FORWARD AND BACKWARD FUZZY ECONOMIC ORDER QUANTITY MODELS CONSIDERING LEARNING THEORY

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FACULTY OF ENGINEERING
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Forward and Backward Fuzzy Economic Order Quantity Models Considering Learning Theory

Field of Study: **Manufacturing Management (Engineering and Engineering Trades)**

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ABSTRACT

Inventory systems deal with any activities to manage inventory of raw materials, work in process, finished products, spares, and equipment. As uncertainty is an inherent part of the real world, during these processes, the formulated inventory system should come up with uncertain data. Due to the capability of analyzing real situations, fuzzy inventory systems assist decision-making processes and provide a better understanding of the behavior of production and inventory environments.

In this research, for the first time, a comprehensive literature review is conducted in the state-of-the-art of fuzzy inventory models where more than 120 papers are carefully and completely investigated according to the previous works. The fuzzy inventory systems that are based on the economic order/production quantity (EOQ/EPQ) settings are reviewed, so as to systematically analyze the fuzzy characteristics involved in capturing the uncertainty. Thereafter, to fill the identified gaps, two fuzzy EOQ models are developed.

A fully fuzzy forward EOQ model for items with imperfect quality based on two different holding costs and learning considerations with triangular fuzzy numbers (TFNs) is extended. According to this model, the effect of learning and fuzziness on an inventory system are analyzed simultaneously. The lot size is obtained in nth shipment when learning occurs optimizing the total cost function using Karush-Kuhn-Tucker (KKT) conditions. It is concluded the optimal lot size directly depends on the amount of uncertainty. The inventory system that is completely fuzzified is compared with the partially fuzzified one. It is shown that the real inventory situation that is affected by uncertainty can be captured if more elements are fuzzified. Application of the model is shown and explained through a real case from the automobile industry. This provides the opportunity to examine the behavior of optimal lot size and optimal total profit in such a situation to make the best strategy.

Moreover, a fuzzy reverse (backward) inventory model where the recoverable manufacturing process which is affected by the learning theory is discussed applying two popular defuzzification methods, namely the signed distance (SD) technique and the graded mean integration representation (GMIR) method. The proposed model is optimized with the inclusion of the fuzzy demand rate of the serviceable products and the fuzzy collection rate of the recoverable products from customers It is shown that when the levels of fuzziness are similar and the optimal number of orders are equal, the percentage changes of the optimal recovery lot size in the GMIR method are negative as compared to the SD method. The model is used to solve a problem in a supply chain network in the milk industry.

ABSTRAK

Sistem-sistem inventori menguruskan aktiviti-aktiviti yang berkaitan dengan inventori bahan, kerja dalam proses, produk siap, alat gantian dan perkakasan. Oleh kerana ketidakpastian adalah bahagian bawaan dari dunia sebenar, semasa proses ini, sistem inventori yang diformulasi selalunya menghasilkan data yang tidak pasti. Oleh kerana kebolehannya menganalisa situasi sebenar, sistem inventori kabur dapat membantu proses membuat keputusan dan memberi kefahaman yang lebih tentang tingkah laku suasana produksi dan inventori.

Untuk pertama kali, soroton kajian yang menyeluruh dilakukan dalam bidang model-model inventori kabur di mana lebih daripada 120 kertas ilmiah telah disiasat secara teliti dan lengkapnya. Di mana dalam sorotan kajian ini, sistem-sistem inventori kabur yang berdasarkan suasana-suasana kuantiti pesanan/pembuatan ekonomi (EOQ/EPQ) adalah disorot, untuk menganalisa karakter-karakter kabur yang telah digunakan untuk menguruskan ketidakpastian dengan secara sistematik. Oleh itu, untuk menutup jurang-jurang ilmu yang ditemui, dua model kabur telah dibina.

