms = 11.89558

time = 31.0166666666667 hour
No of crystals in tyler mesh 65 = 10974
No of crystals in tyler mesh 48 = 2610
No of crystals in tyler mesh 35 = 2622
No of crystals in tyler mesh 28 = 2189
No of crystals in tyler mesh 20 = 1594
No of crystals in tyler mesh 14 = 1019
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.1285
cmwtfr in tyler mesh 35 = 97.29533
cmwtfr in tyler mesh 28 = 92.15073
cmwtfr in tyler mesh 20 = 80.37537
cmwtfr in tyler mesh 14 = 56.15508
Sandler A-test = .292055495276171
rms = 12.26792

time = 32.0166666666667 hour
No of crystals in tyler mesh 65 = 10865
No of crystals in tyler mesh 48 = 2634
No of crystals in tyler mesh 35 = 2664
No of crystals in tyler mesh 28 = 2236
No of crystals in tyler mesh 20 = 1531
No of crystals in tyler mesh 14 = 1053
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.16982
cmwtfr in tyler mesh 35 = 97.39096
cmwtfr in tyler mesh 28 = 92.32141
cmwtfr in tyler mesh 20 = 80.48339
cmwtfr in tyler mesh 14 = 57.89549
Sandler A-test = .295282737578631
rms = 12.81382

time = 33.0166666666667 hour
No of crystals in tyler mesh 65 = 10855
No of crystals in tyler mesh 48 = 2718
No of crystals in tyler mesh 35 = 2626
No of crystals in tyler mesh 28 = 2237
No of crystals in tyler mesh 20 = 1546
No of crystals in tyler mesh 14 = 1049
cmwtrf in tyler mesh 65 = 100  
cmwtrf in tyler mesh 48 = 99.1958  
cmwtrf in tyler mesh 35 = 97.36531  
cmwtrf in tyler mesh 28 = 92.40554  
cmwtrf in tyler mesh 20 = 80.66828  
cmwtrf in tyler mesh 14 = 58.18551  
Sandler A-test = .295947630431022  
rms = 12.95247

time =34.0166666666667 hour  
No of crystals in tyler mesh 65 = 10876  
No of crystals in tyler mesh 48 = 2636  
No of crystals in tyler mesh 35 = 2635  
No of crystals in tyler mesh 28 = 2248  
No of crystals in tyler mesh 20 = 1517  
No of crystals in tyler mesh 14 = 1105  
cmwtrf in tyler mesh 65 = 99.99999  
cmwtrf in tyler mesh 48 = 99.21347  
cmwtrf in tyler mesh 35 = 97.49135  
cmwtrf in tyler mesh 28 = 92.68999  
cmwtrf in tyler mesh 20 = 81.35741  
cmwtrf in tyler mesh 14 = 60.17699  
Sandler A-test = .300315472150746  
rms = 13.74321

time =35.0166666666667 hour  
No of crystals in tyler mesh 65 = 10938  
No of crystals in tyler mesh 48 = 2561  
No of crystals in tyler mesh 35 = 2661  
No of crystals in tyler mesh 28 = 2254  
No of crystals in tyler mesh 20 = 1503  
No of crystals in tyler mesh 14 = 1103  
cmwtrf in tyler mesh 65 = 100  
cmwtrf in tyler mesh 48 = 99.20908  
cmwtrf in tyler mesh 35 = 97.57296  
cmwtrf in tyler mesh 28 = 92.81255  
cmwtrf in tyler mesh 20 = 81.55589  
cmwtrf in tyler mesh 14 = 60.82286  
Sandler A-test = .301426798210528  
rms = 14.00264

time =35.5166666666667 hour  
No of crystals in tyler mesh 65 = 10869  
No of crystals in tyler mesh 48 = 2644  
No of crystals in tyler mesh 35 = 2628  
No of crystals in tyler mesh 28 = 2241  
No of crystals in tyler mesh 20 = 1514  
No of crystals in tyler mesh 14 = 1115  
cmwtrf in tyler mesh 65 = 100  
cmwtrf in tyler mesh 48 = 99.23483  
cmwtrf in tyler mesh 35 = 97.58866  
cmwtrf in tyler mesh 28 = 92.91881  
cmwtrf in tyler mesh 20 = 81.80238  
cmwtrf in tyler mesh 14 = 61.15747  
Sandler A-test = .301837614028042
rms = 14.17565

time = 36.01666666666667 hour
No of crystals in tyler mesh 65 = 10923
No of crystals in tyler mesh 48 = 2653
No of crystals in tyler mesh 35 = 2581
No of crystals in tyler mesh 28 = 2275
No of crystals in tyler mesh 20 = 1521
No of crystals in tyler mesh 14 = 1113
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.22429
cmwtfr in tyler mesh 35 = 97.57133
cmwtfr in tyler mesh 28 = 93.03802
cmwtfr in tyler mesh 20 = 81.85957
cmwtfr in tyler mesh 14 = 61.4355
Sandler A-test = 0.302526398842713
rms = 14.28759


time = 37.01666666666667 hour
No of crystals in tyler mesh 65 = 10896
No of crystals in tyler mesh 48 = 2699
No of crystals in tyler mesh 35 = 2563
No of crystals in tyler mesh 28 = 2254
No of crystals in tyler mesh 20 = 1563
No of crystals in tyler mesh 14 = 1102
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.25972
cmwtfr in tyler mesh 35 = 97.59444
cmwtfr in tyler mesh 28 = 93.18676
cmwtfr in tyler mesh 20 = 82.33935
cmwtfr in tyler mesh 14 = 61.68658
Sandler A-test = 0.302473477023103
rms = 14.49697


time = 38.01666666666667 hour
No of crystals in tyler mesh 65 = 10948
No of crystals in tyler mesh 48 = 2628
No of crystals in tyler mesh 35 = 2603
No of crystals in tyler mesh 28 = 2214
No of crystals in tyler mesh 20 = 1611
No of crystals in tyler mesh 14 = 1105
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.26936
cmwtfr in tyler mesh 35 = 97.68858
cmwtfr in tyler mesh 28 = 93.31765
cmwtfr in tyler mesh 20 = 82.97797
cmwtfr in tyler mesh 14 = 62.15949
Sandler A-test = 0.302955588809474
rms = 14.81103


time = 39.01666666666667 hour
No of crystals in tyler mesh 65 = 10802
No of crystals in tyler mesh 48 = 2605
No of crystals in tyler mesh 35 = 2607
No of crystals in tyler mesh 28 = 2215
No of crystals in tyler mesh 20 = 1621
No of crystals in tyler mesh 14 = 1133
cmwtfr in tyler mesh 65 = 99.99999
cmwtfr in tyler mesh 48 = 99.29192
cmwtfr in tyler mesh 35 = 97.76755
cmwtfr in tyler mesh 28 = 93.48762
cmwtfr in tyler mesh 20 = 83.35524
cmwtfr in tyler mesh 14 = 62.76393
Sandler A-test = .303550028311803
rms = 15.10743

time =40.0166666666667 hour
No of crystals in tyler mesh 65 = 10805
No of crystals in tyler mesh 48 = 2602
No of crystals in tyler mesh 35 = 2547
No of crystals in tyler mesh 28 = 2201
No of crystals in tyler mesh 20 = 1604
No of crystals in tyler mesh 14 = 1156
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.2887
cmwtfr in tyler mesh 35 = 97.77763
cmwtfr in tyler mesh 28 = 93.60469
cmwtfr in tyler mesh 20 = 83.72418
cmwtfr in tyler mesh 14 = 63.60073
Sandler A-test = .305635732914936
rms = 15.45935

time =41.0166666666667 hour
No of crystals in tyler mesh 65 = 10755
No of crystals in tyler mesh 48 = 2633
No of crystals in tyler mesh 35 = 2543
No of crystals in tyler mesh 28 = 2234
No of crystals in tyler mesh 20 = 1565
No of crystals in tyler mesh 14 = 1184
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.31725
cmwtfr in tyler mesh 35 = 97.84011
cmwtfr in tyler mesh 28 = 93.83126
cmwtfr in tyler mesh 20 = 84.08232
cmwtfr in tyler mesh 14 = 64.98148
Sandler A-test = .308270888559233
rms = 15.99199

time =42.0166666666667 hour
No of crystals in tyler mesh 65 = 10691
No of crystals in tyler mesh 48 = 2649
No of crystals in tyler mesh 35 = 2556
No of crystals in tyler mesh 28 = 2200
No of crystals in tyler mesh 20 = 1551
No of crystals in tyler mesh 14 = 1208
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.33851
cmwtfr in tyler mesh 35 = 97.86464
cmwtfr in tyler mesh 28 = 93.88072
cmwtfr in tyler mesh 20 = 84.32304
cmwtrf in tyler mesh 14 = 65.71479
Sandler A-test = .30992519204701
rms = 16.27786

time =43.016666666667 hour
No of crystals in tyler mesh 65 = 10728
No of crystals in tyler mesh 48 = 2623
No of crystals in tyler mesh 35 = 2600
No of crystals in tyler mesh 28 = 2227
No of crystals in tyler mesh 20 = 1522
No of crystals in tyler mesh 14 = 1199
cmwtrf in tyler mesh 65 = 100
cmwtrf in tyler mesh 48 = 99.35044
cmwtrf in tyler mesh 35 = 97.89392
cmwtrf in tyler mesh 28 = 93.88297
cmwtrf in tyler mesh 20 = 84.20128
cmwtrf in tyler mesh 14 = 65.94925
Sandler A-test = .31031778206736
rms = 16.32368

time =44.016666666667 hour
No of crystals in tyler mesh 65 = 10941
No of crystals in tyler mesh 48 = 2556
No of crystals in tyler mesh 35 = 2607
No of crystals in tyler mesh 28 = 2181
No of crystals in tyler mesh 20 = 1532
No of crystals in tyler mesh 14 = 1203
cmwtrf in tyler mesh 65 = 100
cmwtrf in tyler mesh 48 = 99.33035
cmwtrf in tyler mesh 35 = 97.93145
cmwtrf in tyler mesh 28 = 93.94152
cmwtrf in tyler mesh 20 = 84.67615
cmwtrf in tyler mesh 14 = 66.61213
Sandler A-test = .311983075230852
rms = 16.64419

time =44.516666666667 hour
No of crystals in tyler mesh 65 = 10989
No of crystals in tyler mesh 48 = 2564
No of crystals in tyler mesh 35 = 2592
No of crystals in tyler mesh 28 = 2170
No of crystals in tyler mesh 20 = 1506
No of crystals in tyler mesh 14 = 1204
cmwtrf in tyler mesh 65 = 99.99999
cmwtrf in tyler mesh 48 = 99.32237
cmwtrf in tyler mesh 35 = 97.91112
cmwtrf in tyler mesh 28 = 93.89577
cmwtrf in tyler mesh 20 = 84.62498
cmwtrf in tyler mesh 14 = 66.86152
Sandler A-test = .313084638674993
rms = 16.70193

-----------------------------------steady state-----------------------------------

time =45.016666666667 hour
No of crystals in tyler mesh 65 = 11009
No of crystals in tyler mesh 48 = 2633
No of crystals in tyler mesh 35 = 2607
No of crystals in tyler mesh 28 = 2174
No of crystals in tyler mesh 20 = 1523
No of crystals in tyler mesh 14 = 1197
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.33537
cmwtfr in tyler mesh 35 = 97.90461
cmwtfr in tyler mesh 28 = 93.87723
cmwtfr in tyler mesh 20 = 84.6563

cmwtfr in tyler mesh 14 = 66.82814
Sandler A-test = .312984021821086
rms = 16.69601

\[ \text{time} = 46.0166666666667 \text{ hour} \]
No of crystals in tyler mesh 65 = 11028
No of crystals in tyler mesh 48 = 2608
No of crystals in tyler mesh 35 = 2548
No of crystals in tyler mesh 28 = 2241
No of crystals in tyler mesh 20 = 1497
No of crystals in tyler mesh 14 = 1162
cmwtfr in tyler mesh 65 = 99.99999

cmwtfr in tyler mesh 48 = 99.32229
cmwtfr in tyler mesh 35 = 97.87884
cmwtfr in tyler mesh 28 = 93.95437
cmwtfr in tyler mesh 20 = 84.28938

cmwtfr in tyler mesh 14 = 66.33758
Sandler A-test = .311557978223198
rms = 16.47125

\[ \text{time} = 47.0166666666667 \text{ hour} \]
No of crystals in tyler mesh 65 = 10966
No of crystals in tyler mesh 48 = 2623
No of crystals in tyler mesh 35 = 2547
No of crystals in tyler mesh 28 = 2182
No of crystals in tyler mesh 20 = 1525
No of crystals in tyler mesh 14 = 1159
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.31033
cmwtfr in tyler mesh 35 = 97.8652

cmwtfr in tyler mesh 28 = 93.92946
cmwtfr in tyler mesh 20 = 84.49644

cmwtfr in tyler mesh 14 = 66.42615
Sandler A-test = .312082695973558
rms = 16.54102

\[ \text{time} = 48.0166666666667 \text{ hour} \]
No of crystals in tyler mesh 65 = 10878
No of crystals in tyler mesh 48 = 2673
No of crystals in tyler mesh 35 = 2575
No of crystals in tyler mesh 28 = 2170
No of crystals in tyler mesh 20 = 1524
No of crystals in tyler mesh 14 = 1143
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.31684

cmwtfr in tyler mesh 35 = 97.84576

251
cmwtf in tyler mesh 28 = 93.84772
cmwtf in tyler mesh 20 = 84.36368
cmwtf in tyler mesh 14 = 66.01564
Sandler A-test = .311230930788754
rms = 16.37314

time = 49.0166666666667 hour
No of crystals in tyler mesh 65 = 10851
No of crystals in tyler mesh 48 = 2677
No of crystals in tyler mesh 35 = 2572
No of crystals in tyler mesh 28 = 2172
No of crystals in tyler mesh 20 = 1538
No of crystals in tyler mesh 14 = 1155
cmwtf in tyler mesh 65 = 100
cmwtf in tyler mesh 48 = 99.34068
cmwtf in tyler mesh 35 = 97.86553
cmwtf in tyler mesh 28 = 93.91071
cmwtf in tyler mesh 20 = 84.55933
cmwtf in tyler mesh 14 = 66.33419
Sandler A-test = .311620897234251
rms = 16.52488

time = 50.0166666666667 hour
No of crystals in tyler mesh 65 = 10957
No of crystals in tyler mesh 48 = 2642
No of crystals in tyler mesh 35 = 2604
No of crystals in tyler mesh 28 = 2171
No of crystals in tyler mesh 20 = 1553
No of crystals in tyler mesh 14 = 1134
cmwtf in tyler mesh 65 = 100
cmwtf in tyler mesh 48 = 99.32742
cmwtf in tyler mesh 35 = 97.83487
cmwtf in tyler mesh 28 = 93.78671
cmwtf in tyler mesh 20 = 84.40119
cmwtf in tyler mesh 14 = 65.83154
Sandler A-test = .310885896825434
rms = 16.31674

time = 51.0166666666667 hour
No of crystals in tyler mesh 65 = 10952
No of crystals in tyler mesh 48 = 2590
No of crystals in tyler mesh 35 = 2652
No of crystals in tyler mesh 28 = 2121
No of crystals in tyler mesh 20 = 1535
No of crystals in tyler mesh 14 = 1129
cmwtf in tyler mesh 65 = 99.99999

cmwtf in tyler mesh 48 = 99.32768

cmwtf in tyler mesh 35 = 97.9052

cmwtf in tyler mesh 28 = 93.77283
cmwtf in tyler mesh 20 = 84.63011
cmwtf in tyler mesh 14 = 66.17296
Sandler A-test = .311481348318927
rms = 16.47523

time = 52.0166666666667 hour
No of crystals in tyler mesh 65 = 10923
No of crystals in tyler mesh 48 = 2635
No of crystals in tyler mesh 35 = 2557
No of crystals in tyler mesh 28 = 2152
No of crystals in tyler mesh 20 = 1472
No of crystals in tyler mesh 14 = 1159
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.33311
cmwtfr in tyler mesh 35 = 97.87908
cmwtfr in tyler mesh 28 = 93.91793
cmwtfr in tyler mesh 20 = 84.79914
cmwtfr in tyler mesh 14 = 67.39142
Sandler A-test = .314688175776879
rms = 16.90607

time = 53.0166666666667 hour
No of crystals in tyler mesh 65 = 10988
No of crystals in tyler mesh 48 = 2607
No of crystals in tyler mesh 35 = 2577
No of crystals in tyler mesh 28 = 2210
No of crystals in tyler mesh 20 = 1475
No of crystals in tyler mesh 14 = 1155
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.3308
cmwtfr in tyler mesh 35 = 97.92008
cmwtfr in tyler mesh 28 = 94.03738
cmwtfr in tyler mesh 20 = 84.79341
cmwtfr in tyler mesh 14 = 67.44263
Sandler A-test = .314166773420531
rms = 16.93865

time = 54.0166666666667 hour
No of crystals in tyler mesh 65 = 10912
No of crystals in tyler mesh 48 = 2693
No of crystals in tyler mesh 35 = 2598
No of crystals in tyler mesh 28 = 2228
No of crystals in tyler mesh 20 = 1464
No of crystals in tyler mesh 14 = 1131
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.33791
cmwtfr in tyler mesh 35 = 97.88251
cmwtfr in tyler mesh 28 = 93.92976
cmwtfr in tyler mesh 20 = 84.47625
cmwtfr in tyler mesh 14 = 67.18937
Sandler A-test = .314032777684063
rms = 16.77295

time = 55.0166666666667 hour
No of crystals in tyler mesh 65 = 10860
No of crystals in tyler mesh 48 = 2706
No of crystals in tyler mesh 35 = 2634
No of crystals in tyler mesh 28 = 2227
No of crystals in tyler mesh 20 = 1497
No of crystals in tyler mesh 14 = 1131
cmwtfr in tyler mesh 65 = 99.99999
cmtwfr in tyler mesh 48 = 99.34525
cmtwfr in tyler mesh 35 = 97.8949

cmtwfr in tyler mesh 28 = 93.9264
cmtwfr in tyler mesh 20 = 84.57015

cmtwfr in tyler mesh 14 = 67.53493
Sandler A-test = .314934109786897
rms = 16.90144

