AN ALTERNATIVE APPROACH TO NORMAL PARAMETER REDUCTION ALGORITHMS FOR DECISION MAKING USING A SOFT SET THEORY

SANI DANJUMA

FACULTY OF COMPUTER SCIENCE AND INFORMATION TECHNOLOGY

UNIVERSITY OF MALAYA

KUALA LUMPUR

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SANI DANJUMA

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Abstract

The soft set theory is a mathematical tool that deals with uncertainty, imprecise and vagueness in decision systems. It has been widely used to identify irrelevant parameters and make reduction of parameters for decision making, in order to bring out the optimal choices of the decision systems. Many normal parameter reduction algorithms exist to handle parameter reduction and maintain consistency of decision choices. However, they require much time to repeatedly run the algorithms to reduce unnecessary parameters using either parameter important degree or oriented parameter sum. This study will firstly review the different parameter reduction and decision making techniques for soft set and hybrid soft sets under unpleasant set of hypothesis environment as well as performance analysis of their derived algorithms. Consequently, the summary of the current literature in those areas of research were given, pointed out the limitations of previous works and areas that require further research works. Secondly, an alternative algorithm for parameter reduction and decision making based on soft set theory was proposed. The proposed algorithm showed that it can reduce the computational complexity and run time compared to baseline algorithms. Finally, to evaluate the proposed algorithm, thorough experimentation on both real life and synthetic binary-valued data set were performed. The experimental result shows that the proposed algorithm was feasible and has relatively reduced the computational complexity and running time with an average of 56 percent compared with the existing algorithms. In addition, the algorithm was relatively easy to understand compare to the state of the art of normal parameter reduction algorithm. The proposed algorithm was able to avoid the use of parameter important degree, decision partition and finding the multiple of the universe within the sets. This study contributes significantly in reducing the computational complexity and running time as compared

with Normal Parameter Reduction algorithm (NPR) and New Efficient Normal Parameter Reduction algorithm (NENPR).

Keyword: Normal parameter reduction, Soft set, Decision making, Computational complexity, Rough set.

Abstrak

Teori set lembut adalah satu alat matematik yang berdepan dengan ketidakpastian, tidak tepat dan kekaburan di dalam sistem keputusan. Ia telah digunakan secara meluas untuk mengenal pasti parameter yang tidak relevan dan membuat pengurangan parameter supaya dapat membuat keputusan, serta mengeluarkan pilihan optimum sistem keputusan. Kebanyakkan normal algoritma pengurangan parameter yang wujud untuk mengendalikan pengurangan parameter dan mengekalkan konsistensi pilihan keputusan. Walaubagaimanapun, mereka memerlukan banyak masa untuk berulang kali menjalankan algoritma untuk mengurangkan parameter yang tidak perlu menggunakan sama ada parameter darjah parameter atau jumlah parameter yang berorientasi. Dalam kajian ini, gambaran keseluruhan untuk teknik-teknik pengurangan parameter yang membuat keputusan berbeza untuk set lembut dan set lembut hibrid di bawah set menyenangkan persekitaran hipotesis serta analisis prestasi algoritma terbitan mereka. Oleh itu, ringkasan sastera semasa di kawasan-kawasan penyelidikan telah diberikan, menunjukkan had berkhidmat kerja-kerja dahulu dan kawasan-kawasan yang memerlukan kerja-kerja penyelidikan lanjut. Justeru, penyelidik boleh menggunakan kaedah ini dengan lebih cepat untuk mengenal pasti kawasan yang menerima pengecil atau tiada perhatian daripada penyelidik bagi mencadangkan kaedah dan aplikasi yang novel. Kedua, cadangan satu algoritma alternatif untuk pengurangan parameter dan membuat keputusan berdasarkan teori set lembut. Kajian ini menunjukkan bahawa algoritma yang dicadangkan boleh mengurangkan kerumitan pengiraan dan menjalankan perbandingan algoritma asas. Untuk menilai algoritma yang dicadangkan itu, penyelidik melakukan eksperimen menyeluruh pada kedua-dua kehidupan sebenar dan binari bernilai sintetik pada set data. Hasil eksperimen menunjukkan bahawa algoritma yang dicadangkan dilaksanakan dan telah mengurangkan kerumitan pengiraan dan masa berjalan dengan secara purata 56% berbanding dengan algoritma yang sedia ada. Di samping itu, algoritma yang agak mudah difahami berbanding dengan keadaan seni algoritma pengurangan parameter normal. Algoritma yang dicadangkan dapat mengelakkan penggunaan parameter ijazah penting, bahagian keputusan dan mencari pelbagai alam semesta dalam set. Kajian ini menyumbang di dalam mengurangkan kerumitan pengiraan dan tempoh sah antara algoritma (NPR) *Normal Parameter Reduction* dan algoritma (NENPR) *New Efficient Normal Parameter Reduction.*

Keyword: Normal parameter reduction, Soft set, Decision making, Computational complexity, Rough set.

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TABLE OF CONTENTS

Abstract iii

Abstrak v

Acknowledgements vii

Table of Contents viii

List of Figures xii

List of Tables xiii

List of Symbols and Abbreviations xv

CHAPTER 1: INTRODUCTION 1

1.1 Introduction 1

1.2 Problem Statement 4

1.3 Research Objectives 6

1.4 Thesis Contributions 7

1.5 Research Scope 7

1.6 Thesis Organization 7

1.7 Chapter Summary 8

CHAPTER 2: LITERATURE REVIEW 9

2.1 Introduction 9

2.2 Rudiment 10

2.2.1 Rough Set Theory 10

2.2.2 Soft set Theory 12

2.3 Hybrid Soft Sets 14

2.4 Parameter Reduction and Decision Making for Soft Set 16

2.5 Parameter Reduction and Decision Making of Hybrid Soft Set 23

2.5.1 Fuzzy Soft Set-based Approach for Parameter Reduction and Decision Making 24

2.5.2 Interval-valued Fuzzy Soft set-based Approach for Parameter Reduction and Decision Making 29

2.5.3 Intuitionistic Fuzzy Soft Set-based Approach for Parameter Reduction and Decision Making 32

2.5.4 A Hybrid Soft Set with Fuzzy and Rough Set for Parameter Reduction and Decision Making 36

2.6 Real World Applications of Soft Sets 39

2.6.1 Soft Set Theory for Association rules mining 39

2.6.2 Soft Set Theory for Medical Analysis 40

2.6.3 Soft Set Theory for Incomplete Data Analysis 41

2.6.4 Soft Set Theory for Data Mining 42

2.7 Uncertainty in Decision making 46

2.8 Other Parameter Reduction Techniques 48

2.8.1 Principal Component Analysis (PCA) 49

2.8.2 Stochastic neighbour embedding 49

2.8.3 Isomap 51

2.8.4 t-Distributed Stochastic Neighbour Embedding (t-SNE) 52

2.8.5 Hessian LLE 52

2.8.6 Sammon mapping 53

2.8.7 Multilayer autoencoders 53

2.8.8 Locally linear Coordination 54

2.8.9 Manifold charting 55

2.8.10 Diffusion map 56

57

2.9 Summary and Conclusion of the Chapter 57

CHAPTER 3: METHODOLOGY 59

3.1 Introduction 59

3.2 Parameter reduction techniques 59

3.3 System Requirement 60

3.4 Methodology 60

3.4.1 Input of Boolean Dataset 61

3.4.2 Processing Stage 61

3.4.3 Reduced sets 61

3.5 Evaluation 62

3.5.1 Big O notation 63

3.6 Summary of the chapter 63

CHAPTER 4: ALTERNATIVE APPROACH TO NORMAL PARAMETER REDUCTION ALGORITHM 64

4.1 Introduction 64

4.2 Normal Parameter Reduction Algorithm 64

4.3 New Efficient Normal Parameter Reduction 69

4.4 The Proposed Algorithm of ANPR 73

4.4.1 Proposed Technique 73

4.4.2 The difference between the proposed algorithm and the algorithm by (Ma et al., 2011) 76

4.4.3 The comparison result 76

4.4.4 Performance Analysis of the Proposed Algorithm 77

4.5 Summary of the chapter 81

CHAPTER 5: APPLICATION OF ANPR ALGORITHM AND CONFLICT ANALYSIS OF SOFT SET 83

5.1 Introduction.. 83

5.2 Selection of Scholarship Award by Kano State Government................................ 83

5.3 Application medical diagnosis for patient with a hiatal hernia 91

5.4 Application of soft set theory in Conflict Analysis 95

5.4.1 Conflict Representation in Multi-Soft Sets 96

5.4.2 Alliance, Neutrality, and Conflict 96

5.4.3 Support, Strength, Certainty and Coverage 96

5.4.3.1 Tutorial Example 98

5.4.4 A Soft Set Approach for Conflict Analysis of Urban Planning in Nigeria 104

5.4.4.1 Urban Planning in Nigeria Problems 104

5.4.4.2 Conflict Analysis 105

106

5.4.4.3 Results and Discussion 106

5.4.4.4 Comparison Results 113

5.5 Summary of the chapter 116

CHAPTER 6: CONCLUSION AND FUTURE WORK 117

6.1 Introduction.. 117

6.2 Reappraisal of the Research Objectives 117

6.3 Discussion.. 118

6.4 Future Direction 119

6.5 Conclusion 120

References 121

List of Publications and Papers Presented 134

LIST OF FIGURES

Figure No.		Title	Page
3.1	:	General research methodology	59
3.2	:	Flow chart for ANPR algorithm	61
4.1	:	Normal Parameter Reduction algorithm	67
4.2	:	New efficient normal parameter reduction algorithm	71
4.3	:	Alternative approach to normal parameter reduction algorithm	73
4.4	:	Number of Iteration perform from the three algorithm	76
5.1	:	Conflict (in general)	95
5.2	:	The proposed soft set approach algorithm for conflict	97
5.3	:	Multi soft set representation of Table 5.5	100
5.4	:	Flow graph of Table 5.5	101
5.5	:	Approximate graph	102
5.6	:	Conflict graph of Table 5.5	103
5.7	:	Urban planning in Nigeria (study area)	104
5.8	:	Flow graph of urban planning	110
5.9	:	Approximate graph of urban planning	111
5.10	:	Conflict graph of urban planning	112
5.11	:	Computational time of computing support	113
5.12	:	Computational time of computing strength	113
5.13	:	Computational time of computing certainty	114
5.14	:	Computational time of computing coverage	114

LIST OF TABLES

Table No.		Title	Page
2.1	:	Representation of a Soft set	14
2.2	:	Parameter Reduction and decision making of soft sets	20
2.3	:	Parameter Reduction and decision making of Hybrid soft sets	27
2.4	:	Parameter Reduction and decision making of Interval-valued fuzzy soft sets	31
2.5	:	Parameter Reduction and decision making of Intuitionistic fuzzy soft sets	35
2.6	:	Parameter Reduction and decision making of Hybrid soft sets with rough soft sets	38
2.7	:	The Summary of real world Application of soft sets and Hybrid soft sets	43
2.8	:	Properties of techniques for dimensionality/parameter reduction	56
4.1	:	A Soft Set Table (F, E)	65
4.2	:	Normal parameter reduction from (F, E) in Table 4.1	68
4.3	:	Soft set (F, E')	70
4.4	:	New efficient normal parameter reduction from (F, E) in Table 4.1	70
4.5	:	Alternative Normal parameter reduction from (F, E) in Table 4.1	75
4.6	:	The comparison results of the three algorithms	80
5.1	:	A soft set (F, E) for Scholarship selection	84
5.2	:	A soft set (F, E') for $F_A(h_1) = F_A(h_2) \cdots$	86
5.3	:	A soft set (F, E') for $F_B(h_1) = F_B(h_2) \cdots$	87
5.4	:	ANPR of a soft set (F, E) for Table 5.1	88
5.3	:	A soft set (F, E) for patient with hiatal hernia	90

5.4	:	ANPR of a soft set (F,E) for patient with hiatal hernia	92
5.5	:	Tabular representation of conflict among agents	97
5.6	:	Boolean value information system representing o_1	98
5.7	:	Boolean value information system representing o_2	99
5.8	:	Boolean value information system representing o_3	99
5.9	:	Boolean value information system representing o_4	99
5.10	:	Values of strength, certainty and coverage	101
5.11	:	Decision Table of Public engagement program for town planning	105
5.12	:	The conflict situation with agents (participate in survey)	106
5.13	:	The Decomposition of user group response into multi soft set	107
5.14	:	The Decomposition of professional town planners response into multi soft set	107
5.15	:	The Decomposition of pressure group response into multi soft set	108
5.16	:	The Decomposition of business group response into multi soft set	108
5.17	:	Strength, Certainty and Coverage of the survey	109
5.18	:	Computational time improvement	115
6.1	:	Performance Rate of the Parameter Reduction Algorithms	117

LIST OF SYMBOLS AND ABBREVIATIONS

ANPR	:	An Alternative approach to Normal Parameter Reduction
NPR	:	Normal Parameter Reduction
NENPR	:	New Efficient Normal Parameter Reduction
PR	:	Parameter Reduction
BFE	:	Business Failure Prediction
MPA	:	Mean Potentiality Approach
FPFS	:	Fuzzy Parameterized Fuzzy Soft
MFSS	:	Multi-Fuzzy Soft Set
ULFSS	:	Uncertain Linguistic Fuzzy Soft Set
IVFS	:	Interval Value Fuzzy Set
IFSM	:	Intuitionistic Fuzzy Soft Matrix
MAGDM	:	Multiple Attribute Group Decision Making
IFSS	:	Intuitionistic Fuzzy Soft Set
ILI	:	Influenza-Like Illness
BFE	:	Business Failure Prediction
AIFSS	:	Atanassov's Intuitionistic Fuzzy Soft Sets
KNN	:	K-Nearest Neighbor
PCA	:	Principal Component Analysis
SNE	:	Stochastic Neighbour Embedding
t-SNE	:	t-distributed stochastic neighbour embedding
HLLE	:	Hessian LLE
RBMs)	:	Restricted Boltzmann Machines
LLC	:	Locally Linear Coordination
MoFA	:	Mixture of *m* Factor Analysers

DM	:	Diffusion maps
MDS	:	Multi-dimensional Scaling

CHAPTER 1: INTRODUCTION

1.1 Introduction

The process of making decision by identifying the choices is what mainly referred as decision making. Decision making is an outcome of mental processes which are fundamentally cognitive in nature prompting to the determination of a course of action among various choices. Following the orderly procedure in decision-making can help to make a thoughtful decision, proper deliberation and produce an effective final selection. The output can yield an action or an opinion of a decision.

Moreover, the decision-making process required time and effort for coming out with good choices. Therefore, it involves many activities such as identifying the problem, collection and verification of relevant information, finding the decision alternatives, informing the concern people or authority of the decision rationale, making the decision, implementation of the selected choices and then evaluation of the consequences of the decision choice. (Clemen & Kwit, 2001; Howard, 1980, 1988; Skinner, 1999).

Many decision-making problems often appear in an uncertainty environment, that is to say, several practical and complicated issues in the field of engineering, economics, social sciences, medicine that involves uncertainty. However, many mathematical tools were developed to handle uncertainty, for example the theory of probability by (Kolmogorov, 1950), the theory of fuzzy set by (Zadeh, 1965), and the rough set theory (Pawlak, 1982), Howeve, these theories suffer from inherent difficulties as stated by (Molodtsov, 1999). The soft set theory was initiated by Molodtsov a Russian mathematician to overcome the problem of the existing mathematical tools for handling uncertainties. The soft set simply defined as a parameterized family of the subset of the universal set. It is a context dependent fuzzy sets. Consequently, the major merits of the soft set over other theories are that it absents from inherent difficulties and also free from setting the membership function like fuzzy and rough sets, which makes it easier to apply to decision-making.

The applications of the soft set are progressing rapidly; (Maji et al., 2002, Roy et al., 2005, Kong et al., 2009, Feng et al., 2010, Yang et al., 2009, Jiang et al., 2010, Qin et al., 2011a, Ma et al. 2011). many researchers are developing the algorithm to solve many problems while soft set provides the analysis of the uncertainty. More operations, properties, hybridization and definitions of the soft set were also given attention by many scholars globally. It also proved that soft set is directly associated with the rough sets (Herawan & Deris, 2009a). The study of the soft set is also extended to soft groups, idealistic soft BCK/BCI-algebras, soft semirings, idealistic soft semirings, soft matrices and soft ideals, have been studied. Moreover, soft sets are extended to fuzzy soft sets, Interval-valued intuitionistic fuzzy soft set, Interval-valued fuzzy soft set and intuitionistic fuzzy soft sets. On inaction of the applications of soft set theory and an increase of the popularity of its models, significant progress is achieved. Data analysis approaches and data filling techniques in an incomplete information in soft set theory are also considered. There are several successful application of soft set theory in data mining. An alternative approach for mining standard association rules and maximal association rules from the transactional dataset using soft set theory was presented (Herawan & Deris, 2011). Choosing the suitable parameterization, procedures such as mappings, real numbers and functions make soft set theory very convenient and feasible for decision-making applications. This motivated some researchers to use soft set theory for classification of the textures. The fuzzy soft set was also proposed to solve a combined forecasting approach by (Xiao, Gong, & Zou, 2009).

Applications of soft sets with its extended models in the decision-making are one of the most important practical applications. Maji, Roy, & Biswas, (2002) firstly showed an application of soft sets in decision-making. The novel methods of object recognition from an uncertain multi-observer data to handle decision-making based on the fuzzy soft set (Roy & Maji, 2007) was presented. Feng et al., (2010) viewed a fuzzy soft and decision-making using level soft sets and presented a flexible approach to fuzzy soft set. Several achievements are recorded on problems about the reduction of soft sets in the decision-making problems. The conclusion of soft set reduction offered in (Maji et al., 2002) clearly shows that it was incorrect, and therefore, a new notion of parameterization reduction in soft sets is presented in comparison with the definition of the related concept of attributes reduction in rough set theory. The idea of standard parameter reduction was introduced in (Kong et al., 2008), which overcomes the problem of suboptimal choice and added parameter set of soft sets. Ma et al., (2011) presented another notion of normal parameter reduction algorithm which is an improvement of (Kong et al., 2008) based on soft sets oriented parameter sum. The authors used oriented parameter sum without considering the parameter reduction degree and decision partition to reduce the parameter. The comparative results on the dataset show that the New Efficient Normal Parameter Reduction algorithm (NENPR) based on the soft set was found to have relatively less computation complexity compare to the algorithm proposed by (Chen et al., 2005; Kong et al., 2008). However, in reviewing their algorithms, they require much time to repeatedly run an algorithm to reduce unnecessary parameters using either parameter important degree or oriented parameter sum. Kong et al., (2015) applied the concept of Particle Swarm Optimization (PSO) to reduce parameter in the soft set. Han, (2016) gives some improvement in (Ma et al., 2011) using linear programming method. They defined dispensable core in the soft set and solved the normal parameter reduction related to the dispensable core. An alternative approach to parameter reduction was proposed in this thesis.

1.2 Problem Statement

The process of choosing a logical choice among the available alternatives are referred as the decision making which is fundamentally cognitive in nature. Each decision-making process is expected to produce a final choice. The outcome can either yield an action or an opinion of choice. Many researchers discussed decision making in the literature (Bazerman & Moore, 2008; Bellman & Zadeh, 1970; Janis & Mann, 1977; Plous, 1993; Ramser, 1993; Zeleny & Cochrane, 1973). However, the inherent feature revolving all decision-making problems is the vagueness or uncertainty aspects. In order to overcome this issue, a number of theories are used such as probability theory (Kolmogorov, 1950), fuzzy sets (Zadeh, 1965), rough sets (Pawlak, 1982), intuitionistic fuzzy sets and interval-valued fuzzy sets (Alsina, Trillas, & Valverde, 1983; Atanassov & Gargov, 1989; Atanassov, 1986) which can be considered as mathematical tools for modeling vagueness are applied into decision making. These theories has their inherent difficulties as pointed out in (Molodtsov, 1999) which includes: In the theory of probability it only deals with the stochastically stable phenomena without going into mathematical details", In fuzzy set the main problem is how to set the membership function in each particular case" While interval mathematics is not sufficiently adaptable for problem with different uncertainties". Molodtsov initiated the concept of soft set theory which is considered to be another new mathematical tool for dealing with uncertainties in 1999. In contrast to all these theories, soft set theory is free from these challenges and has no issue of setting the membership function, which makes it and its extended models suitable and very easy to apply into decision-making (Maji et al., 2002, Roy et al., 2005, Kong et al., 2009, Feng et al., 2010, Yang et al., 2009, Jiang et al., 2010, Qin et al., 2011a, Ma et al. 2011).

Maji et al., (2002) firstly presented an application of rough set-based dimensionality reduction (Pawlak & Skowron, 2007) in decision-making problem to a soft set. They used few parameters after reduction to select optimal choice of objects for decision making, and the decision value was computed on condition parameters. However, the results of their techniques have some problems because they firstly computed the reduction of rough set and then compute the choice value to find the optimal object for decision-making. It shows that after the dimensionality reduction of the rough set the optimal choice object could be changed. Chen et al., (2005) presented an approach based on soft set for parameter reduction (PR) to find optimal decisions on global Boolean datasets as an improvement of (Maji et al., 2002). They improved the application of soft set theory in a decision-making problem. However, method by Chen et al., (2005) faced the problem in determining all level of sub-optimal choices. Consequently, Kong et al., (2008) proposed the concept of Normal Parameter Reduction (NPR) to find optimal decisions as well as all level of suboptimal decisions and added parameter set of sets. The methods of (Chen et al., 2005; Maji et al., 2002) only considered the optimal choice, they did not consider all level of suboptimal choices. In NPR, the data of optimal objects can be deleted directly from the normal parameter reduction, and the next optimal choice can be obtained exactly from normal parameter reduction in which the data of optimal objects were deleted. It was evident from the performance analysis that NPR had shown the ability to reduce the dimensionality significantly, but the performance of their algorithm in term of computational time is still an outstanding issue. Zou & Chen, (2008) proposed an algorithm of parameter reduction based on invariability of the optimal choice object; the algorithm is applied to efficiently decrease the number of the parameter used for evaluation and cut down the workload. The authors applied basic operation of relational algebra to realise the algorithm using Structural Query Language (SQL). The result shows that it is effective as compared to Chen et al., (2005). Herawan, Rose, & Deris, (2009) proposed a new approach to reduce attribute in multi-valued information system using soft set. The AND operations were used for dimensionality reduction. The result of the experiment found that the reductions values obtained are equivalent to the Pawlak's rough reducts technique (Pawlak & Skowron, 2007). Rose, Herawan, & Deris, (2010) presented a technique of decision making by parameterization reduction to determined maximal supported sets from Boolean value information system based on soft set theory and also provide consistency decision making. The approach ensured that any process of attribute elimination in convert complex database into a smaller database so as to make a decision. The result of the experiment shows that this technique is better than that method proposed by (Chen et al., 2005; Maji et al., 2002) because it maintains the optimal and all level of sub-optimal objects. Çağman & Enginoğlu, (2010) initiated a novel soft set approach based decision-making called uni-int decision-making. This technique could decrease an extensive set of choices to its subset of optimal objects based on the criteria given by decision maker. It has achieved the optimal decision from the experiment but still the computational complexity remain the outstanding issue. Ma et al., (2011) presented another approach called a New Efficient Normal Parameter Reduction algorithm (NENPR) based on soft sets oriented parameter sum. The authors used oriented parameter sum without parameter reduction degree and decision partition to reduce the parameter. The comparative results on the dataset show that the NENPR algorithm based on the soft set was found to have relatively less computational complexity, easy implementation and easy to understand than that the algorithms proposed by (Chen et al., 2005; Kong et al., 2008). In reviewing these algorithms of NPR and NENPR, this thesis points out their shortcomings: the algorithms involve high computation and it is time consuming due to its complexity of finding the multiple of the universe of the set, decision partition and parameter importance degree and decision partition which are the primary importance in the algorithms. However, based on these problems, there is a need for improving those techniques and developing an optimal normal parameter reduction algorithm of soft sets to achieve lower computational complexity.

1.3 Research Objectives

. In this research, the following objectives are addressed.

- To analyse the existing algorithms of parameter reduction and decision making in soft sets.
- To propose an alternative technique for parameter reduction and decision making using a soft set approach.
- To derive related algorithm for parameter reduction that has the ability to achieve better computational time (lower complexity).

- To validate the developed algorithm by comparing it with the baseline algorithms on the real-life datasets.

1.4 Thesis Contributions

The contribution of this research work includes:

- Proposed an alternative algorithm based on a soft set theory for normal parameter reduction that can improve computational complexity and run time and perform better than the state of the art normal parameter reduction algorithms.
- Presented some new definitions of normal parameter reduction.
- Applied the proposed algorithm for decision making to shows our contribution for dealing with real life applications.
- Validated algorithm for achieving optimal choice parameter reduction, and apply the algorithm into some real life.
- Highlight the advances of hybrid soft sets in parameter reduction and decision-making

1.5 Research Scope

The scope of this research lies in Normal Parameter Reduction of soft set in decision making and application of soft set.

1.6 Thesis Organization

This thesis is organized into six chapters; Chapter 2 reviews the related work on soft set-based parameter reduction and decision making. Chapter 3 provides a general discussion of the research methodology that is employed in carrying out the research. An alternative approach to normal parameter reduction algorithm is presented in Chapter 4. Chapter 5 shows the application of ANPR algorithm into some real-life data set. Finally, Chapter 6 summarises and concludes the research findings.

1.7 Chapter Summary

In this chapter, the motivation behind using a soft set as a mathematical tool to handle uncertainty in normal parameter reduction was discussed. The problems this research intends to address i.e. computational complexity and run time were clearly defined. Research objectives and scope were also outlined. The next chapter is an overview of the concept of parameter reduction and the detail of the literature that highlights the strength and weakness of each technique in soft sets.

CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

Many mathematical theories exist to deal with uncertainties, imprecise and vagueness involving the collection of data in information systems. One of the most popular and recent mathematical theories to handle the problem of uncertainties is a soft set theory which is introduced by Molodtsov, (1999). Molodtsov defines the soft set theory as a tuple which is associated with a set of parameters and a mapping from a parameter set onto the power set of a universal set. Unlike the other existing mathematical theories for dealing with uncertainties, such as probability theory (Kolmogorov, 1950), Fuzzy set theory (Zadeh, 1965), intuitionistic fuzzy set theory (K. T. Atanassov, 1986), Rough set theory (Pawlak, 1982), vague set (Gau & Buehrer, 1993), grey set theory (Julong, 1989), and etc. that all these theories lack parameterization tools. Although the soft set has parameterization tool, it requires hybridization in cases that involving non-Boolean datasets which could establish larger paradigms, so that, one can choose any parameters, which enormously explain the decision-making process and compose the procedure more proficient from available data. The major advantage of the soft set theory is that it need not bother with any additional information about the data such as the probability of statistic or possibility value in fuzzy set theory. The soft set theory uses parameterization as its main vehicle for developing theory and its applications. The soft set is progressing rapidly, and many researchers (Maji et al., 2002, Roy et al., 2005, Kong et al., 2009, Feng et al., 2010, Yang et al., 2009, Jiang et al., 2010, Qin et al., 2011a, Ma et al. 2011) are developing the algorithm to solve many practical problems. The significant problem of parameter reduction and decision making is becoming fascinating to be useful approaches to dealing with uncertainties. Parameterization reduction and decision making problems in soft set theory are interesting to be explored. However, a review that summarised recent advances in the applications of the soft set theory in parameter reduction is scarce in the literature despite the significance of the subject matter.

In this chapter, a review of soft set-based parameter reduction and decision making which aims to provide the depth and breadth of the state of the art issues in this research area were presented. In summary, the review has explored the following:

a. It gives recent advances on soft set-based parameter reduction and decision making.
b. It highlights the advances of hybrid soft sets in parameter reduction and decision making.
c. It provides analysis and synthesis of the published research outputs with insight on findings. Additionally, a summary of applications of the soft set and hybrid soft set in real-life problems were also discussed.
d. It points out unresolved problems and future research direction in the applications of the soft set and hybrid soft sets in parameter reduction and decision making.

2.2 Rudiment

The fundamental concept of rough set theory via constructive approach and soft set theory via descriptive approach were presented.

2.2.1 Rough Set Theory

The theory of Rough set was proposed by (Pawlak, 1982) as a result of a long-term basic research project in developing a new mathematical model for computer science. It can be referred as one of alternative set theories for analysing data. The idea of the rough set theory is about approximation of imprecise set using lower and upper approximations. In this sub-section, the rough set theory is presented from the point of view via constructive approach. The starting point of rough set theory for data analysis is due to the occurrence of uncertainty in an information system. The concept of an information system (or information table) is defined as follow.