Sebuah model penuh EOQ kehadapan untuk barang-barang yang kurang berkualiti berdasarkan dua kos pemegangan dan pertimbangan pembelajaran dengan nombor-nombor kabur bersegi tiga (TFNs) adalah dibina. Berdasarkan model ini, kesan pembelajaran dan kekaburan ke atas sistem inventori di analisa secara selari. Saiz lot di dapati pada penghantaran yang ke- n, apabila pembelajaran dioptima fungsi kos total mengunakan kaedah Karush-Kuhn-Tucker (KKT). Adalah disimpulkan saiz lot yang optimal bergantung kepada kepada jumlah ketidakpastian. Sistem inventori yang kabur sepenuhnya adalah dibanding dengan separuh kabur. Adalah didapati bahawa keadaan inventori yang sebenar yang terkesan dengan ketidakpastian dapat dijana jika makin banyak elemen-elemen dapat dikaburkan. Aplikasi model ini ditunjuk dan diterangkan melalui kes kajian sebenar di industri automotif. Ini menyediakan peluang untuk

mengkaji tingkah laku saiz lot optimal dan keuntungan total optimal dalam keadaan terbaik untuk membuat keputusan.

Tambahan pula, model inventori kabur balikan dimana proses pembuatan terkembalikan yang terkesan oleh teori pembelajaran adalah dibincangkan- mengaplikasikan dua kaedah pengeyahkaburan, iaitu teknik "signed distance" (SD) dan "graded mean integration representation" (GMIR). Model yang dicadangkan ini adalah dioptima dengan mengambil kira kadar pemintaan kabur bagi produk-produk yang boleh diservis dan kadar kutipan kabur dari produk-produk yang boleh-dikembalikan dari pelanggan. Ditunjukkan apabila takat-takat kekaburan adalah sama dan bilangan permintaan yang optimal adalah sama, kadar perubahan peratusan bagi saiz lot terkembalikan yang optimal dengan kaedah GMIR adalah negatif berbanding dengan kaedah SD. Model ini digunakan untuk menyelesaikan masalah rangkai rantaian bekalan di industry tenusu.

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TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF SYMBOLS AND ABBREVIATIONS

EOQ	:	Economic Order Quantity
EOQB	:	Fuzzy Economic Order Quantity Model with Backorder
EO/PQEQ	:	Quality Based Studies of EO/PQ
EOQED	:	Studies with Delay in Payment
EOQEO	:	Other Extensions of EOQ
EO/PQEQI	:	Mix Quality Multi-Item Studies of EO/PQ
EO/PQMI	:	Multi-Item Models of EO/PQ
EPQ	:	Economic Production Quantity
EPQEO	:	Other Extensions of EPQ
EPQS	:	Shifting in the Production Status
EPQW	:	Rework Based Studies
FAGP	:	Fuzzy Additive Goal Programming
FDCP	:	Fuzzy Dependent Chance Programming
FEOQ	:	Fuzzy Economic Order Quantity
FEPQ	:	Fuzzy Economic Production Quantity
FEV	:	Fuzzy Expected Value
FNLP	:	Fuzzy Non-Linear Programming
FSC	:	Forward Supply Chain
FST	:	Fuzzy Set Theory
GMIR	:	Graded Mean Integration Representation
KKT	:	Karush-Kuhn-Tucker
PSO	:	Particle Swarm Optimization
RSC	:	Reverse Supply Chain
SD	:	Signed Distance
TFN	:	Triangular Fuzzy Number

LIST OF APPENDICES

CHAPTER 1: INTRODUCTION

1.1 Research Background

Inventory control and management is one of the most important fields in supply chain management. In today's highly competitive business markets, taking the importance of the safety stocks into account is one of the key factors for organizations to compete with their rivals. On the other hand, designing an appropriate inventory model help to gain more benefit, and at the same time, satisfy the customers' demands. Since the first inventory system proposed by Harris (1913), who introduced the basic model of inventory systems called economic order quantity model (EOQ), many versions have accordingly been suggested. These include economic production quantity model (EPQ) (Taft, 1918), inventory control systems (i.e. periodic/continuous review inventory models), joint economic lot size model (JEL) (Glock, 2012), just to name a few. To reflect the real world conditions, these models have been extended with other characteristics such as imperfect quality items (Khan et al., 2011b), inflation and discount (Maity & Maiti, 2008), and delay in payment (Chen & Ouyang, 2006).

One of the most important extensions of these models is studying them in uncertain situations. In chapter 2, the related literature that deals with the fuzzy EOQ/EPQ models is comprehensively reviewed.

