THE EFFECT OF HUMAN LEARNING AND FORGETTING ON FUZZY EOQ MODEL WITH BACKORDERS

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Field of Study: Manufacturing Management

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ABSTRACT

Inventory planning is a repetitive task, often characterized with inventory planners experiencing high levels of uncertainty. When estimating the key cost parameters of an inventory model the experience and learning capabilities of the planners affect efficiency of the inventory system. Fuzzy set theory has been used to model inventory parameters subject to uncertainty, where determining uncertain parameters depends upon the subjective opinions of the decision makers. Due to the repetitive nature of inventory planning, the inventory planner has to perform planning tasks repetitively, and consequently, s/he becomes more familiar with the tasks over time. Familiarity with the tasks suggests that learning takes place during inventory planning. Even though the operator's learning over time may improve his/her efficiency, prior research on fuzzy inventory management completely overlooked the effect of human learning and learning transfer in their models. To close the research gap in this area, this thesis aims to present fuzzy economic order quantity (EOQ) models with backorders, with the objective of formulating the planner's learning in estimating fuzzy parameters. After a comprehensive and systematic literature review, it was identified that the studies in the literature lacked the empirical evidence on the existence of human learning. Hence, the methodology of the thesis starts with a set of semi-structured interviews with six industrial staff members from different companies. The interviews helped us to gain insights into how human learning is observed in inventory planning under uncertainty. The main themes that emerged from the interviews were summarized as four propositions, which later assisted us in formulating assumptions for the inventory models. Subsequently, through extending an earlier study in the literature, four fuzzy EOQ models with backorders that took account of human learning and forgetting over the planning cycles were developed. The models suggested situations in which the operator applies the acquired knowledge over the cycles in setting the imprecise parameters at the beginning of every planning cycle.

The learning ability of the planner was formulated using the log-linear learning curve and the learning curve with the cognitive and motor capabilities of a human being. In order to optimize the models and derive solutions, an optimization algorithm was developed for the first model and applied later throughout the study. Finally, the developed models were examined using both primary and secondary data sets. In the first step, the models were tested using the data obtained from a study in the literature. Next, a case study company in the manufacturing industry was selected and the related data was collected form the inventory system. The models were optimized for the collected data to derive optimal policies for the case study company, highlighting the gap between the current and optimized policies. The results of the study show that learning and forgetting are relevant in inventory management under uncertainty, and that human learning could improve the performance of an inventory system. Incorporating human learning into decision leads to increasing the number of orders, which tends to decrease the batch sizes and increase the maximum amount of inventory.

ABSTRAK

Perancangan inventori merupakan suatu kaedah berulang yang kerap kali dikaitkan dengan perancang inventori yang mempunyai tahap ketidakpastian yang tinggi apabila membuat anggaran parameter kos utama. Teori set fuzzy telah digunakan untuk merangka parameter inventori yang tertakluk kepada ketidakpastian, di mana tugas menentukan parameter-parameter yang tidak pasti bergantung kepada pendapat subjektif pembuat keputusan. Perancang inventori telah melaksanakan tugas-tugas perancangan secara berulang-ulang di sebabkan oleh ciri-ciri kekerapan di dalam perancangan inventori. Lantas, menjadikan kebiasaan kepada Perancang inventori menjalankan tugas dari semasa ke semasa. Pembelajaran berlaku dimana terdapat kelaziman di dalam menguruskan tugas dalam perancangan inventori. Pembelajaran bagi mengendalikan sesuatu dari semasa ke semasa mungkin juga dapat meningkatkan kecekapan pengendali. Walaubagaimanapun, penyelidikan di awal pembelajaran mengenai masalah saiz-lot kabur telah terlepas pandang kesan pembelajaran manusia dan pemindahan pembelajaran dalam model mereka dan kesannya ke atas prestasi pengendali. Untuk merapatkan jurang penyelidikan dalam bidang ini, tesis ini adalah bertujuan untuk memberikan beberapa kabur kuantiti pesanan ekonomi (EOQ) model dengan tempahan yang kurang dan dengan objektif untuk merangka pembelajaran jururancang bagi menganggarkan parameter kabur. Model-model mencadangkan keadaan di mana pengendali telah menerapkan pengetahuan yang diperolehi dalam tempoh kitaran dalam menetapkan parameter kabur di awal tiap kitaran perancangan. Keupayaan pembelajaran perancang itu termasuk keupayaan pembelajaran yang mudah dan pembelajaran dengan keupayaan kognitif dan keupayaan motor daripada seseorang manusia. Selain proses pembelajaran, tesis ini juga memberi penerapan kepada dua model bagi kes-kes di mana nilai kitaran pengetahuan yang diperolehi oleh pengendali menurun apabila berjauhan daripada tugas-tugas perancangan. Selain daripada itu,tesis ini juga memberi penerapan kepada empat model

matematik yang mengambil kira pembelajaran manusia yang mudah termasuk dua peringkat pembelajaran dan lupa terhadap kitaran perancangan yang telah berkembang. Selepas itu, empat model matematik yang mengambil kira pembelajaran manusia yang mudah dan dua peringkat dan melupakan lebih kitaran perancangan dibangunkan. Setelah penyelidikan kajian-kajian lepas dilakukan dengan menyeluruh dengan menggunakan kaedah yang sistematik, didapati bahawa kajian-kajian terdahulu tidak mempunyai bukti empirikal yang cukup tentang kewujudan proses pembelajaran manusia. Justeru, metodologi tesis ini dimulakan dengan satu set temuramah separa-berstruktur dengan enam orang pekerja industri daripada beberapa buah syarikat. Temuramah tersebut membantu kami mendapatkan maklumat tentang bagaimana proses pembelajaran manusia diperhatikan dalam perancangan inventori yang mengandungi ketidakpastian. Tema-tema utama yang diperhatikan daripada temuramah tersebut diringkaskan kepada empat usul, yang kemudiannya membantu kami memformulakan beberapa andaian untuk model-model inventori tersebut. Seterusnya, melalui pelanjutan kajian, empat model EOQ kabur dengan tempahan lambat, yang mengambil kira faktor pembelajaran manusia dan sifat lupanya sepanjang kitaran perancangan, telah dihasilkan. Model-model tersebut menggambarkan situasi di mana pengendali menggunakan pengetahuan yang diperoleh sepanjang kitaran dalam menentukan parameter tidak tepat pada permulaan setiap kitaran perancangan. Kemampuan belajar perancang tersebut diformulakan menggunakan keluk pembelajaran log-linear dan keluk pembelajaran dengan kemampuan motor dan kognitif seorang manusia. Untuk mengoptimumkan model tersebut dan mendapatkan penyelesaian, satu algoritma pengoptimuman telahpun dibina untuk model pertama, dan terus digunakan sepanjang kajian ini. Akhirnya, model-model yang dibina telah pun diperiksa menggunakan data primer dan sekunder. Pada langkah pertama, model-model tersebut diuji dengan menggunakan data yang diperoleh daripada penyelidikan kajian-kajian terdahulu. Seterusnya, satu kajian kes terhadap syarikat yang terlibat dalam industri pembuatan telah dipilih, dan data berkaitan telah diambil daripada sistem inventori syarikat tersebut. Model-model tersebut dioptimumkan untuk data yang dikumpul, untuk menghasilkan polisi yang optimum untuk kajian kes syarikat, sekali gus menonjolkan jurang antara polisi semasa dengan polisi yang telah dioptimumkan. Hasil kajian menunjukkan bahawa pembelajaran dan alpa adalah relevan dalam pengurusan ketidakpastian inventori dan pembelajaran manusia dapat meningkatkan prestasi dalam sistem inventori. Penggabungan pembelajaran manusia ke dalam membuat keputusan membawa kepada peningkatan bilangan pesanan, yang cenderung untuk mengurangkan saiz kumpulan dan meningkatkan jumlah maksimum inventori.

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LIST OF ABBREVIATIONS

ACI Pars Co.	Auto Chassis International Pars Company
BFEOQ	Basic Fuzzy EOQ Models
DFLC	Dual Phase Learning Curve
EDI	Electronic Data Interchange
ELSP	Economic Lot Scheduling Problem
EOQ	Economic Order Quantity
EOQ-S	EOQ Model with Shortages
EOT	Economic Order Time
EPQ	Economic Production Quantity
FEOQ	Fuzzy EOQ Models
FEOQB	Fuzzy EOQ Models with Backorders
FEOQE	Extensions of Fuzzy EOQ Model
FEOQED	Fuzzy EOQ Models with Delay in Payment
FEOQEO	Other Extensions of Fuzzy EOQ Models
FEOQEQ	Fuzzy EOQ Models with Quality
FEOQEQI	Mix Quality-Multi-Item Fuzzy EOQ Models
FEOQMI	Multi-item Fuzzy EOQ Models
FEPQ	Fuzzy EPQ Model
FEPQEMI	Multi-Items Fuzzy EPQ Models
FEPQEQ	Quality Based Fuzzy EPQ Models
FEPQES	Fuzzy EPQ Models with Shifting in Production
FEPQEW	Fuzzy EPQ Models with Rework
FJS	Fuzzy Joint/Supply Chain Models

FJSME	Fuzzy Joint/Supply Chain Models with Multi Echelons
FJSTE	Fuzzy Joint/Supply Chain Models with Two Echelons
FJSTEE	Extensions of Fuzzy Joint/Supply Chain Models with Two Echelons
FJSTEEQ	Fuzzy Two-Echelon Joint/Supply Chain Models with Quality
FJSTEES	Fuzzy Two-Echelon Joint/Supply Chain Models with Stochastic Demand and/or Stochastic Lead time
GA	Genetic Algorithm
GMIR	Graded Mean Integration Representation
GRG	Generalized Reduced Gradient
IDRO	Industrial Development Renovation Organization
ISI	Institute for Scientific Information
JELS	Joint Economic Lot Size
JGLC	Jaber-Glock Learning Curve
LFCM	Learn–Forget Curve Model
LFCMCM	Learn–Forget Curve Model with Cognitive and Motor Learning
LR	Learning Rates
MRP	Material Requirement Planning
MSMD	Multiple Setups Multiple Deliveries
SPR	Système de Production Renault
SVSB	Single-Vendor Single-Buyer
PHF	Prodiuts Hors Fabrication
PID	Power Integration Diffusion
POE	Pièces Ouvrées Extèrieures
PSO	Particle Swarm Optimization
RC	Recency Model
WOS	Web of Science

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CHAPTER 1: INTRODUCTION

1.1 Importance of inventory management in global business

In today's turbulent and fast-moving business environment, companies are facing intense competition in both local and global markets. For every product or service on the market, there are several companies supplying the products with the superior quality, at very competitive prices, and with the highest level of customer service. Market competition in terms of cost, quality, and delivery are not the only challenges companies facing; they are also dealing with rapid business changes, which is a menace to their survival. In fact, companies have to work in an environment characterized by complex technology, increasing customer demands and uncertainty. With this ever-increasing pressure on companies to adapt to the new business environment, they must improve their products and services to gain a better position in the market. If they reconcile slowly or do not embrace the changes, their businesses will most likely fail.

In this context, companies have started to realize the effect internal and external operations management has on their performance in terms of customer satisfaction. Inventory management is one of the main activities of companies in managing their operations (Muckstadt & Sapra, 2010; Cárdenas-Barrón et al., 2014). The importance of managing inventories properly in the contemporary commercial world is currently being highlighted more than it was in the past. For example, retail stores take advantage of maintaining inventory to fulfill their customers' demand. Inventory in its various types (including raw materials, works in process, and finished goods) also plays a pivotal role in manufacturing companies. Inventory holding and management is not limited merely to its application in commercial sectors. Governmental sectors must also keep accurate stock of inventory to ensure adequate supplies in emergencies, such as vaccines in case of biological attacks or military equipment in case of terrorist attacks.

Because companies tend to store a specific amount of inventory to prevent supply disruptions, a large portion of their capital is usually tied up in inventory, which illustrates the importance of efficient inventory management to business success. For instance, according to the results of a US survey, the total value of all inventories (e.g., raw materials, finished or semi-finished products) in different parts of the US economy are valued at more than 12 trillion dollars (Jorgenson et al., 2014). This amounts to more than $4,000 for every man, woman, and child in the country (Jorgenson et al., 2014). This study also showed that the costs incurred for the storage of inventory in the US run into the hundreds of billions of dollars annually. This demonstrates how proper inventory management could lead to a massive cost savings for the economy.

While large amounts of capital are required to maintain inventories, producing, refining, or improving any kind of goods and delivering them promptly to customers requires stocking inventories at some stage in the process (Björk, 2009). Inventory storage and management is a viable strategy in the supply chain of a product, as it ensures that whenever an order is received from a customer, the customer's needs are satisfied as quickly as possible. Preventing stock-out situations is of major importance to companies, since studies show that only 15% of customers who encounter stock-out situations will wait for the product supply, whereas the remaining 85% will buy the product from other sources or do not buy the product at all (Bijvank & Vis, 2011). Appropriate inventory management could also decrease product storage and spoilage costs, thus increasing company profit. Clearly, an effective inventory management system could make a substantial contribution to increase company profit and provide a high level of customer service.

1.2 Lot-sizing problem

In an efficient inventory management system, it is essential to make decisions about the size and timing of orders. Specifically, decisions should be made about the scale of the inventory replenishment order and how often the inventory should be replenished (Glock et al., 2013; Bushuev et al., 2015). These two main factors are the fundamental inputs to an inventory system, and determining their optimal quantities typically leads to a mathematical function, which aims at minimizing the total cost or maximizing total profit of the system (Bushuev et al., 2015). The first mathematical model developed to assist managers in determining the size and replenishment cycles of inventories dates back to the earliest decades of the previous century, when Harris (1913) laid the foundations for the first economic order quantity (EOQ) model. Another pioneering and major contribution to the inventory management theory was made by Taft (1918), who developed the economic production quantity (EPQ) model by relaxing the assumption of instantaneous replenishment.

Since the rise of the classical EOQ and EPQ models, also familiar as lot-sizing models/problems, they have received extensive attention from both scholars and practitioners (Andriolo et al., 2014; Glock et al., 2014). Although the classical lot-sizing models proposed by Harris (1913) and Taft (1918) proved to be simple and efficient tools for reducing excessive inventory in companies (Dobson, 1988; Stadtler, 2007), they have always received a great deal of criticism in the inventory management literature on the basis that they use assumptions that have significant shortcomings in real-world contexts (Jaber et al., 2008; Jaber et al., 2009). W.J. Selen (1987), for example, noted that the common costs of the lot-sizing models are difficult to estimate, meaning that any miscalculation or misunderstanding of the inventory parameters may result in inaccurate outcomes and thus incurs additional inventory costs. Another example is Rumyantsev and Netessine (2007), who cautioned against some insights obtained from classical models by pointing out that they ignore the complexities and uncertainty inherent to real businesses. Hence, the application of lot-sizing models for determining replenishment lot sizes can lead to an inappropriate estimation for the inventory system, which can be costly. To overcome these drawbacks, building upon the formula developed by Harris (1913) as a fundamental mathematical model, a plethora of research has been done to extend lot-sizing models under realistic conditions. These models provide a mathematical framework that closely complies with the practical aspects of managing inventory systems, and take account of factors that are never addressed in basic models.

1.3 Fuzzy lot-sizing problem

One area in which the classical models fall short in real situations is when the information for an inventory system is uncertain, unstable or incomplete over the planning horizon. This is important because, owing to today's turbulent business environment, it is common that much of the relevant input data in an inventory planning system are either imprecise or unavailable due to insufficient information (Soni & Joshi, 2013; Yadav et al., 2013; Das et al., 2015). For instance, the inventory cost for a stocked product is likely to vary from what was planned, because of the variable costs, such as repairs, financial interest and storage costs, which are attached to the inventory holding cost. In addition, companies make modifications in their products or they encounter a situation in which market demand alters regularly. These factors highlight the degree of uncertainty that inventory planners should take into account when planning.

Based upon whether the parameters of the inventory model are precise or not, mathematical inventory models can be broadly categorized into two types: deterministic (classical or crisp models) and non-deterministic models (Guiffrida, 2009). Earlier papers were initially formulated non-deterministic models using stochastic methods. Applying probabilistic distributions in uncertainty modelling requires observation of the past performance of activities, and this is quite often impossible as either the activity has not previously been performed or the related data are characterized by uncertainty, ambiguity or a lack of perfect information (Vijayan & Kumaran, 2008; Halim et al., 2009; Kumar & Goswami, 2015). Obviously, uncertainty could not be treated using probabilistic theory in real applications, but it usually can be captured using the inventory planner's opinion (Vujošević et al., 1996; Mahata & Mahata, 2011). Fuzzy set theory provides an appropriate tool for quantifying uncertain data that can only be described subjectively by human experts (Yazgı Tütüncü et al., 2008). In other words, the main advantage of the fuzzy set theory lies in its inherent ability to deal with imprecise information based on human judgment, without any need for predictable regularities or posterior frequency distributions.

1.4 Problem Statement

Production and operations activities are inherently performed in a manned environment, where human characteristics or abilities may influence the outcome of decisions and the efficiency of the system. However, traditional operations management models are not practical when the task is performed by humans. Clearly, not taking into account human capabilities in such cases will result in unrealistic operation management models, which will in turn lead to underestimating or overestimating the planning outcomes (Givi et al., 2015). This being so, many researchers have emphasized that human factors have to be taken into account when planning production and operations management activities (Boudreau et al., 2003; Gino & Pisano, 2008; Lodree et al., 2009; Neumann & Dul, 2010; Neumann & Village, 2012; Grosse & Glock, 2013; Grosse et al., 2015). Similar to other operations management fields, inventory management is usually a human-dependent task in practice and humans, whether as decision-makers or operators, play pivotal roles by collecting, retrieving, analysing and processing information. More specifically, the inventory system becomes more dependent on human when inventory systems should rely on decision makers, due to lack of data, who should describe his/her opinion using his/her subjective abilities. On the basis of the role that human factors play in fuzzy inventory systems, one may assume that a considerable number of academic research articles on fuzzy inventory management have already been published that investigate human interactions with the system. Even though the impact of human factors on production and operations management has already been well-documented in the literature (Jaber, 2006; Anzanello & Fogliatto, 2011; Jaber & Bonney, 2011), a meticulous overview of the works that studied fuzzy inventory management shows, surprisingly, no study accounted for human capabilities in their models (see chapter 2). As it is apparent from real practice and in light of the evidences from production and operations management literature, in an inventory management process in which a human repeats inventory planning over cycles, he becomes more familiar with it through time; a decision maker's familiarity suggests learning. There is permanently an interaction between the individual and inventory planning process, and because the human performance is changing over time, his performance could subsequently affect the planning. Hence, it is straightforward that individual characteristics, in particular those that are not stable during the planning horizon (for instance, learning from previous cycles or forgetting the information acquired from the prior cycles), affect the efficiency of the planning and the outcome. Therefore, taking the individual characteristics of a planner into account in fuzzy lot-sizing problems is indispensable.

1.5 Significance of the study

It is not surprising the lot-sizing problem has received massive attention given the crucial role of inventory in the current global economy. Inventory management is among the most important activities of industries and trading companies (Vastag & Montabon, 2001). Furthermore, as described before, managing inventory properly and defining the right policies in this regard could considerably reduce costs for an individual company or an economy as a whole. However, besides inventory management's advantages, its side effects could be significant if the conditions in which the planner is making decision does not properly estimated. The importance of this subject in inventory management is stressed by many authors. For instance, W.J. Selen (1987) noted that the input costs of the EOQ model are not very easy to estimate. Therefore, any miscalculation or misunderstanding of the inventory data may result in an inaccurate outcome which may impose additional costs to the inventory system. In this regards, Woolsey (1990) wrote, "If you continue to love and use the EOQ without knowing how much it costs for you, I can only suggest that you deserve each other." As to using the exact value instead of the imprecise value in the model Shekarian et al. (2014) pointed out that assuming crisp values of the input parameters while the model parameters are imprecise, as in the classical models, results in costly and erroneous inventory policies. It is clear that efficiency of inventory models highly depends on formulating the condition of the system in a way that truly reflects the circumstances in which decision maker are operating. Therefore, it is worth mentioning that if human factors are not considered in estimating the model's parameters, planning process could lead to an erroneous in choosing the right policy, which can be costly for the system. In addition, given the fact that human capabilities are not constant over time, treating them as a constant factor in decisions making for inventory management is, undoubtedly, in contrast to the real situation, which trivially makes the available models in the literature impractical.

1.6 Research question

What would be the impact of human learning and learning transfer in imprecise parameters on a fuzzy EOQ model with backorders?

The above research question will be addressed by fulfilling the following research objectives:

1.7 Research objectives

1- To develop four fuzzy EOQ models with backorders to account for human learning and learning transfer over the planning horizon.
2- To develop appropriate learning curves for each model.
3- To evaluate the effect of different learning curves and learning transfers on fuzzy EOQ with backorders.
4- To derive insights for managers and practitioners who are working in environments that inventory planning is highly dependent on human workforce.
5- To validate the developed models by comparing them with the one in the literature.

1.8 Scope

In this study, the intention is to keep the developed models as simple as possible to build a cornerstone for further studies. Therefore, the basic model of this study will be chosen from the fuzzy EOQ models, which are generally the simplest models in inventory management literature. It is obvious that other types of inventory models such as EPQ, supply chain or inventory control models will be excluded from this study. In order to validate the model and be able to compare the results with the literature to gain insights, all the models in this study will be developed on the basis of the previous studies. To investigate human learning and learning transfer in a fuzzy EOQ model, a fuzzy EOQ model with backorders and fuzzy demand and lead times will be developed to take account of human learning and learning transfer over the planning horizon.

1.9 Research process

This section will briefly review the process that will be used in each step of the thesis to achieve the objectives defined in section 1.7.

1- This study will be started with a comprehensive analyze of the literature, which will be achieved by means of a systematic literature review. The literature review section will comprise of four main steps: material collection, descriptive analysis, category selection, and material evaluation. First, the characteristics of the earlier reviews in this research stream will be discussed to declare the need to conduct this review and then the steps mentioned above will be implemented successively to review the literature.

2- To achieve objectives 1 and 2, a mixture of qualitative and quantitative approaches will be adopted. To this aim, first a semi-structured interview process will be conducted within a number of companies in Malaysia and Iran, which could be of help in the extension of the understanding as to the learning process in practice. The result of the interview process will be summarized into a number of propositions, which will later aid in formulating the assumptions of the models. Subsequently, the comprehensive literature review conducted in the previous step will help to identify the appropriate and basic model for this study. In order to study a fuzzy EOQ model with learning-based improvement and learning transfer, an EOQ model with fuzzy lead times and demand, developed by Björk (2009), will be considered as the basic model. At the initial step, following the learning curve developed by Wright (1936), the fuzzy EOQ model will be developed to account for the situation where the operator can improve his estimates of the uncertain inventory input parameters due to learning. This will be implemented by integrating the modified version of Wright's learning function, adapted to the model of this study, into the fuzzy EOQ. Two different scenarios for transfer of the operator's experience are modelled: (i) total transfer of learning, and (ii) partial transfer of learning, which is equal to the case where the operator forgets a part of the obtained experience. The formulated models are non-linear total cost functions which are minimized using a developed algorithm to determine the optimal order quantity and the maximum inventory level. The models of the earlier step will be developed afterward to afford the possibility that the learning ability of the operator includes cognitive and motor skills to analyze the inventory data.

3- To achieve objectives 3 and 4, the developed models will be analyzed applying primary and secondary data sets. The secondary data sets will be obtained from the model taken from literature, i.e. the model which were extended over the study, and the primary data set will be collected from a company whose is active in the automotive industry in Iran. Using both types of data sets, the models will be numerically analyzed and compared with the basic models in the literature to realize the effect of operator learning on the fuzzy EOQ models with backorders. The result will be finally summarized and synthesized to propose some recommendations for the instances where inventory planning depends on human workforce.

1.10 Thesis layout

This thesis is laid out in six chapters. In Chapter 2, the studies in the field of fuzzy inventory management and learning will be reviewed to identify the major research gaps and then to position this study into the literature. In Chapter 3, the research methodology adopted in this study will be presented. This chapter is followed by Chapter 4 where the details of the developed the mathematical models will be discussed. In the next chapter, Chapter 5, the developed models will be numerically examined to evaluate the effect of learning and forgetting in fuzzy parameters on the inventory policy, complemented with a case study to gain empirical point of views. The final part of the thesis, Chapter 6, will summarize the findings and present some insights for managers and practitioners.

CHAPTER 2 : LITERATURE REVIEW

The aim of this chapter is to comprehensively present the literature review and the research gap identified by the author based on a systematic literature analysis, which helped in formulation of the thesis topic, research question and objectives. As the topic formulated in this thesis includes two distinctive research streams, fuzzy inventory management and human learning, the literature review is divided into two different sections, and each section gives an overview of the most important studies implemented in that particular research stream. The final part of this section consolidates both subjects into a single frame and synthesizes the literature to identify the gaps. Finally, it is explained how the identified gaps are going to be addressed in the study.

2.1 Fuzzy set theory

The concept of fuzzy set theory was initially proposed by Zadeh (1965), and recognized as a robust way of expressing and dealing with imprecise information quantitatively (Vijayan & Kumaran, 2008; Björk, 2009; Björk, 2012). Fuzzy logic furnishes mathematical power for the emulation of the thoughts and perception processes (Ko et al., 2010; Tettamanzi & Tomassini, 2013). To deal with qualitative, inexact, uncertain and complicated processes, fuzzy logic can be well-adopted since it exhibits a human-like thinking process (Du & Wolfe, 1997; Ko et al., 2010). Since the emergence of fuzzy set theory, it has been applied extensively to solve various types of problems relative to the uncertainty in finance, health science, engineering and business, and has resulted in a vast literature body in operations and research management (Ko et al., 2010, Wong & Lai, 2011; Mardani et al., 2015). The major idea of fuzzy set theory is that each element is associated with a so-called membership degree, which is a unique value indicating to what extent the element belongs to the fuzzy set. All membership degrees constitute a membership function that maps out each element to a number in [0,1].

2.2 Fuzzy set theory in the inventory management

One of the restrictions of inventory models that has received substantial attention in inventory management literature is the imprecision of the inventory model's parameters or variables, which are usually owing to incomplete or unobtainable information, an uncertain decision-making environment, or variation of the values over the planning horizon. This happens frequently in real-world inventory problems when, for example, companies have modifications in their products, or they encounter a situation where market demand alters regularly. On the other hand, with being influenced by uncertainty, the supply chain of the companies is increasingly becoming uncertain and dynamic, where the uncertainty emerges so as the data/information required for inventory planning are not certain, or even available over the planning horizon. Having these dynamic and uncertain situations make inventory planning more troublesome and challenging as the decision makers are unable to define exact values for the inventory system. In such cases, it is possible that the decision objects have a fluctuation from their bases or could be defined orally, such as: "ordering cost is substantially less than x" or "set up cost is approximately of value y" (Vujošević et al., 1996; Glock et al., 2012). In the literature, fuzzy set theory has been recognized as a useful tool to tackle this kind imprecision, which allows converting the oral expressions or approximate estimations to mathematical relations (Vujošević et al., 1996; Glock et al., 2012). These mathematical expressions could be combined into inventory problems and could be particularly helpful in providing a flexible model which could facilitate formulating imprecise data. The difference between fuzzy set theory and other methods (i.e. deterministic or stochastic methods) in modelling the uncertainty is that unlike the deterministic inventory models in which the decision maker attempts to assign a unique and constant value to each inventory parameter, or the stochastic models that utilize the randomness and probability concepts to quantify uncertain parameters, fuzzy set theory helps in formulating the imprecise data mathematically- something that the two aforementioned models often fail to achieve.

The mathematical models within the inventory management literature are distinguishable as to whether or not the parameters of the inventory model are precise. Accordingly, they can be categorized into two types: deterministic (classical or crisp models) and non-deterministic models (Guiffrida, 2009). Deterministic models typically refer to a class of models in which there is no imprecision in the formulation of the model or in defining the model's parameters. Non-deterministic models, in contrast, are a variant of classical models in which imprecision is considered in the process of modeling the inventory problem (Goyal & Satir, 1989; Aloulou et al., 2014). Previously, studies initially approached non-deterministic models by using stochastic methods (Yano & Lee, 1995; Grosfeld-Nir & Gerchak, 2004; Winands et al., 2011). However, there are some drawbacks in using stochastic methods in practice. Due to the inherent characteristics of stochastic methods, the subsequent stages require the pattern from the previous stages, which then should undergo statistical analysis in order to predict the outcome. Nevertheless, the processes do not frequently happen in real manufacturing as expected by stochastic methods. It is trivial that probabilistic approaches are not helpful in such cases. Speaking broadly, stochastic methods are not mostly proper tools in handling the inventory decision process, and instead, the decisions with qualitative data, which are usually taken by subjective opinions, are preferred to absorb the uncertainty. Since fuzzy set theory is mostly able to capture the qualitative inventory planner’s opinions and experiences, it is by far preferred to the stochastic method in operating the decision-making process under uncertainty. Therefore, the main advantage of the fuzzy set theory lies in its inherent ability to deal with imprecise information based on human judgment without any need to predict regularities or posterior frequency distributions (Björk, 2009; Kazemi et al., 2010; Soni & Joshi, 2013).

2.3 Previous reviews

In spite of the vast body of the literature on fuzzy inventory management, only few studies are available that reviewed the paper published in this area. The majority of these studies contributed merely to a small portion of fuzzy inventory management models in their review (Guiffrida & Nagi, 1998). However, among the review papers published so far, there is a study by Guiffrida (2009) which analyzed and categorized fuzzy inventory management papers. Despite presenting a fairly extensive review, including 160 papers, his study was not implemented using a systematic methodology, and thus the study lacks presenting quantitative, qualitative and bibliographical analysis, and research gaps. Therefore, there is a need to present a systematic literature review on fuzzy inventory management to structure the main research development, point out the emerged themes, and identify the most important gaps in this area.

2.4 Fuzzy inventory management literature review

As the success of every research project depends heavily on a deep knowledge of the existing literature, this part of the thesis consists of a comprehensive and systematic literature review that will identify publications that study or investigate how fuzzy set theory is treated and dealt with in inventory management. In this part of the thesis, the aim is to synthesize the existing research on the fuzzy inventory management problem and human learning, with the intention to identify research gaps in the literature and to summarize the important findings that have been made so far. The literature review regarding fuzzy inventory management will mainly focus on the models that extended classical models to account for fuzzy input parameters.

2.4.1 Systematic literature review

To identify the works that are relevant to this research and to explore the research gaps, a systematic literature review was carried out. The prominent characteristic of a systematic literature review is to utilize a systematic methodology in reviewing the literature, and to present results in a transparent, objective and reproducible way (Hochrein & Glock, 2012). The procedures which are taken for the systematic review are discussed in the following.

2.4.2 Literature search and selection strategy

This section deals with the materials of the review and discusses how the related sample papers were identified. In an initial exploratory search phase, three databases, namely Google Scholar, Scopus and Web of Science (WOS) were selected and explored to identify the relevant journals and papers. Bringing WOS along with the other two databases increases the reliability of the paper selection process, as the papers that identified in Scopus could Google Scholar underwent a cross-checking process in WOS to ensure that the selected papers published in an ISI-listed journal.

To search through the selected databases, the necessary keywords were defined, which help in identifying relevant papers. According to Glock et al. (2014) and Grosse et al. (2015), three groups of the keywords were selected. The first group included the keywords pertaining to lot-sizing and inventory management problems, hence the words were : ‘economic order quantity”, “EOQ”, “economic production quantity”, “EPQ”, “economic lot scheduling problem”, “ELSP”, “lot-size”, “lot sizing”, “inventory management”, “inventory model”, “lot”, “inventory”. The second group included the keywords related to learning such as: “human learning”, “learning”, ”human factor” and finally the keywords belonging to the third group were : “fuzzy set theory”, “fuzzy set’, “fuzzy”, “impreciseness”, “fuzzy number”, “membership function”. All these keywords were combined with the keywords of another group to create the keyword lists. The lists helped in searching the databases. In order to form the initial search sample, three databases introduced before were comprehensively searched using the final list of the keywords to identify the papers that carrying the keywords. During the search phase, the language of the papers was set as English, and document type was limited to papers. Therefore, other types of documents like conference papers and book chapters were excluded from the search. In the next phase, the papers were searched in terms of the relevance of their title, abstract or keyword. Hence, the papers were added to the initial sample only when they were relevant as per the search criteria. In the second search phase, the references of the collected papers in the first phase were checked. The references were checked to find whether they contain the aforementioned keywords or not. Subsequently, the papers explored in the second search phase were united with the first sample to form the final sample. In addition, the databases were also updated intermittently on a regular basis to find new and relevant publications.

Before the descriptive analysis phase, the sample papers were subjected to further read to examine their content. To be included in the final sample, the papers must carry the following criteria:

- ❖ The sole focus of the developed model must be on inventory management. Therefore, the papers which studied the combinations of other topics with inventory management, e.g. location-inventory problem, maintenance-inventory problem, are removed from the final sample.
- ❖ The paper must discuss the impreciseness in parameters or variables of the model, but not the impreciseness in solution procedure. Hence, the models which presented deterministic models with fuzzy solution producer were kept out from the analysis.

- Determining lot-sizes must be one of the objectives of the paper. As a result, the models which the concentration was not on the determination of lot sizes were taken as unrelated.

Finally, to decrease the risk of missing any important publication, the papers in the sample were consulted with a number of researchers in the related fields. This was done through showing the spreadsheet containing the list of sample papers to a set of researchers from the relevant area and asking them to evaluate whether there is any important study that did not provide in the sample list. In addition, during the evaluation process, different aspects of analysis were performed using excel software to reduce the probability of errors.

2.4.3 Descriptive analysis

The result of the literature search is outlined in the so-called review protocol shown in Table 2.1. Totally, 251 papers collected through the first search phase, where 26 papers were recognized to be irrelevant or duplicated. In addition, the second search phase using the forward and backward snowball approach resulted in 16 additional hits, where 12 papers excluded in the latter stage by careful consideration. The literature searches resulted from the two previous stages were combined to establish a single sample, where five papers were found to be duplicated. Afterward, the sample was subjected to an in-depth read to analyze the content of the papers. In the final step, 14 papers were identified to be irrelevant in terms of their content, and therefore, were excluded from the analysis, which led to the total sample of 210 papers.

Table 2.1: Review protocol

Refine criteria	Description	Result
Inclusion criteria	Document type: journal articles (excluded conference papers, chapter books) Journal type: indexed in ISI Language: English Search time: not specific	
Defined keywords	Group A: "economic order quantity", "EOQ", "economic production quantity", "EPQ", "economic lot scheduling problem", "ELSP", "lot-size", "lot sizing", "inventory management", "inventory model", "lot", "inventory". Group B: "human learning", "learning", "human factor" Group C: "fuzzy set theory", "fuzzy set', "fuzzy", "impreciseness", "fuzzy number", "membership function".	
Database search	The papers containing at least one keyword from the defined keywords above were included. Some papers found irrelevant by going through their title, abstract and keywords.	251 26
Snowball approach	Finding additional papers by subjecting the references of the first sample to manual analyze. Refining irrelevant or duplicated papers.	16 12
Combination of the samples	Integrating two samples to establish a single sample. Eliminating duplicated papers.	229 5
Content analysis	Analyzing the papers comprehensively in terms of their focus and content to ensure their relevance to this review. The irrelevant papers are excluded.	14
Final sample	The final sample was carefully examined to categorize and summarize the contribution of the papers.	210

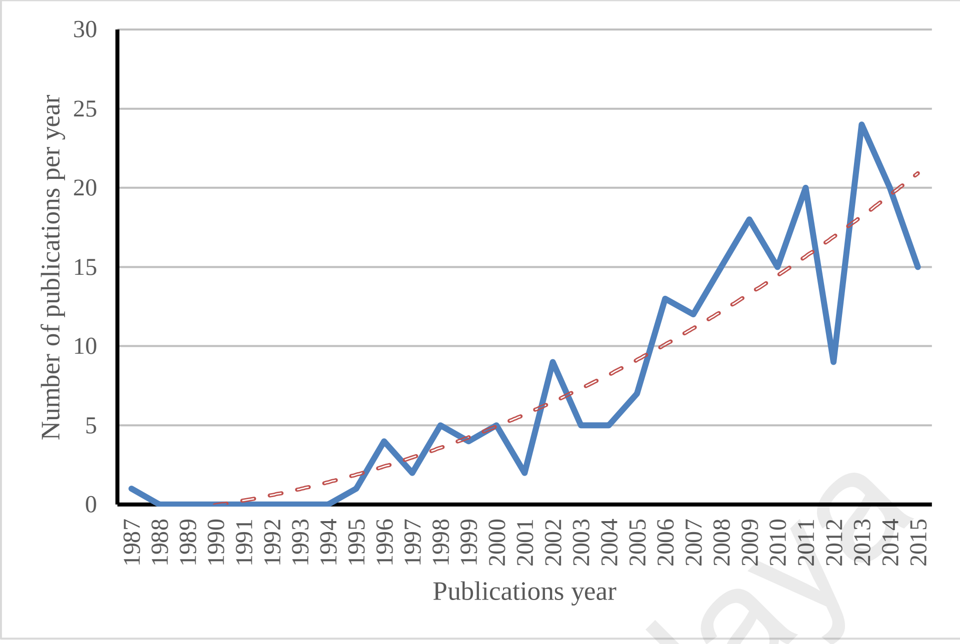

Figure 2.1: Distribution of the published papers per year over the investigated time interval

A total sample of 210 papers were included in the descriptive analysis. Fig. 2.1 illustrates the distribution of the sample papers over the years. Overall, the trend line indicates that the topic of fuzzy inventory management has become increasingly popular throughout the years. The years on which the highest number of papers were published were 2013, 2011 and 2014, with 24, 20 and 20 papers respectively. Interestingly, even though the first paper published in 1987, this area was unnoticed for 7 years until the second paper published in 1995. Moreover, roughly 75% of the paper published throughout the recent 10 years, highlighting the importance of this domain for researchers. Fig. 2.2 ranks the academic peer-reviewed journals in terms of the number of papers published about this topic. As can be seen, European Journal of Operational Research, Computers and Industrial Engineering, Applied Mathematical Modelling, and International Journal of Production Economics are the four primary journals published the models in the area of fuzzy inventory management, which cover almost 36% of the entire papers. To retain the length of this chapter within a reasonable extent, discussing all 210 papers in details is avoided afterward, but the analyze is curbed to those papers that added a major contribution to the literature.

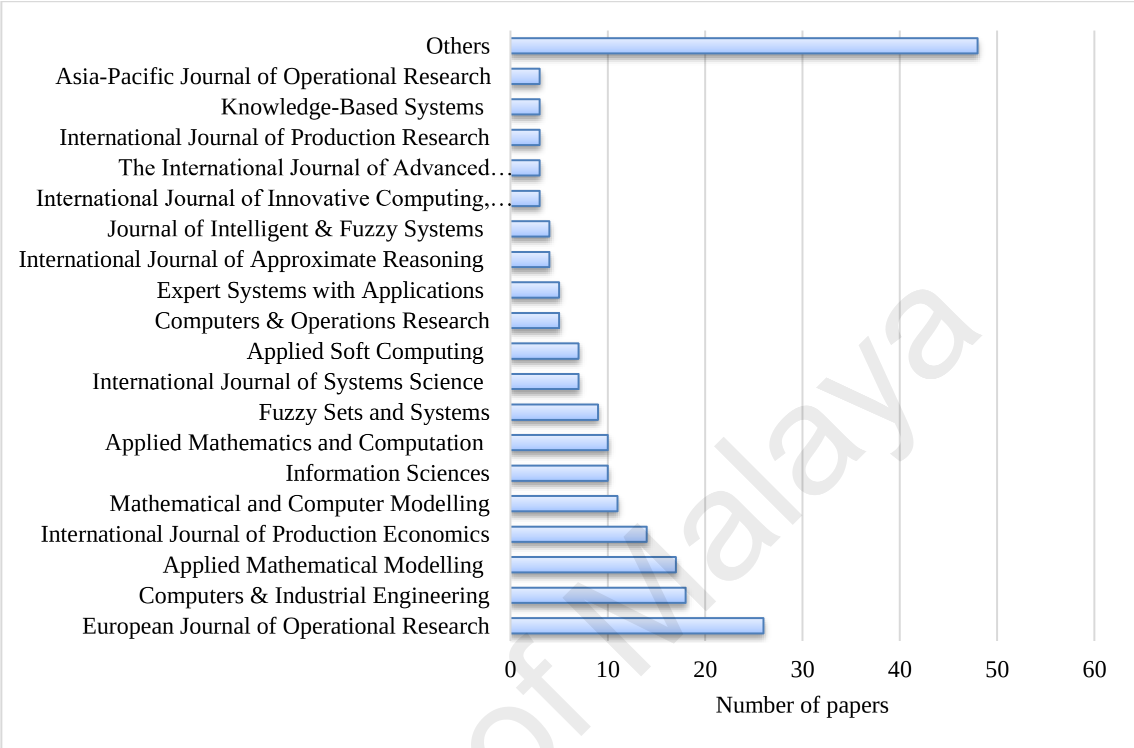

Figure 2.2: The major journals published fuzzy inventory management topic

2.4.4 Classification framework

In this section, the papers grouped in the final sample will be classified along several dimensions. By going over the sample papers, it is apprehended that the papers on fuzzy inventory management can be fallen into different categorizations. In addition, the literature review reveals that there is not any unique approach in classifying the inventory models and the available studies mostly considered a specific aspect of the inventory management problem. Therefore, in order to maintain the universality, the intention is to present a generic framework in classifying the models, which facilitates categorizing the models into main classes.

Category selection for the sample papers was carried out in two steps, where initially the main classes were determined, and afterward the papers in the main classes were explored to find the joint characteristics. In order to analyze the papers with regard to the main categories, rather than using a deductive approach where a classification scheme is determined before sample analysis, it is preferred to use an inductive approach which is based on evaluating the sample papers to achieve the most appropriate categorization. However, to extract the sub-categories, both deductive and inductive approaches were used. Analyzing the structural dimension of this study was performed by the researcher and the findings were discussed several times with other colleagues and the supervisors to evaluate and synthesize the results. The review papers on the inventory management area were of help in this stage as the authors observed different categorizations and then benchmarked with analytics of the sample papers. After concluding the findings, the sample papers were classified into the following main themes:

- EOQ models: the type of the models that try to determine the optimal quantity from buyer/retailer's perspective by minimizing the inventory costs.
- EPQ models: The models that aim to determine the right quantity of a product that should be manufactured through minimizing the inventory costs.
- Joint/supply chain models: A category of models with the objective to coordinate inventories among entities in a supply chain.
- Inventory control models: a class of models whose the aim is to support the requisition processing, and physical inventory reconciliation, which ensures gaining maximum profit (or minimum cost) and maximum use of inventory.
- Newsvendor models: this group of the models are special types of inventory problem dealing with short life products and uncertain demand where the decision should be made as to how much should be ordered in order to minimize the total cost of the system.

In this step, using each of the aforementioned categories, sample papers were assigned a unique code, which represents the category that they belonged to. Next, the sample papers were entered into an Excel sheet and assigned to the right code according to their content. It is worth noting that the focus at this stage was not on the methodology of the paper, instead the main themes were explored and specified. Furthermore, the intention was to assign each paper to a specific class; however, whenever a conflict was seen between categorizes, the paper was assigned to the nearest class.

In the second phase of the structural dimension, the review papers by Andriolo et al. (2014) and Glock et al. (2014) aided drawing out the general characteristics of inventory models. The sample papers in each category were listed in a different Excel sheet according to their code and then were deeply assessed as to whether they included the predetermined characteristics. To increase the reliability of this part, this process is executed by two researchers apart. After terminating the process, the results were cross-checked and compared to identify inconsistencies. The inconsistencies were further analyzed to discover the root cause and finally the opinions of the researchers were consolidated.