Time = 56.01666666666667 hour
No of crystals in tyler mesh 65 = 10845
No of crystals in tyler mesh 48 = 2617
No of crystals in tyler mesh 35 = 2617
No of crystals in tyler mesh 28 = 2260
No of crystals in tyler mesh 20 = 1513
No of crystals in tyler mesh 14 = 1126
cmtwfr in tyler mesh 65 = 99.99999

cmtwfr in tyler mesh 48 = 99.3266

cmtwfr in tyler mesh 35 = 97.90559
cmtwfr in tyler mesh 28 = 93.95223
cmtwfr in tyler mesh 20 = 84.40979
cmtwfr in tyler mesh 14 = 66.96484
Sandler A-test = .313226610976668
rms = 16.69275

Time = 57.01666666666667 hour
No of crystals in tyler mesh 65 = 10764
No of crystals in tyler mesh 48 = 2699
No of crystals in tyler mesh 35 = 2586
No of crystals in tyler mesh 28 = 2224
No of crystals in tyler mesh 20 = 1569
No of crystals in tyler mesh 14 = 1135
cmtwfr in tyler mesh 65 = 100
cmtwfr in tyler mesh 48 = 99.3584
cmtwfr in tyler mesh 35 = 97.93477
cmtwfr in tyler mesh 28 = 94.07123
cmtwfr in tyler mesh 20 = 84.8345
cmtwfr in tyler mesh 14 = 67.03375
Sandler A-test = .312509684702367
rms = 16.82736

Time = 58.01666666666667 hour
No of crystals in tyler mesh 65 = 10887
No of crystals in tyler mesh 48 = 2666
No of crystals in tyler mesh 35 = 2608
No of crystals in tyler mesh 28 = 2217
No of crystals in tyler mesh 20 = 1580
No of crystals in tyler mesh 14 = 1110
cmtwfr in tyler mesh 65 = 99.99999

cmtwfr in tyler mesh 48 = 99.3609
cmtwfr in tyler mesh 35 = 97.92674
cmtwfr in tyler mesh 28 = 94.03017
cmtwfr in tyler mesh 20 = 84.79861
cmtwfr in tyler mesh 14 = 66.62698
Sandler A-test = .311494980242167
rms = 16.68854
time = 59.016666666667 hour
No of crystals in tyler mesh 65 = 11044
No of crystals in tyler mesh 48 = 2556
No of crystals in tyler mesh 35 = 2690
No of crystals in tyler mesh 28 = 2218
No of crystals in tyler mesh 20 = 1595
No of crystals in tyler mesh 14 = 1111
cmwtrfr in tyler mesh 65 = 99.999999
cmwtrfr in tyler mesh 48 = 99.35316
cmwtrfr in tyler mesh 35 = 97.99371
cmwtrfr in tyler mesh 28 = 94.02881
cmwtrfr in tyler mesh 20 = 84.87241
cmwtrfr in tyler mesh 14 = 66.52699
Sandler A-test = .310841569763063
rms = 16.67824

time = 60.016666666667 hour
No of crystals in tyler mesh 65 = 11131
No of crystals in tyler mesh 48 = 2572
No of crystals in tyler mesh 35 = 2622
No of crystals in tyler mesh 28 = 2199
No of crystals in tyler mesh 20 = 1559
No of crystals in tyler mesh 14 = 1126
cmwtrfr in tyler mesh 65 = 100
cmwtrfr in tyler mesh 48 = 99.33383
cmwtrfr in tyler mesh 35 = 97.97628
cmwtrfr in tyler mesh 28 = 94.0544
cmwtrfr in tyler mesh 20 = 84.97659
cmwtrfr in tyler mesh 14 = 67.02853
Sandler A-test = .312476957541544
rms = 16.85774

time = 61.016666666667 hour
No of crystals in tyler mesh 65 = 10984
No of crystals in tyler mesh 48 = 2730
No of crystals in tyler mesh 35 = 2534
No of crystals in tyler mesh 28 = 2226
No of crystals in tyler mesh 20 = 1530
No of crystals in tyler mesh 14 = 1145
cmwtrfr in tyler mesh 65 = 99.999999
cmwtrfr in tyler mesh 48 = 99.34772
cmwtrfr in tyler mesh 35 = 97.92094
cmwtrfr in tyler mesh 28 = 94.1347
cmwtrfr in tyler mesh 20 = 85.07236
cmwtrfr in tyler mesh 14 = 67.63218
Sandler A-test = .314199178577868
rms = 17.07307

time = 62.016666666667 hour
No of crystals in tyler mesh 65 = 10910
No of crystals in tyler mesh 48 = 2751
No of crystals in tyler mesh 35 = 2528
No of crystals in tyler mesh 28 = 2261
No of crystals in tyler mesh 20 = 1501
No of crystals in tyler mesh 14 = 1120
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.33391
cmwtfr in tyler mesh 35 = 97.84097
cmwtfr in tyler mesh 28 = 94.0424
cmwtfr in tyler mesh 20 = 84.57938
cmwtfr in tyler mesh 14 = 67.07618
Sandler A-test = .313578976711623
rms = 16.77387

time =63.0166666666667 hour
No of crystals in tyler mesh 65 = 10939
No of crystals in tyler mesh 48 = 2710
No of crystals in tyler mesh 35 = 2641
No of crystals in tyler mesh 28 = 2241
No of crystals in tyler mesh 20 = 1499
No of crystals in tyler mesh 14 = 1139
cmwtfr in tyler mesh 65 = 99.99999
cmwtfr in tyler mesh 48 = 99.34596
cmwtfr in tyler mesh 35 = 97.90733
cmwtfr in tyler mesh 28 = 94.0383
cmwtfr in tyler mesh 20 = 84.73317
cmwtfr in tyler mesh 14 = 67.37117
Sandler A-test = .314519400969609
rms = 16.96455

time =64.0166666666667 hour
No of crystals in tyler mesh 65 = 10986
No of crystals in tyler mesh 48 = 2714
No of crystals in tyler mesh 35 = 2719
No of crystals in tyler mesh 28 = 2186
No of crystals in tyler mesh 20 = 1525
No of crystals in tyler mesh 14 = 1129
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.34044
cmwtfr in tyler mesh 35 = 97.88425
cmwtfr in tyler mesh 28 = 93.77081
cmwtfr in tyler mesh 20 = 84.47691
cmwtfr in tyler mesh 14 = 66.89059
Sandler A-test = .313664613604091
rms = 16.65959

time =65.0166666666667 hour
No of crystals in tyler mesh 65 = 11035
No of crystals in tyler mesh 48 = 2694
No of crystals in tyler mesh 35 = 2683
No of crystals in tyler mesh 28 = 2183
No of crystals in tyler mesh 20 = 1541
No of crystals in tyler mesh 14 = 1122
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.3227
cmwtfr in tyler mesh 35 = 97.8414
cmwtfr in tyler mesh 28 = 93.73537
cmwtfr in tyler mesh 20 = 84.48708
cmwtfr in tyler mesh 14 = 66.3995
Sandler A-test = .312745537533213
rms = 16.50344

time = 66.0166666666667 hour
No of crystals in tyler mesh 65 = 11035
No of crystals in tyler mesh 48 = 2701
No of crystals in tyler mesh 35 = 2661
No of crystals in tyler mesh 28 = 2211
No of crystals in tyler mesh 20 = 1532
No of crystals in tyler mesh 14 = 1123
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.33969
cmwtfr in tyler mesh 35 = 97.87347
cmwtfr in tyler mesh 28 = 93.8506
cmwtfr in tyler mesh 20 = 84.5909
cmwtfr in tyler mesh 14 = 66.55086
Sandler A-test = .31242753904466
rms = 16.59077

time = 67.0166666666667 hour
No of crystals in tyler mesh 65 = 10976
No of crystals in tyler mesh 48 = 2618
No of crystals in tyler mesh 35 = 2709
No of crystals in tyler mesh 28 = 2236
No of crystals in tyler mesh 20 = 1497
No of crystals in tyler mesh 14 = 1135
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.3345
cmwtfr in tyler mesh 35 = 97.92648
cmwtfr in tyler mesh 28 = 93.85755
cmwtfr in tyler mesh 20 = 84.49099
cmwtfr in tyler mesh 14 = 66.84332
Sandler A-test = .312981143141941
rms = 16.66194

time = 68.0166666666667 hour
No of crystals in tyler mesh 65 = 11019
No of crystals in tyler mesh 48 = 2642
No of crystals in tyler mesh 35 = 2667
No of crystals in tyler mesh 28 = 2262
No of crystals in tyler mesh 20 = 1455
No of crystals in tyler mesh 14 = 1148
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.34064
cmwtfr in tyler mesh 35 = 97.94502
cmwtfr in tyler mesh 28 = 93.96114
cmwtfr in tyler mesh 20 = 84.54438
cmwtfr in tyler mesh 14 = 67.743
Sandler A-test = .315162424959963
rms = 16.96774

time = 69.0166666666667 hour
No of crystals in tyler mesh 65 = 11000
No of crystals in tyler mesh 48 = 2713
No of crystals in tyler mesh 35 = 2679
No of crystals in tyler mesh 28 = 2295
No of crystals in tyler mesh 20 = 1491
No of crystals in tyler mesh 14 = 1167
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.35182
cmwtfr in tyler mesh 35 = 97.94057
cmwtfr in tyler mesh 28 = 93.9974
cmwtfr in tyler mesh 20 = 84.71622
cmwtfr in tyler mesh 14 = 68.06862
Sandler A-test = 0.315916113364729
rms = 17.11202

time = 70.0166666666667 hour
No of crystals in tyler mesh 65 = 10868
No of crystals in tyler mesh 48 = 2733
No of crystals in tyler mesh 35 = 2681
No of crystals in tyler mesh 28 = 2344
No of crystals in tyler mesh 20 = 1512
No of crystals in tyler mesh 14 = 1162
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.36913
cmwtfr in tyler mesh 35 = 97.94108
cmwtfr in tyler mesh 28 = 94.02653
cmwtfr in tyler mesh 20 = 84.56218
cmwtfr in tyler mesh 14 = 67.81499
Sandler A-test = 0.314935819447598
rms = 17.00281

time = 71.0166666666667 hour
No of crystals in tyler mesh 65 = 10972
No of crystals in tyler mesh 48 = 2654
No of crystals in tyler mesh 35 = 2676
No of crystals in tyler mesh 28 = 2334
No of crystals in tyler mesh 20 = 1524
No of crystals in tyler mesh 14 = 1142
cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.34394
cmwtfr in tyler mesh 35 = 97.93798
cmwtfr in tyler mesh 28 = 93.99815
cmwtfr in tyler mesh 20 = 84.34079
cmwtfr in tyler mesh 14 = 67.16167
Sandler A-test = 0.313322427357324
rms = 16.74637

time = 72.0166666666667 hour
No of crystals in tyler mesh 65 = 10981
No of crystals in tyler mesh 48 = 2679
No of crystals in tyler mesh 35 = 2679
No of crystals in tyler mesh 28 = 2287
No of crystals in tyler mesh 20 = 1565
No of crystals in tyler mesh 14 = 1124
cmwtfr in tyler mesh 65 = 99.999999
cmwtfr in tyler mesh 48 = 99.35983
cmwtfr in tyler mesh 35 = 97.97022
cmwtfr in tyler mesh 28 = 94.02898
cmwtfir in tyler mesh 20 = 84.71387
cmwtfir in tyler mesh 14 = 67.25992
Sandler A-test = .313100254803892
rms = 16.86613

time = 73.016666666667 hour
No of crystals in tyler mesh 65 = 10998
No of crystals in tyler mesh 48 = 2690
No of crystals in tyler mesh 35 = 2659
No of crystals in tyler mesh 28 = 2240
No of crystals in tyler mesh 20 = 1574
No of crystals in tyler mesh 14 = 1109
cmwtfir in tyler mesh 65 = 100
cmwtfir in tyler mesh 48 = 99.36318
cmwtfir in tyler mesh 35 = 97.95901
cmwtfir in tyler mesh 28 = 94.05848
cmwtfir in tyler mesh 20 = 84.95378
cmwtfir in tyler mesh 14 = 67.41994
Sandler A-test = .313485033799109
rms = 16.97286

time = 74.016666666667 hour
No of crystals in tyler mesh 65 = 10991
No of crystals in tyler mesh 48 = 2615
No of crystals in tyler mesh 35 = 2646
No of crystals in tyler mesh 28 = 2307
No of crystals in tyler mesh 20 = 1573
No of crystals in tyler mesh 14 = 1116
cmwtfir in tyler mesh 65 = 100
cmwtfir in tyler mesh 48 = 99.35324
cmwtfir in tyler mesh 35 = 97.98764
cmwtfir in tyler mesh 28 = 94.13931
cmwtfir in tyler mesh 20 = 84.81032
cmwtfir in tyler mesh 14 = 67.22337
Sandler A-test = .312534771647884
rms = 16.89222

time = 75.016666666667 hour
No of crystals in tyler mesh 65 = 10982
No of crystals in tyler mesh 48 = 2666
No of crystals in tyler mesh 35 = 2635
No of crystals in tyler mesh 28 = 2233
No of crystals in tyler mesh 20 = 1582
No of crystals in tyler mesh 14 = 1123
cmwtfir in tyler mesh 65 = 100
cmwtfir in tyler mesh 48 = 99.35835
cmwtfir in tyler mesh 35 = 97.98483
cmwtfir in tyler mesh 28 = 94.13574
cmwtfir in tyler mesh 20 = 85.07947
cmwtfir in tyler mesh 14 = 67.44095
Sandler A-test = .313135407011662
rms = 17.01934

time = 76.016666666667 hour
No of crystals in tyler mesh 65 = 10935

259
No of crystals in tyler mesh 48 = 2709
No of crystals in tyler mesh 35 = 2578
No of crystals in tyler mesh 28 = 2259
No of crystals in tyler mesh 20 = 1604
No of crystals in tyler mesh 14 = 1103

cmwtfr in tyler mesh 65 = 100
cmwtfr in tyler mesh 48 = 99.36588
cmwtfr in tyler mesh 35 = 97.95306
cmwtfr in tyler mesh 28 = 94.19566
cmwtfr in tyler mesh 20 = 85.05192
cmwtfr in tyler mesh 14 = 66.93319
Sandler A-test = .311600307073731
rms = 16.86372

time = 76.51666666666667 hour
No of crystals in tyler mesh 65 = 10987
No of crystals in tyler mesh 48 = 2662
No of crystals in tyler mesh 35 = 2601
No of crystals in tyler mesh 28 = 2284
No of crystals in tyler mesh 20 = 1573
No of crystals in tyler mesh 14 = 1097

cmwtfr in tyler mesh 65 = 99.99999
cmwtfr in tyler mesh 48 = 99.35903
cmwtfr in tyler mesh 35 = 97.95693
cmwtfr in tyler mesh 28 = 94.15465
cmwtfr in tyler mesh 20 = 84.76763
cmwtfr in tyler mesh 14 = 66.8354
Sandler A-test = .311495931219554
rms = 16.76381

