DETERMINATION OF HELIOSTAT NORMAL USING MATLAB

NORAIN BINTI IDRIS

INSTITUTE OF GRADUATE STUDIES
UNIVERSITY OF MALAYA
KUALA LUMPUR

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THESIS SUBMITTED IN FULFILMENT
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ABSTRACT

The second explosion in doing research in solar power was in 1997 as consequence to Kyoto Protocol. This protocol outlined the effect of greenhouse emission which endangers the earth. As the result, research in solar power field started to take its path again. This work is a part of solar project to build the first concentrating solar tower (CST) model in South East Asia and was aimed to develop a template using MATLAB programming for the calculation of the heliostat position with respect to the heat absorber mounted at the top of the CST. This template serves as the calculation platform to control the movement of the heliostat using a two-axis motion system so that the sun light is redirected perfectly to the absorber all day long. Since the heliostat normal vector depends on sun position vector, both vectors were calculated by the program and were set as the output of the program. The input from the user will be the Cartesian coordinate of the heliostat and absorber by taking the absorber tower frontal surface and its base as the origin and also the date. The result will be in vector form and will change automatically according to the sun movement. These values will be programmed in the micro controller which will control the motion system of the heliostat, which will be done by a control team. The program´s functionality was proved via several analytical, numerical and three-dimensional graphical verifications and its accuracy which is 0.0005 metre is stated and verified via comparison with four analytical calculations and a graphical verification using SolidWorks. From the verifications, it can be seen that difference of the numerical and analytical results varied from 0.0000 to 0.0005 m which validates the statement of minimum accuracy of the numerical calculated results is 5/10,000 m.

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LIST OF SYMBOLS

GREEK SYMBOLS

SYMBOL	DESCRIPTION
θ	Angle of Sun ray incidence on heliostat
δ	Celestial declination/inclination
ω	Hour angle
α_S	Solar altitude angle
ϕ	Latitude
γ_S	Azimuth angle
γ_S'	Pseudo azimuth angle
θ_z	Zenith angle
ω_{WE}	Hour angle when the Sun is due to the east or west

FORMULA SYMBOLS

SYMBOL	DESCRIPTION
D or n	Day of the year or Julian day of the year
E_0	Earth-Sun distance
$\hat{\imath}$	Unit vector in z-direction
$\hat{\jmath}$	Unit vector in y-direction
$\hat{k}$	Unit vector in x-direction
$\vec{m}$	Mirror/heliostat vector
$\vec{N}$	Heliostat normal vector
$\vec{r}$	Receiver/absorber vector
$\vec{R}$	Redirection of Sun ray vector
$\vec{S}$	Sun position vector

INDICES

INDEX	DESCRIPTION
n	northern
e	eastern
z	In zenith direction

LIST OF ABBREVIATIONS

INSTITUTIONS/UNIVERSITIES ABBREVIATIONS

ABBREVIATION	DESCRIPTION
CRIM	Centre for research and Innovation Management
DESERTEC	A foundation who propose a concept to make use of solar and wind energy from Sahara desert
DLR	Deutsches Luft- und Raumfahrt (German Aerospace Centre)

MUST	Malaysia University of Science and Technology
STJ	Solar Tower Juelich
UM	University of Malaya
UPM	Universiti Putra Malaysia
UTAR	Universiti Tunku Abdul Rahman
UTeM	Universiti Teknikal Malaysia Melaka
UTM	Universiti Teknologi Malaysia
UTP	Universiti Teknologi Petronas

TECHNOLOGICAL ABBREVIATIONS

ABBREVIATION	DESCRIPTION
CRS	Central Receiver System
CSP	Concentrating Solar Power
CST	Concentrating Solar Tower
DNI	Direct Normal Irradiance
FiTs	Fit-in-Tariff
GPS	Global Positioning System
GUI	Graphic User Interface
HTF	Heat Transfer Fluid
HVDC	High Voltage Direct Current
LFR	Linear Fresnel
MATLAB	Matrix Laboratory

PV	Photovoltaic
RES	Renewable Energy Sources
TES	Thermal Energy Storage

LIST OF APPENDICES

1. INTRODUCTION

1.1. OVERVIEW

Research on solar energy started in 1860´s, but due to increasing usage of fossil fuel, the research in this field has stalled. The next explosion in doing research in solar power was in 1997 as consequence to Kyoto Protocol (Aliman et al., 2007).

This protocol outlined the effect of greenhouse emission which endangers the earth. As result, research in solar power field started to take its path again. The chronology of solar power technology development could be summarized in the following table; Table 1.1:

Table 1.1: Development of Solar Power Technology (Commercial Solar Power Industry History, 2010)

Year	Scientist/Engineers	Achievement
1767	Horace De Saussure	Created first world solar cooker which could reach temperature of almost 190˚F
1816	Robert Stirling	Patent for his solar dish system which created electricity
1866	Auguste Mouchout	Used a parabolic trough to produce steam for the first steam engine
1872	John Ericsson	Developed a solar thermal Stirling Dish concentrated-solar-powered devices for irrigation, refrigeration and locomotion
1878	Augustine Mouchout	First to combine the oven heat trap and burning mirrors concepts to create solar oven
1886	Alessandro Battalia	First patent for solar collector
1901	Aubrey Eneas, John Ericsson	Construction of the world first solar thermal dish with dual access tracking
1907	Frank Schuman	Completes construction of the “direct acting solar engine” with maximum temperature output of 202˚F
1913	Frank Schuman	Finished a 55HP “The No. 1 Sun Engine” parabolic solar thermal station
1936	Charles G. Abbot	Created solar system comprised of three parabolic throughs with tracking system

'Table 1.1, continued'

1968	Professor Giovanni Francia	Designed and built the first solar concentrated plant which entered in operation, able to produce 1MW with superheated steam at 100 bars and 500˚C
2004	Schaich Bergermann	Designed and supervised the construction of a 10kW Stirling EuroDish System
2009	Abengoe	The completion of PS20 20MW power tower

Source: Aliman et al., 2007

Sunlight that reaches the earth is spread out over such a large area. Basically, the Sun does not deliver that much energy to one place at one time (Aliman et al., 2007). Solar energy received at a place depends on several conditions. They actually act as limitations in solar energy capturing process. These include the time of the day, the season of the year, the latitude of area and the clearness or cloudness of sky. Via Concentrating Solar Power (CSP) technology, the solar energy is multiplied from several to few thousand times as it is concentrated onto a certain size of area before it is converted into electric power (which is around 600 to 1000 times). This produces very high temperature. The temperature is in the range of 800°C to 1000°C (Machinda et al., 2011).

The basic CSP components are mirror field, heat exchanger, boiler, turbine, generator and condenser. However, heat exchanger can be omitted if steam is produced directly in the receiver (Van Voorthuysen, 1995). The Concentrating Solar Power system can be divided into two types of system which are line focusing and point focusing. The point focusing system produces higher temperature concentration as sun irradiation is focused on one point compared to line. It also has higher efficiency compared to its counter system (Machinda et al., 2011).

However there are still a lot of research is going on over the world to improve CSP efficiency. For example, some US researchers examine the value of CSP and thermal energy storage (TES) in a number of regions in the southwestern United States (Sioshansi & Denholm, 2010). Another example is demonstrative Solar Tower Jülich (STJ), which is used by Solar-Institut Jülich and Deutsches Luft- und Raumfahrt (DLR) or also known as German Aerospace Centre to improve the efficiency of solar tower receiver or absorber (Idris, 2010). South Africa is in a virtue of using CSP technollogy as it seems viable in this country due to high average annual Direct Normal Irradiance (DNI) as experienced in places such as the Northern Cape in South Africa (Machinda et al., 2011).

Apart from that, there is a non-profit foundation called DESERTEC, proposes a concept to make use of solar and wind energy from the Sahara desert (Machinda et al., 2011, Esmeralda & Moreno, 2011). The objective of the project is to supply 15% of the European electricity demand by 2050 from renewable energy sources in the middle east and north africa (MENA) region through high voltage direct current (HVDC) lines (Esmeralda & Moreno, 2011). The expected investment of the project is roughly 400 billion Euro over 40 years. Concentrated Solar Power Systems location in the Sahara desert is also under the DESERTEC proposal.

Basically, CSP system contains four main elements which are:

a. Concentrator

b. Receiver

c. Some form of transport or storage

d. Power conversion

(Aliman et al., 2007)

Concentrator is a device which is used to concentrate solar irradiation at a receiver. The captured energy in form of heat is then transported to heat water to produce steam which is then used to turn the turbine in order to generate electricity. Apart from that, the captured heat is also used to heat heat transfer fluid (HTF) like molten salt to store the thermal energy for later use.

There are several types of concentrated solar thermal power (CSP) technologies which can be categorized as follows:

a) Linear Fresnel (LFR)

b) Parabolic Trough

c) Dish Stirling

d) Concentrating Solar Power Tower (CST)

Linear Fresnel and parabolic through are categorized as line-focusing systems, while dish stirling and CST are point-focusing systems. However, this work is only focused on CST development in Malaysia.

The Concentrating Solar Tower (CST) system is a Central Receiver System (CRS) which consists of several components known as CRS components. The components are the tower, heliostats, heat transfer fluid (HTF) and receiver (Idris, 2010).

There are several different mediums that can be used as HTF such as molten salt, sodium, sand and air. As an example, Demonstration Solar Tower Jülich uses air to transfer captured heat in a volumetric absorber, which is used as its system receiver (Idris, 2010). The heliostats which are field of distributed mirrors individually track the movement of the sun and focuses the sun irradiation on the receiver or known as absorber on the top of the tower (Machinda et al., 2011, Idris, 2010). The absorber, which is also called as receiver, acts as heat exchanger where HTF collects the captured heat and transfers the thermal energy to heat water to produce steam. The steam is then used to turn a turbine which is connected to a generator to generate electricity (Idris, 2010).

Solar tower is quite a new technology, but despite of this fact, there are several commercial solar towers already built like PS10 and PS20 in Spain ("Solar power tower," 2012). There is also research and demonstration power plant built like Solar Tower Jülich which is used for research purposes ("Solar power tower," 2012). This is a top-down research which means the tower is firstly built to understand how the system functions. From the facing problem,the system is further refined and improved to optimize the existing system.

Table 1.2: Examples of solar tower power plant

Power plants	Installed capacity (MW)	Yearly production (GWh)	Country	Developer/ Owner	Completed
Crescent Dunes Solar Energy Project (in construction)	*110 (U/C)*	500	United States	SolarReserve	2013
Ivanpah Solar Power Facility (in construction)	*392 (U/C)*	1,080	United States	BrightSource Energy	2013
Jülich Demo Solar Tower (Demonstration Plant)	1.5		Germany	Kraftanlagen Muenchen DLR (24)	2008
PS10 solar power tower (Commercial)	11	24	Spain	Abengoa	2006
PS20 solar power tower (Commercial)	20	44	Spain	Abengoa	2009

‘Table 1.2, continued’

Sierra SunTower (Commercial)	5		United States	eSolar	2009
Solar Tres Power Tower (Commercial)	17	100	Spain	Sener	2011

Source: “Solar power tower,” 2012

Apart from that, there are also CST plants in construction phase which are 110MW-Crescent Dunes which is predicted to supply to 75000 homes (“Crescent Dunes, Tonopah, Nevada,” n.d.) and 392 MW-Ivanpah in United States. Both plants are predicted to be completed in 2013. Some examples of solar tower power plant from year 2006 to 2013 can be seen in Table 1.2.

There is still neither demonstrative nor commercial CST implemented in Malaysia yet, as CSP technology in Malaysia is as yet at an early research stage. However, Malaysia is now trying to catch the pace. One of the most important CSP component is concentrator, which is known as heliostat for the case of CST. Some universities in Malaysia is actively involved in research of this area to optimize the concentration of the Sun irradiation by investigating different modes of sun-tracking.

The researches were carried out for investigating the best sun-tracking mode to cut down the cost, optimizing the whole sun-tracking process and to find the best concentration of the sun irradiation resulting in higher efficiency of CST system.

One of the involved universities is Universiti Teknikal Malaysia Melaka (UTeM), which proposes to generate a mini CST model for demonstration purposes. This is a scale-down project before actually constructing the real system in bigger scale. The bigger-scale system will then be used as a demonstrative plant for top-down research for further optimization of CST plant.

The scope of this work is developing a programme by using MATLAB for heliostat normal calculation, which depends on sun position vector. This means that the sun position vector calculation is included in the code. The developed code will then be utilized by Department of Control of Electric and Electronic Engineering Faculty of UTeM to realize the azimuth-elevation mode of sun tracking of the heliostat of mini CST project. Therefore, this work is a platform for controlling the heliostat which is one of most important components for CST technology. In other words, this work is a contributor towards the realization of the first CST model in south east Asia. Work flow and main focus of this work is shown in Figure 1.1:

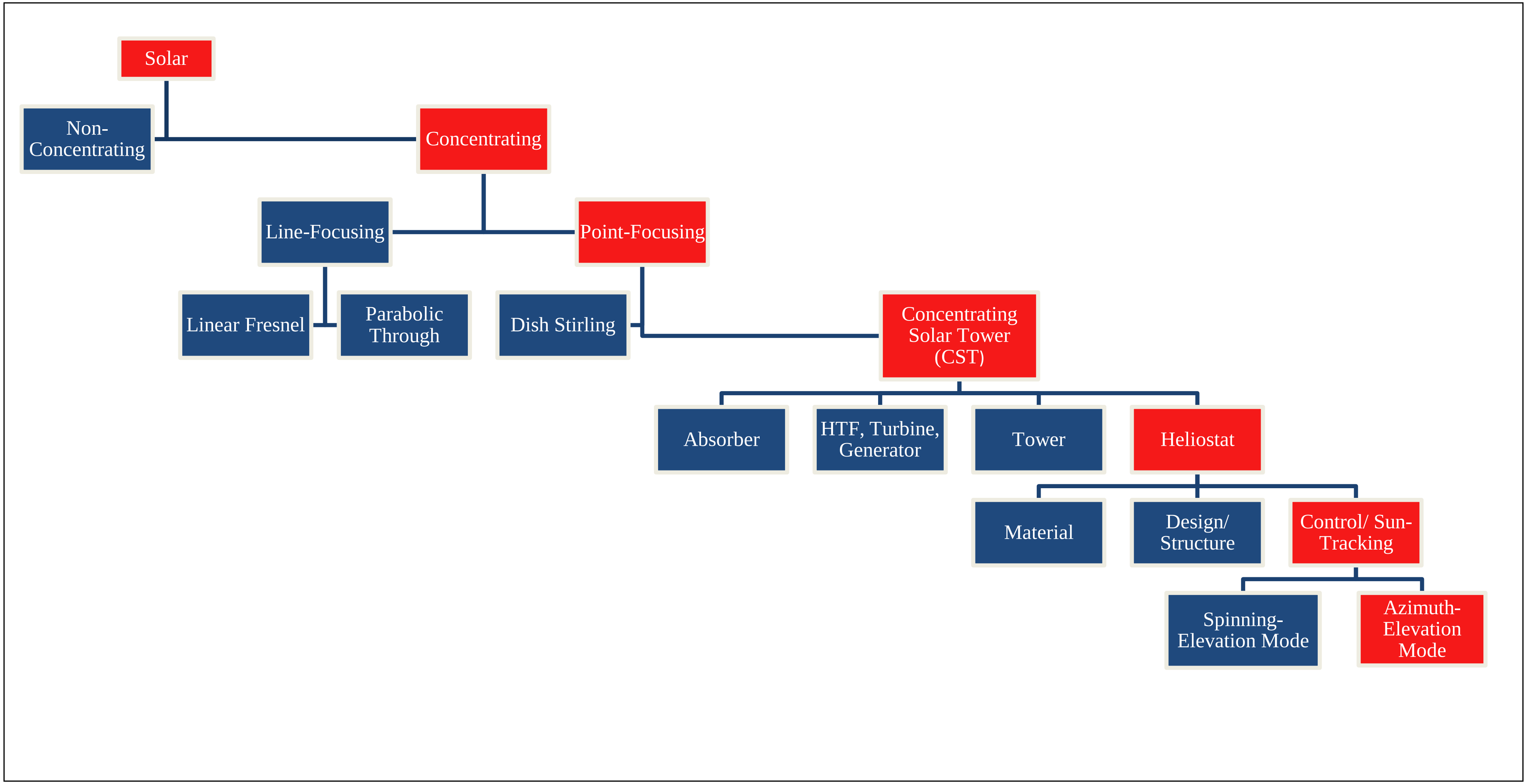


Figure 1.1: Focus area of this work

1.2. PROBLEM STATEMENT AND OBJECTIVES

With a vast solar irradiation in Malaysia especially in Malacca this free energy is unluckily still not being tapped to the maximum. Steps have been taken by the government together with the industrial players to use this energy using photovoltaics (PV) and also solar heater for domestic use. With the limited efficiency and roof space available these technologies are still insufficient to promote green energy and also they are very costly.