Fig. 2.3 presents the assortment of the models in this area. As to the subclasses of EOQ and EPQ models, they can be further divided into "basic models", "model with backorders" and "extended models". The basic models refer to a class of models whose consider only the basic costs such as ordering, holding or set up, while extended models cover a set of papers that investigated additional aspects such as multiple products, product quality or process deterioration, delay in payment or their combination.

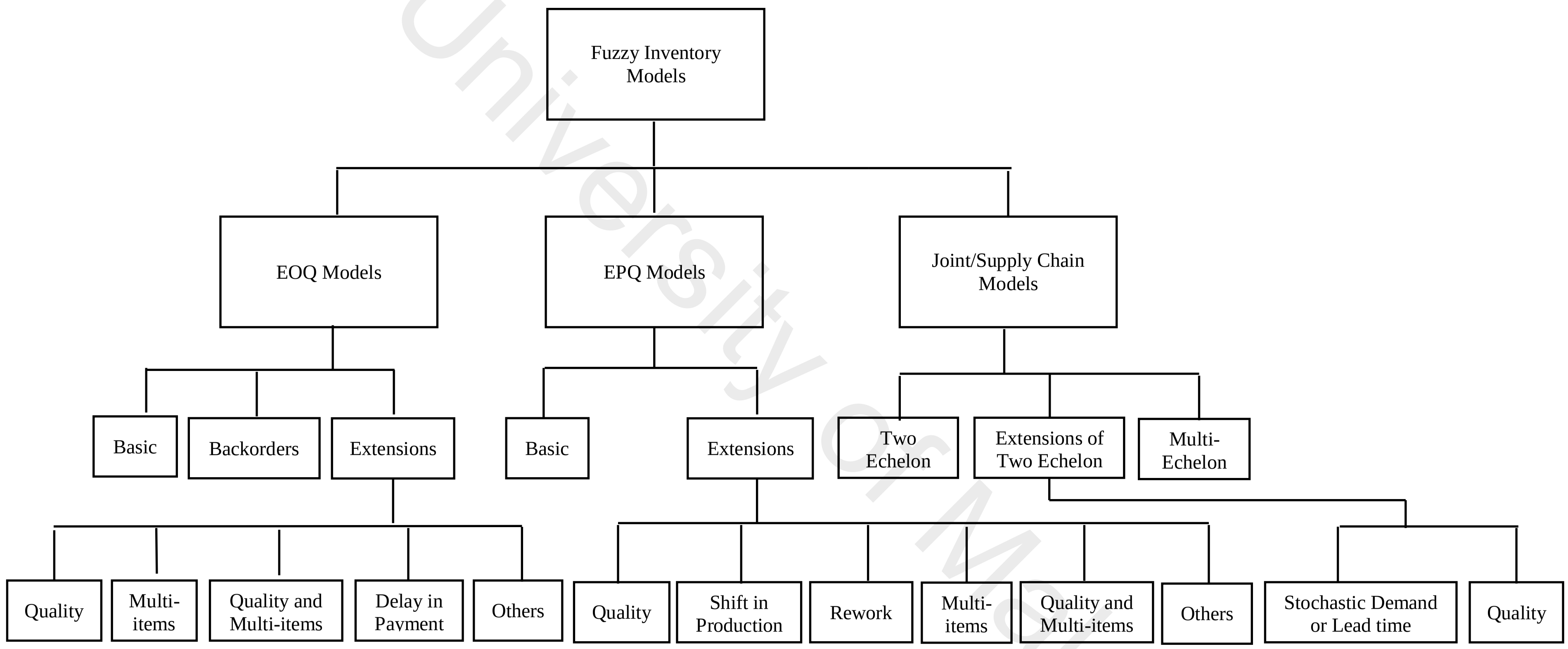

Figure 2.3: Classification of fuzzy inventory management models

Concerning joint/supply chain models, the models can be additionally broken down into smaller groups based on the distinctive feature of the number of stages they considered in their model. Specifically, they can be subdivided with respect to whether they studied two echelon models with one or more entities in each echelon, or whether they investigated multi-echelon type models. Two echelon models also showed the capability to decompose further on the basis of quality or stochastic demand or lead time aspects. This type of classification can be found in the review of Glock (2012a). Finally, inventory control models can be represented as continuous review and periodic review.

2.4.5 Content Analysis

In what follows, the papers contained in the sample will be reviewed according to the classification structure explained before. To retain the length of this chapter within a reasonable extent, it is avoided discussing the papers by giving more details, but it is tried to present the studies in short sentences by summarizing their major contributions and findings.

2.4.5.1 Fuzzy EOQ models (FEOQ)

(a) ***Basic fuzzy EOQ models (BFEOQ)***

Incorporating fuzziness into inventory models was initially addressed in the EOQ type models in the literature. Park (1987) was the first author who addressed the problem of EOQ model carrying fuzzy parameters. In this paper, the basic EOQ model given by Harris is reconsidered by replacing with fuzzy input parameters, where the author assumed ordering and holding costs were fuzzy numbers. The next earlier work in this research stream was the study by Vujošević et al. (1996), who addressed the same problem as in Park (1987), but using different methodology. In contrast to Park (1987), they used fuzzy arithmetic operations for applying fuzzification process along with center of gravity for defuzzification. The fuzzy EOQ problem coupled with fuzzy decision variable was offered by Lee and Yao (1999), who assumed that the order quantity (as a variable of the model) is a triangular fuzzy number. The method derived by them seemed too complicated, although it was appropriate to achieve the optimal planning in fuzzy cases. Similar problem was studied by Yao et al. (2000) also, where order quantity as well as demand was assumed as a fuzzy number. This phenomenon was described using triangular fuzzy membership function. Later, the case of fuzzy demand and holding cost in an EOQ model was analyzed by Yao and Chiang (2003). They discussed which defuzzification method should be used by the decision makers depending upon the different values which fuzzy numbers can adopt. The comparison between statistical and fuzzy methods for carrying out parameter estimation was drew by Hojati (2004), where the author explored the shortcomings of using probabilistic theory in Lowe and Schwarz (1983) and fuzzy set theory in Vujošević et al. (1996). The author proved that the result of the statistical model could serve as a good solution for the fuzzy model. In two similar studies, Syed and Aziz (2007) and Lee and Lin (2011) investigated a fuzzy EOQ model using Singed Distance as a defuzzification method. Considering holding, ordering and backorder costs as fuzzy variables, Samal and Pratihar (2014) developed a chance programming model to solve a fuzzy EOQ problem. They showed that, in solution procedure, Particle Swarm Optimization (PSO) algorithm outperformed Genetic Algorithm (GA) when the complexity of models, with and without backorder, was increased.

(b) ***Fuzzy EOQ model with backorders (FEOQB)***

Chen et al. (1996) studied one of the first EOQ models with backorders under fuzziness, who investigated an inventory system with fuzzy demand and fuzzy costs. The authors concluded that using Extension Principle changes the membership function; however, when Function Principle is utilized, it can preserve the trapezoidal shape of the fuzzy numbers in defuzzification process. An EOQ model with backorders and fuzzy order quantity was investigated by Yao and Lee (1996). The author examined some solution approaches aiming at finding the optimal solutions for order quantity and maximum inventory stock, as decision variables, for the case where maximum inventory stock is lower than the elements of fuzzy order quantity. On the lines of the previous model constituted by them, Lee and Yao (1999) revisited the basic EOQ model under the circumstance that the order quantity is a fuzzy number. The authors also explored how an optimal solution for the formulated problem could be obtained, and concluded that if lead time changes, then an initial point should be considered for fuzzy order quantity. Similar to Yao and Lee (1996) and Lee and Yao (1999) models, Chang et al. (1998) studied an extension of the EOQ model with backorders to a fuzzy case. While Yao and Lee (1996) assumed that order quantity is a fuzzy variable and hence maximum inventory and the other parameters of the model are deterministic, Chang et al. (1998) reversed this case and assumed that maximum inventory is fuzzy and other model's variable and parameters were assumed deterministic. The findings of this study were similar to the works reviewed before. The fuzzy EOQ problem with backorders was also studied by Yao and Su (2000), who considered three different possibilities deeming demand as an imprecise parameter. The benefit of doing this exercise was that it gave flexibility to the impreciseness of the model, especially whenever the elements of membership function could not be described precisely. A paper complementing the previous studies proposed by Wu and Yao (2003), who relaxed an assumption made by Chang et al. (1998) and proposed a model having fuzzy order quantity and fuzzy maximum inventory level. The difference between the model of Wu and Yao (2003) and Chang et al. (1998) is that the model given by Chang et al. (1998) considered one variable of the model fuzzy at a time, whereas Wu and Yao (2003) model took both variables simultaneously into practice. Another variation of the EOQ model provided by

Björk (2009), who investigated a model with fuzzy lead time and fuzzy demand, considering the fuzzy number modelled as triangular type. As a result, the author recommended a larger lot size for the uncertain case compared to the crisp one. The model given by Björk (2009) extended later by Kazemi et al. (2010) wherein the researchers took into account complete imprecise input parameters and variables of the model and provided a different analytical solution. The model was tested against both triangular and trapezoidal fuzzy numbers, and it was found that a linear relation prevails between the deviation in fuzzy parameters and order size. Thereafter, as one of the few case study models, the model of Björk (2009) used and simulated by Milenkovic and Bojovic (2014) to find the empty car inventories in Serbian rail network.

2.4.5.2 Extensions of fuzzy EOQ model (FEOQE)

(a) ***Quality based studies (FEOQEQ)***

By extending the study of Porteus (1986), Chang (2003) developed an EOQ model with the opportunity for quality improvement and further assumed that the imprecision attributed to the opportunity cost can be represented using a statistic-fuzzy number. The model developed by Salameh and Jaber (2000) revisited by Chang (2004) through relaxing the assumption that a number of parameters of the model are imprecise rather than being deterministic. A set of numerical examples was conducted to compare fuzzy and crisp models for several cases where fuzzy numbers vary. The case of a deteriorating model in which stock is dependent on demand studied by Roy et al. (2007), who assumed that deterioration rate is fuzzy over the planning horizon, assumed to be a stochastic parameter with an exponential distribution. A combination of simulation approach and GA was employed as a solution procedure to maximize the profit of the retailer. Wang et al. (2007) developed an EOQ model with imperfect quality items and divided the parameters and variables of the model into two groups, where the first group represented using fuzzy variable while the second group illustrated using fuzzy random variable. Rong et al. (2008b) offered an EOQ model with deteriorating items for a retailer who manages two warehouses, one called an owned warehouse and another one a rented warehouse. They constructed the model with the assumptions that the items are delivered from the owned warehouse to the rented warehouse and that holding cost at the rented warehouse has the inverse relation to the distance of the warehouse from the market. A similar model to that of Roy et al. (2007) proposed by Roy et al. (2009), entailing fuzzy time value of money and inflation in their study. A fuzzy EOQ model with quality aspect also studied by Yadav et al. (2012), who considered an imperfect supply process with learning in imperfect quality items where demand assumed to be proportional to advertisement frequency. The numerical study showed that as learning become faster, the number of defective items and the order quantity drop, whereas the retailer's profit and backorder level rise. Furthermore, an analogous case for an imperfect supply process to that of Salameh and Jaber (2000) was studied by Hsu (2012). Finally, Mahata and Goswami (2013) followed the modelling approach in Kazemi et al. (2010) and developed a fuzzy EOQ model with imperfect quality items in a fully fuzzy form.

(b) ***Multi-item models (FEOQMI)***

Das et al. (2000) developed a multi-item fuzzy EOQ model similar to Roy and Maiti (1998)'s approach, where inventory costs and unit purchase/production costs were assumed to be proportional to the average inventory and inversely related to the demand, respectively. Mondal and Maiti (2002) developed a multi-item non-linear inventory model with fuzziness on objective and constraints and focused on the solution procedure. Yao et al. (2003) discussed price and inventory planning problem for two interchangeable commodities in two different markets, one took monopoly and another competitive into account. Das et al. (2004b) formulated two types of multi-item multi-constraints inventory problem in fuzzy and fuzzy stochastic cases considering that shortage occurs in the system and that inventory costs are proportional to demand. Yadavalli et al. (2005) surveyed a fuzzy multi-item EOQ problem by utilizing a number of membership functions and concluded that linear membership function is the best among the examined membership functions and leads to achieving the highest aspiration level. Baykasoğlu and Göçken (2007) and Baykasoğlu and Göçken (2011) solved a multi-item fuzzy EOQ problem by employing PSO method and four different fuzzy ranking approaches (i.e. Signed Distance, Integral Value, Possibility Programming, and Expected Intervals). They showed that the integral value method generates better results than the other types, and claimed that fuzzy optimization problems can be easily solved through applying a compound of fuzzy ranking and metaheuristics methods. Their suggestion attributes to the fact that, using the suggested approach, the fuzzy models do not necessarily need to be transformed into a crisp equivalent model. Maiti and Maiti (2007) suggested a possibility/necessity based optimization technique as a solution for a fuzzy stochastic multi-item inventory model with two warehouses. Maiti (2008) analyzed a multi-item fuzzy EOQ models with two warehouses and transportation of the items between them using a policy called basic policy. Panda and Maiti (2008) formulated an inventory model with hybrid inventory costs in a fuzzy-stochastic environment where unit cost is taken to be dependent on demand. Huang (2011) extended an earlier model in the literature, which investigated a multilevel lot-sizing problem, to a fuzzy case and compared the performance of Signed Distance and Centroid methods in defuzzification process. A modification of a joint replenishment model using fuzzy concept offered by Wang et al. (2013), who formulated a chance constraint programming with the help of credibility measures. Mousavi et al. (2014) studied a multi-period multi-item inventory system for seasonal products comprising of shortage, lost sales, discount on order quantities, and constraints on transportation and number of pallets. The problem was formulated in two different cases based on different objectives of the decision makers and then solved exerting intelligence algorithms.

(c) *Mix quality-multi-item studies (FEOQEQI)*

Roy and Maiti (1998) discussed the necessity of defining decision goals and resource constraints in a fuzzy mode and therefore formulated two fuzzy multi-item multi-objective inventory problems with the respective fuzzy traits. The study of Roy and Maiti (1998) was later extended by Xu and Liu (2008) to account for fuzziness and randomness of the inventory data. Roy et al. (2008) analyzed a multi-item deteriorating inventory system subject to constraints on available budget and space and included the impreciseness of inventory costs and the available budget into their model. The case of an inventory model for deteriorating items with constraints on shortage cost examined by Wee et al. (2009). Their model implemented joint replenishment policy with an additional assumption that demand is dependent on stock. Saha et al. (2010) and Guchhait et al. (2010) brought up the problem of breakable items and formulated fuzzy multi-item inventory models with stock dependent demand. Whilst in the first study the number of damaged items was taken to be dependent on the current stock level, both linearly and non-linearly, in the second study, it assumed to increase linearly with stock and non-linearly with time. Moreover, Chakraborty et al. (2013) formulated an inventory model for a wholesaler who acquisitions a number of products from a couple of established suppliers and sells them to some showrooms, which are located in different places. Their model also takes into account a random planning horizon and different discount schemes offered by suppliers. Finally, the effects of the time value of money and inflation were considered by Jana et al. (2014) in developing multi-item fuzzy inventory models for deteriorating items in a random planning horizon.

(d) ***Studies with delay in payment (FEOQED)***

De and Goswami (2006) analyzed an inventory system with permissive delay in payment and discussed three cases where the beginning of shortage time is smaller, equal or bigger than the permissible delay period for settling accounts. Chen and Ouyang (2006) extended a fuzzy version of the model of Jamal et al. (1997), who investigated an inventory system with deteriorating items, and allowed shortage and deferment in payment. Mahata and Goswami (2007) examined an EOQ model for deteriorating items in which not only the supplier offers credit period to the retailer, but also customers could receive deferment in the payment offer from the retailer. Ouyang et al. (2010) proposed a fuzzified version of the model developed by Chang et al. (2003) through fuzzifying all the parameters related to financial credit. Mahata and Goswami (2009) formulated an inventory model for a condition where a retailer holds a stronger position dealing with its supplier and thus benefites from a full trade credit, whereas the retailer just offers partial trade credit to its customer. In order to purchase precious raw materials, Taleizadeh et al. (2011) and Guchhait et al. (2015) analyzed a fuzzy multi-item EOQ model in a rough environment and formulated the problem as a mixed integer nonlinear programming. Next, to resemble a more realistic case, Soni and Joshi (2013) extended the model of Mahata and Goswami (2009) by assuming that demand is proportionate to selling price and by adding more fuzzy parameters to the model. Guchhait et al. (2014) offered a variation of fuzzy EOQ model with deteriorating items under the condition in which supplier to retailer and the retailer to customer present permissive delay in payment. They further assumed that the retailer could benefit from a discount if s/he pays in cash swiftly after purchase. Yadav et al. (2015) investigated a payment scheme in which the buyer can pay either immediately or with a delay after the credit period overs, which in the latter case he must pay interest on the outstanding amount for the delayed interval. The authors came to a conclusion that the retailer could boost the profit by ordering lower quantity. Guchhait et al. (2015) analyzed a fuzzy EOQ model where the supplier offers a couple of payment methods based on the order quantity of the retailer and where the retailer also offers a predetermined partial credit to the customers to elevate demand.

(e) ***Other extensions of EOQ (FEOQEO)***

Roy and Maiti (1997) studied a fuzzy EOQ model with the assumption that demand is dependent on unit price and set up cost is directly related to the order quantity. Later on, the solution procedure in Roy and Maiti (1997) criticized and enhanced by Chou et al. (2009). Liu (2008) studied an alternation of EOQ model with fuzzy demand and fuzzy unit cost and discussed the solution procedure for the developed model using Extension Principle and Duality Theorem. Vijayan and Kumaran (2009) proposed a fuzzy version of the so-called Economic Order Time (EOT) model, a model which the focus is on the time period, by illustrating some components of the models as fuzzy numbers. A fuzzy EOQ model for pricing and marketing planning suggested by Sadjadi et al. (2010), who formulated the problem as a fuzzy possibility geometric programming approach. A fuzzy EOQ problem dealing with time-dependent ramp-type demand with fuzzy lead time and fuzzy planning horizon surveyed by Bera et al. (2012). Through developing a fuzzy EOQ model to maximize the total profit of a retailer, Samadi et al. (2013) assumed that demand directly appertains to price, service and marketing expenses, and the unit cost is inversely dependent on the order quantity. Ketsarapong et al. (2012) developed an uncapacitated lot-sizing model with fuzzy parameters and argued that their model is appropriate for the planning case in which there is not enough qualitative data for decision making. Yadav et al. (2013) fuzzified demand in the model developed by Lin (2008), who brought up a continuous review inventory model with lead time reduction, and further supposed that shortage occurs but partially backorders. De and Sana (2013a) studied an intuitionistic fuzzy EOQ model with time horizon and infinite replenishment rate and backlogging, and developed a solution method based on α-cut concept. De and Sana (2013b) included Promotional Index concept, i.e. a type of strategy aiming at promoting demand, into a fuzzy EOQ model with shortage and defined it as a fuzzy variable. De et al. (2014) offered an alternation of a fuzzy EOQ model where demand is promoted and shortages are permitted, having an inverse effect on demand. Finally, Panda et al. (2014) suggested a fuzzy EOQ model with two warehouses considering demand during lead time as a fuzzy random variable.

2.4.5.3 Fuzzy EPQ model (FEPQ)

(a) ***Basic fuzzy EPQ models***

One of the first EPQ models with fuzzy parameters developed by Lee and Yao (1998), who replaced deterministic values of production rate and demand in the basic EPO model with triangular fuzzy numbers. Since the generated problem was complex, they applied numerical examples rather than providing an analytical approach to find the optimal solution. In a similar paper, Chang (1999) applied the methodology proposed by Lee and Yao (1998) and analyzed a condition that production quantity is a triangular fuzzy number. Therefore, he deducted that fuzzy and crisp approach lead to the same result. A similar issue addressed by Lin and Yao (2000), as they assumed that production quantity is a trapezoidal fuzzy number. Hsieh (2002) examined two scenarios of the EPQ model. In the first one, all the parameters of the model were taken into account as trapezoidal fuzzy number, while keeping the production quantity as a crisp variable. In the second scenario, the author considered fuzzy production quantity as a fuzzy variable and added it to other fuzzy parameters in case one. The work of Hsieh (2002) criticized by Yang (2011), who proposed an easier to implement approach for optimizing the model. In addition, Yang (2011) analyzed the effect of two defuzzification methods, Graded Mean Integration Representation (GMIR) and Median Rule, on the model developed by Hsieh (2002). The author illustrated that the two defuzzification methods result in identical optimal solutions. He also proved that there was no much difference between defuzzification methods, in before and after deriving the fuzzy total cost function.

2.4.5.4 Extensions of fuzzy EPQ model

(a) ***Quality based studies (FEPQEQ)***

Several authors went a further step. They formulated an EPQ model in a fuzzy setting and considered a more complex problem. Maity and Maiti (2005) were the first to address the deteriorated items in an inventory model. They proposed a production-inventory model, assuming demand and production rates as functions of time. Next, Mahapatra and Maiti (2006) offered a production-inventory model with the discussion of different cases concerning the occurrence of shortage in production cycles. Moreover, Maity and Maiti (2007) suggested a dynamic production-inventory model with a time-dependent demand, production rate, and shortage level. Chen and Chang (2008a) proposed two different optimization systems so as to cope with the formulated problem, whereas Maity and Maiti (2008) surveyed a multi-item production-inventory system with time-dependent demand, where the demand could fall or rise under the influence of sale degradation and advertising policy, respectively. Xu and Zhao (2008) formulated a multi-objective fuzzy-rough production-inventory model of imperfect quality, whose rate of production they assumed to be the same as the rate of rework. Their model tried to simultaneously optimize the total profit and total waste cost.

As the fuzziness and randomness effects may concurrently influence the demand pattern, Bag et al. (2009) introduced a fuzzy random variable concept to an imperfect production system, taking into consideration the reliability of the production system. Das et al. (2011) addressed a production system’s production of defective items because of machine failure; they took the machine failure rate to be random and permitted shortages during the production failure. The case of ramp-up demand in an EPQ model with a fuzzy setting analyzed by Pal et al. (2014), who assumed that demand is a function of time, and that the items in stock could deteriorate, with the deterioration rate following a Weibull distribution. The authors, interestingly, found that the fuzzy model returns a lower total cost value than the crisp model for some values of the degree of optimism and for total replenishment cycles. Their model also recommended a shorter production cycle in order to achieve a lower total cost. Pal et al. (2014) formulated a fuzzy EPQ problem with imperfect quality and production process reliability in order to maximize the total profit of a manufacturer. In order to validate the derived solution, it was compared with Genetic and Simulated Annealing algorithms, and the results of the three approaches were nearly identical. In this context, a model with ramp-type demand and deterioration of product under inflation offered by Pal et al. (2015). The authors observed that the total cost in the fuzzy and crisp cases could be equal when the decision maker was semi optimistic.

(b) ***Studies with shifting in production (FEPQES)***

Some authors studied a production system that may experience two states: in control and out of control, where imperfect quality items are produced during out of control states. Halim et al. (2009) focused on a production system that produces imperfect items due to shifting to out of control states, and they examined two different scenarios for an imprecise fraction of the defective items. The authors recommended that although it is difficult to recognize which model (fuzzy or crisp) performs better, it is more appropriate to use the fuzzy model when the fraction of the defective items fluctuates. Zhang et al. (2009) studied a production process which starts with in control state, producing good quality items, and then may change to out of control state during the production cycle, which produces a fixed fraction of defective items. As to the randomness and fuzziness feature of some parameters, the production system was modelled using fuzzy and random fuzzy concepts, followed by the extensions of the model to take account of different possibilities for the percentage of defective items, defined as fuzzy variable, fuzzy linear and fuzzy exponential function. A similar condition formulated in Wang and Tang (2009b) with the difference that the time until the production shifts to the out of control state is fuzzy variable instead of being fuzzy random variable. A simpler problem than that of Zhang et al. (2009) and Wang and Tang (2009b) was investigated by Hu et al. (2010), who, in contrast to Zhang et al. (2009) and Wang and Tang (2009b), considered set up and holding costs as crisp parameters. An identical production system which produces imperfect quality items was studied by Kumar and Goswami (2015) and Mahata (2015). The models of Kumar and Goswami (2015) and Mahata (2015) are very similar in structure, with the difference that the model of Mahata (2015) assumes some more parameters to be fuzzy variables, resulting in a different and more complex solution procedure. Kumar and Goswami (2015) studied a type of model of this category, where the number of defective items produced during the out of control period is independent of the shifting time. They also integrated a shortage case into their model.

(c) ***Rework based studies (FEPQEW)***

Some authors assumed a production-inventory system with imperfect quality, in which the imperfect items could be 'as-good-as' the perfect quality items after rework. Roy et al. (2009) investigated an imperfect production system where a portion of the imperfect items can be remanufactured to as-good-as perfect quality to satisfy customer demand, while the remaining items are irreparable and consequently are disposed of. It was illustrated that the relation between the total profit and the fuzzy confidence level of the defective rate is opposite. Another study that addressed a fuzzy EPQ model with remanufacturing of imperfect quality items was carried out by Guchhait et al. (2013), who formulated their model using fuzzy differential equation and fuzzy Riemannintegration. Following this line of thought, Mondal et al. (2013) also develop an EPQ model in a fuzzy rough environment in which imperfect quality items can be repaired and become the same as a perfect quality item. They identified that the existence of either uncertainty or inflation has a negative impact on the total profit, and also suggested that the repairing process should start up from the second cycle when the repairing rate is a dynamic control variable. Mondal et al. (2014) formulated a fuzzy EPQ problem with two storage areas, where the rework process starts after the production cycle to rework imperfect quality items. Additionally, Shekarian et al. (2014) presented an extension of an EPQ model in the literature in a fuzzy environment, so that all the model's parameters were taking fuzzy into account. As using the trapezoidal fuzzy number increases the dimension of the problem, they showed that formulating a fuzzy model with trapezoidal membership function leads to a higher total cost compared to triangular membership function. The model in Shekarian et al. (2014) without backorder and different fuzzy settings studied in Shekarian et al. (2014), who applied and compared two defuzzification methods in transforming the fuzzy total cost to its corresponding crisp function. The comparison illustrated that using Signed Distance method leads to a larger lot size, which tends to decrease the total cost of the model.

(d) ***Multi-items fuzzy EPQ studies (FEPQEMI)***

One of the first models dealing with the extension of the EPQ problem to include multi-products is the model of Pappis and Karacapilidis (1995), who studied the problem of determining the optimal production runs in a batch production system. They analyzed a production system where all the products have a common production cycle time. Mahapatra and Maiti (2005) developed single and multi-objective models with and without a shortage to maximize the total profit of a manufacturer. Their analysis shows that the profit gained from a single-objective model is more than that of the multi-objective, and that model with shortages gives higher profit. Mandal et al. (2005) formulated a multi-product multi-objective EPQ problem with shortage and the constraints on storage area, production cost and number of orders. Mandal and Roy (2006a) optimized a multi-item inventory model with a demand-dependent inventory level and shelf-space constraint under three different fuzzy numbers. The extension of Islam and Roy (2006) offered by Islam and Roy (2007), who developed the multi-item version of the model in Islam and Roy (2006) with a different solution procedure. A similar but more complex problem as in Islam and Roy (2006) and Islam and Roy (2007) studied by Panda and Maiti (2009). Whereas in Islam and Roy (2006) and Islam and Roy (2007) unit production cost was assumed to be dependent on demand, in Panda and Maiti (2009) unit production cost was considered to be dependent on the stock level as well as demand, which was given dependent on unit selling price. The fuzzy production-inventory situation where the time interval between the decision to produce and the real time of production is variable studied by Mandal et al. (2011). As the production with preparation time is more costly, the author stressed that the decision for production should be made as early as possible to reduce the production cost. Björk (2012) referred to the supply chain of the paper industry and rationalized that it is imperative to consider an imprecise production cycle time when calculating the optimal production quantity. The author came up with the result that greater cycle times are better to set for the uncertain case; however, this would not have much effect on the total cost. This model given by Björk (2012) extended by Mezei and Björk (2015) to account for backorders. Another study that considered multi-products in a fuzzy EPQ problem carried out by Jana et al. (2013), who offered a variation of the EPQ model in which demand is dependent on the stock level as well as unit production cost, which considered to be in relation to the production rate. Through a numerical test the authors found that there is not much difference between the result of the model solving with triangular and Parabolic fuzzy numbers, and that Generalized Reduced Gradient (GRG) approach gives lower profit than necessity approach.

(e) ***Mix quality-multi-item fuzzy EPQ studies (FEPQEQI)***

Mandal and Roy (2006b) used hybrid numbers (numbers that simultaneously contain fuzziness and randomness properties) to present a model with imperfect quality under uncertainty. The authors found that when the weight of the objective function increases the value of objective function decreases as a result. The case of the imperfect production process where the constraints are stochastic or fuzzy studied by PandaKarMaity et al. (2008). They concluded that modeling budget and shortage constraints using possibility measure gives the greatest total profit among all possible combinations. Xu and Zhao (2010) included the fuzzy rough set theory in a multi-objective programming problem and additionally showed the application of their model in a manufacturing company in China. Mandal et al. (2010) studied an imperfect production system with the fuzzy time period and analyzed their model under quadratic, linear and constant production rate. The author recommended using constant production because of the lower total production cost. The first researchers who applied fuzzy inequality as well as fuzzy objective functions in this line of research was Maity (2011). He addressed a fuzzy EPQ model in a system including a single machine and multiple products. Mandal et al. (2011) developed a multi-item production-inventory model with two warehouses and uncertain constraints, and assumed that inventory level and production and demand rates are functions of time. Das and Maiti (2013) formulated a fuzzy EPQ model with a fuzzy stochastic constraint on storage space and an optimistic fuzzy equality for the budget, and discussed the solution approach for the model proposed.

(f) Other extensions

Islam and Roy (2006) offered a fuzzy EPQ model considering investment for reducing set up cost and quality improvement process. The authors added the storage area as a constraint to the model and then formulated a fuzzy EPQ model with fuzzy objective function and constraint. Mahapatra et al. (2011) discussed a simpler variation of the model of Islam and Roy (2006) without storage space. Chang and Chang (2006) analyzed an EPQ inventory system by accounting for the relative cost of the inventory system generated from inventory holding and production. Their model tried to include a variety of costs into an EPQ inventory system, which other models failed to account for. Chang et al. (2006) surveyed the problem of fuzzy demand in economic lot-size scheduling problem by applying a triangular fuzzy number to address demand fluctuation. Through comparing the fuzzy and crisp cases, the authors suggested that fuzzy demand should be considered in economic lot-size scheduling problem. Maity et al. (2008) developed a fuzzy reverse logistics model under the situation in which the used items are collected for recycling or disposal, which are treated as-good-as-new after recycling. Pal et al. (2009) developed a type of an EPQ model with the assumption that demand can be promoted by a discounted price offer, and the life time of the product can be described by a fuzzy number. They added an additional feature to the model and assumed that the production and set up costs can be reduced with the help of learning. Numerical studies further indicated that the maximum profit could be gained if possibility measure is used. Wang and Tang (2009a) modeled a fuzzy EPQ problem with fuzzy variable costs, and derived the equivalent value of the fuzzy total cost function, which led to a complex structure to solve analytically. Soni and Shah (2011) argued that, similar to the lead times in the EOQ model, the preparation time before production start-up in an EPQ model usually depends on various factors. So, it is logical that the pre-production time is taken fuzzy into account. The numerical example showed that when demand is taken a trapezoidal fuzzy number into account, the optimal expected total cost, production quantity and cycle length will be higher than the case of demand defined as an interval fuzzy number. Chakrabortty et al. (2013) proposed a new solution approach based on intuitionistic fuzzy sets to solve a fuzzy EPQ problem. The proposed solution approach proved to be a strong pareto-optimal solution using pareto-optimality test, since the obtained value of the objective function found to be quite small. Yaghin et al. (2013) formulated a non-linear fuzzy mathematical programming for a production-inventory model which confronts different demands from several market sectors. A fuzzy EPQ problem with multiple periods and machines studied by De and Sana (2014). Examining general fuzzy and intuitionistic fuzzy optimization methods in solving the model illustrated that the intuitionistic fuzzy optimization method performs better than the general fuzzy optimization. Finally, Kumar and Goswami (2015) explored the effect of stochastic and fuzzy stochastic demand on a reorder level of a production-inventory model, which worked under continuous review inventory control policy.

2.4.5.5 Fuzzy joint/supply chain inventory models (FJS)

Even though inventory management operations could be beneficial for a single entity, the required synergy may not be found when the organization should coordinate the inventory process along the supply chain. Hence, rather than assuming an individual entity in the fuzzy inventory decision, some researchers broaden their view and surveyed the fuzzy inventory problem from an integrated (supply chain) point of view, where more actors are included in the model.

(a) Fuzzy joint/supply chain models with two echelons (FJSTE)

One of the first models to integrate the fuzzy set theory into the joint economic lot size (JELS) models was the model of Lam and Wong (1996), who proposed a solution procedure based on the fuzzy membership function concept for a single-vendor single-buyer (SVSB) system. Mahata et al. (2005) extended an earlier JELS model in the literature to take account of fuzzy order quantity and discussed the solution procedure under several conditions. Xu and Zhai (2008) formulated a SVSB model in which retailer faces imprecise market demand, and proposed a solution approach to maximize the profit of the supply chain under cooperative and non-cooperative policies. Pirayesh and Yazdi (2010) analyzed a model from this category in a two-level supply chain by fuzzifying customer demand, adopting continuous review policy. Yu and Jin (2011) studied an optimal return policy, a mechanism that supplier agrees to collect the unsold products at the end of the cycle for refunding retailer, between the players of a two-level supply chain with fuzzy demand and developed a model for the instances that information sharing follows symmetric or asymmetric policies. They discovered that the uncertainty in demand and price affects the supply chain profit in both symmetric and asymmetric information sharing scenarios. Chang and Yeh (2013) studied the effect of fuzzy demand and return policy on the total profit of a SVSB model in both centralized and decentralized cases. They indicated that the positive change of the fuzzy demand from the base value could decrease the total profit of the supply chain in both centralized and decentralized policies. A vendor-managed inventory model for a SVSB system offered by Nia et al. (2014), who developed a multi-constraint EOQ model with the assumption that the vendor and the buyer agree on the number of pallets, delivery numbers and order quantities. Kumar et al. (2014) established a SVSB model in a fuzzy random environment and assumed that the buyer’s order quantity is an integer multiplier of the manufacturer, and that the supply chain works under the centralized coordination mechanism.

2.4.5.6 Extensions of fuzzy joint/supply chain models with two-echelon (FJSTEE)

(a) ***Fuzzy two-echelon joint/supply chain models with product quality or product/process deterioration (FJSTEEQ)***

A multi-objective model for a SVSB system with deteriorating items offered by Das et al. (2004a), who demonstrated that, under different preferences of the supply chain, the fuzzy model leads to a better result than the crisp model if the weights assigned to the buyer's cost and the vendor's benefit are unequal. Ouyang et al. (2006) included fuzzy defective rate into a SVSB model, and, besides modelling fuzzy defective rate with a triangular fuzzy number, applied the stochastic method to define the fuzzy defective rate. Numerical study proved that the total cost of the system will be greater for the fuzzy model compared to the corresponding crisp model if the defective rate is formulated using an asymmetric triangular fuzzy number. They also illustrated that the result will be inversed if the asymmetric triangular fuzzy number is utilized. Hu et al. (2010) developed a SVSB model dealing with imperfect quality items in both centralized and decentralized coordination mechanisms. The author assumed that in the decentralized case the manufacture buys the supplied imperfect items from the retailer at a cheaper price. Xu (2014) formulated three fuzzy random mathematical models for a SVSB model with trade credit. Chakraborty et al. (2015) presented a fuzzy SVSB model with stock dependent demand in fuzzy and bifuzzy environments.

(b) ***Fuzzy two-echelon joint/supply chain models with stochastic demand and/or stochastic lead time (FJSTEES)***

A SVSB model where demand during lead time at the buyer follows a normal distribution surveyed by Pan and Yang (2008). The authors identified that the total cost of the fuzzy models in most cases is greater than that of the crisp model. Rong et al. (2008a) surveyed an alternative demand during lead time in a supply chain, including one wholesaler and n retailers by. Their model assumed that demand during lead time is either random or fuzzy random in nature. Taleizadeh et al. (2013) developed a chance-constraint model for a SVSB system with stochastic demand and fuzzy lead times and proposed a heuristic solution approach. Soni and Patel (2015) explored a fuzzy random SVSB model with stochastic demand, controllable lead time and service level constraint. They supposed that there is a relation between lead time and ordering cost reduction, and further showed that the supply chain's cost increases when uncertainty is accounted for.

(c) ***Fuzzy joint/supply chain models with multi echelons (FJSME)***

In a couple of successive papers, Petrovic et al. (1998), Petrovic et al. (1999) and Petrovic (2001) brought up a serial supply chain comprised of raw material, semi-finished and final product inventory, and argued how the performance of the supply chain could be improved using analytical and simulation tools. Building upon the model of Petrovic et al. (1998), Petrovic et al. (1999) and Petrovic (2001), Xie et al. (2006) analyzed a two level serial supply chain model with multiple facilities/actors and discussed a coordination mechanism between facilities with fuzzy demand. Das et al. (2007) modelled a supply chain comprising of raw material suppliers, a manufacturer and a retailer and some warehouses in between in a fuzzy case. A fuzzy single wholesaler and multiple retailers supply chain model developed by Das et al. (2008), who assumed that the items at the wholesaler are sold immediately after an inventory replenishment, but at the retailers are sold over the planning horizon. Petrovic et al. (2008) considered the same solution structure as in Xie et al. (2006) for coordinating a system incorporating a single warehouse and *n* retailers, and divided the total problem into some the sub-problems. Wang (2009) formulated a fuzzy multi-objective, multi-echelon integrated supply chain model and supported their study using the data adopted from three multi-echelon retail types of chain-store supermarkets in Taiwan. The authors compared the characteristics of their models with the earlier models and stressed that their model contains some attributes, e.g. time value of money, multi-objective nature and flexibility in decision making, which other models failed to consider. To determine the inventory policy for each entity, Mahnam et al. (2009) developed a multi-echelon assembly supply chain inventory model with multiple suppliers, following the periodic review policy. To do so, they adopted a policy termed as partially centralized, i.e. supply chain entities determine their policy based on the reliability of their suppliers, and developed a bi-objective model to optimize the total cost and the service rate measures concurrently. Ryu and Yücesan (2010) presented a fuzzy newsvendor supply chain model containing one manufacturer and one retailer with different coordination strategy, and then developed their model to encompass multiple retailers. The authors numerically compared different coordination policies with non-coordination policy to show how the supply chain could achieve a better coordination. Sadeghi et al. (2014) extended a JELS model in the literature, which follows vendor-managed policy, to take account of fuzzy demand and finding the shortest path for delivery, proved to be an NP-hard problem. To meet different decision maker's objectives, Chen and Cheng (2014) surveyed a multi-echelon serial supply chain, pursuing periodic-review inventory control, and ascertained that the total cost of the supply chain increases as fuzzy demand grows. Das et al. (2015) worked on a coordination policy between a manufacturer and multiple markets with the assumption that the manufacturer could receive a credit period from its own supplier, and the market could make a part-payment to the manufacturer during the production period. Sadeghi and Niaki (2015) established a fuzzy model with single vendor and multiple retailers, where the vendor implemented vendor-managed policy and whose faces some limitations on budget and number of orders.

2.4.6 Concluding remarks and research gaps

In this section, the research papers on fuzzy inventory management are reviewed. As shown in this section, fuzzy inventory management has received extensive attention by researchers over the past decades. Examining the distribution of the papers over the years has illustrated that there is an ascending trend in developing the models that apply fuzzy concept in diverse problems of inventory management. The main attempt of the researchers in this research stream has been to employ and model the effect of imprecise input parameters and variables to appropriately represent this aspect of industrial problems.

It has been identified that fuzzy inventory models have extended along several categories, covering a fairly extensive range of problem formulation and solutions. Whereas the analysis shows several areas has thoroughly been studied by researchers, there are still some areas that has been unnoticed thus far, which requires more attention to build more fruitful models in inventory planning under uncertainty. The following main research gaps have been identified.

❖ One aspect uncovered when surveying the literature is the assumption attributed to the majority of the developed models. The developed models mostly assume that the uncertain parameters can be determined relying on the policy maker’s expertise. Whereas researchers highlighted the role of human in their models, they overlooked human abilities and traits, e.g. human learning, knowledge depreciation, human error, in their models. It is apparent that model’s parameters and model’s efficiency are influenced by human characteristics, and thus not considering these aspects in fuzzy inventory models contrasts with practical situations. Therefore, the high relevance of human factors necessitates future research to develop models that incorporate and analyze the effect of human characteristics in fuzzy inventory models.

- The literature survey further designated that the majority of the models had theoretical concentration, and there exists the lack of the empirical observations on how the inventory management under uncertainty is treated and implemented in practice. It is clear that the benefits or the weakness of these types of models cannot be fully realized until they are implemented using real data, or adopted to real cases. Thus, future research are required to conduct case study research that helps gaining insights from practical aspects.
- As to molding approach in the literature, another aspect that arose from literature review is quantitative nature of the current models, showing that the researchers totally overlooked qualitative approaches in modelling. Combining qualitative (like questionnaires and interviews) and quantitative research aids in gaining an in-depth understanding of the fundamental factors affecting the inventory planning under uncertainty, which subsequently can help developing more precise mathematical models.
- A further issue which is recognized relates to the structure of fuzzy inventory models, where in most cases were identified very complex. This contributes to the fact that utilizing fuzzy numbers commonly increases the dimension and hereupon the complexity of the problem. Future research could address the question that whether it is possible to develop simpler and more user-friendly tools so as even the non-specialist users, who has little information about fuzzy set theory, can employ it.

2.4.7 Human Learning

2.4.7.1 Human learning concept

Learning is one of the human characteristics that has been studied in the literature extensively. According to psychologist perspectives, learning is defined as the act of obtaining new knowledge and/or skill or improving the past knowledge and/or skill, which may comprise of different sources of information (Hart, 1983; Shanks & St John, 1994; Melton, 2014). The researchers in psychology area are substantially unanimous that learning incurs an improvement trend in the tasks/processes the learner is involved, which is the consequence of practicing (Jaber, 2006). In operations and production management, definition of learning is the same, entailing that learning is the improvement in performance when a person or an organization is involved in a repetitive task (Jaber, 2006; Jaber & Bonney, 2011). Many researchers stressed the importance of the learning for companies. Kapp (1999), for example, wrote: "Many experts believe that the only sustainable advantage an organization will have in the future is its potential to learn faster than its opponents. This competitive advantage can be accomplished by transforming the organization into a learning organization" (p. 74). In addition, Cunningham (1980) wrote that: "Companies that have neglected the learning-curve principles fall prey to more aggressive manufacturers" (p. 48). Due to the importance and the wide applicability of the learning topic, it has been studied in many industrial sections, including automotive, construction, chemical, healthcare, energy, military, information technologies, education, design, and banking (Anzanello & Fogliatto, 2011; Jaber & Glock, 2013). Just to underline an interesting effect of learning in semi-conductor industry, Webb (1994) observed that the efficiency caused by learning led to a price fall up to 30 % yearly.

Like other industrial areas, learning can occur in many areas in inventory and production settings. For example, when a shop floor worker in a working station performs a task repetitively, s/he could learn over time, lead to decreasing the time required to perform the task. Furthermore, performing cross-trainings programs for workers, assigning them to different working stations with different tasks, and restructuring and reorganizing works due to increasing flexibility or productivity of the tasks are the reasons that result in learning. Learning could not solely occur for those who are working in production lines. It could also occur for staff positions, in accomplishing their daily stuff. For example, the staff working in sales department could use their experience and learning from their past experience to improve the customer demand prediction.

