# DESIGN OF A DYNAMOMETER-ENGINE COUPLING SHAFT

**MOHD HASNUN ARIF BIN HASSAN**

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Name of Candidate: **MOHD HASNUN ARIF BIN HASSAN**

(I.C/Passport No:

Registration/Matric No: **KGH100002**

Name of Degree: **MASTER OF ENGINEERING**

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# Abstract

In measuring the power output of an engine, the engine has to be coupled to a load device known as dynamometer. The coupling is done by means of a solid shaft. The proper couplings and shaft are required for the connection to avoid any failure to the engine or the dynamometer. Unsuitable selection could lead to undesired problems such as torsional vibrations, vibration of the engine and dynamometer, whirling of the coupling shaft, damage of the bearings, engine starting problem or immoderate wear of the shaft line components. The commonly encountered problem is the resonance in torsional vibration, which results in disastrous failure of the shaft due to excessive vibration. This project is aimed to study the appropriate design of the shaft to be used in the dynamometer-engine coupling to prevent the system from undergoing unwanted problems. The theoretical calculations involve in the design are presented. The dimension of the coupling shafts for engines with various maximum torques are estimated. It is shown that the diameter of the shaft is proportional to the maximum torque of the engine given that the same coupling is used for every system, whereas the length of the shaft is almost equal for every engine. The diameter of the shaft is a vital parameter compared to its length. For engines with the maximum torque vary from 40 to 200 Nm, the same shaft length of 500 mm can be used but with increasing shaft diameter as the maximum torque increases. For a 40 Nm engine, the shaft diameter of 20 mm generated acceptable result. The shaft diameter was increased by 5 mm as the maximum torque increases and acceptable results were obtained. On the other hand, by using aluminium instead of steel as the material of the shaft, lower critical engine speed is obtained given that the same dimension of the shaft is used. This is due to the fact that aluminium possesses lower modulus of rigidity in comparison to steel.

# Abstrak

Di dalam mengukur kuasa yang dijana oleh sesebuah enjin, enjin perlu disambungkan kepada sebuah mesin dikenali sebagai dinamometer. Penyambungan dilakukan dengan menggunakan syaf yang padat. Syaf dan perangkai yang sesuai diperlukan untuk mengelakkan sebarang kerosakan pada enjin atau dinamometer. Pemilihan yang tidak bersesuaian boleh mengakibatkan berlakunya masalah-masalah yang tidak diingini seperti getaran kilasan, getaran pada enjin dan dinamometer, pemusingan pada syaf perangkai, kerosakan pada galas, masalah untuk menhidupkan enjin dan kerosakan teruk pada komponen-komponen syaf. Masalah yang paling biasa dihadapi ialah resonan pada getaran kilasan yang boleh mengakibatkan kerosakan teruk pada syaf disebabkan oleh lebihan getaran. Projek ini disasarkan untuk mengkaji tentang rekabentuk syaf yang sesuai untuk diaplikasikan di dalam sistem dinamometer-enjin bagi mengelakkan sistem daripada dilanda masalah yang tidak diingini. Pengiraan secara teori yang terlibat didalam proses merekabentuk dipersembahkan didalam kajian ini. Dimensi syaf perangkai bagi enjin-enjin yang berlainan nilai tork maksimum adalah dianggarkan. Kajian ini menunjukkan bahawa diameter syaf berkadar terus dengan nilai tork maksimum enjin, dengan semua system menggunakan perangkai yang sama, tetapi panjang syaf adalah hampir sama bagi semua enjin. Ini menunjukkan diameter syaf adalah lebih penting daripada panjangnya. Bagi enjin-enjin dengan nilai tork maksimum berbeza daripada 40 hingga 200 Nm, panjang syaf yang sama iaitu 500 mm boleh digunakan tetapi dengan diameter syaf bertambah bagi setiap peningkatan nilai tork maksimum. Bagi engine dengan 40 Nm tork, diameter syaf sebesar 20 mm menghasilkan keputusan yang boleh diterima. Diameter syaf dibesarkan sebanyak 5 mm dengan nilai tork maksimum enjin meningkat dan keputusan yang memuaskan diperoleh. Dalam pada itu, dengan menggunakan syaf yang diperbuat daripada aluminium berbanding besi, kelajuan kritikal enjin yang lebih rendah diperolehi dengan

menggunakan syaf yang berdimensi sama. Ini kerana aluminium mempunyai modulus ketegaran yang lebih rendah berbanding besi.

# Acknowledgement

This research report would have not been possible without the guidance and help of several individuals, who in one way or another contributed and extended their valuable assistance in the preparation and completion of this study.

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Last but not the least, my family and the one above all of us, the omnipresent God, for answering my prayers and giving me the strength to plod on despite my constitution wanting to give up and throw in the towel, thank you so much Dear Allah, the Almighty.

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# List of Symbols and Abbreviations

| | | |
|---|---|---|
| $n$ | Frequency of torsional vibration | [cycles/min] |
| $n_c$ | Critical frequency of torsional vibration | [cycles/min] |
| $C_s$ | Stiffness of coupling shaft | [Nm/rad] |
| $I_e$ | Rotational inertia of the engine | [kg m$^2$] |
| $I_b$ | Rotational inertia of the dynamometer | [kg m$^2$] |
| $T$ | Torque | [Nm] |
| $T_{ex}$ | Amplitude of exciting torque | [Nm] |
| $\theta$ | Amplitude of torsional vibration | [rad] |
| $\theta_0$ | Static deflection of shaft | [rad] |
| $M$ | Dynamic magnifier | |
| $M_c$ | Dynamic magnifier of critical frequency | |
| $N_0$ | Order of harmonic component | |
| $N_{cyl}$ | Number of cylinders | |
| $M_{mean}$ | Mean turning moment | [Nm] |
| $imep$ | Indicated mean effective pressure | [bar] |
| $B$ | Cylinder bore | [mm] |
| $S$ | Stroke | [mm] |
| $T_m$ | Component of tangential effort | [Nm] |
| $T_v$ | Amplitude of vibratory torque | [Nm] |
| $N_c$ | Engine speed corresponding to $n_c$ | [rev/min] |
| $\tau$ | Maximum shear stress in shaft | [N/m$^2$] |
| $N_w$ | Whirling speed of shaft | [rev/min] |
| $N_t$ | Transverse critical frequency | [cycles/min] |
| $C_c$ | Dynamic torsional stiffness of coupling | [Nm/rad] |
| $\psi$ | Damping energy ratio | |
| $E$ | Modulus of elasticity | [Pa] |
| $G$ | Modulus of rigidity | [Pa] |

# CHAPTER 1 : Introduction

## 1.1 Overview

In the automotive industry, there are a wide variety of tests conducted on the engine to measure the performance, responsiveness for acceleration/deceleration, emissions, fuel economy, durability, noise and vibration. Parameters affecting an engine's performance include the basic engine design, compression ratio, valve timing, ignition timing, fuel, lubricant and temperature [Gitano, 2008c]. Therefore, the development of vehicle cannot be realised without engine testing. However, some of these targets or parameters often work against each other [Tominaga, 2010]. Hence there are a number of specialist systems and control systems that requires isolated execution of the test.

The most commonly used prime mover in an automotive vehicle is the internal combustion engine. It produces the power through the conversion of the chemical energy in the fuel into heat followed by the conversion of the heat into mechanical work [Klingebiel & Dietsche, 2007; Pulkrabek, 2004]. This conversion takes place by means of combustion. The conversion of thermal energy into mechanical work occurs through a transmission of the energy to a working medium, which hereupon increases its pressure and subsequently produce power [Klingebiel & Dietsche, 2007]. Hence, it can be said that the internal combustion engine is an energy transformer [Crolla, 2009].

The ability of an internal combustion engine to do work is measured by a quantity known as torque. Torque is defined as the force acting at a moment distance and it is measured in newton metres (Nm). The engine creates torque and uses it to spin the crankshaft. During the power stroke, the crankshaft moves 180° from the top dead centre (TDC) to the bottom dead centre (BDC). During the movement, the effective radius of the crank-arm increases from zero (at TDC) to the maximum value in the region of mid-stroke and decreases to zero again at the end of the stroke (at BDC). The movement is illustrated in Figure 1.1 where:

- *p* is the cylinder gas pressure,
- *F* is the connecting-rod thrust,
- *R* is crank-throw,
- *r* is the effective crank radius, and
- *T* is the turning-effort or torque.

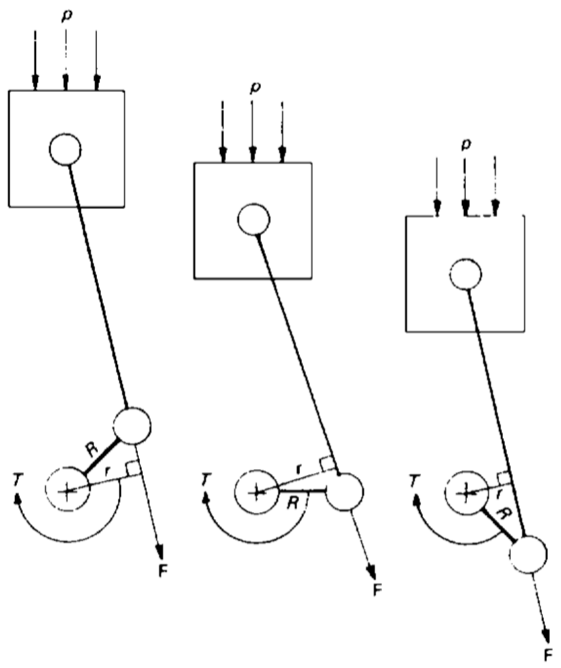


Figure 1.1 : Torque variation during power stroke. [Heisler, 1998]

This shows that the torque produced varies during the power stroke, whereas during the idling stroke, there is no useful torque generated [Heisler, 1998]. The maximum torque of an engine is known as the maximum brake torque speed (MBT). Most of the modern automobiles possess the maximum torque within the range of 200 – 300 Nm at the engine speed of 4000 – 6000 RPM [Pulkrabek, 2004].

The torque produced by an internal combustion engine is measured using a device known as dynamometer. The dynamometer resists the torque produced by the engine connected to it and measures the torque [Martyr & Plint, 2007a]. In engine testing, it is important to recreate the actual on-road situation in the most effective way in order to obtain accurate data. However, the test conducted must be safe and repeatable, thus the engine can be tested with different desired conditions [Atkins, 2009].

## 1.2 Background of the study

As mentioned in previous section, the internal combustion engine has to be coupled to a dynamometer in order to measure its torque. Figure 1.2 shows the illustration of dynamometer-engine setup. The engine is connected to the dynamometer by means of a shaft. The shaft has to be properly designed since a poorly designed shaft could lead to serious impairments not only to the engine, the dynamometer or the shaft, but also to the human conducting the test.

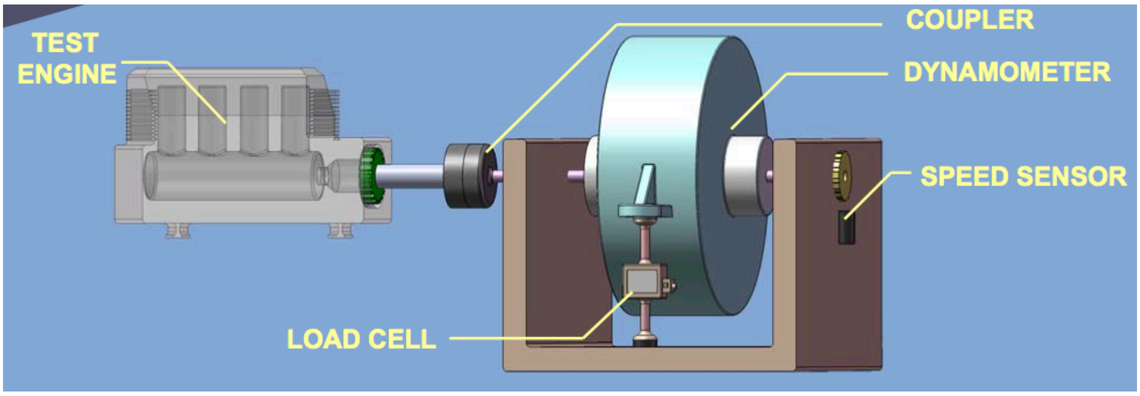


Figure 1.2 : Dynamometer-engine setup. [Gitano, 2008b]

Figure 1.3 depicts a broken shaft and a broken coupling as a result of improper designing prior to the test. The inappropriate design of the shaft could lead to various problems, namely excessive torsional vibrations, whirling of the coupling shaft, bearing damages, excessive wear of the shaft line components, engine starting problem and so forth [Martyr & Plint, 2007b]. The most commonly encountered problem in a dynamometer-engine system is the resonance of torsional vibration. The frequency of the torsional vibration depends on the inertia of the engine and dynamometer, and the stiffness of the coupling shaft. Therefore, it is vital to design a shaft with the proper stiffness to avoid the problem.

(a)

(b)

Figure 1.3 : (a) A broken shaft. (b) A broken coupling. [DynoTech-Research, 2010]

## 1.3 Objectives of the study

This project is aimed to study the design shaft to be used in a dynamometer-engine system. As stated in [Martyr & Plint, 2007b], the design of the shaft for different engines may differ. The objectives of this study are as follows:

1. To study the relation between the maximum torque of an engine and the design of the shaft to couple it to a dynamometer.
2. To investigate the importance of the dimensions of the shaft (i.e. shaft diameter and length) to the dynamometer-engine coupling.
3. To study the impact of using different material such as aluminium instead of steel in fabricating the shaft.

## 1.4 Scope and limitation

In this study, five hypothetical engines with different maximum torque values (i.e. 40, 80, 120, 160 and 200 Nm) were used. The shafts to couple these engines to a dynamometer were virtually designed to investigate the impact of maximum torque value on the shaft design. From the results, the dimensions of the shaft for each case will be observed. This study only involves theoretical calculations and the actual shafts are not fabricated. The study was conducted analytically rather than experimentally. Since hypothetical engines were used in the calculations, some parameters such as the displacement volume, bore, stroke and the moment of inertia of the engines were estimated.

# CHAPTER 2 : Literature review

## 2.1 Introduction

There are only a few studies that were conducted regarding this topic. One of the relevant journals was published in year 2004 [Jayabalan, 2004]. In addition to it, there is a book entitled Engine Testing that contains a chapter dedicated to the dynamometer-engine coupling [Martyr & Plint, 2007b]. However, no comparison can be made since the design of the shaft differs for every dynamometer-engine setup (i.e. different engines and/or different types of dynamometer). In this chapter, the theories involve in this topic, namely the engine dynamometer and its operating mechanism, the torsional vibration/oscillation, damping and coupling, and so forth, are presented and described. They were reviewed from various journals, books, articles and brochures from manufacturers.

## 2.2 Engine dynamometer

To measure the torque and power output of an engine in a laboratory, the engine is coupled directly to a device known as engine dynamometer. It introduces variable loading conditions on the engine under test across the range of engine speeds and durations. Hence, the torque and power output of the engine can be accurately measured [Atkins, 2009]. Direct coupling means that the dynamometer shaft is connected to the driveshaft or propeller shaft of the engine under test resulting both the engine and the dynamometer running at the same speed [Gitano, 2008a]. In addition, since the dynamometer rotor is coupled to the shaft, its speed is also identical to the speed of engine crankshaft [Atkins, 2009].

William Froude introduced the first modern dynamometer when he designed a dynamometer for HMS Conquest, a C-class light cruiser of the Royal Navy [Atkins, 2009]. Nowadays, there are many types of dynamometers used in the industry. Each of them has its own advantages and disadvantages over its counterparts. Commonly used dynamometers in the industry include [Gitano, 2008a]:

- Frictional (brake) dynamometer,
- Hydraulic (water brake) dynamometer,
- Eddy current dynamometer,
- Generator type dynamometer, and so forth.

### 2.2.1 Frictional (brake) dynamometer

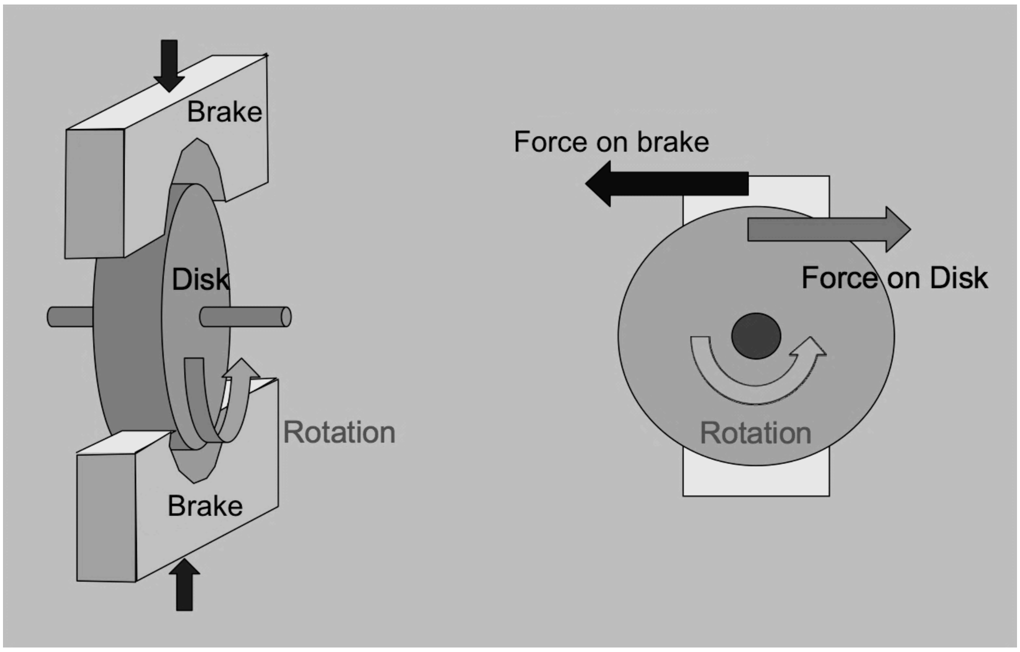


Figure 2.1 : Frictional dynamometer. [Gitano, 2008a]

Frictional (brake) or dry friction dynamometer is the oldest type of dynamometer. It contains mechanical braking device such as belt or frictional ‘shoe’ as shown in Figure 2.1. It operates with the shaft spins the disk or drum. The braking device then applies force to resist the rotating disc or shaft. The force applied by the brake is equal to the force on the disk and acts in the opposite direction.