Research has been started by the Fakulti Kejuruteraan Mekanikal, Universiti Teknikal Malaysia Melaka (UTeM) to explore the heat from the sunlight for the electricity generation. Budget allocation for the feasibility studies as well as for the modeling of the solar thermal power plant has been approved by the Center for Research and Innovation Management (CRIM) of UTeM. The project will cover the calculation of solar irradiation, the heliostat configuration and control for the first CST in South-East Asia.

This work is part of the project and the scope of the study is to develop a template using MATLAB programming for the calculation of the heliostat position with respect to the heat absorber mounted at the top of the CST. Verification of functionality of the code is also included in the scope. This template will serve as the calculation platform to control the movement of the heliostat using a two-axis motion system so that the sun light will be redirected perfectly to the absorber all day long.

The reasons for the study are to join Malaysian researchers to diversify solar energy harvest method and to help the Control Department team of UTeM to realize heliostat sun-tracking to the point of programming of sun position and heliostat normal calculation. The tasks after installation such as experimental investigation and simulation are not included in this work. They will be executed by the control team.

The input from the user will be the Cartesian coordinate of the heliostat and absorber by taking the absorber tower frontal surface and its base as the origin and also the date. The result will be in vector form and will change automatically according to the Sun movement. These values, which will be then used by the Control Department of UTeM to control the motion system of the heliostat, will be programmed in the micro controller using C programming. It will be connected to servomotor installed on heliostat to control its axis motion.

The objectives of this master dissertation are outlined as follows:

1. To calculate and to develop the corresponding code of sun position vector by using MATLAB.
2. To calculate and to develop the corresponding code of heliostat normal vector based on the calculated sun position by using MATLAB.
3. To verify the functionality of the developed code and to check accuracy of the numerical generated vector compared with analytical calculated vector.

2 LITERATURE REVIEW

There are a lot of ongoing research in Concentrated Solar Thermal Power in Malaysia. This gives a good sign that Malaysia is in the right path to make together use of clean and always-available energy, which is solar energy.

Malaysian government has taken some measurements to promote the development of renewable energy specifically solar energy such as fit-in-tariff (FiTs), law and economic encouragement (Fayaz et al., 2011). The government also supports research by allocating budget for this purpose via grant. This promotes rapid research in this sector.

Amongst the researchers is Universiti Teknologi Malaysia (UTM), which had developed and demonstrated a prototype of solar (heat) engine in 2007. It is an adoption of modifying a commercial internal combustion engine into a Stirling engine driven by concentrated solar energy. The system which consisted of a new proposed rotation elevation tracking mode heliostat is mechanically designed and developed for sun tracking and a computer designed optical receiver. The system also use fabricated multi-fold cones with mirror arrays to further concentrate sunlight which is specially developed for related experiment for their research (Aliman et al., 2007).

Other than that, in 2003, Universiti Teknologi Petronas (UTP) designed a solar thermal cylindrical parabolic trough concentrator by simulation (Mahinder Singh, 2003). There are also some other universities which are actively taking part in this area of research.

There is also review paper written by Universiti Putra Malaysia (UPM) which is published in 2010 on methodology to design and fabricate solar dish concentrator and represents efficiency parameters that can be altered to increase the performance under Malaysia´s climate (Kadir & Rafeeu, 2010). Apart from this, methods for deriving solar radiation from satellite data in Malaysia is also developed (Applasamy, 2011). This is very important as wrong peak sunshine data will lead to wrong equipment sizing for the solar projects (Ahmed & Sulaiman, 2003).

Those are examples of ongoing research and also paper written which helps CSP development and realization of CSP commercialization in the near future. One of the feature that is need to be considered in commercializing the CSP system is storage system to make energy stored during the day dispatchable as daily peak consumption takes place after the sunset (Cabeza et al., 2012).

But since this work only focuses on one type of CSP technology which is concentrated solar tower or CST and only on one of its component, which is heliostat, let us look at the overview of heliostat development. Since this technology is quite new and still not mature, bigger regional scope is used here i.e. the heliostat development is not only in Malaysia, but also in other asien countries is observed.

Here are some reviews of about ten years back and recent development of heliostat in Malaysia and other Asian countries. Researches to improve the existing heliostat prototype are actively carried out until today.

In 2001, UTM in Skudai, Johor has suggested a design for a single layer sun-ray concentrator, a non-imaging focusing heliostat. The design use several small mirrors to form a heliostat to reduce first order image defect, however this design faces several problems such as significant high weight, setting control of slave mirrors and cost-effective issue. The university is now actively doing research to solve the problems.

Prototype of the design is used so far as Stirling Engine power source by making full use of always-available solar energy (Chen et al., 2001).

The image defect of heliostat attracts interest of several universities in Malaysia. Other than UTM, Malaysia University of Science and Technology (MUST) also working on the same area, but instead doing research for first order image defect (aberration), they working on residual aberration for non-imaging focusing heliostat in 2003. Higher order of aberration is caused by fabrication and inheritance. The university, which situated in Petaling Jaya, Malaysia tried to amend the image defect to certain degree by adjusting the pivot point of mirrors and their responding tilting angles (Chen et al., 2003).

Japan also show its interest as an institute in Tokyo; The Institute of Applied Energy has designed a heliostat for usage in any country at Equator in 2006, which tracks the Sun using photo sensor. The developed heliostat is not only well function in clear weather but also in cloudy condition by using additional cloud sensor, which makes it stable to function under this condition. Carried tests show that this heliostat has relatively small error during operation (Aiuchi et al., 2006).

Universiti Tunku Abdul Rahman (UTAR) contributed in heliostat research by deriving general formula of system for all existing-method on-axis sun-tracking system in 2009. The formula derivation is not only used for tracking the Sun, but also utilized to improve persistence and reduce installation error of the mentioned solar collector (K. K. Chong & Wong, 2009).

In 2010, the Laboratory of Solar Thermal Energy and Photovoltaic System of Chinese Academy of Sciences, Beijing has presented something on spinning-elevation method in its paper. The paper consists of basic tracking formulas, formula of elevation angle for heliostat with a mirror-pivot offset, and a more general formula for the biased elevation angle. The formulas are tested to ensure their accuracy in operation. This paper also presented several tracking tactics in practice (Guo et al., 2010).

The most active Asian country in this research area, which is China, has once more published a paper on design and analysis of heliostat field layout code for CST in 2010. In the paper, they developed the code by setting the CST components such as tower as boundary conditions and the code are tested for toroidal and also spherical heliostat field and comparison between those two layouts is made. They also tried to use the land, where the heliostats are placed, to the maximum by developing method for calculating the annual distribution of sunshine duration on the land surface for appropriate crop growing purpose (Wei et al., 2010).

In 2011, once more UTAR came out with a paper related to heliostat. In the paper, UTAR did comparison between two methods of sun tracking, which are Azimuth-Elevation and Spinning-Elevation methods. The scopes covered in the paper are details of application of both methods and also annual total angles of a single heliostat and also field of heliostats for both Azimuth-Elevation and Spinning-Elevation methods (K. Chong & Tan, 2011).

One of institutes of Chinese Academy of Sciences, China appointed equations of sun irradiation tracking for CST in 2011. By using the derivation as guidelines, a module, which is used to analyze an asymmetric-surface heliostat is generated and included in HFLD code. For validation purpose, a heliostat with toroidal surface is designed and modeled. The resulting image from the heliostat is calculated by using modified code of the HFLD code and result comparison is also done by calculating the output by means of commercial software; Zemax. The results show good coincidence with each other (Wei et al., 2011).

In 2011, an institute in China derived altitude-azimuth tracking inclination formulas from their previous derivation of spinning-elevation tracking geometry, which they claimed to be very accurate and almost-error-free. Therefore, due to the accuracy, normal of the centre of mirror surface is also claimed as accurate for any dual-axis tracking heliostat and can be utilized to derive general formulas of altitude-azimuth tracking inclination for pivot-adjusted-mirror heliostat and also geometrically default heliostat. Validation procedures are carried out experimentally and also done via tests (Guo et al., 2011). In 2012, they once more published a paper related to the topic, by taking zero-angle-position into account and therefore, the previous formulas are slightly modified. They used some parameters, which are found from the least-square-fit- and the classical-Hartley-Meyer-solution-algorithm-based conducted experiments (Guo et al., 2012).

Summary of the development of CSP technology and heliostat in Malaysia are tabulated in Table 2.1 and Table 2.2 respectively.

Table 2.1: Concentrating solar power (CSP) technology development in Malaysia

Researcher	Contribution	Year
UTP	Designed a solar thermal cylindrical parabolic trough concentrator by simulation	2003
UTM	Developed and demonstrated a prototype of solar (heat) engine	2007
UPM	Worked on methodology to design and fabricate solar dish concentrator and efficiency parameters alteration under Malaysia's climate	2010

From Table 2.1, it can be concluded that Malaysia started to diversify solar-harnesting devices other than photovoltaic. For example, by using parabolic trough and solar dish which are included in CSP technology. But, since the major focus of the work is heliostat for concentrating solar tower (CST) technology, Table 2.2 is more of interest.

Table 2.2: Heliostat development in Malaysia

Researcher	Contribution	Year
UTM	Suggested a design for a single layer sun-ray concentrator, an non-imaging focusing heliostat	2001
MUST	Worked on residual aberration for non-imaging focusing heliostat.	2003
UTAR	Derived a system general formula for all existing-method on-axis sun-tracking system	2009
UTAR	Published a paper on comparison between two sun-tracking methods	2011

Based on Table 2.2, it can be concluded that the major focus of heliostat development in Malaysia lie in correcting heliostat aberration but it is not applied for CST technology. Instead, it is applied inter alia for Stirling engine. There is however some research done in sun-tracking field, but contribution from the research is only theoretical and they are not proved experimentally.

Therefore, this work helps to fill the gap to realize an experimental heliostat sun-tracking system to the point of sun position and heliostat normal calculation via MATLAB programming. It is important to be clear that further programming into controller board which connects to mechanical devices for the heliostat sun-tracking realization is out of the work scope. It will be done by a control team.

3 METHODOLOGY

3.1 RESEARCH PROCEDURES

The task is tackled by firstly understanding the theory behind sun-tracking procedures and its corresponding calculation. All related calculations inter alia sun position, azimuth angle, heliostat normal, the sun irradiation redirection vector are understood.

Second step is an analytical calculation of a sample case. This is done to give an overview and act as reference values to ensure that the output values from to-be-developed code are correct. Only after achieving this, the code started to be developed.

Next step is the most time-consuming procedure; developing the code. Every calculation step which is taken from the theory part should be converted to MATLAB code. The input should be the date (no. of month and no. of day are entered separately), the Cartesian coordinate of heliostat, the Cartesian coordinate of the tower. Latitude is set as a fixed value in this code, which is the latitude of UTeM. Whereas output are sun position and heliostat mirror normal vector. Since the name 'MATLAB' itself means 'Matrix Laboratory', the output are in matrix form. The code is developed for the utilization between 7 a.m. and 7 p.m. as the sun is assumed to be only available during this time range.

After the developed code is finished, the verification of the result is done by testing the code of several different dates and comparing the outputs with analytical calculation for several cases of different dates with selected solar time. The accuracy of the calculated numerical results will be found and stated afterwards by comparing them with analytical calculated results. After the outputs coincide with the calculated results, the graphical verification is then executed. The code is then ready to be used after all the verification processes are done. For simpler clearer picture and for better understanding of the methodology used, please refer next figure; Figure 3.1:

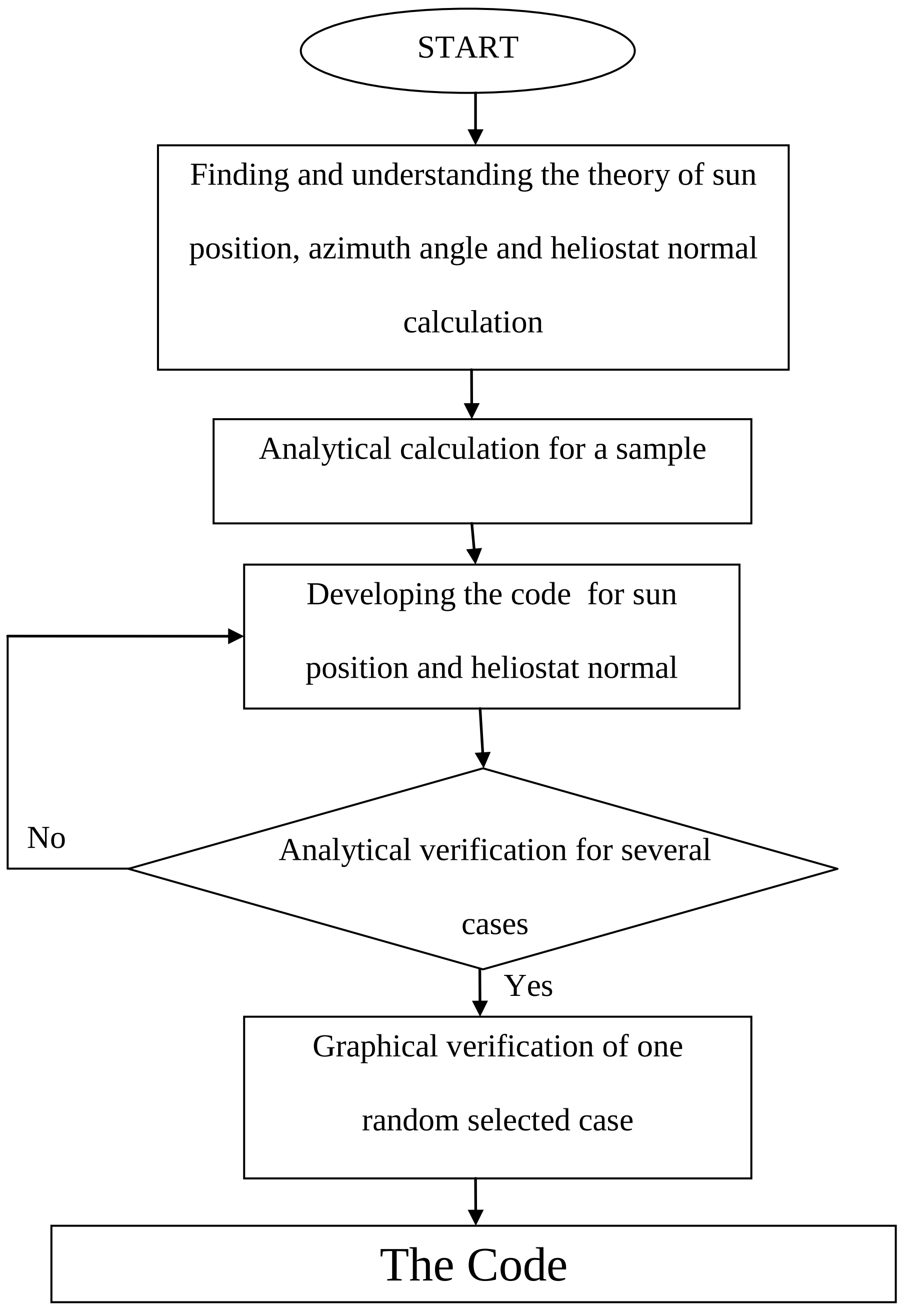


Figure 3.1: The methodology flowchart

3.2 THEORY OF SUN POSITION AND HELIOSTAT NORMAL CALCULATION

In this subtopic, all related components to calculate the end result, which is the main objective of developing the code; heliostat normal vector , are shown step by step. The components are inter alia sun position $\vec{S}$ and redirection of sun ray vector $\vec{R}$ and angle of sun ray incidence on mirror and angle of redirection of the sun ray towards the receiver (since angle of incidence and reflection have the same value) or denoted as θ. Please refer below figure; Figure 3.2 for clearer picture of geometric relationships of those vectors.