The attempts made to predict and monitor the performance of individuals or a group performing a task resulted in the development of learning curves, which are usually a mathematical relationship between the time and the number of the produced products. The first observation of learning phenomenon in an industrial setting was made by Wright (1936), who found that there existed a relation between an individual task and a unit production cost. The results of his study revealed that the unit production cost decreased proportional to the cumulative number of the units produced by a worker in conformance with a power-form learning curve. According to the trend line of the data studied, Wright (1936) fitted the data and scattered points around the trend line using a geometric form mathematical relation, also called as log-linear, which is given by

$$T_i = T_1 i^{-l} \tag{2.1}$$

where T_i is the performance at the time of ith repetition, T_1 is the performance at the beginning of the planning period, i is the number of repetitions, and l $(0 \leq l < 1)$ is the learning exponent. The learning exponent is the slope of the learning curve and represents the individual's learning rate (Teplitz, 1991; Badiru, 1992; Dar-El, 2000; Jaber & Bonney, 2011). Fig. 2.4 illustrates the behavior of the learning curve in

Eq. (2.1). The learning exponent in Eq. (2.1) can be calculated using the expression $l = -log(LR)/log(2)$, where LR is the learning rate and can be described as a percentage ranging from 50% to 100%. If the learning rate is 100%, the learning exponent adopts 0 in the equation, while in case the learning rate is 50%, the learning exponent is equal to 1.The higher value of l means faster learning while the lower value signifies slower learning (Jaber & Bonney, 2011).

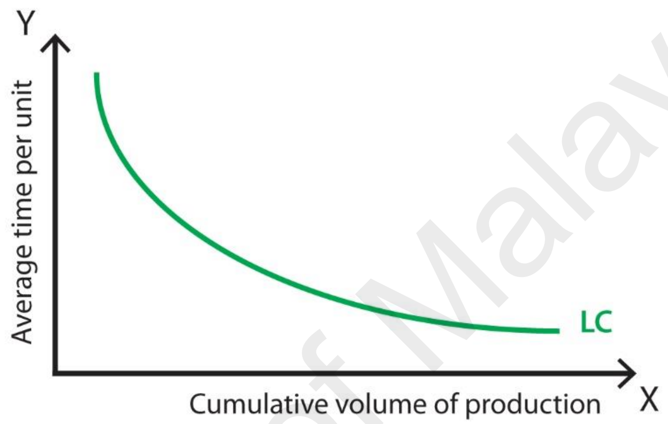

Figure 2.4: The behavior of the Wright's learning curve

2.4.7.2 Learning curve models

Since development of the first learning curve model, its variant forms have debated or extended, aiming at proposing a version that is capable of fitting the real data. In addition, Wright's learning curve has been extended to take account of different individual's learning processes in real practice (Yelle, 1979; Badiru, 1992; Badiru & Ijaduola, 2009; Anzanello & Fogliatto, 2011). All the developed models have commonly argued that the performance improves with the repetition of tasks. One of the first modifications of Wright's model is the Stanford-B model, developed by Carlson (1973), which includes worker's prior experience. The Stanford-B learning curve is as the following:

$$T_i = T_1(i+B)^{-l} \tag{2.2}$$

In Eq. (2.2), B is corresponding to the number of units of experience that have already been gained. This model was applied to the assembly line of Boeing 707, and to fit the data for order picking activities (Yelle, 1979; Badiru, 1992; Nembhard & Uzumeri, 2000; Grosse & Glock, 2013). De Jong (1957) developed the model for machinery assembly line as follows:

$$T_i = T_1[M + (1-M)i^{-l}] \tag{2.3}$$

In this learning curve, M, which adopts a value over the interval [0,1], is a coefficient indicating the percentage of the automation of a task. There is an inverse relation between the automation of a task and the learning process, entailing that the more the process is mechanized the less the improvement through learning. The integration of the Stanford-B model and De Jong (1957)'s model was also developed with the purpose of including both factors, i.e. prior experience of the worker and the automation of the process. This learning curve was called S-curve, which is given by

$$T_i = T_1[M + (1-M)(i+B)^{-l}] \tag{2.4}$$

De Jong (1957) measured the time of the production in a mix-model assembly lines and suggested the following learning curve

$$T_i = T_1 i^{-b} Q \tag{2.5}$$

In Eq. (2.5), Q is the extra time required to process additional models over an equivalent single model line (Jaber, 2006). Suitable for a new process, a type of learning function, termed as adaptation function, that helps organizations to improve their performance was offered by Levy (1965), having the following form:

$$R(i) = P - (P - \frac{i^b}{T_1}) \tag{2.6}$$

where $R(i)$ is the production rate after producing i units, P is the target production rate the firm aims to achieve, and other parameters are like the ones of the log-linear model. To predict the production time in long production runs, Knecht (1974) developed an exponential learning curve by combining exponential and log-linear functions

$$T_i = T_1 i^{-b} e^{\alpha i} \tag{2.7}$$

where α is a coefficient that helps to come to a more precise estimation of the time in the long production run. Following the work of Knecht (1974), a family of exponential learning curves appeared in the literature to deal with different production situations. Among these learning curves, the constant time learning curve suggested by Bevis et al. (1970), which modified later by Hackett (1983), with the form as:

$$T_i = T_c + T_f(1 - e^{-t_i/\beta}) \tag{2.8}$$

where T_c is the performance of the worker at the startup, measured as the number of produced items per unit time, T_f is the maximum performance of the worker after learning period is finished, t_i is the unit time, measures by the day, β is a time constant for the learning curve. Yelle (1979) proposed a learning curve with n disparate stages, each of which including different learning rate, which is of the form

$$T_i = T_1 i_1^{-l_1} + T_2 i_2^{-l_2} + \cdots + T_n i_n^{-l_n} \tag{2.9}$$

However, Howell (1980) criticized the model as not being able to estimate the production rate precisely. Another form of the learning curve was obtained by integrating the exponential form with the power form function of the log-linear model. Dar-el et al. (1995), discussed that an industrial task, like routine life, comprises of two phases, namely cognitive and motor, and suggested that Wright's learning curve should be modified to include both phases. Building on the power form model of the Wright, their learning curve has an aggregated form, which is some of both cognitive and motor learning curves. This learning curve is called dual phase learning curve (DFLC) and is of the form

$$T_i = T_1^c i^{-l_c} + T_1^m i^{-l_m}, \tag{2.10}$$

where and l_c and l_m represent the learning exponents for the cognitive (with subscript c) and the motor (with subscript m) part of a task, and T_1^c and T_1^m indicate the initial time for performing the motor and the cognitive parts, respectively. The extension of the DFLC was offered by Jaber and Glock (2013) (called JGLC), who replaced the initial time for the cognitive and the motor part of a task in Eq. (2.10), T_1^c and T_1^m, by αT_1 and $(1-\alpha)T_1$, respectively.

$$T_i = \alpha T_1 i^{-l_c} + (1-\alpha) T_1 i^{-l_m} = T_1[\alpha(i^{-l_c} - i^{-l_m}) + i^{-l_m}], \tag{2.11}$$

In Eq. (2.11), α is the percentage (weight) that T_1 could be divided into the cognitive and motor components. In fact, α provides a mechanism on how many percentages of a task could be implemented using the cognitive and how many percentages could be done using the motor ability. Jaber and Glock (2013) demonstrated that their model outperforms the log-linear learning curve and DFLC in fitting the data.

The review of the above learning curves brings up the question of which learning curve is recommended to use. Actually, there is a continuing and ongoing debate among researchers about the question of which learning curve is appropriate for which application. However, the researchers are not unanimous about a unique form of the learning curve that suits the applications. This is the main reason why several forms of the learning curves have emerged to adapt to different problems under study. But, it has been, by far, accepted that Wright‘s learning curve is the most popular model that have received a favorable attention, and the one that demonstrated to fit empirical data quite well (Jaber, 2006). For further study about the forms of the learning curves, readers are referred to Yelle (1979), Belkaoui (1986), and Badiru (1992) and Anzanello and Fogliatto (2011).

2.4.8 Forgetting

Forgetting, also termed as knowledge depreciation or knowledge decay, is defined as a phenomenon that is the mirror image of learning, which leads to loss of knowledge. It is based on the fact that the knowledge and experience that are learnt might be lost if learning process is interrupted, for example because the process is ceased for a period. The empirical evidences demonstrated that knowledge depreciation may happen in practice and hinder the learning process (Argote et al., 1990; Argote, 1993; Dar-el et al., 1995).

Although the literature on learning theory is fertile, the empirical and mathematical investigation of forgetting models are not as mature as learning models. This is probably because obtaining data on forgetting process over time is more difficult compared to the learning models (Globerson, 1987). There are plethora reasons why forgetting occurs. In production and inventory literature, operation break is stressed as the main cause of forgetting (Jaber, 2006).

The existing studies on forgetting phenomenon in inventory management can be fallen into three main classes. The first group of the studies usually deals with modelling forgetting process using empirical data gained from laboratory setting, while the second group are the models with practical data obtained from industrial areas. The last class of the models is the one that the main aim is to develop a general mathematical model describing a relation between human forgetting and time. As in this study we intend to model the forgetting process mathematically, we thus review the third group.

2.4.8.1 Learning and forgetting curve models

Reviewing the literature on mathematical models of forgetting indicates that there are only three promising models, which studied the learn-forget process mathematically. These models are: learn–forget curve model (LFCM) developed by Jaber and Bonney (1996b), the recency model (RC) developed by Nembhard and Uzumeri (2000), and the power integration diffusion (PID) developed by Sikström and Jaber (2002). These models are constituted based on the log-linear model. In the following, we will review these models and highlight their main characteristics.

(a) ***LFCM***

Jaber and Bonney (1996b) developed the LFCM based on the log-linear model and assumed that learning is transferred partially. Therefore, they modified log-linear model as

$$T_i = T_1(u_i + n_i)^{-l} \tag{2.12}$$

where u_i is the accumulated experience remembered at the beginning of cycle i, and n_i is the performance in ith cycle. The term u_i in ith cycle can be computed as

$$u_{i+1} = (u_i + n_i)^{(1+f_i/l)} S_i^{-\frac{f_i}{l}} \tag{2.13}$$

where f_i is the forgetting rate in cycle i and S_i is the amount of experience that would have been accumulated if the process was not stopped for d_i units of time. According to Jaber and Bonney (1996b), f_i and S_i can be computed as

$$f_i = \frac{l(l-1)\log(u_i + n_i)}{\log\{1 + B/t(u_i + n_i)\}} \tag{2.14}$$

$$S_i = \left\{\frac{1-l}{T_1}[t(u_i + n_i) + d_i]\right\}^{\frac{1}{1-l}} \tag{2.15}$$

In Eq. (2.14), B is a time to which total forgetting occurs, and $t(u_i + n_i)$ is the time for $u_i + n_i$ experience in ith cycle. It is necessary to note that in Eq. (2.14), due to the forgetting effect, the learning transfers partially among cycles, which entails that $0 \leq u_i \leq \sum_{j=1}^{i-1} n_i$ and $u_1 = 0$. On the other hand, full transmission of learning occurs when $u_i = \sum_{j=1}^{i-1} n_i$.

(b) ***RC Model***

Nembhard and Uzumeri (2000) modified the three parameter hyperbolic learning function given by Mazur and Hastie (1978) and developed RC model. They suggested the term "recency of experiential learning", indicated by R_x. In their model, they computed R_x for every unit of cumulative production, x, by dividing the elapsed time observed for x unit of cumulative production by the elapsed time viewed for the latest produced unit. According to Nembhard and Uzumeri (2000), R_x could be determined as

$$R_x = 2\frac{\sum_{i=1}^{x}(t_i - t_0)}{x(t_x - t_0)} \tag{2.16}$$

where x is the aggregated number of produced unit, t_x is the performance at the time of xth repetition, t_0 is the performance at the beginning of the production and t_i is the performance at the time of ith repetition. By modifying and adopting the log-linear model to this case, the performance of the worker in producing the first unit after break will be as

$$\hat{T}_{ii}^{RC} = T_1(xR_x^{\alpha})^{-l} \tag{2.17}$$

where α is the forgetting rate of the worker and could be determined through fitting the learning data.

(c) ***PID model***

Sikström and Jaber (2002) developed PID model to form a mathematical model for forgetting process of a worker in production line. The idea of PID model was based on memory trace concept, which entails that every time a task is repeated, memory traces the experience. However, due to break in production, the ability of the worker in tracing the experience depreciates in conformance with a power function over time. In addition, the time required for performing a task is assumed to be determined using the diffusion concept, which is related to the strength of memory in tracing. As suggested by Sikström and Jaber (2002), the ability of a memory in tracing over a short time interval could be calculated as:

$$S_t' = S_0 t^{-\alpha} dt \tag{2.18}$$

where α is forgetting rate and $0 \leq \alpha \leq 1$, S_0 is scaling parameter and dt is the short time period. Sikström and Jaber (2002) then calculated the memory strength for an extended time interval as

$$S(t_{e,1}, t_{e,2}) = \int_{t_{e,1}}^{t_{e,2}} S_t' \, dt = \frac{S_0}{1-a}\left[t_{e,2}^{1-a} - t_{e,1}^{1-a}\right] \tag{2.19}$$

where $t_{e,1}$ and $t_{e,2}$ are elapsed time since the start and end of encoding of e unit, respectively. As a result, the strength of memory in tracing during M time interval is given by

$$S(t_{e,1}, t_{e,2}) = \frac{S_0}{1-a}\sum_{e=1}^{M}\left[t_{e,2}^{1-a} - t_{e,1}^{1-a}\right] \tag{2.20}$$

The performance of the worker on producing a unit has an inverse relation with memory strength, as given in Eq (20). Sikström and Jaber (2002) further assumed that the performance of the worker included diffusion process, which is added to the time for producing a unit, and is given as

$$T(t_r) = S(t_{e,1}, t_{e,2})^{-1} + t_0 = \frac{1-a}{S_0}\left\{\sum_{e=1}^{M}\left[t_{e,2}^{1-a} - t_{e,1}^{1-a}\right]\right\}^{-1} + t_0$$

$$= S_0'\left\{\sum_{e=1}^{M}\left[t_{e,2}^{a'} - t_{e,1}^{a'}\right]\right\}^{-1} + t_0 \tag{2.21}$$

where $S_0' = \frac{1-a}{S_0}$, and $a' = 1 - a$. Sikström and Jaber (2002) proved that log-linear learning curve is a special case of PID model as

$$T_i = dt(i)/di = \left\{[(1+a')S_0']^{1/(1+a')}\right\}(i^{-a'/1+a'})/(1+a') \tag{2.22}$$

where $t(i) = T_1 \sum_{n=1}^{i} n^{-l}$.

2.5 Learning in lot-sizing literature review

Learning is one of the most important considerations for the lot-sizing problems due to the effect that the workers' learning has on the production outcome. Learning in the lot-sizing problem entails that whenever the worker starts a new process, i.e. replacing the tools or machine that she/he is working with, changing the work procedure, starting a new production, the performance would improve over time. The more she/he is proficient, the more is capable of improving the performance, which performance suggests the time required for production. It is trivial that planning the optimal batch size without learning is completely nonsense and impractical. As a result of this fact, several researchers studied lot-sizing problems with the effect of the workers' learning. Regarding the application of learning in lot-sizing problems, the available models in the literature could fall into the following categorizations.

2.5.1 Lot-sizing models with learning in set up

Set up time is one of the factors that increases production time, but since it repeats through time, it could be reduced using the learning of workers. The problem of learning in set up appeared in the literature in various forms. For example, Rachamadugu (1994) addressed the problem of the learning in set up and assumed that set up cost reduces over time due to learning. Rachamadugu and Schriber (1995) considered a similar problem and assumed that set up cost lessened over time due to the continuous improvement, the workers' learning, and incremental process improvements. The authors suggested two heuristic approaches for the circumstances in which the trend in set up cost was not known. Following these papers, Rachamadugu and Tan (1997), in a similar study, addressed the problem of determining lot-sizing in a finite planning horizon under the condition of learning and continues improvement, which led to decrease in set up cost. They suggested a lot-sizing policy such that having information about the subsequent set up costs was not necessary. Maity et al. (2009) studied a production system with recycling facilities where customer demand was satisfied using the produced and recycled items. The manufacturing set up cost was assumed to reduce because of the learning. Hung and Chen (2010) addressed the problem of replenishment policy in an EPQ model with finite planning horizon and learning in set up cost. Finally, the case of a production-inventory model with two storage locations, deteriorating items and inflation in a random planning horizon was offered by Das et al. (2012), who assumed that set up, production and selling undergo learning process. Due to the complex nature of the formulated problem, the author developed a hybrid genetic algorithm to derive the optimal solution.

2.5.2 Lot-sizing models with learning in production

When a product is produced and the volume of production is increased as a result, it is possible that the worker learns over time, which has a direct effect on the production time. That is, less time is required for producing in the latter stages of the production compared to the earlier stages. Several researchers studied the impact of worker's learning (and forgetting as an opposite effect) in production on lot-sizing models. Among the papers that studied the effect of learning in lot-sizing, the majority fall in this category, which shows that the learning in production gained favorable attention from researchers. Salameh et al. (1993) developed a production-inventory model with learning in production, which the learning was assumed to be in conformance with the log-linear learning curve. Li and Cheng (1994) studied this problem and assumed that as production goes up, learning affects the direct labor time required for production, causing to its reduction over time. The model of Salameh et al. (1993) revisited by Jaber and Bonney (1996b), who incorporated forgetting phenomenon, besides learning, into the EPQ model and analyzed the changes in the optimal production quantity and the optimal total cost due to the learning and forgetting. The work of Salameh et al. (1993) was further extended in Jaber and Salameh (1995) by assuming that shortages are permitted and could be backordered. Jaber and Bonney (1997) studied learning in a production situation in which the improvement in the performance might be slower because of the difficulty of the task under operation. The analysis of their model showed that the model under partial transfer of learning returns larger cycles and consequently greater labor and total inventory costs. Jaber and Bonney (1996a) proposed a closed-form solution for the total holding cost of a model in the literature, which developed a lot-sizing problem with two different types of the learning curves. Zhou and Lau (1998) declared that there is an error in the model of Jaber and Bonney (1996a). In their paper, they reconsidered the corrected version of the model of Jaber and Bonney (1996a) and permitted the occurrence of shortage in the model, which was assumed to be fully backordered. Chiu (1997) developed an optimal dynamic lot-sizing model with a time-varying demand and incorporated the effect of worker's learning and forgetting into the model, where the forgetting rate assumed to be a ratio or an exponential function of the break period. Chiu and Chen (1997) reconsidered the model of Chiu (1997) by integrating the impact of time value of money, and identified the substantial effect that discount rate, learning and forgetting have on determination of lot sizes. Eroglu and

Ozdemir (2005) criticized the model developed by Chiu and Chen (1997) for the use of Wagner-Whitin algorithm. They claimed that the algorithm developed by Chiu and Chen (1997) cannot achieve the optimal solution. Therefore, they suggested a recurrence relationship to correct the solution algorithm. Jaber and Bonney (1998) studied the effect that learning and forgetting have on an intermittent production system in both finite and infinite production planning horizons. The main intention of their model was to investigate how learning and forgetting influence determination of the production lot in both planning horizons. Their study suggests that when the partial transfer of learning occurs between cycles, the optimal policy is to stock less inventory in the subsequent runs, and to increase the cycle length. Jaber and Bonney (1998) analyzed the concurrent effect of learning and continuous time discounting on a lot-sizing problem and addressed the question of whether continuous time discounting could be relinquished when the production system benefits from the cost reduction caused by learning. They concluded that ignoring continuous time discounting may, however, result in the non-optimal solution, but its effect on the optimal total cost is not much significant, therefore it gives flexibility to decision makers if they aim to ignore it. The situation where production facilities are randomly unavailable, i.e. due to maintenance, was addressed in Jaber and Abboud (2001), who integrated the learning and forgetting phenomenon into their models. The optimal policy in this case is to produce smaller lots with higher frequency for the learning case, while producing in larger lots with less frequency is the observed outcome when forgetting is taken into account.

Ben-Daya and Hariga (2003) offered a variation of a lot-sizing problem and studied a continuous review model with controllable lead time. In their model, the processing time of lead time, which considered to be composed of various elements such as set up time, processing time, and non-productive time, assumed to reduce with the help of learning. Another lot-sizing model with learning effect proposed by Balkhi (2003), who assumed that the items are exposed of deterioration in production or storage process. Chiu et al. (2003) included learning and forgetting in set up and production, and assumed that the forgetting rate is dependent on two elements: 1- the level of experience gained before break 2- the break period. In addition, to avoid inventory repletion, they also assumed that each batch could be produced in a time that is as close as possible to delivery. They identified a relation between learning and forgetting in set up and concluded that learning has a significant impact on the total production cost. The same problem was treated by Chiu and Chen (2005), who incorporated learning and forgetting effects into a dynamic lot-sizing problem, with the intention to determine the optimal production planning in a finite planning horizon.

Alamri and Balkhi (2007) surveyed a lot-sizing problem with infinite planning horizon, deteriorating items and learning and forgetting effects on production, where the authors assumed that the total forgetting period is variable. Jaber and Bonney (2007) discussed the assumption of steady learning slope and insinuated that it is illogical to assume the constant learning slope when the learning changes according to production numbers. To address this research gap, they combined the cognitive and motor capabilities of the worker into the model of Salameh et al. (1993). The numerical analysis showed that disregarding the cognitive and motor abilities of the worker could result in an erroneous policy in determining the production batch sizes. Some drawbacks of the developed models in the literature in terms of learning exponent and holding cost was addressed in Jaber and Guiffrida (2007). Teyarachakul et al. (2008) investigated a production system with learning and forgetting in production under the assumption that production varies between two limits when the production rate approach infinity, called alternative convergence. In their subsequent paper, Teyarachakul et al. (2011) added the assumption of delayed forgetting and studied a case where forgetting commences slowly with a lower rate initially and turns to a faster forgetting through time. They suggested producing in smaller lot-size in the presence of learning and forgetting. A production system with imperfect quality, shortage and learning in production proposed by Chen and Chang (2008b). The authors assumed that production system may produce defective items, both over in control and out of control states. Jaber et al. (2009) extended the work of Jaber and Bonney (1998) by integrating the concept of entropy cost to their model and investigated the opposite effect of learning on one hand and forgetting and entropy cost on the other hand on the production lot size. They found that improper control policy increases the system's cost and leads to producing larger batch size to mitigate the opposite effect of entropy costs. To support supplier selection decision with learning condition, Glock (2012b) analyzed a situation where a buyer faces a sourcing decision between single or double sourcing and where the candidate supplier/s experience/s learning in their production process. The analysis of the model suggested that not only the learning process at the supplier, but also the total cost of the system could be affected by the learning process.

The learning of workers was also studied in the reverse logistics system, which involves learning in remanufacturing, recycling and repairing processes. A similar system as in Maity et al. (2009) can be found in Tsai (2012), who assumed that the production time in the recovery process is affected by learning. Jaber and El Saadany (2011) modeled learning process of a worker in a reverse logistics model comprising of production, remanufacturing and waste disposal stages, and assumed that operator's learning occurs in production and remanufacturing processes in an environment that learning requires capital investment. They observed that faster learning rate in production decreases the collection rate of the used items, and consequently concluded that if the company is pushed by the legislation to mount the collection rate, increasing learning rate should not be the optimal policy. Eventually, Teng et al. (2014) established an EPQ model for a manufacturer (seller), who offers a trade credit to his buyer to promote the purchasing quantities with the assumption that the production cost of the manufacturer declines as a result of the learning effect.

2.5.3 Lot-sizing models with learning in quality

The quality of products is one of the areas that could be affected by the worker's learning process. The activities associated with quality like inspection process and rework are inherently repetitive tasks over time, which incur learning process to improve the activities. This fact attracted several researchers to study the problem of learning in quality settings. For instance, Jaber and Bonney (2003) studied the relation between learning and forgetting in set up and quality and the economic lot size. They hypothesized that as production increased, the time needed for reworking defective items reduced in conformance with a learning curve. They additionally supposed that forgetting had a negative effect on quality due to interruption in process. Jaber and Guiffrida (2004) developed a composite learning curve for a production process with defective items and rework. The proposed learning curve summed up two different parts, one for the production and another one for rework. This work was revisited and extended by Jaber and Guiffrida (2008) to a case that production process can be ceased to restore the quality of the product. With the data taken from automotive industry, Jaber et al. (2008) demonstrated that the imperfect quality items, receiving in lots by the buyer, reduces in conformance to a learning curve. They fitted the real data and consequently proved that the S-shaped logistic learning curve can be fitted more appropriately. The situation in which the time of inspection subjects to learning in an imperfect supply process was investigated by Khan et al. (2010). They investigated a scenario that inspection rate was lower than demand rate, and therefore the inventory system encountered shortage, which was treated as backorders or lost-sales. The learning in inspection scenario in their paper included three possible alternatives, which were: 1- no transfer of learning 2- total transfer of learning, and 3- partial transfer of learning. The work of Jaber and Guiffrida (2004) and Jaber and Guiffrida (2008) was finally revisited by Jaber and Givi (2015) to account for learning and forgetting.

2.5.4 JELS models with learning

Like other areas discussed before, the application of learning was also extended to incorporate more players in supply chain, with the aim to investigate the JELS and supply chain coordination problems under learning. One of the first papers that presented JELS model with learning was the one of Nanda and Nam (1992), who developed a SVSB model with learning and forgetting effects in production. The faster learning was identified to have the potential of reducing the total joint cost considerably. The model of Nanda and Nam (1992) was extended by Nanda and Nam (1993) to a model with multiple retailers. Kim et al. (2008) countered the assumption of earlier works and developed a model assuming that manufacturer can adopt his own policy, apart from the buyer's order's policy. They considered the learning as a result of multiple set ups in MSMD (multiple set ups multiple deliveries) policy. Jaber et al. (2010) analyzed a three level supply chain, including individual supplier, manufacturer and retailer, where the manufacturer experienced learning and forgetting in set up, quality and capacity utilization. The average cost reduction of the supply chain was observed to be higher when learning occurred concurrently in set up, production and rework following centralized policy.

Tsai (2011) is believed to be the first researcher who developed a JELS model with the integration of deteriorating items and learning effect. In his models, he took into account a two-stage supply chain model consisting of single manufacturer and single buyer under a circumstance where the production process at the manufacturer followed a log-linear learning process, and where the stored items at the buyer exposed to deterioration. Tsao and Sheen (2012) formulated a two-stage supply chain and assumed that there is a competition between retailers because of the shortage substitution, and further supposed that the sales activities at the retailers followed a learning curve. Zanoni et al. (2012) developed a two level supply chain model under consignment agreement between the vendor and the buyer with the assumption that the manufacturing process at the vendor underwent learning and forgetting. Khan et al. (2012) surveyed the presence of human factors, i.e. learning in production and quality error in screening defective quality items, in the supply side of the supply chain by developing a multi supplier-single manufacturer model with two different coordination mechanism. Whereas the first mechanism assumes that the players of the supply chain work with equal cycle time, the second mechanism assumes that supplier's cycle time is a an integer-multiplier of the manufacturer's cycle time. Pursuing their previous research, Khan et al. (2014) presented a mathematical model for a two-level supply chain with imperfect quality items considering the learning in production at the vendor and error in screening the items at the buyer. The learning effect was identified to be useful in reducing the total cost of the supply chain. In this context, Chen and Tsao (2014) considered a manufacture supplying imperfect quality items to a retailer with the assumption that the rework process at the manufacture is affected by learning of workers due to the repetitive nature of the task. A two-period two-level supply chain model incorporating single manufacturer and single retailer offered by Li et al. (2015), who assumed that the manufacturer experiences learning in his production process, which results in production cost reduction. They considered a structure in which the manufacturer could gain experience in the first period and could then apply the accumulated experience in the second period of the production.

2.6 Syntheses of both research streams

In this chapter, a deep analysis of the literature in fuzzy inventory management and human learning illustrated that both research streams were studied frequently in the past. It is, however, surprising that researchers dealt with both topics mostly independently. As described before, inventory management activities are inherently carried out in a manned environment, where human characteristics may affect the decision outcome and the system's efficiency. The involvement of human in inventory planning could be more highlighted for the cases where the inventory system must rely on the experience of experts due to uncertainty. Therefore, it is worth mentioning here that if human factors are not considered in modelling fuzzy inventory models, then the developed models could lead to an erroneous or perhaps costly decision making. On the other hand, given the fact that human capabilities are not constant over time, treating them as constant factors in fuzzy inventory decision making is, undoubtedly, in contrast to real situations, which trivially makes the available models in the literature impractical.

In Section 2.4.6, a number of research gaps resulted from the systematic literature review are listed. As this study intends to cover the identified research gaps in the literature, the structure of the model will be built such that it covers a number of the research gaps pointed out in Section 2.4.6. To this aim, this thesis covers the research gaps in the following ways:

1- It addresses the research gap through developing fuzzy inventory models that formulate human factors, i.e. learning and forgetting, in the modelling process.

2- It covers the research gap by conducting a qualitative study, which subsequently helps in developing model's assumptions.

3- It conceals the research gap by implementing a case study for validation of the model.

2.7 Summary of the chapter

The intention of this chapter was to present a comprehensive review on the studies in the fields of fuzzy inventory management and learning. Since fuzzy inventory management lacks a comprehensive and systematic literature review, a systematic literature review is adopted to review and analyze the papers in the respective research stream and to highlight the research gaps. There are several aspects that became apparent from the review. First and foremost, the review revealed that even though the human role in fuzzy inventory management is completely common in practice, the available studies overlooked this effect in their model. Secondly, the available models deemed to have a certain focus on developing theoretical models, without paying attention to the practical aspects. As to the third issue, it is comprehended that the available models are purely quantitative in terms of the structure of the models. Reviewing the aforementioned research gaps highlights the necessity of conducting a study to integrate these aspects into a single research framework to cover the gaps and gain further insights.

CHAPTER 3: RESEARCH DESIGN AND METHODOLOGY

This chapter seeks to present the research design and methodology adopted to answer the research questions and achieve the objectives defined in Chapter 1. The qualitative and quantitative features of the study will be discussed in detail and the specific approach employed in the research methodology will be justified. Additionally, the model that forms the foundation of this study will be clarified and the data analysis method implemented to evaluate the models will be documented.

3.1 Research design

Research design refers to the methodology and processes applied by a researcher to conduct research, and it includes the entire process from formulating the problem of the study to developing research questions and collecting and analyzing the data (Yin, 2013). To achieve the objectives of this study, after careful examination of the literature, research questions, and research objectives, it was determined that a combination of qualitative and quantitative research methods are appropriate to achieve the objectives of the study. Although the objectives of this study could be attained using only a quantitative method, a general understanding of the learning phenomenon and what happens in reality could not be achieved comprehensively. Using a combination of these methods allowed to acquire more information and to conduct a more accurate and concrete evaluation (Jick, 1979; Neuman, 2005; Creswell & Clark, 2007; Creswell, 2013).

The view gained from the literature survey legitimizes adopting both qualitative and quantitative techniques. The survey indicated that the literature lacks empirical observation on the application of human learning in fuzzy inventory planning. Therefore, it is important to obtain insights on how learning is implemented in practice by performing a qualitative analysis, and then to combine the obtained views into quantitative analysis in order to explore the effects of human learning in fuzzy inventory planning. The steps taken to conduct this study are outlined in Figure 3.1.

According to the diagram depicting the research process, this study begins with a systematic literature review. The systematic literature review helps the researcher to analyze the available publications in a systematic way and to formulate the relevant research gaps accordingly. The systematic state-of-the-art review could also be of help in picking out the pilot model for this study (i.e., the model in the literature that is most appropriate for extension), which is possible by a thorough investigation and comparison of the developed mathematical models. As depicted in Fig. 3.1, the data-collection process occurs in two steps in the research process. The first step involves carrying out semi-structured interviews with the experts in the inventory management field. As stated before, this step helps the researcher to gain more insights into how learning phenomena and learning transfer take place in practical situations and assists in summarizing the experts' divergent point of views, which afterward provides a basis for formulating the assumptions of the mathematical models. For the purpose of model development, after the step in which the assumptions are formulated, four mathematical models are developed to consider different human learning and forgetting processes in fuzzy inventory management. In the second step, the developed models are analyzed using data from the literature and a case study. The tools applied by the researcher in this study are presented in the following section.

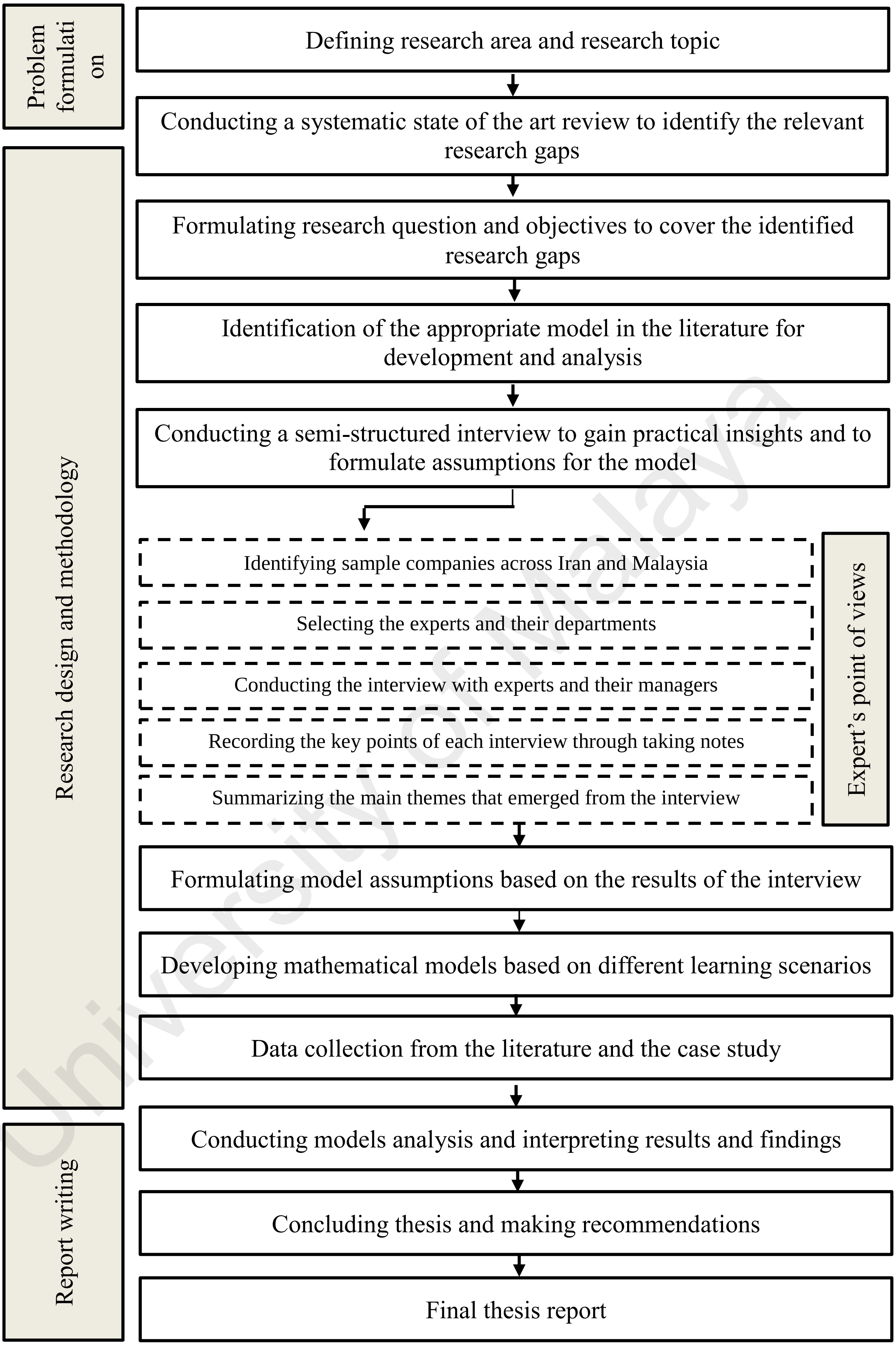

Figure 3.1: Research process diagram

3.1.1 Systematic literature review

Literature reviews provide researchers with an advantageous guide to a specific subject and can give an overview of the topics and findings in a particular area of research. The strengths and weaknesses of the available studies can be distinguished through a literature review, and the analysis can reveal the current research directions (Hochrein et al., 2014; Hochrein et al., 2015).

Based on the approach used to review the body of existing literature in a specific field, literature reviews can be categorized into three types: narrative reviews, systematic reviews, and meta-analyses (Hochrein & Glock, 2012). A narrative literature review is a type of review that critically narrates and discusses the existing publications on a particular subject or theme from a theoretical and contextual standpoint. Narrative literature reviews do not explicitly point out the methodology, such as the selected databases for conducting the search, the inclusion and exclusion criteria, or the search methods used for collecting sample papers for review (Green et al., 2006). A systematic literature review uses a systematic and transparent methodology to go over the studies on clearly defined research questions, and it aims to critically identify, select, and evaluate the relevant primary studies to draw out data and synthesize the findings. A meta-analysis, on the other hand, is a special kind of systematic review that uses quantitative methods to extract information from the available studies and standardize statistical methods to report research findings.

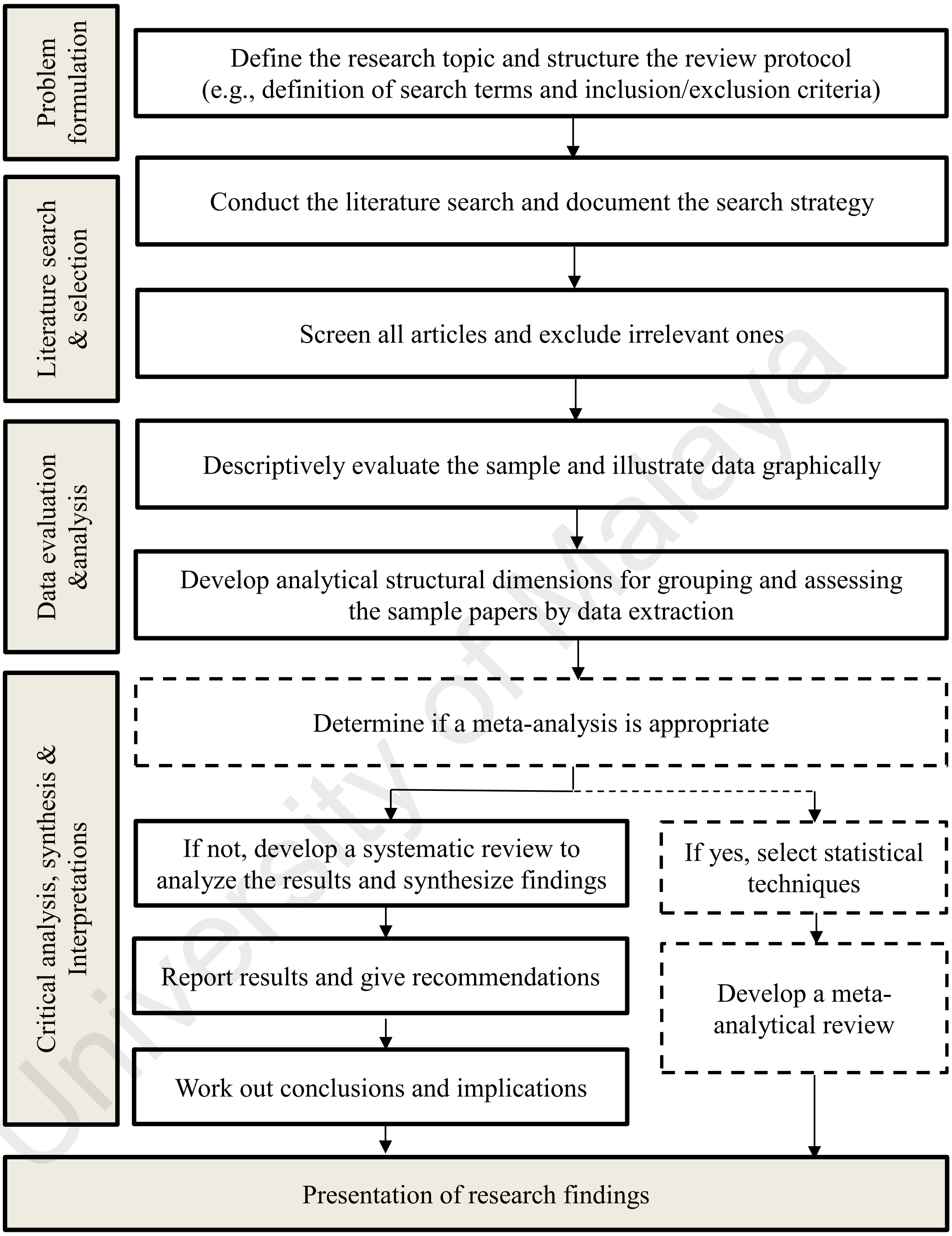

Figure 3.2: The methodology of the systematic literature review adopted in this thesis (Hochrein & Glock, 2012)

The systematic review is usually a more trustworthy and solid approach when compared to the narrative review because the outcomes furnished by systematic review contain fewer errors (Hochrein & Glock, 2012). This is due to the reason that a systematic review prevents producing methodical errors by applying a structured literature assessment and by examining all primary studies in a lucid, objective, and thereby consistent and impartial way (Hochrein et al., 2014; Hochrein et al., 2015). Therefore, in this study a systematic literature review is adopted to review the studies in the fuzzy inventory management field. The methodology used for this purpose is according to the one described in Tranfield et al. (2003), Cooper (2009), Hochrein and Glock (2012), Glock and Hochrein (2011) and Glock et al. (2014), which follows a standard systematic literature review process. Fig 3.2 illustrates this procedure.

3.1.2 Data collection

Data collection is a part of the research process where researchers collect data form the concerned population in a systematic fashion so that they could be able to answer research questions and objectives (Bickman & Rog, 2008). As to the type of research data, they can be divided into qualitative and quantitative types. Qualitative data are a kind of data that are generally difficult to measure or quantify. Quantitative data, contrariwise, are a type of data that can be quantified, are measurable or can be transformed into measurable quantities. While qualitative data are solely suitable for the cases in which there is a need to gain a deep understanding of fundamental factors and reasons affecting the particular area of study, quantitative data are mostly suitable for generalizing results from a sample to a population (Mertens, 1998; Neuman, 2005; Creswell, 2013). Qualitative data could be an appropriate research method for attaining insights into the settings of the problem under study and it frequently generates ideas for the subsequent quantitative research (Creswell & Clark, 2007).

Qualitative data collection is adopted in the first phase of the study to understand learning in imprecise inventory management from expert’s perspective. As the evidence on the prevalence of human learning and learning transfer in inventory management is qualitative in nature and, in addition, since the available literature does not support a solid observance on this matter, qualitative method, like questionnaire or interview, could be implemented to gain more insights. Qualitative data for this study are obtained through the semi-instructed interviews, which are preferred to standardized questionnaire or structured interview. Obtaining data using standardized questionnaire or structured interview requires that the interviewer has predetermined conceptions and codes to classify questions so as interviewees are not allowed to criticize and discuss questions (Louise Barriball & While, 1994). However, in a semi-instructed interview process, the interviewer does not utilize closed-form questions, though instead gives liberty to interviewees to broaden, modify, or criticize the topic, which permits the researcher to probe the findings until saturation of the theme. This enables the interviewee to spell out their expertise details in their own phrases. The unscripted characteristic, in addition, makes it possible for interviewee to discuss concerns, events and activities that researcher might not exactly have anticipated or perhaps envisaged through an outsider's viewpoint, nevertheless could be the main factors to comprehending an individual's experience (Patton, 1990; Neuman, 2005).

It is necessary to indicate that the intention by conducting a semi-structured interview is not neither to find a sample from a population, nor to measure or quantify something, but is rather to enhance understanding of learning and learning transferring phenomenon in imprecise inventory management through obtaining information from professionals.

In order to assess the developed models in the second phase of this study, quantitative type data are employed, which include both primary and secondary data. Primary data are a type of the data collected by a researcher for a specific area, whereas secondary data are a kind that are gathered by someone else for another area, not necessarily the same as the field which the researcher is working on. For quantitative research purpose, initially, the developed models will be optimized numerically using data taken from the literature, which will help to compare the result with a basic model from the literature. Secondly, primary data will be obtained from a car manufacturing company. This not only will assist in running the model using a real case data set, but also will help in realizing how far the current inventory management of the case company is from the optimized situation.