### 2.2.2 Hydraulic (water brake) dynamometer

Hydraulic dynamometer is fundamentally a hydraulic pump. The engine rotates the shaft, which hereupon spins the impeller. Water is pumped from a reservoir through a hydraulic circuit via a throttling valve as shown in Figure 2.2. Hydraulic drag induced by the water resists the motion of the impeller. The load is varied through opening and closing of the valve. Hydraulic dynamometers typically have the highest power densities.

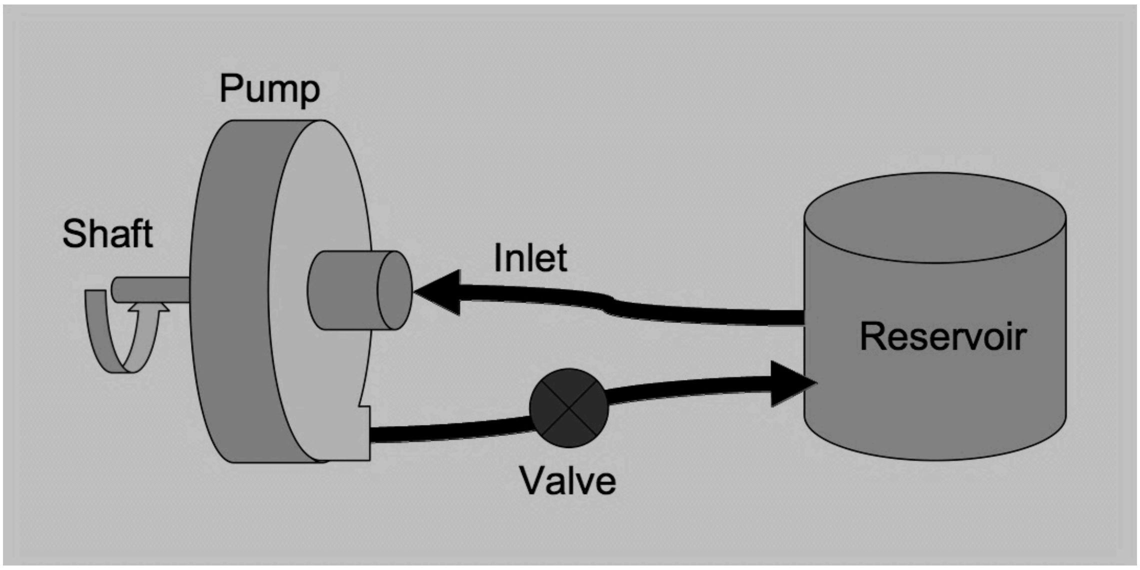


Figure 2.2 : Hydraulic dynamometer. [Gitano, 2008a]

### 2.2.3 Eddy current dynamometer

Eddy current dynamometer is an electromagnetic load device consists of a disk placed inside its housing. The coupling shaft spins the disk, which contains large electromagnetic coils as shown in Figure 2.3. This initiates electric current. As the current passes through the coils that surround the disk, a strong magnetic field is induced. The magnetic field creates a so-called ‘eddy current’ in the disk that resists its rotation. This produces a torque between the housing and the disk. Varying the current varies the torque generated as well as the load on the engine.

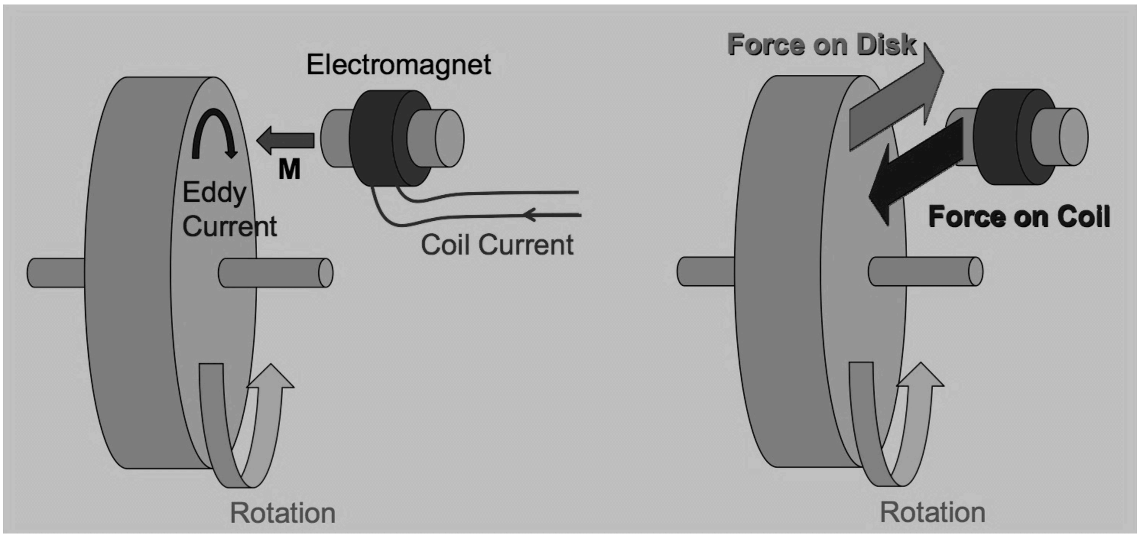


Figure 2.3 : Eddy current dynamometer. [Gitano, 2008a]

### 2.2.4 Generator type dynamometer

In a system comprising of generator type dynamometer, the coupling shaft spins the rotor of a generator as depicted in Figure 2.4. Electrical load is applied to the output of the generator creating an electromagnetic force.

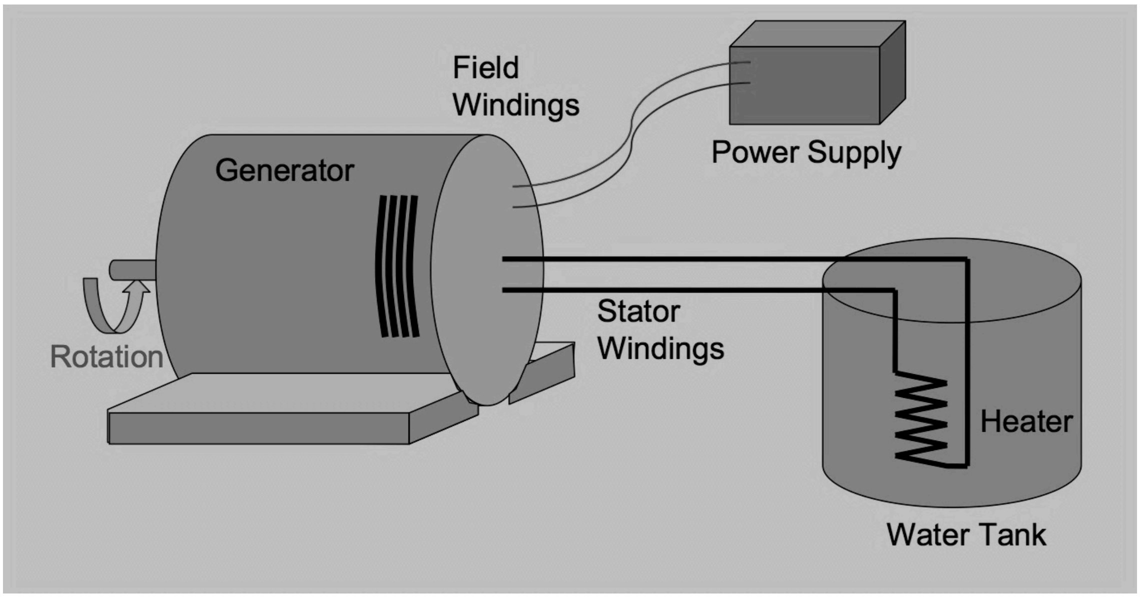


Figure 2.4 : Generator type dynamometer. [Gitano, 2008a]

This force resists the motion of the rotor. A resistor bank (heater) is commonly used as the load, which is either air or water-cooled. In order to vary the mechanical load, the field winding current is controlled.

### 2.2.5 Different types of dynamometer

In previous sections, four common dynamometers are described. Following table lists the advantages and disadvantages of different types of dynamometer. This table was reproduced from some literatures. [Crolla, 2009; Martyr & Plint, 2007a].

Table 2.1 : Pros and cons of different types of dynamometer. [Martyr & Plint, 2007a]

| Dynamometer type | Advantages | Disadvantages |
|---|---|---|
| Froude sluice plate | Obsolete, but many cheap and reconditioned models in use worldwide, robust | Slow response to change in load. Manual control not easy to automate |
| Variable fill water brakes | Capable of medium speed load change, automated control, robust and tolerant of overload. Available for largest prime-movers | ‘Open’ water system required. Can suffer from cavitation or corrosion damage |
| ‘Bolt-on’ variable fill water brakes | Cheap and simple installation. Up to 1000 kW | Lower accuracy of measurement and control than fixed machines |
| Disc type hydraulic | Suitable for high speeds | Poor low speed performance |
| Hydrostatic | For special applications, provides four quadrant performance | Mechanically complex, noisy and expensive. System contains large volumes of high pressure oil |
| D.C. electrical motor | Mature technology. Four quadrant performance | High inertia, commutator may be fire and maintenance risk |
| Asynchronous motor (A.C.) | Lower inertia than DC. Four quadrant performance | Expensive. Large drive cabinet needs suitable housing |
| Permanent magnet motor | Lowest inertia, most dynamic four quadrants. Small size in cell | Expensive. Large drive cabinet needs suitable housing performance |
| Eddy current | Low inertia (disc type air gap). Well adapted to computer control. Mechanically simple | Vulnerable to poor cooling supply. Not suitable for sustained rapid changes in power (thermal cycling) |

| | | |
|---|---|---|
| Friction brake | Special purpose applications for very high torques at low speed | Limited speed range |
| Air brake | Cheap. Very little support services needed | Noisy. Limited control accuracy |
| Hybrid | Possible cost advantage over sole electrical machine | Complexity of construction and control |

In this study, the eddy current dynamometer is used throughout the analysis. Different results will be obtained, given that other type of dynamometer is used.

## 2.3 Operating mechanism of a dynamometer

The operation of a dynamometer can be simulated by a spring balance, anchored to the ground, with a rope attached to the top eye and wrapped around a drum with a slipknot as shown in Figure 2.5. As the drum rotates, the slipknot tightens, tensioning the rope. The tension is indicated as a weight by the spring balance.

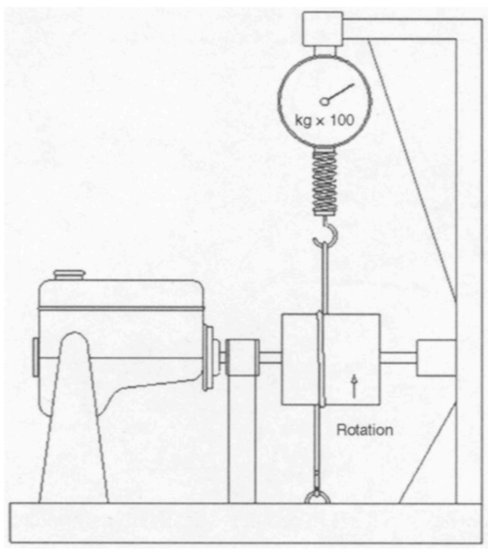


Figure 2.5 : Dynamometer operation simulated by a spring balance. [Atkins, 2009]

There is a friction between the rope and the drum, which slows down the motion of the drum and its driving engine until a certain speed, for example ‘X’ RPM, and the spring balance shows a reading of ‘Y’ kg. This shows that the weight lifted is ‘Y’ kg, and therefore the speed of the drum or the engine recorded is used to calculate the horsepower. In real application, the engine is clamped on a test bed with a drive shaft coupled to it. The other end of the drive shaft is coupled to the dynamometer, which replaces the system containing the drum and the spring balance as described previously [Atkins, 2009].

### 2.3.1 Principle of operation

The operating principle of a dynamometer is illustrated in Figure 2.6. Depending on the type of dynamometer, the rotor is coupled to a stator electromagnetically, hydraulically or by mechanical friction. The stator is supported in low-friction bearings. It is stationary balanced with the rotor via static calibration. By balancing it with weights, springs or pneumatic means, the torque exerted on it with the rotor turning can be measured [Atkins, 2009].

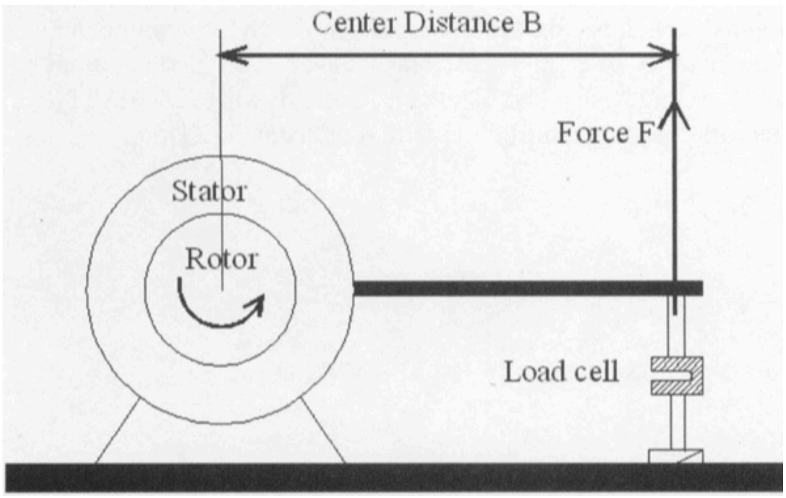


Figure 2.6 : The mechanism of torque measurement [Atkins, 2009].

Given that the torque $T$ is exerted, then it can be calculated as follows:

$$T = FB \tag{2.1}$$

On the other hand, the power P generated by the engine under test is as the matter of fact the product of torque and angular speed as given by following equation:

$$P = 2\pi NT \tag{2.2}$$

where $N$ is the engine speed in revolution per minute (RPM).

As previously mentioned, torque denotes the ability of an engine to do work, whereas power indicates the rate at which the work is done. The power calculated from Equation ( 2.2 ) is known as brake power, designated as $P_b$. This is the useful power delivered by the engine to the applied load. Basically, the dynamometer applies a resistive force to oppose the rotation of the drive shaft (or the torque of the engine's crankshaft). This causes the engine to work harder to retain its rotational speed.

### 2.3.2 Operating quadrants

Figure 2.7 depicts the four quadrants, which the dynamometer may be operated.

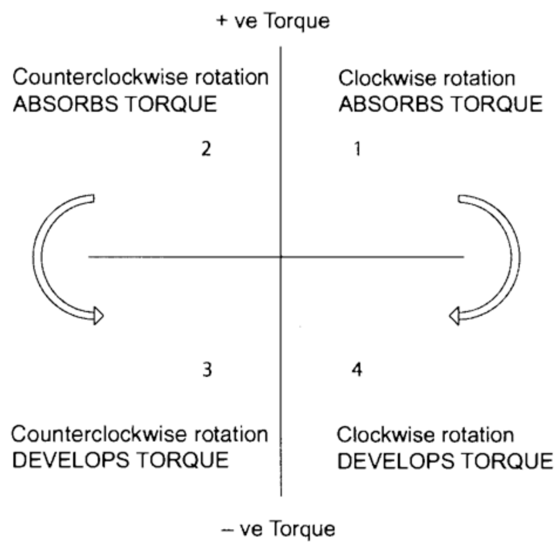


Figure 2.7 : Operating quadrants of a dynamometer. [Atkins, 2009]

In general, most of the engines testing takes place in the first quadrant with the engine running counter-clockwise if viewed from the flywheel end. All types of dynamometer are normally able to operate in the first or second quadrant [Atkins, 2009; Martyr & Plint, 2007a].

A dynamometer needs to operate in third and fourth quadrants when it is required to produce power as well as to absorb it. However, the choice is limited since only DC machines, AC machines, hydrostatic and hybrid dynamometers are able to operate in such quadrants. These dynamometers are reversible, thus able to operate in all four quadrants. An eddy-current dynamometer is also basically reversible. Nevertheless, a hydraulic dynamometer is normally designed for one directional rotation, albeit it could be operated in reverse at low fill state without damage.

In present, the transient testing (very rapid load changes and torque reversals) is growing resulting an increase in demands for four-quadrant operation. A notable feature of a four-quadrant dynamometer is its ability to start the engine. Table 2.2 lists some common types of dynamometer and their particular operating quadrant, which is reproduced from [Atkins, 2009].

