THE GEOMETRY OF CR-SUBMANIFOLDS

CLOSED STACKS

by

NG YEN YEN

A dissertation submitted to the
Faculty of Science of the
University of Malaya, Kuala Lumpur
in fulfilment of the requirements
for the degree of
Master of Science

1995

ABSTRACT

The concept of a CR-submanifold of an almost Hermitian manifold was introduced by A. Bejancu. Since then, the geometry of a CR-submanifold in a Kaehler manifold has been extensively studied by numerous mathematicans. In this dissertation, we showed that with a few alterations, some of the results of a CR-submanifold of a Kaehler manifold can be generalized to a nearly-Kaehler manifold. For instance, B.Y. Chen proved that a CR-submanifold of a Kaehler manifold is a CR-product if and only if the condition $A_{JD}{}^{\perp}D = 0$ is satisfied. We managed to prove a similar result for a CR-submanifold of a nearly-Kaehler manifold. Furthermore, we also improved on a theorem of A. Bejancu which characterizes a CR-submanifold of an almost Hermitian manifold.

In addition, we also obtained some interesting results for a CR-submanifold of a quasi-Kaehler manifold. This includes a theorem on the necessary and sufficient condition for the distribution $D^{\perp}$ to be integrable when the CR-submanifold is totally umbilical.

ABSTRAK

Konsep submanifold-CR dalam manifold hampir Hermitian telah diperkenalkan oleh A. Bejancu. Semenjak itu, geometri submanifold-CR dalam manifold Kaehler telah banyak dikaji oleh ramai ahli matematik. Dalam dissertasi ini, kami menunjukkan bahawa dengan sedikit pengubaisuaian, setengah daripada keputusan submanifold-CR dalam manifold Kaehler boleh diperkembangkan kepada manifold dekat Kaehler. Sebagai contoh, B.Y. Chen telah membuktikan bahawa submanifold-CR dalam manifold Kaehler merupakan hasildaraban-CR jika dan hanya jika syarat $A_{JD^{\perp}}D = 0$ dipenuhi. Disini, kami membuktikan suatu keputusan yang hampir serupa dengan keputusan B.Y. Chen bagi submanifold-CR dalam manifold nearly-Kaehler. Tambahan juga, kami telah berjaya memperbaiki suatu theorem yang dibuktikan oleh A. Bejancu tentang cirian submanifold-CR dalam manifold hampir Hermitian.

Di samping itu, kami juga mendapat beberapa keputusan bagi submanifold-CR dalam manifold quasi-Kaehler. Ini meliputi suatu theorem mengenai pengamiran taburan $D^{\perp}$ apabila submanifold-CR ini adalah umbilik keseluruh.

ACKNOWLEDGEMENT

I would like to express my heartfelt thanks to my supervisor, Dr. Tan Sin Leng for his invaluable advice, guidance and patience, without which this dissertation would never have been completed.

I would also like to express my sincere thanks to Prof. Chong Kong Ming and the Department of Mathematics, for granting me a teaching assistantship and for allowing me to use the facilities needed.

My sincere thanks goes to Dr. Kon Song How, for his kind assistance and Mr. Tu Yong Eng for his many helpful suggestions.

I am extremely grateful to my family members for their love and support. Pa, Ma, Chen, Boon, Ching,..I love you all.

Special thanks goes to Dr. Toh and the Tutor Gang: Chong Mun, Wai Leong, Wee Keong, Thean Seng, Lee Meng and especially Hse Tzia, my roommate-cum-secret conspirator. I really treasure the many happy and crazy moments we had together. Way to go, gang!

I am also indebted to my BGF and BCS friends for their love and encouragement throughout these years. Three Sadhu's to all of you. What a gem you all have been.

I would like to express my sincere gratitude to Rev. U. Vimala (Cirasaddho) for his spiritual guidance and support, without which I will never be the person I am today. Thank you Rev., for your loving and kind thoughts.

And, lastly, ... Eng, thank you for coming into my life at the right time and at the right moment. I really appreciate the beautiful moments we share together.

CONTENTS

Introduction

This dissertation is devoted to the study of CR-submanifolds of Kaehler, nearly-Kaehler and quasi-Kaehler manifolds. It consists of four chapters. Chapter One introduces the basic concepts which will be used in the other chapters of the dissertation.

The concept of a CR-submanifold of an almost Hermitian manifold as introduced by Bejancu [1] is discussed in Chapter Two. We are able to improve Bejancu's characterization theorem (Theorem 2.1, p.27) by dropping one of his conditions (Proposition 2.1, p.27). We also obtain another characterization for a CR-submanifold of an almost Hermitian manifold (Proposition 2.2, p.30). A section on the geometry of CR-submanifold of a Kaehler manifold is also included in this chapter.

Chapter Three is focused on CR-submanifolds of nearly-Kaehler manifolds. We begin by surveying some known results on the integrability conditions for the distributions D and $D^{\perp}$ and on the geometry of a totally umbilical CR-submanifold of a nearly-Kaehler manifold. We also have a section on CR-products of nearly-Kaehler manifolds. In this section, we obtain a necessary and sufficient condition for a CR-submanifold to be a CR-product. The last section of this chapter is on the D-parallel normal section of a CR-submanifold. Here, we obtain a characterization for the D-parallel normal section which belongs to the normal subbundle $JD^{\perp}$ (Proposition 3.7, p.71).

In Chapter Four, we will discuss some integrability conditions of the distribution D of a CR-submanifold of a quasi-Kaehler manifold. We also include a section on totally umbilical CR-submanifolds of a quasi-Kaehler manifold. Here, we obtain a necessary and sufficient condition for the integrability of the distribution $D^{\perp}$ (Proposition 4.4, p.86). We also proved that the mean curvature vector H lies in $JD^{\perp}$ (Proposition 4.5, p.87). A short section on mixed totally geodesic CR-submanifolds of a quasi-Kaehler manifold is also included in this last chapter.