3.1.2.1 Semi-structured interview

Semi-structured interview is recognized as a well-established method in gathering qualitative information (Fontana & Frey, 2000; Ayres, 2008). This type of interview is generally worthy of dealing with small samples and is ideal for investigating particular cases or for validation of information resulting from other sources. Furthermore, Semi-structured interviews are efficient in cases in which it is vital to obtain perceptions straight into the problems that are not instantly tangible, but yet entail the problem in certain sectors of a population (Fontana & Frey, 2000; Ayres, 2008). According to Irvine, Drew, and Sainsbury (2013), semi-structured interview is a suitable choice when the interviewer does not have chance to meet interviewee more than once.

Semi-structured interview includes a topic, an interview guide and a list of questions, often in a distinct sequence, that need to be addressed during a dialogue. In contrast to a questionnaire that a list of questions is designed beforehand, a semi-structured interview initiates with more common questions where the majority of questions are generated throughout the interview. Figure 3.3 illustrates the steps of a semi-structured interview in detail.

The first step is to identify a target population including those who have enough expertise concerning the problem under study due to their responsibilities and activities, and then determine the key informants among the population. Determining the precise number of interviewees to get enough information with regard to the subject under study is usually a challenging task, nevertheless, if semi-structured interviews could possibly be applied as a tool for supplementing other data collection approaches, it could be adequate to carry out just a few interviews. In most studies, a purposive study is a better choice than selecting a random sample from a population when the researcher studies a narrow but deep area.

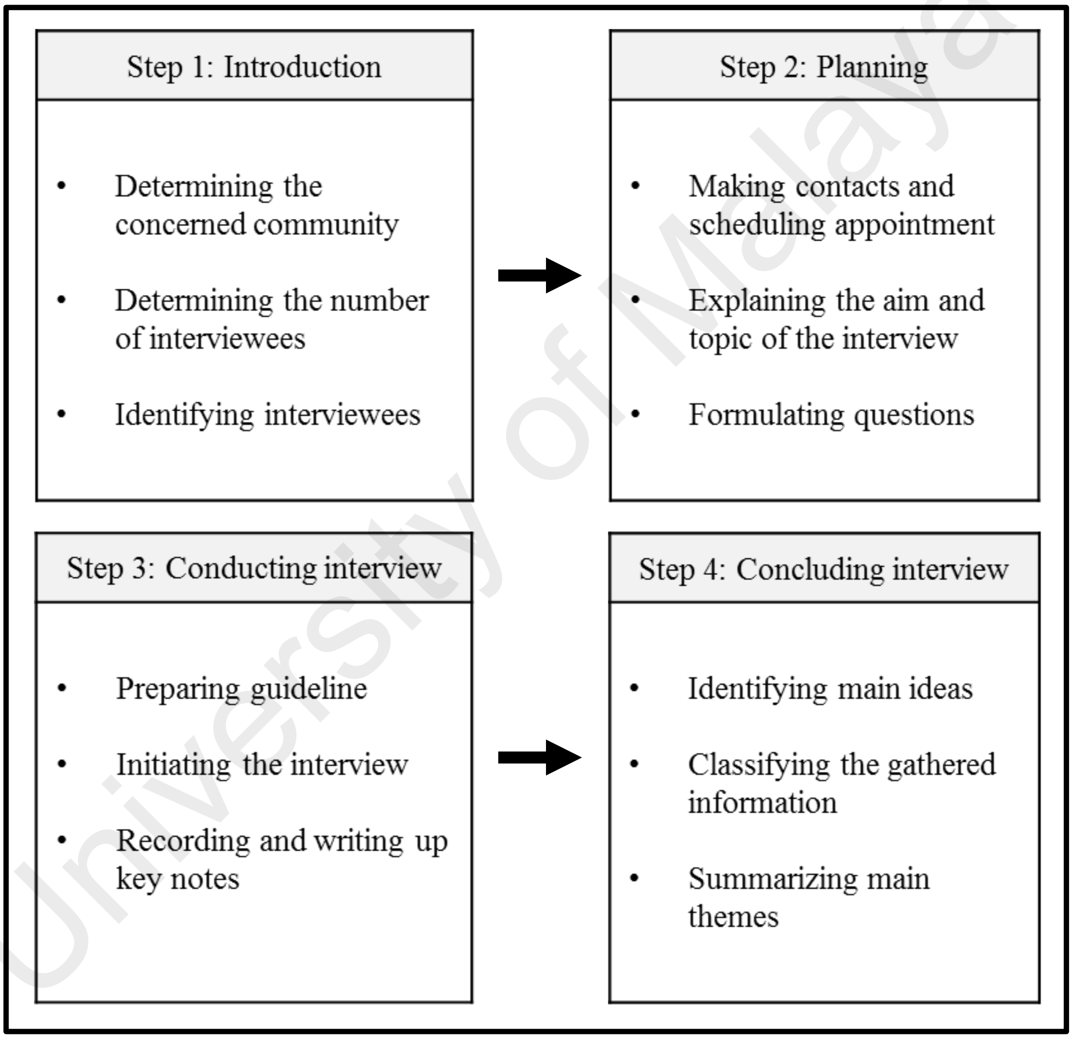

Figure 3.3: Semi-structured interview

After the first step has been done, the interviewer should contact respondents to discuss the objective of the study, ask for permission, plan a meeting, and decide about the location where they are going to meet. In the meantime, the interviewer should think of the questions which are going to be discussed over the interview day. When conducting the interview, the interviewer should act according to the interview guidelines which had already been prepared. The interview starts with some introductory questions, which are easy to respond and comprises of some general questions about the respondent, i.e. job title, age, company name. The interview proceeds with core questions about the topic derived from research's central subjects and goals. In this study, core questions relate to the perspectives of learning and learning transfer from the expert's standpoint. All the questions asked by the interviewer are open-ended types, following, in some cases, inquiries to explore more in-depth and contextual data. The respondents should be given the opportunity to change or criticize the questions whenever they feel questions are on wrong track. Over the interview process, the key information should be recorded to grasp the main themes in interviewee's' responses, which could be written down using a recorder or by taking notes on paper, whatever is possible.

Once the interview has been completed, all the data should be transcribed into a file. As transcription of all the details discussed in the course of the interview process is difficult, the solution is to ease the process through inscribing the main themes extruded from the interview process. Doing so attributes to the fact that after breaking down the general text, smaller units are easily manageable. The prevalent procedure for this purpose is to tag key information and quote them as an indicator for themes and then pick them out from the respective theme.

3.1.3 Model development

At this point of the thesis, the steps taken for development of the mathematical models in this study will be explained. The preliminaries and the mathematical concepts required for the model will be presented in details.

3.1.3.1 EOQ model with backorders

The EOQ model with planned shortages (EOQ-S), or backorders, is perhaps the first extension of the model of Harris (1913). The EOQ-S model deals with an inventory system in which encounters shortages, yet the customer can wait until the system can satisfy the shortages. In this case, the shortages are completely backordered; however, the inventory system incurs shortage penalty for the items which are not delivered to the customer on time. The EOQ-S model constitutes based on the following assumptions:

1- The inventory system manages only a single item during the planning horizon.
2- All the inventory's parameters remain unchanged over the planning horizon.
3- The customer demand receives ceaselessly and at a fixed rate.
4- The intervals that the system receives order are constant and deterministic.
5- The length of the planning horizon is infinite.

The primary objective of the inventory system working under EOQ-S condition is to determine the size of the order, which aims at minimizing the total cost of the inventory. In doing so, it is required to determine how much an order should be and how much inventories are allowed to be backordered to be assure that the system incurs the minimum inventory cost. Fig. 3.4 depicts the behavior of inventory over time for this model.

As Fig. 3.4 clearly shows, when an order arrives, all the backordered demand is satisfied immediately in the EOQ-S model. The total cost function of the EOQ-S model, including the order setup, the inventory holding and the shortage penalty costs, is given as:

$$TCU(Q,M) = \frac{KD}{Q} + \frac{M^2h}{2Q} + \frac{(Q-M)^2p}{2Q}, \tag{3.1}$$

where Q is the batch size (in units), M is the maximum inventory level (just after replenishment and measured in units), K is the fixed cost per order, D is the demand rate (units/unit of time), h is the unit holding cost (Dollar/unit/unit of time), and p is the penalty cost due to shortages (Dollar/ unit/unit of time). The optimal policy of the model, including the optimal maximum inventory storage and the optimal batch size, can be calculated as follows:

$$M^* = \sqrt{\frac{2KD}{h}} \cdot \sqrt{\frac{p}{h+p}}, \tag{3.2}$$

$$Q^* = M^* \left(\frac{h+p}{p}\right) = \sqrt{\frac{2KD(h+p)}{hp}} = \sqrt{\frac{2KD}{p} + \frac{2KD}{h}}, \tag{3.3}$$

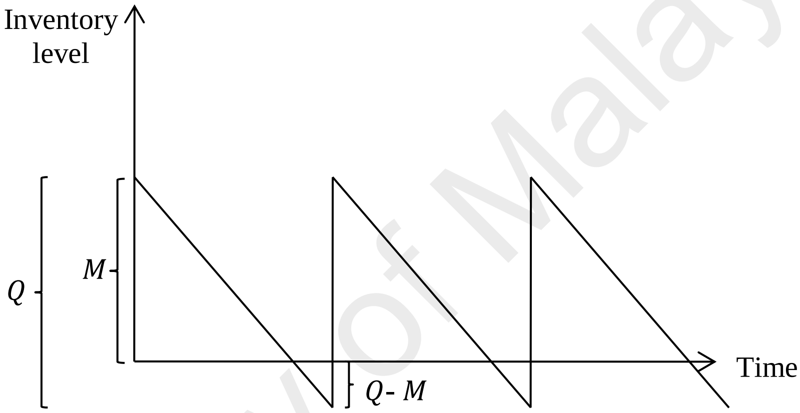

Fig. 3.4: Inventory patterns during the planning period

When p approaches a very large value the optimal batch size in Eq. (3.3) reduces to that of the classical EOQ without backorders, which is as:

$$Q^* = \lim_{p\to\infty} \sqrt{\frac{2KD(h+p)}{hp}} = \sqrt{\frac{2KD}{h}} \,. \tag{3.4}$$

3.1.3.2 Fuzzy EOQ model with backorders

In this section, the basic model of the thesis, which will be discussed, extended and analyzed throughout the paper, will be reviewed. Furthermore, a selection procedure to pick out a suitable model from literature will also be presented.

As indicated in the literature review section, various categories of the problems in the context of fuzzy inventory management suggested in the literature, covering a fairly extensive range of problem formulation and solutions. A number of the categories are presented some of which extended the basic models, and others extended a more complex fuzzy mathematical modelling.

Since the intention of this study is to cover the drawbacks of the previous studies, the models are therefore built on the principles of a previous study in the literature. Thus, to identify a model that is appropriate for this study, a set of selection criteria are defined as the following:

- Since this study is trying to build a stepping stone for future studies, it is therefore tried to keep the models as simple as possible. Therefore, the focus will be on the fuzzy EOQ model without or with backorder, as, according to literature review, these two categories are mostly simpler than other extensions.
- To be a candidate model for this study, the focus of the model should be on developing an analytical model, i.e. the solution of the model should lead to deriving an exact function. Hence, the numerical models in the category of fuzzy EOQ model without or with backorder will be excluded from this study.
- The study should use the real world data (or at least should make a relation of the data with a real case) and not solely a numerical example. Therefore, any model which is tested on the basis of test data will not be considered.

Table 3.1 evaluates the fuzzy EOQ models without or with backorders with respective to the defined criteria above.

Table 3.1: Categorizing fuzzy EOQ models without or with backorders according to the defined criteria

Reference	Model category		Type of the model		Type of the data	
	EOQ	EOQ-S	Analytical	Numerical	Empirical	Test
Park (1987)	√		√			√
Vujošević, Petrović, and Petrović (1996)	√		√			√
Lee and Yao (1999)	√			√		√
Yao and Lee (1999)	√	√		√		√
Yao, Chang, and Su (2000)	√			√		√
Yao and Chiang (2003)	√			√		√
Hojati (2004)	√		√			√
Syed and Aziz (2007)	√		√			√
Lee and Lin (2011)	√		√			√
Samal and Pratihar (2014)	√	√	√			√
Chen, Wang, and Arthur (1996)		√	√			√
Yao and Lee (1996)		√		√		√
Lee and Yao (1999)		√		√		√
Chang, Yao, and Lee (1998)		√		√		√
Yao and Su (2000)		√		√		√
Wu and Yao (2003)		√		√		√
Björk (2009)		√	√		√	
Kazemi, Ehsani, and Jaber (2010)		√	√		√	
Milenkovic and Bojovic (2014)		√	√		√	

As it is clear from Table 3.1, only three studies including Björk (2009), Kazemi et al. (2010), and Milenkovic and Bojovic (2014) cover all the criteria specified above. Among these studies, Kazemi et al. (2010) extended the model of Björk (2009) by defining more imprecise parameters, resulted in a more complex mathematical model. Likewise, Milenkovic and Bojovic (2014) extended the model given by Björk (2009) to the rail freight car inventory problem in which the mathematical model showed to be more complex than that of Björk (2009). Since the model developed by Björk (2009) build the foundation for other two models, this model is consequently chosen as the basic model of this study. In the next chapter, this model will be presented in detail.

3.1.3.3 Formulating assumptions

Upon determining the system under which the mathematical model is going to be developed, it is firstly required to build a fundamental framework. This indicates the credence about the function of the system and can be translated into some underlying assumptions. In this study, developing mathematical models for the inventory system is linked with the results of the semi-structured interview process where it is identified that learning and its transfer are of relevance in inventory management under uncertainty, and they should therefore be taken into account in fuzzy inventory management models. Pursuant to the final step of the interview, the results derived from the interview will be coded and summarize to find relations between the quotes. Next, these codes will be integrated and summarized in some propositions reflecting the summary of the interview. Finally, the assumptions of the mathematical models will be formulated with the help of the propositions.

3.1.3.4 Developing learning and forgetting curves

In this section of the thesis, it will be discussed how learning and forgetting curves, as a major part of the thesis, will be developed and further discuss how two basics learning curves are selected from literature.

As discussed in Chapter 2, after the seminal work of Wright (1936a), various forms and extensions of learning curves have been suggested in the literature, covering fairly extensive applications. According to Jaber (2006), authors have not yet been unanimous as to which learning curve has to be applied in which application. Thus, there is no universal learning curve to be used in different applications. Further, Grosse, Glock, and Müller (2015) pointed out that each time learning effects are considered, researchers and practitioners deal with the issue that which learning curve suits their model.

Two factors can be of help in selecting the most appropriate learning curves for an application. The first one is the power of the learning curve in fitting well with empirical data (Jaber & Glock, 2013; Grosse et al., 2015). That is, the higher the ability of a learning curve in fitting with real data, the better the learning curve is for predicting the performance of a worker. In this context, the long liner model of Wright (1936b) has demonstrated to be one of the best learning curves. Yelle (1979), Lieberman (1987) and Jaber (2006) expressed that, despite its simple mathematics, the log-linear model is still, surprisingly, the widely accepted and mostly applied learning curve, owing to its capability to fit the data in various application areas. Additionally, to highlight the importance of log-linear model, Globerson (1987) and Vits and Gelders (2002) stressed that many types of operations could take advantage of using this model with rather high precision, while benefiting from its non-complex and easy-to-use mathematical structure.

Besides its power in predicting performance, the log-linear model of Wright (1936b) has extensively been applied in various areas. For example, Baloff (1971) and Dar-El

(2013) examined the applicability of the model in the automotive industry. Some researchers like Spence (1981), Teplitz (1991), Rea and Kerzner (1997) and Teng and Thompson (1996) utilized this model to find out the relation between learning and cost reduction. In successive studies, Gruber (1992); Gruber (1994); Gruber (1996) and Gruber (1998) investigated workers' learning in the production process of semiconductor memory chips with the help of Wright's learning curve and its modifications. With a case study in a building company, Blancett (2002) analyzed the performance of the workforces from different sectors of a company in the course of developing a product from the first to the end phase. Furthermore, the log-linear learning curve was studied extensively in renewal energy, which are included, but not limited to: Duke (2002); van der Zwaan and Rabl (2003); Van der Zwaan and Rabl (2004); Nemet (2006); Papineau (2006); Lundvall (2009); Kim and Chang (2012); Hong et al. (2015).

The log-linear learning curve was also broadly studied in production settings. To scheme a tool to enhance production planning and control, Yelle (1980), Yelle (1983), Kortge (1993), and Kortge, Okonkwo et al. (1994) used the log-linear learning curve to study the correlation between product life cycle and the learning curve. Biskup (1999) applied the modified version of the log-linear learning curve in a single-machine scheduling problem. Learning was assumed to take place by the repetition of processing time. Pramongkit, Shawyun, and Sirinaovakul (2000) applied a form of the log-linear learning curve combined with Cobb-Douglas function to estimate the learning curve of the manufacturing companies in Thailand. The similar research was conducted by Karaoz and Albeni (2005) for predicting the learning curve of the Turkish manufacturing industry.

Due to the justifications provided above, log-linear model will be employed in this study to formulate the learning curves for decision maker's learning process. Throughout the study, the learning curve will be modified according to the features of the developed model. Moreover, to model the forgetting process, LFCM will be used, which is an extended version of the log-linear model to account for knowledge depreciation (See section 2.4.8).

The second part of the mathematical modelling will be developing learning and forgetting curves with cognitive and motor abilities of the inventory operator. Cognitive capabilities of human being are brain-based skills that deal with the mechanism of how a person learns, remembers, or solves a problem. Unlike the cognitive skills, motor skills are the ones that help a person to employ the skills obtained with the help of the cognitive ability. Because of the high relevance of both cognitive and motor abilities in inventory planning, the effects of these skills are very important in any planning process, and thus should be considered in modelling inventory planning process.

As illustrated in the review in section 2.4.7.2, two models can be tracked in the literature that developed learning curves for cognitive and motor capabilities, which both are developed building upon the log-linear model. The first model is DFLC, developing a two-stage learning curve with cognitive and motor abilities for an industrial task, and the second one is JGLC, which is the extended version of DFLC model. Referring to the discussion in section 2.4.7.2, JGLC outperforms the log-linear model and DFLC and has the capability of fitting real data, with returning the least error. Therefore, JGLC will be adopted in this study as the basic learning curve in order to model the inventory planning under learning with cognitive and motor abilities condition. For the sake of this purpose, JGLC will be extended to account for the assumptions of the mathematical models. In the next step, to develop the model to account for forgetting process, JGLC will be extended to develop a learning curve with learning, forgetting, cognitive and motor features. This would be achievable through integrating JGLC and LFCM.

3.1.3.5 Developing an optimization algorithm

Following the step of developing the models, the obtained total cost function will be optimized to derive the optimal inventory policies. In the optimization step of this study, the result of the developed model will be an unconstrained nonlinear programming with a continuous and an integer variables. The general form of an unconstrained nonlinear programming is as:

$$\min_{x \in \mathcal{R}^n} f(X)$$

Subject to

$$g_i(X) = 0 \qquad i = 1, 2, \dots, m_e$$

$$g_i(X) \leq 0 \qquad i = m_e + 1, \dots, m$$

$$X = x_1, x_2, \dots, x_n \tag{3.5}$$

where $f(X)$ or $g_i(X)$ are nonlinear functions. In a particular case, if m=0 the problem is called unconstrained nonlinear programming. Reviewing the literature illustrates that a variety of different algorithms and optimization techniques have been used and developed to solve unconstrained nonlinear programming problems some of which specialized for the models, whereas others are general type optimization techniques. In this context, analytical techniques and particularly derivation showed to be very prevalent methods in coping with optimization process. Besides being helpful in dealing with analytical or exact models, classical derivation falls short when it comes to solving large-scale and more complicated mathematical models. Therefore, researchers applied other types of solution approaches to cope with the more complex problems.

Concerning the optimization technique in this study, it is not possible to prove convexity of the model by applying derivation techniques due to complexity of the structure of the total cost function. Hence, an algorithm will be developed for optimizing the mathematical models and finding the optimal values through integrating the analytical method and a search technique. This algorithm is a prevalent way to analytically optimize a function with one continuous and one integer variables.

3.1.4 Data analysis

This section of the thesis is designed in such a way to answer the research questions and cover the objectives of the study. The type of data analysis that will be undertaken for this study includes both primary and secondary data analysis. The analysis section will be started with secondary data analysis and will be followed by primary data analysis. As stipulated by Vartanian (2010), in case the secondary data analysis is carried out carefully, it could help in acquiring broader insights into the research questions and in designing the subsequent primary data analysis in a better way. Secondary data analysis, in addition, could furnish a basis with which researchers can compare the results with that of primary data analysis. Another reason that secondary data analysis is adopted attributed to the fact that this study develops a model already available in the literature. Hence, it is logical that data analysis section starts up with comparing the developed model with that of the literature using the same data set, as applied in the literature. For this purpose, numerical analysis will be conducted using the same data set as utilized by Björk (2009) and compare the result of both studies. Doing so makes it possible to compare the models with and without learning and gain insights into how operator's learning changes the optimal policy of the inventory system.

In order to evaluate the model further, the second section of data analysis will be devoted to evaluate the model through primary data, which were collected from Renault Iran Company. This part is a complementary to the first step of data analysis and will allow to study the model features in-depth and draw concrete results. To implement data analysis, Microsoft Excel was used over the analysis process where all the formulas were written in different sheets and numerically tested by means of the data. The results of the numerical example were drawn and presented in tables and bar graphs.

3.2 Summary of the chapter

The main aim of this chapter was to explain the research methodology of this study and how it is set up for achieving the research objectives. In each step of this chapter, the characteristics of the study along with the adopted methodology are discussed, followed by justification on why the chosen method is the right choice for this study. Specifically, the qualitative and quantitative nature of this study is discussed and the methods adopted for data collection and data analysis are argued. The subsequent chapter furnishes more detail on data collection through the semi-structured interview process, the result of semi-structured interview process, formulation of the assumptions, and also development of the models.

CHAPTER 4 : METHODOLOGY

The aim of this chapter is to present fuzzy EOQ models with backorders and learning and forgetting effects on fuzzy parameters. The basic model of the study that investigated the effect of fuzzy lead times and demand on an EOQ-S models will be initially reviewed. Following this, the models under study in this thesis will then be presented. Development of the models will start with the semi-structured interview process, where the results of the interviews will aid the formulation of assumptions on the mathematical models. Next, four fuzzy EOQ-S models with learning and forgetting effects on fuzzy parameters will be developed and optimized to find the optimal policies.

4.1 Review of the basic model

4.1.1 The justification of imprecise lead times and demand by Björk (2009)

Björk (2009) did not explicitly state the justification on why demand and lead times are considered fuzzy in his study. However, his study was in line with a few studies conducted before, such as: Björk et al. (2004), Björk and Carlsson (2005a), Björk and Carlsson (2005b), Björk and Carlsson (2006), where the supply chain of the paper industry was investigated. In the following, the case studied will be reviewed to see why the lead times and demand are necessary to be taken imprecise into account in the developed models.

The studies derived from a larger project aiming at identifying the sources of the bullwhip effects in the supply chain of the forest products in Nordic countries. The cases were conducted on the supply chain of fine paper, including the manufacturer and the distributor, the supply chain of tissue paper, including the manufacturer, as well as the supply chain of the plywood, comprising of the plywood manufacturer. However, the focus was on the distribution side of the supply chain in the case study. The distribution side of the paper in the Nordic countries often faces the problem of confliction between customers and producers. Whereas, the producers are willing to produce larger batch sizes, due to the reduced production costs, customers, in contrast, prefer receiving the orders in smaller batches.

In this case, there are two different mechanisms through which the customers could order the required amounts of the paper. In the first way, the customers could send their orders in smaller quantities to the distributor, this entails the lead times being quite short, usually a couple of hours up to a day. While in the second case, the customer could send the order directly to the producer, this requires a greater lead time for the orders to be delivered. Besides variation of the lead times, the supply chain also suffers from the lack of sufficient information from the customer's side to forecast the demand. The studies show that although the supply chain benefits from a vertical integration, the producer, at the first stage of the chain, does not often receive the correct information since the information may often be distorted from the distributor and retailers' end. Furthermore, the facilities required for production in the paper industry are quite expensive, which requires an appropriate scheduling to utilize them in an efficient way. Therefore, it is of great importance to determine the batch size between producer and retailers considering the imprecise lead time and demand. The next section presents a brief review of the model of Björk (2009), which is an analytical model among the aforementioned studies. The following notations will be used throughout the paper.

D demand during planning period

M maximum inventory level

Q order quantity

R reorder point

L lead time

K	fixed ordering cost
h	holding cost per unit per planning period
p	shortage penalty cost per unit per planning period
n	number of orders per planning period
$\Delta_{\boldsymbol{l}}^{\boldsymbol{D}}$	lower fuzzy deviation value of demand
$\Delta_{\boldsymbol{h}}^{\boldsymbol{D}}$	upper fuzzy deviation value of demand
$\Delta_{\boldsymbol{l}}^{M}$	lower fuzzy deviation value of maximum inventory
$\Delta_{\boldsymbol{h}}^{M}$	upper fuzzy deviation value of maximum inventory
$\Delta_{\boldsymbol{l}}^{\boldsymbol{L}}$	lower fuzzy deviation value of lead time
$\Delta_{\boldsymbol{h}}^{\boldsymbol{L}}$	upper fuzzy deviation value of lead time
$d(\tilde{A}, \tilde{B})$	The distance between fuzzy numbers A and B
l	learning exponent of log-linear learning curve
l_c	learning exponent for cognitive task
l_m	learning exponent for motor task
S_i^c	the accumulated cognitive experience over i orders if the learning process was not interrupted
S_i^m	the accumulated motor experience over i orders if the learning process was not interrupted
u_i^c	the cognitive experience accumulated over i orders
u_i^m	the motor experience accumulated over i orders
f_i^c	the cognitive forgetting rate of the operator in ith order
f_i^m	the motor forgetting rate of the operator in ith order
i	order counter
T	planning period per unit time
$u_{x,i}^j$	the experience regarding jth parameter, $j = D, M$, accumulated over i orders

	$(x = l, h)$
$u_{j,i}^{c}$	the cognitive experience regarding jth parameter, $j = D, M$, accumulated over i orders
$u_{j,i}^{m}$	the motor experience regarding jth parameter, $j = D, M$, accumulated over i orders
$n_{x,i}^{j}$	time of the ith order for jth parameter $(x = l, h)$
$n_{j,i}^{c}$	time of the ith order (cognitive learning curve) for jth parameter $(x = l, h)$
$n_{j,i}^{m}$	time of the ith order (motor learning curve) for jth parameter $(x = l, h)$
$S_{x,i}^{j}$	the accumulated experience of jth parameter, $j = D, M$, in the time of ith order if the learning process was not interrupted $(x = l, h)$
$S_{x,i}^{j,c}$	the accumulated cognitive experience of jth parameter, $j = D, M$, in the time ith order if the learning process was not interrupted $(x = l, h)$
$S_{x,i}^{j,m}$	the accumulated motor experience of jth parameter, $j = D, M$, in ith order if the learning process was not interrupted $(x = l, h)$
$f_{x,i}^{j}$	forgetting rate of the operator for jth parameter, $j = D, M$, in ith order $(x = l, h)$
$f_{x,i}^{j,c}$	the cognitive forgetting rate of the operator for jth parameter, $j = D, M$, in ith order $(x = l, h)$
$f_{x,i}^{j,m}$	the motor forgetting rate of the operator for jth parameter, $j = D, M$, in ith order $(x = l, h)$
$t_{x,i}^{j}$	the performance of the jth parameter, $j = D, M$, at the time of ith order
φ_i	period of interruption
P_t	performance in tth unit of the time
P_1	performance at the beginning of the planning period
α	the weight representing the percentage that a task could be divided into

	cognitive and motor components
$\hat{T}_i$	Performance on the forgetting curve in ith order
$\hat{T}_{1,i}$	intercept of the forgetting curve
ϑ	the interruption time to which total forgetting takes place
$\Delta_{l,i}^{j}$	the lower fuzzy deviation value of ith order for $j = D, M$ in the planning horizon
$\Delta_{h,i}^{j}$	the upper fuzzy deviation value of ith order for $j = D, M$ in the planning horizon
$\Delta_{\boldsymbol{l},\mathbf{1}}^{\boldsymbol{j}}$	the first lower fuzzy deviation value for $j = D, M$ in the planning horizon
$\Delta_{\boldsymbol{h},\mathbf{1}}^{\boldsymbol{j}}$	the first upper fuzzy deviation value for $j = D, M$ in the planning horizon
TCU	crisp total cost function
$T\tilde{C}U$	fuzzified total cost function
TCU_F	fuzzy total cost function without learning
TCU_{FL_i}	fuzzy total cost function with learning for ith order
TCU_{FL}	fuzzy total cost function with learning for n orders

4.1.2 A brief review of the EOQ-S model with fuzzy demand and fuzzy lead times

Following his successive studies, Björk (2009) developed a real inventory planning problem at the distributors, considering uncertainty in the lead times and demand. The aim of the model was to assist decision makers at the distributors in determining the appropriate quantities of orders and the maximum inventory. To tackle uncertainty in lead times and demand, they were specified within intervals of which their values most probably vary over the planning horizon. The intervals were in turn formulated using triangular fuzzy numbers (see Appendix A for reviewing fuzzy numbers) as follows:

$$\tilde{L} = (L - \Delta_l^L, L, L + \Delta_h^L)$$

$\widetilde{M} = (M - \Delta_l^M, M, M + \Delta_h^M)$

$\widetilde{D} = (D - \Delta_l^D, D, D + \Delta_h^D)$

where $\Delta_l^M = \Delta_h^L D,\ \Delta_h^M = \Delta_l^L D,\ 0 < \Delta_l^i < i, i = L, D, M$ and $\Delta_h^i > 0, i = L, D, M$. Here Δ_l^i is the deviation from the base value on the negative side (lower deviation value) and Δ_h^i is the deviation from the base value on the positive side (upper deviation value). The $\Delta_h^i + \Delta_l^i$ illustrates the total deviation and is called the degree of fuzziness. It is necessary to note that the impact of uncertain lead times could not be instantly found in the formulation; however, due to the equation $M = R - LD + Q$, it affects the maximum inventory level in the formulation of the problem. Therefore, the maximum inventory level is taken fuzzy into account. Replacing fuzzy demand and fuzzy maximum inventory in Eq. (3.1), the total cost function of the model becomes:

$$T\tilde{C}U(Q, \widetilde{M}) = \frac{K\widetilde{D}}{Q} + \frac{\widetilde{M}^2 h}{2Q} + \frac{(Q - \widetilde{M})^2 p}{2Q} \tag{4.1}$$

For the purpose of deriving the equivalent crisp formulation of the total cost function, the Signed Distance method (see Appendix A) was applied. Using the signed distance method, the distance of TCU from 0 was computed as:

$$d(T\tilde{C}U, \tilde{0}) = \frac{K.d(\widetilde{D}, \tilde{0})}{Q} + \frac{d(\widetilde{M}^2, \tilde{0}).h}{2Q} + \frac{d((Q - \widetilde{M})^2, \tilde{0}).p}{2Q} \tag{4.2}$$

where according to the properties of the signed distance method (see appendix A) the distances in Eq. (4.2) are given by:

$$d(\widetilde{D}, \tilde{0}) = \frac{1}{4}\ [(D - \Delta_l^D) + 2D +\ (D + \Delta_h^D)] = D + \frac{1}{4}\Delta_h^D - \frac{1}{4}\Delta_l^D \tag{4.3}$$

$$d(\widetilde{M}^2, \tilde{0}) = \frac{1}{2}\int_0^1 [(M^2)_L(\alpha) + (M^2)_H(\alpha)]d\alpha =$$

$$\frac{1}{2}\int_0^1 [(M-\Delta_l^M+\Delta_l^M\alpha)^2+(M+\Delta_h^M-\Delta_h^M\alpha)^2]d\alpha = M^2-\frac{1}{2}M\Delta_l^M+\frac{1}{2}M\Delta_h^M$$

$$+\frac{1}{6}\Delta_l^{M^2}+\frac{1}{6}\Delta_h^{M^2} \tag{4.4}$$

$$d((Q-\widetilde{M})^2,\tilde{0})=\frac{1}{2}\int_0^1[(Q-M)_L^2(\alpha)+(Q-M)_H^2(\alpha)]d\alpha=$$

$$\frac{1}{2}\int_0^1[(Q-M+\Delta_l^M-\Delta_l^M\alpha)^2+(Q-M-\Delta_h^M+\Delta_h^M\alpha)^2]d\alpha=Q^2-2QM+M^2$$

$$-\frac{1}{2}M\Delta_l^M+\frac{1}{2}M\Delta_h^M+\frac{1}{2}Q\Delta_l^M-\frac{1}{2}Q\Delta_h^M+\frac{1}{6}\Delta_l^{M^2}\Delta_h^{M^2} \tag{4.5}$$

By replacing Eqs. (4.3), (4.4) and (4.5) into Eq. (4.2), the deffuzified total cost function of Eq. (4.1) can be calculated as:

$$TCU(Q,M)=\frac{KD}{Q}+\frac{K\Delta_h^D}{4Q}-\frac{K\Delta_l^D}{4Q}+\frac{M^2h}{2Q}+\frac{\Delta_l^{M^2}h}{12Q}+\frac{\Delta_h^{M^2}h}{12Q}+\frac{M^2p}{2Q}+\frac{\Delta_l^{M^2}p}{12Q}+\frac{Qp}{2}$$

$$+\frac{\Delta_h^{M^2}p}{12Q}+\frac{M\Delta_h^Mh}{4Q}-\frac{M\Delta_l^Mh}{4Q}+\frac{M\Delta_h^Mp}{4Q}-\frac{M\Delta_l^Mp}{4Q}+\frac{\Delta_l^Mp}{4}-\frac{\Delta_h^Mp}{4}-pM \tag{4.6}$$

In order to optimize the total cost function derived in Eq. (4.6), the convexity test should be initially implemented. A Hessian matrix was used to analytically prove the convexity of the total cost function. For this purpose, the derivatives of the total cost function with respect to the two variables should be initially calculated, which was given by:

$$\frac{\partial TCU(Q,M)}{\partial Q}=-\frac{KD}{Q^2}-\frac{K\Delta_h^D}{4Q^2}+\frac{K\Delta_l^D}{4Q^2}-\frac{M^2h}{2Q^2}-\frac{\Delta_l^{M^2}h}{12Q^2}-\frac{\Delta_h^{M^2}h}{12Q^2}-\frac{M^2p}{2Q^2}+\frac{p}{2}$$

$$-\frac{\Delta_l^{M^2}p}{12Q^2}-\frac{\Delta_h^{M^2}p}{12Q^2}-\frac{M\Delta_h^Mh}{4Q^2}+\frac{M\Delta_l^Mh}{4Q^2}-\frac{M\Delta_h^Mp}{4Q^2}+\frac{M\Delta_l^Mp}{4Q^2} \tag{4.7}$$

$$\frac{\partial TCU(Q,M)}{\partial M} = \frac{Mh}{Q} + \frac{Mp}{Q} + \frac{\Delta_h^M h}{4Q} - \frac{\Delta_l^M h}{4Q} + \frac{\Delta_h^M p}{4Q} - \frac{\Delta_l^M p}{4Q} - p \tag{4.8}$$

$$\frac{\partial^2 TCU(Q,M)}{\partial Q^2} = \frac{2KD}{Q^3} + \frac{K\Delta_h^D}{2Q^3} - \frac{K\Delta_l^D}{2Q^3} + \frac{M^2 h}{Q^3} + \frac{\Delta_l^{M^2} h}{6Q^3} + \frac{\Delta_h^{M^2} h}{6Q^3} + \frac{M^2 p}{Q^3} + \frac{\Delta_l^{M^2} p}{6Q^3}$$

$$+ \frac{\Delta_h^{M^2} p}{6Q^3} + \frac{M\Delta_h^M h}{2Q^3} - \frac{M\Delta_l^M h}{2Q^3} + \frac{M\Delta_h^M p}{2Q^3} - \frac{M\Delta_l^M p}{2Q^3} \tag{4.9}$$

$$\frac{\partial^2 TCU(Q,M)}{\partial Q^2} = \frac{2KD}{Q^3} + \frac{K\Delta_h^D}{2Q^3} - \frac{K\Delta_l^D}{2Q^3} + \frac{M^2 h}{Q^3} + \frac{\Delta_l^{M^2} h}{6Q^3} + \frac{\Delta_h^{M^2} h}{6Q^3} + \frac{M^2 p}{Q^3} + \frac{\Delta_l^{M^2} p}{6Q^3}$$

$$+ \frac{\Delta_h^{M^2} p}{6Q^3} + \frac{M\Delta_h^M h}{2Q^3} - \frac{M\Delta_l^M h}{2Q^3} + \frac{M\Delta_h^M p}{2Q^3} - \frac{M\Delta_l^M p}{2Q^3} \tag{4.10}$$

$$\frac{\partial^2 TCU(Q,M)}{\partial M^2} = \frac{h}{Q} + \frac{p}{Q} \tag{4.11}$$

$$\frac{\partial^2 TCU(Q,M)}{\partial M \partial Q} = -\frac{Mh}{Q^2} - \frac{Mp}{Q^2} - \frac{\Delta_h^M h}{4Q^2} + \frac{\Delta_l^M h}{4Q^2} - \frac{\Delta_h^M p}{4Q^2} + \frac{\Delta_l^M p}{4Q^2} \tag{4.12}$$

With the help of the above derivations, the hessian matrix were calculated as follows:

$$H = \begin{pmatrix} \frac{\partial^2 TCU(Q,M)}{\partial M^2} & \frac{\partial^2 TCU(Q,M)}{\partial M \partial Q} \\ \frac{\partial^2 TCU(Q,M)}{\partial M \partial Q} & \frac{\partial^2 TCU(Q,M)}{\partial Q^2} \end{pmatrix} \tag{4.13}$$

$$Det(H) = \frac{h+p}{2} \begin{pmatrix} \frac{2KD}{Q^3} + \frac{K\Delta_h^D}{2Q^3} - \frac{K\Delta_l^D}{2Q^3} + \frac{M^2 h}{Q^3} + \frac{\Delta_l^{M^2} h}{6Q^3} + \frac{\Delta_h^{M^2} h}{6Q^3} + \frac{M^2 p}{Q^3} \\ + \frac{\Delta_l^{M^2} p}{6Q^3} + \frac{\Delta_h^{M^2} p}{6Q^3} + \frac{M\Delta_h^M h}{2Q^3} - \frac{M\Delta_l^M h}{2Q^3} + \frac{M\Delta_h^M p}{2Q^3} - \frac{M\Delta_l^M p}{2Q^3} \end{pmatrix}$$

$$-\left(-\frac{Mh}{Q^2} - \frac{Mp}{Q^2} - \frac{\Delta_h^M h}{4Q^2} + \frac{\Delta_l^M h}{4Q^2} - \frac{\Delta_h^M p}{4Q^2} + \frac{\Delta_l^M p}{4Q^2}\right)^2 \tag{4.14}$$

After computing the determinant in Eq. (4.14) and some modifications, the author proved that the derived term is strictly positive and consequently Eq. (4.6) is strictly convex in Q. Therefore, the closed-form solution for the optimal policy of the model was obtained according to the following expression:

$$Q^* = \begin{pmatrix} \frac{2KD}{p} + \frac{2KD}{h} + \frac{K\Delta_h^D}{2p} + \frac{K\Delta_h^D}{2h} - \frac{K\Delta_l^D}{2p} - \frac{K\Delta_l^D}{2h} + \frac{5\Delta_h^{M^2}h}{48p} + \frac{5\Delta_h^{M^2}}{24} \\ + \frac{5\Delta_h^{M^2}p}{48h} + \frac{5\Delta_l^{M^2}h}{48p} + \frac{5\Delta_l^{M^2}}{24} + \frac{5\Delta_l^{M^2}p}{48h} + \frac{6\Delta_h^M\Delta_l^M h}{48p} + \frac{6\Delta_h^M\Delta_l^M}{24} \\ + \frac{6\Delta_h^M\Delta_l^M p}{48h} \end{pmatrix}^{0.5} \tag{4.15}$$

$$M^* = \frac{1}{4}(\Delta_l^M - \Delta_h^M)$$

$$+ \frac{p}{h+p}\begin{pmatrix} \frac{2KD}{p} + \frac{2KD}{h} + \frac{K\Delta_h^D}{2p} + \frac{K\Delta_h^D}{2h} - \frac{K\Delta_l^D}{2p} - \frac{K\Delta_l^D}{2h} + \frac{5\Delta_h^{M^2}h}{48p} + \frac{5\Delta_h^{M^2}}{24} \\ + \frac{5\Delta_h^{M^2}p}{48h} + \frac{5\Delta_l^{M^2}h}{48p} + \frac{5\Delta_l^{M^2}}{24} + \frac{5\Delta_l^{M^2}p}{48h} + \frac{6\Delta_h^M\Delta_l^M h}{48p} + \frac{6\Delta_h^M\Delta_l^M}{24} \\ + \frac{6\Delta_h^M\Delta_l^M p}{48h} \end{pmatrix}^{0.5} \tag{4.16}$$

For the sake of brevity, it is avoided discussing the basic model in details, but only the fundamental elements required for better understanding of the developed models are discussed above. For more discussion, the readers are referred to Björk (2009).

4.2 Development of the models

4.2.1 Learning and learning transfer in inventory management with imprecise parameters: An empirical observation

In Chapter two, it was widely discussed why human learning is relevant in inventory management under uncertainty by reviewing the literature and providing some theoretical foundations. As evident from the literature, the available studies on fuzzy inventory models lack the empirical evidence on the existence of human learning and its transfer in inventory planning. Even though the impact of learning and learning transfer has been ignored in the pertaining studies thus far, the researchers and industrial practitioners are well aware that learning by doing is entirely common in practice. As it is obvious, since learning process in an uncertain environment is highly dependent on the state of the knowledge of the operator, gathering data in order to process human learning is not applicable in this research. However, semi-structured interviews are suited as the required information is qualitative and cannot immediately be accessed or is not easily perceptible. As described in the previous Chapter, semi-structured interviews permit interviewees to not solely limit themselves to the questions that are being asked by the interviewer, but also describe their own thoughts and views according to their experiences (Roulston, 2010; Patton, 2005). This ensures that undiscovered facts, which might be hidden from the researcher, can be prospected (Thomas, 2006). It is necessary to note that the interviews conducted for this study do not aim at finding a sample from a population, instead trying to support the development of the analytical model (Grosse & Glock, 2014).

Hence, in order to build a solid foundation for the mathematical models in this study, a set of semi-structured interviews with industrial staffs from different companies in Malaysia and Iran was conducted, which helped to gain insights into how human learning is observed in an uncertain inventory planning.

4.2.2 Interviews

The procedure for the interviews adopted for this study is according to standardized semi-structured interview procedure and is similar to that of DiCicco-Bloom and Crabtree (2006), Whiting (2008) and Grosse and Glock (2014). Following the standard procedure, the interviews extended across the sample industrial companies in Malaysia and Iran, where they were selected according to the relevance of their operations to this study. The selection process of companies was implemented through the review of companies' websites, their profiles and magazines. In particular, the sample companies were selected as to whether they implement inventory management system and indeed whether they have at least one expert devoted to the inventory operation having enough experience. The experts selected for the interview were mainly from Industrial Engineering, Supply Chain, Logistics or Operations Management departments of the sample companies, who their responsibilities were considerably concerned with material requirement planning or inventory management/planning. In cases where it was discovered that the candidate expert had recently been assigned to the position and did not have enough experience with the position, it was tried to find an individual who held the position formerly, if the person was still working with the company. Besides encompassing a number of experts in the interviews, it was tried to set up an interview (wherever was possible) with the direct manager of the experts, who supervise and monitor their performance. This ensured that the gained information through the answers was impartial. After identifying the target companies and the sample of the experts, discussion topics and a set of questions for the interviews were worked out. Each interviewee was initially contacted by providing a thorough explanation about the objective of the study and their role as an expert, and further they were informed about how the interview process will be conducted and that how their information will be used in the project. Those experts who were available and consented to take part in the interview were reached through telephone, Skype or in person meet up, whatever was possible.