Table 2.2 : List of dynamometers and their operating quadrants. [Atkins, 2009]

| **Type of Machine** | **Operating Quadrant(s)** |
|---|---|
| Hydraulic sluice plate | 1 or 2 |
| Variable fill hydraulic | 1 or 2 |
| Hydrostatic | 1, 2, 3, 4 |
| DC electrical | 1, 2, 3, 4 |
| AC electrical | 1, 2, 3, 4 |
| Eddy current | 1 and 2 |
| Friction brake | 1 and 2 |

## 2.4 Torsional vibration

### 2.4.1 Overview

A dynamometer-engine system can be considered as identical to a system comprises of two rotating masses connected by a flexible shaft as illustrated in Figure 2.8. Both masses possess a tendency to vibrate 180° out of phase about an arbitrary point located along the connecting shaft. The oscillatory movement is superimposed on any steady rotation of the shaft. Hence, such system tends to generate torsional vibrations [Martyr & Plint, 2007b]. The twisting of the shaft while the engine rotates is known as torsional vibration. It occurs due to the periodical nature of actuating torque [Meirelles et al., 2007]. Excessive amount of torsional vibration can bring about failures of the crankshaft, couplings, engine dampers and so forth [Feese & Hill, 2009].

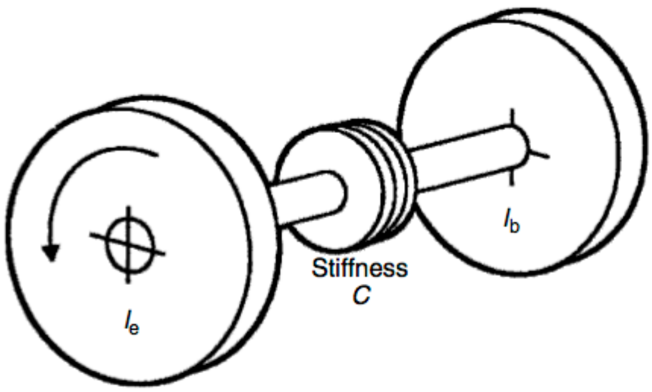


Figure 2.8 : Two mass system. [Martyr & Plint, 2007b]

### 2.4.2 Literature review

The importance of the knowledge and understanding of torsional vibration has led to publishing of many journals and articles regarding the subject. A method to predict the behaviour of the torsional vibrations in internal combustion engines at transient and steady state regime by the modal superposing method was developed in 1987 [Johnston & Shusto, 1987]. In some systems, excessive vibrations are exhibited on particular speeds [Draminski, 1988]. On the other hand, it was deduced that the variable inertia characteristics of the crank-mechanism that caused the unpredicted large angular displacements in multiples of the engine speed [Hesterman & Stone, 1995].

Before the awareness of the effect of variable inertia in internal combustion engine arose, it was often ignored and even considered as negligible in the calculation. After many studies were conducted, it was concluded that this effect is able to bring about structural failures to the crankshaft. It was concluded that the interaction of the secondary forces was profoundly dangerous for the crankshaft [Pasricha, 2001]. The calculation of torsional vibration including the mentioned effect was conducted in 1997. The calculation executed taking into account the variation of the inertia over the crank throw angular position, primarily for engines with large displacement (the masses of the pistons and connecting rod are significantly large) [Brusa et al., 1997].

Moreover, the effect of torsional and axial vibration coupling at the crankshafts was also studied. It was concluded that the situation where the axial and torsional natural frequencies are equal generates large angular displacements. Besides, this phenomenon also takes place when the one of the natural frequencies is two times larger than the other [Song et al., 1991].

A model to study the torsional vibration of four-cylinder gasoline engine was developed in 1987. In the model, the journals were connected to the main bearings taking the elastic properties of the oil film into consideration [Lacy, 1987]. The same model was also used to study the noise generated by the vibrations, in which the dynamical rigid body influence of all involved inertias on a multi-body model was included [Boysal & Rahnejat, 1997].

The coefficient of torsional damping in internal combustion engine was first estimated many years ago by researchers, however its accuracy was uncertain [Hartog, 1985; Wilson, 1965]. Another model was studied and presented to estimate the damping coefficient. This model was developed taking into account analytical relations between the damping and other measurable parameters of the engines [Iwamoto & Wakabayashi, 1985]. An accurate estimation of the absolute damping of a single-cylinder engine powered by an electric motor was also proposed [Wang & Lim, 2000].

To reduce the vibratory effects, the implementation of rubber damper was proposed. The study was conducted on a six-cylinder diesel engine, taking into consideration the transition state matrix methodology. It was observed that the torsional stiffness of the rubber damper was a vital parameter in comparison to the engine internal damping or the rubber damping. The stiffness was determined by the geometry of the rubber and its chemical properties [Honda & Saito, 1987]. Furthermore, the variation of the excitation load through the cylinders due to the wear of piston rings and liner was studied in 1992 [Margaronis, 1992].

### 2.4.3 The resonant frequency

Figure 2.9 depicts the relation of the engendered oscillation $\theta$ and the ratio $n/n_c$ of a system without damping similar to the system described in Section 2.4.1, which is subjected to an exciting torque of constant amplitude, $T_{ex}$ and frequency $n$. The combined amplitude of both masses is equivalent to the static deflection of the shaft under the influence of the exciting torque ($\theta_0 = T_{ex}/C_s$) at low frequency. The combined amplitude increases proportional to the frequency. At the frequency of $n = n_c$ (the system frequency is equal to the resonant frequency), it is theoretically infinite as observed in Figure 2.9. This could lead to massive damage to the shaft. As the frequency increases further, the amplitude decreases and at $n - \sqrt{2n_c}$ , it is down to the level of static deflection. The amplitude decreases continually as the frequency increases [Jayabalan, 2004; Martyr & Plint, 2007b].

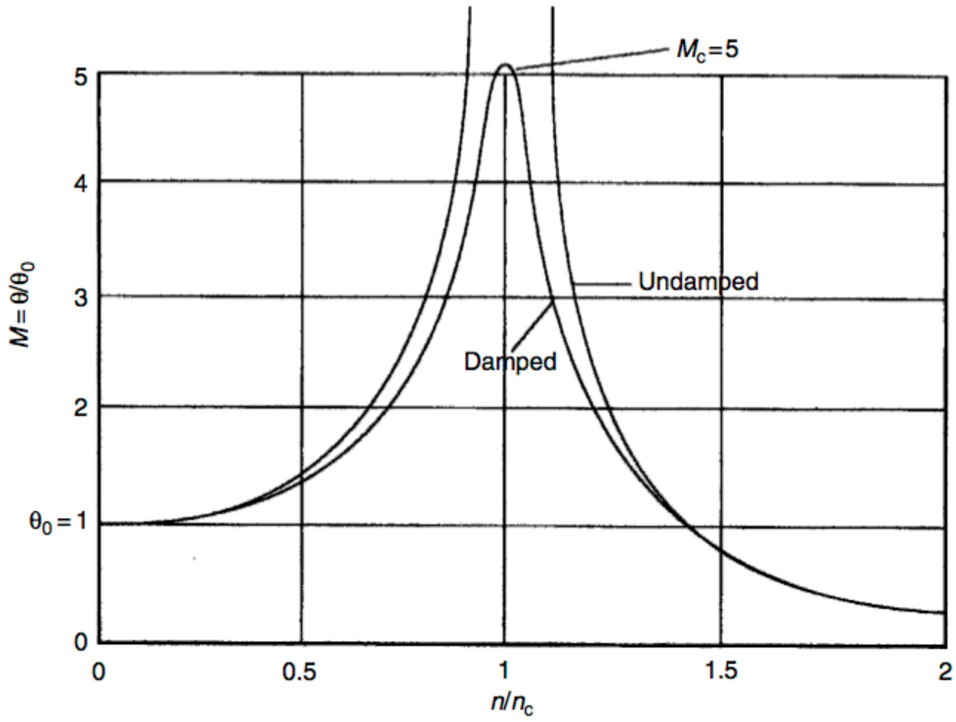


Figure 2.9 : Effect of damped and undamped vibration. [Jayabalan, 2004]

The behaviour of a damped system is also depicted in Figure 2.9. The implementation of an appropriate damping prevents the combined amplitude from being infinite at the resonance frequency, hence avoiding the damage of the shaft. The ratio $\theta/\theta_0$ is known as the dynamic magnifier $M$. The significant parameter is the value of the dynamic magnifier at the resonant frequency, denoted as $M_c$ [Martyr & Plint, 2007b].

### 2.4.4 The harmonic components

The engine torque varies corresponding to the pressure cycles in the individual cylinders, which causes the excitation of the torsional vibration. Figure 2.10 shows the variation of the torque of a single-cylinder four-stroke engine. The periodic torque curve is obtained from the combination of a series of harmonic components in form of sine waves with different amplitudes. The frequency of each wave corresponds to the multiple or sub-multiple of the engine speed. Figure 2.10 illustrates the first six harmonic components [Jayabalan, 2004; Martyr & Plint, 2007b].

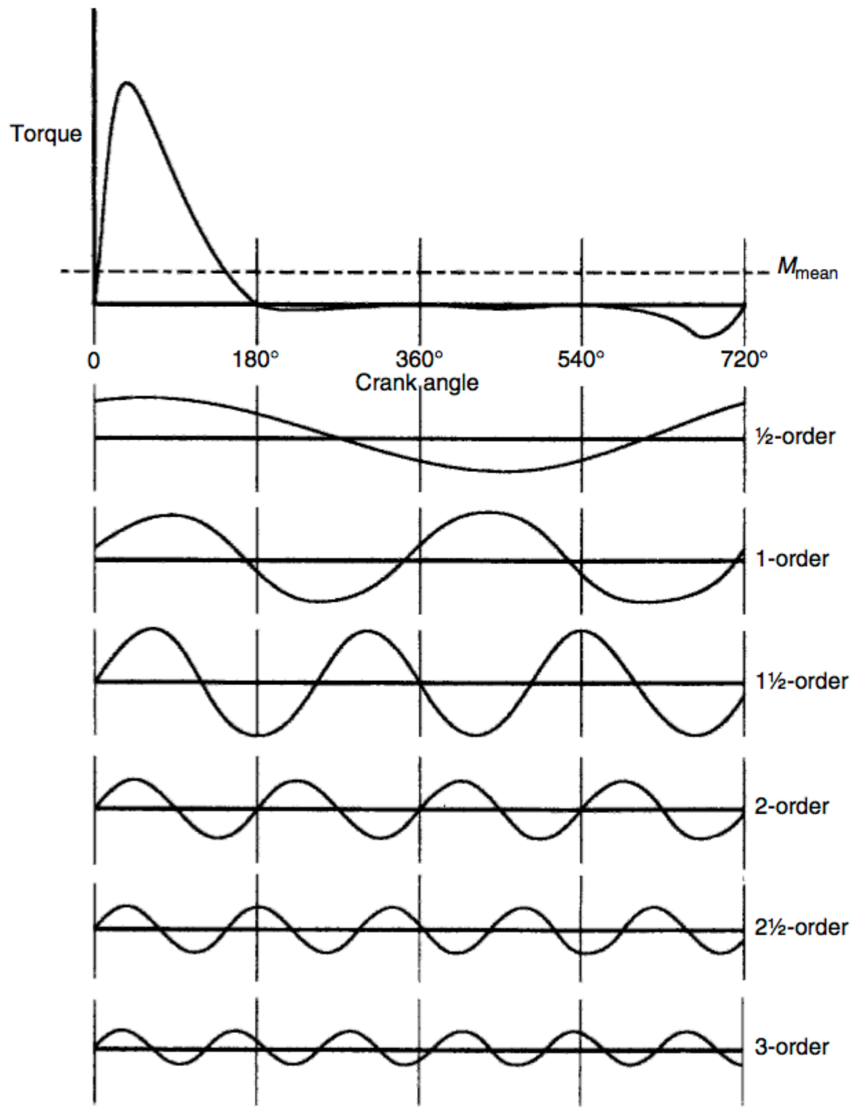


Figure 2.10 : Harmonics of engine exciting torque. [Jayabalan, 2004]

The multiple is defined by the order of the harmonics. A component of order $N_0$ = 1/2 occupies two revolutions of the engine, while $N_0$ = 1 occupies one revolution and so forth. In the case of a four cylinder four-stroke engine, there are two firing strokes per revolution of the crankshaft and the turning moment curve is repeated at intervals of 180°. The major critical speeds are important since at these speeds, the exciting torque $T_{ex}$ of all individual cylinders will act in phase. Further on the major critical speed is discussed in Chapter 3 - Torsional critical speed.

The torsional behaviour of a multi-cylinder engine must be completely analysed to prevent any unwanted damage. Computer program can be used to reduce the required endeavour. In some cases, the engine needs to be run close to or exactly at the critical speeds. Torsional vibration dampers (TVD) are often used to deal with the situation, where the viscous shearing absorbs the energy that is delivered into the system by the exciting forces. In engine testing, the TVD are normally installed close to the flywheel or at the dynamometer. The effectiveness of the damping at critical frequency depends on the proper selection of the damper. In selecting the right damper, the calculation of the energy supplied into the system per cycle with the energy absorbed by viscous shear in the damper need to be performed. The calculation will produce an estimation of the magnitude of the oscillatory stresses at the critical speed [Martyr & Plint, 2007b].

In dealing with the torsional vibration, there are some important points that should be considered as mentioned in [Martyr & Plint, 2007b]:

- It is recommended to avoid running the engine under power at the speed between 0.8 and 1.2 times the critical speed. If in the case that the mentioned situation is inevitable, it should be executed off load and as quickly as possible.
- In the case of high inertia dynamometer, the safety measure of the driveline components should be increased since the transient vibratory torque may well exceed the mechanical capacity of the driveline.
- It will also be problematic if the inertia of the dynamometer is much higher than that of the engine. This is the situation when it is required to couple a very small engine to a dynamometer with very high rated capacity. If the factor exceeds 2, a thorough torsional analysis should be carried out.
- The mentioned two-mass approximation of a dynamometer-engine system is insufficient when it involves large engine. This could result in overestimation of the critical speed.

## 2.5 Couplings

In rotating machineries that transmit power, misalignments can lead to harmful consequences, such as vibration, noise, failure of the shaft and bearings, and so on. Flexible couplings are widely implemented to allow a certain degree of misalignment between the shafts. One notable function of couplings is to compensate for the unavoidable misalignment in the co-axial shaft assembly. The misalignment is either parallel, angular, axial or the combination of these. It is stated in [Tadeo & Cavalca, 2003] that couplings can be divided into to basic groups, namely the one for mechanical misalignment and the other for the bending misalignment.

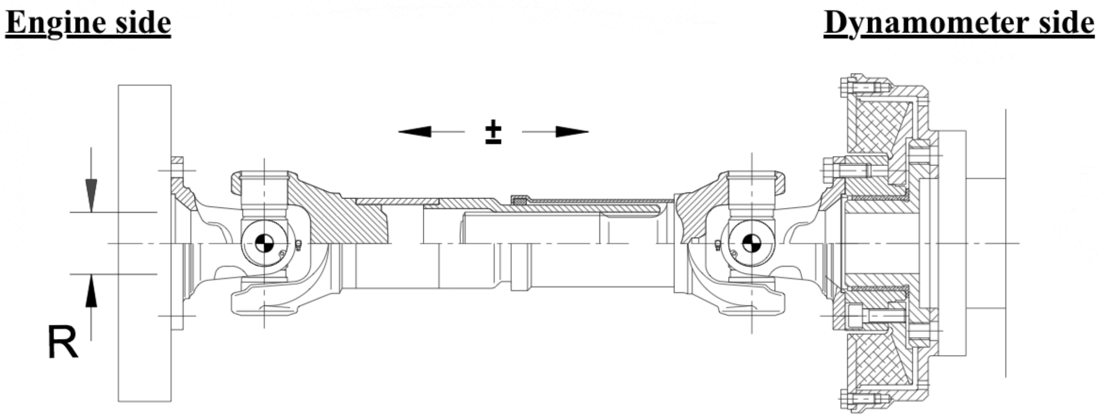


Figure 2.11 : Couplings installed on a shaft. [Ringfeder-Corp., 2006]

A dynamometer-engine system is very sensitive to torsional vibration of elements, such as the shaft, couplings, dynamometer and the engine itself. If improperly chosen, the coupling between the engine and the dynamometer can induce a serious problem during an engine test [Cruz-Peragón et al., 2009]. In a dynamometer-engine system, the couplings are normally installed at both ends as illustrated in Figure 2.11. It is not easy to choose the right coupling for a particular application. Most of the problems in the driveline are originated by unsuitable coupling.

Some common types of coupling are briefly described in following list as presented in [Martyr & Plint, 2007b].

Quill shaft with integral flanges and rigid couplings

This is a simple and reliable solution in the case of a driven machine is permanently coupled to the power source. However, it cannot tolerate relative vibration and misalignment. Hence, it is not suitable to be used in the test bed.

Quill shaft with toothed or gear type couplings

Unlike the previous coupling, the gear coupling is capable of dealing with relative vibration and some degree of misalignment, which makes it very suitable for high powers and speeds. Nevertheless, it must be carefully controlled to prevent problems involving wear and lubrication. Lubrication is vital since rapid deterioration may occur once local tooth-to-tooth seizure takes place and the impact can be catastrophic. Moreover, this shaft is stiff in torsion.

Conventional ‘cardan shaft’ with universal joints

Figure 2.12 shows a cardan shaft with universal joints. This type of shaft is the preferred solution in most cases. In general, it is readily available from the manufacturers. However, when run at larger speeds in comparison to that in automotive applications, this standard type shaft can be troublesome. To prevent fretting of the needle rollers, it is necessary for the shaft to be designed with a correct degree of misalignment.

Figure 2.12 : Cardan shaft with universal joints. [Gitano, 2008a]

Multiple membrane couplings

Figure 2.13 depicts a multiple membrane coupling. It is torsionally stiff, but it is capable to tolerate moderate degree of misalignment and relative axial displacement. It can be used in very high-speed applications.