260
Appendix III.3

Sample Program for Transient CSD in FC Crystallizer

Declaration

Dim wtfr(0 To 6) As Single
Dim cmwtfr(0 To 6), Ben(0 To 6) As Single
Dim wet(0 To 6) As Single
Dim cr(1 To 8000, 0 To 6) As Integer
Dim wt(1 To 8000, 0 To 6) As Single
Dim tau, Tscan, t1, t2, nt, Num, size As Single
Dim top, bottom, count, A, rms As Single
Dim cryswt, Vplug, Vmixed, r, tp, tm As Single

Main Program

Sub main()

Vplug = Val(mixed.Text1.Text)
tau = Val(mixed.Text2.Text) * 60
G = Val(mixed.Text3.Text) / 60
Num = Val(mixed.Text4.Text)

Open "sim_fc" For Output As #1
Print #1, "Results of transient MC Simulation"
Print #1, "PFR of total volume = " & Vplug * 100; " %"
Print #1, "Residence time in an imperfectly mixed crystallizer =" & tau / 60;
" hr"
Print #1, "Average growth rate of a crystal = " & G * 60; " mm/hr"
Print #1, "Compared with Run No. " & Num; " Bennett and Van Buren"

nt = 60
Print #1, "Nucleation rate =" & nt; " nuclei/min"

Ben(0) = 98.3
Ben(1) = 90.5
Ben(2) = 74.4
Ben(3) = 44.4
Ben(4) = 18.2
Ben(5) = 8.2
Ben(6) = 4.3
Vmixed = 1 - Vplug
r = Vplug / Vmixed
tp = tau * r / (r + 1)
tm = tau / (r + 1)

Tscan = 13 * tau
For t1 = 1 To Tscan
For n = 1 To n t
    t2 = -tm * Log(Rnd) + tp
    ttot = t1 + t2
    If ttot > Tscan Then ttot = Tscan
    For times = t1 To ttot Step 1
        size = G * (times - t1)
        If size > .147 Then
            GoTo 250
        Else
            s = 0
            GoTo 310
        End If
    250 If size > .208 Then
        GoTo 260
    Else
        s = 1
        GoTo 310
    End If
    260 If size > .295 Then
        GoTo 270
    Else
        s = 2
        GoTo 310
    End If
    270 If size > .417 Then
        GoTo 280
    Else
        s = 3
        GoTo 310
    End If
    280 If size > .589 Then
        GoTo 290
    Else
        s = 4
        GoTo 310
    End If
    290 If size < .833 Then
        s = 5
    End If
End If
End For
End For
Else
    s = 6
End If
310  cr(times, s) = cr(times, s) + 1
    wt(times, s) = wt(times, s) + (size ^ 3)
Next
n5 = n5 + 1
Debug.Print n5
sumrmd = 0
Next
Next
For times = 1 To Tscan Step 30
    For j = 0 To 6
        wet(j) = wt(times, j)
        cryswt = cryswt + wet(j)
    Next
    If cryswt = 0 Then
        GoTo 4
    Else
        wtfr(6) = (wt(6) / cryswt) * 100
        cmwtfr(6) = wtfr(6)
        For j = 5 To 0 Step -1
            wtfr(j) = (wt(j) / cryswt) * 100
            cmwtfr(j) = cmwtfr(j + 1) + wtfr(j)
        Next
        For j = 0 To 6
            top = top + (cmwtfr(j) - Ben(j)) ^ 2
            bottom = bottom + (cmwtfr(j) - Ben(j))
            count = count + 1
        Next
        A = top / bottom ^ 2
        rms = (top / count) ^ (1 / 2)
        Print #1, "time = " & times / 60; " hour"
        Print #1, " No of crystals in tyler mesh 100 = " & cr(times, 0)
        Print #1, " No of crystals in tyler mesh 65 = " & cr(times, 1)
        Print #1, " No of crystals in tyler mesh 48 = " & cr(times, 2)
        Print #1, " No of crystals in tyler mesh 35 = " & cr(times, 3)
        Print #1, " No of crystals in tyler mesh 28 = " & cr(times, 4)
        Print #1, " No of crystals in tyler mesh 20 = " & cr(times, 5)
        Print #1, " No of crystals in tyler mesh 14 = " & cr(times, 6)
        Print #1, " cmwtfr in tyler mesh 100 = " & cmwtfr(0)
        Print #1, " cmwtfr in tyler mesh 65 = " & cmwtfr(1)
        Print #1, " cmwtfr in tyler mesh 48 = " & cmwtfr(2)
        Print #1, " cmwtfr in tyler mesh 35 = " & cmwtfr(3)
        Print #1, " cmwtfr in tyler mesh 28 = " & cmwtfr(4)
        Print #1, " cmwtfr in tyler mesh 20 = " & cmwtfr(5)
Print #1, " cmwtfrr in tyler mesh 14 = " & cmwtfrr(6)
Print #1, "Sandler A-test = " & A
Print #1, "rms = " & rms
Print #1,
cryswt = 0
End If
4 top = 0: bottom = 0: count = 0
Next

End Sub
Appendix IV.1
Outline of Simulation Algorithm for Transient CSD in DTB and FC
Crystallizers under Stochastic Dispersion Effects

START

READ TAU, GO, GM, KO, KM, LM, NO, NM, (N), [R]

TCOUNT ← 1 MIN

CALCULATE NRATE

NCOUNT ← 1

{CALCULATE N SUMMATION OF RANDOM NUMBER}
{CALCULATE TM, TP}
GENERATE RANDOM NUMBER AND
CALCULATE TI, KI, LI, GI

T ← TCOUNT
TTOT ← TCOUNT + T

IS TTOT > TSCAN

YES

TTOT ← TSCAN

NO

COMPUTE DI (T), W(T)

FIND THE SIZE CLASS
OF THE CRYSTAL

CRYSN0(T,SIZE) ← CRYSN0(T,SIZE) + 1
CRYSW(T,SIZE) ← CRYSW(T,SIZE) + W(T)

IS T < TTOT

YES

T ← T + 1 MIN

NO

IS NCOUNT < NRATE

YES

NCOUNT ← NCOUNT + 1

NO

IS TCOUNT < TSCAN

YES

TCOUNT ← TCOUNT + 1 MIN

NO

PRINT CRYSN0(T,SIZE), CRYSW(T,SIZE)
FOR ALL SIZE CLASSES AT SELECTED
TIME INTERVAL

STOP

{} apply only to DTB crystallizer
[] apply only to FC crystallizer

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Appendix IV.2

Sample Program for Transient CSD in DTB Crystallizer under Stochastic Dispersion Effects

**Declaration**

Dim wtfr(0 To 6) As Single  
Dim cmwtfr(0 To 6) As Single  
Dim wt(0 To 6) As Single  
Dim cr(1 To 780, 0 To 6) As Integer  
Dim wt(1 To 780, 0 To 6) As Single  
Dim tau, gmin, gmax, kmin, kmax, nmin, nmax, lm As Single  
Dim t1 As Integer

**Main Program**

Sub main ()

nstage = Val(stage.Text1.Text)  
Open "dtb" For Output As #1  
Print #1, "Results of transient MC Simulation"  
Print #1, "Number of stage=" & nstage  
tau = 60  
gmin = .001  
gmax = .0015  
kmin = .5238  
kmax = 1  
nmin = 50  
nmax = 70  
lm = .02  
Print #1, "Tau=" & tau  
Print #1, "Go=" & gmin  
Print #1, "Gm=" & gmax  
Print #1, "Ko=" & kmin  
Print #1, "Km=" & kmax  
Print #1, "No=" & nmin  
Print #1, "Nm=" & nmax  
Print #1, "Lm=" & lm

sdg = (gmax - gmin) / 6

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\[ \text{sdk} = (\text{kmax} - \text{kmin}) / 6 \]
\[ \text{mg} = (\text{gmax} + \text{gmin}) / 2 \]
\[ \text{mk} = (\text{kmax} + \text{kmin}) / 2 \]

For \( t1 = 1 \) To 780

Randomize

\[ \text{nt} = \text{nmin} + (\text{nmax} - \text{nmin}) \times \text{Rnd} \]

For \( n = 1 \) To \( n_t \)

If \( n_{stage} = 1 \) Then
\[ t2 = -\tau \times \log(\text{Rnd}) \]
Else

For \( st = 1 \) To \( n_{stage} \)
\[ \text{sumrnd} = \text{sumrnd} + \log(\text{Rnd}) \]
Next
\[ t2 = (-\tau / n_{stage}) \times \text{sumrnd} \]
End If

\[ \text{tg} = (-2 \times (\text{sdg}^2) \times \log(\text{Rnd}))^{.5} \]
\[ \text{tk} = (-2 \times (\text{sdk}^2) \times \log(\text{Rnd}))^{.5} \]
\[ x = \text{Rnd} \]

If \( x < .5 \) Then
\[ g = \text{mg} - \text{tg} \]
\[ k = \text{mk} - \text{tk} \]
Else
\[ g = \text{mg} + \text{tg} \]
\[ k = \text{mk} + \text{tk} \]
End If

\[ \text{ttot} = t1 + t2 \]

If \( \text{ttot} > 780 \) Then \( \text{ttot} = 780 \)
\[ l = 2 \times \text{ln} \times \text{Rnd} \]

For \( \text{times} = t1 \) To \( \text{ttot} \) Step 1
\[ \text{size} = 1 + g \times (\text{times} - t1) \]
If \( \text{size} > .074 \) Then
    GoTo 260
Else
    \[ s = 0 \]
    GoTo 310
End If

260

If \( \text{size} > .147 \) Then
    GoTo 270
Else
\[ s = 1 \]
    GoTo 310
End If

270

If \( \text{size} > .208 \) Then
    GoTo 280
Else
\[ s = 2 \]
GoTo 310
End If
280  If size > .295 Then
      GoTo 290
Else
   s = 3
      GoTo 310
End If
290  If size > .417 Then
      GoTo 300
Else
   s = 4
      GoTo 310
End If
300  If size < .589 Then
      s = 5
Else
   s = 6
End If
310  cr(times, s) = cr(times, s) + 1
    wt(times, s) = wt(times, s) + k * (size ^ 3)
    Next
    n5 = n5 + 1
    Debug.Print n5
    sumrnd = 0
    Next
    Next
For times = 30 To 780 Step 30
    For j = 0 To 6
      wet(j) = wt(times, j)
      cswrt = cswrt + wet(j)
      Next
wtftr(0) = wet(0) / cswrt
    cmwtftr(0) = wfr(0)
    For j = 1 To 6
      wfr(j) = wet(j) / cswrt
      cmwtftr(j) = cmwtftr(j - 1) + wfr(j)
      Next
Print #1, "time=" & times
Print #1, " No of crystals in tyler mesh 200 = " & cr(times, 0)
Print #1, " No of crystals in tyler mesh 100 = " & cr(times, 1)
Print #1, " No of crystals in tyler mesh 65 = " & cr(times, 2)
Print #1, " No of crystals in tyler mesh 48 = " & cr(times, 3)
Print #1, " No of crystals in tyler mesh 35 = " & cr(times, 4)
Print #1, " No of crystals in tyler mesh 28 = " & cr(times, 5)
Print #1, " No of crystals in tyler mesh 20 = " & cr(times, 6)
Print #1, "cmwtfir in tyler mesh 200 = " & cmwtfir(0)
Print #1, "cmwtfir in tyler mesh 100 = " & cmwtfir(1)
Print #1, "cmwtfir in tyler mesh 65 = " & cmwtfir(2)
Print #1, "cmwtfir in tyler mesh 48 = " & cmwtfir(3)
Print #1, "cmwtfir in tyler mesh 35 = " & cmwtfir(4)
Print #1, "cmwtfir in tyler mesh 28 = " & cmwtfir(5)
Print #1, "cmwtfir in tyler mesh 20 = " & cmwtfir(6)
cryswt = 0
Next

End Sub
Appendix IV.3

Sample Program and Results for Transient CSD in FC Crystallizer under Stochastic Dispersion Effects

Declaration

Dim wtfr(0 To 6) As Single
Dim cmwtfr(0 To 6) As Single
Dim wet(0 To 6) As Single
Dim cr(1 To 780, 0 To 6) As Integer
Dim wt(1 To 780, 0 To 6) As Single
Dim tau, gmin, gmax, kmin, kmax, nmin, nmax, lm As Single
Dim Vplug, Vmixed, r
Dim t1 As Integer

Main Program

Sub main ()

Vplug = Val(ratio.Text1.Text)
Open "FC" For Output As #1
Print #1, "Results of transient MC Simulation"
Print #1, "Ratio of PFR to Total Volume=" & Vplug
tau = 60
gmin = .001
gmax = .0015
kmin = .5238
kmax = 1
nmin = 50
nmax = 70
lm = .02
Print #1, "Tau=" & tau
Print #1, "Gm=" & gmin
Print #1, "Gm=" & gmax
Print #1, "Ko=" & kmin
Print #1, "Km=" & kmax
Print #1, "No=" & nmin
Print #1, "Nm=" & nmax
Print #1, "Lm=" & lm

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sdg = (gmax - gmin) / 6
sdk = (kmax - kmin) / 6
mg = (gmax + gmin) / 2
mk = (kmax + kmin) / 2
Vmixed = 1 - Vplug
r = Vplug / Vmixed
tp = tau * r / (r + 1)
tm = tau / (r + 1)
For t1 = 1 To 780
    Randomize
    nt = nmin + (nmax - nmin) * Rnd
    For n = 1 To nt
        t2 = -tm * Log(Rnd) + tp
        tg = (-2 * (sdg ^ 2) * Log(Rnd))^.5
        tk = (-2 * (sdk ^ 2) * Log(Rnd))^.5
        x = Rnd
        If x < .5 Then
            g = mg - tg
            k = mk - tk
        Else
            g = mg + tg
            k = mk + tk
        End If
        ttot = t1 + t2
        If ttot > 780 Then ttot = 780
        l = 2 * lm * Rnd
        For times = t1 To ttot Step 1
            size = 1 + g * (times - t1)
            If size > .074 Then
                GoTo 260
            Else
                s = 0
                GoTo 310
            End If
    260 If size > .147 Then
        GoTo 270
    Else
        s = 1
        GoTo 310
    End If
    270 If size > .208 Then
        GoTo 280
    Else
        s = 2
        GoTo 310
    End If
End If

280  If size > .295 Then
     GoTo 290
   Else
     s = 3
     GoTo 310
   End If
290  If size > .417 Then
     GoTo 300
   Else
     s = 4
     GoTo 310
   End If
300  If size < .589 Then
     s = 5
   Else
     s = 6
   End If
310  cr(times, s) = cr(times, s) + 1
     wt(times, s) = wt(times, s) + k * (size ^ 3)
   Next
  n5 = n5 + 1
  Debug.Print n5
  Next
  Next
For times = 30 To 780 Step 30
   For j = 0 To 6
     wet(j) = wt(times, j)
     cryswt = cryswt + wet(j)
   Next
  wtrfr(0) = wet(0) / cryswt
  cmwtrfr(0) = wtrfr(0)
  For j = 1 To 6
     wtrfr(j) = wet(j) / cryswt
     cmwtrfr(j) = cmwtrfr(j - 1) + wtrfr(j)
   Next
  Print #1, "time=" & times
  Print #1, " No of crystals in tyler mesh 200 = " & cr(times, 0)
  Print #1, " No of crystals in tyler mesh 100 = " & cr(times, 1)
  Print #1, " No of crystals in tyler mesh 65 = " & cr(times, 2)
  Print #1, " No of crystals in tyler mesh 48 = " & cr(times, 3)
  Print #1, " No of crystals in tyler mesh 35 = " & cr(times, 4)
  Print #1, " No of crystals in tyler mesh 28 = " & cr(times, 5)
  Print #1, " No of crystals in tyler mesh 20 = " & cr(times, 6)
  Print #1, "cmwtrfr in tyler mesh 200 = " & cmwtrfr(0)
  Print #1, "cmwtrfr in tyler mesh 100 = " & cmwtrfr(1)
  Print #1, "cmwtrfr in tyler mesh 65 = " & cmwtrfr(2)

