FACTORS AFFECTING DIFFERENTIAL EQUATION PROBLEM SOLVING ABILITY OF PRE-UNIVERSITY LEVEL STUDENTS IN A SELECTED PROVINCE IN PAKISTAN

AISHA BIBI

FACULTY OF EDUCATION
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THESIS SUBMITTED IN FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

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ABSTRACT

The role of differential equations (DEs) is very important in the modern technological era to inter-relate and solve a variety of routine daily life problems. Several approaches (algebraic, numerical and graphical) have been developed and more are being developed to make DEs course more effective and valuable. Several studies also have well elaborated the students' epistemological math problem solving beliefs, goal orientations and self-regulated learning (SRL) towards DEs problem solving. However, in spite of the great importance of these factors, no study had related these four factors. Therefore, this quantitative correlational study was designed to relate and model these three factors particularly for DEs problem solving. The purpose of this study was to explore the factors affecting DEs problem solving, particularly epistemological math problem solving beliefs, usefulness, goal orientations and self-regulated learning strategies at pre-university level students in a selected province in Pakistan. Specifically, the objectives of this study were i) to investigate the direct effect of epistemological math problem solving beliefs, usefulness, goal orientations and self-regulatory learning (SRL) strategies towards differential equation problem solving and; ii) to examine the mediating role of goal orientations and self-regulatory learning (SRL) strategies. Three different types of the adapted questionnaires along with an assessment test containing five self-developed non-routine differential equation tasks were distributed to 430 pre-university students, studying in public and private institutions. Collected data were analyzed using SPSS and SmartPLS software. Both direct and indirect effects of the selected factors on DE problem solving were measured. The analysis of the direct paths revealed that epistemological math problem solving beliefs, self-regulated learning strategies, and goal orientations strongly affected the DE problem solving. In the second phase of the study, mediation roles were identified. For this, initially the mediation effects of goal orientations (mastery, performance and avoidance goals) were considered. The findings revealed that epistemological math problem solving beliefs strongly affected the DE problem solving via mastery, performance, but the effect of avoidance goal was non-significant and negative. While considering the mediation effect of self-regulated learning strategies (critical thinking and elaboration), results revealed that epistemological math problem solving beliefs strongly affected the DE problem solving via elaboration, however, through critical thinking no significant effects were observed. Finally, findings have shown that elaboration had played the role of mediation for master and performance goals, while no such effect was observed for avoidance. Overall it can be concluded that epistemological math problem solving beliefs, usefulness, goal orientations (both mastery and performance goals) and elaboration can be effectively employed to boost the students' ability to solve DE problems and to ensure that teaching and learning of differential equation may become more effective and meaningful.

ABSTRAK

Peranan persamaan pembezaan (*differential equation*) adalah sangat penting di dalam era berteknologi moden untuk menghubungkait dan menyelesaikan pelbagai masalah rutin harian kehidupan. Pelbagai kaedah (algebra, berangka dan grafik) telah dibangunkan dan lebih banyak kaedah sedang dibangunkan untuk menjadikan kursus DEs lebih berkesan dan bernilai. Beberapa kajian juga telah menghuraikan mengenai kepercayaan pelajar mengenai epistemologi penyelesaian masalah matematik, kecenderungan matlamat dan pembelajaran kendiri (*self regulated learning*) terhadap penyelesaian masalah persamaan pembezaan. Walaubagaimanapun, di sebalik kepentingan yang tinggi mengenai faktor-faktor ini, tiada kajian yang mengaitkan ketiga-tiga faktor ini. Maka, kajian korelasi kuantitatif telah direkabentuk untuk mengaitkan dan memodelkan ketiga-tiga faktor ini, khasnya bagi penyelesaian masalah persamaan pembezaan. Matlamat kajian ini adalah untuk menyelidik faktor-faktor yang mempengaruhi penyelesaian masalah persamaan pembezaan, khasnya kepercayaan epistemologi penyelesaian masalah matematik, kecenderungan matlamat dan strategi pembelajaran kendiri pelajar peringkat pra-universiti wilayah terpilih di Pakistan. Khususnya, objektif kajian ini adalah: i) untuk menyelidik kesan langsung bagi faktor-faktor terpilih iaitu kepercayaan epistemologi penyelesaian masalah matematik, kebergunaan, kecenderungan matlamat dan strategi pembelajaran kendiri terhadap penyelesaian masalah persamaan pembezaan dan; ii) untuk meneliti peranan pengantaraan bagi kecenderungan matlamat dan strategi pembelajaran kendiri. Tiga jenis soal selidik yang berbeza telah diadaptasi bersama ujian penilaian yang mengandungi lima tugasan bukan rutin membabitkan persamaan pembezaan yang dibangunkan sendiri telah diedarkan kepada 430 pelajar pra-universiti, di institusi awam dan swasta. Data yang dikumpulkan telah dianalisa menggunakan perisian SPSS dan SmartPLS. Kedua-dua kesan langsung dan tidak langsung bagi faktor-faktor tersebut ke atas penyelesaian masalah persamaan pembezaan telah diukur. Analisa

laluan langsung menunjukkan bahawa kepercayaan epistemologi penyelesaian masalah matematik, strategi pembelajaran kendiri dan kecenderungan matlamat amat mempengaruhi penyelesaian masalah persamaan pembezaan. Di dalam fasa kedua kajian, peranan pengantaraan telah dikenalpasti. Untuk ini, kesan pengantaraan bagi kecenderungan matlamat (penguasaan, prestasi dan matlamat penghindaran) telah dipertimbangkan. Hasil kajian menunjukkan bahawa kepercayaan epistemologi penyelesaian masalah matematik amat mempengaruhi penyelesaian masalah persamaan pembezaan melalui penguasaan, prestasi, tetapi kesan matlamat penghindaran adalah negatif dan tidak ketara. Di samping mempertimbangkan kesan pengantaraan bagi strategi pembelajaran kendiri (pemikiran kritikal dan penghuraian), hasil kajian menunjukkan bahawa kepercayaan epistemologi penyelesaian masalah matematik amat mempengaruhi penyelesaian masalah DE melalui penghuraian. Walaubagaimanapun, tiada kesan yang ketara melalui pemikiran kritikal telah diperhatikan. Akhirnya, hasil kajian telah menunjukkan bahawa penghuraian telah memainkan peranan sebagai pengantaraan bagi penguasaan dan matlamat prestasi, manakala tiada kesan telah diperhatikan bagi penghindaran. Secara keseluruhannya, boleh disimpulkan bahawa kepercayaan epistemologi penyelesaian masalah matematik, matlamat kecenderungan (penguasaan dan matlamat prestasi) dan penghuraian boleh digunapakai dengan berkesan untuk merangsang kebolehan pelajar untuk menyelesaikan masalah persamaan pembezaan dan juga untuk memastikan bahawa pengajaran dan pembelajaran persamaan pembezaan boleh menjadi lebih berkesan dan bermakna.

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LIST OF SYMBOLS AND ABBREVIATIONS

AVE	:	Average variance extracted
AV	:	Avoidance goal
AV1	:	Avoidance item 1
AV2	:	Avoidance item 2
AV3	:	Avoidance item 3
AV4	:	Avoidance item 4
AV5	:	Avoidance item 5
AV6	:	Avoidance item 6
β	:	Beta value
CMR	:	Composite reliability
CFI	:	Confirmatory fit index
CR	:	Critical thinking
CR1	:	Critical thinking item1
CR2	:	Critical thinking item2
CR3	:	Critical thinking item 3
CR4	:	Critical thinking item 4
CR5	:	Critical thinking item 5
DEs	:	Differential equations
DEPS	:	Differential equation problem solving
DP	:	Duration of problems
EL	:	Elaboration
EMB	:	Epistemological math problem solving beliefs
EF	:	Effort
IO-DE	:	Inquiry oriented differential equation

MA	:	Mastery goal
MA1	:	Mastery item 1
MA2	:	Mastery item 2
MA3	:	Mastery item 3
MA4	:	Mastery item 4
MA5	:	Mastery item 5
MA6	:	Mastery item 6
PLS	:	Partial Least Squares
PR	:	Performance goal
PR1	:	Performance item 1
PR2	:	Performance item 2
PR3	:	Performance item 3
PR4	:	Performance item 4
PR5	:	Performance item 5
SE	:	Standard estimate
SEM	:	Structure equation modeling
SPSS	:	Statistical package for the social sciences
SRL	:	Self-regulated learning strategies
ST	:	Steps
IMBS	:	Indiana mathematics belief scale
KMO	:	Kaiser-Meyer-Olkin
MSLQ	:	Motivated strategies for learning questionnaire
WP	:	Word problems
UF	:	Usefulness
UN	:	Understanding

CHAPTER 1: INTRODUCTION

Teaching and learning of differential equations (DEs) has a prominent role in all the fields of education, which allows the formulation of phenomena from other disciplines (such as physics, chemistry, biology, economics, etc.) into mathematical language. In spite of its prominence and frequent applications, teaching and learning of DE course is still considered as one of the most difficult, particularly at pre-university level. This is because, the topic of differential equation along with differentiation and integration is only introduced first time at the 12th year of study or at pre-university level, and the students have no previous knowledge and understandings of this topic (Rehman & Masud, 2012). This current study was designed to explore the different factors affecting differential equation problem solving ability of pre-university level students. A conceptual model was developed to provide firm implications for teachers to boost up students' conceptual understandings required to deal and solve differential equations problems. Beside this, comparative study of the different problem solving approaches (such as algebraic and graphical) and their yielded results towards differential equations problem solving were also considered.

1.1 Background

Differential equations (DEs) have been at the center of calculus for centuries and play a prominent role in mathematics. They provide description of many real-life situations (e.g. motions of heavenly bodies, bridge designs and interactions among neurons), and thus allow the formulation of phenomena from other disciplines (such as mechanics, astronomy, physics, chemistry, biology, and economics) into mathematical language. The study of DEs provides an excellent opportunity to demonstrate the application of mathematics to real life and also expose learners to the nature of contemporary research in mathematics (Arslan, 2010a). Therefore, the study of DEs has been included in various courses in different departments including college level (Blumenfeld, 2006).

A DE is an equation which involves an independent variable t (usually denoting time), a dependent variable y, and the first derivative of y with respect to t. Equation 1.1 illustrates a DE. In most of the DE class, the time (t) is considered as the independent variable to add a dynamical aspect to the subject (Habre, 2000).

$$y'(t) = f\ (t, y) \qquad 1.1$$

To solve a DE, means finding a function y (t) that satisfies that equation. Quantitatively, this requires expressing y (t) implicitly or explicitly in terms of t. In a classical ordinary DE course, equations are classified as separable, linear, exact, and others. For each class of equations, an analytical method of solution is presented to the students, and integration is fundamental to the solution process. Thus, a student who shows proficiency in the quantitative approach has simply shown proficiency in calculus. However, an appreciation of the solution requires a qualitative approach and this is achieved by a sketch of the direction or slope field. A DE gives a formula for the slope of a solution at a given point. A sketch of the directions of a solution through any point of the ty-plane constitutes the direction or slope field. Starting at any point and flowing through the field gives a picture of a solution through that point (Habre, 2000).

Teaching and learning of DE is generally classified as procedural and conceptual in mathematics. First category of teaching DE focuses on teaching definitions, symbols, and isolated skills in an expository way. It does not focus on building deep and connected meaning to support those concepts, therefore, procedural methods are unable to enhance conceptual understanding (R. R. Skemp, 1987). On the other hand, teaching for conceptual knowledge commenced with posing problems that requires student's logics and reasoning ability. Through the solution process, students try to make connections to what they already know. Thus, they utilize their previous knowledge by extending and transferring it to new situations (Engelbrecht, Harding et al., 2005; Reston, 2000a). Recently, development in the technology has integrated these categories into single approach. Inquiry oriented based approaches have further added the positive effect of environment, epistemological and motivational beliefs on the DEs learning. By improving the conceptual knowledge of differential equations, students would be able to understand and deal with the real-life problems and processes.

Regarding these two categories, Kwon, Rasmussen et al. (2005) conducted a follow-up study conducted on the retention effect (one year after instruction) of conceptual and procedural knowledge, inquiry oriented differential equation (IO-DE) exhibited a key difference than the traditional counterparts. Further, Rasmussen, Kwon et al. (2006) investigated students' beliefs, skills, and understandings in inquiry oriented differential equation (IO-DE) classes and traditional approaches. Assessment of conceptual understanding favored project student as compared to comparison group, while there was no substantial difference regarding the evaluation of routine skills between two groups.

1.2 Factors affecting the differential equation problem solving

There are three major cognitive and contributing factors, including knowledge, control (metacognition) and beliefs, which enable students to solve mathematics problem and also to overcome difficulties (Kroll & Miller, 1993). Among these factors, beliefs are the most essential components to generate meaning and set up overall intention that define the context for learning mathematics (Cobb, 1986).

Generally, mathematics educators agree that the formal mathematics education has crucial influence on the development of student's mathematics beliefs. However, social or cultural processes are also important when accounting for students' mathematical growth (Cobb & Bauersfeld, 1995; Lave, 1988; Rogoff, 1990). Several mathematics educators have focused primarily on the individual psychological aspects of learning undergraduate mathematics (Harel & Sowder, 1998; Tall & Vinner, 1981). In this context, Yackel, Rasmussen et al. (2000) well supported the mathematics educators and suggested that students' individual beliefs about their own role, others role, and the general nature of mathematical activity and the classroom social norms are mutually constitutive. Author analyzed social interaction patterns, social and socio mathematical norms, to explore the effect of these norms towards differential equations problem solving (Yackel et al., 2000). Similar observations were revealed in few other studies. It was concluded that students' evolving beliefs regarding their capability to create mathematics and the role of explanation and reasoning are intuitively related to the social and socio-mathematical norms of their classroom settings (Yackel & Rasmussen, 2002).

In similar context, Ju and Kwon (2007) documented the change in students' mathematics beliefs especially for the case of differential equation, about their relation to mathematics, and their roles in the classroom practice. Discourse analysis showed that students portray a shift from third person perception to first person perception as a way to presume changes in students' beliefs. Consequently, transformation of students' beliefs depends on classroom learning environment, including students own cognitive assists, the role of teacher and also teaching resources.

Recently, several other researchers also observed a strong correlation between beliefs about mathematics and mathematical performance / achievement (Beghetto & Baxter, 2012; Schommer- Aikins, Duell et al., 2005; Schommer-Aikins & Duell, 2013a). Focusing on the students' beliefs in relation to science and especially math problem solving remained a highly promising area of investigation. Likewise, McLeod (1992) have same opinion that mathematics beliefs enhance or weaken individual's mathematical and problem solving ability. These beliefs further affect students learning approaches. Several researchers introduced self-regulated learning (SRL) theory and studied the epistemological beliefs into the study of mathematical problem solving (Hofer, 1999; Muis, 2004, 2008; Stockton, 2010). Epistemological beliefs affect students learning strategies and automatically their mathematics achievement. Numerous studies also correlated the implication of students' self-regulated learning skills with goals and goal orientation beliefs (Pintrich, 1991). Muis (2007) interlinked the epistemological beliefs, goal orientation, learning strategies, and achievement. In addition to these three constructs, (Schommer-Aikins et al., 2013a) also reported that the belief about the usefulness of mathematics strongly effects mathematics problem solving.

It may be concluded that if these four constructs affect general mathematics problem solving, then these may have potential to solve differential equation problems. Beside this, literature also reveals that selection and employment of the problem solving approach (such as algebraic, graphical or numerical) also effect problem solving. Mostly, algebraic approaches are being used to solve differential equation problems. While, Graphical based solutions show the real understandings of the students but difficult to construct, particular at pre-university levels (Arslan, 2010b; M Artigue, 1989).

Overall, four constructs "epistemological math problem solving beliefs, usefulness, self-regulated learning strategies (SRL) and goal orientations have great potential to solve differential equation problem. In addition, choice of suitable problem-solving approach may enhance differential equation problem solving.

1.3 Differential equation problem solving issues in Pakistan

A “problem” specifies a challenge, and to tackle this challenge one need more studies and investigations (Farooq, 1980). The term “problem solving” is defined as the schema within which creative thinking and learning is ensured (Skinner, 1984). According to National Council of Supervisors of Mathematics (NCSM), problem solving is the process of applying previous knowledge to new and unfamiliar situations (Carl, 1989). Therefore, mathematics problem solving, particularly differential equation problem solving is an innovative task.

In Malaysia, problem solving is one of the major aspects in mathematics curriculum. However, students lack many mathematical skills and cognitive abilities in learning and these deficiencies obstruct the mathematics problem solving. Researchers highlighted some reasons why mostly students fail to solve problems successfully. One of the major issue is that some students are unable to create an appropriate image fitting for the problem’s context (Novak, 1990). Other students cannot sustain the original problem while processing part of it (Campbell, Collis et al., 1995). Several researchers Koontz (1996), also reported that some students don’t have logical thinking skills or they are unable to exploit them to problem situation.

In Pakistan, like many other countries, students have difficulties in mathematics problem solving. One of major reason is that Pakistan education system focus only on attaining mathematical skills and strategies to solve mathematical problems. Therefore, they have totally ignored the application of those problem in the real word and also in other subjects. Although, it is quite possible to pass examinations by seeking, grasping or memorizing some procedural techniques with slight understanding of their meaning. Mostly rules and algorithm dominate and hence, the concept of mathematics became difficult to understand.

According to Feynman and Sackett (1985) comments “so you see they could pass the examinations, and ‘learn’ all this stuff, and not known anything at all, except what they had memorized”. Another author Akhter, Akhtar et al. (2015) further argued that it looks a good description of the Pakistan education system. Because in this system, current teaching methodology focuses on to solve exercise problem rather than making them clear of the basic concepts. Moreover, traditional tendency emphasized to gain a right answer. Therefore, it focuses students attention towards rote learning of the textbooks (Ali, 2008). Author feel danger that conceptual understanding is totally ignored which may lead to failure while applying mathematical skills in unfamiliar situation.

As Bay (2000) clarify that teaching about problem solving is the teaching of strategies, or approaches to solve problems. However, problem solving teaching methods are less valued in the mathematics class room, because mostly teachers argued that Pakistan educational setting are less likely to apply them. Because problem solving teaching method is more time-consuming than the traditional teaching method. Also taught procedures are usually traditionally followed by the teachers, hence it became problematic to teach using problem solving.

Mostly teachers face the problems regarding curriculum and examination system in Pakistan. Few researchers portrayed a picture of current situation of Pakistan that the teachers lack either confidence or support that a curriculum can provide. As a result, quality of teaching became diminished (Ali, 2008).

Ali (2008) highlighted some more reason that our assessment systems rely on massive examination only and the current curriculum that covers text book only. Therefore, most part of the world including Pakistan, curriculum reforms now strongly recommended problem solving approach. Moreover, Akhter et al. (2015) evidently proved that teachers are more passionate with problem solving strategy. However, the implementation of this method is not possible, until curriculum, the text books and especially, the assessment or examination system reflects the value of this approach.

Regarding to curriculum, national and international researchers are agreed that the current curriculum has less potential to prepare teachers for the challenges of 21^{st} century. Because there are massive gaps between the curriculum of teacher training programs and class room environment (Khan, 2012). Moreover, Kiani, Malik et al. (2012) recommended that even though most of the teachers have professional qualifications such as B.Ed., (Bachelor of Education) and M.Ed. (Master of Education), even though curriculum and training programs may be reviewed time to time for the teachers. Furthermore, lack of training and resource limitations can also make it difficult to implement.

1.4 Problem statement

Teaching and learning of differential equation is most difficult part of the mathematics course, particularly at pre-university level. This is because, the topic of differential equation along with differentiation and integration is introduced first time at 12^{th} year of study, and the students have no previous knowledge and understandings of this topic (Rehman et al., 2012). In addition to it, students' special attention, efforts and learning strategies are required to solve problems containing differential equations, particularly non-routine problems. Since these problems are generally concerned with unforeseen and unfamiliar solutions (Polya, 1962; Rehman et al., 2012), even successful calculus students are unable to solve non-routine problems (Dawkins & Epperson, 2014). As a result, it is common for students to avoid the essential part of mathematics, which leads to sever understanding problems at higher levels of education, when they correlate the real-life problems.

From the teaching point of view, finding effective strategies for the teaching of a differential equation course remained a focus of recent researchers in the field of mathematics education. Various proposals has been emerged in the case of ordinary differential equations for addressing the concepts related to them (Raychaudhuri, 2008). Generally, three different approaches (algebraic, numerical and graphical) are employed to solve differential equations (Arslan, 2010b; M Artigue, 1989).

In traditional differential equation teaching and learning, algebraic approach predominates. But both numerical and graphical approaches are generally emphasized to facilitate conceptual learning of differential equations. Selahattin (2010b) discovered that nature of students' learning in traditional differential is procedural and is limited to mastering and applying some algebraic techniques. M Artigue, (1989) expanded these consequence in a sense, that students have misconceptions and learning difficulties about DEs (Boyce, 1994; Rasmussen, 2001). The reason for the students' inability to comprehend the issue may be the content and instruction of differential equations courses (Blanchard, 1994; Boyce, 1994). The main difficulties that students found when handling the algebraic based solutions of differential equations are related to the unsuitable choice of the method of solution or an incorrect process of integration (Camacho-Machín, Perdomo-Díaz et al., 2012c).

Graphical based solutions are considered as qualitative approach and show the real understandings of the students. However, in graphical based solutions, different functions such as linear, exponential, and trigonometric and hyperbolic functions are difficult to represent and retrieve (Camacho-Machín, Perdomo-Díaz et al., 2012a). In addition to these, transition from the algebraic to the graphical register is quite hard and students often make mistakes during this conversion (Camacho-Machín et al., 2012a).

Geometric representation can be incorporated to give meaning to solution methods for ordinary differential equation when the students model different phenomena (Camacho-Machín & Guerrero-Ortiz, 2015b; Rowland & Jovanoski, 2004). However, it is a challenging task for the students to adapt a geometrical approach. Habre (2003) observed that idea of solving an ordinary differential equation using geometrical approach did not appeal to the interviewees even though the instructor of the section had geared up the course in a qualitative direction.

The reform movement in teaching and learning differential equation was stimulated in the mid-1980s due to increased accessibility of technology and also by calculus reform. Use of technological advances as a reform movement has initiated to analyze ordinary differential equations involving graphical, numerical and algebraic representations (Camacho-Machín et al., 2015b; Hubbard & West, 2012). At higher levels of education, this moment yielded better results. However, at initial or pre university levels, it is still a great challenge to determine how students interact with the digital tools and representation registers associated with ordinary differential equations to give meaning to parameters associated with it (Rowland, 2006; Rowland et al., 2004), and how to develop instruction strategies to promote student learning (Rasmussen, 2001). These reform movement also goes a step further by introducing the inquiry-oriented instruction method to the teaching and learning of differential equations (Ju et al., 2007). The inquiry-oriented class for differential equation learning is a constructive learning setting in which students participate, explicit meaning negotiation, discover, argument and assist their mathematical understanding to accomplish the formal mathematics (Rasmussen & King, 2000).

Apart from these reforms, Cobb (1985) argued for the incorporation of students' belief systems, because there is a strong correlation between beliefs about mathematics and mathematical performance / achievement (Beghetto et al., 2012; Schommer- Aikins et al., 2005; Schommer-Aikins et al., 2013a). Likewise, McLeod (1992) had same opinion that mathematics beliefs enhance or weaken individual's mathematical and problem solving ability.

Another remarkable belief "useful of mathematics" was highlighted by Schommer-Aikins and Duell (2013b). Author investigated the relationship among belief about usefulness of mathematics, epistemological beliefs and mathematics problem solving, and reported that these beliefs strongly effect math problem solving. Ju et al. (2007) extended their evaluation beyond the cognitive aspects in students' beliefs and also highlighted the role of student's self-regulated learning (SRL). Several other studies strongly supported these findings in terms of problem solving abilities and performance (Muis & Franco, 2009; Schommer-Aikins, 2004; Stockton, 2010). Beside these beliefs and SRL, role of goal orientation beliefs (part of self-motivational beliefs) were also found as an energizing agent for an individual's self-regulatory behaviors and influence the implementation of self-regulatory knowledge and skills (Kingir, Tas et al., 2013). Because, multiple component of interrelated beliefs and self-directed strategies influence students mathematics learning (Abdulwahed, Jaworski et al., 2012).

Wolters, Shirley et al. (1996) investigated the association between three goal orientations and student self-regulated learning focusing the subject mathematics. Author concluded that student's goal orientations are related in predictable and consistent ways to motivational and cognitive process and actual development. Marcou (2005) also examined motivational beliefs and self-regulated learning in the context of mathematical problem solving. Findings showed that students who tend to use self-regulated strategies while solving a mathematical task, are more probable to had increased mathematics beliefs. These findings are align with the results of (Fadlelmula, Cakiroglu et al., 2015).

Overall it may be concluded that epistemological math problem solving beliefs, usefulness, goal orientations and self-regulated learning (SRL) strategies may able to enhance students' differential equation problem solving ability. However, up to researcher knowledge, no one had combined these four for the differential equations problem solving, particularly non-routine problems. Therefore, in this work, effect of these factors was studied to analyze students' differential equation problem solving ability. Focus was given to student perceptions about differential equation, their perceptions to achieve task and their learning strategies to solve differential equation, so that researcher was able to find out the nature of the difficulties they had with differential equation and possible solutions for these difficulties. Beside this, it was revealed that several studies had correlated usefulness with mathematics achievements and problem solving, however, indirect effects via goal orientations and self-regulated learning (SRL) problem solving ability are not explored up to researcher knowledge. Therefore, this indirect relation was also considered in this study.

1.5 Conceptual frame work

Mathematical problem solving is at the heart of students' learning of mathematics. However, knowing appropriate facts, algorithms, and procedures are not sufficient to guarantee success in solving problems. Instead, there are some others factors which depend on much more than the prerequisite mathematical content knowledge. These factors including employment of different learning strategies, the emotions (like anxiety, frustration, enjoyment), and the beliefs about mathematical tasks strongly influence the direction and outcome of one's performance (Garofalo, 1989; Schoenfeld, 1985a). Beliefs are further classified as domain general (epistemological beliefs), and domain specific (epistemological mathematical problem solving beliefs) including usefulness (part of domain specific belief). Several studies well confirmed that both types of beliefs play an important role in many aspects of cognitive and problem solving performance (Schommer- Aikins et al., 2005; Schommer-Aikins et al., 2013a; Schommer-Aikins & Hutter, 2002).

Epistemological beliefs affect students learning strategies and consequently their learning outcomes (Schommer, 1990). Several researchers extended their studies to analyze the relationships between beliefs and SRL. Findings show that SRL processing and epistemological beliefs are interrelated constructs (Bråten & Strømsø, 2005; Hofer & Pintrich, 1997; Muis et al., 2009; Schommer-Aikins, 2004). Schommer-Aikins (2004) hypothesized reciprocal relationship between epistemological beliefs and self-regulated learning (SRL) strategy. However, experimental results shown that strong correlation exist between self-regulated learning strategy and epistemological beliefs exist in multiple contexts (Bråten et al., 2005; Hofer, 1999; Muis, 2008).

Later on, few researchers induced current self-regulated learning (SRL) theory and epistemological beliefs into the study of mathematical problem solving (Hofer, 1999; Muis, 2004, 2008; Stockton, 2010). Findings revealed that mathematical problem solving, self-regulated learning strategy and epistemological beliefs are interrelated constructs and these interrelated constructs are responsible for student learning. Typically, successful problem solver exerts control over the problem space and have availing epistemological beliefs (Muis, 2008; Perels, Gürtler et al., 2005; Schoenfeld, 1983, 1985b, 1989). Beside this, the role of goal orientation beliefs and SRL were also remained prominent in analyzing mathematical problem solving skills (Pintrich, 1991).

Goal orientations are a part of self-motivational beliefs (Zimmerman, Boekarts et al., 2000) and these beliefs act as an energizing agent for an individual's self-regulatory behaviors and influence the implementation of self-regulatory knowledge and skills (Kingir et al., 2013; Montalvo & Torres, 2004). Students' goal orientations are further categorized as mastery, performance and avoidance goals (Ames, 1992; Kadioglu & Kondakci, 2014). Students who adopt a mastery orientation are highly motivated to report using cognitive strategies such as elaboration and organizational strategies which reflect deeper levels of cognitive processing (Pintrich & Schrauben, 1992). In addition, mastery orientation is also positively related to metacognitive (part of self-regulated learning strategy) such as planning, monitoring, and regulating learning (García & Pintrich, 1991; Pintrich & De Groot, 1990; Pintrich, Roeser et al., 1994). In spite of several successful findings, there were some inconsistencies in the literature regarding to the role of goal orientations.

Recently, Fadlelmula et al. (2015) examined the interrelationship among goal orientation, use of self-regulated strategies and mathematics' achievement. Findings showed that only mastery goal was related to SRL strategies and math achievement. Among SRL, only elaboration was significant predictor of math achievement. These findings were partially supporting the previous studies, in which it was reported that both mastery goal and performance goal were positive predictors of self-regulated learning strategies and can generate adaptive outcomes (Liem, Lau et al., 2008).

Most of the researchers who had investigated the trichotomous goal frame work reported that mastery and performance-oriented learners have shown more tendency towards self-regulation than avoidance goal one (Wolters, 2004). These researchers further argued that both mastery and performance goal orientations can be adopted by students and can provide students with important guides for interpreting feedback and regulating their learning (Butler & Winne, 1995; García et al., 1991).

As contrary to mastery and performance goal, achievement goal theory proposed that avoidance goal is basically based on negative beliefs (i.e. fear of failure or rejection). Therefore, avoidance goal oriented students mostly give up when they face difficult and uninteresting task (Liem et al., 2008). Many studies reported that avoidance goal has negative effect on math achievement (Elliot & McGregor, 2001; Elliot, McGregor et al., 1999; Wolters, 2004). Rastegar (2006), and Hejazi, et al.’s (2008) observed similar indirect effects of avoidance goal on mathematics performance via cognitive strategies.

Students’ perceptions may differ due to various domains that may influence the relationship between goal orientation and self-regulated learning strategy (Grossman & Stodolsky, 1995). To further elaborate these facts, Wolters et al. (1996) studied the relationship between three goal orientations (mastery, performance and avoidance goal) and student self-regulated learning and replicate findings across three different academic subject area Math, English and Social study. Afterward, same scheme was used for chemistry course (Kadioglu et al., 2014; Kadioglu, Uzuntiryaki et al., 2011; Kadioglu, 2009). Findings have illustrated that both master and performance approach goal significantly predicted students SRL.

Muis (2007) prolonged these constructs and theoretically interlinked the epistemological beliefs, goal orientation, SRL, and achievement. Same group of authors had used empirically test to examine these factors (Muis et al., 2009). Findings revealed that epistemological beliefs influenced the adopted goals, as a result these adopted goal stimulate the learning strategies, which they use in their achievement. In addition, achievement goals have shown mediating role between epistemological beliefs and self-regulated learning strategy. Similarly, self-regulated learning strategies mediated the relation between goal orientation and achievement.

An another remarkable effort was noticed by Rastegar, Jahromi et al. (2010), who had considered the mediating role of goal orientations, mathematics self-efficacy, and cognitive engagement, while investigating the relationship between epistemological beliefs and mathematics achievement. Findings clearly confirmed that achievement goals, mathematics self-efficacy, and cognitive engagement had mediating role between dimensions of epistemological beliefs and math achievement.

Overall, literature reveals that epistemological beliefs, usefulness, goal orientation, and self-regulated learning strategies have significant role towards mathematics achievement as well as problem solving. However, researcher could not able to see any study, showing the combined effect of these four factors towards mathematics problem solving. In addition, it may be hypothesized that if afore cited positively affect the mathematics problem solving, similarly it can also affect the differential equation based problem solving. Therefore, in the present study the effect of four factors, epistemological belief, usefulness, goal orientation and self-regulated learning strategies on the differential equation problem solving was investigated. Efforts were furnished to examine direct effect of each factor individually, as well as through mediating factors (such as goal orientation and/or self-regulated learning).

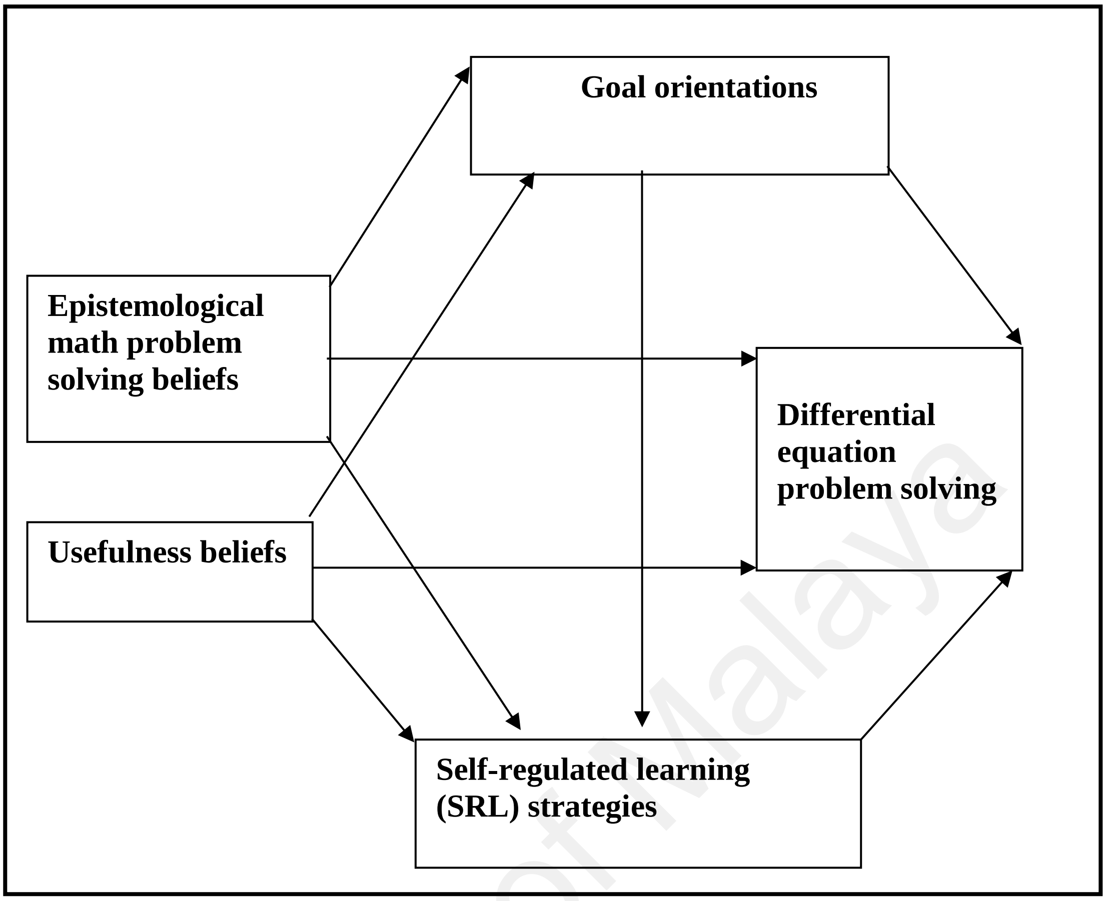

Figure 1.1: The proposed conceptual model

1.6 Research purpose

The purpose of this study was to explore the factors affecting differential equation problem solving ability, specifically epistemological math problem solving beliefs, usefulness, goal orientations and self-regulated learning (SRL) strategies, at pre-university level. Besides these, selection and employment of different problem-solving approaches (such as algebraic and/or graphical) were also investigated and comparatively analyzed.

1.7 Research objectives

To achieve the desired purpose, following objectives were finalized for this study,

1. To examine whether epistemological math problem solving beliefs, usefulness, self-regulated learning (SRL) strategies and goal orientations directly affect students' differential equation problem solving ability.
2. To examine whether goal orientations play a mediating role between epistemological math problem solving beliefs and differential equation problem solving ability.
3. To examine whether goal orientations play a mediating role between usefulness and differential equation problem solving ability.
4. To examine whether self-regulated learning (SRL) strategies play a mediating role between epistemological math problem solving beliefs and differential equation problem solving ability.
5. To examine whether self-regulated learning (SRL) strategies play a mediating role between usefulness and differential equation problem solving ability.
6. To examine whether self-regulated learning (SRL) strategies play a mediating role between goal orientations and differential equation problem solving ability.
7. To comparatively analyze the algebraic and graphical problem solving approaches for differential equation problem solving.

1.8 Research questions

From a constructivist view point, it was hypothesized that engagement in mathematical problem solving could potentially lead to learning. Hence, advancement of our understanding of the factors involved in both successful and unsuccessful student's differential equation problem-solving engagement must lead to pedagogical initiatives intended to enhance students learning. This study investigated these issues by answering the following research questions:

1. Do epistemological math problem solving beliefs, usefulness, self-regulated learning (SRL) strategies and goal orientations directly affect differential equation problem solving ability?
2. Do goal orientations play a mediating role between epistemological math problem solving beliefs and differential equation problem solving ability?
3. Do goal orientations play a mediating role between usefulness and differential equation problem solving ability?
4. Do self-regulated learning (SRL) strategies play a mediating role between epistemological math problem solving beliefs and differential equation problem solving ability?
5. Do self-regulated learning (SRL) strategies play a mediating role between usefulness and differential equation problem solving ability?
6. Do self-regulated learning (SRL) strategies play a mediating role between goal orientations and differential equation problem solving ability?
7. Does algebraic approach yield better results than graphical approach for differential equation problem solving?

1.9 Hypothesis of the study

This study was designed specifically to answer the above questions, and was summarized into the following hypotheses for statistical purpose:

1. Epistemological math problem solving beliefs, usefulness, goals orientations and self-regulated learning strategies (SRL) have direct effects on differential equation problem solving ability.

To evaluate the first hypothesis, it was further divided into following four sub-hypotheses.

H-1.1 Epistemological math problem solving beliefs have positive direct effects on differential equation problem solving ability.

H-1.2 Usefulness has positive direct effects on differential equation problem solving ability.

H-1.3 Goals orientations including mastery, performance and avoidance goals directly affect the differential equation problem solving ability.

H-1.4 Self-regulated learning strategies (SRL) including elaboration and critical thinking have positive direct effects on differential equation problem solving ability.

2. Epistemological math problem solving beliefs have indirect effect on differential equation problem solving ability via goal orientations.

To evaluate the second hypothesis, it was divided into following three sub-hypotheses;

H-2.1 Mastery goal play a mediating role between epistemological math problem solving beliefs and differential equation problem solving ability.

H-2.2 Performance goal play a mediating role between epistemological math problem solving beliefs and differential equation problem solving ability.

H-2.3 Avoidance goal play a mediating role between epistemological math problem solving beliefs and differential equation problem solving ability.

3. Usefulness has indirect effect on differential equation problem solving via goal orientations.

To evaluate the third hypothesis, it was divided into following three sub-hypotheses;

H-3.1 Mastery goal play a mediating role between usefulness and differential equation problem solving ability.

H-3.2 Performance goal play a mediating role between usefulness and differential equation problem solving ability.

H-3.3 Avoidance goal play a mediating role between usefulness and differential equation problem solving ability.

4. Epistemological math problem solving beliefs have indirect effect on differential equation problem solving via self-regulated learning (SRL) strategies.

To evaluate the third hypothesis, it was divided into following two sub-hypothesis;

H-4.1 Epistemological math problem solving beliefs have positive indirect effect on differential equation problem solving via elaboration.

H-4.2 Epistemological math problem solving beliefs have positive indirect effect on differential equation problem solving via critical thinking.

5. Usefulness has indirect effect on differential equation problem solving ability via self-regulated learning (SRL) strategies.

To evaluate the fifth hypothesis, it was divided into following two sub-hypothesis;

H-5.1 Usefulness has positive indirect effect on differential equation problem solving ability via elaboration.

H-5.2 Usefulness has positive indirect effect on differential equation problem solving ability via critical thinking.

6. Goals orientations have indirect effect on differential equation problem solving ability via self-regulated learning (SRL) strategies.

To evaluate the fourth hypothesis, it was divided into following six sub-hypotheses;

H-6.1 Mastery goal has positive indirect effect on differential equation problem solving ability via elaboration.

H-6.2 Mastery goal has positive indirect effect on differential equation problem solving ability via critical thinking.

H-6.3 Performance goal has positive indirect effect on differential equation problem solving ability via elaboration.

H-6.4 Performance goal has positive indirect effect on differential equation problem solving ability via critical thinking.

H-6.5 Avoidance goal has negative indirect effect on differential equation problem solving ability via elaboration.

H-6.6 Avoidance goal has negative indirect effect on differential equation problem solving ability via critical thinking.

7. Algebraic approach yields better results than graphical approach for differential equation problem solving.

1.10 Rationale for the study

The study came as a response to the following:

This research has extended the differential equation studies by looking at non-routine problems and focusing on the understandings that students normally use in college level mathematics classes. Mostly, differential equation problem solving studies were about university level students and limited findings were available relevant to pre-university level. Therefore, the results of this dissertation have added an underrepresented body of research.

Students' epistemological math problem solving beliefs, usefulness, goal orientations and self-regulated learning strategies were used to relate students' non-routine problem-solving performance. Therefore, this work was an extension of previous studies (Muis, 2008; Schoenfeld, 1985b; Stockton, 2010). Additionally, this work has provided a formative groundwork for the development of pedagogical interventions for developing students' availing mathematical beliefs, self-regulated learning strategies and goal orientation in a differential equation course.

Cultural environment is also very important while considering academic motivation because in western countries it may operate in different ways as compared to Asian contexts (Ho & Hau, 2008). Most of the existing studies relevant to differential equation were done in West countries and East West Country (Turkey). There was no reliable literature available, particularly from developing countries. Therefore, the findings of this work have provided additional guidelines for developing countries.

1.11 Significance of the study

This study has great potential for the students of developing countries particularly Pakistan in the areas of mathematics education. This research was carefully designed to investigate the students' difficulties and misconceptions during learning of differential equation course because it is common for students to avoid this essential part of mathematics, which leads to sever understanding problems at higher levels of education, when they correlate the real-life problems. Therefore, in this research, effects of different psychological factors were studied to boost up the students' understandings, so that teaching and learning of differential equation became meaningful.

The results of this study have provided a new avenue to educators and teachers in Pakistan to overcome the students' problem by boosting the students' mind psychological. This may be of value for the authorities to take into consideration and, also to enhance the positive factors and to avoid those factors that influence mathematics students negatively.

It is anticipated that the result of this study would provide the Ministry of Education in Pakistan with current data that would aid the ministry in making better policy decisions and applying educational strategies with greater certainty the implementation of curriculum in colleges.

The results of this study have been positively added to the literature and attempts were made to fill the research gap generally and particularly in Pakistan as a foundation for the research community to proceed with further research on the curriculum implementation, and teaching and learning of differential equation effectively.

1.12 Limitations and delimitations

The study had following major limitations that might influence the collection and interpretation of data collected from that particular context. This research had used non-routine differential equation tasks to investigate students' problem, which strongly needed students' special attention, efforts and learning strategies to solve them (Polya, 1962; Rehman et al., 2012). Beside this the assessment test was an informal exam or test having no short-term incentives for them. Hence, participants' lack of cooperation was possible. The adapted instruments containing epistemological math problem solving beliefs, goal orientations and self-regulated learning (MSLQ) questionnaires were used to assess participants' motivation and their use of cognitive strategies. All of these adapted questionnaires were based on the theories and findings of developed countries. As Pakistan is a developing country, so due to change in the resources, infrastructure, teachers and students' abilities and findings might not be same as compared with hypothesized. In addition, a limitation relevant to self-reported instruments was possible to have occurred including participants' lack of cooperation in providing information about themselves.

Results of this small-scale study were another important limitation it might not be a generalized data for all secondary mathematics students of all provinces in Pakistan. This research was carried out in a limited number of institutes located in one province of Pakistan. Results in other provinces or states might be different due to students' learning capacity, teacher training and availability, infrastructure, cultural and regional constraints. Other major limitations were investigations of a limited range of strategies, tasks and problem solving approaches. In self-regulated learning strategies, only two cognitive strategies (elaboration and critical thinking) were analyzed in this study. Similarly, five non-routine tasks involving only two problem solving approaches (algebraic and graphical) were considered here.

To reduce the limitation of using non-routine differential equation problems to assess the students' problem and to overcome the issue of participants' lack of cooperation, non-routine tasks exhibiting daily life problems were the best option. Daily life based problems were able to capture the students' attention. In addition, sufficient efforts were carried out to give them shape of non-routine problems with adequate hidden data to analyze different factors. In addition, by reducing the number of tasks up to five had helped students to solve these tasks without feeling bored.

To overcome the issue of adapted instrument's validity for the developing countries, an additional questionnaire (in addition to research instrument) was designed for the field experts (educators/ teachers), in which, consents of the experts were assessed with respect to different parameters, such as suitability of the country/province, selected factors, their inter-connection in the present study, clarity of representations. Responses of the experts were also analyzed for the final data collection.

A suitable choice of sample size and random sampling from both public and private sectors as well as from urban and rural areas may enable a generalization of the findings of this research to other Pakistani students studying at secondary level from similar context. The sample was taken from the province of the Khyber Pakhtunkhwa and contained approximately 52 percent of participants who were enrolled in rural areas colleges. In addition, by comparing and confirming the demographic information with the institutional data about participants remained helpful to delimitate the error in self-reporting data.

1.13 Operational definitions of key terms

In this section, operational definitions of major constructs have been presented. These definitions provide clarification about different terms which were frequently appeared or used in this study. A more detailed and extensive description and analysis of each construct and its component parts may be found in Chapter 2: Review of Relevant Literature.

1.13.1 Beliefs

Twenty five years ago, Pajares (1992) stated: " defining beliefs is at best a game of player's choice". However, Abelson (1979) defined beliefs in terms of people manipulating knowledge for a particular purpose or under a necessary circumstance. An individual's knowledge stems from the interaction between the individual and the environment (Tang, 2010). However, distinguishing knowledge from belief is a daunting job. Nespor (1987) differentiated between the concept of beliefs and knowledge and suggested that beliefs have stronger affective and evaluative components than knowledge. Further author recommended that these beliefs affect typically operates independently of the cognition associated with knowledge.

1.13.2 Epistemological Beliefs

Epistemological beliefs can be defined as how individual comes to know, the concepts and beliefs they hold about knowing, and the manner in which such epistemological premises are a part of and an influence on the cognitive processes of thinking and reasoning (Hofer, 1997). Based on multi-dimensional view point, general and mathematics-specific epistemological beliefs are the two significant categories of beliefs.

This study has investigated only mathematics-specific epistemological beliefs. The epistemological mathematical problem solving beliefs have been further categorized into six dimensions including; beliefs on duration of problem solving, steps in problem solving, understanding concepts, solving word problems, effort in solving problems, and beliefs on usefulness of mathematics (Kloosterman & Stage, 1992). Operational definition of each dimension is given below.

1.13.2.1 Beliefs on duration of problem solving

It is a student's perception about time interval required for the mathematical problems solving. On the basis of this perception a student decides about task as solvable or difficult (Kloosterman et al., 1992).

1.13.2.2 Steps in problem solving

It is a belief based on the solutions of problems via procedural method with several steps or rules to solve a computational or words problem (Kloosterman et al., 1992).

1.13.2.3 Understanding concepts

Belief based on understanding concept is the student's perception to get the correct answer and also why the answer is correct (Kloosterman et al., 1992).

1.13.2.4 Word problems

Belief based on importance of word problems is the degree to which students connect their mathematical perceptions to the attainment of computational skills (Kloosterman et al., 1992).

1.13.2.5 Effort in solving problems

Belief based on effort involves student's perception that anyone can improve mathematics ability with adequate effort (Kloosterman et al., 1992).

1.13.3 Usefulness of mathematics

Belief about usefulness of mathematics is based on students' perception, that mathematics is useful. It increases their motivation and consequently achievements (Stockton, 2010).

To assess students' beliefs, adapted Indiana Mathematics Belief Scale (IMBS) was used. All responses were measured on a 5-point scale (1= strongly disagree, 2 = disagree, 3= uncertain, 4 = agree, and 5 = strongly agree). Total 36 items were considered and investigated for beliefs system. All of these scales consist of three positively-oriented items and three negatively oriented items. Several researchers criticize the Indiana Mathematics Scale (IMBS) because this does not have option-"no response". Then participants are being forced to give a reason they do not hold thus diluting the validity. However, other researchers believe that IMBS scales have an option "uncertain".

Therefore, validity of these scales cannot be diminished. Fennema-Sherman's (1976) reported Cronbach's alpha value of usefulness as 0.86.

1.13.4 Goal Orientations

Goal orientations are the integrated motivational pattern of beliefs that lead to different ways of approaching, engaging in and responding to solve tasks (Ames, 1992). There are three main types of goal orientations, including; performance, mastery and avoidance goal orientations (Rastegar et al., 2010).

1.13.4.1 Mastery goal orientation

A type of motivational beliefs, that causes to engage and respond a student to achieve a desired goal. In the present case the desired goal is differential equation problem solving (Rastegar et al., 2010).

1.13.4.2 Performance goal orientation

Performance goal orientation is a motivational belief that provokes a student to perform well to prove himself a better candidate in front of other students and teachers (Rastegar et al., 2010).

1.13.4.3 Avoidance goal orientation

Avoidance goal orientation is a negative motivational belief that causes a student to avoid lack of skills as compare to their peers and class fellows (Rastegar et al., 2010).

These motivational beliefs were measured by an adopted scale based on the Patterns of Adaptive Learning Survey (Midgley, Maehr et al., 2000). All responses were measured using a 5-point scale (1= not at all true, 2 = not true, 3= somewhat true, 4= true, and 5 = very true). Total 17 items were considered and investigated for goal orientations. Several researchers criticize the adaptive learning survey scale because this also not have option-"no response". Then participants are being forced to give a reason they do not hold thus diluting the validity. However, other researchers believe that as like IMBS scales, this scale has an option "uncertain/ somewhat true". Therefore, validity of these scales cannot be diminished. The reported reliability for the mastery, performance and avoidance goals were 0.86, 0.86 and 0.75, respectively (Carol Midgley, Martin L. Maehr et al., 2000).

1.13.5 Self-regulated learning (SRL) strategies

Self-regulated learning strategies (SRL) is a self-directive procedure through which students switch their mental abilities into academic skills or to achieve their goal (Zimmerman et al., 2000). For current study, personal goal/task is to solve differential equation task. Self-regulated learning strategy use is composed of cognitive strategies and meta-cognitive strategies. In this research, only two dimensions; elaboration and critical thinking (part of cognitive strategies) were used.

These dimensions were measured by using an adaptive subscale of Motivated Strategies for Learning Questionnaire (MSLQ) (García et al., 1991). All responses were measured using a 7-point scale (1= not at all true, 2 = not true, 3= somewhat not true, 4= uncertain, and 5 = somewhat true, 6= true, 7= very true). Total 11 items including 6 items for elaboration and 5 items for critical thinking were considered. Similar to IMBS, several researchers also criticize the MSLQ scale because this also not have option-"no response". Therefore, participants are being forced to give a reason they do not hold thus diluting the validity. However, other researchers believe that as like IMBS scales, this scale has an option "uncertain". Therefore, validity of these scales cannot be diminished. The reported reliability values for elaboration and critical thinking were 0.75 and 0.80 (Duncan & McKeachie, 2005).

1.13.6 Differential equation

Differential equation is used to illustrate the rate of change into the language of mathematics using a continuous model means setting up an equation containing a derivative. For the current study, two type of differential equation autonomous and pure time differential equation were used.

First order equations in which the independent variable does not appear explicitly are called autonomous equations (Boyce, DiPrima et al., 1992). Normally, this type of equation is illustrated as below;

$$dy/dt = f(y) \tag{1.2}$$

Autonomous equations are mostly used in the context of the growth or decline of the population of a given species, and also effectively used to highlight and describe issues in different fields ranging from medicine to global economics.

Pure-time differential equations are those whose non-derivative part of the equation involved time (independent variable) explicitly, but not y (depend variable) explicitly. For this study, five non-routine differential equation tasks were used to assess differential equation problem solving.

1.13.7 Differential equation problem solving

A "problem" specifies a challenge, and to tackle this challenge one need more studies and investigations Farooq (1980). The term "problem solving" is defined as the schema within which creative thinking and learning is ensured (Skinner, 1984). According to National Council of Supervisors of Mathematics (NCSM), problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations (Carl, 1989). Therefore, differential equation problem solving is an innovative task.

1.13.8 Non-routine differential equation tasks

Gilfeather and Regato (1999) defined routine problems as well defined, well-structured tasks that use already known steps and procedures in order to find a solution. While, non-routine tasks involve different types of strategies such as guessing and checking, looking for a pattern, drawing a diagram for their solution (Gilfeather et al., 1999; Lee & Chen, 2016; Polya, 1957; Robinson, 2016; Schoenfeld, 1992).

For present study, five non-routine differential equation tasks were developed. These tasks covered different aspects of the differential equations to relate and solve daily life problems. Task 1, task 2 and task 3 were about population growth, projectile motion and compound interest, respectively. Similarly, task 4 and task 5 were about polio infected people (health and disease), cooking of bakery items (a particular application of Newton's law of heating/cooling).

An adapted analytic scale for problem solving based on scale of Charles, Lester et al. (1987) was used to score these tasks. Authors proposed three categories as understanding, planning and getting answer (Charles et al., 1987). The understanding stage, students needed to interpret or retrieve hidden data. In case of full understanding they were assigned 2 marks otherwise 1. The next phase was planning, in which students had to plan the whole steps, procedures, formulas, and strategies. Students who were successful in their planning phase they were assigned 2 marks, otherwise they were considered partial planner and were assigned 1 mark. In getting an answer phase, the answer of the task, students who used the correct procedure but not completed the solution or made a sign or unit mistakes they were assigned one marks and vice versa.

1.13.9 The reliability of the instrument

Reliability refers to the consistence of a study's dealings/actions and the stability of responses to multiple coders of data sets (Creswell & Clark, 2007). According to Cho and Kim (2015) "*Cronbach's alpha is the degree to which participants' responses are consistent across the items with in questionnaire construct*" (Chew, Kueh et al., 2017).

Cronbach's alpha is commonly-used to test the extent to which multiple indicators for a latent variable belong together. Cronbach's alpha value depends on the correlation between items, therefore, as the number of items involved in an instrument increases, the Cronbach's alpha also increases. The reliability values of the all of the instruments were in permissible range. Individual reliability coefficient (Cronbach's alpha) value for diverse scale should be above the threshold value of 0.7 (Gliem & Gliem, 2003).

1.13.10 Construct validity

Construct validity is the degree to which a test measures what it claims to be measuring. To find construct validity, usually exploratory factor analysis (EFA) is exploited. The use of exploratory factor analysis, explains the consistency of instrument and also verify that the questioned items are strongly deem to be able to measure what is to be measured (Adarlo & Jackson, 2017; O'Brien, Pan et al., 2017; Said, Badru et al., 2011).

1.13.11 Structural equation modeling

Structural equation modeling (SEM), is a general and a very powerful multivariate technique that uses a conceptual model, path diagram, and system of linked regression equation to capture complex and dynamic relationships within a web of observed and unobserved variables (Gunzler, Chen et al., 2013). There are two types of variables included in SEM models, exogenous variable and endogenous variable. Exogenous variables are always independent variable in SEM models, while exogenous variables represent a dependent variable in at least one of the SEM. These exogenous variables may become independent variable in other equations within the structural equation model.

SEM models are best represented by path diagrams, in which nodes represent variables and arrow shows relationship among these variables. By convention, latent variables are represented by a circle or ellipse, whereas, observed variables are represented by a rectangle or square. Generally, arrows are used to represent the relationship between variables. Two types of arrow exist in SEM. A single straight arrow indicates a causal relationship from the base of the arrow to the head of the arrow. Two straight single-headed arrows in opposing directions connecting two variables indicate a reciprocal relationship. A curve two-headed arrow indicates there may be some association between the two variables (Gunzler et al., 2013). SEM can be used when extending a mediation process to multiple independent variables, mediators or outcomes (MacKinnon, 2008).

1.13.12 PLS-SEM model evaluation

PLS is a well-established, second generation multivariate technique which can simultaneously evaluate the measurement model and the structural model with the aim of minimizing the error variance (Byrne, 2013b; Rabe-Hesketh, Skrondal et al., 2007; Sarstedt, Ringle et al., 2014). Moreover, PLS is a powerful data analysis technique that does not make relative assumptions regarding data distribution (Chin, 2010; Chin, Marcolin et al., 2003; Chin & Newsted, 1999). PLS analyses can be performed using Smart PLS software which computed the estimates of standardized regression coefficient of the paths of the model, factor loadings for the indicators, and the amount of variance account for the dependent variables (Ringle, Wende et al., 2005). Generally, this software makes it possible to test the hypothesized relationships between independent and dependent variables depicted in the model. Another important application of Smart PLS software is that it computes several types of reliabilities (Cronbach's alpha and composite reliability coefficient) and validities (convergent and divergent) statistics, which can be used to assess the quality of the model (Jaouadi, Zorgui et al., 2017). In addition, SEM measures latent, unobserved concepts with multiple observed indicators. In SEM, two types of models including measurement model and structural model are embedded. Measurement model also known as outer model describes relationship between latent variables and their measures (indicators). Further, measurement model can be reflexive or formative or their combination depending upon the nature of constructs and variables. Whereas, structural model also known as inner model and it determines the relationships between the determinants.

1.14 Chapter Summary

This chapter has provided a brief description of teaching and learning of differential equation problem solving, followed by highlighting the numbers of factors contributing towards differential equation problem solving ability. This has enabled researcher to select factors that affect differential equation problem solving. Four important factors "epistemological math problem solving beliefs, usefulness, self-regulated learning strategies (SRL) and goal orientations were identified as the most influencing factors. In addition to these factors, utilization of a specify problem solving approach also had potential to effect differential equation problem solving. Based on these justifications, research objectives and hypothesis were developed.

CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

This chapter provides an overview of the learning of differential equations, theoretical perspectives and their problem solving approaches, followed by three most important factors affecting the learning of differential equations. In addition, this chapter also covers the theoretical framework employed for the four constructs of the study epistemological math problem solving beliefs, usefulness, self-regulated learning (SRL) strategies and achievement goal orientations, and a review of the relevant literature. It also describes and discusses the relationship of these four constructs of the study with each other and their relevance with differential equation problem solving ability. Figure 2.1 illustrates the overview of literature review scheme for current study.

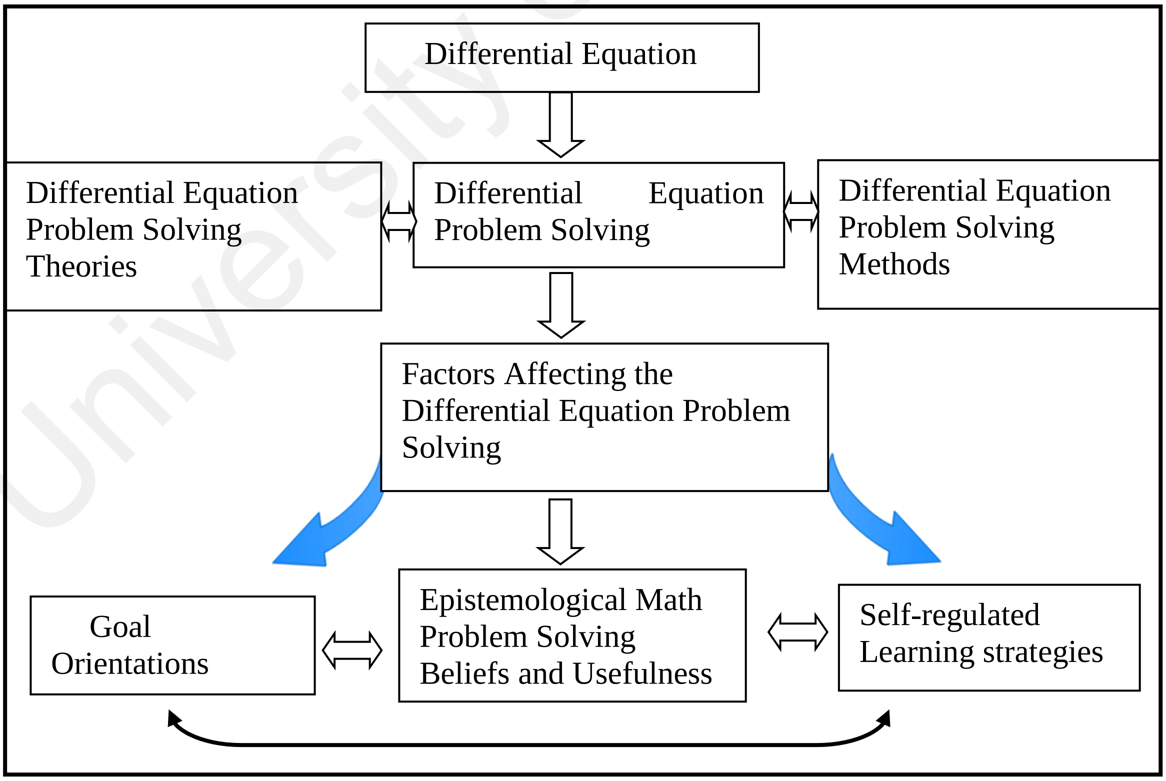

Figure 2.1: Framework for Literature Review.

2.2 Differential equation problems solving

Differential equations (DEs) have an essential role in mathematics and have been at the center of calculus for centuries. The study of differential equations started in the late 17th century with Sir Isaac Newton, who sought information about motion of planets indirectly through the analysis of rate of change equations (Rasmussen et al., 2000). The concept of DE is used to model and understand real life problems. Therefore, these provide opportunities to formulate the application of phenomena from other discipline of science and social science fields such as Physics, Astronomy, Biology, and Economics.

Differential equation course is usually taught as separate course or part of calculus appears in all types of university scientific, engineering, and social science curricula (Camacho-Machín, Perdomo-Díaz et al., 2012b; Lin & Thomas, 2017a). A typical science student begins their university studies with a year of calculus, generally followed by differential equations in the sophomore year with little technology use and emphasizing mainly analytic techniques (Rasmussen, 1996).

Teaching and learning of differential equation is the most difficult part of the mathematics course, particularly at pre-university level. This is because, the topic of differential equation along with differentiation and integration is introduced first time at 12^{th} year of study, and the students have no previous knowledge and understandings of this topic (Rehman et al., 2012). In addition to it, students' special attention, efforts and learning strategies are required to solve problems containing differential equations, particularly non-routine problems because these problems are typically concerned with unanticipated, unusual, and strange solutions (Polya, 1962; Rehman et al., 2012). Even talented calculus problem solver sometimes became unable to solve these non-routine problems (Dawkins et al., 2014). So, it is common for students to avoid the essential part of mathematics, which leads to sever understanding problems at higher levels of education, when they correlate the real-life problems (Czocher, 2017; González & Vázquez, 2017).

From the teaching point of view, most important aspects of the recent studies in mathematics education were to find competent strategies for teaching differential equation course (Caro, Lenkeit et al., 2016; Lin & Thomas, 2017b). Within the case of differential equation, various kinds of scheme have been come out to deal with the concepts related to them (Raychaudhuri, 2008). Usually, differential equations are categorized as autonomous, separable, linear, exact, and others. For each of equations analytical methods of solution are taught (Habre, 2003). Generally, three different approaches (algebraic, numerical and graphical) are employed to solve differential equations (Arslan, 2010b; Michele Artigue, 1989). In traditional differential equation teaching and learning, algebraic approach predominates, while recently, graphical and numerical approaches are more emphasized to facilitate the conceptual learning of differential equations.

In a traditional differential equation environment, Selahattin (2010) reported that nature of students' learning is procedural and is restricted to mastering and applying a few algebraic techniques. Michele Artigue (1989) supported these results in a sense, so as to learner do not have understanding of differential equation concept (Boyce, 1994; Rasmussen, 2001). The main difficulties that students found when handling the algebraic based solutions of differential equations are related to the unsuitable choice of the method of solution or an incorrect process of integration (Camacho-Machín et al., 2012c). Unfortunately, traditional methods of instruction did not encourage students from creating their own strategies and solving problems (Allen, 2006).

Moving away from the conventional approach, more recently conceptual learning of differential equation is boost up by emphasizing the other two approaches (graphical and numerical) (Allen, 2006; Leong, Kaur et al., 2017; Pandya, 2017). Graphical based solutions are considered as qualitative approach and show the real understandings of the students. However, in graphical based solutions, different functions such as linear, exponential, and trigonometric and hyperbolic functions are difficult to represent and retrieve (Camacho-Machín et al., 2012a). In addition to these, transition from the algebraic to the graphical register is quite hard and students often make mistakes during this conversion (Camacho-Machín et al., 2012a). Geometric representation can be incorporated to provide sense to the solution methods for ordinary differential equation while modeling different phenomena (Camacho-Machín & Guerrero-Ortiz, 2015a; Rowland et al., 2004). However, it has proven to be critical for the learners to adapt a geometrical approach for solving differential equations and their solutions.

Likewise qualitative methods, numerical methods offer alternate solutions for DE which cannot be solved using analytic methods (Rasmussen, 1998a). Numerical methods give access to approximate solutions of differential equations, while qualitative or graphical methods views the differential equations geometrically as well as analyzing the differential equations itself, hence these methods provide overall information about solutions (Rasmussen, 1998a). Several researchers described and compared that how numerical methods are used to analyze differential equations (Blanchard, Devaney et al., 1996; Borrelli & Coleman, 1996; Coombes, Stuck et al., 1995; Kostelich & Armbruster, 1996; Lomen & Lovelock, 1996; West, 1996).

Rasmussen (1998a) investigated students' understanding and difficulties while using qualitative and numerical methods to analyze differential equations. Findings showed that difficulties influencing students understanding includes function dilemma, trend of overgeneralization such as, overgeneralization of the autonomous term, and interference from informal concepts, and the complications with graphical interpretations. Several researches extended this concept and also stated that in order to support the development of understandings, the rule of three algebraic, numerical, and graphical should be replaced with rule of four, where writing in mathematics has an important role (Habre, 2002). Several other researchers (Habre, 2002; Schurle, 1991) examined whether writing helps students in learning differential equations and evidently proved that students were more adapted to the idea of writing.

2.3 Differential equation problem solving issues in Pakistan

A “problem” specifies a challenge, and to tackle this challenge one need more studies and investigations Farooq (1980). The term “problem solving” is defined as the schema within which creative thinking and learning is ensured (Skinner, 1984). According to National Council of Supervisors of Mathematics (NCSM), problem solving is the process of applying previously acquired knowledge to new and unfamiliar situations (Carl, 1989). Therefore, problem solving, particularly differential equation problem solving is an innovative task.

In Malaysia, problem solving is one of the major aspects in mathematics curriculum. However, students lack many mathematical skills and cognitive abilities in learning and these deficiencies obstruct the mathematics problem solving. Researchers highlighted some reasons why mostly students fail to solve problems successfully. One of the major issue is that some students are unable to create an appropriate image fitting for the problem’s context (Novak, 1990). Other students cannot sustain the original problem while processing part of it (Campbell et al., 1995). Several researchers Koontz (1996), also reported that some students don’t have logical thinking skills or they are unable to exploit them to problem situation.

In Pakistan, like many other countries, students have difficulties in mathematics problem solving. One of major reason is that Pakistan education system focus only on acquiring mathematical skills and strategies to solve mathematical problems. Therefore, they have totally ignored the application of those problem in the real word and also in other subjects. Although, it is quite possible to pass examinations by seeking, grasping or memorizing some procedural techniques with slight understanding of their meaning. Mostly rules and algorithm dominate and hence, the concept of mathematics became difficult to understand.