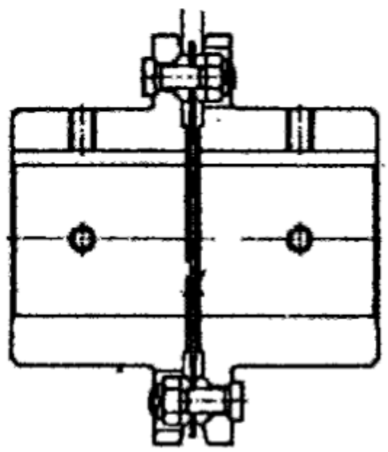

Figure 2.13 : Multiple steel disc type flexible coupling. [Martyr & Plint, 2007b]

Elastomeric element couplings

Another type of couplings is the elastomeric element coupling. A myriad of designs are available in the market. However, it is difficult to choose the right one. A notable advantage of this coupling is the capability to widely vary its torsional stiffness, which is done by changing the elastic elements.

# CHAPTER 3 : Methodology

## 3.1 Introduction

In designing the engine-dynamometer coupling shaft, certain procedures and calculations are to be performed to ensure the appropriate shaft is employed to avoid future problems to the engine, dynamometer or the shaft itself. The procedures to design the engine-dynamometer coupling as stated in [Martyr & Plint, 2007b] are as follows:

1. Prior to the test, the speed range and torque characteristics of the engine have to be ascertained. Moreover, it is to be determined whether the engine will run on load throughout the range.
2. Initial suggestions of the dimension (i.e. the length and diameter) as well as the material of the shaft are to be made before continuing with the analysis. The maximum allowable speed and the shaft stresses also need to be checked.
3. Calculation of the stiffness of the proposed shaft followed by the computation of the torsional critical speed.
4. In case of unacceptable critical speed, couplings are to be installed and the calculations are repeated, taking into account the couplings used.
5. The vibratory torque at critical speed is determined and it is to be checked with the maximum allowable continuous vibratory torque of the couplings.
6. Calculation of the whirling speed, transverse critical frequency and combined whirling speed.
7. Specification of alignment requirements.
8. Design of the shaft guard.

In this chapter, the details of each procedure are discussed and explained.

## 3.2 Bore and stroke

In this study, the maximum torque of the engine was varied to investigate the impact of the maximum torque of the engine on the geometry and dimensions of the shaft to be used to connect the engine and the dynamometer. Since only hypothetical engines were used in the analysis, the cylinder bore and stroke were estimated prior to designing the shaft. The estimation of the cylinder bore and stroke can be done with the known value of the displacement volume of the engine. The displacement volume, $V_{\mathrm{d}}$ of the engine in cubic metres ($m^3$) was calculated using the following equation:

$$V_{\mathrm{d}} = N_{\mathrm{cyl}} \cdot \frac{\pi}{4} B^2 S \qquad (3.1)$$

It was assumed that the engine was square with $B = S$. Hence, the bore and stroke can be easily calculated using the Equation ( 3.1 ).

The displacement volume $V_{\mathrm{d}}$ can be estimated given the brake mean effective pressure (*bmep*) was known. Mean effective pressure is a parameter that neither independent of engine's size nor speed. The *bmep* is defined as the work per cycle divided by the cylinder volume displaced per cycle. The typical values of *bmep* as stated in [Heywood, 1988] are as follows:

Table 3.1 : Typical brake mean effective pressure of engines. [Heywood, 1988]

| **Engine type** | **Maximum *bmep* [kPa]** |
|---|---|
| Naturally aspirated spark-ignition engine | 850 – 1050 |
| Turbocharged spark-ignition engine | 1250 – 1700 |
| Naturally aspirated four-stroke diesel engine | 700 – 900 |
| Turbocharged four-stroke diesel engine | 1000 – 1200 |
| Turbocharged after-cooled engine | 1400 |

With the typical *bmep* values from Table 3.1, the displacement volume $V_d$ of the engine in litres can be calculated using following equation:

$$bmep = \frac{6.28 \cdot n_g \cdot T}{V_d} \tag{3.2}$$

With the displacement volume obtained from Equation ( 3.2 ), the bore and stroke of the engine can be determined using Equation ( 3.1 ).

## 3.3 Speed range and torque characteristics of the engine

Prior to the analysis, the type of the engine, the number of cylinders and the type of dynamometer involve are to be ascertained. These parameters is needed to determine the service factors for dynamometer-engine combinations from Table 3.2 as mentioned in [Martyr & Plint, 2007b].

Table 3.2 : Service factors for engine-dynamometer setup. [Martyr & Plint, 2007b]

| **Dynamometer type** | **Number of cylinders** | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | **Diesel** | | | | | **Gasoline** | | | | |
| | 1/2 | 3/4/5 | 6 | 8 | 10+ | 1/2 | 3/4/5 | 6 | 8 | 10+ |
| Hydraulic | 4.5 | 4.0 | 3.7 | 3.3 | 3.0 | 3.7 | 3.3 | 3.0 | 2.7 | 2.4 |
| Hyd. + Dyno. (Start) | 6.0 | 5.0 | 4.3 | 3.7 | 3.0 | 5.2 | 4.3 | 3.6 | 3.1 | 2.4 |
| Eddy current (EC) | 5.0 | 4.5 | 4.0 | 3.5 | 3.0 | 4.2 | 3.8 | 3.3 | 2.9 | 2.4 |
| EC + Dyno. (Start) | 6.5 | 5.5 | 4.5 | 4.0 | 3.0 | 5.7 | 4.8 | 3.8 | 3.4 | 2.4 |
| D.C. + Dyno. (Start) | 8.0 | 6.5 | 5.0 | 4.0 | 3.0 | 7.2 | 5.8 | 4.3 | 3.4 | 2.4 |

With the service factor obtained from Table 3.2, the design torque was calculated by multiplying the maximum torque of the engine and the particular service factor according to the dynamometer-engine setup.

## 3.4 Shaft geometry, material and stress

The next procedure in designing the coupling shaft was to make a preliminary selection of possible dimensions and material of the shaft to be used in the dynamometer-engine connection. The diameter, *D* as well as the length, *L* of the shaft is the dimensions to be estimated prior to the calculation. Moreover, the modulus of rigidity, *G* of the shaft has to be known and will be used in further calculation. This depends on the material of the shaft, which in case of steel shaft, the G is equal to 80 GPa, while for aluminium shaft, the value of G is 26 GPa.

After the preliminary suggestion of the dimensions of the shaft is made, the shear stress can be determined. For a shaft of diameter *D*, the maximum shear stress in Pascal (Pa) induced by a torque *T* is given by following equation as presented in [Martyr & Plint, 2007b]:

$$\tau = \frac{16T}{\pi D^3} \tag{3.3}$$

For a tubular shaft with bore diameter *d*, the maximum shear stress induced in Pascal (Pa) is given by:

$$\tau = \frac{16TD}{\pi(D^4 - d^4)} \tag{3.4}$$

## 3.5 Shaft stiffness

The critical frequency of the shaft must lie outside the normal operating frequency range of the engine. Thus, the shaft connecting the engine and the dynamometer has to be designed with a suitable stiffness $C_s$ to avoid unwanted problems. Apart from that, the shaft also needs to be designed with a suitable degree of damping to prevent the initiation of dangerous level of torsional vibration should the unit is run through the critical speed [Martyr & Plint, 2007b]. As stated in [Martyr & Plint, 2007b], the torsional stiffness of a solid shaft of diameter $D$ and length $L$ in (cycles/minute) is given by:

$$C_s = \frac{\pi D^4 G}{32L} \tag{3.5}$$

whereas for a tubular shaft with bore $d$, the torsional stiffness is calculated as follows:

$$C_s = \frac{\pi (D^4 - d^4)}{32L} \tag{3.6}$$

Suppose that there is a combination of several elements in series, the combined torsional stiffness according to [Martyr & Plint, 2007b] is:

$$\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n} \tag{3.7}$$

It is very important to design the connecting shaft with the appropriate stiffness to ensure the critical engine speed $N_c$ does not lie within the speed range required by the engine to develop power [Martyr & Plint, 2007b]. The critical engine speed is discussed in following section.

## 3.6 Torsional critical speed

As mentioned in previous chapter, the dynamometer-engine connection tends to develop torsional vibrations. Hence, a preliminary calculation of torsional critical speed was carried out. If the critical engine speed obtained lies within the speed range of the engine, some modifications have to be made to the system to prevent it from undergoing any serious damage or unwanted problem. As stated in [Martyr & Plint, 2007b] and [Jayabalan, 2004], the resonant or critical frequency of torsional vibration of the system can be determined by following equation:

$$n_{\mathrm{c}} = \frac{60}{2\pi}\sqrt{\frac{C_{\mathrm{c}}(I_{\mathrm{e}} + I_{\mathrm{b}})}{I_{\mathrm{e}} I_{\mathrm{b}}}} \tag{3.8}$$

To calculate torsional critical speed, the critical frequency and the first major critical speed are required. Hence, after obtaining the critical frequency, the major critical speeds are to be determined. It is important to calculate the major critical speeds since the exciting torque, $T_{\mathrm{ex}}$ of all individual cylinders acts in phase at these speeds [Jayabalan, 2004; Martyr & Plint, 2007b]. The first major critical speed for a four-stroke, multi-cylinder, inline engine is:

$$N_0 = \frac{N_{\mathrm{cyl}}}{2} \tag{3.9}$$

Thus, in the case of a four cylinder, four-stroke engine the major critical speeds are of order 2, 4, 6, etc., while in the case of a six cylinder engine, they are of order 3, 6, 9, etc. Nevertheless, the minor critical speeds should not be completely ignored, especially in the case of large multi-cylinder engine [Jayabalan, 2004; Martyr & Plint, 2007b].

The first harmonic is generally the most significant in the excitation of torsional vibrations. For engines of moderate size such as passenger vehicle engines, it is generally sufficient to calculate the critical frequency from Equation ( 3.8 ) followed by the calculation of the corresponding critical engine speed. The corresponding critical engine speed for the critical frequency obtained is given by:

$$N_{\mathrm{c}} = \frac{n_{\mathrm{c}}}{N_0} \tag{3.10}$$

As recommended in [Jayabalan, 2004], it is best to ensure the first major critical speed lies below the minimum speed of the engine (i.e. lower than the idle speed of the engine which is around 600 – 1000 RPM). In this study, the target was to reduce the critical speed below 750 RPM in order to obtain acceptable critical engine speed. This important to avoid damages to the shaft and the system. If the first major critical speed falls within the engine speed range, it is considered as unacceptable. If that is the case, the flexible couplings should be introduced into the dynamometer-engine setup.

## 3.7 Vibratory torque

After the acceptable critical engine speed is obtained, the probable amplitude of any torsional vibration at the critical speed needs to be checked. The exciting torque, $T_{ex}$ is obtained from the multiplication of the value of the mean turning moment developed by the cylinder, $M_{mean}$ and the *p*-factor as stated in [Hartog, 1985]. The values of *p*-factor can be found in Table 3.3 which is taken from [Hartog, 1985; Martyr & Plint, 2007b].

Table 3.3 : *p*-factors. [Hartog, 1985]

| **Order** | ½ | 1 | 1½ | 2 | 2½ | 3… | 8 |
|---|---|---|---|---|---|---|---|
| ***p*-factor** | 2.16 | 2.32 | 2.23 | 1.91 | 1.57 | 1.28 | 0.08 |

To determine the $M_{mean}$, the indicated mean effective pressure *imep* is required. The *imep* is the indicated work for one cycle divided by the displacement volume. It is mentioned in [Martyr & Plint, 2007b] that the *imep* of the engine under the no-load condition is approximately 2 bar and this value was used in the analysis. With the known value of the *imep*, the $M_{mean}$ is calculated as follows:

$$M_{\text{mean}} = imep \cdot \frac{B^2 S}{16} \quad (3.11)$$

Hence, the exciting torque, $T_{ex}$ for one cylinder is given by:

$$T_{\text{ex}} = p \cdot M_{\text{mean}} \quad (3.12)$$

The exciting torque obtained from Equation ( 3.12 ) is only for one cylinder. Thus, the summation of the exciting torque for all cylinders is given by:

$$\sum T_{ex} = N_c \cdot T_{ex} \tag{3.13}$$

The vector summation of the exciting torque for all cylinders, $\sum T_{ex}$ induces the vibratory torque in the connecting shaft. In order to determine the vibratory torque, the dynamic magnifier has to be known. Dynamic magnifier is the measure of the susceptibility of the engine-dynamometer system to torsional vibration. The value of dynamic magnifier can be obtained from Table 3.4 as presented in [Jayabalan, 2004].

Table 3.4 : Dynamic magnifier, *M*. [Jayabalan, 2004]

| **Shore hardness** | 50/55 | 60/65 | 70/75 | 75/80 |
|---|---|---|---|---|
| **Dynamic magnifier** | 10.5 | 8.0 | 5.2 | 2.7 |

In the case involving several components such as two identical rubber couplings, the combined dynamic magnifier, $M_c$ has to be calculated instead. This is given by following equation:

$$\left(\frac{1}{M_c}\right)^2 = \left(\frac{1}{M_1}\right)^2 + \left(\frac{1}{M_2}\right)^2 + \left(\frac{1}{M_3}\right)^2 + \ldots + \left(\frac{1}{M_n}\right)^2 \tag{3.14}$$

Finally, the vibratory torque, $T_v$ can be calculated as follows:

$$T_v = \frac{\sum T_{ex} \cdot M_c}{\left(1 + \frac{I_e}{I_b}\right)} \tag{3.15}$$

The engine tested will develop both steady as well as vibratory torques. It is important that the coupling shaft designed to be able to withstand both torques.

## 3.8 Whirling speed

To reduce the torsional stiffness of the shaft, the universal joint or flexible coupling are normally installed at each end of the shaft. This could result in a whirling to occur at a rotational speed $N_w$. To conclude the calculation of the dynamometer-engine coupling shaft design, the whirling speed of the shaft needs to be calculated. The whirling speed in RPM (rev/min) of a solid shaft of length $L$ is given by:

$$N_w = \frac{30\pi}{L^2}\sqrt{\frac{E\pi D^4}{64W_s}} \quad (3.16)$$

where $W_s$ is the mass of the shaft per unit length. Since the shaft is made of steel, the mass of the shaft is determined by multiplying the density of steel, which is 7850 $kgm^{-3}$ with the volume of the shaft.

The maximum speed of the engine tested should not exceed 80% of the whirling speed $N_w$. It is recommended to allow for radial flexibility in the case where the rubber flexible couplings are used or otherwise the whirling speed will be greatly reduced. In high-speed applications, occasionally the self-aligning rigid steady bearings are mounted at the centre of the flexible couplings. However this method is unfavourable since it could lead to fretting problems [Martyr & Plint, 2007b].

It is pointed in [Martyr & Plint, 2007b] that the whirling speed of a shaft and its natural frequency of transverse oscillation are identical. Therefore, it is recommended to calculate the transverse critical frequency of the shaft plus two half couplings in order to include the effect of transverse coupling flexibility. The transverse critical frequency of the shaft plus two half couplings $N_t$ is determined by following equation:

$$N_{\mathrm{t}} = \frac{30}{\pi}\sqrt{\frac{k}{W}} \tag{3.17}$$

where $W$ = mass of the shaft + half couplings and $k$ = combined radial stiffness of the two couplings. Taking this effect into account, the combined whirling speed $N$ is given by:

$$\left(\frac{1}{N}\right)^2 = \left(\frac{1}{N_{\mathrm{w}}}\right)^2 + \left(\frac{1}{N_{\mathrm{t}}}\right)^2 \tag{3.18}$$

All calculations in this study were done using Microsoft Excel. All spreadsheets involved can be found in the Appendix section.

# CHAPTER 4 : Results and Discussion

## 4.1 Introduction

In this section, the results obtained from the analysis are presented in form of tables and graphs. The calculation has been conducted on hypothetical engines with maximum torque varies from 40 Nm to 200 Nm. It was assumed that all engines are square, i.e. the bore and stroke of the engines are equal. Since only hypothetical engines involved, the geometry and dimensions as well as the weight of the flywheel were unknown. Those were required to calculate the moment of inertia of the engines. Thus, the moment of inertia of the engine, $I_e$ was estimated according to engine's size as follows:

Table 4.1 : Estimated moment of inertia of the engines.

| **Engine torque, $T$ [Nm]** | **Moment of inertia, $I_e$ [kgm$^2$]** |
|---|---|
| 40 | 0.09 |
| 80 | 0.18 |
| 120 | 0.27 |
| 160 | 0.36 |
| 200 | 0.45 |

Various analyses were carried out to investigate the importance of each parameter such as shaft diameter and length in designing the shaft for dynamometer-engine connection. Apart from that, the comparison of steel and aluminium shaft was also presented. Two multi-bush couplings were used to reduce the torsional stiffness. Table 4.2 shows the characteristics of the multi-bush coupling used, which were obtained from manufacturer's catalogue.

Table 4.2 : Characteristics of the multi-bush coupling.

| | |
|---|---|
| **Maximum torque** | 814 Nm |
| **Rated torque** | 170 Nm |
| **Maximum continuous vibratory torque** | ± 136 Nm |
| **Shore (IHRD) hardness** | 50/55 |
| **Dynamic torsional stiffness** | 8400 Nm/rad |

This was applied for all calculations. Utilisation of other types of coupling might generate different results, which are not presented in this study. The analysis was carried out for the coupling of an engine and an eddy current dynamometer with the dynamometer starting.

## 4.2 Shaft diameter and length

Following table shows the estimated diameter and length of steel shaft, which can be used in the dynamometer-engine connection for various engines with different maximum torque value. The values were obtained through calculations using equations presented in CHAPTER 3 : Methodology. A proper shaft dimension is important to ensure the engine critical speed falls below 750 RPM and the whirling speed of the shaft stays higher than the maximum speed of the engine, i.e. 7000 RPM and above.