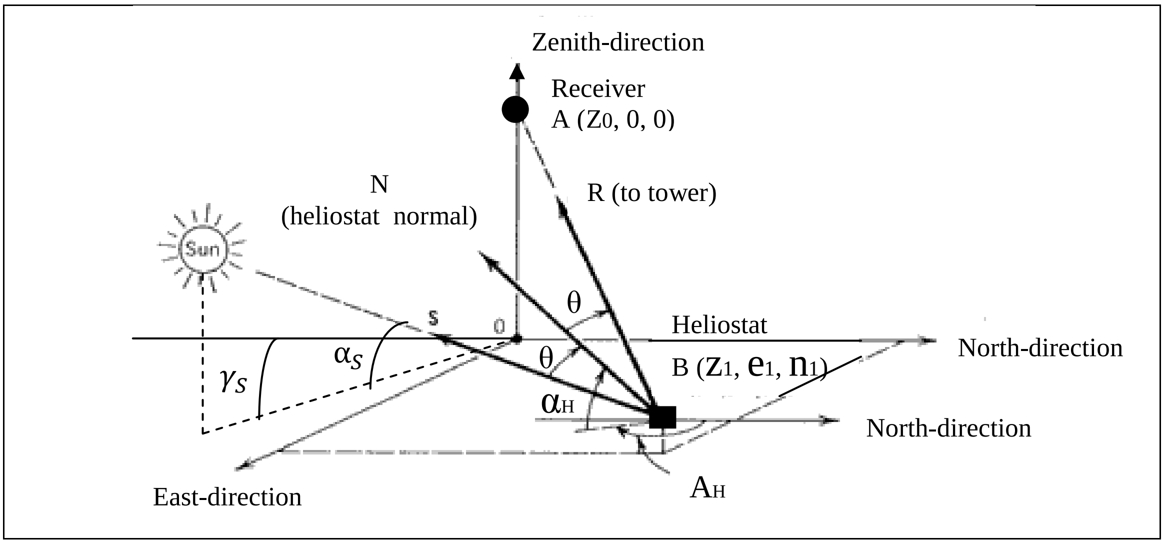


Figure 3.2: Geometric relationships between heliostat and receiver in a zenith-east-north Cartesian coordinate system (Foster et al., 2010)

It is important to note the coordinate system used before calculations are proceeded. Based on the Figure 3.3, the heliostat or mirror and receiver or also known as absorber vectors utilize the tower base as reference. The vectors are denoted as $\vec{m}$ and $\vec{r}$ respectively and these vectors are user's input. The x-axis lies right under the tower base itself, whereas z-axis coincide with the front surface of the tower, if both axes are observed from the side view of the tower. Y-axis lies in the middle of the tower if it is observed from bird's eye view.

It also can be extracted from the Figure 3.3 that sun position vector $\vec{S}$ and heliostat normal vector $\vec{N}$ take the center of the heliostat as their origin. These two vectors are the output of the developed program. In other words, there are two different origin used in the calculation. The program user and reader must be clear about the utilized coordinate system to avoid confusion. Please refer next Figure 3.3:

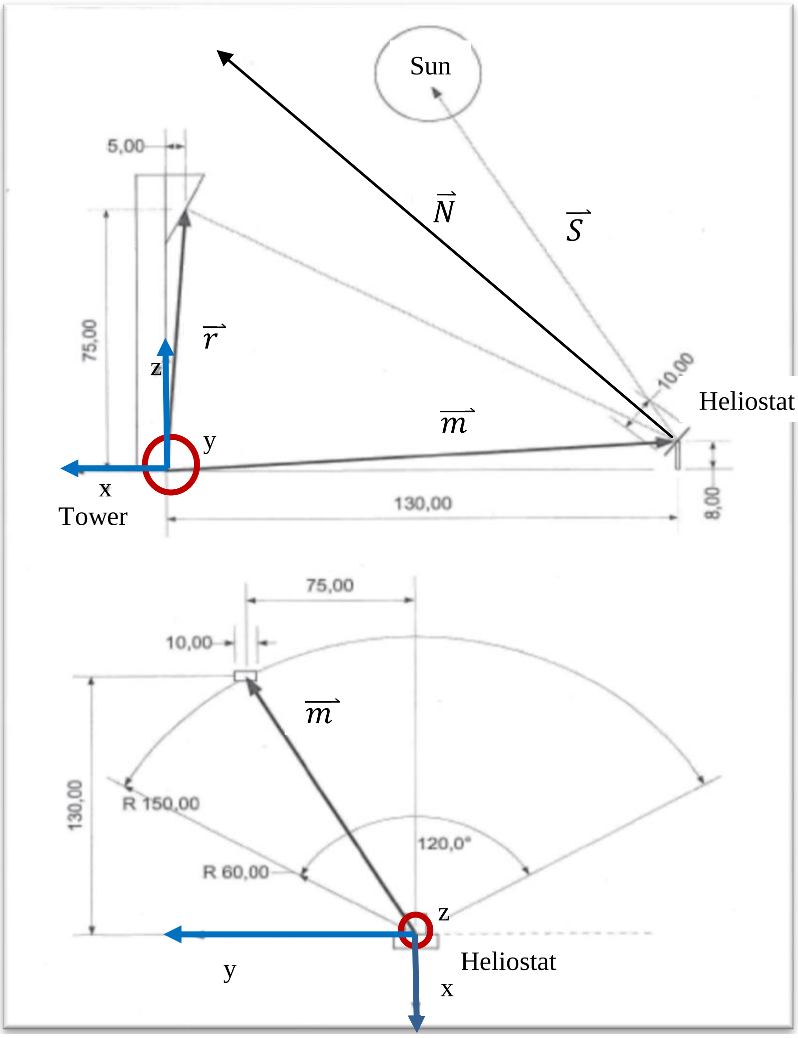


Figure 3.3: Example sketch of side and top view of CST and its coordinate systems

In next subtopics, sun position vector $\vec{S}$, redirection of sun ray vector $\vec{R}$, angle of incidence θ and heliostat normal vector $\vec{N}$ are shown step by step.

3.2.1 SUN POSITION VECTOR $\vec{S}$

To calculate sun position for a certain day, the date of the day must be known and be provided to determine the n_{th} day of the year. If the day of the month which is denoted by i is known, then the n_{th} day of the year can be calculated by using the third column of the following Table 3.1:

Table 3.1: Declination and Earth-Sun Distance of the Representative Averaged Days for Months

*i*th day of the month	Month	*n* or *D* for *i*th day of the month	Julian Day of the year *n*	Declination δ in degrees	Earth-Sun distance *E0* in AU
17	January	*i*	17	-20.92	1.03
16	February	31 + *i*	47	-12.95	1.02
16	March	59 + *i*	75	-2.42	1.01
15	April	90 + *i*	105	9.41	0.99
15	May	120 + *i*	135	18.79	0.98
11	June	151 + *i*	162	23.09	0.97
17	July	181 + *i*	198	21.18	0.97
16	August	212 + *i*	228	13.45	0.98
15	September	243 + *i*	258	2.22	0.99
15	October	273 + *i*	288	-9.60	1.01
14	November	304 + *i*	318	-18.91	1.02
10	December	334 + *i*	344	-23.05	1.03

In the developed code, *n* is denoted by *D*, the day of the year.

To avoid confusion, the below and afterwards mathematical formulation involving 'n' or 'the day of the year', the symbol 'n' will be replaced by 'D'.

The true position of the sun observed from any horizontal surface at any point on earth can be determined via calculation if the coordinate system of the earth is projected to the celestial sphere. The celestial sphere is defined as a hypothetical sphere with infinite radius and with the earth as its center. The stars are also projected on the celestial sphere. Refer Figure 3.4 to have a better understanding of the celestial sphere (Foster et al., 2010).

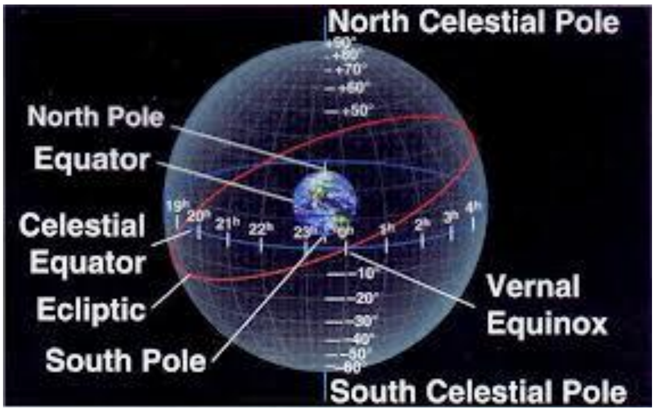


Figure 3.4: The Celestial Sphere (“The Celestial Sphere,” n.d.)

The north and south celestial poles are in the same alignment of north and south poles of the earth and so does its equator. Similar to concept of earth latitude, the angle δ of the celestial sphere is measured in the north or south direction from the celestial equator plane (Foster et al., 2010). Several methods inter alia Spencer, Perrin and Cooper are available to calculate the declination δ, but the code is developed based on Cooper’s equation as follows:

$$\delta = 23.45 * \sin\left(\frac{360}{365} * (D + 284)\right) \tag{3.1}$$

The celestial declination δ will be later needed for solar altitude α_s calculation.

Before we proceed any further, let us understand the meaning of solar altitude α_s, zenith angle θ_z, azimuth angle γ_s and hour angle ω. The solar altitude is measured in degrees from horizon of the projection of the radiation beam to the position of the Sun (when the Sun is overhead, $\alpha_s = 90^\circ$ and when the sun is over the horizon, $\alpha_s = 0^\circ$). Whereas, zenith angle is the angle of the sun relative to a line perpendicular to the earth's surface. The meaning of solar azimuth is the angle on the horizontal plane between the projection of the beam radiation and the north-south direction line ($+\gamma_s$ = The sun is west of south and $-\gamma_s =$ The sun is east of south). Last but not least of importance is the hour angle, which is defined as angular distance between the sun's position at a particular time and its highest position for that day when crossing the local meridian at the solar noon.

Other than δ, hour angle, which is denoted as ω and latitude ϕ, are also needed for the solar altitude α_s calculation. The components of the sun position with respect to the horizontal surface of any point on the earth can be then calculated by using the calculated solar altitude α_s.

Latitude ϕ must be known and it is encoded in the code, which shows the angular distance of a place north or south of the earth's equator, usually expressed in degrees and minutes. In this case, latitude of UTeM location is +2.309357˚. However, for validation purposes, random latitude which is +37.000000˚, is used.

For the hour angle ω calculation, it is important to note that the sun moves through the sky 15˚ each hour daily. At solar noon (local meridian), the value of the hour angle is zero and takes negative values during mornings and positive values in the afternoons. In other words, the time representatives or time factor from 7 a.m. to 7 p.m. are listed as follows in Table 3.2:

Table 3.2: Time factor and hour angle ω for respective times

Time	Time Factor	Hour Angle ω
7 a.m.	-5	-5 * 15˚ = -75˚
8 a.m.	-4	-4 * 15˚ = -60˚
9 a.m.	-3	-3 * 15˚ = -45˚
10 a.m.	-2	-2 * 15˚ = -30˚
11 a.m.	-1	-1 * 15˚ = -15˚
12 p.m.	0	0 * 15˚ = 0˚
1 p.m.	+1	+1 * 15˚ = +15˚
2 p.m.	+2	+2 * 15˚ = +30˚
3 p.m.	+3	+3 * 15˚ = +45˚
4 p.m.	+4	+4 * 15˚ = +60˚
5 p.m.	+5	+5 * 15˚ = +75˚
6 p.m.	+6	+6 * 15˚ = +90˚
7 p.m.	+7	+7 * 15˚ = +105˚

Therefore, to calculate the hour angle ω, corresponding value which represents the time must be multiplied with 15˚. For example, at 3 p.m. the hour angle of that particular time is (+3 * 15˚) which gives +45˚.

Upon the completion of the celestial declination δ and the hour angle ω, both angles are used to determine solar altitude α_s. The equation of solar altitude α_s is as follows:

$$\sin \alpha_s = \sin \phi \, \sin \delta + \cos \phi \, \cos \delta \, \cos \omega \tag{3.2}$$

And since $sin^2\,\alpha_s + cos^2\,\alpha_s = 1$;

$$\cos\alpha_s = (1 - sin^2\,\alpha_s)^{1/2} \tag{3.3}$$

$$\alpha_s = \text{atan}\left(\frac{\sin\alpha_s}{\cos\alpha_s}\right) \tag{3.4}$$

It is important to follow above mentioned steps to calculate the solar altitude α_s to ensure that no failure occurs at any point due to arcsine of negative number.

Other than the solar altitude α_s, another important angle is azimuth angle γ_s, which is needed for calculation of the sun position vector $\vec{S}$. The equation is as follows:

$$\gamma_s = C_1 C_2 \gamma_s' + C_3\left(\frac{1 - C_1 C_2}{2}\right) 180^{\circ} \tag{3.5}$$

where γ_s' is a pseudo solar azimuth angle for the first or fourth quadrant

$$\gamma_s' = \text{arcsin}\left(\frac{\sin\omega\cos\delta}{\sin\theta_z}\right) \tag{3.6}$$

And to calculate the pseudo azimuth γ_s', zenith angle θ_z must be first calculated using below equation:

$$\cos\theta_z = \sin\phi\,\sin\delta + \cos\,\phi\,\cos\,\delta\,\cos\,\omega \tag{3.7}$$

Moreover, to determine the azimuth angle, C_1, C_2 and C_3 must be first determined like following:

If $\omega < \omega_{WE}$, then C_1 = 1, else C_1 = -1 (3.8) where $\cos\omega_{WE} = {}^{\tan\delta}/_{\tan\phi}$; (3.9)

ω_{WE} is hour angle if the sun position is due to east or west direction.

If ϕ (ϕ - δ) > 0, then C_2 = 1, else C_2 = -1 (3.10)

If ω > 0, then C_3 = 1, else C_3 = -1 (3.11)

The constant C_1 is calculated to determine whether or not the Sun is within the first or fourth quadrants and above the horizon. On the other hand, C_2 includes the variables of latitude and declination and C_3 is calculated to define whether or not the Sun has passed the local meridian to identify whether it is morning or afternoon.

With the known solar altitude α_s and azimuth angle γ_s the sun position vector $\vec{S}$ with respect to horizontal surface at any point on the Earth can be determined using below formula:

$$S_z = S_z = \sin\alpha_s \tag{3.12}$$

$$S_e = S_y = \cos\alpha_s \sin\gamma_s \tag{3.13}$$

$$S_n = S_x = \cos\alpha_s \cos\gamma_s \tag{3.14}$$

3.2.2 REDIRECTION OF SUN RAY VECTOR $\vec{R}$

The law of specular reflection is applied in the redirection of the Sun ray calculation. This law states that the angle of incidence is equal to the angle of reflection.