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Print #1, "cmwtfir in tyler mesh 48 = " & cmwtfir(3)
Print #1, "cmwtfir in tyler mesh 35 = " & cmwtfir(4)
Print #1, "cmwtfir in tyler mesh 28 = " & cmwtfir(5)
Print #1, "cmwtfir in tyler mesh 20 = " & cmwtfir(6)
cryswt = 0
Next

End Sub

Sample of results (for 0% PFR)

Results of transient MC Simulation
Ratio of PFR to Total Volume=0
Tau=60
Go=.001
Gm=.0015
Ko=.5238
Km=1
No=50
Nm=70
Lm=.02

time=30
No of crystals in tyler mesh 200 = 1389
No of crystals in tyler mesh 100 = 3
No of crystals in tyler mesh 65 = 0
No of crystals in tyler mesh 48 = 0
No of crystals in tyler mesh 35 = 0
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
cmwtfir in tyler mesh 200 = .9861985
cmwtfir in tyler mesh 100 = 1
cmwtfir in tyler mesh 65 = 1
cmwtfir in tyler mesh 48 = 1
cmwtfir in tyler mesh 35 = 1
cmwtfir in tyler mesh 28 = 1
cmwtfir in tyler mesh 20 = 1

time=60
No of crystals in tyler mesh 200 = 1838
No of crystals in tyler mesh 100 = 433
No of crystals in tyler mesh 65 = 0
No of crystals in tyler mesh 48 = 0
No of crystals in tyler mesh 35 = 0
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
cmwtfir in tyler mesh 200 = .3889922
cmwtfir in tyler mesh 100 = 1
cmwtfir in tyler mesh 65 = 1
cmwtfir in tyler mesh 48 = 1
cmwtfir in tyler mesh 35 = 1
cmwtfir in tyler mesh 28 = 1

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cmwtrf in tyler mesh 20 = 1

time=90
No of crystals in tyler mesh 200 = 1854
No of crystals in tyler mesh 100 = 936
No of crystals in tyler mesh 65 = 12
No of crystals in tyler mesh 48 = 0
No of crystals in tyler mesh 35 = 0
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
cmwtrf in tyler mesh 200 = .1602252
cmwtrf in tyler mesh 100 = .9645727
cmwtrf in tyler mesh 65 = 1
cmwtrf in tyler mesh 48 = 1
cmwtrf in tyler mesh 35 = 1
cmwtrf in tyler mesh 28 = 1
cmwtrf in tyler mesh 20 = 1

time=120
No of crystals in tyler mesh 200 = 1794
No of crystals in tyler mesh 100 = 1086
No of crystals in tyler mesh 65 = 185
No of crystals in tyler mesh 48 = 0
No of crystals in tyler mesh 35 = 0
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
cmwtrf in tyler mesh 200 = 8.542478E-02
cmwtrf in tyler mesh 100 = .6421517
cmwtrf in tyler mesh 65 = 1
cmwtrf in tyler mesh 48 = 1
cmwtrf in tyler mesh 35 = 1
cmwtrf in tyler mesh 28 = 1
cmwtrf in tyler mesh 20 = 1

time=150
No of crystals in tyler mesh 200 = 1857
No of crystals in tyler mesh 100 = 1117
No of crystals in tyler mesh 65 = 348
No of crystals in tyler mesh 48 = 38
No of crystals in tyler mesh 35 = 0
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
cmwtrf in tyler mesh 200 = 5.251395E-02
cmwtrf in tyler mesh 100 = .4159707
cmwtrf in tyler mesh 65 = .8849602
cmwtrf in tyler mesh 48 = 1
cmwtrf in tyler mesh 35 = 1
cmwtrf in tyler mesh 28 = 1
cmwtrf in tyler mesh 20 = 1

time=180
No of crystals in tyler mesh 200 = 1816
No of crystals in tyler mesh 100 = 1107
No of crystals in tyler mesh 65 = 393
No of crystals in tyler mesh 48 = 119
No of crystals in tyler mesh 35 = 0
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 3.958658E-02
cmwtfr in tyler mesh 100 = .3086667
cmwtfr in tyler mesh 65 = .7046975
cmwtfr in tyler mesh 48 = 1
cmwtfr in tyler mesh 35 = 1
cmwtfr in tyler mesh 28 = 1
cmwtfr in tyler mesh 20 = 1

time=210
No of crystals in tyler mesh 200 = 1807
No of crystals in tyler mesh 100 = 1072
No of crystals in tyler mesh 65 = 367
No of crystals in tyler mesh 48 = 194
No of crystals in tyler mesh 35 = 7
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 3.011664E-02
cmwtfr in tyler mesh 100 = .2339615
cmwtfr in tyler mesh 65 = .5366954
cmwtfr in tyler mesh 48 = .9641281
cmwtfr in tyler mesh 35 = 1
cmwtfr in tyler mesh 28 = 1
cmwtfr in tyler mesh 20 = 1

time=240
No of crystals in tyler mesh 200 = 1784
No of crystals in tyler mesh 100 = 1075
No of crystals in tyler mesh 65 = 375
No of crystals in tyler mesh 48 = 223
No of crystals in tyler mesh 35 = 38
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 2.469984E-02
cmwtfr in tyler mesh 100 = .1895512
cmwtfr in tyler mesh 65 = .4372618
cmwtfr in tyler mesh 48 = .8385384
cmwtfr in tyler mesh 35 = .9999999
cmwtfr in tyler mesh 28 = .9999999
cmwtfr in tyler mesh 20 = .9999999

time=270
No of crystals in tyler mesh 200 = 1787
No of crystals in tyler mesh 100 = 1056
No of crystals in tyler mesh 65 = 349
No of crystals in tyler mesh 48 = 224
No of crystals in tyler mesh 35 = 58
No of crystals in tyler mesh 28 = 0
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 2.202494E-02
cmwtfr in tyler mesh 100 = .1752478
cmwtfr in tyler mesh 65 = .3820049
cmwtfr in tyler mesh 48 = .7646667
cmwtrfr in tyler mesh 35 = 1
cmwtrfr in tyler mesh 28 = 1
cmwtrfr in tyler mesh 20 = 1

\text{time}=300
\text{No of crystals in tyler mesh 200} = 1850
\text{No of crystals in tyler mesh 100} = 1036
\text{No of crystals in tyler mesh 65} = 365
\text{No of crystals in tyler mesh 48} = 215
\text{No of crystals in tyler mesh 35} = 62
\text{No of crystals in tyler mesh 28} = 1
\text{No of crystals in tyler mesh 20} = 0
\text{cmwtrfr in tyler mesh 200} = 2.210675E-02
\text{cmwtrfr in tyler mesh 100} = .1597621
\text{cmwtrfr in tyler mesh 65} = .3693035
\text{cmwtrfr in tyler mesh 48} = .7238129
\text{cmwtrfr in tyler mesh 35} = .9909256
\text{cmwtrfr in tyler mesh 28} = 1
\text{cmwtrfr in tyler mesh 20} = 1

\text{time}=330
\text{No of crystals in tyler mesh 200} = 1849
\text{No of crystals in tyler mesh 100} = 1093
\text{No of crystals in tyler mesh 65} = 339
\text{No of crystals in tyler mesh 48} = 196
\text{No of crystals in tyler mesh 35} = 79
\text{No of crystals in tyler mesh 28} = 5
\text{No of crystals in tyler mesh 20} = 0
\text{cmwtrfr in tyler mesh 200} = 2.050627E-02
\text{cmwtrfr in tyler mesh 100} = .1545391
\text{cmwtrfr in tyler mesh 65} = .3389382
\text{cmwtrfr in tyler mesh 48} = .6323854
\text{cmwtrfr in tyler mesh 35} = .9523973
\text{cmwtrfr in tyler mesh 28} = 1
\text{cmwtrfr in tyler mesh 20} = 1

\text{time}=360
\text{No of crystals in tyler mesh 200} = 1923
\text{No of crystals in tyler mesh 100} = 1049
\text{No of crystals in tyler mesh 65} = 342
\text{No of crystals in tyler mesh 48} = 199
\text{No of crystals in tyler mesh 35} = 70
\text{No of crystals in tyler mesh 28} = 5
\text{No of crystals in tyler mesh 20} = 0
\text{cmwtrfr in tyler mesh 200} = .0225617
\text{cmwtrfr in tyler mesh 100} = .1645152
\text{cmwtrfr in tyler mesh 65} = .3465231
\text{cmwtrfr in tyler mesh 48} = .651548
\text{cmwtrfr in tyler mesh 35} = .9535317
\text{cmwtrfr in tyler mesh 28} = 1
\text{cmwtrfr in tyler mesh 20} = 1

\text{time}=390
\text{No of crystals in tyler mesh 200} = 1800
\text{No of crystals in tyler mesh 100} = 1076
No of crystals in tyler mesh 65 = 361
No of crystals in tyler mesh 48 = 163
No of crystals in tyler mesh 35 = 74
No of crystals in tyler mesh 28 = 10
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 1.998275E-02
cmwtfr in tyler mesh 100 = .1543041
cmwtfr in tyler mesh 65 = .3507973
cmwtfr in tyler mesh 48 = .5983913
cmwtfr in tyler mesh 35 = .8941116
cmwtfr in tyler mesh 28 = .9999999
cmwtfr in tyler mesh 20 = .9999999

time=420
No of crystals in tyler mesh 200 = 1811
No of crystals in tyler mesh 100 = 1054
No of crystals in tyler mesh 65 = 348
No of crystals in tyler mesh 48 = 182
No of crystals in tyler mesh 35 = 77
No of crystals in tyler mesh 28 = 9
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 1.968133E-02
cmwtfr in tyler mesh 100 = .1534835
cmwtfr in tyler mesh 65 = .3386047
cmwtfr in tyler mesh 48 = .5938872
cmwtfr in tyler mesh 35 = .900502
cmwtfr in tyler mesh 28 = .9999999
cmwtfr in tyler mesh 20 = .9999999

time=450
No of crystals in tyler mesh 200 = 1784
No of crystals in tyler mesh 100 = 1031
No of crystals in tyler mesh 65 = 364
No of crystals in tyler mesh 48 = 217
No of crystals in tyler mesh 35 = 78
No of crystals in tyler mesh 28 = 11
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 1.828448E-02
cmwtfr in tyler mesh 100 = .1385517
cmwtfr in tyler mesh 65 = .3177183
cmwtfr in tyler mesh 48 = .6078454
cmwtfr in tyler mesh 35 = .8979965
cmwtfr in tyler mesh 28 = 1
cmwtfr in tyler mesh 20 = 1

time=480
No of crystals in tyler mesh 200 = 1865
No of crystals in tyler mesh 100 = 1064
No of crystals in tyler mesh 65 = 352
No of crystals in tyler mesh 48 = 217
No of crystals in tyler mesh 35 = 77
No of crystals in tyler mesh 28 = 19
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 1.688894E-02
cmwtfr in tyler mesh 100 = .1296194
No of crystals in tyler mesh 65 = 361
No of crystals in tyler mesh 48 = 163
No of crystals in tyler mesh 35 = 74
No of crystals in tyler mesh 28 = 10
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 1.998275E-02
cmwtfr in tyler mesh 100 = .1543041
cmwtfr in tyler mesh 65 = .3507973
cmwtfr in tyler mesh 48 = .5983913
cmwtfr in tyler mesh 35 = .8941116
cmwtfr in tyler mesh 28 = .9999999
cmwtfr in tyler mesh 20 = .9999999 

time=420
No of crystals in tyler mesh 200 = 1811
No of crystals in tyler mesh 100 = 1054
No of crystals in tyler mesh 65 = 348
No of crystals in tyler mesh 48 = 182
No of crystals in tyler mesh 35 = 77
No of crystals in tyler mesh 28 = 9
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 1.968133E-02
cmwtfr in tyler mesh 100 = .1534835
cmwtfr in tyler mesh 65 = .3386047
cmwtfr in tyler mesh 48 = .5938872
cmwtfr in tyler mesh 35 = .900502
cmwtfr in tyler mesh 28 = .9999999
cmwtfr in tyler mesh 20 = .9999999

time=450
No of crystals in tyler mesh 200 = 1784
No of crystals in tyler mesh 100 = 1031
No of crystals in tyler mesh 65 = 364
No of crystals in tyler mesh 48 = 217
No of crystals in tyler mesh 35 = 78
No of crystals in tyler mesh 28 = 11
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 1.828448E-02
cmwtfr in tyler mesh 100 = .1385517
cmwtfr in tyler mesh 65 = .3177183
cmwtfr in tyler mesh 48 = .6078454
cmwtfr in tyler mesh 35 = .8979965
cmwtfr in tyler mesh 28 = 1
cmwtfr in tyler mesh 20 = 1

time=480
No of crystals in tyler mesh 200 = 1865
No of crystals in tyler mesh 100 = 1064
No of crystals in tyler mesh 65 = 352
No of crystals in tyler mesh 48 = 217
No of crystals in tyler mesh 35 = 77
No of crystals in tyler mesh 28 = 19
No of crystals in tyler mesh 20 = 0
cmwtfr in tyler mesh 200 = 1.688894E-02
cmwtfr in tyler mesh 100 = .1296194
cmwtf in tyler mesh 65 = .2928881
cmwtf in tyler mesh 48 = .5745622
cmwtf in tyler mesh 35 = .8305749
cmwtf in tyler mesh 28 = 1
cmwtf in tyler mesh 20 = 1

steady state

time=510
No of crystals in tyler mesh 200 = 1893
No of crystals in tyler mesh 100 = 1040
No of crystals in tyler mesh 65 = 339
No of crystals in tyler mesh 48 = 197
No of crystals in tyler mesh 35 = 85
No of crystals in tyler mesh 28 = 21
No of crystals in tyler mesh 20 = 1
cmwtf in tyler mesh 200 = 1.724211E-02
cmwtf in tyler mesh 100 = .1204618
cmwtf in tyler mesh 65 = .2630246
cmwtf in tyler mesh 48 = .5040336
cmwtf in tyler mesh 35 = .7852369
cmwtf in tyler mesh 28 = .9788637
cmwtf in tyler mesh 20 = 1

time=540
No of crystals in tyler mesh 200 = 1867
No of crystals in tyler mesh 100 = 1049
No of crystals in tyler mesh 65 = 327
No of crystals in tyler mesh 48 = 180
No of crystals in tyler mesh 35 = 83
No of crystals in tyler mesh 28 = 24
No of crystals in tyler mesh 20 = 0
cmwtf in tyler mesh 200 = 1.734064E-02
cmwtf in tyler mesh 100 = .1275765
cmwtf in tyler mesh 65 = .2703285
cmwtf in tyler mesh 48 = .4940478
cmwtf in tyler mesh 35 = .7696309
cmwtf in tyler mesh 28 = 1
cmwtf in tyler mesh 20 = 1

time=570
No of crystals in tyler mesh 200 = 1889
No of crystals in tyler mesh 100 = 1033
No of crystals in tyler mesh 65 = 362
No of crystals in tyler mesh 48 = 181
No of crystals in tyler mesh 35 = 71
No of crystals in tyler mesh 28 = 18
No of crystals in tyler mesh 20 = 2
cmwtf in tyler mesh 200 = 1.722681E-02
cmwtf in tyler mesh 100 = .1254049
cmwtf in tyler mesh 65 = .2885264
cmwtf in tyler mesh 48 = .527334
cmwtf in tyler mesh 35 = .7977287
cmwtf in tyler mesh 28 = .9626451
cmwtf in tyler mesh 20 = 1

time=600
No of crystals in tyler mesh 200 = 1886
No of crystals in tyler mesh 100 = 1071
No of crystals in tyler mesh 65 = 345
No of crystals in tyler mesh 48 = 181
No of crystals in tyler mesh 35 = 65
No of crystals in tyler mesh 28 = 18
No of crystals in tyler mesh 20 = 4
cmwtfr in tyler mesh 200 = .0174655
cmwtfr in tyler mesh 100 = .130412
cmwtfr in tyler mesh 65 = .2881138
cmwtfr in tyler mesh 48 = .5075626
cmwtfr in tyler mesh 35 = .7468447
cmwtfr in tyler mesh 28 = .9119091
cmwtfr in tyler mesh 20 = 1

time=630
No of crystals in tyler mesh 200 = 1855
No of crystals in tyler mesh 100 = 1076
No of crystals in tyler mesh 65 = 345
No of crystals in tyler mesh 48 = 194
No of crystals in tyler mesh 35 = 59
No of crystals in tyler mesh 28 = 21
No of crystals in tyler mesh 20 = 1
cmwtfr in tyler mesh 200 = .0187367
cmwtfr in tyler mesh 100 = .1355076
cmwtfr in tyler mesh 65 = .2966362
cmwtfr in tyler mesh 48 = .5446144
cmwtfr in tyler mesh 35 = .7540574
cmwtfr in tyler mesh 28 = .9681376
cmwtfr in tyler mesh 20 = 1

time=660
No of crystals in tyler mesh 200 = 1793
No of crystals in tyler mesh 100 = 1092
No of crystals in tyler mesh 65 = 369
No of crystals in tyler mesh 48 = 192
No of crystals in tyler mesh 35 = 64
No of crystals in tyler mesh 28 = 16
No of crystals in tyler mesh 20 = 2
cmwtfr in tyler mesh 200 = 1.685518E-02
cmwtfr in tyler mesh 100 = .1322738
cmwtfr in tyler mesh 65 = .2989801
cmwtfr in tyler mesh 48 = .548808
cmwtfr in tyler mesh 35 = .7744268
cmwtfr in tyler mesh 28 = .937535
cmwtfr in tyler mesh 20 = 1

time=690
No of crystals in tyler mesh 200 = 1842
No of crystals in tyler mesh 100 = 1048
No of crystals in tyler mesh 65 = 359
No of crystals in tyler mesh 48 = 195
No of crystals in tyler mesh 35 = 59
No of crystals in tyler mesh 28 = 15
No of crystals in tyler mesh 20 = 1