Table 4.3 : Shaft diameter and length.

| **Torque, *T* [Nm]** | **Shaft diameter, *D* [mm]** | **Shaft length, *L* [mm]** |
|---|---|---|
| 40 | 20 | 450 - 500 |
| 80 | 25 | 350 - 550 |
| 120 | 30 - 35 | 300 - 600 |
| 160 | 30 - 80 | 200 - 700 |
| 200 | 30 - 80 | 200 - 750 |

It was observed that the larger the maximum torque of the engine, the wider the range of shaft diameter and length that can be used in a dynamometer-engine coupling. A too long shaft is not good since it will reduce the whirling speed of the shaft, whereas a shaft with too large in diameter is also not preferable for the reason that it is neither compact nor economical. Hence, the average values of diameter and length were determined from Table 4.3.

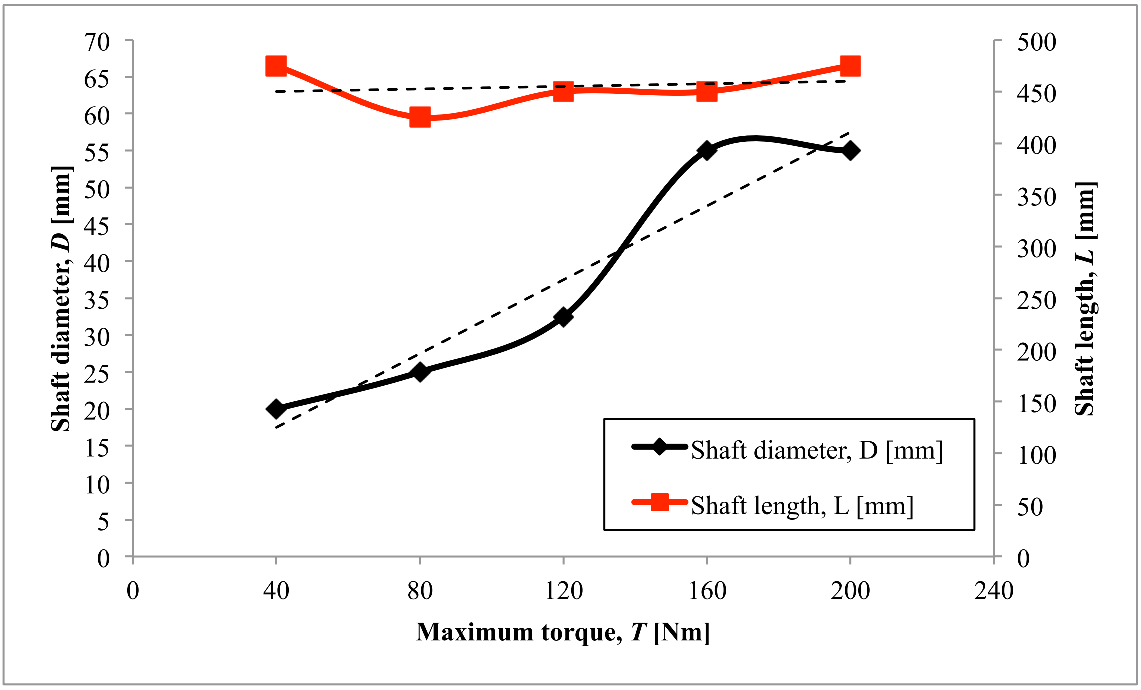


Figure 4.1 : Shaft and diameter length.

Figure 4.1 depicts the average shaft diameter and length for each engine. It can be seen that the shaft diameter increases linearly as the maximum torque of the engine increases. This shows that the shaft diameter is proportional to the maximum torque of the engine. However, it was observed that the shaft length was almost equal for each engine. To further investigate this matter, the analysis was repeated using constant shaft length for each engine. This is presented in following section.

## 4.3 Critical and whirling speed of the shaft with an equal length

As observed in previous section, the length of the shaft is almost equal for engines with the maximum torque vary from 40 Nm to 200 Nm. To prove that constant shaft length can be used for all engines regardless of its maximum torque value, the analysis was repeated with the length of the shaft for all engines was fixed at 500 mm, whereas the diameter was increased as the maximum torque increases as shown in Table 4.4.

Table 4.4 : Critical and whirling speed of the shafts with constant length.

| **Torque, $T$ [Nm]** | **Shaft diameter, $D$ [mm]** | **Critical engine speed, $N_c$ [rev/min]** | **Combined whirling speed, $N$ [rev/min]** |
|---|---|---|---|
| 40 | 20 | 719.56 | 8522.90 |
| 80 | 25 | 710.81 | 9991.98 |
| 120 | 30 | 711.74 | 11088.91 |
| 160 | 35 | 704.73 | 11820.56 |
| 200 | 40 | 694.00 | 12227.21 |

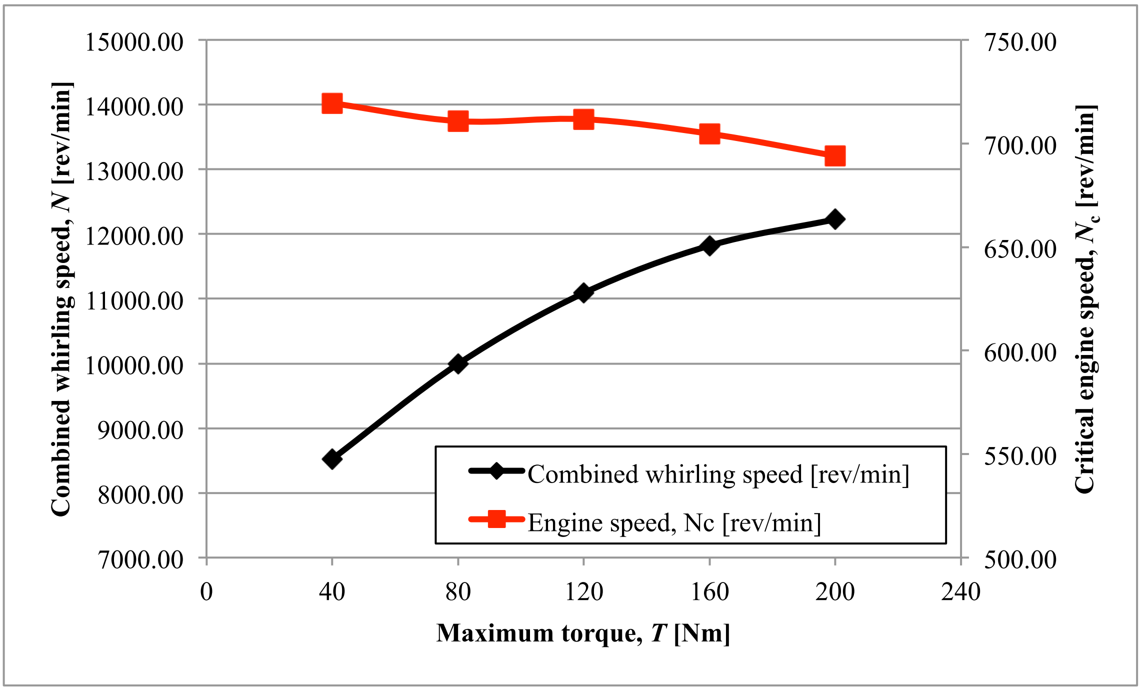


Figure 4.2 : Critical and whirling speed of the shafts with constant length.

From the calculation, it was proved that even though the length of the shaft was kept constant for each engine, the critical speed of the engine, $N_c$ stays below 750 RPM, whereas the whirling speed of the shaft lay above 7000 RPM and was considered as acceptable as shown in Figure 4.2. It was observed that for the engines with lower maximum torque, the whirling speed of the shaft was lower than the engines with higher maximum torque. To increase the whirling speed of the shaft, slightly shorter shaft can be used. However, if the shaft was too short, the critical engine speed will exceed 750 RPM and this has to be avoided.

## 4.4 Critical and whirling speed of the shaft with an equal diameter

To investigate the importance of the shaft diameter in the design of dynamometer-engine coupling shaft, the analysis was repeated for shaft with constant diameter. The diameter of the shaft was fixed at 40 mm for every engine, whereas the shaft length was increased linearly with increasing maximum torque of the engine as shown in Table 4.5. The same calculations were carried out to obtain the critical engine speed and the whirling speed of the shaft. The results obtained from the analysis were tabulated in Table 4.5.

Table 4.5 : Critical and whirling speed of the shafts with constant diameter.

| **Torque, $T$ [Nm]** | **Shaft length, $L$ [mm]** | **Critical engine speed, $N_c$ [rev/min]** | **Combined whirling speed, $N$ [rev/min]** |
|---|---|---|---|
| 40 | 400 | 1129.77 | 14530.51 |
| 80 | 500 | 877.84 | 12227.21 |
| 120 | 600 | 773.78 | 10012.18 |
| 160 | 700 | 714.48 | 8116.32 |
| 200 | 800 | 675.11 | 6594.95 |

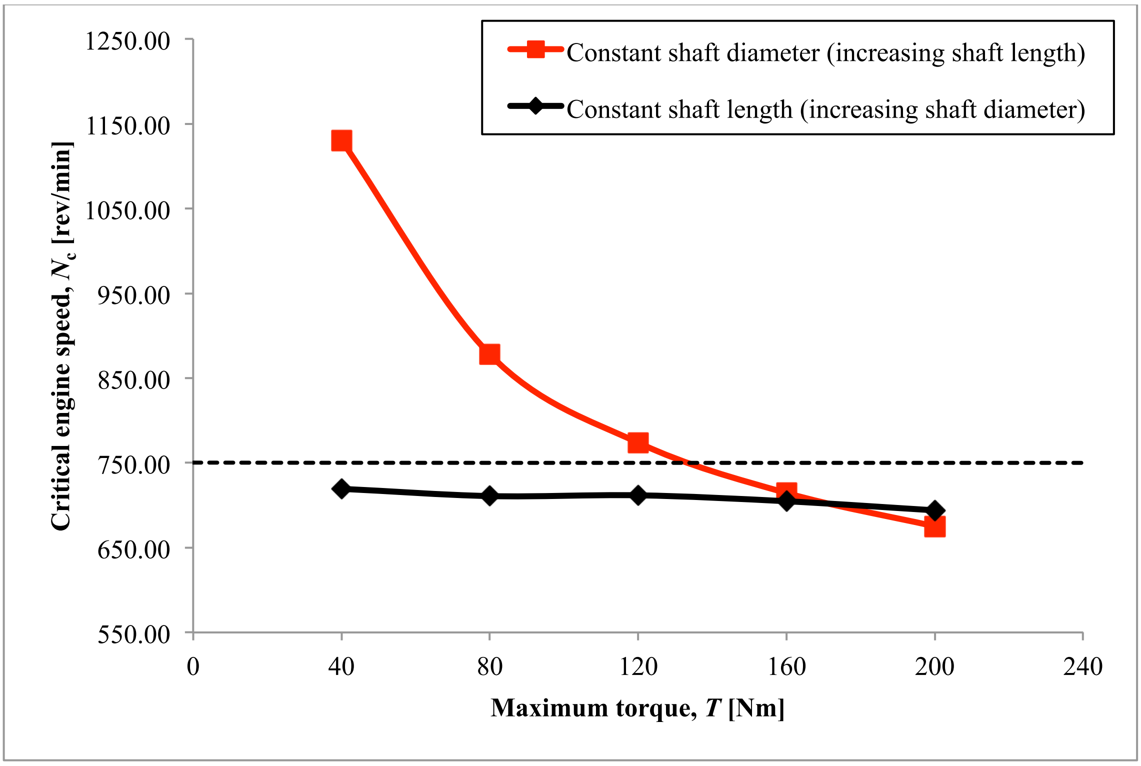


Figure 4.3 : Critical and whirling speed of the shafts with constant diameter.

Figure 4.3 depicts the critical engine speed for the case of shafts with constant diameter as well as shafts with constant length connecting the engine with various maximum torques to the dynamometer. It is important that the critical engine speed to stay below the idle speed of the engine or about 750 RPM. It was observed that for the case of shafts with constant length while the diameter was increased as the maximum torque increases, the critical engine speed of every engine lay below 750 RPM. On the other hand, for the shafts with constant diameter while the length was increased with increasing maximum torque of the engine, the critical engine speeds for engines with maximum torque of 40 Nm, 80 Nm and 120 Nm exceeded the specified limit. This is an unfavourable situation and it should be avoided to prevent any serious damage to the system or any injury to the personnel.

Apart from that, the whirling speed for the engine with maximum torque of 200 Nm for the case of constant shaft diameter is 6595 RPM as shown in Table 4.5, which is unacceptable. This results show that the shaft diameter is a vital parameter in a dynamometer-engine coupling shaft design and has to be chosen properly. It cannot be fixed to any value for a variety of engines with different maximum torques. The shaft diameter should be dimensioned and optimised according to the maximum torque of the engine. Nevertheless, the shaft length plays less important role compared to the diameter, whereby the same shaft length can be used for engines with different maximum torques under the circumstances that the diameter is increased as the maximum engine torque increases.

## 4.5 Dynamometer coupling shaft design for CamPro IAFM engine

To further understand the procedures in designing the coupling shaft for dynamometer-engine connection, a step-by-step example of the analysis is presented in this section. The system involves a four-cylinder four-stroke CamPro IAFM gasoline engine, which was coupled to an eddy current dynamometer with the dynamometer starting. The specifications of the engine are as follows:

Table 4.6 : CamPro IAFM engine specifications.

| | |
|---|---|
| **Engine displacement, $V_d$** | 1597 $cm^3$ |
| **Bore, *B* & Stroke, *S*** | 76 × 88 mm |
| **Maximum torque, *T*** | 148 Nm at 4000 RPM |
| **Maximum power, *P*** | 82 kW |
| **Maximum speed** | 6500 RPM |
| **Estimated moment of inertia, $I_e$** | 0.34 $kgm^2$ |

The dimension and geometry of the engine's flywheel was unknown, hence the moment of inertia of the engine was estimated as 0.34 $kgm^2$. The moment of inertia of an eddy current dynamometer, $I_d$ was 0.30 $kgm^2$.

Step 1 : Service factor and design torque

The service factor for the dynamometer-engine combination was determined from Table 3.2, which in this case (gasoline engine and an eddy current dynamometer with the dynamometer starting) indicated a service factor of 4.8. Thus, the design torque was given by:

$$148 \text{ Nm} \times 4.8 = 710.4 \text{ Nm}$$

Step 2 : Shaft geometry, material and stress

The dimensions and the material of the shaft to be used to connect the engine and the dynamometer are to be proposed by the designer prior to further calculations. A steel shaft was used in the design. Therefore, according to the graph obtained from previous results as shown in Figure 4.1, the proposed dimensions and material properties of the shaft were as follows:

- Shaft diameter $D = 40$ mm
- Shaft length $L = 450$ mm
- Modulus of rigidity $G = 80 \times 10^9$ Pa
- Modulus of elasticity $E = 200 \times 10^9$ Pa

The torsional stress was calculated using Equation ( 3.3 ):

$$\tau = \frac{16 \times (710.4 \text{ Nm})}{\pi \times (0.04 \text{ m})^3} \times 10^{-6}$$

$$\tau = 56.53 \text{ MPa}$$

The torsional stress, $\tau$ of 56.53 MPa was a lot smaller than the modulus of rigidity of the shaft, hence it was considered as acceptable.

Step 3 : Shaft stiffness and critical speed

The stiffness of the shaft was calculated using Equation ( 3.5 ), which yield:

$$C_s = \frac{\pi \times (0.04\ \text{m})^4 \times 80 \times 10^9\ \text{Pa}}{32 \times 0.45\ \text{m}}$$

$$C_s = 44680\ \text{Nm/rad}$$

With the torsional stiffness of the shaft was known, the critical frequency of torsional vibration was determined using Equation ( 3.8 ).

$$n_c = \frac{60}{2\pi}\sqrt{\frac{(44680\ \text{Nm/rad}) \times (0.34\ \text{kgm}^2 + 0.3\ \text{kgm}^2)}{0.34\ \text{kgm}^2 \times 0.3\ \text{kgm}^2}}$$

$$n_c = 5056\ \text{cycles/min}$$

In order to calculate the first major critical speed, the order of the first major critical speed, $N_0$ has to be determined. Since it was a four-cylinder, four-stroke engine, the first major critical speed occurred at:

$$N_0 = \frac{4\ \text{cylinders}}{2} = 2$$

Therefore, the first major critical speed, $N_c$ was calculated using Equation ( 3.10 ) as follows:

$$N_c = \frac{5056\ \text{cycles/min}}{2}$$

$$N_c = 2528\ \text{rev/min}$$

The first major critical speed, $N_c$ = 2528 rev/min fell within the speed range of the engine. This was an unacceptable situation and would cause the shaft to break in less than an hour during the testing. This is a result of a dynamometer-engine connection done without any flexible coupling involves. As mentioned in previous chapter, the resonant speed should lie below the idle speed of the engine or approximately at 750 RPM, which corresponds to a critical frequency of torsional vibration, $n_c$ = 1500 cycles/min.

Step 4 : Flexible coupling

Flexible couplings need to be installed in order to reduce the resonant speed. In this case, two multi-bush couplings were used, each having the stiffness of 8400 Nm/rad. The characteristics of the flexible coupling can be found in Table 4.2. The combined torsional stiffness involving the combination of these elements was calculated using Equation ( 3.7 ).

$$\frac{1}{C_s} = \frac{1}{44680 \text{ Nm/rad}} + \frac{1}{8400 \text{ Nm/rad}} + \frac{1}{8400 \text{ Nm/rad}}$$

$$= 2.605 \times 10^{-4}$$

$$C_s = 3839 \text{ Nm/rad}$$

With the new torsional stiffness of 3839 Nm/rad, calculations in Step 3 were repeated to determine the critical frequency of torsional vibration and the corresponding engine speed.

$$n_c = \frac{60}{2\pi}\sqrt{\frac{(3839 \text{ Nm/rad}) \times (0.34 \text{ kgm}^2 + 0.3 \text{ kgm}^2)}{0.34 \text{ kgm}^2 \times 0.3 \text{ kgm}^2}}$$

$$n_c = 1482 \text{ cycles/min}$$

This corresponds to an engine speed of:

$$N_c = \frac{1482 \text{ cycles/min}}{2}$$

$$N_c = 741 \text{ rev/min}$$

which was considered as acceptable.