Receiver vector, $\vec{r} = \begin{pmatrix} n_0 \\ e_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix}$;

Mirror vector, $\vec{m} = \begin{pmatrix} n_1 \\ e_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$;

Redirection of the sun ray vector,

$$\vec{R} = \left[(z_0 - z_1)\,\hat{\imath} + (e_0 - e_1)\,\hat{\jmath} + (n_0 - n_1)\,\hat{k}\right] / \sqrt[2]{[(z_0 - z_1)^2 + (e_0 - e_1)^2 + (n_0 - n_1} \tag{3.15}$$

but both receiver and heliostat vectors could be reset so that the mirror vector $\vec{m} = \begin{pmatrix} 0 \\ 0 \\ z_1 \end{pmatrix}$ for simplification of calculation. This results as following:

$$\vec{R} = \left[(z_0 - z_1)\,\hat{\imath} - e_1\,\hat{\jmath} + - n_1\,\hat{k}\right] / \sqrt[2]{[(z_0 - z_1)^2 + e_1{}^2 + n_1{}^2]} \tag{3.16}$$

Therefore,

$$R_n = R_x = - n_1 / \sqrt[2]{[(z_0 - z_1)^2 + e_1{}^2 + n_1{}^2]} \tag{3.17}$$

$$R_e = R_y = - e_1 / \sqrt[2]{[(z_0 - z_1)^2 + e_1{}^2 + n_1{}^2]} \tag{3.18}$$

$$R_z = R_z = (z_0 - z_1) / \sqrt[2]{[(z_0 - z_1)^2 + e_1{}^2 + n_1{}^2]} \tag{3.19}$$

3.2.3 ANGLE OF INCIDENCE θ

The scalar point between the vectors of $\vec{S}$ and $\vec{R}$ results in cosines of two-time angle of incidence θ like in following expression:

$$\cos 2\theta = \vec{S} . \vec{R} \tag{3.20}$$

The angle of incidence or reflection can be determined if the position of the sun and receiver relative to the heliostat are known:

$$\cos 2\theta = R_z \sin \alpha_s + R_e \cos \alpha_s \sin \gamma_s + R_n \cos \alpha_s \cos \gamma_s \tag{3.21}$$

3.2.4 HELIOSTAT NORMAL VECTOR $\vec{N}$

$\vec{N}$ is a normal vector of heliostat at a moment, at which sun ray arrives parallel to the vector $\vec{S}$ and the ray is reflected onto the receiver or absorber. It can be calculated via addition of incidence $\vec{S}$ and reflection vectors $\vec{R}$ and via division of the added vector values with the suitable scalar quantity:

$$\vec{N} = \frac{\left[(R_z + S_z)\hat{\imath} + (R_e + S_e)\hat{\jmath} + (R_n + S_n)\hat{k}\right]}{2\cos\theta} \tag{3.22}$$

where component *x* is denoted by unit vector $\hat{k}$, component *y* is denoted by unit vector $\hat{\jmath}$ and component *z* is denoted by unit vector $\hat{\imath}$ along north, east and zenith direction respectively.

Before proceeding to generate the aimed MATLAB programming code, it does make sense to precalculate a random selected case analytically to guide the program generation process. The analytical results can then be compared with the results that will be calculated by the program numerically. Functionality of the code can then be justified.

3.3 ANALYTICAL SAMPLE CASE

Below data are used to analyze the case analytically. The results should be used to guide and support the development of the code. The data are as follows:

No. of the month = 7

No. of the day = 24

The Cartesian coordinate of heliostat = [-130, 75, 10]

The Cartesian coordinate of the tower = [-5, 0, 75]

The analytical verification:

By using the mathematical theory in the previous subtopic 'THEORY OF SUN POSITION AND HELIOSTAT NORMAL CALCULATION', the case is calculated. By using (3.1), the declination/inclination of the case can be calculated:

$$\delta = 23.45 * \sin\left(\frac{360}{365} * \ (D \ + 284)\right)$$

From the Table 3.1, the day of the year *D* is the 205fh day of that year, since *D* = 181 + 24 = 205. Therefore;

$$\delta = 23.45 * \sin\left(\frac{360}{365} * \ (205 + 284)\right)$$

$$\underline{\underline{\delta = 19.8^{\circ}}}$$

Latitude ϕ = +37.000000˚

From Table 3.2, the hour angle ω for 3 p.m. is +45˚. To calculate the sinus component of solar altitude, (3.2) is used as follows:

$$\sin \alpha_s = \ \sin\phi \ \sin\delta + \ \cos \ \phi \ \cos \ \delta \ \cos \ \omega$$

$$\sin \alpha_s = (\sin 37^{\circ}) \ (\sin 19.8^{\circ}) + \ (\cos \ 37^{\circ}) \ (\cos \ 19.8^{\circ}) \ (\cos \ 45^{\circ})$$

$$\underline{\underline{\sin \alpha_s = \ Sz = 0.7352}}$$

And since $sin^2 \ \alpha_s + \ cos^2 \ \alpha_s = 1$;

$\cos \alpha_s = (1 - 0.7352^2)^{1/2}$

$\cos \alpha_s = 0.6779$

$\alpha_s = \text{atan} \left(\frac{0.7352}{0.6779}\right)$

$\underline{\underline{\alpha_s = 47.32^\circ}}$

By using (3.7), the cosinus component of zenith angle can be calculated as follows:

$\cos \theta_z = \sin \phi \ \sin \delta + \cos \ \phi \ \cos \ \delta \ \cos \ \omega$

$\cos \theta_z = (\sin 37^\circ)(\sin 19.8^\circ) + (\cos \ 37^\circ) (\cos 19.8^\circ) (\cos \ 45^\circ)$

$\cos \theta_z = 0.7352$

$\underline{\underline{\theta_z = 42.68^\circ}}$

Pseudo azimuth angle is calculated by using (3.6):

$\gamma_s' = \arcsin \left[\frac{\sin \omega \cos \delta}{\sin \theta_z}\right]$

$\gamma_s' = \arcsin \left[\frac{(\sin 45^\circ)(\cos 19.8^\circ)}{(\sin 42.68^\circ)}\right]$

$\underline{\underline{\gamma_s' = 78.94^\circ}}$

By using (3.9);

$\cos \omega_{WE} = \frac{\tan \delta}{\tan \phi}$

$\cos \omega_{WE} = \frac{\tan \ (19.8^\circ)}{\tan \ (37^\circ)}$

$\cos \omega_{WE} = 0.4778$

$\underline{\underline{\omega_{WE} = 61.46^{\circ}}}$

According to (3.8), since ω = 45° < $\omega_{WE} = 61.46^{\circ}$, then $\underline{\underline{C_1 = 1}}$

And according to (3.10), since $37^{\circ} * (37^{\circ} - 19.8^{\circ}) > 0$, then $\underline{\underline{C_2 = 1}}$

And according to (3.11), since ω = +45° > 0, then $\underline{\underline{C_3 = 1}}$

Azimuth angle is calculated by using (3.5) as follows:

$$\gamma_s = C_1 C_2 \gamma_s' + C_3 \left(\frac{1 - C_1 C_2}{2}\right) 180^{\circ}$$

$$\gamma_s = 1 * 1 * 78.94^{\circ} + 1 * \left(\frac{1 - 1 * 1}{2}\right) 180^{\circ}$$

$$\underline{\underline{\gamma_s = 78.94^{\circ}}}$$

By referring the (3.12), (3.13) and (3.14), the components of sun position vector are as follows:

$$S_z = S_z = \sin \alpha_s$$

$$\underline{\underline{S_z = S_z = 0.7352}}$$

$$S_e = S_y = \cos \alpha_s \sin \gamma_s$$

$$S_e = S_y = (\cos 47.32^{\circ}) * (\sin 78.94^{\circ})$$

$$\underline{\underline{S_e = S_y = 0.6653}}$$

$$S_n = S_x = \cos \alpha_s \cos \gamma_s$$

$$S_n = S_x = (\cos 47.32^{\circ}) * (\cos 78.94^{\circ})$$

$$\underline{\underline{S_n = S_x = 0.1300}}$$

Therefore;

$$\begin{pmatrix} S_n \\ S_e \\ S_z \end{pmatrix} = \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix} = \begin{pmatrix} 0.1300 \\ 0.6653 \\ 0.7352 \end{pmatrix}$$

Receiver vector, $\vec{r} = \begin{pmatrix} n_0 \\ e_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \\ 75 \end{pmatrix}$;

Mirror vector, $\vec{m} = \begin{pmatrix} n_1 \\ e_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} -130 \\ 75 \\ 10 \end{pmatrix}$;

After resetting both vectors for simplification of calculation i.e. resetting the *x*-axis by changing the position of *x*-origin from the frontal face of tower base to the centre of receiver/ absorber. The respective vectors are as follows:

Receiver vector, $\vec{r} = \begin{pmatrix} n_0 \\ e_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 75 \end{pmatrix}$;

Mirror vector, $\vec{m} = \begin{pmatrix} n_1 \\ e_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} -125 \\ 75 \\ 10 \end{pmatrix}$

To calculate the sun ray redirection vector in a simplified case, please refer (3.16):

$$\vec{R} = \left[(z_0 - z_1)\,\hat{\imath} - e_1\,\hat{\jmath} + -n_1\,\hat{k}\right] / \sqrt[2]{[(z_0 - z_1)^2 + e_1{}^2 + n_1{}^2]}$$

$$\vec{R} = \left[(75 - 10)\,\hat{\imath} - 75\,\hat{\jmath} + 125\,\hat{k}\right] / \sqrt[2]{[(75 - 10)^2 + 75^2 + (-125)^2]}$$

$$\vec{R} = \left[65\,\hat{\imath} - 75\,\hat{\jmath} + 125\,\hat{k}\right] / 159.608897$$

$$\vec{R} = 0.407\,\hat{\imath} - 0.470\,\hat{\jmath} + 0.783\,\hat{k}$$

$$R_n = R_x = 65/159.608897$$

$$R_e = R_y = -75/159.608897$$

$R_z = R_z = 125/159.608897$

$$\begin{pmatrix} R_n \\ R_e \\ R_z \end{pmatrix} = \begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix} = \begin{bmatrix} 0.783 \\ -0.470 \\ 0.407 \end{bmatrix}$$

By applying the (3.21), the angle of incidence or reflection can be calculated as follows:

$$\cos 2\theta = R_z \sin \alpha_s + R_e \cos \alpha_s \sin \gamma_s + R_n \cos \alpha_s \cos \gamma_s$$

$$\cos 2\theta = 0.407 * \sin (47.32^\circ) - 0.470 * \cos (47.32^\circ) * \sin (78.94^\circ) + 0.783 * \cos (47.32^\circ) * \cos (78.94^\circ)$$

$$\cos 2\theta = 0.08834$$

$$2\theta = 84.93^\circ$$

$$\underline{\underline{\theta = 42.47^\circ}}$$

The most important output from the program which is the heliostat normal vector is calculated by using the (3.22) like following:

$$\overrightarrow{N} = \frac{[(R_z + S_z)\hat{\imath} + (R_e + S_e)\hat{\jmath} + (R_n + S_n)\hat{k}]}{2\cos\theta}$$

$$\overrightarrow{N} = \frac{[(0.407 + 0.7352)\hat{\imath} + (-0.470 + 0.6653)\hat{\jmath} + (0.783 + 0.1300)\hat{k}]}{2 * \cos (42.47^\circ)}$$

$$\overrightarrow{N} = 0.7742\,\hat{\imath} + 0.1324\,\hat{\jmath} + 0.6189\,\hat{k}$$

$$\begin{pmatrix} N_n \\ N_e \\ N_z \end{pmatrix} = \begin{pmatrix} N_x \\ N_y \\ N_z \end{pmatrix} = \begin{bmatrix} 0.6189 \\ 0.1324 \\ 0.7742 \end{bmatrix}$$

$\vec{N}$ is normal vector of heliostat at the moment, which sunlight arrives parallel to the vector $\vec{S}$, is reflected onto the receiver.

4 RESULTS AND DISCUSSION

4.1 CODE GENERATION

At the very beginning of the code, '*clc*' command is used to clear the screen and to clear the previous result from the program memory so that the program can calculate the new set of results, the command '*clear all*' is applied.

The input of the program should be the date (no. of month m and no. of day of the month d are entered separately), the Cartesian coordinate of heliostat $\vec{m}$ and the Cartesian coordinate of the receiver $\vec{r}$. Based on Figure 3.3, please note that the coordinate origin is located at the base of the receiving tower. *X*-axis is in the north-, *Y*-axis in the east- and *Z*-axis in the zenith direction. The input data use following command:

```
X = input ('Enter X: ')
```

User must enter the heliostat $\vec{m}$ and receiver $\vec{r}$ vector in square bracket which cointains value *x*, *y* and *z* in separated commas as per below:

```
[x,x,x]
```

Latitude ϕ is set as a fixed value or known as a constant in this code, which is the latitude of UTeM.

```
latitude_UteM = +2.309357
```

The output of the program are sun position and heliostat mirror normal vector. The code is developed for the utilization between 7 a.m. and 7 p.m. as the sun is assumed to be only available during this time range.

The day of the year *D* can be calculated as denoted in the Table 3.1. To achieve this calculation, '*if*' and '*elseif*' commands are used like in the following command lines for the purpose:

```
    if m==1
D=d;
    elseif m==2
D=31+d;
    elseif m==3
       .
       .
       .
    else
D=334+d;
    end
```

Calculation of the hour angle for 7 a.m. to 7 p.m. can be referred to Table 3.2. From the table, the time factor t is set as a fix constant and is entered in vector form as follows:

```
t=[-5;-4;-3;-2;-1;0;+1;+2;+3;+4;+5;+6;+7]
```

It is to be noted that t is zero at 12 p.m. (noon) and t takes positive value after noon and negative before noon. Therefore, the result for hour angle ω which is denoted by 'omega' in the code, is also in a vector form (1 x 13 matrix) which means the end results or rather the output which are sun position and heliostat normal vector are for the time range from 7 a.m. to 7 p.m..

Next is the calculation of celestial inclination or declination δ or denoted by 'delta' in the code. The 'delta' uses value of the resulted D which is calculated beforehand. The code line is as follows:

```
delta=23.45*sin(360/365*(D+284)*pi/180)
```

Since all components which are `latitude_UTeM`, `delta` and `omega`, are available for the solar altitude angle α_S calculation, the code lines for the α_S or known as SA in the code can be developed. Please refer 'THEORY OF SUN POSITION AND HELIOSTAT NORMAL CALCULATION' for more info on how to calculate α_S. Here, sinus and cosinus components of the α_S are first calculated before the end product of α_S is generated.

It is again to be noted that it is important to follow the mentioned steps in the theory to calculate the solar altitude α_s to ensure that no failure occurs at any point due to arcsine of negative number. But as the calculation is quite complicated, the calculation is broken into several further steps and the resulting components are as listed below:

```
squaresinussolaraltitude

cosinussolaraltitude

productofsinussolaraltitudeandcosinussolaraltitude

atanofsinussolaraltitudeandcosinussolaraltitude

SA
```

Next step, pseudo azimuth angle γ_s', zenith angle θ_z and omegaWE ω_{WE} are calculated. OmegaWE ω_{WE} is an hour angle whenever the position of the Sun is either due to the east or west direction. Zenith angle θ_z is needed for the γ_s' calculation, ω_{WE} for a constant or rather C_1 calculation and both C_1 and γ_s' are then used for azimuth angle γ_s calculation.

Other two constants which are C_2 and C_3 depend on ω, δ and ϕ. Command 'if' and 'else' are used to determine all the three constants i.e. C_1, C_2 and C_3. The code lines for the constants are as denoted below:

```
for i=1:13
if omega(i,1)<omegaWE
    C1(i,1)=1;
else
    C1(i,1)=-1;
end
if omega(i,1)>=0
    C3(i,1)=1;
else
    C3(i,1)=-1;
end
end
if latitude_UTeM*(latitude_UTeM-delta)>=0
    C2=1;
else
```

```
    C2=-1;
End
C1
C2
C3
```

By using the available results of γ_s', C_1, C_2 and C_3 from the program, code for azimuth angle γ_s can now be developed and γ_s can be calculated. The cosinus and sinus components of γ_s (known as `azimuthangle` in the code) and α_S (known as `SA` in the code) are then used to calculate the sun position vector $\vec{S}$. This means the first desired output is considered done.

Next measurement is calculating the vector of redirection of the Sun ray towards point A at the receiver $\vec{R}$. The vector $\vec{R}$ can be calculated and developed by using two other vectors which are $\vec{r}$ and $\vec{m}$. The vector $\vec{R}$, together with the sinus and cosinus components of α_S (known as `SA`) and γ_s (known as `azimuthangle`) will be used to develop the code for angle of incidence θ (known as `theta` in the code) calculation.

The angle of incidence θ, component of the vector $\vec{R}$ and also component of the first output which is sun position vector $\vec{S}$ can be utilized to develop the code of heliostat normal vector. In other words, the second desired output which the heliostat normal vector is completed and successfully achieved.