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cmwtfr in tyler mesh 200 = .0178348
cmwtfr in tyler mesh 100 = .13652
cmwtfr in tyler mesh 65 = .3080125
cmwtfr in tyler mesh 48 = .5719891
cmwtfr in tyler mesh 35 = .7856463
cmwtfr in tyler mesh 28 = .9674219
cmwtfr in tyler mesh 20 = 1

time=720
No of crystals in tyler mesh 200 = 1828
No of crystals in tyler mesh 100 = 1051
No of crystals in tyler mesh 65 = 357
No of crystals in tyler mesh 48 = 188
No of crystals in tyler mesh 35 = 72
No of crystals in tyler mesh 28 = 10
No of crystals in tyler mesh 20 = 2
cmwtfr in tyler mesh 200 = 1.897358E-02
cmwtfr in tyler mesh 100 = .1421815
cmwtfr in tyler mesh 65 = .3144156
cmwtfr in tyler mesh 48 = .5710347
cmwtfr in tyler mesh 35 = .8349816
cmwtfr in tyler mesh 28 = .95425
cmwtfr in tyler mesh 20 = 1

time=750
No of crystals in tyler mesh 200 = 1799
No of crystals in tyler mesh 100 = 1053
No of crystals in tyler mesh 65 = 357
No of crystals in tyler mesh 48 = 186
No of crystals in tyler mesh 35 = 82
No of crystals in tyler mesh 28 = 12
No of crystals in tyler mesh 20 = 3
cmwtfr in tyler mesh 200 = 1.733694E-02
cmwtfr in tyler mesh 100 = .1313016
cmwtfr in tyler mesh 65 = .3010784
cmwtfr in tyler mesh 48 = .5420061
cmwtfr in tyler mesh 35 = .8360603
cmwtfr in tyler mesh 28 = .9313347
cmwtfr in tyler mesh 20 = 1

time=780
No of crystals in tyler mesh 200 = 1853
No of crystals in tyler mesh 100 = 1063
No of crystals in tyler mesh 65 = 339
No of crystals in tyler mesh 48 = 202
No of crystals in tyler mesh 35 = 70
No of crystals in tyler mesh 28 = 19
No of crystals in tyler mesh 20 = 2
cmwtfr in tyler mesh 200 = 1.752982E-02
cmwtfr in tyler mesh 100 = .1293155
cmwtfr in tyler mesh 65 = .2850104
cmwtfr in tyler mesh 48 = .5422521
cmwtfr in tyler mesh 35 = .7779839
cmwtfr in tyler mesh 28 = .9388199
cmwtfr in tyler mesh 20 = 1
iii) Dominant size (on a weight basis) occurs at $x = 3$

iv) Fractional breakage at $x = 3$ relative to product removal,

$$q = \frac{\beta_m 3^m}{y} \quad \text{or} \quad \beta_m = \frac{q}{3^m} \quad \text{(AV.1.5)}$$

v) Boundary conditions:

- For large $x$, $y(2x) \ll y(x)$; therefore
  suppress the birth term and consider
  only the death term.
- $y(0) = 1$

the following differential equation results:

$$\dot{y} + (1 + \beta_m x^m) y = g(x) \quad \text{(AV.1.6)}$$

where

$$g(x) = 2^{m+1} \beta_m x^m \exp\left[2x + \frac{\beta_m}{m+1} (2x)^{m+1}\right] \quad \text{(AV.1.7)}$$

Cumulative and differential weight distributions can be obtained from Eq. (AV.1.6) using the transformations

$$w = \int_0^x y \, d\dot{y} \quad \text{(AV.1.8)}$$

$$\dot{w} = x^3 y \quad \text{(AV.1.9)}$$

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\[ \ddot{w} = 3x^2y + x^3\dot{y} \]  

(AV.1.10)

resulting in the final second-order differential equation for cumulative weight distribution,

\[ \ddot{w} = \left[ \frac{3}{x} - 1 - \beta_n x^n \right] \dot{w} + x^3g(x) \]  

(AV.1.11)
Appendix V.2
Outline of Simulation Algorithm for Steady State CSD with Random Breakage Model

START

I ← 1
k ← 10

READ TAU, G, L(k), m, q, sample

x(k) ← L(k) / (G.TAU)
Beta ← q / 3^m

Generate RND

Ti ← TAU log (RND)
size ← G.Ti

Find the mesh interval correspond to size

crys wt ← crys wt + size^3
w(k) ← w(k) + size^3
cr(k) ← cr(k) + 1

I ← I + 1

Is i = sample?

yes

wf(k) ← w(k) / crys wt

D(k) ← Beta.cr(k).x(k)^m

Is D(k) > cr(k)?

yes

D(k) ← cr(k)

no

cr_2(10) ← cr(10) - D(10)
cr_2(9) ← cr(9) - D(9)
cr_2(8) ← cr(8) - D(8)
cr_2(7) ← cr(7) - D(7)
cr_2(6) ← cr(6) - D(6)
cr_2(5) ← cr(5) + 2D(10) - D(5)
cr_2(4) ← cr(4) + 2D(9) + 2D(8) - D(4)
cr_2(3) ← cr(3) + 2D(7) + 2D(6) - D(3)
cr_2(2) ← cr(2) + 2D(5) + 2D(4) - D(2)
cr_2(1) ← cr(1) + 2D(3) + 2D(2) - D(1)
cr_2(0) ← cr(0) + 2D(1) - D(0)

wt_2(k) ← (cr_2(k) * w(k)) / cr(k)
crys wt_2 ← crys wt_2 + wt_2(k)
wf_2(k) ← w(k) / crys wt_2

PRINT wf(k); wf_2(k)
cr(k); cr_2(k)

STOP
Appendix V.3

Sample Program and Results for Steady State CSD with Random Breakage

Model

Declaration

Dim wtf0r(0 To 10), wtf_2(0 To 10) As Single
Dim wt(0 To 10), wt_2(0 To 10) As Single
Dim cr(0 To 10), cr_2(0 To 10) As Single
Dim cum(0 To 10), cum_2(0 To 10) As Single
Dim L(0 To 11), D(0 To 10) As Single
Dim x(0 To 10), cons(0 To 10) As Single
Dim sample, q, m, g, tau, ti, size, tem, tem_2 As Single
Dim crys wt, crys wt_2, crys no, crys no_2 As Single
Dim k As Integer

Main Program

Sub main()

sample = Val(randolph.Text1.Text)
q = Val(randolph.Text2.Text)
m = Val(randolph.Text3.Text)
g = .00125
tau = 60

Open "randolph" For Output As #1
Print #1, "Results of steady state MC Simulation"
Print #1, "Sample size at steady state=" & sample
Print #1, "Breakage fraction=" & q
Print #1, "m=" & m
Print #1, "Growth rate=" & g; " mm/min"
Print #1, "Average residence time of a crystal in the crystallizer=" & tau; " min"
Print #1,
L(0) = 0
L(1) = .075
L(2) = .15
L(3) = .225
L(4) = .3
L(5) = .375
L(6) = .45
L(7) = .525
L(8) = .6
L(9) = .675
L(10) = .75
L(11) = .825

For i = 1 To sample
  Randomize
  ti = -tau * Log(Rnd)
  size = g * ti
  If size > .075 Then
    GoTo 250
  Else
    k = 0
    GoTo 410
  End If
250 If size > .15 Then
  GoTo 260
Else
  k = 1
  GoTo 410
End If
260 If size > .225 Then
  GoTo 270
Else
  k = 2
  GoTo 410
End If
270 If size > .3 Then
  GoTo 280
Else
  k = 3
  GoTo 410
End If
280 If size > .375 Then
  GoTo 290
Else
  k = 4
  GoTo 410
End If
290 If size > .45 Then
  GoTo 300
Else
  k = 5
GoTo 410
End If
300 If size > .525 Then
    GoTo 310
Else
    k = 6
    GoTo 410
End If
310 If size > .6 Then
    GoTo 320
Else
    k = 7
    GoTo 410
End If
320 If size > .675 Then
    GoTo 330
Else
    k = 8
    GoTo 410
End If
330 If size > .75 Then
    GoTo 340
Else
    k = 9
    GoTo 410
End If
340 k = 10
410 cr(k) = cr(k) + 1
    wt(k) = wt(k) + (size ^ 3)
Next
For k = 0 To 10
    crysno = crysno + cr(k)
    cryswt = cryswt + wt(k)
Next
For k = 0 To 10
    x(k) = L(k) / (g * tau)
Next
For k = 0 To 10
    cons(k) = (q / (3 ^ m)) * x(k) ^ m
Next
Rem cons(5) to cons(10) is greater than 1 which result in
Rem the death particle, D(5) to D(10), greater than its
Rem initial particle number, cr(5) to cr(10), if Randolph Rem Breakage Model is to apply. "Number imbalance!!"
Rem Therefore:

For k = 5 To 10
    D(k) = cr(k)
Next

Rem Assume non-multiple breakage
Rem D(4) = cons(4) * (cr(4) + 2 * D(9) + 2 * D(8))
Rem D(3) = cons(3) * (cr(3) + 2 * D(7) + 2 * D(6))
Rem D(2) = cons(2) * (cr(2) + 2 * D(5) + 2 * D(4))
Rem D(1) = cons(1) * (cr(1) + 2 * D(3) + 2 * D(2))
Rem D(0) = 0

D(4) = cons(4) * cr(4)
D(3) = cons(3) * cr(3)
D(2) = cons(2) * cr(2)
D(1) = cons(1) * cr(1)
D(0) = 0

cr_2(10) = cr(10) - D(10)
cr_2(9) = cr(9) - D(9)
cr_2(8) = cr(8) - D(8)
cr_2(7) = cr(7) - D(7)
cr_2(6) = cr(6) - D(6)
cr_2(5) = cr(5) + 2 * D(10) - D(5)
cr_2(4) = cr(4) + 2 * D(9) + 2 * D(8) - D(4)
cr_2(3) = cr(3) + 2 * D(7) + 2 * D(6) - D(3)
cr_2(2) = cr(2) + 2 * D(5) + 2 * D(4) - D(2)
cr_2(1) = cr(1) + 2 * D(3) + 2 * D(2) - D(1)
cr_2(0) = cr(0) + 2 * D(1) - D(0)

For k = 0 To 10
    wt_2(k) = (cr_2(k) / cr(k)) * wt(k)
Next

For k = 0 To 10
    crysno_2 = crysno_2 + cr_2(k)
    cryswt_2 = cryswt_2 + wt_2(k)
Next

For k = 0 To 10
    wtfr(k) = (wt(k) / cryswt) * 100
    wtfr_2(k) = (wt_2(k) / cryswt_2) * 100
Next

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tem = 0
tem_2 = 0
For k = 0 To 10
    cum_2(k) = tem_2 + wtfr_2(k)
    tem_2 = cum_2(k)
    cum(k) = tem + wtfr(k)
    tem = cum(k)
Next

Print #1, "Total number of crystals = " & crysno
For k = 0 To 10
    Print #1, "For x ("; k + 1; "); mass fraction under-size ="; wtfr(k)
    Print #1, "cumulative mass fraction ="; cum(k)
    Print #1, "NO. OF CRYSTALS ="; cr(k)
Next

Print #1,
Print #1, "Total number of crystals including breakage = " & crysno_2
For k = 0 To 10
    Print #1, "For x ("; k + 1; "); mass fraction undersize ="; wtfr_2(k)
    Print #1, "cumulative mass fraction ="; cum_2(k)
    Print #1, "NO. OF CRYSTALS ="; cr_2(k)
Next

End Sub

Sample of results (for m=4 and q=0.2)

Results of steady state MC Simulation
Sample size at steady state=100000
Breakage fraction=.2
m=4
Growth rate=.00125 mm/min
Average residence time of a crystal in the crystallizer=60 min

Total number of crystals = 100000
For x ( 1 ), mass fraction under-size = 1.87917693212762
cumulative mass fraction = 1.87917693212762
NO. OF CRYSTALS = 63271
For x ( 2 ), mass fraction under-size = 12.2246079490638
    cumulative mass fraction = 14.1037848811914
NO. OF CRYSTALS = 23173
For x ( 3 ), mass fraction under-size = 21.0311248412267
    cumulative mass fraction = 35.1349097224182
NO. OF CRYSTALS = 8647
For x ( 4 ), mass fraction under-size = 20.3870342264851
    cumulative mass fraction = 55.5219439489032

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NO. OF CRYSTALS = 3023
For x (5), mass fraction under-size = 17.0891888560525
    cumulative mass fraction = 72.6111328049557
NO. OF CRYSTALS = 1191
For x (6), mass fraction under-size = 11.401246711006
    cumulative mass fraction = 84.0123795159617
NO. OF CRYSTALS = 428
For x (7), mass fraction under-size = 7.28952908359445
    cumulative mass fraction = 91.3019085995562
NO. OF CRYSTALS = 169
For x (8), mass fraction under-size = 4.0796034250265
    cumulative mass fraction = 95.3815120245827
NO. OF CRYSTALS = 60
For x (9), mass fraction under-size = 2.51863625795787
    cumulative mass fraction = 97.9001482825406
NO. OF CRYSTALS = 25
For x (10), mass fraction under-size = 1.06700049093263
    cumulative mass fraction = 98.9671487734732
NO. OF CRYSTALS = 8
For x (11), mass fraction under-size = 1.0328512265268
    cumulative mass fraction = 100

NO. OF CRYSTALS = 5
Total number of crystals including breakage = 102451.3
For x (1), mass fraction under-size = 2.771816
    cumulative mass fraction = 2.771816
NO. OF CRYSTALS = 63385.43
For x (2), mass fraction under-size = 19.42438
    cumulative mass fraction = 22.1962
NO. OF CRYSTALS = 25008.2
For x (3), mass fraction under-size = 38.19913
    cumulative mass fraction = 60.39532
NO. OF CRYSTALS = 10667.05
For x (4), mass fraction under-size = 28.56125
    cumulative mass fraction = 88.95657
NO. OF CRYSTALS = 2876.4
For x (5), mass fraction under-size = 10.65121
    cumulative mass fraction = 99.60779
NO. OF CRYSTALS = 504.1703
For x (6), mass fraction under-size = .3922118
    cumulative mass fraction = 100
NO. OF CRYSTALS = 10
For x (7), mass fraction under-size = 0
    cumulative mass fraction = 100
NO. OF CRYSTALS = 0
For x (8), mass fraction under-size = 0
    cumulative mass fraction = 100
NO. OF CRYSTALS = 0
For x (9), mass fraction under-size = 0
    cumulative mass fraction = 100
NO. OF CRYSTALS = 0
For x (10), mass fraction under-size = 0
    cumulative mass fraction = 100
NO. OF CRYSTALS = 0
For x (11), mass fraction under-size = 0
    cumulative mass fraction = 100
cumulative mass fraction = 100
NO. OF CRYSTALS = 0
APPENDIX VI
Appendix VI.1

Experimental Procedure

1. Prepare a saturated solution of brine by heating a saturated solution along with excess NaCl and subsequently cooling it. Use granular NaCl\(^\text{\textdegree}\) (99.5% purity).

