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THE GEOMETRY OF CR-SUBMANIFOLDS

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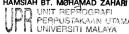
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ABSTRACT

The concept of a CR-submanifold of an almost Hermitian manifold was introduced by A. Bejancu. Since then, the geometry of a CR-submanifold in a Kaehler manifold has been extensively studied by numerous mathematicans. In this dissertation, we showed that with a few alterations, some of the results of a CR-submanifold of a Kaehler manifold can be generalized to a nearly-Kaehler manifold. For instance, B.Y. Chen proved that a CR-submanifold of a Kaehler manifold is a CR-product if and only if the condition $A_{\rm JD}{}^{\rm LD}=0$ is satisfied. We managed to prove a similar result for a CR-submanifold of a nearly-Kaehler manifold. Furthermore, we also improved on a theorem of A. Bejancu which characterizes a CR-submanifold of an almost Hermitian manifold.

In addition, we also obtained some interesting results for a CR-submanifold of a quasi-Kaehler manifold. This includes a theorem on the necessary and sufficient condition for the distribution D^{\perp} to be integrable when the CR-submanifold is totally umbilical.

ABSTRAK

Konsep submanifold-CR dalam manifold hampir Hermitian telah diperkenalkan oleh A. Bejancu. Semenjak itu, geometri submanifold-CR dalam manifold Kaehler telah banyak dikaji oleh ramai ahli matematik. Dalam dissertasi ini, kami menunjukkan bahawa dengan sedikit pengubaisuaian, setengah daripada keputusan submanifold-CR dalam manifold Kaehler boleh diperkembangkan kepada manifold dekat Kaehler. Sebagai contoh, B.Y. Chen membuktikan bahawa submanifold-CR dalam manifold Kaehler merupakan hasildaraban-CR jika dan hanya jika syarat A_{tn} LD = 0 dipenuhi. Disini, kami membuktikan suatu keputusan yang hampir serupa dengan keputusan B.Y. Chen bagi submanifold-CR dalam manifold nearly-Kaehler. Tambahan juga, kami telah berjaya memperbaiki suatu theorem yang dibuktikan oleh A. Bejancu tentang cirian submanifold-CR dalam manifold hampir Hermitian.

Di samping itu, kami juga mendapat beberapa keputusan bagi submanifold-CR dalam manifold quasi-Kaehler. Ini meliputi suatu theorem mengenai pengamiran taburan D^{\perp} apabila submanifold-CR ini adalah umbilik keseluruh.

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CONTENTS

ACKNOWLEDGEMENTS		i	
CONTENTS	5		iii
INTRODU	CTION		v
CHAPTER	1.	DEFINITIONS AND PRELIMINARIES	1
	1.1.	Differentiable manifolds and tensor	
		fields	1
	1.2.	Linear connections on a manifold	7
	1.3.	Submanifolds of a Riemannian manifold	10
	1.4.	Distributions on a manifold	13
	1.5.	Almost Hermitian manifold	17
CHAPTER	2.	CR-SUBMANIFOLDS	
	2.1.	Introduction	21
	2.2.	Examples of CR-submanifolds	22
	2.3.	Characterization of a CR-submanifold	26
	2.4.	Integrability conditions of distributions	
		on a CR-submanifold	33
	2.5.	CR-submanifolds of a Kaehler manifold	35
CHAPTER	3.	CR-SUBMANIFOLDS OF A NEARLY-KAEHLER	
		MANIFOLD	48
	3.1.	Introduction	48
	3.2.	Integrability of distributions of a	
		CR-submanifold of a nearly-Kaehler manifold	50

	3.3.	lotally umbilical CK-submanifolds of a	
		nearly-Kaehler manifold	60
	3.4.	CR-product of a nearly-Kaehler manifold	65
	3.5.	D-parallel normal section on a CR-submani-	
		fold	70
CHAPTER	4.	CR-SUBMANIFOLDS OF A QUASI-KAEHLER MANIFOLD	75
	4.1.	Introduction	75
	4.2.	Integrability of the holomorphic	
		distribution of a CR-submanifold of a	
		quasi-Kaehler manifold	76
	4.3.	Minimal distribution	82
	4.4.	Totally umbilical CR-submanifold of a	
		quasi-Kaehler manifold	86
	4.5	Mixed totally geodesic CR-submanifold	89
REFERENC	CES		91

Introduction

This dissertation is devoted to the study of CR-submanifolds of Kaehler, nearly-Kaehler and quasi-Kaehler manifolds. It consists of four chapters. Chapter One introduces the basic concepts which will be used in the other chapters of the dissertation.

The concept of a CR-submanifold of an almost Hermitian manifold as introduced by Bejancu [1] is discussed in Chapter Two. We are able to improve Bejancu's characterization theorem (Theorem 2.1, p.27) by dropping one of his conditions (Proposition 2.1, p.27). We also obtain another characterization for a CR-submanifold of an almost Hermitian manifold (Proposition 2.2, p.30). A section on the geometry of CR-submanifold of a Kaehler manifold is also included in this chapter.

Chapter Three is focused on CR-submanifolds of nearly-Kaehler manifolds. We begin by surveying some known results on the integrability conditions for the distributions D and D¹ and on the geometry of a totally umbilical CR-submanifold of a nearly-Kaehler manifold. We also have a section on CR-products of nearly-Kaehler manifolds. In this section, we obtain a necessary and sufficient condition for a CR-submanifold to be a CR-product. The last section of this chapter is on the D-parallel normal section of a CR-submanifold. Here, we obtain a characterization for the D-parallel normal section which belongs to the normal subbundle JD¹ (Proposition 3.7, p.71).

In Chapter Four, we will discuss some integrability conditions of the distribution D of a CR-submanifold of a quasi-Kaehler manifold. We also include a section on totally umbilical CR-submanifolds of a quasi-Kaehler manifold. Here, we obtain a necessary and sufficient condition for the integrability of the distribution D^1 (Proposition 4.4, p.86). We also proved that the mean curvature vector H lies in JD^1 (Proposition 4.5, p.87). A short section on mixed totally geodesic CR-submanifolds of a quasi-Kaehler manifold is also included in this last chapter.