Now, a test of functionality is conducted by running the program and entering the input data from the previous sample case; the ANALYTICAL SAMPLE CASE and the numerical output from the program are compared with the analytical results.

At 3 p.m.:

Table 4.1: Numerical and analytical results comparison of the selected sample case

	Heliostatsnormal Vector	Sunposition Vector
Numerical Output	[0.6186 0.1324 0.7744]	[0.1295 0.6652 0.7353]
Analytical Output	$\begin{pmatrix} 0.6189 \\ 0.1324 \\ 0.7742 \end{pmatrix}$	$\begin{pmatrix} 0.1300 \\ 0.6653 \\ 0.7352 \end{pmatrix}$

From the Table 4.1, it can be extracted that the numerical output is set to be a row vector. It is only the transformation of the analytical output which is a column vector. Both should mean the same result.

The end numerical results from the developed code agree with the analytically calculated results for the selected case. It shows that numerical results differs from analytical results only from 0.01 to 0.05%. This indicates that the code is working correctly. However, further numerical and analytical verifications are needed to verify and qualify that the code can be utilized in time range from 7 a.m. to 7 p.m since the sun is assumed to be available in this time period. In addition, the verifications are also needed to ensure that the code can also be used for any date. The generated code is shown in Appendix A.

4.2 RESULT/ CODE VERIFICATIONS VIA ANALYTICAL CALCULATION

4.2.1 FIRST VERIFICATION

The user's input in this case are taken as follows:

No. of the month = 7

No. of the day = 24

The Cartesian coordinate of heliostat = [-130, 75, 10]

The Cartesian coordinate of the tower = [-5, 0, 75]

<u>The numerical output (from the developed program):</u>

At 7 a.m.:

Heliostatsnormal Vector			Sunposition Vector		
0.4941	-0.7506	0.4387	0.1243	-0.9087	0.3985

<u>The analytical verification:</u>

By using (3.1), the declination/inclination of the case can be calculated:

$$\delta = 23.45 * \sin\left(\frac{360}{365} * (D + 284)\right)$$

From the Table 3.1, the day of the year D is the 205th day of that year, since D = 181 + 24 = 205. Therefore;

$$\delta = 23.45 * \sin\left(\frac{360}{365} * (205 + 284)\right)$$

$\underline{\underline{\delta = 19.8^{\circ}}}$

Latitude ϕ = +37.000000°

From Table 3.2, the hour angle ω for 7 a.m. is -75°. To calculate the sinus component of solar altitude, (3.2) is used as follows:

$$\sin \alpha_s = \sin \phi \ \sin \delta + \cos \ \phi \ \cos \ \delta \ \cos \ \omega$$

$$\sin \alpha_s = (\sin 37°)\ (\sin 19.8°) + (\cos \ 37°)\ (\cos \ 19.8°)\ (\cos \ -75°)$$

$$\underline{\underline{\sin \alpha_s = Sz = 0.3983}}$$

And since $sin^2 \ \alpha_s + \ cos^2 \ \alpha_s = 1$;

$$\cos \alpha_s = (1 - \ 0.3983^2)^{1/2}$$

$$\cos \alpha_s = 0.9172$$

$$\alpha_s = \text{atan} \ \left(\frac{0.3983}{0.9172}\right)$$

$$\underline{\underline{\alpha_s = 23.47°}}$$

By using (3.7), the cosinus component of zenith angle can be calculated as follows:

$$\cos \theta_z = \sin \phi \ \sin \delta + \cos \ \phi \ \cos \ \delta \ \cos \ \omega$$

$$\cos \theta_z = (\sin 37°)(\sin 19.8°) + (\cos \ 37°)\ (\cos 19.8°)\ (\cos \ -75°)$$

$$\cos \theta_z = 0.3983$$

$$\underline{\underline{\theta_z = 66.53°}}$$

Pseudo azimuth angle is calculated by using (3.6):

$$\gamma_s' = \text{arcsin} \ \left[\frac{\sin \omega \cos \delta}{\sin \theta_z}\right]$$

$$\gamma_s' = \arcsin\left[\frac{(\sin -75^{\circ})(\cos 19.8^{\circ})}{(\sin 66.53^{\circ})}\right]$$

$$\underline{\underline{\gamma_s' = -82.22^{\circ}}}$$

By using (3.9);

$$\cos \omega_{WE} = \frac{\tan \delta}{\tan \phi}$$

$$\cos \omega_{WE} = \frac{\tan\ (19.8^{\circ})}{\tan\ (37^{\circ})}$$

$$\cos \omega_{WE} = 0.4778$$

$$\underline{\underline{\omega_{WE} = 61.46^{\circ}}}$$

According to (3.8), since ω = -75° < $\omega_{WE} = 61.46^{\circ}$, then $\underline{\underline{C_1 = 1}}$

And according to (3.10), since 37° * ($37^{\circ} - 19.8^{\circ}$) > 0, then $\underline{\underline{C_2 = 1}}$

And according to (3.11), since ω = -75° < 0, then $\underline{\underline{C_3 = -1}}$

Azimuth angle is calculated by using (3.5) as follows:

$$\gamma_s = C_1 C_2 \gamma_s' + C_3 \left(\frac{1 - C_1 C_2}{2}\right) 180^{\circ}$$

$$\gamma_s = 1 * 1 * -82.22^{\circ} - 1 * \left(\frac{1 - 1*1}{2}\right) 180^{\circ}$$

$$\underline{\underline{\gamma_s = -82.22^{\circ}}}$$

By referring the (3.12), (3.13) and (3.14), the components of sun position vector are as follows:

$$S_z = S_z = \sin \alpha_s$$

$$\underline{\underline{S_z = S_z = 0.3983}}$$

$S_e = S_y = \cos\alpha_s \sin\gamma_s$

$S_e = S_y = (\cos 23.47°) * (\sin -82.22°)$

$\underline{\underline{S_e = S_y = -0.9088}}$

$S_n = S_x = \cos\alpha_s \cos\gamma_s$

$S_n = S_x = (\cos 23.47°) * (\cos -82.22°)$

$\underline{\underline{S_n = S_x = 0.1242}}$

Therefore;

$$\begin{pmatrix} S_n \\ S_e \\ S_z \end{pmatrix} = \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix} = \begin{pmatrix} 0.1242 \\ -0.9088 \\ 0.3983 \end{pmatrix}$$

Receiver vector, $\vec{r} = \begin{pmatrix} n_0 \\ e_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \\ 75 \end{pmatrix}$;

Mirror vector, $\vec{m} = \begin{pmatrix} n_1 \\ e_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} -130 \\ 75 \\ 10 \end{pmatrix}$;

After resetting both vectors for simplification of calculation i.e. resetting the x-axis by changing the position of x-origin from the frontal face of tower base to the centre of receiver/ absorber. The respective vectors are as follows:

Receiver vector, $\vec{r} = \begin{pmatrix} n_0 \\ e_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 75 \end{pmatrix}$;

Mirror vector, $\vec{m} = \begin{pmatrix} n_1 \\ e_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} -125 \\ 75 \\ 10 \end{pmatrix}$;

To calculate the sun ray redirection vector in a simplified case, please refer (3.16):

$$\vec{R} = \left[(z_0 - z_1)\,\hat{\imath} - e_1\,\hat{\jmath} + -n_1\,\hat{k}\right] / \sqrt[2]{[(z_0 - z_1)^2 + e_1{}^2 + n_1{}^2]}$$

$$\vec{R} = \left[(75 - 10)\,\hat{\imath} - 75\,\hat{\jmath} + 125\,\hat{k}\right] / \sqrt[2]{[(75 - 10)^2 + 75^2 + (-125)^2]}$$

$$\vec{R} = \left[65\,\hat{\imath} - 75\,\hat{\jmath} + 125\,\hat{k}\right] / 159.608897$$

$$\vec{R} = 0.407\,\hat{\imath} - 0.470\,\hat{\jmath} + 0.783\,\hat{k}$$

$$R_n = R_x = 65/159.608897$$

$$R_e = R_y = -75/159.608897$$

$$R_z = R_z = 125/159.608897$$

$$\begin{pmatrix} R_n \\ R_e \\ R_z \end{pmatrix} = \begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix} = \begin{bmatrix} 0.783 \\ -0.470 \\ 0.407 \end{bmatrix}$$

By applying the (3.21), the angle of incidence or reflection can be calculated as follows:

$$\cos 2\theta = R_z \sin \alpha_s + R_e \cos \alpha_s \sin \gamma_s + R_n \cos \alpha_s \cos \gamma_s$$

$$\cos 2\theta = 0.407 * \sin\ (23.47^\circ) - 0.470 * \cos\ (23.47^\circ) * \sin\ (-82.22^\circ) + 0.783 * \cos\ (23.47^\circ) * \cos\ (-82.22^\circ)$$

$$\cos 2\theta = 0.6864$$

$$2\theta = 46.65^\circ$$

$$\underline{\underline{\theta = 23.33^\circ}}$$

The most important output from the program which is the heliostat normal vector is calculated by using the (3.22) like following:

$$\overrightarrow{N} = \frac{[(R_z + S_z)\hat{\imath} + (R_e + S_e)\hat{\jmath} + (R_n + S_n)\hat{k}]}{2\cos\theta}$$

$$\overrightarrow{N} = \frac{[(0.407 + 0.3983)\hat{\imath} + (-0.470 - 0.9088)\hat{\jmath} + (0.783 + 0.1242)\hat{k}]}{2 * \cos(23.33^{\circ})}$$

$$\overrightarrow{N} = 0.4385\,\hat{\imath} - 0.7508\,\hat{\jmath} + 0.4940\,\hat{k}$$

$$\begin{pmatrix} N_n \\ N_e \\ N_z \end{pmatrix} = \begin{pmatrix} N_x \\ N_y \\ N_z \end{pmatrix} = \begin{bmatrix} 0.4940 \\ -0.7508 \\ 0.4385 \end{bmatrix}$$

Table 4.2: Numerical and analytical results comparison for the first verification

	Heliostatsnormal Vector	Sunposition Vector
Numerical Output	[0.4941 -0.7506 0.4387]	[0.1243 -0.9087 0.3985]
Analytical Output	$\begin{pmatrix} 0.4940 \\ -0.7508 \\ 0.4385 \end{pmatrix}$	$\begin{pmatrix} 0.1242 \\ -0.9088 \\ 0.3983 \end{pmatrix}$

From Table 4.2, the end numerical results for the first verification from the developed code agree with the analytically calculated results. It shows that numerical results differs from analytical results only from 0.01 to 0.08%. This verifies that the code is also working correctly for the same date as the ANALYTICAL SAMPLE CASE, but different time which is in this case for 7 a.m. (whereby results for the sample case is calculated for 3 p.m.).

4.2.2 SECOND VERIFICATION

The user's input in this case are taken as follows:

No. of the month = 7

No. of the day = 24

The Cartesian coordinate of heliostat = [-130, 75, 10]

The Cartesian coordinate of the tower = [-5, 0, 75]

<u>The numerical output (from the developed program):</u>

At 7 p.m.:

Heliostatsnormal Vector			Sunposition Vector		
0.5173	0.6205	0.5894	-0.4173	0.9087	0.0096

<u>The analytical verification:</u>

By using (3.1), the declination/inclination of the case can be calculated:

$$\delta = 23.45 * \sin\left(\frac{360}{365} * (D + 284)\right)$$

But since the date for the subtopic 'FIRST VERIFICATION' and 'SECOND VERIFICATION' are the same, the resulted delta are also the same.

Therefore, from previous verification, value of the delta is as follows:

$$\underline{\underline{\delta = 19.8^{\circ}}}$$

Latitude ϕ = +37.000000˚

From Table 3.2, the hour angle ω for 7 p.m. is +105˚. To calculate the sinus component of solar altitude, (3.2) is used as follows:

$$\sin \alpha_s = \sin \phi \ \sin \delta + \cos \ \phi \ \cos \ \delta \ \cos \ \omega$$

$$\sin \alpha_s = (\sin 37^\circ) \ (\sin 19.8^\circ) + (\cos \ 37^\circ) \ (\cos \ 19.8^\circ) \ (\cos \ 105^\circ)$$

$$\underline{\underline{\sin \alpha_s = \ Sz = 0.0094}}$$

And since $sin^2 \ \alpha_s + \ cos^2 \ \alpha_s = 1$;

$$\cos \alpha_s = \ (1 - 0.0094^2)^{1/2}$$

$$\underline{\underline{\cos \alpha_s = \ 0.99996}}$$

$$\alpha_s = \text{atan} \ \left(\frac{0.00940}{0.99996}\right)$$

$$\underline{\underline{\alpha_s = 0.5372^\circ}}$$

By using (3.7), the cosinus component of zenith angle can be calculated as follows:

$$\cos \theta_z = \ \sin \phi \ \sin \delta + \ \cos \ \phi \ \cos \ \delta \ \cos \ \omega$$

$$\cos \theta_z = \ (\sin 37^\circ)(\sin 19.8^\circ) + \ (\cos \ 37^\circ) \ (\cos 19.8^\circ) \ (\cos \ 105^\circ)$$

$$\cos \theta_z = 0.0094$$

$$\underline{\underline{\theta_z = \ 89.46^\circ}}$$

Pseudo azimuth angle is calculated by using (3.6):

$$\gamma_s' = \text{arcsin} \ \left[\frac{\sin \omega \cos \delta}{\sin \theta_z}\right]$$

$$\gamma_s' = \arcsin\left[\frac{(\sin 105^\circ)(\cos 19.8^\circ)}{(\sin 89.46^\circ)}\right]$$

$$\underline{\underline{\gamma_s' = 65.34^\circ}}$$

By using (3.9);

$$\cos\omega_{WE} = \frac{\tan\delta}{\tan\phi}$$

$$\cos\omega_{WE} = \frac{\tan\ (19.8^\circ)}{\tan\ (37^\circ)}$$

$$\cos\omega_{WE} = 0.4778$$

$$\underline{\underline{\omega_{WE} = 61.46^\circ}}$$

According to (3.8), since ω = 105° > $\omega_{WE} = 61.46^\circ$, then $\underline{\underline{C_1 = -1}}$

And according to (3.10), since $37^\circ * (37^\circ - 19.8^\circ)$ > 0, then $\underline{\underline{C_2 = 1}}$

And according to (3.11), since ω = $+105^\circ$ > 0, then $\underline{\underline{C_3 = 1}}$

Azimuth angle is calculated by using (3.5) as follows:

$$\gamma_s = C_1 C_2 \gamma_s' + C_3\left(\frac{1 - C_1 C_2}{2}\right) 180^\circ$$

$$\gamma_s = -1 * 1 * 65.34^\circ + 1 * \left(\frac{1 + 1 * 1}{2}\right) 180^\circ$$

$$\underline{\underline{\gamma_s = 114.66^\circ}}$$

By referring the (3.12), (3.13) and (3.14), the components of sun position vector are as follows:

$$S_z = S_z = \sin\alpha_s$$

$$\underline{\underline{S_z = S_z = 0.0094}}$$

$$S_e = S_y = \cos\alpha_s \sin\gamma_s$$

$$S_e = S_y = (\cos 0.5372^{\circ}) * (\sin 114.66^{\circ})$$

$$\underline{\underline{S_e = S_y = 0.9088}}$$

$$S_n = S_x = \cos\alpha_s \cos\gamma_s$$

$$S_n = S_x = (\cos 0.5372^{\circ}) * (\cos 114.66^{\circ})$$

$$\underline{\underline{S_n = S_x = -0.4172}}$$

Therefore;

$$\begin{pmatrix} S_n \\ S_e \\ S_z \end{pmatrix} = \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix} = \begin{pmatrix} -0.4172 \\ 0.9088 \\ 0.0094 \end{pmatrix}$$

Receiver vector, $\vec{r} = \begin{pmatrix} n_0 \\ e_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \\ 75 \end{pmatrix}$;

Mirror vector, $\vec{m} = \begin{pmatrix} n_1 \\ e_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} -130 \\ 75 \\ 10 \end{pmatrix}$;