According to Feynman et al. (1985) comments “so you see they could pass the examinations, and ‘learn’ all this stuff, and not known anything at all, except what they had memorized”. Another author Akhter et al. (2015) further argued that it seems a good description of the Pakistan education system. Because in this system, current teaching methodology focuses on to solve exercise problem rather than making them clear of the basic concepts. Moreover, traditional tendency emphasized to gain a right answer. Therefore, it leads students towards rote learning of the textbooks (Ali, 2008). Author feel danger that conceptual understanding is totally ignored which may lead to failure while applying mathematical skills in unfamiliar situation.

As Bay (2000) clarify that teaching about problem solving is the teaching of strategies, or approaches to solve problems. However, problem solving teaching methods are devalued in the class room, because mostly teachers argued that educational setting in Pakistan are less likely to apply them. Because problem solving teaching method is more time-consuming as compared to the traditional teaching method. Also taught procedures are usually traditionally followed by the teachers, hence it became problematic to teach using problem solving.

Mostly teachers face the problems regarding curriculum and assessment system in Pakistan. Several researchers portrayed a picture of current situation regarding examination and curriculum in Pakistan that the teachers lack either confidence or support that a curriculum can provide. As a result, quality of teaching became diminished (Ali, 2008).

Ali (2008) highlighted some more reason that our assessment systems rely on massive examination only and the current curriculum that covers text book only. Therefore, most part of the world including Pakistan, curriculum reforms now strongly recommended problem solving approach. Moreover, Akhter et al. (2015) evidently proved that teachers are more passionate with problem solving strategy. However, the implementation of this method is not possible, until curriculum, the text books and especially, the assessment system reflects the value of this approach.

Regarding to curriculum, national and international researchers are agreed that the current curriculum has less potential to prepare teachers for the challenges of 21st century. Because there are massive gaps between the curriculum of teacher training programs and class room environment (Khan, 2012). Moreover, Kiani et al. (2012) recommended that even though most of the teachers have professional qualifications such as B.Ed. (Bachelor of Education) and M.Ed. (Master of Education), even though curriculum and training programs may be reviewed time to time for the teachers. Furthermore, lack of training and resource limitations can also make it difficult to implement.

2.4 Mathematics Education Research in the area of differential equations

Mathematics educators categorized learning as procedural and conceptual learning. Procedural learning refers to the rote memorization of mathematics concept without understanding the basics. Consequently, person must have good memory for procedural learning. (R. Skemp, 1987; Skemp, 2002) further elaborated that person having good memory is well able to take in information, organize it, store it, and retrieve from his large memory store just what he needs at any particular time. Whereas, conceptual learning involves understanding, interpretation and the relation between concepts (Arslan, 2010b). Moreover, both procedural and conceptual learning are essential and complementary because latter configure a basic for the former (Ukoha, 2017).

In a study of Arslan (2010b), the relationship between procedural and conceptual learning in differential equation course was clarified. Author reported that students' learning was procedural in their traditional differential equation (DE) course and was limited to apply some algebraic techniques. Further, Camacho-Machín, Perdomo-Díaz et al. (2008) also reported that students idea to solve differential equation is restricted to the application of appropriate solution methods to a certain type of differential equation. These findings are in line with the previous studies (Artigue, 1989b; Boyce, 1994; Rasmussen, 2001) that are more concerned with student's misunderstanding, and also highlighted their learning difficulties about differential equation concept.

On the other hand, understanding of a differential equation concept involves various stages, such as students comprehend concept definition, algorithmic use of concept, and identification of concepts as an instrument to solve mathematics problems (Camacho-Machín et al., 2008). Hence, author recommended that instructor should authorized to make use of several other representation systems in which they can expose various aspects connected with the differential equation concept, solution methods or procedures, and the corresponding logic and correlation among those representations.

Regarding to differential equation, five primary area in differential equation were remained the focus of mathematics educators researchers (Allen, 2006). Students' reasoning about single DEs, student thinking about graphical and numerical solutions, their understanding of systems to DEs, technology in differential equations, and student learning in the context of an inquiry-based DEs classroom (Whitehead & Rasmussen, 2003). In first area, Artigue (1992) identified that although students reason accurately about graphical solutions to differential equations in certain situation, but it may not signify correct conceptualizations at some other times. Author further elaborated this situation that students may give appropriate answer reasoning from ideas from calculus, they might not have correct understandings of solutions to certain types of differential equations (Allen, 2006). These findings were well supported by several other researchers. For example, a few studies evidently proved that behind students correct answer, there might exist an incorrect concept (Rasmussen, 1997; Rasmussen, 2001). In his studies, author noticed that students conceptualize the existence of equilibrium solution is whenever the differential equation is zero. Although this concept is true for autonomous differential equation, but generally it is not true for all differential equations. Students either over-generalizing the concept to situations in which it does not apply, or not seeing equilibrium solutions as a subset of solutions to a differential equation (Zandieh & McDonald, 1999). Several researchers recommended that they can develop a scheme to understand and find solutions to first order differential equations. For instance, Donovan (2004) reported one scheme, in which high achieving students thought of differential equations as functions and as “objects to be solved” at the same time. These results were aligned with previous findings (Rasmussen, 2001).

Thinking of differential equations and their solutions as a function is important for students as they reason about what meaning the curves one sketches from a slope field. Students who have used technology to study solutions of differential equations (or qualitative graphical solutions) exhibit a more conceptual understanding than students who did not (Slavit, LoFaro et al., 2002). Finding the qualitative graphical solutions has become an important course of action in current differential equations classes, primarily due to the introduction of sophisticated technology that provides sketches of these solutions. Therefore, students reasoning about graphical and numerical representation became another important research area in mathematics education.

Rasmussen (2001) examined the connections students make between graphical and algebraic representations, while providing autonomous differential equation and the corresponding graph of function and its derivative graph. Based on findings of the study, author concluded that targeted student of the study had learned a graphical approach to determine stability in which the graphs they constructed did not carry the deliberate conceptual logic. Consequently, author recommended that just adding graphical techniques to a differential equations class may not increase conceptual understanding (Rasmussen, 2001), and may not necessarily develop better conceptualizations (Allen, 2006). They may use graphical techniques in ways that are not connected to meaning and may have determined the notions of what graphs look like which they utilize when looking at graphical solutions to differential equations (Rasmussen, 2001). Unfortunately, students do not prefer to use graphical methods and techniques and almost always choose analytic solutions when possible. Habre (2000) also realized the disadvantage of favoring only analytic or algebraic representation. Author noticed that student's dominant perception about solution remained in the analytic realm although main amount of class time involved learning qualitative methods. This result shows that few more environmental factors, such as student's mathematical culture can be responsible for it (Artigue, 1992). However, more work in graphical setting, learning environment and norms can potentially contribute to greater conceptual understanding.

For that reason, mathematics educators started to examine students' understanding of systems of differential equations. Trigueros (2004) examined students' understanding of straight line solutions to a linear system of DEs. Author reported that only one student had a complete understanding of straight line solutions. Whereas, others students had faced the interpreting problems regarding to the meaning of equilibrium. Further, author highlighted that these students were unable to interpret the meaning of a point in phase space and seeing the dependence of time in the phase space while learning of systems of differential equations (Trigueros, 2000).

To overcome these difficulties, Whitehead et al. (2003) anticipated that mental operations can be used to reason about and to develop conceptualizations for systems of differential equations. Later on, Allen (2006) suggested that advent of technology provide a venue for mathematics educators and mathematicians, to conduct research about students attitudes or their thinking, reasoning about graphical solutions and numerical approximations and, also their understanding of systems to DEs. These technology advancements enhance student's graphical representations, because thinking visually makes higher cognitive demands than thinking algorithmically (Eisenberg & Dreyfus, 1991). Consequently, it promotes better understanding in a differential equation class.

For instance, Habre (2000) exploit computer modules in his study that were intended for specific course goals. Based on the study findings, author recommended that these computer modules might have been supportive to students in their development of mathematics in the graphical setting. Also, these issues deserve more attention and required further elaborations.

In another study, it was studied the development of flexibility in thinking in a class studying differential equations by using the CAS Derive (computer algebra systems) course (Keene & Rasmussen, 2013). The use of Derive was a positive influence in students' ability to develop flexibility, particularly in their movement between types of representation. Stroup (2002) also supported this flexibility that reasoning symbolically and non-symbolically are both useful and one does not necessarily build on the other.

Students moved from quantitative to qualitative reasoning or qualitative to quantitative reasoning while solving problem (Keene, 2007), hence both these perspectives has value for students in mathematics courses (Goldenberg, Lewis et al., 1992).

Mathematics education reform movement including use of technology has been remained influential. Also, it can be used as a teaching tool to convey a stronger emphasis on mathematics reasoning from a qualitative point of view (Reston, 2000b). Moreover, the introduction of technology gradually made it possible to study the ideas of change over time at the K-12 school level and students can reason about mathematical situations involving time. Learners' conceptions of time are a component of mathematical reasoning within the broader mathematical domain "mathematics of change and variation" [MCV] (Kaput, 1999a).

Mathematics education researchers are encouraging the prominent place of MCV in the K-12 curriculum (Confrey & Smith, 1994; Kaput, 1999b; Stroup, 2002), because students at all levels need to be exposed to the ideas of dynamic behavior in mathematics (Kaput, 1999b). The concept of time and how quantities behave and interact as time passes, is central to dynamical systems (Allen, 2006). However, real world dynamical systems are situations where varying quantities have mutual effects on each other as time passes, e.g. phenomenon of two animal populations is an example of dynamical system (Williamson, 1997). To better understand the complex dynamical systems, and to extend the MCV, parametric or mathematical reasoning were found helpful (Allen, 2006). Parameters play an important role behind the mathematical situation of the change as it may influence and even define other quantities and might be received relatively little attention in mathematics education. Furthermore, the use of parametric reasoning occurs when students use their understanding of time to reason about mathematical quantities (Allen, 2006).

The mathematics of dynamical systems approaches in differential equation can generally be classified into one of three categories; systems of differential equations that model continuous phenomena using two or more rate of change equations, difference equations that model discrete phenomena and other systems that cannot be modeled with either of the other two types of systems of equations. The system of two differential equations that models the population of a parasite population and its host population, where both populations are continually changing over time and affecting the rate of change of the other population and hence the size of the population (Williamson, 1997). The increase in mathematicians' interest in dynamical systems, which relates directly to the MCV, is one factor contributing to pedagogical and curricular changes in differential equations.

2.4.1 Changes in differential equation courses pedagogy and curricula

A dynamical systems approach to teaching and learning differential equations has become more common in North American universities and is being integrated into the reform efforts for many differential equation classes. These changes are the result of three primary influences; mathematicians' interest and contemporary work in dynamical systems, K-12 mathematics and calculus reform, and new technology that makes it possible to investigate solutions to systems that were not solvable with analytic methods (Allen, 2006).

The first influence that has supported the new differential equation course curricula and pedagogy was to study differential equations with a dynamical systems approach. The tools to investigate scientific phenomena by mathematically modeling them as dynamical systems, in order to understand them qualitatively and numerically in considerable detail are relatively new.

The second influence was calculus reform, which began in the 1980's. It involves changes to the standard curriculum and instructional strategies, which include a decreased emphasis on symbolic differentiation and integration and the inclusion of the "rule of three." In the rule of three, graphical, numerical, and algebraic techniques for differentiation and integration and technology such as computer algebra systems and graphing calculators are all components of instruction. Further, influences from K-12 mathematics education reform have impacted university classrooms through increased emphasis on cooperative learning, more active involvement by students, and written communication as an important aspect of learning.

The third stimulus for the differential equations reform was the introduction of new technology with its power to investigate graphically and numerically phenomena that were previously not accessible. As Blanchard (1994) explains "Technology serves as a vehicle for changing the nature of the course from one where students passively receive information to one where students actively participate in their education". In summary, advances in technology that allow for the contemporary investigation of dynamical systems, K-12 mathematics education reform and calculus reform have contributed to differential equation reform and the move to the dynamical systems approach to teaching differential equations. These reform movements such as, use of technology and dynamical approach to teaching DE (as a part of this movement) is explained in the section below.

2.4.1.1 Advancements in differential equation problem solving

The reform movement within the region of differential equations was stimulated in the mid-1980s by the improved accessibility of technology and by calculus reforms for the effective teaching and learning differential equations (Ju et al., 2007). Use of technological advances was initiated to analyze ordinary differential equations by combining algebraic, numeric, and graphical representations (Camacho-Machín et al., 2015a; Hubbard et al., 2012). Examples of technology incorporations are Mathlets; a java applet, Computer Algebra System including Maxima, Maple, and Math lab and Interactive Differential equation. These software pogrammes are employed to visualize graph and also to understand the connection between graph and equations (Azman & Ismail, 2013; Dana-Picard & Kidron, 2008). These developments have changed the entire setting and opened a new evenues for assembling the concepts and their connections to the real world situations (West, Strogatz et al., 1997). As a result, variety of real world problems, including moving object velocities/accelrations, temperature/pressures changes, fluid flows and aerodymomics were identified and mechanisms were deveolped to solve these problems though solving their related differential equations (Aravind, Valluvan et al., 2013; Moore, Miller et al., 2013; Pollak, 2015; Yurtseven Avci, Vasu et al., 2014).

Inspite of the many advanatges of technology tools, modeling a physical problem through differential equation became a problematic situation in reality. Because, students emerging from a conventional differential equation courses have slight understanding of what solutions of differential equation represent in an applicable conditions (Habre, 2000). Hence, several educationalist considered the importance of qualitative approach and it should be adopted for differential equation course. Although in past, adoptation of such approach was not accepted due to the difficulties related to visual aspects. However, incorporation of computer graphics has provided extra-ordinary visual capabilities to the teachers and learners and give advantage in the visualization of complex relationship that student often found difficult to understand.

Visualization has a very important role regarding to the understanding of dynamical aspects of an basics differential equations. It is used to assists in understanding the derivative as the slope of a curve;and also help to interprate graphs,and reading informationfrom these graph such as existence of an equilibrium state and long-term behaviour of the solutions (Borrelli & Coleman, 1999). In this respect, Gollwitzer (1991) considered the direction field as an important tool that is used to encourage students to think about visual component in differential equation.

Two main processes that are more worthy to mention in the study of ordinary differential equationare; identification of equilibrium solution and the recognition of the values where the slope field exists (Camacho-Machín et al., 2015a). Habre (2000) analysed how students use the slope field to solve first order differential equation and how they extract information from these field in a reform setting. Findings were highly encourging. Howerver, student work with ease with single representation and they found it difficult to cope with different representations simultaniously (Habre, 2000).

Rowland et al. (2004) explored that teaching and learning become more problemetic to handle different register of representation in the context of ordinary differential equation. This problem is associated with students understanding of how mathematical model are interelated with the real context and how they interpret the parameters. Camacho-Machín et al. (2015a) revealed that different digital tools gave students self-confidence to represent the same phenomena and also encouraged them to empower the information using graphical and as well as numerical representation of solution. Furthermore, by considering the relationship between ordinary differential equation and context, and with the addition of digital tool can assist studentsin their understanding, in many countries. Accordingly, differential equations curriculum has been changed at introductory level (Rasmussen, 1996).

Despite of technology advantages, at initial or pre university levels, it is a great challenge to determine students interaction by means of the digital tools and representation guidance to provide sense to parameters connected with it (Rowland, 2006; Rowland et al., 2004). Also, it is difficult task to find out how to construct instruction strategies to promote student learning, particularly in the developing countries where utilization of the technological based methods are still challenging (Rasmussen, 2001).

The second reform movement was initiated, when international commission on mathematics instruction on teaching and learning of mathematics realized that universities are now facing newest challenges. The commission further argued that community needs to respond in innovative and theoretically grounded ways (Swanson & Holton, 2001). To overcome these challenges, one response is to develop new curricular and instructional approaches that might be based on current theories of learning and instructional intentions. One such innovative approach, referred to as the Inquiry-oriented differential equations (IO-DE) projects. Moreover, Keeping in mind the dynamical systems standpoint of (Artigue & Gautheron, 1983; Blanchard, Devaney et al., 2002), the function of IO-DE project is to handle differential equations as system that elucidate how functions develop and change over time.

The function of this approach is to utilize on advances within the discipline of mathematics and of mathematics education, such as instructional design theory of RME approach and the social negotiation of meaning at K-12 and tertiary level (Rasmussen, Kwon, et al., 2006). The focus of IO-DE is to integrate analytical, graphical (qualitative method), and numerical methods.

An important pillar in constructivist pedagogy is contextualizing learning using real-world examples and an authentic environment (Abdulwahed et al., 2012). Differential equations are a beautiful application of the ideas and techniques of calculus to solve various real life problems (Habre, 2002). However, regarding to authentic environment, teachers should identify and valued those characteristics of classroom settings that have strong contribution towards students learning. Unfortunately, traditional instructions discourage students from creating their own problem solving strategies.

One promising alternative to traditional curricular approaches is the realistic mathematics education approach (RME). Wherein, students' learning is positioned in experientially real situations (Rasmussen et al., 2000; Yackel, Stephan et al., 2003). In RME setting, students are provided with various opportunities through diverse learning context. So that, they can reinvent solution strategies for certain types of problems. As a result, this assists students to create their own methods of reasoning. Teachers can examine student's informal strategies and historical trends. Further, based on this examination they can design mechanism (composed of activities) towards assisting students' towards re-inventing mathematical knowledge through their experience with mathematics. More detail of IO-DE is provided in the section below.

2.4.1.2 Inquiry-Oriented based approach for learning differential equations

The inquiry-oriented class for differential equation learning is an constructive learning environment where student participate, explicit meaning negotiation, create, discuss, and cooperate by integrating students' mathematical understandings to attain the formal mathematics (Gado, 2005; Rasmussen et al., 2000). Therefore, inquiry enables students to learn mathematics through engagement in authentic reasoning and also make them as a authorize learner to glance mathematics as a human activity as well as they are capable to reinvent mathematics (Rasmussen & Kwon, 2007).

This approach was established on the recommendations of international commission on teaching and learning of mathematics at university level to overcome the newest challenges. Among these, one major issue is the accommodation of much large and diverse group of students (Holton, 2001).

Through inquiry-oriented differential equations (IO-DE) approach students are able to learn most up-to-date mathematics via inquiry, which engrossed in engaging in mathematical conversation, creating and subsequently conjecturing, exposing and defending ideas and their approach to solve innovative problems. In contrast to it, in traditional environment, instructions design discourages individuals as of generating their own problem solving strategies. Instructors in this project coordinates and assist students in their mathematical exploration. In this way, they can also get opportunity to learn incredible about particular mathematics ideas, considering students thinking and better situate them to build on students thinking by posing new questions and tasks.

In light of these characteristics, this project was claimed to provide a model for those who were fascinated in exploring the view points and potentials of improving undergraduate mathematics education (Rasmussen & Kwon, 2007).

On the other hand, to implement such reform practices of instructions in real class rooms requires a specific type of knowledge, different from the mathematical content knowledge, pedagogical content knowledge, and pedagogical knowledge. Therefore, for the first time, the IO-DE curriculums proposed an informative portrait of knowledge (Rasmussen & Kwon, 2007). Inquiry-oriented based teaching also incorporated the concept of realistic mathematics education approach (RME) which is an alternative approach to the traditional curriculum approach, in which students' learning is based on practical real condition (Rasmussen et al., 2000; Yackel et al., 2000). Instructional design theory of RME was well adapted as key stone of inquiry-oriented differential equation projects (Rasmussen & Blumenfeld, 2007), in which students' were asked to find out solution methods and make interaction with teacher as well as with class mates (Kwon, 2002).

The study of Rasmussen and Blumenfeld (2007) has a significant contribution towards RME emergent model, in which the researchers elaborated the emergent model for student reinvention of system of linear differential equations solutions. In addition to it, Rasmussen and Marrongelle (2006) also explained the function of graphs and gestures for the reinvention of the Euler method for differential equation and emphasized how these functions change in students' subsequence use of the Euler method to approximate system of differential equations. This study provided a dictionary of student gestures and also how they are associated with students' reinvention and use of algorithm of Euler method.

A second corner stone of inquiry-oriented differential equations (IO-DE) project is a research area whose focal point is student thinking and teacher knowledge (Rasmussen & Kwon, 2007). Regarding to student thinking, Keene, Glass et al. (2011) observed students' ideas about the use of time as dynamic quantity and the way time-based reasoning can promote understanding of solution function. Author identified five different technique where students had integrated time as a varying quantity as their understanding of differential equation increased. Wagner, Speer et al. (2007) argued that reform practice of instructions include knowledge apart from mathematics pedagogical knowledge, content knowledge and pedagogical content knowledge supporting traditional instructions design.

Literature from quantitative studies has also evidently proved that inquiry oriented differential equation approach enhances desirable students learning outcomes as compared to traditional assessment methods (Rasmussen & Kwon, 2007). Regarding quantitative assessments of IO-DE learning, Rasmussen, Kwon, et al. (2006) evidently proved that there was no significant difference on student performance on the procedurally-oriented items, although analytic solutions were the main focus of the comparison groups. However, IO-DE group students performed better on the conceptually-oriented than the traditional differential equation students' group. Since, IO-DE project instructional approach and course design emphasized on developing multiple strategies both concurrently and with equal substance (Kwon et al., 2005). Therefore, author concluded that IO-DE students retain both qualitative and analytic techniques, and utilize them more flexibly as compared to traditional differential equation students.

In similar context, Kwon et al. (2005) conducted a follow-up study one year after instruction for a subset of the students from the comparison study on the retention effects of conceptual and procedural knowledge. Researchers concluded that IO-DE enabled students to emphasize on variety of strategies both simultaneously and with equal importance, due to which, these students retained multiple ways to approach problems and performed better even after one year (Kwon et al., 2005). Other evaluation studies also proved the positive outcome of IO-DE approach on students' conceptual understanding, problem solving, retention and justification (Ju & Kwon, 2004; Kim & Kwon, 2006; Kwon, Cho et al.; Kwon et al., 2005; Rasmussen, Kwon, et al., 2006).

2.4.1.3 Beliefs based systems for learning differential equations

In addition to mathematics conceptions, Cobb (1985) argued for the assessment of students' belief systems apart from these reforms. There are few other important aspects of class room settings that have essential role in transforming student's beliefs, such as instructional resources, students own cognitive resources, and also the role of teacher. Ju et al. (2007) illustrated the effect of an inquiry oriented differential equation course on the enhancement of student beliefs about mathematics. Authors extended the evaluation of the inquiry oriented differential course model beyond the cognitive aspects of mathematics learning and investigated transformation of students' beliefs about mathematics as well as their relation to the discipline.

Concerning the students' mathematical growth, several researchers also highlighted the significance of social or cultural processes (Cobb, 1995; Lave, 1988; Rogoff, 1990; Saxe, 1991). An another remarkable approach was carried out by Yackel et al. (2000). Authors extended the analysis of social interaction patterns, social and socio-mathematical norms regarding explanation and how these norms were characterized in differential equation class. The study of Yackel et al. (2000) directed attention towards explicit social aspects of reform-oriented university-level differential equation.

These social aspects, in addition to instructional materials, use of technology and course content, are elements which instructors can control and provide theoretical goal of explicating the usefulness of the constructs of social and socio-mathematical norms for analyzing university-level mathematics teaching and learning. A psychological construct incorporated students' individual beliefs about their own role, others' roles, and the general nature of mathematical activity, and the sociological construct, the classroom social norms are mutually constitutive (Yackel et al., 2000).

Yackel and Rasmussen (2002) documented how students emerging beliefs about their ability to create mathematics and also the role of explanation and justification are reflexively connected with the social and socio-mathematical norms of their classroom communities. Findings revealed that student's way of talking about mathematics changed from the view of mathematics as self-determined, exterior and superior to students mind, to that of mathematics as a product of their own engagements. Several studies were carried out to further elaborate these aspects in promoting student learning at the K- 12 level to inquiry oriented university level teaching (Rasmussen et al., 2000; Stephan & Rasmussen, 2002). Researchers reported how these emerging norms cultivate a shift in student's beliefs about their own and their teacher role, and about the general nature of mathematical activity. Moreover, results revealed that these beliefs transformed from considering their role as passive learner to active participants in knowledge creation. Based on these findings, Yackel et al. (2003) reported that such class rooms support students beliefs emergence for better developments. Also, author concluded that such class room environment support in enhancing student's beliefs about mathematics more compatible with the discipline itself.

Several researches also claimed that student learning is located inside the interconnected constructs of problem solving, epistemology, and self-regulated learning (Muis, 2007; Muis et al., 2009; Schommer-Aikins, 2004; Stockton, 2010). Typically, successful problem-solvers exert control over the problem space and have availing epistemological beliefs (Muis, 2008; Perels et al., 2005; Schoenfeld, 1983, 1985b, 1989). Schommer-Aikins (2004) hypothesized reciprocal relationship between epistemological beliefs and self-regulated learning. However, experimental results have shown that a relationship exists between SRL and epistemological beliefs in multiple contexts (Bråten et al., 2005; Hofer, 1999; Muis, 2008). Several studies also highlighted thefunction of self-efficacy beliefs, motivation, engagement, and attitudes towards mathematics learning (Abdulwahed et al., 2012; Alpaslan, Yalvac et al., 2016; Fadlelmula et al., 2015; Velayutham, Aldridge et al., 2012). Beside these, role of goal orientation beliefs (part of self-motivational beliefs) were also found as an energizing agent for an individual's self-regulatory behaviors and influence the implementation of self-regulatory knowledge and skills (Kingir et al., 2013; Montalvo et al., 2004). Wolters et al. (1996) studied the relationship between three goal orientations and student self-regulated learning focusing the subject mathematics.

Literature reveals that, epistemological math problem solving beliefs, self-regulated learning (SRL) strategies and goal orientations may play an important role towards the students' math problem solving ability, particularly, in problem solving of differential equation. Therefore, for the current study, the relevant theoretical perspective of differential equation problem solving along with these three factors has been reviewed and is placed in the following sections.

2.5 Theoretical prospective of non-routine differential equation problem solving

Problem solving is a principal component and heart of mathematics and mathematics education. It is a mechanism that serves as a vehicle for learning new mathematical ideas and skills. A problem is only a problem if you don't know how to solve it. Whereas, a problem that can be solved easily by familiar procedures is known as exercise (Schoenfeld, 2014). National council of teachers of mathematics (NCTM) adopted a more constructivist perspective about problem solving. According to NCTM, problem solving can be a type of learning, given that students are involved in the upgrading of their own knowledge.

During last two decades, a greater emphasis has been placed on student's mathematical problem solving. In spite of various research studies and books written about problem solving, mathematics educators shows little understanding of problem-solving skills (Reston, 1989, 2000b). In fact, Lester (1994) grieved over that problem solving is probably the least understood topic. The reason is that the focus of these research is on narrow theoretical perspectives (both general and particular) mathematical problem solving (McGinn & Boote, 2003).

Mathematics problem solving field has its origins in Polya's (Polya, 1957) "*How to solve it*". In this influential work, author described a broad framework of four stages of mathematics problem solving, understanding, devise a plan, carry out the plane and look back (Leong, Toh et al., 2012). This framework assimilates diverse cognitive processes required for successful problem solving.

According to Schoenfeld (2014), there are four aspects that have contribution to problem-solving performance, basic mathematical knowledge and resources, heuristics knowledge, factors affect the way the problem solver views problem solving, and decision-making skills associated with monitoring and self-evaluation of problem solving process. Thus, the framework allows describing the students' problem solving process in terms of relevant mathematical process and resources (Camacho-Machín et al., 2012b).

However, problem solving is a very complex phenomenon that is affected by several factors such as mathematical ability, selection of strategy, self-regulation, and motivational factors (Robinson, 2016; Schoenfeld, 2014). Much of the research on mathematical problem solving, examines one particular factor in depth, but there are rarely studies that explore several factors collectively. In spite of several factors, mostly researcher investigated one specific factor in depth, but there are rarely studies that explored several factors collectively affecting math problem solving. For example, several researchers focus on how content knowledge affects problem solving (Canobi, Reeve et al., 2003; Hecht & Vagi, 2012), some studies take a close look on specific problem solving strategies and how the strategies play an important role in problem solving (Cai, 2000; Cai & Hwang, 2002; Huntley & Davis, 2008; Lannin, Barker et al., 2006; Montague, 1992). Several researchers concentrate on belief and affective factors such as motivation for math problem solving (Areepattamannil, 2014; Areepattamannil, Freeman et al., 2011; Halawah, 2006; Lepola, Niemi et al., 2005) and mathematics anxiety during math problem solving (Kyttälä & Björn, 2014; Ramirez, Gunderson et al., 2013; Vukovic, Kieffer et al., 2013). Recently, Perveen (2010) examined the effect of problem-solving approach on academic achievement of students in mathematics at secondary level in Pakistan. Author compared the effect of expository strategy and problem solving approach of teaching mathematics. Findings showed that presenting mathematical concept via problem solving sequence cause the learner to incorporate the content conceptually. As a result, students can retain it more rapidly as compared to expository strategy.

Although majority of the research studies focused on several factors affecting mathematical problem solving, however, Robinson (2016) feel regret that but they do not concentrate on how students perform on non-routine and unconstrained tasks. Non-routine tasks involve different type of strategy such as guessing and checking, looking for a pattern, drawing a diagram (Gilfeather et al., 1999; Polya, 1957; Schoenfeld, 1992). Whereas, Gilfeather et al. (1999) defined routine problems as well defined, well-structured tasks that use already known steps and procedures in order to find a solution.

Other researcher also agreed that problem-solving ability cannot be monitored from the advancement of general mathematical skills. Teacher educate their students to solve mathematics problems by some standard method provided by the text book at secondary level (Foshay & Kirkley, 2003). As a result, these students have great difficulty in solving non-standard problems (non-routine) that require the application of domain knowledge and routines. They also had lack of motivation and confidence in attempting unfamiliar non-routine problems (Kaur & Yap, 1998; Kaur, Yap et al., 1996).

Besides this, students became uneasy, nervous, and extremely uncomfortable because they are unable to recall and apply learned procedures in a simple way (Yeo, 2009). Moreover, Yeo (2009) revealed that students non-routine problem solving difficulties are, lack of comprehension of the problem posed, lack of strategy knowledge, unable to translate the problem into mathematical form, and incompetence to use the correct mathematics. Consequently, the low correlation was observed between problem-solving abilities and academic achievement (Joseph, 2011). Author recommended that instructor should explore methods to incorporate non-routine problems into their mathematical curriculum with the intention of getting desired outcomes of understanding and thinking.

Table 2.1: Three steps of problem solving

Steps	Types of actions	Description of actions	Description of actions
1	Find the link and the relationship.	Students look for ways to reduce the problem into smaller and simpler parts so that it is easier to solve.	Able to translate it into math form/make a mathematical modeling of the problem $\frac{dv}{dt} = -g$
2	Plan	Students try for ways to solve the word problem and plan out the steps.	Integrate the problem Find the constant of integration then put back value of constant in the original equation. To find out the height of the ball they need to take the derivative of velocity.
3	Work it out	Students carry out the computations to solve the word	Execute the procedure for velocity and then for height of ball.

Polya (1957) an originator of math problem solving, also recommended that solving a routine problem did not contribute in mental development of student. As a result, authors recommended that non-routine problem should be employed to develop higher-order thinking in the process of understanding, analysis, analysis, exploration and application of mathematical concepts.

Based on the Poly's works, Schoenfeld (1985b) mathematical problem solving framework included the aspect of control, which he further subdivided into reading, analyzing, exploring, planning, implementing, and verifying. Although the popularity of the four stages experienced ebb and flow over the last 50 years, a significant number of research (Ho & Hedberg, 2005; Muir, Beswick et al., 2008; Weber, Radu et al., 2010) and mathematics teacher education program (Lee & Lee, 2009; Namukasa, Gadanidis et al., 2009) continue to make reference to the model of the four stages as a theoretical basis.

For this study, Polya's three stages frame work for problem solving including understand a plan, devise a plan and carry out a plan, was used to evaluate the differential equation problem-solving ability of the students with non-routine differential equation tasks. Detail for steps is provided in Table 2.1. The relevant literature of epistemological math problem solving beliefs, self-regulated learning (SRL) strategies and goal orientations is placed in the following sections. Detail literature review framework for epistemological beliefs is provided in Figure 2.2.

2.6 Epistemological beliefs

Epistemology is a branch of philosophy concerned with the nature and scope of knowledge (Edwards, 1967). Whereas, Twenty five years ago, Pajares (1992) stated: " defining beliefs is at best a game of player's choice". However, Abelson (1979) defined beliefs in terms of people manipulating knowledge for a particular purpose or under a necessary circumstance. An individual's knowledge stems from the interaction between the individual and the environment (Tang, 2010). However, distinguishing knowledge from belief is a daunting job. Nespor (1987) differentiated between the concept of beliefs and knowledge and suggested that beliefs have stronger affective and evaluative components than knowledge. Further author recommended that these beliefs affect typically operates independently of the cognition associated with knowledge.

Individuals' views and concepts about knowledge and knowing remained focal point of several researchers (Buehl & Alexander, 2001; Burr & Hofer, 2002; Hofer, 2004; Schraw & Sinatra, 2004). Educational psychologists identified the three major categories of epistemology, a developmental perspective, a system of personal beliefs, and an alternative concept design.

Developmental perspective is based on students' personal expectations (Belenky, 1986; King & Kitchener, 1994; Kuhn, 1991; Magolda, 1992; Perry, 1970). Perry (1970) further elaborated it as a configuration in which individuals understands the nature, origins of knowledge, value, and responsibility in a chronological and coherent process. A system of personal beliefs is a belief system comprised of several but more or less independent dimensions, such as as the stability of knowledge, the structure of knowledge, the source of knowledge, the speed of knowledge acquisition, and the control of knowledge acquisition (Schommer, 1990).

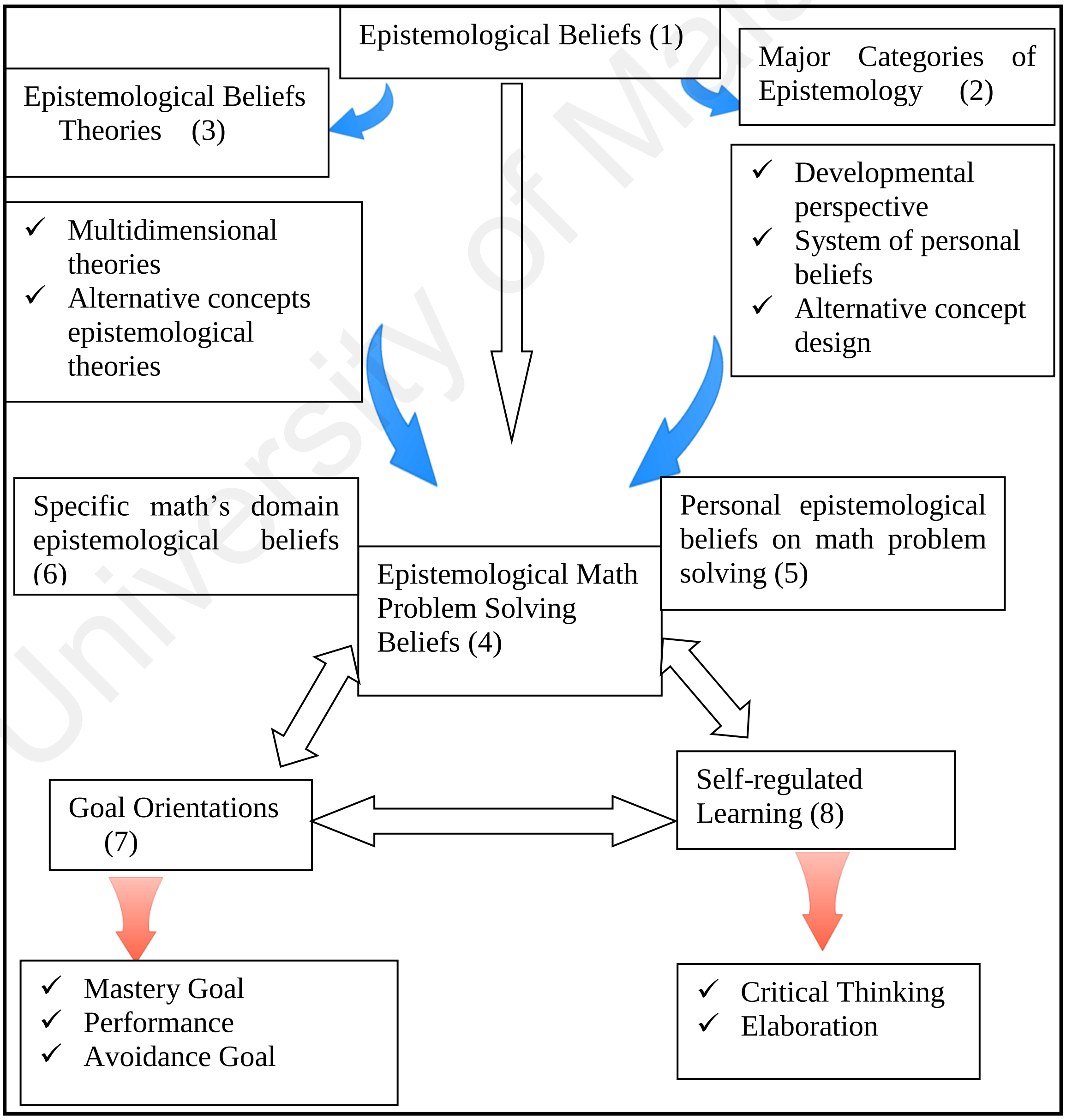

Figure 2.2: Literature review framework for epistemological beliefs

In the third category (alternative concepts of epistemology), usually researchers examine individuals' epistemic beliefs about the definition of knowledge, and is concerned with how knowledge is constructed, evaluated, where knowledge reside, and how knowing occurs (Hofer, 2001). It includes epistemological theories (Hofer et al., 1997) and epistemological resources (Hammer & Elby, 2002). Further detail of these three perspectives is presented below.

2.6.1 Review model of the development perspective

Concept of development perspective was first time introduced by Perry (1970). Later on, several researchers also proposed different models. The different models used in development perspective include ways of knowing (Belenky, 1986), epistemological reflection (Magolda, 1992), and reflective judgment (King et al., 1994). The details of these models and schemes are provided in the following sections.

2.6.1.1 The Perry scheme

The movement of developmental epistemology started with Perry's work (Perry, 1970). The focus of Perry's work was to analyze students' intellectual development and to examine how students interpret their experiences. Author inspired others with his work by determining college student's transformation in their views about nature of knowledge. At college level, mostly students think that knowledge is simple, certain, and allotted by authority. Whilst they accomplish graduation, many of these students recognize that knowledge is complex, tentative, and derived through reason and evidence (Schommer-Aikins et al., 2013a).

Although Perry's work revealed several aspects of development perspectives, however, it was based only on the perspective of white educated males. Therefore, it could not to be applied to students with different educational backgrounds, ages, gender and life circumstances. To overcome this issue, Belenky (1986) interviewed women with different educational backgrounds, ages, and life circumstances. On the basis of the findings, Belenky classified five main epistemological perspectives: silence, received knowing, subjective knowledge, procedural knowledge, and knowledge construction.

Magolda (1992) developed epistemological reflection model to elaborate the development perspective. The focus of this model was to analyze how students conjecture about the nature, limits, and the certainty of knowledge developed. According to epistemological reflection model, absolute knowing, transition knowing, independent knowing, and contextual knowing are four major patterns (Magolda, 1992). Soon after, King et al. (1994) developed reflective judgment model (RJM), that depicted the development of complex reasoning in late adolescents and adults. Further, it also explored how the assumptions people hold are related to the way they make judgments about controversial issues (King & Kitchener, 2004). In the beginning stages of this model, individuals see knowledge as absolute; however, as individuals progress through the stages, their beliefs evolve into temporarily uncertain knowledge. In later stages, individuals begin to see multiple perspectives of knowledge and conclude that knowledge is subjective. In the final stage, individuals believe that knowledge is a continuing process of inquiry and only approximates reality (King et al., 1994).

For many years researchers studied epistemological beliefs with Perry's uni-dimensionality paradigm as the underlying assumption (Kitchener & King, 1981; Magolda, 1992). Kuhn (1991) extended Perry's conceptualization with ill-structured problems. According to Kuhn (1991), absolutist, multiplist, and evaluativist are three main epistemological views. People with multiplist posture admit other views, while evaluativist recognize the uncertainty of knowledge then compare and explore all views according to their relative situation (Jung, 2011).

Several researchers investigated young children's epistemological beliefs about intelligence independently from Perry's work (Dweck & Bempechat, 1983; Dweck & Leggett, 1988). Dweck's theory suggested that some children considered that the learning capabilities are fixed at birth and academic assignment are just used to document their intelligence. Therefore, these children have a tendency to exhibit weak behavior/performance when faced with challenging task. In contrast, other children believe that learning abilities are improvable over time and with experience. They also considered that the function of an academic task is to enhance their intelligence. Therefore, these children tend to confront diverse strategies and show persistence in their efforts to learn when faced with hard task.

Research has also shown definitely the importance of a growth mindset- the belief that intelligence grows and the more you learn, the smarter you get (Boaler, 2015). Author further focused the mindset of math failure and suggested that to erase mathematics failure, we need students to have growth beliefs about themselves and accompany them with growth beliefs about the nature of mathematics and their role in relation to it. Children need to see mathematics as a conceptual, growth subject, that they should think about and make sense of.

2.6.2 A system of beliefs

In 1990s, Schommer (1990) criticized the Perry's general concepts of developmental and sequential stages. The author anticipated a totally different perspectives about personal epistemology from those of other researchers in terms of dimension and progression of stages (Jung, 2011). The concept of personal epistemology is attributed with initiating the study of epistemology through the lens of independent and multidimensional beliefs. Schommer (1990) proposed that personal epistemology is a belief system that is composed of several but more or less independent dimensions.

Schommer anticipated that more than one belief encompassed personal epistemology, and by more-or-less independent author meant that each belief may or may not develop at a different rate (Schommer, 1990). It means one cannot assume that if individuals are mature in one belief then they are necessarily mature in all of their epistemological beliefs (Schommer-Aikins, Duell et al., 2003).

Besides this, Schommer argued that the development of epistemological beliefs may be recursive rather than sequential that is, beliefs are revisited, reviewed, and refined throughout life (Schommer, 1994b). The author hypothesized five beliefs, the stability of knowledge (tentative to unchanging), the structure of knowledge (isolated bits to integrated concepts), the source of knowledge (handed down by authority to gathered from observation and reason); the speed of knowledge acquisition (quick-all to gradual learning), and the control of knowledge acquisition (fixed at birth to life-long improvement) (Schommer-Aikins et al., 2002).

To estimate these five hypothesized beliefs, Schommer (1990) developed Schommer'sepistemological belief questionnaire (EBQ), which consistently yielded four factors. These factors are simple knowledge, fixed ability, quick learning, and certain knowledge (Hofer et al., 1997). Schommer (1990) further classified beliefs along each dimension, such as beliefs as being naive or sophisticated. For example, an individual's belief regarding tothe certainty of knowledge can range from believing that knowledge is absoluteto believing that knowledge is tentative. Believing that knowledge is absolute is considered naive, while belief about tentative /certainity of knowledge is considered as sophisticated belief. These sophisticated beliefs further support high-quality study strategies, comprehension, interpretation, and high-quality problem solving (Schommer-Aikins et al., 2002).

2.6.3 Alternative conceptions

Alternative concepts of epistemology include epistemological theories (Hofer et al., 1997) and epistemological resources (Hammer et al., 2002). Although the perspectives on epistemology are totally different, researchers usually examine individuals' epistemic beliefs, including beliefs about the definition of knowledge, how knowledge is constructed, how knowledge is evaluated, where knowledge resides, and how knowing occurs (Hofer, 2001).

2.7 Theories of epistemological beliefs system and alternative concepts

Belief system is govened by multidimensional theory while alternative concepts of epistemology mainly involve epistemological theories (Hofer et al., 1997) and epistemological resources (Hammer et al., 2002). The detail of each theory is provided in the following sections.

2.7.1 Multidimensional theory of epistemological beliefs

Epistemological beliefs turned out to be a fastest growing area of research after the inspirational work of Perry (1970), whose focus was the development of students' views of knowledge. Several researchers studied epistemological beliefs with Perry's uni-dimensional paradigm as the underlying assumption (Chandler, 1987; Kitchener et al., 1981; Magolda, 1992). Over the past quarter century, researchers recognized that knowledge is a multidimensional and multilayered construct, and also some knowledge has superiority power in a certain situation over other knowledge (Alexander, Schallert et al., 1991; Prawat, 1989).

Starting in 1990, Schommer (1990) introduced a new paradigm for epistemological beliefs. Author hypothesized that epistemological beliefs are considered as a system of beliefs that are more-or-less independent beliefs. These beliefs are system of small number of uncoordinated dimensions that are more or less independent developing not necessarily synchrony. Individuals' epistemological beliefs are the components of a complex and sophisticated belief system and are reflective of the nature of knowledge itself (Schommer, 1990).

From a multi-dimensional epistemological beliefs perspective, students may hold diverse, even sometimes contradictory beliefs depends on both contextual as well as domain-related issues (Stockton, 2010). For example, few researchers (Hynd & Guzzetti, 1998; Vosniadou & Brewer, 1992) noticed that prior knowledge can assist or be a hindrance in specific context or situation. Regarding to domain-related issues, students beliefs about mathematics knowledge may be varied from those beliefs they implicate to history. Due to this, students may hold specific and diverse epistemological beliefs about school physics versus everyday physical phenomena. Therefore, Buehl and Alexander (2006) proposed that beliefs about knowledge have several of these similar characteristics such as multidimensional, multilayered, interactive, situational, and developmental.

Several researchers (Hofer et al., 1997; Schommer, 1990), specifically explored a general approach to the assessment of epistemological beliefs by addressing multidimensional nature of personal epistemology. However, Hofer (2000), investigated the multidimensionality of epistemological beliefs within specific academic domains (Burr et al., 2002; Hofer, 2000). Even though many researchers clarified the nature and structure of epistemological beliefs, still there is lack of consensus on the domain-general and/or domain-specific nature of epistemological beliefs (Buehl et al., 2006; Chan* & Elliott, 2004; Op't Eynde, De Corte et al., 2006; Schommer-Aikins, 2004; Youn, 2000). These issues deserve more attentions, because there is significant relationship between domain-general and domain-specific beliefs (Burr et al., 2002; Hofer, 2000). Buehl and

Alexander (2005) explored students' epistemological belief profiles for both history and mathematics. Within domain, author exposed the possibility of interactions across the dimensions of beliefs, for instance, certainty of knowledge beliefs dimensions are relatively consistent each profile group (Buehl et al., 2006).

In addition to the interactions across belief dimensions, these epistemological beliefs are also related to constructs and process central to learning. For instance, these beliefs are interrelated to student's academic performance and motivation (Buehl, 2003; Hofer, 2000; Schommer-Aikins, Mau et al., 2000), the strategies they employed (Kardash & Howell, 2000), and also their reasoning and problem-solving abilities (Schraw, Dunkle et al., 1995).

These theoretical and empirical endeavors established a new way towards the development of a most recent system of epistemological beliefs; simplicity of knowledge, its certainty, sources and justification for knowing (Hofer, 2000; Hofer et al., 1997). It was revealed that students can differentiate among beliefs based on domain because they hold domain-general epistemological beliefs. Besides this, researchers also tried to validate Schommer's instrument that was basically based on multidimensional belief system. Hofer (2000) employed discipline-focused epistemological beliefs questionnaire (DFEBQ) to measure students' epistemological beliefs specifically in educational psychology and science. Author consistently investigated the domain specificity and dimensionality of epistemological beliefs by means of think aloud protocol (Hofer, 2004). In spite of the methodology concern, findings of the study provided evidence of above cited four dimensions of beliefs. Likewise, cultural background has also an essential role while studying epistemological beliefs. Therefore, Tang (2010) developed epistemological beliefs questionnaire about mathematics (EBQM) in china as well as for eastern countries. The results supported the previous studies.

Overall, entirely studies were further filtered down into two main categories; the implications of students' personal epistemological beliefs on mathematical problem solving (Kloosterman et al., 1992; Muis, 2004, 2008; Schoenfeld, 1985b, 1988, 1989, 1992) and the domain specificity of epistemological beliefs (Hofer et al., 1997; Muis, Bendixen et al., 2006). Detail is provided in the section below.

2.7.1.1 Implications of students' personal epistemological beliefs on mathematical problem solving

Epistemological beliefs have critical educational implications towards thinking, learning, and problem solving. Generally, students came across with new information in the class room and may proceed the learning practice quite differently depending upon how they view knowledge (Burr et al., 2002). The epistemological beliefs influence students reasoning, use of knowledge strategies, and their cognitive information processing (Hammer et al., 2002). Therefore it may be concluded that student's beliefs are the essential components that considerate their learning, strongly influencing and mediating the development of learning and its outcome (Hofer, 2001; Muis, 2007, 2008; Schommer- Aikins et al., 2005).

The influence of epistemological beliefs on both learning process and problem solving behavior was also investigated in numerous studies (Schommer, 1994a). The study of Schraw et al. (1995) revealed that belief in simple knowledge is associated with study strategies and comprehension of complex text, whereas, belief in simple and certainty of knowledge both are interrelated with students problem solving of ill-structured contents (Schraw et al., 1995). Similarly, author noticed that beliefs in quick learning predict students problem solving in well-structured contents and also related to their grade point average/performance. Hofer (2001) further explored the combined impact of epistemological beliefs on thinking, learning, and problem solving. Findings were supporting the previous studies. In the same line, few more studies were conducted to further elaborate impact of beliefs on test comprehension and meta-comprehension, conceptual change and problem solving (Mason, 2003). Literature reveals that problem solving was particularly elaborated by Polya (1957). The author has a lot of contributions to problem solving, focusing on teacher-student discourse, adaptation and application of heuristic strategy. However, author did not describe and relate the problem solving with personal epistemology. D'Ambrosio (2003) argued that this matter is consistent with historical reviews of mathematics education, which referred early to mid-twentieth century as an era of the philosophy of knowledge transfer.

Schoenfeld (1983, 1985b, 1992) was the first who formally introduced personal epistemology to the mathematical problem solving discourse. The author believed that students' mathematical viewpoints are major components to the successful completion of mathematics and mathematics problem-based tasks (Schoenfeld, 1985b). Author considered both students and experts for her studies related to mathematical problem solving (Stockton, 2010). The findings revealed that mathematics instructors are also competent to obtain mathematical information even though they are unable to memorize it (Schoenfeld, 1983, 1985a). In case of college level mathematics, it was observed that students had a tendency to solve problems from an empirical viewpoint. Similar context, Royce and Mos (1980) also acknowledged the exhibition of rationalist-based approaches of the mathematics professors while solving mathematics problems.

Schommer (1994a) categorized mathematical problem solving beliefs as rational and empirical beliefs. Rational beliefs involve logical and analytical approaches, whereas, empirical beliefs include observational and perceptual approaches. Typically, empirical belief solvers hold some non-availing mathematical assumptions. For instance, formal mathematics is not necessary during math problem solving and also, they believe that math problems are quickly solved or not at all. These problem solvers more believe on the existence of a unique solution and, also the availability of algorithm, procedure for all mathematics problems (Schoenfeld, 1985a, 1992). These beliefs are congruent with Schommer (1990) quick learning and low-level beliefs from source of knowledge and simplicity of knowledge dimensions (Hofer et al., 1997). In contrast to empirical beliefs, rational problem solvers usually exploit supplementary control, and so they are more successful as compare to their empirical counterparts.

Similar to Schommer (1994b) classification, mathematics educators also have dichotomized the types of beliefs, appropriate and inappropriate beliefs. These two types of beliefs are based on how students' beliefs influence learning and their learning outcomes. Appropriate beliefs are positively correlated with learning outcomes such as academic achievement, understanding of mathematical concepts and also associated with effective study strategies and problem solving, as contradictory to inappropriate beliefs (Schoenfeld, 1988, 1989). Regarding to mathematics problem solving, Romberg (1992) explained mathematics epistemology from a process point of view in contrast to an acquisition perspective. For many, to know means, to recognize basic concepts and procedures of the discipline. For those who are much familiar with the discipline, to know mathematics, is to do mathematics. Such person accumulates, discover, or construct knowledge in the course of some activity having a purpose. Author suggested that students can make sense if emphasis is put on the process of doing mathematics.

Further, based on mixture of mathematical-based personal epistemology research, Muis (2004) also identified system of mathematical beliefs. For instance, nature of mathematics knowledge, justifications of mathematics knowledge, sources of mathematics knowledge, and acquisition of mathematics knowledge. Within this mixture, students' epistemological beliefs influenced students cognition and motivation, which anticipated her future work connecting personal epistemology and self-regulated learning (SRL) (Muis, 2004). The author further characterized mathematical beliefs into two main categories availing and non-availing beliefs. Availing beliefs are positively correlated to both quality learning and achievement, while non-availing beliefs do not affect in a positive way. Therefore, non-availing beliefs are generally inadequate to mathematics learning and achievement. Other researchers also proposed that students at all level grasp non-availing beliefs. For example, students believe that mathematics knowledge is reflexively supplied by some authority, educators and text book writer. A number of students believe that those who are competent of doing mathematics are born with mathematics genes (Tang, 2010). Therefore, Mason (2003) recommended that instructor should offer interventions to enhance students availing mathematical beliefs, and also planned instruction, tasks, and their evaluation in placement with such availing mathematical beliefs.

In addition to problem solving, several researchers also investigated the influence of epistemological beliefs about knowledge and learning on educational process (Schommer- Aikins et al., 2005; Schommer-Aikins et al., 2003; Schommer-Aikins et al., 2002; Schommer, 1990, 1993b). For example, students who believes in quick learning have a tendency to construct overgeneralized conclusions, acquire poor results, and became overconfidence on test (Schommer, 1990, 1993b).

Similarly, students who believe certainty of knowledge probably generate absolute conclusions (Schommer, 1990), whereas, students holding uncertain believes admit multiple perspectives and willingly revise their thinking (Schommer-Aikins et al., 2002). In addition, strong believers in quick and fixed beliefs do not employ study strategies and are expected to believe that mistakes expose their inadequacy (Schommer- Aikins et al., 2005). Consequently, these students feel trouble and are more likely to hang up in facing difficult problems, because strong believers in fixed ability are anticipated to believe that mistake expose their inadequacy. As a result, they may feel more perturbance and more probable hang up in the face of difficulty (Schommer, 1998).

2.7.2 The domain specificity of epistemological beliefs

The domain specificity of the epistemological beliefs remained one of the core issue faced during the whole journey of epistemology research. For many years research has been conducted as if epistemological beliefs were domain general which means they can apply across all domains (Kitchener et al., 1981; Magolda, 1992; Perry, 1970; Schommer, 1990). Although, Muis et al. (2006) supported both domain-general and domain specific views of epistemological beliefs.Since, epistemological beliefs vary with respect to domain. Therefore, author suggested that domain must considered when developing contextually dependent studies involving personal epistemology.

Domain-specificity is a key factor in the study of students' epistemological beliefs (Hofer et al., 1997; Muis, 2004; Muis et al., 2006). By domain-specificity of epistemological beliefs mean that they can be applicable to specific academic domains such as mathematics, history, and social sciences (Schommer-Aikins et al., 2013a). Students' perceptions differ along with diverse domains. For instance, beliefs about mathematics usually involve perceived level of personal abilities, while beliefs about social studies referred to the level to which the contents are appealing (Schommer, 1990; Stodolsky, Salk et al., 1991). Further, the study of Stodolsky et al. (1991) also supported the existence of domain-specific beliefs about knowledge. Author noticed that students possess different attitude towards mathematics versus social science and, also, they have different concepts of learning for both of these domains. Findings revealed that students believe that they need some support to explicitly solve mathematics. Whereas, regarding to social studies, these students believe that they can learn by themselves if proper material is provided. Therefore, this study had opened a new avenue to the existing four categories of beliefs. These were beliefs about mathematics teaching (Thompson, 1984), beliefs about mathematics, mathematical learning, and problem solving (Schoenfeld, 1985a), beliefs about social context (Cobb, Yackel et al., 1989), beliefs about self in the context of mathematics learning and problem solving (Kloosterman, Raymond et al., 1996). Whereas, Garofalo (1989) demonstrated a different class of students' beliefs influencing mathematical achievement. For instance; the level of problem difficulty is due to the size and quantity of numbers, mathematical problems can be solved by performing one/two computational operations, the operation to be performed usually is determined by problem keywords, students' decisions to revise and to check what has been done depends on the availability of time.

Literature also reveals that several studies conducted for students' beliefs about mathematics and mathematical learning, remained unsuccessful. The main reasons were the restrictions of time-consuming interviews and observations, due to which mathematics instructor and researchers were not able to get desired results, related to students mathematics problem solving beliefs (Kloosterman et al., 1992). This highlighted the need of an effective instrument to measure beliefs and allow instructor to determine student's beliefs and to modify the instructions to improve student's beliefs. To overcome this issue, Kloosterman et al. (1992) developed the Indiana Mathematics Belief Scales (IMBS) to measure students' beliefs about mathematics as a subject and how mathematics is learned. This scale was partially based on Schoenfeld's work. Authors further explored the dimensionality of mathematics problem solving beliefs. Through a series of studies, five dimensions/scales were validated. These dimensions were; rational/empirical, unique/arbitrary, duration of problem solving, procedural/conceptual approach, and effort/inherent mathematical ability (Kloosterman et al., 1992; Muis, 2004; Royce et al., 1980; Schoenfeld, 1985a).

Originally, these five scales comprised of ten items. However, multiple stages of testing for scale validity reduced each scale to six items. An additional sixth scale usefulness of mathematics contains items from Fennema-Sherman's (1976), was also integrated with these five scales. This scale measures students' beliefs about the usefulness of mathematics (Stockton, 2010). Although, Kloosterman et al. (1992) developed the scale to measure beliefs which are related to motivation, but author did not measure the extent of usefulness of mathematics scale because the usefulness scale of mathematics was already existed (Fennema & Sherman, 1976). Detail of IMBS is presented below.

(a) Indiana mathematics belief scale (IMBS)

Indiana mathematics belief scale (IMBS) measures student's beliefs about mathematics problem solving. IMBS is evaluated by the following six scales; 1) duration of problem, 2) steps, 3) understanding, 4) word problems, 5) effort. All these scales consist of three positively-oriented items and three negatively-oriented items except scale (5). All six items from this scale are positively-oriented. Detail of these scales is provided in the below section.

Duration of problem solving engagement (time-consuming mathematic problem solving) belief is based on the findings "several students believe that mathematical problems should be solved quickly or not at all" (Schoenfeld, 1985a). This scale involves the perceived capability to solve time-consuming mathematics problems. Many college students assumed that all mathematics problems can be completed in five minutes or less (Schoenfeld, 1985b, 1988). For this reason, these students quit those problems which cannot be completed in five minutes (Kloosterman et al., 1992). Beside this, most precollege text book words problems are one or two step variety that can be easily be solved in less time (Nibbelink, Stockdale et al., 1987). Students with no motivation to solve problems quickly will have difficulty in college-level mathematics courses. Therefore, it is important to consider students 'beliefs about their ability to solve problems which take more than a minute or two to complete.

Belief based on the solutions of problems via procedural method involves the steps or rules to follow to solve computational or words problems. The theoretical basis is that good problem solvers are more likely to be motivated to solve problems even when no apparent algorithm applies and vice versa. Usually in elementary and middle school mathematics textbook, students are taught to seek key words, so as to reduce a problem to apply a rule (Kloosterman et al., 1992). As a result, they view mathematics as a rigid system of externally dictated rules administered by standards of accuracy, speed and memory, when they became socialized by school and society (National Academy of Sciences-National Research Council, 1989). However, to solve non-routine words problems, these rules are impossible. In such situations, students who believe in following the available rules will hand over or try to apply inappropriate strategy (Kloosterman et al., 1992).

Few researchers argued that true mathematical problem solving, students do not need any rule to follow (Charles & Lester, 1982). In addition, it is also important that students understand why the rules they follow actually work. Good problem solver must be motivated to solve problems for which there are no memorized rules to follow. To measure their belief that words problems that cannot be solved with simple step-by-step procedures, Kloosterman et al. (1992) developed this scale.

Belief based on understanding concept, is also important for the student to get the correct answer and also why the answer is correct. This scale measures the degree to which students have the availing beliefs that conceptual understanding is important, as opposite to their non-availing beliefs that merely applying algorithm procedures leads to successful problem solving. Unfortunately, forty-eight percent student agreed that learning mathematics is memorizing and getting the correct answer is more important instead of knowing the reason why answer is correct (Dossey, 1988).

Likewise, it was observed that college students think that they are unable to create mathematics, therefore they need to accept procedures without trying to understand how they work (Schoenfeld, 1985a, 1988). The study of Lester, Garofalo et al. (1989) also noticed that students believe teachers and textbooks are the authorities and dispensers of mathematical knowledge. These students absolutely rely on their teachers and textbook knowledge. Therefore, they believe that all mathematics can be solved by applying facts, rules, formulas, and procedures the teacher has taught or as presented in a textbook. Similarly, DÍaz-Obando, Plasencia-Cruz et al. (2003) found that secondary school student believe that school mathematics is based on rules and memorization and mostly driven by procedures rather than concepts. Even mathematically talented students believed that mathematics is computation and learning mathematics involves memorizing arithmetic facts and algorithms (Frank, 1988).

Students dependent on memorized techniques for solving mathematics problem, they have very little motivation to solve real mathematics problem. At the same time as memorizing distinct bits of mathematics information is insufficient for keeping record of everything on which one is going to be tested (Tobias, 1993). Consequently these students set themselves up for eventual failure (Kloosterman et al., 1992). Therefore, author focused these points during the creation of his scale to measure students' beliefs about the importance of understanding in mathematics.

Belief based on importance of word problems is the degree to which students connect their mathematical perceptions to the attainment of computational skills. It involves students perception about importance of words problem, because many students learn to compute with whole numbers instead of solving problems (Carpenter, Lindquist et al., 1988). The curriculum and evaluation standard for math's by National Council of Teachers Reston (1989), also stressed on the importance of teaching problem solving while diminishing paper-and-pencil computation.

In addition, mathematics problem-solving skills are compulsory for college level students. Because students who believe that problem-solving skills are important, they have more motivation to become a good problem solver (Kloosterman et al., 1992). Therefore, Kloosterman et al. (1992) introduced a scale to measure students problem-solving skills perceptions about the importance of words problem.

Belief based on effort measures the degree to which students have availing beliefs about effort yielding positive results in obtaining mathematical skills. It involves student's perception that anyone can improve mathematics ability with adequate effort. These students are more probable motivated to do work and will accelerate their problem solving ability (Kloosterman et al., 1992). In contrast to it, some students believe that they should not be expected to excel in mathematics because they lack a mathematical mind (Tobias, 1993). Furthermore, Kloosterman et al. (1992) also discussed the relation between beliefs about ability and motivation to solve problems, that's why author make scale to measure students perception that effort can increase their mathematics ability.

Further, Kloosterman et al. (1992) included six items from the Fennema et al. (1976) usefulness of mathematics scale because of relations between availing beliefs about the usefulness of mathematics in daily life and motivation to learn. Belief about usefulness of mathematics is based on students perception, that mathematics is useful, it will increase their motivation and consequently achievement (Kloosterman et al., 1992). Moreover, Fennema and Carpenter (1981) studied the effects of an intervention program on female's intent to enroll in optional mathematics class. This intervention stressed the usefulness of mathematics outside the school. National Academy of Sciences-National Research Council (1989) observed that students' impassive and ungainly attitude towards mathematics are shifting to acknowledge the important role of mathematics in today's society. Due to this observation, beliefs about usefulness of mathematics get prominence. Overall, the IMBS was a suitable fit to the theoretical framework developed for the current study.

Indiana mathematics belief scale (IMBS) have been utilized extensively to explore mathematics beliefs. The study of Mason (2003) used IMBS to measure high school students beliefs. Results indicated that as students' progress through high school, their beliefs that all problems may be solved via routine means begin to diminish. Non-availing beliefs emerged during the high school years as students' beliefs that they can solve difficult problems and their beliefs in the usefulness of mathematics vary with time. Additionally, the importance of word problems scale revealed low reliability. Difficulty in generating reliable ratings can be students' confusion with the term word problems as reported by Kloosterman et al. (1992). However, four scales predicted student achievement to varying degree from strongest to weakest: duration of problem-solving engagement, solutions of problems via procedural means, usefulness of mathematics, and importance of conceptual understanding. Subsequently, all these results show the importance of mathematical beliefs as an important factor in students' mathematics education.

In mathematics education, McLeod (1992) categorized mathematics beliefs that are concerned with wide variety of beliefs that students have and also, their potential effects on learning. The first category, mathematics is difficult, belief that mathematics is useful lie in this category. The second category, beliefs about self, includes self-confidence in learning mathematics and acknowledgment for success or failure in mathematics. The third category, beliefs about teaching, contains beliefs about what a teacher should do to assist their students in learning mathematics such as teaching approaches and classroom environment. The fourth category includes, beliefs about social context, cover the beliefs that mathematics learning is competitive and that parents and others outside the school have a significant influence on one's mathematics learning.

Recently, Abedalaziz and Akmar (2012) investigated student's epistemological beliefs about mathematical problem solving and academic achievement in Malaysia. Findings from multiple regression revealed that the five scales are able to predict mathematical achievement significantly. However, the strongest predictor was belief regarding to the role of effort in increasing mathematical ability because students were convinced and motivated enough to do their best.

Mathematics educators primarily explored the relationships between students' beliefs and their environmental factors (Schoenfeld, 1988), and also examined sources of influence on beliefs (Jehng, Johnson et al., 1993; Schommer, 1990, 1993a). Since, the focus of epistemological beliefs is on the way individuals come to know, their beliefs about knowing, and how those beliefs are a part of and influence cognitive processes (Hofer et al., 1997). Consequently, there are some others factors such as formal educational practices like engagement in problem solving and learning, teacher and peer influences and life experiences such as home environment influenced the development and change in epistemological beliefs (Muis, 2004). Schommer and others also have identified early home environment matters (Schommer, 1993b), precollege schooling experiences (Schommer, 1994a) and the level and nature of postsecondary educational experiences (Jehng et al., 1993; Schommer, 1993a). All these factors are associated with development and change of epistemological beliefs.

Several researchers theorized that epistemological beliefs are both domain general and domain specific (Buehl, Alexander et al., 2002; Hofer, 2000; Muis et al., 2006; Schraw, Bendixen et al., 2002). Similarly, Schommer- Aikins et al. (2005) also hypothesized that general epistemological beliefs can be linked to the mathematical problem-solving beliefs and assessed using epistemological belief questionnaire (EB), Indiana Mathematics Belief Scale (Kloosterman et al., 1992) and the Usefulness of Mathematics Scale (Fennema et al., 1976). Findings showed that quick/fixed learning and studying aimlessly were the two strongest general epistemological beliefs. In addition to these two general epistemological beliefs, effortful math, useful math, math confidence, and understand math concepts were four strongest mathematical problem-solving beliefs. Furthermore, less the students believe in quick/fixed learning, the more probable they will believe that problem solving is effortful and useful, requires understanding of concepts, and have confidence in their ability to solve problems. Finally, path analysis indicated that beliefs in quick/fixed learning, useful mathematics, math confidence, and understand math concepts had a significant effect on overall academic performance. Therefore, author concluded that both general epistemological beliefs and mathematical problem solving beliefs contribute towards students' problem solving performance.