Fuzzy set theory that was firstly introduced by Zadeh (1965) has vastly been applied in almost all area of supply chain management as well as inventory models to depict models that can feed back the real condition. Because firms are facing the uncertainty in the inventory process, ignoring this theory results the biased and incorrect policies. When there is shortcoming of historical data in the inventory system, in contrast to the probability theory, fuzzy set theory could be helpful while it uses the previous experiences of the decision makers and managers. Hence, it is becoming increasingly difficult to ignore developing inventory models in fuzzy environment.

In this study, inventory models in fuzzy environment regarding some characteristics for the model which are mainly based on the learning theory and the rate of return of used products in the inventory systems are tried to be developed. Some policies when these characteristics are integrated in an uncertain environment are suggested.

1.2 Research Gaps

Although the fuzzy set theory has widely been investigated in inventory models, there are still a lot of potential rooms for research. One of the research agenda can be studying the depth of fuzziness of an inventory system. It answers to what will occur if an inventory system is extended and considered in a forward supply chain (FSC) with a complete uncertain situation? Usually the previous researches deal with an inventory system in a partially uncertain condition. It means that they fuzzified the whole of the system to some special levels. The more the level and the depth of fuzziness, the more complex the system is. As optimizing the system is very hard in a fully fuzzy environment, researches to date have tended to focus on fuzzification of a part of the inventory system while the other part remains as crisp one.

In this study, the behavior of an EOQ system which is completely fuzzy is attempted to be analyzed. Besides, formulated model includes other characteristics such as learning process that usually affects inventory systems.

Literature has emerged that most of the related researches have tended to integrate fuzzy set theory on forward inventory management while using this theory in backward (reverse) inventory systems is still in its infancy. Moreover, as a reverse cycle occurs in such systems, the importance of the combination of this system with fuzziness even could be more. In order to design a more effective reverse inventory system through a reverse supply chain (RSC), this theory undoubtedly is one of the best tools.

In another part of this study, a reverse EOQ inventory system that is studied under fuzziness of some important factors regarding the learning process is surveyed. Effects of well-known defuzzification methods on the mentioned model that have not been discussed on the previous reverse inventory systems are compared.

1.3 Problem Statement

Establishing the derived policies in inventory systems generally and usually is based on the factors which are assumed being precise and accurate. However, this assumption does not work in an uncertain context that the inventory systems are planned to meet the goals of organizations in which should be efficient and cost-effective. Especially some parameters such as demand naturally are difficult to be predicted because of the lake of statistical data. On the other hand, uncertainty is inherent in costs of inventory systems such as setup and ordering costs. Therefore, these uncertainties are an integral part of the inventory systems (Guiffrida, 2009).

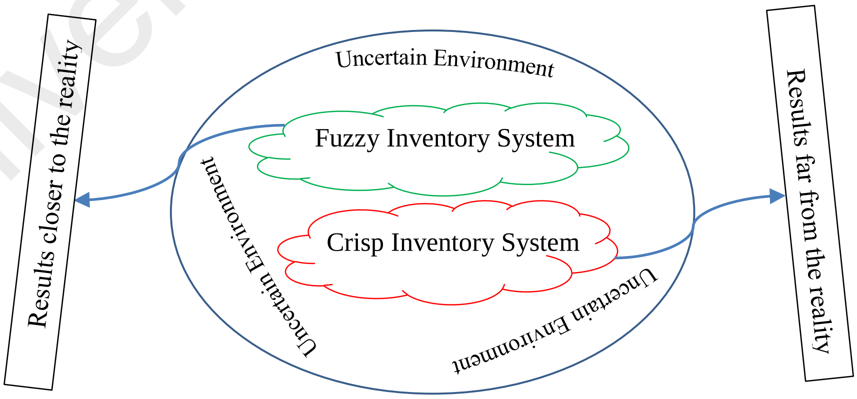

Figure 1.1: Designing of the inventory system

A serious weakness of the crisp inventory systems is that they lead to improper optimal policies such as incorrect economic order quantities which are far from the reality as it is depicted in Figure 1.1. A system which is designed as a crisp model while it is surrounded by uncertainties logically leads to overestimated or underestimated results that affect the decision-making, and consequently the organization.