Of various experts and their managers who were contacted, six individuals, including four experts and two managers from four different companies, accepted to take part in the interview. Table 4.1 shows the details of the companies and the experts. The experts and managers were reached by interview, and each interview lasted 30 to 45 minutes, including the field observation in two cases. All interviewees were treated the same by asking open-ended questions to keep the interview process away from any prejudice and to compose a conversation that supported unpremeditated and lengthy responses (Roulston, 2010). When it was clear that there was not mutual understanding or the expert trying to give a short answer, the questions were phrased in a way that the detailed answer of the experts could be accessible. During the interview, the experts were also allowed to cease the interview or criticize the question by giving their own opinions, if they were not agreed with the type or structure of the question. In Appendix B, the summary of the interview with one of the experts is provided.

Table 4.1: The details of the interviewed experts and their companies

Company	Location	Core business	Size	Number of the interviewed experts	Position of the expert
KTL Sdn. Bhd.	Malaysia	Electrical products	<1000 personnel	2	Operations manager and inventory executive
Renault Iran	Iran	Automobile parts	>1000 personnel	2	Logistic Manager and MRP executive
Iranmed	Iran	Hygienic products	<1000 personnel	1	Inventory planner
Elite Traffic	Iran	Logistics and transportation	<1000 personnel	1	Logistics executive

With emphasize on the responses given by the interviewees, the key points of each interview were recorded through taking notes. Once the interview process had been completed, the notes were attentively reviewed and transcribed on word-processed document, which led to summarizing the main themes that emerged from the interview (Miles & Huberman, 1994). The final transcripts, which were fertile and contextualized texts, were analyzed to identify the common ideas expressed by interviewees and to classify them.

After consolidating the results, the following propositions were concluded:

Proposition 1. *Uncertainty in planning decreases as a result of learning.*

The first and foremost fact identified is about the effect of learning on imprecision in inventory planning. All experts jointly expressed that the estimation of planning parameters, i.e. demand, lead-times, safety stock, order quantity, could be further enhanced over time as they increase their experience with the planning case. They insinuated several instances in which learning-based improvement can help them to make a more precise estimation. Working with a new supplier is the most common case that was emphasized by the experts, particularly in conditions in which the supplier is not local. When a new supplier is added to their supplier panel, they typically consider a larger estimation for inventory parameters to avoid any service interruption. They also benchmark, if possible, the case of the new supplier with the similar cases in the past to achieve a rough estimation about the items. As their experience with supplier heighten over time, they are able to revise the planning parameters to achieve a more precise estimation.

Proposition 2. *Operators with more experience are more dominant in inventory planning under uncertainty than operators with less or no experience.*

The interviewees pointed out that experience is an important criterion in their job, and usually more experienced experts have more knowledge about planning stuffs. According to their statements, those staffs who are new on the job usually have to invest more time to get to know their duties, the suppliers who they are dealing with and the planning tasks. The experiences they gain over time help them in doing planning stuff more easily and precisely.

Proposition 3. *The acquired experience is helpful and used when planning inventory.*

The experts revealed it is prevalent that they employ their or other staff's experience in planning inventories. During operations, they usually need to make regular internal contacts (for example, with other departments within company) and/or external ones (like supplier and third party logistics) to gather information about the latest status and then replenish and manage inventories. When doing so, they utilize their experience gained before to manage the operation properly.

Proposition 4. *The acquired experience might be depreciated sometimes.*

The experts signalized that their performance when they are dealing with the task regularly is better than their performance when the planning is an intermittent case. According to their declarations, when planning inventory for an item is their day to day operation, they are more acquainted with the planning tasks and parameter estimation; however, when the planning is interrupted and they are away for a long period, they believe they are not much familiar as compared to their daily operations. As noted, this mostly happens in new projects, where inventory planning is not a daily task, or in multi-source cases where a supplier is only referred to for an emergency supply.

In a nutshell, the result of interviews along with the observation of the cases illustrated the high relevance of learning in inventory operations under uncertainty. Relying on this fact, it is obvious that human learning should be considered in developing the mathematical models under uncertainty. In view of this finding, in the next section, mathematical models will be developed that consider human learning and forgetting in estimation of uncertain parameters.

4.2.3 The fuzzy EOQ-S model with learning effect on fuzzy parameters and full transfer of learning: the case of log-linear learning curve

In this section, the model developed by Björk (2009) is extended to account for human learning in fuzzy parameters and full transfer of learning over the cycles. To do so, the total cost function developed by Björk (2009), Eq. (4.6), will be the starting point of the models and will be developed in each section according to the assumptions of the model. For this purpose, the concept of learning and forgetting theory discussed in Section 2.4.7 will be used throughout the development of the models.

In addition to the assumptions stated for the EOQ model with backorders (see Section 3.1.3.1), the following assumptions are additionally made for developing the models of this study. These assumptions are derived on the basis of the propositions expressed in the previous section. These assumptions will be used throughout the study unless their modifications or extra assumptions are pointed out in the related section.

Assumptions

- Learning occurs with every order and all the related inventory information is influenced by increasing the number of orders.
- The knowledge attained from the previous stages affects the adjustment of the fuzzy parameters in the latter stages.
- The learning will result in reduction in fuzziness.
- The learning pattern for all fuzzy parameters is identical.
- All the fuzzy parameters are modified with the same learning rate.
- In this model, forgetting does not impede learning. That is, the operator does not forget the information gained from the earlier planning stages.

The first and third assumptions are made based on proposition 1, as it is understood that uncertain parameters are affected by learning and are decreased over time. As the first assumption additionally shows, it is assumed that the operator learns with every order. Hence, based on this assumption the operator could gain more familiarity with the fuzzy parameters by gathering more experience. The second assumption is formulated according to proposition 3, since it is expressed that the attained knowledge by the operator is used in the next planning stages. The fourth and fifth assumptions are made for the simplicity of the mathematical models. Finally, the last assumption limits the model of this section to solely consider learning in planning (without forgetting).

According to the assumptions, the model resembles a situation where the operator plans inventories for the subsequent cycle at the beginning of every cycle. Hence, the operator could be able to adjust parameters more precisely when he/she plans for the subsequent stages based on what he/she learnt from the previous planning stages. Since it is assume that learning occurs with the number of orders and not with the size of the order, therefore the total cost of the fuzzy EOQ-S model in Eq. (4.16) should be modified to account for the number of orders. By this modification, the EOQ-S model will in turn become an inventory model with variable number of orders and maximum inventory level. Considering n orders placed per planning horizon, the total cost function of the fuzzy EOQ-S model in Eq. (4.16) could be rewritten as below:

$$TCU(n,M) = nK - pM + \frac{Dp}{2n} + \frac{nM^2h}{2D} + \frac{nM^2p}{2D} + \frac{nK\Delta_h^D}{4D} - \frac{nK\Delta_l^D}{4D} + \frac{n\Delta_l^{M^2}h}{12D} + \frac{n\Delta_h^{M^2}h}{12D}$$

$$+ \frac{n\Delta_l^{M^2}p}{12D} + \frac{n\Delta_h^{M^2}p}{12D} + \frac{nM\Delta_h^M h}{4D} - \frac{nM\Delta_l^M h}{4D} + \frac{nM\Delta_h^M p}{4D} - \frac{nM\Delta_l^M p}{4D} + \frac{\Delta_l^M p}{4} - \frac{\Delta_h^M p}{4} \quad (4.17)$$

In order to integrate the effect of the operator's learning into the fuzzy model, an appropriate learning curve needs to be developed first to reflect the learning process of the operator. To develop a suitable learning curve, the modified version of the log-linear learning curve is presented and used, which is adapted to the problem defined in this paper. The reason why the log-linear learning curve is adopted in this study is thoroughly explained in Section 3.1.3.4. Therefore, the log-linear learning curve is modified to adopt it to this study.

It is apparent that adjusting the value of the fuzzy parameters in every planning cycle depends upon the knowledge of the operator, who learns over time. As discussed before, the operator learns in conformance with a learning curve. On the other hand, referring to the formula of the long-linear learning curve, the initial value of the fuzzy parameter must be multiplied by the learning curve of the operator up to that stage. Since in the EOQ-S models, the orders receive in every $\frac{T}{n}$ unit of time (T is the duration of planning period), subsequently the accumulated experience of the operator up to the $(i-1)$th order at the time of the ith order can be formulated as $\left[(i-1)\frac{T}{n}\right]^{-l}$. Therefore, for $j = L, D, M$ the value of jth lower and upper deviation values at the time of ith order will be as:

$$\Delta_{l,i}^{j} = \begin{cases} \Delta_{l,1}^{j} & i = 1 \\ \Delta_{l,1}^{j} \left[(i-1)\dfrac{T}{n}\right]^{-l} & i > 1 \end{cases} \tag{4.18}$$

$$\Delta_{h,i}^{j} = \begin{cases} \Delta_{h,1}^{j} & i = 1 \\ \Delta_{h,1}^{j} \left[(i-1)\dfrac{T}{n}\right]^{-l} & i > 1 \end{cases} \tag{4.19}$$

where $\Delta_{l,1}^{j}$and $\Delta_{h,1}^{j}$, $j = L, D, M,$ are the initial lower and upper deviation values for the fuzzy inventory parameters. Eqs. (4.18) and (4.19) show that for the first order, no learning occurs and the fuzzy model adopts the initial estimated values. Beginning from the second up to the nth order, the operator starts to learn, which allows deviation values to drop according to his/her learning rate. If all the deviation values change by the same learning rate, the expected total cost for order $1 < i \leq n, n \geq 2$ are given as:

$$TCU_{FL_i}(n,M) = K - \frac{pM}{n} + \frac{Dp}{2n^2} + \frac{M^2h}{2D} + \frac{M^2p}{2D} + \frac{K}{4D}\left[(i-1)\frac{T}{n}\right]^{-l}$$

$$-\frac{K}{4D}\Delta_{l,1}^{D}\left[(i-1)\frac{T}{n}\right]^{-l} + \frac{h}{12D}\left(\Delta_{l,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l}\right)^2 + \frac{h}{12D}\left(\Delta_{h,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l}\right)^2$$

$$+\frac{p}{12D}\left(\Delta_{l,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l}\right)^2 + \frac{p}{12D}\left(\Delta_{h,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l}\right)^2 + \frac{Mh}{4D}\Delta_{h,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l}$$

$$-\frac{Mh}{4D}\Delta_{l,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l} + \frac{Mp}{4D}\Delta_{h,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l} - \frac{Mp}{4D}\Delta_{l,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l}$$

$$+\frac{p}{4n}\Delta_{l,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l} - \frac{p}{4n}\Delta_{h,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l} \tag{4.20}$$

The expected total cost for n orders is the summation of costs for the first order up to nth order ($n \geq 2$), which after summation would be as below:

$$TCU_{FL}(n,M) = nK - pM + \frac{Dp}{2n} + \frac{nM^2h}{2D} + \frac{nM^2p}{2D}$$

$$+\frac{K}{4D}\left(\Delta_{h,1}^{D} - \Delta_{l,1}^{D}\right)\left[1 + \sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l}\right] + \frac{h}{12D}\left(\Delta_{h,1}^{M^2} + \Delta_{l,1}^{M^2}\right)\left[1 + \sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-2l}\right]$$

$$+\frac{p}{12D}\left(\Delta_{h,1}^{M^2} + \Delta_{l,1}^{M^2}\right)\left[1 + \sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-2l}\right] + \frac{Mh}{4D}\left(\Delta_{h,1}^{M} - \Delta_{l,1}^{M}\right)\left[1 + \sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l}\right]$$

$$+\frac{Mp}{4D}\left(\Delta_{h,1}^{M} - \Delta_{l,1}^{M}\right)\left[1 + \sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l}\right] + \frac{p}{4n}\left(\Delta_{l,1}^{M} - \Delta_{h,1}^{M}\right)\left[1 + \sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l}\right] \tag{4.21}$$

After rearrangement and further computations, the total cost function of the EOQ-S model with human learning and full transfer of learning will be given by:

$$TCU_{FL}(n,M) = nK - pM + \frac{Dp}{2n} + \frac{nM^2(h+p)}{2D}$$

$$+\frac{1}{4D}\left[K\left(\Delta_{h,1}^{D}-\Delta_{l,1}^{D}\right)+(\frac{nM(h+p)-Dp}{n})\left(\Delta_{h,1}^{M}-\Delta_{l,1}^{M}\right)\right]\left[1+\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l}\right]$$

$$+\left(\frac{h+p}{12D}\right)\left(\Delta_{h,1}^{M^2}+\Delta_{l,1}^{M^2}\right)\left[1+\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-2l}\right] \tag{4.22}$$

Prior to obtaining the optimal solution for the model, the convexity of the total cost function with respect to decision variables should be tested. Since variable n appears in the upper limit of the summations, it is very difficult to analytically prove that Eq. (4.22) is convex in both n and M. However, since n only adopts integer values, examining the convexity in one variable helps finding an optimal solution using an iterative algorithm. The following Lemma is helpful in finding the optimal solution.

Lemma 1. Finding a global optimal solution for $TCU_{FL}(n,M)$ can be reduced to search for finding a local optimal solution for n.

Proof: To prove Lemma 1, it is required to show that $TCU_{FL}(n,M)$ is convex in M. For a given value of n, the first and second order partial derivative of Eq. (4.22) with respect to M are given by:

$$\frac{\partial TCU_{FL}(n,M)}{\partial M}=\frac{nMh}{D}+\frac{nMp}{D}-p$$

$$+\left[1+\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l}\right]\left[\frac{h}{4D}\left(\Delta_{h,1}^{M}-\Delta_{l,1}^{M}\right)+\frac{p}{4D}\left(\Delta_{h,1}^{M}-\Delta_{l,1}^{M}\right)\right] \tag{4.23}$$

and

$$(\partial^2/\partial M^2)TCU_{FL}(n,M)=\frac{nh}{D}+\frac{np}{D} \tag{4.24}$$

Eq. (4.24) is obviously greater than zero for every value that n,h,p,D adopts. This is the sufficient condition for convexity and implies that $TCU_{FL}(n,M)$ is convex in M.

This indicates that for a fixed value of n, there exists a unique value of M that minimizes $TCU_{FL}(n, M)$. Therefore, to find the optimal maximum inventory, its first derivative with respect to n is equated to zero, which gives the following result:

$$M^* = \frac{pD - \left(1 + \sum_{i=2}^{n}[(i-1)\frac{T}{n}]^{-l}\right)\left[\frac{h}{4}\left(\Delta_{h,1}^{M} - \Delta_{l,1}^{M}\right) + \frac{p}{4}\left(\Delta_{h,1}^{M} - \Delta_{l,1}^{M}\right)\right]}{n^*(h+p)} \tag{4.25}$$

In order to find the optimal total annual cost and optimal number of orders from Eqs. (4.22) and (4.25), the following linear search is applied to find a unique n over the interval [2,∞) such that the inequality $TCU_{FL}(n-1, M) \geq TCU_{FL}(n, M) \leq TCU_{FL}(n+1, M)$ is satisfied. Although the aforementioned steps taken in finding the optimal solution for the problem developed are straightforward, it is preferred to provide a step-by-step algorithm that aids in better understanding the solution procedure. The steps of algorithm are given as:

1- Start the algorithm by setting n =2.

2- Insert the value of n in Eq. (4.25) and obtain $M_{(n)}$.

3- Compute $TCU_{FL}(n, M_{(n)})$ in Eq. (4.22).

4- If $TCU_{FL}(n, M_{(n)}) < TCU_{FL}(n-1, M_{(n)})$ then set $n = n+1$ and go to step 2, otherwise go to the next step

5- Record $n^* = n-1$ and $M^* = M_{(n-1)}$ and compute $TCU^*{}_{FL}(n^*, M^*)$.

The algorithm's loop examines the successive values of n over the interval [2,∞) to identify for which n total cost function changes from descending to ascending. As soon as the total cost function starts to go up for a specific value of n, this value will be recorded as the optimal one, and then optimal solution will be derived by substituting the optimal value of n and M in TCU.

4.2.4 The fuzzy EOQ-S model with forgetting effect on fuzzy parameters: the case of log-linear learning curve

In previous sections, it is discussed that being involved in gathering and processing information regarding the imprecise input parameters enables the inventory operator to learn over time and thus to improve his/her performance in parameter estimation. However, this is not the only scenario that happens in practice. There are other situations that affect the performance of the inventory operator in setting the imprecise parameters. Even though the inventory operator accumulates experience about the imprecise parameters over time, the obtained expertise may subject to depreciation, for instance, due to the break in the process or being away from the task for a period of time. It is clear that while the performance of the inventory operator improves as the operator learns, it aggravates while the operator forgets a part of the experience. Therefore, it is necessary to extend the model with full transfer of learning to a case with the forgetting effect in order to account for the detrimental impact of forgetting on operator performance and the inventory system as a result.

To formulate a mathematical model for the fuzzy EOQ-S model with forgetting/knowledge depreciation effect on setting the fuzzy parameters, the assumption of full transfer of learning is countered in the model developed in Section 4.2.3 by assuming that the experience of the operator transfers partially because of the loss of knowledge. To do so, the LFCM described in Section 2.4.8.1 is incorporated into the model. It is required to note that other assumptions stated for the models in Section 4.2.3 are true and will be applied in this section. The LFCM was proved to appropriately capture the characteristics of learning and forgetting process (Jaber & Bonney, 1996; Jaber & Bonney, 1997; Jaber & Bonney, 2003), was shown to fit experimental data very well (Sikström & Jaber, 2002), and was illustrated to be easy to apply and flexible in mathematics (Jaber & Guiffrida, 2004). Therefore, the LFCM is adopted in this section to formulate learning and forgetting process of the operator. The same scenario as given in Section 4.2.3 is considered for the EOQ-S model with fuzzy parameters. That is, an inventory system includes n orders, which are placed over the planning horizon. Due to uncertainty over the planning horizon, the operator prefers to specify lower and upper intervals for lead times and demand, which are consequently defined as fuzzy parameters. The operator determines the boundaries at the beginning of every planning cycle based on the experience gained from the prior cycles. However, the experienced gained by the operator is depreciated since the operator has been away from the task for a specific period of time. The experience here is measured as the time that the operator can remember from the initial stage of the planning. If the experience of the operator is affected by forgetting, then the value of jth interval parameter at the time of ith order is defined be as follows:

$$\Delta_{l,i}^{j}=\begin{cases}\Delta_{l,1}^{j} & i=1\\ \Delta_{l,1}^{j}(u_{l,i}^{j}+n_{l,i}^{j})^{-l} & i>1\end{cases} \tag{4.26}$$

$$\Delta_{h,i}^{j}=\begin{cases}\Delta_{h,1}^{j} & i=1\\ \Delta_{h,1}^{j}(u_{h,i}^{j}+n_{h,i}^{j})^{-l} & i>1\end{cases} \tag{4.27}$$

Through comparing Eqs. (4.26) and (4.27) with Eqs. (4.18) and (4.19), it can be found that the experience transferred over the cycles for the forgetting case is $u_{x,i}^{j}+n_{x,i}^{j}$, $x=l,h$, which is replaced by $[(i-1)\frac{T}{n}]^{-l}$ in Eqs. (4.18) and (4.19). Here, $u_{x,i}^{j}$, $x=l,h$, is the accumulated experience over $i-1$ cycles for jth parameter, $j=L,D,M$, that the operator remembers at the time of ith repetition/order, and $n_{x,i}^{j}, x=l,h,$ is the time of ith cycle. In order to calculate the experience transferring between cycles in Eqs. (4.26) and (4.27), the LFCM is modified to adapt to the problem defined in this study. Accordingly, for $x=l,h$ and $j=L,D,M$, the experience for jth parameter that would have been accumulated if the learning process was not interrupted can be calculated as:

$$S_{x,i}^{j} = \left\{\frac{1-l}{\Delta_{x,1}^{j}}\left[t_x^{j}\left(u_{x,i}^{j}+n_{x,i}^{j}\right)+\varphi_i\right]\right\}^{\frac{1}{1-l}} \tag{4.28}$$

where

$$t_x^{j}\left(u_{x,i}^{j}+n_{x,i}^{j}\right) = \sum_{i=1}^{u_{x,i}^{j}+n_{x,i}^{j}} \Delta_{x,1}^{j}\left[(i-1)\frac{T}{n}\right]^{-l} \cong \int_{1}^{u_{x,i}^{j}+n_{x,i}^{j}} \Delta_{x,1}^{j}\left[(i-1)\frac{T}{n}\right]^{-l} di$$

$$= \Delta_{x,1}^{j}\left(\frac{T}{n}\right)^{-l}\left[\left(u_{x,i}^{j}+n_{x,i}^{j}-1\right)^{1-l}-(-1)^{1-l}\right]/1-l \tag{4.29}$$

Following the definition of LFCM and Eqs. (4.28) and (4.29), the modified forgetting rate and the accumulated experience for jth parameter are given as:

$$f_{x,i}^{j} = \frac{l(1-l)\log(u_{x,i}^{j}+n_{x,i}^{j})}{\log\left\{1+\left(v(1-l)n^{-l}/\Delta_{x,1}^{j}T^{-l}\left[\left(u_{x,i}^{j}+n_{x,i}^{j}-1\right)^{1-l}-(-1)^{1-l}\right]\right)\right\}} \tag{4.30}$$

and

$$u_{x,i+1}^{j} = (u_{x,i}^{j}+n_{x,i}^{j})^{(1+f_{x,i}^{j}/l)} S_{x,i}^{j\ -\frac{f_{x,i}^{j}}{l}} \tag{4.31}$$

Recalling the total cost function developed in Eq. (4.17) and considering the definition of Δ parameters in Eqs. (4.26) and (4.27), the total cost function of the fuzzy EOQ-S model in forgetting situation for the first cycle is given by:

$$TCU_{FL_1}(n,M) = K+\frac{pD}{2n^2}-\frac{pM}{n}+\frac{M^2h}{2D}+\frac{M^2p}{2D}+\frac{nK\Delta_{h,1}^{D}}{4D}-\frac{nK\Delta_{l,1}^{D}}{4D}-\frac{p\Delta_{h,1}^{M}}{4}$$

$$+\frac{nh\Delta_{l,1}^{M^2}}{12D}+\frac{p\Delta_{l,1}^{M}}{4}+\frac{nh\Delta_{h,1}^{M^2}}{12D}+\frac{np\Delta_{l,1}^{M^2}}{12D}+\frac{np\Delta_{h,1}^{M^2}}{12D}+\frac{nMh\Delta_{h,1}^{M}}{4D}-\frac{nMh\Delta_{l,1}^{M}}{4D} \tag{4.32}$$

$$+\frac{nMp\Delta_{h,1}^{M}}{4D}-\frac{nMp\Delta_{l,1}^{M}}{4D}$$

Accordingly, following the developed Δ functions in Eqs. (4.26) and (4.27), the cost function of the inventory system for ith cycle, $i \in [2, n]$, is defined as:

$$TCU_{FL_i}(n, M) = K + \frac{pD}{2n^2} - \frac{pM}{n} + \frac{M^2 h}{2D} + \frac{M^2 p}{2D} + \frac{K}{4D}\Delta_{h,1}^{D}\left(u_{h,i}^{D} + n_{h,i}^{D}\right)^{-l}$$

$$-\frac{K}{4D}\Delta_{l,1}^{D}\left(u_{l,i}^{D} + n_{l,i}^{D}\right)^{-l} + \frac{h}{12D}\Delta_{l,1}^{M^2}\left(u_{l,i}^{M} + n_{l,i}^{M}\right)^{-2l} + \frac{h}{12D}\Delta_{h,1}^{M^2}\left(u_{h,i}^{M} + n_{h,i}^{M}\right)^{-2l}$$

$$+\frac{p}{12D}\Delta_{l,1}^{M^2}\left(u_{l,i}^{M} + n_{l,i}^{M}\right)^{-2l} + \frac{p}{12D}\Delta_{h,1}^{M^2}\left(u_{h,i}^{M} + n_{h,i}^{M}\right)^{-2l} + \frac{Mh}{4D}\Delta_{h,1}^{M}\left(u_{h,i}^{M} + n_{h,i}^{M}\right)^{-l}$$

$$-\frac{Mh}{4D}\Delta_{l,1}^{M}\left(u_{l,i}^{M} + n_{l,i}^{M}\right)^{-l} + \frac{Mp}{4D}\Delta_{h,1}^{M}\left(u_{h,i}^{M} + n_{h,i}^{M}\right)^{-l} - \frac{Mp}{4D}\Delta_{l,1}^{M}\left(u_{l,i}^{M} + n_{l,i}^{M}\right)^{-l}$$

$$+\frac{p}{4}\Delta_{l,1}^{M}\left(u_{l,i}^{M} + n_{l,i}^{M}\right)^{-l} - \frac{p}{4}\Delta_{h,1}^{M}\left(u_{h,i}^{M} + n_{h,i}^{M}\right)^{-l} \tag{4.33}$$

The total cost function of the inventory system is derived by summing up the cost functions over n cycles, which the result will be as:

$$TCU_{FL}(n, M) = \sum_{i=1}^{n} TCU_{FL_i}(n, M) = TCU_{FL_1}(n, M) + \sum_{i=2}^{n} TCU_{FL_i}(n, M)$$

$$= nK - pM + \frac{pD}{2n} + \frac{nhM^2}{2D} + \frac{npM^2}{2D} + \frac{K}{4D}\Delta_{h,1}^{D}\left[1 + \sum_{i=2}^{n}(u_{h,i}^{D} + n_{h,i}^{D})^{-l}\right]$$

$$-\frac{K}{4D}\Delta_{l,1}^{D}\left[1 + \sum_{i=2}^{n}(u_{l,i}^{D} + n_{l,i}^{D})^{-l}\right] + \frac{\Delta_{l,1}^{M^2}}{12D}(h + p)\left[1 + \sum_{i=2}^{n}(u_{l,i}^{M} + n_{l,i}^{M})^{-2l}\right]$$

$$+\frac{\Delta_{h,1}^{M^2}}{12D}(h + p)\left[1 + \sum_{i=2}^{n}(u_{h,i}^{M} + n_{h,i}^{M})^{-2l}\right] + \Delta_{h,1}^{M}\left(\frac{Mh}{4D} + \frac{Mp}{4D} - \frac{p}{4n}\right)\left[1 + \sum_{i=2}^{n}(u_{h,i}^{M} + n_{h,i}^{M})^{-l}\right]$$

$$-\Delta_{l,1}^{M}\left(\frac{Mh}{4D}+\frac{Mp}{4D}-\frac{p}{4n}\right)\left[1+\sum_{i=2}^{n}(u_{l,i}^{M}+n_{l,i}^{M})^{-l}\right] \tag{4.34}$$

The same optimization procedure as the one presented for the function in Eq. (4.22) could be adopted here. Similarly, due to complexity of the total cost function in Eq. (4.34), it is not possible to prove convexity of the function analytically, for example with the help of Hessian matrix. However, it is possible to show that finding a global optimal solution for $TCU_{FL}(n,M)$ in Eq. (4.34) is equal to search for finding a local optimal solution for n. To do so, it is necessary to show that $TCU_{FL}(n,M)$ is a convex function in M. Taking the second order derivative of Eq. (4.34) with respect to M gives $n(h+p)/D$, which is greater than zero for a given n,h,p and D. For a given n, the optimal solution for M is derived by setting $\frac{\partial TCU_{FL}(n,M)}{\partial M}$ to zero, which gives:

$$M=\frac{pD-0.25(h+p)\begin{pmatrix}\Delta_{h,1}^{M}\left[1+\sum_{i=2}^{n}(u_{h,i}^{M}+n_{h,i}^{M})^{-l}\right]\\-\Delta_{l,1}^{M}\left[1+\sum_{i=2}^{n}(u_{l,i}^{M}+n_{l,i}^{M})^{-l}\right]\end{pmatrix}}{n(h+p)} \tag{4.35}$$

In order to find the optimal value of n, the same iterative algorithm as given for the first model can be applied here. Therefore, readers are referred to the optimization algorithm for the model in Section 4.2.3.

4.2.5 The fuzzy EOQ-S model with learning effect on fuzzy parameters and full transfer of learning: the case of two-stage learning curve

In two previous sections, learning effect is investigated in inventory management problem under fuzzy parameters so that the learning occurs in its simple form, which is one stage and does not consider other abilities of human being. However, this is not the only scenario that may happen in real applications. In fuzzy inventory planning, the operator sometimes requires information from previous planning cycles that helps him to shape his knowledge base and estimate the uncertain inventory parameters more precisely. Therefore, he/she should devote time to obtain, process and analyze information during the execution of the planning. A portion of the time required for obtaining information and building-up knowledge constitutes the first phase of the planning, where the operator prepares the necessary knowledge for the next phase. After the phase where the knowledge is looked up by the operator and the prerequisite knowledge is built up, the planning phase starts, which is the second step of the planning. This phase is referred to as the knowledge retrieval step (Dar-El, 2000). As stated by Jaber and Glock (2013), learning may occur in both process steps, but the operator's learning in both steps of the planning is different. After the operator becomes familiar with the planning by doing it several times, less time is required for looking up the information, and the planning steps could be performed faster. In the learning literature, the first stage is called cognitive learning, whilst the second one is termed as motor learning.

Referring to the cases described above, the model in this section intends to formulate the situation in a fuzzy inventory planning in which the operator's learning process includes cognitive and motor capabilities of human being. In the model presented herein, the cognitive ability of the operator helps him/her to analyze the characteristics of the fuzzy parameters, whereas the motor ability is of help in adjusting the most accurate quantities for the parameters using the processed information obtained from the cognitive learning stage. Therefore, cognitive and motor capabilities of the operator will be integrated into the fuzzy model to reflect these abilities of the operator in inventory management. Besides assumptions 1 to 5 in section 4.2.3 and according to the above-mentioned scenario, the following assumption are further made:

- The learning is fully transferred within the cycles. That is, the inventory planners do not forget the information gained from the earlier cycles;

- The learning in every cycle occurs in two phases where the operator learns with cognitive and motor abilities of human being.

The same scenario is followed as proposed for the two earlier models. First, the relevant decision variable of the model should be the number of orders instead of the size of the order, as it is assumed that the operator learns with every order. Therefore, Eq. (4.6) can be modified to account for the number of orders as the decision variable by replacing $n = \frac{D}{Q}$ in Eq. (4.6), with the result given the same as Eq. (4.17).

The next step is to formulate the impact of operator's learning with cognitive and motor capabilities on fuzzy parameters. To define cognitive and motor capabilities of the operator, JGLC is applied due to the reasons provided in Section 3.1.3.4. As the learning affects Δ values, their values change over the planning period in conformance with JGLC. The variation of JGLC, adopted to the problem defined in this paper, is of the form:

$$P_t = \alpha P_1 t^{-l_c} + (1 - \alpha) P_1 t^{-l_m} = P_1[\alpha(t^{-l_c} - t^{-l_m}) + t^{-l_m}], \tag{4.36}$$

where P_t is the performance in the tth unit of the time, P_1 is the performance in the first unit of the time (i.e. the beginning of the planning period in the problem defined), α is the percentage (weight) that P_1 could be divided into the cognitive and motor components, and l_c and l_m represent the learning exponents for the cognitive and motor task, respectively. If full transfer of learning occurs between the cycles, considering that EOQ-S model follows equal lot-size replenishment policy which lot of size Q receives in equal intervals over the planning horizon, the value of jth deviation parameter at the time of ith order will be as follows:

$$\Delta_{l,i}^{j}=\begin{cases}\Delta_{l,1}^{j} & i=1\\ \alpha\Delta_{l,1}^{j}\left[(i-1)\frac{T}{n}\right]^{-l_c}+(1-\alpha)\Delta_{l,1}^{j}\left[(i-1)\frac{T}{n}\right]^{-l_m}= & \\ \Delta_{l,1}^{j}\left\{\alpha\left(\left[(i-1)\frac{T}{n}\right]^{-l_c}-\left[(i-1)\frac{T}{n}\right]^{-l_m}\right)+\left[(i-1)\frac{T}{n}\right]^{-l_m}\right\} & i>1\end{cases} \tag{4.37}$$

$$\Delta_{h,i}^{j}=\begin{cases}\Delta_{h,1}^{j} & i=1\\ \alpha\Delta_{h,1}^{j}\left[(i-1)\frac{T}{n}\right]^{-l_c}+(1-\alpha)\Delta_{h,1}^{j}\left[(i-1)\frac{T}{n}\right]^{-l_m}= & \\ \Delta_{h,1}^{j}\left\{\alpha\left(\left[(i-1)\frac{T}{n}\right]^{-l_c}-\left[(i-1)\frac{T}{n}\right]^{-l_m}\right)+\left[(i-1)\frac{T}{n}\right]^{-l_m}\right\} & i>1\end{cases} \tag{4.38}$$

In Eqs. (4.37) and (4.38), the first term in the summations represents the cognitive component of the planning task and the second expression indicates the motor component, while α represents a mechanism that planning could be broken down into two elements. Due to the natural capability of the human, who tend to recall the process faster with every repetition, the cognitive component in Eqs. (4.37) and (4.38) reduces at a faster rate comparing to the motor element (Jaber & Glock, 2013). At the beginning of the planning horizon, when the first cycle starts, the deviation values adopt their largest possible limit over the planning horizon, which are set by the operator. As the process continues by every repetition, caused by order shipments, the deviation values diminish according to the learning curve, with the effect of both cognitive and motor characteristics of the operator.

Thereupon, to derive the total cost function for the EOQ-S model with the two-stage learning effect, the cost of every planning cycle is initially computed, and then the total cost of the inventory system over the planning horizon is determined by integrating the total costs over n cycles. Considering the definition of the deviation values in Eqs. (4.37) and (4.38), the cost function of the first cycle will be as follows:

$$TCU_{FL_1}(n,M) = K + \frac{pD}{2n^2} - \frac{pM}{n} + \frac{M^2h}{2D} + \frac{M^2p}{2D} + \frac{nK\Delta_{h,1}^{D}}{4D} - \frac{nK\Delta_{l,1}^{D}}{4D} + \frac{nh\Delta_{l,1}^{M^2}}{12D}$$

$$+ \frac{p\Delta_{l,1}^{M}}{4} + \frac{nh\Delta_{h,1}^{M^2}}{12D} + \frac{np\Delta_{l,1}^{M^2}}{12D} + \frac{np\Delta_{h,1}^{M^2}}{12D} + \frac{nMh\Delta_{h,1}^{M}}{4D} - \frac{nMh\Delta_{l,1}^{M}}{4D} + \frac{nMp\Delta_{h,1}^{M}}{4D}$$

$$- \frac{nMp\Delta_{l,1}^{M}}{4D} - \frac{p\Delta_{h,1}^{M}}{4} \tag{4.39}$$

Following the definitions as given in Eqs. (4.37) and (4.38), the cost function of the inventory system for ith cycle, $i \in [2, n]$, will be as:

$$TCU_{FL_i}(n,M) = K + \frac{Dp}{2n^2} - \frac{Mp}{n} + \frac{M^2h}{2D} + \frac{M^2p}{2D}$$

$$+ \frac{K}{4D}\left[\begin{array}{l} \left\{\alpha\Delta_{h,1}^{D}\left[(i-1)\frac{T}{n}\right]^{-l_c} + (1-\alpha)\Delta_{h,1}^{D}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right\} - \\ \left\{\alpha\Delta_{l,1}^{D}\left[(i-1)\frac{T}{n}\right]^{-l_c} - (1-\alpha)\Delta_{l,1}^{D}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right\} \end{array}\right]$$

$$+ \frac{h}{12D}\left[\begin{array}{l} \left\{\alpha\Delta_{h,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l_c} + (1-\alpha)\Delta_{h,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right\}^2 + \\ \left\{\alpha\Delta_{l,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l_c} + (1-\alpha)\Delta_{l,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right\}^2 \end{array}\right]$$

$$+ \frac{p}{12D}\left[\begin{array}{l} \left\{\alpha\Delta_{h,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l_c} + (1-\alpha)\Delta_{h,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right\}^2 + \\ \left\{\alpha\Delta_{l,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l_c} + (1-\alpha)\Delta_{l,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right\}^2 \end{array}\right]$$

$$+ \frac{Mh}{4D}\left[\begin{array}{l} \left\{\alpha\Delta_{h,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l_c} + (1-\alpha)\Delta_{h,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right\} - \\ \left\{\alpha\Delta_{l,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l_c} - (1-\alpha)\Delta_{l,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right\} \end{array}\right]$$

$$+ \frac{Mp}{4D}\left[\begin{array}{l} \left\{\alpha\Delta_{h,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l_c} + (1-\alpha)\Delta_{h,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right\} - \\ \left\{\alpha\Delta_{l,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l_c} - (1-\alpha)\Delta_{l,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right\} \end{array}\right] \tag{4.40}$$

$$+\frac{p}{4n}\left[\begin{array}{l}\left\{\alpha\Delta_{l,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l_c}+(1-\alpha)\Delta_{l,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right\}- \\ \left\{\alpha\Delta_{h,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l_c}-(1-\alpha)\Delta_{h,1}^{M}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right\}\end{array}\right]$$

The total cost function of the inventory system is determined by summing up Eqs. (4.39) and (4.40) over the entire n cycles, resulting in the following expression:

$$TCU_{FL}(n,M)=\sum_{i=1}^{n}TCU_{FL_i}(n,M)=TCU_{FL_1}(n,M)+\sum_{i=2}^{n}TCU_{FL_i}(n,M)$$

$$=nK-pM+\frac{Dp}{2n}+\frac{nM^2h}{2D}+\frac{nM^2p}{2D}$$

$$+\frac{K}{4D}\Delta_{h,1}^{D}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right]\right\}$$

$$-\frac{K}{4D}\Delta_{l,1}^{D}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right]\right\}$$

$$+\frac{Mh}{4D}\Delta_{h,1}^{M}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right]\right\}$$

$$-\frac{Mh}{4D}\Delta_{l,1}^{M}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right]\right\}$$

$$+\frac{Mp}{4D}\Delta_{h,1}^{M}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right]\right\}$$

$$-\frac{Mp}{4D}\Delta_{l,1}^{M}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right]\right\}$$

$$+\frac{p}{4n}\Delta_{l,1}^{M}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right]\right\}$$

$$-\frac{p}{4n}\Delta_{h,1}^{M}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right]\right\}$$

$$+\frac{h}{12D}{\Delta_{h,1}^{M}}^{2}\left\{\begin{array}{c}\left[1+\alpha^2\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-2l_c}\right]+\left[1+(1-\alpha)^2\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-2l_m}\right]\\+\left[1+2\alpha(1-\alpha)\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_c-l_m}\right]\end{array}\right\}$$

$$+\frac{h}{12D}{\Delta_{l,1}^{M}}^{2}\left\{\begin{array}{c}\left[1+\alpha^2\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-2l_c}\right]+\left[1+(1-\alpha)^2\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-2l_m}\right]\\+\left[1+2\alpha(1-\alpha)\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_c-l_m}\right]\end{array}\right\}$$

$$+\frac{p}{12D}{\Delta_{h,1}^{M}}^{2}\left\{\begin{array}{c}\left[1+\alpha^2\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-2l_c}\right]+\left[1+(1-\alpha)^2\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-2l_m}\right]\\+\left[1+2\alpha(1-\alpha)\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_c-l_m}\right]\end{array}\right\}$$

$$+\frac{h}{12D}{\Delta_{l,1}^{M}}^{2}\left\{\begin{array}{c}\left[1+\alpha^2\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-2l_c}\right]+\left[1+(1-\alpha)^2\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-2l_m}\right]\\+\left[1+2\alpha(1-\alpha)\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_c-l_m}\right]\end{array}\right\} \tag{4.41}$$

After rearrangement, Eq. (4.41) reduces to the following:

$$TCU_{FL}(n,M)=nK-pM+\frac{Dp}{2n}+\frac{nM^2h}{2D}+\frac{nM^2p}{2D}$$

$$+\left(\begin{array}{c}\left\{\begin{array}{c}\frac{K}{4D}\Delta_{h,1}^{D}-\frac{K}{4D}\Delta_{l,1}^{D}+\frac{Mh}{4D}\Delta_{h,1}^{M}-\frac{Mh}{4D}\Delta_{l,1}^{M}+\frac{Mp}{4D}\Delta_{h,1}^{M}-\frac{Mp}{4D}\Delta_{l,1}^{M}\\+\frac{p}{4n}\Delta_{l,1}^{M}-\frac{p}{4n}\Delta_{h,1}^{M}\end{array}\right\}\\\times\left\{\left[1+\alpha\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right]\right\}\end{array}\right)$$

$$+\left(\begin{array}{c}\left\{\begin{array}{c}\left[1+\alpha^2\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-2l_c}\right]+\left[1+(1-\alpha)^2\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-2l_m}\right]\\ +\left[1+2\alpha(1-\alpha)\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_c-l_m}\right]\end{array}\right\}\\ \times\left\{\frac{h}{12D}{\Delta_{h,1}^{M}}^{2}+\frac{h}{12D}{\Delta_{l,1}^{M}}^{2}+\frac{p}{12D}{\Delta_{h,1}^{M}}^{2}+\frac{p}{12D}{\Delta_{l,1}^{M}}^{2}\right\}\end{array}\right) \tag{4.42}$$

The structure of the function in Eq. (4.42) is similar to that of the previous models, thus their optimization procedure is followed to obtain the optimal solution of the model. In order to run the convexity test, the first and second order derivations of the total cost function with respect to M should be calculated. The derivations gives:

$$\frac{\partial TCU_{FL}(n,M)}{\partial M}=-p+\frac{nMh}{D}+\frac{nMp}{D}$$

$$+\left(\begin{array}{c}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right]\right\}\times\\ \times\left\{\frac{h}{4D}\Delta_{h,1}^{M}-\frac{h}{4D}\Delta_{l,1}^{M}+\frac{p}{4D}\Delta_{h,1}^{M}-\frac{p}{4D}\Delta_{l,1}^{M}\right\}\end{array}\right) \tag{4.43}$$

$$\frac{\partial^2 TCU_{FL}(n,M)}{\partial^2 M}=\frac{nh}{D}+\frac{np}{D} \tag{4.44}$$

Because $\forall\, n,h,p,D, \frac{\partial^2 TCU_{FL}(n,M)}{\partial^2 M}>0$, it can be concluded that the total cost function is convex in M, and therefore the optimal value of M can be calculated through the term $\frac{\partial^2 TCU_{FL}(n,M)}{\partial M}=0$, which gives the result as:

$$M=\frac{pD-\left(\begin{array}{c}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[(i-1)\frac{T}{n}\right]^{-l_m}\right]\right\}\times\\ \times\left\{\frac{h}{4}\left(\Delta_{h,1}^{M}-\Delta_{l,1}^{M}\right)+\frac{p}{4}\left(\Delta_{h,1}^{M}-\Delta_{l,1}^{M}\right)\right\}\end{array}\right)}{n(h+p)} \tag{4.45}$$

As discussed before, due to the trait of the variables of the total cost function, finding the global optimum for the total cost function is equivalent to finding a local optimum for n. Hence, the linear search algorithm developed in Section 4.2.3 can be adopted here to find the optimal solution for n.

4.2.6 The fuzzy EOQ-S model with forgetting effect on fuzzy parameters and full transfer of learning: the case of two-stage learning curve

In the former section, a model was developed with cognitive and motor human learning where the operator's attained knowledge is transferred fully cycle by cycle. However, this model does not account for the cases in which the gained experience is depreciated. In this section, the model in the earlier section will be extended to account for the operator's forgetting phenomenon due to break from the task. It is obvious that, in case forgetting occurs and the operator loses a part of his knowledge, the model with the only learning phenomenon cannot be effective and therefore it may produce error in estimation of the imprecise parameters of the model.

In the models developed so far, the learning curves already existed in the literature are applied and modified them to fit the case of the models. Nonetheless, there is no learning and forgetting curve model in the literature considering the cognitive and motor capabilities of human being. Specifically, there is no learning curve which is taken to be an extended version of JGLC that includes forgetting phenomenon. To address this problem in the model, the JGLC will be developed initially to account for the effect of forgetting, and then the developed learning curve will be integrate into the model formulated in the previous section.

