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Introduction

This dissertation is devoted to the study of CR-submanifolds of Kaehler, nearly-Kaehler and quasi-Kaehler manifolds. It consists of four chapters. Chapter One introduces the basic concepts which will be used in the other chapters of the dissertation.

The concept of a CR-submanifold of an almost Hermitian manifold as introduced by Bejancu [1] is discussed in Chapter Two. We are able to improve Bejancu's characterization theorem (Theorem 2.1, p.27) by dropping one of his conditions (Proposition 2.1, p.27). We also obtain another characterization for a CR-submanifold of an almost Hermitian manifold (Proposition 2.2, p.30). A section on the geometry of CR-submanifold of a Kaehler manifold is also included in this chapter.

Chapter Three is focused on CR-submanifolds of nearly-Kaehler manifolds. We begin by surveying some known results on the integrability conditions for the distributions D and D^\perp and on the geometry of a totally umbilical CR-submanifold of a nearly-Kaehler manifold. We also have a section on CR-products of nearly-Kaehler manifolds. In this section, we obtain a necessary and sufficient condition for a CR-submanifold to be a CR-product. The last section of this chapter is on the D -parallel normal section of a CR-submanifold. Here, we obtain a characterization for the D -parallel normal section which belongs to the normal subbundle JD^\perp (Proposition 3.7, p.71).

In Chapter Four, we will discuss some integrability conditions of the distribution D of a CR-submanifold of a quasi-Kaehler manifold. We also include a section on totally umbilical CR-submanifolds of a quasi-Kaehler manifold. Here, we obtain a necessary and sufficient condition for the integrability of the distribution D^\perp (Proposition 4.4, p.86). We also proved that the mean curvature vector H lies in JD^\perp (Proposition 4.5, p.87). A short section on mixed totally geodesic CR-submanifolds of a quasi-Kaehler manifold is also included in this last chapter.