An interesting characteristic that affects the inventory system is the learning process which causes the improvement of the decision-making by passing the time (Jaber et al., 2008). However, as it was discussed the behavior of this process is different when it occurs in uncertain environment. Because it is a part of the whole system. Other characteristics such as imperfect quality items (Salameh & Jaber, 2000) and return rate of the used products which is important part of the reverse inventory system (Govindan et al., 2015) not only influence the inventory problem in the crisp status but also are of interest in fuzzy situation.

In this study, effects of fuzziness in forward and backward EOQ models that exploit the effect of learning process simultaneously are tried to be analyzed. Although the learning theory could be extensively applied in operation management, there is a little contribution in its application with fuzzy set theory simultaneously.

1.4 Aim and Objectives

This study aims to find the optimal policies in fuzzy inventory models regarding some important characteristics such as the learning process, imperfect quality items and the return rate of used products. The purpose is to achieve to the following research objectives:

- **Objective 1:** To develop a fully fuzzy EOQ inventory model in a forward supply chain to optimize the whole system integrating the concept of fuzziness, learning and imperfect quality items;
- **Objective 2:** To analyze the effect of learning and fuzziness on an inventory system simultaneously;

- **Objective 3:** To develop a partial-fuzzified reverse EOQ inventory model in which includes the concept of learning and fuzzy set theory in a reverse inventory model;
- **Objective 4:** To investigate the effect of different defuzzification methods on the fuzzified parameters and the obtained results in a fuzzy reversed EOQ-based model.

1.5 Research Questions

The following questions which are based on the mentioned objectives are attempted to be answered:

- **Question 1:** How a fully fuzzy inventory system can be developed with imperfect quality items and learning?
- **Question 2:** What are the simultaneous effects of fuzziness and learning on the inventory system?
- **Question 3:** How a partial-fuzzified reverse EOQ inventory model can be developed with the concept of learning and fuzzy set theory?
- **Question 4:** How is the performance of the different defuzzification methods on the fuzzified model in a fuzzy reversed EOQ-based inventory system?

1.6 Research Framework

The applied methodology in Figure 1.2 has been explained. Generally, it includes 10 steps in which each step has some details.

After a comprehensive review of the related literature, two inventory models are selected to develop in the uncertain environment. These models have some special characteristics. The first model is a well-known inventory control model which includes the imperfect quality items in a forward supply chain management. The second one considers an inventory system that analyses the effect of learning process in a reverse inventory system.

In the next step, elements (i.e. parameters and variables) that are intended to study the effect of uncertainty on them are determined and discussed. The first model is developed in a fully fuzzy environment which is the first one in the literature. The second one is studied in a partial-fuzzy situation considering some fuzzified parameters which is the first work in a fuzzy reverse inventory system with learning.

Steps 3 and 4 are the processes of fuzzification where fuzzified models are derived regarding the calculations on the selected fuzzy numbers. In these steps, models are extended and entered to the uncertain environment and the behavior of them with imprecise conditions is analyzed.