2. Dry the grinded granular sodium chloride at 40°C for 24 hours.

3. Sieve it at room temperature and note the screen fraction. Select the sieved salt with size fraction between 1400 to 1800µm.

4. Reconstitute the original NaCl by mixing the selected size fractions.

5. Fill the mixing rig with saturated NaCl solution (from step 1) and add the reconstitute NaCl to achieve 10% slurry.

6. Control the stirrer rpm at 200rpm.

7. Take a sample at an interval of 15min.

8. Allow the stirrer to run for 1 hour.

9. Repeat the run (steps 7 and 8) for different rpm, namely 300, 400 and 500rpm.

\(^\text{Source: J. T. Baker Inc. (CAS No. 7647-14-5; MERCK Index: 11, 8544)}\)
Appendix VI.2

Sample Program and Results for Simulation of the Experimental Results

Generated from the Coulter Counter -LS Particle Size Analyzer

Declaration

Dim L(1 To 92), y(1 To 92), yw(1 To 92) As Single
Dim ywf(1 To 92), ywb(1 To 92), ywc(1 To 92) As Single
Dim xb(1 To 92), xf(1 To 92), xc(1 To 92) As Single
Dim rpm, t, Lc, Lf, Lmax, top, bottom, statis As Single
Dim count, rms, tem As Single
Dim i, j As Single

Main Program

Sub main()

rpm = (Coulter.Text1.Text)
t = Val(Coulter.Text2.Text)
statis = Val(Coulter.Text3.Text)
Lf = 200
Lc = 500
Lmax = 1822
top = 0
bottom = 0
count = 0
tem = 1000

Open "exp2" For Output As #1
Print #1, "rpm=" & rpm
Print #1, "Duration=" & t; " hr"
Print #1, "Lf=" & Lf; " mu"
Print #1, "Lc=" & Lc; " mu"
Print #1, "Lmax=" & Lmax; " mu"

L(1) = .375: y(1) = 0
L(2) = .412: y(2) = 0
L(3) = .452: y(3) = 0
L(4) = .496: y(4) = 0
L(5) = .545: y(5) = 0

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L(6) = .598: y(6) = 0
L(7) = .657: y(7) = 0
L(8) = .721: y(8) = 0
L(9) = .791: y(9) = 0
L(10) = .869: y(10) = 0
L(11) = .953: y(11) = 0
L(12) = 1.047: y(12) = 0
L(13) = 1.149: y(13) = 0
L(14) = 1.261: y(14) = 0
L(15) = 1.385: y(15) = 0
L(16) = 1.52: y(16) = 0
L(17) = 1.669: y(17) = 0
L(18) = 1.832: y(18) = 0
L(19) = 2.01: y(19) = 0
L(20) = 2.207: y(20) = 0
L(21) = 2.423: y(21) = 0
L(22) = 2.66: y(22) = 0
L(23) = 2.92: y(23) = 0
L(24) = 3.206: y(24) = 0
L(25) = 3.519: y(25) = 0
L(26) = 3.862: y(26) = 0
L(27) = 4.241: y(27) = 0
L(28) = 4.656: y(28) = 0
L(29) = 5.111: y(29) = 0
L(30) = 5.611: y(30) = 0
L(31) = 6.158: y(31) = 0
L(32) = 6.761: y(32) = 0
L(33) = 7.421: y(33) = 0
L(34) = 8.147: y(34) = 0
L(35) = 8.944: y(35) = 0
L(36) = 9.819: y(36) = 0
L(37) = 10.78: y(37) = 0
L(38) = 11.83: y(38) = 0
L(39) = 12.99: y(39) = 0
L(40) = 14.26: y(40) = 0
L(41) = 15.65: y(41) = 0
L(42) = 17.18: y(42) = 0
L(43) = 18.86: y(43) = 0
L(44) = 20.7: y(44) = 0
L(45) = 22.73: y(45) = 0
L(46) = 24.95: y(46) = 0
L(47) = 27.38: y(47) = 0
L(48) = 30.07: y(48) = 0
L(49) = 33: y(49) = 0
L(50) = 36.24: y(50) = 0
L(51) = 39.77: y(51) = 0

294
L(52) = 43.66: y(52) = 0
L(53) = 47.93: y(53) = 0
L(54) = 52.63: y(54) = 0
L(55) = 57.77: y(55) = 0
L(56) = 63.41: y(56) = 0
L(57) = 69.62: y(57) = 0
L(58) = 76.43: y(58) = 0
L(59) = 83.9: y(59) = 0
L(60) = 92.09: y(60) = 0
L(61) = 101.1: y(61) = 0
L(62) = 111: y(62) = 0
L(63) = 121.8: y(63) = 0
L(64) = 133.7: y(64) = .0004
L(65) = 146.8: y(65) = .00304
L(66) = 161.2: y(66) = .023
L(67) = 176.8: y(67) = .071
L(68) = 194.2: y(68) = .136
L(69) = 213.2: y(69) = .212
L(70) = 234.1: y(70) = .296
L(71) = 256.8: y(71) = .394
L(72) = 282.1: y(72) = .518
L(73) = 309.6: y(73) = .679
L(74) = 339.8: y(74) = .882
L(75) = 373.1: y(75) = 1.128
L(76) = 409.6: y(76) = 1.431
L(77) = 449.7: y(77) = 1.83
L(78) = 493.6: y(78) = 2.409
L(79) = 541.9: y(79) = 3.309
L(80) = 594.9: y(80) = 4.719
L(81) = 653: y(81) = 6.889
L(82) = 716.9: y(82) = 10.049
L(83) = 786.9: y(83) = 14.409
L(84) = 863.9: y(84) = 20.099
L(85) = 948.2: y(85) = 27.139
L(86) = 1041: y(86) = 35.449
L(87) = 1143: y(87) = 44.839
L(88) = 1255: y(88) = 55.089
L(89) = 1377: y(89) = 65.939
L(90) = 1512: y(90) = 77.179
L(91) = 1660: y(91) = 88.639
L(92) = 1822: y(92) = 99.999

For kf = .001 To .1 Step .001
For kb = .001 To .1 Step .001
For i = 1 To 92
If y(i) > 0 And kb > kf Then
Sample of results (for 1 hour duration and 200 rpm)

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Sample of results (for 1 hour duration and 300 rpm)

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**Sample of results (for 1 hour duration and 400 rpm)**

\[ rpm = 400 \]
\[ Duration = 1 \text{ hr} \]
\[ L_f = 200 \text{ mu} \]
\[ L_c = 500 \text{ mu} \]
\[ L_{max} = 1822 \text{ mu} \]

\[ A = 0.161380377769634 \]
\[ \text{rms} = 9.99819373084631 \]
\[ kb = 9.70000000000001E-02 \]
\[ k_f = 0.016 \]

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APPENDIX VII
Appendix VII.1
Outline of Simulation Algorithm for CSD of Fragments due to Attrition

START

input Wp

sample ← 1
sample ← sample + 1

sample > 50 ?

yes

no

randomize \( \alpha \)
calculate \( \Gamma / K_r \)
calculate \( N \)

I > N ?

yes

I ← 1
I ← I + 1

no

calculate \( L_{\text{min}} \)
calculate \( L_{\text{max}} \)
calculate \( r_{\text{max}} \)
calculate \( a \)

randomize \( r \)
calculate \( L \)

find size interval \( J \) corresponding to \( L \)

\( w_I \leftarrow w_I + L^3 \)
\( w(J) \leftarrow w(J) + L^3 \)
\( cr(J) \leftarrow cr(J) + 1 \)

\( w_f(J) \leftarrow w(J) / w_I \)

print \( w_f(J), cr(J) \)

End
Appendix VII.2

Sample Program and Results for CSD of Fragments due to Attrition

Substances = MS

Declaration

Dim x(1 To 9) As Single
Dim cr(1 To 8), q(1 To 8) As Single
Dim wt(1 To 8), wtr(1 To 8) As Single
Dim substances, Hv, m, Wc, Wp, ratio As Single
Dim i, j As Single
Dim FracR, N As Single
Dim Amin, Amak, Aa, Ak, A, ran1 As Single
Dim Lmin, Lmak, L, rmin, rmak, r, slope, cons, konst, Va As Single
Dim crysnq, cryswt As Single
Dim xm, ym, xy, xmean, xsqrt, ymean, top, bottom As Single
Dim k, count As Integer

Main Program

Sub main()

substances = (mersmann.Text1.Text)
Hv = Val(mersmann.Text2.Text) * 1000000
m = Val(mersmann.Text3.Text) * 1000000000
Wc = Val(mersmann.Text4.Text) * .0000000001
Wp = Val(mersmann.Text5.Text) * .000001
ratio = Val(mersmann.Text6.Text)

Open "mer" For Output As #1
Print #1, "Substances=" & substances
Print #1, "Vicker Hardness=" & Hv; " Pa"
Print #1, "Quasi-isotropic Shear Modulus=" & m; " Pa"
Print #1, "Critical Work to form cracks=" & Wc; " J"
Print #1, "Impact Energy=" & Wp; " J"
Print #1, "Ratio of Efficiency Constant=" & ratio

Amin = .5236

301
Amak = 1
Aa = (Amak + Amin) / 2
Ab = (Amak - Amin) / 6

FracR = (ratio * Wc ^ (1 / 3) * Hv ^ (5 / 3)) / (5.2 * m)
Lmin = ((32 * m * FracR) / (3 * Hv ^ 2)
rmín = ((Lmin * Wp ^ (4 / 3) * Hv ^ (2 / 3)) / (3 * m * FracR)) ^ (1 / 4))
rmáx = ((Wp ^ (4 / 3) * Hv ^ (2 / 3)) / (2 * 3 * m * FracR)) ^ (1 / 3)
Lmak = rmax / 2
konst = ((2 / 3) * (m ^ 3) * (FracR ^ 3)) / (((.0007 ^ (4 / 3)) * (Hv ^ 6))
cons = 13 * Log(rmin)
x(1) = Lmin
x(9) = Lmak
Rem For i = 2 To 9
Rem x(i) = x(i - 1) + (Lmak - Lmin) / 9
Rem Next
x(2) = .000003
x(3) = .000005
x(4) = .00001
x(5) = .000015
x(6) = .00002
x(7) = .000025
x(8) = .00003

For i = 1 To 50
 Debug.Print i
 Randomize
 Ak = (-2 * (Ab ^ 2) * Log(Rnd)) ^ .5
 randl = Rnd
 If randl < .5 Then
   A = Aa - Ak
 Else
   A = Aa + Ak
 End If
 N = (.0007 * Wp * (Hv ^ 5)) / (A * m ^ 3 * FracR ^ 3)
 Debug.Print N
 For j = 1 To N
 Randomize
 200 r = Exp((cons - Log(Rnd)) / 13)
   L = (3 * m * FracR * r ^ 4) / (Wp ^ (4 / 3) * Hv ^ (2 / 3))
 If L > x(1) Then
   GoTo 250
 Else
   GoTo 200
 End If
 250 If L > x(2) Then
GoTo 260
Else
  k = 1
  GoTo 1000
End If
260 If L > x(3) Then
  GoTo 270
Else
  k = 2
  GoTo 1000
End If
270 If L > x(4) Then
  GoTo 280
Else
  k = 3
  GoTo 1000
End If
280 If L > x(5) Then
  GoTo 290
Else
  k = 4
  GoTo 1000
End If
290 If L > x(6) Then
  GoTo 300
Else
  k = 5
  GoTo 1000
End If
300 If L > x(7) Then
  GoTo 310
Else
  k = 6
  GoTo 1000
End If
310 If L > x(8) Then
  GoTo 320
Else
  k = 7
  GoTo 1000
End If
320 If L > x(9) Then
  GoTo 200
Else
  k = 8
  GoTo 1000

303
End If

1000cr(k) = cr(k) + 1
wt(k) = wt(k) + L ^ 3
Next
Va = Va + konst * (N * A) ^ (4 / 3)
Next

For k = 1 To 8
   crysno = crysno + cr(k)
cryswt = cryswt + wt(k)
Next

For k = 1 To 8
   q(k) = (cr(k) / crysno) * 100
   wtfr(k) = (wt(k) / cryswt) * 100
Next

Print #1, "Total number of fragments for 50 samples =" & crysno
Print #1, "Total volume removed by attrition per sample =" & Va / 50; " m3"
Print #1, "Lmin =" & x(1); " rmin =" & rmin
Print #1, "Lmax =" & x(9); " rmax =" & rmax
Print #1,
For k = 1 To 8
   Print #1, "For x (", k; ",") ="; x(k); ", mass fraction above-size ="; wtfr(k)
   Print #1, "Number fraction above-size ="; q(k)
   If cr(k) = 0 Then
      Print #1, "No. of fragments = 0"
   Else
      Print #1, "No. of fragments ="; cr(k); ", Ln = "; Log(cr(k))
   End If
Next
Print #1,

For k = 1 To 8
   If cr(k) = 0 Then
      GoTo 1
   Else
      count = count + 1
      xm = xm + Log(x(k))
xsqrt = xsqrt + Log(x(k)) ^ 2
      ym = ym + Log(cr(k))
      xy = xy + Log(x(k)) * Log(cr(k))
   End If
   Count
   xmean = xm / count

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Sample of results (for impact energy 1E-4J and normally distributed shape factor)

Substances=MS
Vicker Hardness=649000000 Pa
Quasi-isotropic Shear Modulus=9060000000 Pa
Critical Work to form cracks=.0000000048 J
Impact Energy=.0001 J
Ratio of Efficiency Constant=.5
Total number of fragments for 50 samples =1080871
Total volume removed by attrition per sample =2.93957E-11 m³
Lmin=1.998301E-06 rmin=7.36170880796748E-05
Lmax=9.721006E-05 rmax=1.9442012384607E-04

For x ( 1 ) = 1.998301E-06 , mass fraction above-size = 15.94178
Number fraction above-size = 73.40636
No. of fragments = 793428 , Ln = 13.5841180775917
For x ( 2 ) = .000003 , mass fraction above-size = 17.75184
Number fraction above-size = 21.55835
No. of fragments = 233018 , Ln = 12.3588709827828
For x ( 3 ) = .000005 , mass fraction above-size = 20.66718
Number fraction above-size = 4.507013
No. of fragments = 48715 , Ln = 10.7937422698594
For x ( 4 ) = .00001 , mass fraction above-size = 10.7206
Number fraction above-size = .3918136
No. of fragments = 4235 , Ln = 8.35113860708615
For x ( 5 ) = .000015 , mass fraction above-size = 6.506121
Number fraction above-size = 8.261856E-02
No. of fragments = 893 , Ln = 6.7945865808765
For x ( 6 ) = .00002 , mass fraction above-size = 4.964769
Number fraction above-size = 2.895813E-02
No. of fragments = 313 , Ln = 5.74620319054015
For x ( 7 ) = .000025 , mass fraction above-size = 3.83447
Number fraction above-size = .0118423
No. of fragments = 128 , Ln = 4.85203026391962
For x ( 8 ) = .00003 , mass fraction above-size = 4.977971
Number fraction above-size = 8.141582E-03
No. of fragments = 88 , Ln = 4.47733681447821
For x ( 9 ) = .00004 , mass fraction above-size = 7.465596
Number fraction above-size = 4.070791E-03
No. of fragments = 44 , Ln = 3.78418963391826
For x ( 10 ) = .00006 , mass fraction above-size = 7.169665
Number fraction above-size = 8.326618E-04
No. of fragments = 9 , Ln = 2.19722457733622
Slope = -3.39262492684397