4.2.6.1 Development of learn-forget curve model with cognitive and motor capabilities

As noted before, JGLC extends and improves the dual phase learning curve of Dar-el et al. (1995), however, it does not suggest a way to take account of forgetting, which is the fact that sometimes happens in empirical cases. In order to develop a learning-forgetting model with cognitive and motor skills of the operator, JGLC is integrated into LFCM, from now on called LFCMCM. LFCM can capture the forgetting phenomenon in an appropriate manner, however, it assumes learning occurs in its simplest form. The LFCM model utilizes a learning curve with a fix learning rate such that it curbs the model to capture the effect of improvement in cognitive and motor skills whenever the learning includes these capabilities of the operator. Combining both models could build a learning curve that benefits from the desirable traits of both models and further makes it possible for the policy makers to estimate the imprecise parameters more precisely.

For this purpose, JGLC model is considered as the basic learning curve that is going to be extended. Following formulation of LFCM in Jaber and Bonney (1996), JGLC when it accounts for forgetting will be as:

$$\hat{T}_i = \alpha \hat{T}_{1,i} i^{f_i^c} + (1-\alpha)\hat{T}_{1,i} i^{f_i^m} \tag{4.46}$$

where $\hat{T}_i$ is the same as the notation defined in the second model, $\hat{T}_{1,i}$ is the equivalent time for the first unit of forgetting curve. Furthermore, f_i^c and f_i^m are forgetting exponents for cognitive and motor forgetting curves, respectively, and can be calculated as:

$$f_i^c = \frac{l_c(1-l_c)\log(u_i^c + n_i)}{\log\{1 + \vartheta/t(u_i^c + n_i)\}} \tag{4.47}$$

$$f_i^m = \frac{l_m(1-l_m)\log(u_i^m + n_i)}{\log\{1 + \vartheta/t(u_i^m + n_i)\}} \tag{4.48}$$

In Eqs. (4.47) and (4.48), u_i^c and u_i^m are the accumulated experience transferred (remembered) at the initial of the ith cycle for cognitive and motor curves, respectively, and l_c is the learning rate for the cognitive part of the task, while l_m is the learning rate for the motor part of the task. The notations ϑ and n_i are the same as their definition in

LFCM model. In addition, $t(u_i^c + n_i)$ is the time to produce $u_i^c + n_i$ units on the cognitive learning curve, and $t(u_i^m + n_i)$ has the same definition for motor learning curve. The values of u_i for cognitive and motor learning curves are calculated as:

$$u_{j,i+1}^c = (n_i + u_i^c)^{(1+f_i^c/l_c)} S_i^{c^{-\frac{f_i^c}{l_c}}} \tag{4.49}$$

$$u_{j,i+1}^m = (n_i + u_i^m)^{(1+f_i^m/l_m)} S_i^{m^{-\frac{f_i^m}{l_m}}} \tag{4.50}$$

where u_1^c and u_1^m are zero. In expressions (49) and (50), S_i^c shows the number of units that could have been produced using cognitive skill if the production have not been ceased, while S_i^m indicates the same value, but for motor skill. The terms S_i^c and S_i^m can be calculated using the following formulas:

$$S_i^c = \left\{\frac{1-l_c}{T_{1,i}}\left[t(u_i^c + n_i) + \varphi_i\right]\right\}^{\frac{1}{1-l_c}} \tag{4.51}$$

$$S_i^m = \left\{\frac{1-l_m}{T_{1,i}}\left[t(u_i^m + n_i) + \varphi_i\right]\right\}^{\frac{1}{1-l_m}} \tag{4.52}$$

where all the terms have the definition noted before. Similar to the second model, when forgetting occurs, since experience transfers partially, the expression $0 \leq u_i \leq \sum_{j=1}^{i-1} n_i$ holds. It is clear that when experience transfers completely $u_i = \sum_{j=1}^{i-1} n_i$.

4.2.6.2 Model development

In this subsection, a fuzzy EOQ-S model is developed that includes cognitive and motor learning and forgetting effect in setting fuzzy parameters. To do so, the LFCMCM developed in the earlier section will be integrated into the fuzzy model developed by Björk (2009). The similar procedure is followed as taken for the second model and assumed that the cognitive and motor learning and forgetting affect the deviation values of the fuzzy parameters. The deviation values of the fuzzy parameters when the LFCMCM is accounted for are described as:

$$\Delta_{l,i}^{j} = \begin{cases} \Delta_{l,1}^{j} & i = 1 \\ \alpha\Delta_{l,1}^{j}\left[u_{l,i}^{j,c} + n_{l,i}^{j}\right]^{-l_c} + (1-\alpha)\Delta_{l,1}^{j}\left[u_{l,i}^{j,m} + n_{l,i}^{j}\right]^{-l_m} = & \\ \Delta_{l,1}^{j}\left\{\alpha\left(\left[u_{l,i}^{j,c} + n_{l,i}^{j}\right]^{-l_c} - \left[u_{l,i}^{j,m} + n_{l,i}^{j}\right]^{-l_m}\right) + \left[u_{l,i}^{j,m} + n_{l,i}^{j}\right]^{-l_m}\right\} & i > 1 \end{cases} \tag{4.53}$$

$$\Delta_{h,i}^{j} = \begin{cases} \Delta_{h,1}^{j} & i = 1 \\ \alpha\Delta_{h,1}^{j}\left[u_{h,i}^{j,c} + n_{h,i}^{j}\right]^{-l_c} + (1-\alpha)\Delta_{h,1}^{j}\left[u_{h,i}^{j,m} + n_{h,i}^{j}\right]^{-l_m} = & \\ \Delta_{h,1}^{j}\left\{\alpha\left(\left[u_{h,i}^{j,c} + n_{h,i}^{j}\right]^{-l_c} - \left[u_{h,i}^{j,m} + n_{h,i}^{j}\right]^{-l_m}\right) + \left[u_{h,i}^{j,m} + n_{h,i}^{j}\right]^{-l_m}\right\} & i > 1 \end{cases} \tag{4.54}$$

In expressions (53) and (54), the experience transferred over cycles should be calculated first. Subsequently, the LFCMCMs developed in the former section is modified to adopt them to the definitions of the model. For $x = l, h$ and $j = D, M$, the accumulated experience for cognitive and motor learning curves if the process was not stopped is of the form:

$$S_{x,i}^{j,c} = \left\{\frac{1-l_c}{\Delta_{x,1,i}^{j}}\left[t_x^j\left(u_{x,i}^{j,c} + n_{x,i}^{j}\right) + \varphi_i\right]\right\}^{\frac{1}{1-l_c}} \tag{4.55}$$

$$S_{x,i}^{j,m} = \left\{\frac{1-l_m}{\Delta_{x,1,i}^{j}}\left[t_x^j\left(u_{x,i}^{j,m} + n_{x,i}^{j}\right) + \varphi_i\right]\right\}^{\frac{1}{1-l_m}} \tag{4.56}$$

where the term t_x^j for both cognitive and motor curves are determined as:

$$t_x^j(u_{x,i}^{j,c}+n_{x,i}^j) = \sum_{i=1}^{u_{x,i}^{j,c}+n_{x,i}^j} \alpha\Delta_{x,1,i}^j \left[(i-1)\frac{T}{n}\right]^{-l}$$

$$\cong \int_0^{u_{x,i}^{j,c}+n_{x,i}^j} \alpha\Delta_{x,1,i}^j \left[(i-1)\frac{T}{n}\right]^{-l} di$$

$$= \alpha\Delta_{x,1,i}^j \left(\frac{T}{n}\right)^{-l} \left[\left(u_{x,i}^{j,c}+n_{x,i}^j-1\right)^{1-l} - (-1)^{1-l}\right]/1-l \qquad (4.57)$$

$$t_x^j(u_{x,i}^{j,m}+n_{x,i}^j) = \sum_{i=1}^{u_{x,i}^{j,m}+n_{x,i}^j} (1-\alpha)\Delta_{x,1,i}^j \left[(i-1)\frac{T}{n}\right]^{-l}$$

$$\cong \int_0^{u_{x,i}^{j,m}+n_{x,i}^j} (1-\alpha)\Delta_{x,1,i}^j \left[(i-1)\frac{T}{n}\right]^{-l} di$$

$$= (1-\alpha)\Delta_{x,1,i}^j \left(\frac{T}{n}\right)^{-l} \left[\left(u_{x,i}^{j,m}+n_{x,i}^j-1\right)^{1-l} - (-1)^{1-l}\right]/1-l \qquad (4.58)$$

The forgetting indexes should be modified to include the obtained value for $t_x^j(u_{x,i}^{j,c}+n_{x,i}^j)$ and $t_x^j(u_{x,i}^{j,m}+n_{x,i}^j)$, which yield:

$$f_{x,i}^{j,c} = \frac{l_c(1-l_c)\log(u_{x,i}^{j,c}+n_{x,i}^j)}{\log\left\{1+\left(v(1-l)n^{-l}/\alpha\Delta_{x,1,i}^j T^{-l}\left[\left(u_{x,i}^{j,c}+n_{x,i}^j-1\right)^{1-l}-(-1)^{1-l}\right]\right)\right\}} \qquad (4.59)$$

$$f_{x,i}^{j,m} = \frac{l_m(1-l_m)\log(u_{x,i}^{j,m}+n_{x,i}^j)}{\log\left\{1+\left(v(1-l)n^{-l}/(1-\alpha)\Delta_{x,1,i}^j T^{-l}\left[\left(u_{x,i}^{j,m}+n_{x,i}^j-1\right)^{1-l}-(-1)^{1-l}\right]\right)\right\}} \qquad (4.60)$$

where the transferred experience can be obtained as

$$u_{x,i+1}^{j,c} = (n_i+u_{x,i}^{j,c})^{(1+f_{x,i}^{j,c}/l_c)} S_{x,i}^{j,c^{-\frac{f_{x,i}^{j,c}}{l_c}}} \qquad (4.61)$$

$$u_{x,i+1}^{j,m} = (n_i + u_{x,i}^{j,m})^{(1+f_{x,i}^{j,m}/l_m)} {S_{x,i}^{j,m}}^{-\frac{f_{x,i}^{j,m}}{l_m}} \tag{4.62}$$

Incorporating the LFCMCM into the fuzzy total cost function will be the same as the earlier models. First, the order quantity variable will be replaced by the number of orders, and then the total cost function will be split up into n functions where the cost function is calculated for every order considering transfer of learning. The cost functions will subsequently sum up to form the total cost function of the inventory system. According to this procedure, the cost function for the first order is as given in Eq. (4.39) and the cost function for orders $i \geq 2$ will be given as:

$$TCU_{FL_i}(n,M) = K + \frac{Dp}{2n^2} - \frac{Mp}{n} + \frac{M^2h}{2D} + \frac{M^2p}{2D}$$

$$+\frac{K}{4D}\left\{\begin{aligned}&\left(\alpha\Delta_{h,1}^{D}\left[u_{h,i}^{D,c}+n_{h,i}^{D}\right]^{-l_c}+(1-\alpha)\Delta_{h,1}^{D}\left[u_{h,i}^{D,m}+n_{h,i}^{D}\right]^{-l_m}\right)\\&-\left(\alpha\Delta_{l,1}^{D}\left[u_{l,i}^{D,c}+n_{l,i}^{D}\right]^{-l_c}-(1-\alpha)\Delta_{l,1}^{D}\left[u_{l,i}^{D,m}+n_{l,i}^{D}\right]^{-l_m}\right)\end{aligned}\right\}$$

$$+\frac{h}{12D}\left\{\begin{aligned}&\left(\alpha\Delta_{h,1}^{M}\left[u_{h,i}^{M,c}+n_{h,i}^{M}\right]^{-l_c}+(1-\alpha)\Delta_{h,1}^{M}\left[u_{h,i}^{M,m}+n_{h,i}^{M}\right]^{-l_m}\right)^2\\&-\left(\alpha\Delta_{l,1}^{M}\left[u_{l,i}^{M,c}+n_{l,i}^{M}\right]^{-l_c}+(1-\alpha)\Delta_{l,1}^{M}\left[u_{l,i}^{M,m}+n_{l,i}^{M}\right]^{-l_m}\right)^2\end{aligned}\right\}$$

$$+\frac{p}{12D}\left\{\begin{aligned}&\left(\alpha\Delta_{h,1}^{M}\left[u_{h,i}^{M,c}+n_{h,i}^{M}\right]^{-l_c}+(1-\alpha)\Delta_{h,1}^{M}\left[u_{h,i}^{M,m}+n_{h,i}^{M}\right]^{-l_m}\right)^2\\&-\left(\alpha\Delta_{l,1}^{M}\left[u_{l,i}^{M,c}+n_{l,i}^{M}\right]^{-l_c}+(1-\alpha)\Delta_{l,1}^{M}\left[u_{l,i}^{M,m}+n_{l,i}^{M}\right]^{-l_m}\right)^2\end{aligned}\right\}$$

$$+\frac{Mh}{4D}\left\{\begin{aligned}&\left(\alpha\Delta_{h,1}^{M}\left[u_{h,i}^{M,c}+n_{h,i}^{M}\right]^{-l_c}+(1-\alpha)\Delta_{h,1}^{M}\left[u_{h,i}^{M,m}+n_{h,i}^{M}\right]^{-l_m}\right)\\&-\left(\alpha\Delta_{l,1}^{M}\left[u_{l,i}^{M,c}+n_{l,i}^{M}\right]^{-l_c}-(1-\alpha)\Delta_{l,1}^{M}\left[u_{l,i}^{M,m}+n_{l,i}^{M}\right]^{-l_m}\right)\end{aligned}\right\}$$

$$+\frac{Mp}{4D}\left\{\begin{aligned}&\left(\alpha\Delta_{h,1}^{M}\left[u_{h,i}^{M,c}+n_{h,i}^{M}\right]^{-l_c}+(1-\alpha)\Delta_{h,1}^{M}\left[u_{h,i}^{M,m}+n_{h,i}^{M}\right]^{-l_m}\right)\\&-\left(\alpha\Delta_{l,1}^{M}\left[u_{l,i}^{M,c}+n_{l,i}^{M}\right]^{-l_c}-(1-\alpha)\Delta_{l,1}^{M}\left[u_{l,i}^{M,m}+n_{l,i}^{M}\right]^{-l_m}\right)\end{aligned}\right\}$$

$$+\frac{Mp}{4D}\left\{\begin{array}{c}\left(\alpha\Delta_{l,1}^{M}\left[u_{l,i}^{M,c}+n_{l,i}^{M}\right]^{-l_c}+(1-\alpha)\Delta_{l,1}^{M}\left[u_{l,i}^{M,m}+n_{l,i}^{M}\right]^{-l_m}\right)\\-\left(\alpha\Delta_{h,1}^{M}\left[u_{h,i}^{M,c}+n_{h,i}^{M}\right]^{-l_c}-(1-\alpha)\Delta_{h,1}^{M}\left[u_{h,i}^{M,m}+n_{h,i}^{M}\right]^{-l_m}\right)\end{array}\right\} \tag{4.63}$$

As noted before the total cost function could be derived through the summation of all $i, 1 \le i \le n$, total cost functions, giving the following result:

$$TCU_{FL}(n,M)=\sum_{i=1}^{n}TCU_{FL_i}(n,M)=TCU_{FL_1}(n,M)+\sum_{i=2}^{n}TCU_{FL_i}(n,M)$$

$$=nK-pM+\frac{Dp}{2n}+\frac{nM^2h}{2D}+\frac{nM^2p}{2D}$$

$$+\frac{K}{4D}\Delta_{h,1}^{D}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[u_{h,i}^{D,c}+n_{h,i}^{D}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[u_{h,i}^{D,m}+n_{h,i}^{D}\right]^{-l_m}\right]\right\}$$

$$-\frac{K}{4D}\Delta_{l,1}^{D}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[u_{l,i}^{D,c}+n_{l,i}^{D}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[u_{l,i}^{D,m}+n_{l,i}^{D}\right]^{-l_m}\right]\right\}$$

$$+\frac{Mh}{4D}\Delta_{h,1}^{M}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[u_{h,i}^{M,c}+n_{h,i}^{M}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[u_{h,i}^{M,m}+n_{h,i}^{M}\right]^{-l_m}\right]\right\}$$

$$-\frac{Mh}{4D}\Delta_{l,1}^{M}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[u_{l,i}^{M,c}+n_{l,i}^{M}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[u_{l,i}^{M,m}+n_{l,i}^{M}\right]^{-l_m}\right]\right\}$$

$$+\frac{Mp}{4D}\Delta_{h,1}^{M}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[u_{h,i}^{M,c}+n_{h,i}^{M}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[u_{h,i}^{M,m}+n_{h,i}^{M}\right]^{-l_m}\right]\right\}$$

$$-\frac{Mp}{4D}\Delta_{l,1}^{M}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[u_{l,i}^{M,c}+n_{l,i}^{M}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[u_{l,i}^{M,m}+n_{l,i}^{M}\right]^{-l_m}\right]\right\}$$

$$+\frac{p}{4n}\Delta_{l,1}^{M}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[u_{l,i}^{M,c}+n_{l,i}^{M}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[u_{l,i}^{M,m}+n_{l,i}^{M}\right]^{-l_m}\right]\right\}$$

$$-\frac{p}{4n}\Delta_{h,1}^{M}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[u_{h,i}^{M,c}+n_{h,i}^{M}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[u_{h,i}^{M,m}+n_{h,i}^{M}\right]^{-l_m}\right]\right\}$$

$$+\frac{h}{12D}{\Delta_{h,1}^{M}}^{2}\left\{\begin{array}{c}\left[1+\alpha^2\sum_{i=2}^{n}\left[u_{h,i}^{M,c}+n_{h,i}^{M}\right]^{-2l_c}\right]+\left[1+(1-\alpha)^2\sum_{i=2}^{n}\left[u_{h,i}^{M,m}+n_{h,i}^{M}\right]^{-2l_m}\right]\\+\left[1+2\alpha(1-\alpha)\sum_{i=2}^{n}\left(\left[u_{h,i}^{M,c}+n_{h,i}^{M}\right]^{-l_c}\left[u_{h,i}^{M,m}+n_{h,i}^{M}\right]^{-l_m}\right)\right]\end{array}\right\}$$

$$+\frac{h}{12D}{\Delta_{l,1}^{M}}^{2}\left\{\begin{array}{c}\left[1+\alpha^2\sum_{i=2}^{n}\left[u_{l,i}^{M,c}+n_{l,i}^{M}\right]^{-2l_c}\right]+\left[1+(1-\alpha)^2\sum_{i=2}^{n}\left[u_{l,i}^{M,m}+n_{l,i}^{M}\right]^{-2l_m}\right]\\+\left[1+2\alpha(1-\alpha)\sum_{i=2}^{n}\left(\left[u_{l,i}^{M,c}+n_{l,i}^{M}\right]^{-l_c}\left[u_{l,i}^{M,m}+n_{l,i}^{M}\right]^{-l_m}\right)\right]\end{array}\right\}$$

$$+\frac{p}{12D}{\Delta_{h,1}^{M}}^{2}\left\{\begin{array}{c}\left[1+\alpha^2\sum_{i=2}^{n}\left[u_{h,i}^{M,c}+n_{h,i}^{M}\right]^{-2l_c}\right]+\left[1+(1-\alpha)^2\sum_{i=2}^{n}\left[u_{h,i}^{M,m}+n_{h,i}^{M}\right]^{-2l_m}\right]\\+\left[1+2\alpha(1-\alpha)\sum_{i=2}^{n}\left(\left[u_{h,i}^{M,c}+n_{h,i}^{M}\right]^{-l_c}\left[u_{h,i}^{M,m}+n_{h,i}^{M}\right]^{-l_m}\right)\right]\end{array}\right\}$$

$$+\frac{h}{12D}{\Delta_{l,1}^{M}}^{2}\left\{\begin{array}{c}\left[1+\alpha^2\sum_{i=2}^{n}\left[u_{l,i}^{M,c}+n_{l,i}^{M}\right]^{-2l_c}\right]+\left[1+(1-\alpha)^2\sum_{i=2}^{n}\left[u_{l,i}^{M,m}+n_{l,i}^{M}\right]^{-2l_m}\right]\\+\left[1+2\alpha(1-\alpha)\sum_{i=2}^{n}\left(\left[u_{l,i}^{M,c}+n_{l,i}^{M}\right]^{-l_c}\left[u_{l,i}^{M,m}+n_{l,i}^{M}\right]^{-l_m}\right)\right]\end{array}\right\}$$

(4.64)

Eq. (4.64) is reduced to the following formula:

$$TCU_{FL}(n,M)=nK-Mp+\frac{Dp}{2n}+\frac{nM^2h}{2D}+\frac{nM^2p}{2D}$$

$$+\Delta_{l,1}^{M}\left\{\begin{array}{c}\left[1+\alpha\sum_{i=2}^{n}\left[u_{l,i}^{M,c}+n_{l,i}^{M}\right]^{-l_c}\right]\\+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[u_{l,i}^{M,m}+n_{l,i}^{M}\right]^{-l_m}\right]\end{array}\right\}\left\{\frac{p}{4n}-\frac{Mh}{4D}-\frac{Mp}{4D}\right\}$$

$$+\Delta_{h,1}^{M}\left\{\begin{array}{c}\left[1+\alpha\sum_{i=2}^{n}\left[u_{h,i}^{M,c}+n_{h,i}^{M}\right]^{-l_c}\right]\\+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[u_{h,i}^{M,m}+n_{h,i}^{M}\right]^{-l_m}\right]\end{array}\right\}\left\{\frac{Mh}{4D}+\frac{Mp}{4D}-\frac{p}{4n}\right\}$$

$$+\frac{K}{4D}\Delta_{h,1}^{D}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[u_{h,i}^{D,c}+n_{h,i}^{D}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[u_{h,i}^{D,m}+n_{h,i}^{D}\right]^{-l_m}\right]\right\}$$

$$-\frac{K}{4D}\Delta_{l,1}^{D}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[u_{l,i}^{D,c}+n_{l,i}^{D}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[u_{l,i}^{D,m}+n_{l,i}^{D}\right]^{-l_m}\right]\right\}$$

$$+\frac{h}{12D}{\Delta_{h,1}^{M}}^{2}\left\{\begin{array}{c}\left[1+\alpha^{2}\sum_{i=2}^{n}\left[u_{h,i}^{M,c}+n_{h,i}^{M}\right]^{-2l_c}\right]+\left[1+(1-\alpha)^{2}\sum_{i=2}^{n}\left[u_{h,i}^{M,m}+n_{h,i}^{M}\right]^{-2l_m}\right]\\+\left[1+2\alpha(1-\alpha)\sum_{i=2}^{n}\left(\left[u_{h,i}^{M,c}+n_{h,i}^{M}\right]^{-l_c}\left[u_{h,i}^{M,m}+n_{h,i}^{M}\right]^{-l_m}\right)\right]\end{array}\right\}$$

$$+\frac{p}{12D}{\Delta_{l,1}^{M}}^{2}\left\{\begin{array}{c}\left[1+\alpha^{2}\sum_{i=2}^{n}\left[u_{l,i}^{M,c}+n_{l,i}^{M}\right]^{-2l_c}\right]+\left[1+(1-\alpha)^{2}\sum_{i=2}^{n}\left[u_{l,i}^{M,m}+n_{l,i}^{M}\right]^{-2l_m}\right]\\+\left[1+2\alpha(1-\alpha)\sum_{i=2}^{n}\left(\left[u_{l,i}^{M,c}+n_{l,i}^{M}\right]^{-l_c}.\left[u_{l,i}^{M,m}+n_{l,i}^{M}\right]^{-l_m}\right)\right]\end{array}\right\} \quad (4.65)$$

In order to prove the convexity of the total cost function, according to the algorithm developed in the former models, the first and second order derivations of the function with respect to M should be derived, which are obtained as:

$$\frac{\partial TCU_{FL}(n,M)}{\partial M}=\frac{nMh}{D}+\frac{nMp}{D}-p$$

$$-\frac{\Delta_{l,1}^{M}}{4D}\{h+p\}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[u_{l,i}^{M,c}+n_{l,i}^{M}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[u_{l,i}^{M,m}+n_{l,i}^{M}\right]^{-l_m}\right]\right\}$$

$$+\frac{\Delta_{h,1}^{M}}{4D}\{h+p\}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[u_{h,i}^{M,c}+n_{h,i}^{M}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[u_{h,i}^{M,m}+n_{h,i}^{M}\right]^{-l_m}\right]\right\} \quad (4.66)$$

$$\frac{\partial^2 TCU_{FL}(n,M)}{\partial^2 M} = n(h+p)D^{-1} \tag{4.67}$$

Eq. (4.67) is strictly greater than zero since $n, h, p, and\ D > 0$. This entails that the total cost function is convex in M and it can be optimized with respect to this variable. Thus, by letting the first derivative with respect to M to zero, with the result is given as follows:

$$\frac{\partial TCU_{FL}(n,M)}{\partial M} = 0 \rightarrow \frac{nMh}{D} + \frac{nMp}{D} =$$

$$p + \frac{\Delta_{l,1}^{M}}{4D}\{h+p\}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[u_{l,i}^{M,c}+n_{l,i}^{M}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[u_{l,i}^{M,m}+n_{l,i}^{M}\right]^{-l_m}\right]\right\}$$

$$-\frac{\Delta_{h,1}^{M}}{4D}\{h+p\}\left\{\left[1+\alpha\sum_{i=2}^{n}\left[u_{h,i}^{M,c}+n_{h,i}^{M}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[u_{h,i}^{M,m}+n_{h,i}^{M}\right]^{-l_m}\right]\right\} \tag{4.68}$$

Which after few steps of mathematical computation the following expression is derived:

$$M^* = \frac{pD}{n(h+p)}$$

$$+\frac{0.25}{n}\left\{\begin{array}{l} \Delta_{l,1}^{M}\left(\left[1+\alpha\sum_{i=2}^{n}\left[u_{l,i}^{M,c}+n_{l,i}^{M}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[u_{l,i}^{M,m}+n_{l,i}^{M}\right]^{-l_m}\right]\right) \\ -\Delta_{u,1}^{M}\left(\left[1+\alpha\sum_{i=2}^{n}\left[u_{u,i}^{M,c}+n_{u,i}^{M}\right]^{-l_c}\right]+\left[1+(1-\alpha)\sum_{i=2}^{n}\left[u_{u,i}^{M,m}+n_{u,i}^{M}\right]^{-l_m}\right]\right) \end{array}\right\} \tag{4.69}$$

Having the optimal quantity of the maximum inventory, the optimal quantity of the number of orders can be derived using the algorithm developed for the previous models. Since the algorithm is similar to the one developed for the previous models, readers are referred to the previous models.

4.3 Summary of the chapter

This chapter addressed the mathematical development of the fuzzy EOQ-S model with learning and forgetting of the inventory operator in estimating the imprecise parameters. Totally, four models were developed that included the learning and forgetting traits of the operator in adjusting the parameters, where the learning capability of the operator follows the log-linear learning curve and the curve with cognitive and motor learning. The assumptions of the inventory models formulated on the basis of the result obtained from the semi-structured interview with a number of experts in the inventory management area. Eventually, the mathematical models were optimized through developing an optimization algorithm to derive the optimal solutions of the models.

CHAPTER 5: RESULTS AND DISCUSSIONS

In this chapter, the models developed in previous chapter will be comprehensively examined using both numerical analysis and case study. Firstly, they will be tested against the secondary data obtained from the literature. This will help evaluate the models in comparison to an earlier model in the literature and allow beneficial insights on how learning and forgetting on imprecise parameters can affect inventory planning. The second part of this chapter will include an empirical study, which is conducted in a case company of automobile industry. For this, the current status of the inventory system of the company will be analyzed. Subsequently, the models for the case company will be utilized to derive the optimal policy in consideration with the learning capability of the planning staff.

5.1 Numerical illustration

To illustrate the effect of human learning and learning transfer on the optimal policy of the fuzzy EOQ-S model, the developed models for this thesis are examined using a test problem. The values of the parameters for the numerical analysis are adopted from the study of Björk (2009), which was later used by Kazemi et al. (2010). This allows to compare the result of the developed models with the past literature and gain insights on the effect of human learning. The data used in the study of Björk (2009) is adopted from a paper distributor company. Consider an inventory system with backorders for a paper distributor where demand is 50000 kg annually with the deviation of $\Delta_l^D = 5000$ kg/year and $\Delta_h^D = 10000$ kg/year. The cost of each paper is 1 euro/kg and for every order the distributor spends 200 Euros. The holding price for storing papers at the distributor is 0.25 of the purchase price, which is equal to 0.25 euro/kg/annum. The penalty cost for paper shortage is 5 euro/kg/annum indicating a delivery rate about 95%. The supply lead times are 10 days with the deviation $\Delta_l^L = 5$ and $\Delta_u^L = 10$ days. According to the assumption made in developing the models, the information available at the beginning of planning period is the ones provided above, but the parameters for the ongoing planning should be calculated. Throughout the planning horizon, the required data about the imprecise parameters could be collected and analyzed, and therefore the information will be updated at the beginning of every planning cycle. To plan inventory, the operator refers to the expertise and the knowledge gained from the earlier planning cycles and adjusts the imprecise parameters accordingly. In order to facilitate the computation process, all the computations were implemented with the help of Microsoft Excel 2013. As four different models were developed, the model's analyses is divided into four different subsections in the following where each model will be analyzed separately. Whenever there is a necessity to compare the models jointly, they will be compared together to gain further insights. It is necessary to note that the numerical analysis section is designed in order to respond to the research question of the thesis.

For the models with learning effect, 9 different learning scenarios are considered and the learning rates (LR) are set from very slow learning (95%, which is equal to learning exponent 0.074) to very fast (55%, which is equal to learning exponent 0.862). Readers are referred to Jaber (2006) for a comprehensive discussion on the classification of learning rates.

5.2 Analyzing the model with log-linear learning curve and full transfer of learning

In Table 5.1, the impact of different learning rates on the optimal policies of the model (including the total cost, the batch size and the maximum inventory level) is examined, and the results are compared with the fuzzy (the model developed by Bjork, 2009) and crisp models. The optimal policy for the crisp model is calculated using Eqs. (3.1), (3.2) and (3.3), for the fuzzy model is calculated using Eqs (4.6), (4.15) and (4.16), and for the model with learning effect is calculated using Eqs. (4.22) and (4.25).

The result of the example shows that considering human learning in the fuzzy EOQ model changes the optimal policy significantly compared to the crisp and fuzzy models. As observed from the results, apart from the matter that with which learning rate the operator learns, bringing operator's learning into consideration leads to decreasing the total cost function. The effect of learning on the total cost of the system tells that the model with learning is more profitable for the buyer as it helps to reduce the total cost of the inventory system. Since the total cost of inventory systems increases as a result of imprecise input parameters, adopting learning could be a viable strategy to counter the detrimental effect of imprecise environment on the system's performance.

Table 5.1: Comparing the fuzzy EOQ-S model with log-linear learning curve with fuzzy (Bjork, 2009) and crisp models

Fuzzy EOQ-S model with human learning						**Fuzzy EOQ-S model (Bjork, 2009)**			
$\boldsymbol{l}$	$\boldsymbol{LR}$(%)	$\boldsymbol{n}^*$	$\boldsymbol{Q}^*$	$\boldsymbol{M}^*$	$\boldsymbol{TCU_{FL}}$	$\boldsymbol{Q}^*$	$\boldsymbol{M}^*$	$\boldsymbol{TCU_F}$	$\boldsymbol{n}^*$
0.074	95	5	10000	9652.16	2264.22	9699.16	9408.53	2341.07	5
0.152	90	5	10000	9621.50	2240.48	**Basic EOQ-S model**			
0.234	85	5	10000	9600.05	2228.44	$\boldsymbol{Q}^*$	$\boldsymbol{M}^*$	$\boldsymbol{TCU}$	$\boldsymbol{n}^*$
0.322	80	5	10000	9585.09	2222.25	9165.15	8728.72	2182.18	5
0.415	75	5	10000	9575.06	2219.13				
0.515	70	5	10000	9568.42	2217.50				
0.621	65	5	10000	9564.21	2216.66				
0.737	60	5	10000	9561.54	2216.20				
0.862	55	5	10000	9560	2215.96				

Looking at the number of orders, the order quantities and the maximum inventory level, it is noted that while there is no difference between the number of orders for the fuzzy models with and without learning, the order quantities and the maximum inventory levels increase as a result of learning. However, through further analysis it is noticed that the optimal policy of the fuzzy EOQ-S model with human learning is highly dependent on demand and deviation values of fuzzy parameters. Therefore, an additional numerical analysis is conducted to gain further insights into how the inventory policy of the model with learning changes with different model parameters. For this purpose, demand and its deviation values are doubled (demand is annually 100,000 kg with the deviation of $\Delta_l^D=$ $10{,}000$ kg/year and $\Delta_h^D= 20{,}000$ kg/year). The results of the changes in demand and the related parameters are brought in Table 5.2 and compared to the fuzzy and crisp models.

By increasing demand value, the total inventory cost is found to drop steadily similar to the previous set of data. However, in contrast to what observed before, the frequency of orders changes with learning as it increases by learning rate, which, consequently, leads to decreasing the batch size and the optimal maximum inventory. So, one could conclude that incorporating learning into fuzzy EOQ-S model leads to increasing the number of orders, while decreasing the batch sizes, the maximum level of stock and the total cost function. The rational for this result is that while retaining the earlier knowledge in planning, the operator catches up more with the characteristics of imprecise parameters over time and tends to order and keep stock more.

Table 5.2: The effect of doubling demand on the fuzzy EOQ-S model with log-linear learning curve

Fuzzy EOQ-S model with human learning						**Fuzzy EOQ-S model (Bjork, 2009)**			
l	LR(%)	n^*	Q^*	M^*	TCU_{FL}	Q^*	M^*	TCU_F	n^*
0.074	95	7	14286	13857	3273	14286.24	13948.42	3401.49	7
0.152	90	8	12500	12088	3207	**Basic EOQ-S model**			
0.234	85	8	12500	12040.80	3170	Q^*	M^*	TCU	n^*
0.322	80	8	12500	12008.04	3151	12961.48	12344.27	3089.74	8
0.415	75	8	12500	11985.99	3142.36				
0.515	70	8	12500	11971.26	3138				
0.621	65	8	12500	11961.84	3135.91				
0.737	60	8	12500	11955.81	3134.84				
0.862	55	8	12500	11952.17	3134				

A closer look at Table 5.2 further reveals that when the learning rate is slow (when learning exponent is closer to 0.074), larger batches should be ordered, which incurs a higher total annual cost. In contrast, when learning becomes faster (learning exponent increases from 0.074 to 0.862) the system tends to order smaller batches, incurring lower expected total annual cost. Figure 5.1 illustrates the impact of different learning rates on the total cost and compares it with fuzzy and crisp models.

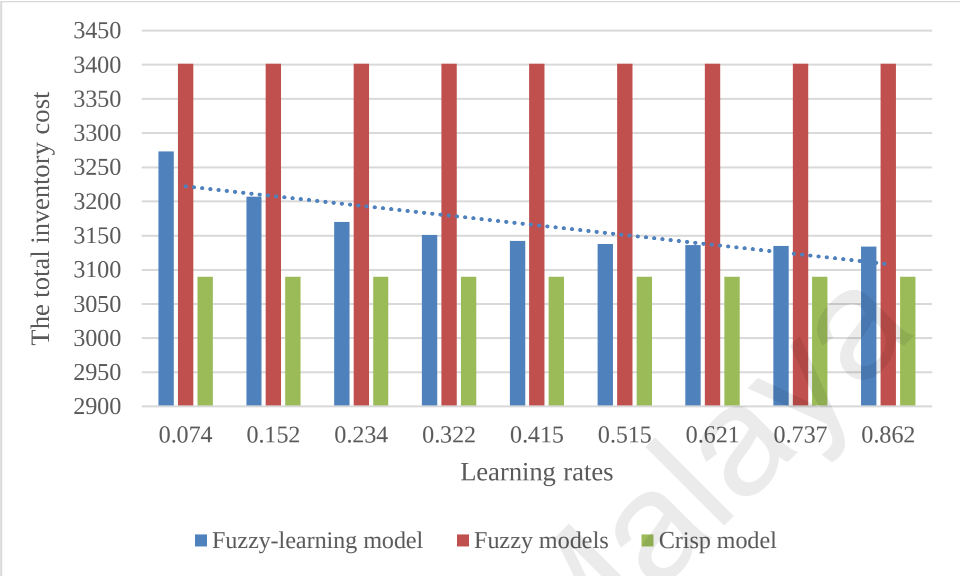

Figure 5.1: Comparing the effect of learning on the total cost of the three models

5.3 Analyzing the model with log-linear learning curve and forgetting

In this section of the numerical example, the behavior of the model when forgetting phenomenon is introduced into the system will be examined. The primary objective is to figure out what changes the loss of experience will incur compared to the case where the operator remembers all the gained experience. In order to have better benchmark and be able to derive conclusions, the same data as given in Section 5.2 are used and the results are compared with the case of full transfer of learning. Besides the data introduced earlier, it is further assumed that total forgetting occurs after 500 days and the duration of cessation in each cycle is 10 days. Table 5.3 summarizes the result of numerical example for forgetting scenario, with the learning rates arranged from very slow to very fast, and also compares the optimal policies with full transfer of learning situation, discussed in Section 5.2. Table 5.3 indicates that there is no change in the batch sizes and number of orders, however, oppositely, the maximum inventory level and the total cost change under forgetting. When the operator tines a part of his knowledge, it leads him to determine a lower amount for the total maximum inventory, which in turn decreases the total cost of the inventory system. In contrast to what expected, forgetting case is a profitable option for the buyer as the buyer can save more cost in this case. This can be interpreted in the way that since a part of the knowledge is lost due to forgetting and the operator does not have full knowledge about the characteristics of the inventory data, the operator tends to consider smaller quantities for the stocked inventory. Like the case of full transfer of learning, the buyer can benefit from faster learning in forgetting situation, as when learning becomes faster, the buyer can reduce inventory cost more. The behavior of the maximum inventory in both cases is found the same as they tend to decrease as the learning rate increases.

Table 5.3: Comparing the fuzzy models with full transfer of learning and forgetting (B=500 days)

Fuzzy EOQ-S model with full transfer of learning						**Fuzzy EOQ-S model with forgetting**					
$\boldsymbol{l}$	$\boldsymbol{LR}$(%)	$\boldsymbol{n}^*$	$\boldsymbol{Q}^*$	$\boldsymbol{M}^*$	$\boldsymbol{TCU_{FL}}$	$\boldsymbol{l}$	$\boldsymbol{LR}$(%)	$\boldsymbol{n}^*$	$\boldsymbol{Q}^*$	$\boldsymbol{M}^*$	$\boldsymbol{TCU_{FL}}$
0.074	95	5	10000	9652.16	2264.21	0.074	95	5	10000	8994	2147
0.152	90	5	10000	9621.50	2240.48	0.152	90	5	10000	8971	2137
0.234	85	5	10000	9600.05	2228.44	0.234	85	5	10000	8954.69	2132
0.322	80	5	10000	9585.09	2222.25	0.322	80	5	10000	8943.53	2129
0.415	75	5	10000	9575.06	2219.12	0.415	75	5	10000	8936.06	2127.03
0.515	70	5	10000	9568.42	2217.50	0.515	70	5	10000	8931.08	2126
0.621	65	5	10000	9564.21	2216.66	0.621	65	5	10000	8927.91	2125.50
0.737	60	5	10000	9561.54	2216.20	0.737	60	5	10000	8925.89	2125.18
0.862	55	5	10000	9560	2215.96	0.862	55	5	10000	8924.66	2124.99

Comparing the results in Table 5.3 with Table 5.1 also makes it possible to draw more conclusions. To help in realizing the trend of the data in both tables, the maximum inventory and the total cost of four models, crisp, fuzzy, fuzzy model with full transfer of learning, and fuzzy model with forgetting, are plotted in Fig. 5.2 and Fig. 5.3, respectively. Looking at Fig. 5.2, it can be clearly seen that the crisp model has the lowest maximum inventory among the four models, followed by the forgetting model being in the second rank, with both having the figure around 8800. The figures for the model with full transfer of learning is closer to that of the fuzzy model and exceeds both of forgetting and crisp models. As discussed earlier, the similar pattern is observed for the models with full transfer of learning and forgetting, which is to decline with faster learning. However, as it is clear visually, the model with forgetting degrades with a fairly faster rate than the model with full transfer of learning.

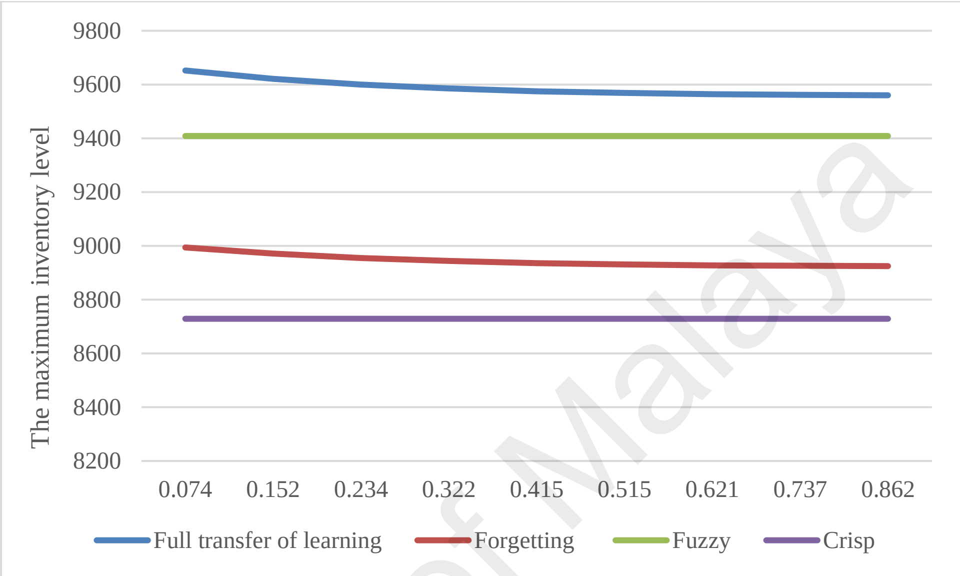

Figure 5.2: Comparison of the maximum inventory model of the four models

Figure 5.3 also reveals some further findings. As it is apparent, interestingly, forgetting model gives the lowest total cost among the compared models, even lower than the crisp model. Furthermore, it can be seen that the inventory system under both learning scenarios performs better than the fuzzy model, which highlights the importance of retaining knowledge in an imprecise inventory system. Generally, one can conclude that the more the knowledge in imprecise parameters is kept, the more the benefit of the inventory system.