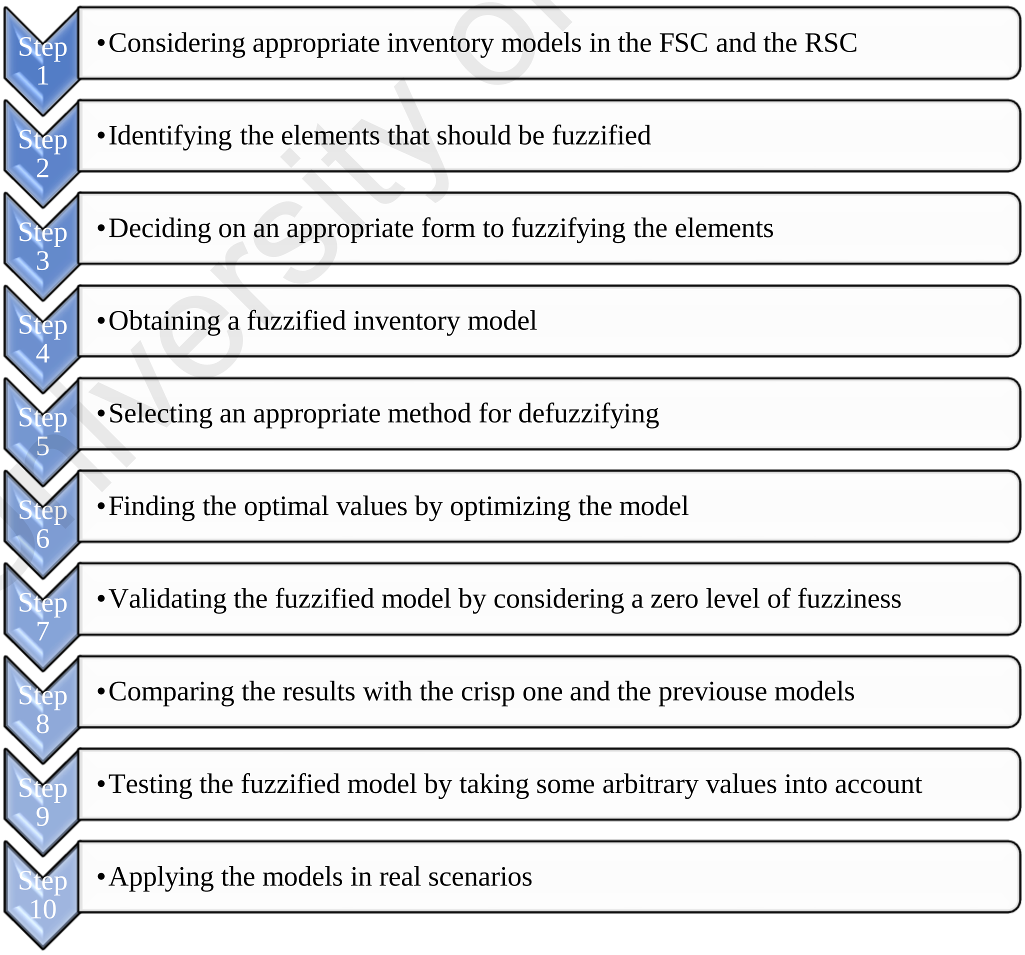

Figure 1.2: Research framework

As correct policies have to be reported to the decision makers precisely, therefore, the next step is devoted to transform the imprecise models to the certain ones with an appropriate method. The results extracted from the process of defuzzification can be applied in the real business environment.

Derived models from the previous steps are discuss how to be optimized in the next stage. Regarding the literature, there are many optimization techniques to obtain the optimal policies. However, it depends on the level of the complexity of the model which method is appropriate and results the best solution. Optimization of the first model is based on the Karush-Kuhn-Tucker theorem (Taha, 1997) while the second one is optimized using a suggested algorithm.

In the next three stages, models with some arbitrary levels of fuzziness and data are validated, compared and tested. Finally, the application of the models is shown in the real case studies.

1.7 Thesis Layout

In this chapter, general framework of the research is overviewed. In the next chapter, EPQ and EOQ models in the literature that are developed in fuzzy situation are comprehensively overviewed. Chapter three is devoted to explain the methodology and techniques that are used to develop and solve the fuzzy inventory models. In chapter 4, the first fuzzy EOQ models are discussed and their characteristics are analyzed. The effect of fuzziness on the second model which is a reverse inventory system is discussed in chapter 5. To show the applicability of the developed models, they are illustrated applying real scenarios in chapter 6. Finally, the research is concluded in the last chapter.

CHAPTER 2: LITERATURE REVIEW

2.1 Introduction

To place the contribution of the proposed fuzzy inventory models in the right perspective, a systematic literature review is conducted and the previous literature categorizing the related papers in some classifications is studied. In fact, a systematic literature review is an approach to summarize the body of existing research on a specific topic, which aims at analyzing the conceptual content of the field, identifying patterns and research streams, and discovering the strengths and weaknesses of selected literature (Hochrein & Glock, 2012). Fuzzy inventory models through two main categories are divided: (1) economic order quantity (EOQ) models, and (2) economic production quantity (EPQ) models. In the next step, some subcategories are considered for the mentioned categories.

Contents of the investigated papers are analyzed in this chapter. Some tables are provided to compare the investigated studies from the fuzzified elements and characteristics points of view. Other tables in the next chapter are presented while the research methodology of our study is explained technically.