Definition 1(See (Pawlak, Z. (1981)). *An information system is a 4-tuple (quadruple)* $S=(U,A,V,f)$, *where* $U=\{u_1,u_1,\cdots,u_{|U|}\}$ *is a non-empty finite set of objects,* $A=\{a_1,a_1,\cdots,a_{|A|}\}$ *is a non-empty finite set of attributes,* $V=\bigcup_{a\in A}V_a$, V_a *is the domain (value set) of attribute a,* $f=U\times A\rightarrow V$ *is an information function such that* $f(u,a)\in V_a$ *, for every* $(u,a)\in E\times A$ *, called the information function.*

In an information system $S=(U,A,V,f)$ if $V_a=\{0,1\}$, for every $a\in A,$ then *S* is referred to as a *Boolean-valued information system* i.e. $S=(U,A,V_{\{0,1\}},f)$.

An information system is said to be a finite data table and many concepts like object-attribute tuple, the dependency of the attribute, etc. are close to the same concepts in a relational database. The following definition presents the notion of indiscernible (similar) objects in an information system.

Definition 2 (See (Pawlak, Z. (1981)). *Let* $S=(U,A,V,f)$ *, be an information system and let B be any subset of A. Two objects* $x,y\in U$ *are said to B-indiscernible if and only if both x and y have the same feature on B i.e.*

$$f(x,a)=f(y,a),\ \forall a\in B$$

From the notion of indiscernible objects above, then the rough-approximating set is constructed via lower and upper approximations of a subset *X* of *U*. The following definition presents the concept of them.

Definition 3 (See (Pawlak, Z. (1981)). *Let* $S=(U,A,V,f)$ *, be an information system and let B be any subset of A and let X be any subset U. The B-lower and B-upper approximations of X, respectively denoted by* $\underline{B}(X)$ *and* $\overline{B}(X)$*, are defined by*

$$\underline{B}(X)=\{x\in U|[x]_B\subseteq X\} \text{ and } \overline{B}(X)=\{x\in U\|[x]_B\cap X\neq\phi\}.$$

From Definition 3, the $\underline{B}(X)$ is a collection of all elements of *U* which can be certainly classified as a member of *X* using *B*. Meanwhile, the $\overline{B}(X)$ is the set of all elements in *U* which can be possibly classified as *X* using *B*. It is clear that a rough-approximating set is not a (crisp) set, when $\underline{B}(X)\neq\overline{B}(X)$. The following sub-section, the notion of the soft set theory was presented via a descriptive approach.

2.2.2 Soft set Theory

A soft set can be defined as a parameterization of the subset of the universal set which is introduced by Molodtsov, (1999). Maji, Biswas, & Roy, (2003) presented the theory of soft sets introduced by (Molodtsov, 1999), they defined equality of two soft sets, the complement of a soft set, null soft set and absolute soft set with examples. Let *U* be an initial universe set and let *E* be set of parameters about the object in *U*. If the set $P(U)$ denotes the power set of *U*, then formally the definition of a soft set is given as follow:

Definition 4 (See (Molodtsov, 1999)). *A pair* (F,E) *is called a soft set over U, where F is a mapping given by* $F:E\to P(U)$.

In other words, the soft set is parameterized family of subsets of the set *U*. Every set $F(e)$, for $e\in E$ from this family may be considered as the set of *e*-element of the soft set (F,E) or as the set of *e*-approximate elements of the soft set. It is clear that a standard soft set is not a crisp set. As an illustration of a soft set, following example is presented.

Example 1

A soft set describes the doctor in a hospital to handle patients suspected with Thrombocythemia disease. The set *U* is the set of patients and there are 6 patients under consideration i.e. $U=\{p_1, p_2, p_3, p_4, p_5, p_6\}$.

The set E is the set of parameters describing the 8 Thrombocythemia symptoms of all patients. Each parameter is a word or sentence, i.e. *E = {Headache, Dizziness, Weakness, Fainting, Numbness or tingling in your feet or hands, Throbbing, Change in vision, Chest pain}* which can be represented as $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$.

Are consider as a mapping $F : E \to P(U)$ which is defined as follows:

$$F(e_1)=\{p_1, p_3, p_4, p_5\}, F(e_2)=\{\ \}, F(e_3)=\{p_1, p_2, p_3, p_4, p_5, p_6\}, F(e_4)=\{p_1, p_2,\ p_4, p_5,\},$$

$$F(e_5)=\{p_1, p_3, p_4, p_5\}, F(e_6)=\{p_1, p_3, p_4, p_5, p_6\}, F(e_7)=\{p_2, p_3, p_6\}, F(e_8)=\{p_1, p_2, p_4, p_5, p_6\}$$

Thus, the soft set has:

$$(F,E)=\left\{\begin{array}{l} \text{Headache } =\{p_1,p_3,p_4,p_5\} \\ \text{Dizziness } =\{\ \} \\ \text{Weakness } =\{p_1,p_2,p_3,p_4,p_5,p_6\} \\ \text{Fainting } =\{p_1,p_2,\ p_4,p_5,\} \\ \text{Numbness } =\{p_1,p_3,p_4,p_5\} \\ \text{Throbbing } =\{p_1,p_3,p_4,p_5,p_6\} \\ \text{Change in vision } =\{p_2,p_3,p_6\} \\ \text{Chest pain } =\{p_1,p_2,p_4,p_5,p_6\} \end{array}\right\}$$

The soft set is deemed to be a mapping from parameter to the clearly defined subset of the universe *U*. The representation of a soft set (F, E) in a finite complete Boolean-valued information system as defined by the proposition 1.

Proposition 1 (See (Feng et al., (2010)). *If* (F, E) *is a soft set over the universe U, then* (F, E) *is a Boolean-valued information system* $S = (U, A, V_{\{0,1\}}, f)$.

Proof

Let (F,E) be a soft set over the universe *U*; we define mapping

$f = \{f_1, f_2, \cdots, f_n\}$,

Where

$f_i : U \to V_i$ and $f(x) = \begin{cases} 1, x \in F(a_i) \\ 1, x \notin F(a_i) \end{cases}, 1 \le i \le |A|$.

Hence, if $A = E$, $V = \bigcup_{a \in A} V_a$, where $V_a = \{0,1\}$, then a soft set (F,E) can be considered as a Boolean-valued information system $S = (U, A, V_{\{0,1\}}, f)$

Thus, a soft set (F,E) in Example 1 can be represented in the form of Boolean-valued information system as shown in Table 2.1 below:

Table 2.1: Representation of a Soft Set of Example 1

U/E	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8
p_1	1	0	1	1	1	1	0	1
p_2	0	0	1	1	0	0	1	1
p_3	1	0	1	0	1	1	0	0
p_4	1	0	1	1	1	1	0	1
p_5	1	0	1	1	1	1	0	1
p_6	0	0	1	0	0	1	1	1

2.3 Hybrid Soft Sets

This section presents some definitions of the marriage of soft set theory with other existing set theories. The concept of fuzzy set is given as follow:

Definition 5. *Let U be a non-empty set of the universe. A fuzzy set is a pair* (U,m), *where* $m : U \to [0,1]$, *for each,* $x \in U$ *the value* $m(x)$ *is called the grade membership of x in* (U,m)

From Definition 5, the concept of a hybrid fuzzy set and the soft set is given in Definition 6 as follow:

Definition 6 (See (Son, 2007)). *Let* $F(U)$ *be the set of all subsets in-universe U and E be a set of parameters. A pair* $(\tilde{F}, E)$ *is called a fuzzy soft set over U, where* $\tilde{F}$ *is a mapping given by* $\tilde{F} : E \rightarrow F(U)$.

From Definition 6, it is clear that a fuzzy soft set is a special case of a soft set. The Definition 7 below presents the approach of an interval-valued fuzzy set.

Definition 7 (See (Ma, Sulaiman, & Rani, 2011)). *An interval value fuzzy set Y on a universe U is a mapping, where* $Int[0,1]$ *stands for the set of all closed subintervals of* $[0,1]$ *the set of all interval-valued fuzzy set on U is denoted by* $I(U)$.

From Definition 7, the notion of a hybrid internal-value fuzzy set and the soft set is given in Definition 8 as follow:

Definition 8 (See (Ma, Sulaiman, & Rani, 2011)). *Let U be an initial universe and E set of parameters. A pair* $(\tilde{G}, E)$ *is called an interval-value fuzzy soft set over U, where* $\tilde{G}$ *is a mapping given by* $\tilde{G} : E \rightarrow I(U)$.

From Definition 8, it is clear that an interval-value fuzzy soft set over a universe *U* is a special case of a soft set. The Definition 9 below presents the notion of an intuitionistic fuzzy set.

Definition 9 (See (Maji, Biswas, & Roy, 2001)). *An intuitionistic fuzzy set (IFS) A in E is defined as an object of the form* $A = \langle x, \mu_A(x), V_A(x) | x \in E \rangle$, *where* $\mu_A : E \rightarrow [0,1]$ *and* $V_A : E \rightarrow [0,1]$ *are respectively defined as the degree of membership and non-membership of the element* $x \in E$ *. For every* $x \in E$ *, the following property is hold*

$$0 \le x, \mu_A(x), V_A(x) \le 1 .$$

From Definition 9, the concept of a hybrid intuitionistic fuzzy set and the soft set is given in Definition 10 as follow:

Definition 10 (See (Maji et al., 2001)). *Consider U and E as a universe set and a set of parameters respectively. Let* $P(U)$ *denotes the set of all intuitionistic fuzzy set of U. A pair* $(\tilde{H}, E)$ *is called an intuitionistic fuzzy soft set over U, where* $\tilde{H}$ *is a mapping given by* $\tilde{H} : E \to I(U)$.

From Definition 10, it is clear that an intuitionistic fuzzy soft set over a universe *U* is a special case of a soft set. Next section presented a review of parameter reduction and decision making of the soft set.

2.4 Parameter Reduction and Decision Making for Soft Set

Parameter reduction of soft set focuses on how to efficiently reduce the number of parameters from a data set while improving several aspects such as storage, speed, and accuracy of the decision-making process. In the literature, there are number of successful implementation of parameter reduction and decision making using soft set theory. Maji et al., (2002) firstly presented an application of rough set-based dimensionality reduction (Pawlak & Skowron, 2007) in decision-making problem to a soft set. They used few parameters after reduction to select optimal choice of objects for decision making and the decision value was computed with respect to condition parameters. However, the results of their techniques have some problems because firstly it computed the reduction of rough set and then computed the choice value to select the optimal object for decision making. This shows that the optimal choice object could be changed after the dimensionality reduction of the rough set. Chen et al., (2005) presented an approach based on soft set for parameter reduction (PR) to find optimal decisions on a general Boolean datasets as an improvement of (Maji et al., 2002). They improved the application of soft set theory in a decision-making problem. However, Chen, *et al*. (2005) method faced the problem in determining all level of sub-optimal choices. Consequently, Kong et al., (2008) proposed a notion of Normal Parameter Reduction (NPR) to find optimal decisions as well as all level of suboptimal decisions and added parameter set of sets. This is because the methods of (Chen et al., 2005; Maji et al., 2002) only considered the optimal choice, they did not consider all level of suboptimal choices. In NPR, the data of optimal objects can be deleted directly from the normal parameter reduction, and the next optimal choice can be obtained exactly from normal parameter reduction in which the data of optimal objects were deleted. It was evident from the performance analysis that NPR had shown the ability to significantly reduce the dimensionality, but the performance of their algorithm in term of computational time is still an outstanding issue. Zou & Chen, (2008) proposed an algorithm of parameter reduction based on invariability of the optimal choice object, to efficiently decrease the number of the parameter used for evaluation and cut down the workload. The authors applied basic operation of relational algebra to realize the algorithm using Structural Query Language (SQL). The result shows that it is effective as compared to (Chen et al., 2005). Herawan et al., (2009) proposed an approach that reduced attribute in multi-valued information system using soft set. The AND operations were used for dimensionality reduction. The results of the experiment showed clearly that the reduction obtained are equivalent to the Pawlak's rough reducts technique (Pawlak & Skowron, 2007). Rose et al., (2010) presented technique of decision making by parameterization reduction to determined maximal supported sets from Boolean value information system based on soft set theory and also provide consistency decision making. The approach ensured that any process of attribute elimination in convert complex database into a smaller database so as to make decision. The result of the experiment shows that this technique is better than that method proposed by (Chen et al., 2005; Maji et al., 2002) because it maintains the optimal and all level of sub-optimal objects. Çağman & Enginoğlu, (2010) proposed a novel soft set based decision-making called uni-int decision making. This method could decrease an extensive set of alternatives to its subset of optimal objects based on the criteria given by decision maker. It has achieved the optimal decision from the experiment. Ma et al., (2011) proposed a new efficient normal parameter reduction algorithm based on soft sets oriented parameter sum. The authors used oriented parameter sum without considering parameter reduction degree and decision partition in order to reduce the parameter. The comparative results on the dataset shows that the new efficient normal parameter reduction algorithm based on the soft set was found to have relatively less computational complexity, easy implementation and easy to understand than that the algorithms proposed by (Chen et al., 2005; Kong et al., 2008). Meanwhile, some researchers continue to give their contribution in parameter reduction of a soft set and decision making either directly or by means of expanding the soft set to include other mathematical models. With the aim of reducing parameter, attribute reduction and dimensionality reduction. Ali, (2012) discussed the idea of reduction of parameters based on soft sets and studied approximation space of Pawlak's rough set model associated with the soft set. The work deletes parameter only one at a time in order to avoid the deep searching. The method of reduction of the parameter is similar to the reduction of the attribute in the rough set. The result of the experiment proved its efficiency. Jothi & Inbarani, (2012) proposed a new soft set based on unsupervised feature selection so as to reduce the time taken to find a minimal subset of the feature using soft set. The reduction algorithm, when tested for the feature selection, shows that it is close to rough set based on reduction and more effective in terms of speed and performance compare with the rough set. Hence, it reduces the computational time. Deng & Wang, (2012) studied the relationship between any two objects in a soft set, exploring sufficient and necessary conditions on characterizing pseudo parameter reductions. Also, to normal parameter reductions and notion of parameter significance, in order to obtained pseudo parameter reductions and normal parameter reductions simultaneously. The result of the experiment shows that it has generally low computational cost compare with the PR and NPR. Feng, Li, & Çağman, (2012) extended the work of (Çağman & Enginoğlu, 2010) by proposing two new concepts of choice value soft sets and *k*-satisfaction relations. They presented deeper insights into the principle of *uni–int* decision making and show its limitations. Finally, they proposed several new soft decision-making schemes. Yuan, Liu & Shi, (2003) proposed an alternative approach to parameter reduction using soft set-based data mining concept according to the importance degree of each factor in decision making system. They claimed that the proposed solution is more widely used in decision analysis systems and obtains a reduction parameter set. They defined an impact factor of parameter and sub-sequentially presented a new indiscernibility relation and decision making priority. Their derived algorithm has been used in a house purchase system to show some valuable information and give a feasible result. Han & Geng, (2013) developed a pruning method which filters out objects that cannot be an element of any optimal decision set. The authors enhanced the (Feng et al., 2012) *int m-int n* decision-making scheme and the experimental result of the method shows that it has a higher efficiency in computing the optimal solution. Kumar & Rengasamy, (2013) studied parameter reduction using soft set theory for better decision making. Hence, sample dataset can be converted into binary valued data and shows that the number of the parameter can be reduced without the loss of any original information in order to make decision making better. However, the dealing of optimal and suboptimal choice was ignored. Xu et al., (2014) introduced a parameter reduction approach using soft set theory to pick out financial ratios for business failure prediction. The method integrates statistical logistic regression into a soft set and the result demonstrates superior forecasting performance in terms of accuracy and stability compare to NPR. Han & Li,

(2014) proposed a method of compiling all the normal parameter reduction of a soft set into a proposition of parameter Boolean variables. The method was used to provide an implicit representation of NPR and it improved the retrieval performance. Li, Xie, & Wen, (2015) introduced soft coverings and investigated its parameter reduction by means of knowledge on the attribute reduction for covering information system. The algorithm of the soft covering parameter reduction has been applied in knowledge discovery and the result shows that the evaluation system of the dataset used is efficient and feasible and thus, save much time. Kong et al., (2015) applied the algorithm of Particle Swarm Optimization (PSO) to reduce parameter in the soft set. They defined dispensable core in the soft set and solve the normal parameter reduction related to the dispensable core. The experiment shows that the method is feasible and fast. Xie, (2016) presented an alternative algorithm for the parameter reduction of soft sets. The parameter reduction of soft sets is investigated by means of the attribute reduction in information systems and its related algorithm is derived. Table 2.2 summarizes the various parameter reduction and decision making methods of soft sets.

Table 2.2: Parameter Reduction and decision making of soft sets

Author	Purpose	Description	Advantages
Maji et al., 2002	Parameter reduction & decision making	Applied the concept of soft sets theory to solve a decision-making problem which used rough mathematics.	It uses few parameters to select optimal objects for decision-making.
Chen et al., 2005	Parameter reduction & decision making	Presented some definition parameter reduction of soft sets and contrast with attribute reduction in rough set theory.	Finding optimal decisions on global Boolean datasets and the difference among all objects according to the parameter in A does not influence the final decision.
Kong et al., 2008	Parameter reduction &	Proposed the concept of NPR to find optimal decisions as well as suboptimal decision	Optimal choice can be obtained exactly from

Table 2.2:, Continued

	decision making	and added parameter set of sets.	normal parameter reduction.
Zou & Chen, 2008	Parameter reduction	Proposed algorithms for parameter reduction based on invariability of the optimal choice object, to efficiently decrease the number of the parameter used for evaluation and cut down the workload.	Decreasing the number of the parameter used for evaluation and cut down the workload.
Herawan et al., 2009	Attribute reduction	Presented an attribute reduction in multi-valued information system to decrease the attribute using soft set.	Considered multi-valued information system to reduced parameters.
Rose et al., 2010	Parameter reduction	Described technique of decision making through parameterization reduction to determined maximal supported sets.	Determined maximal supported sets from the Boolean value information system.
Çağman & Enginoğlu, 2010	Parameter reduction	Initiated a new soft set based decision-making scheme, called uni-int decision making. This approach could decrease an extensive set of alternatives based on the criteria given by decision maker.	Uses the decision maker criteria to reduced set optimal choices.
Ma et al., 2011	Parameter reduction & decision making	Proposed an NENPR of soft sets based on the oriented parameter sum. Moreover, it uses oriented parameter sum without considering parameter reduction degree and decision partition.	Reduced the computational complexity by ignoring parameter reduction degree and decision partition to reduce the parameter.
Ali, 2012	Parameter reduction	Discuss the idea of reduction of parameters using soft sets. It shows that in the studied there is an approximation space of Pawlak associated with a soft set.	A technique is developed for the reduction of parameters.
Jothi & Inbarani, 2012	Feature selection	Proposed a new soft set based on unsupervised feature selection so as to reduce the time taken to find a minimal subset of the feature using soft set.	Reduced the time taking to find minimal subset.

Table 2.3: Continuation

Table 2.2: Continued

Deng & Wang, 2012	Parameter reduction	Studied the relationship between any two objects in a soft set Sufficient and necessary conditions on characterising pseudo parameter reductions and normal parameter reductions on the parameter significance and used it to reduce the parameter.	Uses the two algorithms to evaluate pseudo parameter and NPR concurrently directly.
Feng et al., 2012	Decision making	Proposed two new concepts of choice value soft sets and *k*-satisfaction relations and several new soft decision-making schemes.	Improves the classical *uni–int* decision-making approach
Yuan et al., 2013	Parameter reduction & decision making	Proposed an alternative approach of parameter reduction using soft set-based data mining concept according to the importance degree of each factor in decision-making system.	More widely used in decision analysis systems and obtains a reduction parameter set.
Han & Geng, 2013	Parameter reduction	Developed a pruning method which filters out objects that cannot be an element of any optimal decision set.	Remove the object that cannot be an element of optimal decision.
Kumar & Rengasamy, 2013	Classification	Discussed how a sample dataset can be converted into binary valued data and how the number of parameters is reduced without loss of any original information.	Converting any dataset into binary valued and reduced it without losing information.
Xu et al., 2014	Parameter reduction	Presented a parameter reduction method that uses soft set theory to choose financial ratios for business failure prediction (BFP).	Integrate statistical logistic regression into a soft set.
Han & Li, 2014	Parameter reduction	Proposed a method of compiling all the normal parameter reduction of a soft set into a proposition of parameter Boolean variables.	Use proposition representation that provides an implicit representation of NPR.
Li et al., 2015	Attribute reduction	Proposed soft covering and obtained the lattice structure of soft set induced on them and investigate the parameter reduction of soft covering using attribute reduction.	Attribute reduction for covering information.

Table 2.2:, Continued

Kong et al., 2015	Parameter reduction	Applied particle swarm optimisation algorithm to reduce parameter in the soft set. They defined dispensable core in the soft set and solved the normal parameter reduction related to the dispensable core.	Applying PSO technique to solve NPR is easier than developing new algorithms.
Xie, 2016)	Parameter reduction	Present an alternative algorithm for the parameter reduction of soft sets.	The parameter reduction of soft sets is investigated using the attribute reduction in information systems, and its related algorithm is derived.

From Table 2.2 several algorithms for parameter reduction of the soft set for decision making and their successful implementation was discussed. It is clear that while some of the scholars are given emphasis on sufficient and necessary condition for the reduction, others only considered optimal choice without referring to suboptimal choice. The issue of normal parameter reduction that examined the suboptimal choice has been reviewed by (Kong et al., 2008; and Ma et al., 2011). These two methods have their inherent difficulties such as computational complexity, speed and storage. Based on these challenges the new proposed algorithm was proposed in this study.

In the next section, a review of parameter reduction and decision making of the hybrid soft set with other set theories was presented.

2.5 Parameter Reduction and Decision Making of Hybrid Soft Set

The notion of the soft set can be merged with other mathematical models to deal with uncertainty, and this combination is referred to as hybridization. Numerous successful hybridizations of the soft set with the fuzzy set and rough set have been studied. Also, interval-valued fuzzy soft set and the concept of the intuitionistic fuzzy soft set were also considered by researchers to handle uncertainty and imprecision in decision making and other applications.

2.5.1 Fuzzy Soft Set-based Approach for Parameter Reduction and Decision Making

With this point of view, Yang et al. (2007) expanded the traditional soft set to fuzzy soft set to improve its quality, and also discussed some immediate outcomes. To continue the investigation on hybridization of the soft set with the fuzzy set, Roy & Maji, (2007) present a novel techniques of object recognition from an imprecise multi-observer data to decrease the problem of imprecision in decision-making using fuzzy soft set. It has used a method that involves the formation of a comparison table from the resultant fuzzy soft established in a parametric sense; the experimental results demonstrate the feasibility of the methods. Kong, Gao, & Wang, (2009) argued that the algorithm proposed in (Roy & Maji, 2007) was incorrect because using the algorithm will not result in the right choice in general. Therefore, used a counter example to illustrate the feasible of the technique and proved the efficiency of their techniques compare to (Roy & Maji, 2007). Feng, Jun et al., (2010) proposed an adjustable method to fuzzy soft set based decision-making using level soft sets. It is an improvement of Roy's and Maji's method. The weighted fuzzy soft set notion was also introduced and applied to decision-making problem.

Cagman, Enginoglu, & Citak, (2011) present fuzzy parameterized fuzzy soft sets and gives some operation to create fuzzy parameterized fuzzy soft decision-making method that allows development of decision processes. The result of experiment demonstrates that the method can be applied to many uncertainties problem. Jiang et al., (2011) presented a fuzzy soft set theory (combination of the fuzzy and soft set) by implementing the idea of fuzzy description logics to serve as the parameters of fuzzy soft sets. The authors give some definitions of the operations for the extended fuzzy soft sets. Additionally, the authors proved the De Morgan's laws in the extended fuzzy soft set theory. Kong, Wang, & Wu, (2011) presented an algorithm for multiple evaluations for fuzzy soft set decision problem based on grey relational analysis, to combine multiple evaluations into a single evaluation value and use the evaluation to make a decision. The result of the experiment proved that the new algorithm is efficient for solving decision problem. Kong et al., (2012) proposed a new parameter reduction which combined fuzzy and soft set called fuzzy soft set to improve the condition of redundant parameters because it is so strict that the number of deleting parameters is very few. The authors introduce some definitions of proximate parameter reduction in the fuzzy soft set. The result of the experiment demonstrated the effectiveness of the proposed method. Basu, Mahapatra, & Mondal, (2012) presented a new approach called Mean Potentiality Approach (MPA) to get a proportional solution of a fuzzy soft set based decision-making problem. Moreover, the reduced parameter based on relational algebra with the help of MPA algorithm. The method demonstrates that it is feasible compared to Feng's method and NPR. Deng & Wang, (2013) introduced the concept of complete distance between two objects and relative control the degree between two parameters, based on an object parameter to predict unknown data in incomplete fuzzy soft sets. The effectiveness of the method was demonstrated by given an example and proved the feasibility of the method compare with the investigation of classical predicted method. Yang, Tan, & Meng, (2013) introduced multi-fuzzy soft set using combining the multi-fuzzy set and soft set model. Some operation and properties are defined and proved. An illustrative example shows the validity of the method as feasible. Li, Wen, & Xie, (2015) presented techniques for a fuzzy soft set in decision-making by combining grey relational analysis and Dempster-Shaper theory of evidence to calculate grey mean relational degree which can be used to compute the uncertain degree of various parameters. The result demonstrates the effectiveness and feasibility of the method. Tao et al., (2015) developed a novel concept of Uncertain Linguistic Fuzzy Soft Set (ULFSS). The decision algorithm used the external aggregation process that is TOPSIS technique for collection decision information aggregate from the individual. The result demonstrates its feasibility. Tang, (2015) employed Grey relational analysis and Dempster–Shafer theory of evidence in proposing a novel fuzzy soft set approach in decision making. Firstly, in calculating the grey mean relational degree, he used the uncertain degrees of various parameters which are determined via Grey relational analysis. Secondly, according to the uncertain degree, the proper mass functions of different independent alternatives with various parameters are given. Thirdly, Dempster's rule of evidence combination is applied to aggregate the alternatives into a collective alternative. Finally, the alternatives decisions are ranked, and the best alternatives decisions are considered. The effectiveness and feasibility of this proposed approach are demonstrated by comparing with that the mean potentiality approach. Wang, Li, & Chen, (2015) presented hybrid hesitant fuzzy set and fuzzy soft sets. They presented algebraic properties of the proposed hybrid approach and further applied it to multi-criteria group decision-making problems. They showed the applicability of their hybrid model to calculate a numerical example. Das & Kar, (2015) presented a hybrid hesitant fuzzy set and soft set. They studied distances measurement procedure and aggregate functions on their hybrid model. Finally, they applied it to a medical analysis involving multiple attribute decision making. Aktaş & Çağman, (2016) presented a combination of fuzzy sets and soft sets. They used a matrix representation of the soft sets which is very useful for computations of the proposed combination method in decision making involving uncertainties. Majumdar, (2016) presented some hybrid soft sets i.e. fuzzy parameterized soft set and vague soft set. They studied algebraic properties of those two hybridizations and presented their application in decision making. Table 2.3 below summarises the various parameter reduction and decision making methods of hybrid fuzzy and soft sets discussed above.

Table 2.3: Parameter Reduction and Decision Making of Hybrid Fuzzy Soft Sets

Authors	Purpose	Description	Advantages
Yang et al., (2007)	Consolidating soft set with fuzzy set	Consolidated traditional soft set to fuzzy set to improve its quality, and discussed some immediate outcomes of the fuzzy soft set.	Expands traditional soft set to fuzzy set to improve the efficiency of decision making.
Roy & Maji, (2007)	Parameter reduction & Decision making	Presented an approach of object recognition from an imprecise multi-observer data to reduce the problem of uncertainty in decision making using fuzzy soft set.	Reduces uncertainty from multi-observer data
Feng et al., (2010)	Decision making	Presented level soft sets and weighted fuzzy soft set for decision making.	Adjustable approach for decision making in fuzzy environment.
Cagman et al., (2011)	Parameter reduction	Presented fuzzy parameterized fuzzy soft (FPFS) sets and its operation and form FPFS-decision making.	Reduces parameter by using fuzzy parameterized fuzzy soft set.
Jiang et al., (2011)	Parameter reduction	Presented an extended fuzzy soft set theory by using the approach of fuzzy description logics to serve as the parameters of fuzzy soft sets.	Use the fuzzy description logic to reduced parameter.
Kong et al., (2011)	Decision making	Presented an algorithm for multiple evaluation bases for fuzzy soft set and decision problem based on Grey relational analysis.	Combines multiple evaluation method into single evaluation to make a decision.
Kong et al., (2012)	Parameter reduction	Presented a new parameter reduction for the fuzzy soft set, to improve the condition of redundant parameters because it is so strict that the number of deleting parameters is very few.	Uses the fuzzy soft set to reduced redundant parameters.
Basu et al., (2012)	Decision making	Presented a new approach called mean potentiality approach (MPA) to get a proportional solution of a fuzzy soft set based decision-making problem.	Reduce parameter based on relational algebra using MPA algorithm.