Substances = PA

Declaration

Dim x(1 To 9) As Single
Dim cr(1 To 8), q(1 To 8) As Single
Dim wt(1 To 8), wtfr(1 To 8) As Single
Dim substances, Hv, m, Wc, Wp, ratio As Single
Dim i, j As Single
Dim FracR, N As Single
Dim Amin, Amak, Aa, Ak, A, ran1 As Single
Dim Lmin, Lmak, L, rmin, rmak, r, slope, cons, konst, Va As Single
Dim crysvno, cryswt As Single
Dim xn, ym, xy, xmean, xsqrt, ymean, top, bottom As Single
Dim k, count As Integer

Main Program

Sub main()

substances = (mersmann.Text1.Text)
Hv = Val(mersmann.Text2.Text) * 1000000
m = Val(mersmann.Text3.Text) * 1000000000
Wc = Val(mersmann.Text4.Text) * .0000000001
Wp = Val(mersmann.Text5.Text) * .000001
ratio = Val(mersmann.Text6.Text)

Open "mer" For Output As #1
Print #1, "Substances=" & substances
Print #1, "Vicker Hardness=" & Hv; " Pa"
Print #1, "Quasi-isotropic Shear Modulus=" & m; " Pa"
Print #1, "Critical Work to form cracks=" & Wc; " J"
Print #1, "Impact Energy=" & Wp; " J"
Print #1, "Ratio of Efficiency Constant=" & ratio

Amin = .5236
Amak = 1
Aa = (Amak + Amin) / 2
Ab = (Amak - Amin) / 6
FracR = (ratio * Wc ^ (1 / 3) * Hv ^ (5 / 3)) / (5.2 * m)
Lmin = (32 * m * FracR) / (3 * Hv ^ 2)
rmin = (((Lmin * Wp ^ (4 / 3) * Hv ^ (2 / 3)) / (3 * m * FracR)) ^ (1 / 4))
rmax = ((Wp ^ (4 / 3) * Hv ^ (2 / 3)) / (2 * 3 * m * FracR)) ^ (1 / 3)
Lmak = rmax / 2
konst = ((2 / 3) * (m ^ 3) * (FracR ^ 3)) / ((.0007 ^ (4 / 3)) * (Hv ^ 6))
cons = 13 * Log(rmin)
x(1) = Lmin
x(9) = Lmak
Rem For i = 2 To 9
Rem x(i) = x(i - 1) + (Lmak - Lmin) / 9
Rem Next
x(2) = .000003
x(3) = .000005
x(4) = .00001
x(5) = .000015
x(6) = .00002
x(7) = .000025
x(8) = .00003

For i = 1 To 50
Debug.Print i
Randomize
Ak = (-2 * (Ab ^ 2) * Log(Rnd)) ^ .5
rand1 = Rnd
If rand1 < .5 Then
    A = Aa - Ak
Else
    A = Aa + Ak
End If
Rem A = .5236
N = (.0007 * Wp * (Hv ^ 5)) / (A * m ^ 3 * FracR ^ 3)
Debug.Print N
For j = 1 To N
Randomize
200  r = Exp((cons - Log(Rnd)) / 13)
    L = (3 * m * FracR * r ^ 4) / (Wp ^ (4 / 3) * Hv ^ (2 / 3))
If L > x(1) Then
    GoTo 250
Else
    GoTo 200
End If
250  If L > x(2) Then
    GoTo 260
Else
    k = 1
GoTo 1000
End If

260 If L > x(3) Then
   GoTo 270
Else
   k = 2
   GoTo 1000
End If

270 If L > x(4) Then
   GoTo 280
Else
   k = 3
   GoTo 1000
End If

280 If L > x(5) Then
   GoTo 290
Else
   k = 4
   GoTo 1000
End If

290 If L > x(6) Then
   GoTo 300
Else
   k = 5
   GoTo 1000
End If

300 If L > x(7) Then
   GoTo 310
Else
   k = 6
   GoTo 1000
End If

310 If L > x(8) Then
   GoTo 320
Else
   k = 7
   GoTo 1000
End If

320 If L > x(9) Then
   GoTo 200
Else
   k = 8
   GoTo 1000
End If

1000 cr(k) = cr(k) + 1
\[ wt(k) = wt(k) + L^3 \]

Next

\[ Va = Va + \text{konst} \times (N \times A)^{(4/3)} \]

Next

For \( k = 1 \) To 8
\[ cyrsno = cyrsno + cr(k) \]
\[ cryswt = cryswt + wt(k) \]
Next

For \( k = 1 \) To 8
\[ q(k) = (cr(k) / cyrsno) \times 100 \]
\[ wtfr(k) = (wt(k) / cryswt) \times 100 \]
Next

Print #1, "Total number of fragments for 50 samples =" & cyrsno
Print #1, "Total volume removed by attrition per sample =" & Va / 50; " m3"
Print #1, "Lmin =" & x(1); " rmin =" & rmin
Print #1, "Lmax =" & x(9); " rmax =" & rmax
Print #1,
For \( k = 1 \) To 8
Print #1, "For x ("; k; ") ="; x(k); ", mass fraction above-size ="; wtfr(k)
Print #1, "Number fraction above-size ="; q(k)
If cr(k) = 0 Then
Print #1, "No. of fragments = 0"
Else
Print #1, "No. of fragments ="; cr(k); ", Ln = "; Log(cr(k))
End If
Next
Print #1,
For \( k = 1 \) To 8
If cr(k) = 0 Then
    GoTo 1
Else
    count = count + 1
    xm = xm + Log(x(k))
    xsqrt = xsqrt + Log(x(k)) \times 2
    ym = ym + Log(cr(k))
    xy = xy + Log(x(k)) \times Log(cr(k))
End If
1 Next
xmean = xm / count
ymean = ym / count
bottom = xsqrt - count \times xmean \times 2
top = xy - count \times xmean \times ymean

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slope = top / bottom
Print #1, "Slope ="; slope

End Sub

Sample of results (for impact energy 1E-4J and normally distributed shape factor)

Substances=PA
Vicker Hardness=754000000 Pa
Quasi-isotropic Shear Modulus=7960000000 Pa
Critical Work to form cracks=.0000000007 J
Impact Energy=.0001 J
Ratio of Efficiency Constant=.5
Total number of fragments for 50 samples =7628530
Total volume removed by attrition per sample =4.80697E-11 m3
Lmin =1.000547E-06 rmin =7.00276821630033E-05
Lmax =1.145268E-04 rmax =2.29053650390034E-04

For x ( 1 ) = 1.000547E-06 , mass fraction above-size = 34.78629
Number fraction above-size = 97.18503
No. of fragments = 7413789 , Ln = 15.8188522026169
For x ( 2 ) = .000003 , mass fraction above-size = 13.17693
Number fraction above-size = 2.279495
No. of fragments = 173892 , Ln = 12.0661896958339
For x ( 3 ) = .000005 , mass fraction above-size = 15.36704
Number fraction above-size = .4786505
No. of fragments = 36514 , Ln = 10.5054510276735
For x ( 4 ) = .00001 , mass fraction above-size = 7.862236
Number fraction above-size = 4.146277E-02
No. of fragments = 3163 , Ln = 8.05927622330565
For x ( 5 ) = .000015 , mass fraction above-size = 5.257358
Number fraction above-size = 9.451362E-03
No. of fragments = 721 , Ln = 6.58063913728495
For x ( 6 ) = .00002 , mass fraction above-size = 3.616263
Number fraction above-size = 3.014998E-03
No. of fragments = 230 , Ln = 5.4380793089232
For x ( 7 ) = .000025 , mass fraction above-size = 3.306294
Number fraction above-size = 1.428847E-03
No. of fragments = 109 , Ln = 4.69134788222914
For x ( 8 ) = .00003 , mass fraction above-size = 4.096673
Number fraction above-size = 9.044993E-04
No. of fragments = 69 , Ln = 4.23410650459726
For x ( 9 ) = .00004 , mass fraction above-size = 5.638319
Number fraction above-size = 4.194779E-04
No. of fragments = 32 , Ln = 3.46573590279973
For x ( 10 ) = .00006 , mass fraction above-size = 4.217582
Number fraction above-size = 1.179782E-04
No. of fragments = 9 , Ln = 2.19722457733622
For x ( 11 ) = .00008 , mass fraction above-size = 2.675012
Number fraction above-size = 2.621737E-05
No. of fragments = 2 , Ln = .693147180559945
Slope =-3.40685649605822

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OVERALL CONCLUSIONS AND RECOMMENDATIONS
OVERALL CONCLUSIONS

Mathematical analysis of transient CSD and attrition phenomena is a difficult task. The advantage of the MC simulation lies in the fact that the CSD can be computed in terms of weight fraction of the crystals, which is mathematically difficult by usual moment transformation of population balance. The scheme can also be easily implemented on a personal computer, as the demand on its memory is very small.

In transient CSD analysis, the simulation scheme is attractive due to its simple algorithm, i.e. free from iterative calculations and related convergence problems. The simulated transient CSD can account for size-dependent growth. It could also incorporate improper mixing conditions under all possible dispersion effects. The agreement among the predicted results and available experimental data confirm the validity of the proposed scheme.

In the attrition process, MC offers an alternative to the conventional deterministic approach in solving the Random Breakage model, described by a second order ODE, as proposed by RANDOLPH (1969). In addition, a comparison between experimental CSD results under attrition conditions with the MC simulated CSD, which employed the model proposed by MAZZAROTTA (1992) and BISCANS et al. (1996), indicate good agreement. To further enhance its credibility, MC methods was also successfully incorporated into GAHN and MERSMANN (1997) model for generating
the fragment size distribution resulted from attrition. The simulated results account for volume shape factor dispersion which GAHN and MERSMANN (1997) assumed a value of 1.

From the practical aspects, the work done has contributed to the design of industrial crystallizer that is still teeming with empiricism due to lack of reliable knowledge on the crystal size distribution. Industrial and commercial products such as sugar, common salt, urea and other fertilizers, which are marketed in the crystal form, have to be produced now in various size ranges to meet varied consumer demand. Detailed knowledge on crystal size distribution and the factors that influencing it will allow the engineers to design and operate crystallizers successfully and economically. As it is not always possible to perform lab or pilot-scale studies to test out a chosen configuration, simulation schemes such as the present work are used. The economy and time factor in such scheme make them attractive to designers and operators. The findings in the present work show that MC technique in association with the appropriate physical and mathematical models can be used to predict the CSD reliably for various crystallizer configurations and operating conditions. The results indicate that the MC technique simulates closely the randomness of the crystallization process.
RECOMMENDATIONS

Some of the recommendations that could be thought off to ensure the continuity of the current work are outlined below:

1. More experimental work needs to be carried out as there are limited published data being reported in the field of crystallization. The experimental work should emphasized on the transient CSD in which the MC simulation results could be tested and compared against the actual transient data.

2. In addition to attrition, the effect of agglomeration on the CSD should be considered in the simulation schemes, as one could not discount it in the actual crystallization processes.

3. The sensitivity of solubility on the CSD should be explored and incorporated into the MC simulation schemes.
NOMENCLATURE
NOMENCLATURE

$A'$  
growth constant (m/s)

$a$  
constant

$a'$  
constant

$a''$  
defined in Eq. (1.139)

$a_i$  
defined in Eq. (1.124)

$a$  
characteristic size of the plastic zone (m)

$a_p$  
liquid-solid contact area of a particle (m$^2$/m$^3$)

$B$  
break terms representing crystal breakage

$B'$  
constant

$b$  
ASL parameter

$C$  
constant

$c$  
concentration, kg solute/kg solution

$C$  
constant, [Chap. 7]

$c^*$  
equilibrium concentration, kg solute/kg solution

$c_1, c_2, c_3$  
constant

$D$  
diameter of a crystal (m, mm)

death term representing crystal breakage, [Chap 5]

$D_G$  
growth rate diffusivity (m$^2$/s)

$D_i$  
diffusivity (m$^2$/s)

$D_f$  
fluid phase diffusivity (m$^2$/s)
$D_i$  

diameter of the $i$ th. crystal (m, mm)

$D_{ik}$  

diameter of the $i$ th. crystal, measured after the $k$ th. crystallizer (m, mm)

$D_{il}$  

diameter of the $i$ th. crystal, formed by nucleation in the $l$th. crystallizer and measured after the $k$ th. crystallizer (m, mm)

$E$  

step operator

$E$  

mean

$F(S)$  

function of supersaturation in Eqs (1.96), (1.97) and (1.100).

$F_1, F_2$  

growth rate parameters defined in Eqs. (1.25) and (1.26)

$f_1(\phi), f_2(\phi), f_3(\phi)$  

function defined in Eqs. (AI.6.3), (AI.6.7) and (AI.6.8) respectively

$f(g_i), f(G_i)$  

function defined by Eqs. (1.28) and (1.29)

$f(j)$  

function defined by Eqs. (1.112) and (1.113)

$f(x)$  

density function as defined by Eqs. (AI.3.1)

$f_1, f_2$  

defined in Eqs. (1.20) and (1.21)

$f_y$  

defined in Eq. (1.78)

$f_i$  

defined in Eq. (1.79)

$G(L)$  

function of crystal length in Eq. (1.96)

$G$  

growth rate of a crystal (m/s, mm/hr)

linear crystal growth rate, [Chap. 5] (mm/min)

$G$  

average growth rate of a crystal (m/s, mm/hr)

$G_M$  

maximum growth rate (m/s, mm/hr, mm/min)

$G_{mk}$  

maximum growth rate of a crystal in the $k$ th. crystallizer (m/s, mm/hr)


$G_O$  
minimum growth rate (m/s, mm/hr, mm/min)

$G_{ok}$  
minimum growth rate of a crystal in the $k$ th. crystallizer (m/s, mm/hr)

$G_i$  
growth rate of the $i$ th. crystal (m/s, mm/hr, mm/min)

$G_{ik}$  
growth rate of the $i$ th. crystal in the $k$ th. crystallizer (m/s, mm/hr)

$G_o$  
growth rate of crystals at zero size (m/s)

$g_i$  
instantaneous growth rate of the $i$ th. crystal (m/s)

$H(j)$  
function of states $j$ in Eqs (1.96), (1.101), (1.107) and (1.109)

$H$  
dynamic hardness (Pa)

$H_v$  
Vickers hardness (Pa)

$h$  
step size

$I$  
value of the integral defined in Eq. (1.73)

$i$  
index for a crystal

$j$  
index for mesh size interval

$I,j,k,k_1,k_2,...$  
index for piece [Eqs. (1.74) through (1.79)]

different states in the Markov process [Eqs. (1.82) through (1.90), Eqs(A.4.1) through A.4.8)]

$K$  
volume shape factor of a crystal

as defined in Eq. (1.71)

$\bar{K}$  
average volume shape factor of a crystal

$K_1,K_2$  
reaction rate constants

$K_M$  
maximum volume shape factor

$K_{mk}$  
maximum volume shape factor in the $k$ th. crystallizer
\( K_0 \) minimum volume shape factor

\( K_{ok} \) minimum volume shape factor in the \( k \) th. crystallizer

\( K_i \) volume shape factor of the \( i \) th. crystal

\( K_{ik} \) volume shape factor of the \( i \) th. crystal in the \( k \) th. crystallizer

\( K_r \) efficiency of stress field created by the crystal-impeller impact

\( k \) constant

\( k \) number of crystallizers in series

\( k' \) constant

\( k^* \) as defined in Eq. (1.1)

\( k_1, k_2, k_3, k_4 \) reaction rate constants \([\text{Eqs. (1.97) through (1.101), Eqs (1.108), (1.110) and (1.111)}] \)

\( k_b \) fraction of attrition given by breakage mechanism

\( k_c \) fraction of attrition given by abrasion mechanism generating coarse fragments

\( k_d \) volume diffusion rate constant (m/s)

\( k_f \) fraction of attrition given by abrasion mechanism generating fine fragments

\( k_n \) no. of pieces in a given size range

\( L \) birth size of a crystal (m)

crystal size, \([\text{Eqs. (1.68), (1.70), (1.72) through (1.75), (1.96), (1.107), (1.109), Chap. 2 and 6} (m)\]

linear crystal dimension, \([\text{Chap. 5} (mm)\]
fragment size, [Chap. 7] (m)

$L'$
characteristic size for Broadbent-Callcott expressions, corresponding to
maximum size distribution under examination (μm)

$L'$
characteristic size of a crystal (m)

$L_{m}$
mean (arithmetic mean) birth size of a crystal (mm)

$L_i$
birth size of the $i$ th. crystal (mm, m)

size of the $i$ th. crystal, [Eq. (1.80), Chap. 2] (m)