2.2 Inventory Management

Inventory control and management is one of the most important fields in supply chain management. It becomes more and more important for the enterprises in the real-life situations. Inventory problems are common in manufacturing, remanufacturing, maintenance service and business operations in general. Inventory is one of the costliest operating expenses through the other activities of a supply chain for many manufacturing industries. A proper control and analyzing of inventory systems can significantly enhance a company's profit (Wang et al., 2007a). In fact, the management of inventory is a vital issue in optimizing productivity and profitability in many industries. For this reasons, investigating inventory management is a critical point for many service and manufacturing industries and many world-wide scholars are interested to find solutions for the inventory management problems using different mathematical point of view.

In recent years, many inventory models including economic order quantity (EOQ) model, economic production quantity (EPQ) model, joint economic lot size (JEL) model, newsvendor model, multi-period inventory model and multi-item inventory model have been developed. These models have been extended by applying other techniques and methods. In the next two sections, studies concerning the EO/PQ models considering the fuzzy set theory are reviewed.

2.3 Fuzzy Set Theory

In order to cope with the ambiguity of input parameters in the realistic environment, the Fuzzy Set Theory (FST) introduced by Zadeh (1965) is recognized as a powerful tool which has received considerable attentions from researchers. The FST becomes a proper method to handle decision-making and reduces risky decision-making where there are vague conditions or no data is available. The property which differentiates the FST from other methods is its capability to utilize decision maker's opinion in an inventory system where its characteristics and data are very complicated. There are many situations in industry where the attributes of the input parameters are complicated so that they can be determined based on the experiences of policy makers and represented by the FST. The application of the FST can permit flexibility in defining the vague parameters which enables the model to handle uncertainties.

2.4 Fuzzy Inventory Management

As the parameters and variables in an inventory model may be derived and estimated by uncertain datum, it is very hard to define a real inventory system exactly using precise terms. Although, in this case, probability theory can be used to estimate the system, some issues make it difficult to estimate the probability distribution of the elements of an inventory system because of (1) absence of historical data, and (2) imprecise nature of data. On one hand, estimation of parameters in the cost functions using traditional econometric methods which use probability theory is not always possible due to insufficient historical data specially, for newly launched products (Guchhait et al., 2014). On the other hand, ignoring uncertainty in the inventory management usually leads to some results that are not compatible with the real world.

Uncertainty of inventory models is a well-established phenomenon in recent years. Extensive research works have been made on stochastic inventory models (Sox et al., 1999; Winands et al., 2011). Since 1965 when Zadeh (1965) introduced fuzzy set theory to cope quantitatively with uncertain information in making proper decisions, inventory models have been extended by applying of this theory. The associated problem becomes a fuzzy inventory model. Fuzzy set theory gives an opportunity to handle inventory models containing imprecise linguistic terms and vagueness in real life situations. In fact, it provides relaxation in fitting of the probability distribution function to deal with these types of situations (Kumar & Goswami, 2015b).

The first fuzzy inventory models that appeared in the literature is believed to be that of Sommer (1981) and Kacprzyk and Stanieski (1982) who used a fuzzy dynamic programming approach and fuzzy set theory to solve a real world production-inventory scheduling problem with capacity constraints, and problem of controlling inventory over an infinite planning horizon, respectively. In the first work, linguistic statements such as "*the stock should be at best zero at the end of the planning horizon*" and "*diminish production capacity as continuously as possible*" were utilized to explain management's fuzzy aspirations for inventory and production capacity reduction in a planned withdrawal from a market. In the second one, considering a fuzzy inventory level and fuzzy replenishment as the output and input, respectively, where demand and system constraints on replenishment are also fuzzy, an inventory system was designed. They presented an algorithm to find the optimal time-invariant strategy for determining the replenishment to current inventory levels that maximizes the membership function for the decision.

2.5 Methodology of Literature Review

The general framework to do the literature review of this research is based on the method designed by Mayring (2010). Four steps are considered including gathering the related papers, descriptive analysis, classification of the works, and content evaluation of the gathered studies as depicted in Figure 2.1. They are explained in the following. In the next stage, the content of the investigated studies is analyzed.