Table 2.3:, Continued

Deng & Wang, (2013)	Parameter reduction	Introduced the concept of complete distance between two objects and relative control the degree between two parameters, based on an object parameter to predict unknown data in incomplete fuzzy soft sets.	Prediction of an unknown data in the fuzzy soft set.
Yang et al., (2013)	Combine multi-fuzzy with soft set	Introduced multi-fuzzy soft set using combining the multi-fuzzy set and soft set model.	Combines MFSS approach with the soft set to handle imprecision.
Li et al., (2015)	Parameter reduction decision making	Presented another technique to fuzzy soft set in decision making by combining grey relational analysis and Dempster-Shaper theory of evidence.	Uses relational algebra and theory of evidence to make a decision.
Tao et al., (2015)	Parameter reduction	Developed a novel concept of the uncertain linguistic fuzzy soft set (ULFSS).	Uses the external aggregation process that is TOPSIS technique for collection decision information aggregate from individual
Tang, (2015)	Decision making	Employed grey relational analysis and Dempster–Shafer theory of evidence in proposing a new approach fuzzy soft set in decision making	The method adopted is efficient and feasible by comparing it with that the mean potentiality approach
Wang et al., (2015)	Decision making	Introduced a hybrid hesitant fuzzy set and fuzzy soft sets.	Workable to be applied to multi-criteria group decision-making problems
Das & Kar, (2015)	Decision making	Introduced a hybrid hesitant fuzzy set and soft set.	Workable to be applied to a medical analysis involving multiple attribute decision making.
Aktaş & Çağman, (2016)	Decision making	Introduced a combination of fuzzy sets and soft sets and presented a matrix representation of the soft sets.	Very useful for computations of the proposed combination method in decision making

Table 2.3:, Continued

			involving uncertainties.
Majumdar, (2016)	Decision making	Introduced hybrid models of fuzzy parameterized soft set and vague soft set.	Workable to be applied to decision-making problems.

Some scholars presented hybridization of soft with other set theory to deal with uncertainty in decision making. Although the implementation was successful, it was applied in fewer real life problem. Based on this issue interval-valued fuzzy soft set approach was presented by researchers that combined interval-value fuzzy with the soft set as discussed in the next sub-section.

2.5.2 Interval-valued Fuzzy Soft set-based Approach for Parameter Reduction and Decision Making

The interval-valued fuzzy soft set was also studied by researchers to handle uncertainty and imprecision in decision making. Yang et al., (2009) introduced the idea of the interval-valued fuzzy soft set which combined interval-valued fuzzy and soft set to obtain a new model for the soft set which is free from the inadequacy of parameterized tools. They have also defined many algebraic operations such as the complement, “AND”, “OR” operations of the interval-valued fuzzy soft set. The authors used an example to prove the validity of the interval-valued fuzzy soft set in decision-making problem. Khalid & Abbas, (2015) initiated the study of distance measures for the interval-valued fuzzy soft set (combination of interval-valued fuzzy and soft set), and Hausdorff metric-based measures for the intuitionistic fuzzy soft set. Some operation and properties are introduced and also proposed some application on medical diagnosis and prove that the performance is effective. Feng, Li, & Leoreanu-Fotea, (2010) introduced the level soft sets of fuzzy soft sets concept (combination of the fuzzy and soft set) and developed an adjustable decision-making approach using fuzzy soft sets. The author's considered appropriate reduct fuzzy soft sets and level soft set to reduce too much simpler crisp soft set. It is evidently that Feng's soft rough set model in some cases provides best approximations than classical rough sets. Qin et al., (2011) presented an adjustable approach to interval-valued intuitionistic fuzzy soft sets based on decision making by using reduct intuitionistic. The authors compute the reduct intuitionistic fuzzy soft set and interval value intuitionistic fuzzy soft set and the converted it into a crisp soft set by using a level soft set of an intuitionistic fuzzy soft set. The performance of the algorithm shows that it is more efficient than the algorithm developed by Jiang et al., (2011). Alkhazaleh, Salleh, & Hassan, (2011b) present the generalisation of the fuzzy soft set to possibility fuzzy soft set and shows some application of this approach in decision-making. Moreover, the concept of parameterized interval-valued fuzzy soft set was also introduced. The result shown is feasible. Alkhazaleh, Salleh, & Hassan, (2011a) present the concept of parameterized interval-value fuzzy soft set where the mapping is defined from the fuzzy soft set parameters to an interval-value fuzzy subset of the universal set. Furthermore, they gave an application of this approach in decision making. Finally, the outcomes demonstrated that the technique could be applied to many problems with uncertainties. Ma et al., (2014) proposed the parameter reduction of the interval- valued fuzzy soft set to deal with uncertainty and imprecision in decision making. They developed heuristic algorithms of parameter reduction that reduce redundant parameters. The result of the experiment demonstrated the effectiveness of the algorithms. Alkhazaleh, (2015) introduced multi-fuzzy soft set by consolidating the concept with the interval-value fuzzy set. Some operation of complement, intersection and union operations were presented on the multi-interval-valued fuzzy soft set and its properties and finally gave the application of this concept in decision-making, and the result was clear as feasible. Mukherjee & Sarkar, (2014) presented some similarity measures of a hybrid interval-valued fuzzy set and soft set. Further, they applied the hybridization of interval-valued fuzzy soft set in decision-making problems. Tripathy, Sooraj, & Mohanty, (2017) presented a new method of interval-valued fuzzy soft sets using membership function approach. They also presented its application in decision-making. Table 2.4 below summarises the various parameter reduction and decision-making methods of hybrid interval-valued fuzzy and soft sets discussed above.

Table 2.4: Parameter Reduction and Decision Making of Hybrid Interval-Value Fuzzy Soft Sets

Authors	Purpose	Description	Advantages
Yang et al., (2009)	Decision-making	Initiated interval-valued fuzzy soft set in making decisions.	First attempt and workable.
Khalid & Abbas, (2015)	Decision-making	Initiated the study of distance measures for the interval-valued fuzzy soft set and Hausdorff metric-based measures for the intuitionistic fuzzy soft set.	Uses distance measure to combine interval-value fuzzy and soft set.
Feng, Li, & Leoreanu-Fotea, (2010)	Parameter reduction & decision-making	Presented the idea of level soft sets of fuzzy soft sets to develop an adjustable decision-making approach using fuzzy soft sets.	Considers appropriate reduct fuzzy soft sets and level soft set IVFS to be reduced.
Qin et al., (2011)	Parameter reduction & decision-making	Presented an adjustable approach to interval-valued intuitionistic fuzzy soft sets based on decision-making using reduct intuitionistic.	Computes the reduct intuitionistic fuzzy soft set and interval value intuitionistic fuzzy soft set and the converted it into the crisp soft set.
Alkhazaleh et al., (2011b)	Parameter reduction & decision-making	Presented the generalisation of the fuzzy soft set to possibility fuzzy soft set.	Achieves possibility fuzzy soft set in decision making.
Alkhazaleh et al., (2011a)	Parameter reduction	Proposed the notion of parameterized interval-value fuzzy soft set where the defined mapping from the fuzzy soft set.	Defines the mapping from fuzzy soft set parameters to an interval-value fuzzy subset of the universal set.
Ma et al., (2014)	Parameter reduction & decision-making	Proposed the parameter reduction of the interval-valued fuzzy soft set to deal with uncertainty and	Considers invariable rank of decision choice of parameter reduction.

Table 2.4:, Continued

		imprecision in decision making.	
Alkhazaleh, (2015)	Decision-making	Introduced multi-fuzzy soft set in which this approach and the interval-value fuzzy set were combined.	Uses the multi-interval-valued fuzzy soft set in decision making.
Mukherjee & Sarkar, (2014)	Decision-making	Introduced some similarity measures for a hybrid interval-valued fuzzy set and soft set.	Workable as an alternative approach for decision-making problems.
Tripathy et al., (2017)	Decision-making	Introduced interval-valued fuzzy soft sets using membership function approach.	More useful than the individual components for decision-making problems.

Several researchers presented Interval-valued fuzzy soft set which combined interval-valued fuzzy and soft set to obtain a new model of the soft set that is free from insufficient of parameterized tools. This is because the existing algorithms did not consider the existing methods that considered only binary and real values, the interval value and intuitionistic data. In the following subsection intuitionistic fuzzy set for parameter reduction and decision making was discussed.

2.5.3 Intuitionistic Fuzzy Soft Set-based Approach for Parameter Reduction and Decision Making

Subsequently, the intuitionistic fuzzy soft set was also studied to give more in the hybridization so as to achieve higher efficiency in parameterization and decision making. Maji, (2009) introduced new operation on intuitionistic fuzzy soft set based on a combination of soft set model and intuitionistic fuzzy set and also studied its properties. Similarity measurement method was used to made decision selection. A simple example was used to show the feasibility of the method. Jiang, Tang, & Chen, (2011) presented an adjustable method to intuitionistic fuzzy soft set using level soft sets of an intuitionistic fuzzy soft set to extend the decision-making approach. Decision criteria function was used as a threshold on membership value and non-member ship value. It evidently from the result of the experiment shows that the adjustable feature makes the Intuitionistic Fuzzy Soft Set (IFSS) approach more appropriate in many real life applications. Mao, Yao, & Wang, (2013) presented a group decision-making technique based on Intuitionistic fuzzy soft matrix and uses median and p-quartile to compute threshold vectors. The authors introduced aggregate operators in the intuitionistic fuzzy soft matrix and established a criterion that the expert has a high weight if his evaluation value is close to the mean value and small weight if it is far from the mean value. An illustrative example demonstrates the efficiency of the method. Das & Kar, (2014) proposed an algorithmic approach on an intuitionistic fuzzy soft matrix to investigate a specific disease that is reflecting the agreement of all experts. The approach is guided by group decision-making model and uses cardinal intuitionistic fuzzy soft set. The approach demonstrates its effectiveness in sample case study. Das, Kar, & Pal, (2014) proposed a technique for making multiple attribute group decision-making problems. It uses interval-valued intuitionistic fuzzy soft matrix and confident weight to the expert based on cardinals of the intuitionistic fuzzy soft set. The confident weight is assigned to each expert based on their opinion. The result shows the effectiveness of using weight assigning method. Deli & Çağman, (2015) proposed intuitionistic fuzzy parameterized soft sets for dealing with uncertainties, based on both soft sets and intuitionistic fuzzy sets. Hence, the decision acquired by using the operations of intuitionistic fuzzy sets and soft sets that make this sets useful and applicable in practically. The result of numerical example demonstrated that the method is effective. Shanthi & Naidu, (2015) proposed a similarity measure of hybrid interval-valued intuitionistic fuzzy set and soft set of root type. Further, they applied it for decision making. Chen, (2015) presented the inclusion-based method for order preference by similarity to ideal solution technique based on interval-valued intuitionistic fuzzy sets. They also addressed multiple criteria group decision-making problems in the framework of interval-valued intuitionistic fuzzy sets. Tripathy, Mohanty, & Sooraj, (2016b) introduced a hybrid intuitionistic fuzzy set and soft sets. By using the notion of a characteristic function, they applied it to decision-making problem. Tripathy, Mohanty, & Sooraj, (2016a) presented another work on hybrid intuitionistic fuzzy set and soft sets and its application to group decision making. Jia et al., (2016) proposed a sequence of hybrid intuitionistic fuzzy soft sets and its related properties. Further, they explored it for decision-making problem. Karaaslan & Karataş, (2016) presented algebraic operations i.e. OR and AND-products of intuitionistic fuzzy parameterized fuzzy soft sets. Furthermore, they presented a decision-making method so-called $\wedge$-aggr based on intuitionistic fuzzy parameterized fuzzy soft sets. Deli & Karataş, (2016) presented a hybrid interval-valued intuitionistic fuzzy parameterized set and soft set. By using soft level sets, they constructed a parameter reduction method to obtain a better decision making. They gave a numerical example to show that the proposed hybrid method is working successfully for decision-making problems containing uncertain data. Wu & Su, (2016) presented a hybrid group generalised interval-valued intuitionistic fuzzy soft sets which contain the basic description by interval-valued intuitionistic fuzzy soft set on the alternatives and a group of experts' evaluation of it. Furthermore, a multi-attribute group decision-making model, based on group generalised interval-valued intuitionistic fuzzy soft sets weighted averaging operator is built. It is employed to solve the group decision-making problems in the more universal interval-valued intuitionistic fuzzy environment. Table 2.5 below summarises the various parameter reduction and decision-making methods of hybrid intuitionistic fuzzy and soft sets discussed above.

Table 2.5: Parameter Reduction and Decision Making of Hybrid Intuitionistic Fuzzy Soft Sets

Authors	Purpose	Description	Advantages
Maji, (2009)	Parameter reduction & decision-making	Introduced new operation on intuitionistic fuzzy soft set based on a merge of the soft set model with the intuitionistic fuzzy set.	Made decision selection with the help of similarity measurement method.
Jiang, Tang, & Chen, (2011)	Parameter reduction & decision-making	Presented an adjustable method to intuitionistic fuzzy soft set using level soft sets of the intuitionistic fuzzy soft set to handle decision-making.	Used decision criteria function as a threshold on membership value and non-member ship value.
Mao et al., (2013)	Decision-making	Presented a group decision-making method based on Intuitionistic fuzzy soft matrix and uses median and p-quantile to compute threshold vectors.	Used median and p-quantile to compute threshold vectors.
Mao et al., (2013)	Parameter reduction & decision-making	Proposed an algorithmic method on the intuitionistic fuzzy soft matrix (IFSM) to investigate a particular disease that mirroring or reflecting the agreement of all experts.	The approach is guided by group decision-making model and uses cardinal IFSS
Das & Kar, (2014)	Decision-making	Proposed a technique for making multiple attribute group decision making (MAGDM) problem.	Assigned a confident weight to each expert based on their opinion of making a decision.
Deli & Çağman, (2015)	Parameter reduction & decision-making	Proposed intuitionistic fuzzy parameterized soft sets for dealing with uncertainties based on intuitionistic fuzzy sets and soft sets.	The decision is obtained by using the operations of soft sets and intuitionistic fuzzy sets.
Shanthi & Naidu, (2015)	Decision-making	Proposed a similarity measure of the hybrid interval-valued intuitionistic fuzzy set and soft set of root type.	The proposed similarity measure can be used for decision making.
Chen, (2015)	Decision-making	Presented the inclusion-based method for order preference by similarity to ideal solution method with	Can be used for solving multiple criteria decision making.

Table 2.5:, Continued

		interval-valued intuitionistic fuzzy sets	
Tripathy et al., (2016a, 2016b)	Decision-making	Introduced a hybrid intuitionistic fuzzy set and soft sets	Decision making using characteristic functions and group decision making
Jia et al., (2016)	Decision-making	Proposed a sequence of hybrid intuitionistic fuzzy soft sets.	Workable for decision making.
Karaaslan & Karataş, (2016)	Decision-making	Proposed ∧-aggr decision making based on intuitionistic fuzzy parameterized fuzzy soft sets.	Workable for decision making.
Deli & Karataş, (2016)	Parameter reduction & decision-making	Presented a hybrid interval-valued intuitionistic fuzzy parameterized set and soft set.	Successfully for parameter reduction and decision making problems containing uncertain data.
Wu & Su, (2016)	Decision-making	Presented a hybrid group generalised interval-valued intuitionistic fuzzy soft sets.	More universal than the interval-valued intuitionistic fuzzy environment.

Several parameter reductions and decision-making approaches of hybrid intuitionistic fuzzy and soft sets were discussed in which intuitionistic fuzzy soft set is combined with intuitionistic fuzzy set and soft set approach. This is to reduce parameters in real life applications that involves interval-valued intuitionistic fuzzy soft sets data. In the following subsection a hybrid soft set with the fuzzy and rough set was discussed.

2.5.4 A Hybrid Soft Set with Fuzzy and Rough Set for Parameter Reduction and Decision Making

Another important aspect in parameter reduction and decision making of a hybrid soft set is consolidating the soft fuzzy rough set to tackle uncertainty and imprecision in decision-making Feng et al., (2010) presented a hybrid model called rough soft sets to provide the framework for consolidating fuzzy sets, rough set, and soft sets. The study presents a preliminary concept of hybridization of the soft fuzzy rough set which could help researchers to establish whether the notion could lead to a fruitful output or not. Hu, An, & Yu, (2010) developed a new model called soft fuzzy rough sets (combination of soft set, fuzzy and rough set) to reduce the influenced of noise because data sets in a real-world application are contaminated by noise. The membership is calculated by k^{th} sample, where k is determined by the tradeoff between the number of misclassified samples and the argumentation membership. The result of their experiment conducted to test the robustness of model in feature selection, and evaluation has proved to be effective. Meng, Zhang, & Qin, (2011) discussed the combination of fuzzy, rough and soft sets and presented the new algorithm soft rough set model. They have proven some theorems and algebraic properties of fuzzy, rough and soft sets. Subsequently, the fuzzy soft set was employed to granulate the universe of discourse. Ali, (2011) presented the notion of an approximation space associated with each parameter in a soft set and an approximation space related to the soft set is defined. The properties of the model are proved. Zhang, (2013) introduced two concepts of reduct and exclusion which can be used to find the reduct or exclusion of a set of parameters. The authors use "smallest" fuzzy soft set that produces the same lower soft fuzzy rough approximation operators and the same "upper" soft fuzzy rough approximation operators. As such, the necessary and sufficient conditions were obtained which demonstrate the effectiveness of their approach using approximation approach. Zhang & Shu, (2015) presented a generalisation of a hybrid interval-valued fuzzy set and rough set which is based on constructive and descriptive (axiomatic) approaches. For constructive approach, they employed an interval-valued fuzzy residual implicator and its dual operator, meanwhile, for descriptive approach, generalised interval-valued fuzzy rough approximation operators are defined by axioms. Finally, they illustrated the proposed model in decision making. Zhang, Shu, & Liao, (2015) presented a hybrid hesitant fuzzy set and rough set over two universes. It is based on a constructive approach, and the union, the intersection and the composition of hesitant fuzzy approximation spaces are proposed, and some properties are also investigated. They further applied the proposed hybrid hesitant fuzzy set and rough set in decision-making problem. Zhan, Liu, & Herawan, (2016) presented a hybrid novel soft set with the rough set, and further, they combined with hemirings, so-called soft rough hemirings. They use the proposed hybrid soft rough hemirings for corresponding multi-criteria group decision making. Table 2.6 below summarises the various parameter reduction and decision making methods of the hybrid soft set with fuzzy and rough sets discussed above.

Table 2.6: Parameter Reduction and Decision Making of Hybrid Soft set with Fuzzy and Rough Sets

Authors	Purpose	Description	Advantages
Feng et al., (2010)	Hybridization	Presented a hybrid model called rough soft sets to provide a framework to consolidate fuzzy sets, rough set and soft sets all together.	Consolidation of fuzzy set, rough set and soft set.
Hu et al., (2010)	Feature selection	Developed a new model called soft fuzzy rough sets to reduce the influenced of noise because datasets in a real-world application are contaminated by noise.	Reduced the noise of dataset.
Meng et al., (2011)	Hybridization	Discussed the combination of fuzzy, rough and soft sets and proposed a new soft rough set model and its properties.	Employed soft rough set model to segment decision making.
Ali, (2011)	Hybridization	Presented the concept of an approximation space associated with each parameter in a soft set.	Can be used further for hybridization and decision making.
Zhang, (2013)	Parameter reduction	Introduced two concepts of reduct and exclusion which can be used to find the reduct or exclusion of a set of parameters.	Obtained the necessary and sufficient conditions for parameter reduction.
Zhang & Shu, (2015)	Decision-making	Presented a generalisation of a hybrid interval-valued fuzzy set and rough set which is based on constructive and descriptive (axiomatic) approaches.	Efficient for decision making.

Table 2.6:, Continued

Zhang et al., (2015)	Decision-making	Presented a hybrid hesitant fuzzy set and rough set over two universes based on constructive approach.	Can be used for practical applications in decision making
Zhan et al., (2016)	Decision-making	Proposed a novel approach to soft rough set and soft rough hemirings.	Useful for corresponding multi-criteria group decision making.

Hybrid soft set involves consolidating the soft fuzzy rough set to handle uncertainty and imprecision in decision-making, the key idea is to handle all the difficulties that fuzzy and rough set suffer especially regarding parameterization. In the following subsection, the real world applications of soft sets and hybrid soft sets were discussed.

2.6 Real World Applications of Soft Sets

Following the previous successful existing set theories in solving real world problems e.g. (Pawlak, Słowiński, & Słowiński, 1986; Sanchez, 1979; Slowinski, 1992), the soft set theory has also been applied in many areas successfully. The major goal for parameter reduction algorithm is to improve the efficiency, many approaches have been developed. This section shows the various applications of soft set theory.

2.6.1 Soft Set Theory for Association rules mining

The soft set theory has been used for mining interesting association rules from transactional databases. Herawan & Deris, (2011) proposed the idea of association rules mining based on soft set approach. It uses the concept of co-occurrence of parameters in defining support and confidence of association rules from Boolean-valued transactional dataset. Vo et al., (2016) presented another approach in which soft set theory was applied for mining maximal association rules in text data. They have shown that the proposed method effectively used for mining interesting association rules which are not obtained by using methods for conventional association rule mining. Recently, Feng et al., (2016) proposed a new model of the soft set for association rule mining as an improvement of

Herawan & Deris, (2011). They refined several existing concepts to improve the generality and clarity of previous definitions.

2.6.2 Soft Set Theory for Medical Analysis

From the domain of medical analysis, the soft set theory has been extensively explored. Chetia & Das, (2010) presented a hybrid interval-valued fuzzy and soft sets. The author used Sanchez's approach for medical diagnosis in interval-valued fuzzy environment for analysing patients with fever and malaria. Herawan, (2010) presented an application of soft set based on decision-making through a Boolean-valued information system from the patient suspected with Influenza-Like Illness (ILI). The application explores how the technique can be applied to reduce the number of dispensable symptoms and make an optimal decision. The result shows that the technique can reduce the complexity of making a medical decision without loss of information. Kumar, Inbarani, & Kumar, (2013) presented a novel approach based on bijective soft sets for the generation of classification rules from the data set. The novel approach showed to be a valuable tool as compared the well-known decision tree classifier algorithm and Naïve bayes. Yuksel et al., (2013b) applied soft set theory to a medical diagnosis, to reduce parameter of the unnecessary biopsies in patients undergoing medical evaluation for prostate cancer. Later, the doctor can calculate the percentage of prostate cancer risk. If the recommendation of the soft expert system for the risk percentage is greater than 50% then biopsy is necessary. Lashari & Ibrahim, (2013) developed a framework for medical images classification using soft set to improve the physician ability to detect and analyzed pathologies to achieve better performance in terms of accuracy, precision, and computational speed. The result demonstrated that the algorithm can help to improve the physician ability to analyze and detect pathology. Çelik & Yamak, (2013) presented an application of a hybrid fuzzy soft set theory through well-known Sanchez's approach to medical diagnosis. The data is taken form collection of patients with symptoms stomach problem, temperature, cough and headache. Alcantud, Santos-García, & Hernández-Galilea, (2015) presented a soft set-based decision making procedure for glaucoma diagnosis. They used an automated combination and analysis of information from structural and functional diagnostic techniques in order to obtain an enhanced Glaucoma detection in the clinic.

2.6.3 Soft Set Theory for Incomplete Data Analysis

From Proposition 1 in Feng et al., (2010), it is shown that a "standard" soft set is equivalent to a Boolean-valued information system. In real world application, we sometime find incomplete data table. There are existing approaches based on a soft set theory for handling incomplete Boolean-valued information system. Zou & Xiao, (2008) initiated data analysis techniques of soft sets under incomplete information. The decision value of an object that has incomplete information is computed by weighted average of all possible choice values of the object, and each possible choice value weight is determined by the distribution of other objects. (Qin et al., (2012) presented a novel data filling approach for an incomplete soft set as an improvement of Zou & Xiao, (2008). It is based on the association degree between the parameters when a more substantial association exists between the parameters or regarding the distribution of other available objects when no more substantial association exists between the parameters. Recently, Khan et al., (2016a) proposed an alternative data filling approach for prediction of missing data in soft sets. The technique is based on the authenticity of association among parameters in the soft set. They have shown that the proposed technique achieves better accuracy as compared to (Qin et al., 2012; Zou & Xiao, 2008). Alcantud & Santos-García, (2016) presented two related techniques in analysing incomplete soft sets as new solutions for decision-making problems.

2.6.4 Soft Set Theory for Data Mining

This sub-section presents applications of soft set theory for data mining and knowledge discovery. Mushrif, Sengupta, & Ray, (2006) classified texture using soft-set theory based classification algorithm. Experimental results show the superiority of the proposed approach compared with some existing methods. Xiao, Gong, & Zou, (2009) use forecasting accuracy as a criterion for fuzzy membership function and proposed a combined forecasting approach based on fuzzy soft set. The result of the approach improves forecasting performance as compared to combining forecasting rough set theory. Senan et al., (2010) developed the application of soft set theory for feature selection of traditional Malay musical instrument sounds. This approach was tested and the obtained features of the proposed model are more efficient compared to the traditional method. Qin et al., (2012) presented a novel soft set approach in selecting clustering attribute. It is based on their proposed technique which is equivalent to the same concept in rough set approximation. They proved that their technique achieves lower computational time and higher accuracy compared to rough set-based approach (Herawan & Deris, 2009b). Handaga, Herawan, & Deris, (2012) improved the soft set-based classification technique in (Mushrif et al., 2006) by proposing an algorithm for classifying numerical data based on a hybrid fuzzy soft set theory. Mamat, Herawan, & Deris, (2013) improved the complexity and accuracy of the attribute selection techniques by (Herawan & Deris, 2009b; Qin et al., 2012) using maximum attribute relative of the soft set. Wang, Liu, Du, & Huang, (2013) proposed an application oriented mobile cloud computing Adaptive Mobile Cloud Computing Middleware (AMCCM). The author used cloud computing to combine the hardware resources control with the application layer software services in the system to automatically adjust the hardware resources and uses the fuzzy soft set as a virtual startup problem. The results were found to be effective and successful. Kumar, Inbarani, & Kumar, (2014) classified large gene expression data based on an improvement of the bijective soft set. To show the efficiency of the proposed model, they compared the performance to fuzzy-soft-set-based classification algorithms, Fuzzy KNN, and k-nearest neighbour approach. Ma & Qin, (2014) applied a New Efficient Parameter Reduction (NENPR) algorithm into a real life e-shopping in Blackberry mobile phone data. The result of the experiment proves that the NENPR is feasible for dealing with e-shopping. Li & Xu, (2015a) proposed a new method to measure airport importance to seek out the most vulnerable nodes based on fuzzy soft set. The result of the experiment from the traffic data of the airport shows that the evaluation method is accurate. Herawan et al., (2015) presented an alternative soft set-based approach based on the maximum degree of domination for educational data mining. It uses for clustering student assessment datasets. Sutoyo et al., (2016) used multi-soft sets as a decomposition of a multi-valued information system into many Boolean-valued information systems for conflict analysis. They proposed an alternative approach for measuring support, strength, certainty and coverage of conflict situation. They showed through experimentation that their approach performs less computation as compared to rough set-based conflict analysis model. Khan et al., (2016) applied data analysis under incomplete soft set for a virtual community detection. It is based on the association between prime nodes in online social networks. Furthermore, they applied it to ranking algorithms discussed above. Table 2.7 below summarises the various real-world applications of the soft set and hybrid soft sets discussed above.

Table 2.7: The Summary of Real World Applications of Soft Set and Hybrid Soft Sets

Author	**Application domain**	**Methods**	**Results**
Herawan & Deris, (2011)	Association rules mining	The concept of co-occurrence of parameters in defining support and confidence of association rules	First attempt and workable. However, it cannot be applied in general transactional data.

Table 2.7:, Continued

Vo et al., (2016)	Association rules mining	Alternative methods which are not obtained by using methods for regular association rule mining	It can be used for mining maximal association rules in text data
Feng et al., (2016)	Association rules mining	Refined several existing concepts from (Herawan & Deris, 2011)to improve the generality and clarity of former definitions.	The approach can be used for mining association rules in transactional data
Chetia & Das, (2010)	Medical diagnosis	Presented a hybrid interval-valued fuzzy set and soft set.	Use Sanchez's approach for medical diagnosis in interval-valued fuzzy environment for analysing patients with fever and malaria.
Herawan, (2012b)	. Medical diagnosis	Soft set-based technique for medical decision making.	The technique can be used to reduce the number of dispensable symptoms and make an optimal decision.
Kumar et al., (2013)	Medical diagnosis	A novel approach based on bijective soft sets for the generation of classification rules from the data set.	The novel approach showed to be a valuable tool as compared the well-known decision tree classifier algorithm and Naïve Bayes.
Yuksel et al., (2013a)	Medical diagnosis	Fuzzy membership functions.	If the percentage is greater than 50%, then a biopsy is necessary.
Lashari & Ibrahim, (2013)	Medical diagnosis	Framework for medical image classification based on soft set.	Achieve higher performance comparing to baseline techniques.
Çelik & Yamak, (2013)	Medical diagnosis	A hybrid fuzzy soft set theory through well-known Sanchez's approach.	The hybrid model can be used to medical diagnosis.
Alcantud et al., (2015)	Medical diagnosis	A soft set-based decision making the procedure for medical diagnosis.	Uses an automated combination and analysis of information from structural and functional diagnostic techniques to obtain an enhanced Glaucoma detection.
Zou & Xiao, (2008)	Incomplete data analysis	Computing the weighted-average of all possible choice values of the object.	First attempt and workable for data filing of the incomplete soft set.