Schommer-Aikins et al. (2013b) continued to determine the influence of both domain general and domain specific mathematical problem solving epistemological beliefs on mathematics problem solving. Findings showed that the indirect effect of domain general belief on cognitive depth and mathematical performance was mediated by student's mathematics background and mathematics problem solving belief such as usefulness. On the other hand, mathematics problem solving belief (usefulness) directly affects cognitive depth and mathematical problem solving. Author illustrated that students believing that mathematics takes time and is useful, enhanced their cognitive depth and mathematical problem solving. Logically, beliefs which are specific to a domain strongly affect the performance in that domain.

2.7.2.1 Role of instructor in measured beliefs

Mathematics educators and instructors have a major role in developing student's epistemological beliefs. They can provide them a good and highly motivated environment. Garofalo (1989) also recommended that to develop more practical and healthy belief about mathematics, there is a need to change classroom environment.

Students will be motivated if they believe that what they are learning has value to them and also, effort can boost their learning. These positive beliefs are very essential for the development of good problem-solving skills (Eccles, Wigfield et al., 1993; Kloosterman et al., 1996). Instructor should also assist students to overcome their negative beliefs that hinder their learning abilities (Kloosterman et al., 1992).

To overcome student's negative beliefs, instructor can utilize counter examples. For example, students who are unable to solve time-consuming problems, needs to engage them getting success at solving such problems. Besides this, students relying on memorized rule, instructor can offer them some common sense problem or challenging tasks (Kloosterman et al., 1992). Further, students having negative beliefs about effort, instructor should clarify that anybody who struggle can learn mathematics. In addition, small group discussion can overcome their fear.

2.7.2.2 Implications to learning and instruction

Epistemological beliefs are one of the most critical components of understanding students learning because they deeply influencing and mediating the learning process and the learning outcome. These epistemological beliefs are like an invisible hand, deeply hiding behind an individual's behavioral expression, cognitive processes and emotional experience (DeBacker & Crowson, 2006; Hofer, 2001; Muis, 2004, 2007; Schommer-Aikins et al., 2005; Tang, 2010). Since Perry published his book *"Forms of Intellectual and Ethical Development in the College Years"*, several researchers became interested in the role of epistemology in learning and instruction (Schommer-Aikins, 2004; Schraw et al., 1995).

Educational psychologists are interested in the effects that students' epistemologies have on cognition, affect, and ultimately student achievement and learning (Stockton, 2010), because they effects on how individuals comprehend, monitor their comprehension, solve problems, and persist in the face of challenging tasks.

In learning mathematics, problem solving procedures are different and are dependent on the type of problem because there are various types of problems ranging from logical problems, such as puzzles to dilemmas (Jonassen, 2000). However, students explain the cause of problems differently and try to solve the problems in differently because of their personal epistemic belief (Gagné, 1965). These beliefs have an important role in solving ill-defined problems (King et al., 1994; Schraw et al., 1995), while well-defined problems can be solved without making epistemic assumptions (King et al., 1994).

In a school setting, students often solve well-defined problems but after their graduation they will face ill-defined real-world problems. The experience of solving well-defined problems does not help students to solve ill-defined problems (Schraw et al., 1995). Personal epistemological assumptions help all individuals to reach solutions. Their personal epistemology affects the processes used to reach a solution as well as the legitimacy of the solution when they solve ill-defined problems (Schraw et al., 1995). Also, it influence students' behavior and processing of information (Garner & Alexander, 1994).

Epistemology can be applied not only to school learning but also to life-long learning in and out of school (Hofer, 2001). It allows us to understand how individuals resolve competing knowledge claims and evaluate new information, and make fundamental decisions that affect their own and others' lives (King et al., 1994; Kuhn, 1991).

Educational experiences can facilitate development of epistemology, however, a limited study available about the connection between approaches and sorts of instruction and epistemology (Hofer, 2001). Fewer studies are found from the study of a belief system, which are about instructional implications because the process of belief acquisition and belief change is unclear (Hofer, 2001).

Schommer (1990) suggested that students should provide the fact that knowledge is integrated and more than one right answer exist. In addition, instructor should provide conceptual understanding of the concepts instead of just teaching facts, challenging tasks that take time, and create test questions that have several possible answers (Schommer, 1993b). More attention is needed about the role of instruction is required (Perry, 1970).

Researchers in mathematics education have taken a social-constructivist approach to belief change that accounts for the development of mathematical beliefs in terms of socio mathematical norms (Yackel & Cobb, 1996). They argued that if classroom practices are a major factor in the development of beliefs, it is plausible that significantly altering those environments can foster positive mathematics-related beliefs. Hence, Verschaffel, De Corte et al. (1999) recommended teachers to implement more constructivist-oriented classroom environment.

In addition to student's beliefs, teacher's beliefs also affect the learning process. Pajares (1992) emphasized that a teacher's beliefs ultimately impact how he teach. Teachers should respect students' assumptions about knowledge regardless of the students' epistemology level and give appropriate feedback to foster the learning process (Hofer, 2001).

2.7.3 Epistemological theories for alternative concepts

Hofer et al. (1997) examined models ranging from developmental model (Perry Jr, 1999) to multi-dimensional model (Schommer, 1990). Epistemological theories include dimensions of both developmental model and Schommer's belief model. Hofer et al. (1997) acknowledged the Schommer (1990) work of theoretical developments and also her contribution of formulating a questionnaire for measuring personal epistemology. Authors argued that fixed ability beliefs concern the nature of intelligence as a personal, psychological trait of an individual and should, therefore, be considered a separate construct from epistemological beliefs. Hofer and Pintrich pointed out whether or not epistemological beliefs can be measured via questionnaire. These general epistemological beliefs dimensions provided a framework for analyzing student beliefs during any learning episode and may be applied to domain-specific investigations. Authors proposed that individuals' beliefs about knowledge and knowing can be organized into personal theories, as structures of interrelated propositions that are interconnected and coherent (Hofer, 2001). Researchers further argued that personal epistemology should be restricted to dimensions concerning the nature of knowledge and the process of knowing, and that each dimension can be expressed as a continuum (Hofer, 2001). In addition, authors also suggested a general framework for epistemological beliefs in which nature of knowledge includes certainty of knowledge and simplicity of knowledge, while nature of knowing includes sources of knowledge and justification of knowledge.

2.7.4 Epistemological resources

Epistemological resources are the another important perspectives about epistemology, highlighted by Hammer et al. (2002). These resources are more fine-grained than a theory and more context-specific than any of the available models. However, a step forward, Schommer-Aikins (2004) stated that *"The need for an embedded systemic model of epistemological beliefs; a model that includes many other aspects of cognition and affect, comes from the assumption that epistemological beliefs do not function in a vacuum"* (p. 23). It means that only epistemological beliefs are not enough to measure mathematical problem solving ability. Therefore, Bråten et al. (2005) suggested that epistemological beliefs should be included in models of self-regulated learning. Based on these findings, researchers have sought to integrate a multidimensional model of epistemological beliefs with other cognitive and affective models of learning (Hofer, 2004; Hofer et al., 1997; Muis, 2007; Schommer-Aikins, 2004). Similar suggestions were also projected by Schommer (1998). Author linked these beliefs with motivational and cognitive factors (Schommer, 1990; Schommer, Crouse et al., 1992). Students who hold more availing epistemological beliefs are more likely to adopt a mastery goal orientation to learning and engage in material more deeply (Schutz, Pintrich et al., 1993). The study of Hofer (1999) also showed that students' beliefs were related to cognitive, motivational, and achievement factors. There is a positively correlation between beliefs and with intrinsic motivation, self-efficacy, and self-regulation, as well as with course grades.

Due to great importance of both goal oreintations and self-regulated learning, this study has been extended to include these important perspectives. The details of goal oreintations and self-regulated learning linking with problem solving, particularly differential equation problem solving are provided in the following sections.

2.7.5 Belief about usefulness of mathematics

In addition to epistemological math problem solving beliefs, belief about usefulness of mathematics has an important role towards math problem solving. Fennema et al. (1981) stressed the usefulness of mathematics outside the school. National Academy of Sciences-National Research Council (1989) also observed that students' impassive and ungainliness attitude about mathematics are shifting to acknowledge the important role of mathematics in today's society. Due to this observation, beliefs about usefulness of mathematics get prominence. Schommer-Aikins et al. (2013b) investigated the belief about usefulness of mathematics and reported that usefulness strongly effected math problem solving. Although several studies have correlated usefulness with mathematics achievements and problem solving, however, indirect effects via goal orientations or self-regulated learning (SRL) problem solving ability was not explored up to researcher knowledge. Therefore, the present study has focused indirect or mediation effects of usefulness via self-regulated learning (SRL) and goal orientations to explore students' problem-solving ability.

2.8 Goal orientations

Goal orientations comprised of an integrated type of beliefs, which are able to direct towards diverse engaging, approaching, and responding to achieve certain goals (Ames, 1992). The idea of goal orientations usually indicates the motive for doing goals or tasks (Bråten & Strømsø, 2004; Rastegar et al., 2010). These goals are the forms of different outcomes for which students pursue their learning environment (Coutinho, 2007; Dweck & Henderson, 1989).

Usually, there are three types of goal orientations, including mastery, performance and avoidance. Mastery goal orientation also labeled as task or learning goal orientation.

Across these different labels, basic construct is same with minor theoretical differences attached to them. However, in mastery goal orientations focus of the students is on mastery of the subject matter. Whereas, performance goal orientations often known as ability or ego orientation, where students are provoked to show their performance as compared to the other students (Ames, 1992; Anderman & Maehr, 1994; Dweck et al., 1988; Nicholls, 1984a; Pintrich & Schunk, 1996; Urdan & Maehr, 1995). The nature of both mastery goal and performance goal is different from each other. Due to dissimilar nature, both these beliefs influence outcomes differently. In contrast, avoidance goal basically mediate students to quit from learning so as to avoid illuminating their incapability in front of others.

Research on goal orientation is not only most commonly used in education, but also is common studied in sport psychology, health psychology, social psychology (Corno & Anderman, 2015). Researchers have also studied the potential relationship between several variables and goal orientation within a variety of test groups (Boyd, 2017). However, findings of these studies provided conflicting results for the variables that affect the goal orientation of an individual.

Regarding to sport psychology, researchers investigated the predictive role of goal orientation regarding to eating disorder in athletes, because these motivation-based factors may be an aspect that can lead an individual to engage in eating disorder. Wahl (2017) also investigated the predictive role of goal orientation in exercise with respect to eating disorder in sports athletes. Findings showed that task and ego orientations (subscale of goal orientations) were the significant predictors of an eating disorder. However, task orientation was positive predictor of eating disorder, whereas, ego orientation was negatively associated with eating disorder symptomology. However, with respect to gender, there was no significant difference in predicting eating disorder symptomology. Though in previous research, researchers (Duda, 1989; Hanrahan & Biddle, 2002) has evidently proved that female athletes were significantly more task-oriented than male athletes, whom were significantly more ego oriented examined the role. Author further argued that females considered mastery of the task and co-operation with team members to be the most important in their sports, whereas, males emphasize the competitiveness, social status, and potential for higher status career opportunities. On the other hand, Li, Harmer et al. (1996) reported that in a physical education class room, ego orientation was the significant predictor of score, whereas, no significant difference was noted between genders for task orientation. Detail of the influence of mastery goal performance goal and avoidance goal is provided in the proceeding section.

2.8.1 Mastery goal orientation

Master goal orientation has a strong correlation with positive motivational beliefs such as, high level of self-efficacy, more adaptive characteristics and perceived competence (Ames, 1992). As it is highly correlated to optimistic self-efficacy beliefs, therefore thoughts of anxiety became diminished (Pintrich et al., 1996).

Generally, master goal orientation is also linked to a wide range of academic outputs (Boyd, 2017; Lamm, Sheikh et al., 2017; Lee & Turner, 2017; Wahl, 2017), such as use of self-regulatory strategies (cognitive), self-efficacy and achievement (Ames, 1992; Patrick, Ryan et al., 1999; Pintrich et al., 1992). In addition, Pintrich et al. (1996) also reported a significant connection between mastery goal and quality of students' cognitive engagement and cognitive processes, respectively. These results were well supported by several others researchers (Graham & Golan, 1991; Nolen, 1988; Pintrich et al., 1990). It was revealed that mastery orientated students prefer cognitive strategies including, organizational strategies and elaboration. Both of these cognitive strategies show deep level of cognitive processing. Therefore, these students more able to utilize self-regulatory strategies and memory recall. As a result, these students show better text comprehension.

In addition, these trends of mastery goal were further investigated by several other researchers (Hall, 2015; Kayis & Ceyhan, 2015; Weinberg & Gould, 2014). And the study results of these researchers were well supportive with the findings of previous research. For instance, Hall (2015) reported that master goal oriented students are much more likely to attempt harder tasks and also ensure more effort to achieve a higher level of knowledge. These students are highly motivated to improve their ability and knowledge by reviewing learning material as an opportunity to improve (Hall, 2015; Kayis et al., 2015).

Gender difference is also important while considering mastery goal orientation (Hjertø & Paulsen, 2017). It was surprisingly observed that females are more mastery oriented as compared to male (Meece & Holt, 1993; Nolen, 1988), whereas, Ryan and Pintrich (1997) rejected this gender differences in his study. In contrast to it, in modern American society, female students are more task oriented as compared to male, because female stereotype are highly motivated as compared to male. Therefore, female perform very well in their academic settings than male counterpart (Kayis et al., 2015). Author further argued that female students are more likely motivated to complete their home task/assignment, study for exam, and therefore, they became successful in an educational setting.

Several researchers interrelated this to school subjects. Students' goal orientations can be functioned differently due to different subjects (Stodolsky et al., 1991). Regarding to the subject English, Anderman and Midgley (1997) revealed that females are more mastery goal-oriented as compared to male, whereas, Patrick et al. (1999) observed no dissimilarity in goal orientation for the mathematics subject.

2.8.2 Performance goal orientation

Performance goal orientation is concerned with institutional grades and other rewards instead of interest in that subject or any intrinsic value (Ames, 1992; Dweck et al., 1988). Performance orientated students have less adaptive motivational beliefs such as lesser awareness of competence and self-efficacy (Ames, 1992; Dweck et al., 1988; Nolen, 1988; Pintrich et al., 1992). Pintrich et al. (1992) proposed that performance goal orientation linked with low level of cognitive engagement which directs students towards surface processing strategies like rehearsal, instead of using deeper self-regulatory strategies. Beside this, performance goal orientation can produce negative cognitive and motivational processes associated with negative performance outputs (Pintrich et al., 1996). Therefore, Midgley and his co-authors further classified performance goal orientations as extrinsic and relative ability goal orientations (Anderman et al., 1994; Midgley, Maehr et al., 1996).

Extrinsic goal orientated students seek rewards including school grade, praise from parents and teachers. Other main reason might be to avoid the external sanctions such as punishment or penalty. While in relative ability goal orientation, social comparison is the main driving force, students do not want to perform less than other and compete with others to be the best (Anderman et al., 1994; Baloglu, Abbassi et al., 2017; Midgley et al., 1996).

In addition, Hall (2015) also noticed that performance oriented choose an easier task so that they feel comfortable and may found more chances of success (Hall, 2015). Therefore, they tend to be competitive in achieving more success as compared to their class mates or peer, as a result these students prefer superficial learning strategies while studying (Hall, 2015; Kayis et al., 2015).

2.8.3 Avoidance goal orientation

Avoidance goal oriented students emphasize on avoiding lack of skills as compare to their peers and class fellows (Rastegar et al., 2010). Since, the goal setting of these students is to avoid failure. Therefore, they realize incompetency as compared to others (Elliot & Harackiewicz, 1996). As a result, avoidance goal oriented students show negative outcomes. Due to this, students show slight interest during task engagement (Elliot et al., 1996), hesitated to look for help during schoolwork (Middleton & Midgley, 1997), reduced intrinsic motivation (Elliot & Church, 1997), and low achievement (Winne, Muis et al., 2004).

Moreover, Kayis et al. (2015) also reported that avoidance goal oriented students feel fear of being considered dull as compared to other class mate due to their lower performance at a certain task. Kayis et al. (2015) highlighted some common characteristics of goal orientation as, avoiding complicated tasks, mismanagement, and leaving tasks incomplete. Therefore, they are low achiever as compared to performance goal oriented students.

Overall, it can be concluded that approach goal oriented students are motivated to achieve a higher level of academic success than avoidance goal oriented (Kayis et al., 2015).

2.8.4 Inconsistencies in literature

There are some inconsistencies in the literature regarding to goal orientations. The study of Wolters et al. (1996) demonstrated that mastery goal orientation is interrelated with achievement, however, few researchers reported a null relationship between these two variables (Elliot et al., 1997; Skaalvik, 1997). Beside this, there are few conflicts concerned with the association of performance goal and academic achievement. The study of Middleton et al. (1997) and Elliot et al. (1997) revealed that performance goal oriented students tend to orient themselves to do well, hence they show better performance. On the other hand, few more studies rejected this relationship (Butler, 1993; Button, Mathieu et al., 1996; Coutinho, 2007).

Similarly, conflicts in relationship between avoidance goal orientation and achievement were observed. In several studies, a negative relationship between avoidance goal orientation and achievement was reported (Elliot et al., 2001; Skaalvik, 1997). However, some others researchers claimed a null relationship between them (Elliot et al., 1997; Kingir et al., 2013).

There is a possibility that interest in goal orientations may led more than one goal may be simultaneously operative and may exist separately from each other. Accordingly, it allows students to engage in multiple goals concurrently (García et al., 1991; Meece et al., 1993; Nolen, 1988).

There is one more possibility that student may adopt one task, or can adopt both goals simultaneously, with being primary and secondary goal, respectively (Coutinho, 2007). Researchers anticipated that these students may probable score high or low on each type of goal (Ames, 1992; Meece, 1994; Meece et al., 1993; Seifert, 1995; Suárez Riveiro, Cabanach et al., 2001). Their combined impact may differ from the individual effects (Fox, Goudas et al., 1994; Wentzel, 1992), because cognitive and self-regulatory processes depend partly on the joint and interactive effects of goals more precisely than on single goals (Suárez Riveiro et al., 2001). Cultural environment is also very important while considering academic motivation because in western countries it may operate in different ways as compared to Asian contexts (Ho et al., 2008).

In the study of motivation, self-efficacy theory, achievement goal theory, intrinsic motivation theories are most important social cognitive theories (Pintrich & Schunk, 2002). These dominating social cognitive theories also known as Bandura's social-cognitive theory (Bandura, 1997), expectancy-value theory (Wigfield & Eccles, 2000), and achievement goal theory (Dweck et al., 1988; Nasiriyan, Azar et al., 2011), respectively.

Self-efficacy is one the major construct in Bandura's social-cognitive theory. Researchers describe self-efficacy as, student's perceptions about their capabilities to master new skills, usually in a specific academic domain like mathematics (Nasiriyan et al., 2011; Pajares & Miller, 1994). In addition, expectancy-value theory proposed that student' passion to learn depends on expectations for achievement and the value attributed to task (Nasiriyan et al., 2011). After that, achievement goal theory came out as a prominent framework intended for explaining individuals' achievement status, their experience, and also their reaction in competent situation (class room and working environments) (Van Yperen, Elliot et al., 2009).

In current educational psychology, achievement goal theory is mainly leading and prominent approach to academic achievement as compared to remaining two theories (Bråten et al., 2004). Further detail of achievement goal theory is provided in the section below.

2.8.5 Achievement goal theory

Achievement goal theory has acquired vast approval among researchers, rising as a new direction for reviewing the construct of motivation (Midgley, Kaplan et al., 1998). The theory of achievement goal focuses on achievement goals, perceived ability, and achievement behavior, because these are three important factors that contribute to an individual's motivation (Weinberg et al., 2014). The primary focus of this theory is on understanding an individual's achievement goals. It conceptualized learners' motivation as goal-directed actions employed to complete an authentic learning task (Elliot et al., 1999). The concept of achievement goal usually point students' purpose for doing tasks (Rastegar et al., 2010). In a specific context, achievement goal is a situational explicit orientation reflecting the motive to obtain, buildup, and boost up the capabilities (Harackiewicz, Barron et al., 1997).

The traditional achievement goal theory was proposed by Nicholls (1984b) and anticipated that, individuals goal orientation and perceived ability contribute to affective outcomes in a given achievement setting (Wang, Liu et al., 2010). Goal orientation is one of the most important construct of this theory, concerned with the motive that students have for engaging in achievement task. With miscellaneous intentions imply that students show their capabilities and success in different ways (Bråten et al., 2004).

2.8.5.1 Goal theory: expanding dichotomous goals into a trichotomous framework

Achievement goals are classified into mastery goal and performance goal. These two different types depend on whether learning is perceived and esteemed as an end in itself or as a mean to other reason (Meece, Blumenfeld et al., 1988).

Dweck and colleagues articulated the proposal by expanding the dichotomous goal framework. The authors reported that mastery-performance simply represents a fundamental and simplified conceptual framework (Bergen & Dweck, 1989). Therefore, Elliot et al. (1997) did not change the traditional mastery goal but classified performance goals into two dimensions, including approach and avoidance goal (Wolters, 2004). The focus of mastery goal is to develop one's aptitude employing mastery over the task.

Similarly, performance-approach shows one's inclination towards involvement in the task for the sake of performing well to show other, while performance avoidance goals cause to avoid task, so as not to show incompetency to classmates and teachers. Elliot et al. (1999) conceptually and empirically analyzed, and reported that approach goal is the outperform to illustrate competence over other fellows, whereas the avoidance is to hide incompetence. Therefore, this approach revealed that students' differences in choosing different goals are correlated to their achievements, which show links to cognitive, motivational, emotional and behavior outputs.

Both mastery and performance goals are the positive motivational factors those swift students to spend their consistent efforts. Overall, achievement motivation inspires and actively engages students in their tasks. However, in case of avoidance goal, opposite results to achievement motivation were observed in most of the studies. An avoidance goal represents an impassive and negative motivational attitude that may impose destructive effects on learning. For this reason, an avoidance goal shows pessimistic aptitude that may result into destructive effects during learning (Wolters, 2004).

Later on, motivational theorists (Elliot et al., 2001; Pintrich, 2000b) proposed a 2 x 2 achievement goal framework that fully integrates the mastery-performance and approach-avoidance distinctions. Crossing these two dimensions yields four types of achievement goal; mastery-approach, mastery-avoidance, performance-approach and performance-avoidance. The focus of mastery-approach and mastery-avoidance is on task-based or interpersonal competence and incompetence, respectively. Similarly, performance-approach focused on normative competence, whereas, performance-avoidance intended on normative incompetence (Wang et al., 2010). Consequently, both mastery-approach and performance-approach has positive contribution towards achievement and consequences. In contrast, mastery-avoidance and performance-avoidance anticipated fewer adaptive motivational pattern (Elliot et al., 2001; McGregor & Elliot, 2002).

A person may endorse multiple goal perspectives. Therefore, looking at the independent effect of each goal may not reveal a complete picture of the person's achievement motivation (Wang, Lim et al., 2008; Wang, Biddle et al., 2007). Therefore, Wang and Liu (2007) found four clusters of students moderate achievement goals, low achievement goals, high achievement goals and final mastery achievement goals with homogenous characteristics based on their achievement goals. Moreover, goal orientation, self-efficacy and intrinsic interest include in self-motivational beliefs (Kingir et al., 2013; Zimmerman et al., 2000) and act as a mediator that stimulate students self-regulatory behaviors. These motivational beliefs properly implement self-regulatory knowledge and skills by improving student's motivations for learning and by the employment of learning strategies (Montalvo et al., 2004).

2.9 Self-regulated learning (SRL)

Self-regulated learning (SRL) is basically self-generated beliefs, emotions, and actions that are intended and frequently adapted for the accomplishment of individual goals (Zimmerman et al., 2000). It is self-directive procedure through which students switch their mental abilities into academic skills.

Basically, the concept of self-regulated learning is generated from concept of inner speech (Vygotsky, 1978) and social cognitive theory (Bandura, 1986). The study of Ormrod (2008) depicted the relationship between Vygotsky's inner speech and SRL. According to Vygotsky's, self-talk (part of inner speech) has an essential role in cognitive maturity. Eventually, it transforms into inner speech in which individuals mentally "talk" to themselves instead of loud. Students involved in inner speech are actually applying self-monitoring process of SRL, by reviewing the efficiency of learning goals and cycling back through the SRL processes if needed. Therefore, Stockton (2010) entitled inner speech as self-regulated learning process.

Generally, self-regulatory processes comprised of three recurring phases; forethought, performance, and self-reflection. The first phase forethought involves actions and beliefs that emerged before efforts to learn. Performance phase includes actions that are taken place during the endeavor of performance, and these actions control students act and interest. Finally, self-reflection phase refers to the actions that come up after each learning effort and influence students reaction (Kingir et al., 2013).

Self-regulated learning is also associated with two types of learning approaches; deep approaches and surface approaches. These approaches are considered as firm practices, originally deliberately applied, but normally go through mechanization as a result of development and practice (Schneider & Weinert, 1990). In deep learning approaches, students attempt to make a connection between previous learned knowledge and newly knowledge. Whereas, in surface approaches, students learning mode rely on the rote memorization (Cavallo, Potter et al., 2004).

Another aspect of self-regulated learning strategy use is cognitive strategies and meta-cognitive strategies. Typically, cognitive strategies are composed of three main strategies; rehearsal, elaboration and organizational (Weinstein, Mayer et al., 1986). Rehearsal strategy involves in repeating the subject matter frequently, so as to memorize it. Whereas, elaboration strategy is associated with reviewing the material. Lastly, organizational strategies are concerned with clarification of the material (Garcia & Pintrich, 1994; Weinstein et al., 1986). Despite of dissimilarity in these cognitive strategies, students reported that they used all three types of strategies in similar ways.

This reporting results in a single factor of cognitive strategy (Patrick et al., 1999; Pintrich et al., 1990).

The second aspect of self-regulated learning strategy use is meta-cognitive strategies. Planning, monitoring and regulating cognition are three main components of meta-cognitive strategies. In planning phase, students try to set goals for themselves during study. While, monitoring is a central part of metacognition and it refers to using self-questioning strategies to check learning and comprehension. During monitoring phase, students are aware about their understanding of the matter. Consequently, they can utilize it to take decisions regarding to the regulation of their cognitive. For instance, students can make decision to continue using their original strategy, whether to stop or revise the issue to be learned. Therefore, self-regulated learning has found to be positively correlated to achievement, with highly self-regulated students being more motivated to use planning, organizational, and self-monitoring strategies than low self-regulated students (Pintrich et al., 1990).

Critical thinking which is an important construct of self-regulation is directly associated with problem solving process. Barman in 1999 has suggested that critical thinking is important not only for academic environment, but in any environment involving problem solving. For that reason, one of the most important objective of engineering education is to develop critical thinking skills for the solution of engineering problems (Siller, 2001). Several other researchers also reported that the employment of SRL strategies results into improved learning and achievement (Patrick et al., 1999; Pintrich et al., 1994; Weinstein et al., 1986). Lack of SRL strategy potential may also affect the talented students (Ablard & Lipschultz, 1998). On the other hand, Ablard et al. (1998) argued that the relationship between SRL and achievement is very complex.

Because few high-achieving students showed better results, even though, they did not employ SRL strategies.

Overall, self-regulated learning strategies has major contribution towards achievement, specifically, math problem solving and math achievement. Perels et al. (2005) revealed that self-regulatory strategies and practicing of problem-solving may be helpful to improve problem-solving. Therefore, authors suggested that a combined training of self-regulatory and problem-solving strategies is effective for enhancing self-regulatory competences. Due to great importance, certain empowerment programs (SREP) were also designed. The function of this program was to provide students a self-regulated learning coach (SRC). The role of coach was to recognize academic flaws, offer instruction to foster SR cyclic processing and also to formulate continuous, instant feedback. Moreover, Cleary and Zimmerman (2004) also assessed the implementation of this program for case study analysis. The purpose was to assist educators in promoting positive, self-inspirited learning practices for students.

2.9.1 Characteristics of a self-regulated learner

Self-regulated learner takes an active part in their learning process from the metacognitive to motivational and behavioral view point (Sadi & Dağyar, 2017; Zimmerman, 2002; Zimmerman & Schunk, 2001). Moreover, the characteristics accredited to self-regulated persons are alike with those of high performance, high capability, as compared to low performer showing deficiency in these variables (Rivas, Reyero et al., 2003; Zimmerman, 1998). In general, several researchers define self-regulated learner in different way. Detail is provided in the section below;

Self-regulated learners know how to apply a series of cognitive strategies such as rehearsal, elaboration, organization for organization, elaboration and recovery purpose

(Winne, 1995; Zimmerman et al., 2001). They are familiar and know how to plan, regulate and direct their mental process such as metacognition to attain their personal goals (Corno, 1989).

Other researchers (Weinstein, Husman et al., 2000; Zimmerman, 2002) defined self-regulated learner as a learner who shows motivational beliefs and adaptive emotions for example, development of self-efficacy beliefs, evolution of positive emotion towards tasks (e.g. satisfaction, passion, bliss) along with the ability to control and adjust these emotions according to the needs of the specific task and learning situation. Moreover, they have ability to plan and control the time and effort to be spent on tasks. Also, these self-regulated learners can not only create but also structure favorable environments, for example finding a proper place to study and seeking help from peers and teachers in case of difficult task (Corno, 1989; Winne, 1995; Zimmerman et al., 2001).

2.9.2 Self-regulation theory

Self-regulated learning (SRL) theory is an advanced theory in educational psychology that is employed to depict student's control of learning. The main frame work of this theory is based on how students regulate their learning.

The concern of SRL theory is, "How and why learners involve themselves in the learning process". In other words, individual learn how to direct their own learning process, and how to choose appropriate cognitive, metacognitive, and behavioral strategies as an sufficient effort to achieve their already set goals. Therefore, Pintrich (1991) define self-regulated learner as a learner having necessary knowledge and skills so that they can choose and apply cognitive, metacognitive, and behavioral strategies. Further, Zimmerman (1998) define self-regulated learner as "motivationally, metacognitively, and behaviorally active participants in their own learning process". A large number of definitions for self-regulatory learning have been proposed, however regarding to students' academic achievement, Saad, Boroomand et al. (2012) highlighted three most important component. These components are; cognitive strategies, meta cognitive strategies and management and control. Cognitive strategies such as rehearsal, elaboration and organization are used to learn, recall, and completely comprehend curricular concepts, whereas, meta-cognitive strategies are utilized to plan, monitor and reorganize their cognition. Another most important component is learners management and control of their attempts to do assignment /task both from curricular and academic ideas. Pintrich (1991) considered these three as the basis component of self-regulated learning.

Pintrich et al. (1990) proposed that self-regulation has been deal as a comprehensive construct, comprised of four key components. For instance; cognitive, metacognitive, behavioral (including management and control of efforts), and motivational components (such as; goals, self-efficacy, and task values) (Pintrich et al., 1990). These four components are interrelated with each other in order to provoke self-generate learning (Wolters, 2004). This framework anticipated that motivations and learning strategies are the dynamic traits of students. Because, student's motivations vary from course to course, depends on student's interest and their performance competency in the course. Whereas, learning strategies can be learned and students can take control over them (Duncan et al., 2005).

Keeping in mind this theoretical framework, the motivated strategies for learning questionnaire (MSLQ) was designed to measure college undergraduates' motivation and self-regulated learning as they relate to a specific course. Different theoretical frameworks were examined concurrently to provide a more complete understanding of human motivation (Wang et al., 2010). To describe SRL processing, multiple models have been developed. However, Stockton (2010) used Zimmerman's model and integrated particular salient constructs from other model (Butler et al., 1995; Garcia et al., 1994; Pintrich, 2000b; Winne & Hadwin, 1998). Details of these models are provided in following sections.

2.9.2.1 Zimmerman's (2000) cyclic model

Zimmerman et al. (2000) cyclic model involves three major phases of self-regulation; forethought, performance control, and self-reflection. The focus of first, forethought phase is to keep developing goal and forecast activities to complete learning task. Further, goal may be classified as mastery or performance approach focus and an avoidance focused goal. Whereas, planning stage refers to the selection of appropriate strategies for learning task that is based on goal-motivated ideas for learning (Pintrich, 2000b). During performance control phase, Zimmerman et al. (2000) acknowledged that learners exploit their self-observation and self-control practices such as self-coaching, imagery, task and attention focusing plans to replicate the forethought phase plane. Lastly, students in self-reflection phase, perform self-assessment of learning task and also imagine acknowledgement in terms of their performance (Zimmerman et al., 2000). During self-evaluation process, students exploit normative criterion, in which they compare their performance with others.

Overall, Zimmerman (1998) considered SRL as a kind of learning in which individuals start and direct their efforts to attain knowledge and skills without getting any assistant from teachers, peers and others. Further, author elaborated that it is a series of cyclic stages in which learner information and their primary beliefs are the starting point. Self-regulated learner uses their information, beliefs and knowledge and in this way, they can judge the qualifications and tools required for performing activities. Based on their evaluation, they determine their goals. Finally, they use their strategies to achieve their pre-set goal, resulting cognitive, effective, and behavioral outcomes. Through regulating their progress and also through monitoring process, learner can find an internal feedback by which they can evaluate those activities. And also, can be aware of what kind of approach they should use to perform those activities.

Muis (2007) extended the previous models of Winne et al. (1998) and Pintrich (2000a). The author proposed an extension of these previous models by integrating epistemological beliefs within self-regulated learning framework. Detail of Muis model is provided in the section below.

2.9.2.2 Muis' (2007) model

Muis (2007) proposed that self-regulated learning comprised of four phases; task definition, planning and goal setting, enactment and evaluation. In first phase of this model, learner makes a perception about the task that is influenced by both external situations such as context, and internal settings including previous knowledge and personal epistemological beliefs (Muis, 2007). Author further argued that during this phase, schemes for task knowledge together with beliefs about knowledge and knowing became activated. Hence, these schemas also stimulate those beliefs to influence other aspects of self-regulated learning.

Planning phase is concerned with choosing the appropriate learning as well as metacognitive strategies to accomplish task. In addition, the function of this phase is to sort out different types of information and also to determine the level to which the accuracy of information should be assessed. Enactment phase become stimulated when students accomplished their task by executing the selected learning and metacognitive strategies. Eventually, in evaluation phase, students became involved in reflection and reaction to approximate success or failure of each phase or their perceptions about self or context (Muis, 2007). Additionally, author proposed that there are four main areas that may be regulated during self-regulated learning. For example; 1) cognition such as strategies knowledge and its activation, 2) motivation and affect including achievement goals, self-efficacy, 3) behavior like time, and 4) effort includes resources and social context.

Literature presented above describes the theoretical and empirical analysis of epistemological belief, usefulness, goal orientation and self-regulated learning strategy and their effect on the mathematics achievements as well as problem solving. Results are highly encouraging towards mathematics problem solving. However, it has been noticed that limited study is available with respect to differential equation based problem solving (Stockton, 2010). In addition, several factors, such as achievement goals, and self-regulated strategy may have important mediating roles, which need further elaboration. In the following sections, interrelation of these factors and their mediating role has been discussed.

2.9.3 General frame work of self-regulated learning model

Different models of self-regulated learning have already been discussed above, however there are four common assumptions of self-regulated learning. These assumptions are shared by all models. There is one common assumption that is followed from a general cognitive perspective. This common assumption might be known as active, constructive assumption. According to this assumption, learners are active constructive participants in the learning process in all models (Wolters, Pintrich et al., 2005). Learners are not just passive receiver of information from external as well as internal environment, but rather they are assumed to actively construct their own meanings, setting their own strategies and goal.

Second general assumption is the potential for control assumption. According to this assumption, learners has some potential to monitor, control, and regulate their certain aspects of cognitive motivation, behavioral as well as some features of their environments.

Third assumption is known as goal, criterion, or standard assumption. All models of self-regulated assumes that there is some kind of standard or criteria also known as goal or reference value is set. The purpose of this standardized value is to make comparison so as to whether the procedure should as it is or there is a need of necessary change (Wolters et al., 2005). Author further elaborated this assumption by giving a common-sense example of thermostat operation for the heating and cooling of a house. Once a desired temperature is set, the thermostat monitors the temperature of the house and then switched on or off the heating or air conditioning units in order to reach and maintain the standard. In the same way, individual in this general assumption of learning can set goals to strive for in their learning, monitor and then regulate their cognitive, motivation and behavior so as to attain their set goals.

Fourth general assumption of self-regulated learning is based on the mediating role of self-regulated activities between personal and contextual characteristics and performance (Wolters et al., 2005). Author further elaborated this assumption by saying that neither individuals culture, demographic, or personality characteristics, nor just the contextual characteristics of the class room environment are just enough to effect learner as well as their achievement. But learner’s self-regulation of their cognitive, motivation, and behavioral mediates the relationship between the learner, context, and achievement.

Overall, these four-general assumptions define self-regulated learning as an active, constructive process in which learning goals are established and then effort is made to monitor, regulate, and control their cognitive, motivation, and behavior, controlled by their goal and the context (Wolters et al., 2005). SRL can be hypothesized as the learner ability to use metacognitive strategies or to control cognition (Saad et al., 2012). Therefore, Schoenfeld (2009) viewed as the learners ability to use both metacognitive and cognitive learning strategies. Further Pintrich (1991), categorized main metacognitive strategies as planning, monitoring and regulating, whereas cognitive strategies includes rehearsal, elaboration, and organizational strategies.

In addition to it, Tanner and Jones (2003) reported another important view of SRL, that it integrate motivation along with cognitive and metacognitive strategies of learning. Later on, Rheinberg, Vollmeyer et al. (2000) also suggested that motivation is associated with SRL and is helpful in promoting and sustaining SRL.

2.10 The relationship between epistemological beliefs and goal orientations

Epistemological beliefs have significant association with the motivational constructs such as goal orientation (Madjar, Weinstock et al., 2017). Paulsen and Feldman (1999) further sustained this claim and revealed that student's epistemological beliefs influence students goal orientation and self-efficacy beliefs (part of motivational constructs). Findings showed that among four dimensions of epistemological beliefs only three dimensions were positively correlated with several motivational constructs such as, intrinsic and extrinsic goal orientation, task value, self-efficacy, control of learning and, anxiety. Among motivational constructs, goal orientation is highly interrelated with epistemological beliefs, as these beliefs influence the type of goal learners establish for themselves. The imperative relationship of epistemological beliefs and goal orientation also enhanced other important facet of formal education for instance, development (Paulsen et al., 1999).

Epistemological beliefs have been considered as an originator and one of the most important mediator for achievement goal orientation (Bråten et al., 2004, 2005; Ghorban-Jahromi, 2007; Rastegar et al., 2010). The study of Kizilgunes, Tekkaya et al. (2009) also approved the positive association of epistemological beliefs with goal orientation, except certainty. Besides this, several other researchers were interested to examine their relationship towards science achievement (Buehl, 2003; Cavallo et al., 2004; Paulsen et al., 1999; Paulsen & Feldman, 2005). Finding were summarized as, among goal orientation, previous knowledge, attitude, learning approaches, self-efficacy, and reasoning ability were significantly correlated with the achievement of science students. Other studies examined the constructivist nature of epistemological beliefs and its connection with goal orientation. Findings showed that constructivist epistemological beliefs are positively associated with mastery goal, whereas, less constructivist epistemological beliefs are correlated with performance goal (Bråten et al., 2004; DeBacker et al., 2006; Muis et al., 2009; Huy P Phan, 2008). Accordingly, Paulsen et al. (1999) recommended that students motivation can be boosted by empowering motivating and creative epistemological beliefs. For this purpose, the role of class teacher is very important. They can also develop a new concept that knowledge is emergent and complex. It will influence students' motivation to learn.

Similarly, several researchers (Dweck, 1999; Dweck, 1986; Dweck et al., 1988) argued that advancement of different goal orientations may be due to the beliefs about the nature of intelligence in the academic domain. There are two types of implicit theories of intelligence entity belief and incremental belief of intelligence, that support the types of goals adopted. According to an entity belief, intelligence is fixed and uncontrollable characteristic that may foster an ego/ performance orientation because pursuing such a goal favors positive judgment of ability (Dweck et al., 1988). While an incremental belief, that intelligence is flexible and controllable quality that promote ability of task orientation because it provides the opportunity for learning and improvement (Dweck, 1999; Spray, Wang et al., 2006).

Moreover, Wang et al. (2010) examined the relationships between the cluster of approach-avoidance dimension and mastery-performance dimension of achievement goals, implicit theory of intelligence, and behavioral regulations among engineering students. Author evidently proved five clusters of achievement goal including, high mastery approach/ moderate performance group, high mastery-approach/low performance group, low mastery approach/ high performance, high mastery/high performance and low mastery/low performance. Students with high mastery-approach goals have relatively higher incremental beliefs, feeling of autonomy, value, exert more effort and enjoy learning. Therefore, author suggested that a mastery-approach goal is best motivator for learning. To increase the mastery goal structures in the classroom Epstein (1988) and Ames (1992) proposed some practical suggestions using the target principles (task, authority, recognition, grouping, evaluation and time (Deemer & Hanich, 2004; Liu, Wang et al., 2009).

2.11 The relationship between epistemological beliefs and self-regulated learning strategies

Epistemological beliefs play an essential role towards students self-regulated learning strategies particularly, their learning approaches and subsequently influence their learning outcomes (Schommer, 1990). The study of Holschuh (1998) addressed this issue by analyzing the correlation between epistemological beliefs and use of learning approaches (deep learning and surface learning). Results revealed that students having more sophisticated beliefs make use of deep learning approaches. Whereas, students with immature or naïve beliefs exploit surface learning strategies.

Further, Chan (2003) supported this claim in a sense that, deep approach is determined by the sophisticated beliefs that are; knowledge can be accomplished by effort, understanding and integration process, and also by reasoning instead of depending on the allotment of authorities. In contrast to it, surface approach rely on fixed ability belief, source of knowledge and certainty belief, that are part of naïve beliefs (Kizilgunes et al., 2009). More recently, Huy P Phan (2009) also declared the same result that sophisticated epistemological beliefs of university students positively associated with deep learning approaches through effort expenditure. Further, Cano (2005) extended the previous study and examined the relationship between epistemological beliefs and academic achievement. Also, author investigated the mediating role of learning strategies. Findings revealed that epistemological beliefs and use of learning strategies were found to influence academic achievement and have contribution towards improving students' biology course grade and also their GPA. Based on the findings of study, author concluded that learning strategies mediate the relationship between epistemological beliefs and academic achievement.

Moreover, several other researchers also studied the relationship between epistemological beliefs and self-regulated learning strategies (Bråten et al., 2005; Hofer et al., 1997; Muis et al., 2009; Schommer-Aikins, 2004). Results showed that both these constructs are emerging as interrelated constructs. Based on preceding studies, Muis (2007) projected a model by assimilating epistemological beliefs into Winne et al. (1998) self-regulated learning strategies model. Specifically, the author hypothesized that epistemological belief are anticipated at the definition of task phase and may stimulate the standards set for a task, which directly impact on the goals that student set. In turn, these standards influence evaluation of strategies that are used to complete the task, and also it have an effect on student's performance. Researcher also agreed that while selecting appropriate learning strategies, few epistemological beliefs may have more dominant and significant affect as compared to others. For example, Dahl, Bals et al. (2005) reported in his study that among epistemological beliefs, beliefs about structure of knowledge and the ability to control learning are related with us of cognitive and metacognitive learning strategies. Recently, Kingir et al. (2013) reported a positive significant relationships among epistemological beliefs, constructivist learning environment perceptions, and learning approaches.

Additionally, it was also realized that there is need to consider this relationship at elementary and secondary level students. Keeping in mind, both epistemological beliefs and self-regulated learning (SRL) were examined at elementary and secondary level. Findings showed that the epistemological beliefs of high school students are no more developed, hence these students shows poor SRL processing in physics than elementary students. Besides this, their goal orientation also shows a weak prediction for SRL processing. Neber and Schommer-Aikins (2002) conceded this weak prediction may be due to the utilization of domain-general questionnaire in a domain specific study.

Regarding to domain specific study, mathematics researchers and theorists recommended that issues of control and mathematics beliefs are related with successful completion of math problems (Polya, 1957; Schoenfeld, 1983, 1985a, 1988, 1989). Recently, few researchers induced advanced self-regulated learning (SRL) theory and epistemological beliefs theory into mathematics problem solving (Hofer, 1999; Muis, 2004, 2008).

Muis (2008) further considered this proposal and tried to theoretically and empirically prove it. Author conducted a two-part study to examine the association of student's epistemological beliefs and self-regulated learning strategies processing in mathematics problem solving. The author proposed her frame work of SRL from the Schoenfeld (1985b) perspective of problem solving control and epistemological beliefs were based on previous perspective of empirical and rational epistemic approach (Royce et al., 1980). Findings showed that rational students scored higher on the metacognitive self-regulation subscale of the MSLQ as compare to other groups. After that, second half of the study was conducted, in which 24 mathematics students were considered from the original sample of the participant. Results from this half also revealed rational students showed higher usage of self-regulated learning strategies including planning, monitoring, and metacognitive control as compared to other two groups. Overall, these results confirmed the findings of previous studies that rational problem solvers are successful as compared to empirical one.

Based on Muis (2008) model, Stockton (2010) explored the relationship of mathematical problem solving beliefs and self-regulated learning practices (SRL) of advanced mathematics students, while solving calculus mathematics problem solving. Findings revealed that among math problem solving beliefs; unique/arbitrary, procedural/conceptual and empirical/rational beliefs are associated with various aspects of SRL processing. Besides this, author noticed that utilization of different SRL strategies rely on cognitive load of problem-solving tasks. Moreover, heuristic strategy and math problem solving performance was also linked with mathematics problem solving beliefs.

Students epistemological beliefs also influence teaching approaches (Brownlee, 2001). As a result, if teachers recognize their belief system, it helps them in enhancing their preparation and teaching practices (Epler, 2011).

2.12 The relationship between self-regulatory learning and goal orientations

Social cognition theorists has initiated an idea about the identification of key factors that are affecting students cognitive engagement in successful settings (Broadbent, 2017; Kizilcec, Pérez-Sanagustín et al., 2017; Runisah, Herman et al., 2017; Wolters, 2004;

Won, Wolters et al., 2017). Among these factors, motivation beliefs has an essential role in student's use of self-regulated learning strategies including metacognitive skills and effort regulation (Pintrich, 1999). These motivational beliefs significantly affect the use of metacognitive strategies that are utilized by the students (Al-Ansari, 2005; Dembo & Eaton, 2000; Neber et al., 2002; Pintrich et al., 1990; Shih, 2002; Wolters, 2004). For this reason, Bandura (1993) provide suggestion that aims of formal education should be to facilitate students with motivational beliefs and metacognitive skills. In this way, they can educate themselves throughout their lives.

Researchers in education and educational psychology, begins to explore it and depicted that motivational variables specifically goal orientations are highly correlated to students learning (Dembo et al., 2000; Neber et al., 2002; Pintrich et al., 1996; Pintrich, 2000b; Pintrich, Smith et al., 1993; Wigfield et al., 2000). Goal orientation as a part of self-motivational beliefs act upon as an inspiring mediator for students self-regulatory behaviors (Kingir et al., 2013; Won et al., 2017; Zimmerman et al., 2000). The function of goal orientation is to enhance students' learning incentives and also the quality of the selection and the employment of learning strategies. In this way, it effect the implementation of self-regulatory knowledge and skills appropriately (Broadbent, 2017; Butler, Perry et al., 2017; Lao, Cheng et al., 2017; Montalvo et al., 2004).

Moreover, goal orientation also explain the reason why few students cling to a task while other students do not (Sungur, 2007). Because, highly-motivated students struggle to learn in spite of complexity of learning task and also utilize various cognitive strategies. The focus of goal oriented students is on learning and considering course material important, useful, and interesting. These students believe that to study with effort influence in mastering of course material, therefore, they frequent utilize metacognitive strategies. Hence, goal orientation and self-regulated learning strategies are necessary for successful outcomes. Marcou (2005) also agrees and adopted a view that both these factors are compulsory for the successful interpretation of learning outcomes. Highly motivated students would not be able to accomplish their academic targets if they lack self-regulated learning strategies and vice versa. Author proposed another possibility, that sometimes high motivated students who are familiar with cognitive and metacognitive strategies are unable to use them due to their lack of volitional strategies (Pape & Wang, 2003). Volitional strategies are basically representing knowledge and skills that are required to create and support an intention until goal is accomplished (De Corte, Verschaffel et al., 2000). Therefore, Marcou (2005) studied the relationship between motivational beliefs such as mastery goal orientation and SRL behavior (including volitional strategies) with respect to mathematics problem solving. Results showed that students who exploit cognitive, metacognitive and volitional strategies are more likely to feel more efficient about their ability to well during mathematics problem solving procedure.

Fadlelmula et al. (2015) investigated the relationships among students' motivational beliefs such as goal orientation, self-efficacy, and perception of class room goal structure, self-regulated learning strategies and achievement in mathematics. Author concluded that motivational factors alone are not enough for enhancing student's mathematics achievement. Rather, it is the use of deep learning strategies which mediate the link between motivational factors and mathematics achievement.

Further in goal orientation, Ames (1992) and Pintrich et al. (1992) given stress on the part of mastery goal orientation, because mastery goal (dimension of goal orientation) is connected to an intrinsic interest in and value for learning and, also with quality of students cognitive arrangement. Since, they are more likely to process the material to be memorized at a deeper level for instance, for example elaboration and organizational strategies that reflect deeper level of cognitive processing (Graham et al., 1991; Pintrich et al., 1992). Therefore, these students try to plane their work and monitor and evaluate their understanding. Through deeper cognitive processing, mastery goal orientation student also shows high memory recall, high-quality text comprehension and better use of self-regulated learning strategies (Graham et al., 1991; Nolen, 1988; Pintrich et al., 1990), and metacognitive strategies (Dupeyrat & Mariné, 2005; Elliot et al., 2001).

Moreover, Flavell (1992) metacognition has a fundamental role in reading, comprehension, writing, memory, problem solving, and other areas of learning. For that reason, researchers targeted the role of metacognition and mastery goal orientation. They investigated the mediating role of metacognitive between mastery goal orientation and math achievement Ames and Archer (1988) and Dweck et al., (1988) together with math self-efficacy (Elliot et al., 1999; Middleton et al., 1997; Rastegar et al., 2010). Results showed that mastery goal oriented learners commence more self-regulated learning strategies as compare to performance goal oriented (Nolen & Haladyna, 1990; Pintrich et al., 1990).

Diseth (2011) also investigated the direct as well as indirect relationship between achievement goal, learning strategies, and academic achievement. Findings of this study integrated previous studies findings by showing that mastery goal, performance goal and learning strategies was positively correlated with student's examination grade. Moreover, author reported that learning strategies play a mediation role between the achievement goal and academic achievement.

In addition, mastery goal also put forth a positive effect on critical thinking (construct of self-regulated learning strategies) and enhance students understanding of knowledge and development skills (Huy Phuong Phan, 2008, 2009). Although, critical thinking is one of the important construct of self-regulation, unfortunately, few research studies have explored this construct (Huy Phuong Phan, 2008, 2009). Consequently, research work concerned with achievement goal and critical thinking is limited to few studies and is still in its early days.

In short, mastery goal was found to be associated with positive outcomes. For example, help seeking (Ryan & Pintrich, 1998), long term retention of informtaion (Elliot et al., 1999), persistance (Pintrich, 2000b), use of deep processing strategies (including meaning making) and shallow strategies (rehearsal) (DeBacker et al., 2006) and high achievemnt outcomes (DeBacker et al., 2006; Elliot et al., 1997).

In contrast to mastery-approach goals, performance goal is determined by lower level of cognitive engagement, for example, employment of more surface processing strategies like, rehersal (Nolen, 1988; Pintrich et al., 1992) and low self-efficacy (Skaalvik, 1997). Also, performance goal oriented mostly avoid help seeking (Middleton et al., 1997; Ryan et al., 1998), and these students shows few negative outcomes such as test anxiety (Elliot et al., 1999; Middleton et al., 1997).

On the other hand, some other researchers reported that performance goal oriented has positive association with cognitive strategies (Dupeyrat et al., 2005; Elliot et al., 2001; Simons, Dewitte et al., 2004), cognitive engagement such as high attitude of persistence (Pintrich, 2000b). Since, performance goal orientation showed correlation with positive factors such as absorption during task involvement (Elliot et al., 1996), therfore, high achievemnt outcomes were been observed (Elliot et al., 1997).

In contrast to (Meece et al., 1993; Pintrich, 1991) studies, the study of Bouffard, Boisvert et al. (1995) is a correlation studies that merely noticed positive influence of both mastery and performance orientation. Results showed that highly mastery and performance goal oriented students' reveals highest level of cognitive strategy use, self-regulation, and course grade. For this reason, Pintrich (1991) recommended that there may be an evolution for students to be relatively high on both goal orientations. Regarding to performance-avoidance goals orientation, it have been apparently influenced by their negative motivational characteristics to become self-defensive about their self-esteem (Wolters, 2004).

Consequently, Pintrich (2000a) presumed that students with mastery-avoidance goal may use less adaptive monitoring processes, because of their focus on not making mistakes. Therefore, they have been positively associated with using low level and cognitive strategies (Elliot et al., 1999; Kadioglu et al., 2014; Simons et al., 2004).

2.13 The relationship between epistemological beliefs, goal orientation, self-regulated learning

Students epistemological beliefs, goal orientations, and their selection of learning approaches are essential determinants of students achievement (Cleary & Kitsantas, 2017; de Jesus Silva, Lay et al., 2017; Kizilgunes et al., 2009). These goals and epistemological beliefs are imperative for substantial and shallow cognitive engagement. Students holding naïve belief, such as belief that knowledge is simple, certain, and quickly acquired from authorities, shows shallow or surface processing (Ravindran, Greene et al., 2005).

Several other researchers conducted studies to investigate the connection between two or three of these four constructs; epistemological beliefs, goal orientations, and their self-regulated learning strategies and achievement. Schommer et al. (1992) investigated the relationship between epistemological beliefs self-regulated learning strategies and achievement. Findings were encouraging. However, other researchers examined the correlation between epistemological beliefs, goal orientations, and their self-regulated learning strategies and achievement (DeBacker et al., 2006; Huy P Phan, 2008).

The study of Muis et al. (2009), identified the combined relationship logic of students epistemological beliefs, goal orientations, and their self-regulated learning strategies with their achievement. Author noticed that epistemological beliefs control goal orientations, which in turn affect use of self-regulated learning strategies. Subsequently, these learning strategies influence student outcomes or their achievement. Statistical findings of this study showed that students holding more constructivist beliefs, including complexity and uncertainty of knowledge are most probably adopt mastery goal and use deep processing strategies (such as, elaboration, critical thinking, metacognitive self-regulation). Consequently, these students show positive outcomes or high-level achievement. In contrast, students who believe that knowledge is simple and certain (holding less constructivist beliefs) usually espouse performance goal and also employ shallow or surface processing strategies, such as rehearsal. As a result, these students show poor performance and hence, they are lower level achiever (Muis et al., 2009).

These findings are consistent with the study of Kizilgunes et al. (2009), which revealed that students holding beliefs about the certainty of knowledge have low level learning strategies, and hence they are performance goal oriented. Whereas, students holding strong beliefs about justification of knowledge, for example supporting the ideas that comes from reasoning, thinking, and experimenting are efficacious in their learning, hence they are less performance goal oriented (Kizilgunes et al., 2009).

Further, narrowing of the literature review revealed that epistemological beliefs, goal orientations, and their selection of learning approaches are related to mathematics problem solving (Hofer, 1997; Muir et al., 2008). These beliefs affect critical thinking and problem solving (Hofer, 2000; Hofer et al., 1997; Muis, 2004, 2007, 2008). Detail is provided in the below section.

2.13.1 The relationship among epistemological belief, goal orientation, self-regulated learning and mathematics

Most of the existing theoretical and empirical studied revealed a significant relationship between epistemological beliefs, use of self-regulated learning approaches . Schommer et al. (1992) investigated the mediating role of study strategies while beliefs effects on learning and also examined the prediction of simplicity beliefs on mathematical text comprehension. Statistical results revealed a negative prediction of simplicity beliefs on mathematical text comprehension and meta-comprehension. Based on findings of the study, author concluded that students believing in simplicity of knowledge are more likely to be engaged in memorization strategies. As a result, these students were unable to summarize important concepts. Moreover, epistemological beliefs directly and indirectly influence achievement, hence the roles of epistemological beliefs were found to be more robust.

After that, Hofer et al. (1997) hypothesized that there exist a relationship among epistemological beliefs, goal orientation, use of self-regulated learning approaches (during learning and problem solving) and academic achievement. According to author hypotheses, epistemological beliefs generate a specific type of achievement goal for learning. These goals further can influence the type of learning and meta-cognitive strategies that student can utilize during their learning and problem solving. Subsequently, these types of learning and meta-cognitive strategies affect their academic outcome.

For the current study, we hypothesized direct and indirect relationship of four factors; epistemological math problem solving beliefs, usefulness, goal orientation and self-regulated learning strategies during differential equation problem solving. Previous finding supporting the direct relationships of these factors, have already been discussed earlier. For the ease of readers, indirect or the role of mediation is explained in coming section.

2.14 Mediating relationship among epistemological belief, goal orientation and self-regulated learning

Mediators are intermediate variables that explain how or why an independent variable influence an outcome. It explain the mechanism through which an intervention affects an outcome by assuming both casual and temporal relations (Gunzler et al., 2013). However, Baron and Kenny (1986) distinguished the concept of mediation from moderation, in which a third variable affects the strength or direction of the relationship between an independent variable and an outcome. Typically, moderator is either part of an interaction term or a grouping variable in multigroup analyses. For example, if males are known to react differently than females to a particular intervention for lowering cholesterol, in a gender by treatment interaction effect, gender is a moderator.

In contrast to it, for a treatment study, identification of a mechanism by which intervention achieves its effect mediation process is used. By investigating mediational processes, pathology of the disease and the mechanism of treatment can be diagnosed. Also, alternative and more efficient intervention strategies can be identified through mediation process. For example, a tobacco prevention programme may educate participants how to stop taking smoking breaks at work (the intervention) which changes their social norms about tobacco use (the intermediate mediator). Consequently it leads to a reduction in smoking behavior which is study outcome (MacKinnon & Fairchild, 2009). Furthermore, Conner, Gunzler et al. (2011) also employed the mediation analysis to evaluate the hypothesis that greater drinking intensity leads to higher level of depression, which in turn, leads to suicidal ideation.

Moreover, the investigation of association between independent and dependent variable through some mechanism is also important in many discipline including child growth (MacKinnon, 2008; MacKinnon et al., 2009; MacKinnon, Fairchild et al., 2007). Child growth is one the most important area and has many examples of hypothesized processes (Gable, Krull et al., 2012; Judd & Kenny, 1981). For example, girl's weight is related to mathematics performance through its relationship with interpersonal skills (Gable et al., 2012), Iron deficiency anemia in children is related to levels physical activity through its relationship with cognitive development intervention programs promote healthy behavior by changing norms that are associated with those behavior (Judd et al., 1981). Further, Gunzler et al. (2013) suggested that based on strong theory and with appropriate context, mediation analysis is helpful in providing motivation for future intervention. In this way, more efficacious and cost- efficient alternative therapies may be developed.

Therefore, importance of mediator in any research study can't be denied. Generally, a variable in any research study acts as a mediator when it holds three conditions. First condition is that the variation in the level of independent variable explained variations in the presumed mediator. Secondly, variation in the assumed mediator significantly composed variation in the dependent variable. Lastly, a significant relationship between dependent and independent variables is no more significant (Baron et al., 1986).

Literature reveals that goal orientation and self-regulated learning play a mediating role. DeBacker et al. (2006) investigated the association among epistemological beliefs, achievement goals, need for answer to question (need for closure), and learning strategies. Results showed that student holding beliefs about complexity and uncertainty, and personal construction of knowledge. After that, these students adopt mastery goal orientations as compared to performance and avoidance goal orientations. Consequently, these mastery goal oriented students will utilize both deep learning (meaning making) and shallow learning strategies (memorization). Therefore, author concluded that among goal orientation, mastery goal mediates the relationship between epistemological and learning strategies, which are directly related. In contrast, students holding less constructivist epistemological beliefs demonstrated negative prediction of deep processing strategies. Whereas, these students explained positively predicted towards surface/shallow processing strategies.

In line with these studies, Muis et al. (2009) empirically tested the previously established theoretical interrelation of epistemological beliefs, goal orientation, learning strategies and achievement (Muis, 2007). Results revealed that epistemological beliefs have a control over the type of goal students adopted, which in turn affect the selection of appropriate learning strategies. Consequently, at the end all this whole process influences student's course grade and achievement. Moreover, author also noticed that an achievement goal orientation mediates the relationship between epistemological beliefs and learning strategies. Besides this, these learning strategies mediate the relationship between achievement goal orientation and achievement. For instance, elaboration (dimension of learning strategies) not only positively predicted achievement, but also mediates the relationship between mastery goal orientation and achievement (Muis et al., 2009). Similar results showed by performance goal. In case of performance goal, critical thinking (subscale of learning strategies) mediates the relationship between goal orientation and achievement.

Rastegar et al. (2010) also realized the importance of mediating role of goal orientation, and learning strategies. Therefore, author conducted a study to examine the relationship between epistemological beliefs and math achievement, as for the mediating role of goal orientation, cognitive engagement, and math self-efficacy. Results revealed that all these variables mediate the relationship between epistemological beliefs and math achievement. Likewise, indirect positive influence of mastery goal on math achievement by means of metacognitive strategies and self-efficacy are consistent with the findings of several studies (Elliot et al., 1999; Middleton et al., 1997; Mohsenpour, 2006; Rastegar, 2006). Moreover, performance goal showed indirect negative effect on math achievement via cognitive strategies. These finding are consistent with the findings of Rastegar (2006). Overall, it can be concluded that mediating role of goal orientation and self-regulated learning strategies towards math problem solving is worth noting.

Overall, it was revealed from the critical analysis of literature that there was a limited range of data available from developing countries (e.g. Pakistan) with respect to differential equations at pre-university level. In this context, mostly available data from developed counties and research were based on general mathematics problem solving or qualitative analysis except Kwon et al. (2005), which is a quantitative study. In addition to these, no work was found showing the combined effect of these four parameters (epistemological beliefs, usefulness, goal orientations, and SRL) for the differential equations problem solving, particularly non-routine problems. However, the combining these factors and mediation analysis is a tricky job that needed a comprehensive review of techniques for the mediation analysis. Details are provided in the following sections.

2.15 Mediation Analysis a tricky job

In social and behavioral sciences, statistical mediation analysis is mostly used to assess evidence of a mediation process that describe how or through what mechanism, an independent variable is an associated with a dependent variable. The mediation model is an Structural equation modeling (SEM) in which X causes M, M causes Y. SEM can be used to simultaneously estimate regression model (Card, 2012; MacKinnon & Valente, 2014). SEM provides a very general and flexible framework for performing mediation analysis. It is used to test mediation hypotheses, because it is designed to test these more complicated mediation models in a single analysis.

Moreover, in social and behavioral sciences, attributes such as attitude, IQ, personality traits, political liberalism, socio-economic status, etc., cannot be observed directly (Yuan & Bentler, 2006). These unobserved variables are often called latent variables (Bollen, 2002). Their affect can be assessed through multiple indicators that are subject to measurement errors (Byrne, 2001; Sarstedt et al., 2014). The idea of multiple indicator is from factor analysis (Spearman, 1904; Thurstone, 1947). Moreover, due to these measurement error, conventional statistical methodology such as ANOVA/MANOVA and regression cannot be directly used to analyze the relationships among social and behavioral science attributes. Detail of Structural equation modeling (SEM) is provided below.

2.16 Structural equation modeling

Structural equation modeling (SEM) world was developed to examine the hypothesized relationship (Wold, 1975, 1985). Structural equation modeling (SEM), is a general and a very powerful multivariate technique that uses a conceptual model, path diagram, and system of linked regression equation to capture complex and dynamic relationships within a web of observed and unobserved variables (Gunzler et al., 2013). There are two types of variables included in SEM models, exogenous variable and endogenous variable. Exogenous variables are always independent variable in SEM models, while exogenous variables represent a dependent variable in at least one of the SEM. These exogenous variables may become independent variable in other equations within the structural equation model.

SEM models are best represented by path diagrams, in which nodes represent variables and arrow shows relationship among these variables. By convention, latent variables are represented by a circle or ellipse, whereas, observed variables are represented by a rectangle or square. Generally, arrows are used to represent the relationship between variables. Two types of arrow exist in SEM. A single straight arrow indicates a causal relationship from the base of the arrow to the head of the arrow. Two straight single-headed arrows in opposing directions connecting two variables indicate a reciprocal relationship. A curve two-headed arrow indicates there may be some association between the two variables (Gunzler et al., 2013). SEM can be used when extending a mediation process to multiple independent variables, mediators or outcomes (MacKinnon, 2008).

2.16.1 Path coefficients analysis through structural equation modeling

The SEM approach is usually carried out to examine the direct and indirect effects of the variables. Consequently, SEM also estimated the size of the total effect of each independent, on dependent variable in the multi-stage path model, by providing both direct and indirect effects (Henseler, Ringle et al., 2015; Kline, 2015; Marsh, Morin et al., 2014; Reise, Scheines et al., 2013). Normally, the direct effects demonstrate the strength of the direct path from predictor variable to the particular dependent variables, as indicated by path coefficient. Whereas, the indirect effects register the strength of indirect path from a predictor variable to a dependent variable through mediator (Byrne, 2001; Rabe-Hesketh et al., 2007; Sarstedt et al., 2014).

2.16.2 Analysis of mediating roles

The analysis of the mediators in the proposed research model is carried out after assessment of the direct paths. Normally, three conditions exists in the casual relationship or direct effect analysis; complete mediation, partial mediation and no mediation. The mediating effect is considered prominent when the effect is reduced after the mediating variable enters the model (Awang, 2012). To interpret this condition, Awang (2012) has noticed that; if it is reduced but still significant, then the partial mediation is achieved. However, if the effect is reduced up to the level where it is no longer significant, then it is considered that complete mediation has achieved.

To test mediation, one should estimate the three following regression equations; first, regressing the mediator on the independent variable; second, regressing the dependent variable on the independent variable; and third, regressing the dependent variable on both the independent variable and on the mediator (Baron et al., 1986). Authors recommended that testing the significance of the indirect path a * b by the Sobel z-test. Equivalently, z tests whether the difference between the total effect and the direct effect should be statistically significant (Zhao, Lynch et al., 2010).

2.16.3 Softwares and techniqes for structural equation modeling

Over the past few decades, significant advances have been made in the theory, application and associated software development in the context of mediation analysis. For example, LISREL (Byrne, 2013b; Joreskog, Sorbom et al., 1979), MPLUS (Muthén & Muthén, 2012), EQS (Bentler, 2006), and AMOSS (Arbuckle, 2010). In addition to these specialized packages, procedures for general purpose statistical packages such as R, STATA, SAS, and statistica are also available. These software are also suitable for SEM purpose (Gunzler et al., 2013). However, PLS-SEM is a versatile method for estimating structural equation models (Hair, Ringle et al., 2013b; Hair, Sarstedt et al., 2012; Sarstedt et al., 2014). Because of its increasing popularity, PLS-SEM path modeling approach has been recently used in different fields, such as, business research (F. Hair Jr, Sarstedt et al., 2014), marketing research (Hair et al., 2012), family, business research (Sarstedt et al., 2014) and academic research studies (Ahmed, Umrani et al., 2017; Hair Jr & Lukas, 2014).

