ABSTRACT

Several computational methods have been proposed to solve optimal control problems. These methods are classified either as direct or indirect methods. This thesis is based on solving optimal control problems by using both direct and indirect methods.

Orthogonal functions have been used to solve various problems of dynamic systems. A typical example is the Haar wavelet function, which is used in this work to convert the underlying differential equations in an optimal control problem into a system of linear algebraic equations.

To establish an indirect method, we propose a novel feedback control method that uses only linear systems to solve affine nonlinear control system with quadratic cost function and infinite time horizon. This method is a numerical technique that is based on the combination of Haar wavelets operational matrices and successive Generalized Hamilton-Jacobi-Bellman (GHJB) equation. This method improves the closed-loop performance of stabilizing controls and reduces the problem of solving a nonlinear Hamilton-Jacobi-Bellman (HJB) equation to solve the corresponding GHJB equation. An interesting fact is that when the process of improving the controls and solving GHJB equation is iterated, the solution to the GHJB equation converges uniformly to the solution of the HJB equation which is in the form of the gradient of the Lyapunov function $\nabla \mathbf{V}(x)$. The Lyapunov function $\mathbf{V}(x)$ is the measure of the performance index, which can be determined by integrating $\nabla \mathbf{V}(x)$ parallel to the axes. In the process of establishing this novel feedback control method, we have to define new operational matrices of integration for a chosen stabilizing domain $[-\tau, \tau)$ and a new operational matrix for the product of two dimensional Haar wavelet functions.

To establish a direct method, an efficient new algorithm is proposed to solve nonlinear optimal control problems with a finite time horizon under inequality constraints. In this technique, we parameterize both the states and the controls by using Haar wavelet functions and Haar wavelet operational matrix. The nonlinear optimal control problem is converted into a quadratic programming (QP) problem through the quasilinearization iterative technique. The inequality constraints for trajectory variables are transformed into quadratic programming constraints by using the Haar wavelet collocation method. The quadratic programming problem with linear inequality constraints is then solved by using standard QP solver.

Both proposed numerical methods have been applied to several examples. The proposed methods obtain better or comparable results compared with other established methods. Moreover, the methods are attractive, stable, convergent and easily coded.

The direct method has been applied in this thesis to solve a practical optimal control problem. This problem is the multi-item production-inventory model with stock-dependent deterioration rates and deterioration due to self-contact and the presence of the other stock. The problem is addressed by using four different types of demand rates namely, constant, linear, logistic and periodic demand rates. The solution to the model is discussed numerically and displayed graphically. By enhancing the resolution of the Haar wavelet, we can improve the accuracy of the states, controls and cost. Simulation results were also compared with those obtained by other researchers.

ABSTRAK

Beberapa kaedah pengiraan telah dicadangkan untuk menyelesaikan masalah kawalan optimum. Kaedah-kaedah ini dikelaskan sama ada sebagai kaedah langsung atau kaedah tidak langsung. Tesis ini adalah berdasarkan kepada menyelesaikan masalah kawalan optimum dengan menggunakan kedua-dua kaedah langsung dan tidak langsung.

Fungsi ortogon telah digunakan untuk menyelesaikan berbagai masalah dalam sistem dinamik. Satu contoh yang biasa adalah fungsi gelombang kecil Haar yang digunakan dalam tesis ini untuk menukar asas persamaan pengamiran sandaran dalam masalah kawalan optimum kepada suatu sistem persamaan aljabar linear.

Bagi mewjudkan kaedah langsung, kami mencadangkan kaedah kawalan suap balik yang asli yang menggunakan hanya sistem linear untuk menyelesaikan sistem kawalan afin tak linear dengan fungsi kuadratik dan ufuk masa tak terhingga. Kaedah ini adalah satu teknik berangka yang berasaskan gabungan matriks operasi gelombang Haar dan persamaan teritlak Hamilton-Jacobi-Bellman (HJB) berturutan. Kaedah ini meningkatkan prestasi gelung tertutup kawalan stabil dan menurunkan masalah menyelesaikan persamaan tak linear Hamilton-Jacobi-Bellman (HJB) kepada menyelesaikan persamaan GHJB yang sepadan. Satu fakta menarik ialah apabila proses meningkatkan kawalan dan menyelesaikan persamaan GHJB dilelarkan, penyelesaian untuk persamaan GHJB itu menmpu secara seragam ke penyelesaian bagi persamaan HJB dalam bentuk kecerunan fungsi Lyapunov $\nabla\mathbf{V}(x)$. Fungsi Lyapunov $\mathrm{V}(x)$ adalah ukuran bagi indeks prestasi, yang boleh ditentukan dengan mengkanirkan $\nabla\mathbf{V}(x)$ selari kepada paksi- paksi. Dalam proses mewujudkan kaedah kawalan suap balik yang asli ini kami perlu menentukan matriks operasi pengamiran yang untuk domain perstabilan $[-\tau,\tau)$ menstabilkan yang dipilih dan matriks operasi baru bagi hasil darab fungsi gelombang kecil Haar dalam dua dimensi.