Table 2.7:, Continued

Qin, Ma, Herawan et al., (2012)	Incomplete data analysis	Association degree between the parameters.	Higher accuracy as compared to Zou & Xiao, 2008).
Khan et al., (2016)	Incomplete data analysis	The reliability of association among parameters in soft set	Higher accuracy as compared to Zou & Xiao, 2008).
Alcantud & Santos-García, (2016)	Incomplete data analysis	Presented two related techniques in analysing incomplete soft sets	New solutions for decision-making problems
Mushrif et al., (2006)	Data mining	Soft-set theory based texture classification algorithm	Better performance as compared to existing texture classification algorithms.
Xiao et al., (2009)	Data mining	Combined forecasting approach based on rough sets and each forecast.	The approach improves forecasting accuracy to combining forecasting rough set theory.
(Senan et al., 2010)	Data mining	Soft set-based feature selection method.	It generates the best feature set for Malaysian musical sound dataset.
Qin, Ma, Zain et al., (2012)	Data mining	A novel soft set approach in selecting clustering attribute.	Higher accuracy and less computation as compared to rough set-based approach.
Handaga et al., (2012)	Data mining	A hybrid fuzzy soft set theory for numerical data classification.	Better computational results.
Mamat et al., (2013)	Data mining	An alternative soft set approach in selecting clustering attribute.	Higher accuracy and lower computational time as compared to soft set-based approach (Senan et al., 2010)
Wang et al., (2013)	Data mining	A fuzzy soft set approach in adaptive Mobile Cloud Computing Middleware.	The results of the proposed approach were found to be efficient and fruitful.
Kumar et al., (2014)	Data mining	An improved of the bijective soft set.	Can be used to classify large gene expression data.
Ma & Qin, (2014)	Data mining	Normal parameter reduction and decision making of soft sets.	Experimental results demonstrate that this algorithm is feasible for dealing with the online shopping.

Table 2.7:, Continued

Li & Xu, (2015b)	Data mining	Integrate important indices information over different time.	The comparison of AIFSS, Degree and Closeness methods indicates that the accuracy of AIFSS is much better.
Herawan et al., (2015)	Data mining	The maximum degree of dominations in soft sets.	Can be applied for educational data mining.
Sutoyo et al., (2016)	Data mining	Multi-soft sets for conflict analysis.	Efficient in computing support, strength, certainty and coverage of conflict situation.
Khan et al., (2016)	Data mining	Soft set-based for incomplete data.	Improves the ranking algorithms in online social networks.

From Table 2.7 some successful implementation of real world problem using soft set were highlighted. Yet, the application of soft sets received little attention by researchers this study shows it was successfully implemented in different fields such as medical diagnosis, data mining, etc.

2.7 Uncertainty in Decision making

Increasing in data size lead to poor memory usage and reduce the effectiveness of processor utilisation. Therefore the reduction of data is necessary to increase the efficiency and performance of memory size and processor time (Padmanabhan et al., 2001). However, a careful handling of reduction is required to ensure the completeness of information does not lose and avoid ambiguity (Chen et al., 2009).

In managing uncertainty in decision-making, several issues have to put into consideration. Firstly, rich information with proper reduction process is crucial toward the success of buying the decision. (Chen et al., 2009). The risk of making a decision without adequate reduction and complete data boundary can lead to ambiguity in decision making (Jiang et al., 2009). Complete data boundary avoids either exceed boundary or incomplete boundary which result in data consistency. The data consistency is paramount in all organisation to avoid ambiguities that occur continuously and faced by everyone. (Feng et al., 2012) For example, in many cases, people ask the similar question with the same answer repeatedly all the time which is lead to inefficiency (Puccinelli et al., 2009). Uncertainties are involved in lots of real life problems, handling such data inconsistency become imperative for example in the field of medical, engineering, medical sciences, social, etc. (Maji et al., 2002). Secondly, in business, the performance will be based on lowest prices and best quality not only in domestic as well as around the globe to satisfy customer satisfaction. Thus, to satisfy different customers' needs, the product features ought to have the capacity to customised and reduced in light of their needs. By disposing of unnecessary features for a specific group of the customer can decrease the cost, consequently, increase customer satisfaction (Hoque et al., 2013). For instance, consider that Mr X needs to purchase a house with restricted decisions which under developments, what is the house that he or she ought to purchases? It is essential to give Mr X the privilege and adequate information about the important features and additional features to help him to make the right decision (Puccinelli et al., 2009). The decision cannot be based on passionate impact and bad quality promotions because of their low demand (Puccinelli et al., 2009). Thus, there was a wide range in the personal day by day life, where information processing is very vital to support them to make a decision at different levels to guarantee accuracy and consistency in basic decision making (Chen et al., 2009). In decision-making processing, there are two sorts of variables involved which is independent and dependent variables, where independent variables focus on external factors whereas the dependent variable focuses on main factors in looking for decision quality.

Moreover, information processing, accuracy, and data integrity are extremely imperative before processing can be possible. One of the major issues of accuracy and integrity of data is the eradication of redundancy data similarity. Considering the idea of 'Garbage in garbage out', data similarities in the satisfied boundary ought to be managed at the initial stage. Many techniques in managing the data similarities exist such as clusters or AND and OR operations can all be used to minimize the differences in soft data. For instance, to manage data similarity in cluster classified data, hierarchical method or partitioning methods can be utilized. However, to reduce time complexity, a hierarchical method can be more appropriate while if the volume of the data is large, the partitioning method could be more suitable (Herawan et al., 2014). To some extent, there is no need to use the aggregate data in decision making (Ibrahim and Yusuf, 2012). If much information governs the process of decision making, then issues of lengthy processing time and inefficiencies will occur. Thus, in information processing, it is very critical to process just sufficient data and deletes all unnecessary data to reduce processing time.

2.8 Other Parameter Reduction Techniques

Parameter reduction techniques or dimensionality reduction, in other words, allow the user to obtain exact data representations of a given dimensionality, improving the process of reducing parameters in decision making. Several techniques have been proposed for dimensionality reduction, its includes PCA, Multi-dimensional Scaling (MDS), swarm optimization method, Genetic algorithm, Isomap, t-Distributed Stochastic Neighbor Embedding (t-SNE), Laplacian Eigenmaps, Stochastic neighbor embedding, Hessian LLE, Sammon mapping, Local Tangent Space Analysis, multilayer autoencoders, Locally Linear Coordination, a manifold charting etc. All these techniques will be discussed in the following subsection:

2.8.1 Principal Component Analysis (PCA)

Principal components analysis (PCA) is another popular dimensionality reduction technique. The aim of PCA is to find linear subspace, for instance, a set of data on n dimensions is given; then the task is to find d such that d is lower than *n* such and the data points lie mainly on this linear subspace. In other words, the principal component analysis performs a linear mapping to reduce the dimension space in such a way that the variance of the reduce data is maximised. Practically, the covariance matrix of the data is created and compute the Eigenvectors of the matrix. The Eigenvectors that correspond to the largest eigenvalues can be utilised to reproduce a vast fraction of the variance of the original data. Besides, the first few Eigen vectors can frequently be interpreted as far as the scale of the physical behaviour of the system. The dimension of the point is decreased data loss. Thus, the significant variance is retained with a couple of Eigenvectors.

Mathematically, Principal component analysis (PCA) is used to solve the linear mapping N that maximise Cov(X). Moreover, the linear mapping is formed by the d that is, principal Eigen vectors (simply principal component) of the covariance of the zero mean data. Subsequently, principal component analysis tackles the Eigenvector. To solve the PCA, we can find the Cov(X) N = λN. The dimension d is solved for the principal Eigenvalues λ. (Maaten, Postma, & Herik, 2009).

2.8.2 Stochastic neighbour embedding

The Stochastic Neighbour Embedding (SNE) attempts to put the objects into a low-dimensional space to keep neighbourhood identity feasibly, and it can naturally be stretch put to allow multiple different low-d images of each object.

For every object, *i* and every potential neighbour, *j* we firstly calculate the asymmetric probability, *pij* that *i* would pick *j* as its neighbour:

$$p_{ij} = \frac{\exp(-d^2{}_{ij})}{\sum_{k \neq j} \exp(-d^2{}_{ik})}$$

The dissimilarities, $d^2{}_{ij}$, can be used to compute the dissimilarity that is, the two distance of high-dimensional points.

$$d^2{}_{ij} = \frac{\left\| x_i - x_j \right\|^2}{2\sigma_i^2}$$

Where σ_i can be found using binary search manually, and the value of σi that makes the entropy of the distribution equal to log k over the neighbours. In this case, k is the considered to be some local neighbours and is selected manually. In low-dimensional space Gaussian neighbourhoods were used with a fixed of the induced probability qij that point i picks point j as its neighbour is a function of the low-dimensional images x_i of all the objects and is given as:

$$q_{ij} = \frac{\exp(-\left\| x_i - x_j \right\|^2)}{\sum_{k \neq i} \exp(-\left\| x_i - x_j \right\|^2)}$$

To match the two distributions, it was embedded. It can be obtained by minimising the cost function i.e. The sum of Kullback-Leibler divergence between the original (p_{ij}) and induced (q_{ij}) distributions over neighbours for each object:

$$\mathrm{C} = \sum_i \sum_j p_{ij} \log \frac{p_{ij}}{q_{ij}} = \sum_i KL\,(P_{i \| Q_i})^1$$

The dimensionality of y space is picked manually. So that, the qij is large when pij is small the portion of probability mass in the q distribution is squandered, this implies that there would be a cost associated with the modelling of large distance in the high dimensional space together with the low dimensional space. Therefore, SNE can be considered as an improvement over methods such as LLE or SOM.

2.8.3 Isomap

Classical scaling always considered two data points as nearby points, and the distance is larger than the inter-point of the manifold. Tenenbaum et al., 2000 define Isomap as a technique that is used to resolve this issue by endeavouring to preserve pairwise geodesic distances between data points. Geodesic distance is said to be a distance measure between two points by manifold.

Classical scaling has been applied successfully in many applications, but one of its limitation is its inability to considered the distribution of neighbouring data points this because it always retains the pairwise Euclidean distances. The geodesic distances between the data point in Isomap are calculated by formulating graph G which each data point x_i is associated with nearest neighbours k, and x_{ij} $(j = 1, 2, . . . , k)$ in the dataset X (Tenenbaum et al., 2000). Dijkstra's or Floyd's algorithm can be used to calculate the shortest path between two points on the graph so as to find the geodesic distance. Once the geodesic distance between all the data point computed and the pairwise geodesic distance matrix are formed. At that point, the low dimensional space Y can likewise be processed by applying classical scaling on the subsequent pairwise geodesic distance matrix. Though, the major downsides of the Isomap algorithm are topological irregularity (Balasubramanian & Schwartz, 2002). Isomap may easily develop erroneous connections. Another disadvantage of Isomap is that the manifold can be affected with ''holes' (Lee & Verleysen, 2005). If the manifold is nonconvex, the Isomap may also fail (Tenenbaum, 1998).

2.8.4 t-Distributed Stochastic Neighbour Embedding (t-SNE)

The t-distributed stochastic neighbour embedding (t-SNE) is considered to be one of the machine learning algorithms that can be used for dimensionality reduction initiated by (Maaten & Hinton, 2008). It is more suitable for embedding high-dimensional data space to two or more dimensions for dimensionality reduction. The t-SNE algorithm has two principle stages. Firstly, t-SNE plans a probability distribution over sets of high-dimensional objects in a way that similar objects have a high probability of being chosen, while different points have a less probability of being chosen. Secondly, t-SNE characterise a similar probability distribution over the points in the low-dimensional space; the divergence has minimised the divergence by Kullback-Leibler between the two distributions. T-SNE has been widely applied in many applications such as biomedical signal processing, computer security research, music analysis, cancer research, etc.

2.8.5 Hessian LLE

Hessian LLE is an alternative of LLE that is used to minimise the ‘curviness’ of the manifold high dimensionality so as to embed it in a low-dimensional space, and the low-dimensional data representation is locally isometric (Donoho & Grimes, 2003). It can use Eigen analysis of a matrix which defined the curviness of manifold around the data points. To measure the curviness of the manifold at every point usually Local Hessian is used, and local tangent space can represent it. It can be demonstrated that the coordinates of the low-dimensional representation can be realised using Eigen analysis of an estimator H. The Hessian LLE begins by distinguishing the k closest neighbours for every data point xi. It is always assuming the local linearity of the manifold. Thus, PCA is used to find the basis for local tangent space on its k nearest neighbours’ x_{ij}.

Hessian Hi estimation is always given as the transpose d (d+1) of the last. The information of curviness of the high dimension data manifold is represented by matrix H.

to solve the low-dimensional that minimise the curviness an Eigen analysis is used. Hessian LLE is successfully applied in sensor localisation by (Patwari & Hero, 2004).

2.8.6 Sammon mapping

Sammon mapping adjusts the traditional scaling cost work by weighting the commitment of the pair *(i, j)* to the cost function in the opposite of their pairwise remove in the high dimensional space d_{ij}. Along these lines, the cost work appoints parallel weight to hold each of the pairwise distance, and in this way, the local data structure is held superior to anything. The function of Sammon cost is given as

$$\varphi(x) = \frac{1}{\sum_{ij} d_{ij}} \sum_{i \neq j} \frac{\left(d_{ij-} \left\| x_{i-x_j} \right\|^2\right)}{d_{ij}}$$

Where d_{ij} represent the Euclidean distance between the high-dimensional data points x_i and x_j, and to simplify the gradient of cost function a constant is added in front. The Sammon cost function minimization is performed using a pseudo-Newton method (Cox & Cox, 2000). It has been shown that Sammon mapping is usually used for visualisation purposes (Martin-Merino & Munoz, 2004). The major drawback of Sammon mapping is that it assigned a higher weight to keep a distance of say, 10^{-5} than to keep a distance of, say, 10^{-4} Sammon mapping has been applied gene data and geospatial data successfully.

2.8.7 Multilayer autoencoders

The techniques of Multilayer autoencoders uses neural systems that have an odd number of hidden layers, and then the weights between the top and base layers are shared. The output layer is denoted with D nodes, and the middle hidden layer was denoted with d nodes respectively. Additionally, the network minimises the mean squared error between input and output of the network despite the fact that the input and output are the same. Preparing the neural network on the data points xi leads toward to a network in which the middle hidden layer gives a d-dimensional structure of the data points that keeps as much structure in the dataset X as possible. The low-dimensional representations y_i can be gotten by removing the node values in the middle hidden layer when data point x_i is utilised as input. Considering the nonlinear mapping between low and high dimensional data the used of Sigmoid is considered to be essential. One of the shortcomings of multilayer auto-encoder is that backpropagation techniques converge steadily and are most likely going to slow down in local minima (Hinton, Osindero, & Teh, 2006). To overcome this limitation, the use of learning procedure was employed, and it has been categorised into three stages. Firstly, the network layers that is the layers from X to Y are recognised one-by-one using Restricted Boltzmann Machines (RBMs) (Welling, Rosen-Zvi, & Hinton, 2004). Using unsupervised learning methods, the RBMs can be prepared effectively to minimise contrastive divergence. Secondly, layers from Y to X of the network are unrolled. Thirdly, used of backpropagation to fine tune in a supervised way was adopts. The limitation of partially addressed recently in deep learning. It was applied successfully in HIV analysis and data imputation.

2.8.8 Locally linear Coordination

Locally Linear Coordination is used to locate various locally linear models and execute the global alignment of all the linear models (Roweis, 2002). There are two steps involves in this process, firstly is computing the mixture of local linear models on the data using EM-algorithm and secondly, is calculating a fusion of local linear models by means of an EM-algorithm on the data and applying a variant of LLE to modify the local linear models so as to have low dimensionality data representation.

The mixture of local linear models can be used to develop *n* data representations of z_{ij} and their related responsibilities. These responsibilities r_{ij} depict to what degree data point x_i can be compared regarding the model. The responsibility-weighted data representations

$u_{ij} = r_{ij}z_{ij}$ is computed using local models and the corresponding responsibilities. LLC adjusts the local models by solving the generalised Eigenproblem

$$Xv = \lambda Yv,$$

The application of LLC in face images expression and handwritten digits demonstrate its effectiveness (Roweis, 2002).

2.8.9 Manifold charting

Manifold charting is practically similar to LLC it develops a representation of low-dimensional data by harmonising the MoFA or MoPPCA model. The only difference between Manifold with LLC is that in Manifold charting the cost function that corresponds with other techniques of dimensionality reduction is not minimise like LLE cost function. (Brand, 2003). The convex cost function is minimise using Manifold charting that shows the estimated amount of disagreement between the linear models of the data points. It minimises using an Eigen problem by firstly the EM algorithm is applied to compute the mixture of factor analysers.

$$\varphi(Y) = \sum_{i=1}^{n}\sum_{j=1}^{m} r_{ij}\left\|y_i - y_j\right\|^2$$

Whenever there are two models the perception behind the cost function is that the data points of the two models most agree on the final coordinate. The cost function is

$$\varphi(Y) = \sum_{i=1}^{n}\sum_{j=1}^{m}\sum_{k=1}^{m} r_{ij}r_{ik}\|y_i - y_{ik}\|^2$$

The cost function can be redeveloped in the form of a Rayleigh quotient. The Rayleigh quotient can be developed by the definition of a block-diagonal matrix B with m blocks by

$$B = \begin{pmatrix} B_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & B_m \end{pmatrix}$$

Where B_j is the weighted sum of covariance z_{ij}.

Thus,

$$B_j = \sum_{i=1}^{n} r_{ij}\, cov\,([z_j \quad 1])$$

2.8.10 Diffusion map

The framework of diffusion maps (DM) is originated from the dynamical systems field. Diffusion mapping can be performed based on Markov random on the graph. The random walk for various time steps, a measure of the proximity of the data points is gotten (Coifman & Lafon, 2006). Given this, the diffusion distance is defined. Low-dimensional representation always retained the pairwise diffusion. The key idea is to integrate all the paths through the graph. In the framework of the diffusion maps, a graph of the data is always constructed first and the uses Gaussian kernel function to compute the weight of the edges in the graph. Which can give matrix W.

Table 2.8: Properties of techniques for dimensionality/parameter reduction.

Dimensionality Reduction Techniques	Parametric	Parameters	Computation
Sammon mapping	No	*none*	$O(ik^2)$
Classical Scaling	No	*none*	$O(p^3)$
Isomap	No	*p*	$O(p^3)$
Autoencoders	Yes	*net size*	$O(inw)$
LLE	No	*l*	$O(pq^2)$
Laplacian Eigenmaps	No	*l, σ*	$O(pm^2)$

Table 2.8:, Continued

PCA	Yes	*none*	$O(p^3)$
Diffusion maps	No	σ, s	$O(p^3)$
LLC	Yes	n, l	$O(imd^3)$
Manifold charting	Yes	n	$O(pnd)$
PR	Yes	n	$O(p^4)$
NPR	Yes	pq	$O(p^3)$

From these different dimensionality reduction algorithms, it shows that all these techniques can only be used depending on the type of data set and task one has to solve. While some techniques require real numbers data set others can only use binary data set. In this study, Normal parameter Reduction algorithm was studied and improved using binary data set.

2.9 Summary and Conclusion of the Chapter

A soft set is an alternative tool for handling uncertainty and vagueness problems. Its effectiveness in dealing with uncertainties problems is as a result of its parameterized tools. In this study, we have presented and clarified the numerous works in this area within a short time, and several algorithms exist for parameter reduction, decision making, applied research of soft set and hybrid soft set with other set theories. We have reviewed different research on parameter reduction and decision making in soft set theory and hybrid soft set with other set theories. Several researchers have contributed different type of algorithm for computing parameter reduction by considering different cases like inconsistency, missing attribute value of decision making and information system. Researchers can use this review to quickly identify areas that require improvement so as to proposed novel approach. With the current advances in soft set theory, some research issue is arising which requires attention from researchers this includes multiset in parameter reductions, multi-criteria decision-making problem in an uncertain environment. Moreover, it is clear that all the approaches presented above so far in this area have their respective advantages and disadvantages. This required attention from researchers to develop a more general approach for parameter reduction of the soft set.

Additionally, other techniques for dimensionality reduction were also discussed. Only few research work on extending ontology based soft set were found in the literature. A very limited practical application of soft set theory can be found in existing literature and many of them still facing the problem of the performance of their models or algorithms. Thus, these applied soft set-based researchers also require more attention to future *soft setters*. The next chapter discussed the general methodology for the development of proposed algorithm.

CHAPTER 3: METHODOLOGY

3.1 Introduction

In this chapter, the general methodology for the elaboration of an Alternative Approach to Normal Parameter Reduction algorithm for soft sets (ANPR) will be discussed. In the previous chapter (Chapter 2), the various techniques that are used for parameter reduction of soft sets and hybrid soft set with other set theories were reviewed. The literature review depicts the success in a different type of algorithms for computing parameter reduction by considering different cases like inconsistency, missing attribute value of decision making and information system. Hence, this work only focuses on parameter reduction and application of soft set.

3.2 Parameter reduction techniques

This technique proposed a parameter reduction algorithm to circumvent the unnecessary or repeated object in reduction based on minimum weight satisfaction. We focus on true objects in parameter reduction, but false object reduction also will be maintained in the decision-making process. The reason behind reduction is to improve the precision of information reasoning without losing the original characteristic. From literature, the limitation of soft set parameter reduction encourages us to develop a new model in filtering this soft noise then transforms it into useful knowledge. In the new model, there are two components. Firstly, is to ensure that the knowledge is complete. Secondly, is to maintain the basic original features to have knowledge value, to solve poor decisions problem. The parameter reduction is achieved when the reduction techniques operate on the high-quality soft sets.

3.3 System Requirement

The alternative approach of normal parameter reduction algorithm is implemented in Matlab programming version R2011b on an Intel® Core™ 2 Duo with the main memory of 4 gigabytes, and the operating system is window 64-bit operating system.

3.4 Methodology

The general methodology that is used in this study is divided into stages as shown in Figure 3.1 that consist of the Review of state of the art to the formulation of research objectives and then the design of ANPR algorithm.

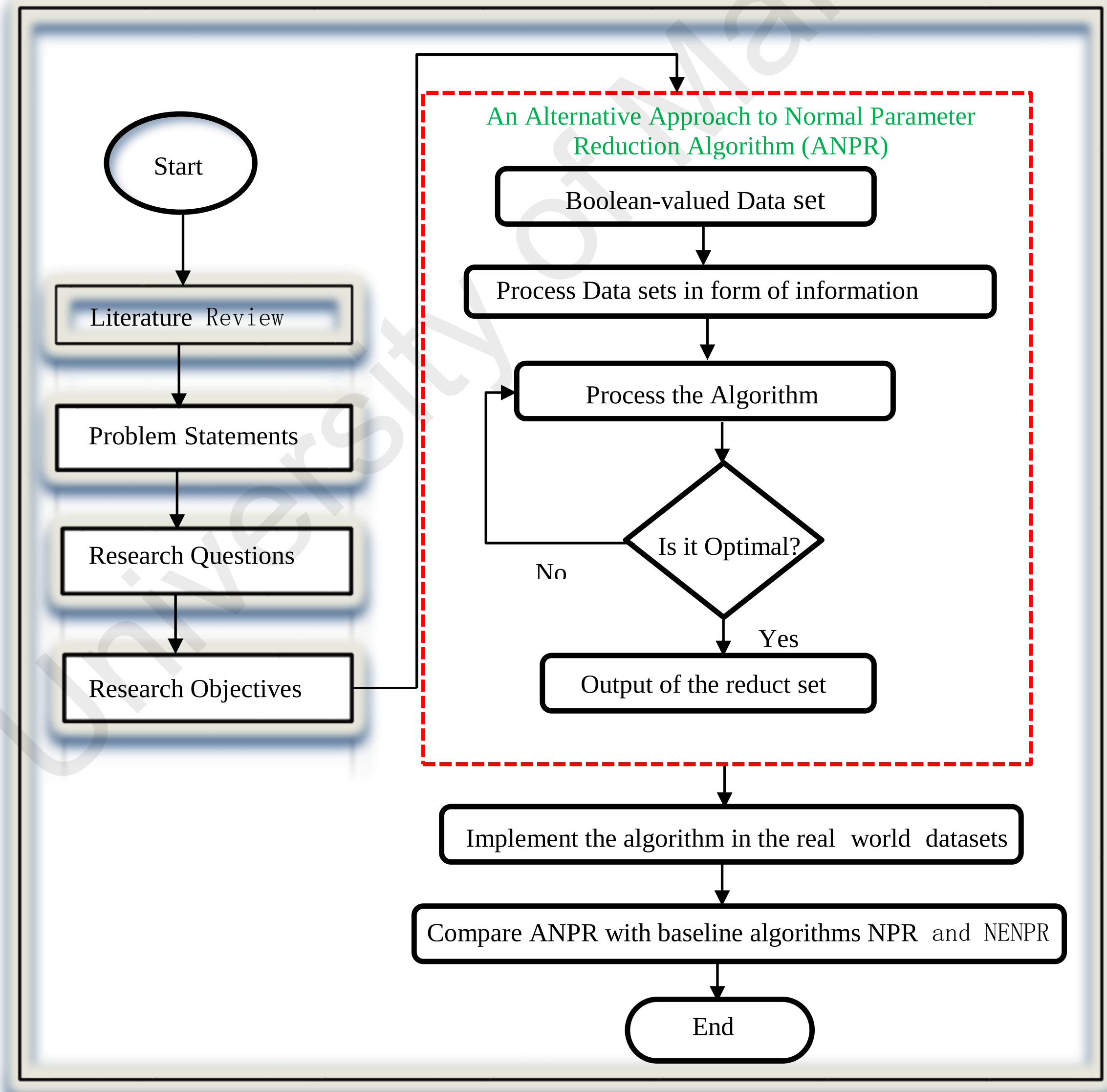

Figure 3.1: General research methodology

3.4.1 Input of Boolean Dataset

In the input stage, only the Boolean data set are required in this stages as an input for parameter reduction. This research study requires only Boolean data set which can be processed by the algorithm.

3.4.2 Processing Stage

Processing the input data by the algorithm to determine the reduct subset, it starts by checking the existence of equal parameters values in a column in the set and the entries that all have zeros or ones. Then the algorithm finds if there exist subsets *A* and *B* in the feasible parameter reduction and it then computes the intersection of A and B and put it into the reduct set. The aim is to reduce the parameters and in the meantime enhance the performance of speed and accuracy of the decision-making process. Figure 3.2 shows the flow chart for the proposed algorithms in which the initial data were to process and checked for its feasibility; the reduction algorithm was applied to reduce the noise that is unnecessary parameters and produces the outcomes for decision making.

3.4.3 Reduced sets

To handle data adequately, the used of dimensionality reduction techniques became more important; dimensionality reduction can simply be used to find the minimum numbers of the significant parameters. The objective is to identify the false frequent data which affects the reduction process negatively and what is their types, really we focus on three factors which are cost of choice, inconsistency and inheritances of decision characteristics

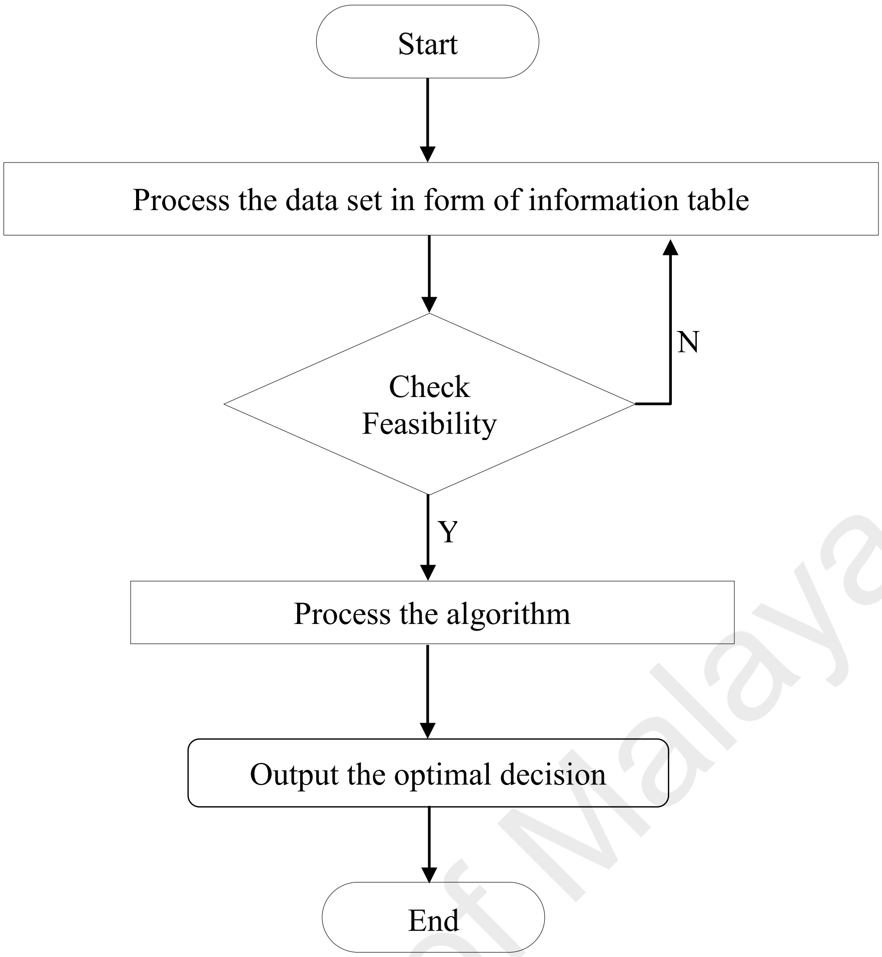

Figure 3.2: Flow chart for ANPR algorithm

3.5 Evaluation

The evaluation of the algorithm for identifying optimal subset for decision making is a critical issue. Performance appraisal allows comparing different algorithms and also yields constructive feedback to improve these algorithms further. Performance evaluation is usually the foremost step once a new technique is developed. In many cases, benchmark datasets or models are used to evaluate such techniques on a common ground so they can be fairly compared. One method to evaluate the performance and compare it with other algorithms is to run them on randomly generated inputs from a distribution (based on actual inputs or a theoretical model of inputs) and measure the average running time of the algorithms. However, in this study, we employed the used of Big O notation to evaluate the performance and complexity of the proposed algorithm.