2.16.3.1 PLS-SEM model evaluation

PLS is a well-established, second generation multivariate technique which can simultaneously evaluate the measurement model and the structural model with the aim of minimizing the error variance (Byrne, 2013b; Rabe-Hesketh et al., 2007; Sarstedt et al., 2014). Moreover, PLS is a powerful data analysis technique that does not make relative assumptions regarding data distribution (Chin, 2010; Chin et al., 2003; Chin et al., 1999). PLS analyses can be performed using Smart PLS software which computed the estimates of standardized regression coefficient of the paths of the model, factor loadings for the indicators, and the amount of variance account for the dependent variables (Ringle et al., 2005). Generally, this software makes it possible to test the hypothesized relationships between independent and dependent variables depicted in the model. Another important application of Smart PLS software is that it computes several types of reliabilities (Cronbach's alpha and composite reliability coefficient) and validities (convergent and divergent) statistics, which can be used to assess the quality of the model (Jaouadi et al., 2017). In addition, SEM measures latent, unobserved concepts with multiple observed indicators. In SEM, two types of models including measurement model and structural model are embedded. Measurement model also known as outer model describes relationship between latent variables and their measures (indicators). Further, measurement model can be reflexive or formative or their combination depending upon the nature of constructs and variables. Whereas, structural model also known as inner model and it determines the relationships between the determinants.

2.16.3.2 Measurement model and structural modeling

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Initially, measurement models are required to be run, which enable the evaluation of reliability and validity of the construct measures. This step also ensures the quality of measurement model prior to hypothesis testing. Main steps of PLS-SEM are summarized (Table 2.2).

Table 2.2: Systematic evaluation of PLS-SEM (Hair et al., 2013)

Steps	Evaluation	
1	Evaluation of measurement model	
	1 (a) Reflective measurement model	1 (a) Formative measurement model
	Internal consistency	Collinearity among indicators
	Convergent validity	Outer weight significance and relevancy
	Discriminant validity	
2	Analyzing the research model and validating the second order constructs	
3	Evaluation of structural model	
	Significance and relevancy of the structural model path coefficients	
	Coefficients of determination (R^2)	
	Effect size (f^2)	
	Predictive relevancy (Q^2) and q^2 effect size	

Several assessments are required to be performed by evaluating the significance and the relevance of the structural model path coefficients, testing coefficients of determination R^2, assessing f^2 effect sizes, and the evaluating the predictive relevance $Q^2$and q^2 effect size.

2.17 Summary of the differential equation problem solving approaches

The detail summary including differential equation problem solving approaches, sample size and related major findings has been provided in the tabulated form in the following sections.

Table 2.3: Use of different approaches; algebraic, numerical, graphical and writing skills in teaching and learning differential equations

S.NO.	Reference	Objectives	Methodology	Sample
1	(Arslan, 2010b)	Explored the nature of students' learning in traditional differential equations (DEs) courses, also clarify the relationship between procedural and conceptual learning between procedural and conceptual learning	An achievement test wherein eight questions assess procedural learning and five questions for the evaluation of conceptual learning	77 candidate teachers
2	(Arslan, 2010a)	Examined understanding, difficulties and weakness of successful algebraic problem solver, associated with the concept of DEs and their solutions	13 open-ended DE questions from; algebraic solutions, curves of particular solutions, graphical interpretation, DE and its particular solution	77 candidates teacher and finally 61 students were chosen based on highest score
3	(Camacho-Machín et al., 2008)	Documented and analyzed students' type of behavior while solving differential equation problems that are presented as of different outlook from instruction process	Questionnaire of 11 problems of four types; solution concept, use of logical reasoning, representation and interpretion information from a real context	21 students, 10 from mathematics and 11 from physics
4	(Camacho-Machín et al., 2012b)	Examined and documented the type of knowledge that university students demonstrate to deal with basic issues that they had studied in a first ordinary DE	Questionnaire and a task-based interview was designed	21 science undergraduate students
5	(Rowland, 2006)	Investigated students' understanding of the units of factors and terms used in modeling context in an ordinary differential equation	Designed diagnostic quiz question and inquired students units and physical interpretation of differential equation terms	108 first year undergraduate engineering students
6	(Rowland et al., 2004)	Explored the students' interpretational difficulties in the modelling context of first order ordinary differential equation (ODE)	Used diagonstic quiz question, short answer exam question and one-on-one interviews	59 first year undergraduate students
7	(Camacho-Machín, Perdomo-Díaz et al., 2012d)	Documented the way wherein students think of and deal with ideas around the solution concept, and also analyzed the extent to which they show consistent preference to select and use a set of resources to deal with problems	Designed questionnaire and a task-based interview	21 students

Table 2.2: Continued

S.NO.	Reference	Objectives	Methodology	Sample
8	(Rasmussen, 1998b)	Investigated students' understanding as well as obstacles with numerical and qualitative approches for analysing differential equations (DE)	Audio-video tapes classroom session, Classroom observation, field notes, copies of quiz exams, computer assignment, questionnaire, instructor and faculty interviews, semi-structured interviews-6 students	6 undergraduate students for scientist and engineers
9	(Rasmussen, 1996)	Explored student (Amy) understanding of qualitative method of analysis for first order differential equation	Two semi-structured interviews, think aloud technique, data triangulation through analysis of documents of exam sheet and mathematical problem set	First year graduate student (Amy) in fishery science
10	(Keene et al., 2011)	Developed a framework for the classification of conceptual knowledge when solving ODE, in the domain of three simple methods including separation of variables,solving linear DEand Euler's method taught in elementry DE classes. Also measured conceptual understanding, hence designed multiple choice and true/false problems	Audio taped,written artifacts of Interviews of 5 mathematicians and mathematics educators, face-to-face and online human tutoring interface, interviews, think aloud for piloting items and 5 DE expertise for assessment of items	Assessment questions were administered to 39 students in one class
11	(Machín, Díaz et al., 2009)	Reported the extent to which university student make use of variou representation and mathematical process to understand and take action to set of questions that involve basic concept in the study of DE	Questionnaire of 11 problems and consist of activities in which use of algebraic, graphic (separately and together are required	21 students among them,10 were studying mathematics and 11 were studying physics
12	(Habre, 2003)	Investigated students' approval of solving a differential equation geometrically	Students exam papers, assignments, questionnaire and photocopyies of students answer sheet, semi structured interviews question- 6 students	36 students with engineering, math education, chemistry
13	(Habre, 2012)	Explored the result of writing on increasing student understanding of DE and examined students' writing skills	Copies of relevant exam problems	43 students enrolled,2 dropped and three failed
14	(Habre, 2002)	Assessed writing tool for analyzing adifferential equation task and its solution in a reform differential equation class	Verbal discussions, writing a short paragraph after analyzing problems geometrically and assignments from the workbook	Engineeing students

Table 2.4 : Technology advancement as an effective tool for teaching and learning differential equations

S.NO.	Reference	Objectives	Methodology	Sample
1	(Habre, 2000)	Examined whether students use slope field as a mean to solve first orde ODEs and are capable to extract information from these field. Moreover, explored students abilities converting graphical information into symbolic ones and vice versa	Classroom and lab session observation,students exam and IDE assignment copies,questionnaire,semi structured interview-9 students	Undergraduate students -26, major in Math, Biometery, Statistics, Chemistry Biology,Economics
2	(Camacho-Machín et al., 2015b)	Explored how the students make a relatiomships between different representations of the same object while examining the model's characteristics with the aid of digital mechanism	Written records of solved questions, team discussions audio recordings, video recording of group discussion and teachers presentation, audio and video recordings transcriptions	7 MA students of mathematics education pursuing engineering, computer science and mathematics
3	(Habre, 2002)	Reported the employment of writing tool to analyse differential equations problem and its solution	Two computer software program ODE and IDE were used, computer homework assignments, various models were geometrically solved and discussed in a verbal manner	Reform differential equation class of engineering students
4	(Azman et al., 2013)	The objectives are to introduce new teaching methodology and to investigate the issues in teaching and learning differenrial equation	Qualitative metasynthesis approach	Reviewed 8 action researches to explore the development of learning differential equation

Table 2.5 : Inquiry-oriented approach to differential equations (IO-DE)

S.NO.	Reference	Objectives	Methodology	Sample
1	(Rasmussen, Kwon, et al., 2006)	Presented an outline of the inquiry-oriented differential equations (IO-DE) approach and reported compared results on beliefs, understanding and abilities in inquiry-oriented differential equations and traditional students	Designed 8 routine assessment items and 8 conceptual assessment items. Also adapted belief instrument, views about mathematics survey (VAMS)	For routine problems (IO-DE n=65, Trad-DE n=83) For conceptual problems (IO-DE n=30, Trad-DE n=42) engineering and mathematics students
2	(Rasmussen & Kwon, 2007)	Highlited theoretical back ground for the IO-DE and charecteristics of inquiry-oriented differential equations (IO-DE) project as a case example of how undergraduate maths can be constructed on theoretical and instrutional progress begins at K-12 to create and retain powerful learning environment at undergaduate level	Highlighted theoretical setting for the IO-DE project and summaries quantitative studies that review this project	Theoratical study
3	(Rasmussen et al., 2000)	Reported on the realistic mathematics education (RME) instructional design perspective, as a resource to situate a starting point for an instructional sequence for DE	Video recording of class room session and of interviews, written work copies, record of meeting project s	12 students
4	(Rasmussen & Blumenfeld, 2007)	Elaborated the instructional design heuristics of emergent mode of realistic mathematics education to the undergraduate setting	Video recording of group discussion, photo copy of students' notes, homework copies, discussion board postings, and examinations., semi-structured interviews-21	37 students major in mathematics and science
5	(Wagner et al., 2007)	Explored knowledge beyond the content knowledge required by a mathematician in his initial use of an inquiry-oriented curriculum for teaching DEs at undergraduate level	Field notes, observations, video recording of mathematician debriefing session, class content notes and written self-report of mathematician	19 students major or minor in maths and/or biology, physics or chemistry, 2 mathematics educaions researchers and mathematician,
6	(Ju et al., 2007)	Explored the impact of IO-DE on students' mathematical beliefs	Whole class discussion followed small-group discussion, video-recorded class room session for discourse analysis	19 students, mostly freshman in mathematics education
7	(Keene, 2007)	Characterization of dynamic reasoning and supporting this type of reasoning to enhance students comprehension in time related areas of mathematics	Transcribtion of entire class discussion, small group discussion,interviews-6 students	10 engineering students

Table 2.4 : Continued

S.NO.	Reference	Objectives	Methodology	Sample
8	(Kwon, 2002)	Discussed current developemental research attempt to adapt realistic mathematics education (RME) in teaching and learning of differential equation	Video tapes of group work,field notes, records of instructional activities, copies of class work and homework assignments,electronic journal entries	43 First year students majoring in mathematics education
9	(Kwon, Cho et al., 2004; Kwon & Rasmussen)	Provided the theoratical and empirical groundings for the IO-DE approach, as a case example of how undergraduate mathematics can draw on theoratical and instructional progresss at K-12 level and maintain powerful learning environment	Theoratical background of inquiry oriented differential equation and also summarise the quantitatve assessment of inquiry IO-DE	Theoratical study
10	(Kwon et al., 2005)	Examined retention of students mathematical knowledge and skills in traditional and inquiry-oriented classes	Examined post-test and delayed post-test scores on CSAT (competitive exam)	IO-DE-15 freshaman TRAD-DE-20 majors in mathematics
11	(Yackel et al., 2000)	Extended the analysis of social interaction pattern such as social and sociomathematical norms regarding explanation and how these norms characterized in differential equation class	Small group work followed by whole class discussion,interpretations and thinking,Planning and debriefing meetings regardin to discussing these aspects	12 students major in mathematics, science and engineering
12	(Whitehead et al., 2003)	Reported on research conduted to understand graduate students' mental resources and way of reasoning about system of differential equation	Videotaped, task-based , semi-structured interviews-6 students	6 engineering students
13	(Rasmussen & Marrongelle, 2006)	Addressed the challenge of teaching in a manner consistent with reform suggestion by the developing the notion of pedagogical content tool	Video recording of classroom sessionand interviews,audio recording of projects meeting, copies of written work	Class room A- 12 students Classroom B- 45 students
14	(Kwon & Shin, 2004)	Students retention effect of RME-based instruction in differential equations	Developed and administered an instrument to two groups to measure students understanding	Group1: RME-DE class Group2: TRD-DE class

Table 2.6 : Others perspective connected with teaching and learning of differential equations

S.NO.	Reference	Objectives	Methodology	Sample
1	(Afamasaga-Fuata'i, 2004)	Investigated the impact of using meta-cognitive tool such as concept maps and vee diagram on students' comprehension of differential equation	Progresssive vee diagram and concept maps , final report,written responses of students perception about concept map and vee diagram	Case study (Nat)
2	(Mallet & McCue, 2009)	Elaborated discovery-based, constructive teaching and learning approach in which students apply their existing skills for finding the solution of ODE	Methodology is based on Five E's of constructivism framework- engage, explore, explain, elaborate and evaluate	First year, first semester class of mathematics students
3	(Sánchez, 2012)	Investigated the use of methodology based on error analysis of ordinary differential equation s enhance students acedemic performance	Three phases are; Pre-test phase , intervention phase posttest phase, used t-test and levene's test for analysis	18 students majors in technical engineering
4	(Raychaudhuri, 2007)	Students usage and interpretation of the existence and uniqueness theorems of first order ordinary differential equations	Not available	Not available
5	(Raychaudhuri, 2008)	Develpoped a frame work that makes explicit the inherent dynamics structure of certain mathematical definitions by means of four facets of context-entity-process-object,to capture specific aspects of first order differential equation	Not available	Not available
6	(Raychaudhuri, 2013a)	Applied reduce abstraction frame work to clarify students' cognitive processes while constructing the concept of solution to DEs and also explain different nuances of the interrelationship between mathematics structure and human thoughts	Classroom notes and observation, quiz and exam work, questionnaire, clinical interview, think aloud protocol	6-Hetrogenous group of student major in chemical engineer, physics, chemistry, mechanical engineer, and mathematics
7	(Raychaudhuri, 2013b)	Proposed a framework to classify student based on their individual approach towards learning DEs-determined by their effort to resolve a conflicts, preserve and reconstruct structures	Classroom notes and observation, quiz and exam work, questionnaire, clinical interview, think aloud protocol , 5 set of interviews,	6-Hetrogenous group of student major in chemical engineer, physics, chemistry, mechanical engineer, and mathematics

2.18 Chapter summary

Finding effective strategies for differential equation problem solving is an important part of mathematics education. Generally, three different approaches (algebraic, numerical and graphical) are employed to solve differential equations. In traditional differential equation teaching and learning, algebraic approach predominates, while recently numerical and graphical approaches are more emphasized and in this way the conceptual learning of differential equations is facilitated. Relevant literature of differential equation problem solving reveals that besides use of other advancements, epistemological math problem solving beliefs, usefulness, goal orientations and self-regulated learning strategies significantly may enhance the students' understanding to solve mathematical problems. However, available studies correlated either one factor or combination of two factors with mathematical problem solving ability. In this study, effect of these four factors; epistemological math problem solving beliefs, usefulness, goal orientations and self-regulated learning strategies, were particularly investigated towards differential equations problems solving ability. Non-routine differential equations tasks along with other research instruments were used to analyze differential equations problems solving ability at pre-university level in a selected province (Khyber Pakhtunkhwa) of Pakistan.

CHAPTER 3: METHODOLOGY

3.1 Introduction

This chapter presents a detailed description of the methodology and procedures that were employed in this study. First, the research design has been explained, followed by an explanation of research procedures, including the development of the instruments, data collection techniques and the data analysis used in this study.

The aim of current research work was to develop a conceptual model with four factors; epistemological math problem solving beliefs, usefulness, goal orientations, and self-regulated learning (SRL) strategies, which affect the differential equation problem solving for Pakistani pre-university level/college level mathematics students. In addition, this research has also investigated the inter-relationship of the selected factors while students were engaged in solving differential equation problem.

This chapter has outlined the research plan, sought to answer the following research questions:

1. Do epistemological math problem solving beliefs, usefulness, goal orientations and self-regulated learning (SRL) strategies directly affect differential problem solving ability?

To evaluate the first question, first hypothesis "Epistemological math problem solving beliefs, usefulness, goals orientations and self-regulated learning strategies (SRL) have direct effects on differential equation problem solving ability" was divided into following four sub-hypotheses.

H-1.1 Epistemological math problem solving beliefs have positive direct effects on differential equation problem solving ability.

H-1.2 Usefulness has positive direct effect on differential equation problem solving ability.

H-1.3 Goals orientations including mastery, performance and avoidance goals directly affect the differential equation problem solving ability.

H-1.4 Self-regulated learning strategies (SRL) including elaboration and critical thinking have positive direct effects on differential equation problem solving ability.

2. Do goal orientations play a mediating role between epistemological math problem solving beliefs and differential equation problem solving ability?

To evaluate the second research question, second hypothesis "Epistemological math problem solving beliefs have positive indirect effect on differential equation problem solving ability via goal orientations" was divided into following three sub-hypotheses;

H-2.1 Mastery goal play a mediating role between epistemological math problem solving beliefs and differential equation problem solving ability.

H-2.2 Performance goal play a mediating role between epistemological math problem solving beliefs and differential equation problem solving ability.

H-2.3 Avoidance goal play a mediating role between epistemological math problem solving beliefs and differential equation problem solving ability.

3. Do goal orientations play a mediating role between usefulness and differential equation problem solving ability?

To evaluate the third research question, proposed third hypothesis “usefulness has indirect effect on differential equation problem solving ability via goal orientations” was divided into following three sub-hypotheses;

H-3.1 Mastery goal play a mediating role between usefulness and differential equation problem solving ability.

H-3.2 Performance goal play a mediating role between usefulness and differential equation problem solving ability.

H-3.3 Avoidance goal play a mediating role between usefulness and differential equation problem solving ability.

4. Do self-regulated learning (SRL) strategies play a mediating role between epistemological math problem solving beliefs and differential equation problem solving ability?

To evaluate the fourth research question, proposed fourth hypothesis “epistemological math problem solving beliefs have indirect effect on differential equation problem solving ability via self-regulated learning (SRL) strategies” was divided into following two sub-hypothesis;.

H-4.1 Epistemological math problem solving beliefs have positive indirect effect on differential equation problem solving via elaboration ability.

H-4.2 Epistemological math problem solving beliefs have positive indirect effect on differential equation problem solving via critical thinking ability.

5. Do self-regulated learning (SRL) strategies play a mediating role between usefulness and differential equation problem solving ability?

To evaluate the fifth research question, proposed fifth hypothesis "usefulness have indirect positive effect on differential equation problem solving ability via self-regulated learning (SRL) strategies" was divided into following two sub-hypotheses;

H-5.1 Usefulness has positive indirect effect on differential equation problem solving ability via elaboration.

H-5.2 Usefulness has positive indirect effect on differential equation problem solving ability via critical thinking.

6. Do self-regulated learning (SRL) strategies play a mediating role between goal orientations and differential equation problem solving ability?

To evaluate the sixth research question, proposed sixth hypothesis "goal orientations have indirect effect on differential equation problem solving ability via self-regulated learning strategies" was divided into following six sub-hypotheses;

H-6.1 Mastery goal has positive indirect effect on differential equation problem solving ability via elaboration.

H-6.2 Mastery goal has positive indirect effect on differential equation problem solving ability via critical thinking.

H-6.3 Performance goal has positive indirect effect on differential equation problem solving ability via elaboration.

H-6.4 Performance goal has positive indirect effect on differential equation problem solving ability via critical thinking.

H-6.5 Avoidance goal has negative indirect effect on differential equation problem solving ability via elaboration.

H-6.6 Avoidance goal has negative indirect effect on differential equation problem solving ability via critical thinking.

7. Does algebraic approach yield better results than graphical approach for differential equation problem solving?

To evaluate the seventh hypothesis “algebraic approach yields better results than graphical approach for differential equation problem solving” was critically compared with marks obtained in the assessment test containing five non-routine differential equation tasks.

Overall to answer these research questions, five non-routine problems, containing first order pure time and autonomous differential equations were developed. These types of tasks required students’ special attention, efforts and learning strategies during differential equation problem solving.

To identify the factors affecting student’s differential equation problem solving, three adapted questionnaires; epistemological math problem solving beliefs and usefulness, goal orientations and self-regulated learning strategies have been used. In order to analyze the direct and indirect effect of above mentioned three factors on differential equation problem solving SEM (structural equation modeling) was used. SEM is basically an advanced statistical technique that is used to analyze complex relationship between variables in several disciplines, especially in education. Detail of analysis is provided in Table 3.1.

Table 3.1: Methods and approaches adopted for the selected factors

No	Activities carried out	Mode (direct/Indirect)	Research Question	Analysis
1	The effect of EMB on DEPS	Direct	111	SEM
2	The effect of UF on DEPS			
3	The effect of SRL (EL and CR) on DEPS			
4	The effect of goals (MA, PR, and AV) on DEPS			
5	The mediation role of goals (MA, PR, and AV) between EMB and DEPS	Mediation / Indirect	2	SEM
6	The mediation role of goals (MA, PR, and AV) between UF and DEPS	Mediation / Indirect	3	SEM
7	The mediation role of SRL (EL and CR) between EMB and DEPS	Mediation / Indirect	4	SEM
8	The mediation role of SRL (EL and CR) between UF and DEPS	Mediation / Indirect	5	SEM
9	The mediation role of SRL (EL and CR) between goals (MA, PR, and AV) and DEPS	Mediation / Indirect	6	SEM
10	Applying different problem-solving techniques (Algebraic or Graphic) effect differential equation problem solving	Comparison of strategies (Charles, Lester, & O'Daffer, 1987)	7	Modified version of the Analytic Scale for problem solving

EMB: Epistemological math problem solving beliefs, DEPS: Differential equation problem solving, UF: Usefulness, SRL: self-regulated learning EL: elaboration CR: Critical thinking, MA: Mastery PR: Performance, AV: avoidance

3.2 Research design

This is a quantitative correlational study. Generally, the correlational study design is used to test the hypothesis regarding expected relationship (Gay, 1981). Creswell et al. (2007) proposed that the correlational study well describes and measures the degree of relationship between two or more variables. Therefore, this research has considered correlational study mode and four different factors including epistemological math problem solving beliefs, usefulness, goal orientations, and self-regulated learning strategies affecting differential equation problem solving ability at pre-university level/college level mathematics.

Based on these factors, a priori model was developed with three endogenous variables and two exogenous variables as shown in Figure 3.1. Exogenous variable is a type of variable that is not caused by another variable in the model. Usually this variable causes one or more variables in the model. As epistemological math problem solving beliefs and usefulness are not caused by another variable, so these two beliefs were considered exogenous variable in the model. Endogenous variables are those variables that are caused by one or more variable in the model and may also cause another endogenous variable in the model (Kline, 2015). In present work, others factors such as goal orientations, self-regulated learning strategies and differential equation problem solving ability were affected by epistemological math problem solving beliefs and usefulness, therefore these factors were considered endogenous as shown in Figure 3.1.

To investigate different factors of the model, four different types of the instruments were finalized and used. These instruments were further investigated through the survey of a large and diverse number of mathematics students. During selection of survey method, different kinds of survey methods were assessed and among these methods, cross-sectional type survey was found suitable for this study. This survey deals with how people perceive their role and can be administered to one or more group of subjects by questionnaire, and test without involving treatment. Usually, this type is helpful to assess interrelationships among different variables within a population and is ideally suited for descriptive and predictive purposes (Shaughnessy, 1990).

On the basis of this survey, researcher discovered the effect of individual variable and the relationship among various variables towards differential equation problem solving. This has provided a full and in-depth picture of problems being studied and the students' concern with differential equation subject matter. Recently, Creswell (2012), suggested that there are certain steps for this kind of survey; designing instruments for data collection, sampling from population, collecting data through questionnaire. Detail of sampling, instrument development and research procedure is provided in next section.

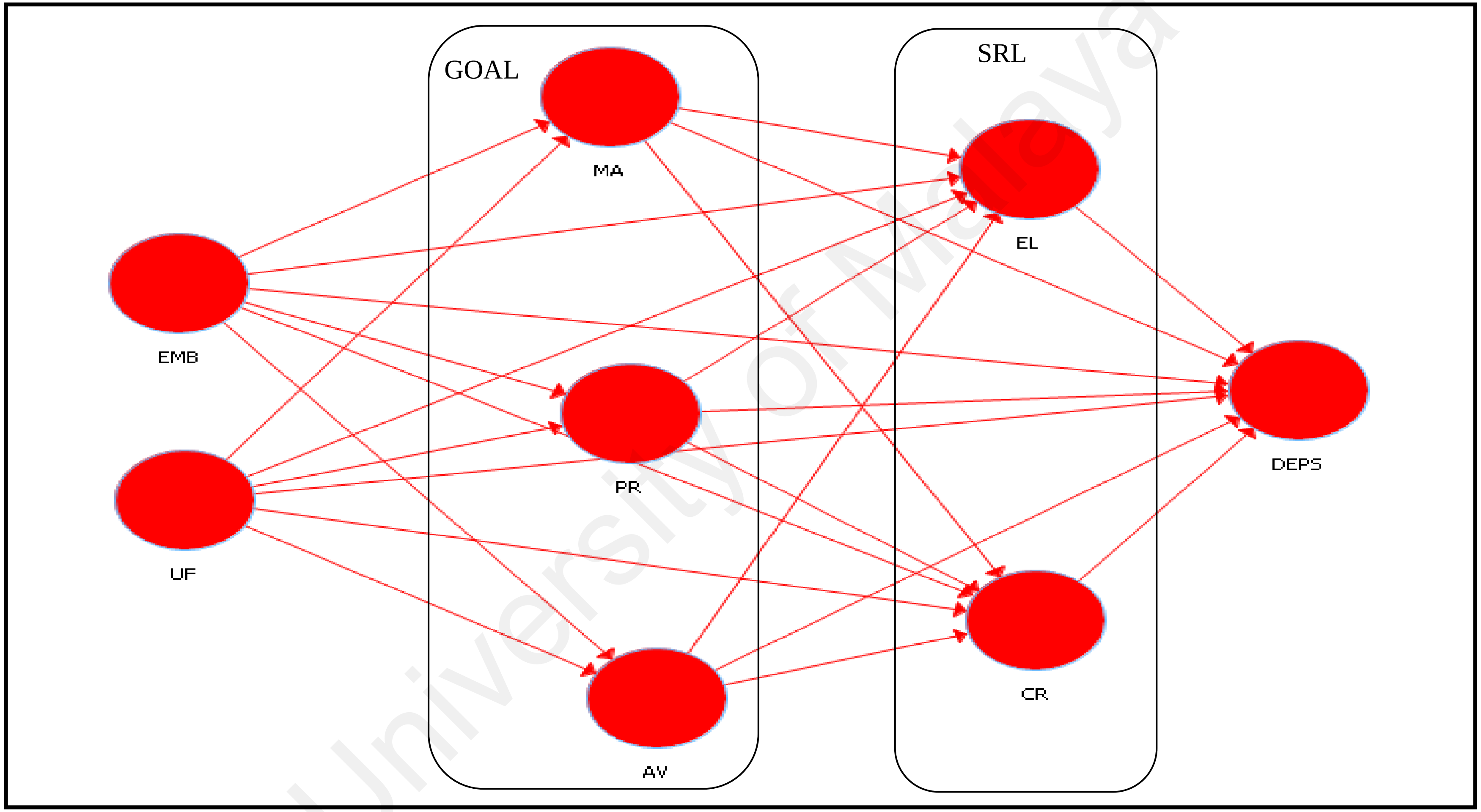

Figure 3.1: A priori model showing three endogenous and two exogenous variables

3.2.1 Sampling

An appropriate sample size is the essential part of the precise measurements for hypothesis testing (Welkowitz, Cohen et al., 2006). In fact an adequate sample size may able to represent the population truly (Gay & Airasian, 2000). Findings of small sample size may be true for the particular sample only, however it may not be a true representative of the population (Gay et al., 2000). Similarly, the larger sample size minimizes the error, and increases the reliability of findings (Welkowitz et al., 2006), however, it makes the method too sensitive (Hair, Anderson et al., 1998). In the same context, Rex B Kline (2005) suggested that a sample size of 220 or large could be necessary for a complicated model, concluding that more complex models with numerous parameters, require larger samples. The sample size for this study was determined through Robert and Daryle (1970) table, which is mostly used for the sample size selection.

In addition to sample size, sampling technique is also important toward the reliable results. Probability sampling and non- Probability sampling are two basic sampling techniques. Probability sampling is concerned with the known probability of being selected, whereas, non- probability sampling does not have known probability of being selected. Probability sampling is further categorized as simple random sampling, stratified sampling, cluster sampling, systematic sampling and multistage sampling. In most of the cases, random sampling, stratified sampling, cluster sampling, techniques are used (Gay, 1981). Among these sampling techniques, random sampling is mostly used. Because, in random sampling approach all individuals in the defined population have an equal and independent chance of being selected for the sample (Gay, 1981). Though this any bias in the population is equally distributed among the people chosen (Creswell, 2008). For current study, cluster random sampling was found appropriate, because this approach selects group, not individual (Gay, 1981).

In this research, research instruments were distributed in 430 (12 percent more than predicted 381) students among science students from different government and private sectors, studying at pre-university level in Pakistan. Average class size is 45; therefore, 9 institutes are required for this sample size. Moreover, keeping in mind the ratio of population in government and private sectors, 6 institutes from government and 3 from private sector were considered for this study. Detail of population and sample size is shown in Table 3.2.

Table 3.2: Estimation of sample size using Robert and Daryle (1970) table

Population in govt and private institutes in Khyber Pakhtunkhwa (Pakistan)								
Institutes			**Population in institutes**			**Sample size (estimated)**		
Govt	Private	Total	Govt	Private	Total	Govt	Private	Total
462	402	864	29,952	15,152	45,104	254 (From 6 Institutes)	127 (From 3 Institutes)	381 (From 9 Institutes)

3.2.2 Development of the research instruments

This research was based on quantitative mode to collect responses from the mathematics students about their concern with differential equation topic, and to examine the full picture of the problems being studied. Therefore, the instruments of this research included five non-routine word problems, and three different types of questionnaires for detail analysis. Table 3.4 illustrates the detail of the three instruments along with endogenous and exogenous variables.

3.2.2.1 Development of differential equation tasks

Literature reveals that choosing suitable mathematical tasks for the study is also an important factor towards the representation of the population. Mandler (1989) emphasized the importance of analyzing a task and also given some perceptions to learner that surprises, errors and missteps can be handled by some alternatives routes, substitute actions, or a rewording of the tasks. Usually, tasks are categorized as routine and non-routine based problems (Yeo, 2009).

Solving a routine problem cannot contribute to the mental development of a student. On the other side, non-routine based tasks require the problem solver to use their heuristic strategies to approach the problem, to understand it and proceed toward a suitable solution. Consequently, these non-routines based tasks enhance student's higher-order thinking during evolution of understanding, analysis, exploration and application of mathematical concepts. But, mostly students feel fear to solve these problems because of the unexpected and unfamiliar results (Polya, 1962; Rehman et al., 2012). Nature of present study required non-routine problems to measure the desired objectives. Keeping in mind, the level of the required population, five non-routine problems contain first order differential equations were developed and used for the current study.

It was quite obvious that nature of this research was demanding the development of non-routine tasks, however, as Pakistan is a developing country, so due to change in the resources, infrastructure, teachers and students' abilities it was quite difficult job. And also findings might not be same as compared with hypothesized. In addition, a limitation relevant to self-reported instruments was possible to have occurred including participants' lack of cooperation in providing information about themselves.

Several major points were considered at time of tasks development. Topic of differential equation along with differentiation and integration is introduced first time at 12^{th} year of study, and the students have no previous knowledge and understandings of this topic (Rehman et al., 2012). Most of the practice problem were of routine nature. This research had to use non-routine differential equation tasks to investigate students' problem, which strongly needed students' special attention, efforts and learning strategies to solve them (Polya, 1962; Rehman et al., 2012). Beside this the assessment test was an informal exam or test having no short-term incentives for them. Hence, participants' lack of cooperation was possible. The adapted instruments containing epistemological math problem solving beliefs, goal orientations and self-regulated learning (MSLQ) questionnaires were to be used to assess participants' motivation and their use of cognitive strategies. All of these adapted questionnaires were based on the theories and findings of developed countries. As Pakistan is a developing country, so due to change in the resources, infrastructure, teachers and students' abilities and findings might not be same as compared with hypothesized. In addition, a limitation relevant to self-reported instruments was possible to be occurred including participants' lack of cooperation in providing information about themselves.

Results of this small-scale study were be the another important limitation it might not be a generalized data for all secondary mathematics students of all provinces in Pakistan. This research had to be carried out in a limited number of institutes located in one province of Pakistan. Results in other provinces or states might be different due to students' learning capacity, teacher training and availability, infrastructure, cultural and regional constraints. Other major limitations were investigations of a limited tasks and problem solving approaches. Similarly, five non-routine tasks involving only two problem solving approaches (algebraic and graphical) had to be considered here.

To reduce the limitation of using non-routine differential equation problems to assess the students' problem and to overcome the issue of participants' lack of cooperation, non-routine tasks exhibiting daily life problems were the best option. Daily life based problems were able to capture the students' attention. In addition, sufficient efforts were carried out to give them shape of non-routine problems with adequate hidden data to analyze different factors. In addition, by reducing the number of tasks up to five had helped students to solve these tasks without feeling bored.

To overcome the issue of adapted instrument's validity for the self-developed tasks, an additional questionnaire (in addition to research instrument) was designed for the field experts (educators/ teachers), in which, consents of the experts were assessed with respect to different parameters, such as suitability of the selected tasks, their inter-connection in the present study, clarity of representations. Responses of the experts were also analyzed for the final data collection.

The tasks were developed considering the level of students' level/ education in province of the Khyber Pakhtunkhwa. A suitable choice of sample size and random sampling from both public and private sectors as well as from urban and rural areas may enable a generalization of the findings of this research to other Pakistani students studying at secondary level from similar context. Atleast 52 percent of participants who were enrolled in rural areas colleges had to be solve these tasks. In addition, by comparing and confirming the demographic information with the institutional data about participants remained helpful to delimitate the error in self-reporting data.

3.2.2.2 Self-developed non-routine differential equation tasks

For present study, five non-routine differential equation tasks were developed. These tasks covered different aspects of the differential equations to relate and solve daily life problems. Task 1, task 2 and task 3 were about population growth, projectile motion and compound interest, respectively. Similarly, task 4 and task 5 were about polio infected people (health and disease), cooking of bakery items (a particular application of Newton's law of heating/cooling). Details are provided below;

1. In a research laboratory, a researcher studied the growth of bacteria culture. Normally, the bacteria population increases at the rate proportional to the size of bacteria present. The researcher found the number of bacteria increases six fold (times) in 10 hours. Initially number of bacteria is 10.

    a. Write differential equation that describes population increase for bacteria.
    b. Assuming normal growth, how long did it take for their population to double?

2. In a playground, a football player has to hit the ball vertically upward, or at certain angel to pass it to another player or hit the goal. During a match, a player hit the ball vertically upward with a velocity of 15 m/s.

    a. Write differential equation (model) that describes the velocity of the ball with respect to time (Neglecting air resistance).
    b. Find the equation for the distance (height) travelled by the ball in any time "t".
    c. Find the height of the ball after one second.

3. In Pakistan as well as in the world, banks provide incentives in the term of interest on the deposited money. Ayaan deposited an amount of 1000 Rupees in the bank Islami, which has an interest rate (dM/dt) of 5.0% per year. He did not draw any money and interest compounds continuously. How much amount he will have after 5 years? What you think, the amount of money increased, decreased or remained constant?

4. In the district Mansehra, district health center, monitors and records the polio cases of the whole district. On the basis of 10 years recorded data, the center has formulated a rate equation

$$\frac{dA}{dt} = 100t \qquad 3.1$$

Where A is number of infected people at the start of 2005, and t is time measured in year. Initially there were 200 people infected. Make a sketch of derivative graph and its function graph at time 0≤t≤5.

5. In the bakeries, cakes and biscuits are normally prepared using different food ingredients and finally cooked in the baking oven. In a bakery, a cake was removed from baking oven at 400K. Five minutes later, its temperature was decreased to 200K, which can be well described by Newton law of cooling;

$$\frac{dT}{dt} = K(T - T1) \qquad 3.2$$

Where T-T1 is change in temperature and t is time. What would be the general solution of this differential equation?

3.2.2.3 Analysis of differential equation tasks

For the current study, five tasks were developed using the guidelines of Polya (1957) and were also tried to make these tasks aligned with guidelines of Schoenfeld (1985a), which had more focused on the distinction between a mathematical problem and a mathematical task. In addition, these tasks were reflecting non-routine based differential equation problems.

While preparing the task items, their cognitive demands were considered to make sure that they reflect low, moderate, and high levels of complexity. According to Webb (1999) classification system, cognitive complexity level of items is associated with its depth of knowledge rather than ability of students. A low complexity of a question requires students to recall a previously learned concept, whereas, moderate complexity requires more critical thinking, in which students are expected to use reasoning and problem solving strategies, and bring together skills and knowledge from various domains. In addition, a high complexity question involves solving non-routine problems that requires multiple steps and decision point. Based on Webb (1999) classification system, each task was reflecting low, moderate and high complexity level.

Present study had to involve five non-routine differential equation tasks. These tasks covered different aspects of the differential equations to relate and solve daily life problems. Task 1, task 2 and task 3 were about population growth, projectile motion and compound interest, respectively. Similarly, task 4 and task 5 were about polio infected people (health and disease), cooking of bakery items (a particular application of Newton's law of heating/cooling).

To analyze the students' problem solving ability an analytic scale for problem solving based on scale of Charles et al. (1987) was adapted. Detail of scoring rubric to assess differential equation problems is illustrated in Table 3.3. In the assessment test, each task was given a total six marks, with a further distribution of two marks for each stage involved. On the basis of these developed tasks, questionnaires were adapted and administered to collect desired data from participants. Three questionnaires covering students' epistemological math problem solving and usefulness beliefs, goal orientations, and self-regulated learning strategies were adapted. Details of each instrument and their dimensions are provided in the below section.

These five non-routine differential equation tasks were covering different aspects of the differential equations to relate and solve daily life problems. Task 1, task 2 and task 3 were about population growth, projectile motion and compound interest, respectively. Similarly, task 4 and task 5 were about polio infected people (health and disease), cooking of bakery items (a particular application of Newton's law of heating/cooling).

Algebraic or procedural based approaches were needed to solve task 1, task 2 and task 3. Task 4 was particularly designed to evaluate the differential equation solving skills using graphical methods. Similarly, in the task 5, major trick was to give excess data (about general and particular solutions) to make it puzzle problem, and analyze how students handle this situation.

An adapted analytic scale for problem solving based on scale of Charles et al. (1987) was used to score these tasks. Authors proposed three categories as understanding, planning and getting answer (Charles et al., 1987). The understanding stage, students needed to interpret or retrieve hidden data. In case of full understanding they were assigned 2 marks otherwise 1. The next phase was planning, in which students had to plan the whole steps, procedures, formulas, and strategies. Students who were successful in their planning phase they were assigned 2 marks, otherwise they were considered partial planner and were assigned 1 mark. In getting an answer phase, the answer of the task, students who used the correct procedure but not completed the solution or made a sign or unit mistakes they were assigned one marks and vice versa.

There was one task, in which students need to construct graph of differential equation tasks. During this phase, students who have done correct procedure, complete the solution without making a sign or unit mistakes they were assigned full marks (2 marks), only if they were successful in graphical representation as well. Overall, in each category, the students with zero marks mean that they could not do or understand it completely.

Similarly, the students with one (1) and two (2) marks mean that they did/understood it partially and completely, respectively. Furthermore, students were graded based on the grading system of the Khyber Pakhtunkhwa (KPK), Pakistan.

Table 3.3: Detail of scoring rubric to assess differential equation problems

Stages	Scores	Characteristics	Description
Understanding	0	Complete miss-understanding	Lack of comprehension problem Not able to identify important given data
	1	Partial understanding	Misinterpreted some part of problem partially understand data, partially understand goals and hidden data
	2	Complete understanding	Ability to take information and to translate it in the mathematical model, fully retrieve given and hidden data and symbolically specify relevant known and un known variables, formulate proper equation
Planning a solution	0	No attempt/ inappropriate plan	Wrong Integration procedure, not able to put constant of integration,
	1	Partially correct plan	Correct interpretation up to a certain point, but strategy remain major flawed
	2	Plan lead to a correct solution	Execute the plane, translate plane into series of appropriate mathematical action, successful findings
Getting answers	0	No answer	Can't execute integration steps
	1	Copying error, computer error	Mathematical/computational error
	2	Correct answer, correct label	Solution complete, No error in answer

Adopted Scoring rubric for non-routine words problem based on Analytic Scoring Scale (Charles et al., 1987)

3.2.2.4 Epistemological beliefs questionnaire administration

The second instrument "epistemological math problem solving beliefs" was measured with an adapted scale taken from Indiana mathematics beliefs scale (IMBS; (Kloosterman et al., 1992). Indiana mathematics belief scale (IMBS) measures student's beliefs about mathematics problem solving. This scale was evaluated by the following five dimensions; 1) duration of problem, 2) steps, 3) understanding, 4) word problems, 5) effort.

Duration of problem comprised of six items, such as, "Differential equation word problems that take long time don't disturb me." The measure of steps composed of six items, such as, "Differential equation word problems can be solved without remembering formula." The measure of understanding included six items, such as, "Time used to examine why a solution to differential equation works is considerably not time passed". In addition, the measure of word problems consisted of six items, such as, "A student who can't solve word problems really can't understand and solve differential equations". Similarly, effort scale also comprised of six items, such as, "Practice can improve one's ability to solve differential equations". Details of the each belief and its items are provided below;

Belief 1: There are differential equation word problems that cannot be solved in specific duration of time

\+ I can solve time-consuming differential equation word problems.

\+ Differential equation word problems that take long time don't disturb me.

\+ I think I can do differential equation word problems that take a long time to finish.

\+ I find I can do difficult differential equation word problems if I just hang in there.

\- If I can't do a differential equation word problem in a limited time/couple of minutes, I probably can't do it at all.

\- If I can't solve differential equation word problems rapidly, I quit trying.

\- I am not very good at solving differential equation word problems that take some time to understand.

Belief 2: There are differential equation word problems that cannot be solved with simple step by step procedure.

+ There are differential equation word problems that just can't be solved by following a pre-defined sequence of steps.

+ Differential equation word problems can be solved without remembering formulas.

+ Remembering steps is not that useful for learning to solve differential equation word problems.

- Any differential equation word problems can be solved if you know the proper step to follow.

- Most differential equation word problems can be solved by using the suitable step by step procedure.

- Learning to do differential equation word problems is mostly an issue of remembering the correct step to follow.

Belief 3: Understanding Differential equation problems are essential in mathematics

+ Time used to examine why a solution to differential equation works is considerably time passed.

+ An individual who does not understand why an answer to differential equation is right has not really solved the problem.

+ In addition to obtaining a correct answer in differential equation solution, it is essential to understand why the answer is right.

- It is not important to know why a differential equation process works as long as it provides a right answer.

- To get the correct answer in differential equation solution is more essential than understanding why the response works.

- It does not actually matter if you understand a differential equation problem if you can find the right response.

Belief 4: Word problems are important in differential equation course

+ A student who can’t solve word problems really can't understand and solve differential equation.

+ If you don’t use computational skills to solve word problems in the differential equation, it means they are not so important.

+ If you don’t use computational skills in differential equation relating real life situation, it means they are useless.

- To learn computational skills for solving differential equation solution is more important than learning word problems

- Mathematics classes should not emphasize differential equation word problems.

- Word problems are not necessarily part of differential equation course

Belief 5: Effort can increase ability to solve differential equation problems

+ Doing more hard work a person became efficient in differential equation problems.

+ Practice can improve one's ability to solve differential equation problems.

+ I can get smarter in differential equation problems by trying hard.

+ When one studies hard, differential equation problems solving ability are increased.

+ More hard work can increase one's ability to solve differential equation problems.

+ If I do hard work I become efficient in differential equation problems

All these scales consist of three positively-oriented items and three negatively-oriented items except scale (5). All six items from this scale were positively-oriented. All responses were made on a 5-point scale ranging from 1 (strongly disagree) to 5 (strongly agree). The reported reliability for the five subscales is 0.73 (Kloosterman et al., 1992).

3.2.2.5 Usefulness questionnaire administration

The third instrument "Usefulness" was measured with an adapted scale taken from Fennema et al. (1976) scale. Usefulness scale included six items, such as, "Differential equation problems are worthy and compulsory". Details of the belief and its items are provided below;

Usefulness belief: Differential equation problems are useful in daily life

+ I like to study differential equation problems because I know how it is useful

+ Knowing differential equation problems will help me earn a living

+ Differential equation problems are worthy and compulsory

- Differential equation problems will not be important to me in my daily life.

- Differential equation problems have no relevance to my life

- To study differential equation problems is a waste of time

All responses were made on a 5-point scale ranging from 1 (strongly disagree) to 5 (strongly agree). All these scales consist of three positively-oriented items and three negatively oriented items. Several researchers criticize the Indiana Mathematics Scale (IMBS) because this does not have option-"no response". Then participants are being forced to give a reason they do not hold thus diluting the validity. However, other researchers believe that IMBS scales have an option "uncertain". Therefore, validity of these scales cannot be diminished. Fennema-Sherman's (1976) reported Cronbach's alpha value of usefulness as 0.86.

3.2.2.6 Goal orientation questionnaire administration

In fourth instrument, goal orientations were grouped into three dimensions, mastery goal, performance goal, and avoidance goal orientation. These goals were measured with scales taken from the Patterns of Adaptive Learning Survey (Midgley et al., 2000). The measure of mastery goal comprised of six items, such as, "In differential equation class, understanding the work is more important to me than the grade I get." The measure of performance goal included five items such as, "Doing better than other students in differential equation class is important to me". While, measure of avoidance goal included six items, such as, "It's very important to me that I don't look stupid in my differential equation class". Details of the goal orientations and its items are provided below;

Mastery Goal Orientation

a) I like differential equation class work that I'll learn from even if I make a lot of mistakes.
b) I do my differential equation class work because I like to learn new things.
c) I like differential equation class work best when it really makes me think.

d) I do my work in differential equation class because it is important for me to get better at it.
e) I do my differential equation class work is because I enjoy to do it.
f) I do my differential equation class work because I’m interested in it.

Performance Goal Orientation

a) I would feel really good if I were the only one who could answer the teacher’s questions in differential equation class.
b) I want to do well than other students in my differential equation class.
c) I would feel successful in differential equation class if I did better than most of the other students.
d) I’d like to show my teacher that I’m smarter than the other students in my differential equation class.
e) Doing better than other students in differential equation class is important to me.

Avoid Goal Orientation

a) It's very important to me that I don't look stupid in my differential equation class.
b) An essential reason I do my differential equation class work is so that I don’t embarrass myself.
c) I do my differential equation class work is, that my teacher doesn't think I know less than others.
d) I do my differential equation work is so that the others won't think I'm dumb.
e) One of my main goals is to avoid looking like I can't do my work.
f) I would not participate in differential equation class because I want to avoid looking stupid.

All responses were made on a 5-point scale ranging from 1 (not at all true) to 5 (very true). Several researchers criticize the adaptive learning survey scale because this also not have option-“no response”. Then participants are being forced to give a reason they do not hold thus diluting the validity. However, other researchers believe that as like IMBS scales, this scale has an option “uncertain”. Therefore, validity of these scales cannot be diminished. The reported reliability for the mastery, performance and avoidance goals were 0.86, 0.86 and 0.75, respectively (Carol Midgley et al., 2000).

3.2.2.7 Self-Regulated Learning questionnaire administration

The fifth instrument, self-regulated learning strategies was assessed with a Norwegian adaptation of the Motivated Strategies for Learning Questionnaire (MSLQ) (Pintrich, 1991). This scale was further separated into two main categories; a motivation section and a learning strategies section. Overall, MSLQ has 15 sub-scales, in which six subscales lie inside the motivation section and nine within the learning strategies section. For present study, only two dimensions critical thinking and elaboration were chosen from learning strategies section. The elaboration strategy has six items such as “When reading for differential equation class, I try to make a connection with my previous knowledge”. The measure of critical thinking included five items such as “I treat the differential equation course material as a starting point and try to cultivate my own thoughts about it”.

Critical Thinking

a) I often find myself questioning things I hear or read in the differential equation course to decide if I find them believable
b) When a theory, interpretation, or conclusion is presented in differential equation class or in the readings, I try to decide if there is good supporting evidence.

c) I treat the differential equation course material as a starting point and try to cultivate my own thoughts about it.
d) I attempt to play around with my own ideas related to what I am learning in differential equation course.
e) Whenever I read or hear an argument or conclusion in differential equation class, I think about possible solutions.

Elaboration

a) When I study for differential equation class, I organize information from various/multiple sources, such as lectures, readings, and discussions.
b) I try to relate differential equation ideas in math's subject to those in other courses like physics, biology, chemistry whenever possible
c) When reading for differential equation class, I try to make a connection with my previous knowledge.
d) When I study for differential equation course, I write brief summaries of the main ideas from the readings and my class notes.
e) I try to understand the contents of the differential equation course by making connections between the readings and the concepts from the lectures.
f) I try to apply ideas from course readings in other class activities such as lecture and discussion.

All responses were made on a 7-point scale ranging from 1 (not at all true) to 7 (very true). Similar to IMBS, several researchers also criticize the MSLQ scale because this also not have option-"no response". Therefore, participants are being forced to give a reason they do not hold thus diluting the validity. However, other researchers believe that as like IMBS scales, this scale has an option "uncertain". Therefore, validity of these scales cannot be diminished. The reported reliability values for elaboration and critical thinking were 0.75 and 0.80 (Duncan et al., 2005). Details of each instrument and their dimensions are provided in Table 3.4. To confer the credibility of instrument, validation was performed by the educators and the mathematicians who were teaching in inter-colleges and university.

Table 3.4: Research variables and selected dimensions

Variable Name	**Variable Type**	**Instrumentation Type**	**Selected Dimensions**	**Number of items**
EMB	Exogenous	EMB and UF beliefs questionnaire	DP	6
			ST	6
			UN	6
			WP	6
			EF	6
UF	Exogenous		UF	6
Goal orientations	Endogenous	Goal orientations questionnaire	MA	6
			PR	5
			AV	6
SRL	Endogenous	SRL questionnaire	CR	5
			EL	6
DEPS	Endogenous	DE tasks (non-routine word problems)	Population growth based	1
			Velocity and acceleration based	1
			Compound interest	1
			Health and diseases	1
			Newton law of cooling	1
Total no of items				**69**

They were apparently told to look into the contents and face validities of the instruments. Further, they inspected the suitability of the non-routine differential equation tasks and decided whether or not they tested the problem solving ability as defined in this study. Details are provided in the following sections.

3.2.2.8 Development of a questionnaire for field experts and teachers

To overcome the issue of adapted instrument's validity for the self-developed tasks, an additional questionnaire (in addition to research instrument) was designed for the field experts (educators/ teachers), in which, consents of the experts were assessed with respect to different parameters, such as suitability of the selected tasks, their inter-connection in the present study, clarity of representations. Responses of the experts were also analyzed for the final data collection.

Considering the level of education systems in province of the Khyber Pakhtunkhwa and a suitable choice of sample size from both public and private sectors educators/teachers may enable a generalization of the findings of this research to other Pakistani educators teaching at secondary level from similar context. In addition, the demographic information along with the institutional data about participants were also important to delimitate the errors. Therefore, a brief demographic information including teacher Name, institute name, gender (Male/Female), highest degree (B.Sc./ M.Sc./ M. Phil/Ph.D), teaching, experience, present designation (Subject Specialist/ Lecturer/ Assistant Professor/ Other), induction (Direct appointment on the present post/ promoted) were asked. In addition to these, teaching experience areas (Urban /Rural/ Both) and teaching sectors (Govt/ Private/ Both) were also requested, so that on the basis of demographic information, research instruments might be validated.

In addition to the detailed demographic information, two sections of the questionnaire were keenly designed. In the first part , suitability of the selected tasks, their inter-connection in the present study, clarity of representations were asked. While the in the second part, educators' beliefs about teaching and learning of differential equation were assessed.

In the first part, again it was divided into two sub-sections. First sub-section had included characteristics of differential equations containing tasks, their relevancy, and clarity. While second sub-section was about adapted questionnaires including clarity, organization and Urdu translation. Details of different items used are provided below;

- Clarity of phrasings and wordings
- Relevancy to the course and level of the students
- Challenging and non-routine as they are not easily solvable by using a previously taught simple algorithms and procedures
- Able to promote active involvement of students
- Allow multiple approaches and solutions
- Able to make connection of differential equations to other mathematical concepts and real-world problems

Details of different items used for adapted questionnaires including clarity, sequential organization, and interpretation level of Urdu translation are provided below,

- How you will rank the clarity and unambiguousty of this questionnaire?
- Are questionnaire items are logically and sequentially well organized?
- How you will rank the interpretation level of Urdu translation?

All responses were made on a 5-point scale ranging from 1 (Poor) to 5 (Excellent). In the second part, educators' beliefs about teaching and learning of differential equation, and understandings and special efforts required were assessed. Details of different items used are provided below;

a) Do you believe that teaching and learning of differential equation is a difficult part of the mathematics at inter-college level as compared to other parts like algebra, trigonometry and etc?

b) Do you think that high level of conceptual understandings and special efforts are required to solve differential equations based problems?

c) Do you agree that differential equation based problems, particularly of non-routine nature can be used to correlate the realworld problems.?

d) Do you agree that at present, less attention is given to the non-routine based problems containing differential equation at inter college level?

e) Should policy makers increase non-routine differential equation problems in the new mathematics curriculum?

f) Do you agree that teachers should be properly equipped and trained, so that they may educate non-routine as well as routine based problems containing differential equation?

g) Do you agree that students psyche can also boost up the differential equation based problem solving?

h) Do you think that students' motivations can enhance the understandings as well as the solution of differential equation based problems?

i) Do you think that self-regulated learning strategies, such as critical thinking and elaboration can affect positively for the students to solve differential equation based problems?

j) Do you think that epistemological beliefs about problem solving can affect the differential equation based problem solving?

k) Do you think that combination of epistemological beliefs about problem, motivations and self-regulated learning strategies can significantly contribute toward differential equation based problem solving?

l) Do you think that this type of research is useful for both of teachers as well as students?

All responses were made on a 5-point scale ranging from 1 (strongly disagree) to 5 (strongly agree).

3.3 Validity and reliability of research instruments

The entire scales epistemological math problem solving beliefs, usefulness, goal orientations and self-regulated learning strategies underwent stages of developments. Although, literature shows that these scales have proved to be trustworthy. However, number of items merely measured the intended construct of school learners rather than in particular subject-matter area. Therefore, to assure the focus of this current study emphasizing differential equation learning situations, items were revised. Moreover, all items and differential equation tasks were written in Urdu for readability purpose. Therefore, for instrument pilot testing was an essential step.

In this work, instruments validation was done by the mathematics educators, psychology expertise and the mathematicians who taught in college and university. They were explicitly told to look into the content and face validities of the instruments. Furthermore, they also examined the appropriateness of the non-routine differential equation problems and whether or not they tested problem-solving ability as defined in this study.

In addition to these, all items were written in Urdu, hence the translated instruments (in Urdu) were distributed among several participants and researchers who are good in both Urdu and English languages to test their readability. They were asked to re-translate the instruments into English or vice versa, to ensure that translation was accurate. Back translation is the common technique used to translate instruments in a cross-nation research (Harkness, Pennell et al., 2004; Smith, 2003; Sousa & Rojjanasrirat, 2011; Su & Parham, 2002).

Deutscher (1973) claimed that this technique may cover the language problem. Word choices and sentence structures of items were replaced and simplified based on students' reflections. These instruments have been further piloted to a differential equation college class that constituted 250 Pakistani students. The main purpose of pilot study was to assess the readability of instruments (both developed non-routine DEs tasks and questionnaires) from student's viewpoint.

3.3.1 Pilot study

The self-developed differential equation tasks and questionnaires were specifically designed for the purpose of this study. Therefore, a trial was essential to check clarity of questions and statements, choice of words, missing items, completeness of response sets, and also to estimate the amount of time it would take to complete. Moreover, to establish the reliability, content validity and construct validity, pilot study was compulsory. In current study, pilot study was conducted by selecting 250 respondents. The respondents involved were current government and private college student's science students. According to their feedback, no incomprehensible, imprecise, or unclear items were existed. After that instruments were finalized.

3.3.2 The reliability of the instrument

Reliability refers to the consistence of a study's dealings/actions and the stability of responses to multiple coders of data sets (Creswell et al., 2007). In this aspect, the researcher made use of the estimates of internal consistency or reliability (Byrne, 2010; R.B Kline, 2005). Cronbach's alpha is commonly-used to test the extent to which multiple indicators for a latent variable belong together. Further, Pedhazur and Schmelkin (1991) suggested that the Cronbach's alpha value depends on the correlation between items, as the number of items involved in an instrument increases, the Cronbach's alpha increases as well. However, individual reliability coefficient (Cronbach's alpha) value for diverse scale should be above the threshold value of 0.7 (Gliem et al., 2003). Several researchers also recommended a cut-off values of Cronbach's alpha >0.6 as criteria of internal consistency (Chew et al., 2017; Cho et al., 2015). Findings were quite satisfactory (Chew et al., 2017; Cho et al., 2015). Therefore, for the present work, these cut-off values were employed. On the other hand, to improve the coefficient substantially, some of the items may be eliminated. Table 3.6 shows the internal consistency of the instrument of this study and is based on data collected from the pilot study. The reliability values of the all of the instruments were in permissible range.

3.3.2.1 Construct validity

Construct validity is the degree to which a test measures what it claims to be measuring. To find construct validity, usually exploratory factor analysis (EFA) is exploited. The use of exploratory factor analysis, explains the consistency of instrument and also verify that the questioned items are strongly deem to be able to measure what is to be measured (Said et al., 2011). Moreover, Researchers historically rely on exploratory factor analysis (EFA) to identify and distinguish between key psychological constructs (Marsh et al., 2014).

Pilot study provided encouraging results and no incomprehensible, imprecise, or unclear items were found. Therefore, instruments were finalized and efforts were made to collect actual data.

3.4 Pilot study data analysis and findings

For the current study, questionnaire was piloted among 250 pre-engineering and science students. Pilot study is the best way to purify the questionnaire items by testing the reliability and validity. Exploratory factor analysis was carried out to identify the construct validity of the questionnaire. The analysis steps are presented in the following sections.

3.4.1 Data coding and cleaning

Data coding was the primary step to prepare data for empirical researches and also facilitate the insertion of the collected data in statistical programs (e.g., SPSS). Researcher inserted all the responses in a systematic way following the items code that was predefined and entered into the SPSS program. Survey questionnaire comprises 69 items and each item was given a code as a representation for data analysis. SPSS was further used for initially data screening, factor analysis and preparation for model testing.

Missing data is the main issue in data screening that occurs when a respondent intentionally or unintentionally does not respond to one or more questions. Missing data create problems during data analysis and automatically may have dramatic effects on the study results (Elshaer, 2012). According to Hair, Ringle et al. (2013a), the questionnaire became inappropriate if the missing data exceeds 15 percent missing data in one questionnaire, hence they might be removed from the data base. SPSS provides three methods to tackle with the missing data: exclude cases list wise, exclude cases pair wise and replace with mean. Hair et al. (2013a) further recommended the option of mean value replacement when there is less than 5 percent of value missing per indicator. The percentage of missing data value was less than 2 percent with no apparent pattern. Therefore, the missing data was imputed using mean value replacement method.

Another issue regarding to data cleaning was, identification of outliers. Hair et al. (2013a) considered outliers as an extreme response to a particular question or to all question. These outliers can create undesired effect on the correlation coefficient (Pallant, 2010). Outliers were detected using the SPSS program by visually screening the histogram, normal Q-Q plot, or box plot for each construct. Next, normality was evaluated. Normality is one of the most crucial assumptions in multivariate analysis. Hair, Black et al. (2010) defined normality, as the degree to which the distribution of the sample data corresponds to a normal distribution. Normality can be represented by two measures: skewness and kurtosis. Skewness describes the balance of the distribution; if the shape is unbalanced it will be shifted to either left or the right side. When the data set values clustered to the left-hand side of the graph, then it is called positive skewness. While cluster of data set values on the right- hand side of the graph represent negative skewness (Pallant, 2007). Whereas, kurtosis refers to the peakedness or flatness of the distribution. Hair et al. (2010) recommended that if the empirical z-values lies between ±2.58 at (0.01 significant level); or ± 1.96, at (0.05), the distribution of the data is considered normal. Examination of possible outliers revealed that some of the cases were three standard deviations away from the mean of its distribution. Thus, those cases were deleted and 228 cases out of 250 were retained for the further analysis. Table 3.5 shows the values for skewness and kurtosis for the pilot study.

Participant's scores from the subscales of epistemological problem solving beliefs, goal orientations and motivated self-regulated questionnaire (MSLQ) were calculated. These calculations were further used to determine their self-reported level of the sub scales of math problem solving beliefs, usefulness, motivation and use of cognitive strategies (elaboration and critical thinking). Table 3.5 presents the mean scores and standard deviations for each subscale of beliefs, goal orientation and MSLQ.

Besides this, Traynor, Mactier et al. (2006) recommended testing the reliability of the data from a pilot study prior to actual data collection. Cronbach's alpha is usually used to verify these scales for internal consistency or reliability (Nunnally & Bernstein, 1994). Therefore, Cronbach's alpha was calculated for each scale.

Table 3.5: Normality of the survey questionnaire

Construct	Mean	SD	Z Skewness	Z Kurtosis
DP	21.2898	4.35427	-1.08	-0.17
ST	21.3369	3.87231	-0.25	-0.54
UN	19.8604	4.25432	-0.43	-0.35
WP	19.1836	4.51491	-0.06	-0.31
EF	19.4504	3.87728	0.74	-0.22
UF	23.3435	5.02360	-1.81	-0.12
MA	17.1781	5.55560	0.47	-0.78
PR	14.3438	4.12547	0.53	-0.73
AV	17.8107	5.10615	-0.21	-0.22
CR	19.4328	7.21327	0.34	-1.59
EL	25.0229	8.46425	-0.56	-1.42
DEPS	6.7368	2.74962	-0.74	-1.19

DEPS; Differential equation problem solving, UF; Usefulness, MA; Mastery goal, PR; performance goal, AV; Avoidance goal, CR; Critical thinking, EL; Elaboration

The reliability coefficient for each subscale might above the threshold value 0.7 (Gliem et al., 2003). Table 3.6 illustrates the Cronbach's alpha results for four proposed constructs, epistemological problem solving beliefs, usefulness, goal orientation and self-regulated learning strategies. Results showed that all these scales were considered reliable as exceeds approximately the least threshold value of 0.7.

The next step was to examine the factor analysis which is a statistical technique. This technique was employed to reduce the number of variables used to explain the relationship. Exploratory factor analysis and confirmatory factor analysis are two main types of factor analysis (Pallant, 2007; Tabachnick & Fidell, 2007).

Table 3.6: Reliability analysis for the evolutionary survey questionnaire constructs

Instrument	Scale	No of items	Cronbach's alpha value	Reported Cronbach's alpha value	Reference
Differential equation task	DPS	5	0.80		
Epistemological belief about mathematical problem solving	DP	6	0.83	0.75	(Kloosterman et al., 1992)
	ST	6			
	UN	6			
	WP	6			
	EF	6			
Usefulness belief	UF	6	0.79	0.86	(Kloosterman et al., 1992)
Goal orientations	MA	6	0.84	0.86	(Midgley et al., 1996)
	PR	6	0.69	0.86	
	AV	6	0.79	0.75	
Self-regulated learning strategy(SRL)	CR	5	0.85	0.75	(Pintrich, 1991)
	EL	6	0.86	0.80	

DEPS; Differential equation problem solving, UF; Usefulness, MA; Mastery goal, PR; performance goal, AV; Avoidance goal, CR; Critical thinking, EL; Elaboration

Exploratory factor analysis is used to explore the loadings of variables to try to achieve the best model (Byrne, 2010). On the other hand, confirmatory factor analysis (CFA) is a precise method that is used to test the dimensionality and validity of the measurements (Hair, Black et al., 2006). For current study, both exploratory factor analysis and confirmatory factor analysis were employed for pilot study. Detail of exploratory factor analysis and confirmatory factor analysis is provided below.

3.4.2 Exploratory factor analysis

Exploratory Factor Analysis (EFA) determined items that assimilate to a factor in a multiple factor structure. SPSS was used to analyze EFA, as the outcome extracted the number of factors and their associated items. Also, it was used to report the factor loadings of each item on the respective factors (Foster & Inglis, 2017; Wang & Hsieh, 2017). Generally, EFA is normally analyzed through two methods including principal component analysis (PCA) and common factor analysis (Mohr-Schroeder, Jackson et al., 2017). However, principal component analysis is the most prominent method (Halai, Woollams et al., 2017; Kruse, Williams et al., 2017). According to Ho (2013), principal component analysis is appropriate if the purpose is no more than to reduce data in order to obtain the minimum number of factors needed to represent the original set of data. Therefore, PCA was employed in the current study with aiming at empirically revealing and demonstrating the hypothesized, underlying structure of the preliminary model of the questionnaire.

Prior to PCA, a preliminary assessment of inter-item correlation was necessary. Usually, bivariate correlation matrix is used to visually inspect the inter-item correlation. According to the interpretation of Davis descriptors (negligible = 0.00 to 0.09; low = 0.10 to 0.29; moderate = 0.30 to 0.49; substantial = 0.50 to 0.69; very strong = 0.70 to 1.00), all correlation coefficients range from substantial to negligible. In this case, all items were fairly independent.

In addition to this, an important assumption of factor analysis is sample adequacy (Hair, 2010). Therefore, the results of the Kaiser-Meyer-Olkin (*KMO*) measure of sampling adequacy and Bartlette's test of sphericity were calculated before conducting factor analysis. Kaiser-Meyer-Olkin (*KMO*) measure of sampling adequacy is a ratio of the sum of the squared correlations to the sum of the squared correlations plus squared partial correlation (Ho, 2013). According to Tabachnick et al. (2007), data is factorable when the Bartlette's test of sphericity is significant (p-value< 0.05) and the *KMO* value must be at least 0.60. Further,Tabachnick et al. (2007) recommended the adaptation of exploratory factor analysis, testing different number of factors until a satisfactory solution is found. To assist in the decision regarding the sustainability of number of factors, researchers suggested several techniques such as, percentage of variance, Kaiser's criterion (eigen value), communalities, and a scree plot (Hair et al., 2006; Pallant, 2007;

Tabachnick et al., 2007). Therefore, all these were the next key concepts of factor analysis and are explained below.

The percentage of variance criterion is statistical technique that is used to achieve a specified cumulative percentage of total variance extracted by successive factors (Hair et al., 2006). Author further recommended that 60 percent or less of the total variance is a satisfactory cutoff point to accept an exploratory factor solution in social science. In contrast, the satisfactory cutoff point is 95 % in natural science, because their information is more precise.

Communalities, Kaiser's criterion (eigen value), and a scree plot are next key concept of factor analysis. Communality is normally represented by the sum of squared loadings for a variable. It represents the correlation between an original variable and all other variables in the analysis (Hair et al., 2006). Hair et al. (2006) further suggested that communality value must lies between 0 and 1, where 0 indicates that the common variance factor explain none of the variance, and 1 represents the variance explained by the common factor. However, several researchers recommended 0.5 and >0.25 as cut-off values for factor loading and communality, respectively (Chew et al., 2017; Hair, Black et al., 1998; Kline, 2015). Beside these, eigen value rule is most reliable technique of exploratory factor analysis and is employed to retain only factors with an eigen value 1.0 or more (Pallant, 2007). Author further defined eigen value of a factor as the amount of the total variance explained by that factor. Therefore, Eigen value represents the strength of factor.