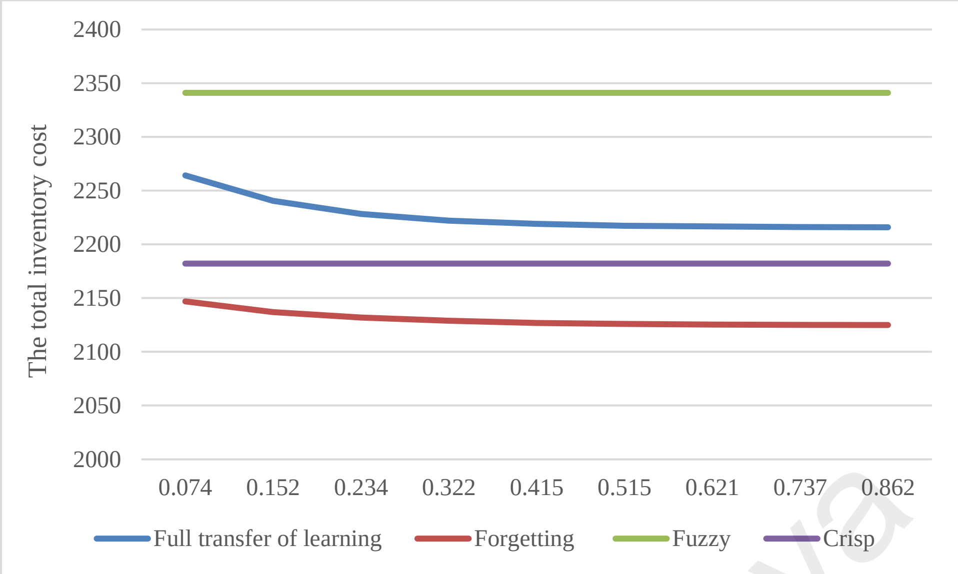

Figure 5.3: Comparison of the total cost of the four models

Table 5.4: Comparing the fuzzy models with full transfer of learning and forgetting when demand is doubled (B=500 days)

Fuzzy EOQ-S model with full transfer of learning						Fuzzy EOQ-S model with forgetting					
l	LR(%)	n^*	Q^*	M^*	TCU_{FL}	l	LR(%)	n^*	Q^*	M^*	TCU_{FL}
0.074	95	7	14286	13857	3273	0.074	95	7	14286	12850	3045
0.152	90	8	12500	12088	3207	0.152	90	7	14286	12818	3029
0.234	85	8	12500	12040.80	3170	0.234	85	7	14286	12796	3020
0.322	80	8	12500	12008.04	3151	0.322	80	7	14286	12779	3016
0.415	75	8	12500	11985.99	3142.36	0.415	75	7	14286	12768	3013
0.515	70	8	12500	11971.26	3138	0.515	70	7	14286	12761	3011
0.621	65	8	12500	11961.84	3135.91	0.621	65	7	14286	12756	3011
0.737	60	8	12500	11955.81	3134.84	0.737	60	7	14286	12752	3010
0.862	55	8	12500	11952.17	3134	0.862	55	7	14286	12750	3010

In the former numerical analysis, it has seen that the batch sizes and the number of orders have not changed using the data set given in Björk (2009) due to the value of demand, and therefore additional numerical study is conducted with another set of data. Likewise, the same set of data have applied for forgetting scenario, with the result provide in Table 5.4. With this set of data, the results are slightly different from the data set given in Björk (2009). When forgetting is accounted for in this case, the order quantities rise (compared to the full transfer of learning model), as the buyer replenishes the orders in smaller number of orders. When it comes to the level of stock the policy changes as the buyer tends to keep higher level of inventory compared to full transfer of learning. Therefore, according to the results, when demand is doubled the optimal policy in forgetting case is to decrease the number of orders, which as a result increases the order quantity and reduces the total cost.

5.4 Analyzing the model with cognitive and motor learning curve and full transfer of learning

In this section, the effect of cognitive and motor learning on the fuzzy EOQ-S model will be evaluated. Therefore, similar to the two previous sections, a numerical analysis is conducted whereby the model developed in Chapter 4 is investigated using different rates of learning and varying weights for both cognitive and motor components. As it was empirically observed by Dar-el et al. (1995) and emphasized later by some studies such as Jaber and Kher (2002) and Jaber and Glock (2013), it is common in the real cases that, with every repetition, humans call up the process or steps learnt before at a faster rate. Thus, it is logical that the learning rate for the cognitive part of a task is higher than that of the motor part. For this purpose, the assumption $l_c > l_m$ is made throughout the numerical analysis of this and the next section. Table 5.5 indicates two different sets of the results for the model applying $\alpha = 0.2$ and $\alpha = 0.8$, which the first one represents the task that is mostly motor while the second figure refers to the task that is mostly cognitive. The learning rates are also set so that they cover a wide range of the operator's learning, following the inequality $l_c > l_m$. The first impression of the model's outcome is that the number of orders and the order quantities are still remained fix at 5 and 10000, respectively, like the learning scenario with the log-linear learning curve. Furthermore, when the planning task is mainly cognitive, the operator tends to set up lower amount for the maximum inventory at hand, decreasing the total cost as a result. To have a better comparison with the basic fuzzy model, Figs. 5.4 and 5.5 are plotted to compare the result of the model with the one of Björk (2009).

Table 5.5: The results of fuzzy EOQ-S model with cognitive and motor learning

Fuzzy EOQ-S model with cognitive and motor learning $(\alpha = 0.2)$					Fuzzy EOQ-S model with cognitive and motor learning $(\alpha = 0.8)$				
(l_c, l_m)	n^*	Q^*	M^*	TCU_{FL}	(l_c, l_m)	n^*	Q^*	M^*	TCU_{FL}
(0.152, 0.074)	5	10000	9680.27	2302.39	(0.152, 0.074)	5	10000	9661.88	2288.49
(0.234, 0.074)	5	10000	9675.98	2298.90	(0.234, 0.074)	5	10000	9644.72	2277.99
(0.322, 0.074)	5	10000	9672.99	2296.54	(0.322, 0.074)	5	10000	9632.75	2272.09
(0.415, 0.074)	5	10000	9670.99	2295.00	(0.415, 0.074)	5	10000	9624.73	2268.78
(0.515, 0.074)	5	10000	9669.66	2293.99	(0.515, 0.074)	5	10000	9619.42	2266.87
(0.621, 0.074)	5	10000	9668.28	2292.97	(0.621, 0.074)	5	10000	9616.04	2265.78
(0.737, 0.074)	5	10000	9668.28	2292.97	(0.737, 0.074)	5	10000	9613.91	2265.14
(0.862, 0.074)	5	10000	9667.97	2292.73	(0.862, 0.074)	5	10000	9612.65	2264.77
(0.515, 0.415)	5	10000	9607.98	2263.62	(0.515, 0.415)	5	10000	9604.00	2262.71
(0.621, 0.415)	5	10000	9607.14	2263.41	(0.621, 0.415)	5	10000	9600.63	2262.05
(0.737, 0.415)	5	10000	9606.61	2263.29	(0.737, 0.415)	5	10000	9598.50	2261.68
(0.862, 0.415)	5	10000	9606.29	2263.21	(0.862, 0.415)	5	10000	9597.23	2261.47
(0.862, 0.737)	5	10000	9595.47	2261.22	(0.862, 0.737)	5	10000	9594.52	2261.09

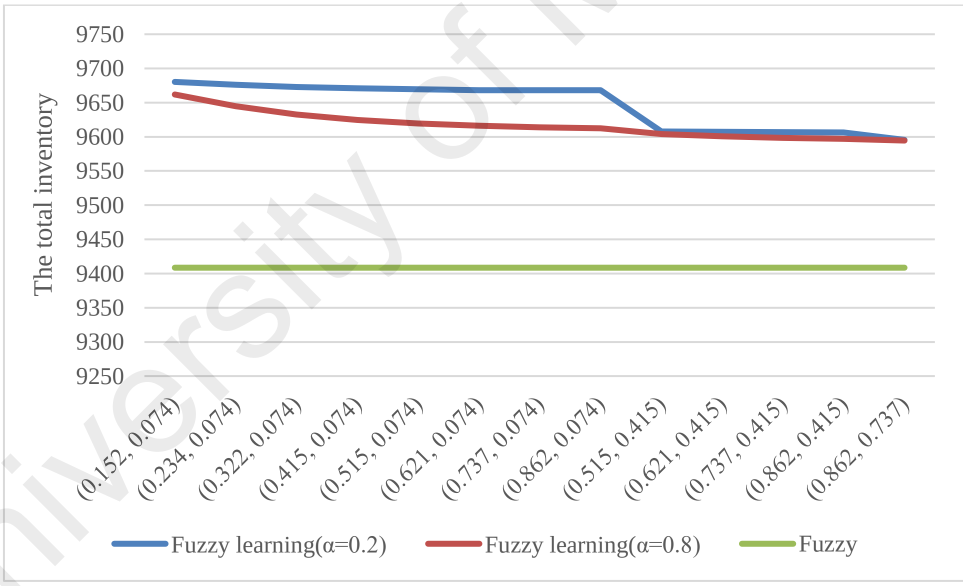

Figure 5.4: Comparing the fuzzy learning model with cognitive and motor capabilities with fuzzy model (Bjork, 2009) in terms of the maximum inventory level

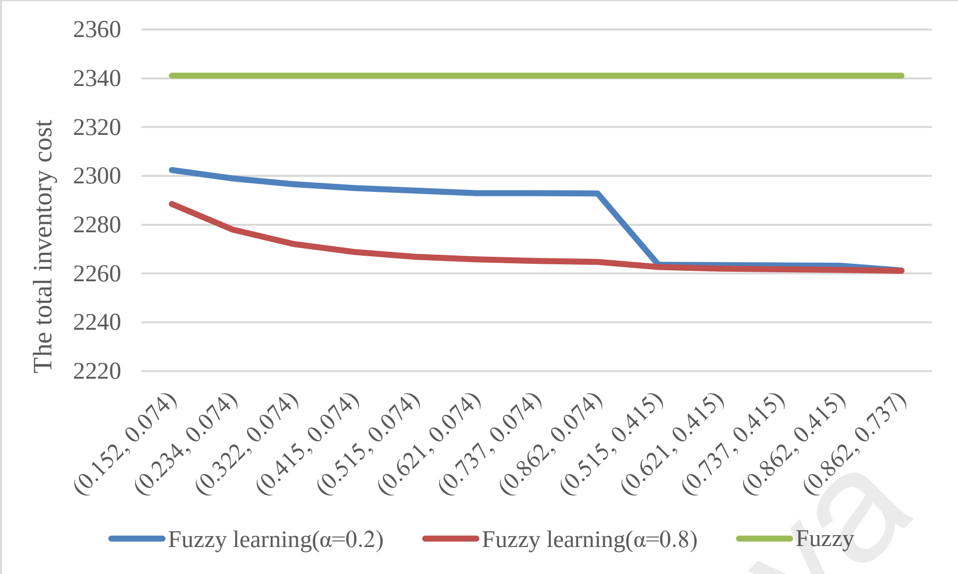

Figure 5.5: Comparing the fuzzy learning model with cognitive and motor capabilities with fuzzy model (Bjork, 2009) in terms of the total cost

According to Fig. 5.4, the learning model with cognitive and motor capabilities gives a higher amount for the maximum inventory level, irrespective of either the learning curve is cognitive or motor. As to comparing the cognitive and motor parts of the model, although the model with mostly motor capability stays upper for some learning rates, both models become the same as the difference between l_c and l_m becomes lower.

As to the total cost, Fig. 5.5 demonstrates that the performance of the model after introducing the learning with cognitive and motor capabilities is improved. Therefore, like the log-linear learning curve, this type of learning can also help the inventory operator to improve the performance and overcome the uncertainty. Furthermore, the inventory system can benefit more from the learning if the planning task becomes more cognitive (see the red line in figure 5.5).

Like the numerical analysis set up for other sections and similar to the computations in Table 5.4, the model is run when demand is increased two times while keeping other parameters constant at their value. The results are tabulated in Table 5.6. The results show that, when demand is doubled, it is beneficial for the buyer to order with higher frequency, which consequently lowers the batch size. Even though increasing the number of orders affords higher ordering cost, the results of the model suggest that this can save inventory cost for the buyer compared to the fuzzy model without learning. Regarding the impact of the increase in demand on the maximum inventory level, the general policy is to stock lower amount of inventory, however the model with the dominant cognitive capability recommends keeping slightly lower inventory than the model with the dominant motor capability. Similarly, the total cost of the inventory with the dominant cognitive capability has the potential to decrease more than the model with the dominated motor capability, which coincides with the results obtained earlier.

Table 5.6: The results of the fuzzy EOQ-S model with cognitive and motor learning when demand is doubled

Fuzzy EOQ-S model with cognitive and motor learning ($\boldsymbol{\alpha = 0.2}$)					Fuzzy EOQ-S model with cognitive and motor learning ($\boldsymbol{\alpha = 0.8}$)				
(l_c, l_m)	n^*	Q^*	M^*	TCU_{FL}	(l_c, l_m)	n^*	Q^*	M^*	TCU_{FL}
(0.152, 0.074)	5	10000	13892.43	3340.56	(0.152, 0.074)	5	10000	13853.19	3302.53
(0.234, 0.074)	5	10000	13883.26	3330.91	(0.234, 0.074)	5	10000	12106.29	3269.85
(0.322, 0.074)	5	10000	13876.84	3324.40	(0.322, 0.074)	5	10000	12080.08	3251.52
(0.415, 0.074)	5	10000	13872.52	3320.13	(0.415, 0.074)	5	10000	12062.44	3241.43
(0.515, 0.074)	5	10000	13869.65	3317.34	(0.515, 0.074)	5	10000	12050.66	3235.70
(0.621, 0.074)	5	10000	13867.81	3315.57	(0.621, 0.074)	5	10000	12043.12	3232.45
(0.737, 0.074)	5	10000	13866.64	3314.45	(0.737, 0.074)	5	10000	12038.30	3230.55
(0.862, 0.074)	5	10000	13865.94	3313.78	(0.862, 0.074)	5	10000	12035.38	3229.46
(0.515, 0.415)	5	10000	12025.85	3226.70	(0.515, 0.415)	5	10000	12017.02	3224.25
(0.621, 0.415)	5	10000	12023.97	3226.12	(0.621, 0.415)	5	10000	12009.48	3222.51
(0.737, 0.415)	5	10000	12022.76	3225.77	(0.737, 0.415)	5	10000	3256.79	3221.58
(0.862, 0.415)	5	10000	12022.03	3225.55	(0.862, 0.415)	5	10000	12001.74	3221.08
(0.862, 0.737)	5	10000	11997.89	3220.54	(0.862, 0.737)	5	10000	11995.71	3220.26

The numerical analysis in the prior section has illustrated that the model is sensitive to variation of α. Therefore, to have a better perception with regard to the impact of α on the optimal policy of the model, its value is varied over the interval [0, 1] and its effect is evaluated on the total cost and the maximum inventory. In addition, four learning rates are selected and used for the analysis so as one side of the learning curve, cognitive or motor, is more dominant at a time. Fig.5.6 represents the behavior of the total cost function described by Eq. (4.42) for four different pairs of learning rates and different weights of cognitive tasks (and as a result motor tasks).

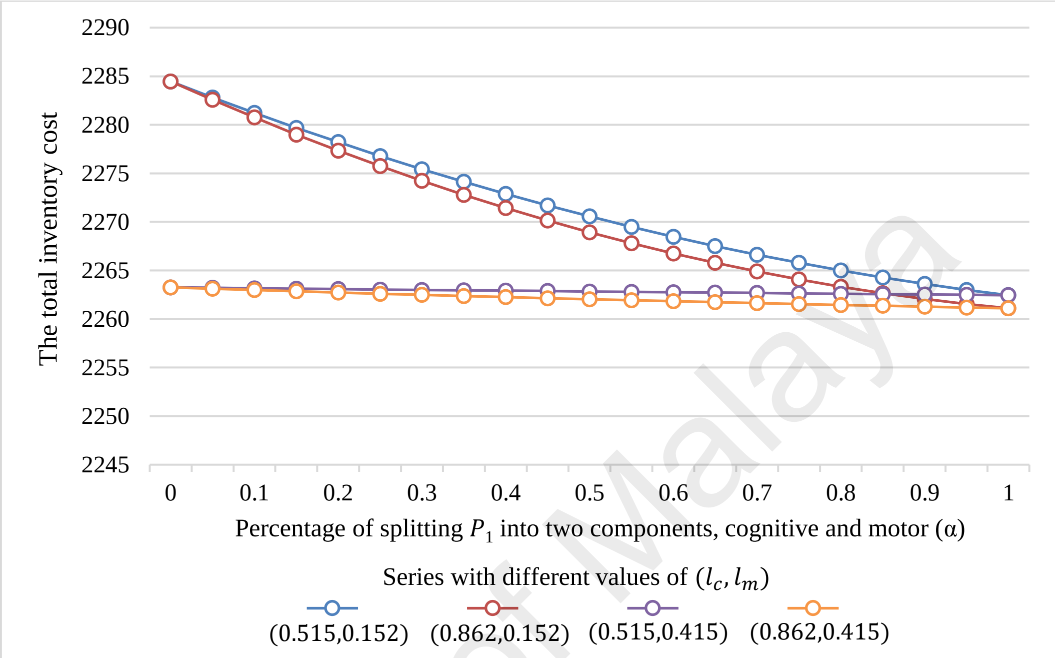

Figure 5.6: Comparing the effect of different values of α and cognitive and motor learning exponents on the total cost function

The pattern of the total cost in Fig.5.6 shows that, irrespective of the learning rate, as the task becomes more cognitive, the system's performance in terms of the total cost improves. Nevertheless, the total inventory cost showed a more profound change by faster learning in the cognitive part rather than faster learning in the motor part. According to the result, bringing in workforce learning ability into consideration in fuzzy EOQ-S model gives clear managerial and operational implications highlighting the importance that training programs for the workforce have on the total cost of the inventory system. This will be discussed later in Chapter 6.

Another impression from Fig.5.6 is the change observed by faster learning. That is, the inventory system benefits from faster learning in either cognitive or motor form as the total inventory cost decreases in both cases. However, the reduction caused by faster learning in the motor part of the task is much more significant than the one resulted from the cognitive part. The effect of human learning on reducing the total cost of the inventory in deterministic models has already been reported in the literature (see e.g. Jaber et al., 2008; Zanoni et al., 2012; Grosse & Glock, 2013). The finding here is in line with the literature on deterministic models and shows that for both deterministic and fuzzy lot-sizing problems, personal training for the purpose of fostering learning among workforces could be a useful tool for managers, who wish to decrease the total cost of the system.

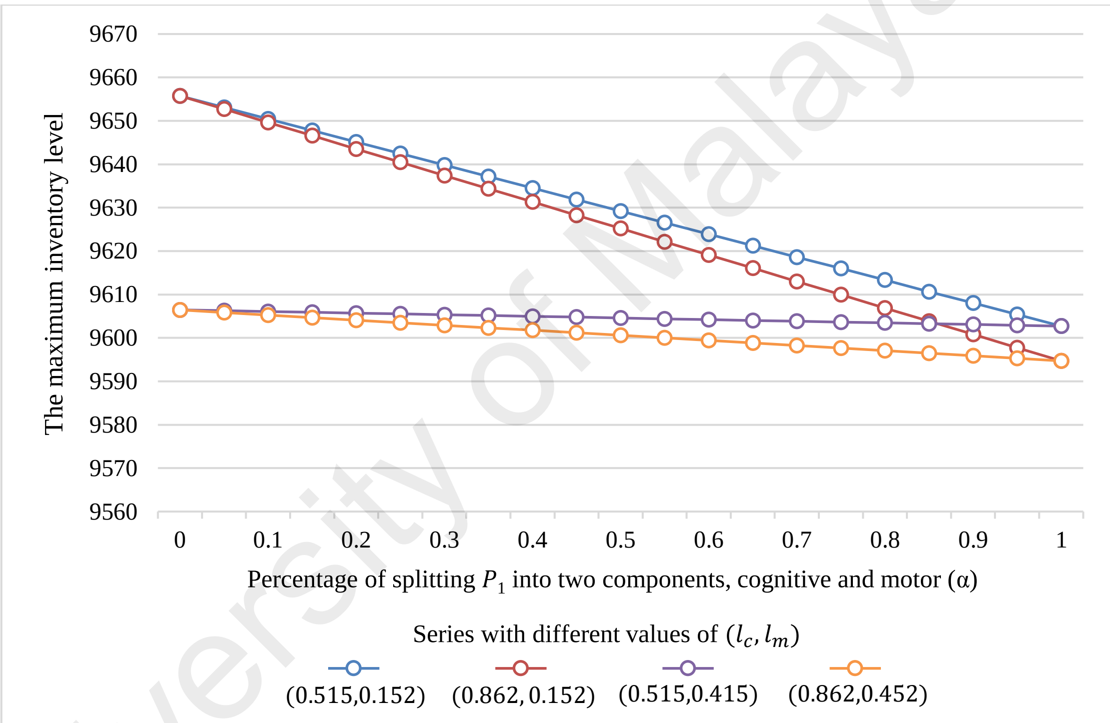

Figure 5.7: Comparing the effect of different values of α and cognitive and motors learning exponents on the maximum inventory level

To gain further insights into the optimal policy of the model, the question of how the two-stage learning affects the maximum inventory level is addressed. Fig. 5.7 depicts the variation of the maximum inventory by varying the amount of α stepwise over an interval from 0 to 1, with the learning rates set the same as in Fig. 5.6. Similar to the total annual inventory cost, which is observed to be lower when the operation is more cognitive, the same could be perceived from the figures for the maximum inventory level. The finding implies that the higher the cognitive task, the more the maximum inventory could reduce.

Moreover, through faster learning, obtained by an increasing learning exponent from 0.514 to 0.814 for l_c and from 0.152 to 0.452 for l_m, the inventory system tends to hold less inventory. Nevertheless, the impact of learning on the cognitive and motor elements is found to be unequal. Strictly speaking, while faster learning leads to a slight change in the maximum stock level when the cognitive part of the task is considered, this phenomenon results in a dramatic drop in the maximum stock level when the motor part of the task is taken into consideration (blue line vs. green, red line vs. purple).

5.5 Analyzing the model with cognitive and motor learning curve and forgetting

The numerical study continues in this section by evaluating the model with cognitive and motor learning abilities while forgetting affects transfer of learning. The settings of the learning parameters in this section are similar to the ones used in Section 4.4. According to the justifications provided in Section 4.4, the assumption $l_c > l_m$ is made when setting learning rates. On the other hand, the outcome of the model is computed for two states, i.e. when the operator's cognitive or motor ability is dominant, one at a time.

Tables 5.7 and 5.8 compare the impact of full transfer of learning on the model with the one of forgetting. The same behavior as observed for other models is repeated here for the order quantity and the number of orders. The results in Tables 5.7 and 5.8 show that there is an opportunity to save the total cost when forgetting is introduced, which is the same when is compared against the model with log-linear learning and forgetting. However, the maximum inventory showed to be equal for the models with and without learning, which is in contrast to what observed before for the model with log-linear learning and forgetting. Furthermore, the performance of the model with $\alpha = 0.8$ (model with mostly cognitive ability in learning) is better than the model with $\alpha = 0.2$ (model with mostly motor ability in learning) which gives a clue to investigate the effect of α on the performance of the model.

Table 5.7: Comparing the models with cognitive and motor learning and forgetting when cognitive learning is dominant

Fuzzy EOQ-S model with cognitive and motor learning and full transfer of learning $(\alpha = 0.2)$					Fuzzy EOQ-S model with cognitive and motor learning and forgetting $(\alpha = 0.2)$				
(l_c, l_m)	n^*	Q^*	M^*	TCU_{FL}	(l_c, l_m)	n^*	Q^*	M^*	TCU_{FL}
(0.152, 0.074)	5	10000	9680.27	2302.39	(0.152, 0.074)	5	10000	9680.27	2279.25
(0.234, 0.074)	5	10000	9675.98	2298.90	(0.234, 0.074)	5	10000	9675.98	2275.50
(0.322, 0.074)	5	10000	9672.99	2296.54	(0.322, 0.074)	5	10000	9672.99	2272.97
(0.415, 0.074)	5	10000	9670.99	2295.00	(0.415, 0.074)	5	10000	9670.99	2271.31
(0.515, 0.074)	5	10000	9669.66	2293.99	(0.515, 0.074)	5	10000	9669.66	2270.23
(0.621, 0.074)	5	10000	9668.28	2292.97	(0.621, 0.074)	5	10000	9668.28	2269.56
(0.737, 0.074)	5	10000	9668.28	2292.97	(0.737, 0.074)	5	10000	9668.28	2269.13
(0.862, 0.074)	5	10000	9667.97	2292.73	(0.862, 0.074)	5	10000	9667.97	2268.87
(0.515, 0.415)	5	10000	9607.98	2263.62	(0.515, 0.415)	5	10000	9607.98	2249.27
(0.621, 0.415)	5	10000	9607.14	2263.41	(0.621, 0.415)	5	10000	9607.14	2248.98
(0.737, 0.415)	5	10000	9606.61	2263.29	(0.737, 0.415)	5	10000	9606.61	2248.79
(0.862, 0.415)	5	10000	9606.29	2263.21	(0.862, 0.415)	5	10000	9606.29	2248.68
(0.862, 0.737)	5	10000	9595.47	2261.22	(0.862, 0.737)	5	10000	9595.47	2247.14

Table 5.8: Comparing the models with cognitive and motor learning and forgetting when motor learning is dominant

Fuzzy EOQ-S model with cognitive and motor learning and full transfer of learning $(\alpha = 0.8)$					Fuzzy EOQ-S model with cognitive and motor learning and forgetting $(\alpha = 0.8)$				
(l_c, l_m)	n^*	Q^*	M^*	TCU_{FL}	(l_c, l_m)	n^*	Q^*	M^*	TCU_{FL}
(0.152, 0.074)	5	10000	9661.88	2288.49	(0.152, 0.074)	5	10000	9661.88	2271.17
(0.234, 0.074)	5	10000	9644.72	2277.99	(0.234, 0.074)	5	10000	9644.72	2263.41
(0.322, 0.074)	5	10000	9632.75	2272.09	(0.322, 0.074)	5	10000	9632.75	2258.89
(0.415, 0.074)	5	10000	9624.73	2268.78	(0.415, 0.074)	5	10000	9624.73	2256.27
(0.515, 0.074)	5	10000	9619.42	2266.87	(0.515, 0.074)	5	10000	9619.41	2254.71
(0.621, 0.074)	5	10000	9616.04	2265.78	(0.621, 0.074)	5	10000	9616.04	2253.79
(0.737, 0.074)	5	10000	9613.91	2265.14	(0.737, 0.074)	5	10000	9613.91	2253.24
(0.862, 0.074)	5	10000	9612.65	2264.77	(0.862, 0.074)	5	10000	9612.65	2252.93
(0.515, 0.415)	5	10000	9604.00	2262.71	(0.515, 0.415)	5	10000	9604.00	2248.79
(0.621, 0.415)	5	10000	9600.63	2262.05	(0.621, 0.415)	5	10000	9600.63	2248.26
(0.737, 0.415)	5	10000	9598.50	2261.68	(0.737, 0.415)	5	10000	9598.50	2247.95
(0.862, 0.415)	5	10000	9597.23	2261.47	(0.862, 0.415)	5	10000	9597.23	2247.78
(0.862, 0.737)	5	10000	9594.52	2261.09	(0.862, 0.737)	5	10000	9594.52	2247.08

Figure 5.8 compares the total cost of four cases investigated in Tables 5.7 and 5.8 with that of the fuzzy model. As the figure clearly shows the total cost of the four models stood

below the fuzzy model indicating the cognitive and motor capabilities the operator can help to save the total cost of the inventory. Since the model with learning outperforms the pure fuzzy model, therefore, adopting learning strategy can guarantee that the performance of the inventory system could improve. Furthermore, the model with forgetting and dominant cognitive learning showed to be the best model among the four models in terms of the total cost.

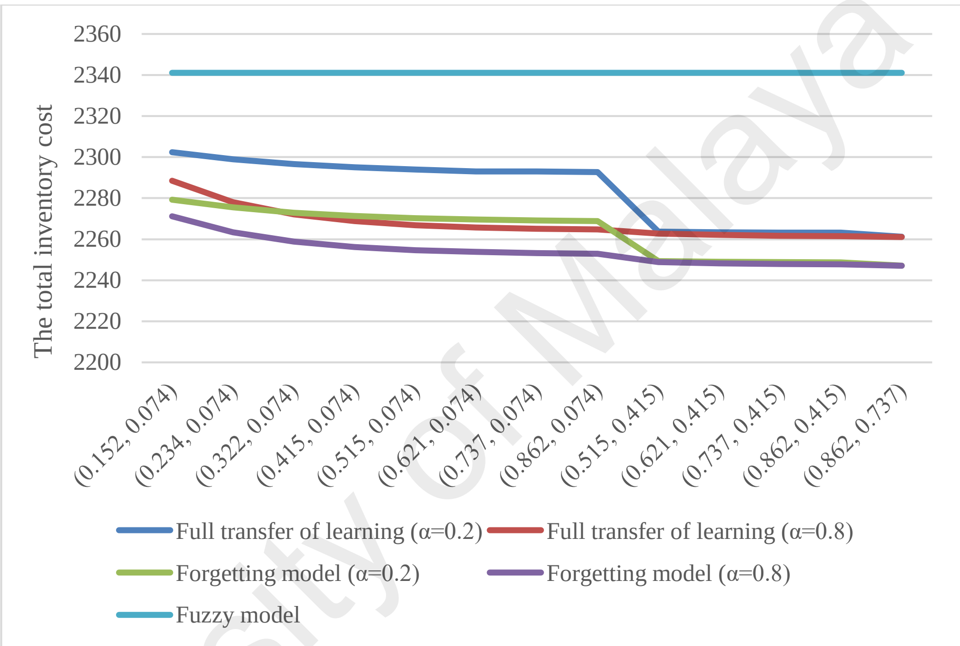

Figure 5.8: Comparing the total cost of the models with cognitive and motor learning and forgetting with fuzzy model

In Figure 5.9, the trend of the total cost function is examined when α is varied stepwise over the interval [0, 1]. The learning rates are selected such that one part of the learning is fixed and another part is changed at a time. So, it is easier to observe the behavior of the model when one learning rate remains fix and another one changes. Totally, six pairs of learning rates are adopted. The main found result is that the more the task is cognitive (the more α value) the more inventory cost could be decreased. This is in line with the result of the model with full transfer of learning. Hence, as the planning task becomes more cognitive it is expected that the inventory system can operate better, and this is unrelated to the fact that whether the operator can remember all the experience or a part of it.

In addition to this fact, however, the effect of faster learning rate in the motor part is higher than the faster learning rate in the cognitive part, as faster learning rate in the motor part can decrease the total cost more than faster learning rate in the cognitive part. This is the same as what it is found in the model with full transfer of learning. Hence, unrelated to the learning ability of the operator, working on his/her motor abilities can be a more effective strategy to improve the performance of the inventory system.

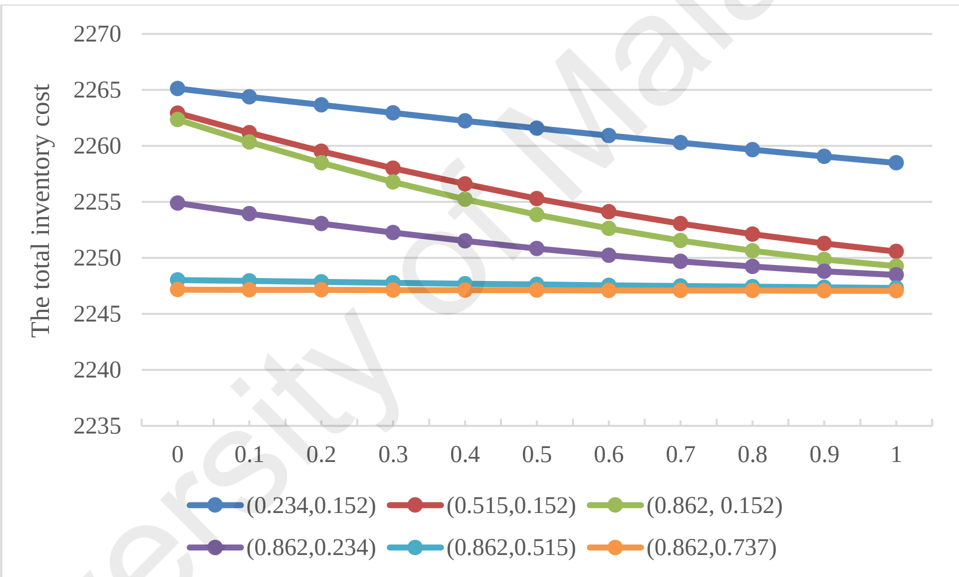

Figure 5.9: Comparing the total cost of the models with cognitive and motor learning and forgetting while varying α

5.6 Case study

In order to show the application of the model in a real-world inventory problem, the models developed in this study are examined on a real case of a manufacturing company, Renault Iran, whose is active in the automotive industry. In the following, an overview of the pilot company under study will be initially presented and the problem of the inventory system will then be described. This is followed by the description on how the model is implemented for the case study.

5.6.1 Renault Group introduction

Established in 1899 by Louis Renault, Marcel Renault, Fernand Renault, Renault Group is a French multinational automobile manufacturer company. The Renault group produces a wide range of products from cars and vans to truck and auto rail vehicles. Referring to the survey of the Organisation Internationale des Constructeurs d'Automobiles in 2013, Renault ranked eleventh among the car manufacturers of the World in terms of the volume of the production. In 1999, Renault and Nissan formed a corporation, Renaul-Nissan Alliance, with Renault taking 54% of Nissan's share, which is now the fourth biggest automotive group in the World. Renault has made partnership with various countries and companies all over the World.

5.6.2 Renault in Iran

Renault's activities in Iran date back to almost four decades ago when the first model was manufactured in December 1976. After that, Renault encountered a lot of restrictions for manufacturing locally and therefore stopped its activity until 1990, when a local manufacturer started to produce Renault 5 and Renault 21.

Since Iran's car market is an emerging one and has been growing enormously over the past years, Renault decided to build a stronger partnership with Iranian main automakers to benefit from its potential and also to transfer the latest technology to Iran. Therefore, Renault and Iran's Industrial Development Renovation Organization (IDRO) established a joint venture in 2004, namely Renault Pars. Renault Pars started its activity in Iran by supplying component parts for local manufacturers, Iran Khodro and Iran Saipa's Pars Khodro. Under the partnership at the time being, Renault Pars is supporting the production of L-90, U-90 partially, with some parts supplying from local manufacturers, while for other models like Koleos Renault Pars is responsible for the full component supply.

5.6.3 Presentation and description of the company

Parallel to establishment of a joint venture company, Renault constituted another company, namely Auto Chassis International Pars Company (ACI Pars Co.), to assemble chassis parts and chassis systems of the Renault's products in Iran. ACI Pars Co. is the only manufacturer of Renault in Iran and is working based on the Renault Production System (SPR), a production system ensuring that Renault is performing at the highest possible performance level. Fig 5.10 illustrates the business area of this company and its position along the supply chain of L-90 and U-90 products. As it is clear from the supply chain of ACI Pars Co., it is not the direct supplier of the car manufacturers, but rather supplies the parts to its sister company Renault Pars.

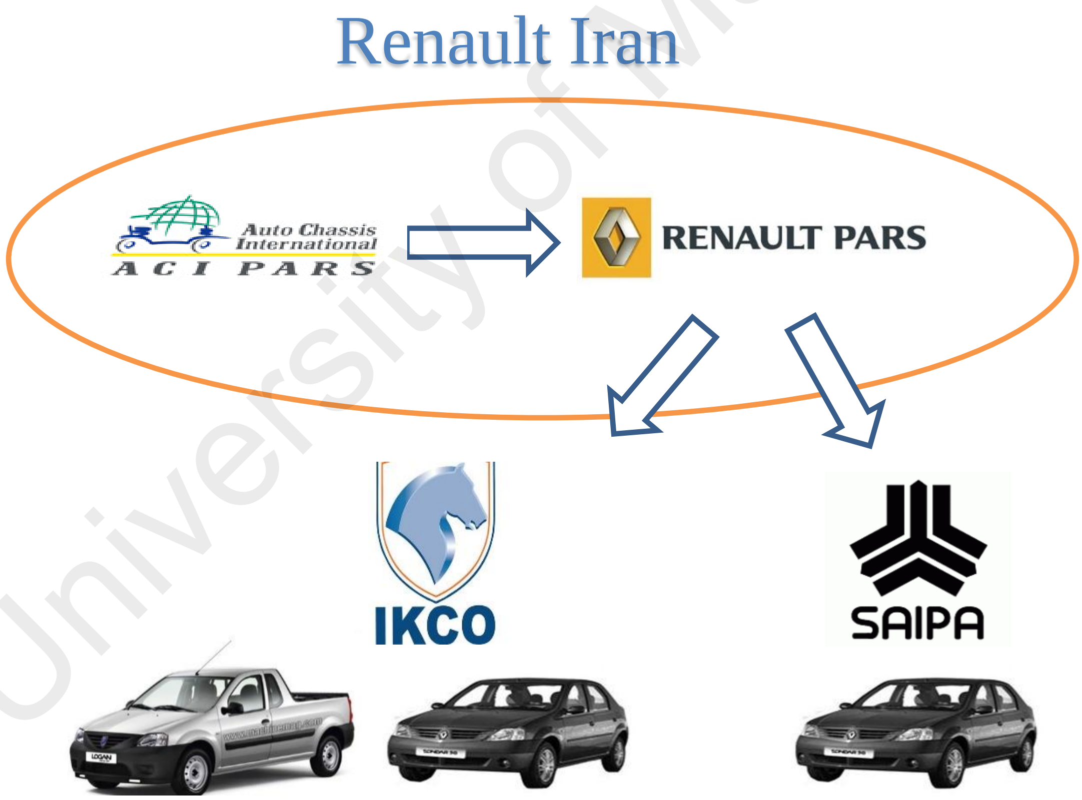

Figure 5.10: ACI Pars Co. business area

The ACI Pars Co.'s plant is located 48 km away from the capital of Iran, Tehran, in Baharestan industrial complex. The plant is in a manufacturing complex jointly used with Robat Machine Co. The company has almost 200 employees. ACI Pars Co. has a production capacity of 200,000 vehicles per year and its main operations include welding, assembly, painting and logistics. Figure 5.11 shows the diagram of the manufacturing processes in ACI Pars Co. and provides the parts which undergo the specific manufacturing process. The plant chiefly manufactures parts for Renault's products for the local market, nonetheless the company sometimes serves as a supplier for other factories of Renault worldwide.

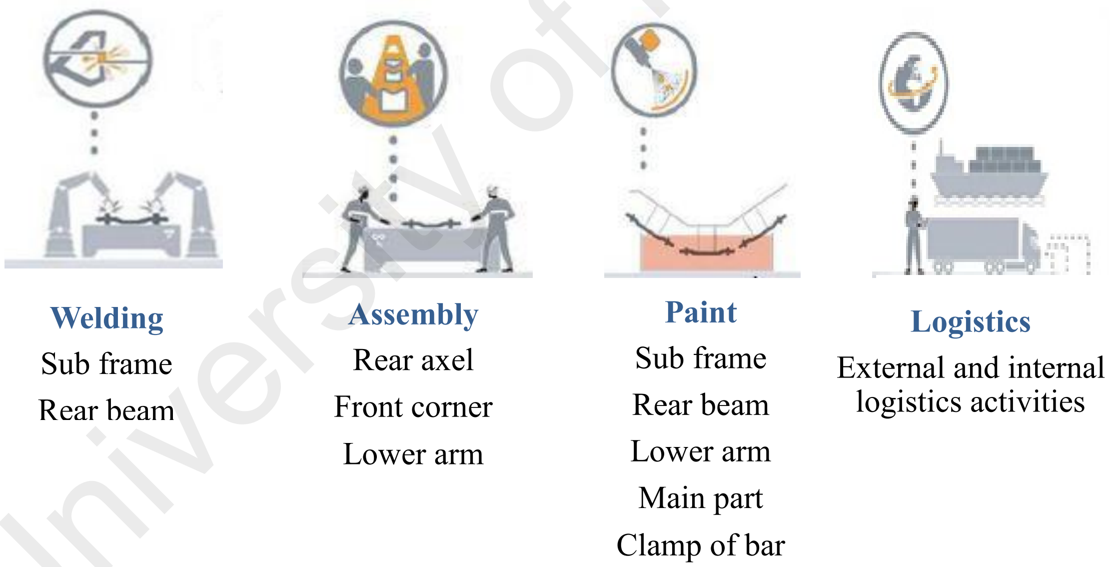

Figure 5.11: Diagram of manufacturing process

5.6.4 Data collection

The aim of studying this company is to present the fuzzy inventory model with human learning and learning transfer in order to decrease the total cost of the inventory system and improve its performance. As the author had been working for the company for almost four years before starting the doctoral study, it was easier to collect data because of the relations with the former colleagues. Working for the company also made it easy for the author to have a complete picture of the current status and get access to the data more easily. To collect data for the study, in cases where it was necessary to define the situation the author referred back to his experience of working in the company and sometimes consulting with the colleagues whenever it was necessary. For collecting quantitative data, they were extracted from the documents which were available to the author. If the file and documents were old, the author requested the updated version from his colleagues.

5.6.5 Supply chain department structure

The company has 10 different departments, including: Production, Supply Chain, Projects, Human Resources, Engineering, SPR[1], Quality, Purchasing, Finance and IT&IS. The inventory management, which is the main focus of this thesis, is implemented and managed by the Supply Chain department. The Supply Chain department itself is divided into a number of divisions such as: Customer Service, Internal Logistic, External Logistics, Customs, PHF[2], Warehouses. The main operations of each subsection are:

- Customer Service: Customer Service section responsibility is to plan incoming and out coming orders and to ensure that orders are delivered on time to its customers.
- Internal Logistics: The Internal Logistics section is responsible for planning, managing and controlling the entire logistic activities and information flows occurring within the company such as production feed up, warehouse operations and improving the flow of material.

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[1] Système de Production Renault (SPR)

[2] Prodiuts Hors Fabrication (PHF)

- External Logistics: External Logistics division is committed to ensure that the entire logistic activities outside of the company performed very well. This section is responsible for coordinating tasks with the third party logistics, relationship with suppliers and managing them, planning inventories and order management.
- Customs: Custom logistics section overtakes the responsibility of all the processes pertained to exporting and importing the CKD and spare parts along with the parts imported from Renault worldwide at the Iranian customs.
- PHF: The parts that are for indirect support of the production lines and are not directly used in the production lines are called PHF parts. PHF division, with the help of warehouse staffs, are responsible for planning and managing this type of parts.
- Warehouse: This division is responsible for receiving the parts from the suppliers, storing and managing the parts in the warehouses and picking and shipping the parts as per requests.

Figure 5.12 depicts the structure of the Supply Chain department and its divisions.

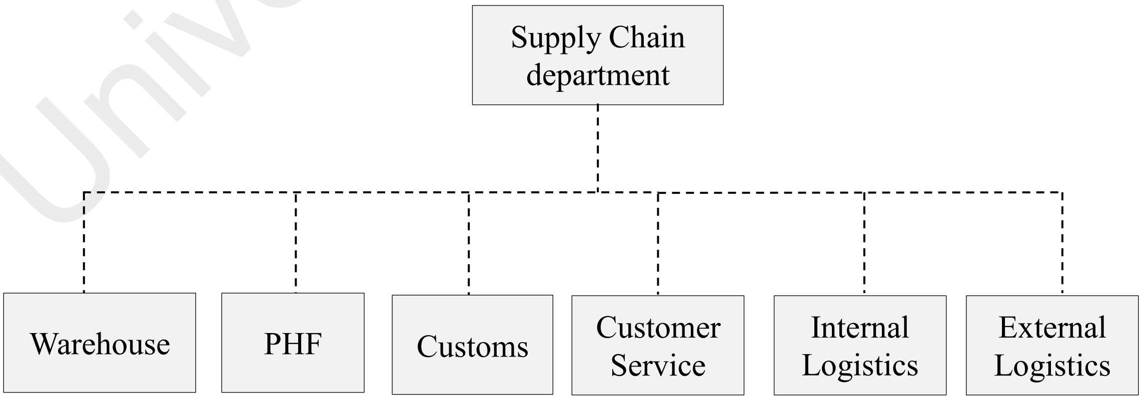

Figure 5.12: Diagram of Supply Chain department

5.6.6 Parts structure

The parts in ACI Co. are divided into two categories according to the mother company's structure in France. Therefore, they are treated in different ways. The first group of parts is called POE[3], a French term used to specify the parts that are directly used in production lines. POE parts are jointly managed by Purchasing-POE and External Logistics divisions. Whereas purchasing-POE division is responsible for contracting and dealing commercially with suppliers, External Logistics division mainly manages the ordering, delivery and inventory management process. However, both divisions sometimes work together supportively on the described tasks is emergency cases.

PHF parts, on the contrary, are a type that plays a supportive role in production, but are not directly used in manufacturing parts. Some items that can be brought here as example are spare parts used for the machineries and welding robots, stationaries for staff, and safety shoes. The supply and planning of PHF parts are jointly accomplished by purchasing-PHF and logistics-PHF divisions, where the first division manages the parts commercially, while the second division performs planning and supply tasks.