Step 5 : Vibratory torque

The next step was to check the probable amplitude of any torsional vibration at the critical speed. The *imep* of the engine under no-load condition was around 2 bar or

200000 Pa as stated in [Martyr & Plint, 2007b]. Hence, the mean turning moment, $M_{\text{mean}}$ was calculated using Equation ( 3.11 ).

$$M_{\text{mean}} = (200000\ \text{Pa}) \cdot \frac{(0.076\ \text{m})^2 \times (0.088\ \text{m})}{16}$$

$$M_{\text{mean}} = 6.35\ \text{Nm}$$

The first major critical occurs at $N_0 = 2$. Thus the *p*-factor was given as 1.91, which was obtained from Table 3.3. The exciting torque for one cylinder was then calculated using Equation ( 3.12 ) as follows:

$$T_{\text{ex}} = 1.91 \times 6.35\ \text{Nm}$$

$$T_{\text{ex}} = 12.14\ \text{Nm}$$

For a four-cylinder engine, the summation of the exiting torques for all cylinders was given by:

$$\sum T_{ex} = 4 \times 12.14\ \text{Nm}\ =\ 48.54\ \text{Nm}$$

Table 3.4 shows the dynamic magnifier, *M*. In this case, the couplings with shore hardness of 50/55 were used and the value of *M* obtained from Table 3.4 is 10.5. Since two rubber couplings were used, the combined dynamic magnifier, $M_{\text{c}}$ was determined. It was calculated using Equation ( 3.14 ) as follows:

$$\left(\frac{1}{M_{\text{c}}}\right)^2 = \left(\frac{1}{10.5}\right)^2 + \left(\frac{1}{10.5}\right)^2$$

$$M_{\text{c}} = 7.42$$

Using Equation ( 3.15 ), the vibratory torque was then calculated.

$$T_{\text{v}} = \frac{(48.54\ \text{Nm}) \times 7.42}{\left(1 + \dfrac{0.34\ \text{kg} \cdot \text{m}^2}{0.30\ \text{kg} \cdot \text{m}^2}\right)}$$

$$T_{\text{v}} = 169\ \text{Nm}$$

This actually exceeds the maximum continuous vibratory torque of the coupling, which is ±136 Nm. However, the multiple bush couplings are able to tolerate a short period of overload, but the engine needs to be run very quickly through the critical speed. Therefore this solution was considered as acceptable. Another alternative is to use the same coupling but with shore hardness of 60/65. This results in reduction of the dynamic magnifier from 10.5 to 2.7, which leads to reduction of vibratory torque, $T_v$.

<u>Step 6 : Whirling speed of the shaft</u>

The final step in designing the shaft is to determine the whirling speed of the shaft. The whirling speed needs to be greater than the speed at which the shaft will be tested. It is recommended to ensure the whirling speed lies above 7000 RPM. With the mass of the shaft per unit length, $W_s$ is 9.86 kg/m, the whirling speed of the shaft was calculated using Equation ( 3.16 ).

$$N_w = \frac{30\pi}{(0.45\text{ m})^2}\sqrt{\frac{(200\times10^9\text{ Pa})\times\pi\times(0.04\text{ m})^4}{64(9.86\text{ kg/m})}}$$

$$N_w = 23492\text{ rev/min}$$

Apart from the whirling speed, the transverse critical frequency, $N_t$ needs to be calculated. With the mass of the shaft and half couplings of 11.54 kg and the combined radial stiffness of the couplings, $k$ of 33.6 MN/m, the transverse critical frequency was determined using Equation ( 3.17 ).

$$N_t = \frac{30}{\pi}\sqrt{\frac{33.6\times10^6\text{ N/m}}{11.54\text{ kg}}}$$

$$N_t = 16295\text{ rev/min}$$

Combining the whirling speed $N_w$ and transverse critical frequency $N_t$, the combined whirling speed $N$ was calculated using Equation ( 3.18 ) as follows:

$$\left(\frac{1}{N}\right)^2 = \left(\frac{1}{23492 \text{ rev/min}}\right)^2 + \left(\frac{1}{16295 \text{ rev/min}}\right)^2$$

$$N = 13389 \text{ rev/min}$$

The combined whirling speed of 13389 RPM was good enough, thus the design was considered as acceptable.

## 4.6 Critical engine speed of an aluminium shaft

Besides the dimension of the shaft, the material also plays an important role in designing the proper shaft to be used in a particular dynamometer-engine connection. Besides steel, aluminium is also widely used as the material of the shaft by many manufacturers. Aluminium is a lightweight material, which possesses lower modulus of rigidity as well as modulus of elasticity in comparison to steel. This gives an advantage to the aluminium over steel shaft since lower modulus of rigidity results in lower torsional stiffness of the shaft.

Table 4.7 : Critical engine speed of steel and aluminium shaft.

| **Torque, $T$ [Nm]** | **Shaft diameter, $D$ [mm]** | **Critical engine speed, $N_c$ [rev/min]** | |
|---|---|---|---|
| | | *Steel shaft* | *Aluminium shaft* |
| 40 | 20 | 719.56 | 474.53 |
| 80 | 25 | 710.81 | 523.46 |
| 120 | 30 | 711.74 | 578.16 |
| 160 | 35 | 704.73 | 614.76 |
| 200 | 40 | 694.00 | 634.48 |

Table 4.7 shows the critical engine speed for steel and aluminium shaft given that the length of the shafts is equal to 500 mm, whereas its diameter is increasing as the maximum torque of the engine increases as shown in Table 4.7. It was observed that critical engine speed of aluminium shaft was lower than steel shaft for every maximum torque value given that the same dimension of the shaft was used.

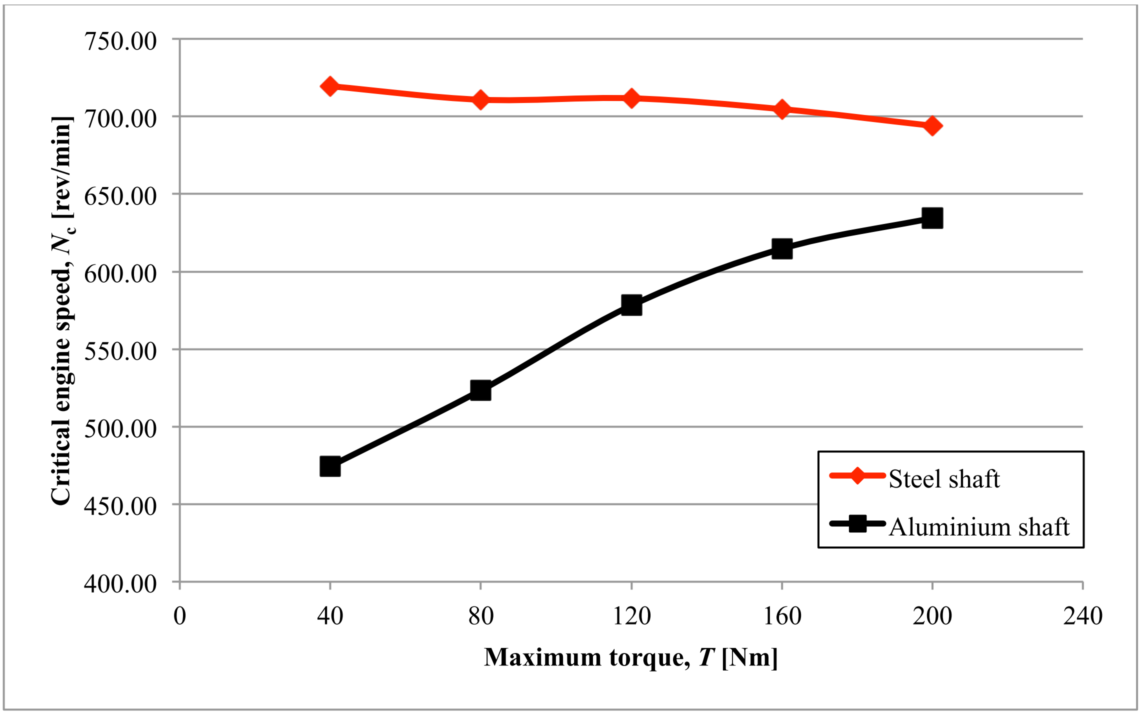


Figure 4.4 : Critical engine speed of steel and aluminium shaft.

Figure 4.4 depicts the curves of steel and aluminium shaft against the maximum torque of the engine so that the difference of the critical engine speed between steel and aluminium shaft can be clearly observed. As mentioned earlier, lower critical engine speed of aluminium shaft is a result of smaller torsional stiffness due to lower modulus of rigidity. This is the reason why many manufacturers have used aluminium instead of steel in producing drive shafts.

# CHAPTER 5 : Conclusion and Recommendation

## 5.1 Conclusion

It was observed in this study that the dimensions of the shaft play an important role in obtaining the right design. The diameter of the shaft is a vital parameter and should be dimensioned properly according to the output of the engine and the type of dynamometer used. On the other hand, the length of the shaft is not as important as its diameter. In every case, the shaft length of 500 mm can be used and acceptable results were obtained. Another factor that influence the design is the material of the shaft. It was observed that aluminium shaft performs better as a dynamometer-engine coupling shaft in comparison to steel. This is supported by the fact that most shaft manufacturers use aluminium in their products. It is important to perform the step-by-step procedures and calculations as presented in this study in designing the shaft. To minimise the designing endeavour, the usage of computer program such as Microsoft Excel is recommended.

## 5.2 Recommendation

This study was conducted analytically due to the limitation of time and funding. It is recommended to design a real shaft for a real dynamometer-engine system. A study consisting of both analytical and experimental part is preferable, in which the accuracy of the result obtained analytically can be proved. Moreover, a study involving hypothetical engines is disadvantageous since the geometry of the engine's flywheel is unknown. This could lead to error, although the magnitude of the error is uncertain. A study using a real engine is recommended where the actual flywheel can be measured and its exact moment of inertia can be determined.

# APPENDIX

## A) Steel shaft (Constant shaft length = 500 mm)

### A1) Engine maximum torque = 40 Nm

| | | | | |
|---|---|---|---|---|
| **Displacement volume, V_d [Liter]** | 0.53 | | | |
| **Bore, B [m]** | 0.055 | **Maximum shear stress, τ [MPa]** | 122.23 | |
| **Stroke, S [m]** | 0.055 | **Torsional stiffness, Cs [cycles/min]** | 2513.27 | |
| **Maximum torque, T [Nm]** | 40 | | | |
| **Service factor** | 4.8 | | | |
| **Design torque, T [Nm]** | 192 | **Critical frequency of torsional oscillation, nc [cycles/min]** | 1819.46 | |
| **Shaft diameter, D [m]** | 0.02 | | | |
| **Shaft length, L [m]** | 0.5 | | | |
| **Shaft volume [m^3]** | 1.57E-04 | | | **Engine speed [rev/min]** |
| **Modulus of rigidity, G [Pa]** | 8.00E+10 | **First major critical** | 2 | 909.73 |
| **Moments of inertia of engine, Ie [kgm^2]** | 0.09 | **Second major critical** | 4 | 454.86 |
| **Moments of inertia of dyno, Id [kgm^2]** | 0.3 | **Third major critical** | 6 | 303.24 |
| **Number of cylinders** | 4 | **Fourth major critical** | 8 | 227.43 |
| | | | | |
| **Number of couplings** | 2 | **Combined torsional stiffness [Nm/rad]** | 1572.37 | |
| **Couplings stiffness [Nm/rad]** | 8400 | **NEW critical frequency, nc [cycles/min]** | 1439.13 | |
| | | **NEW engine speed, Nc [rev/min]** | 719.56 | |
| | | | | |
| **imep [Pa]** | 200000 | **M_mean [Nm]** | 2.10 | |
| **p-factor** | 1.91 | **T_ex [Nm]** | 4.02 | |
| | | **∑T_ex [Nm]** | 16.08 | |
| | | | | |
| **Dynamic magnifier, M** | 10.5 | **Combined dynamic magnifier, M** | 7.42 | |
| | | **Vibratory torque, T_v [Nm]** | 91.81 | |
| | | | | |
| **Mass of shaft [kg]** | 1.23 | | | |
| **Mass of the shaft/unit length, W_s [kg/m]** | 2.47 | **Whirling speed, N_w [rev/min]** | 9514.40 | |
| **Modulus of elasticity, E [Pa]** | 2.00E+11 | **Transverse critical frequency, N_t [rev/min]** | 19175.13 | |
| **Mass of shaft + half couplings, W [kg]** | 8.33 | **Combined whirling speed, N [rev/min]** | 8522.90 | |
| **Combined radial stiffness, k [N/m]** | 3.36E+07 | | | |

## A2) Engine maximum torque = 80 Nm

| | | | | |
|---|---|---|---|---|
| **Displacement volume, V_d [Liter]** | 1.06 | | | |
| **Bore, B [m]** | 0.070 | **Maximum shear stress, τ [MPa]** | 125.16 | |
| **Stroke, S [m]** | 0.070 | **Torsional stiffness, Cs [cycles/min]** | 6135.92 | |
| **Maximum torque, T [Nm]** | 80 | | | |
| **Service factor** | 4.8 | | | |
| **Design torque, T [Nm]** | 384 | **Critical frequency of torsional oscillation, nc [cycles/min]** | 2230.16 | |
| **Shaft diameter, D [m]** | 0.025 | | | |
| **Shaft length, L [m]** | 0.5 | | | |
| **Shaft volume [m^3]** | 2.45E-04 | | | **Engine speed [rev/min]** |
| **Modulus of rigidity, G [Pa]** | 8.00E+10 | **First major critical** | 2 | 1115.08 |
| **Moments of inertia of engine, Ie [kgm^2]** | 0.18 | **Second major critical** | 4 | 557.54 |
| **Moments of inertia of dyno, Id [kgm^2]** | 0.3 | **Third major critical** | 6 | 371.69 |
| **Number of cylinders** | 4 | **Fourth major critical** | 8 | 278.77 |
| | | | | |
| **Number of couplings** | 2 | **Combined torsional stiffness [Nm/rad]** | 2493.33 | |
| **Couplings stiffness [Nm/rad]** | 8400 | **NEW critical frequency, nc [cycles/min]** | 1421.63 | |
| | | **NEW engine speed, Nc [rev/min]** | 710.81 | |
| | | | | |
| **imep [Pa]** | 200000 | **M_mean [Nm]** | 4.21 | |
| **p-factor** | 1.91 | **T_ex [Nm]** | 8.04 | |
| | | **∑T_ex [Nm]** | 32.15 | |
| | | | | |
| **Dynamic magnifier, M** | 10.5 | **Combined dynamic magnifier, M** | 7.42 | |
| | | **Vibratory torque, T_v [Nm]** | 149.20 | |
| | | | | |
| **Mass of shaft [kg]** | 1.93 | | | |
| **Mass of the shaft/unit length, W_s [kg/m]** | 3.85 | **Whirling speed, N_w [rev/min]** | 11893.00 | |
| **Modulus of elasticity, E [Pa]** | 2.00E+11 | **Transverse critical frequency, N_t [rev/min]** | 18423.71 | |
| **Mass of shaft + half couplings, W [kg]** | 9.03 | **Combined whirling speed, N [rev/min]** | 9991.98 | |
| **Combined radial stiffness, k [N/m]** | 3.36E+07 | | | |

## A3) Engine maximum torque = 120 Nm

| | | | | |
|---|---|---|---|---|
| **Displacement volume, V_d [Liter]** | 1.59 | | | |
| **Bore, B [m]** | 0.080 | **Maximum shear stress, τ [MPa]** | 108.65 | |
| **Stroke, S [m]** | 0.080 | **Torsional stiffness, Cs [cycles/min]** | 12723.45 | |
| **Maximum torque, T [Nm]** | 120 | | | |
| **Service factor** | 4.8 | | | |
| **Design torque, T [Nm]** | 576 | **Critical frequency of torsional oscillation, nc [cycles/min]** | 2857.38 | |
| **Shaft diameter, D [m]** | 0.03 | | | |
| **Shaft length, L [m]** | 0.5 | | | |
| **Shaft volume [m^3]** | 3.53E-04 | | | **Engine speed [rev/min]** |
| **Modulus of rigidity, G [Pa]** | 8.00E+10 | **First major critical** | 2 | 1428.69 |
| **Moments of inertia of engine, Ie [kgm^2]** | 0.27 | **Second major critical** | 4 | 714.35 |
| **Moments of inertia of dyno, Id [kgm^2]** | 0.3 | **Third major critical** | 6 | 476.23 |
| **Number of cylinders** | 4 | **Fourth major critical** | 8 | 357.17 |
| | | | | |
| **Number of couplings** | 2 | **Combined torsional stiffness [Nm/rad]** | 3157.66 | |
| **Couplings stiffness [Nm/rad]** | 8400 | **NEW critical frequency, nc [cycles/min]** | 1423.47 | |
| | | **NEW engine speed, Nc [rev/min]** | 711.74 | |
| | | | | |
| **imep [Pa]** | 200000 | **M_mean [Nm]** | 6.31 | |
| **p-factor** | 1.91 | **T_ex [Nm]** | 12.06 | |
| | | **∑T_ex [Nm]** | 48.23 | |
| | | | | |
| **Dynamic magnifier, M** | 10.5 | **Combined dynamic magnifier, M** | 7.42 | |
| | | **Vibratory torque, T_v [Nm]** | 188.46 | |
| | | | | |
| **Mass of shaft [kg]** | 2.77 | | | |
| **Mass of the shaft/unit length, W_s [kg/m]** | 5.55 | **Whirling speed, N_w [rev/min]** | 14271.60 | |
| **Modulus of elasticity, E [Pa]** | 2.00E+11 | **Transverse critical frequency, N_t [rev/min]** | 17615.11 | |
| **Mass of shaft + half couplings, W [kg]** | 9.87 | **Combined whirling speed, N [rev/min]** | 11088.91 | |
| **Combined radial stiffness, k [N/m]** | 3.36E+07 | | | |