Another approach is Catell's scree test. This approach involved in plotting every one of the eigen values of the factors and inspecting the plot to locate a point at which the shape of the curve changes path and becomes horizontal (Hair et al., 2006; Pallant, 2007; Tabachnick et al., 2007).

Therefore, next phase of present study was to categorize the structural dimensions of the items. For current study, exploratory factor analysis (EFA) comprised 64 items distributed into 7 constructs. Details of the results of each construct and its factors are provided in the following sections.

3.4.2.1 Exploratory factor analysis for epistemological math problem solving beliefs

Exploratory factor analysis (EFA) was run for the first construct, epistemological problem solving belief, using principal component analysis (PCA). Prior to PCA, bivariate correlation matrix was visually inspected as a preliminary assessment of inter-item correlation. Accordingly, all correlation coefficients ranged from 0.07 – 0.40, low to substantial (Davis, 1971). This revealed that all scales were fairly independent to be used as independent variables.

Statistical assessments of the correlation matrix for factor analysis was performed using both *KMO* and Bartlett‘s Test of Sphericity. The calculated *KMO* value for the epistemological problem solving beliefs was 0.82, showing excellent sampling adequacy indicating factor analysis was appropriate for the scale. The recommended value for *KMO* is 0.60 or higher to proceed with factor analysis (Tabachnick et al., 2007). Similarly the Bartlett‘s Test of Sphericity was also significant [$\chi2 = 1,741.4$; $p < 0.001$], which rejected the null hypothesis that the correlation matrix was an identity matrix. Hence, inter-item correlation matrix was suitable for factor analysis. A varimax rotation was then undertaken to assists in the interpretation of the factors. Literature reveals that an overall factor loading of greater than 0.50 is significant enough to determine the meaningfulness of the instrument (Hair, 2010; Hair et al., 1998). Several other researchers also recommended 0.5 and >0.25 as cut-off values for factor loading and communality, respectively (Chew et al., 2017; Hair, Black, et al., 1998; Kline, 2015). Therefore, in the present case, all the items with factor loading of 0.50 were considered.

In the present case, two factors were explaining 30 percent of the variance. However, these two factors were highly correlated. The Eigen value 6.44/ 2.41 is equal to approximately 2, and then there is index of uni-dimensionality. From the scree plot (Figure 3.2) and the Kaiser-Guttman rule, factor analysis of results on the 30 items indicated that two factors were interpretable (Epler, 2011). However, there was a doubt of uni-dimensionality.

The presence of a single factor had accounted for a substantial amount of the total variance led to hypothesis that the data may be characterized by a general factor (Epler, 2011; Wheeler, 2007). Uni-dimensionality was further evaluated through PCA and all the procedure done above. The data were reanalyzed by applying PCA to five scale combined. An iterative process was used to refine each theoretical set of scales to a uni-dimensional scale. Corresponding items of each belief were added and the calculation was re-run for five beliefs. The *KMO* value of 0.83 and Bartlett's Test [χ^2= 392.8; $p<0.001$] was significant, which revealed that the conducted factor analysis was appropriate. This has resulted into only one factor explaining 60 percent variance of the total variance. Again a varimax rotation with factor loading of 0.50 has been considered. Both the Kaiser Criterion and scree plot supported a single factor (λ=2.9) that accounted for 60 percent of the total variance.

The hierarchical factor was interpreted as general math problem solving belief. The detail has been provided in the Table 3.7. Scree plot was also used to estimate the number of factors to extract. The scree plot appeared to support only one factor solution (Figure 3.3).

Table 3.7: Factor loadings and other values of epistemological math problem solving beliefs

Factor	Dimensions	Factor loading	communalities	Eigen values	% of variance
EMB	DP	0.78	0.60	2.99	60%
	ST	0.77	0.60		
	UN	0.77	0.59		
	WP	0.83	0.69		
	EF	0.69	0.48		

EMB: Epistemological math problem solving beliefs. DP: Duration of problem, ST: Steps, UN: Understandings, WP: Word problems, EF: Effort

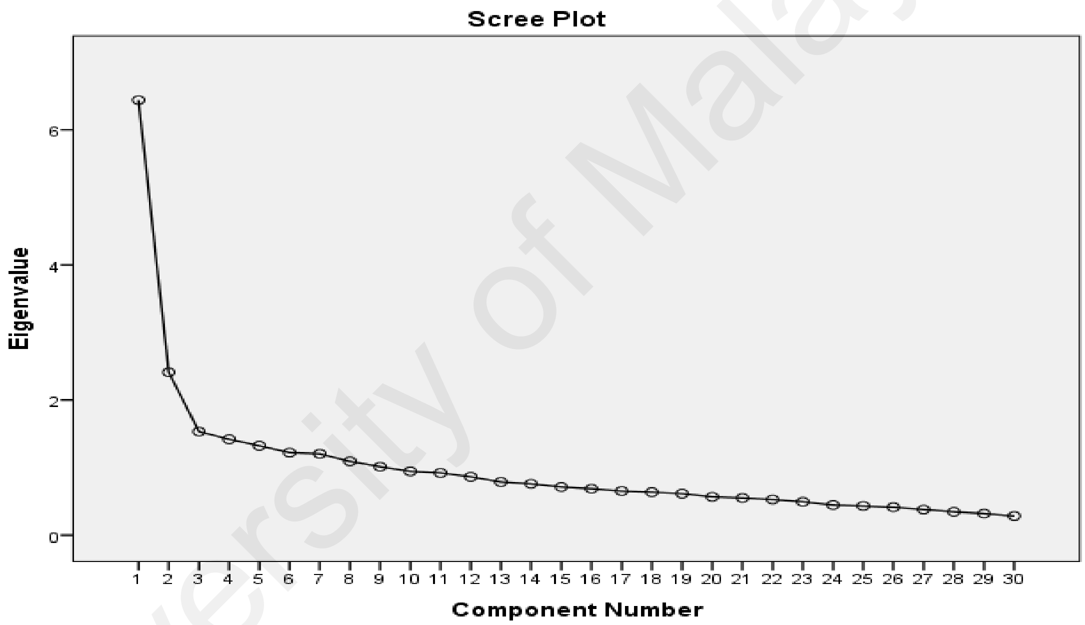

Figure 3.2: Scree plot of for 30 beliefs items.

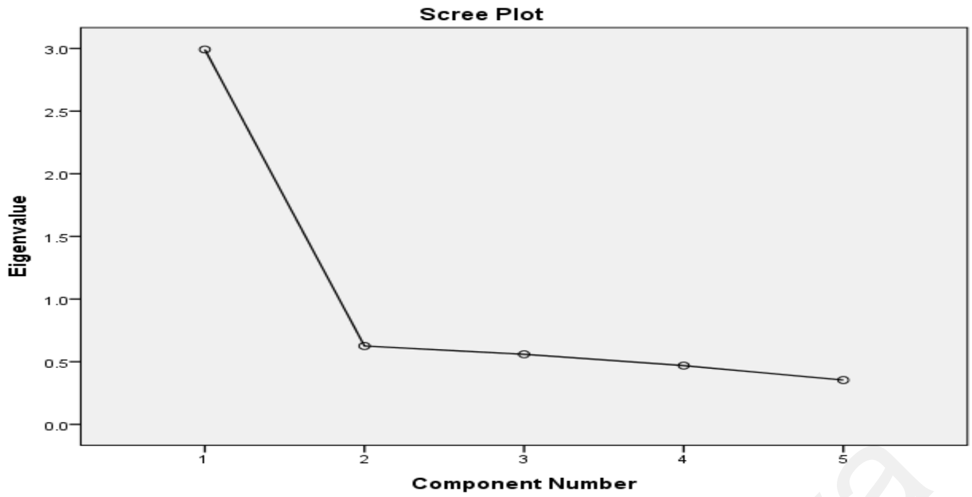

Figure 3.3: Scree plot of for five beliefs items.

3.4.2.2 Exploratory factor analysis for Usefulness (pilot study)

For usefulness (second construct), the *KMO* value for the usefulness belief was 0.76, indicating highly acceptable for appropriate factor analysis. In addition to it, Bartlett‘s Test of Sphericity was also significant [$\chi2 = 433.9$; $p < 0.001$], which rejected the null hypothesis that the correlation matrix was an identity matrix.

Table 3.8: Factor loadings, communalities, eigen value, percent variances explained by Usefulness

Factor	**Item code**	**Loading**	**Communalities**	**Eigen values**	**% of variance**
UF :Usefulness	B51	0.59	0.34	3.03	51 %
	B52	0.65	0.43		
	B53	0.67	0.45		
	B54	0.81	0.67		
	B55	0.80	0.64		
	B56	0.69	0.48		

Figure 3.4: Scree plot of for usefulness belief

Initial results revealed only one factor with Eigen values greater than 1.00. This factor explained 51 percent of the total variance. Table 3.8 illustrates the detail of factor loading, communalities, Eigen values and percentage of variance explained by usefulness. The scree plot was also investigated to select the correct number of factors to be extracted. From the scree plot (Figure 3.4) and the Kaiser-Guttman rule, factor analysis only indicated one factor.

3.4.2.3 Exploratory factor analysis for goal orientations (pilot study)

The *KMO* value for goal orientation was acceptable at 0.83, indicating factor analysis was appropriate for the scale. In addition to it, Bartlett‘s Test of Sphericity was significant [$\chi 2$ = 1,326.3; p <0.001]. Therefore, goal orientation was considered acceptable for factor analytic techniques.

Further factor analysis revealed three factors with Eigen values greater than 1.00. These three-factor structure explained 52 percent of the total variance, with factor 1 contributed 29 percent, factor 2 contributed 13 percent and factor 3 contributed 8 percent.

Since, researchers have recommended 0.5 and >0.25 as cut-off values for factor loading and communality, respectively (Chew et al., 2017; Hair, Black, et al., 1998; Kline, 2015). Therefore, in the present case, all the items with factor loading of 0.50 have been considered. Table 3.9 illustrates detail of rotated component matrix of three components along with their factor loadings and communalities.

Table 3.9: Factor loadings, communalities of Goal orientation

Construct	Item code	Component			Communalities
		1	2	3	
MA	MA1	0.58			0.62
	MA2	0.63			0.64
	MA3	0.69			0.56
	MA4	0.74			0.59
	MA5	0.80			0.67
	MA6	0.67			0.49
PR	PER1			0.59	0.39
	PER2			0.76	0.61
	PER3			0.67	0.50
	PER4			0.41	0.27
	PER5			0.57	0.40
AV	AV1		0.63		0.55
	AV2		0.66		0.58
	AV3		0.72		0.54
	AV4		0.75		0.57
	AV5		0.68		0.47
	AV6		0.60		0.42

MA: mastery goal, PR: performance goal, AV: avoidance goal

In addition to it, the scree plot was also investigated to select the correct number of factors to be extracted. From the scree plot (Figure 3.5) and the Kaiser-Guttman rule, factor analysis of results on the 17 items indicated that three factors were interpretable.

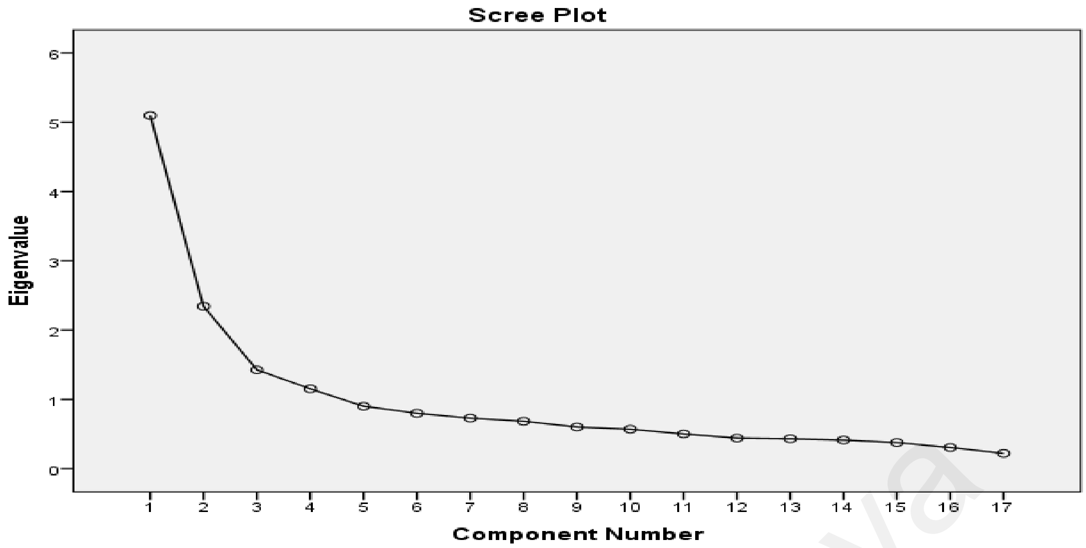

Figure 3.5: Scree plot of goal orientations

3.4.2.4 Exploratory factor analysis for self-regulated learning strategies (pilot study)

For elaboration, the results of the *KMO* (0.8) and Bartlett‘s Test of Sphericity were significant [$\chi2 = 465.9$; $p < 0.001$]. Furthermore, results revealed high communalities ranging from 0.77 to 0.81, and a single factor with Eigen values greater than 1.00. In the literature, researchers have recommended 0.5 and >0.25 as cut-off values for factor loading and communality, respectively (Chew et al., 2017; Hair, Black, et al., 1998; Kline, 2015). Detail of communalities is shown in Table 3.10. In addition, the one factor structure explained 62 percent of the total variance (Figure 3.6).

Table 3.10: Factor loadings, communalities, eigen value, percent variances explained by Self-regulated learning strategies

Factor	Item code	Factor loadings	Communalities	Eigen values	% Variance
CR :Critical thinking	CR1	0.79	0.62	3.1	62%
	CR2	0.81	0.65		
	CR3	0.78	0.61		
	CR4	0.79	0.63		
	CR5	0.77	0.60		

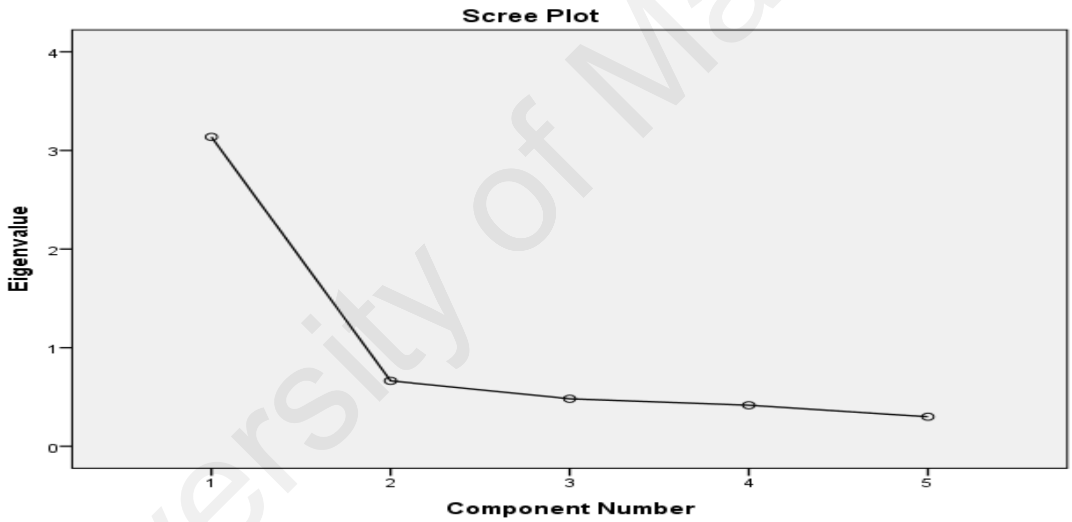

Figure 3.6: Scree plot of critical thinking

Similarly for elaboration, the results of the *KMO* (0.8) and Bartlett's Test of Sphericity were significant [$\chi 2 = 577.5$; $p < 0.001$]. Furthermore, results revealed high communalities ranging from 0.66 to 0.83, and a single factor. In the literature, researchers have recommended 0.5 and >0.25 as cut-off values for factor loading and communality, respectively (Chew et al., 2017; Hair, Black, et al., 1998; Kline, 2015). Details of communalities and factor loading are shown in Table 3.11. Predicted one factor explained 59 percent of the total variance (Figure 3.7).

Table 3.11: Factor loadings, communalities, eigen value, percent variances explained by Self-regulated learning strategies

Factor	Item code	Factor loadings	Communalities	Eigen values	% Variance
EL	EL1	0.66	0.43	3.5	59%
	EL2	0.75	0.57		
	EL3	0.83	0.70		
	EL4	0.79	0.63		
	EL5	0.78	0.61		
	EL6	0.76	0.59		

EL: Elaboration

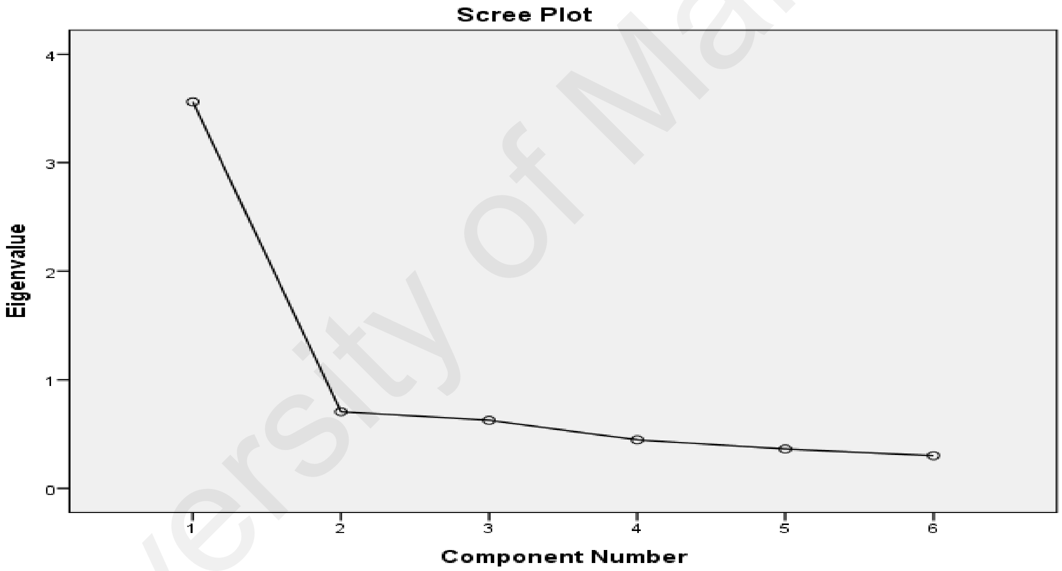

Figure 3.7: Scree plot of elaboration

3.4.3 Confirmatory factor analysis

Confirmatory factor analysis (CFA) is a multivariate statistical technique that is used to confirm or reject the measurement theory. Although, EFA is an important antecedent of CFA/SEM, but seen as less useful (Cudeck & MacCallum, 2012). Because of the misconception that it is purely an "exploratory" method that should be used only when the researchers have no a priori assumptions regarding factor structure. However, advent of CFA/SEM made it possible to conduct systematic tests of measurement invariance (e.g., (Joreskog et al., 1979; Meredith & Horn, 2001)). This may have led to many additional advances, including the analysis of relationships involving latent constructs estimated after correction for measurement error (Choy, Goh et al., 2016; Honan, McDonald et al., 2016; Masuwai, Tajudin et al., 2017; Van den Berg, Harskamp et al., 2016).

Confirmatory factor analysis (CFA) is also imperative to validate a multi-factorial model (Byrne, 2010). Therefore, Said et al. (2011) recommended for CFA in structure equation modeling (SEM), as it gives better results in testing the validity and reliability of an instrument. Therefore, in the present case, a first order CFA was applied to validate factor structure for an appropriate structural model. Moreover, statistical values of the Chi-square test of goodness of fit (χ2), the root mean square error of approximation (RMSEA) and comparative fit index (CFI) were also calculated. The recommended cutoff value for the CFI and RMSEA is 0.95 and 0.6, respectively (Hu & Bentler, 1999). Detail of recommended fit index along with its criteria is presented in the table below.

Table 3.12: Factor Recommended cutoff values for SEM fit indices

Fit index	Cut- off values from literature	References
Chi square/df	≤ 5.0	(Byrne, 2010; Hair et al., 2006)
RMSEA	≤ 0.6	
CFI	≥ 0.90	

3.4.3.1 Confirmatory factor analysis for epistemological problem solving belief

Epistemological problem solving belief is measured through thirty items or observed variables. Through the first order, 30 items of epistemological problem solving beliefs were clustered with one dimension. The first round of the first order confirmatory was performed and a single factor model was not a good fit ($\chi2$ = 762, df = 390, $\chi2$/ df = 1.954, CFI= 0.78, RMSEA= 0.06). Therefore, second order factor analysis was performed. Next, 30 items were clustered in second order of CFA by the dimensions of (difficult problems, steps, understanding, word problems, effort) with a proposed name (DP, ST, UN, WP, EF). Results showed that a factor model with good fit ($\chi2$ = 8.603, df = 5, $\chi2$/ df = 1.72, CFI= 0.99, RMSEA= 0.05) was found to be reasonable. Moreover, $\chi2$ difference test is used to evaluate the fit of one model nested within another. A statistically significant decrease in indicates better model fit (Manning, 2015). Therefore, second order epistemological problem solving beliefs model shown better results.

3.4.3.2 Confirmatory factor analysis for goal orientations

Construct of goal orientation is measured through seventeen items. The first order confirmatory factor analysis was performed. Results showed that ($\chi2$ = 252, df = 114, $\chi2$/ df = 2.2, CFI= 0.90, RMSEA= 0.07). Although root mean square of error approximation was high but according to Brown (1993), RMSEA< 0.08 is acceptable.

3.4.3.3 Confirmatory factor analysis for self-regulated learning strategies

Self-regulated learning strategies is measured through eleven items or observed variables. Findings showed that ($\chi2$ = 163, df = 43, $\chi2$/ df = 3.7, CFI= 0.90, RMSEA= 0.07).

3.5 Analysis of educators' questionnaire

For the current research, self-developed non-routine tasks instrument, and adapted scales of beliefs, goal orientations and SRL were translated into Urdu (National Language of Pakistan). Efforts were been made to make the instrument more understandable and comprehensible using the guidelines from the literature (Harkness et al., 2004; Sousa et al., 2011; Su et al., 2002).

In the developed countries, teaches and educators are well familiar with the role of incorporation of students' belief systems, goal orientations, SRL and their strong correlation for mathematical performance / achievement (Beghetto et al., 2012; Schommer- Aikins et al., 2005; Schommer-Aikins et al., 2013a). This trend is still exists in their present studies whether learning is perceived and esteemed as an end in itself or as a mean to other reason (Lamm et al., 2017; Madjar et al., 2017; Meece et al., 1988; Wahl, 2017). Similar trend may be seen about non-routine tasks (Fernández-Macías & Hurley, 2016; Lee et al., 2016; Robinson, 2016; Szabo & Andrews, 2017; Wijaya, van den Heuvel-Panhuizen et al., 2015). Even in the primary level, several studies were conducted to involve non-routine tasks (Doorman, Drijvers et al., 2007; Marchis, 2012). However, in these directions, a quite limited data was available including belief systems, goal orientations, SRL and their strong correlation for mathematical performance / achievement. Therefore, validity of adapted instruments was also a challenging task (Smith, 2003).

To overcome the issue of adapted instrument's validity for the developing countries, an additional questionnaire (in addition to research instrument) was designed for the field experts (educators/ teachers), in which, consents of the experts were assessed. This questionnaire had two main parts. First part was about the research instruments including clarity of representations and suitability. In the second part of questionnaire, consents of the experts with respect to current research, its suitability for the selected country/province and etc. were assessed. Responses of the experts have been summarized in the graphs and are provided below.

About clarity of phrasings and wordings of research instruments, 20 percent of the total experts had opinion that the level of clarity of phrasings and wordings was fair, while 10 percent of the total experts had opinion that the level of clarity of phrasings and wordings was good. However, 50 percent of the total experts had opinion that the level of clarity of phrasings and wordings was very good, while 20 percent of the total experts had opinion that the level of clarity of phrasings and wordings was excellent. These responses were quite satisfactory to continue research using the developed instruments (Blaich, Wise et al., 2016; Harkness et al., 2004; Sousa et al., 2011; Su et al., 2002).

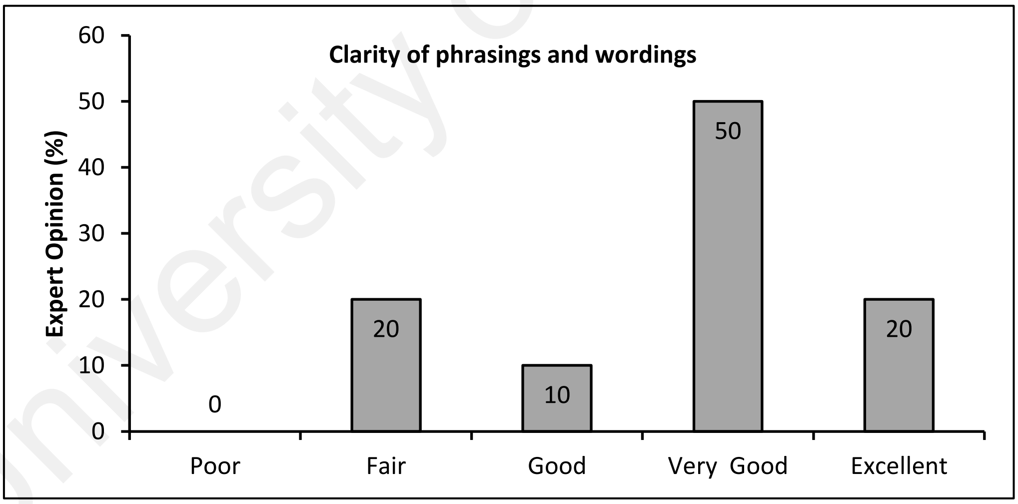

Figure 3.8: Experts feedback Part A, clarity of phrasings and wordings

About research instruments' relevancy to the course and level of the students, 10 percent of the total experts had opinion that the relevancy to the course and level of the students was good. However, 70 percent of the total experts had opinion that the relevancy to the course and level of the students was very good, while 20 percent of the total experts had opinion that the research instruments' relevancy was excellent.

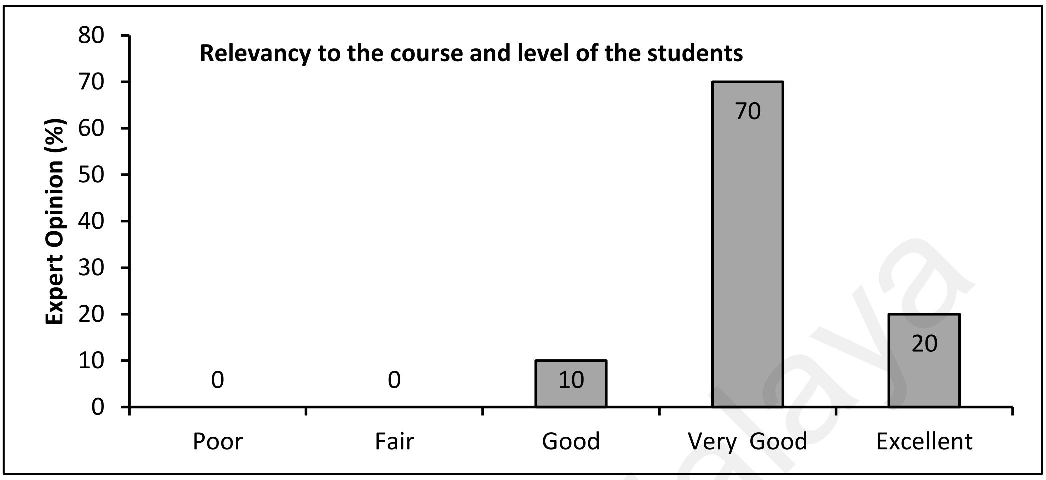

Figure 3.9: Experts feedback Part A, relevancy to the course

When the experts were asked about nature of self-developed non-routine tasks (used in research instruments) as challenging because they cannot easily solvable by using a previously taught simple algorithms and procedures (Fernández-Macías et al., 2016; Lee et al., 2016; Robinson, 2016; Szabo et al., 2017; Wijaya et al., 2015). It was seen that, 50 percent of the total experts had opinion that the self-developed non-routine tasks were good enough, while 40 percent and 10 percent of the total experts considered these as very good and excellent, respectively.

Similarly, when the experts were asked about self-developed non-routine tasks and adapted questionnaires. It was seen that, 50 percent of the total experts had opinion that the self-developed non-routine tasks were good enough, while 40 percent and 10 percent of the total experts considered these as very good and excellent, respectively.

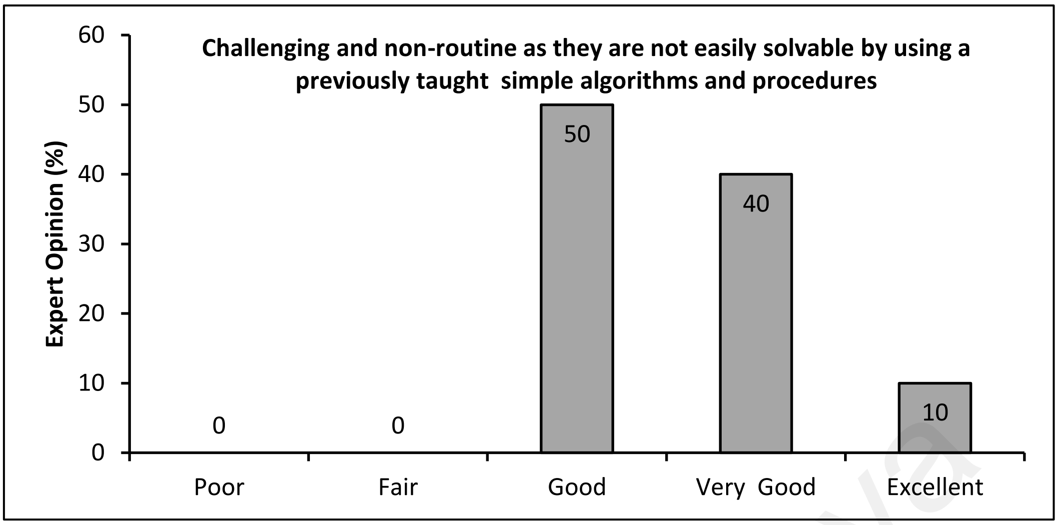

Figure 3.10: Experts feedback Part A, challenging and non-routine

About research instruments' ability to promote active involvement of students, 60 percent of the total experts had opinion that the instrument ability to endorse student 'energetic participation was good. However, 30 percent of the total experts were agreed that the instrument ability to promote active involvement of students was very good, Whereas, 10 percent of the total experts termed it as excellent.

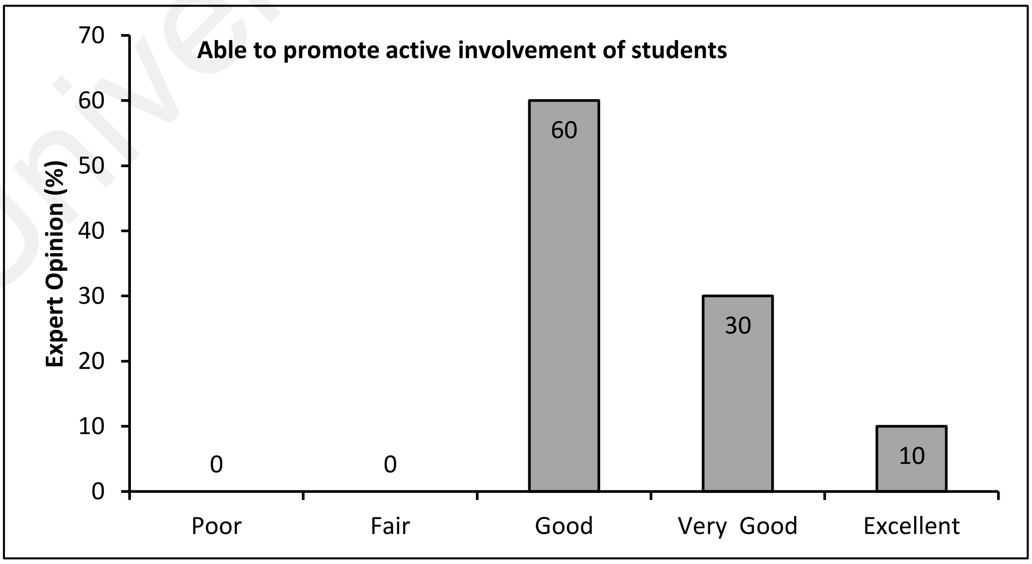

Figure 3.11: Experts feedback Part A, able to promote active involvement

When the experts were asked about multiple approaches and solution of self-developed non-routine tasks (used in research instruments). It was observed that, only 20 percent of the total experts had opinion that the self-developed non-routine tasks were fair. whereas, feedback from 30 percent experts termed it as good. Similarly, 20 percent and 30 percent of the total experts considered these as very good and excellent, respectively. Responses were quite satisfactory to conduct research using these self-developed non-routine tasks for the differential equations problem solving ability judgment along with other chosen factors (Blanchard, 1994; Fernández-Macías et al., 2016; Lin et al., 2017a; Robinson, 2016; Szabo et al., 2017).

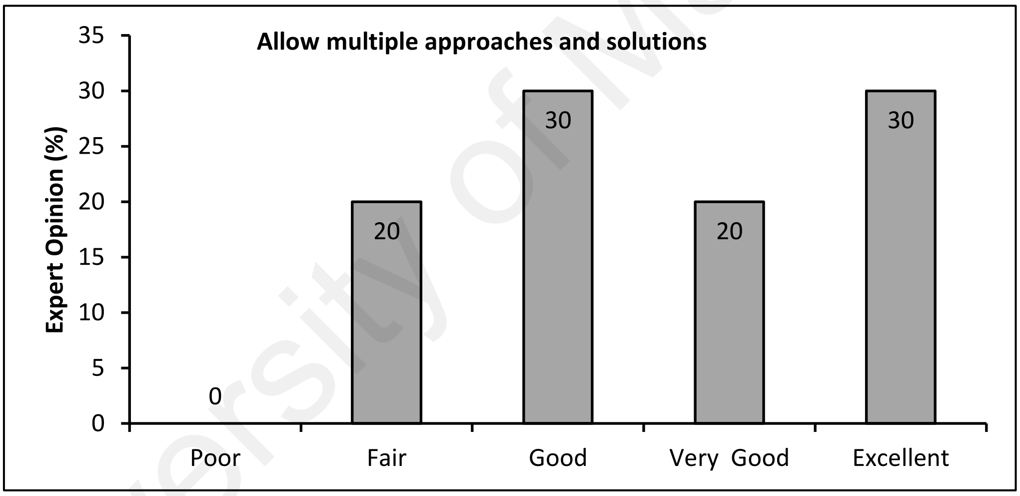

Figure 3.12: Experts feedback Part A, allow multiple approaches and solutions

Educators opinion about the application of differential equation problem solving in real world problems was also considered. It was observed that self-developed instrument had ability to make a bridge between mathematical concepts and real-world problems (Blum & Niss, 1991; He, 2000; Lin et al., 2017a).

Overall, 50 percent and 30 percent experts considered these items as very good and excellent, respectively. Which was a quite satisfactory level and validity indices to explore the combined effect of four parameters (epistemological beliefs, usefulness, goal orientations, and SRL) for the differential equations problem solving, particularly non-routine problems. (Su et al., 2002; Zan, Brown et al., 2006).

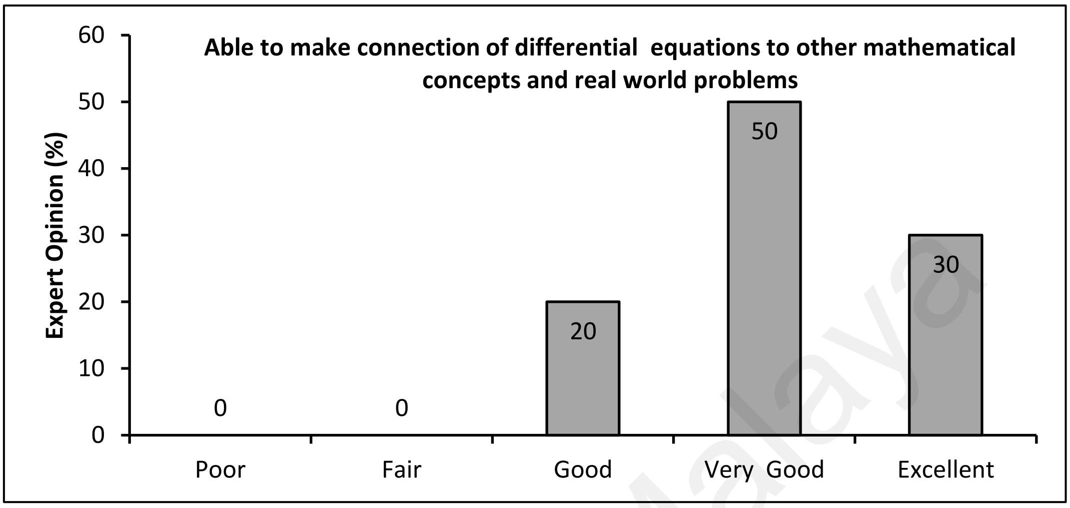

Figure 3.13: Experts feedback Part A, able to make connection of differential equations to other mathematical concepts and real-world problems

Likewise, when the experts were asked about overall clarity of self-developed non-routine tasks as well as adapted questionnaires. It was noticed that, 40 percent and 60 percent of the total experts considered these as good and very good, respectively. The self-developed non-routine tasks as well as adapted questionnaires had satisfactory clarity and validity indices to explore the combined effect of four parameters (epistemological beliefs, usefulness, goal orientations, and SRL) for the differential equations problem solving, particularly non-routine problems. (Leder, Pehkonen et al., 2006; Op't Eynde & De Corte, 2003; Su et al., 2002).

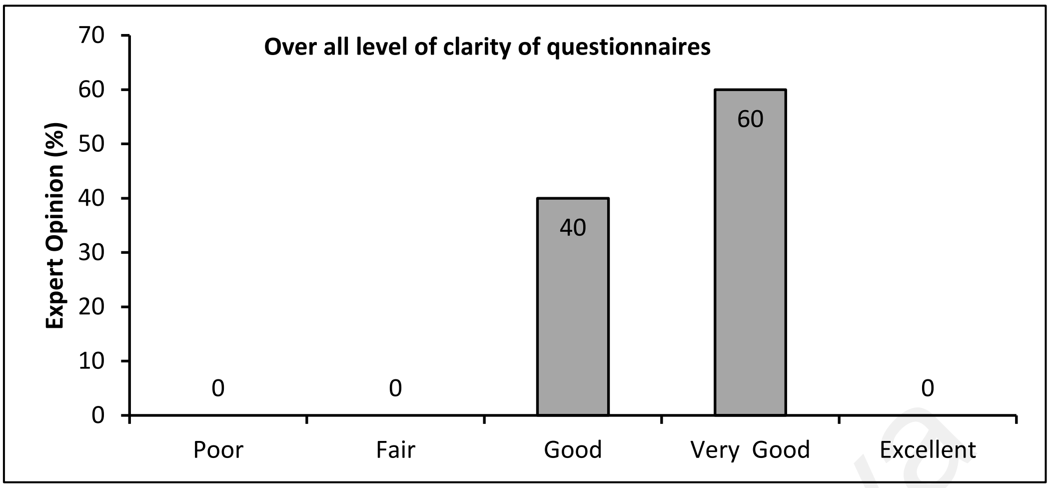

Figure 3.14: Experts feedback Part A, overall level of clarity of questionnaires

Experts' opinion about the order or organization (logical as well as sequential) was also requested. Result showed that 10 percent considered this order fair, where as 60 percent and 30 percent considered the order of instrument very was good and excellent, respectively. Overall, expert's opinion regarding to organization was found satisfactory for cross- cultural research (Harkness et al., 2004; Sousa et al., 2011; Su et al., 2002; Weinberg et al., 2014).

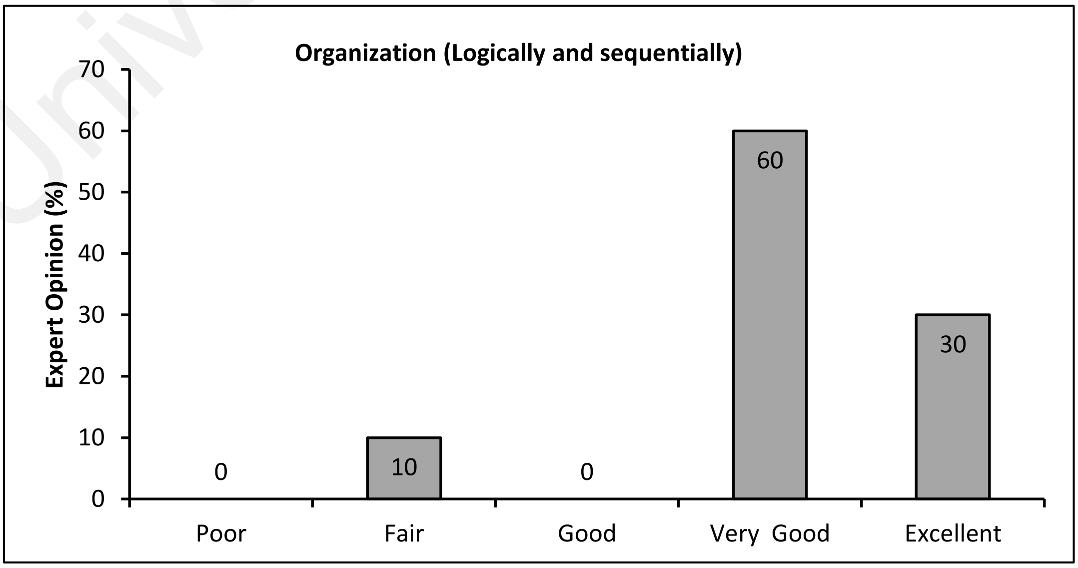

Figure 3.15: Experts feedback Part A, organization (logically and sequentially)

Likewise, tasks instrument, beliefs, goal orientations and SRL instruments were also translated into Urdu (National Language of Pakistan). Efforts were been made to make the instrument more understandable and comprehensible using the guidelines from the literature (Harkness et al., 2004; Sousa et al., 2011; Su et al., 2002). It was observed that only 10 percent perceived it as fair, whereas, 30, 40, and 20 percent documented it as good, very good and excellent, respectively. Overall, expert's opinion regarding to Urdu translation was found satisfactory and met the translation standards and guidelines for cross- cultural research (Harkness et al., 2004; Sousa et al., 2011; Su et al., 2002; Weinberg et al., 2014).

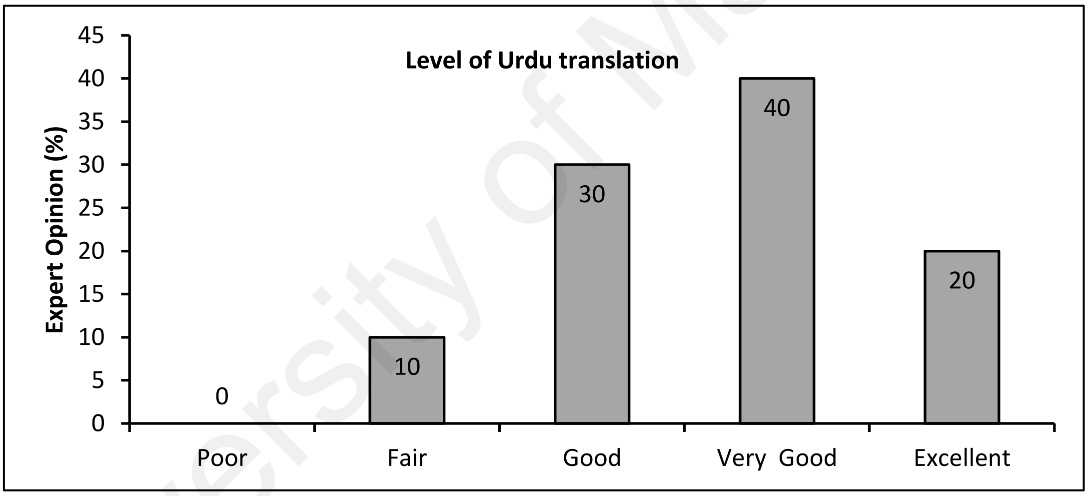

Figure 3.16: Experts feedback Part A, level of Urdu translation

3.5.1.1 Experts' feedback Part B

In the second part of questionnaire, consents of the experts with respect to current research, its suitability for the selected country/province and etc. were assessed. Responses of the experts have been summarized in the graphs and are provided below.

Experts opinion about "teaching and learning of differential equation as a difficult part of the mathematics at inter-college level as compared to other parts" showed that

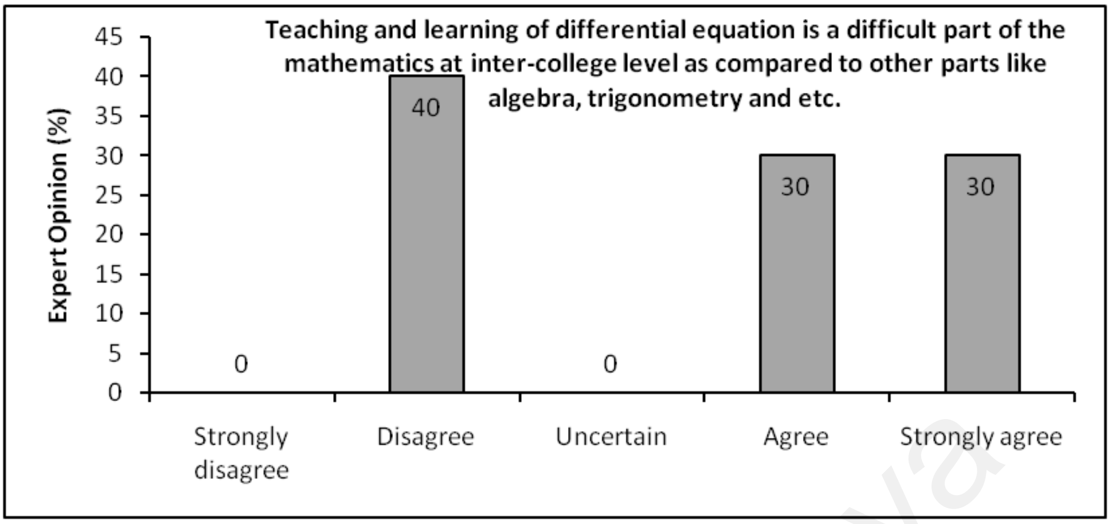

Figure 3.17: Experts feedback Part B, Teaching and learning is a difficult part of the mathematics

30 percent of the total experts were agreed as that the teaching and learning of differential equation is a difficult part of the mathematics. Moreover, 30 percent of the total experts were strongly agreed with it. However, 40 percent of the total experts disagreed that the teaching and learning of differential equation is a difficult part of the mathematics as compared to algebra, trigonometry.

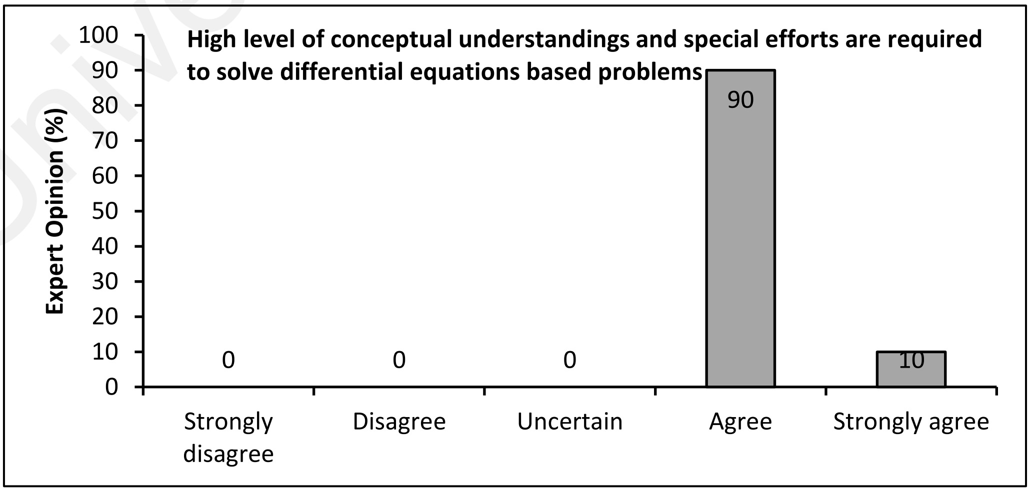

Figure 3.18: Experts feedback Part B, high level of conceptual understandings and special efforts are required to solve differential equations based problems

Conceptual understanding also effect the problem solving ability therefore, experts were also asked that whether differential equation problem solving need high level of conceptual understanding and special efforts. 90 percent were agreed, whereas 10 percent were strongly agreed that high level of conceptual understanding and special efforts is essential for differential equation problem solving.

The application or correlation of non-routine differential equation in real world problem was important (Szabo et al., 2017; Wijaya et al., 2015). In the response, 80 percent of experts were agreed with. Whereas, 10 percent expert were not sure about the correlation of non-routine differential equation in real world problem.

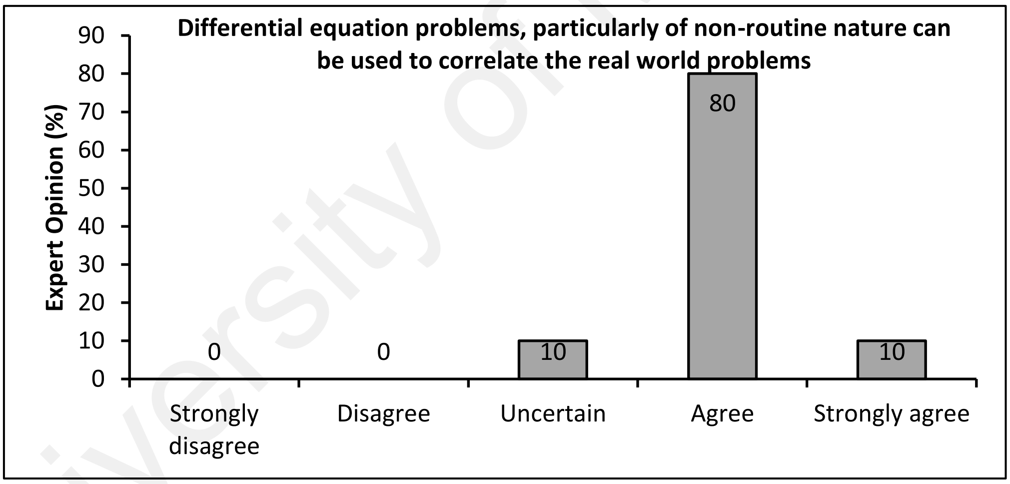

Figure 3.19: Experts feedback Part B, differential equation problems, particularly of non-routine nature can be used to correlate the realworld problems

Although non-routine differential equation problems are considered one of the most important area of calculus. However, these non-routine differential equation problems are considered as difficult because, students' special attention, efforts and learning strategies are required to solve problems containing differential equations, particularly non-routine problems. The reason is that these problems are typically concerned with unanticipated, unusual, and strange solutions (Polya, 1962; Rehman et al., 2012). Alas, these non-routine

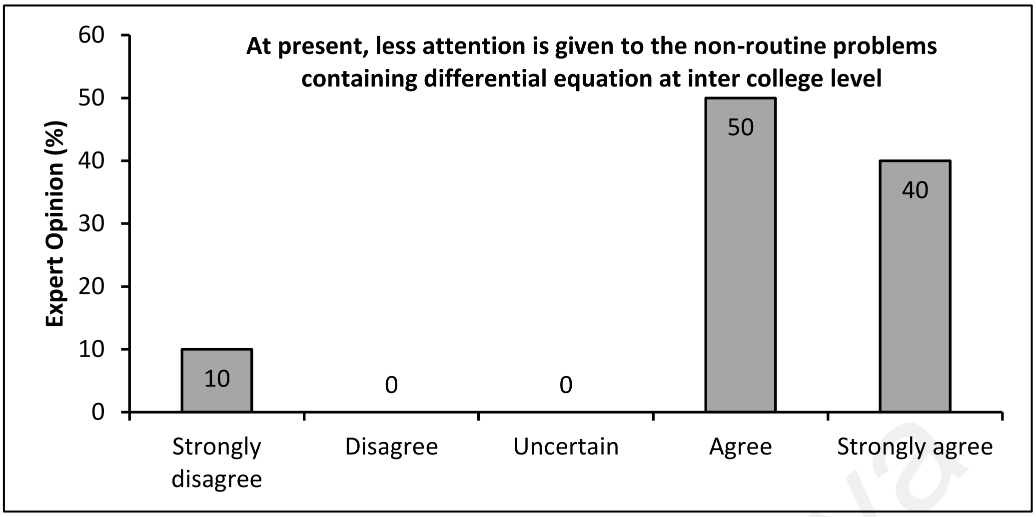

Figure 3.20: Experts feedback Part B, at present, less attention is given to the non-routine problems containing differential equation at inter college level

differential equation problems are avoided at inter level. Keeping in mind this perspective, expert's opinion regarding to the consideration of non-routine problems was asked. It was observed that 50 percent and 40 percent experts were agreed and strongly agreed with it that non-routine differential equation problems are less focused at inter level. However, only 10 percent experts were strongly disagreed with this statement.

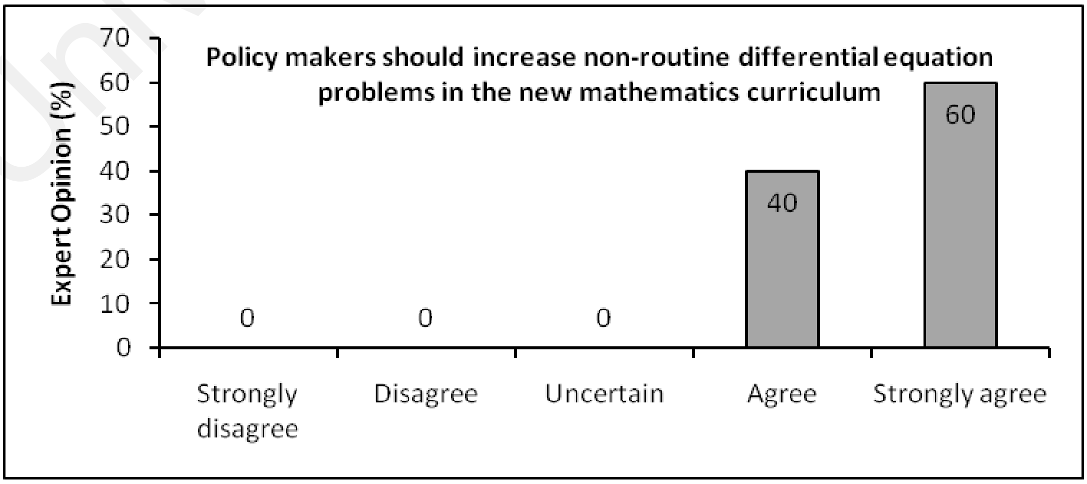

Figure 3.21: Experts feedback Part B, policy makers should increase non-routine differential equation problems in mathematics curriculum

Experts were further asked whether policy makers should add non-routine differential equation problems in new mathematics curriculum. 40 percent and 60 percent experts were in the favor of this statement.

Only addition of non-routine differential equation problem in mathematics curriculum is not enough. But teacher should be properly equipped and trained, so that they may educate both routine as well as non-routine differential equation problem. For this purpose, expert's opinion and concerned were important. Results showed that 60 percent were agreed with it. Moreover, 40 percent of the experts were strongly agreed that teacher should be properly equipped and trained, so that they may educate both routine as well as non-routine differential equation problem.

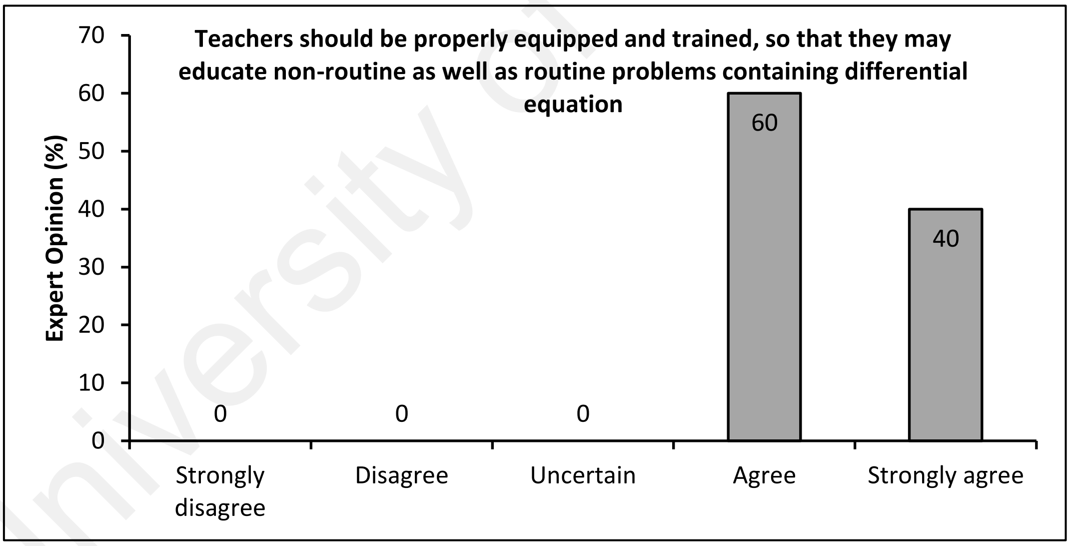

Figure 3.22: Experts feedback Part B, teachers should be properly equipped and trained, so that they may educate non-routine as well as routine problems containing differential equation

In addition, experts were also asked about whether student psyche can boost student's differential equation problems. 40 percent and 50 percent were agreed and strongly agreed with the role of student psyche in enhancing student's differential equation problems ability, respectively. However, 10 percent were not sure about it.

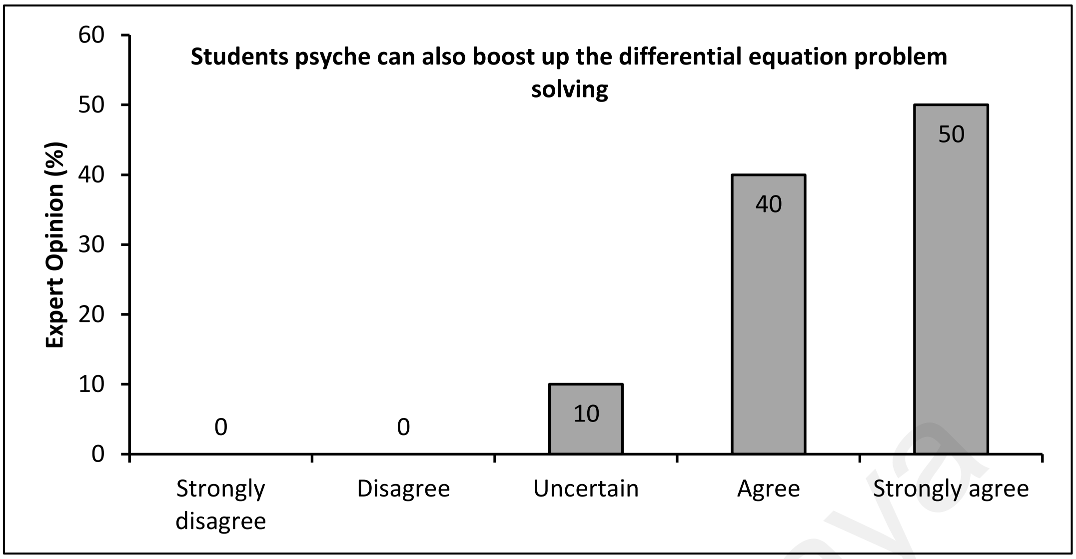

Figure 3.23: Experts feedback Part B, students psyche can also boost up the differential equation problem solving

In the developed countries, educators are well familiar with the role of achievement goals, their classifications and other related dependence, such as whether learning is perceived and esteemed as an end in itself or as a mean to other reason (Lamm et al., 2017; Madjar et al., 2017; Meece et al., 1988; Wahl, 2017). The focus of mastery goal is to develop one's aptitude employing mastery over the task. Similarly, performance-approach shows one's inclination towards involvement in the task for the sake of performing well to show other, while performance avoidance goals cause to avoid task, so as not to show incompetency to classmates and teachers. Elliot et al. (1999) conceptually and empirically analyzed, and reported that approach goal is the outperform to illustrate competence over other fellows, whereas the avoidance is to hide incompetence. Therefore, this approach revealed that students' differences in choosing different goals are correlated to their achievements, which show links to cognitive, motivational, emotional and behavior outputs.

Both mastery and performance goals are the positive motivational factors those swift students to spend their consistent efforts. Overall, achievement motivation inspires and actively engages students in their tasks. However, in case of avoidance goal, opposite results to achievement motivation were observed in most of the studies. An avoidance goal represents an impassive and negative motivational attitude that may impose destructive effects on learning. For this reason, an avoidance goal shows pessimistic aptitude that may result into destructive effects during learning (Wolters, 2004).

Master goal oriented students are much more likely to attempt harder tasks and also ensure more effort to achieve a higher level of knowledge (Hall, 2015). These students are highly motivated to improve their ability and knowledge by reviewing learning material as an opportunity to improve (Hall, 2015; Kayis et al., 2015). Whereas performance oriented choose an easier task so that they feel comfortable and may found more chances of success (Hall, 2015). Therefore, they tend to be competitive in achieving more success as compared to their class mates or peer, as a result these students prefer superficial learning strategies while studying (Hall, 2015; Kayis et al., 2015). On the other hand, avoidance goal oriented feel fear of being considered dull as compared to other class mate due to their lower performance at a certain task. Kayis et al. (2015) highlighted some common characteristics of goal orientation as, avoiding complicated tasks, mismanagement, and leaving tasks incomplete. Overall, it can be concluded that approach goal oriented students are motivated to achieve a higher level of academic success than avoidance goal oriented (Kayis et al., 2015).

Later on, motivational theorists (Elliot et al., 2001; Pintrich, 2000b) proposed a 2 x 2 achievement goal framework that fully integrates the mastery-performance and approach-avoidance distinctions. Crossing these two dimensions yields four types of achievement goal; mastery-approach, mastery-avoidance, performance-approach and performance-avoidance. The focus of mastery-approach and mastery-avoidance is on task-based or interpersonal competence and incompetence, respectively. Similarly, performance-approach focused on normative competence, whereas, performance-avoidance intended on normative incompetence (Wang et al., 2010). Consequently, both mastery-approach and performance-approach has positive contribution towards achievement and consequences. In contrast, mastery-avoidance and performance-avoidance anticipated fewer adaptive motivational pattern (Elliot et al., 2001; McGregor et al., 2002). However, no studies were found from Pakistan. Therefore, validation of instruments in this context were also important. However, when the educators were asked about whether students motivation can enhance student's differential equation problems understanding. 70 percent were agreed with it, whereas, 30 percent were strongly agreed with it that student's motivation can enhance student's differential equation problems understanding. These responses were quite favorable and had well agreement with existing literature (Boyd, 2017; Lamm et al., 2017; Madjar et al., 2017; Wahl, 2017).

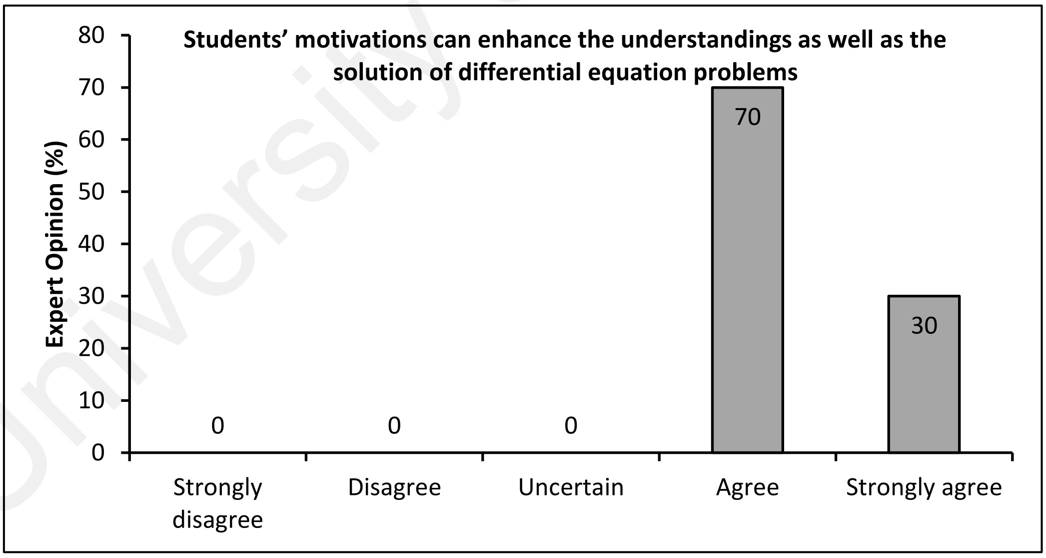

Figure 3.24: Experts feedback Part B, students' motivations can enhance the understandings as well as the solution of differential equation problems

Besides motivation, expert's opinion regarding to the role of self-regulated learning strategies such as critical thinking and elaboration was also observed (Butler et al., 2017; Kizilcec et al., 2017; Won et al., 2017). Self-regulated learner takes an active part in their learning process from the metacognitive to motivational and behavioral view point (Sadi et al., 2017; Zimmerman, 2002; Zimmerman et al., 2001). Moreover, the characteristics accredited to self-regulated persons are alike with those of high performance, high capability, as compared to low performer showing deficiency in these variables (Rivas et al., 2003; Zimmerman, 1998). In general, several researchers define self-regulated learner in different way (Butler et al., 2017; Won et al., 2017). Self-regulated learners know how to apply a series of cognitive strategies such as rehearsal, elaboration, organization for organization, elaboration and recovery purpose (Winne, 1995; Zimmerman et al., 2001). They are familiar and know how to plan, regulate and direct their mental process such as metacognition to attain their personal goals (Butler et al., 2017; Corno, 1989).

Other researchers (Weinstein et al., 2000; Zimmerman, 2002) defined self-regulated learner as a learner who shows motivational beliefs and adaptive emotions for example, development of self-efficacy beliefs, evolution of positive emotion towards tasks (e.g. satisfaction, passion, bliss) along with the ability to control and adjust these emotions according to the needs of the specific task and learning situation. Moreover, they have ability to plan and control the time and effort to be spent on tasks. Also, these self-regulated learners can not only create but also structure favorable environments, for example finding a proper place to study and seeking help from peers and teachers in case of difficult task (Corno, 1989; Winne, 1995; Zimmerman et al., 2001). For the present study, when educators were particularly asked in this regards, it was seen that 80 percent were agreed that self-regulated learning strategies effect student’s differential equation problem solving ability. However, 20 percent were disagreed with it.