3.5.1 Big O notation

Big O notation is employed to sketch the performance or complexity of an algorithm. Big O particularly explains the worst-case scenario, and can also be used to determine the execution time required or the space used by an algorithm. Big O notation can also be used to classify algorithms according to how their running time or space requirements increase as the input size also increase (Danziger 2015). The characteristics functions of big O depend according to their growth rates. Different functions with the same growth rate may be represented using the same O notation. The used of letter O is because of the growth rate of a function, and it is also referred as order function. Several notations are associated with big O using the symbols such as o, Ω, ω, and Θ, to explain other kinds of bounds on asymptotic growth rates. It can also be used in numerous field to provide estimates. In this study, the big O is employed to provide the mathematical estimation of the complexity of the proposed algorithms with the baseline algorithms.

3.6 Summary of the chapter

This chapter described the general methodology which is used in the design and implementation of an alternative approach to normal parameter reduction algorithm. Moreover, the detail contributions and the real life application of the algorithm were also explained in some detail in the subsequent chapters (chapter 4 and 5) respectively.

CHAPTER 4: ALTERNATIVE APPROACH TO NORMAL PARAMETER REDUCTION ALGORITHM

4.1 Introduction

An alternative approach to normal parameter reduction method based on soft set theory to improve computational complexity and reduce running time. The summary of the contributions of this work are:

a. An alternative algorithm based on a soft set theory for normal parameter reduction is proposed that can improve computational complexity and run time.
b. The proposed algorithm is expected to perform better than the state of the art normal parameter reduction algorithms.
c. The proposed algorithm will be applied to real-world data set for decision-making.

4.2 Normal Parameter Reduction Algorithm

The idea of normal parameter reduction and decision making of the soft set was proposed by Kong et al., (2008). The main objectives of normal parameter reduction are to provide consistency in selecting optimal and suboptimal decision of objects. The following definition presents the notion of indiscernibility relation generated by a subset of *E*. which describe the number the partition and rank of the object in the universe.

Suppose $U = \{p_1, p_2, \cdots, p_n\}$ and $E = \{e_1, e_2, \cdots, e_m\}$, where (F, E) is a soft set with tabular representation. We define $f_E(p_i) = \sum_j p_{ij}$, where p_{ij} and entries in the table of (F, E).

Definition 11 (See (Kong et al., 2008)). *Let given a soft set* (F, E) *with a universe U. With every subset of parameters* $B \subseteq E$, *an indiscernibility relation* $IND(B)$ *is defined by*

$$IND(B) = \{(p_i, p_j) \in U \times U : f_B(p_i) = f_B(p_j)\}.$$

For a soft set (F,E) and $U=\{p_1,p_2,\cdots,p_n\}$. The following

$$C_E=\{\{p_1,p_2,\cdots,p_i\}f_1,\{p_{i+1},p_{i+2},\cdots,p_j\}f_2,\cdots\{p_k,\cdots,p_n\}f_s\}$$

Is a decision partition of the object in U, which partitioned and ranks the objects according to the value of $f_E(.)$ based on indiscernibility relation. For subclass

$\{p_v,p_{v+1},\cdots,p_{v+w}\}f_i$, $f_E(p_v)=f_E(p_{V+1})=\cdots=f_E(p_{v+w})=f_i$, and $f_1\geq f_2\geq\cdots\geq f_s$

is the number of subclasses or simply objects in U are classified and ranked according to the value of $f_E(.)$ based on indiscernibility relation.

Based on Definition 11, the following Definition 12 of dispensable parameters is presented.

Definition 12 (See (Kong et al., 2008)). *For a soft set* (F,E) *and* $E=\{e_1,e_2,\cdots,e_m\}$. *If there exists a subset* $E=\{e'_1,e'_2,\cdots,e'_p\}\subset C$ *satisfying* $f_A(p_1)=f_A(p_2)=\cdots=f_A(p_n)$, *then A is dispensable. Otherwise, A is indispensable.*

The subset $B\subset C$ is a *normal parameter reduction* of E if B is indispensable and $f_{E-B}(p_1)=f_{E-B}(p_2)=\cdots=f_{E-B}(p_n)$. This is to say that $E-B$ is the maximal subset of E, which the value of $f_{E-B}(.)$ keeps constant.

Before obtaining normal parameter reduction, the following definition was used to describe the notion of decision partition of deleted e_i.

Definition 13 (See (Kong et al., 2008)). *For a soft set* (F,E) *with parameter set* $E=\{e_1,e_2,\cdots,e_m\}$, $A=\{e'_1,e'_2,\cdots,e'_p\}\subset E$ *and an object set* $U=\{p_1,p_2,\cdots,p_n\}$. *The decision partition and decision deleted* e_i *are defined as*

$C_E=\{E_{f_1},E_{f_2},\cdots,E_{f_s}\}$ *and* $C_E=\{\overline{E-e_i f}_{1'},\overline{E-e_i f}_{2'},\cdots,\overline{E-e_i f}_{s'}\}$, *respectively.*

The importance degree of e_i for the decision partition is defined by

$$r e_i = \frac{1}{|U|}\left(\propto_1, e_i + \propto_2, e_i, + \cdots + \propto_s, e_i\right),$$

where $|.|$ denotes the cardinality of set and

$$\propto_k, e_i = \begin{Bmatrix} |Ef_K - E - e_i f_{2'}|, \text{ there exists } z': f_k = f_{z'} \; 1 \le z' \le s', 1 \le k \le s \\ |Ef_k|, \qquad \text{otherwise} \end{Bmatrix}$$

For a clear description of Definitions 11-13, from Table 4.1 we can compute the decision partition and parameter importance degree as:

Table 4.1: A Soft Set Table (F, E)

U/E	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	$f(.)$
p_1	1	0	1	1	1	1	0	1	6
p_2	0	0	1	1	0	0	1	1	4
p_3	1	0	1	0	1	1	1	0	5
p_4	1	0	1	1	1	1	0	1	6
p_5	1	0	1	1	1	1	0	1	6
p_6	0	0	1	0	0	1	1	1	4

Example 2. Let a soft set (F, E) with Boolean representation as displayed in Table 4.1, $U = \{p_1, p_2, p_3, p_4, p_5, p_6\}$ and $E = \{e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8\}$.

The decision partitions based on Definition 5 are:

$$C_E = \{\{p_1, p_4, p_5\}_6, \{p_3\}_5, \{p_2, p_6\}_4\}.$$

Therefore, $s = 3$. Thus, we have the following sets:

$$C_E - e_1 = \{\{p_1, p_4, p_5\}_5, \{p_2, p_3, p_6\}_4\},$$

$$C_E - e_2 = C_E,$$

$$C_E - e_3 = \{\{p_1, p_4, p_5\}_5, \{p_3\}_4, \{p_2, p_6\}_3\},$$

$$C_E - e_4 = \{\{p_1, p_3, p_4, p_5\}_5, \{p_6\}_4, \{p_2\}_3\},$$

$$C_E - e_5 = \{\{p_1, p_4, p_5\}_5, \{p_2, p_3, p_6\}_4\},$$

$$C_E - e_6 = \{\{p_1, p_4, p_5\}_5, \{p_2, p_3\}_4, \{p_6\}_3\},$$

$$C_E - e_7 = \{\{p_1, p_4, p_5\}_6, \{p_3\}_4, \{p_2, p_6\}_3\},$$

$$C_E - e_8 = \{\{p_1, p_3, p_4, p_5\}_5, \{p_2, p_6\}_3\},$$

Now, we show how to compute the parameter important degree of Example 2 based on Table 4.1:

We have:

$$r\,e_i = \frac{1}{|U|}(\propto_1, e_i + \propto_2, e_i, + \cdots + \propto_s, e_i)$$

$$re_1 = \frac{1}{6} \times (3+1+0) = \frac{2}{3}; \qquad \mathrm{re}_2 = \frac{1}{6} \times (0+0+0) = 0;$$

$$\mathrm{re}_3 = \frac{1}{6} \times (3+1+2) = 1; \ \mathrm{re}_4 = \frac{1}{6} \times (3+0+1) = \frac{2}{3};$$

$$\mathrm{re}_5 = \frac{1}{6} \times (3+1+0) = \frac{2}{3}; \ \mathrm{re}_6 = \frac{1}{6} \times (3+1+1) = \frac{5}{6};$$

$$\mathrm{re}_7 = \frac{1}{6} \times (0+1+2) = \frac{1}{2};$$

$$\mathrm{re}_8 = \frac{1}{6} \times (3+0+2) = \frac{5}{6}$$

From Table 4.1, based on the above computation of parameter importance degree *A* could be any of the followings

$A = \{e_2, e_3, e_4, e_5, e_6, e_8\}$ or $A = \{e_1, e_3, e_4, e_5\}$, $A = \{e_1, e_2, e_3, e_4, e_5\}$, and $A = \{e_1, e_7, e_8\}$

Because they satisfied the Theorem 1 i.e.

$re_2 + re_3 + re_4 + re_5 + re_6 + re_8 = 4$; $re_1 + re_2 + re_3 + re_4 + re_5 = 3$. $re_1 + re_7 + re_8 = 2$.

Now check for A that satisfy $f_A(p_1) = f_A(p_2) = \cdots = f_A(p_n)$, we have $A = \{e_1, e_7, e_8\}$. Therefore, the normal parameter reduction of Table 4.1 is $\{e_4, e_5, e_6\}$ as shown in Table 4.2. However, this parameter importance degree (re_i) is the major computation that characterised the normal parameter reduction algorithm as presented by of Flow chart for ANPR algorithm in Figure 4.1.

Algorithm: Normal Parameter Reduction **Input:** Input soft set (F, E) and its parameters E **Output:** Normal Parameter Reduction of (F, E)
1. Compute the parameter importance degree re_i $(1 \le i \le m)$; 2. Select the maximal subset $A = \{e'_1, e'_2, \cdots, e'_p\}$ in E which satisfying that sum of $re_i = (1 \le i \le p)$ is non-negative integer, then put the A into a feasible parameter reduction set; 3. Check A, if $f_A(p_1) = f_A(p_2) = \cdots = f_A(p_n)$, then $E - A$ is the normal parameter reduction, and A is saved in the feasible parameter reduction set. Otherwise, A is deleted from the feasible parameter reduction set. 4. Find the maximum cardinality of A in feasible parameter reduction set. 5. Compute $E - A$ as the optimal normal parameter reduction

Figure 4.1: Normal Parameter Reduction algorithm in (Kong et al. 2008)

Figure 4.1 above shows the normal parameter reduction algorithm which described the steps involved. The input is soft set (F, E) while the output after processing the algorithm is the reduct sets. Firstly, the algorithm will compute the parameter importance degree re_i. Then, select the subset A of E in which the sum of it degree is a non-negative integer and then put A into feasible parameter reduction set. Lastly, the algorithm computes the maximum cardinality i.e. $E - A$ which is the normal parameter reduction.

Table 4.2: Normal parameter reduction from (F, E) in Table 4.1

U/E	e_4	e_5	e_6	$f(.)$
p_1	1	1	1	3
p_2	1	0	0	1
p_3	0	1	1	2
p_4	1	1	1	3
p_5	1	1	1	3
p_6	0	0	1	1

It simply shows that the normal parameter reduction in Table 4.2 has the optimal object is $\{p_1, p_4, p_5\}$. The objects p_3 are the suboptimal choices and p_2 or p_6 are the last choices.

Remark 1:

The normal parameter reduction keeps the classification power after reduction from the original data and maintains invariant final choice object rank in decision making. However, this requires much computation, and it is hard to understand due to its nature of its complexity as such (Ma et al., 2011) improved the algorithm with new efficient normal parameter reduction algorithm to reduce the level of computation complexity and make it easy to understand.

4.3 New Efficient Normal Parameter Reduction

The new efficient normal parameter reduction algorithm was presented by (Ma et al., 2011). The main objective of the algorithm is to improve the computational complexity of the Normal Parameter reduction algorithm (NPR) by (Kong et al., 2008).

In this section, we briefly analyse the new efficient normal parameter reduction and Figure 2 present the steps of the algorithm. The following Definitions 14, 15, and 16 present the notion of object-oriented sum, oriented parameter sum and the overall sum of the entries of the soft set (F, E), respectively.

Definition 14 (See (Ma et al., 2011)). *For soft sets* (F,E), $U=\{p_1,p_2,\cdots,p_n\}$, $E=\{e_1,e_2,\cdots,e_m\}$, *we denote* $f(p_i)=\sum_j p_{ij}$ *as an oriented-object sum.*

Definition 15 (See (Ma et al., 2011)). *For soft sets* (F,E), $U=\{p_1,p_2,\cdots,p_n\}$, $E=\{e_1,e_2,\cdots,e_m\}$, *we denote* $S(e_j)=\sum_i p_{ij}$ *as an oriented-parameter sum.*

Definition 16 (See (Ma et al., 2011)). *For soft sets* (F,E), $U=\{p_1,p_2,\cdots,p_n\}$, $E=\{e_1,e_2,\cdots,e_m\}$, *we denote* $S_A=\sum_j S(e_j)$ *as the overall sum of A.*

Based on Definitions 14, 15 and 16, Definitions 17 and 18 are presented to check the parameters that have same entries, and it will be kept as reduce set.

Definition 17 (See (Ma et al., 2011)). *For soft sets* (F,E), $U=\{p_1,p_2,\cdots,p_n\}$, $E=\{e_1,e_2,\cdots,e_m\}$, *for* $e_j \in E$, *if* $p_{1j}=p_{2j}=\cdots=p_{nj}=1$, *we denote* e_j *as* e_j^1.

Definition 18 (See (Ma et al., 2011)). *For soft sets* (F,E), $U=\{p_1,p_2,\cdots,p_n\}$, $E=\{e_1,e_2,\cdots,e_m\}$, *for* $e_j \in E$, *if* $p_{1j}=p_{2j}=\cdots=p_{nj}=0$, *we denote* e_j *as* e_j^0.

For a clear description of the above definitions, we now illustrate the new efficient normal parameter reduction algorithm (NENPR) based on Table 4.1.

Example 3

Suppose a soft set (F,E) with Boolean representation as shown in Table 4.1. Let given $U=\{p_1,p_2,p_3,p_4,p_5,p_6\}$ and $E=\{e_1,e_2,e_3,e_4,e_5,e_6,e_7,e_8\}$.

Step 1: Based on the Definitions 9 and 10, there exists e_j^0 and e_j^1 which are e_2^0 and e_3^1 from Table 1, so put them into reduced parameter set denoted by *C*. Now, we may have new soft set (F,E') as shown in Table 4.3.

Table 4.3: Soft set (F, E')

U/E	e_1	e_4	e_5	e_6	e_7	e_8	$f(.)$
p_1	1	1	1	1	0	1	5
p_2	0	1	0	0	1	1	3
p_3	1	0	1	1	1	0	4
p_4	1	1	1	1	0	1	5
p_5	1	1	1	1	0	1	5
p_6	0	0	0	1	1	1	3
							25

Step 2: Compute the oriented-parameter sum $S(e'_j)$ of e'_j as shown in Table 4.3.

Step 3: find the subset $A \subset E'$ in which S_A is a multiple of $|U| = 6$. This gives so many subsets among which are: $\{e_1, e_2, e_3, e_4, e_5\}$, $\{e_1, e_3, e_4, e_5\}$, $\{e_4, e_5, e_6, e_8,\}$ etc. and put them in candidate parameter reduction set.

Step 4: Now filter the candidate parameter set satisfying $f_A(p_1) = f_A(p_2) = \cdots = f_A(p_n)$ and delete the reminders. In this case, we have $\{e_5, e_7, e_8\}$

Step 5: Finally finding the maximum cardinality of A in the candidate parameter reduction set in which we have $E - A - C = \{e_1,\ e_4,\ e_6\}$ which could be considered as efficient normal parameter reduction as shown in Table 4.4.

Table 4.4: New efficient normal parameter reduction from (F, E) in Table 4.1

U/E	e_1	e_4	e_6	$f(.)$
p_1	1	1	1	3
p_2	0	1	0	1
p_3	1	0	1	2
p_4	1	1	1	3
p_5	1	1	1	3
p_6	0	0	1	1

Algorithm: New efficient Normal Parameter Reduction algorithm **Input:** Input soft set (F,E) and its parameters E **Output:** Normal Parameter Reduction of (F,E)
1. If there exist e_j^0 and e_j^1, they would be put into the reduced parameter set denoted by C and a new soft set (F, E') will be established without e_j^0 and e_j^1 where $U=\{p_1,p_2,\cdots,p_n\}, E'=\{e_1,e_2,\cdots,e_m\}$; 2. For the soft set (F,E'), calculate $S(e_{j'})$ of $e_{j'}$ (that is oriented-parameter sum) for $j'=1',2',\cdots,t'$; 3. Find the subset $A\subset E'$ in which S_A is a multiple of $\|U\|$, then put A into a candidate parameter reduction set; 4. Check every A in the candidate parameter reduction set, if $f_A(p_1)=f_A(p_2)=\cdots=f_A(p_n)$, it will be kept; otherwise it will be omitted; 5. Find the maximum cardinality of A in the candidate parameter reduction set, then $E-A-C$ *as the optimal normal parameter reduction.*

Figure 4.2: New efficient normal parameter reduction algorithm

Figure 4.2 above presents the step by step method of new efficient normal parameter reduction algorithm and the major task in this algorithm are checking for the existing of unique entries, computing oriented parameter sum and then finding the subset which is multiple of the universe set. Moreover, finally, determine the maximum cardinality of A in the candidate parameter set and considered $E-A-C$ as the optimal parameter reduction.

Remark 2:

The NENPR algorithm has some setback this includes:

(a) The algorithm does not consider the general purpose of the data set in trying to reduce the noise, for instance, the issue $e_i = e_j$ which should be considered as repetition.

(b) The algorithms only focused on reducing the amount of computational complexity of normal parameter reduction algorithm without considering the generality of running the algorithm in a different dataset.

4.4 The Proposed Algorithm of ANPR

In this section, we present our proposed algorithm of alternative approach normal parameter reduction (ANPR).

4.4.1 Proposed Technique

Suppose a soft set (F,E) with tabular representation $U=\{p_1,p_2,\cdots,p_n\}$ is the object set, $E=\{e_1,e_2,\cdots,e_m\}$ is the parameter set, p_{ij} are the entries in the table of (F,E). The following Definition checks for the equal parameters among E so as to put kept it into reduct parameter set.

Definition 19. *For a soft set* (F,E), $U=\{p_1,p_2,\cdots,p_n\}$ *and* $E=\{e_1,e_2,\cdots,e_m\}$. *If there exist* $e_i = e_j$ *and* $p_{1j}=p_{2j}=\cdots=p_{nj}$, *then* e_i *or* e_j *is a special entry denoted as Q.*

The following Definition 12 defines the dialectic subset.

Definition 20. *For a soft set* (F,E), $U=\{p_1,p_2,\cdots,p_n\}$ *and* $E=\{e_1,e_2,\cdots,e_m\}$. *A dialectic subset are* $A,B\subset E$ *such that if* $A=\{e_1',e_2',\cdots,e_h'\}$ *and* $B=\{e_1'',e_2'',\cdots,e_q''\}$ *satisfying*

$$f_A(p_1)=f_A(p_2)=\cdots=f_A(p_n) \text{ and } f_B(p_1)=f_B(p_2)=\cdots=f_B(p_n).$$

The following Definition defines the intersection of the dialectic subset *A* and *B*.

Definition 21. *For a soft set* (F,E), $U=\{p_1,p_2,\cdots,p_n\}$ *and* $E=\{e_1,e_2,\cdots,e_m\}$. *If there exist a dialectic subset A and B, then* $(A\cup B)-(A\cap B)=(F,E'')$.

From Definitions 19-21, the proposed algorithm steps are shown in Figure 4.3. Figure 4.3 shows the alternative approach to normal parameter reduction algorithm. The input to the algorithm is the given soft set (F,E) in tabular representation, and the output is the normal parameter reduction algorithm of the given soft sets. It starts by checking the existence of equal parameters values in a column in the set and the entries that have all have zeros or ones. Then the algorithm computes the oriented parameter sum and finds if there exist a subsets *A* and *B* in the feasible parameter reduction as shown in steps 2 and 3 of Figure 4.3. Also, the algorithm computes the intersection of the subsets *A* and *B* which is the reduce set. Finally, the algorithm will establish the maximum cardinality of the candidate parameter reduction set $E - C - D - Q$ as the normal parameter reduction.

Algorithm: ANPR

Input: Input soft set (F, E) and its parameters *E*

Output: Normal Parameter Reduction of (F, E)

1. Check if there exist $e_i = e_j$ choose one of them and put it in *Q*, if there exist e_j^1 *and* e_j^0 keep into reduced parameter denoted as *C*. and a new soft set (F, E') will be established without one of the $e_i = e_j$, e_j^0 and e_j^1 where $U = \{p_1, p_2, \cdots, p_n\}$, $E' = \{e_1, e_2, \cdots, e_m\}$;
2. For the soft set (F, E'), calculate $S(e_{j'})$ of $e_{j'}$ (that is oriented-parameter sum), for $j' = 1', 2', \cdots, t'$;
3. Check every *A* in the candidate parameter reduction set, if $f_A(p_1) = f_A(p_2) = \cdots = f_A(p_n)$, and also if there exists B in the candidate parameter reduction set such that $f_B(p_1) = f_B(p_2) = \cdots = f_B(p_n)$.
4. Compute the $(A \cup B) - (A \cap B)$ put the intersections into reduced parameter sets denoted by *D*;
5. Find the maximum cardinality of candidate parameter reduction set, then $E - C - D - Q$ *as the alternative normal parameter reduction.*

Figure 4.3: Alternative approach to normal parameter reduction algorithm

To have a clear understanding of the algorithm above, we give an example from the Boolean dataset in Table 4.1.

Example 4

Suppose a soft set (F, E) with tabular representation displayed in Table 4.1. Let $U = \{p_1, p_2, \cdots, p_n\}$ and $E = \{e_1, e_2, \cdots, e_m\}$

Step 1: Input soft set (F, E) and its parameters *E*;

Step 2: Check if there exist $e_i = e_j$ choose one of them and if there exist e_j^1 and e_j^0 keep into reduced parameter denoted as *C*. From Table 4.1, it shows that there exists $e_i = e_j$ and also e_j^1 and e_j^0. This are $e_1 = e_5$, and there exists e_j^0 and e_j^1 which are e_2^0 and e_3^1. Therefore we can put them in the reduce parameter denoted as *C and Q* respectively.

Step 3: For the soft set (F, E'), calculate $S(e_{j'})$ of $e_{j'}$ (that is oriented-parameter sum), for $j' = 1', 2', \cdots, t'$;

Step 4: Check every A in the candidate parameter reduction set, if $f_A(p_1) = f_A(p_2) = \cdots = f_A(p_n)$, and also if there exists B in the candidate parameter reduction set such that $f_B(p_1) = f_B(p_2) = \cdots = f_B(p_n)$. From Table 4.1 $\{e_1, e_7, e_8\}$ and $\{e_4, e_6, e_7\}$ satisfied the condition above

Step 5: Compute the $(A \cup B) - (A \cap B)$ take the intersections it will be kept; otherwise it will be omitted from table *E*. $A = \{e_1, e_7, e_8\}$ and $B = \{e_4, e_6, e_7\}$. Therefore, $A \cap B = \{e_7\}$ which could be could also be put in another reduced parameter denoted as *D*.

Step 6: Find the maximum cardinality in the candidate parameter reduction set, then $E - C - D - Q$ *as the alternative normal parameter reduction*. In this case, the optimal parameter reduction of the soft set would be $\{e_1, e_4, e_6, e_8\}$ as shown in Table 4.5.

Table 4.5: Alternative Normal parameter reduction from (F, E) in Table 4.1

U/E	e_1	e_4	e_6	e_8	$f(.)$
p_1	1	1	1	1	4
p_2	0	1	0	1	2
p_3	1	0	1	0	2
p_4	1	1	1	1	4
p_5	1	1	1	1	4
p_6	0	0	1	1	2

4.4.2 The difference between the proposed algorithm and the algorithm by (Ma et al., 2011)

The proposed algorithm differs from the algorithm presented by (Ma et al., 2011) in the following ways:

a. In proposed algorithm, we considered that if there exists, e_j^1 and e_j^0 put into parameter reduce set which reduces the number of soft sets to (F, E'). As such, it clearly indicates that the proposed algorithm reduce the computation compare to (Ma et al., 2011).

b. The proposed algorithm also considered the intersection of A and B in the candidate parameter set that satisfied $f_A(p_1) = f_A(p_2) = \cdots = f_A(p_n)$, and $f_B(p_1) = f_B(p_2) = \cdots = f_B(p_n)$ then compute the intersection. Subsequently, reduced it into reduce parameter set, this could also reduce the number of accessing the candidate parameter set. Hence, it reduces time.

4.4.3 The comparison result

The proposed algorithm is compared with the algorithm of Ma et al., (2011), regarding the consideration of general-purpose data set that may comprise different category and then compare it in term of computational complexity. The computational complexity of the algorithm of (Ma et al., 2011) has the following basic operation:

a. Finding the oriented parameter sum and

b. Finding the subset $A \subset E'$ in which the sum of A is a multiple of the $|U|$.

Based on these two operations, we can deduce that the number of element to be accessed once in every iteration which is *m.n* so comparing this with our proposed algorithm in which no such computation is required. We can say that the amount of computation is drastically reduced. The number of iteration perform by each algorithm in Figure 4.4 determine the complexity ratio, as the parameter increase the number of iterations in NPR and NENPR are increase while in ANPR algorithm it is drastically reduced.

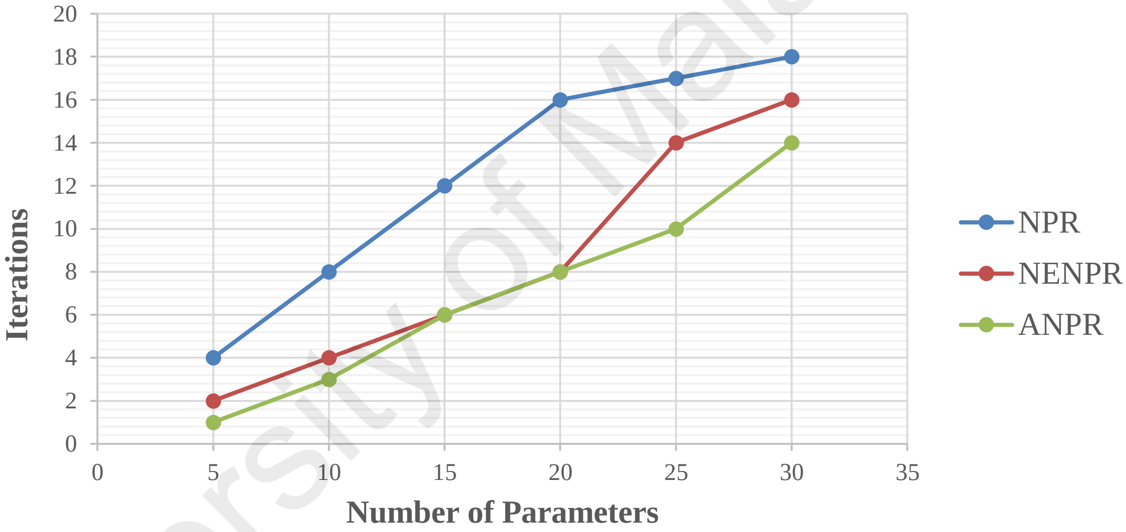

Figure 4.4: Number of Iteration perform from the three algorithm

4.4.4 Performance Analysis of the Proposed Algorithm

Parameter reduction of soft set focuses on how to efficiently reduce the number of parameters while improving several aspects of storage, speed, and accuracy of the decision-making process. Different algorithms exist to handle the reduction of parameters, but it seems each and every one of the existing algorithm did not solve the issues of optimality in which every data set could be able to be reduced.