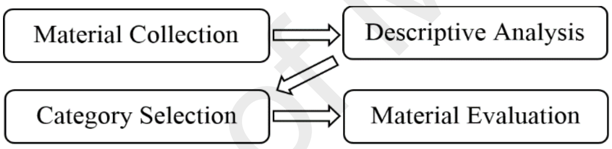

Figure 2.1: Research methodology framework (Mayring, 2010)

2.5.1 Material Collection

Journals which are indexed in Social Sciences Citation Index and Web of Science previously known as Web of Knowledge, the “Web of Science™ tore Collection” including Science Citation Index Expanded (SCI-EXPANDED) and Social Sciences Citation Index (SSCI) are chosen. Keywords such as “fuzzy”, “EOQ”, “EPQ”, “inventory”, “economic order quantity”, “economic production quantity”, and “model” were searched through these databases separately or with each other. All papers that have been published since 1980 are considered. It should be noted that because of the vast number of publications in this area, our review is limited to consider only the papers published in ISI journals (indexed in Journal Citation Reports by Thomson Reuters). Later, to ensure the coverage of all related papers, other well-known databases including

Scopus, Elsevier, Springer, Taylor & Francis, Wiley, IEEE Xplore, Emerald, Inderscience, and ABI/INFORM Complete to find relevant studies published in journals that are indexed by Institute for Scientific Information (ISI) are searched.

2.5.2 Descriptive Analysis

The sample of 130 papers in the previous step was subject to an assessment in terms of descriptive analysis. Figure 2.2 illustrates the yearly distribution of the papers over 29 years' time frame.

Overall, the trend line indicates that the topic of fuzzy inventory management with the focus of fuzzy EOQ (FEOQ) and fuzzy EPQ (FEPQ) has become increasingly popular throughout the years. The years on which the higher number of papers was published were 2009, 2011 and 2013, with 13, 12 and 15 papers respectively. Interestingly, even though the first paper published in 1987, this area was unnoticed for 7 years until the second paper published in 1996. Moreover, roughly 82% of the papers published throughout the recent 10 years, highlights the importance of this domain for researchers.

Figure 2.3 ranks the academic peer-reviewed journals as to the number of papers that they published about this topic. Due to the variety of the journals in this realm, only the journals with four or more publications are provided in this figure, and other journals are grouped in a general category namely "others". As can be seen, *Computers & Industrial Engineering (CAIE)*, *European Journal of Operational Research (EJOR)*, and *Applied Mathematical Modelling (AMM)* are the four primary journals published the models on fuzzy inventory management, which account for almost 31% of the entire papers.

2.5.3 Category Selection

After finalizing the collected papers, these sample papers were categorized regarding two main categories including EOQ and EPQ models.

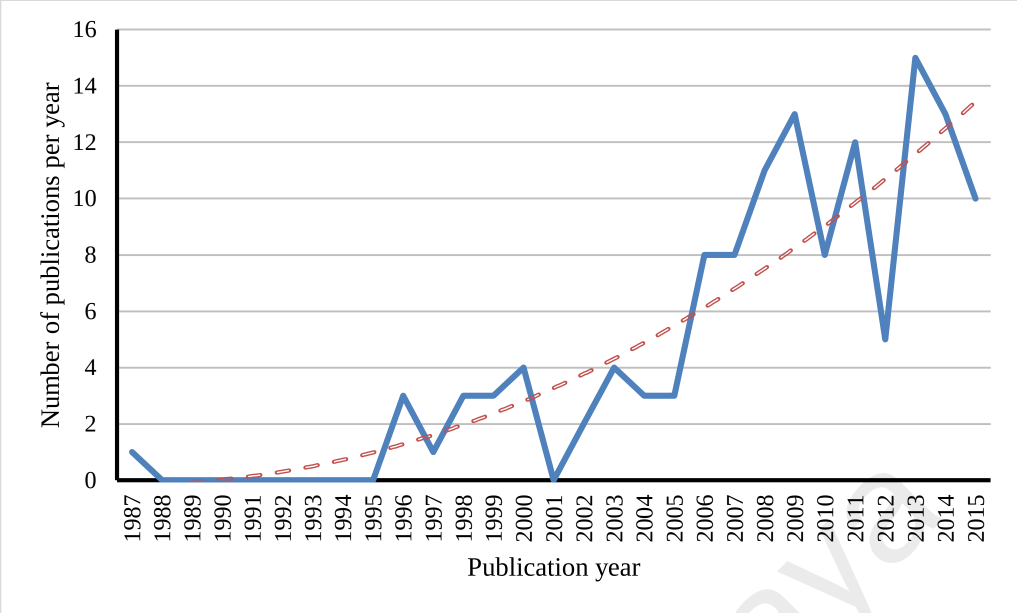

Figure 2.2: Distribution of the published papers per year over the investigated time interval