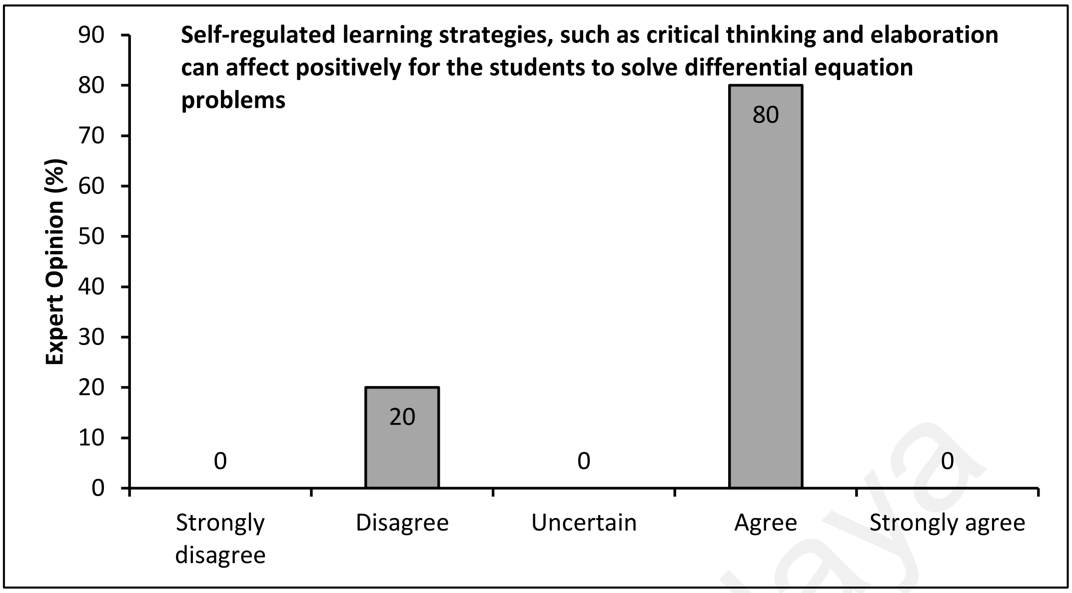

Figure 3.25: Experts feedback Part B, self-regulated learning strategies can affect positively for the students to solve differential equation problems

In the developed countries, teaches and educators are well familiar with the role of incorporation of students' belief systems, and their strong correlation between beliefs about mathematics and mathematical performance / achievement (Beghetto et al., 2012; Schommer- Aikins et al., 2005; Schommer-Aikins et al., 2013a). Likewise, McLeod (1992) had same opinion that mathematics beliefs enhance or weaken individual's mathematical and problem solving ability. However, no studies were found from Pakistan. Therefore, validation of instruments in this context were important. 70 percent experts were agreed that epistemological beliefs effect differential equation problem solving, whereas 10 percent were strongly agreed with it. However, 20 percent were not sure about the role of epistemological beliefs in differential equation problem solving.

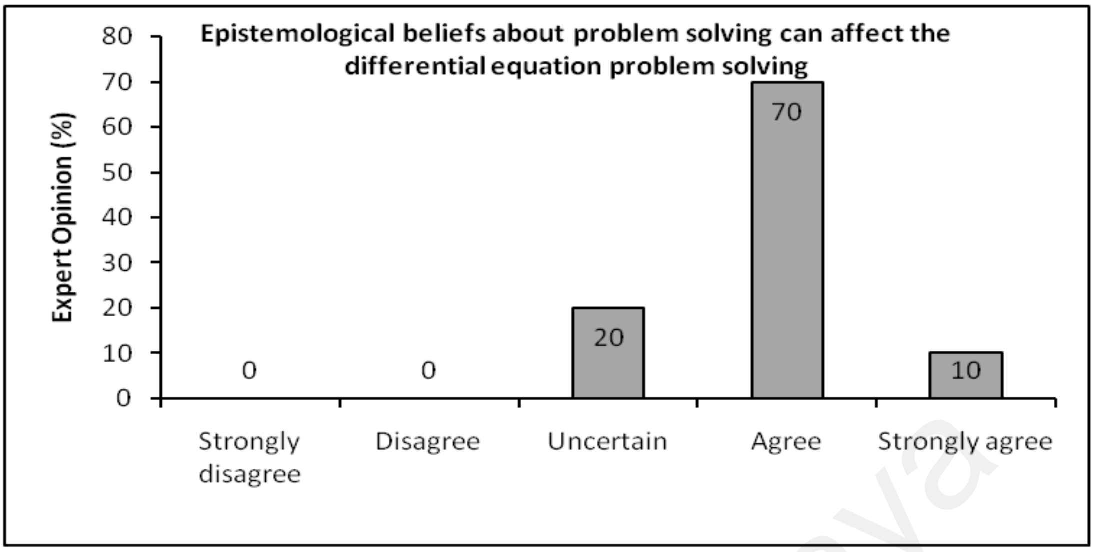

Figure 3.26: Experts feedback Part B, epistemological beliefs can affect positively for the students to solve differential equation problems

In addition to their individual effects of epistemological beliefs motivation and self-regulated learning strategies, expert's opinion regarding to their combined effect was also considered. As mediators are intermediate variables that explain how or why an independent variable influence an outcome. It explain the mechanism through which an intervention affects an outcome by assuming both casual and temporal relations (Gunzler et al., 2013). However, Baron et al. (1986) distinguished the concept of mediation from moderation, in which a third variable affects the strength or direction of the relationship between an independent variable and an outcome. Typically, moderator is either part of an interaction term or a grouping variable in multigroup analyses. For example, if males are known to react differently than females to a particular intervention for lowering cholesterol, in a gender by treatment interaction effect, gender is a moderator.

In contrast to it, for a treatment study, identification of a mechanism by which intervention achieves its effect mediation process is used. By investigating mediational processes, pathology of the disease and the mechanism of treatment can be diagnosed. Also, alternative and more efficient intervention strategies can be identified through mediation process. For example, a tobacco prevention programme may educate participants how to stop taking smoking breaks at work (the intervention) which changes their social norms about tobacco use (the intermediate mediator). Consequently it leads to a reduction in smoking behavior which is study outcome (MacKinnon et al., 2009). Furthermore, Conner et al. (2011) also employed the mediation analysis to evaluate the hypothesis that greater drinking intensity leads to higher level of depression, which in turn, leads to suicidal ideation.

Moreover, the investigation of association between independent and dependent variable through some mechanism is also important in many discipline including child growth (MacKinnon, 2008; MacKinnon et al., 2009; MacKinnon et al., 2007). Child growth is one the most important area and has many examples of hypothesized processes (Gable et al., 2012; Judd et al., 1981). For example, girl's weight is related to mathematics performance through its relationship with interpersonal skills (Gable et al., 2012), Iron deficiency anemia in children is related to levels physical activity through its relationship with cognitive development intervention programs promote healthy behavior by changing norms that are associated with those behavior (Judd et al., 1981). Further, Gunzler et al. (2013) suggested that based on strong theory and with appropriate context, mediation analysis is helpful in providing motivation for future intervention. In this way, more efficacious and cost- efficient alternative therapies may be developed.

In present study, results showed that 50 percent were agreed that these three parameters epistemological beliefs, motivation and self-regulated learning strategies effects differential equation problem solving, whereas, 40 strongly agreed with it. 10 percent were not sure about the combined role of these three parameters.

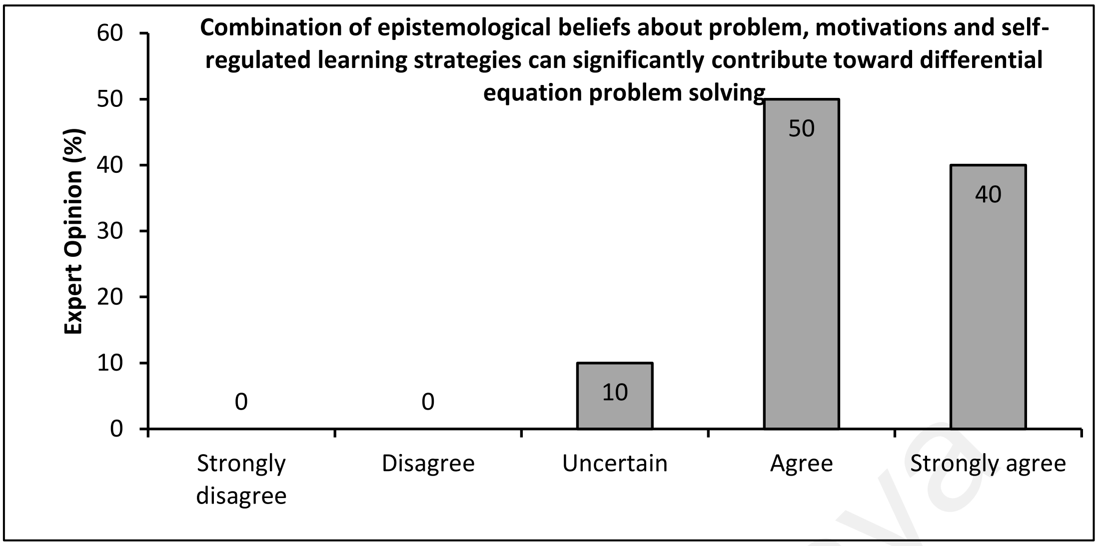

Figure 3.27: Experts feedback Part B, combination of epistemological beliefs, motivations and self-regulated learning strategies can significantly contribute toward differential equation problem solving

Moreover, expert's opinion about the usefulness of current research was also noticed. It was observed that 30 percent were agreed that current research is beneficial both for teachers as well as for students. Moreover, 70 percent strongly agreed with it. Overall, all experts considered that current research is very useful.

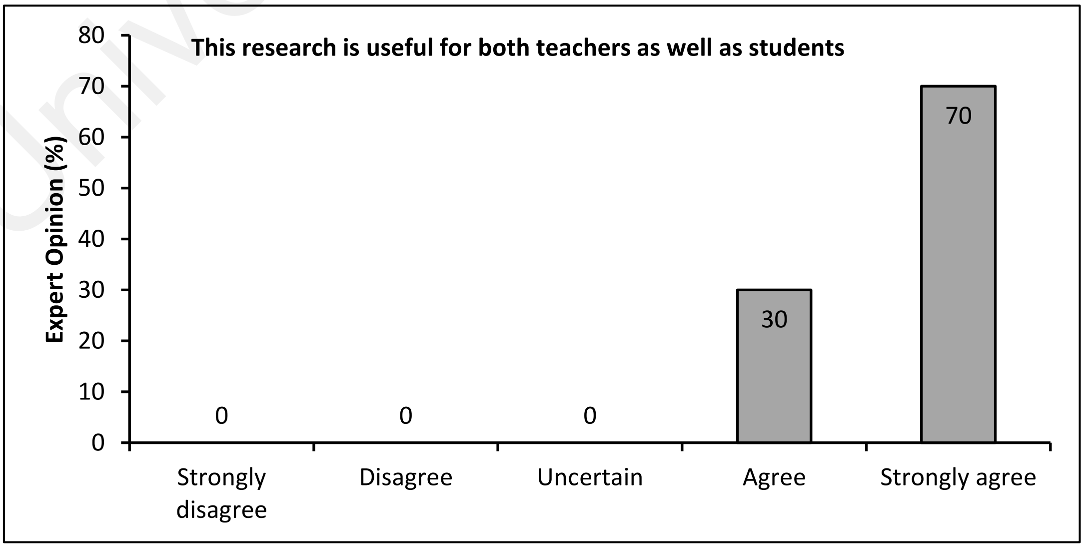

Figure 3.28: Experts feedback Part B, this research is useful for both teachers and students

3.6 Major changes adapted for actual study

After piloting the research instruments, careful analysis of the feedback from students as well as experts was done and a few changes in the research plan were made. These are summarized as follow,

1) The whole research instruments were distributed into three sections, including tasks section, beliefs section (epistemological beliefs and usefulness) and goal-SRL section (goal orientations and SRL strategies).
2) To capture more attention, an introductory title page was also prepared and through these students were been told that this assessment test would be helpful for their university's entrance exam. In Pakistan, most of the universities conduct entrance exam before students' enrolment. On the basis of this entrance test and their pre-university level grades or marks, universities decide their merits for different disciplines (e.g. engineering, computer science and etc.).
3) During pilot study, it was observed that many students were not aware about several terms such as epistemological math problem solving beliefs. To overcome this issue, before starting each section, a short 10 mints briefing was added for the students, so that they might be able to understand research instruments properly.
4) During pilot study, one and half hour was given to students to solve tasks and fill the questionnaires. Many students had complaint about shortage of time, because most of them have given feedback that non-routine tasks were taking more time as compared to routine tasks. To overcome this issue, assessment time was increased up to 2 hours. In addition, ten minutes break was also added between two consecutive sections, so that they might take some rest and respond properly. This recess time was addition to ten minutes briefing for each section.

After making changes and finalizing the instruments, concerned authorities of the randomly selected institutes had been requested and all the necessary documentations were completed for the final data collection. Consent detail is provided in the Appendix A. After that, visiting institutes one by one, participants were asked to solve assessment test and respond to adapted questionnaires. Collected responses were further analyzed using numerous steps. Detail of these steps is provided in the following sections.

3.7 Data collection for actual study

The data was collected from a total 430 participants, in which 52 percent were enrolled in rural area colleges while, 48 percent were studying in urban college. Collected data was used to make a SEM model and its evaluation.

3.8 Data analysis and model evaluation

This research involved multiple statistical techniques to analyze the quantitative data. To analyze the actual study data, two software SPSS and Smart PLS were used in two phases. In the phase 1, similar to pilot study, 394 responses selected from 430 returned responses and were initially analyzed in SPSS to calculate descriptive statistics, data screening and also preparation for model testing. In addition, the exploratory factor analysis (EFA) was also performed in SPSS, to identify the construct validity of the questionnaire. For readers ease, this segment was separated as phase 1. In the second phase, SmartPLS version (3.2.0) was employed for confirmatory factor analysis (CFA) as well as for the evaluation of structure equation modeling (SEM). SEM is an advanced statistical method that is used to measure latent, unobserved concepts with multiple observed indicators (Hair et al., 2013a).

To assume multivariate normality, the extent and shape of normality, outliers and skewness and kurtosis are determined (Hair et al., 2010; Tabachnick et al., 2007).

Moreover, scatterplots are used to test the linear relationship between variable. Also, if the variables are highly correlated, then there is a chance of multicollinearity and thus the variables did not meet the assumption of SEM. Regarding to requirement of indicators for each construct, there is no agreement on the literature (Byrne, 2013a). However, some researcher prefer several indicators for each construct, whereas others prefer using the smallest number of indicators to adequately represent a construct (Hair et al., 2006). Further author suggested that good practice dictates at least three items per each factor, preferably four to produce good results and also to avoid any model identification problems.

In addition to these assumptions, SEM is further categorized as covariance based structure equation model (CB-SEM) and least square structure equation model (PLS-SEM). Covariance based structure equation model represents constructs through factors, whereas, least square structure equation model is concerned with constructs through components (Lowry & Gaskin, 2014). However, PLS-SEM has advantages over CB-SEM in term of several statistical techniques such as principal component analysis, multiple regression, multivariate analysis of variance. These statistical techniques are not part CB-SEM (Hair Jr, Hult et al., 2016).

In the literature, both AMOS (analysis of moment structure) and smart-PLS can be used for CFA (Hair et al., 2013a). However, several studies claimed that Smart-PLS results are more reliable and valid than AMOS (Afthanorhan, 2013). In addition, literature also revealed that second-order constructs could be properly modeled using partial least square structural equation modeling (PLS-SEM) (Wetzels, Odekerken-Schröder et al., 2009). Therefore, for the current study, PLS-SEM scheme was finalized. Detail of analysis is provided in the below section.

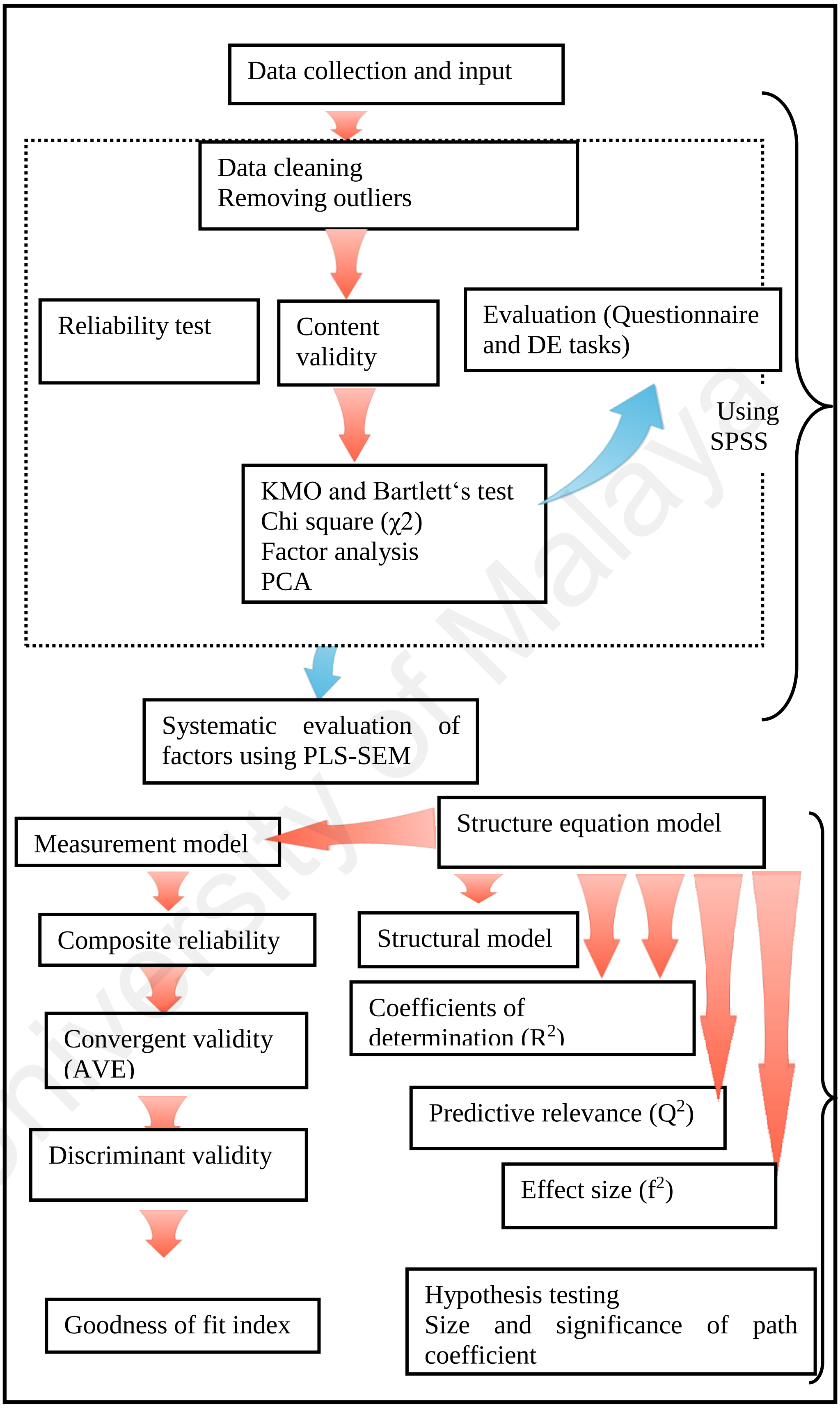

Figure 3.29: Schematic flow diagram for data analysis

3.8.1 PLS-SEM model evalution

To describe PLS path model, measurement model and construct model were created. Measurement model specifies the indicators of each construct and enables researchers to assess validity of the construct (Hair et al., 2006). Whereas, a structural model represents the relationships among latent variables.

Validity of the measurement model was measured using convergent validity and discriminant validity steps. Convergent validity was analyzed by calculating factor loadings, composite reliability (CMR), and average variance extracted (AVE) (Wynne W Chin, 1998). Similarly, discriminant validity was confirmed via calculating the square root of AVE values for each construct, and comparing it with guidelines. it should be higher than the correlation of pair of any latent variable (Gefen & Straub, 2005). Similarly, the structural model was evaluated with significance of path coefficients and R^2 (variance) of latent variables. Detail of analysis is discussed in chapter 4.

3.8.1.1 Path coefficients analysis

The SEM approach had been carried out to examine the direct and indirect effects of the variables. Consequently, SEM also estimated the size of the total effect of each independent, on dependent variable in the multi-stage path model, by providing both direct and indirect effects. Normally, the direct effects demonstrate the strength of the direct path from predictor variable to the particular dependent variables, as indicated by path coefficient. Whereas, the indirect effects register the strength of indirect path from a predictor variable to a dependent variable through mediator.

3.8.1.2 Analysis of mediating roles

The analysis of the mediators in the proposed research model was carried out after assessment of the direct paths. There were three conditions achieved in the casual relationship or direct effect analysis; complete mediation, partial mediation and no mediation. The mediating effect considered prominent when the effect was reduced after the mediating variable enters the model (Awang, 2012). To interpret this condition, Awang (2012) has noticed that; if it is reduced but still significant, then the partial mediation is achieved. However, if the effect is reduced up to the level where it is no longer significant, then it is considered that complete mediation has achieved.

To test mediation, one should estimate the three following regression equations; first, regressing the mediator on the independent variable; second, regressing the dependent variable on the independent variable; and third, regressing the dependent variable on both the independent variable and on the mediator (Baron et al., 1986). Authors recommended that testing the significance of the indirect path a * b by the Sobel z-test. Equivalently, z tests whether the difference between the total effect and the direct effect is statistically significant (Zhao et al., 2010).

3.9 Chapter Summary

This chapter has illustrated the detailed methodology for correlational study, currently carried out with five non-routine developed tasks and three adapted questionnaires covering students' epistemological math problem solving beliefs and usefulness, goal orientations, and self-regulated learning strategies. This chapter also explained the detailed about data collection, analysis and development of structural equation model (SEM).

CHAPTER 4: RESEARCH RESULTS

4.1 Introduction

This chapter presents the results of the analysis of quantitative data. The target population for this study was limited to pre-university level students studying in the public and private sectors of Khyber Pakhtunkhwa (A big province of Pakistan). Five self-developed differential equation tasks, containing non-routine problems and three adapted self-reported questionnaires, were distributed among 430 students. These 430 students were randomly selected with equal frequency of male and female from nine colleges (from three districts) of Pakistan. Both rural as well as three urban areas were considered to get real and generalized data. Among 430 participants, 52 percent were enrolled in rural areas colleges while 48 percent were studying in urban areas. Their ages were in the range of 18-19 years. In Pakistan, the differential equation course is introduced at pre-university level, which is mostly the 12th year of study (Rehman et al., 2012). Therefore, students in the same class mostly have the same age group. However, few students, studied from other systems (i.e. O level) or countries, may have the age of 19. Therefore, for this study, average age is taken as 18.5 years and no other specific demographic parameter with respect to different age group was considered. Table 4.1 illustrates the descriptive statistics of sample.

The returned responses were initially screened and 394 responses have been selected for the further analysis. To analyze the survey data, suitable techniques and software's were used. SPSS and SmartPLS software were used for the further data analysis. SPSS was used to calculate descriptive statistics; data screening and also preparation for model testing. In addition, the exploratory factor analysis (EFA) was performed in SPSS, to identify the construct validity of the questionnaire. For readers ease, this segment was separated as phase 1.Whereas, Smart PLS software Version 3.2.4 was used for the evaluation the survey data (Ringle et al., 2005). PLS is a well-established technique for estimating path coefficient in structural models and has become increasingly popular in the past decade because of its ability to model latent constructs under conditions of non-normality and suitability for small to medium

Table 4.1: Descriptive statistics of demographic variables

Demographic variable	n	percent
Age		
Average Age 18.5		
Area		
Rural	224	52
Urban	206	48
Sector		
Private	172	40
Govt	258	60
Field (Selected/Desired)		
Engineering /Pre-Engineering	172	40
Computer Science	129	30
Geology	107	25
Education	13	3
Economics	9	2
Problem solving approaches		
Algebraic approach	Graphical approach	

sample sizes (Ali, Zhou et al., 2016). This software was used for the accomplishment of confirmatory factor analysis (CFA) purpose, and also to evaluate hypothesized differential equation problem solving model. For current study, hypotheses were tested based on structural equation modeling using the Partial Least Squares (PLS) method. To make it easier for readers, this segment was called phase 2. In the next step, findings of the currently analyzed data were evaluated against the proposed research questions and available literature.

4.2 Data analysis (Phase 1)

In the phase 1, the 394 responses selected from 430 returned responses were initially analyzed in SPSS to calculate descriptive statistics, data screening and also preparation for model testing. The reason for the using SPSS in phase 1 was of its effectiveness and straightforwardness to clean data and removal of outliers followed reliability. In addition, the exploratory factor analysis (EFA) was performed in SPSS, to identify the construct validity of the questionnaire. For readers ease, this segment was separated as phase 1. However, keeping in mind the model complexity, smart-PLS was employed in the second phase for the complete model development.

4.2.1 Data screening

Initially, SPSS was used to input collected data, data screening, factor analysis and preparation for model testing. After careful analysis, few cases were deleted and 394 cases out of 430 were retained for the further analysis. The percentage of missing data value was less than 2 percent with no apparent pattern. Hair et al. (2013a) recommended mean value replacement option, when there are less than 5 percent missing value per indicator. Therefore, the missing data was imputed using a mean value replacement method. Since other steps such as EFA and their results were similar to pilot study data. Therefore, to avid repetitions, these results have been summarized and are provided in the Appendix A.

Multicollinearity reflects the relationship between the independent variables. Author further suggested that the precense of multicollinearity affects the quality and the results of the regression model. Variance of inflation factor (VIF) and Tolerance index (TI) are two tehniques that are used to check the degree multicolinearity (Hair et al., 2010; Pallant, 2010). If the Tolerance index value is less than 0.10, and the VIF value more than 10, it indicates that the two variables are highly correlated. Table 4.2 summarises all the VIF and TI value of all proposed independent variable under the costruct. Variance Inflation Factor (VIF) values are below the threshold value of 5.0, hence indicating no multicollinearity problem.

The next phase of this study was to evaluate the hypothesized research model using Smart PLS. Detail of further procedure for the evaluation of research model is provided in next section.

Table 4.2: Multicollinearity results

Model	**Collinearity Statistics**	
	Tolerance	**VIF**
MA	0.339	2.951
PR	0.533	1.876
AV	0.979	1.021
CRT	0.885	1.131
UF	0.505	1.980
EMB	0.388	2.576

4.3 Evaluation of research model (Phase 2)

In the phase 1, after removing outliers, the selected 394 responses were initially analyzed using SPSS to calculate descriptive statistics, and exploratory factor analysis (EFA). In phase 2, Partial least square structural equation modeling (PLS-SEM), a type structural equation modeling was employed for the model development and evaluation. PLS-SEM is particularly appropriate for second-order constructs (Wetzels et al., 2009).

PLS is a well-established, second generation multivariate technique which can simultaneously evaluate the measurement model and the structural model with the aim of minimizing the error variance. PLS analyses were performed using Smart PLS software which computed the estimates of standardized regression coefficient of the paths of the model, factor loadings for the indicators, and the amount of variance account for the dependent variables (Ringle et al., 2005). Generally, this software makes it possible to test the hypothesized relationships between independent and dependent variables depicted in the model. Another important application of Smart PLS software is that it computes several types of reliabilities (Cronbach's alpha and composite reliability coefficient) and validities (convergent and divergent) statistics, which can be used to assess the quality of the model. In addition, SEM measures latent, unobserved concepts with multiple observed indicators. In SEM, two types of models including measurement model and structural model are embedded. Measurement model also known as outer model describes relationship between latent variables and their measures (indicators). Further, measurement model can be reflexive or formative or their combination depending upon the nature of constructs and variables. Whereas, structural model also known as inner model and it determines the relationships between the determinants.

For the current research, analysis had been carried out using a PLS's two-stage analytical procedures. The results obtained in PLS-SEM were evaluated in two stages. In first stage, reliability and validity of measurement model (outer model) were assessed. In the second stage, the structural model (inner model), the hypothesized relationships between the independent (exogenous) and dependent (endogenous) variables were evaluated (Howell et al., 2007).

Initially, measurement models were run, which have enabled the researcher to evaluate the reliability and validity of the construct measures. This step also ensured the quality of measurement model prior to hypothesis testing. Overall model steps have been summarized (Table 4.3) and were used to evaluate and test the differential equation problem solving ability in the present study.

Table 4.3: Systematic evaluation of PLS-SEM results (Hair et al., 2013)

Steps	**Evaluation**	
1	Evaluation of measurement model	
	1 (a) Reflective measurement model	1 (a) Formative measurement model
	Internal consistency	Collinearity among indicators
	Convergent validity	Outer weight significance and relevancy
	Discriminant validity	
2	Analyzing the research model and validating the second order constructs	
3	Evaluation of structural model	
	Significance and relevancy of the structural model path coefficients	
	Coefficients of determination (R^2)	
	Effect size (f^2)	
	Predictive relevancy (Q^2) and q^2 effect size	

As the purpose of the present study was to analyze overall effect of the four factors (epistemological math problem solving beliefs, usefulness, goal orientations and self-regulated learning strategies), and also their mediation role on the differential equation problem solving, therefore both direct and indirect effects were considered. Several assessments were also performed by evaluating the significance and the relevance of the structural model path coefficients, testing coefficients of determination R^2, assessing f^2 effect sizes, and the evaluating the predictive relevance $Q^2$and q^2 effect size. The detail of each step and relevant terminologies are provided in further next section. For the ease of readers, measurement models were calculated separately followed by the summarized structural estimates of all direct effects. Figure 4.1 illustrates the flowchart for the direct and indirect paths. This scheme has enabled to observe few changes in the path values when the individual paths were merged into an overall model. To further analyze these changes, mediation or indirect paths were analyzed, in the next phase, and discussed in the following sections.

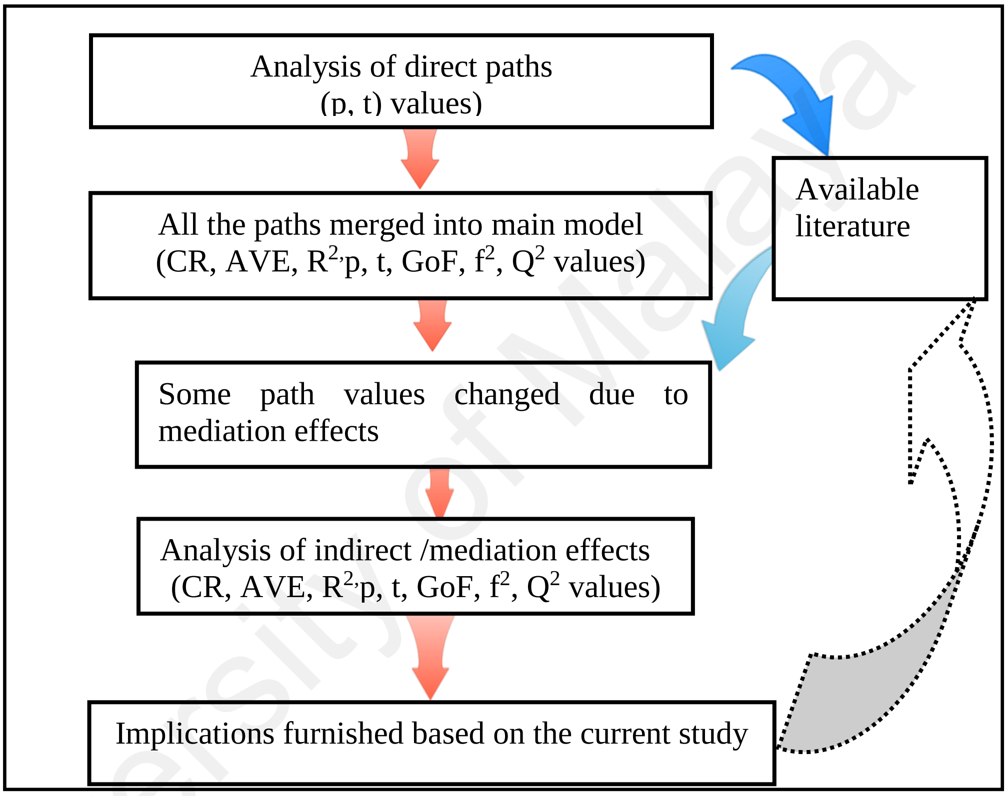

Figure 4.1: Framework for direct and indirect path analysis

Prior to model evaluation, Hair et al. (2013a) recommended the specification of the nature of constructs in the measurement and structural model. The specifications of the nature of the constructs of measurement and structural model are explained below.

4.3.1 Specifying measurement model in Smart PLS

The measurement model refers to the relationship between measures and their related constructs (Jarvis, MacKenzie et al., 2003). Specification of measurement model are categorized as, formative and reflective measurement model. A reflective measurement model shows an arrow pointing from constructs to its measures. While a formative measurement model is based on the assumptions that the indicators cause the constructs. Therefore, formative measurement models are identified by arrows pointing from indicators to the constructs. Furthermore, Jarvis et al. (2003) suggested four main decision rules to identify formative and reflective constructs. Based on these rules, researcher made a decision about the model (scales and sub-scales) and the overall constructs whether it is reflective or formative constructs.

In current study, epistemological problem solving beliefs were the perception about differential equation problem solving. After running the factor analysis, there were five sub-stages namely: duration of problems, steps, understanding, word problems and effort. To confirm the uni-dimensionality of this construct, factor analysis was re-run. Results showed that this construct was second order hierarchical construct. Based on the decision rules and construct measures analysis, as shown in Table 4.4, the current study hypothesized that epistemological problem solving beliefs was a second-order reflective construct comprising of five dimensions (sub-scales); duration of problems (DP), steps (ST), understanding (UN), word problems (WP) and effort (EF).

Similarly, second construct of proposed model was usefulness. Based on the Jarvis et al. (2003) rule (Table 4.4), usefulness was considered a first order reflective construct. Regarding goal orientations, three subscale mastery goal, performance goal and avoidance goal were used for this work. Mastery goal, performance and avoidance goal were first order reflective as recommended by Jarvis et al. (2003). Similarly, other constructs, critical thinking, elaboration and differential equation problem solving also best fit the category of first order reflective constructs (Jarvis et al., 2003).

Table 4.4: Decision rule to identify the sub-stages of construct

Rule	Description	Decision Formative/ Reflective
Rule 1	Direction of causality from construct to measured implied by the conceptual definition	Reflective
Rule 2	Interchangeability of the indicators/items	Reflective
Rule 3	Covariance among the indicators	Reflective
Rule 4	Nomo logical net of the construct indicators	Reflective

The next and second phase of specification was to identify the nature of constructs in the structural model. For specifying the structural model, there were two types of structural model; first order component model and higher order component. Usually, hierarchal latent variable models are characterized by the number of levels in the model (Rindskopf & Rose, 1988) and the relationships (formative vs. reflective) between the constructs in the model (Hair et al., 2012; Jarvis et al., 2003; Wetzels et al., 2009). Coltman, Devinney et al. (2008) claimed that the formative and reflective constructs are distinct, and hence should not be treated in the same way in the measurement model.

Reflective constructs are applicable to be assessed for reliability and validity by conducting CFA using PLS-SEM. Regarding to the reliability and validity of formative constructs, Petter, Straub et al. (2007) suggested that reliability for formative construct is irrelevant, therefore, only validity test might be conducted for formative construct. Miss-specified measurement models may lead to measurement errors that in turn affect the structural model validity (Jarvis et al., 2003).

Table 4.5: Measurement of constructs of proposed model

First order	No of items	Type	Second order	Type
DP	6	Reflective	EMB	Reflective
ST	6			
UN	6			
WP	6			
EF	6	Reflective		Not Found
UF	6			
MA	6			
PR	5			
AV	6			
CR	5			
EL	6			

In case of hierarchical latent variable, several researchers differentiate four types of models dependent on the relationship among the first order latent variables and their manifest variables and second order latent variable and the first order latent variables (Hair et al., 2012; Jarvis et al., 2003). These four types are; reflective-reflective type, reflective-formative type, formative-reflective type, formative-formative type.

For the current study reflective-reflective type was used because in this type, lower order constructs can be reflectively measured themselves that can be distinguished from each other but are correlated (Becker, Klein et al., 2012).

According to earlier discussion, each construct was assigned construct type and hierarchical order along with number of items (Table 4.5). All the constructs were measured using multiple items.

4.3.2 Confirmatory factor analysis using Smart PLS

Confirmatory factor analysis (CFA) is also imperative to validate a multi-factorial model (Byrne, 2010), as it is used for validating the correlation between items and factors. Said et al. (2011) recommended that CFA using SEM gives better results in testing the validity and reliability of an instrument. Therefore, considering these recommendations, CFA was carried out to validate all scale in term of the convergent validity and discriminant validity (Byrne, 2013a; Hair et al., 2006). According to Ringle et al. (2005), Smart PLS is a free tool for path modeling.

The validity of measurement model was done in two ways: convergent validity and discriminant validity. According to Frankfort-Nachmias and Nachmias (2007), convergent validity is concerned with measuring the degree of a positive relationship among scale items developed to measure the same concept/construct. Therefore, the purpose of convergent validity is to confirm that measures that should be theoretically related are in reality related. While discriminant validity indicates that a measure does not correlate with another measure which no theoretical relationships are expected. Discriminant validity is evaluated, by comparing the square root of *AVE* with the correlation between the variables (Hair Jr et al., 2016). In addition, for a distinct variable, correlation between the variable must be lower than the square root of the AVE (Hulland & Business, 1999). Further detail of both convergent and discriminant validity is provided below.

4.3.2.1 Convergent validity

Convergent validity is assessed by confirmatory factor analysis (CFA) through three main criteria, factor loading, composite reliability (CMR), and average variance extracted (*AVE*). Composite reliability measure overall reliability of a set of heterogenous but similar indicators, whereas, the average variance extracted represents the overall amount of variance in the manifest variables accounted for by the latent construct (Hair et al., 2006). Furthermore, Hair (2010) recommended that values for outer loading, *AVE* and *CMR* must be greater than 0.5, 0.5 and 0.7 respectively. In addition, for a distinct variable, correlation between the variable must be lower than the square root of the *AVE* (Hulland et al., 1999).

4.3.3 Measurement model assessments

This section discussed the findings for the measurement model. As discussed earlier, the subscale or dimensions of proposed model were reflective measurement model. Therefore, following the steps of evaluating reflective measurement model, the sub-scales of the model were evaluated. Detail is provided in the following section.

4.3.3.1 Reliability of model sub-scales

To investigate the reliability of reflective constructs (sub-scales), Cronbach's alpha and composite reliability measures can be extracted by PLS-SEM. The current model was conceptually based on the determinants epistemological math problem solving beliefs, usefulness, goal orientations and self-regulated learning strategies. Table 4.6 showed that overall results of the items exceeding the value of 0.70 (Götz, Liehr-Gobbers et al., 2010). Similarly, the Cronbach's alpha value was 0.90, which was quite acceptable. All of the reflective items were found acceptable and reliable. Each construct was also briefly elaborated and discussed, individually. Detail is included in next section.

4.3.3.2 Measurement model for epistemological math problem solving beliefs

In the present study, epistemological math problem solving beliefs were considered as second order hierarchal factor. Therefore, it was necessary to confirm the convergent and discriminant validity for both first order and second order. The convergent validity of the second order differential equation problem solving belief measurement model was assessed through factor loadings, composite reliability (*CMR*) and average variance extracted (*AVE*) (Hair Jr et al., 2016). Factor loading demonstrates the score of the variance shared among an item and factor. (Teker, 2011)recommended that the factor loading of 0.55 or bigger are acceptable range for convergent validity. Factor loading of 0.55 can explains 30 percent of the variance by its latent variable (Falk & Miller, 1992). However, Henseler, Ringle et al. (2009) suggested 0.7 as a cutoff value for acceptable loading. It means that a latent variable should explain about half of the variance in its indicator variables.

In the present case, factor loadings for epistemological problem solving beliefs were in the range of 0.82-0.86. These high values had demonstrated a strong evidence for the convergent validity of the model. Next step was the estimations of *CMR* and *AVE* values. Usually, *CMR* value depicts the degree to which the construct indicators reflect the latent construct, while *AVE* reveals the overall amount of variance in the indicators accounted for by the latent construct. In present study, *CMR* and *AVE* values, were well above the recommended values of 0.7 and 0.5, respectively (Hair Jr et al., 2016). Table 4.6 illustrated the factor loadings, *CMR*, *AVE* and values.

Table 4.6: Reliability of reflective constructs (sub-scales)

Construct	Items	Outer loadings	Cronbach's alpha	CMR	AVE
Epistemological problem solving belief					
DP	6	0.83			
ST	6	0.85			
UN	6	0.86	0.90	0.93	0.72
WP	6	0.86			
EF	6	0.82			

4.3.3.3 Measurement model for usefulness

The convergent validity for usefulness measurement model was also assessed through factor loadings, Cronbach's alpha, *CMR* and *AVE* .The Cronbach's alpha, *CMR* and *AVE* values were 0.93, 0.95 and 0.77, respectively. Table 4.7 shows that all values were well in agreement with recommended values (Wynne W. Chin, 1998).

Table 4.7: Construct reliability and validity of usefulness

Subscale	Item code	Loadings	Cronbach's alpha	CMR	AVE
UF	B51	0.90	0.93	0.95	0.77
	B52	0.89			
	B53	0.90			
	B54	0.86			
	B55	0.87			
	B56	0.84			

4.3.3.4 Measurement model for goal orientations

For goal orientations, factor loadings were in the range of 0.75-0.93, greater than recommended value of 0.6 (Wynne W. Chin, 1998). Similarly, higher values Cronbach's alpha, *CMR* and *AVE* values confirmed the convergent validity (Hair 2006). Table 4.8 shows detail of these values.

Table 4.8: Construct reliability and validity of goal orientation

Subscale	Item code	Loadings	Cronbach's alpha	CMR	AVE
MA	MA1	0.89	0.93	0.95	0.74
	MA2	0.87			
	MA3	0.84			
	MA4	0.87			
	MA5	0.85			
	MA6	0.87			
PR	PER1	0.85	0.89	0.92	0.70
	PER2	0.85			
	PER3	0.85			
	PER4	0.87			
	PER5	0.75			
AV	AV1	0.88	0.95	0.96	0.82
	AV2	0.90			
	AV3	0.90			
	AV4	0.92			
	AV5	0.93			
	AV6	0.90			

4.3.3.5 Measurement model for self-regulated learning strategies

For self-regulated learning strategies, factor loadings were in the range of 0.74-0.85, greater than recommended value of 0.6 (Chin 1998). Likewise other constructs, higher Cronbach's alpha, *CMR* and *AVE* values confirmed the convergent validity (Hair 2006). Table 4.9 shows detail of these values.

Table 4.9: Construct reliability and validity of self-regulated learning strategies

Subscale	Item code	Loadings	Cronbach's alpha	CMR	AVE
CR	CR1	0.77	0.81	0.87	0.58
	CR2	0.78			
	CR3	0.75			
	CR4	0.74			
	CR5	0.77			
EL	EL1	0.80	0.91	0.93	0.69
	EL2	0.84			
	EL3	0.84			
	EL4	0.85			
	EL5	0.82			
	EL6	0.83			

4.3.3.6 Discriminant validity

After confirming the convergent validity, the next process was to assess the discriminant validity. To test discriminant validity, CFA provide two most common techniques. First, the correlation between any two specific constructs can be fixed as equal to one; in essence it is the same as identifying that the items that structure two constructs might just as well make only one construct. If the fit of the two-construct model is different from that of the one-construct model, then discriminant validity is accepted (Byrne, 2010). But, Hair et al. (2006) evidently proved that practically this method does not offer strong evidence of discriminant validity, because strong correlation, sometimes as high as 0.9, can still create significant difference in fit between the two models. As a result, a second alternative test was provided by Fornell and Larcker (1981) and Hair et al. (2006) comparing the average variance extracted (AVE) value for any two construct with the square of the correlation estimates between the same two constructs.

To verify the discriminant validity, usually Fornell-Larcker criterion (1981) is used. The function of Fornell-Larcker criterion (1981) is to compare the square root of Average Variance Extracted (*AVE*) with the latent variable correlations. Because, if the correlations between the variables are lower than the square root of the *AVE* the variables can be considered as distinct theoretical entities (Hulland et al., 1999). Table 4.10 shows that the square root of each construct's *AVE* value was higher than to its correlation with another construct and also each item load was highest on its associated construct (Fornell et al., 1981).

The second phase of PLS's two-stage analytical procedure was to examine the structural model in order to test the hypothesized relationship within the model. To determine the significance levels of the loadings, weight and path coefficients, Hair Jr et al. (2016) recommended a bootstrapping method with at least 5000 samples. The detail of the structural assessments is provided in next section.

Table 4.10: Discriminant validity of whole model constructs

Construct	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**
AV	**0.91**							
CR	0.03	**0.76**						
DEPS	-0.26	0.33	Single item					
EL	0.04	0.44	0.61	**0.83**				
EMB	-0.01	0.40	0.65	0.72	**0.85**			
MA	-0.01	0.33	0.68	0.68	0.73	**0.86**		
PR	0.07	0.37	0.53	0.61	0.61	0.64	**0.83**	
UF	-0.08	0.28	0.66	0.52	0.62	0.67	0.42	**0.88**

4.3.4 Structural model assessments

The measurement models of epistemological math problem solving beliefs, usefulness, goal orientations, and self-regulated learning strategies were examined in terms of reliability and validity. Second-order reflective-reflective construct was also validated by providing and comparing the second order construct with alternative models. After assessments of the measurement models, the next step was to evaluate the structural model (overall model).

The evaluation of structural model, also called inner model, shows the hypothesized predictive or causal relationship between the latent variables in the study (Tenenhaus et al., 2005). Predictive relationships between the exogenous and the endogenous latent variables are represented through single-headed arrows. Variables that have arrows pointed towards them are called endogenous variables and variables that do not receive any arrow are called exogenous variables. Hair et al. (2013a) recommended criteria for the evaluation of structural model in PLS-SEM. It includes estimations of significance of the path coefficients, the level of R^2, the f^2 effect size, predictive relevance Q^2 and the q^2effect size. Detail of each criterion is provided in below section.

4.3.5 Significance and the relevance of the structural model path coefficients

The assessment of structural model requires the execution of bootstrapping. Using bootstrapping option, results of path coefficient, t-value and significance level were calculated for current study. After bootstrapping, obtained t-value was compared with critical t-value at a certain selected level. It is criteria that if t-value is higher than the critical t-value then the coefficient is significantly different from zero (Hair Jr et al., 2014).

Table 4.11 illustrates the results of path coefficient, t-value and significance level for all constructs. Evaluation of the significance relevance of the proposed model was carried out after evaluating the significance of the relationships between the constructs. The relevance of the structural model relationship is essential for the interpretation of results (Hair et al., 2013). To test the structural estimates, significance of path coefficient using t-values were calculated for epistemological math problem solving belief, usefulness, goal orientations and self-regulated learning strategy and discussed briefly.

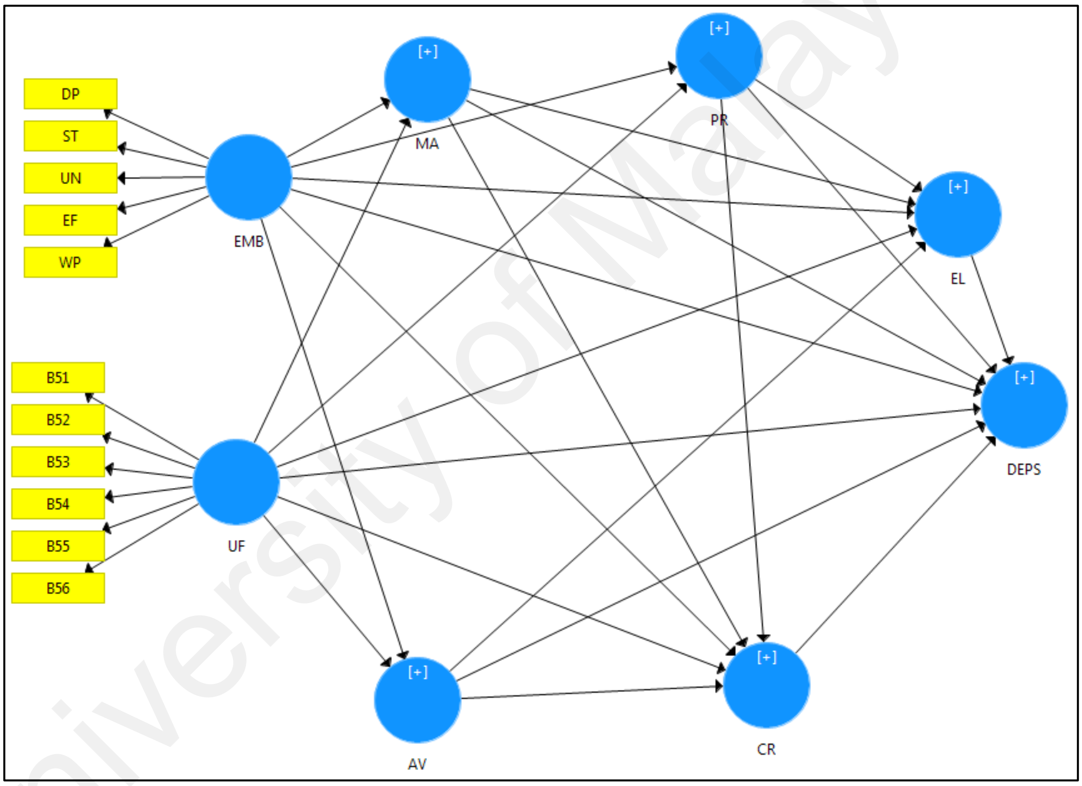

Figure 4.2: Overall structural model for epistemological math problem solving beliefs, usefulness, goal orientations and self-regulated learning strategies

Avoidance goal had shown highest but negative contribution (β = -0.25, t = 8.10, P < 0.01), followed by usefulness belief (β = 0.29, t = 4.05, P < 0.01). It was also revealed that among remaining constructs, mastery goal (β = 0.22, t = 3.52, P < 0.01) and elaboration (β = 0.16, t = 3.12, P < 0.01) had relatively greater influence on differential equation problem solving ability as compared to performance goal (β = 0.10, t = 2.20, P < 0.05) and epistemological problem solving beliefs (β = 0.12, t = 2.01, P < 0.05).

Table 4.11: Significance testing results of the structural model path coefficients

Path	Path coefficients	t-values	p values	Significance level
AV -> CR	0.03	0.56	0.58	P > 0.05
AV -> DEPS	**-0.25**	**8.10**	**0.00**	**P < 0.01**
AV -> EL	0.04	1.43	0.15	P > 0.05
CR -> DEPS	0.03	0.91	0.37	P > 0.05
EL -> DEPS	**0.16**	**3.12**	**0.00**	**P < 0.01**
EMB -> AV	0.07	1.12	0.26	P > 0.05
EMB -> CR	**0.27**	**1.99**	**0.05**	**P < 0.05**
EMB -> DEPS	**0.12**	**2.01**	**0.04**	**P < 0.05**
EMB -> EL	**0.40**	**3.76**	**0.00**	**P < 0.01**
EMB -> MA	**0.51**	**5.95**	**0.00**	**P < 0.01**
EMB -> PR	**0.57**	**7.17**	**0.00**	**P < 0.01**
MA -> CR	-0.03	0.35	0.73	P > 0.05
MA -> DEPS	**0.22**	**3.52**	**0.00**	**P < 0.01**
MA -> EL	**0.25**	**2.23**	**0.03**	**P < 0.05**
PR -> CR	**0.20**	**2.19**	**0.03**	**P < 0.05**
PR -> DEPS	**0.10**	**2.20**	**0.03**	**P < 0.05**
PR -> EL	**0.19**	**2.34**	**0.02**	**P < 0.05**
UF -> AV	**-0.12**	**2.09**	**0.04**	**P < 0.05**
UF -> CR	0.06	0.54	0.59	P > 0.05
UF -> DEPS	**0.29**	**4.05**	**0.00**	**P < 0.01**
UF -> EL	0.03	0.37	0.71	P > 0.05
UF -> PR	0.07	0.96	0.34	P > 0.05
UF -> MA	**0.35**	**4.11**	**0.00**	**P < 0.01**

AV: Avoidance goal, MA: Mastery goal, PR: Performance goal, UF: Usefulness, DEPS: Differential equation problem solving, CR: Critical thinking, EL: Elaboration, EMPB: Epistemological problem solving belief.

Regarding to the effect of exogenous variables, both epistemological math problem solving beliefs (β = 0.51, t = 5.95, $P < 0.01$) and usefulness (β = 0.35, t = 4.11, $P < 0.01$) showed significant direct effect on mastery goal. Similarly, epistemological math problem solving beliefs (β = 0.57, t = 7.17, $P < 0.01$) had shown direct significant effect on performance goal, whereas, usefulness did not show direct significant effect towards performance. Results also revealed that regarding to avoidance goal, only usefulness (β = -0.12, t = 2.09, $P < 0.05$) showed negative but significant direct path.

Analyzing the direct path towards elaboration, it was observed that epistemological math problem solving beliefs (β = 0.40, t = 3.76, $P < 0.01$), mastery goal (β = 0.25, t = 2.23, $P < 0.05$), and performance goal (β = 0.19, t = 2.34, $P < 0.05$) had shown significant path coefficients. Whereas, remaining paths were not significant towards elaboration.

4.3.6 Coefficient of determination R^2

The coefficient of determination R^2 is considered as a measure of a model's predictive accuracy, and is calculated as the squared correlation between dependent construct and predicted values (Hair et al., 2013). Furthermore, the value of R^2 refers to an explanatory power of the predictor variables on the respective construct. Normally, the endogenous latent variables are classified as substantial, moderate or weak based on R^2 values of 0.67, 0.33 or 0.19, respectively (Chin 2008).

For the current study, R^2 value for the differential equation problem solving construct was 0.64 (moderate), which indicated that 64 percent of the variance in this construct is explained by factors: beliefs, usefulness, mastery, performance, avoidance goal, elaboration and critical thinking.

Further analysis revealed that epistemological problem solving beliefs and usefulness explained 61 percent towards mastery goal. Similarly, both of these (beliefs and usefulness) had explained 38 percent performance goal. Therefore, both mastery goal and performance goal with R^2 value 0.62 and 0.38 were described as moderator. Avoidance goal explained very small percent. Furthermore, R^2 values for elaboration and critical thinking were 0.59 and 0.19, representing moderate and weak effects. Overall, model explains 64 percent of the variance for differential equation problem solving, which is described as nearer to substantial endogenous latent variable.

Table 4.12: Systematic evaluation of PLS-SEM results

Construct	AVE	R^2	GoF=$\sqrt{(AVE*R^2)}$
AV	0.82	0.009	0.55
CR	0.58	0.188	
DEPS	1.00	0.64	
EL	0.69	0.591	
MA	0.74	0.611	
PR	0.70	0.377	
UF	0.77	-	
EMB	0.72		

Figure 4.3 shows the R^2values. PLS path modeling does not optimize any global scalar function so it naturally lacks of an index that can provide the user with a global validation of the model (Tenenhaus, Vinzi et al., 2005). Further Sen, Roy et al. (2015) argued that complete structural equation models is usually demonstrated by acceptable level of absolute fit also known as Goodness-of-fit index (GoF). Moreover, GoF provides an operational solution to this problem to validate the PLS model globally. Several other researchers also recommended the calculation of goodness of fit (GoF), prior to the structural model estimates (Anderson & Gerbing, 1988). As PLS cannot generate itself overall goodness of fit indices. Therefore, a diagnostic tool known as the GoF is normally used to assess the model fit (Tenenhaus et al., 2005).

The GoF is typically measured using the geometric mean of the average communality (AVE) and the average R^2 (for endogenous constructs). Equation 4.1 represents the GoF

$$GoF = \sqrt{AVE \times R^2} \qquad 4.1$$

Reported cutoff values for assessing the results of the GoF analysis are : GoF_{small} = 0.1; GoF_{medium} = 0.25; GoF_{large} = 0.36 (Hoffmann & Birnbrich, 2012).The current model yielded a (GoF) value of 0.55 which indicated a large model fit as shown in Table 4.12.

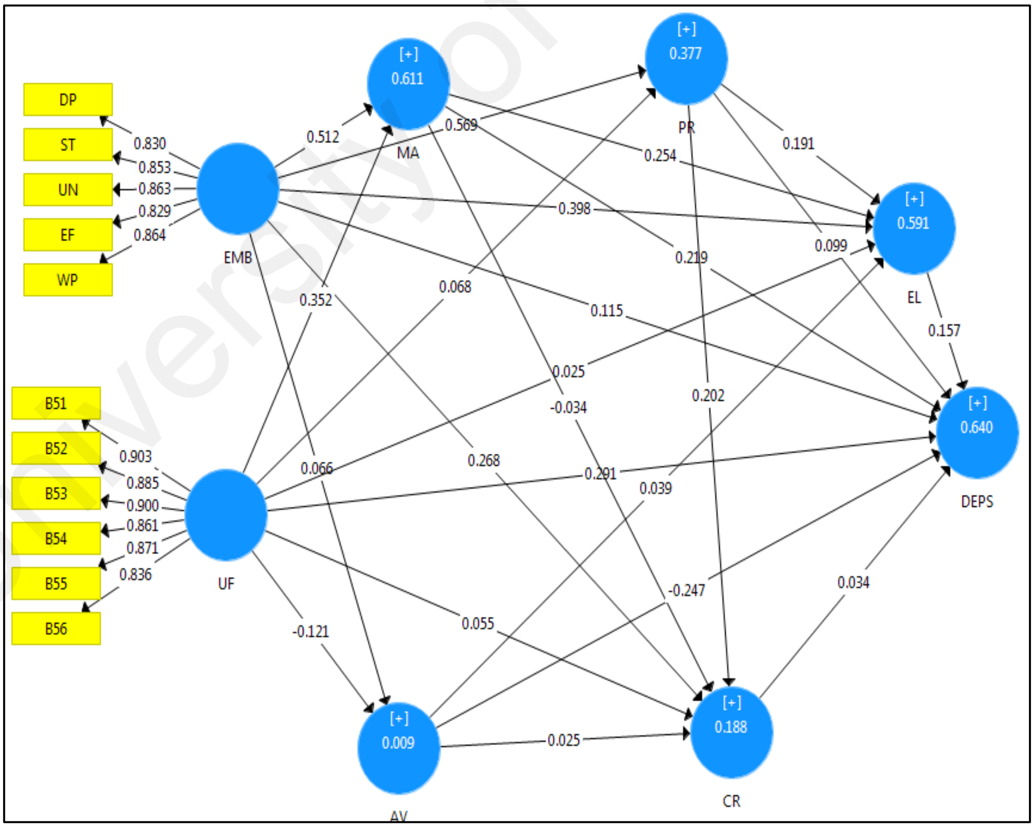

Figure 4.3: Overall structural model showing path coefficients and R^2values

In addition to it, the predictive sample reuses technique was also used for predictive relevance (Q^2) (Akter, D'Ambra et al., 2011; Chin, 2010). Based on the blindfolding procedures, Q^2 evaluates the predictive validity of a complex model by omitting data for a given block of indicators and then predicts the omitted part based on the calculated parameters. Blindfolding procedure can be regarded as a re-sampling process that specifies and deletes data points of the indicators in a systematic way to predict the measurement model of reflective dependent constructs (Hair et al., 2013). Since, Q^2 value can be extracted and calculated for reflective dependent constructs only, that's way researcher used the blind folding to specify the omission distance of (D= 7). For this study, Q^2 was obtained using cross-validated redundancy procedures as suggested by Chin (2010). As shown in Table 4.13, the Q^2values for avoidance goal, mastery goal, performance goal, elaboration, critical thinking and DE problem solving were 0.007, 0.448, 0.259, 0.40, 0.09 and 0.62, respectively. Earlier, Fornell et al. (1981) suggested that a Q^2 value greater than 0 means the model has predictive relevance, whereas a Q^2 value less than 0 means otherwise. Comparing current study values with Fornell et al. (1981) guidelines, it was revealed that all of these values were above zero; indicating acceptable predictive relevance.

4.3.7 Estimation of effect size (f^2)

The effect size (f^2) is the assessment of R^2 in a case when a particular independent construct is removed from the model. It evaluates the impact size of the removed independent construct on the dependent construct (Hair et al., 2013). Since in the present model, dependent/endogenous variables were predicted by more than one predicting/exogenous variable. In such a situation, effect size was important. For effect size calculation, following equation was used;

$$f^2 = (R^2_{included} - R^2_{excluded}) / (1 - R^2_{included}) \qquad 4.2$$

According to Cohen (1988), a f^2 value up to 0.02 shows a small effect, a f^2 value of 0.15 shows a medium effect and a f^2 value of 0.35 shows a large effect. In the current model, the predicted f^2vales for the DE problem solving were 0.02, 0.118, 0.043, 0.02, 0.167, 0.02 and 0.002 for beliefs, usefulness, mastery goal, performance goal and avoidance goal, elaboration and critical thinking respectively. From the f^2 values it was revealed that avoidance goal has the medium effect while the usefulness effect is nearer to medium effect. Whereas the math beliefs, mastery and performance goals have relatively very small effects.

Goodhue, Lewis et al. (2007) suggested that small f^2 does not necessarily imply an unimportant effect. If there is a possibility of occurrence for the extreme moderating conditions and the resulting beta changes are meaningful, then it is important to take these situations into account. Similarly, when the f^2 values of problem solving beliefs and usefulness towards the mediating variables (mastery goal, performance goal and avoidance goal) were considered. It was revealed that in predicting mastery goal and performance goal, the effect of problem solving beliefs is large with f^2 values of 0.412and 0.318 (Table 4.13). However, in case of avoidance goal, the small effect of problem solving beliefs was observed (0.03). Similarly, usefulness has contributions of 0.195 towards mastery goal. Moreover, small size contributions were observed from usefulness towards performance and avoidance goals (Table 4.13). For the elaboration and critical thinking (considering as mediating variables), f^2 values of beliefs were 0.154 and 0.03 respectively. Likewise, for usefulness have relatively very small effects on elaboration and critical thinking.

In the next step, a recommended bootstrapping with 5,000 iterations was performed to examine the statistical significance of the weights of sub-constructs and path coefficient (chin 2008). Detail of the model is provided in Table 4.13. It was noticed that all direct paths were significant except critical thinking. These results had already been discussed in previous section. After investigating direct effect, mediation paths were analyzed. Detail of indirect or mediation path is provided below.

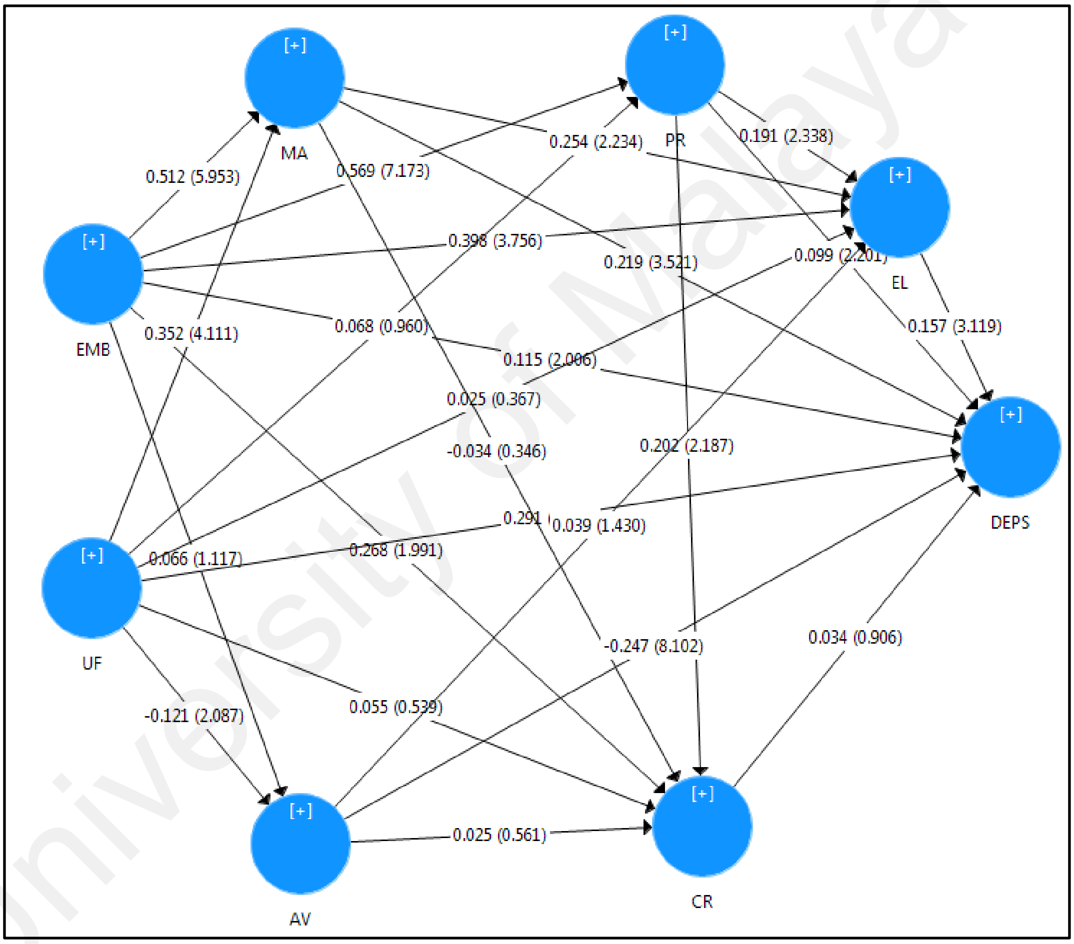

Figure 4.4: Overall structural model for problem solving beliefs, goal orientations and self-regulations

Table 4.13: Predictive relevancy (Q^2) and effect size (f^2)

Constructs	Q^2	f^2 (MA)	f^2 (PR)	f^2 (AV)	f^2 (CR)	f^2 (EL)	f^2 (DEPS)
AV	0.007	-	-	-	0.001	0.004	0.167
CR	0.094	-	-	-	-	-	0.002
DEPS	0.62	-	-	-	-	-	-
EL	0.401	-	-	-	-	-	0.027
MA	0.448	-	-	-	0.00	0.053	0.043
PR	0.259	-	-	-	0.027	0.048	0.02
EMB	-	0.412	0.318	0.003	0.035	0.154	0.02
UF	-	0.195	0.002	0.005	0.002	0.001	0.118

EMB: Epistemological math problem solving beliefs, DEPS: Differential equation problem solving, UF: Usefulness, SRL: Self-regulated learning, EL: Elaboration, CR: Critical thinking, MA: Mastery, PR: Performance, AV: Avoidance

Table 4.14: Structure estimates for direct paths of the complete model

Path name	β	SE	T values	P Values
EMB -> DEPS	0.124	0.057	2.10	0.01
UF -> DEPS	0.29	0.072	4.05	0.00
MA -> DEPS	0.22	0.062	3.52	0.00
PR -> DEPS	0.10	0.045	2.20	0.01
AV -> DEPS	-0.25	0.031	8.10	0.00
CR -> DEPS	0.035	0.037	0.91	0.18
EL -> DEPS	0.16	0.05	3.12	0.00

EMB: Epistemological math problem solving beliefs, DEPS: Differential equation problem solving, UF: Usefulness, SRL: Self-regulated learning, EL: Elaboration, CR: Critical thinking, MA: Mastery, PR: Performance, AV: Avoidance

4.4 Mediation models

Estimates and predictions of the overall structural model were highly encouraging. Hoverer, the model was quite complex due to the various latent variables representing mediating role. To examine the in-depth effects of the each mediator, following mediating models were considered and evaluated.

1. Epistemological math problem solving beliefs and usefulness beliefs → Goal orientations→ Differential equation problem solving ability
2. Epistemological problem solving belief and usefulness→ Self-regulated learning strategies→ Differential equation problem solving ability
3. Goal orientations→ Self-regulated learning strategies→ Differential equation problem solving ability

The detail of each mediating model has been provided in the following sections.