Based on NPR and NENPR to estimate the computational complexity of these algorithms is firstly by counting the number of basic operation. The basic operation in this case is simply the number of times each element is access.

For computing the parameter importance degree

$$r\,e_i = \frac{1}{|U|}\left(\propto_1, e_i + \propto_2, e_i, + \cdots + \propto_s, e_i\right)$$

The following steps are required:

Step 1: Compute $f_E(h_i) = \sum_j h_{ij}$, $1 \le i \le n, 1 \le j \le m.$ for every element access the number of element access is *m.n*

Step 2: Get $C_E = \{E_{f1}, E_{f2}, \cdots, E_{fs}\}$ That is classifying object according to $f_E(h_i)$

Therefore,

$$\begin{pmatrix} f_E(h_i) \\ . \\ . \end{pmatrix} = n$$

Step 3: Find the decision partition by

(a) *Computing* $f_E(h_i) = \sum_j h_{ij}$, $1 \le i \le n, 1 \le j \le m.$

(b) *Find* C_{E-e_j}

Thus, the number of element access are:

n.(m-1) and n

Step 4: Compute the parameter importance degree $r\,e_i$ every element will be access once. Therefore, the number of element access is *n*.

Step 5: Compute $r\,e_i$ for every element $\propto_k, e_i$ every element will be access once. Therefore, the number of element access is *n*.

From Step 4 and 5 the total number of computing one parameter importance degree is:

$$n.m^2+2n$$

Hence, for *m* parameter

$$NEA= m(n.m^2+2n) = m^3n+2n$$

Including step 1 and 2

$$\Rightarrow \; NEA = m^3 n + 2mn + mn + n$$
$$= m^3 n + 3mn + n$$

Suppose:

$$m=n$$

The complexity will be

$$n^4 + 3n^2 + n$$

so taking the big **O** notation, $O\left(\mathrm{n}^4\right)$

For the NENPR algorithm only parameter oriented sum is accessing every entry once and then finding the subset $A = \{e_1', e_2', \cdots e_m'\} \subset \mathrm{E}$ for $1 \le \mathrm{i} \le \mathrm{p}$ is also access once.

Therefore,

$$NEA = m^2 n$$

If $m = n$

The complexity will be

$$O(n^3)\ .$$

For the ANPR algorithm

Computing $A = \{e_1^{'}, e_2^{'}, \cdots e_m^{'}\}$ and $B = \{e_1^{''}, e_2^{''}, \cdots e_s^{''}\}$ every entry is only access once

$$NEA = m.n + n$$

Suppose m=n2

$$n^2 + n$$

Hence, the complexity is $O(n^2)$

Therefore, comparing with $m^3 n + 3mn + n$ in computing parameter importance degree and $m^2 n + 3mn + n$ of computing oriented sum ANPR algorithm decrease the computational complexity.

However, the algorithm of NENPR used the combination that consists of k' columns for the candidate parameter set to test the combination from combination-2 to combination-($\lfloor n/2 \rfloor$). However, in this proposed algorithm, we retrieve all the combination from 1 to *m* and check if there exists other combination k'' that has the same value with the combination k'. Table 4.6 give the summary of the comparison result of the three algorithms based on the operation involved, computational complexity, limitations, parameter reduction and final solution.

Table 4.6: The comparison results of the three algorithms

Comparison	NPR	NENPR	ANPR	Remark
The operation involved	Classification for parameter reduction and some set operation	Addition and set operation	Addition and set operation	All the algorithms require some certain set operation.
Computational complexity	$O(n^4)$	$O(n^3)$	$O(n^2)$	The NENPR and ANPR have very lower computational complexity as compared to the NPR algorithm.
Limitation	Requires the computation of parameter important degree	Considered Suboptimal choice and added parameters, and it does not consider same parameters values.	Does not always consider last choices.	The priority of NPR and NENPR was to maintain invariant final choice objects rank in decision making
Parameter reduction result	Partition and rank of all the choice are maintained and considering the suboptimal choices which may lead to information loss in some cases.	Partition and rank of all the choice are maintained and considering the suboptimal choices	Partition and rank of optimal and suboptimal choice are maintained.	All three algorithms maintain rank and sub-optimal choices
Final solution	Certain	Certain	Certain	All the three algorithms have shown have a certain result.

4.5 Summary of the chapter

In this thesis, we have explained the voluminous work in normal parameter reduction of the soft set. We have presented an algorithm that overcomes the problem of the existing algorithms ranging from computational complexity, difficulty in understanding of the algorithms and implementing it within any data set with ease. The proposed algorithm was also compared with the algorithm of (Kong, 2008; and Ma, et al., 2011) and it evidently shows that the proposed algorithm has reduced the computational time as it evidently shown in comparison Table 4.5.

CHAPTER 5: APPLICATION OF ANPR ALGORITHM AND CONFLICT ANALYSIS OF SOFT SET

5.1 Introduction

In this section, the real life applications of the proposed algorithm to justify that it is not only suitable but rather feasible for real life problems is presented. Firstly, the decision-making problem of scholarship award selection from the data set Kano state scholarship board for selection of students for the foreign scholarship was considered. A second case study is on medical diagnosis in a patient suffering from a hiatal hernia.

5.2 Selection of Scholarship Award by Kano State Government

The Kano state scholarship board is a parastatal under Ministry of Education Kano state that awards a scholarship position to the indigene of the state. The board is saddled with the responsibilities[1] :

a. Awarding the scholarship and improving the welfare of the state-sponsored students for foreign training.
b. Provide guidance and counselling for students
c. Contact with Government establishment, institutes of learning and foreign universities.
d. Conducts the selection criteria in respect of all the applicants
e. Formulation and review of policies governing the award of scholarship.
f. Formal recommendation of suitably qualified applicants for oversea training to the governor of the state through the commissioner of education.

Based on this responsibility we find out that our algorithm will help the board in selecting the eligible applicant for a formal recommendation to the state government, so we, therefore, collect some data and applied it in our proposed algorithm. We find out

[1] www.kssbonline.org/

that it is not only feasible but suitable for choosing the applicants and making an optimal decision.

The data set for sponsoring students into for a foreign scholarship by Kano state government Nigeria is used in making a decision whether the student is appropriate for scholarship award based on certain criteria employed by the state government. Hence there are thirty-five (35) applicants for considerations by the board, we can represent the applicants as $U = \{s_1, s_2, \cdots, s_{35}\}$ and the set of parameters that each candidate can be described to have $E = \{e_1, e_2, \cdots, e_{16}\}$ where e_i means "English proficiency", "Mathematics", "Physics" "Chemistry", "Biology", "Agricultural Sciences", "Hausa language", Islamic Studies", "attended public school", "above 17 years", "has leadership potential", "has ambassadorial potential", "indigene of the state", "healthy condition","has 2.1 in undergraduate" and "complete NYSC", for $i = \{1,2,3,\cdots,16\}$ respectively. Let's define the mapping by $F : E \to P(U)$.

$$F(e_1) = \{s_1, s_2, \cdots, s_{35}\},$$

$$F(e_2) = \{s_1, s_2, \cdots, s_{35}\},$$

$$F(e_3) = \begin{Bmatrix} s_1, s_2, s_4, s_5, s_7, s_8, s_9, s_{12}, s_{14}, s_{15}, s_{16}, s_{17}, s_{18}, s_{19}, \\ s_{20}, s_{21}, s_{22}, s_{23}, s_{25}, s_{26}\, s_{27}, s_{29}, s_{30}, s_{31}, s_{33,} s_{34}, s_{35} \end{Bmatrix}$$

$$F(e_4) = \{s_1, s_2, \cdots, s_{35}\},$$

$$F(e_5) = \begin{Bmatrix} s_1, s_3, s_4, s_5, s_6, s_8, s_9, s_{11}, s_{12}, s_{14}, s_{15}, s_{16}, s_{17}, s_{18}, s_{19}, \\ s_{20}, s_{21}, s_{22}, s_{23}, s_{24}, s_{25}, s_{26}\, s_{27}, s_{29}, s_{30}, s_{31}, s_{32,} s_{34} \end{Bmatrix},$$

$$F(e_6) = \begin{Bmatrix} s_1, s_2, s_3, s_4, s_5, s_6, s_7, s_9, s_{10}, s_{11}, s_{13}, s_{14}, \\ s_{16}, s_{18}, s_{19}, s_{21}, s_{24}, s_{28}, s_{29}, s_{32}, s_{33,} s_{35} \end{Bmatrix},$$

$$F(e_7) = \begin{Bmatrix} s_2, s_3, s_6, s_7, s_8, s_{10}, s_{11}, s_{12}, s_{13}\, s_{14}, s_{15}, s_{16}, s_{17}, s_{18}, \\ s_{19}, s_{20}, s_{21}, s_{22}, s_{23}, s_{24}, s_{25}, s_{26}\, s_{27}, s_{28,} s_{29}, s_{30}, s_{31}, s_{32,} s_{34} \end{Bmatrix},$$

$$F(e_8) = \begin{Bmatrix} s_2, s_4, s_7, s_8, s_{10}, s_{11}, s_{12}, s_{14}, s_{15}, s_{17}, s_{18}, s_{19}, \\ s_{22}, s_{23}, s_{24}, s_{26}\, s_{28}, s_{30}, s_{31}, s_{32,} s_{34}, s_{35} \end{Bmatrix},$$

$$F(e_9) = \{s_1, s_2, s_3, s_5, s_6, s_{20}, s_{23}, s_{24}, s_{25}, s_{27}, s_{28,} s_{29}, s_{33,} s_{34}\},$$

$$F(e_{10})=\left\{\begin{matrix} s_1, s_3, s_4, s_5, s_6, s_8, s_9, \; s_{11}, s_{12}, \; s_{14}, s_{15}, s_{16}, s_{17}, \; s_{18}, s_{19}, \\ s_{20}, s_{21}, s_{22}, \; s_{23}, s_{24}, s_{25}, \; s_{26}\; s_{27}, s_{29}, \; s_{30}, s_{31}, \; s_{32,} s_{34} \end{matrix}\right\},$$

$$F(e_{11})=\{s_1,\, s_3, s_4, s_5,\, s_8, s_9,\, s_{10}, s_{11}, s_{12},\, s_{13}, s_{14}\, s_{16}, s_{17}\; s_{18}, s_{19},\, s_{21}, s_{22}, s_{25}\},$$

$$F(e_{12})=\{s_1,\; s_4,\; s_6, s_7, s_{15}, s_{16}, s_{21}, s_{22},\; s_{23}, s_{24}, s_{27}, s_{28,}\; s_{31},\; s_{32,}\},$$

$$F(e_{13})=\{s_4,\; s_7, s_8, s_9, s_{10}, s_{11},\; s_{13}, s_{15}, s_{16} s_{21}, s_{23}, s_{24}, s_{25},\; s_{26}\; s_{27}, s_{28,} s_{31},\; s_{32,}\},$$

$$F(e_{14})=\left\{\begin{matrix} s_3,\; s_6, s_8, s_9,\; s_{10}, s_{11}, s_{12},\; s_{13}, s_{14}, s_{15}\; s_{16}, s_{17}\;\; s_{18}, \\ s_{19}, s_{20}, s_{21}, s_{22}, s_{25}, s_{26}\; s_{27}, s_{30}, s_{31},\; s_{32,} \end{matrix}\right\},$$

$$F(e_{15})=\left\{\begin{matrix} s_1, s_2, s_3, s_4, s_5, s_6, s_8, s_9,\; s_{10}, s_{11},\; s_{13}, s_{15}, s_{17},\; s_{18}, s_{19}, \\ s_{20}, s_{22},\; s_{23}, s_{24}, s_{25}, s_{26}\; s_{27}, s_{28,}\; s_{30}, s_{31}, s_{32,}\; s_{33,} s_{34}, s_{35} \end{matrix}\right\},$$

$$F(e_{16})=\{s_1, s_2, \cdots, s_{35}\}.$$

The tabular representation of a soft set (F,E) as foreign scholarship applicants is shown in Table 5.1. The board wants to select students who satisfy the selected criteria.

Table 5.1: A soft set (F,E) for Scholarship selection

U/E	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}	e_{16}	*f(.)*
s_1	1	1	1	1	1	1	0	0	1	1	1	1	0	0	1	1	12
s_2	1	1	1	1	0	0	1	1	1	0	0	0	0	0	1	1	9
s_3	1	1	0	1	1	1	1	0	1	1	1	0	0	1	1	1	12
s_4	1	1	1	1	1	1	0	1	0	1	1	1	1	0	1	1	13
s_5	1	1	1	1	1	1	0	0	1	1	1	0	0	0	1	1	11
s_6	1	1	0	1	1	1	1	0	1	1	0	1	0	1	1	1	12
s_7	1	1	1	1	0	1	1	1	0	0	0	1	1	0	0	1	10
s_8	1	1	0	1	1	1	1	1	0	1	1	1	1	1	1	1	14
s_9	1	1	1	1	1	1	0	0	0	1	1	0	1	1	1	1	12
s_{10}	1	1	0	1	0	1	1	1	0	0	1	0	1	1	1	1	11
s_{11}	1	1	1	1	1	0	1	1	0	1	1	1	1	1	1	1	14
s_{12}	1	1	0	1	1	1	1	1	0	1	1	1	1	1	1	1	14
s_{13}	1	1	0	1	0	1	1	1	0	0	1	0	1	1	1	1	11
s_{14}	1	1	1	1	1	1	1	1	0	1	1	0	0	0	0	1	11
s_{15}	1	1	1	1	1	0	1	1	0	1	1	1	1	1	1	1	14
s_{16}	1	1	1	1	1	1	1	0	0	1	1	1	1	1	0	1	13
s_{17}	1	1	1	1	1	0	1	1	0	1	1	0	0	1	1	1	12
s_{18}	1	1	1	1	1	1	0	1	0	1	1	0	0	0	1	1	11
s_{19}	1	1	1	1	1	0	1	1	0	1	1	0	0	1	1	1	12
s_{20}	1	1	1	1	1	0	1	0	1	1	0	0	0	1	1	1	11
s_{21}	1	1	1	1	1	1	1	0	0	1	1	1	1	1	0	1	13
s_{22}	1	1	0	1	1	1	1	1	0	1	1	1	1	1	1	1	14

Table 5.1:, Continued

U/E	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}	e_{16}	*f(.)*
s_{23}	1	1	1	1	1	0	1	1	1	1	1	1	1	0	1	1	14
s_{24}	1	1	1	1	1	0	1	1	1	1	1	1	1	0	1	1	14
s_{25}	1	1	1	1	1	0	1	0	1	1	1	1	1	1	1	1	14
s_{26}	1	1	1	1	1	0	1	1	0	1	0	0	1	1	1	1	12
s_{27}	1	1	1	1	1	0	1	0	1	1	1	1	1	1	1	1	14
s_{28}	1	1	0	1	0	1	1	1	1	0	0	1	1	0	1	1	11
s_{29}	1	1	1	1	1	1	1	0	1	1	0	0	0	0	0	1	10
s_{30}	1	1	1	1	1	0	1	1	0	1	0	0	0	1	1	1	11
s_{31}	1	1	1	1	1	0	1	1	0	1	1	1	1	1	1	1	14
s_{32}	1	1	0	1	1	1	1	1	0	1	1	1	1	1	1	1	14
s_{33}	1	1	1	1	0	1	0	0	1	0	0	0	0	0	1	1	8
s_{34}	1	1	1	1	1	0	1	1	1	1	0	0	0	0	1	1	11
s_{35}	1	1	1	1	0	1	0	1	0	0	0	0	0	0	1	1	8

From Table 5.1 we can see that $\{s_8, s_{11}, s_{12}, s_{15}, s_{22}, s_{23}, s_{24}, s_{25}, s_{27}, s_{31}, s_{32}\}$ are considered as the best choice. Based on algorithm presented in Figure 4.3 we Firstly, input the given Boolean-valued dataset and check the existence of equal parameters values among the parameter set and entries has all zeros, or ones value, in this case, $\{e_1, e_2, e_4, e_{16}\}$ all has one and $\{e_5, e_{10}\}$ has the same entry. Therefore we can keep them into reduce parameter set *C* and *Q*, respectively. Then the algorithm computes the oriented parameter sum and finds if there exist subsets *A* and *B* in the feasible parameter reduction as shown in steps 2 and 3 in Figure 4.3 which are

$$f_A(h_1) = f_A(h_2) = \cdots = f_A(h_n) \text{ and}$$
$$f_B(h_1) = f_B(h_2) = \cdots = f_B(h_n)$$

i.e.

$\{e_3, e_6, e_8, e_9, e_{14}\}$ and $\{e_3, e_6, e_7, e_{15}\}$, respectively. As shown in Table 5.2 and 5.3

Table 5.2: A soft set (F, E') for $F_A(h_1) = F_A(h_2) \cdots$

U/E	e_3	e_6	e_8	e_9	e_{14}	*f(.)*
s_1	1	1	0	1	0	3
s_2	1	0	1	1	0	3
s_3	0	1	0	1	1	3
s_4	1	1	1	0	0	3
s_5	1	1	0	1	0	3
s_6	0	1	0	1	1	3
s_7	1	1	1	0	0	3
s_8	0	1	1	0	1	3
s_9	1	1	0	0	1	3
s_{10}	0	1	1	0	1	3
s_{11}	1	0	1	0	1	3
s_{12}	0	1	1	0	1	3
s_{13}	0	1	1	0	1	3
s_{14}	1	1	1	0	0	3
s_{15}	1	0	1	0	1	3
s_{16}	1	1	0	0	1	3
s_{17}	1	0	1	0	1	3
s_{18}	1	1	1	0	0	3
s_{19}	1	0	1	0	1	3
s_{20}	1	0	0	1	1	3
s_{21}	1	1	0	0	1	3
s_{22}	0	1	1	0	1	3
s_{23}	1	0	1	1	0	3
s_{24}	1	0	1	1	0	3
s_{25}	1	0	0	1	1	3
s_{26}	1	0	1	0	1	3
s_{27}	1	0	0	1	1	3
s_{28}	0	1	1	1	0	3
s_{29}	1	1	0	1	0	3
s_{30}	1	0	1	0	1	3
s_{31}	1	0	1	0	1	3
s_{32}	0	1	1	0	1	3
s_{33}	1	1	0	1	0	3
s_{34}	1	0	1	1	0	3
s_{35}	1	1	1	0	0	3

Based on this result, we compute the intersection of the subsets *A* and *B* which is

$$(A \cup B) - (A \cap B) = \{e_7, e_8, e_9, e_{14}, e_{15}\}$$

Table 5.3: A soft set (F, E') for $F_B(h_1) = F_B(h_2)\cdots$

U/E	e_3	e_6	e_7	e_{15}	*f(.)*
s_1	1	1	0	1	3
s_2	1	0	1	1	3
s_3	0	1	1	1	3
s_4	1	1	0	1	3
s_5	1	1	0	1	3
s_6	0	1	1	1	3
s_7	1	1	1	0	3
s_8	0	1	1	1	3
s_9	1	1	0	1	3
s_{10}	0	1	1	1	3
s_{11}	1	0	1	1	3
s_{12}	0	1	1	1	3
s_{13}	0	1	1	1	3
s_{14}	1	1	1	0	3
s_{15}	1	0	1	1	3
s_{16}	1	1	1	0	3
s_{17}	1	0	1	1	3
s_{18}	1	1	0	1	3
s_{19}	1	0	1	1	3
s_{20}	1	0	1	1	3
s_{21}	1	1	1	0	3
s_{22}	0	1	1	1	3
s_{23}	1	0	1	1	3
s_{24}	1	0	1	1	3
s_{25}	1	0	1	1	3
s_{26}	1	0	1	1	3
s_{27}	1	0	1	1	3
s_{28}	0	1	1	1	3
s_{29}	1	1	1	0	3
s_{30}	1	0	1	1	3
s_{31}	1	0	1	1	3
s_{32}	0	1	1	1	3
s_{33}	1	1	0	1	3
s_{34}	1	0	1	1	3
s_{35}	1	1	0	1	3

and $A \cap B = \{e_3, e_6\}$ it can be kept into reduced parameter set denoted as *D*. Finally; the algorithm will determine the maximum cardinality of the candidate parameter reduction set

$$E - C - D - Q = \{e_5, e_7, e_8, e_9, e_{11}, e_{12}, e_{13}, e_{14}, e_{15}\},$$

which is considered to be reduced set normal parameter reduction as given in Table 5.4.

Based on this reduct set the parameters that are of great important in making decision in this data set are “Biology”, “Hausa language”, Islamic Studies”, “attended public school”, “has leadership potential”, “has ambassadorial potential”, “indigene of the state”, ”healthy condition” and ”has 2.1 in undergraduate”.

Table 5.4: ANPR of a soft set (F, E) for Table 5.1

U/E	e_5	e_7	e_8	e_9	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}	$f(.)$
s_1	1	0	0	1	1	1	0	0	1	5
s_2	0	1	1	1	0	0	0	0	1	4
s_3	1	1	0	1	1	0	0	1	1	6
s_4	1	0	1	0	1	1	1	0	1	6
s_5	1	0	0	1	1	0	0	0	1	4
s_6	1	1	0	1	0	1	0	1	1	6
s_7	0	1	1	0	0	1	1	0	0	4
s_8	1	1	1	0	1	1	1	1	1	8
s_9	1	0	0	0	1	0	1	1	1	5
s_{10}	0	1	1	0	1	0	1	1	1	6
s_{11}	1	1	1	0	1	1	1	1	1	8
s_{12}	1	1	1	0	1	1	1	1	1	8
s_{13}	0	1	1	0	1	0	1	1	1	6
s_{14}	1	1	1	0	1	0	0	0	0	4
s_{15}	1	1	1	0	1	1	1	1	1	8
s_{16}	1	1	0	0	1	1	1	1	0	6
s_{17}	1	1	1	0	1	0	0	1	1	6

Table 5.4:, Continued

U/E	e_5	e_7	e_8	e_9	e_{11}	e_{12}	e_{13}	e_{14}	e_{15}	$f(.)$
s_{18}	1	0	1	0	1	0	0	0	1	4
s_{19}	1	1	1	0	1	0	0	1	1	6
s_{20}	1	1	0	1	0	0	0	1	1	5
s_{21}	1	1	0	0	1	1	1	1	0	6
s_{22}	1	1	1	0	1	1	1	1	1	8
s_{23}	1	1	1	1	1	1	1	0	1	8
s_{24}	1	1	1	1	1	1	1	0	1	8
s_{25}	1	1	0	1	1	1	1	1	1	8
s_{26}	1	1	1	0	0	0	1	1	1	6
s_{27}	1	1	0	1	1	1	1	1	1	8
s_{28}	0	1	1	1	0	1	1	0	1	6
s_{29}	1	1	0	1	0	0	0	0	0	3
s_{30}	1	1	1	0	0	0	0	1	1	5
s_{31}	1	1	1	0	1	1	1	1	1	8
s_{32}	1	1	1	0	1	1	1	1	1	8
s_{33}	0	0	0	1	0	0	0	0	1	2
s_{34}	1	1	1	1	0	0	0	0	1	5
s_{35}	0	0	1	0	0	0	0	0	1	2

Based on the solution obtained from Table 5.4, we can see that the Kano state scholarship board can make its decision by selecting $\{s_8, s_{11}, s_{12}, s_{15}, s_{22}, s_{23}, s_{24}, s_{25}, s_{27}, s_{31}, s_{32}\}$ as the best candidate for the award of scholarship, while the candidate $\{s_3, s_4, s_6, s_{10}, s_{13}, s_{16}, s_{17}, s_{19}, s_{21}, s_{26}, s_{28}\}$ will be considered as the suboptimal choice for the award followed by $\{s_1, s_9, s_{20}, s_{30}, s_{34}\}$. Moreover, with this result, the ministry of education Kano state will have different choices for recommendation to the political authority on either to select the best choice or suboptimal or the last choice for the award. The scholarship awarded will improve the welfare of the state-sponsored students for foreign training.

5.3 Application medical diagnosis for patient with a hiatal hernia

In this subsection, we consider hiatal hernia disease that causes abnormality in the stomach and slide up into the chest cavity and sometimes cause acid reflux or gastroesophageal reflux (GERD). We apply our proposed algorithm to elaborate decision making for a patient suspected with a hiatal hernia. The data collected are from Mariri comprehensive hospital in Kano state, Nigeria. Table 5.3 shows the data set that contain 50 patient with various symptoms as the parameters i.e. $U = \{p_1, p_2, \cdots, p_{50}\}$ and the set of parameters that each patient can be described to have $E = \{e_1, e_2, \cdots, e_{12}\}$ where e_i means "heartburn", "chest pain", "nausea" "vomiting ", "burping", "water brash", "appearance of large amount of saliva ", cough", "difficulty in swallowing", "passing black stool", "abdominal pain", "belching" and "fever" for $i = \{1, 2, 3, \cdots, 13\}$, respectively.

Table 5.5: A soft set (F, E) for patient with hiatal hernia

U/E	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	$f(.)$
p_1	1	1	1	1	0	1	1	0	0	1	0	1	0	8
p_2	0	1	1	1	1	1	1	1	0	1	0	1	0	9
p_3	1	1	0	0	1	1	1	0	0	1	1	0	0	7
p_4	1	1	1	1	0	1	1	1	0	1	0	1	0	9
p_5	0	1	1	1	1	0	1	0	0	1	1	1	0	8
p_6	1	1	0	0	1	1	1	0	0	1	1	0	0	7
p_7	1	1	1	1	1	0	1	1	0	0	1	1	1	10
p_8	1	1	1	1	1	0	1	1	0	0	1	1	1	10
p_9	1	1	1	1	1	1	1	0	0	0	0	1	0	8
p_{10}	0	1	0	0	1	1	1	1	0	1	1	0	1	8
p_{11}	0	1	0	1	1	1	1	1	0	1	0	0	0	7
p_{12}	1	1	1	1	0	0	1	1	0	1	1	1	0	9
p_{13}	1	1	0	1	1	1	1	1	0	0	0	0	0	7
p_{14}	0	0	1	1	1	1	1	1	0	1	1	1	0	9
p_{15}	0	1	1	1	1	1	1	1	0	1	0	1	0	9
p_{16}	1	1	0	0	1	1	1	0	0	1	1	0	0	7
p_{17}	1	1	1	1	0	0	1	1	0	1	1	1	0	9
p_{18}	1	1	1	1	0	0	1	1	0	1	1	1	0	9
p_{19}	1	1	1	1	0	0	1	1	0	1	1	1	0	9

Table 5.5:, Continued

U/E	e_1	e_2	e_3	e_4	e_5	e_6	e_7	e_8	e_9	e_{10}	e_{11}	e_{12}	e_{13}	*f(.)*
p_{20}	1	0	1	1	0	1	1	0	0	1	1	1	0	8
p_{21}	1	1	0	0	1	1	1	0	0	1	1	0	0	7
p_{22}	1	0	1	1	0	1	1	1	0	1	1	1	0	9
p_{23}	1	1	1	1	0	0	1	1	0	1	1	1	0	9
p_{24}	1	1	0	0	1	1	1	0	0	1	1	0	0	7
p_{25}	1	1	1	1	1	1	1	0	0	0	0	1	0	8
p_{26}	1	1	1	1	1	1	1	1	0	0	0	1	1	10
p_{27}	1	1	1	0	1	1	1	0	0	1	1	1	0	9
p_{28}	1	1	0	1	1	1	1	1	0	0	0	0	0	7
p_{29}	1	1	1	1	1	1	1	0	0	0	0	1	0	8
p_{30}	1	1	1	1	1	1	1	1	0	0	0	1	0	9
p_{31}	0	1	1	1	1	0	1	0	0	1	1	1	0	8
p_{32}	1	1	0	0	1	1	1	0	0	1	1	0	0	7
p_{33}	1	1	1	1	1	1	1	0	0	0	0	1	0	8
p_{34}	0	1	1	1	1	0	1	1	0	1	1	1	0	9
p_{35}	1	1	1	1	0	1	1	1	0	1	0	1	0	9
p_{36}	0	0	1	1	1	1	1	0	0	1	1	1	0	8
p_{37}	1	1	0	0	1	1	1	0	0	1	1	0	0	7
p_{38}	1	1	0	0	1	1	1	0	0	1	1	0	0	7
p_{39}	1	1	0	0	1	1	1	0	0	1	1	0	0	7
p_{40}	1	0	1	1	1	1	1	0	0	0	1	1	0	8
p_{41}	0	0	1	1	1	1	1	0	0	1	1	1	0	8
p_{42}	0	1	1	1	1	1	1	0	0	1	0	1	0	8
p_{43}	1	1	0	0	1	1	1	0	0	1	1	0	0	7
p_{44}	0	1	0	1	1	0	1	0	0	1	1	0	0	6
p_{45}	1	0	0	1	0	1	1	1	0	1	1	0	0	7
p_{46}	0	0	1	1	1	1	1	1	0	1	1	1	0	9
p_{47}	1	1	1	1	1	1	1	1	0	0	0	1	1	10
p_{48}	0	1	1	1	1	1	1	1	0	1	0	1	0	9
p_{49}	1	1	0	0	1	1	1	0	0	1	1	0	0	7
p_{50}	1	1	1	1	1	1	1	1	0	0	0	1	1	10

After applying the proposed algorithm, it is clear that the reduct symptoms from Table 5.6 show that $\{p_7, p_8, p_{26}, p_{27}, p_{47}, p_{50}\}$ are the best choice because $\{e_7, e_9\}$ and (e_3) can be kept in reduced parameter set as *C* and *Q*, respectively. Therefore,

$f_A(h_1) = f_A(h_2) = \cdots = f_A(h_n)$ and $f_B(h_1) = f_B(h_2) = \cdots = f_B(h_n)$

are

$\{e_1, e_4, e_5, e_{10}\}$ and $\{e_2, e_4, e_6, e_{11}\}$, respectively.