As it is clear from the definition of the parts, PHF items cover a wide range of references (almost 60000 references), which is far more than POE types (almost 350 references). However, generally, the importance of POE parts is more than PHF parts due to their direct effect on line stoppage and their turn over importance for the company. Due to the importance of the parts for the company, in this thesis the focus will be on POE parts and PHF parts will be excluded from further analysis.

[3] Pièces Ouvrées Extèrieures (POE)

5.6.7 Suppliers

The POE parts are typically supplied by 12 local suppliers along with one supplier from Renault's group, which is located out of Iran. ACI Pars Co. routinely works with a couple of suppliers who have formerly been nominated as Renault's suppliers through a rigorous evaluation process. Renault's preference is often to rely on a single-sourcing strategy to ensure a constant supply and to maintain strong ties with its suppliers. The local suppliers are geographically located in three hubs, which are the northeast, the center and near the capital of Iran. Figure 5.13 shows the location of the local suppliers as well as the external one.

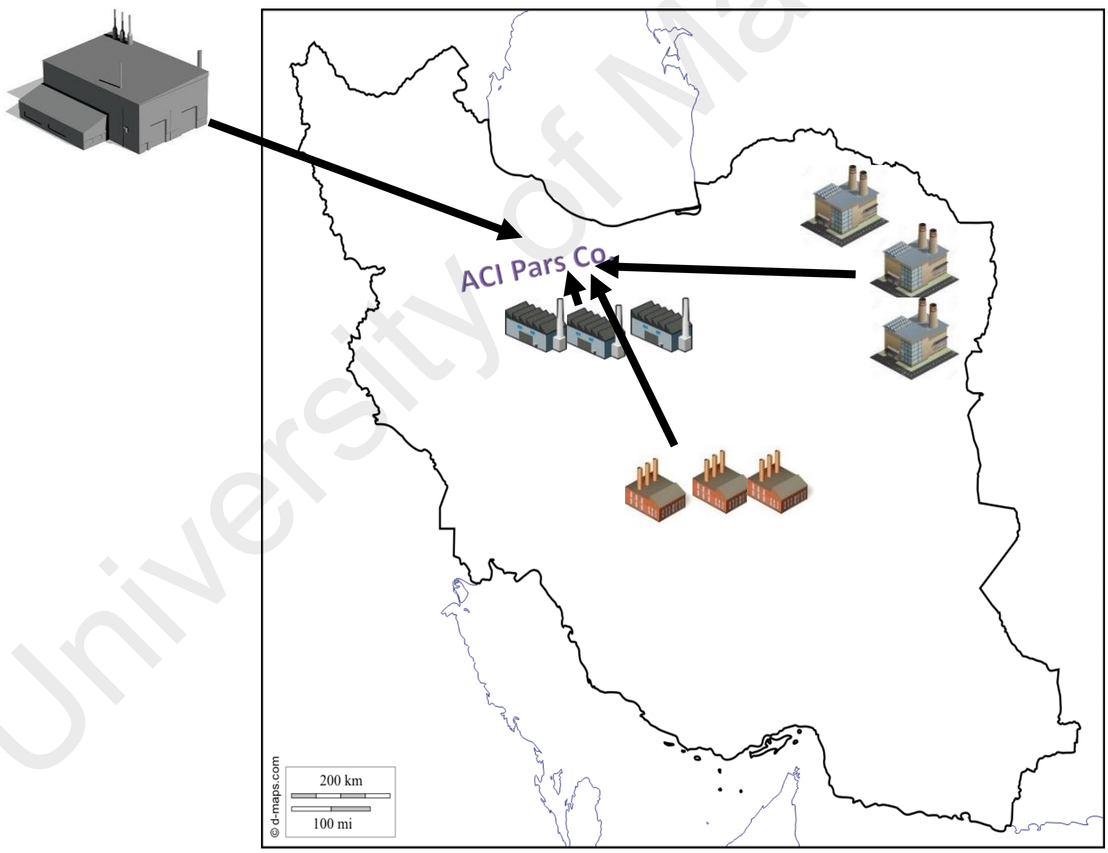

Figure 5.13: ACI Pars Co.'s supplier hub

Because of the technological issues and business condition of Renault in Iran, ACI Pars Co., does get the entire of its part components from the local suppliers. Thus, a portion of the demand comes from its external source, ILN[4], which is a logistic center established by Renault for supporting Renault's companies in Iran and Russia. ILN also serves as a backup supplier for the local suppliers in case they are unable to deliver the order.

5.6.8 Ordering procedure and inventory management structure

To study the current status of the inventory system at ACI Pars Co., it is required to initially have an overview of the ordering process and order fulfillment. At ACI Pars Co, inventory management process starts with forecasting customer demand. Demand forecast procedure in this company is in contrast to other companies that forecast customer demand themselves. ACI Pars Co., receives final customer demand from Renault Pars as a forecast file, showing Renault' production plan in Iran. After receiving the file, External Logistics division considers the weekly volume and calculates the right quantity for each supplier based on the MRP[5] method, which considers the in hand inventories, shipments on the way, and safety stock level determined for each part. Every supplier receives firm order two weeks ahead of the target week using either the Electronic Data Interchange (EDI) system or email. Figure 5.14 illustrates a sample firm order which was sent to a supplier. Suppliers are required to deliver according to the time schedule and the quantities determined in the firm orders.

As to the inventory management procedure, inventories are planned and controlled with the help of SAP software and Excel sheets. SAP is usually used to record the number of physical commodities, while Excel sheets are used for calculating the order quantity and reorder points.

[4] International Logistic Center

[5] Material Requirement Planning

Table 5.23: A firm order sample (source: ACI Pars Co. firm order, 2015)

Supplier	Description	Ref.	Firm Orders					
			Saturday	Sunday	Monday	Tuesday	Wednesday	Thursday
MSTOOS	Brake Disc 259*20,6 painted	7700436567	0		1092	1092	936	1092
	Hub	6040121873	0	0	784	784	0	0
	Spindle Axle Plate	6040134169	0	0	1344	1344	0	0
	Hub Carrier No ABS With Brake Shield L 50%	6040136441	0	0	0	0	0	0
	Hub Carrier No ABS With Brake Shield R 50%	6040136442	0	0	0	0	0	0
	Hub Carrier ABS With Brake Shield L 50%	400151919R	0	2080	320	320	320	160
	Hub Carrier ABS With Brake Shield R 50%	400145329R	0	2080	320	320	320	160
	Rear Hub Drum 8' Painted	6040162590	0	0	792	720	1296	1296

5.6.9 The problem with the current inventory planning

Even though ACI Pars Co. is utilizing SAP for managing inventories, the company is not able to benefit thoroughly from the capabilities of SAP in all the aspects of the planning. This attributes to the reason that SAP was not fully installed and therefor the company is not using all the package that SAP provides for enterprises. Due to this fact, inventory is planned and managed mostly in traditional ways rather than using a mechanized system. Another issue with the planning is the accuracy of forecasting file that ACI Pars Co. receives from Renault Pars. The forecasts are rarely perfectly accurate and fluctuate from the real demand frequently. Hence, ACI Pars Co.'s planning staffs quite often encounter with the problem of how they run the MRP plan with the accurate demand quantity. To deal with demand fluctuations, they employ their experience gained over the years and therefore estimate the demand value based on their experience. As it is clear from the current status of inventory planning, the current state is not efficient and can be improved using a scientific approach to determine the order quantities and decrease the inventory costs consequently. Nonetheless, according to the current status of the inventory management at ACI Pars Co., it is not transparent what is the optimal ordering condition considering holding costs, ordering costs and penalty costs and further how much is the gap between the current state and the ideal one. Since uncertainty and human learning are two fundamental factors in the pilot case, the models developed in this thesis are quite relevant to the pilot case and thus could be of help to improve the inventory system.

5.6.10 Selection of the parts

In order to conduct the model for the pilot study, it is initially required to pick out the suitable parts to be included in the study. As explained before, the POE parts comprise of nearly 350 references, which makes running the model for all parts cumbersome and time-consuming. Instead, a number of parts can be selected as a sample to study their inventory management process. Several criteria can be defined for the parts to be included in or excluded from the study. What is of major importance in this regard is to select parts that are crucial for the company because of their financial impact or their criticality for the production system. Based on the author's experience and consulting with the experts of the company, three criteria are very important for the inventory system: 1- turnover with the supplier 2- the value of the physical inventory in the warehouse, and 3- how reliable is the supplier with regard to the delivery. In order to ensure that the parts that have the mentioned characteristics are selected, a set of criteria are defined. These criteria are derived through consulting with a number of experts within the company and also on the basis of the author’s experience over the years. The criteria are: 1- turnover with the supplier should be more than 6000,000,000 IRR 2- The price of the part should be more than 100,000 IRR, and 3- The service rate of the supplier must be less than 80%. 4- The supplier of the part must be local. Table 5.9 presents and ranks the local suppliers[6] with

[6] The name of the suppliers are not mentioned due to confidentiality purpose.

respect to their turn over. Table 5.10 also represents the supplier service rates over the year 2015.

As to the supplier service rates in Table 5.10, it is required to note that the External Logistics department calculates the service rates weekly. Hence, the figures provided in Table 5.10 are the averages of the weekly service rates. Concerning the part prices, they cannot be declared in the thesis due to confidentiality purpose.

Table 5.9: Supplier turn overs in 2015

Rank	Supplier	Category	Turn over (IRR)
1	A	Foundry	39,373,450,200
2	B	Machining	34,151,132,366
3	C	Forging	24,546,345,120
4	D	Brake System	22,133,002,595
5	E	Stamping	12,133,002,595
6	F	Painting	9,596,178,489
7	G	Stamping	3,477,000,000
8	H	Bending	693,799,200

Looking at Table 5.10, it is clear that Supplier A is the only supplier with the service rate less than 80%. On the other hand, Supplier A meets the criteria defined above and thus is the only supplier who satisfies all the criteria. Supplier A is the supplier of raw brake disk and raw hub drum. Hence, these two parts are considered as the sample parts for analysis.

Table 5.10: Supplier service rates in 2015

Suppliers	Service rates
A	75
B	80
C	85
D	95
E	95
F	85
G	90
H	95

5.6.11 Determining inventory parameters

As discussed before, every efficient inventory management system heavily depends on the values of the inventory parameters. Consequently, it is required to specify inventory parameters for the case study before presenting the analysis. To do so, it is necessary to determine the inventory costs of the two selected parts. In what follows, it will be presented how the inventory cost parameters are calculated for the two chosen parts.

5.6.11.1 Annual demand

The first parameter of the model that should be determined is demand rate. Table 5.11 shows the weekly demand for the two parts in 2015. As can be seen from Table 5.11, some of the rows under Raw Brake Drum are blank indicating that the External Logistics Division did not place any order for these weeks in 2015. At the end of Table 5.11, the yearly demand is calculated using the weekly data. In order to compute the deviation values of yearly demand, the weekly demands are inserted into an Excel sheet and then the average yearly demand are calculated for each part. Afterward, the weekly demand of each part is abstracted from their respective average demand. The resulted negative values show that demand deviates from the average value on the negative side, while the positive figures illustrate that demand deviates from the average value on the positive side. Next, the negative and positive values are added to respectively form the lower and upper deviation values of demand. The steps are done for both parts and the results are obtained as: lower deviation value of Raw Brake Disc: 17302, upper deviation value of Raw Brake Disc: 13344, lower deviation value of Raw Brake Drum: 8284, and upper deviation value of Raw Brake Drum: 8284. These values help to form the fuzzy demand of the parts.

5.6.11.2 Holding cost

The next important parameter of the inventory system is holding cost. The holding cost cannot be directly found in companies' accountant system as theoretical models simply assume (Tersine, 1994). A standard approach in inventory management textbooks is to consider inventory holding cost as a portion of the part price as $H = i.C$, where i is the interest rate that is tied up with the capital and C is the unit price of the part (Heizer et al., 2004). According to consultation with the finance department and also based on the scientific literature, the following costs are identified that affect holding costs (Muller, 2011):

Warehousing cost: warehousing cost refers to any expenditure on storing the goods in a warehouse like renting the warehouse, the maintenance cost, costs for administrating warehouse and warehouse operations costs.

Table 5.11: Weekly demand for Raw Brake Disc and Raw Brake Drum in 2015

Raw Brake Disc				Raw Brake Drum			
Week	Demand (parts)	Week	Demand (parts)	Week	Demand (parts)	Week	Demand (parts)
2	2520	34	2520	2		34	1344
3	1440	36	1440	3	1344	36	1344
6	1440	37	1980	6	1344	37	1344
9	2520	38	2520	9		38	
18	2520	39	540	18	2464	39	2464
19	3960	40	2520	19	3808	40	
20	5040	41	3960	20	2464	41	
21	1440	42	2520	21	1344	42	2464
22	2520	43	5040	22	2464	43	2464
23	1440	44	3960	23	1344	44	1344
25	2520	46	540	25	2464	46	
26	2520	47	1440	26		47	1344
27	2520	48	1980	27	2464	48	1344
29	1440	49	3960	29	1344	49	2646
30	1260	50	3990	30		50	
31	1440	51	2520	31	1344	51	2464
32	2160	52	5040	32	1344	52	
33	540	53	5040	33	2464	53	2464
Total demand in 2015: 90750				Total demand in 2015: 53494			

Insurance costs: the inventories are generally insured to prevent the influence of some unforeseen events.

Stocktaking cost: the inventory in the warehouse should be regularly audited for the discrepant information and physical verification.

Buyback cost: the inventories in a warehouse may become deteriorated or outdated and therefore they should dispose of.

Equipment cost: this cost refers to the equipment needed for storing inventory and their depreciation cost.

Capital cost: holding inventory consumes the capital resource of the company that could be invested elsewhere in case they are not used for inventory.

As each of the cost items above has different interest rates, they should be identified. Table 5.12 provides interest rate for each cost and the average interest rate for holding inventory.

Table 5.12: Interest rates for inventory holding

Cost	Ware.	Insur.	Stockt.	Buyback	Equ.	Cap.	Total	Average
Interest rate	12.6%	1.5%	0.75%	1.45%	1.67%	11%	17.97	0.299

Since warehousing and equipment costs are not dependent on storing one additional unit in the warehouse, thus they should not be included in computing the average interest rate (Heizer et al., 2004). Hence, the inventory holding cost can be calculated as:

$$H = \frac{0.015 + 0.0075 + 0.0145 + 0.11}{12}.C = 0.01225C \tag{5.1}$$

Based on Eq. (5.1), inventory holding costs for Raw Brake Disc and Raw Brake Drum are, respectively, calculated as: 0.01225× 189619.93 = 2322.84 IRR, 0.01225×186318.14 = 2282.4 IRR.

5.6.11.3 Shortage penalty cost

In Renault's supply chain, any interruption in delivery to main manufacturers is not allowed and faces a heavy penalty from the car manufacturers, based upon the contract already signed among the supply chain's partners. ACI Pars Co., in turn, would face a penalty in case it could not deliver the parts on the scheduled time to Renault Pars. This penalty cost is imposed by IKh Co. and Saipa Co. to Renault Pars and Renault Pars as a result charges ACI Pars for line stoppage, which is calculated based on an hourly cost. In ACI Pars Co.'s finance system, no penalty cost for each individual part can be found. Therefore, the penalty cost for the sample parts should be determined. The penalty cost for the parts are computed using the following formula:

$$PCP = \left(\frac{C}{TP}\right) \times \left(\frac{PC.(SH.WD)}{AV}\right) \tag{5.2}$$

where the notations are as:

PCP	penalty cost per part
C	price of the part
TP	total sale price of the ACI Pars Co. to the manufacturer
PC	line stoppage penalty cost per hour determined by the manufacturer
SH	standard daily working hours of the manufacturer
WD	working days per year
AV	annual volume

In Eq. (5.2), the first term in the numerator is the weight of the parts in the basket sold by ACI Pars Co. to IKh and Saipa Cos. The second expression calculates the annual line stoppage cost at IKh and Saipa Cos. The total penalty cost is calculated by summing up the two individual costs obtained for IKh Co. and Saipa Co. Table 5.13 represents the result of the calculation for Raw Brake Disc and Raw Drum Brake. For computing the penalty costs in Table 5.13, the working hours per year is considered 321 days and the standard working hours per day is taken 8 hours.

Table 5.13: Penalty costs for parts

Part	Penalty cost from IKH Co. (IRR/per part)	Penalty cost from Saipa Co. (IRR/per part)	Total annual penalty cost (IRR/per part)
Disc	4471.46	2235.73	6707.18
Drum	7453.56	3726.78	11180.33

Table 5.14 summarizes the parameters of the inventory systems over one year operation in 2015.

It has already been discussed how a number of parameters like annual demand, holding cost and penalty cost are determined, summarized in Table 5.14. In order to derive the annual ordering cost, the cost of each order is initially considered and it is thereupon multiplied by the number of orders per year. It is necessary to note that the values of the order quantities of the parts are the average of their value throughout the year. The quantities of the maximum inventories are treated in the same way, meaning that their average value is taken into account in the table. As to the quantities given for the deviation of the maximum inventories, the deviations of the lead times are determined first and later they are computed using the expressions $\Delta_l^M = \Delta_h^L D$, and $\Delta_h^M = \Delta_l^L D$, which are presented in Chapter 4. Based on the data obtained from the experts of the company, the lead times are eight days on average, which sometimes decreases to up to 4 days and increases to utmost 15 days.

Table 5.14: The list of inventory parameters

Inventory parameter	Parts	
	Raw Brake Disk	Raw Brake Drum
Annual demand (part/year)	90750	53494
Deviation values of demand (part/year)	(17302, 13344)	(8284, 8284)
Holding cost (IRR/year)	2322.84	2282.4
Penalty cost (IRR/year)	6707.18	11180.33
Ordering cost (IRR/order)	1337468	1071125
Ordering cost (IRR/year)	48,148,848	28,920,375
Order quantity (part/order)	2520.83	1981.26
Number of orders (orders/year)	36	27
Maximum inventory (part/period)	9561	6046
Deviation of maximum inventory (part/year)	(1740.41, 994.52)	(2198.38, 586.24)

5.6.12 Inventory system analysis

This section investigates the current status of ACI Pars Co.'s inventory system for the two selected parts and shows how far the current inventory system from the ideal status is by implementing the developed models and comparing the optimal policies. Considering the values given in Table 5.14, the current inventory policies of ACI Pars

Co. for Raw Brake Disc and Raw Brake Drum are compared with the optimal policies using the models developed in this paper. The results are shown in Tables 5.12-Table 5.21.

Table 5.15: Inventory policy for Raw Brake Disc with log-linear learning curve and full transfer of learning

Optimal policy with learning						**Fuzzy model's policy**			
$\boldsymbol{l}$	$\boldsymbol{LR}$(%)	$\boldsymbol{n^*}$	$\boldsymbol{Q^*}$	$\boldsymbol{M^*}$	$\boldsymbol{TCU_{FL}}$	$\boldsymbol{Q^*}$	$\boldsymbol{M^*}$	$\boldsymbol{TCU_F}$	$\boldsymbol{n^*}$
0.074	95	3	30250	54733.78	113586768.5	70037.29	52332.02	120836987.80	2
0.152	90	3	30250	54563.24	112378398.7	**Current policy**			
0.234	85	3	30250	54443.93	111170028.8	$\boldsymbol{Q^*}$	$\boldsymbol{M^*}$	$\boldsymbol{TCU}$	$\boldsymbol{n^*}$
0.322	80	3	30250	54360.72	116942411.7	2520	9561	1778900435.69	36
0.415	75	4	22687.5	54304.93	115600637.8				
0.515	70	4	22687.5	54268.00	115345671.7				
0.621	65	4	22687.5	54244.58	115208763.4				
0.737	60	4	22687.5	54229.73	114983160.8				
0.862	55	4	22687.5	54221.17	114808566.1				

Table 5.15 contains the results of the fuzzy model with log-linear learning curve and complete transfer of learning. For the purpose of better comparison, the result of the fuzzy model and the current policy are provided in the right-hand side of the table. Comparing the four elements of the inventory system with that of the current policy indicates that the current policy of ACIP Co. is not optimal and there is an extensive gap between the optimal and the current policies. The policy that the company is adopting at the time being is to place orders with high frequency and lower order size throughout the year. This costs 1778900435.69 IRR per year. However, the analysis unveils that the system can perform optimally by reducing the number of orders, and increasing lot sizes, leading to reducing the total cost of the inventory. The results show that, with faster learning, the total cost of the system has the potential to reduce up to 15 times (compare 1778900435.69 with 114808566.1), which brings a huge cost saving for ACIP Co. Concerning the optimal inventory level, the optimal model with learning shows almost five times increase than the current policy. The number of order also shows a significant decrease compared to the current policy.

Table 5.16: Inventory policy for Raw Brake Disc with log-linear learning curve and forgetting (B=500 days)

Optimal policy with forgetting						**Fuzzy model's policy**			
l	LR(%)	n^*	Q^*	M^*	TCU_{FL}	Q^*	M^*	TCU_F	n^*
0.074	95	3	30250	47826.78	103872532.8	70037.29	52332.02	120836987.80	2
0.152	90	3	30250	47686.42	103255969.4	**Current policy**			
0.234	85	4	22687.5	47586.46	102945518.8	Q^*	M^*	TCU	n^*
0.322	80	4	22687.5	46994.53	102758548.5	2520	9561	1778900435.69	36
0.415	75	4	22687.5	46948.51	102635484.4				
0.515	70	4	22687.5	46917.78	101362680.6				
0.621	65	4	22687.5	46898.21	101331379.5				
0.737	60	4	22687.5	46885.73	101311339				
0.862	55	4	22687.5	46878.12	101299437.1				

Table 5.16 analyses the inventory policy of ACIP Co. when the experience of the planning staff is depreciated. The general trend of the policy is almost the same as the case where the experience is fully transferred with the exception that the model suggests lower maximum inventory for this case. Similar to the full transfer of learning, the model with knowledge depreciation gives a noticeable saving in the total cost of the system by decreasing the frequency of orders. This cost saving would be realized more if the planning staff learns faster.

The results of another scenario in planning inventories for ACIP Co. are provided in Tables 5.17 and 5.18. In these cases, the cognitive and motor abilities of the planning staff are taken into account. As shown in both tables, the general policies are also similar to the case with the log-linear learning. That is, the models recommend ordering with less frequency but higher lot sizes. Both models can additionally guarantee a huge cost saving for the system. However, the total cost resulted from these cases are higher than the ones with the log-linear learning curve.

Table 5.17: Inventory policy for Raw Brake Disc with cognitive and motor learning and full transfer of learning ($\alpha = 0.2$)

Optimal policy with learning					Fuzzy model's policy			
l	n^*	Q^*	M^*	TCU_{FL}	Q^*	M^*	TCU_F	n^*
(0.152, 0.074)	3	30250	56940.98	115181824.7	70037.29	52332.02	120836987.80	2
(0.234, 0.074)	3	30250	56918.43	114995298.2	**Current policy**			
(0.322, 0.074)	3	30250	56902.70	114868844.3	Q^*	M^*	TCU	n^*
(0.415, 0.074)	3	30250	56892.17	114786187.5	2520	9561	1778900435.69	36
(0.515, 0.074)	3	30250	56885.17	114731917.3				
(0.621, 0.074)	3	30250	56877.90	114677061.3				
(0.737, 0.074)	3	30250	56877.90	114677061.3				
(0.862, 0.074)	3	30250	56876.27	114664146.9				
(0.515, 0.415)	4	22687.5	56558.29	113077427.7				
(0.621, 0.415)	4	22687.5	56553.81	113065832.8				
(0.737, 0.415)	4	22687.5	56550.98	113059206.2				
(0.862, 0.415)	4	22687.5	56549.27	113054788.1				
(0.862, 0.737)	4	22687.5	56491.48	112944786.3				

Through comparing the maximum inventory level in Tables 5.17 and 5.18 with Tables 5.15 and 5.16, it is realized that this component of inventory policy is higher when the planning staff uses his cognitive and motor abilities. However, comparing the number of orders and the order quantities demonstrates that there is relatively no difference between two models, and they suggest placing three or four orders per year, depending on the learning rate of the planning staff.

The inventory system of ACIP Co. is also optimized for Raw Brake Drum under different learning scenarios discussed throughout this study and the results are provided in Tables 5.19, 5.20, 5.21 and 5.22.

Table 5.18: Inventory policy for Raw Brake Disc with cognitive and motor learning and forgetting ($\alpha = 0.2$)

Optimal policy with learning				
l	n^*	Q^*	M^*	TCU_{FL}
(0.152, 0.074)	3	30250	57987.62	113973454.9
(0.234, 0.074)	3	30250	57965.07	113786928.3
(0.322, 0.074)	3	30250	57949.34	113660474.4
(0.415, 0.074)	3	30250	57938.81	113577817.6
(0.515, 0.074)	3	30250	57931.81	113523547.4
(0.621, 0.074)	3	30250	57924.54	113468691.4
(0.737, 0.074)	3	30250	57924.54	113468691.4
(0.862, 0.074)	3	30250	57922.91	113455777
(0.515, 0.415)	4	22687.5	57604.93	111869057.8
(0.621, 0.415)	4	22687.5	57600.45	111857463
(0.737, 0.415)	4	22687.5	57597.62	111850836.3
(0.862, 0.415)	4	22687.5	57595.92	111846418.2
(0.862, 0.737)	4	22687.5	57538.12	111736416.4

Fuzzy model's policy			
Q^*	M^*	TCU_F	n^*
70037.29	52332.02	120836987.80	2

Current policy			
Q^*	M^*	TCU	n^*
2520	9561	1778900435.69	36

Table 5.19: Inventory policy for Raw Brake Drum with log-linear learning curve and full transfer of learning

Optimal policy with learning					
l	$LR(\%)$	n^*	Q^*	M^*	TCU_{FL}
0.074	95	3	17831.3	35316.34	72516205.81
0.152	90	3	17831.3	35206.30	71676451.40
0.234	85	3	17831.3	35129.33	71243722.66
0.322	80	4	13373.5	35075.63	71019423.28
0.415	75	4	13373.5	35039.64	70905893.39
0.515	70	4	13373.5	35015.81	70846454.26
0.621	65	4	13373.5	35000.7	70815788.91
0.737	60	4	13373.5	34991.12	70798986.13
0.862	55	4	13373.5	34985.59	70790216.70

Fuzzy model's policy			
Q^*	M^*	TCU_F	n^*
40439.30	33766.64	76650832.03	2

Current policy			
Q^*	M^*	TCU	n^*
1981.26	6046	844564568.42	27

The suggested policy by the model for Raw Brake Drum is the same as the policy for Raw Brake Disc and the model developed in this study could help the system to reduce the inventory cost significantly. As the pattern of the inventory for both parts are the same, discussing the tables are avoided. Readers are referred to the explanations under Tables 5.15 to 5.19.

Table 5.20: Inventory policy for Raw Brake Drum with log-linear learning curve and forgetting (B=500 days)

Optimal policy with learning					
l	$LR(\%)$	n^*	Q^*	M^*	TCU_{FL}
0.074	95	3	17831.3	32954.25	67422743.04
0.152	90	3	17831.3	32871.70	67031636.83
0.234	85	3	17831.3	32813.17	66834707.89
0.322	80	3	17831.3	32773.12	66716106.53
0.415	75	3	17831.3	32746.31	66638042.99
0.515	70	3	17831.3	32728.43	66597170.45
0.621	65	3	17831.3	32717.06	66577315.13
0.737	60	3	17831.3	32709.81	66564602.82
0.862	55	3	17831.3	32705.39	66557053.08

Fuzzy model's policy			
Q^*	M^*	TCU_F	n^*
40439.30	33766.64	76650832.03	2

Current policy			
Q^*	M^*	TCU	n^*
1981.26	6046	844564568.42	27

Table 5.21: Inventory policy for Raw Brake Drum with cognitive and motor learning and full transfer of learning ($\alpha = 0.2$)

Optimal policy with learning				
l	n^*	Q^*	M^*	TCU_{FL}
(0.152, 0.074)	3		36678.05	73063578.15
(0.234, 0.074)	3		36619.55	72945258.27
(0.322, 0.074)	3		36578.62	72865044.45
(0.415, 0.074)	3		36551.14	72812612.59
(0.515, 0.074)	4		36532.92	72778187.23
(0.621, 0.074)	4		36521.31	72743390.25
(0.737, 0.074)	4		36513.99	72743390.25
(0.862, 0.074)	4		36509.66	72735198.23
(0.515, 0.415)	4		36479.89	71728690.67
(0.621, 0.415)	4		36468.28	71721335.65
(0.737, 0.415)	4		36460.94	71717132.17
(0.862, 0.415)	4		36456.56	71714329.6
(0.862, 0.737)	4		36447.21	71644551.89

Fuzzy model's policy			
Q^*	M^*	TCU_F	n^*
40439.30	33766.64	76650832.03	2

Current policy			
Q^*	M^*	TCU	n^*
1981.26	6046	844564568.42	27

Table 5.22: Inventory policy for Raw Brake Drum with cognitive and motor learning and forgetting ($\alpha = 0.2$)

Optimal policy with learning					**Fuzzy model's policy**			
l	n^*	Q^*	M^*	TCU_{FL}	Q^*	M^*	TCU_F	n^*
(0.152, 0.074)	3	17831.3	36740.52	72272309.84	40439.30	33766.64	76650832.03	2
(0.234, 0.074)	3	17831.3	36725.97	72142563.91	**Current policy**			
(0.322, 0.074)	4	13373.5	36715.82	72054786.81	Q^*	M^*	TCU	n^*
(0.415, 0.074)	4	13373.5	36709.03	71997087.68	1981.26	6046	844564568.42	27
(0.515, 0.074)	4	13373.5	36704.51	71959503.18				
(0.621, 0.074)	4	13373.5	36699.82	71936168.89				
(0.737, 0.074)	4	13373.5	36699.82	71921185.9				
(0.862, 0.074)	4	13373.5	36698.77	71912123.66				
(0.515, 0.415)	4	13373.5	36493.59	71222938.14				
(0.621, 0.415)	4	13373.5	36490.70	71212650.82				
(0.737, 0.415)	5	13373.5	36488.88	71205909.42				
(0.862, 0.415)	5	10698.8	36487.78	71202005.98				
(0.862, 0.737)	4	10698.8	36450.48	71147317.71				

5.7 Summary of the chapter

In this chapter, the results of the analysis performed on the models using both secondary and primary data were presented. In the first section of the chapter, the models were successively examined using the data obtained from an earlier study. The models developed in this study and the previous ones were compared, which helped to draw conclusion about the effect of learning and its transfer on inventory planning under uncertainty. Besides analyzing the models using a secondary data set, a primary data set was collected and analyzed to indicate the application of the models in a real-world case. In both cases, it is identified that, when planning is associated with a certain amount of uncertainty, operator's learning and its transfer is a suitable strategy to reduce the effect of uncertainty on the total cost of the inventory system.

CHAPTER 6: CONCLUSIONS AND DISCUSSIONS

6.1 Conclusions

This thesis contributes to the area of fuzzy inventory management by integrating the impact of human learning and forgetting into the EOQ-S model to account for learning-based improvement and learning transfer over the planning cycles in a fuzzy inventory decision. The main inspiration behind the performance of this study arose from real-life inventory planning situations where learning occurs and uncertainty exists in the estimation of parameters of the underlying inventory models.

In Chapter 2, a comprehensive and systematic literature review was conducted to identify the main gaps in the existing body of literature. The first gap identified in the literature is that qualitative studies are completely overlooked by researchers as almost all existing models are built on a quantitative framework. It was further determined that the existing models in the literature lack consideration of the capabilities of a decision maker as emphasis is primarily placed on the roles of the decision maker. Finally, it was also noticed that there are few empirical studies in the literature and that investigations into the application of fuzzy inventory management are limited.

In Chapter 3, a research methodology was designed such that the study covers all three of the aforementioned research gaps. To address the first gap and to broaden the insights with regard to the relevance of learning and its transfer in inventory planning in contexts of uncertainty, a semi-structured interview was conducted with a number of experts currently active in the area of inventory management. The results of the interviews were synthesised into four propositions that helped formulate the necessary assumptions for the mathematical models.

In Chapter 4, the details of the interview process and the developed models were presented. Six experts from the selected companies in Iran and Malaysia were interviewed by asking some questions about the effect of learning and experience transfer in their daily works. The views of the experts and the result of the interview were summarized into four propositions, which were later linked to the assumptions of the inventory models. It was identified that learning and learning transfer happen in real inventory planning under uncertainty and therefore they should be considered in inventory models with imprecise parameters. Next, the model of Björk (2009) were extended to account for the operator's learning and its transfer, and totally investigated four different scenarios. The first model developed an EOQ model with backorders and learning in fuzzy parameters where learning is fully transferred over the planning cycles. This model assumed that the learning curve of the operator follows the log-linear learning curve. In the second model, the first model was extended by countering the assumption of full transfer of learning and it was assumed that the experience of the operator about the fuzzy parameters is partially transferred over the cycles due to an intermittent planning process. In the two subsequent models, the first and second models were reconsidered and the learning curve of the operator was changed by the learning curve with cognitive and motor capabilities of humans. In the last model, since there is no learning curve with cognitive and motor learning and forgetting in the literature, a learning curve with the specified properties was developed.

In Chapter 5, the developed models were examined using both primary and secondary data. Initially, the four models were tested using the data obtained from Björk (2009) allowed to compare the result of the models with that of Björk (2009) and derive conclusions. In the second stage, a case company in manufacturing industry was selected and its inventory system was analyzed. Using the data collected from the company, the models were optimized to derive optimal policies for the inventory system. In this thesis, a number of new results was achieved that could be useful for researchers and practitioners. The result of this study can be summarized as the following:

1. Apart from the matter with which learning curve the operator learns, bringing operator's learning into consideration in the fuzzy EOQ-S problem leads to increasing the number of orders that should be placed by the operator over the planning horizon, which makes the operator to decrease the batch sizes and increase the maximum amount of inventory that should be stocked.
2. Analysis of all the models further showed that, irrespective of the learning rate of the operator, the operator's learning could result in improving the inventory system's performance in terms of the total cost. However, the inventory system could benefit more when the operator learns faster, as faster learning rate has the potential to reduce the total cost of the system more.
3. The inventory system encounters a lower total cost when the learning curve of the operator follows the log-linear form compared to the learning curve with cognitive and motor components.
4. When experience of the operator is transferred partially (forgetting), it leads the operator to decrease number of orders, leading to adjusting higher values for the maximum inventory and the batch size, which results in the lower total cost of the system.

6.2 Contributions of the work

The thesis at hand covered a number of important research gaps in the area of inventory management under uncertainty. As noted before, while an extensive number of decision support models has published in this area so far, the role of human operator and the effects of human factors on the performance of inventory systems have totally been overlooked by researchers. This study is the first attempt that has been made in the literature with regard to modeling two important human factors, i.e. learning and forgetting, and analyzing their effect on inventory planning under uncertainty. The contributions of this thesis can be summed up as below:

1- This thesis presented a comprehensive and systematic review on the prior research that studied the application of fuzzy set theory in inventory management. To date, no such study is available in the literature to clearly analyze the advancement made in this area and to identify research gaps for possible research opportunities.

2- For the first time in the literature, an empirical study is conducted, which combines a qualitative research with mathematical modeling. The conducted semi-structured interview shed light on the relevance of the learning and forgetting in inventory planning under uncertainty.

3- It extends the model of Björk (2009) to account for two practical scenarios where the inventory operator learns how to adjust the fuzzy parameters by planning repetitively over time, or on contrary, forgets some information gained because of interruption occurred in the planning.

4- It extends the learning curve developed by Jaber and Glock (2013) by incorporating forgetting into their model.

6.3 Discussion and managerial Implications

Inventory planning usually involves the contribution of policy makers or the operators doing the tasks. This becomes more important when the policy maker or the operator uses his state of knowledge for managing uncertainty. Human involvement in planning inventories and managing uncertainties creates confluence between human behavior or factors and inventory system. For instance, every human being is naturally capable of learning after dealing with a task for a long time. As learning changes over time, the performance of human will change as a consequence. Since both human behavior and inventory system interacts, changing human behavior would affect the performance of the inventory system as a result. Despite the fact that human involvement in inventory planning under uncertainty is quite relevant in practice, human abilities were often neglected in modelling the inventory patterns. Doing so makes the available models unrealistic and may lead to underestimated or overestimated inventory outcomes. Therefore, human abilities should be combined into inventory models that are affected by uncertainty. This study clearly showed that the inventory system with fuzzy parameters could benefit from the operator's learning. The benefits of learning provide an important managerial and business guideline. That is, it necessitates providing a supportive learning environment with easy knowledge sharing and knowledge management within an organization so that employees are able to reinforce their concrete learning by practices. For example, during planning process or right after a cycle is completed, the managers may call for post-audit and analysis of inventory data, and the obtained knowledge could then be either shared with others planners engaged in the similar planning task or could be applied in the same ongoing planning cycles. Another possible way that can aid in fostering learning is to utilize and deploy ICT[1] systems. ICT systems and software can be of help in collecting inventory data and processing the related information. Therefore the precision of data estimation can be enhanced through this way. Apart from the inventory systems that merely work using ICT systems, fostering learning is also a useful tool in semi-mechanized environments where the knowledge of human can be integrated into ICT system to facilitate enhancing the performance of the inventory system.

In addition, it was discovered that boosting the cognitive abilities of the operator is useful for the inventory system, because of its higher effect on improving the performance of the system. This fact can give a clear managerial and operational implication, highlighting the importance that training programs for the workforce have on the total cost of the inventory system. In this context, depending on the company's policy, several policies could be adopted. For instance, if the company's policy is to give higher priority

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to reducing total inventory cost, then more investment in worker training programs has reasonable justification. If this is the issue, it is then indispensable to make a trade-off between the amount of the investment the company is willing to spend on the training and the reduction in the total inventory cost that would be achieved. This, in turn, could help the company to prioritize worker's training programs.

6.4 Practical Implications

Besides presenting several functional suggestions for managers, this study also can derive useful practical applications for practitioners in adopting the appropriate policy when facing a remarkable level of uncertainty. If a company encounters a high level of uncertainty in estimation of customer demand and the lead-times for delivery and in case the operator in charge of planning is able to become more familiar with the variation of demand and lead-times throughout time, it is then recommended that the operator issues orders to suppliers with smaller lot size and with more frequency. Increasing the frequency of orders could give a chance to the operator to have time to gather more information about the characteristics of demand and lead time sooner, and will allow him to estimate the demand and lead times of future orders with more precision.

6.5 Limitations of the study

Every study, regardless of how well is conducted, suffer from limitations, and this study is not an exception. There were a number of limitations when formulating the models, which mostly were related to the assumptions of the models. The limitations of this study can be summarized as below:

1- Owing to the lack of experimental studies in the fuzzy lot-sizing problem in one hand and due to the fact that learning in planning is a phenomenon that is not measurable, the learning rates of the operators are taken from the operations management literature. These learning rates are often for the tasks performed in working stations. Even though choosing the learning rates does not affect the generality of the current study, it is clear that more studies, even in laboratory setting or in real applications, are required to observe more evidence from real cases. Doing so can ensure that the estimated parameters are close to what happens in practice.

2- As for most of the mathematical models, the results of this study are highly dependent on the assumptions that were made in formulating the problem, even though the assumptions were linked to real application through interviews. Like other mathematical models, any change in the model assumptions would lead to new formulation of the problem and new results.

3- In order to keep the computational process as simple as possible, it was assumed that the learning curves of the operator for distinctive imprecise parameters are uniform. Nonetheless, the learning curve of the operator may not necessarily be equal for both parameters, implying that the regulation of the parameters may follow different learning curves.

4- Furthermore, and again for simplicity in calculations, it is further assumed that the operator learns equally with respect to the preferences of the imprecise parameters. Hence, equal learning rates for both parameters were considered. However, each parameter might subject to a different learning rate.

6.6 Future research

This thesis is the first study that integrates human characteristics and fuzzy set theory and intends to stimulate additional studies on the interaction between human behaviors and inventory models with imprecise parameters. The following research areas are suggested for future research:

1- In this thesis, two human characteristics, learning and forgetting, were only studied. Future studies could investigate the impact of other human characteristics, e.g. human error, on fuzzy inventory models and could examine the changes in the optimal planning policies under learning condition.
2- In this study, the concentration was on the application of learning and learning transfer in a fuzzy inventory system, but rather different fuzzy settings, like different membership functions and different deffuzification methods, was not surveyed in this study. It is obvious that the models developed in this study still need to be investigated under different fuzzy settings to evaluate the behavior of the model when formulations of uncertain parameters are different.
3- The developed models could be extended and applied in other areas of operations management such as production planning and project management, where uncertainty is a part of decision making. If so, it could be analyzed how learning in fuzziness can reduce the uncertainty in these areas, and what would be its effect on decision outcome.
4- The concept of learning in fuzziness could be studied in supply chain level and it would be worthy to analyze whether fostering learning among the members of the supply chain could reduce the uncertainty and the total cost of the supply chain as a result. In this case, it would be also interesting to investigate how the cooperation of the members would affect transfer of learning, and further what would be its effect on the whole policy of the supply chain.
5- Future studies could also work on relationships between organizational criteria and learning and could integrate it into the model. This provides a basis to study organizational measures and assists to spot how organizations could encourage learning in planning. The aim could be to evaluate the result of promoting

learning using organizational measures on the performance of inventory systems.

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LIST OF PUBLICATIONS

(a) ***Accepted***

1. Kazemi, N., Olugu, E. U., Salwa Hanim, A.-R., & Ghazilla, R. A. B. R. (2015). Development of a fuzzy economic order quantity model for imperfect quality items using the learning effect on fuzzy parameters. Journal of Intelligent & Fuzzy Systems, 28(5), 2377-2389.

2. Kazemi, N., Olugu, E. U., Salwa Hanim, A.-R., & Ghazilla, R. A. B. R. (2016). A fuzzy EOQ model with backorders and forgetting effect on fuzzy parameters: An empirical study. Computers & Industrial Engineering, 96, 140–148.

(b) ***Conference***

1. Kazemi, N., Olugu, E. U., Salwa Hanim, A.-R., Ghazilla, R. A. B. R., & Shekarian, E. (2016). The effect of human learning with cognitive and motor capabilities on a fuzzy EOQ model. Proceedings of the 2016 International Conference on Industrial Engineering and Operations Management, Kuala Lumpur, Malaysia.

(c) ***Under review***

1. Kazemi, N., Salwa Hanim, A.-R., Olugu, E. U., Ghazilla, R. A. B. R., & Fallahpour, A., A cognitive and motor learning-forgetting curve and its effect on a fuzzy EOQ model with backorders, Computers & Industrial Engineering.

(c) ***Related publications***

1. Kazemi, N., Shekarian, E., Cárdenas-Barrón, L. E., & Olugu, E. U. (2015). Incorporating human learning into a fuzzy EOQ inventory model with backorders. Computers & Industrial Engineering, 87, 540–542.

2- Shekarian, E, Olugu, E. U., Salwa Hanim, A.-R., & Kazemi, N. (2016). An economic order quantity model considering different holding costs for imperfect quality items subject to fuzziness and learning, Journal of Intelligent & Fuzzy Systems, 1-13.

3- Shekarian, S., Kazemi, N., Salwa Hanim, A.-R., & Olugu, E. U. (2017). Fuzzy inventory models: A comprehensive review. Applied Soft Computing. In press.

4. Shekarian, E, Olugu, E. U., Salwa Hanim, A.-R., & Kazemi, N. (2016). Analyzing optimization techniques in inventory models: the case of fuzzy economic order quantity problems. Proceedings of the 2016 International Conference on Industrial Engineering and Operations Management, Kuala Lumpur, Malaysia.

5. Kazemi, N., Salwa Hanim, Shekarian, S., A.-R., Bottani, E., & Montanari, R. (2016). A fuzzy lot-sizing problem with two-stage composite human learning. International Journal of Production Research. 54(16), 1-17.