4.4.1 Mediation model 1

To investigate the mediating role of goal orientation between epistemological math problem solving beliefs and differential equation problem solving ability, structural estimates for mediation effects were calculated. Table 4.15 illustrates the estimation results of indirect path between epistemological math problem solving beliefs and differential equation problem solving ability via goal orientations. It was observed that mastery goal ($\beta = 0.12$, $t = 3.03$, $p < 0.01$) and performance goal ($\beta = 0.05$, $t = 2.10$, $p < 0.05$) had partial mediation role with respect to problem solving beliefs. Regarding to avoidance goal, no significant mediation role was observed both for math problem solving beliefs. Furthermore, mastery goals ($\beta = 0.07$, $t = 2.67$, $p < 0.01$) also had a partial mediation role with respect to usefulness, whereas no such effects were observed from performance. Figure 4.5 illustrates the mediation model involving the epistemological math problem solving beliefs, goal orientations and differential equation problem solving.

Further the mediating role of goal orientation between usefulness and differential equation problem solving was also investigated. The findings revealed that in case of usefulness, a partial mediation was observed via mastery and avoidance goal goals. Results showed that mastery goal (β = 0.07, t = 2.67, $p < 0.01$) had relatively more significant mediation results as compared to avoidance goal (β = 0.02, t = 2.01, $p < 0.01$), whereas, no mediation was observed via performance goal.

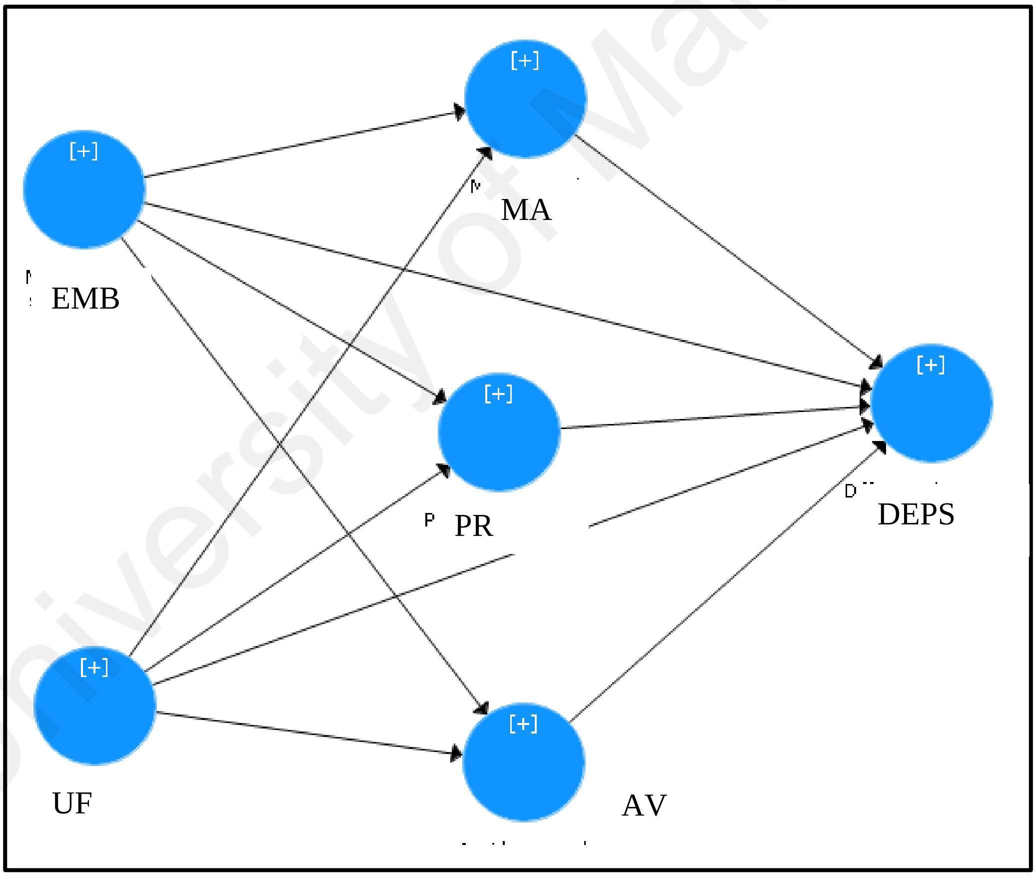

Figure 4.5: A mediation model for beliefs, usefulness and goal orientations

Table 4.15: Structural estimates (hypothesis testing) for mediation model 1

Mediation	β	SE	T Value	p value	Findings
EPB -> MA -> DEPS	0.112	0.037	3.03	0.00	Partial mediation
EPB -> PR -> DEPS	0.056	0.026	2.10	0.01	Partial mediation
EPB -> AV -> DEPS	-0.016	0.014	-1.10	0.13	No mediation
UF -> MA -> DEPS	0.077	0.028	2.67	0.00	Partial mediation
UF -> PR -> DEPS	0.006	0.007	0.87	0.19	No mediation
UF -> AV -> DEPS	0.029	0.014	2.01	0.02	Partial mediation

EMB: Epistemological math problem solving beliefs, DEPS: Differential equation problem solving, UF: Usefulness, SRL: Self-regulated learning, EL: Elaboration, CR: Critical thinking, MA: Mastery, PR: Performance, AV: Avoidance

4.4.2 Mediation model 2

The next step was to find out the mediating role of self-regulated learning strategies between epistemological math problem solving beliefs and differential equation problem solving ability. Table 4.16 illustrates the estimation results. It was observed that elaboration have partial mediation role with respect to problem solving beliefs (β= 0.062, t = 2.40, p < 0.01). However, with respect to usefulness, no such affects were observed from elaboration and critical thinking. Figure illustrates the mediation model involving the epistemological math problem solving belief, self-regulation and differential equation problem solving ability.

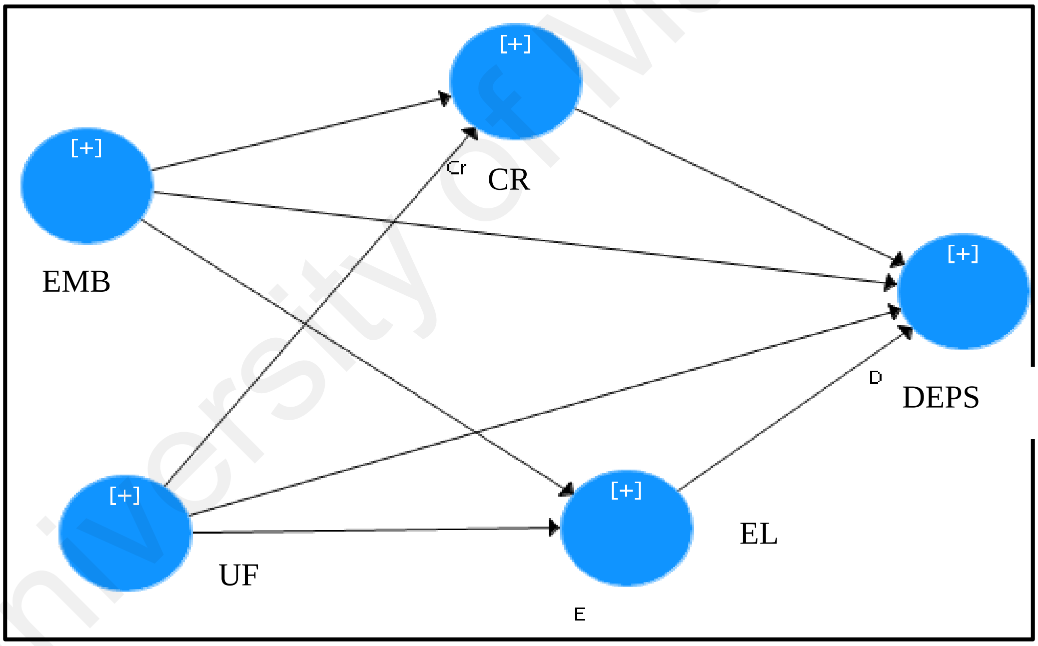

Figure 4.6: Overall structural model for beliefs, self-regulated learning and differential equation problem solving ability

Table 4.16: Structural estimates (hypothesis testing) for mediation model 2

Mediation	β	SE	T Value	p value	Findings
EPB-> EL-> DEPS	0.062	0.025	2.40	0.00	Partial mediation
EPB->CR->DEPS	0.009	0.010	0.83	0.20	No mediation
UF->EL-> DEPS	0.003	0.01	0.35	0.36	No mediation
UF->CR-> DEPS	0.001	0.004	0.46	0.32	No mediation

EMB: Epistemological math problem solving beliefs, DEPS: Differential equation problem solving, UF: Usefulness, SRL: Self-regulated learning, EL: Elaboration, CR: Critical thinking, MA: Mastery, PR: Performance, AV: Avoidance

4.4.3 Mediation model 3

To examine the mediating role of self-regulated learning strategies, mediating model estimations were calculated. This model comprised of goal orientation subscales (mastery, performance and avoidance goal), self-regulated subscales and differential

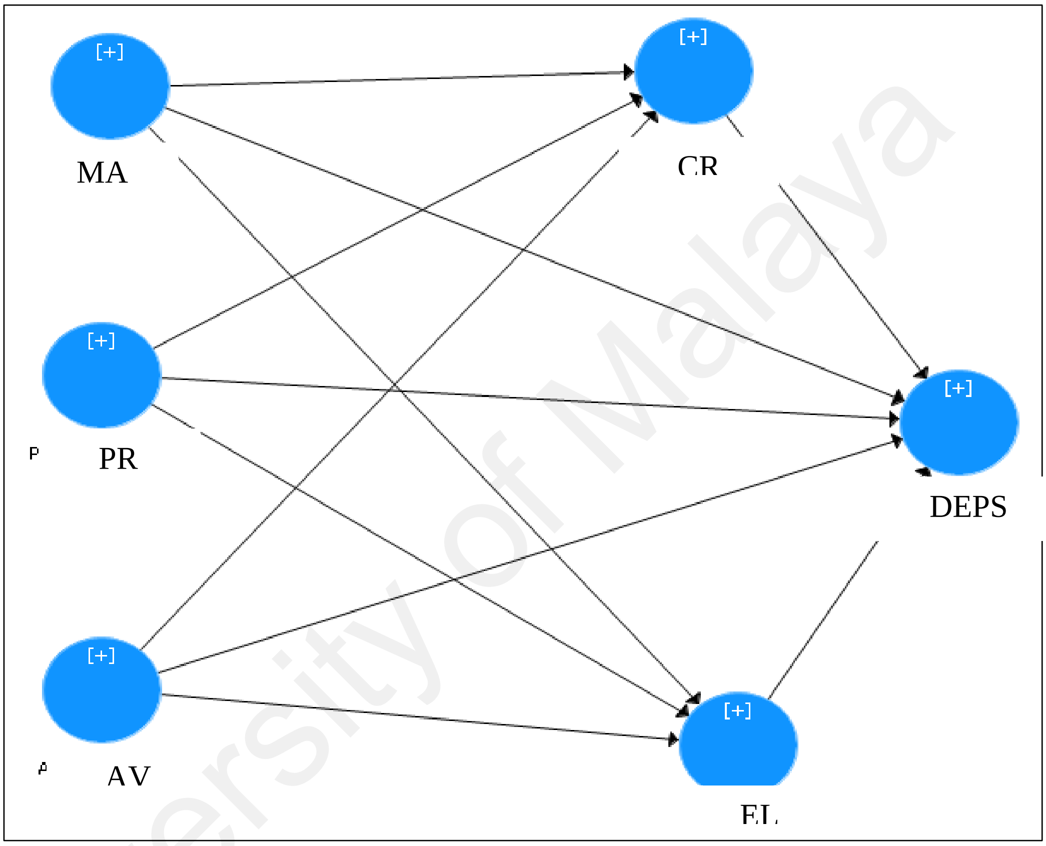

Figure 4.7: Overall structural model for goal orientations, self-regulations and differential equation problem solving

equation problem solving. Figure 4.7 illustrates the mediation model involving the goal orientation, self-regulations and differential equation problem solving. Table 4.17 illustrates the estimation results. It was observed that elaboration ($\beta = 0.03$, $t = 1.96$, $p < 0.05$) had partial mediation role with respect to mastery and performance goal. Moreover, in case of avoidance goal, no such affect were observed from elaboration and critical thinking.

Table 4.17: Structural estimates (hypothesis testing) for mediation model 3

Mediation	β	SE	T Value	p value	Findings
MA -> EL-> DEPS	0.039	0.02	1.82	0.03	Partial mediation
PR -> EL -> DEPS	0.030	0.01	1.96	0.02	Partial mediation
AV -> EL -> DEPS	0.006	0.004	1.32	0.09	No mediation
MA -> CR -> DEPS	-0.001	0.003	-0.32	0.37	No mediation
PR -> CR -> DEPS	0.007	0.008	0.85	0.19	No mediation
AV -> CR -> DEPS	0.008	0.001	0.48	0.31	No mediation

DEPS: Differential equation problem solving, UF: Usefulness, EL: Elaboration, CR: Critical thinking, MA: Mastery, PR: Performance, AV: Avoidance

4.5 Tasks analysis

For the current study, five tasks were developed using the guidelines of Polya (1957) and were also tried to make these tasks aligned with guidelines of Schoenfeld (1985a), which had more focused on the distinction between a mathematical problem and a mathematical task. In addition, these tasks were reflecting non-routine based differential equation problems.

While preparing the task items, their cognitive demands were considered to make sure that they reflect low, moderate, and high levels of complexity. According to Webb (1999) classification system, cognitive complexity level of items is associated with its depth of knowledge rather than ability of students. A low complexity of a question requires students to recall a previously learned concept, whereas, moderate complexity requires more critical thinking, in which students are expected to use reasoning and problem solving strategies, and bring together skills and knowledge from various domains. In addition, a high complexity question involves solving non-routine problems that requires multiple steps and decision point. Based on Webb (1999) classification system, each task was reflecting low, moderate and high complexity level.

These five non-routine differential equation tasks were covering different aspects of the differential equations to relate and solve daily life problems. Task 1, task 2 and task 3 were about population growth, projectile motion and compound interest, respectively. Similarly, task 4 and task 5 were about polio infected people (health and disease), cooking of bakery items (a particular application of Newton's law of heating/cooling).

Algebraic or procedural based approaches were needed to solve task 1, task 2 and task 3. Task 4 was particularly designed to evaluate the differential equation solving skills using graphical methods. Similarly, in the task 5, major trick was to give excess data (about general and particular solutions) to make it puzzle problem, and analyze how students handle this situation.

An adapted Analytic scale for problem solving based on scale of Charles et al. (1987) was used to score these tasks. Authors proposed three categories as understanding, planning and getting answer (Charles et al., 1987). The understanding stage, students needed to interpret or retrieve hidden data. In case of full understanding they were assigned 2 marks otherwise 1. The next phase was planning, in which students had to plan the whole steps, procedures, formulas, and strategies. Students who were successful in their planning phase they were assigned 2 marks, otherwise they were considered partial planner and were assigned 1 mark. In getting an answer phase, the answer of the task, students who used the correct procedure but not completed the solution or made a sign or unit mistakes they were assigned one marks and vice versa.

There was one task, in which students need to construct graph of differential equation tasks. During this phase, students who have done correct procedure, complete the solution without making a sign or unit mistakes they were assigned full marks (2 marks), only if they were successful in graphical representation as well. Overall, in each category, the students with zero marks mean that they could not do or understand it completely. Similarly, the students with one (1) and two (2) marks mean that they did/understood it partially and completely, respectively. Furthermore, students were graded based on the grading system of the Khyber Pakhtunkhwa (KPK), Pakistan. The detail of the grading system is provided in the Table 4.18. The results of five tasks were summarized in the Table 4.19, showing the students marks for each category; understanding, planning and getting answer as suggested by (Charles et al., 1987).

Table 4.18: Grading system of the Khyber Pakhtunkhwa (KPK), Pakistan

Marks Percentage Range	Grade	Remarks
80 % or above	A+	Outstanding
70-79 %	A	Excellent
60-69 %	B	Very good
50-59 %	C	Good
40-49 %	D	Fair
Below 40 %	E	Un-Satisfactory

For task 1, out of 394, 2 percent of the participants were unable to interpret word problems and due to this, they were unable to understand problem completely, whereas, 8 percent participants had shown partial understanding, while 90 percent have shown reasonable understanding. In the next stage, 4 percent entirely failed to plane, 13 percent had partially planed, while 83 percent shown reasonable planning skills. In the third stage, 15 percent entirely failed to get answer, 8 percent had partially answered, while 77 percent had given the correct answer with right units.

Table 4.19: Grading system of the Khyber Pakhtunkhwa (KPK), Pakistan

Percentage Range	Marks Range	Grade	Remarks
80 % or above	25-29	A+	Outstanding
70-79 %	21-24	A	Excellent
60-69 %	18-20	B	Very good
50-59 %	15-17	C	Good
40-49 %	13-14	D	Fair
Below 40 %	0-12	E	Satisfactory

Analysis of the task 2 showed that 2 percent of the participants were unable to understand the problem completely, 5 percent participants had shown partial understanding, while 93 percent had shown reasonable understanding. In the next stage, 3 percent entirely failed to plan, 12 percent had partially planed, while, 86 percent shown reasonable planning skills. In the third stage, 14 percent entirely failed to get answer, 25 percent had partially answered, while 61 percent had given the correct answer with right units. From the analysis of task 3, it was observed that 3 percent of the participants were unable to understand the problem completely, 3 percent participants had shown partial understanding, while 94 percent had shown reasonable understanding. In the next stage, 3 percent entirely failed to plane, 7 percent had partially planed, while 90 percent have shown reasonable planning skills. In the third stage, 10 percent entirely failed to get answer, 68 percent had partially answered, while 22 percent had given the correct answer with right units.

In case of task 4, it was noticed that 5 percent of the participants were unable to understand the problem completely, 12 percent participants had shown partial understanding, while 83 percent shown a reasonable understanding. In the next stage, 6 percent entirely failed to plane, 36 percent had partially planed, while 58 percent have shown reasonable planning skills. In the third stage, 61 percent entirely failed to get answer, 36 percent had partially answered, while 3 percent have given the correct answer with right units.

For task 5, 7 percent of the participants were unable to understand the problem completely, 12 percent participants had shown partial understanding, while 81percent have shown reasonable understanding. In the next stage, 4 percent entirely failed to plane, 13 percent had partially planed, while 83 percent have shown reasonable planning skills. In the third stage, 21 percent entirely failed to get answer, 11 percent had partially answered, while 68 percent have given the correct answer with right units.

Findings of this chapter were further compared and discussed briefly with literature reported values and hypothesized results. Based on these findings, implications were furnished. Detail is provided in the chapter 5.

CHAPTER 5: DISCUSSIONS AND RESEARCH CONCLUSIONS

This chapter presents evaluation and discussions of the results and their relevance with hypothesized research objectives. The present study had seven main research objectives. The main theme of first six objectives was to investigate whether epistemological math problem solving beliefs, usefulness, goal orientations and self-regulated learning strategies (SRL) directly or indirectly affect students DE problem solving ability. To test the predictive ability of the hypothesized model of differential equation problem solving, four research hypotheses were evaluated using Partial Least Square Structural Equation Modeling (PLS-SEM).

In the seventh objective, an additional premise about the problem solving strategies (such as algebraic or graphical) effectively employed in differential equation problem solving at pre- university level, was explored, compared and reported. The detail analysis along with discussions of each hypothesis and sub hypothesis are provided in the next section. The results of this study were elaborated and discussed with respect to relevant literature and previous findings. Finally, analyzing the present study findings, the implications for curriculum and instructional practices were furnished and presented in this chapter. Figure 5.1 illustrates overall framework for this chapter.

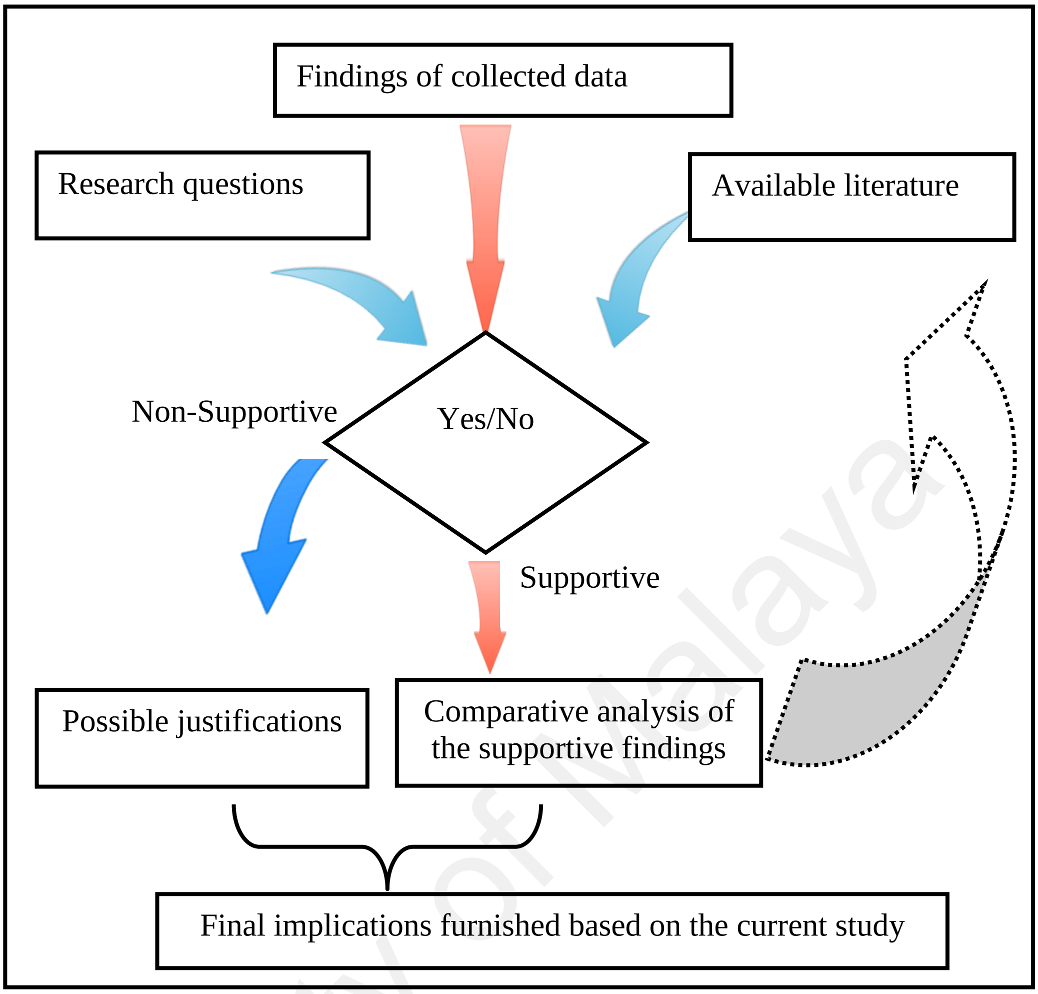

Figure 5.1: Framework for evaluation and discussion of findings with respect to research question

5.1 Evaluation and discussions of the findings with respect to research questions

In this section, findings of each research question have been compared and discussed with available literature results. Findings of the present study were also summarized and provided in the tabulated form.

5.1.1 Research question 1

Do epistemological math problem solving beliefs, usefulness, goal orientations and self-regulated learning (SRL) strategies directly affect differential equation problem solving ability?

To evaluate the first question, first hypothesis "Epistemological math problem solving beliefs, usefulness, goals orientations and self-regulated learning strategies (SRL) have direct effects on differential equation problem solving ability" was divided into following four sub-hypotheses.

H-1.5 Epistemological math problem solving beliefs have positive direct effects on differential equation problem solving ability.

H-1.6 Usefulness has positive direct effect on differential equation problem solving ability.

H-1.7 Goals orientations including mastery, performance and avoidance goals directly affect the differential equation problem solving ability.

H-1.8 Self-regulated learning strategies (SRL) including elaboration and critical thinking have positive direct effects on differential equation problem solving ability.

5.1.1.1 Findings and discussions of first and second sub hypothesis

The findings of first sub hypothesis (H-1.1) revealed that epistemological math problem solving beliefs had strongly affected the differential equation problem solving ability. Detail of these results is provided in Table 5.1. It was observed that epistemological math problem solving beliefs had significant effects (β=0.12, $t = 2.02$, $p< 0.05$). Current results confirmed that students with positive perceptions of problem solving had higher differential equation problem solving ability. The detail results of direct effects are provided in Table 5.1.

The findings of the first sub hypothesis (H-1.1) were well supported by related literature. Several researchers have reported epistemological math problem solving beliefs as a major contributing factor in the study of student's epistemological beliefs (Hofer, 1997; Muis, 2004; Muis et al., 2006). A Student's mathematics related beliefs are the implicitly or explicitly held subjective conceptions that influence their mathematical learning and problem solving (Op't Eynde, De Corte et al., 2002). Mathematics beliefs engage students' and enable them to use different strategies effectively in their problem solving and academic achievement (Lerch, 2004; Schommer- Aikins et al., 2005; Schommer-Aikins et al., 2013a). Other studies also claimed similar findings (Abedlazeez, 2011; Chan, 2003; Stockton, 2010). The students' mathematical problem-solving beliefs were clearly associated with calculus problem solving performance (Stockton, 2010). Differential equation problem solving as a part of calculus might be related to students' problem solving beliefs. Recently, Abedalaziz et al. (2012) examined Malaysians student's epistemological beliefs about mathematical problem solving. Results showed that math problem solving beliefs affect students' achievement.

5.1.1.2 Findings and discussions of second sub hypothesis

The findings of second sub hypothesis (H-1.2) revealed that usefulness had positive effects ($\beta = 0.29$, $t = 4.05$, $p < 0.01$) on differential equation problem solving. Details of these results are provided in Table 5.1. It was observed that usefulness had relatively larger effect on differential equation problem solving ability as compared to epistemological math problem solving beliefs ($\beta = 0.12$, $t = 2.02$, $p < 0.05$). Current results confirmed that students with positive perceptions of usefulness of differential equation problem solving had higher differential equation problem solving ability. The detail results of direct effects are provided in Table 5.1.

The findings of the second sub hypothesis (H-1.2) were also well supported by previous studies (Schommer-Aikins et al., 2013a). In their findings, authors reported that belief in useful of mathematics resulted into better problem solving ability (Schommer-Aikins et al., 2013a). Similar results of belief in useful of mathematics were claimed while studying academic performance such as mathematical problem solving and overall grade point average (Schommer- Aikins et al., 2005). In current study, belief in useful of differential equation problems ($\beta = 0.29$, $t = 4.05$, $p < 0.01$) had significant effect and even shown greater magnitude as compared to math problem solving beliefs.

Students holding sophisticated usefulness beliefs or give value the differential equation problems in daily life were more successful in solving those problems. Overall, these results confirmed that students with positive perceptions about differential equation problem solving had higher ability or skills to solve differential equation problems.

5.1.1.3 Findings and discussions of third sub hypothesis

Similarly, evaluating the third sub hypothesis, it was noticed that both mastery and performance goals positively affected the differential equation problem solving. The detail results of direct effects are provided in Table 4.14, while compared direct effects are provided in Table 5.1. It was observed that mastery goal ($\beta = 0.22$, $t = 3.52$, $p < 0.01$), had relatively larger effect as compared to performance goal ($\beta = 0.10$, $t = 2.20$, $p<0.05$). Similarly, the avoidance goal ($\beta = -0.25$, $t = 8.10$, $p < 0.01$) had also significant but negative estimates towards differential equation problem solving. These results confirmed that students with positive motivations were more successful in differential equation problems solving. Related achievement goal theory and literature had well supported the findings of the third sub-hypothesis.

According to achievement goal theory, goal orientations provide a frame work for interpretation and reaction to tackle a situation or event (Dweck et al., 1988). This framework assumed that people's differences in selecting various goals are related to their achievement behaviors that lead to different emotional, motivational, cognitive and behavioral outcomes. Within this framework, an approach goal further comprises the achievement motivation and active avoidance characteristics. Learners influenced by their achievement motivation are able to learn and complete a task due to their willingness to outperform. Whereas, active avoidance mediates learners simply to withdraw from learning in order to avoid revealing their inability in front of others.

Therefore, both mastery and performance goals represent positive motivational factors that swift learners to invest consistent efforts. While, avoidance goal is a part of active avoidance and it represents an impassive and negative motivational attitude that may impose destructive effects on learning (Wolters, 2004).

Results of current study were well aligned with the study of Wolters et al. (1996), who reported that the adoption of mastery goals positively related to achievement. In addition, these results were also well supported by several others researchers (Kaplan, Middleton et al., 2002; Midgley, Kaplan et al., 2001), in which mastery goals are thought to be most beneficial for all students across socio-emotional, cognitive, and achievement outcomes. Recently, Kayis et al. (2015) also reported that master goal oriented students are highly motivated to improve their ability and knowledge by reviewing learning material as an opportunity to improve. These students attempt harder task and do more hard work to achieve a higher level of knowledge.

However, the findings of a few studies were opposite to this study (Elliot et al., 1997; Skaalvik, 1997). These authors have reported a null relationship between mastery goal and achievements. The reason might be the conjunction of performance and mastery goals had resulted into some positive outcomes such as cognitive engagement but it might not be beneficial for other outcomes (e.g., help seeking). Similarly, the present study revealed that performance goal had positive effect on the differential equation problem solving. The previous findings of several researchers were well supporting the findings of the present study, claiming that performance goal oriented students can perform well (Elliot et al., 1997; Middleton et al., 1997). Recently, Kayis et al. (2015) also observed that performance goal oriented choose an easier task so that they feel comfortable and may found more chances of success. They tend to be competitive in achieving more success as compared to their class mates or peer, as a result these students prefer superficial learning strategies while studying. However, few studies also exist, which reported no relationship between performance goals and performance (Butler, 1993; Button et al., 1996; Coutinho, 2007). In case of avoidance goal, the negative direct effect was observed (Table 5.1) ,which was a good agreement with literature (Elliot et al., 2001; Skaalvik, 1997). On the other hand, these findings also contradicted few studies, reporting a null relationship between these variables (Elliot et al., 1997; Kingir et al., 2013). Overall, researchers concluded that both mastery and approach goal oriented students are motivated to achieve a higher level of academic success than avoidance goal oriented (Hall, 2015; Kayis et al., 2015; Weinberg et al., 2014).

Recently, Kayis et al. (2015) highlighted some common characteristics of avoidance goal orientation as, avoiding complicated tasks, mismanagement, and leaving tasks incomplete. Keeping in mind these characteristics, it can be concluded that avoidance goal oriented students found non-routine differential equation task very difficult.

Therefore, either they tried to avoid those tasks or they left them unfinished. As a result, these students were not successful in getting high score.

Overall, it was confirmed that student's holding positive motivational beliefs about differential equation problem solving had higher ability to solve differential equation problems. In addition, mastery oriented students, were remained more successful in differential equation problem solving as compared to others.

5.1.1.4 Findings and discussions of fourth sub hypothesis

Similarly, evaluating the fourth sub hypothesis, it was noticed that both elaboration and critical thinking also had positive effects on differential equation problem solving. It was also observed that the elaboration had more significant estimates ($\beta = 0.16$, $t = 3.12$, $p < 0.01$) as compared to critical thinking ($\beta = 0.03$, $t < 1.96$, $p > 0.05$). The results are provided in Table 4.14.

In case of elaboration, the results were matched with the study of (Fadlelmula et al., 2015). The authors have reported that elaboration was significantly related to mathematics achievement. Several other researchers also reported that use of different learning strategies (goal setting, self-evaluation, and transforming) serve as an important predictor of their academic performance and mathematics problem solving (Pape et al., 2003; Pintrich et al., 1990; Valle, Núñez et al., 2008).

Similarly, in the present case, findings of critical thinking ($\beta = 0.03$, $t < 1.96$, $p > 0.05$), had shown non-significant results. These findings were supported by literature. Savoji, Niusha et al. (2013) evidently proved that among self-regulated learning strategies, critical thinking has no contribution towards academic achievement. These results of present study were also seen consistent with the study of Fadlelmula et al. (2015), who had claimed that among self-regulated learning strategies only elaboration is significantly related to mathematics achievement. Authors also explained that the applying limited set of learning strategies might contradict the previously reported findings. Therefore, in present study, non-significant value of critical thinking might be appeared because of applying only two strategies (elaboration and critical thinking). Other reason might be the complex nature of relationships between self-regulatory strategies and achievement. Ablard et al. (1998) reported that some high-achieving students succeed without the use of self-regulated learning strategies.

Overall, it was confirmed that students applying self-regulatory strategies had shown higher ability or skills to solve differential equation problems. Moreover, elaboration had yielded more prominent results as compared to critical thinking.

5.1.2 Research question 2

Do goal orientations play a mediating role between epistemological math problem solving beliefs and differential equation problem solving ability?

To evaluate the second research question, second hypothesis "Epistemological math problem solving beliefs have positive indirect effect on differential equation problem solving ability via goal orientations" was divided into following three sub-hypotheses;

H-2.4 Mastery goal play a mediating role between epistemological math problem solving beliefs and differential equation problem solving ability.

H-2.5 Performance goal play a mediating role between epistemological math problem solving beliefs and differential equation problem solving ability.

H-2.6 Avoidance goal play a mediating role between epistemological math problem solving beliefs and differential equation problem solving ability.

The second set of research hypotheses investigated the mediating role of goal orientation among epistemological math problem solving beliefs and differential equation problem solving ability and included three sub-hypotheses. Findings revealed that epistemological math problem solving beliefs strongly affected the differential equation problem solving ability via mastery and performance goals. However, the effect of avoidance goal was noticed non-significant and negative.

As per first sub-hypothesis (H-2.1), mastery goal played a mediating role between epistemological math problem solving beliefs and differential equation problem solving. The mastery goal results were quite significant ($\beta = 0.12$, $t = 3.03$, $p<0.01$) and confirmed that mastery oriented students had affirmative skills to solve differential equation problem solving. Similarly, results of performance goal supported the second sub-hypothesis (H-2.2) and confirmed the mediating role of performance goal between epistemological beliefs and differential equation problem solving ($\beta = 0.05$, $t = 2.10$, $p < 0.05$). Conversely, non-significant estimates were found for avoidance goal between epistemological math problem solving beliefs and differential equation problem solving.

Mediations results were well supported by related literature. Many researchers believed that epistemological beliefs are important predictors of achievement goal orientation (Bråten et al., 2004, 2005; Ghorban-Jahromi, 2007; Rastegar et al., 2010). Kizilgunes et al. (2009) also claimed a positive association of epistemological beliefs and goal orientation. Similar results were reported by Rastegar et al. (2010). Authors have empirically proved the mediating role of achievement goals, mathematics self-efficacy, and cognitive engagement between epistemological beliefs and math achievement. In addition to mediation role, negative and non-significant results of avoidance goal had also supported the findings of present study. Overall, it may be concluded that students with positive perceptions about mathematics had more differential equation problem solving ability.

5.1.3 Research question 3

Do goal orientations play a mediating role between usefulness and differential equation problem solving ability?

To evaluate the third research question, proposed third hypothesis "usefulness has indirect effect on differential equation problem solving ability via goal orientations" was divided into following three sub-hypotheses;

H-3.4 Mastery goal play a mediating role between usefulness and differential equation problem solving ability.

H-3.5 Performance goal play a mediating role between usefulness and differential equation problem solving ability.

H-3.6 Avoidance goal play a mediating role between usefulness and differential equation problem solving ability.

The third set of research hypotheses investigated the mediating role of goal orientation between usefulness and differential equation problem solving. The findings revealed that in case of usefulness, a partial mediation was observed via mastery and avoidance goal goals. Results showed that mastery goal ($\beta = 0.07$, $t = 2.67$, $p < 0.01$) had relatively more significant mediation results as compared to avoidance goal ($\beta = 0.02$, $t = 2.01$, $p < 0.01$), whereas, no mediation was observed via performance goal. Figure 4.5 illustrates the mediation model involving the epistemological math problem solving beliefs, goal orientations and differential equation problem solving. Overall findings had supported hypothesis H-3.1 and H-3.3 but did not support the sub hypothesis H-3.2. The detail results of mediation effects are provided in Table 5.2.

The findings of the hypothesis “H-3.1 and H-3.3” were partial supported by several studies, in which direct effects of belief in useful of mathematics was studied (Schommer-Aikins et al., 2013a). Researchers observed that belief in useful of mathematics had enabled students to solve mathematical problems (Schommer-Aikins et al., 2013a). However, mediating role of belief in useful of mathematics was not explored for problem solving, prior to current study (up to researcher knowledge). Therefore, present study findings about mediation role of belief in useful of mathematics (particularly, for differential equation) were quite novel. Overall it may be concluded that students with positive perceptions about differential equation and its usefulness had more differential equation problem solving ability.

5.1.4 Research question 4

Do self-regulated learning (SRL) strategies play a mediating role between epistemological math problem solving beliefs and differential equation problem solving ability?

To evaluate the fourth research question, proposed fourth hypothesis “epistemological math problem solving beliefs have indirect effect on differential equation problem solving ability via self-regulated learning (SRL) strategies” was divided into following two sub-hypothesis;.

H-4.3 Epistemological math problem solving beliefs have positive indirect effect on differential equation problem solving via elaboration ability.

H-4.4 Epistemological math problem solving beliefs have positive indirect effect on differential equation problem solving via critical thinking ability.

The findings of this study revealed that epistemological math problem solving beliefs ($\beta = 0.062$, $t = 2.40$, $p < 0.01$) strongly affected the differential equation problem solving ability via elaboration. However, via critical thinking no significant effects were observed. In other words, for elaboration both the findings and sub-hypothesis (H-4.1) were aligned. But in case of critical thinking, findings of the current study were different from sub-hypothesis (H-4.2).

These results were also aligned with previous studies (Rastegar et al., 2010). The authors have investigated the effect of epistemological beliefs on mathematics performance through cognitive strategies, achievement goal and mathematics self-efficacy. The authors claimed a meditation role of elaboration between epistemological math problem solving beliefs and the problem solving (Rastegar et al., 2010). Savoji et al. (2013) also reported similar results and proved that the dimensions of epistemological beliefs and motivational strategies predicts academic achievement. In the context of mathematics, Muis (2004) described that beliefs are positively correlated with more effective learning strategies, which subsequently positively influence learning and achievement. In addition to it, Stockton (2010) investigated the relationships between epistemological beliefs and self-regulated learning processing, while students were engaged in solving calculus problem. Results showed that math problem solving beliefs are related to various facets of self-regulated learning processing and these beliefs are related to their problem solving performance.

On the other side, current study results showed that epistemological math problem solving beliefs did not affect the differential equation problem solving via critical thinking. These results were partially supported by the study of Fadlelmula et al. (2015), who had claimed that critical thinking had not affected significantly on mathematics achievement. Authors also explained that the applying limited set of learning strategies might contradict the previously reported findings. Therefore, in present study, non-significant value of critical thinking might be appeared because of applying only two strategies (elaboration and critical thinking).

Present study results contradicted the findings of Schommer (1990), who had suggested that epistemological beliefs are related to self-regulated learning strategies and achievement. These contradictions might be due to the technical and conceptual issues resulted from the self-report measures. This happening was best illustrated by Muis (2004). The researcher stated that the majority of studies relating to epistemological beliefs to cognitive, motivation, and achievement relied solely on self-report measures. Therefore, there might be some technical and conceptual issues that had limited the contribution of studies due to self-reported measures of these constructs. Winne, Jamieson-Noel et al. (2002) also found students error in self-reports about actual studying events.

5.1.5 Research question 5

Do self-regulated learning (SRL) strategies play a mediating role between usefulness and differential equation problem solving ability?

To evaluate the fifth research question, proposed fifth hypothesis "usefulness have indirect positive effect on differential equation problem solving ability via self-regulated learning (SRL) strategies" was divided into following two sub-hypotheses;

H-5.3 Usefulness has positive indirect effect on differential equation problem solving ability via elaboration.

H-5.4 Usefulness has positive indirect effect on differential equation problem solving ability via critical thinking.

In case of usefulness, no mediation was observed via both elaboration and critical thinking. Findings had not supported sub hypotheses "H-5.1 and H5.2". These findings were opposite to current research findings goals, in which partial mediation was observed via mastery and avoidance goal goals. Figure 4.5 illustrates the mediation model involving the usefulness, elaboration and critical thinking and differential equation problem solving. The detail results of mediation effects are provided in Table 5.3. Although direct effect of belief in useful of mathematics was investigated in previous studies (Schommer-Aikins et al., 2013a). Nevertheless, mediating role of self-regulated learning (SRL) between belief in useful of mathematics and problem solving ability was not explored prior to current study (up to researcher knowledge). Therefore, present study findings about mediation role of self-regulated learning (SRL) between belief in useful of mathematics and problem solving ability were quite novel.

These contradictions of no mediation might be occurred due to the technical and conceptual issues resulted from the self-report measures. This happening was best illustrated by Muis (2004), who reported that the majority of studies relating to belief systems to cognitive, motivation, and achievement relied solely on self-reporting measures. For that reason, there might be some technical and conceptual issues that had limited the contribution of studies due to self-reported measures of these constructs. Winne et al. (2002) also reported similar results about self-reporting measures.

Overall, it can be concluded that self-regulated learning (SRL) had not mediating role between usefulness and differential equation problem solving. Summary of mediation results are provided in Table 5.3 and more details are provided in chapter 4.

5.1.6 Research question 6

Do self-regulated learning strategies play a mediating role between goal orientations and differential equation problem solving ability?

To evaluate the sixth research question, proposed sixth hypothesis “goal orientations have indirect effect on differential equation problem solving ability via self-regulated learning strategies” was divided into following six sub-hypotheses;

H-6.7 Mastery goal has positive indirect effect on differential equation problem solving ability via elaboration.

H-6.8 Mastery goal has positive indirect effect on differential equation problem solving ability via critical thinking.

H-6.9 Performance goal has positive indirect effect on differential equation problem solving ability via elaboration.

H-6.10 Performance goal has positive indirect effect on differential equation problem solving ability via critical thinking.

H-6.11 Avoidance goal has negative indirect effect on differential equation problem solving ability via elaboration.

H-6.12 Avoidance goal has negative indirect effect on differential equation problem solving ability via critical thinking.

The findings of this study revealed that elaboration had a mediation role for both mastery and performance goals. However, no such effect was observed for avoidance. The detail results of mediation effects are provided in Table 5.4.

Findings of the current study showed that elaboration (also part of cognitive strategies) partially mediated the relationship between goal orientations (mastery and performance goal) and differential equation problem solving. These results were aligned with the study of Mohsenpour (2006). The author evidently proved that use of cognitive strategies partially mediate the relationship between achievement goals and achievement. Mastery goal perspective acknowledged that the conjunction of performance goal with mastery goal might be adaptive for some outcomes such as cognitive engagement but was not beneficial for other outcomes (e.g., help seeking) (Elliot et al., 1997; Skaalvik, 1997)

In case of mastery goal, Wolters (2004) empirically proved that mastery goal contributed towards self-regulated learning strategies. Recently, Fadlelmula et al. (2015) also investigated the interrelationships among students motivational beliefs such as achievement goal orientations, self-efficacy, perception of class room goal structure), use of self-regulated learning strategies and mathematics achievement. Among achievement goals only mastery was significantly related to use of self-regulated learning strategies and math achievement. Hence, author concluded that when students value learning for its own sake and focus on expanding their skills, they tend to use more learning strategies and hence, became successful in mathematics. However, in present study, performance goal results ($\beta = 0.03$, $t = 1.96$, $p < 0.05$), had shown that it was positively and significantly related to use of elaboration (self-regulated learning strategies) and math achievement as contrary to Fadlelmula et al. (2015). Several other researchers also reported that only mastery goal predicts deeper level strategies, such as elaboration (Elliot et al., 2001; Elliot et al., 1999; Yumusak, Sungur et al., 2007). In contrast to literature, current study results showed that performance goal were also linked to strategy use such as elaboration. Students, who tried to outperform others, used more strategies to achieve better results in differential equation problem solving. These findings are align with the study of Kadioglu et al. (2014), where performance goal was linked to strategy use.

Overall, with respect to elaboration, the results of indirect path were mediated through mastery and performance goal. However, the results of performance goal were more positive and significant as compared to other goals. Findings of current study can be attributed to the common evaluation practices in the Pakistan educational context such as grade focused evaluation, dominance of the entrance exam and also, addition of secondary school score to calculate final entry test results. Therefore, contribution of performance goal in solving differential equation problems model was believable. Beside this, mastery goal results were strongly positive as compared to avoidance goal. It is consistent with previous findings, where mastery oriented learners exhibited more self-regulation than avoidance (Elliot et al., 1999; Skaalvik, 1997).

Mathematics educators also considered the importance of these motivational factors, and hence, suggested that these factors alone are not enough for fostering student's mathematics achievement. Rather it is use of deep learning strategies which mediate the association between motivational factors and mathematics achievement. Furthermore, regarding to indirect relation, Barron and Harackiewicz (2001) suggested that optimal achievement outcomes may occur when student pursue both mastery and performance goals together, because when they have the option of pursuing both types of goals they can better negotiate their achievement experiences by focusing on the achievement goal that is more relevant at a particular time.

On the other hand, critical thinking did not mediate the relationship between performance goal and differential equation problem solving. These results were supporting the study of Fadlelmula et al. (2015), where performance goal did not mediate the relationship between self-regulation and math achievement. Author suggested that students, who tried to outperform other students, might not be able to use more strategies. Therefore, these students were unable to get math achievement.

In case of avoidance goal via (elaboration and critical thinking) no significant effects were observed. As a result, we concluded that self-regulated learning strategies had no mediation role between avoidance goal and differential equation problem solving. These results were consistent with Fadlelmula et al. (2015) research findings. Authors reported that students who avoid looking incompetent may not use more learning strategies nor get achievement in mathematics. Kadioglu et al. (2014) also claimed that avoidance goal is not a significant predictor of learning strategies. Goal theorist posited that once learner adopt an avoidance goal, they became passive and pessimistic about their learning and tend to withdraw from learning, as a result self-regulation does not happen (Wolters, 2004).

Critical thinking is also one of the important construct of self-regulation, but there are only a few research investigations, which have explored this construct (Huy Phuong Phan, 2008, 2009). Therefore, research pertaining to achievement goals and critical thinking is still in its early years and is limited to a few studies. Conversely, it is evidently proved that mastery goals exert a positive effect on critical thinking and facilitates a better understanding of knowledge and in the development of skill improvement (Huy Phuong Phan, 2008, 2009).

Again, via critical thinking no significant effects from mastery, performance, and avoidance goal were observed. Therefore, findings had supported only two sub hypotheses H-6.1and H-6.3. Evaluations of each research questions with the proposed sub hypotheses are summarized in Table 5.2 to Table 5.4.

Table 5.1: Summary of sub-hypotheses for the evaluation of first research question

Path name	Findings (Significance)	Proposed Sub-hypothesis	Literature
EMB -> DEPS	Significant	Supported	Supported
UF-> DEPS	Significant	Supported	Supported
MA -> DEPS	Significant	Supported	Supported
PR -> DEPS	Significant	Supported	Supported
AV -> DEPS	Significant	Supported	Supported
CR -> DEPS	Not-significant	Not supported	Not supported
EL -> DEPS	Significant	Supported	Supported

EMB: Epistemological math problem solving beliefs, DEPS: Differential equation problem solving, UF: Usefulness, SRL: Self-regulated learning, EL: Elaboration, CR: Critical thinking, MA: Mastery, PR: Performance, AV: Avoidance

Research question 1: Do epistemological beliefs about differential equation problem solving, self-regulatory learning (SRL) strategies and goal orientation directly affect differential equation problem solving ability?

Table 5.2: Summary of sub-hypotheses for the evaluation of second and third research questions

Path name	Findings (Mediation)	Proposed Sub-hypothesis	Literature
EMB -> MA -> DEPS	Partial mediation	Supported	Supported
EMB > PR -> DEPS	Partial mediation	Supported	Supported
EMB -> AV -> DEPS	No mediation	Not Supported	Supported
UF -> MA -> DEPS	Partial mediation	Supported	Supported
UF -> PR -> DEPS	No mediation	Not supported	Supported
UF -> AV -> DEPS	No mediation	Not supported	Supported

EMB: Epistemological math problem solving beliefs, DEPS: Differential equation problem solving, UF: Usefulness, SRL: Self-regulated learning, EL: Elaboration, CR: Critical thinking, MA: Mastery, PR: Performance, AV: Avoidance

Research question 2: Do goal orientations play a mediating role between epistemological math problem solving beliefs and differential equation problem solving ability?

Research question 3: Do goal orientations play a mediating role between usefulness and differential equation problem solving ability?

Table 5.3: Summary of sub-hypotheses for the evaluation of fourth and fifth research questions

Path name	Findings (Mediation)	Proposed Sub-hypothesis	Literature
EMB -> EL -> DEPS	Partial mediation	Supported	Supported
EMB -> CR -> DEPS	No mediation	Not supported	Not supported
UF -> EL -> DEPS	No mediation	Not supported	Not supported
UF -> CR -> DEPS	No mediation	Not supported	Not supported

EMB: Epistemological math problem solving beliefs, DEPS: Differential equation problem solving, UF: Usefulness, EL: Elaboration, CR: Critical thinking,
Research question 4: Do self-regulated learning (SRL) strategies play a mediating role between epistemological math problem solving beliefs and differential equation problem solving ability?
Research question 5: Do self-regulated learning (SRL) strategies play a mediating role between usefulness and differential equation problem solving ability?

Table 5.4: Summary of sub-hypotheses for the evaluation of sixth research question

Path name	Findings (Mediation)	Proposed Sub-hypothesis	Literature
MA -> EL -> DEPS	Partial mediation	Supported	Supported
PR -> EL -> DEPS	Partial mediation	Supported	Supported
AV -> EL -> DEPS	No mediation	Not supported	Supported
MA -> CR -> DEPS	No mediation	Not supported	Not supported
PR -> CR -> DEPS	No mediation	Not supported	Supported
AV -> CR -> DEPS	No mediation	Not supported	Supported

DEPS: Differential equation problem solving, SRL: Self-regulated learning, EL: Elaboration, CR: Critical thinking, MA: Mastery, PR: Performance, AV: Avoidance
Research question 6: Do self-regulated learning (SRL) strategies play a mediating role between goal orientation and differential equation problem solving ability?

5.1.7 Research question 7

Does algebraic approach yield better results than graphical approach for differential equation problem solving?

To evaluate the seventh hypothesis "algebraic approach yields better results than graphical approach for differential equation problem solving" was critically compared with marks obtained in the assessment test containing five non-routine differential equation tasks. Findings of the current study had supported hypothesis.

In this assessment test, task 1, task 2 and task 3 were about population growth, projectile motion and compound interest, respectively. Similarly, task 4 and task 5 were about polio infected people (health and disease), cooking of bakery items (a particular application of Newton's law of heating/cooling). Algebraic or procedural based approaches were needed to solve task 1, task 2 and task 3. Likewise, Task 4 was particularly designed to evaluate the differential equation solving skills using graphical methods. Similarly, in the task 5, major trick was to give excess data (about general and particular solutions) to make it puzzle problem, and analyze how students handle this situation. Careful analysis of results has provided an additional premise about the problem solving strategies (such as algebraic or graphical), effectively employed in differential equation problem solving at pre- university level. Findings of the current study had supported hypothesis.

Analysis revealed that in both of the task1 and task 2, it was asked to find the explicit general solution followed by initial value solutions. However, the nature of the problem was different. Task 1 was from the biological based problem interrelating population growth of bacteria, while task 2 was from physics field related to projectile motion. For both of these tasks, algebraic or procedural based approaches were needed to solve. It was observed that during understanding and planning phases, results were improved in task 2 as compared with task 1. However, in getting answer phase the trends were seen opposite to previous phases' findings.

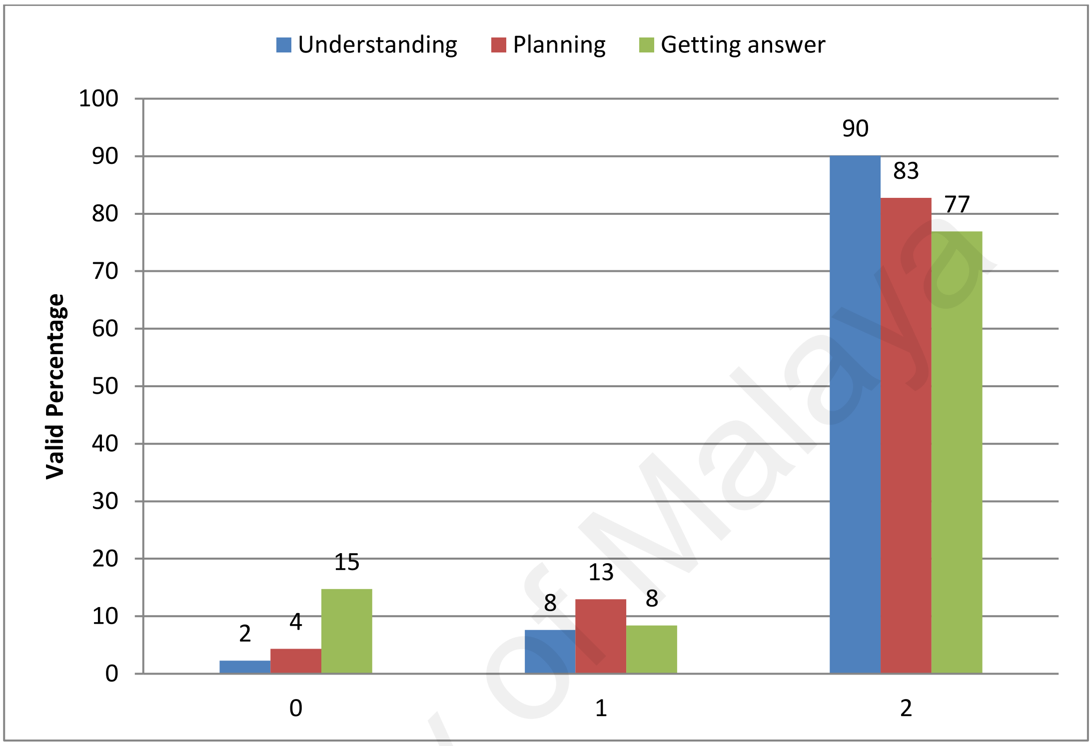

Figure 5.2: Findings of task 1 in terms of percentage success of students while they engaged in DE problem solving

Overall, findings showed high performance in both of these tasks were well supporting the previous studies, which had claimed that algebraic approach predominates in traditional differential equation course, yielding a relatively good performance (Arslan, 2010b; Rowland, 2006). Furthermore, Arslan (2010b) elaborated that procedural learning in traditional differential equation course confines students to mastering and applying some algebraic techniques.

Performance trend was different in first two phases (understanding and planning) as compared with last phase. This may be explained on the basis of previous studies, in which several researchers have suggested "domain-specificity" as a major contributing factor in the study of student's epistemological beliefs resulting into high performance or mathematics achievements because their beliefs vary with respect to the domain of the study (Hofer, 1997; Muis, 2004; Muis et al., 2006; Rowland, 2006).

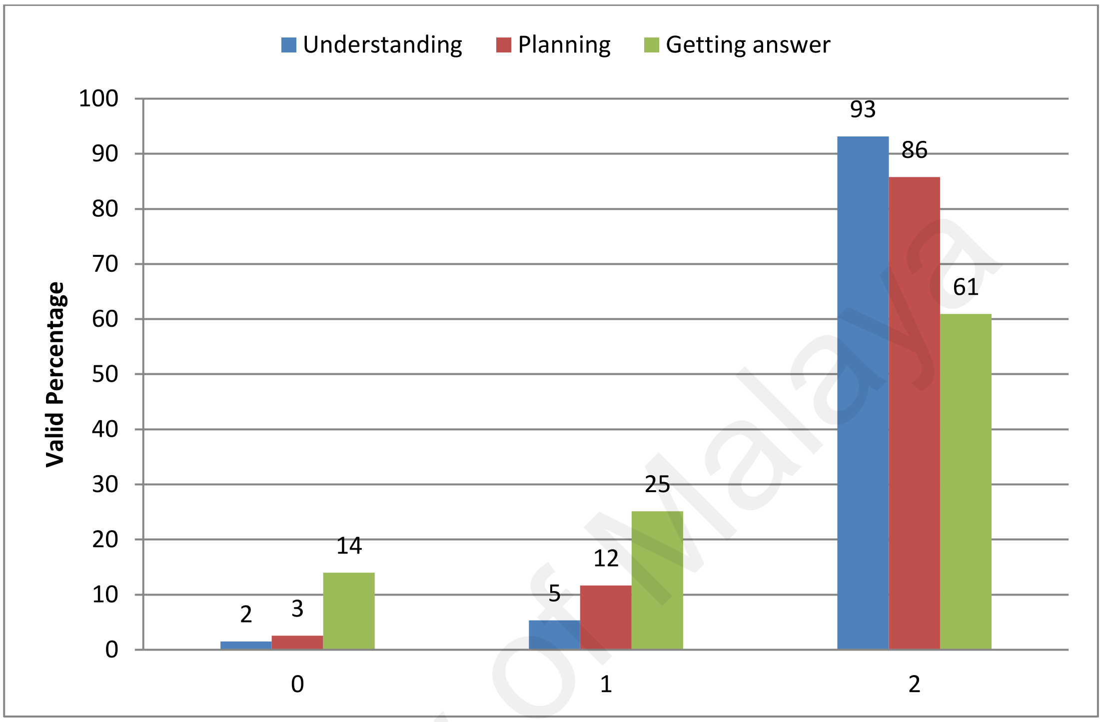

Figure 5.3: Findings of task 2 in terms of percentage success of students while they engaged in DE problem solving

Regarding to the interpretation of algebraic expression, Rowland et al. (2004) have reported that students with physics background were able to recognize dv/dt as representing acceleration and recognized that they have to find out an equation for acceleration instead of velocity. Rowland (2006) further elaborated that students thought in term of rate of change. Similar results are reported by Arslan (2010b). As in the present case, task 2 had specific physics domain, therefore it was easy to understand and solve by mathematics students. In the third phase, opposite trend in getting answer, may be due to the complex nature of answer, calculation errors and different units involved for the task 2 (Camacho-Machín et al., 2012b; Rowland, 2006; Rowland et al., 2004).

For the task 3 (compound interest), similar positive trend was notice. Students showed very high percentage 94 percent and 90 percent both in understanding and planning phases respectively. However, at final phase, only 22 percent were able to give the correct answer with right units. In this task, researcher asked a logical question about function increase, decrease after algebraic calculation. Since the logical questions need high level of understandings, critical thinking and some more tricks to be solved properly. In procedural based learning, critical does not play active role, also cleared from the findings of present study (Figure 5.2), therefore logical part was not solved properly.

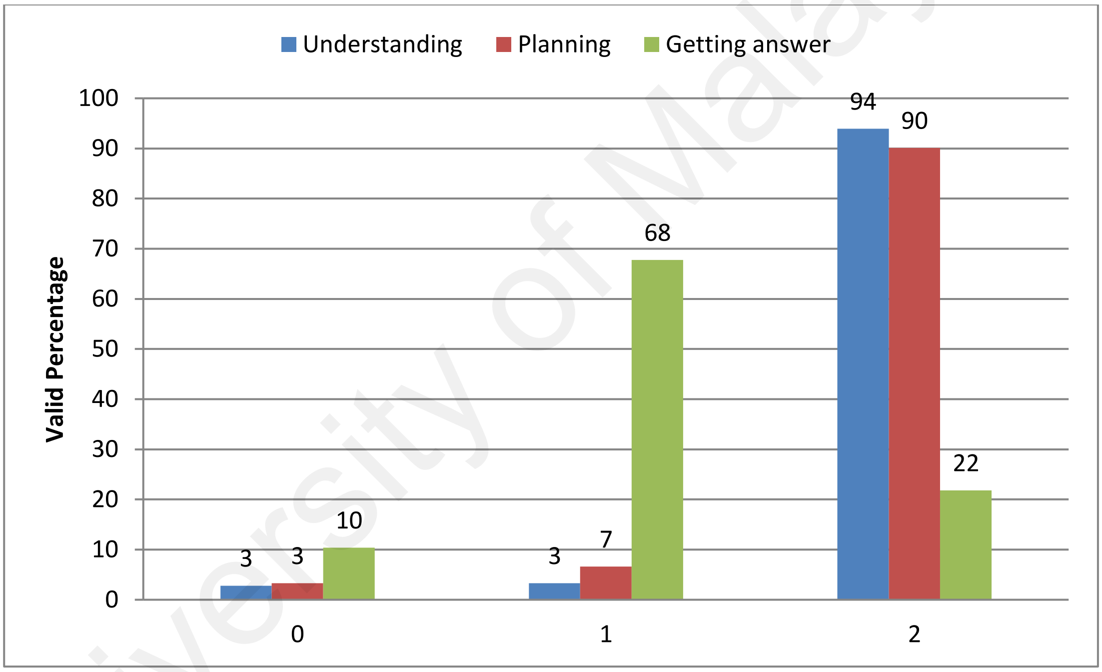

Figure 5.4: Findings of task 3 in terms of percentage success of students while they engaged in DE problem solving

Task 4 had needed graphical or qualitative based skills to be solved. Results were quite attention-grabbing and opposite to findings of previous three tasks. Only 3 percent of the students were observed successful in solving the differential equation graphically and getting appropriate answers. Overall, the student's performance was noticed very low, as compared to all other tasks. The reason for student's low performance in problem solving may be due to their preference of algebraic approach and hence, these students were unable to construct graphical representation. Another, reason can be Pakistan education system, because it focuses only on acquiring mathematical skills and strategies to solve mathematical problems. Therefore, students have totally ignored the application of those problem in the real word and also in other subjects. Although, it is quite possible to pass examinations by seeking, grasping or memorizing some procedural techniques with slight understanding of their meaning. Mostly rules and algorithm dominate and hence, the concept of mathematics became difficult to understand. Therefore, like many other countries, students in Pakistan have also difficulties in mathematics problem solving.

Several other researchers, reported that accepting the system of graphical representation can widen the understanding of differential equation solution concept (Camacho-Machín et al., 2008; Machín et al., 2009). However regarding to the graphical system of representation, Camacho-Machín et al. (2012d) also pointed out that students make mistakes when they have to graphically represent basic functions such as linear, exponential, trigonometric and hyperbolic functions. Therefore, graphical based solutions are difficult to represent and retrieve for the most of the students (Camacho-Machín et al., 2012a). Besides this, Camacho-Machín et al. (2008) also documented that students who used algebraic method to solve DEs, had less skills in graphical representation and mathematical interpretation. In the further study, it was also revealed that the transition from the algebraic to the graphical register is quite hard and students often make mistakes during this conversion (Camacho-Machín et al., 2012a). Therefore, Habre (2000) argued that this is not to say that analytic approach is obsolete. From the findings of task 4, it may be concluded that considered populations had less skills to solve differential equation using graphical based approach. Consequently, Camacho-Machín et al. (2008) proposed that teaching activities must be revised to promote students understandings and they able to utilize several different systems in which they may reflect on different aspects linked to concept itself, procedures, the solution methods and connections and meaning among these representations.

For the task 5 (application of Newton's law of heating/cooling), procedural based approaches were required to identify and handle the excess data. Similar positive trend was observed. Students showed very high percentage 81 percent and 83 percent and 68 percent in understanding, planning and getting answer phases respectively. Among 83 percent, only 68 percent became successful at getting answer phase, because these students became confused with the term t (time) and T (temperature) during integration process. Both these were used at the same time in task 5. In applied mathematics, t (time) and T (temperature) are often used in the same problems to represent different quantities.

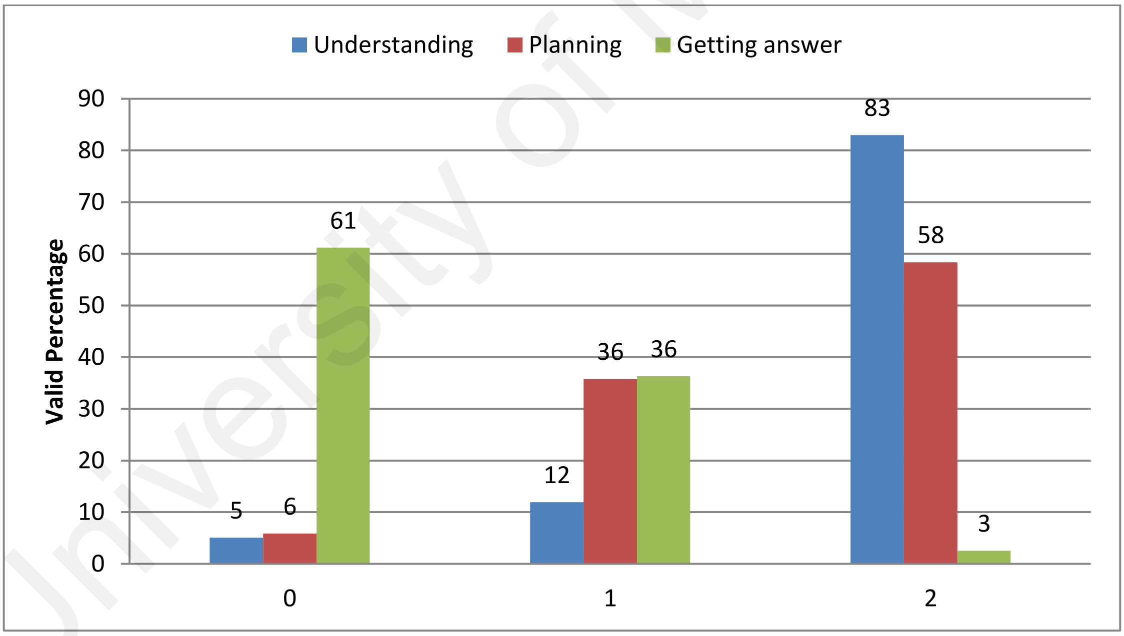

Figure 5.5: Findings task 4 in terms of percentage success of students while they engaged in DE problem solving

Taken as a whole, students showed good performance in task 1, task 2 and task 5 representing; population growth, velocity problem and Newton law respectively. These results were in a good agreement with Heller, Keith et al. (1997). According to author, Context familiar to majority of introductory students through direct experience, newspapers, televisions, or solving standards textbook problems are easier than problems with context unfamiliar to the students.

Since in the present case, task 3 and task 4 are the application of health and disease/ medicines and finance respectively. Therefore, performance was low as compared to other three tasks.

A few students were not successful in all of these five tasks. These results are consistent with Camacho-Machín et al. (2012d), who claimed that most students possess the conceptual resources such as, differentiation, integration, graphical representation of functions, properties of functions and their derivatives, algorithm to solve differential equation, etc.), but they cannot exploit these resources efficiently. Besides this, students might be uneasy, nervous, and uncomfortable. This normally happens when students unable to recall and apply the learned procedures in a simple way (Yeo, 2009). Other reason might be the complex nature of non-routine differential equation tasks. Yeo (2009) reported that lack of comprehension of the problem posed, lack of strategy knowledge, unable to translate the problem into mathematical form, and incompetence to use the correct mathematics negatively affect non-routine problem solving. A few other researchers also observed similar low correlation between problem-solving abilities and academic achievement while considering non-routine problems (Joseph, 2011).

From the critical analysis of all tasks, it may be concluded that algebraic approach is mostly employed in Pakistan at pre-university level. The reason might me the less resources and less availability of the modern technology required for the qualitative based approach. With less training and efforts, teachers can effectively teach the course by applying few procedures and steps.