Hence,

$$(A \cup B) - (A \cap B) = \{e_1, e_2, e_5, e_6, e_{11}\},$$

which then we kept $A \cap B = \{e_4\}$ it into a reduced parameter set denoted as D. Finally we find the maximum cardinality which is $E - C - D - Q = \{e_1, e_2, e_5, e_6, e_8, e_{10}, e_{11}, e_{12}, e_{13}\}$ which is considered to be reduced set normal parameter reduction as shown in Table 5.4.

Table 5.6: ANPR of a soft set (F, E) for patient with a hiatal hernia

U/E	e_1	e_2	e_5	e_6	e_8	e_{10}	e_{11}	e_{12}	e_{13}	$f(.)$
p_1	1	1	0	1	0	1	0	1	0	5
p_2	0	1	1	1	1	1	0	1	0	6
p_3	1	1	1	1	0	1	1	0	0	6
p_4	1	1	0	1	1	1	0	1	0	6
p_5	0	1	1	0	0	1	1	1	0	5
p_6	1	1	1	1	0	1	1	0	0	6
p_7	1	1	1	0	1	0	1	1	1	7
p_8	1	1	1	0	1	0	1	1	1	7
p_9	1	1	1	1	0	0	0	1	0	5
p_{10}	0	1	1	1	1	1	1	0	1	6
p_{11}	0	1	1	1	1	1	0	0	0	5
p_{12}	1	1	0	0	1	1	1	1	0	6
p_{13}	1	1	1	1	1	0	0	0	0	5
p_{14}	0	0	1	1	1	1	1	1	0	6
p_{15}	0	1	1	1	1	1	0	1	0	6
p_{16}	1	1	1	1	0	1	1	0	0	6
p_{17}	1	1	0	0	1	1	1	1	0	6
p_{18}	1	1	0	0	1	1	1	1	0	6
p_{19}	1	1	0	0	1	1	1	1	0	6
p_{20}	1	0	0	1	0	1	1	1	0	5
p_{21}	1	1	1	1	0	1	1	0	0	6
p_{22}	1	0	0	1	1	1	1	1	0	6
p_{23}	1	1	0	0	1	1	1	1	0	6
p_{24}	1	1	1	1	0	1	1	0	0	6

Table 5.6:, Continued

U/E	e_1	e_2	e_5	e_6	e_8	e_{10}	e_{11}	e_{12}	e_{13}	$f(.)$
p_{25}	1	1	1	1	0	0	0	1	0	5
p_{26}	1	1	1	1	1	0	0	1	1	7
p_{27}	1	1	1	1	0	1	1	1	0	7
p_{28}	1	1	1	1	1	0	0	0	0	5
p_{29}	1	1	1	1	0	0	0	1	0	5
p_{30}	1	1	1	1	1	0	0	1	0	6
p_{31}	0	1	1	0	0	1	1	1	0	5
p_{32}	1	1	1	1	0	1	1	0	0	6
p_{33}	1	1	1	1	0	0	0	1	0	5
p_{34}	0	1	1	0	1	1	1	1	0	6
p_{35}	1	1	0	1	1	1	0	1	0	6
p_{36}	0	0	1	1	0	1	1	1	0	5
p_{37}	1	1	1	1	0	1	1	0	0	6
p_{38}	1	1	1	1	0	1	1	0	0	6
p_{39}	1	1	1	1	0	1	1	0	0	6
p_{40}	1	0	1	1	0	0	1	1	0	5
p_{41}	0	0	1	1	0	1	1	1	0	5
p_{42}	0	1	1	1	0	1	0	1	0	5
p_{43}	1	1	1	1	0	1	1	0	0	6
p_{44}	0	1	1	0	0	1	1	0	0	4
p_{45}	1	0	0	1	1	1	1	0	0	5
p_{46}	0	0	1	1	1	1	1	1	0	6
p_{47}	1	1	1	1	1	0	0	1	1	7
p_{48}	0	1	1	1	1	1	0	1	0	6
p_{49}	1	1	1	1	0	1	1	0	0	6
p_{50}	1	1	1	1	1	0	0	1	1	7

From Table 5.6 we can see that $\{p_7, p_8, p_{26}, p_{27}, p_{47}, p_{50}\}$ are considered as the patient that affected with hiatal hernia disease that causes abnormality on the stomach and slides up into the chest cavity and sometimes causes acid reflux or gastroesophageal reflux (GERD). Meanwhile, the suboptimal choice of

$$\begin{Bmatrix} p_2, p_3, p_4, p_6, p_{10}, p_{12}, p_{14}, p_{15}, p_{16}, p_{17}, p_{18}, p_{19}, p_{21}, p_{22}, p_{23}, p_{24}, p_{30}, p_{32}, \\ p_{34}, p_{35}, p_{37}, p_{38}, p_{39}, p_{43}, p_{46}, p_{48}, p_{49} \end{Bmatrix},$$

have the tendency of being affected with hiatal hernia disease if precautions are not being taken.

5.4 Application of soft set theory in Conflict Analysis

In this research, the application of soft set theory in conflict issues in urban planning was also presented, so as, to compare it with the baseline algorithm in terms of efficiency and ease of understanding.

Conflict analysis plays a vital role in the fields of politics, business, terrorism, management, social networks, engineering, and sports. A new computational intelligence approach such as rough and soft set theories have been used to solve conflict situation which has the ability to handle uncertainty. This study presents an alternative idea using soft set theory for conflict analysis of public engagement program for urban planning. Conflict is an important element in urban planning which is considered by the urban planners as one of the issues that affecting the planning process by the government and sometimes lead to lawsuits, protest, etc. involving stakeholders in the planning process the government may avoid such a conflicts, and it will make the process more efficient. At least there are two parties in an urban planning conflict, named agents, where they controvert in some issues. This is a very simple conflict example in Figure 5.1, where this is a conflict between two people. Overall, the agents could be individuals, groups, companies, states, political parties, etc.

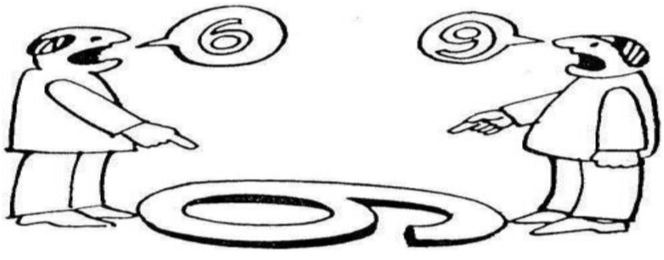

Figure 5.1: Conflict (in general)

5.4.1 Conflict Representation in Multi-Soft Sets

The conflict situation based on multi-soft sets in (Zhu & Zhan, 2015) is modelled as a (F,A) representing a conflict table $S=(U,A)$. On a conflict situation, $S=(U,A)$ is special type of multi-soft sets, since $V_a=\{1,0,-1\}$ simply mean that, Alliance (coalition), neutral, and conflict toward the issue, respectively. Therefore, the conflict model based on multi-soft sets is as follow:

$$(F,A)=(U,A), \text{ for } V_a=\{1,0,-1\},\ a\in A.$$

In the following sub-section, the notions of Alliance (coalition), neutrality, and conflict from the soft set point of view is presented.

5.4.2 Alliance, Neutrality, and Conflict

From the fact that a standard soft set (F,a) can be represented as a Boolean-valued information system $(U,a,V_{\{0,1\}},f)$ or simply as (U,a) (Herawan & Deris, 2011), then the notions of alliance (1), neutrality (0), and conflict (−1) can be:

$$f(x,y)=\begin{cases} 1, & \text{if } f(x)\times f(y)=1 \\ 0, & \text{if } f(x)\times f(y)=0 \\ -1, & \text{if } f(x)\times f(y)=-1 \end{cases}, \text{ for } x,y\in a.$$

5.4.3 Support, Strength, Certainty and Coverage

Firstly, the definition of the notion of occurrence of parameters in soft set theory as in Definition 22 is given.

Definition 22: *Let (F,a) be a soft set over the universe U representing (U,a) and an object $u\in U$. A parameter co-occurrence set of an object u can be defined as following*

$$coo(u)=\{a_i\in a \mid f(u,a_i)=1\}$$

Obviously, $coo(u)=\{a_i\in a \mid f(a_i)=1\}$.

Based on Definition 22, in the following definition, we present the notion of support for an agent in (F,a).

Definition 23: *Let (F,a) be a soft set over the universe U representing (U,a) and an agent $u \in U$. The support of an agent u is defined by*

$$\operatorname{supp}(u) = \operatorname{card}(coo(u)) = \operatorname{card}(\{a_i \in a : f(u,a_i) = 1\}).$$

From Definition 23, we have the following definition of rules strength.

Definition 24: *Let (F,a) be a soft set over the universe U representing (U,a). The strength of a rule $a_i \Rightarrow a_j$ for $a_i, a_j \in a$ denoted by $\sigma_x(a_i, a_j)$ is defined by*

$$\sigma_x(a_i, a_j) = \operatorname{supp}_x(a_i, a_j) / |U|, \textit{ for } x \in U.$$

From Definition 24, the following definition of rules for certainty is presented.

Definition 25: *Let (F,a) be a soft set over the universe U representing (U,a). The certainty of a rule $a_i \Rightarrow a_j$ for $a_i, a_j \in a$ denoted by $cer_x(a_i, a_j)$ is defined by*

$$\begin{aligned}\operatorname{cer}_x(a_i, a_j) &= \operatorname{supp}_x(a_i, a_j) / |A_1(x)| \\ &= \sigma_x(a_i, a_j) / \pi(A_1(x))\end{aligned}$$

where $\pi(a_1(x)) = |a_1(x)| / |U|$.

From Definition 25, the following definition of rules for coverage is presented.

Definition 26: *Let (F,a) be a soft set over the universe U representing (U,a). The coverage of a rule $a_i \Rightarrow a_j$ for $a_i, a_j \in a$ denoted by $\operatorname{cov}_x(a_i, a_j)$ is defined by*

$$\begin{aligned}\operatorname{cov}_x(a_i, a_j) &= \operatorname{supp}_x(a_i, a_j) / |a_2(x)| \\ &= \sigma_x(a_i, a_j) / \pi(a_2(x)),\end{aligned}$$

where $\pi(a_j(x)) = |a_j(x)| / |U|$.

Similarly, $\operatorname{cov}_x(a_i, a_j) = \pi_x(a_i \mid a_j)$.

The algorithm for handling conflict data using multi-soft sets is given in Figure 5.2 as follow.

Algorithm: Soft set for conflict analysis Input: A conflict data set (U, A) Output: Support, strength, certainty and coverage of conflict rules
1. Transfer the conflict data set (U, A) into multi-soft sets (F, A) 2. Determine all rules to the values of alliance, neutral, and conflict. 3. Calculate support for all rules. 4. Calculate strength for all rules. 5. Calculate certainty for all rules. 6. Calculate coverage for all rules.

Figure 5.2: The proposed soft set-approach algorithm

In the following section, a tutorial example of handling a conflict situation in the voting analysis is shown.

5.4.3.1 Tutorial Example

In this study, the concept of alliance, neutrality, and conflict was look over for clear understanding. In conflict, there are at least two parties called agents in a dispute over some issues. The agents may be individuals, political parties, groups, etc. The relationships among each agent to a specific issue are clearly given in Table 5.7. In Table 5.7, there are four agents namely o_i, for $i = 1,2,3,4$. The entry corresponding rows and column represent the opinion of agents about some certain issues. The last column presents the number of support for each agent related to an opinion. For simplicity, the representation is given +, 0, and – as an alliance, neutral, and conflict, respectively.

Table 5.7: Tabular representation of conflicts among agents

U / E	Opinions	Support
o_1	+	10
o_1	0	5
o_1	–	8
o_2	+	10
o_2	0	6
o_2	–	5
o_3	+	9
o_3	0	8
o_3	–	14
o_4	+	5

Table 5.7:, Continued

U/E	Opinions	Support
o_4	0	9
o_4	–	11

By the tutorial example of a conflict data set as shown in Table 5.7 above the method can be used in Section 5.4.4 to illustrate the multi-valued information system $S=(U,A,V,f)$ where the domain objects (universe) and the issues function f are defined as:

$$U=\{(1,o_1),\cdots,(23,o_1),(24,o_2),\cdots,(44,o_2),(45,o_3),\cdots,(75,o_3),(76,o_4),\cdots,(100,o_4)\}$$

and

$$f(1,o_1)=\ldots=f(10,o_1)=+,\ f(11,o_1)=\ldots=f(15,o_1)=0,$$
$$f(16,o_1)=\ldots=f(23,o_1)=-$$

$$f(24,o_2)=\ldots=f(33,o_2)=+,\ f(34,o_2)=\ldots=f(39,o_2)=0,$$

$$f(40,o_2)=\ldots=f(44,o_2)=-$$

$$f(45,o_3)=\ldots=f(53,o_3)=+,\ f(54,o_3)=\ldots=f(61,o_3)=0,$$

$$f(62,o_3)=\ldots=f(75,o_3)=0,$$

$$f(76,o_4)=\ldots=f(79,o_4)=+,\ f(80,o_4)=\ldots=f(88,o_4)=0,$$

$$f(89,o_4)=\ldots=f(100,o_4)=-$$

Based on Figure 5.2, a decomposition of Table 5.7 into four Boolean-valued information systems as shown in Table 5.8–5.11 is given.

Table 5.8: Boolean-valued information system representing o_1

	o_1		
Objects	+	0	–
1	1	0	0
2	1	0	0
⋮	⋮	⋮	⋮
10	1	0	0
11	0	1	0
⋮	⋮	⋮	⋮
15	0	1	0
16	0	0	1

Table 5.8:, Continued

⋮	⋮	⋮	⋮
13	0	0	1

Table 5.9: Boolean-valued information system representing o_2

	o_2		
Objects	+	0	−
24	1	0	0
⋮	⋮	⋮	⋮
33	1	0	0
34	0	1	0
⋮	⋮	⋮	⋮
39	0	1	0
40	0	0	1
⋮	⋮	⋮	⋮
44	0	0	1

Table 5.10: Boolean-valued information system representing o_3

	o_3		
Objects	+	0	−
45	1	0	0
⋮	⋮	⋮	⋮
53	1	0	0
54	0	1	0
⋮	⋮	⋮	⋮
61	0	1	0
62	0	0	1
⋮	⋮	⋮	⋮
75	0	0	1

Table 5.11: Boolean-valued information system representing o_4

	o_4		
Objects	+	0	−
76	1	0	0
⋮	⋮	⋮	⋮
79	1	0	0
80	0	1	0
⋮	⋮	⋮	⋮
88	0	1	0
89	0	0	1
⋮	⋮	⋮	⋮
100	0	0	1

Based on Tables 5.8-5.11, the multi-soft sets representing Table 5.7 as shown in Figure 5.3.

$$(F,E)=\left\{\begin{array}{l}(F,o_1)=\left\{\begin{array}{l}(F,o_1^+)=\{(1,o_1),\cdots,(10,o_1)\}\\(F,o_1^0)=\{(11,o_1),\ldots,(15,o_1)\}\\(F,o_1^-)=\{(16,o_1),\ldots,(23,o_1)\}\end{array}\right.\\(F,o_2)=\left\{\begin{array}{l}(F,o_2^+)=\{(24,o_2),\cdots,(33,o_2)\}\\(F,o_2^0)=\{(34,o_2),\ldots,(39,o_2)\}\\(F,o_2^-)=\{(40,o_2),\ldots,(44,o_2)\}\end{array}\right.\\(F,o_3)=\left\{\begin{array}{l}(F,o_3^+)=\{(45,o_3),\cdots,(53,o_3)\}\\(F,o_3^0)=\{(54,o_3),\ldots,(61,o_3)\}\\(F,o_3^-)=\{(62,o_3),\ldots,(75,o_3)\}\end{array}\right.\\(F,o_4)=\left\{\begin{array}{l}(F,o_4^+)=\{(76,o_4),\cdots,(79,o_4)\}\\(F,o_4^0)=\{(80,o_4),\ldots,(88,o_4)\}\\(F,o_4^-)=\{(89,o_4),\ldots,(100,o_4)\}\end{array}\right.\end{array}\right\}$$

Figure 5.3: Multi soft sets representation of Tables 5.6–5.9.

Based on step 3 in proposed algorithm (Figure 5.2), the support of each issue can be given as follow:

$$(F,o_1)=\left\{\begin{array}{l}\operatorname{supp}(F,o_1^+)=10\\\operatorname{supp}(F,o_1^0)=5\\\operatorname{supp}(F,o_1^-)=8\end{array}\right.$$

$$(F,o_2)=\left\{\begin{array}{l}\operatorname{supp}(F,o_2^+)=10\\\operatorname{supp}(F,o_2^0)=6\\\operatorname{supp}(F,o_2^-)=5\end{array}\right.$$

$$(F,o_3)=\left\{\begin{array}{l}\operatorname{supp}(F,o_3^+)=9\\\operatorname{supp}(F,o_3^0)=8\\\operatorname{supp}(F,o_3^-)=14\end{array}\right.$$

$$(F,o_4)=\left\{\begin{array}{l}\operatorname{supp}(F,o_4^+)=5\\\operatorname{supp}(F,o_4^0)=9\\\operatorname{supp}(F,o_4^-)=11\end{array}\right.$$

Based on steps 4, 5 and 6 of proposed algorithm (Figure 5.2), the strength, certainty and coverage of each issue can be given as follow (Table 5.12):

Table 5.12: Values of strength, certainty and coverage

Fact	Description	Strength	Certainty	Coverage
1	$(o_1,+)$	0.10	0.43	0.29
2	$(o_1,0)$	0.05	0.22	0.18
3	$(o_1,-)$	0.08	0.35	0.21
4	$(o_2,+)$	0.10	0.48	0.29
5	$(o_2,0)$	0.06	0.29	0.21
6	$(o_2,-)$	0.05	0.24	0.13
7	$(o_3,+)$	0.09	0.29	0.26
8	$(o_3,0)$	0.08	0.26	0.29
9	$(o_3,-)$	0.14	0.45	0.37
10	$(o_4,+)$	0.05	0.20	0.15
11	$(o_4,0)$	0.09	0.36	0.32
12	$(o_4,-)$	0.11	0.44	0.29

The following figure present flow graph, approximate graph and conflict graph of above case in Table 5.7.

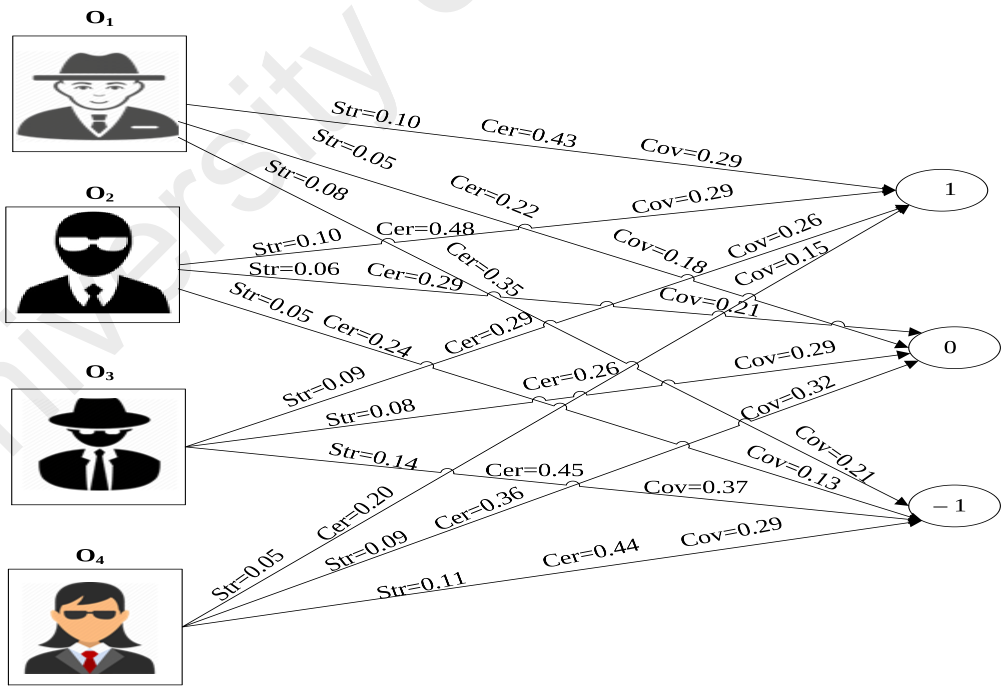

Figure 5.4: Flow graph for Table 5.5

Figure 5.4 depicts the flow graph of Table 5.7 branches of the flow graph represent strength, certainty and coverage, for example, $O \rightarrow 1$ have the strength value as 0.11, certainty value as 0.43 and coverage value as 0.29, respectively. For easy understanding of the conflict situation Figure 5.5 presents the approximate graph

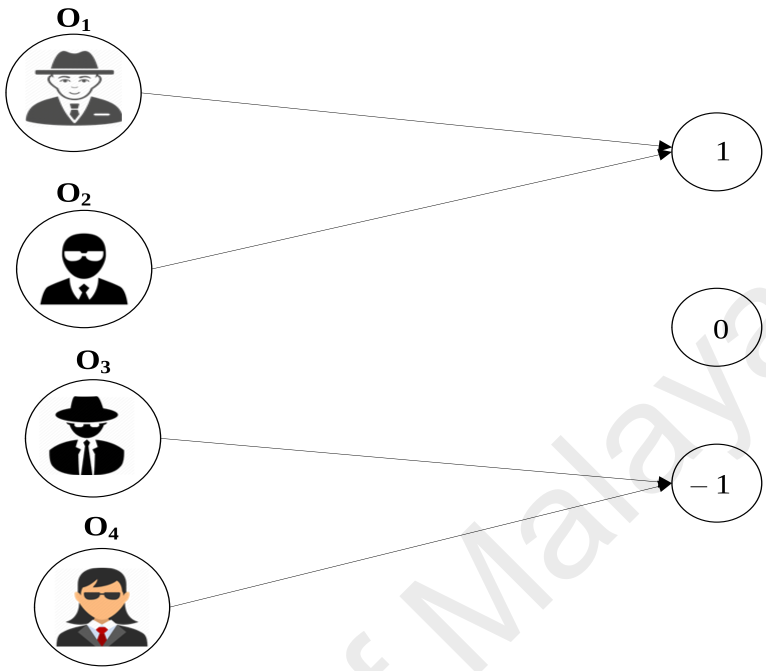

Figure 5.5: Approximate graph

Figure 5.5 clearly shows the approximate graph which indicates that O_1 and O_2 form an alliance, which is in conflict with O_3 and O_4.

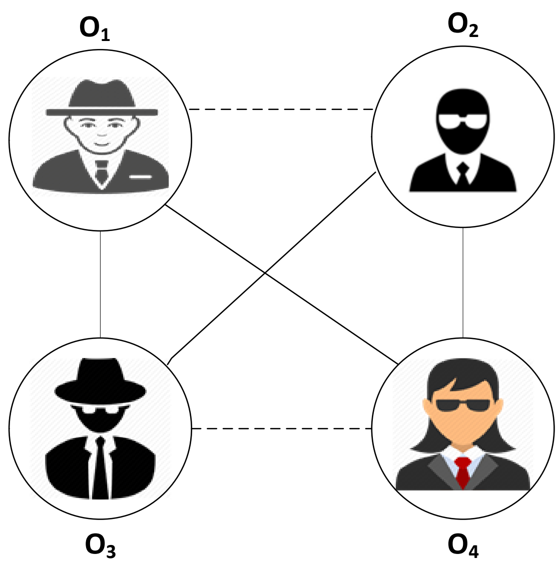

Figure 5.6: Conflict graph of Table 5.5

The corresponding conflict graph shown in Figure 5.6 depicts clearly that object O_1 is in alliance with O_2 and conflicting with O_3 and O_4. While object O_3 is in alliance with O_4 and conflicting with O_1 and O_2.

5.4.4 A Soft Set Approach for Conflict Analysis of Urban Planning in Nigeria

In this section, a soft set approach for conflict analysis a case study of Abuja, the capital city of Nigeria, is presented.

5.4.4.1 Urban Planning in Nigeria Problems

Urban planning is one of the problem facing many cities in Nigeria. In this study, we look at Abuja which is the capital city of Nigeria as our reference area, many of the problems of urban planning in Abuja is as a result of wide spread of illegal and informal development which cause by negligence of the authority to either come with widely accepted policy or lack of enlighten the public. Hence, several development projects conducted in the area does not consider environmental factors, and this leads to potential medical issues and other hazards such as confusion, flooding, blockage, etc.. This has obstructed the extension of not just water, electricity and solid waste collection services, but also sufficient sanitation and road networks system to such areas. The inability of the authority concern to engage public in policy and decision making program also contributed to the poor planning policy. By these overwhelming issues confronting urban arranging notwithstanding fast urban development and urbanisation in Abuja the capital city of Nigeria. In this study, a survey of the conflict that mostly occurred due to the communication gap between authorities and public in urban planning was presented. Figure 5.7 shows the study area where all the opinions are collected and analysed.

Figure 5.7: Urban Planning in Nigeria (study area)

5.4.4.2 Conflict Analysis

The primary objective of a conflict analysis is to find the relationship between objects (agents) that are having some dispute and find the possible resolution of the dispute (Pawlak, 1998). In this study, the public opinions for urban development based on soft set approach is considered in which the public is divided into four interest groups as User group (*P*), Professionals town planners (*Q*), Pressure group (*R*), and Business group (*S*). With this approach, conflict analysis will use mathematical tools to systematically analyse the opinions of public urban planning. Table 5.13 gives the decision table for Public engagement program for town planning which can be used to examine the relationships between interest parties involved in the conflict and find the possible resolution.

Table 5.13: Decision table for public engagement program for town planning

Fact	Interest group	Opinions	Supports
1	P	1	134
2	P	0	48
3	P	–1	36

Table 5.13:, Continued

4	Q	1	43
5	Q	0	5
6	Q	–1	3
7	R	1	17
8	R	–1	52
9	S	1	50
10	S	0	2
11	S	–1	60
	Total		450

5.4.4.3 Results and Discussion

This section illustrate conflicts analysis in public engagement program, a survey with a sample of 450 people around the area of study were conducted, and the participant is classified into four groups as User group (*P*), Professionals town planners (*Q*), Pressure group (*R*), and Business group (*S*). The decision table describes condition attribute as Interest group and decision attribute as opinion, Table 5.13 gives the views of each interest group on engaging public in urban planning. We denoted the opinion as 1, 0 and –1 i.e. Alliance, neutral and conflicts, respectively.

Let the conflict situation be presented in a multi-valued information system (U, A, V, f) where the domain agents and function of interest group are defined as:

$$U = \begin{Bmatrix} (1, user\ group(P)),\ \cdots, (218, user\ group(P)) \\ (219, town\ planners(Q)),\ \cdots, (269, town\ planners(Q)) \\ (270, pressure\ group(R)),\ \cdots, (288, pressure\ group(R)) \\ (289, bu\sin ess\ group(S)),\ \cdots, (350, bu\sin essgroup(S)) \end{Bmatrix}$$

and

$$f(1,P)=\cdots= f(134,P)=1,\ f(135,P)=\cdots= f(182,P)=0,\ f(183,P)=\cdots= f(218,P)=-1,$$

$$f(219,Q)=\cdots= f(261,Q)=1,\ f(262,Q)=\cdots= f(266,Q)=0,\ f(267,Q)=\cdots= f(269,Q)=-1,$$
$$f(270,R)=\cdots= f(286,R)=1,\ f(287,R)=-1,\ f(288,R)=-1\ ,$$

$$f(289,S)=\cdots= f(344,S)=1,\ f(345,S)=0,\ f(346,S)=0,\ f(347,S)=\cdots= f(350,S)=-1,$$

The conflict situation is generated in Table 5.11 and then the decomposition of this table into a soft set where illustrated in Tables 5.12–5.16. In which all the interest group form it soft set.