## A4) Engine maximum torque = 160 Nm

| | | | | |
|---|---|---|---|---|
| **Displacement volume, V_d [Liter]** | 2.12 | | | |
| **Bore, B [m]** | 0.088 | **Maximum shear stress, τ [MPa]** | 91.23 | |
| **Stroke, S [m]** | 0.088 | **Torsional stiffness, Cs [cycles/min]** | 23571.76 | |
| **Maximum torque, T [Nm]** | 160 | | | |
| **Service factor** | 4.8 | | | |
| **Design torque, T [Nm]** | 768 | **Critical frequency of torsional oscillation, nc [cycles/min]** | 3624.33 | |
| **Shaft diameter, D [m]** | 0.035 | | | |
| **Shaft length, L [m]** | 0.5 | | | |
| **Shaft volume [m^3]** | 4.81E-04 | | | **Engine speed [rev/min]** |
| **Modulus of rigidity, G [Pa]** | 8.00E+10 | **First major critical** | 2 | 1812.16 |
| **Moments of inertia of engine, Ie [kgm^2]** | 0.36 | **Second major critical** | 4 | 906.08 |
| **Moments of inertia of dyno, Id [kgm^2]** | 0.3 | **Third major critical** | 6 | 604.05 |
| **Number of cylinders** | 4 | **Fourth major critical** | 8 | 453.04 |
| | | | | |
| **Number of couplings** | 2 | **Combined torsional stiffness [Nm/rad]** | 3564.82 | |
| **Couplings stiffness [Nm/rad]** | 8400 | **NEW critical frequency, nc [cycles/min]** | 1409.45 | |
| | | **NEW engine speed, Nc [rev/min]** | 704.73 | |
| | | | | |
| **imep [Pa]** | 200000 | **M_mean [Nm]** | 8.42 | |
| **p-factor** | 1.91 | **T_ex [Nm]** | 16.08 | |
| | | **∑T_ex [Nm]** | 64.30 | |
| | | | | |
| **Dynamic magnifier, M** | 10.5 | **Combined dynamic magnifier, M** | 7.42 | |
| | | **Vibratory torque, T_v [Nm]** | 217.02 | |
| | | | | |
| **Mass of shaft [kg]** | 3.78 | | | |
| **Mass of the shaft/unit length, W_s [kg/m]** | 7.55 | **Whirling speed, N_w [rev/min]** | 16650.20 | |
| **Modulus of elasticity, E [Pa]** | 2.00E+11 | **Transverse critical frequency, N_t [rev/min]** | 16784.20 | |
| **Mass of shaft + half couplings, W [kg]** | 10.88 | **Combined whirling speed, N [rev/min]** | 11820.56 | |
| **Combined radial stiffness, k [N/m]** | 3.36E+07 | | | |

## A5) Engine maximum torque = 200 Nm

| | | | | |
|---|---|---|---|---|
| **Displacement volume, V_d [Liter]** | 2.64 | | | |
| **Bore, B [m]** | 0.094 | **Maximum shear stress, τ [MPa]** | 76.39 | |
| **Stroke, S [m]** | 0.094 | **Torsional stiffness, Cs [cycles/min]** | 40212.39 | |
| **Maximum torque, T [Nm]** | 200 | | | |
| **Service factor** | 4.8 | | | |
| **Design torque, T [Nm]** | 960 | **Critical frequency of torsional oscillation, nc [cycles/min]** | 4513.52 | |
| **Shaft diameter, D [m]** | 0.04 | | | |
| **Shaft length, L [m]** | 0.5 | | | |
| **Shaft volume [m^3]** | 6.28E-04 | | | **Engine speed [rev/min]** |
| **Modulus of rigidity, G [Pa]** | 8.00E+10 | **First major critical** | 2 | 2256.76 |
| **Moments of inertia of engine, Ie [kgm^2]** | 0.45 | **Second major critical** | 4 | 1128.38 |
| **Moments of inertia of dyno, Id [kgm^2]** | 0.3 | **Third major critical** | 6 | 752.25 |
| **Number of cylinders** | 4 | **Fourth major critical** | 8 | 564.19 |
| | | | | |
| **Number of couplings** | 2 | **Combined torsional stiffness [Nm/rad]** | 3802.81 | |
| **Couplings stiffness [Nm/rad]** | 8400 | **NEW critical frequency, nc [cycles/min]** | 1387.99 | |
| | | **NEW engine speed, Nc [rev/min]** | 694.00 | |
| | | | | |
| **imep [Pa]** | 200000 | **M_mean [Nm]** | 10.52 | |
| **p-factor** | 1.91 | **T_ex [Nm]** | 20.10 | |
| | | **∑T_ex [Nm]** | 80.38 | |
| | | | | |
| **Dynamic magnifier, M** | 10.5 | **Combined dynamic magnifier, M** | 7.42 | |
| | | **Vibratory torque, T_v [Nm]** | 238.72 | |
| | | | | |
| **Mass of shaft [kg]** | 4.93 | | | |
| **Mass of the shaft/unit length, W_s [kg/m]** | 9.86 | **Whirling speed, N_w [rev/min]** | 19028.80 | |
| **Modulus of elasticity, E [Pa]** | 2.00E+11 | **Transverse critical frequency, N_t [rev/min]** | 15957.57 | |
| **Mass of shaft + half couplings, W [kg]** | 12.03 | **Combined whirling speed, N [rev/min]** | 12227.21 | |
| **Combined radial stiffness, k [N/m]** | 3.36E+07 | | | |

## B) Steel shaft (Constant shaft diameter = 40 mm)

### B1) Engine maximum torque = 40 Nm

| | | | | |
|---|---|---|---|---|
| **Displacement volume, V_d [Liter]** | 0.53 | | | |
| **Bore, B [m]** | 0.055 | **Maximum shear stress, τ [MPa]** | 15.28 | |
| **Stroke, S [m]** | 0.055 | **Torsional stiffness, Cs [cycles/min]** | 50265.48 | |
| **Maximum torque, T [Nm]** | 40 | | | |
| **Service factor** | 4.8 | | | |
| **Design torque, T [Nm]** | 192 | **Critical frequency of torsional oscillation, nc [cycles/min]** | 8136.86 | |
| **Shaft diameter, D [m]** | 0.04 | | | |
| **Shaft length, L [m]** | 0.4 | | | |
| **Shaft volume [m^3]** | 5.03E-04 | | | **Engine speed [rev/min]** |
| **Modulus of rigidity, G [Pa]** | 8.00E+10 | **First major critical** | 2 | 4068.43 |
| **Moments of inertia of engine, Ie [kgm^2]** | 0.09 | **Second major critical** | 4 | 2034.21 |
| **Moments of inertia of dyno, Id [kgm^2]** | 0.3 | **Third major critical** | 6 | 1356.14 |
| **Number of cylinders** | 4 | **Fourth major critical** | 8 | 1017.11 |
| | | | | |
| **Number of couplings** | 2 | **Combined torsional stiffness [Nm/rad]** | 3876.13 | |
| **Couplings stiffness [Nm/rad]** | 8400 | **NEW critical frequency, nc [cycles/min]** | 2259.54 | |
| | | **NEW engine speed, Nc [rev/min]** | 1129.77 | |
| | | | | |
| **imep [Pa]** | 200000 | **M_mean [Nm]** | 2.10 | |
| **p-factor** | 1.91 | **T_ex [Nm]** | 4.02 | |
| | | **∑T_ex [Nm]** | 16.08 | |
| | | | | |
| **Dynamic magnifier, M** | 10.5 | **Combined dynamic magnifier, M** | 7.42 | |
| | | **Vibratory torque, T_v [Nm]** | 91.81 | |
| | | | | |
| **Mass of shaft [kg]** | 3.95 | | | |
| **Mass of the shaft/unit length, W_s [kg/m]** | 9.86 | **Whirling speed, N_w [rev/min]** | 29732.49 | |
| **Modulus of elasticity, E [Pa]** | 2.00E+11 | **Transverse critical frequency, N_t [rev/min]** | 16654.89 | |
| **Mass of shaft + half couplings, W [kg]** | 11.05 | **Combined whirling speed, N [rev/min]** | 14530.51 | |
| **Combined radial stiffness, k [N/m]** | 3.36E+07 | | | |

## B2) Engine maximum torque = 80 Nm

| | | | | |
|---|---|---|---|---|
| **Displacement volume, V_d [Liter]** | 1.06 | | | |
| **Bore, B [m]** | 0.070 | **Maximum shear stress, τ [MPa]** | 30.56 | |
| **Stroke, S [m]** | 0.070 | **Torsional stiffness, Cs [cycles/min]** | 40212.39 | |
| **Maximum torque, T [Nm]** | 80 | | | |
| **Service factor** | 4.8 | | | |
| **Design torque, T [Nm]** | 384 | **Critical frequency of torsional oscillation, nc [cycles/min]** | 5709.20 | |
| **Shaft diameter, D [m]** | 0.04 | | | |
| **Shaft length, L [m]** | 0.5 | | | |
| **Shaft volume [m^3]** | 6.28E-04 | | | **Engine speed [rev/min]** |
| **Modulus of rigidity, G [Pa]** | 8.00E+10 | **First major critical** | 2 | 2854.60 |
| **Moments of inertia of engine, Ie [kgm^2]** | 0.18 | **Second major critical** | 4 | 1427.30 |
| **Moments of inertia of dyno, Id [kgm^2]** | 0.3 | **Third major critical** | 6 | 951.53 |
| **Number of cylinders** | 4 | **Fourth major critical** | 8 | 713.65 |
| | | | | |
| **Number of couplings** | 2 | **Combined torsional stiffness [Nm/rad]** | 3802.81 | |
| **Couplings stiffness [Nm/rad]** | 8400 | **NEW critical frequency, nc [cycles/min]** | 1755.69 | |
| | | **NEW engine speed, Nc [rev/min]** | 877.84 | |
| | | | | |
| **imep [Pa]** | 200000 | **M_mean [Nm]** | 4.21 | |
| **p-factor** | 1.91 | **T_ex [Nm]** | 8.04 | |
| | | **∑T_ex [Nm]** | 32.15 | |
| | | | | |
| **Dynamic magnifier, M** | 10.5 | **Combined dynamic magnifier, M** | 7.42 | |
| | | **Vibratory torque, T_v [Nm]** | 149.20 | |
| | | | | |
| **Mass of shaft [kg]** | 4.93 | | | |
| **Mass of the shaft/unit length, W_s [kg/m]** | 9.86 | **Whirling speed, N_w [rev/min]** | 19028.80 | |
| **Modulus of elasticity, E [Pa]** | 2.00E+11 | **Transverse critical frequency, N_t [rev/min]** | 15957.57 | |
| **Mass of shaft + half couplings, W [kg]** | 12.03 | **Combined whirling speed, N [rev/min]** | 12227.21 | |
| **Combined radial stiffness, k [N/m]** | 3.36E+07 | | | |

## B3) Engine maximum torque = 120 Nm

| | | | | |
|---|---|---|---|---|
| **Displacement volume, V_d [Liter]** | 1.59 | | | |
| **Bore, B [m]** | 0.080 | **Maximum shear stress, τ [MPa]** | 45.84 | |
| **Stroke, S [m]** | 0.080 | **Torsional stiffness, Cs [cycles/min]** | 33510.32 | |
| **Maximum torque, T [Nm]** | 120 | | | |
| **Service factor** | 4.8 | | | |
| **Design torque, T [Nm]** | 576 | **Critical frequency of torsional oscillation, nc [cycles/min]** | 4637.20 | |
| **Shaft diameter, D [m]** | 0.04 | | | |
| **Shaft length, L [m]** | 0.6 | | | |
| **Shaft volume [m^3]** | 7.54E-04 | | | **Engine speed [rev/min]** |
| **Modulus of rigidity, G [Pa]** | 8.00E+10 | **First major critical** | 2 | 2318.60 |
| **Moments of inertia of engine, Ie [kgm^2]** | 0.27 | **Second major critical** | 4 | 1159.30 |
| **Moments of inertia of dyno, Id [kgm^2]** | 0.3 | **Third major critical** | 6 | 772.87 |
| **Number of cylinders** | 4 | **Fourth major critical** | 8 | 579.65 |
| | | | | |
| **Number of couplings** | 2 | **Combined torsional stiffness [Nm/rad]** | 3732.22 | |
| **Couplings stiffness [Nm/rad]** | 8400 | **NEW critical frequency, nc [cycles/min]** | 1547.57 | |
| | | **NEW engine speed, Nc [rev/min]** | 773.78 | |
| | | | | |
| **imep [Pa]** | 200000 | **M_mean [Nm]** | 6.31 | |
| **p-factor** | 1.91 | **T_ex [Nm]** | 12.06 | |
| | | **∑T_ex [Nm]** | 48.23 | |
| | | | | |
| **Dynamic magnifier, M** | 10.5 | **Combined dynamic magnifier, M** | 7.42 | |
| | | **Vibratory torque, T_v [Nm]** | 188.46 | |
| | | | | |
| **Mass of shaft [kg]** | 5.92 | | | |
| **Mass of the shaft/unit length, W_s [kg/m]** | 9.86 | **Whirling speed, N_w [rev/min]** | 13214.44 | |
| **Modulus of elasticity, E [Pa]** | 2.00E+11 | **Transverse critical frequency, N_t [rev/min]** | 15341.09 | |
| **Mass of shaft + half couplings, W [kg]** | 13.02 | **Combined whirling speed, N [rev/min]** | 10012.18 | |
| **Combined radial stiffness, k [N/m]** | 3.36E+07 | | | |

## B4) Engine maximum torque = 160 Nm

| | | | | |
|---|---|---|---|---|
| **Displacement volume, V_d [Liter]** | 2.12 | | | |
| **Bore, B [m]** | 0.088 | **Maximum shear stress, τ [MPa]** | 61.12 | |
| **Stroke, S [m]** | 0.088 | **Torsional stiffness, Cs [cycles/min]** | 28723.13 | |
| **Maximum torque, T [Nm]** | 160 | | | |
| **Service factor** | 4.8 | | | |
| **Design torque, T [Nm]** | 768 | **Critical frequency of torsional oscillation, nc [cycles/min]** | 4000.80 | |
| **Shaft diameter, D [m]** | 0.04 | | | |
| **Shaft length, L [m]** | 0.7 | | | |
| **Shaft volume [m^3]** | 8.80E-04 | | | **Engine speed [rev/min]** |
| **Modulus of rigidity, G [Pa]** | 8.00E+10 | **First major critical** | 2 | 2000.40 |
| **Moments of inertia of engine, Ie [kgm^2]** | 0.36 | **Second major critical** | 4 | 1000.20 |
| **Moments of inertia of dyno, Id [kgm^2]** | 0.3 | **Third major critical** | 6 | 666.80 |
| **Number of cylinders** | 4 | **Fourth major critical** | 8 | 500.10 |
| | | | | |
| **Number of couplings** | 2 | **Combined torsional stiffness [Nm/rad]** | 3664.21 | |
| **Couplings stiffness [Nm/rad]** | 8400 | **NEW critical frequency, nc [cycles/min]** | 1428.96 | |
| | | **NEW engine speed, Nc [rev/min]** | 714.48 | |
| | | | | |
| **imep [Pa]** | 200000 | **M_mean [Nm]** | 8.42 | |
| **p-factor** | 1.91 | **T_ex [Nm]** | 16.08 | |
| | | **∑T_ex [Nm]** | 64.30 | |
| | | | | |
| **Dynamic magnifier, M** | 10.5 | **Combined dynamic magnifier, M** | 7.42 | |
| | | **Vibratory torque, T_v [Nm]** | 217.02 | |
| | | | | |
| **Mass of shaft [kg]** | 6.91 | | | |
| **Mass of the shaft/unit length, W_s [kg/m]** | 9.86 | **Whirling speed, N_w [rev/min]** | 9708.57 | |
| **Modulus of elasticity, E [Pa]** | 2.00E+11 | **Transverse critical frequency, N_t [rev/min]** | 14790.95 | |
| **Mass of shaft + half couplings, W [kg]** | 14.01 | **Combined whirling speed, N [rev/min]** | 8116.32 | |
| **Combined radial stiffness, k [N/m]** | 3.36E+07 | | | |