Beside this student also prefer this approach. However, for the in-depth understandings and effective use of differential equation both algebraic and graphical methods should be considered through modern technologies. In addition, contextualizing learning using real world problems or authentic environment examples are also an important pillar in constructivist pedagogy (Abdulwahed et al., 2012).

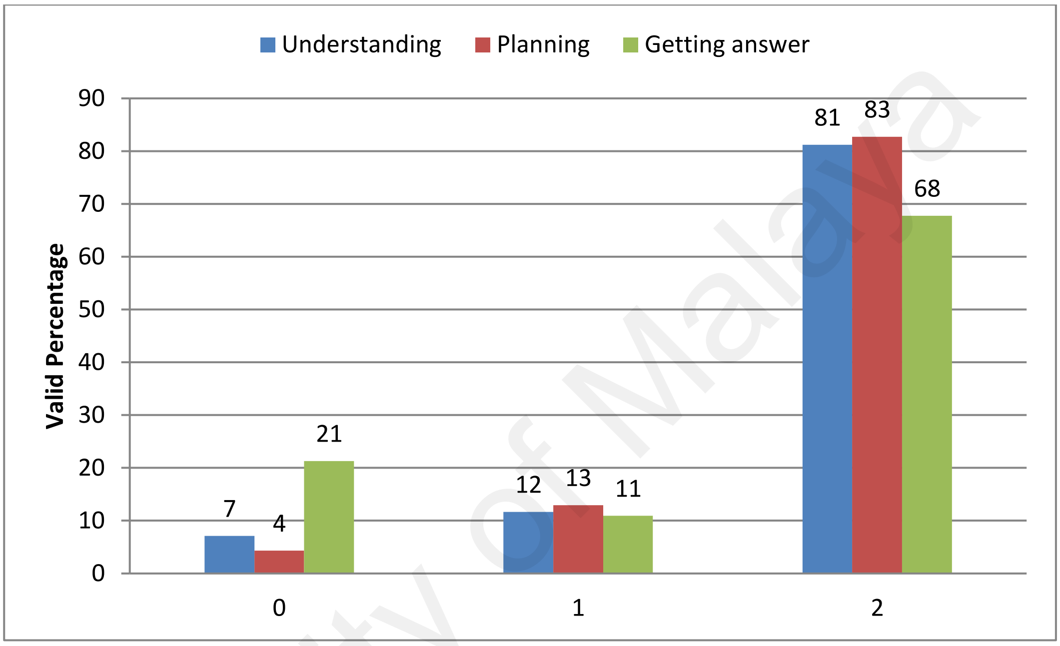

Figure 5.6: Findings of task 5 in terms of percentage success of students while they engaged in DE problem solving

In the developed countries, different methods including novel pedagogies (such as collaborative learning, inquiry/problems/discovery based learning), contextual problems, mathematical software packages (Mathematica, Maple), and online tools (Wikis and web based courses) are being used to facilitate conceptual understanding and constructivist mathematics learning (Abdulwahed et al., 2012). Therefore, to enhance conceptual understanding in differential equation problem solving in the developing countries such as Pakistan, all these tacit should be employed for class room and also actions to be taken with administration (instructors) and policy makers of the educational institute.

5.2 Significance of findings

The nature of the sample imposes limitations upon the findings and their generalization across all of Pakistan, however, the findings may provide directions for those teachers and educators who recognize resonance in the context, issue, and findings that are portrayed in this study.

5.3 Contribution of the study

The main contribution of this research is that it has highlighted the factors affecting differential equation problem solving ability, specifically, epistemological math problem solving beliefs, usefulness, goal orientations and self-regulated learning (SRL) strategies, at pre-university level. This study provided an empirical research that investigated the direct as well as indirect effect of these parameters on differential equation problem solving ability. The contribution of this study can be considered in terms of following area:

This study provided reliable evidence that certain key parameters, epistemological math problem solving beliefs, usefulness, goal orientations (mastery goal) and self-regulated learning (SRL) strategies (elaboration) enhance differential equation problem solving ability. Several studies examined the effect of one or two constructs on math problem solving, but mostly available data is from developed counties and research were based on general mathematics problem solving or qualitative analysis except Kwon et al. (2005), which is a quantitative study. No data was available from developing countries (e.g. Pakistan) with respect to differential equations at pre-university level (HS schools/ Intercollege). Moreover, no one had studied the combined effect of these parameters (epistemological beliefs, Usefulness beliefs, goal orientations, and SRL) for the differential equations problem solving, particularly for non-routine problems. Therefore, this research has extended the differential equation studies by looking at non-routine problems and focusing on the understandings that students normally use in college level mathematics classes.

Findings of the current study have specifically provided evidence, explaining the selection and employment of different problem solving approaches (such as algebraic and/or graphical) can generate a more competitive advantage. Therefore, overall current study contributed in identifying those factors affecting differential equation problem solving ability along with the adaptation of graphical approach to solve differential equation based problems.

This study will also contribute to students of developing countries in the areas of mathematics education. This research has been carefully designed to investigate the students' difficulties and misconceptions during learning of differential equation course because it is common for students to avoid this essential part of mathematics, which leads to sever understanding problems at higher levels of education, when they correlate the real-life problems. Therefore, in this research, effects of different psychological factors will be studied to boost up the students' understandings, so that teaching and learning of differential equation become meaningful and painless.

The results of this study will provide a new avenue to educators and teachers in Pakistan to overcome the students' problem by boosting up the students' mind psychological. This may be of value for the authorities to take into consideration and to enhance the positive factors and to avoid the factors that affects mathematics students negatively.

It is anticipated that the result of this study would provide the Ministry of Education in Pakistan with current data that will aid the ministry in making better policy decisions and applying educational strategies with greater certainty the implementation of curriculum in colleges.

The results of this study have been positively added to the literature and attempts were made to fill the research gap generally and particularly in Pakistan as a foundation for the research community to proceed with further research on the curriculum implementation, and teaching and learning of differential equation effectively.

5.4 Implications of the study

The findings of present study have provided several important implications for the curriculum designers and teachers, particularly in mathematics and science education. The nature of the sample imposes limitations upon the findings and their generalization across all of Pakistan, however, the findings may provide directions into the following two aspects:

5.4.1 Implications for curriculum

In Pakistan, calculus instruction starts in the last year of secondary school with the teaching of limits, function, derivatives. After that, these topics are complemented with a few simple differential equations- separations of variables and linear ordinary differential equations and solving for the general solutions and particular solutions of differential equations. A typical calculus at secondary level or pre-university level course include following major topics;

Derivative part includes derivatives of polynomials, trigonometric, exponential and logarithmic functions, derivatives of sums, differences, products and quotients of functions, derivatives of composite functions, monotonic functions, stationary points (maximum, minimum and inflexion points). Beside this, second order derivatives are also used to discriminate between maxima and minima. The second part includes integration as the reverse of differentiation, properties of integrals and main theorems, integration of polynomials, trigonometric, exponential. In addition, integration by parts and substitution, integration of rational functions, improper integrals and convergence criteria are also included in second part.

In the third part, main topics are introduction of differential equation, formulation of the differential equation from a problem situation or from a graphical representation, differential equation problem solving for the general solutions and particular solutions of differential equations.

In the first-year university calculus courses, they start revisiting topics of secondary school (like functions, limits and derivatives, differential equation, which usually takes half a semester or even more. At that moment students can utilize their previous knowledge of secondary school. However, there are some issues, those deserved to be considered at secondary school level. From the current study, it was observed that presently exams are focused only on routine practical procedures such as calculating limits and derivatives and to solve differential equation, therefore, students considered it useful and just try to earn good marks. Due to this, transition from school to tertiary mathematics teaching and learning becomes more challenging and difficult in terms of real understandings and problem solving. Taking all these facts into account, a typical calculus at secondary course is expected to include:

Applications of non-routine and real-life problems are not very common in calculus secondary courses. More non-routine DE problems and their applications of real life should be included.

The exams could be divided into three parts; one devoted to routine exercises, second one that should evaluate theoretical aspects of differential equation course, and final part should consider relate non-routine real problems.

These non-routine differential equation word problems, routine procedure should also be complemented with graphics of the involved functions. For that reason, teachers must be facilitated with effective training, so that they may able to provide guidance to students during transition from algebraic to graphical mode or vice versa to avoid mistakes.

Accessibility of computers labs can also overcome several issues of algebraic, graphical, numerical methods and their integration into a single approach for effective differential equation based problem solving.

5.4.2 Implications for teaching and learning

Besides course content, teaching style and instructional approaches, perceptions about mathematics and specific concept, learning strategies, purpose or goal of learning also affect teaching and learning mathematics (Biza, Giraldo et al., 2016). Findings of current study also confirmed that epistemological math problem solving beliefs and usefulness (perceptions about mathematics), and motivational beliefs and learning strategies influence the differential equation problem solving. Therefore, teachers/educators must design their instructional strategies by incorporating the students' epistemological and motivational beliefs as well as learning strategies for the effective learning of DE course. Mathematics teachers can utilize the findings of this study in assessing the students' differential equation problem solving ability. In addition, teacher may apply these findings for assessing students' problem solving ability in other parts of calculus or mathematics. Implications for teachers and educators are summarized as below, The adaption of beliefs particularly motivational beliefs during differential equation problem solving is neither easy nor automatic. Many students may have little motivation for mathematics tasks or to pursue a goal, while others depend only on extrinsic motivation. Therefore, differential class room practices should be changed to facilitate adaptive problem solving beliefs, promote interest and appreciation of task and also foster the adoption of motivation and of learning strategies. To enhance student's motivation and interest and utilization of learning strategies, differential equation class environment should be interactive and self-motivated, and also have an atmosphere of inquiring, exploration, and discovery.

From teaching point of view, teachers should emphasize activities that encourage students to explore differential equation topics, develop and refine their own ideas, strategies, and technique. Furthermore, challenging activities should be created and avoid comparison among students. So that students can actively participate in the whole activity. The role of teacher should be a facilitator rather than a dispenser of information. Additionally, differential equation problem solving and reasoning should more emphasize at secondary school mathematics instead of rote manipulations section. For this purpose, connections, applications, verifications, and related differential equation problems must be given priority over rehearsing algorithms.

In addition to it, teachers must give attention to non-routine based problems, related to some specific type and area to give students in-depth understandings. Also, these non-routine problems must be balanced with graphics of the involved functions to assess and enhance student's differential equation problem solving ability. They should educate and smartly trained their students during transition from algebraic to graphical mode or vice versa to avoid mistakes.

An alternative teaching approach is needed towards a reduction of traditional lecturing model. There may be a need to implement of flipped learning approach like (Oh Nam, 2015), in which students attend the short instructions in videos or online courses while face-to-face time is devoted to classes for exercise, activities or discussions. Alternatively, students assist other students in the development of study skills, in peer assisted learning environment (PAL) through flipped learning approach (Biza et al., 2016). This will not only enhance their skills, also boost up their motivation as well as interest for differential equation problem solving.

5.5 Recommendations for future work

This work has enabled the development of a structural equation model (SEM) relating the four factors including epistemological math problem solving beliefs, usefulness, self-regulated learning strategies, and goal orientation beliefs, to support the teaching and learning of differential equation problems at pre-university level students. In addition to SEM, it is anticipated that the results of this study would provide a new avenue to educators and teachers in Pakistan to overcome the students' problem by boosting up the students' mind psychological. Findings of this work have resulted into more than five articles, which have added useful information to available literature to fill the research gap generally and particularly in Pakistan for curriculum implementation, and teaching and learning of differential equation effectively.

Present study has yielded a comprehensive structural equation model which covers several important aspects teaching and learning of differential equation and their problem solving. Efforts were made to make this study as well as developed model more generalized. In the future, research work in the following directions would be helpful to extend and make this work more generalized particularly for developing countries.

In this work, target population was from only one province of Pakistan. In the future, this population can be extended to whole Pakistan and also to several other countries to make a generalized model for the developing countries. The study of culturally diverse populations has the potential to add new insights into the study of epistemological beliefs, usefulness, goal orientations and self-regulated strategies. Cross investigations would be further helpful in understanding the nature of each factor.

In the future study, influence of these factors can be studied at university level to compare the findings at different levels and this would give a more generalized model. Only pre-university level was examined in the current study. High level or university level may add more precision to predictions.

The present study can be extended by including variables such as classroom goal structure, and personal characteristics. Furthermore, teachers' beliefs are also very important, as they affect their instructions. Current study can be expanded by considering teachers beliefs.

In case of self-regulated learning strategies, only two cognitive strategies were analyzed in this study. In future, more learning strategies can be taking into account. Utilization of multiple data collection methods, such as observations or interviews (triangulation method), would be helpful to provide the richer understandings of differential equation problem solving.

5.6 Conclusions

The role of differential equations is very important in the modern technologies to inter-relate and solve a variety of routine or daily life problems. Several approaches have been developed and more are being developed to make DEs course more effective and valuable. Within the case of differential equation, generally, three different approaches (algebraic, numerical and graphical) are employed to solve differential equations at pre-university level. Algebraic approaches are mostly procedural while the others are qualitative based. With advancement in technology (such as modern soft wares, and other electronic tools), algebraic, numerical and graphical were tried to combine. This has yielded better results at higher levels of education. However, at initial or pre-university levels, it is still a great challenge to determine how students interact with the digital tools and representation registers associated with ordinary differential equations to give meaning to parameters associated with it.

In addition to these problem solving approaches, positive effects of development of student epistemological math problem solving beliefs, usefulness, goal orientations and self-regulated learning (SRL) strategies were observed for math achievements and problem solving. Results were encouraging, particularly at initial or pre university levels. However, in spite of the great importance of these factors, no study had related these four factors. Therefore, this correlation study was designed to relate and model these four factors including epistemological math problem solving beliefs, usefulness, goal orientations and self-regulated learning strategies to support the teaching and learning of differential equation problems at pre-university level/college level mathematics students.

To investigate the selected factors of the model, three different types of the self-developed instruments were investigated through the cross-sectional survey of a large and diverse number of mathematics students. Population of this study was the pre university level (12^{th} class) student's, studying in the public and private sectors in Khyber Pakhtunkhwa (one of the largest province of Pakistan).

The collected data was analyzed using SPSS and Smart PLS software. SPSS was used for data cleaning and removal of outliers followed by normality, reliability and factor analysis tests. While Smart PLS methods were used to test the hypotheses through structural equation modeling. Both direct and indirect effects of the selected factors were considered towards the differential equation problem solving.

The analysis of the direct paths revealed that epistemological math problem solving beliefs, usefulness, goal orientation and self-regulated learning strategies strongly affected the differential equation problem solving. Results of direct effects are provided in Table 4.14. It was observed that all the direct path was significant except critical thinking. Moreover, usefulness ($\beta = 0.29$, $t = 4.05$, $p < 0.01$) had relatively larger effect than epistemological beliefs ($\beta = 0.12$, $t = 2.02$, $p < 0.05$).

Similarly, the goal orientations including mastery goal, performance and avoidance goal strongly influence the differential equation problem solving. It was observed that mastery goal ($\beta = 0.22$, $t = 3.52$, $p < 0.01$) had relatively larger effect as compared to performance goal ($\beta = 0.10$, $t = 2.20$, $p < 0.05$). Whereas, Avoidance goal ($\beta = -0.25$, $t = 8.10$, $p < 0.01$) have shown negative contribution towards differential equation problem solving. These results confirmed that students with positive motivations, students had more ability to solve differential equation problems.

In addition, it was noticed that both elaboration and critical thinking also had positive effects on differential equation problem solving. Elaboration had significant estimates ($\beta = 0.16$, $t = 3.12$, $p < 0.01$), while critical thinking had non-significant estimates ($\beta = 0.03$, $t < 1.96$, $p > 0.05$). These findings had well supported by the concerned theories and literature. Elaboration helped learners to make connections between prior knowledge and new knowledge, thus had promoted differential equation problem solving ability.

In the second phase of this study, the mediation roles of goal orientations and self-regulated learning strategies towards differential equation problem solving were identified. For this, initially the mediation effects of goal orientations (mastery, performance and avoidance goals) were considered. The findings revealed that epistemological math problem solving beliefs solving strongly affected the differential equation problem solving via mastery, and performance goals. However, avoidance goal did not show significant mediation.

In case of usefulness, mastery goal ($\beta = 0.07$, $t = 2.67$, $p < 0.01$) and avoidance goal ($\beta = 0.02$, $t = 2.01$, $p < 0.01$) had partial mediations, whereas no such effect was observed from performance goal. There was an agreement in the literature that belief in useful of mathematics had enabled students to solve mathematical problems. However, mediating role of belief in useful of mathematics was not explored for problem solving ability, particularly, differential equation problem solving prior to current study (up to researcher knowledge). Therefore, present study findings about mediation role of belief in useful of mathematics were quite novel. Overall it may be concluded that students with positive perceptions about mathematics and its usefulness had more differential equation problem solving ability

The fourth and fifth set of research hypotheses addressed the mediation role of self-regulated learning strategies (elaboration and critical thinking) among the epistemological math problem solving beliefs, usefulness and differential equation problem solving. Results showed that epistemological math problem solving beliefs strongly affected the differential equation problem solving via elaboration. However, via critical thinking no significant effects were observed. In case of usefulness, no mediation was observed via both elaboration and critical thinking. The sixth set of hypotheses had considered the mediation role of self-regulated learning strategies between the goal orientations and differential equation problem solving. The findings of this study revealed that only elaboration had played the role of mediation for both mastery and performance goals. Critical thinking had not shown mediation effects.

The seventh set of hypotheses had considered the comparative study of the different problem solving approaches (such as algebraic and graphical) and their yielded results towards differential equations problem solving. From the critical analysis of all tasks, it was concluded that algebraic approach was mostly employed in Pakistan at pre-university level. Mostly students can solve many problems using simply few rules and algorithms. The reason might be the less resources and less availability of the modern technology required for the qualitative based approach. Beside this student also had preferred this approach. Regarding to curriculum, national and international researchers are agreed that the current curriculum has less potential to prepare teachers for the challenges of 21st century. Because there are massive gaps between the curriculum of teacher training programs and class room environment (Khan, 2012). With less training and efforts, teachers can effectively teach the course by applying few procedures and steps. Moreover, Kiani et al. (2012) recommended that even though most of the teachers have professional qualifications such as B.Ed., (Bachelor of Education) and M.Ed. (Master of Education), even though curriculum and training programs may be reviewed time to time for the teachers.

In addition, it was also revealed that context familiarity also enhanced the problem solving ability. Even applying same approach (i.e. algebraic) for solving tasks of different areas, results were different. Students had shown better ability of physics related problem solving as compared to biological related problems. Therefore, these factors include utilization of preferred and frequently applied problem solving approach (algebraic or graphical) and the context familiarity of the problems has also contribution towards differential equation problem solving performance.

Overall, it can be concluded that epistemological math problem solving beliefs, usefulness, goal orientations (mastery and performance goals) and elaboration can be effectively employed to boost up the students' understandings, so that teaching and learning of differential equation may become more effective, and meaningful. In addition, this work would be more helpful in the developing countries, where the use of advanced computer based technology is not common.

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APPENDIX A: ACTUAL STUDY SUPPLEMENTARY DATA

For current study, exploratory factor analysis (EFA) had been carried out for each construct of the selected variables. Since the survey evaluation questionnaire comprises 64 items distributed into 7 constructs to measure the effect of these variables on differential equation problem solving. Details of the results of each construct and its factors are provided in the following sections.

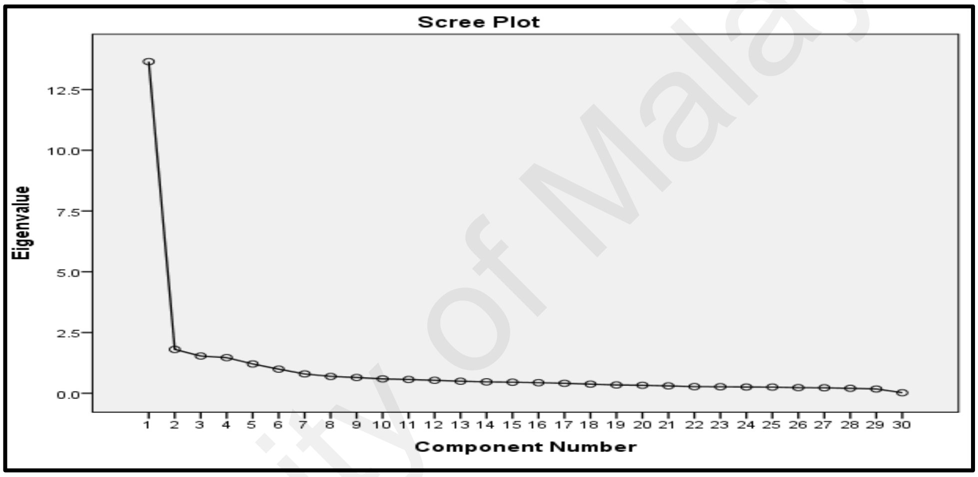


Figure 1B: Scree plot of for 30 beliefs items

The presence of a single factor that accounted for a substantial amount of the total variance led to hypothesis that the data may be characterized by a general factor (Epler, 2011; Wheeler, 2007). Therefore, there was a doubt of uni-dimensionality.

Table 1B: Factor loadings, communalities, eigen value, percent variances explained by epistemological problem solving belief

Factor	Dimensions	Factor loading	communalities	Eigen values	% of variance
EMB	DP	0.83	0.69	3.60	72%
	ST	0.86	0.74		
	UN	0.87	0.75		
	WP	0.82	0.68		
	EF	0.86	0.74		

EMB: Epistemological math problem solving beliefs. DP: Duration of problem, ST: Steps, UN: Understandings, WP: Word problems, EF: Effort

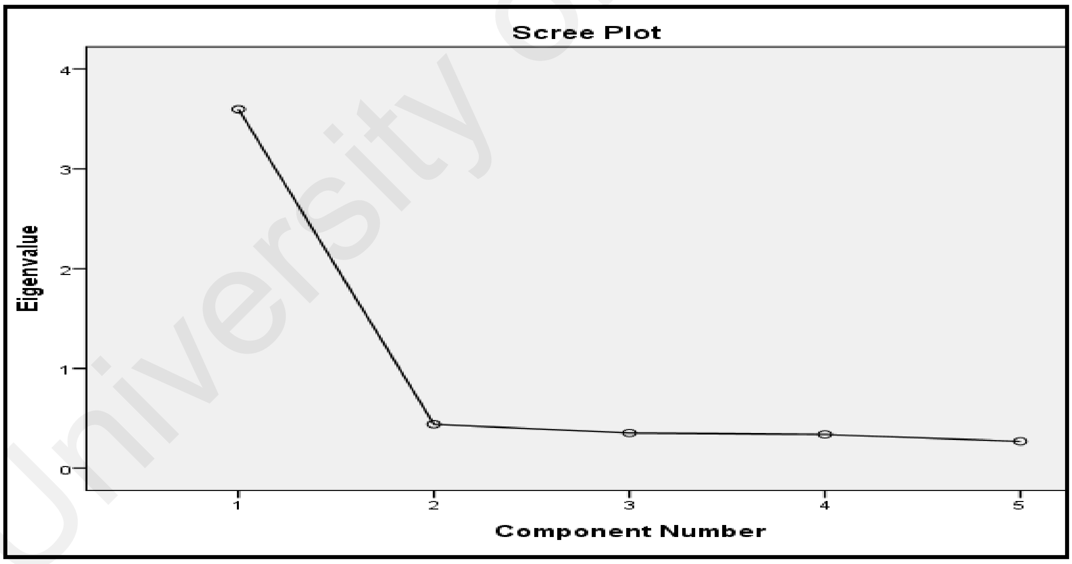


Figure 2B: Scree plot of for general math problem solving beliefs

Exploratory factor analysis for Usefulness (actual study)

Table 2B: Factor loadings, communalities, eigen value, % variances explained by Usefulness

Factor	Item code	Loading	Communalities	Eigen values	% of variance
UF	B51	0.89	0.78	4.39	77 %
	B52	0.87	0.75		
	B53	0.88	0.78		
	B54	0.84	0.70		
	B55	0.85	0.73		
	B56	0.81	0.65		

UF: Usefulness

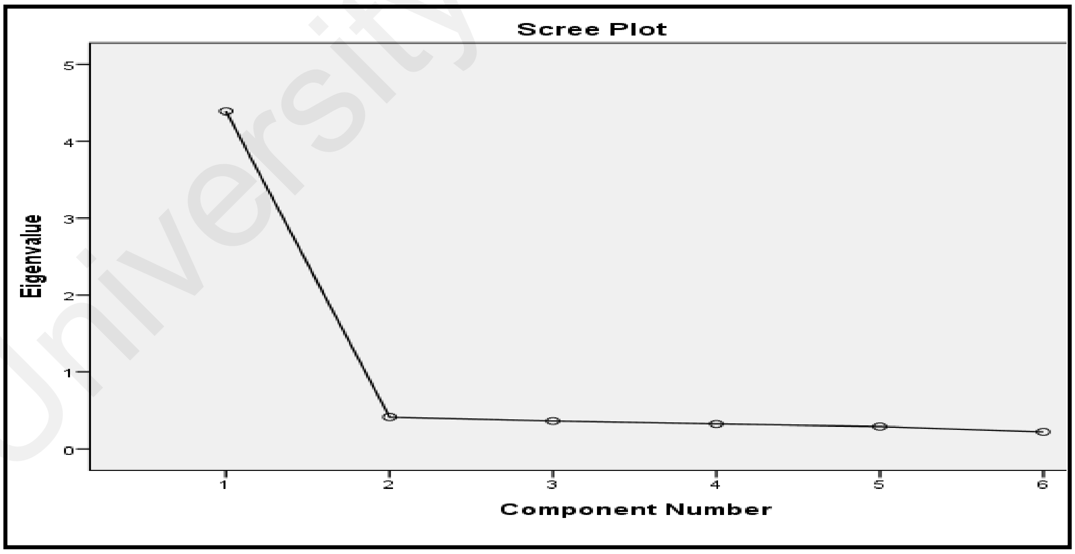


Figure 3B: Scree plot of for usefulness

Exploratory factor analysis for goal orientations (actual study)

Table 3B: Factor loadings, communalities of Goal orientation

Construct	Item code	Component			Communalities
		1	2	3	
MA	MA1		0.82		0.78
	MA2		0.83		0.75
	MA3		0.78		0.69
	MA4		0.83		0.76
	MA5		0.80		0.71
	MA6		0.81		0.75
PR	PER1			0.81	0.74
	PER2			0.78	0.71
	PER3			0.80	0.73
	PER4			0.81	0.75
	PER5			0.66	0.53
AV	AV1	0.89			0.79
	AV2	0.90			0.81
	AV3	0.90			0.81
	AV4	0.92			0.83
	AV5	0.92			0.84
	AV6	0.89			0.80

MA: mastery goal, PR: performance goal, AV: avoidance goal

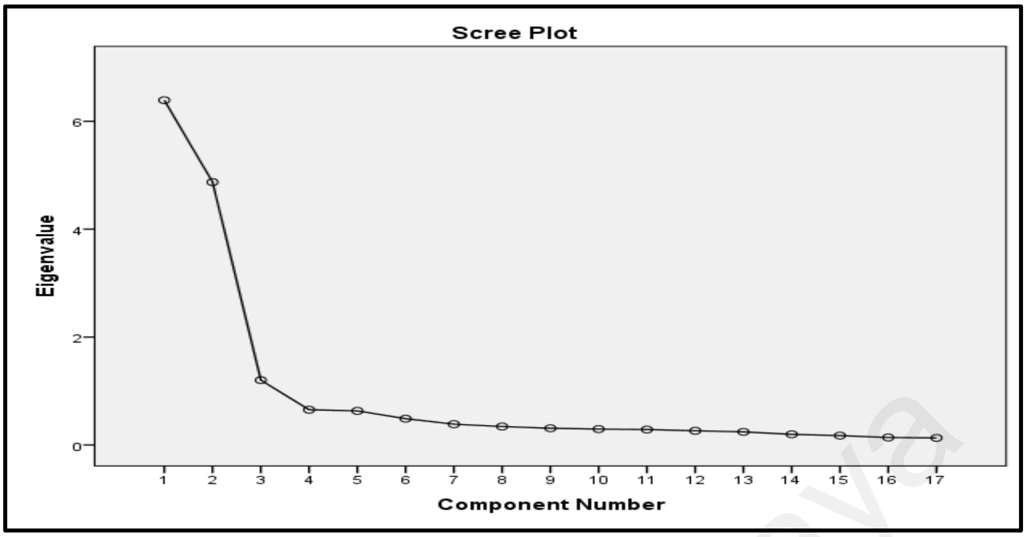


Figure 4B: Scree plot of goal orientations

Exploratory factor analysis for self-regulated learning strategies (actual study)

Table 4B: Factor loadings, communalities, eigen value, percent variances explained by Self-regulated learning strategies

Factor	Item code	Component		Communalities	Eigen values	% Variance
CRT	CR1		0.74	0.58	1.8	17.1
	CR2		0.77	0.62		
	CR3		0.70	0.54		
	CR4		0.73	0.55		
	CR5		0.73	0.57		
EL	EL1	0.76		0.63	5.1	46.7
	EL2	0.82		0.70		
	EL3	0.82		0.70		
	EL4	0.82		0.72		
	EL5	0.80		0.67		
	EL6	0.79		0.69		

CRT: Critical thinking, EL: Elaboration

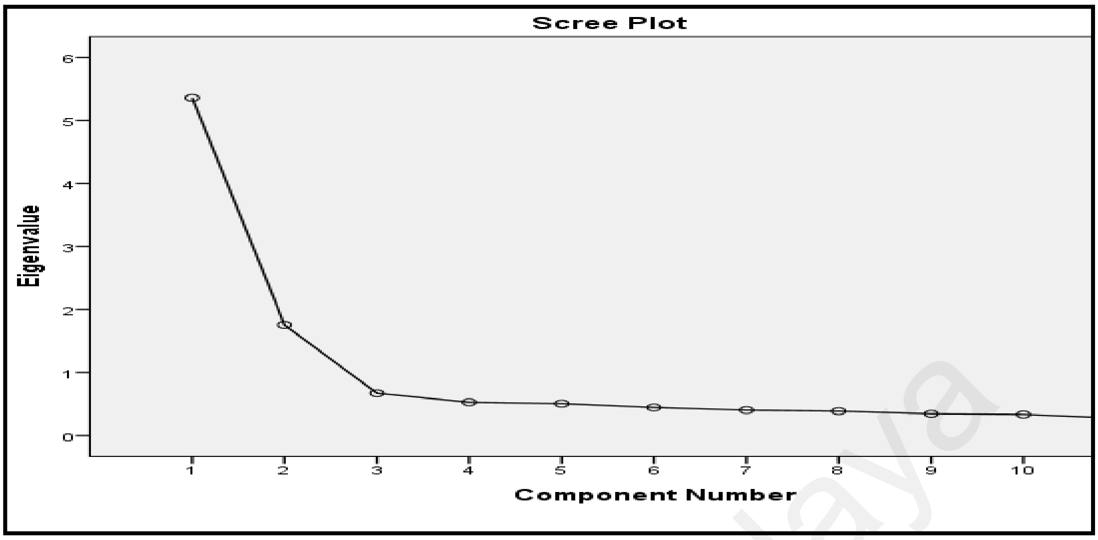


Figure 5B: Scree plot of self-regulated learning strategies

APPENDIX B: CONSENT LETTERS AND QUESTIONNAIRES

In this appendix, questionnaires, acknowledgement letters of the principals and consents letters have been arranged in the following order,

1 Consent letter from faculty of education to collect data
2 Acknowledgement letters of the principals
3 Questionnaires of epistemological math problem solving beliefs
4 Questionnaires of goal orientations
5 Questionnaires of self-regulated learning strategies
6 Differential equation tasks
7 Consent of the original researcher, whose questionnaires were adapted

UNIVERSITI
MALAYA

4th March 2016

To Whom It May Concern

Name	: Aisha Bibi
I/c No./ Passport No.	: UZ3091421
Registration No	: PHA140004
Programme	: Doctor of Philosophy
Specialization	: Mathematics Education

This is to confirm that the above candidate is a student of Faculty of Education, University of Malaya, beginning in semester II, session 2014/2015.

She/he is currently doing research and would require research data which can be obtained from your school/office/institution/university. We would appreciate it if you are able to assist our candidate in his/her research and would like to thank you in advance for your cooperation.

Thank you.

Yours truly,

ANIDA KAMALUDIN
Assistant Registrar (Higher Degrees)
Faculty of Education

FAKULTI PENDIDIKAN
Universiti Malaya, 50603 Kuala Lumpur, Malaysia • http://www.um.edu.my
Pejabat Dekan: (603) 7967 5000 • Faks: (603) 7956 5506 • Timbalan Dekan (603) 7967 5079 / 5080 / 5099 • Faks: (603) 7967 5141 / 5089
Penolong Pendaftar: (603) 7967 5131 / 5001 • Pejabat Am: (603) 7967 5006 / 5133• Faks: (603) 7967 5130
Jabatan Asas Pendidikan dan Kemanusiaan/Jabatan Pendidikan Matematik dan Sains: (603) 7967 5040 • Faks: (603) 7967 5148
Jabatan Pendidikan Bahasa dan Literasi/Jabatan Kurikulum dan Teknologi Pengajaran: (603) 7967 5027 • Faks: (603) 7967 5139
Jabatan Psikologi Pendidikan dan Kaunseling/Jabatan Pengurusan Perancangan dan Dasar Pendidikan: (603) 7967 5036 • Faks: (603) 7967 5010

Date: 18-03-2016

Assistant Registrar (Higher Degrees),
Faculty of Education, University of Malaya,
50603 Kuala Lumpur.

Dear Sir/Madam,

Acknowledgment for Research Data

With reference to your letter dated on 4th March, 2016, the required data has been provided for the Aisha Bibi (PhD student, Matric No. PHA140004) to complete her research in the area of Mathematics Education.

Yours Truly, FARHAN ANJUM

Sig..

(Principal/HOD/ Course I/C)

Name...FARHAN ANJUM.....

Official Stamp ....

Date:

Assistant Registrar (Higher Degrees),
Faculty of Education, University of Malaya,
50603 Kuala Lumpur.

Dear Sir/Madam,

Acknowledgment for Research Data

With reference to your letter dated on 4th March, 2016, the required data has been provided for the Aisha Bibi (PhD student, Matric No. PHA140004) to complete her research in the area of Mathematics Education.

Yours Truly,

22-3-16

Sig..

(Principal/HOD/ Course I/C)

Name...Shaid Muhammad

Principal
Govt. Higher Secondary School
Batta (Mansehra).

Official Stamp ....

Date: 24-03-16

Assistant Registrar (Higher Degrees),
Faculty of Education, University of Malaya,
50603 Kuala Lumpur.

Dear Sir/Madam,

Acknowledgment for Research Data

With reference to your letter dated on 4th March, 2016, the required data has been provided for the Aisha Bibi (PhD student, Matric No. PHA140004) to complete her research in the area of Mathematics Education.

Yours Truly,

Sig...
THE PEACE COLLEGE
(Principal/HOD/ Course I/C)
Name....Naveed Ahmad

Official Stamp ....

Date: 29/3/16

Assistant Registrar (Higher Degrees),
Faculty of Education, University of Malaya,
50603 Kuala Lumpur.

Dear Sir/Madam,

Acknowledgment for Research Data

With reference to your letter dated on 4th March, 2016, the required data has been provided for the Aisha Bibi (PhD student, Matric No. PHA140004) to complete her research in the area of Mathematics Education.

Yours Truly,

Sig...
(Principal/HOD/ Course I/C)
Name...Muhammad Sajid

Official Stamp ....PRINCIPAL GHSS Sherpur (Mansehra)

Date:

Assistant Registrar (Higher Degrees),
Faculty of Education, University of Malaya,
50603 Kuala Lumpur.

Dear Sir/Madam,

Acknowledgment for Research Data

With reference to your letter dated on 4th March, 2016, the required data has been provided for the Aisha Bibi (PhD student, Matric No. PHA140004) to complete her research in the area of Mathematics Education.

Yours Truly,

Sig. ....

(Principal/HOD/ Course I/C)

Name. Muhammad Shehzad

Official Stamp .. PRINCIPAL GHSS PHULRA MANSEHRA

تفرقی مساوات ↕ عبارتی سوال ↕

These tasks would be helpful to those students who are looking for entrance exam of engineering, computer science, MBA and etc

Differential Equation Words Problem, Math Chapter 3, Exercise 3.8

Function: $y = 2x + 5$

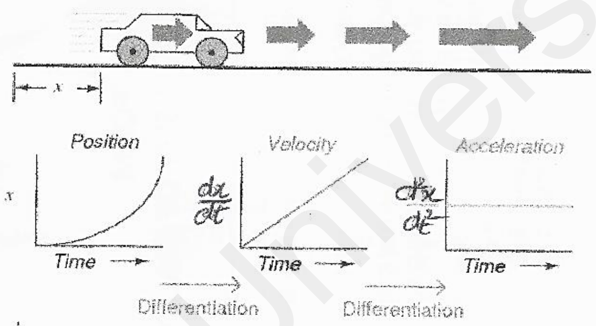
Differentiation: $(\frac{dy}{dx}) = \frac{d}{dx}(2x + 5) = 2$

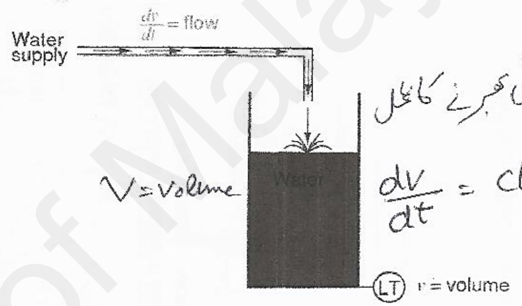
Integration: $\int dy = \int 2dx = 2x + C$

Differential equation =Rate equation = $(\frac{dy}{dx} = mx + C)$

Applications : تفرقی مساوات کا استعمال :

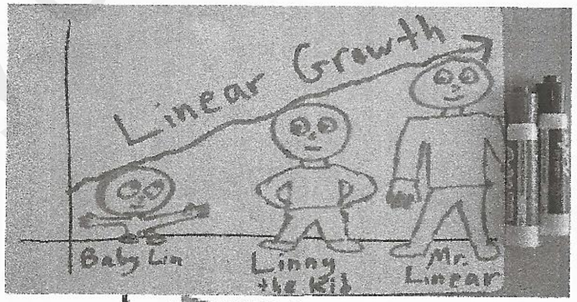
Liquit Flow Rate, car velocity, acceleration Projectile motion of ball, rocket etc



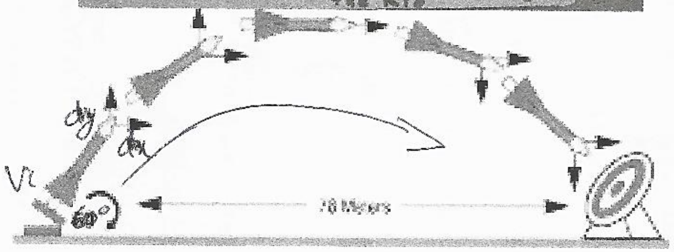


ٹینک میں پانی بھرنے کا عمل

$\frac{dV}{dt}$ = $\frac{\text{Change in volume}}{\text{Time}}$



اونچائی میں اضافہ بلحاظ ٹائم = $\frac{dh}{dt}$



Projectile Motion

(A)

فاصلہ/دوری $s = x$ = distance

ولاسٹی = Velocity = $\frac{dx}{dt}$

اسراع = acceleration = $\frac{d}{dx}(\frac{dx}{dt}) = d^2x/dt^2$

Student Name:	Email ID:
District:	Gender: Male/ Female
Institute/ College Name:	Matric Marks:
Future Liked Field : Engineering/ MBA/ Computer Science/ Finance/ Others ( )	F.Sc Part 1 Marks :
	Math Marks in F.Sc Part 1:

S. NO	Belief 1	Strongly Disagree مکمل غیر متفق	Disagree غیر متفق	Uncertain غیر یقینی	Agree متفق	Strongly Agree مکمل متفق
1.1	Differential equation word problems that take long time don't disturb me. تفرقی مساوات کے عبارتی سوال جو حل میں زیادہ وقت لیں مجھے تنگ نہیں کرتے	1	2	3	4	5
1.2	I think I can do differential equation word problems that take a long time to finish. میرا خیال ہے کہ تفرقی مساوات کے عبارتی سوالات جو حل کرنے میں زیادہ وقت لیں میں ان کو حل کر سکتا ہوں۔	1	2	3	4	5
1.3	I find I can do difficult differential equation word problems if I just hang in there. میں تب بھی تفرقی مساوات کے عبارتی سوال حل کر سکتا ہوں جب مجھے لازمی ان کو حل کرنا ہو	1	2	3	4	5
1.4	If I can't do a differential equation word problem in a limited time/couple of minutes, I probably can't do it at all. اگر مخصوص وقت میں، میں تفرقی سوالات کو حل نہ کر سکوں تو اس کا مطلب ہے میں ان کو حل نہیں کر سکتا۔	1	2	3	4	5
1.5	If I can't solve differential equation word problems rapidly, I quit trying. اگر مجھ سے تفرقی مساوات کا عبارتی سوال حل نہ ہو رہا ہو تو میں اسے چھوڑ دیتا ہوں۔	1	2	3	4	5
1.6	I am not very good at solving differential equation word problems that take some time to understand. تفرقی مساوات کے جس عبارتی سوال کو سمجھنے میں زیادہ وقت لگے میں ان کو حل کرنے میں اچھا نہیں	1	2	3	4	5
	Belief 2					
2.1	There are differential equation word problems that just can't be solved by following a pre-defined sequence of steps. تفرقی مساوات کے کچھ عبارتی سوالات ایسے بھی ہیں جو کہ صرف طے شدہ مراحل (steps) کے ذریعے حل نہیں کیے جا سکتے	1	2	3	4	5

S.No	Sentences	Strongly Disagree مکمل غیر متفق	Disagree غیر متفق	Uncertain غیر یقینی	Agree متفق	Strongly Agree مکمل متفق
2.2	Differential equation word problems can be solved without remembering formulas. تفرقی مساوات کے عباراتی سوال بغیر فارمولا یاد کیے بھی حل کیے جا سکتے ہیں	1	2	3	4	5
2.3	Remembering steps is not that useful for learning to solve differential equation word problems. تفرقی مساوات کے عباراتی سوال کے حل کرنے کے مراحل (steps) کو یاد کرنا مفید نہیں	1	2	3	4	5
2.4	Any differential equation word problems can be solved if you know the proper step to follow. میرا خیال ہے کہ اگر مجھے عباراتی سوال حل کرنے کے مراحل (steps) آتے ہیں تو میں ہر قسم کا سوال حل کر سکتا ہوں	1	2	3	4	5
2.5	Most differential equation word problems can be solved by using the suitable step by step procedure. زیادہ تر تفرقی سوال مناسب ترتیب (steps by steps) کے ذریعے حل کیے جا سکتے ہیں	1	2	3	4	5
2.6	Learning to do differential equation word problems is mostly an issue of remembering the correct step to follow. تفرقی مساوات کے عباراتی سوال کو حل کرنے کا بہترین طریقہ یہ ہے کہ آپ اس کے حل کرنے والے مراحل (steps) یاد کریں	1	2	3	4	5
	Belief 3					
3.1	Time used to examine why a solution to differential equation works is considerably not time passed. کسی بھی تفرقی مساوات کے عباراتی سوال کے جواب کی پڑتال کرنا، وقت ضائع کرنا نہیں۔	1	2	3	4	5
3.2	An individual who does not understand why an answer to the differential equation is right has not really solved the problem. اگر ایک طالبعلم کو عباراتی سوال کے جواب کی سمجھ نہیں کہ درست / ٹھیک ہے تو اس کا مطلب ہے اس نے سوال ہی حل نہیں کیا	1	2	3	4	5
3.3	In addition to obtaining a correct answer in differential equation solution, it is essential to understand why the answer is right. تفرقی مساوات کے درست جواب حاصل کرنے کے ساتھ ساتھ یہ سمجھنا بھی اہم ہے کہ جواب کیوں درست / ٹھیک ہے	1	2	3	4	5
3.4	It is not important to know why a differential equation process works as long as it provides a right answer. یہ جاننا اتنا ضروری نہیں کہ تفرقی مساوات کو حل کرنے کے مراحل کیا ہیں، جتنا کہ اس کا صحیح جواب حاصل کرنا ہے	1	2	3	4	5

S.No	Sentences	Strongly Disagree مکمل غیر متفق	Disagree غیر متفق	Uncertain غیر یقینی	Agree متفق	Strongly Agree مکمل متفق
3.5	To get the correct answer in differential equation solution is more essential than understanding why the responses work. درست جواب کا حصول اہم ہے بنسبت تفرقی سوال کو سمجھنے کے	1	2	3	4	5
3.6	It does not actually matter to understand a differential equation problem if you can find the right response. اگر آپ تفرقی مساوات کے درست جوابات حاصل کرنے میں کامیاب ہو جاتے ہیں تو اس کو سمجھنے کی ضرورت نہیں	1	2	3	4	5
	Belief 4					
4.1	A student who can't solve word problems really can't understand and solve differential equations. جو طالب علم عبارتی سوال کو حل نہیں کرسکتا وہ دراصل تفرقی مساوات کو سمجھنے سے قاصر ہے	1	2	3	4	5
4.2	If you don't use computational skills to solve word problems in the differential equation, it means they are not so important. اگر میں تفرقی مساوات کو حل کرنے حسابی صلاحیت کو بروئے کار نہیں لاتا تو اس کا مطلب ہے یہ حساب سرے سے اچھے نہیں	1	2	3	4	5
4.3	If you don't use computational skills in differential equation relating real life situation, it means they are useless. اگر کوئی تفرقی مساوات کو روزمرہ کی زندگی میں استعمال نہ کرسکے تو اس کا مطلب ہے یہ مفید کورس نہیں	1	2	3	4	5
4.4	To learn computational skills for solving differential equation solution is more important than learning word problems تفرقی مساوات کے عبارتی سوال حل کرنے کی بجائے ریاضی کے دوسرے حصے کو پڑھنا زیادہ بہتر ہے	1	2	3	4	5
4.5	Mathematics classes should not emphasize differential equation word problems. ریاضی کے کورس میں تفرقی مساوات نہیں ہونی چاہیے	1	2	3	4	5
4.6	Word problems are not necessarily part of differential equation course. عبارتی سوالات تفرقی مساوات کا اہم حصہ نہیں	1	2	3	4	5
	Belief 5					
5.1	I like to study differential equation problems because I know how it is useful. مجھے تفرقی مساوات کے سوالات کو حل کرنا اچھا لگتا ہے کیونکہ مجھے پتہ ہے یہ بے حد مفید ہیں	1	2	3	4	5

S.No	Sentences	Strongly Disagree مکمل غیر متفق	Disagree غیر متفق	Uncertain غیر یقینی	Agree متفق	Strongly Agree مکمل متفق
5.2	Knowing differential equation problems will help me earn a living. تفرقی مساوات کے سوالات حل کرنے سے روزمرہ زندگی کے معاملات حل کرنے میں مدد ملتی ہے	1	2	3	4	5
5.3	Differential equation problems are worthy and compulsory. تفرقی مساوات کے سوالات حل کرنا نہایت اہم اور ضروری ہے	1	2	3	4	5
5.4	Differential equation problems will not be important to me in my daily life. تفرقی مساوات کے جوابات روزمرہ زندگی میں اہم نہیں	1	2	3	4	5
5.5	Differential equation problems have no relevance to my life. تفرقی مساوات کے سوالات کا روزمرہ زندگی سے کوئی خاص تعلق نہیں	1	2	3	4	5
5.6	To study differential equation problems is a waste of time. تفرقی مساوات کا پڑھنا وقت کا ضیاع ہے	1	2	3	4	5
	Belief 6					
6.1	Doing more hard work a person became efficient in differential equation problems. زیادہ محنت کرنے والا تفرقی سوالات کو حل کرنے میں تیز ہو جاتا ہے	1	2	3	4	5
6.2	Practice can improve one's ability to solve differential equation problems. زیادہ مشق (Practice) کرنے سے بھی عبارتی سوال حل کرنے کی صلاحیت بڑھتی ہے	1	2	3	4	5
6.3	I can get smarter in differential equation problems by trying hard. زیادہ محنت کرنے سے تفرقی سوالات حل کرنے میں پھرتی/تیزی آجاتی ہے	1	2	3	4	5
6.4	When one studies hard, differential equation problems solving ability are increased. جو بھی زیادہ محنت کرے گا وہ اتنی جلدی تفرقی سوالات کو حل کر سکے گا	1	2	3	4	5
6.5	More hard work can increase one's ability to solve differential equation problems. زیادہ محنت کرنے سے تفرقی سوالات کو حل کرنا آسان ہو جاتا ہے	1	2	3	4	5
6.6	If I do hard work I become efficient in differential equation problems اگر میں زیادہ محنت کروں گا تو تفرقی سوالات کو تیزی سے	1	2	3	4	5

Differential Equation, تفرقی مساوات

S.No	Sentences	Not at all true مکمل غلط	Not true غلط	Somewhat true کسی حد تک درست	True درست	Very true مکمل درست
	Mastery Goal Orientations					
1	I like differential equation class work that I'll learn from even if I make a lot of mistakes. میں مجھے تفرقی مساوات کی کلاس لینا پسند کرتا ہوں کیونکہ باوجود غلطیوں کے اس میں بہت کچھ سیکھتا ہوں	1	2	3	4	5
2	I do my differential equation class work because I like to learn new things. میں تفرقی مساوات کی کلاس کا کام خوشی سے کرتا ہوں کیونکہ مجھے نئی چیزیں سیکھنے کا شوق ہے	1	2	3	4	5
3	I like differential equation class work best when it really makes me think. میں تفرقی مساوات کی اس کلاس کو بے حد پسند کرتا ہوں جو مجھے سوچنے پر مجبور کرے	1	2	3	4	5
4	I do my work in differential equation class because it is important for me to get better at it. میں تفرقی مساوات کی کلاس کا کام خوشی سے کرتا ہوں کیونکہ میرے لیے نئی چیز کو سیکھنا اہم ہے	1	2	3	4	5
5	I do my differential equation class work is because I enjoy to do it. میں تفرقی مساوات کی کلاس کا کام کرنے سے لطف اندوز ہوتا ہوں	1	2	3	4	5
6	I do my differential equation class work because I'm interested in it. مجھے تفرقی مساوات کے کلاس ورک/کام میں دلچسپی ہے	1	2	3	4	5
	Performance-Approach Goal Orientation					
1	I would feel really good if I were the only one who could answer the teacher's questions in differential equation class. مجھے اچھا لگتا ہے جب میں کلاس میں وہ واحد طالب علم ہوتا ہوں جو کہ تفرقی سوالات کے جوابات دے	1	2	3	4	5
2	I want to do well than other students in my differential equation class. میں تفرقی مساوات کی کلاس میں دوسرے طالب علموں سے بہتر پرفارم کرنا چاہتا ہوں	1	2	3	4	5
3	I would feel successful in differential equation class if I did better than most of the other students. اگر میں تفرقی مساوات کی کلاس میں دوسروں کی بنسبت بہتر پرفارم کروں تو مجھے اپنی کامیابی کا احساس ہوتا ہے	1	2	3	4	5
4	I would like to show my teacher that I am smarter than the other students in my differential	1	2	3	4	5

Words Problem: عبارتی سوالات
Differential Equations: تفرقی مساوات

S.No	Sentences	Not at all true مکمل غلط	Not true غلط	Somewhat true کسی حد تک درست	True درست	Very true مکمل درست
	equation class. میں بھی تفرقی مساوات کی کلاس میں اپنے استاد کو دکھانا چاہتا ہوں کہ میں دوسروں کی بنسبت بہتر ہوں					
5	Doing better than other students in differential equation class is important to me. دوسروں کی بنسبت، تفرقی مساوات کے کورس کو بہتر طریقے سے پڑھنا میرے لیے اہم ہے	1	2	3	4	5
	Performance-Avoid Goal Orientation					
1	It's very important to me that I don't look stupid in my differential equation class. میرے لیے یہ بہت اہم ہے کہ میں تفرقی سوالات کی کلاس میں نالائق نہ لگوں	1	2	3	4	5
2	An essential reason I do my differential equation class work is so that I don't embarrass myself. تفرقی سوالات کی کلاس میں کلام کرنا مجھے اس لیے بھی اہم لگتا ہے کہ میں اپنے آپ کو شرمندہ نہ بناؤں	1	2	3	4	5
3	I do my differential equation class work is, that my teacher doesn't think I know less than others. تفرقی سوالات کی کلاس کا کام میں اس لیے بھی کرتا ہوں کہ میرا استاد یہ نہ سمجھے کہ مجھے یہ کام کرنا نہیں آتا	1	2	3	4	5
4	I do my differential equation work is so that the others won't think I'm dumb. تفرقی سوالات کی کلاس کا کام میں اس لیے بھی کرتا ہوں کہ باقی لوگ مجھے نالائق نہ سمجھیں	1	2	3	4	5
5	One of my main goals is to avoid looking like I can't do my work. تفرقی سوالات کی کلاس کا کام کرنے کا اہم مقصد یہ بھی ہے کہ میں ناکارہ نہ لگوں	1	2	3	4	5
6	I would not participate in differential equation class because I want to avoid looking stupid. میں تفرقی سوالات کی کلاس میں اسلیے بھی شرکت نہیں کرتا کیونکہ میں نالائق نہیں لگنا چاہتا	1	2	3	4	5

Differential Equations: تفرقی مساوات

S.No	Sentences	Not at all True of me مکمل غلط	Not True غلط	Some What Not True کسی حد تک غلط	Uncertain غیر یقینی (تذبذب میں)	Some What True کسی حد تک درست	True درست	Very True of me مکمل درست
	Critical Thinking Questionnaires							
1	I often find myself questioning things I hear or read in the differential equation course to decide if I find them believable. میں اکثر اپنے آپ سے سوال کرتا ہوں کہ جو میں نے تفرقی مساوات کے نصاب / کورس کی کلاس میں پڑھا یا سنا وہ قابل یقین ہے ۔	1	2	3	4	5	6	7
2	When a theory, interpretation, or conclusion is presented in differential equation class or in the readings, I try to decide if there is good supporting evidence. جب بھی کوئی نظریہ (theory) ، یا اس کی تشریح یا اس کا خلاصہ کلاس میں پیش کیا جاتا ہے تو میں اس کی حمایت (support) کرتا ہوں	1	2	3	4	5	6	7
3	I treat the differential equation course material as a starting point and try to cultivate my own thoughts about it. میں تفرقی مساوات کو ریاضی کا بنیادی حصہ سمجھتا ہوں اور اس سے نئے نظریات کو ترتیب دیتا ہوں	1	2	3	4	5	6	7
4	I attempt to play around with my own ideas related to what I am learning in the differential equation course. میں جب بھی تفرقی مساوات کی کلاس لیتا ہوں تو میں اپنے خیالات کو اس بات پر فوکس کرتا ہوں جو میں سیکھ رہا ہوں	1	2	3	4	5	6	7
5	Whenever I read or hear an argument or conclusion in differential equation class, I think about possible solutions. میں جب بھی تفرقی مساوات کی کلاس میں کوئی دلائل سنتا ہوں تو میں ممکنہ حل تلاش کرنے کی کوشش کرتا ہوں	1	2	3	4	5	6	7
	Elaboration Questionnaires							
1	When I study for differential equation class, I organize information from various/multiple sources, such as lectures, readings, and discussions. میں جب تفرقی مساوات کی کلاس لیتا ہوں میں مختلف ذرائع مثلاً لیکچر، نوٹس یا سوال پوچھنے سے اپنی معلومات کو بڑھاتا ہوں۔	1	2	3	4	5	6	7

Differential Equation: تفرقی مساوات

S.No	Sentences	Not at all True of me مکمل غلط	Not True غلط	Some What Not True کسی حد تک غلط	Uncertain غیر یقینی	Some What True کسی حد تک درست	True درست	Very True of me
2	I try to relate differential equation ideas in math's subject to those in other courses like physics, biology, chemistry whenever possible جہاں کہیں کی بھی ممکن ہو میں تفرقی مساوات کے نظریات کو دوسرے مضامین جیسے فزکس، کیمسٹری یا بیالوجی کے ساتھ منسلک کرتا ہوں۔	1	2	3	4	5	6	7
3	When reading for differential equation class, I try to make a connection with my previous knowledge. میں جب بھی تفرقی مساوات کا کورس پڑھتا ہوں تو میں اپنا سابقہ (پہلے پڑھا ہوا علم) بھی اس سے منسلک کرتا ہوں۔	1	2	3	4	5	6	7
4	When I study for the differential equation course, I write brief summaries of the main ideas from the readings and my class notes. میں جب بھی تفرقی مساوات کا کورس پڑھتا ہوں تو میں اس کے اہم پوائنٹس / نظریات کا خلاصہ لکھتا ہوں۔	1	2	3	4	5	6	7
5	I try to understand the contents of the differential equation course by making connections between the readings and the concepts from the lectures. میں تفرقی مساوات کے سوالوں کو کلاس لیکچر کے نوٹس اور لیکچر سے سمجھنے کی کوشش کرتا ہوں	1	2	3	4	5	6	7
6	I try to apply ideas from course readings in other class activities such as lecture and discussion. میں پڑھے ہوئے کورس (نصاب) کے اہم نظریے / پوائنٹس کو دوسری سرگرمیوں جیسے لیکچر کے دوران بحث ومباحثے کے درمیان بھی استعمال کرتا ہوں۔	1	2	3	4	5	6	7

1. In a research laboratory, a researcher studied the growth of bacteria culture. Normally, the bacteria population increases at the rate proportional to

تحقیقاتی لیبارٹری میں ایک محقق (Researcher) نے بیکٹیریا کے بڑھنے کا عمل کا مشاہدہ/مطالعہ کیا۔

the size of bacteria present. The researcher found the number of bacteria increases six fold (times) in 10 hours. Initially number of bacteria is 10.

عموماً بیکٹیریا کی نشوونما اس کے موجودہ سائز کے براہ راست متناسب ہوتی ہے۔ اس نے ۱۰ گھنٹوں میں 6 گنا اضافہ دیکھا جبکہ ابتدا میں بیکٹیریا کی تعداد ۱۰ تھی

a. Write differential equation that describes population increase for bacteria.

(a) نارمل نشوونما کو فرض کریں اور بیکٹیریا کے اضافے کی مساوات لکھیں

b. Assuming normal growth, how long did it take for their population to double?

(b) یہ بتائیں کہ ابتدائی تعداد کتنے وقت کے بعد 2 گنا ہو جائے گی؟

2. In a playground, a football player has to hit the ball vertically upward, or at certain angel to pass it to another player or hit the goal. During a match, a player hit the ball vertically upward with a velocity of 15 m/s.

فٹ بال کے میدان میں ساتھی کھلاڑی کو بال/گول پاس کرنے کے لیے کسی بھی کھلاڑی کو مخصوص سمت میں بال کو اچھالنا پڑتا ہے۔ اس میچ کے دوران ایک کھلاڑی نے بال کو 15 میٹر فی سیکنڈ کی ولاسٹی سے اوپر کو اچھالا۔

a. Write differential equation (model) that describes the velocity of the ball with respect to time (Neglecting air resistance).

(a) ہوا کی مزاحمت کو نظر انداز کرتے ہوئے ایک تفرقی مساوات (ماڈل) لکھیں جو ولاسٹی کو بلحاظ وقت ظاہر کرے

b. Find the equation for the distance (height) travelled by the ball in any time "t".

(b) اسی طرح کی ایک مساوات (ماڈل) بھی لکھیں جو اونچائی (h) اور وقت (t) کا تعلق ظاہر کرے

c. Find the height of the ball after one second.

(c) اچھالی گئی بال کی اونچائی ایک سیکنڈ کے بعد کیا ہوگی؟

Solution

3. In Pakistan as well as in the world, banks provide incentives in the term of interest on the deposited money. Ayaan deposited an amount of 1000 Rupees in the bank Islami, which has an interest rate (dM/dt) of 5.0% per year. He did not draw any money and interest compounds continuously. How much amount he will have after 5 years? What you think, the amount of money increased, decreased or remained constant?

پاکستان اور پوری دنیا میں بینکوں میں رقم رکھنے سے، رقم پر منافع ملتا ہے۔ آیان نے بینک اسلامی میں ایک ہزار روپے جمع کیے۔ بینک اسلامی سالانہ 5% منافع دیتا ہے۔ آیان نے 5 سال تک کوئی رقم نہیں نکالی۔ یہ بتائیں کہ 5 سال بعد اس کی رقم کتنی ہو گئی؟
آپ کا کیا خیال ہے کہ جمع شدہ رقم میں اضافہ ہوا، کمی ہوئی یا کوئی تبدیلی وقوع پذیر نہیں ہوئی؟

Solution

4. In the district Mansehra, district health center, monitors and records the polio cases of the whole district. On the basis of 10 years recorded data, the center has formulated a rate equation $\frac{dA}{dt} = 100t$

Where A is number of infected people at the start of 2005, and t is time measured in year. Initially there were 200 people infected. Make a sketch of derivative graph and its function graph at time $0 \leq t \leq 5$.

ضلع مانسہرہ میں، ضلعی مرکز صحت پورے ضلع کے پولیو کیسوں کا مشاہدہ کرنے اور ریکارڈ کرنے کا ذمہ دار ہے
دس سالہ ریکارڈ کی بنیاد پر اس مرکز صحت نے ایک مساوات (ماڈل) ترتیب دی ہے۔ اس میں A
متاثرہ (پولیو سے) لوگوں کی تعداد کو ظاہر کرتا ہے۔ t - ٹائم/وقت کو ظاہر کرتا ہے۔ ابتدائی رپورٹ کے مطابق، ابتدا میں
200 بندے پولیو سے متاثر تھے۔ اس ریکارڈ کو دیکھتے ہوئے، متاثرہ لوگوں (Function) اور
ماخذ (Derivative) مساوات کے گراف بنائیں۔

Solution

5. In the bakeries, cakes and biscuits are normally prepared using different food ingredients and finally cooked in the baking

بیکریوں میں عموماً کیک اور بسکٹ مختلف اجزاء کے مرکب سے بنائے جاتے ہیں۔ جن کو پھر تنور / بھٹی میں پکایا جاتا ہے۔

oven. In a bakery, a cake was removed from baking oven at 400K. Five minutes later, its temperature was decreased to 200K,

ایک بیکری میں، ایک کیک کو 400 ڈگری کیلون پر پکانے کے بعد تنور / بھٹی سے نکالا گیا۔ 5 منٹ کے بعد اس کا درجہ حرارت 200 ڈگری کیلون تک گر گیا۔ جو کہ نیوٹن کے ٹھنڈک کے قانون کے عین مطابق ہے

which can be well described by Newton law of cooling

$$\frac{dT}{dt} = K(T - T1)$$

اس مساوات میں $T-T_1$ درجہ حرارت کی تبدیلی کو ظاہر کرتا ہے۔ جبکہ t وقت کو ظاہر کرتا ہے۔

Where T-T1 is change in temperature and t is time. What would be the general solution of this differential equation?

اس مساوات کا جنرل حل (general solution) کیا ہوگا؟

Solution

Questionnaire for Field Expert And Teachers

Teacher Name	
Institute/ College Name	
Gender	Male/Female
Highest Degree	B.Sc./ M.Sc./ M. Phil/Ph.D
Teaching Experience	Less than 2 years/ 2-4 years/ 5-10 years/other ( )
Present Designation	Subject Specialist/ Lecturer/ Assistant Professor/Other
Induction	Direct Appointment on the Present Post/ Promoted
Teaching Experience Areas	Urban /Rural/ Both
Teaching Sectors	Govt/ Private/ Both

Part-A: Developed Tasks of Differential Equations and Adapted Questionnaire

How you will rank these developed differential equation tasks among the main characteristics of good mathematics problems?

Characteristics of Differential Equations Containing Tasks	Poor	Fair	Good	Very Good	Excellent
Clarity of phrasings and wordings					
Relevancy to the course and level of the students					
Challenging and non-routine as they are not easily solvable by using a previously taught simple algorithms and procedures					

Able to promote active involvement of students					
Allow multiple approaches and solutions					
Able to make connection of differential equations to other mathematical concepts and real world problems					
Adapted Questionnaire					
How you will rank the clarity and unambiguousty of this questionnaire?					
Are questionnaire items are logically and sequentially well organized?					
How you will rank the interpretation level of Urdu translation?					

Part-B: Differential Equation Course for 12th Year of Study

a) Do you believe that teaching and learning of differential equation is a difficult part of the mathematics at inter-college level as compared to other parts like algebra, trigonometry and etc?

(1) Strongly disagree (2) Disagree (3) Uncertain (4) Agree (5) Strongly agree

b) Do you think that high level of conceptual understandings and special efforts are required to solve differential equations based problems?

(1) Strongly disagree (2) Disagree (3) Uncertain (4) Agree (5) Strongly agree

c) Do you agree that differential equation based problems, particularly of non-routine nature can be used to correlate the real world problems.

(1) Strongly disagree (2) Disagree (3) Uncertain (4) Agree (5) Strongly agree

d) Do you agree that at present, less attention is given to the non-routine based problems containing differential equation at inter college level?

(1) Strongly disagree (2) Disagree (3) Uncertain (4) Agree (5) Strongly agree

e) Should policy makers increase non-routine differential equation problems in the new mathematics curriculum?

(1) Strongly disagree (2) Disagree (3) Uncertain (4) Agree (5) Strongly agree

f) Do you agree that teachers should be properly equipped and trained, so that they may educate non-routine as well as routine based problems containing differential equation?

(1) Strongly disagree (2) Disagree (3) Uncertain (4) Agree (5) Strongly agree

g) Do you agree that students psyche can also boost up the differential equation based problem solving?

(1) Strongly disagree (2) Disagree (3) Uncertain (4) Agree (5) Strongly agree

h) Do you think that students' motivations can enhance the understandings as well as the solution of differential equation based problems?

(1) Strongly disagree (2) Disagree (3) Uncertain (4) Agree (5) Strongly agree

i) Do you think that self-regulated learning strategies, such as critical thinking and elaboration can affect positively for the students to solve differential equation based problems?

(1) Strongly disagree (2) Disagree (3) Uncertain (4) Agree (5) Strongly agree

j) Do you think that epistemological beliefs about problem solving can affect the differential equation based problem solving?

(1) Strongly disagree (2) Disagree (3) Uncertain (4) Agree (5) Strongly agree

k) Do you think that combination of epistemological beliefs about problem, motivations and self-regulated learning strategies can significantly contribute toward differential equation based problem solving?

(1) Strongly disagree (2) Disagree (3) Uncertain (4) Agree (5) Strongly agree

l) Do you think that this type of research is useful for both of teachers as well as students?

(1) Strongly disagree (2) Disagree (3) Uncertain (4) Agree (5) Strongly agree

Permission to use adapted questionnaire based on measuring beliefs questionnaires

Dear Aisha,

Copyright law does not allow selling these scales or in any way making a profit from them. As long as you are just using the scales in your research without selling them you are free to use them without any permission. Good luck with your work.

Peter Kloosterman

Martha Lea and Bill Armstrong Chair for Teacher Education, 2010-2015

Professor of Mathematics Education

Indiana University School of Education

201 N. Rose Ave.

Bloomington, IN 47405812-856-8147

Permission to use adapted questionnaire based on Dr. Paul Pintrich¹s work Motivated Strategy learning questionnaire

Dear Aisha Bibi,

This is not my questionnaire; therefore, I am not in position to give you full permission to use it. However, I believed it is in the public domain, though, and so can be used.

Allan Wigfield

Department of Human Development and Quantitative Methodology

College of Education, University of Maryland

College Park, MD 20742, USA