Both receiver and mirror vectors are used to calculate redirection of the Sun ray vector $\vec{R}$. Since the same receiver and mirror vector which are denoted as $\vec{r}$ and $\vec{m}$ respectively, the recalculation of $\vec{R}$ is unnecessary here and can be refer under subtopic 'FIRST VERIFICATION'. The end result of $\vec{R}$ is as follows:

$$\begin{pmatrix} R_n \\ R_e \\ R_z \end{pmatrix} = \begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix} = \begin{bmatrix} 0.783 \\ -0.470 \\ 0.407 \end{bmatrix}$$

By applying the (3.21), the angle of incidence or reflection can be calculated as follows:

$$\cos 2\theta = R_z \sin \alpha_s + R_e \cos \alpha_s \sin \gamma_s + R_n \cos \alpha_s \cos \gamma_s$$

$$\cos 2\theta = 0.407 * \sin (0.5372^\circ) - 0.470 * \cos (0.5372^\circ) * \sin (114.66^\circ) + 0.783 * \cos (0.5372^\circ) * \cos (114.66^\circ)$$

$$\cos 2\theta = -0.74998$$

$$2\theta = 138.5890^\circ$$

$$\underline{\underline{\theta = 69.29^\circ}}$$

The most important output from the program which is the heliostat normal vector is calculated by using the (3.22) like following:

$$\overrightarrow{N} = \frac{[(R_z + S_z)\hat{\imath} + (R_e + S_e)\hat{\jmath} + (R_n + S_n)\hat{k}]}{2 \cos \theta}$$

$$\overrightarrow{N} = \frac{[(0.407 + 0.0094)\hat{\imath} + (-0.470 + 0.9088)\hat{\jmath} + (0.783 + -0.4172)\hat{k}]}{2 * \cos (69.29^\circ)}$$

$$\overrightarrow{N} = 0.5887\,\hat{\imath} + 0.6204\,\hat{\jmath} + 0.5172\,\hat{k}$$

$$\begin{pmatrix} N_n \\ N_e \\ N_z \end{pmatrix} = \begin{pmatrix} N_x \\ N_y \\ N_z \end{pmatrix} = \begin{bmatrix} 0.5172 \\ 0.6204 \\ 0.5887 \end{bmatrix}$$

Table 4.3: Numerical and analytical results comparison for the second verification

	Heliostatsnormal Vector	Sunposition Vector
Numerical Output	[0.5173 0.6205 0.5894]	[-0.4173 0.9087 0.0096]
Analytical Output	$\begin{pmatrix} 0.5172 \\ 0.6204 \\ 0.5887 \end{pmatrix}$	$\begin{pmatrix} -0.4172 \\ 0.9088 \\ 0.0094 \end{pmatrix}$

From Table 4.3, the end numerical results for the second verification from the developed code agree with the analytically calculated results. It shows that numerical results differs from analytical results only from 0.01 to 0.12%. This verifies that the code is also working correctly for the same date as the ANALYTICAL SAMPLE CASE, but different time which is in this case for 7 p.m. (whereby results for the sample case is calculated for 3 p.m.).

4.2.3 THIRD VERIFICATION

The user's input in this case are taken as follows:

No. of the month = 5

No. of the day = 24

The Cartesian coordinate of heliostat = [-130, 75, 10]

The Cartesian coordinate of the tower = [-5, 0, 75]

<u>The numerical output (from the developed program):</u>

At 7 p.m.:

Heliostatsnormal Vector			Sunposition Vector		
0.5037	0.6155	0.6062	-0.4284	0.9034	0.0197

The analytical verification:

By using (3.1), the declination/inclination of the case can be calculated:

$$\delta = 23.45 * \sin\left(\frac{360}{365} * \ (D \ + 284)\right)$$

From the Table 3.1, the day of the year *D* is the 144th day of that year, since *D* = 120 + 24 = 144. Therefore;

$$\delta = 23.45 * \sin\left(\frac{360}{365} * \ (144 + 284)\right)$$

$$\underline{\underline{\delta = 20.73^{\circ}}}$$

Latitude ϕ = +37.000000˚

From Table 3.2, the hour angle ω for 7 p.m. is +105˚. To calculate the sinus component of solar altitude, (3.2) is used as follows:

$$\sin \alpha_s = \sin \phi \ \sin \delta + \cos \ \phi \ \cos \ \delta \ \cos \ \omega$$

$$\sin \alpha_s = (\sin 37^{\circ}) \ (\sin 20.73^{\circ}) + \ (\cos \ 37^{\circ}) \ (\cos \ 20.73^{\circ}) \ (\cos \ 105^{\circ})$$

$$\underline{\underline{\sin \alpha_s = \ Sz = 0.0197}}$$

And since $sin^2 \ \alpha_s + \ cos^2 \ \alpha_s = 1$;

$$\cos \alpha_s = \ (1 - \ sin^2 \ \alpha_s)^{1/2}$$

$$\cos \alpha_s = \ (1 - \ 0.0197^2)^{1/2}$$

$$\cos \alpha_s = \ 0.9998$$

$$\alpha_s = \text{atan} \ \left(\frac{\sin \alpha_s}{\cos \alpha_s}\right)$$

$\alpha_s = \text{atan}\left(\frac{0.0197}{0.9998}\right)$

$\underline{\underline{\alpha_s = 1.129°}}$

By using (3.7), the cosinus component of zenith angle can be calculated as follows:

$\cos\theta_z = \sin\phi \sin\delta + \cos\phi \cos\delta \cos\omega$

$\cos\theta_z = (\sin 37°)(\sin 20.73°) + (\cos 37°)(\cos 20.73°)(\cos 105°)$

$\cos\theta_z = 0.0197$

$\underline{\underline{\theta_z = 88.87°}}$

Pseudo azimuth angle is calculated by using (3.6):

$\gamma_s' = \arcsin\left[\frac{\sin\omega\cos\delta}{\sin\theta_z}\right]$

$\gamma_s' = \arcsin\left[\frac{(\sin 105°)(\cos 20.73°)}{(\sin 88.87°)}\right]$

$\underline{\underline{\gamma_s' = 64.63°}}$

By using (3.9);

$\cos\omega_{WE} = \frac{\tan\delta}{\tan\phi}$

$\cos\omega_{WE} = \frac{\tan(20.73°)}{\tan(37.00°)}$

$\cos\omega_{WE} = 0.5022$

$\underline{\underline{\omega_{WE} = 59.85°}}$

According to (3.8), $\omega = 105° > \omega_{WE} = 59.85°$, then $\underline{\underline{C_1 = -1}}$

And according to (3.10), since $37^\circ * (37^\circ - 20.73^\circ) > 0$, then $\underline{\underline{C_2 = 1}}$

And according to (3.11), since $\omega = +105^\circ > 0$, then $\underline{\underline{C_3 = 1}}$

Azimuth angle is calculated by using (3.5) as follows:

$$\gamma_s = C_1 C_2 \gamma_s' + C_3 \left(\frac{1 - C_1 C_2}{2}\right) 180^\circ$$

$$\gamma_s = -1 * 1 * 64.63^\circ + 1 * \left(\frac{1+1*1}{2}\right) 180^\circ$$

$$\underline{\underline{\gamma_s = 115.37^\circ}}$$

By referring the (3.12), (3.13) and (3.14), the components of sun position vector are as follows:

$$S_z = S_z = \sin \alpha_s$$

$$\underline{\underline{S_z = S_z = 0.0197}}$$

$$S_e = S_y = \cos \alpha_s \sin \gamma_s$$

$$S_e = S_y = (\cos 1.129^\circ) * (\sin 115.37^\circ)$$

$$\underline{\underline{S_e = S_y = 0.9034}}$$

$$S_n = S_x = \cos \alpha_s \cos \gamma_s$$

$$S_n = S_x = (\cos 1.129^\circ) * (\cos 115.37^\circ)$$

$$\underline{\underline{S_n = S_x = -0.4284}}$$

Therefore;

$$\begin{pmatrix} S_n \\ S_e \\ S_z \end{pmatrix} = \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix} = \begin{pmatrix} -0.4284 \\ 0.9034 \\ 0.0197 \end{pmatrix}$$

Receiver vector, $\vec{r} = \begin{pmatrix} n_0 \\ e_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \\ 75 \end{pmatrix}$;

Mirror vector, $\vec{m} = \begin{pmatrix} n_1 \\ e_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} -130 \\ 75 \\ 10 \end{pmatrix}$;

Both receiver and mirror vectors are used to calculate redirection of the Sun ray vector $\vec{R}$. Since the same receiver and mirror vector which are denoted as $\vec{r}$ and $\vec{m}$ respectively, the recalculation of $\vec{R}$ is unnecessary here and can be refer under subtopic 'FIRST VERIFICATION'. The end result of $\vec{R}$ is as follows:

$$\begin{pmatrix} R_n \\ R_e \\ R_z \end{pmatrix} = \begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix} = \begin{bmatrix} 0.783 \\ -0.470 \\ 0.407 \end{bmatrix}$$

By applying the (3.21), the angle of incidence or reflection can be calculated as follows:

$$\cos 2\theta = R_z \sin \alpha_s + R_e \cos \alpha_s \sin \gamma_s + R_n \cos \alpha_s \cos \gamma_s$$

$$\cos 2\theta = 0.407 * \sin (1.129^\circ) - 0.470 * \cos (1.129^\circ) * \sin (115.37^\circ) + 0.783 * \cos (1.129^\circ) * \cos (115.37^\circ)$$

$$\cos 2\theta = -0.7520$$

$$2\theta = 138.76^\circ$$

$$\underline{\underline{\theta = 69.38^\circ}}$$

The most important output from the program which is the heliostat normal vector is calculated by using the (3.22) like following:

$$\overrightarrow{N} = \frac{[(R_z + S_z)\hat{\imath} + (R_e + S_e)\hat{\jmath} + (R_n + S_n)\hat{k}]}{2\cos\theta}$$

$$\overrightarrow{N} = \frac{[(0.407 + 0.0197)\hat{\imath} + (-0.470 + 0.9034)\hat{\jmath} + (0.783 - 0.4284)\hat{k}]}{2 * \cos(69.38°)}$$

$$\overrightarrow{N} = 0.6058\,\hat{\imath} + 0.6153\,\hat{\jmath} + 0.5035\,\hat{k}$$

$$\begin{pmatrix} N_n \\ N_e \\ N_z \end{pmatrix} = \begin{pmatrix} N_x \\ N_y \\ N_z \end{pmatrix} = \begin{bmatrix} 0.5035 \\ 0.6153 \\ 0.6058 \end{bmatrix}$$

Table 4.4: Numerical and analytical results comparison for the third verification

	Heliostatsnormal Vector	Sunposition Vector
Numerical Output	[0.5037 0.6155 0.6062]	[-0.4284 0.9034 0.0197]
Analytical Output	$\begin{pmatrix} 0.5035 \\ 0.6153 \\ 0.6058 \end{pmatrix}$	$\begin{pmatrix} -0.4284 \\ 0.9034 \\ 0.0197 \end{pmatrix}$

From Table 4.4, the end numerical results for the third verification from the developed code agree with the analytically calculated results. It shows that numerical results differs from analytical results only from 0.00 to 0.07%. This verifies that the code is also working correctly for the different date than the ANALYTICAL SAMPLE CASE. This is an important indication that the developed code also functions for other case and can be utilized for different random case.

4.2.4 FOURTH VERIFICATION

The user's input in this case are taken as follows:

No. of the month = 9

No. of the day = 3

The Cartesian coordinate of heliostat = [-130, 75, 10]

The Cartesian coordinate of the tower = [-5, 0, 75]

The numerical output (from the developed program):

At 7 p.m.:

Heliostatsnormal Vector			Sunposition Vector		
0.6880	0.6325	0.3557	-0.2514	0.9588	-0.1323

The analytical verification:

By using (3.1), the declination/inclination of the case can be calculated:

$$\delta = 23.45 * \sin\left(\frac{360}{365} * (D + 284)\right)$$

From the Table 3.1, the day of the year D is the 246th day of that year, since D = 243 + 3 = 246. Therefore;

$$\delta = 23.45 * \sin\left(\frac{360}{365} * (246 + 284)\right)$$

$\underline{\delta = 6.96^{\circ}}$

Latitude ϕ = +37.000000°

From Table 3.2, the hour angle ω for 7 p.m. is +105°. To calculate the sinus component of solar altitude, (3.2) is used as follows:

$$\sin \alpha_s = \sin \phi \ \sin \delta + \cos \ \phi \ \cos \ \delta \ \cos \ \omega$$

$$\sin \alpha_s = (\sin 37^{\circ})\ (\sin 6.96^{\circ}) + (\cos \ 37^{\circ})\ (\cos \ 6.96^{\circ})\ (\cos \ 105^{\circ})$$

$$\underline{\underline{\sin \alpha_s = Sz = -0.1323}}$$

And since $sin^2 \ \alpha_s + \ cos^2 \ \alpha_s = 1$;

$$\cos \alpha_s = (1 - (-0.1323)^2)^{1/2}$$

$$\cos \alpha_s = 0.9912$$

$$\alpha_s = \text{atan} \left(\frac{-0.1323}{0.9912}\right)$$

$$\underline{\underline{\alpha_s = -7.603^{\circ}}}$$

By using (3.7), the cosinus component of zenith angle can be calculated as follows:

$$\cos \theta_z = \sin \phi \ \sin \delta + \cos \ \phi \ \cos \ \delta \ \cos \ \omega$$

$$\cos \theta_z = (\sin 37^{\circ})(\sin 6.96^{\circ}) + (\cos \ 37^{\circ})\ (\cos 6.96^{\circ})\ (\cos \ 105^{\circ})$$

$$\cos \theta_z = -0.1323$$

$$\underline{\underline{\theta_z = 97.60^{\circ}}}$$

Pseudo azimuth angle is calculated by using (3.6):

$$\gamma_s' = \arcsin \left[\frac{\sin \omega \cos \delta}{\sin \theta_z}\right]$$

$$\gamma_s' = \arcsin\left[\frac{(\sin 105^\circ)(\cos 6.96^\circ)}{(\sin 97.60^\circ)}\right]$$

$$\underline{\underline{\gamma_s' = 75.31^\circ}}$$

By using (3.9);

$$\cos\omega_{WE} = \frac{\tan\delta}{\tan\phi}$$

$$\cos\omega_{WE} = \frac{\tan\ (6.96^\circ)}{\tan\ (37^\circ)}$$

$$\cos\omega_{WE} = 0.1620$$

$$\underline{\underline{\omega_{WE} = 80.68^\circ}}$$

According to (3.8), since ω = 105° > $\omega_{WE} = 80.68^\circ$, then $\underline{\underline{C_1 = -1}}$

And according to (3.10), since $37^\circ * (37^\circ - 6.96^\circ)$ > 0, then $\underline{\underline{C_2 = 1}}$

And according to (3.11), since ω = $+105^\circ$ > 0, then $\underline{\underline{C_3 = 1}}$

Then, azimuth angle is calculated by using (3.5) as follows:

$$\gamma_s = C_1 C_2 \gamma_s' + C_3\left(\frac{1 - C_1 C_2}{2}\right) 180^\circ$$

$$\gamma_s = -1 * 1 * 75.31^\circ + 1 * \left(\frac{1 + 1 * 1}{2}\right) 180^\circ$$

$$\underline{\underline{\gamma_s = 104.69^\circ}}$$

By referring the (3.12), (3.13) and (3.14), the components of sun position vector are as follows:

$$S_z = S_z = \sin\alpha_s$$

$$\underline{\underline{S_z = S_z = -0.1323}}$$

$S_e = S_y = \cos\alpha_s \sin\gamma_s$

$S_e = S_y = (\cos -7.603^\circ) * (\sin 104.69^\circ)$

$\underline{\underline{S_e = S_y = 0.9588}}$

$S_n = S_x = \cos\alpha_s \cos\gamma_s$

$S_n = S_x = (\cos -7.603^\circ) * (\cos 104.69^\circ)$

$\underline{\underline{S_n = S_x = -0.2514}}$

Therefore;

$$\begin{pmatrix} S_n \\ S_e \\ S_z \end{pmatrix} = \begin{pmatrix} S_x \\ S_y \\ S_z \end{pmatrix} = \begin{pmatrix} -0.2514 \\ 0.9588 \\ -0.1323 \end{pmatrix}$$