## B5) Engine maximum torque = 200 Nm

| | | | | |
|---|---|---|---|---|
| **Displacement volume, V_d [Liter]** | 2.64 | | | |
| **Bore, B [m]** | 0.094 | **Maximum shear stress, τ [MPa]** | 76.39 | |
| **Stroke, S [m]** | 0.094 | **Torsional stiffness, Cs [cycles/min]** | 25132.74 | |
| **Maximum torque, T [Nm]** | 200 | | | |
| **Service factor** | 4.8 | | | |
| **Design torque, T [Nm]** | 960 | **Critical frequency of torsional oscillation, nc [cycles/min]** | 3568.25 | |
| **Shaft diameter, D [m]** | 0.04 | | | |
| **Shaft length, L [m]** | 0.8 | | | |
| **Shaft volume [m^3]** | 1.01E-03 | | | **Engine speed [rev/min]** |
| **Modulus of rigidity, G [Pa]** | 8.00E+10 | **First major critical** | 2 | 1784.12 |
| **Moments of inertia of engine, Ie [kgm^2]** | 0.45 | **Second major critical** | 4 | 892.06 |
| **Moments of inertia of dyno, Id [kgm^2]** | 0.3 | **Third major critical** | 6 | 594.71 |
| **Number of cylinders** | 4 | **Fourth major critical** | 8 | 446.03 |
| | | | | |
| **Number of couplings** | 2 | **Combined torsional stiffness [Nm/rad]** | 3598.62 | |
| **Couplings stiffness [Nm/rad]** | 8400 | **NEW critical frequency, nc [cycles/min]** | 1350.22 | |
| | | **NEW engine speed, Nc [rev/min]** | 675.11 | |
| | | | | |
| **imep [Pa]** | 200000 | **M_mean [Nm]** | 10.52 | |
| **p-factor** | 1.91 | **T_ex [Nm]** | 20.10 | |
| | | **∑T_ex [Nm]** | 80.38 | |
| | | | | |
| **Dynamic magnifier, M** | 10.5 | **Combined dynamic magnifier, M** | 7.42 | |
| | | **Vibratory torque, T_v [Nm]** | 238.72 | |
| | | | | |
| **Mass of shaft [kg]** | 7.89 | | | |
| **Mass of the shaft/unit length, W_s [kg/m]** | 9.86 | **Whirling speed, N_w [rev/min]** | 7433.12 | |
| **Modulus of elasticity, E [Pa]** | 2.00E+11 | **Transverse critical frequency, N_t [rev/min]** | 14296.04 | |
| **Mass of shaft + half couplings, W [kg]** | 14.99 | **Combined whirling speed, N [rev/min]** | 6594.95 | |
| **Combined radial stiffness, k [N/m]** | 3.36E+07 | | | |

# C) CamPro IAFM Engine

| | | | | |
|---|---|---|---|---|
| Displacement volume, V_d [Liter] | 1.60 | | | |
| Bore, B [m] | 0.076 | Maximum shear stress, τ [MPa] | 56.53 | |
| Stroke, S [m] | 0.088 | Torsional stiffness, Cs [cycles/min] | 44680.43 | |
| Maximum torque, T [Nm] | 148 | | | |
| Service factor | 4.8 | | | |
| Design torque, T [Nm] | 710.4 | Critical frequency of torsional oscillation, nc [cycles/min] | 5056.15 | |
| Shaft diameter, D [m] | 0.04 | | | |
| Shaft length, L [m] | 0.45 | | | |
| Shaft volume [m^3] | 5.65E-04 | | | Engine speed [rev/min] |
| Modulus of rigidity, G [Pa] | 8.00E+10 | First major critical | 2 | 2528.07 |
| Moments of inertia of engine, Ie [kgm^2] | 0.34 | Second major critical | 4 | 1264.04 |
| Moments of inertia of dyno, Id [kgm^2] | 0.3 | Third major critical | 6 | 842.69 |
| Number of cylinders | 4 | Fourth major critical | 8 | 632.02 |
| | | | | |
| Number of couplings | 2 | Combined torsional stiffness [Nm/rad] | 3839.12 | |
| Couplings stiffness [Nm/rad] | 8400 | NEW critical frequency, nc [cycles/min] | 1482.10 | |
| | | NEW engine speed, Nc [rev/min] | 741.05 | |
| | | | | |
| imep [Pa] | 200000 | M_mean [Nm] | 6.35 | |
| p-factor | 1.91 | T_ex [Nm] | 12.14 | |
| | | ∑T_ex [Nm] | 48.54 | |
| | | | | |
| Dynamic magnifier, M | 10.5 | Combined dynamic magnifier, M | 7.42 | |
| | | Vibratory torque, T_v [Nm] | 168.94 | |
| | | | | |
| Mass of shaft [kg] | 4.44 | | | |
| Mass of the shaft/unit length, W_s [kg/m] | 9.86 | Whirling speed, N_w [rev/min] | 23492.34 | |
| Modulus of elasticity, E [Pa] | 2.00E+11 | Transverse critical frequency, N_t [rev/min] | 16295.05 | |
| Mass of shaft + half couplings, W [kg] | 11.54 | Combined whirling speed, N [rev/min] | 13389.35 | |
| Combined radial stiffness, k [N/m] | 3.36E+07 | | | |

## D) Aluminium shaft (Constant shaft length = 500 mm)

### D1) Engine maximum torque = 40 Nm

| | | | | |
|---|---|---|---|---|
| **Displacement volume, V_d [Liter]** | 0.53 | | | |
| **Bore, B [m]** | 0.055 | **Maximum shear stress, τ [MPa]** | 122.23 | |
| **Stroke, S [m]** | 0.055 | **Torsional stiffness, Cs [cycles/min]** | 816.81 | |
| **Maximum torque, T [Nm]** | 40 | | | |
| **Service factor** | 4.8 | | | |
| **Design torque, T [Nm]** | 192 | **Critical frequency of torsional oscillation, nc [cycles/min]** | 1037.25 | |
| **Shaft diameter, D [m]** | 0.02 | | | |
| **Shaft length, L [m]** | 0.5 | | | |
| **Shaft volume [m^3]** | 1.57E-04 | | | **Engine speed [rev/min]** |
| **Modulus of rigidity, G [Pa]** | 2.60E+10 | **First major critical** | 2 | 518.62 |
| **Moments of inertia of engine, Ie [kgm^2]** | 0.09 | **Second major critical** | 4 | 259.31 |
| **Moments of inertia of dyno, Id [kgm^2]** | 0.3 | **Third major critical** | 6 | 172.87 |
| **Number of cylinders** | 4 | **Fourth major critical** | 8 | 129.66 |
| | | | | |
| **Number of couplings** | 2 | **Combined torsional stiffness [Nm/rad]** | 683.82 | |
| **Couplings stiffness [Nm/rad]** | 8400 | **NEW critical frequency, nc [cycles/min]** | 949.06 | |
| | | **NEW engine speed, Nc [rev/min]** | 474.53 | |
| | | | | |
| **imep [Pa]** | 200000 | **M_mean [Nm]** | 2.10 | |
| **p-factor** | 1.91 | **T_ex [Nm]** | 4.02 | |
| | | **∑T_ex [Nm]** | 16.08 | |
| | | | | |
| **Dynamic magnifier, M** | 10.5 | **Combined dynamic magnifier, M** | 7.42 | |
| | | **Vibratory torque, T_v [Nm]** | 91.81 | |
| | | | | |
| **Mass of shaft [kg]** | 0.41 | | | |
| **Mass of the shaft/unit length, W_s [kg/m]** | 0.82 | **Whirling speed, N_w [rev/min]** | 9780.55 | |
| **Modulus of elasticity, E [Pa]** | 7.00E+10 | **Transverse critical frequency, N_t [rev/min]** | 20200.73 | |
| **Mass of shaft + half couplings, W [kg]** | 7.51 | **Combined whirling speed, N [rev/min]** | 8803.03 | |
| **Combined radial stiffness, k [N/m]** | 3.36E+07 | | | |

## D2) Engine maximum torque = 80 Nm

| | | | | |
|---|---|---|---|---|
| **Displacement volume, V_d [Liter]** | 1.06 | | | |
| **Bore, B [m]** | 0.070 | **Maximum shear stress, τ [MPa]** | 125.16 | |
| **Stroke, S [m]** | 0.070 | **Torsional stiffness, Cs [cycles/min]** | 1994.18 | |
| **Maximum torque, T [Nm]** | 80 | | | |
| **Service factor** | 4.8 | | | |
| **Design torque, T [Nm]** | 384 | **Critical frequency of torsional oscillation, nc [cycles/min]** | 1271.38 | |
| **Shaft diameter, D [m]** | 0.025 | | | |
| **Shaft length, L [m]** | 0.5 | | | |
| **Shaft volume [m^3]** | 2.45E-04 | | | **Engine speed [rev/min]** |
| **Modulus of rigidity, G [Pa]** | 2.60E+10 | **First major critical** | 2 | 635.69 |
| **Moments of inertia of engine, Ie [kgm^2]** | 0.18 | **Second major critical** | 4 | 317.85 |
| **Moments of inertia of dyno, Id [kgm^2]** | 0.3 | **Third major critical** | 6 | 211.90 |
| **Number of cylinders** | 4 | **Fourth major critical** | 8 | 158.92 |
| | | | | |
| **Number of couplings** | 2 | **Combined torsional stiffness [Nm/rad]** | 1352.16 | |
| **Couplings stiffness [Nm/rad]** | 8400 | **NEW critical frequency, nc [cycles/min]** | 1046.91 | |
| | | **NEW engine speed, Nc [rev/min]** | 523.46 | |
| | | | | |
| **imep [Pa]** | 200000 | **M_mean [Nm]** | 4.21 | |
| **p-factor** | 1.91 | **T_ex [Nm]** | 8.04 | |
| | | **∑T_ex [Nm]** | 32.15 | |
| | | | | |
| **Dynamic magnifier, M** | 10.5 | **Combined dynamic magnifier, M** | 7.42 | |
| | | **Vibratory torque, T_v [Nm]** | 149.20 | |
| | | | | |
| **Mass of shaft [kg]** | 0.64 | | | |
| **Mass of the shaft/unit length, W_s [kg/m]** | 1.28 | **Whirling speed, N_w [rev/min]** | 12225.69 | |
| **Modulus of elasticity, E [Pa]** | 7.00E+10 | **Transverse critical frequency, N_t [rev/min]** | 19898.61 | |
| **Mass of shaft + half couplings, W [kg]** | 7.74 | **Combined whirling speed, N [rev/min]** | 10416.69 | |
| **Combined radial stiffness, k [N/m]** | 3.36E+07 | | | |

## D3) Engine maximum torque = 120 Nm

| | | | | |
|---|---|---|---|---|
| **Displacement volume, V_d [Liter]** | 1.59 | | | |
| **Bore, B [m]** | 0.080 | **Maximum shear stress, τ [MPa]** | 108.65 | |
| **Stroke, S [m]** | 0.080 | **Torsional stiffness, Cs [cycles/min]** | 4135.12 | |
| **Maximum torque, T [Nm]** | 120 | | | |
| **Service factor** | 4.8 | | | |
| **Design torque, T [Nm]** | 576 | **Critical frequency of torsional oscillation, nc [cycles/min]** | 1628.96 | |
| **Shaft diameter, D [m]** | 0.03 | | | |
| **Shaft length, L [m]** | 0.5 | | | |
| **Shaft volume [m^3]** | 3.53E-04 | | | **Engine speed [rev/min]** |
| **Modulus of rigidity, G [Pa]** | 2.60E+10 | **First major critical** | 2 | 814.48 |
| **Moments of inertia of engine, Ie [kgm^2]** | 0.27 | **Second major critical** | 4 | 407.24 |
| **Moments of inertia of dyno, Id [kgm^2]** | 0.3 | **Third major critical** | 6 | 271.49 |
| **Number of cylinders** | 4 | **Fourth major critical** | 8 | 203.62 |
| | | | | |
| **Number of couplings** | 2 | **Combined torsional stiffness [Nm/rad]** | 2083.65 | |
| **Couplings stiffness [Nm/rad]** | 8400 | **NEW critical frequency, nc [cycles/min]** | 1156.32 | |
| | | **NEW engine speed, Nc [rev/min]** | 578.16 | |
| | | | | |
| **imep [Pa]** | 200000 | **M_mean [Nm]** | 6.31 | |
| **p-factor** | 1.91 | **T_ex [Nm]** | 12.06 | |
| | | **∑T_ex [Nm]** | 48.23 | |
| | | | | |
| **Dynamic magnifier, M** | 10.5 | **Combined dynamic magnifier, M** | 7.42 | |
| | | **Vibratory torque, T_v [Nm]** | 188.46 | |
| | | | | |
| **Mass of shaft [kg]** | 0.92 | | | |
| **Mass of the shaft/unit length, W_s [kg/m]** | 1.84 | **Whirling speed, N_w [rev/min]** | 14670.83 | |
| **Modulus of elasticity, E [Pa]** | 7.00E+10 | **Transverse critical frequency, N_t [rev/min]** | 19547.14 | |
| **Mass of shaft + half couplings, W [kg]** | 8.02 | **Combined whirling speed, N [rev/min]** | 11733.65 | |
| **Combined radial stiffness, k [N/m]** | 3.36E+07 | | | |

## D4) Engine maximum torque = 160 Nm

| | | | | |
|---|---|---|---|---|
| **Displacement volume, V_d [Liter]** | 2.12 | | | |
| **Bore, B [m]** | 0.088 | **Maximum shear stress, τ [MPa]** | 488.92 | |
| **Stroke, S [m]** | 0.088 | **Torsional stiffness, Cs [cycles/min]** | 1256.64 | |
| **Maximum torque, T [Nm]** | 160 | | | |
| **Service factor** | 4.8 | | | |
| **Design torque, T [Nm]** | 768 | **Critical frequency of torsional oscillation, nc [cycles/min]** | 836.83 | |
| **Shaft diameter, D [m]** | 0.02 | | | |
| **Shaft length, L [m]** | 0.325 | | | |
| **Shaft volume [m^3]** | 1.02E-04 | | | **Engine speed [rev/min]** |
| **Modulus of rigidity, G [Pa]** | 2.60E+10 | **First major critical** | 2 | 418.41 |
| **Moments of inertia of engine, Ie [kgm^2]** | 0.36 | **Second major critical** | 4 | 209.21 |
| **Moments of inertia of dyno, Id [kgm^2]** | 0.3 | **Third major critical** | 6 | 139.47 |
| **Number of cylinders** | 4 | **Fourth major critical** | 8 | 104.60 |
| | | | | |
| **Number of couplings** | 2 | **Combined torsional stiffness [Nm/rad]** | 967.24 | |
| **Couplings stiffness [Nm/rad]** | 8400 | **NEW critical frequency, nc [cycles/min]** | 734.17 | |
| | | **NEW engine speed, Nc [rev/min]** | 367.09 | |
| | | | | |
| **imep [Pa]** | 200000 | **M_mean [Nm]** | 8.42 | |
| **p-factor** | 1.91 | **T_ex [Nm]** | 16.08 | |
| | | **∑T_ex [Nm]** | 64.30 | |
| | | | | |
| **Dynamic magnifier, M** | 10.5 | **Combined dynamic magnifier, M** | 7.42 | |
| | | **Vibratory torque, T_v [Nm]** | 217.02 | |
| | | | | |
| **Mass of shaft [kg]** | 0.27 | | | |
| **Mass of the shaft/unit length, W_s [kg/m]** | 0.82 | **Whirling speed, N_w [rev/min]** | 23149.24 | |
| **Modulus of elasticity, E [Pa]** | 7.00E+10 | **Transverse critical frequency, N_t [rev/min]** | 20395.81 | |
| **Mass of shaft + half couplings, W [kg]** | 7.37 | **Combined whirling speed, N [rev/min]** | 15303.38 | |
| **Combined radial stiffness, k [N/m]** | 3.36E+07 | | | |

## D5) Engine maximum torque = 200 Nm

| | | | | |
|---|---|---|---|---|
| **Displacement volume, V_d [Liter]** | 2.64 | | | |
| **Bore, B [m]** | 0.094 | **Maximum shear stress, τ [MPa]** | 611.15 | |
| **Stroke, S [m]** | 0.094 | **Torsional stiffness, Cs [cycles/min]** | 1256.64 | |
| **Maximum torque, T [Nm]** | 200 | | | |
| **Service factor** | 4.8 | | | |
| **Design torque, T [Nm]** | 960 | **Critical frequency of torsional oscillation, nc [cycles/min]** | 797.88 | |
| **Shaft diameter, D [m]** | 0.02 | | | |
| **Shaft length, L [m]** | 0.325 | | | |
| **Shaft volume [m^3]** | 1.02E-04 | | | **Engine speed [rev/min]** |
| **Modulus of rigidity, G [Pa]** | 2.60E+10 | **First major critical** | 2 | 398.94 |
| **Moments of inertia of engine, Ie [kgm^2]** | 0.45 | **Second major critical** | 4 | 199.47 |
| **Moments of inertia of dyno, Id [kgm^2]** | 0.3 | **Third major critical** | 6 | 132.98 |
| **Number of cylinders** | 4 | **Fourth major critical** | 8 | 99.74 |
| | | | | |
| **Number of couplings** | 2 | **Combined torsional stiffness [Nm/rad]** | 967.24 | |
| **Couplings stiffness [Nm/rad]** | 8400 | **NEW critical frequency, nc [cycles/min]** | 700.01 | |
| | | **NEW engine speed, Nc [rev/min]** | 350.00 | |
| | | | | |
| **imep [Pa]** | 200000 | **M_mean [Nm]** | 10.52 | |
| **p-factor** | 1.91 | **T_ex [Nm]** | 20.10 | |
| | | **∑T_ex [Nm]** | 80.38 | |
| | | | | |
| **Dynamic magnifier, M** | 10.5 | **Combined dynamic magnifier, M** | 7.42 | |
| | | **Vibratory torque, T_v [Nm]** | 238.72 | |
| | | | | |
| **Mass of shaft [kg]** | 0.27 | | | |
| **Mass of the shaft/unit length, W_s [kg/m]** | 0.82 | **Whirling speed, N_w [rev/min]** | 23149.24 | |
| **Modulus of elasticity, E [Pa]** | 7.00E+10 | **Transverse critical frequency, N_t [rev/min]** | 20395.81 | |
| **Mass of shaft + half couplings, W [kg]** | 7.37 | **Combined whirling speed, N [rev/min]** | 15303.38 | |
| **Combined radial stiffness, k [N/m]** | 3.36E+07 | | | |

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