Table 5.14: The conflict situation with agents (participate in the survey)

Participant, Interest group	Opinions	Participant, Interest group	Opinions
(1,(User group(P))	1	(267, (Town Planners(Q))	–1
(2,(User group(P))	1	(268, (Town Planners(Q))	–1
…	…	(269, (Town Planners(Q))	–1
(134,(User group(P))	1	(270, (Pressure group(R))	1
(135,(User group(P))	0	…	
(136,(User group(P))	0	(286, (Pressure group(R))	1
…	…	(287, (Pressure group(R))	–1
(182, (User group(P))	0	(338, (Pressure group(R))	–1
(183, (User group(P))	–1	(339, (Business group(S))	1
…	…	…	…
(218, (User group(P))	–1	(388, (Business group(S))	1
(219, (Town Planners(Q))	1	(389, (Business group(S))	0
…	…	(390, (Business group(S))	0
(265, (Town Planners(Q))	1	(391, (Business group(S))	–1
(262, (Town Planners(Q))	0	…	…
…	…	(449, (Business group(S))	–1
(266, (Town Planners(Q))	0	(450, (Business group(S))	–1

Table 5.15: The decomposition of user group response into multi-soft set

	User group (P)		
Participant	1	0	–1
1	1	0	0
2	1	0	0
⋮	⋮	⋮	⋮

Table 5.15:, Continued

133	1	0	0
134	1	0	0
135	0	1	0
⋮	⋮	⋮	⋮
181	0	1	0
182	0	1	0
183	0	0	1
184	0	0	1
⋮	⋮	⋮	⋮
216	0	0	1
217	0	0	1
218	0	0	1

Table 5.16: The decomposition of professional town planner's response into multi-soft set

	Professional town planners (Q)		
Participant	1	0	–1
219	1	0	0
210	1	0	0
⋮	⋮	⋮	⋮
260	1	0	0
261	1	0	0
262	0	1	0
⋮	⋮	⋮	⋮
265	0	1	0
266	0	1	0
267	0	0	1
268	0	0	1
269	0	0	1

Table 5.17: The decomposition of pressure group response into multi-soft set

	Pressure group (R)		
Participant	1	0	–1
270	1	0	0
271	1	0	0
272	1	0	0
273	1	0	0
274	1	0	0

Table 5.17:, Continued

275	1	0	0
276	1	0	0
277	1	0	0
278	1	0	0
279	1	0	0
280	1	0	0
281	1	0	0
282	1	0	0
283	1	0	0
284	1	0	0
285	1	0	0
286	1	0	0
287	0	0	1
…	…	…	…
338	0	0	1

Table 5.18: The decomposition of business group response into multi-soft set

	Business group (S)		
Participant	1	0	-1
339	1	0	0
340	1	0	0
341	1	0	0
342	1	0	0
⋮	⋮	⋮	⋮
387	1	0	0
388	1	0	0
389	0	1	0
390	0	1	0
391	0	0	1
392	0	0	1
…	…	…	…
450	0	0	1

Based on the soft set approach the decision table can be obtained as shown in Table 5.14. The decision table in Table 5.14 above can be associated with the flow graph i.e. a directed, connected, acyclic graph. This illustrates that the strength of the decision rule represents a through the flow of the corresponding branch. The flow graph associated with Table 5.14 shown in Figure 5.8. Branches of the flow graph show the decision rules together with their strength, certainty and coverage factors. The strength which is the percentage of support of fact, the certainty which is the percentage of supports to a particular opinion within a group and coverage which is the percentage of support of a group to a particular opinion among all who support the opinion. The strength, certainty and coverage values are given in Table 5.19.

Table 5.19: Strength, certainty and coverage of the survey

Fact	Strength	Certainty	Coverage
1	0.298	0.615	0.549
2	0.107	0.220	0.873
3	0.08	0.165	0.238
4	0.096	0.843	0.176
5	0.011	0.098	0.091
6	0.007	0.059	0.020
7	0.038	0.246	0.070
8	0.116	0.754	0.344
9	0.111	0.446	0.205
10	0.004	0.018	0.036
11	0.133	0.536	0.397

From the flow graph in Figure 5.8, we can see that the certainty factors are:

- User group (P) show that 61.47% are in alliance with the issue for engaging public in urban planning. While 22.02% are neutral to the issue and 16.51% are in conflict on the issue.
- From the coverage factors for User group (P) is 54.92%, 17.6% from Professional town planners (Q), 7% from Pressure group (R) and 20.5% of Business group (S) are all in alliance for the issue.

From the flow graph, the decision rule $P \rightarrow 1$ has the certainty and coverage valued of 0.615 and 0.549 respectively. Now comparing the certainty and coverage among the interest group the main opinion can be plotted as the arrow as depicted in Figure 5.8. moreover, it can be concluded by saying User group (P) and Professional town planners (Q) form a coalition and Professional town planners (Q) and Business group (S) while

Professional town planners (Q) and Business group (S) also form an alliance, a conflicting issue and it was depicted more clearly in Figure 5.8-5.9.

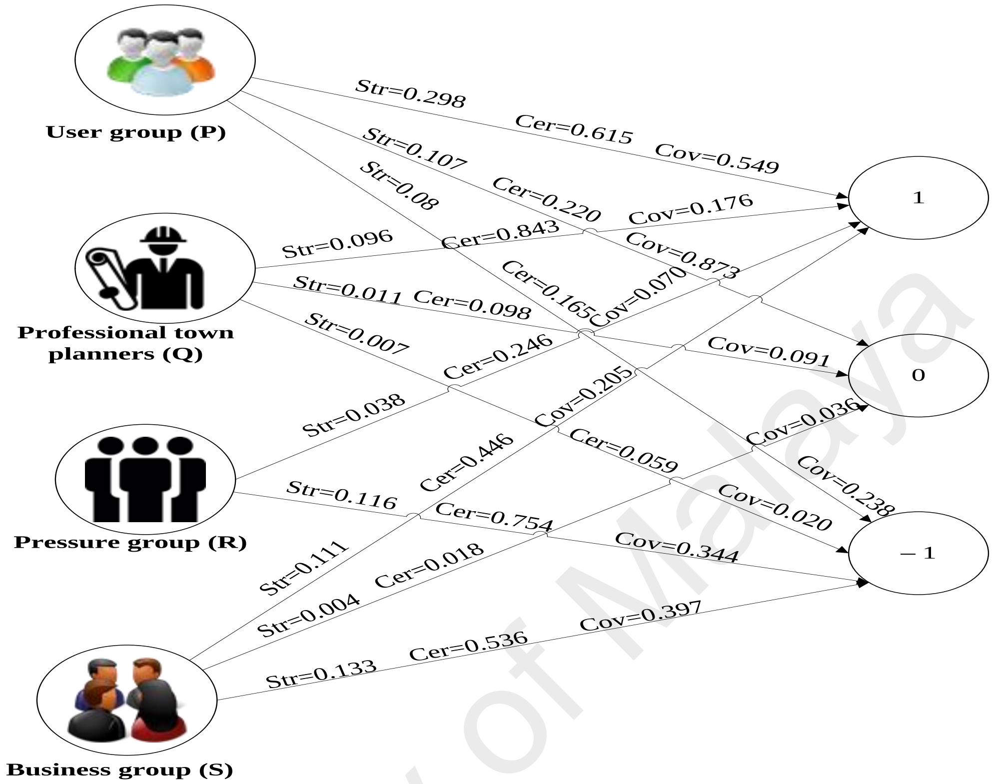

Figure 5.8: Flow Graph of Urban planning conflict

Figure 5.8 depicts the flow graph of Table 5.19 branches of the flow graph represent strength, certainty and coverage for example user group (P) have the strength value as 0.298, certainty value as 0.615 and coverage value as 0.549, respectively. For easy understanding of the conflict situation Figure 5.9 present the approximate graph. Similarly, the number of return opinion from the four different group are not equal.

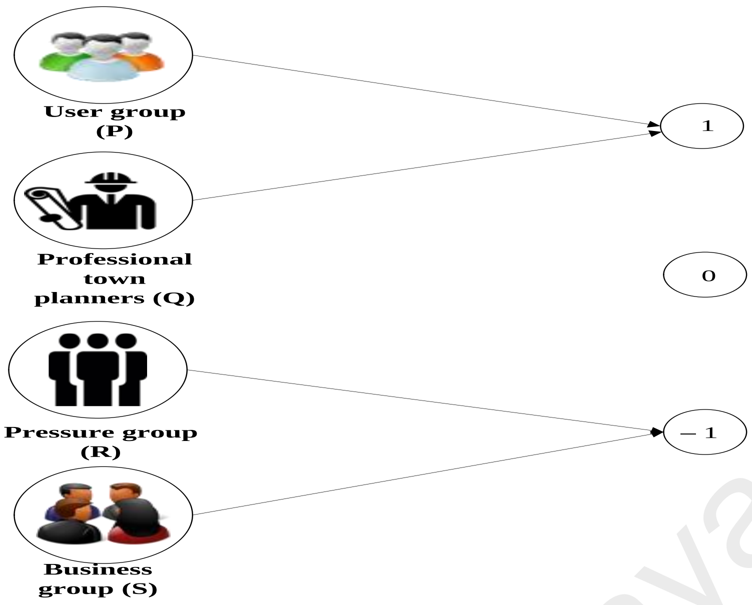

Figure 5.9: Approximate graph of urban planning conflict

Figure 5.9 clearly shows the approximate graph for User group (P) and Professional town planners (Q) form an alliance, which are in conflict with Pressure group (R) and Business group (S). The corresponding conflict graph shown in Figure 10 depict clearly that object User group (P) is in alliance with Professional town planners (Q) and conflicting with Pressure group (R) and Business group (S). While Pressure group (R) and Business group (S) form an alliance and conflicting User group (P) and Professional town planners (Q).

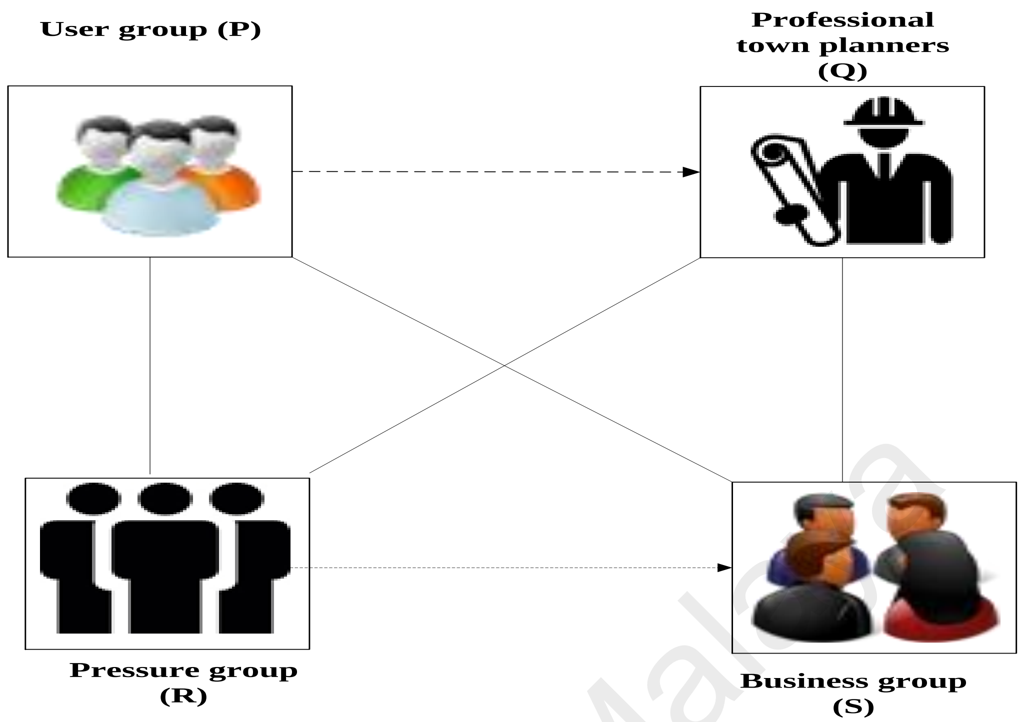

Figure 5.10: Conflict graph of urban planning conflict

5.4.4.4 Comparison Results

The comparison on the computational time of the proposed approach with baseline rough set approach (Pawlak & Skowron, 2007) is presented. Both proposed algorithm and method in (Pawlak & Skowron, 2007) are implemented in Matlab version 7.6.0.324 (R2008a). The execution is done sequentially on an Intel Core i3 CPUs processor. The main memory is 4GB, and the operating system is Windows 10. Figures 5.11, 5.12, 5.13, and 5.14 respectively show the comparison results in term of computational time on computing support, strength, certainty, and coverage for all conflict facts in Table 5.19.

The strength, certainty and coverage factors shown in Table 5.19 above clearly defined the degree of subjective belief on the actual value of a certain issue in this was case engagement of the public in urban planning to reduce the level of conflicts. These factors can be used to show the validity of a fact.

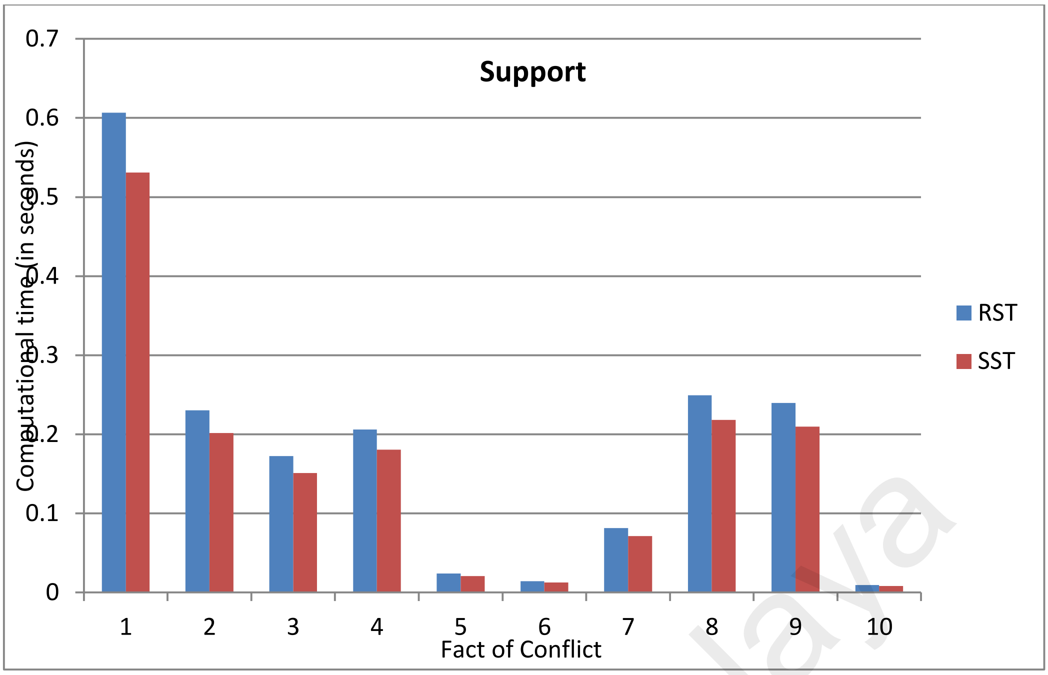

Figure 5.11: Computational time on computing support

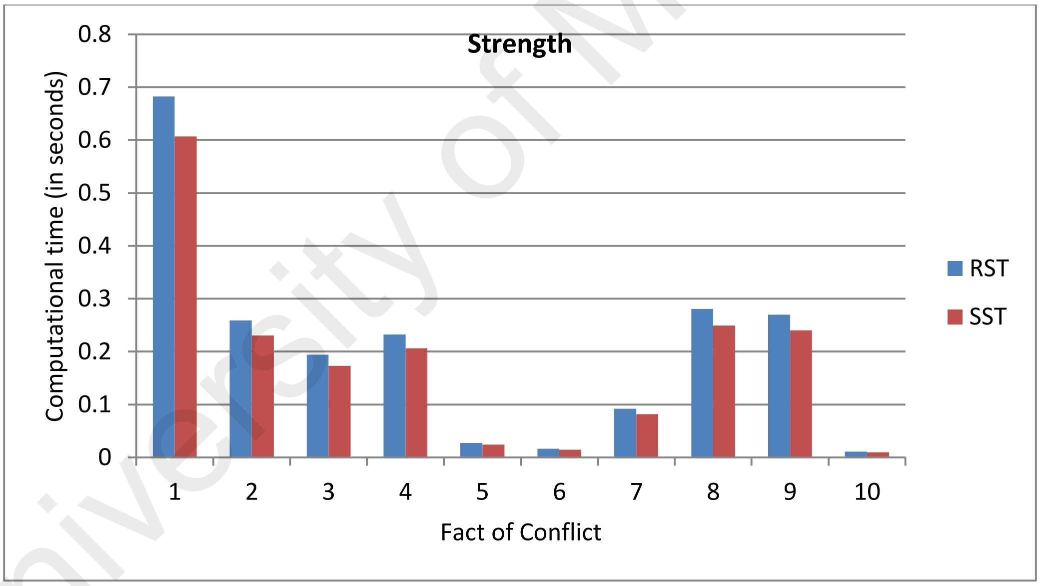

Figure 5.12: Computational time on computing strength

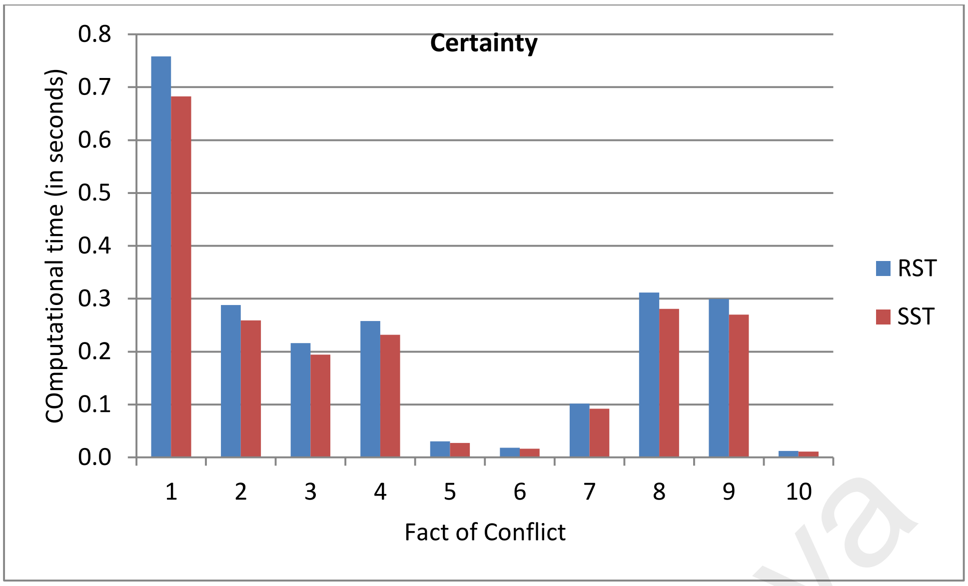

Figure 5.13: Computational time on computing certainty

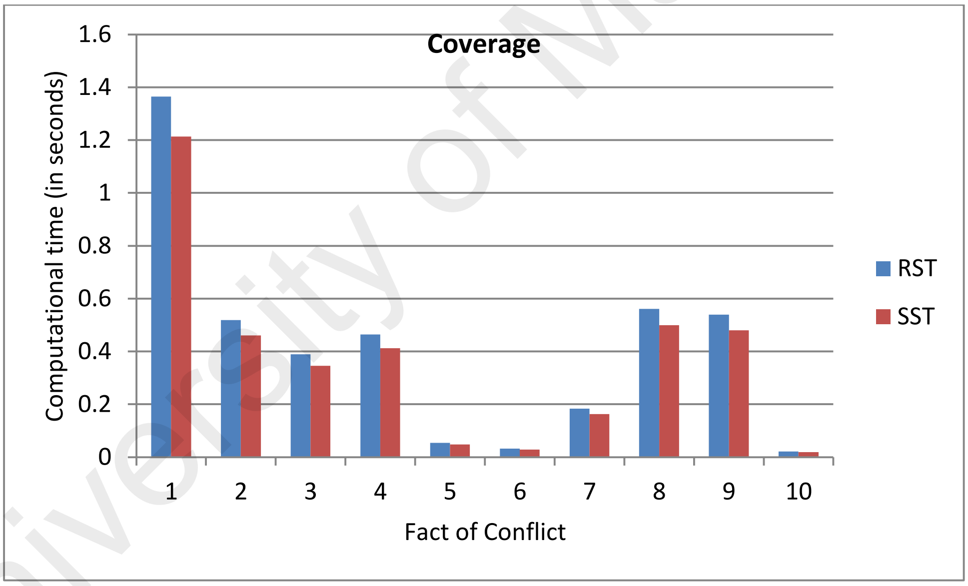

Figure 5.14: Computational time on computing coverage

It can be seen from Table 5.20, that the proposed soft set theory (SST) based approach achieve lower computational time as compared to existing rough set theory based technique (RST) up to 11.18%.

Table 5.20: Computational time improvement

No	Comparison	Average of computational time (in second)		Improvement (%)
		RST	SST	
1	Support	0.1835	0.1605	12.53
2	Strength	0.2064	0.1835	11.09
3	Certainty	0.2293	0.2064	9.99
4	Coverage	0.4128	0.3670	11.09
Average improvement				11.18

5.5 Summary of the chapter

An Alternative Normal Parameter Reduction algorithm was presented that overcome problem of the existing algorithms ranging from computational complexity, difficulty in understanding of the algorithms and implementing it within any data set with ease. The ANPR algorithm was also compared with the algorithm presented by (Ma, et al., 2011), and it evidently shows that the proposed algorithm has reduced the computational complexity. Furthermore, in this research, the application of soft set in were also presented for handling conflict situation of public engagement program for urban planning. Moreover, the interpretations of the result obtained have demonstrated that this technique offers the following merits. The conflict analysis using the traditional survey to show the opinion are sometimes lead to biased from various interest groups more especially those who are eager to voice out their views for political reason or otherwise. With the help of this soft set approach in analysing the opinions, we can easily eliminate biased opinion. Moreover, the approach shows the efficiency of the proposed algorithm in term of lower computational as compared to rough set-based approach. The next chapter gives the summary, conclusion and future direction of the study.

CHAPTER 6: CONCLUSION AND FUTURE WORK

6.1 Introduction

This chapter discussed all the findings of this research in summary and makes the conclusion of the findings as well as providing future direction of state of the art.

6.2 Reappraisal of the Research Objectives

The first objective of this research is to analyse the existing algorithms of parameter reduction and decision making in soft sets. In order to achieve this objective, an extensive literature review was conducted that gives more insight into the soft set theory and its application in parameter reduction and hybridization of the soft set with other set theory. Analysis of different parameter reduction techniques and its applications to the real world was also discussed in detail and point out unresolved problems and future research direction in the applications of the soft set and hybrid soft sets in parameter reduction and decision making. Based on the literature we studied normal parameter reduction and its applications have received diminutive or to say little attention from researchers such as computing parameter reduction by considering different cases like inconsistency, missing attribute value of decision making and information system. This however, motivated us to contribute and propose novel methods and applications for parameter reduction.

The second and third objective is to develop an alternative technique for parameter reduction and decision making using a soft set approach. In order to realise this objective, an algorithm was proposed and tested with both synthetic and real-life data set. The algorithm does not involve decision partition and degree of the parameter but it rather considers same parameters and same entries to reduce the parameter. The alternative algorithm shows better result and can achieve better computational time due to the lower complexity of the algorithm.

The fourth objective is to validate the developed algorithm by comparing it with the baseline algorithms on the real-life datasets. This is achieved by running the proposed algorithm and the two baseline algorithm that is a New Efficient Normal Parameter Reduction algorithm (NENPR) and Normal Parameter Reduction algorithm (NPR). The results are compared in terms of complexity and running time using synthetic and real-life data sets. It is found that the proposed algorithm has a better result and manage to reduced complexity.

6.3 Discussion

Several researchers analysed the situation of vagueness and uncertainty, and soft set provides the tools to handle such vagueness and uncertainties. However, the problem of this uncertainties occurred when the number of parameters is vast which normally increased the candidates' subsets. To solve this problem, it is imperative to study the characteristic of data and discovers their relations to determine the parameters that can be reduced. Moreover, every soft reduction algorithm identifies how the relation between these data goes and how it will affect the decision generation. However, the factor is which algorithm reduces the amount of knowledge and gives important decision quality, of course, it increased when their performance relies on proper functions or properties.

This research focus on removing uncertainty from the dataset using soft set theory, the parameter reduction algorithm where proposed which have excellent strategic in reduction's and its increase accuracy of the decision.

Table 6.1: Performance Rate of the Parameter Reduction Algorithms

Authors	**Reduction Techniques**	**Percentage of Reduction**
Maji et al., 2002	PR	66%
Chen et al., 2005	PR(2)	50%
Kong et al., 2008	NPR	35%
Rose et al., 2010	PR	15%
Ma et al., 2011	NENPR	40%
Rose et al., 2011	Rose et al., 2011	37%
Kumar et al., 2013	Kumar et al., 2013	17%
Danjuma et al., 2016	ANPR	56%

For the purpose of evaluation, the proposed algorithm was compared with the state of the art algorithm and it was found that the proposed algorithm advances the performance of the existing parameter reduction algorithm discussed in the literature i.e. 56% compare to the existing algorithms of normal parameter reduction by (Kong et al. 2008 & Ma et al. 2011) as shown in chapter 4, while other works look only into parameter reduction as shown in Table 6.1. The limitation of NENPR and NPR algorithm is that it requires computing oriented parameter sum, parameter importance degree in which the number of element access as many as possible. The algorithm of NENPR and NPR used the combination that consists of k' columns for the candidate parameter set to test the combination from combination-2 to combinations $(\lfloor n/2 \rfloor)$. However, the ANPR algorithm was only accessing each entry once.

6.4 Future Direction

With the current advances in soft set theory, some research issue is arising which requires attention from researchers this includes multiset in parameter reductions, multi-criteria decision-making problem in an uncertain environment. Moreover, it is clear that all the approaches presented above so far in this area have their respective advantages and disadvantages. This required attention from researchers to develop a more general approach for parameter reduction of the soft set. Furthermore, the application of parameter reduction of the soft set to the practical field has to be explored more by researchers. Only few research work on extending ontology based soft set, manage soft set based decision making under incomplete information in terms of Khan et al., (2016) and application of soft set parameter reduction for classification were found in the literature. A very limited practical application of soft set theory can be found in existing literature and many of them still facing the problem of the performance of their models or algorithms. Thus, these applied that soft set-based research requires more attention by future *soft setters* to develop a more general approach for parameter reduction.

6.5 Conclusion

A soft set is a mathematical tool for handling uncertainty problems. Its efficiency in dealing with uncertainty problems is as a result of its parameterized tools as its main vehicle. In this research, we presented and clarified the voluminous works in this field within a short period of time. Several algorithms exist for parameter reduction, decision making, applied research of soft set and hybrid soft set with other set theories. The different research was reviewed on parameter reduction and decision making in soft set theory and hybrid soft set with other set theories. Several researchers have contributed different type of algorithm for computing parameter reduction by considering different cases like inconsistency, missing attribute value of decision making and information system. Furthermore, an algorithm that overcomes a problem of the existing algorithms ranging from computational complexity, difficulty in understanding of the algorithms and implementing it within any data set with ease was presented. The proposed algorithm was also compared with the algorithm of (Kong et al. 2008; Ma et al. 2011), and it evidently shows that the proposed algorithm has excludes unnecessary parameters and searches for suitable parameters in the feasible sets and finally achieved lower computation.

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LIST OF PUBLICATIONS AND PAPERS PRESENTED

Articles on Research Topic

1. Sani Danjuma, Maizatul Akmar Ismail, and Tutut Herawan. An Alternative Approach to Normal Parameter Reduction Algorithm for Soft Set Theory. IEEE ACCESS (ISI-Cited Publication) Q2. Accepted.

2. Sani Danjuma, Tutut Herawan, Maizatul Akmar Ismail, Adamu I Abubakar, Akram M Zeki, and Haruna Chiroma. A Review on Soft set-based parameter Reduction and Decision Making. IEEE ACCESS (ISI-Cited Publication) Q2. Accepted.

3. Sani Danjuma, Maizatul Akmar Ismail, Khalid Haruna and Tutut Herawan
A Soft Set Approach for Conflict Analysis of Public engagement program for Urban Planning in Nigeria. Journal of Urban planning and Development (ISI-Cited Publication) Q1. (Under Review).

Seminars

1. Postgraduate Research Excellence Symposium (PGRes) Held in Faculty of Computer Science and Information Technology, University Malaya. Malaysia. June, 2015.

Conference

1. Application of Alternative approach Normal Parameter Reduction (ANPR) Algorithm for soft set.