Receiver vector, $\vec{r} = \begin{pmatrix} n_0 \\ e_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} x_0 \\ y_0 \\ z_0 \end{pmatrix} = \begin{pmatrix} -5 \\ 0 \\ 75 \end{pmatrix}$;

Mirror vector, $\vec{m} = \begin{pmatrix} n_1 \\ e_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} -130 \\ 75 \\ 10 \end{pmatrix}$;

Both receiver and mirror vectors are used to calculate redirection of the Sun ray vector $\vec{R}$. Since the same receiver and mirror vector which are denoted as $\vec{r}$ and $\vec{m}$ respectively, the recalculation of $\vec{R}$ is unnecessary here and can be refer under subtopic 'FIRST VERIFICATION'. The end result of $\vec{R}$ is as follows:

$$\begin{pmatrix} R_n \\ R_e \\ R_z \end{pmatrix} = \begin{pmatrix} R_x \\ R_y \\ R_z \end{pmatrix} = \begin{bmatrix} 0.783 \\ -0.470 \\ 0.407 \end{bmatrix}$$

By applying the (3.21), the angle of incidence or reflection can be calculated as follows:

$$\cos 2\theta = R_z \sin \alpha_s + R_e \cos \alpha_s \sin \gamma_s + R_n \cos \alpha_s \cos \gamma_s$$

$$\cos 2\theta = 0.407 * \sin\ (-7.603^\circ) - 0.470 * \cos\ (-7.603^\circ)$$
$$* \sin\ (104.69^\circ) + 0.783 * \cos\ (-7.603^\circ) * \cos\ (104.69^\circ)$$

$$\cos 2\theta = -0.7013$$

$$2\theta = 134.53^\circ$$

$$\underline{\underline{\theta = 67.27^\circ}}$$

The most important output from the program which is the heliostat normal vector is calculated by using the (3.22) like following:

$$\overrightarrow{N} = \frac{[(R_z + S_z)\hat{\imath} + (R_e + S_e)\hat{\jmath} + (R_n + S_n)\hat{k}]}{2 \cos \theta}$$

$$\overrightarrow{N} = \frac{[(0.407 - 0.1323)\hat{\imath} + (-0.470 + 0.9588)\hat{\jmath} + (0.783 - 0.2514)\hat{k}]}{2 * \cos\ (67.27^\circ)}$$

$$\overrightarrow{N} = 0.3555\ \hat{\imath} + 0.6325\ \hat{\jmath} + 0.6879\ \hat{k}$$

$$\begin{pmatrix} N_n \\ N_e \\ N_z \end{pmatrix} = \begin{pmatrix} N_x \\ N_y \\ N_z \end{pmatrix} = \begin{bmatrix} 0.6879 \\ 0.6325 \\ 0.3555 \end{bmatrix}$$

Table 4.5: Numerical and analytical results comparison for the fourth verification

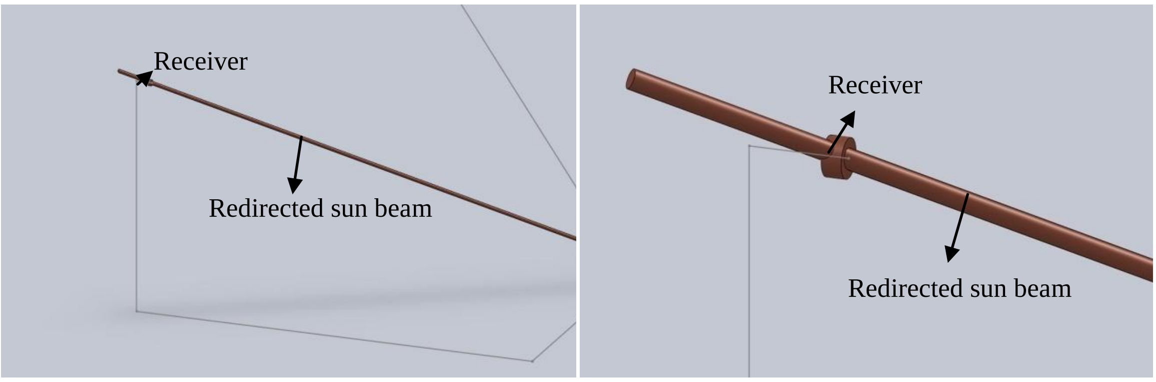
	Heliostatsnormal Vector	Sunposition Vector
Numerical Output	[0.6880 0.6325 0.3557]	[-0.2514 0.9588 -0.1323]
Analytical Output	$\begin{pmatrix} 0.6879 \\ 0.6325 \\ 0.3555 \end{pmatrix}$	$\begin{pmatrix} -0.2514 \\ 0.9588 \\ -0.1323 \end{pmatrix}$

From Table 4.5, the end numerical results for the fourth verification from the developed code agree with the analytically calculated results. It shows that numerical results differs from analytical results only from 0.00 to 0.06%. This verifies that the code is also working correctly for the different date than the ANALYTICAL SAMPLE CASE. This is an important indication that the developed code also functions for other case and can be utilized for different random case. This is the second trial to use a different date than the ANALYTICAL SAMPLE CASE.

As a conclusion to all four analytical verifications, the code can be applied for any single mirror case, with different dates and different time from 7 a.m. to 7 p.m. and with independant heliostat and receiver position.

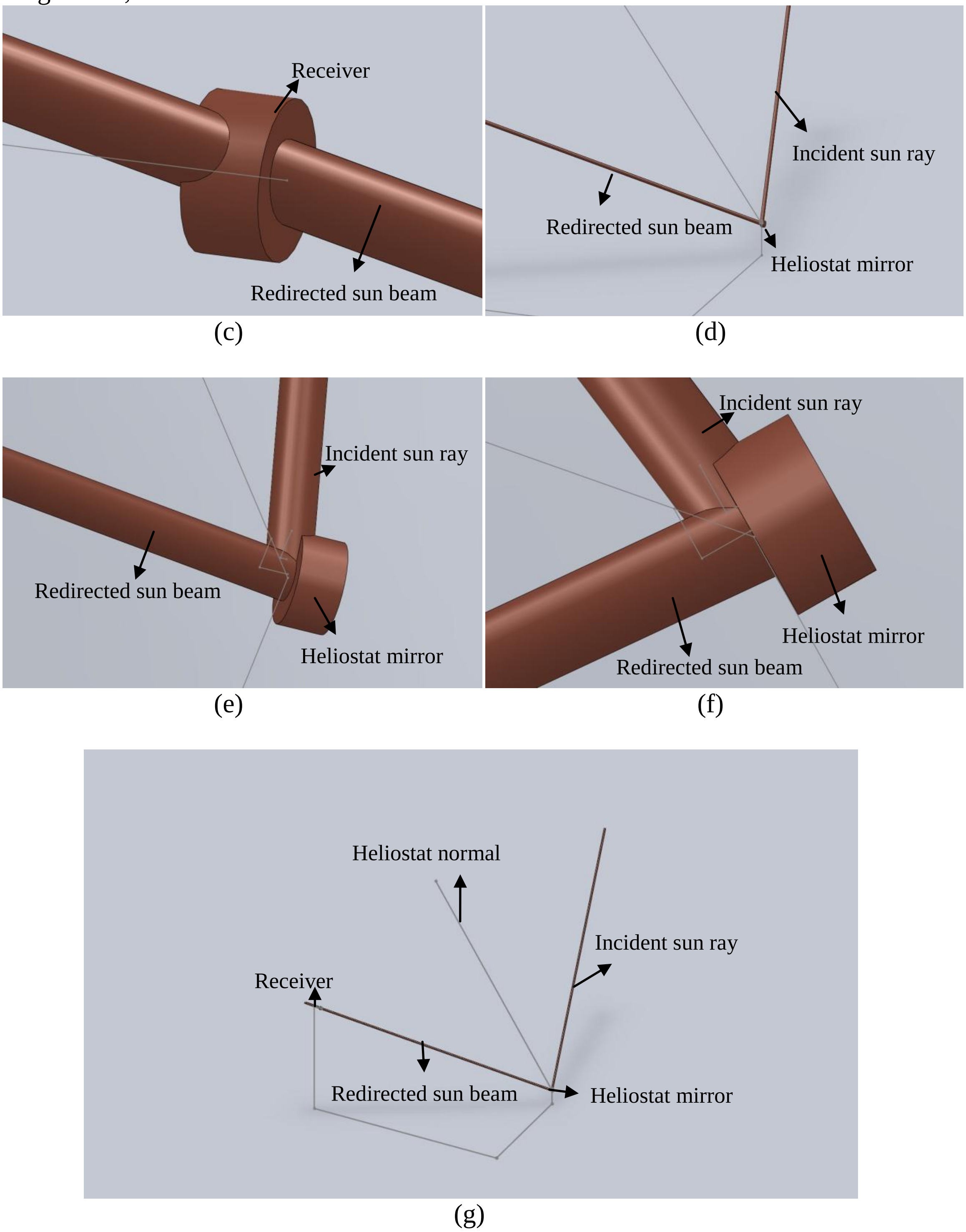
4.2.5 GRAPHICAL VERIFICATION

By using the previous analytical case (See the subtopic ANALYTICAL SAMPLE CASE), an analogical model of receiver, tower, heliostat are designed using SolidWorks to verify that incident sun ray on the heliostat is reflected back exactly onto the receiver by reflecting the ray at the heliostat normal. This verifies that position of the heliostat is correct at the instantenous sun position.



(a) (b)

‘ Figure 4.1, continued’



(c) (d) (e) (f) (g)

Figure 4.1: Image for graphical verification using SolidWorks

Firstly, the heliostat and the receiver are modeled at the respective position according to their respective coordinate point. Then, the sun position which is the first outcome numerically calculated based on the previous analytical sample case, is modeled. Next step is modeling the heliostat normal, which is also a numerically calculated outcome from the developed program. Last but the most essential step is reflecting the sun ray, which connects the sun to the heliostat by using the heliostat normal as reflecting axis, to see whether the ray is reflected exactly on the receiver or not.

Figure 4.1 (a) shows sun ray being reflected at heliostat to the receiver. The angle between sun ray and heliostat normal must be the same angle as the angle between heliostat normal and receiver, whereas Figure 4.1 (b) indicates sun ray hits the receiver exactly in the middle. Figure 4.1 (c) is a closer look of Figure 4.1 (b), which a zoom-in of the sun ray, which hits exactly in the middle of the receiver, whereas Figure 4.1 (d) picturizes the sun ray being reflected at the heliostat. Again Figure 4.1 (e) is a zoom-in of Figure 4.1 (d). It shows the incident sun ray and reflected sun ray at heliostat. The reflected sun ray is so modeled by using the function ‘mirror’ in SolidWorks with the calculated heliostat normal as the reflecting axis. Another angle of view of the Figure 4.1 (e) is shown in Figure 4.1 (f). It is to be noted that the actual mirror IS NOT in the position as indicated in the picture. The mirror should be perpendicular to the heliostat normal. The indicated object is only for design aid purpose, whereas the picture of the whole CAD model is recorded in the Figure 4.1 (g).

From the Figure 4.1, it can be extracted that the sun ray is reflected exactly in the middle of the receiver, which once again verifies the validation of results from the developed code. In other words, the code is convincing to be used for any heliostat normal calculation due to sun position from 7 a.m. to 7 p.m. for any independant single mirror case.

4.3 DISCUSSION

From literature review, it can be extracted that the CST technology is becoming very attractive in Asia and in Malaysia itself nowadays. However, real implementation in Malaysia is unlikely done. Moreover, some of Malaysia universities like UTM only focus on reducing heliostat aberration, but not controlling the heliostat itself. On the other hand, some universities like UTAR did publish some papers regarding the control the heliostat or to be more exact formula derivation of controlling methods, but only theoretically. They did not really did it practically or rather experimentally.

Therefore, it is reasonable that this work focusing on applying the already-derived formulas by converting those formulas into code, which is in this case the MATLAB code, so that it can be applied to a model for experimental purpose. From there, optimization can be done. In other words, this work is a start point before proceeding the project to more serious stage like building a large demonstrative concentrated solar tower power plant.

In developing the code, several important parameters which determine the sun position and the resulting heliostat normal such as the date of a certain day, heliostat and receiver position are set as variables, so that user can determine the input as needed. For example, the date must be keyed in to tell the program what day it is to determine n_{th} day of the year and celestial declination/inclination δ. The celestial declination/inclination δ is a necessary component for sun position vector calculation. Different day gives different sun position. This explains the reason of the date being important to be set as a variable. Whilst heliostat and receiver positions are also set as variable since both vectors depend on the heliostat field design and layout. Therefore, this program can be used for an independant single heliostat, which its position is determined by user. The output of this programme are sun position of a certain day and also the corresponding heliostat normal, so that incident sun ray will be reflected exactly on the receiver.

The developed code or rather programme has its limitations. It is developed for only a single mirror utilization, and calculated results which are the sun position and the corresponding heliostat normal are only hourly (from 7 a.m. to 7 p.m., since the sun is assumed to be available only during this time period).

Following is the summary of the numerically and analytically calculated results with the aid of the developed program. They are tabulated as follows:

Table 4.6: Output from the developed code for all five cases

No. of Verification	Date	Time	Sun Position Vector	Heliostat Normal Vector
Sample Case	24th July	3 p.m.	Numerical: [0.1295 0.6652 0.7353] Analytical: [0.1300 0.6653 0.7352]	Numerical: [0.6186 0.1324 0.7744] Analytical: [0.6189 0.1324 0.7742]
1	24th July	7 a.m.	Numerical: [0.1243 -0.9087 0.3985] Analytical: [0.1242 -0.9088 0.3983]	Numerical: [0.4941 -0.7506 0.4387] Analytical: [0.4940 -0.7508 0.4385]
2	24th July	7 p.m.	Numerical: [-0.4173 0.9087 0.0096] Analytical: [-0.4172 0.9088 0.0094]	Numerical: [0.5173 0.6205 0.5894] Analytical: [0.5172 0.6204 0.5887]
3	25th May	7 p.m.	Numerical: [-0.4284 0.9034 0.0197] Analytical: [-0.4284 0.9034 0.0197]	Numerical: [0.5037 0.6155 0.6062] Analytical: [0.5035 0.6153 0.6058]
4	3rd Sept.	7 p.m.	Numerical: [-0.2514 0.9588 -0.1323] Analytical: [-0.2514 0.9588 -0.1323]	Numerical: [0.6880 0.6325 0.3557] Analytical: [0.6879 0.6325 0.3555]

From Table 4.6, it is shown that the calculated results are only accurate to 0.0005 metre since numerical and analytical results differs up to 5/10,000 m due to some rounding error. However, the result deviation is too small which generates only less than one degree, since the distance range between the heliostat and receiver is normally will not exceed one kilometre (Sánchez & Romero, 2006) (Wei et al., 2007) (J Noone et al., 2011). The novelty of this work is that the first-in-Malaysia ready-to-be-applied experimental code for controlling a single heliostat is generated. This work is a small step towards construction of bigger scale of solar tower power plant and latter a commercial CST.

5 CONCLUSION

This work provides a platform to control sun tracking of a heliostat for a CST project. In other words, this work is a part of contributions to generate the first CST model in South East Asia. Please refer 'APPENDIX C – PICTURES OF THE UTeM CST MODEL PROJECT' for pictures of the CST model.

This work has produced a MATLAB program to calculate the hourly sun position and heliostat normal vector from 7 a.m. to 7 p.m for a given day. User should enter the day of the desired output, heliostat/mirror and also receiver/absorber position vector as input. The program will automatically calculate the related sun position vector and the corresponding heliostat normal vector, at which the sun ray will be redirected onto the receiver hourly for a time range from 7 a.m. to 7 p.m..