Untuk mewujudkan kaedah langsung, satu algoritma baru yang cekap telah dicadangkan untuk menyelesaikan masalah kawalan optimum tak linear dengan tempoh masa yang terhad di bawah kekangan ketahsamaan. Dalam teknik ini, kami berparameter kedua-dua keadaan dan kawalan menggunakan fungsi gelombang kecil Haar dan matriks operasi gelombang kecil Haar. Masalah kawalan optimum tak linear ditukar menjadi masalah pengaturcaraan kuadratik (QP) melalui teknik lelaran kuasilinear. Selain itu, Kekangan ketahsamaan bagi pembolehubah trajektori diubah menjadi kekangan pengaturcaraan kuadratik menggunakan kaedah kolokasi gelombang kecil Haar. Masalah pengaturcaraan kuadratik dengan kekangan ketaksamaan linear kemudiannya diselesaikan menggunakan solver QP yang biasa.

Kedua-dua kaedah berangka yang dicadangkan telah digunakan pada beberapa contoh. Kami mendapati bahawa kaedah yang dicadangkan mendapatkan keputusan yang lebih baik atau yang setanding berbanding dengan kaedah biasa yang lain. Juga, kaedah ini adalah menarik, stabil, menumpu dan mudah dikodkan.

Dalam tesis ini, kaedah langsung telah digunakan untuk menyelesaikan suatu masalah kawalan optimum praktis. Masalahnya ialah model inventori-pengeluaran pelbagai item dengan kadar kemerosotan bergantung kepada stok dan kemerosotan disebabkan saling bersantuhan dan kehadiran stok lain. Masalah ini diatasi dengan menggunakan empat jenis kadar permintaan iaitu kadar permintaan tetap, linear, logistik dan berkala. Penyelesaian kepada model dibincangkan secara berangka dan dipaparkan secara grafik. Dengan meningkatkan resolusi gelombang kecil Haar, kami boleh meningkatkan ketepatan keadaan, kawalan dan kos. Keputusan simulasi juga dibandingkan dengan hasil yang diperolehi oleh penyelidik lain

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LIST OF SYMBOLS AND ABBREVIATIONS

u^*	Optimal control
x_j	Collocation points
x	State variable
x^*	Optimal infinite horizon state trajectory corresponding to V^*
J	Performance index
$\mathbf{x_0}$	Initial condition vector
$\mathbf{x}_f$	Final condition vector
u	An arbitrary admissible control
m	Level of Haar wavelet
t	Time
t_f	Finite time
t_0	Initial time
n_1	Number of state variables
n_2	Number of control variables
$\mathbf{T}$	Transpose operation
$\mathbf{H}_m$	Haar wavelet matrix
$\mathbf{H}_{ess}$	Hessian matrix
$h_0(t)$	Haar scaling function
$h_1(t)$	Haar mother wavelet function
$\mathbf{P}_m$	Haar wavelet operational matrix
$\mathbf{h}_m(t)$	Vector of Haar wavelet functions

c_i	Haar series coefficient
d_i	Haar series coefficient
$\mathbf{Q}$	Positive semi-definite matrix
$\mathbf{I}_n$	Identity matrix
$\mathbf{R}$	Symmetric positive definite matrix
M(c)	Operational matrix of product one dimension
N(**D**)	Operational matrix of product two dimensions
LQP	Linear quadratic programming problem
Eqn.	Equation
HJB	Hamilton-Jacobi-Bellman Equation
GHJB	Generalize Hamilton-Jacobi-Bellman Equation
$\mathrm{V}^{(i)}$	i^{th} performance index in successive GHJB equation
$u^{(i)}$	i^{th} control in successive GHJB equation
V^{*}	Solution of the HJB equation
Ω	Compact subset of $\Re^n$ containing a ball around the origin
$\dfrac{\partial \mathrm{V}}{\partial x}$	Row vector of partial derivatives of V , $\left[\dfrac{\partial \mathrm{V}}{\partial x_1}, \dfrac{\partial \mathrm{V}}{\partial x_2}, \ldots, \dfrac{\partial \mathrm{V}}{\partial x_n}\right]$
SHWCM	Successive Haar wavelet collocation method