$L_{j}$
linear crystal dimension of the $i$ th. crystal, [Chap. 5] (mm)

$L_{j}$
birth size of the $i$ th. crystal in the $l$ th. crystallizer (mm, m)

$L_{c}$
characteristic length of a crystal in state $j$ for the Markov process

$L_{f}$
minimum size of coarse fragments generated by abrasion (μm)

$L_{f}$
maximum size of fine fragments generated by abrasion (μm)

$L_{\text{max}}$
maximum size of a fragment (m)

$L_{\text{min}}$
minimum size of a fragment (m)

$l$
summation index

$l'$
number of trials

$M(t)$
number of available solutes molecules at time $t$

$M$
number of available solutes molecules

$M$
total number of states in the Markov process

defined in Eq. (1.125)

$m$
number of seeds generated in the crystallizer initially

$m(0)$
number of mesh size intervals in which sampled crystals are classified

$m$
number of mesh size intervals in which sampled crystals are classified
mean as defined in Eq. (AI.3.4)

parameter indicative of the breakage rate, [Chap. 5]

mean volume shape factor, [Chap. 7]

\(m_j\)
sample size in the \(j\)th piece

\(N_j(t)\)
number of crystals in state \(j\) at time \(t\)  [Eqs. (1.105) and (1.106), Appendix I.5]

\(N\)
number of crystallization stages

total number of fragments, [Chap. 7]

\(N'\)
number of fluid elements in a CSTR

\(N_M\)
maximum nucleation rate (1/min)

\(N_O\)
minimum nucleation rate (1/min)

\(N_T\)
nucleation rate at time \(T\) (1/min)

\(n(L)\)
crystal population density at crystal size \(L\) (no./kg.m)

\(n(L')\)
crystal population density at crystal size \(L'\) (no./kg.m)

\(n\)
sample size of the crystals

\(n\)-steps in the Markov process [Eqs. (1.82 through (1.90))]

population density, [Chap. 2 and 5] (1/m⁴, 1/mm⁴)

distribution modulus, [Chap. 6]

number of crystals in the Markov process [Appendix I.5]

\(n'\)
size population density (no./kg.m)

\(n'\)
number of crystals in sample, formed by nucleation in the \(l\)th crystallizer

\(n^o\)
nuclei density (1/mm⁴)

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$n'_{j}$  number of crystals formed by nucleation in the $l$th. crystallizer and grown into the $j$th. size interval

$n_{1}, n_{2}, ..., n_{m}$  number of crystals in 1, 2, ..., $m$ mesh size intervals

$n_{1}, n_{2}, ..., n_{M}$  number of crystals in state 1, 2, ..., $M$ in the Markov process [Eqs. (1.102) and (1.103), Appendix I.5]

$n_{b}$  modulus of size distribution generated by breakage mechanism

$n_{c}$  modulus of size distribution generated by coarse fragments generated by abrasion mechanism

$n_{f}$  modulus of size distribution generated by fine fragments generated by abrasion mechanism

$n_{j}$  number of crystals in the $j$th. size interval

$n_{o}$  crystal population density at zero size (no./kg.m)

$P_{j}(t)$ or $P(j,t)$  probability of a crystal to be in state $j$, $j = 1, 2, ..., M$ at time $t$

$P^{n}$  transition probability matrix

$P^{n}$  $n$-steps transition probability matrix

$P_{g}^{n}$  conditional probability that random variable $X$, starting from state $i$ will be in state $j$ after $n$-steps

$P_{g}$  conditional probability that random variable $X$, starting from state $i$ will be in state $j$ after one-step

$p$  size range

$q_{o}(L)$  number density distribution ($m^{-1}$)

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<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>( q(x,t) )</td>
<td>probability function with one-to-one correspondence to ( P(j,t) )</td>
</tr>
<tr>
<td>( q )</td>
<td>fractional breakage at ( x = 3 ) relative to product removal</td>
</tr>
<tr>
<td>( RND )</td>
<td>random number, range 0 to 1</td>
</tr>
<tr>
<td>( r )</td>
<td>distance from the peak of the cone (m)</td>
</tr>
<tr>
<td>( \bar{r} )</td>
<td>mean pore radius</td>
</tr>
<tr>
<td>( r_{\text{max}} )</td>
<td>maximum distance from the peak of the cone to the newly created surface (m)</td>
</tr>
<tr>
<td>( r_{\text{min}} )</td>
<td>minimum pore radius</td>
</tr>
<tr>
<td>( S )</td>
<td>dummy variable of integration</td>
</tr>
<tr>
<td>( s )</td>
<td>standard deviation for volume shape factor</td>
</tr>
<tr>
<td>( T )</td>
<td>non-negative integers</td>
</tr>
<tr>
<td>( T )</td>
<td>residence time of a crystal in a crystallizer (hr, min, s)</td>
</tr>
<tr>
<td>( T_i )</td>
<td>residence time of the ( i ) th. crystal (hr, min, s)</td>
</tr>
<tr>
<td>( T_{ik} )</td>
<td>residence time of the ( i ) th. crystal in the ( k ) th. crystallizer (hr, min, s)</td>
</tr>
<tr>
<td>( t )</td>
<td>time (hr, min, s)</td>
</tr>
<tr>
<td>( u )</td>
<td>dye concentration in a capillary</td>
</tr>
<tr>
<td>( v )</td>
<td>( \nu )-steps in the Markov process</td>
</tr>
<tr>
<td>( V )</td>
<td>total volume of crystallizer (m(^3))</td>
</tr>
<tr>
<td>( V_M )</td>
<td>volume at perfect mixing in crystallizer (m(^3))</td>
</tr>
<tr>
<td>( V_p )</td>
<td>volume at plug flow in crystallizer (m(^3))</td>
</tr>
<tr>
<td>( V_a )</td>
<td>total volume removed by attrition (m(^3))</td>
</tr>
</tbody>
</table>
\( V_{el} \) volume removed in the elastic region (m\(^3\))

\( Var \) variance

\( W \) total weight of \( n \) crystals (g)

\( W_k^j \) mass fraction of crystals in the \( j \)th. size interval measured after the \( k \)th. crystallizer

\( W_{lk}^j \) mass fraction of crystals formed by nucleation in the \( l \)th. crystallizer, grown into the \( j \)th. size interval and measured after the \( k \)th. crystallizer

\( W_c \) critical work to form crack (J)

\( W_i \) weight of the \( i \)th. crystal (g)

\( W_{ik} \) weight of the \( i \)th. crystal in the \( k \)th. crystallization unit (kg)

\( W_p \) impact energy (J)

\( w \) cumulative weight distributions (g)

\( \dot{w} \) differential weight distributions (g)

\( \ddot{w} \) second order differential equation for cumulative weight distributions (g)

\( w_p \) cumulative mass fraction of crystals in the \( p \)th. size range

\( w_T \) total weight of a sample size of crystal (g)

\( WF_j \) weight fraction of the crystals in the \( j \)th. size interval

\( WC_j \) cumulative weight fraction of the crystals up to the \( j \)th. size interval

\( X \) random variable

\( X_1, X_2, \ldots, X_m \) random vector
\( X_t \) or \( X(t) \)  
random variable in the Markov process at time \( t \)

\( x \)  
dimensionless size

realization of the random variable, \( X \), [Eqs. (1.112) through (1.143), (A1.6.1) through (A1.6.8)]

dimensionless size, \( L/L^* \), [Chap. 6]

\( \langle x \rangle \)  
mean of the random variable \( X \)

\( \langle x^2 \rangle \)  
second moment of the random variable \( X \)

\( x_1, x_2, \ldots, x_M \)  
probability mass fraction, [Appendix I.5]

\( y \)  
dimensionless population density

defined in Eq. (1.126). It represents the ratio between the fluctuating and macroscopic components of \( j \) as indicated in Eq. (1.117).

\( \dot{y} \)  

\[ \frac{dy}{dx} \]

\( y_w \)  
overall cumulative size distribution of fragments generated by attrition

\( y_{w,b} \)  
cumulative size distribution of fragments generated by breakage mechanism

\( y_{w,c} \)  
cumulative size distribution of coarse fragments generated by abrasion mechanism

\( y_{w,f} \)  
cumulative size distribution of fine fragments generated by abrasion mechanism
**GREEK LETTERS**

\[ \alpha \]
volume shape factor

\[ \alpha_1, \alpha_2, \ldots, \alpha_k \]
crystal size in the 1, 2, \ldots, k piece

\[ \alpha_{\text{max}} \]
maximum volume shape factor

\[ \alpha_{\text{min}} \]
minimum volume shape factor

\[ \alpha_n \]
normally distributed volume shape factor, ranges from \( \alpha_{\text{min}} \) to \( \alpha_{\text{max}} \)

\[ \beta_m \]
dimensionless parameter indicative of the importance of breakage in the system

\[ \delta \]
segment length in each state for the Markov process

\[ \phi(t) \]
deterministic function of \( t \) representing the macroscopic trajectory of crystal size which gives rise to a sharp maximum in the value of \( P(j,t) \) around the \( \Omega \phi(t) \) [Eqs. (1.112) through (1.143), Appendix I.6]

\[ \phi \]
polar coordinates (rad)

\[ \gamma \]
as defined in Eq. (1.69) (m\(^{-1}\))

\[ \gamma_0, \gamma_1, \gamma_2, \ldots, \gamma_k \]
random numbers

\[ \kappa \]
parameter \( \kappa = 0.5 \)

\[ \lambda \]
transition intensities among the states in the Markov process

\[ \lambda_j(t) \]
transition intensity (probability of migration) in state \( j \) at time \( t \) used in the Markov process

\[ \lambda(L) \]
initial size population density function
\( \lambda_1 \) scale parameter for the initial size population density function

\( \lambda_2 \) shape parameter for the initial size population density function

\( \mu \) shear modulus (Pa)

\( \mu_{VRH} \) quasi-isotropic shear modulus (Pa)

\( o(\Delta t) \) higher-order term of the transition intensities, \( \lambda \)

\( \theta \) polar coordinates (rad)

\( \theta' \) mean residence time of crystals (s)

\( \theta^*(T) \) residence time distribution function

\( \rho \) crystal density (g/m³, g/mm³, kg/mm³)

\( \sigma \) relative supersaturation, \((c - c^*) / c^*\)

mean square deviation of growth rate [Eqs. (1.29), (1.32) and (1.33)] (m²/s²)

stress, [Chap. 7] (Pa)

standard deviation [Eq. (A1.3.5)]

\( \sigma' \) relative supersaturation, \((c - c^*) / c^*\)

\( \sigma_i \) mean square deviation of growth rate of the \( i \) th. crystal (m²/s²)

\( \Gamma \) fracture resistance (J/m²)

\( \tau \) mean residence time of crystals in a crystallizer (hr, min, s)

residence time in an imperfectly mixed crystallizer, [Chap. 3 and 4] (hr, min)

\( \overline{\tau_N} \) mean residence time of crystals in the \( N \) th. stage (min)

\( \tau_M \) mean residence time of crystals in perfectly mixed region (min)
\( \tau_p \)  
mean residence time of crystals in plug flow region (min)

\( \tau_k \)  
mean residence time of crystals in the \( k \) th. crystallizer (hr, s)

\( \psi(G) \)  
growth rate population density function

\( \nu_1 \)  
shape parameter for the growth rate population density function

\( \xi_{i,j} \)  
random number generated for the \( i \) th. crystal in the \( j \) th. piece

random number [Eqs. (1.76) and (1.78)]

\( \xi_{i,N} \)  
random number generated for the \( i \) th. crystal in the \( N \) th. stage

\( \Omega \)  
maximum obtainable crystal size
FLOW CHART

NOTATION
FLOW CHART NOTATION

a  characteristic size of the plastic zone (m)

Beta  dimensionless parameter indicative of the importance of breakage in the system

CRYSN0 (T, SIZE)  counts the number of crystals in a particular size class at time T

CRYSWT(T, SIZE)  records the mass of crystals in a particular size class at time T (g)

cr(J)  number of fragments in the J th. size classes

cr(k)  count the number of crystals in size class k before birth and death function consideration

cr_2(k)  count the number of crystals in size class k after birth and death function consideration

cryswt  total weight of sample crystals before birth and death consideration (g)

cryswt_2  total weight of sample crystals after birth and death consideration (g)

DI(T)  diameter of the i th. crystal at time T (mm)

D(k)  number of crystals death in size class k

Di  diameter of the i th. crystal (mm)

F1  growth rate parameter defined in Eq. (1.25) for $F_1$ (mm$^2$/min)

F2  growth rate parameter defined in Eq. (1.26) for $F_2$ (mm$^{2.5}$/min)

G  average growth rate of a crystal, [Appendix III.1 and IV.1] (mm/hr); growth rate of crystal, [Appendix V.2] (mm/min)

GI  growth rate of the i th. crystal (mm/min)

GM  maximum growth rate (mm/min)

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GO  minimum growth rate (mm/min)
I  counter for the number of crystals, [Appendix V.1];
counter for the number of fragments, [Appendix VII.1]
J  particle size classes
K  shape factor of crystal
KI  shape factor of the $i$ th. crystal
KM  maximum shape factor
KO  minimum shape factor
$K_r$  efficiency of stress field created by the crystal-impeller impact
k  number of size classes or mesh
L  fragment size (m)
LI  birth size of the $i$ th. crystal (mm)
LM  mean (arithmetic mean) birth size of a crystal) (mm)
L(k)  linear crystal dimension in size class $k$ (mm)
Lmin  minimum size of a fragment (m)
Lmak  maximum size of a fragment (m)
m  parameter indicative of the breakage rate
N  number of stages for DTB crystallizers, [Appendix III.1 and IV.1]
$N$  total number of fragments, [Appendix VII.1]
NCOUNT  nucleation counter, range 1 to NRATE
NM  maximum nucleation rate (nuclei/min)
NO  minimum nucleation rate (nuclei/min)
NRATE  nucleation rate (nuclei/min)

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p  crystal density (g/mm$^3$)
q  fractional breakage at $x = 3$ relative to product removal
R  ratio of plug flow regions to total volume of crystallizer for FC crystallizers
RND  random number, range 0 to 1
r  distance from the peak of the cone (m)
rmak  maximum distance from the peak of the cone to the newly created surface (m)
SIZE  size-classes, [Appendix III.1 and IV.1] (mm)
sample  sample size of crystal in a steady state run, [Appendix I.1 and V.1]; number of crystals used in simulation, [Appendix VIII.1]
size  diameter of the $i$th. crystal, [Appendix V.1] (mm)
TAU  average residence time of crystals in a crystallizer (hour, min)
TCOUNT  time counter, range 1 min to TSCAN (min)
TI  residence time of the $i$th. crystal on a continuous time scale, range TCOUNT to TTOT. (TI = -TAU.LOG(RND)), [Appendix III.1 and IV.1] (min)
TM  mean residence time of crystals in perfectly mixed region (min)
TP  mean residence time of crystals in plug flow region (min)
TSCAN  predetermined search time for observation of CSD in a continuous crystallizer. It is arbitrarily fixed at 13 times the average residence time of crystals in the crystallizer (TSCAN = 13.TAU) (min)
TTOT exit time of a crystal on a continuous time scale. If TTOT is greater than TSCAN, it is fixed at TSCAN, otherwise, \( TTOT = TCOUNT + TI \) (min)

Ti residence time of the \( i \) th. crystal, [Appendix I.1 and V.1] (min)

WT total weight of crystals (g)

W(J) weight of crystals in the \( J \) th. size classes (g)

WF(J) weight fraction of the crystals in the \( J \) th. size classes

WI(T) mass of the \( i \) th. crystal at time \( T \) (g)

Wp impact energy (J)

w(J) weight of fragment in the \( J \) th. size classes (g)

wf(J) weight fraction of the fragments in the \( J \) th. size classes

wf(k) weight fraction of the sample crystals in size class \( k \) before birth and death consideration

wf_2(k) weight fraction of the sample crystals in size class \( k \) after birth and death consideration

wt total weight of fragments (g)

wt(k) total weight of sample crystals in size class \( k \) before birth and death consideration (g)

wt_2(k) total weight of sample crystals in size class \( k \) after birth and death consideration (g)

x(k) dimensionless length of crystals in size class \( k \)

\( \alpha \) volume shape factor

\( \Gamma \) fracture resistance (Jm\(^{-2}\))