The program´s functionality has been proved via several verifications and its accuracy which is 0.0005 m has been stated and verified. For example, at 3 p.m. of 24th July, as shown in previous analytical sample case, the analytical and numerical sun position vector and the corresponding heliostat normal vector for that day are as follows :

Sun Position Vector	Heliostat Normal Vector
Numerical: [0.1295 0.6652 0.7353] Analytical: [0.1300 0.6653 0.7352]	Numerical: [0.6186 0.1324 0.7744] Analytical: [0.6189 0.1324 0.7742]

From the example, it can be seen that difference of the numerical and analytical results vary from 0.0000 to 0.0005 metre which validates the statement of minimum accuracy of the numerical calculated results is 5/10,000 m.

As a conclusion, this work has:

1) Calculated sun position vector and generated its corresponding MATLAB code.
2) Calculated heliostat normal vector in corresponding to the sun position vector and generated its MATLAB code.
3) Verified the functionality of the generated code in the time range from 7 a.m. to 7 p.m. for any date.
4) Checked accuracy of the generated code.

However, the generated code is only for single heliostat utilization and also calculates the sun position hourly. In addition, the code is in M-file form. Therefore, future works may include:

1) Calculation not only for a single mirror, but for n mirrors.
2) The calculation may be carried out for every minute instead of hourly to guarantee the more precise sun position and corresponding heliostat normal vector.
3) This work may be converted to ‘Graphic User Interface’ (GUI) form to make it more interactive and user friendly.

6 SUPPLEMENTARY

6.1 APPENDICES

6.1.1 APPENDIX A - DEVELOPED CODE

```
clc
clear all
%User input
%Please pay attention!The coordinate origin is located at the base of
the
%receiving tower.X-axis is in the north-,Y-axis in the east- and Z-
axis in
%the zenith direction
m=input ('Enter the number of month m: ')
d=input ('Enter the number of day d: ')
mirror_coordinate=input ('Enter the coordinate of the center point of
mirror [mx,my,mz]: ')   %Example = [-130,75,10]
receiver_coordinate=input ('Enter the coordinate of point at the
receiver [rx,ry,rz]: ')       %Example = [-5,0,75]
%Fixed entity
%Latitude is +ve for northern and -ve for the southern hemisphere
latitude_UTeM=+2.309357
%Calculated result
%D is the day of the year and d is the day of the month
    if m==1
D=d;
    elseif m==2
D=31+d;
    elseif m==3
D=59+d;
    elseif m==4
D=90+d;
    elseif m==5
D=120+d;
    elseif m==6
D=151+d;
    elseif m==7
D=181+d;
    elseif m==8
D=212+d;
    elseif m==9
D=243+d;
    elseif m==10
D=273+d;
    elseif m==11
D=304+d;
    else
D=334+d;
    end
%D
%t is zero at 12noon,positive afterwards and negative before noon
t=[-5;-4;-3;-2;-1;0;+1;+2;+3;+4;+5;+6;+7];
%Calculation of omega
%omega is an hour angle
omega=t*15;
%t
%omega
%Calculation of delta
```

```
%delta is celestial inclination/declination
delta=23.45*sin(360/365*(D+284)*pi/180);
%delta
%Calculation of solaraltitude angle,SA
%SA is solaraltitude angle
for i=1:13
solaraltitudecomponent(i,1)=sin(latitude_UTeM*pi/180)*sin(delta*pi/180
);
end
%solaraltitudecomponent
sinussolaraltitude=solaraltitudecomponent+cos(latitude_UTeM*pi/180)*co
s(delta*pi/180)*cos(omega*pi/180);
%sinussolaraltitude
for i=1:13
squaresinussolaraltitude(i,1)=sinussolaraltitude(i,1)*sinussolaraltitu
de(i,1);
columnmatrixofone(i,1)=1;
cosinussolaraltitudecomponent(i,1)=columnmatrixofone(i,1)-
squaresinussolaraltitude(i,1);
cosinussolaraltitude(i,1)=cosinussolaraltitudecomponent(i,1)^0.5;
productofsinussolaraltitudeandcosinussolaraltitude(i,1)=sinussolaralti
tude(i,1)/cosinussolaraltitude(i,1);
atanofsinussolaraltitudeandcosinussolaraltitude(i,1)=atan(sinussolaral
titude(i,1)/cosinussolaraltitude(i,1));
SA(i,1)=atanofsinussolaraltitudeandcosinussolaraltitude(i,1)*180/pi;
end
%squaresinussolaraltitude
%cosinussolaraltitude
%productofsinussolaraltitudeandcosinussolaraltitude
%atanofsinussolaraltitudeandcosinussolaraltitude
%SA
%Calculation of pseudo azimuth angle,zenith angle and omegaWE
%omegaWE is hour angle when the sun is due to east or west
for i=1:13
cosinusomega(i,1)= cos(omega(i,1)*pi/180);
cosinuszenith(i,1)=sin(latitude_UTeM*pi/180)*sin(delta*pi/180)+cos(del
ta*pi/180)*cos(latitude_UTeM*pi/180)*cosinusomega(i,1);
zenithangle(i,1)=acos(cosinuszenith(i,1))*180/pi;
sinuszenithangle(i,1)=sin(zenithangle(i,1)*pi/180);
sinusomega(i,1)=sin(omega(i,1)*pi/180);
sinuspseudoazimuth(i,1)=sinusomega(i,1)*(cos(delta*pi/180))/sinuszenit
hangle(i,1);
pseudoazimuthangle(i,1)=asin(sinuspseudoazimuth(i,1))*180/pi;
end
%cosinusomega
%cosinuszenith
%zenithangle
%sinuspseudoazimuth
%pseudoazimuthangle
cosinusomegaWE=(tan(delta*pi/180))/(tan(latitude_UTeM*pi/180));
%cosinusomegaWE
omegaWE=acos(cosinusomegaWE)*180/pi;
%omegaWE
%Calculation of C1,C2 and C3
%C1 is calculated to determine whether or not the Sun is within the
first or fourth quadrants and above the horizon
%C2 includes the variables of latitude and declination
%C3 is calculated to define whether or not the Sun has passed the
local meridian (identify whether it is morning or afternoon)
for i=1:13
if omega(i,1)<omegaWE
    C1(i,1)=1;
else
```

```
    C1(i,1)=-1;
end
if omega(i,1)>=0
    C3(i,1)=1;
else
    C3(i,1)=-1;
end
end
%C1
%C3
if latitude_UTeM*(latitude_UTeM-delta)>=0
    C2=1;
else
    C2=-1;
end
%C2
%Calculation of azimuth angle and sun position vector
for i=1:13
azimuthangle(i,1)=C1(i,1)*C2*pseudoazimuthangle(i,1)+C3(i,1)*((1-
C1(i,1)*C2)/2)*180;
sinusazimuth(i,1)=sin(azimuthangle(i,1)*pi/180);
cosinusazimuth(i,1)=cos(azimuthangle(i,1)*pi/180);
Sz(i,1)=sinussolaraltitude(i,1);
Se(i,1)= cosinussolaraltitude(i,1)* sinusazimuth(i,1);
Sn(i,1)= cosinussolaraltitude(i,1)* cosinusazimuth(i,1);
sunposition(i,1:3)=[Sn(i,1),Se(i,1),Sz(i,1)];
end
%azimuthangle
%Sn
%Se
%Sz
sunposition
%Calculation of vector of redirection of the Sun´s ray toward the
point A at the receiver;vector R
magnitude_r_minus_m=((receiver_coordinate(1,1)-
mirror_coordinate(1,1))^2+(receiver_coordinate(1,2)-
mirror_coordinate(1,2))^2+(receiver_coordinate(1,3)-
mirror_coordinate(1,3))^2)^0.5;
vector_R=[(receiver_coordinate(1,1)-
mirror_coordinate(1,1))/magnitude_r_minus_m,(receiver_coordinate(1,2)-
mirror_coordinate(1,2))/magnitude_r_minus_m,(receiver_coordinate(1,3)-
mirror_coordinate(1,3))/magnitude_r_minus_m];
%magnitude_r_minus_m
%vector_R
%Calculation of heliostatnormal
for i=1:13
cosinusSA(i,1)=cos(SA(i,1)*pi/180);
sinusSA(i,1)=sin(SA(i,1)*pi/180);
cosinusazimuthangle(i,1)=cos(azimuthangle(i,1)*pi/180);
sinusazimuthangle(i,1)=sin(azimuthangle(i,1)*pi/180);
rowsofvector_Rx(i,1)=vector_R(1,1);
rowsofvector_Ry(i,1)=vector_R(1,2);
rowsofvector_Rz(i,1)=vector_R(1,3);
cosinus2theta(i,1)=rowsofvector_Rx(i,1)*cosinusSA(i,1)*cosinusazimutha
ngle(i,1)+rowsofvector_Ry(i,1)*cosinusSA(i,1)*sinusazimuthangle(i,1)+r
owsofvector_Rz(i,1)*sinusSA(i,1);
twotheta(i,1)=acos(cosinus2theta(i,1))*180/pi;
theta(i,1)=twotheta(i,1)/2;
cosinustheta(i,1)=cos(theta(i,1)*pi/180);
rowsoftwo(i,1)=2;
twocosinustheta(i,1)=rowsoftwo(i,1)*cosinustheta(i,1);
heliostatnormal_X(i,1)=(sunposition(i,1)+vector_R(1,1))/twocosinusthet
a(i,1);
```

```
heliostatnormal_Y(i,1)=(sunposition(i,2)+vector_R(1,2))/twocosinusthet
a(i,1);
heliostatnormal_Z(i,1)=(sunposition(i,3)+vector_R(1,3))/twocosinusthet
a(i,1);
heliostatnormal(i,1:3)=[heliostatnormal_X(i,1),heliostatnormal_Y(i,1),
heliostatnormal_Z(i,1)];
end
%cosinus2theta
%twotheta
%theta
%twocosinustheta
Heliostatnormal
```

Apendix A: The generated code

6.1.2 APPENDIX B – NUMERICAL VERIFICATION RESULTS

First Verification

Latitude=+37 Date=24 July Time=3pm

Heliostatsnormal Vector			Sunposition Vector		
0.4941	-0.7506	0.4387	0.1243	-0.9087	0.3985
0.4408	-0.7118	0.5469	0.0123	-0.8147	0.5797
0.493	-0.6132	0.6172	0.1295	-0.6652	0.7353
0.5373	-0.5039	0.6763	0.2195	-0.4704	0.8547
0.5729	-0.3858	0.7231	0.2761	-0.2435	0.9298
0.5991	-0.261	0.7569	0.2954	0	0.9554
0.6156	-0.1316	0.777	0.2761	0.2435	0.9298
0.6221	0.0003	0.783	0.2195	0.4704	0.8547
0.6186	0.1324	0.7744	0.1295	0.6652	0.7353
0.6055	0.2625	0.7513	0.0123	0.8147	0.5797
0.5833	0.3885	0.7133	-0.1243	0.9087	0.3985
0.5532	0.5084	0.66	-0.2708	0.9408	0.2041
0.5173	0.6205	0.5894	-0.4173	0.9087	0.0096

Second Verification

Latitude=+37 Date=24 May Time=7pm

Heliostatsnormal Vector			Sunposition Vector		
0.4994	-0.7454	0.4416	0.137	-0.9034	0.4064
0.4357	-0.7109	0.552	0.0013	-0.81	0.5865
0.4868	-0.613	0.6223	0.1153	-0.6613	0.7412
0.5311	-0.504	0.6812	0.2047	-0.4676	0.8599
0.5665	-0.3863	0.7279	0.261	-0.2421	0.9345
0.5925	-0.2618	0.7618	0.2801	0	0.96
0.6087	-0.1328	0.7822	0.261	0.2421	0.9345
0.6149	-0.0014	0.7886	0.2047	0.4676	0.8599
0.6109	0.1302	0.7809	0.1153	0.6613	0.7412
0.5971	0.2597	0.7589	-0.0013	0.81	0.5865
0.574	0.3851	0.7227	-0.137	0.9034	0.4064
0.5423	0.5042	0.6721	-0.2827	0.9353	0.213
0.5037	0.6155	0.6062	-0.4284	0.9034	0.0197

Third Verification

Latitude=+37 Date=24 July Time=7am

Heliostatsnormal Vector			Sunposition Vector		
0.4941	-0.7506	0.4387	0.1243	-0.9087	0.3985
0.4408	-0.7118	0.5469	0.0123	-0.8147	0.5797
0.493	-0.6132	0.6172	0.1295	-0.6652	0.7353
0.5373	-0.5039	0.6763	0.2195	-0.4704	0.8547
0.5729	-0.3858	0.7231	0.2761	-0.2435	0.9298
0.5991	-0.261	0.7569	0.2954	0	0.9554

0.6156	-0.1316	0.777	0.2761	0.2435	0.9298
0.6221	0.0003	0.783	0.2195	0.4704	0.8547
0.6186	0.1324	0.7744	0.1295	0.6652	0.7353
0.6055	0.2625	0.7513	0.0123	0.8147	0.5797
0.5833	0.3885	0.7133	-0.1243	0.9087	0.3985
0.5532	0.5084	0.66	-0.2708	0.9408	0.2041
0.5173	0.6205	0.5894	-0.4173	0.9087	0.0096

Fourth Verification

Latitude=+37	Date=24 July		Time=7pm		
Heliostatsnormal Vector			Sunposition Vector		
0.4941	-0.7506	0.4387	0.1243	-0.9087	0.3985
0.4408	-0.7118	0.5469	0.0123	-0.8147	0.5797
0.493	-0.6132	0.6172	0.1295	-0.6652	0.7353
0.5373	-0.5039	0.6763	0.2195	-0.4704	0.8547
0.5729	-0.3858	0.7231	0.2761	-0.2435	0.9298
0.5991	-0.261	0.7569	0.2954	0	0.9554
0.6156	-0.1316	0.777	0.2761	0.2435	0.9298
0.6221	0.0003	0.783	0.2195	0.4704	0.8547
0.6186	0.1324	0.7744	0.1295	0.6652	0.7353
0.6055	0.2625	0.7513	0.0123	0.8147	0.5797
0.5833	0.3885	0.7133	-0.1243	0.9087	0.3985
0.5532	0.5084	0.66	-0.2708	0.9408	0.2041
0.5173	0.6205	0.5894	-0.4173	0.9087	0.0096

Fifth Verification

Latitude=+37	Date=3rd September		Time=7pm		
Heliostatsnormal Vector			Sunposition Vector		
0.4688	-0.7964	0.382	0.0579	-0.9588	0.2781
0.5261	-0.71	0.4681	0.2019	-0.8596	0.4693
0.5776	-0.6104	0.5421	0.3257	-0.7019	0.6335
0.6221	-0.4994	0.603	0.4206	-0.4963	0.7594
0.6589	-0.379	0.6497	0.4803	-0.2569	0.8386
0.6873	-0.2516	0.6814	0.5006	0	0.8657
0.707	-0.1192	0.6971	0.4803	0.2569	0.8386
0.718	0.0158	0.6959	0.4206	0.4963	0.7594
0.7208	0.1508	0.6765	0.3257	0.7019	0.6335
0.7164	0.2835	0.6375	0.2019	0.8596	0.4693
0.7068	0.4109	0.5759	0.0579	0.9588	0.2781
0.6952	0.5294	0.4863	-0.0967	0.9926	0.0729
0.688	0.6325	0.3557	-0.2514	0.9588	-0.1323

Apendix B: Numerical verification results

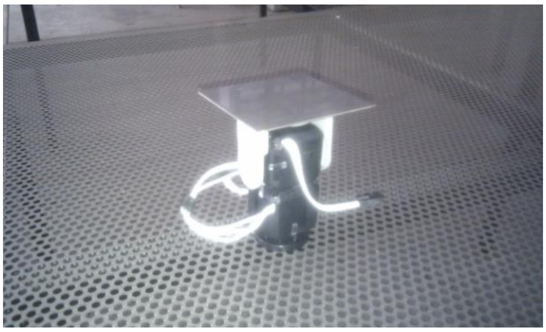
6.1.3 APPENDIX C – PICTURES OF THE UTeM CST MODEL PROJECT

(a)
The whole model of mini CST (an imitate sun, CST tower and a heliostat)

(b)
The sun during the day at 12 noon

(c)
The heliostat in action

(d)
The CST model when there is no sun



(e)
The heliostat

Apendix C: Pictures of the CST model

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