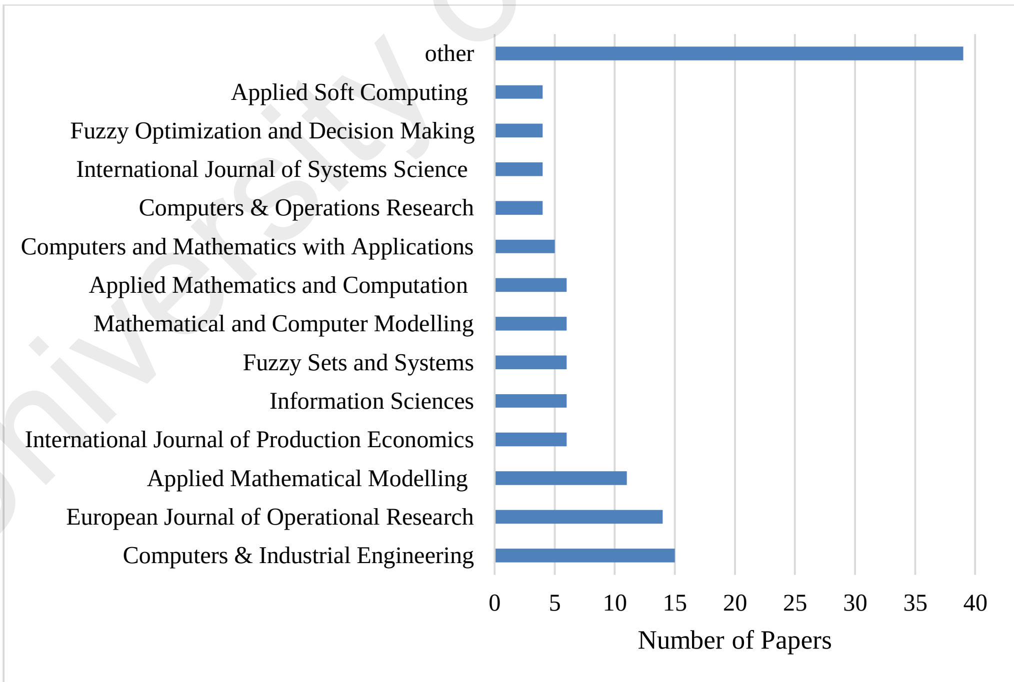

Figure 2.3: Distribution of publications based on different journals

Table 2.1: Classification of the publications according to the considered models

Row	Field of research	No. of Papers	Publications
1.	EOQ	10	Park (1987); Vujošević et al., (1996); Lee and Yao (1999); Yao and Lee (1999)*; Yao et al., (2000); Yao and Chiang (2003); Hojati (2004); Syed and Aziz (2007); Lee and Lin (2011); Samal and Pratihar (2014)*
2.	EOQB	11	Chen et al., (1996); Yao and Lee (1996); Chang et al., (1998); Yao and Lee (1999)*; Yao and Su (2000); Wu and Yao (2003); Björk (2009); Kazemi et al., (2010); Samal and Pratihar (2014)*; Milenkovic and Bojovic (2014); Kazemi et al., (2015)
3.	EOQEQ	9	Chang (2003); Chang (2004); Roy et al., (2007); Wang et al., (2007b); Rong et al., (2008); Mahata and Goswami (2013); Roy et al., (2009a); Hsu (2012); Yadav et al., (2012a)
4.	EOQMI	13	Yadavalli et al., (2005); Wang et al., (2013); Das et al., (2000); Mondal and Maiti (2002); Yao et al., (2003); Das et al., (2004); Baykasoğlu and Göçken (2007); Baykasoglu and Gocken (2011); Maiti and Maiti (2007); Maiti (2008); Panda et al., (2008); Huang (2011); Mousavi et al., (2014)
5.	EOQEQI	8	Roy et al., (2008); Wee et al., (2009); Guchhait et al., (2010); Chakraborty et al., (2013); Roy and Maiti (1998); Xu and Liu (2008); Saha et al., (2010); Jana et al., (2014)
6.	EOQED	12	Chen and Ouyang (2006); De and Goswami (2006); Mahata and Goswami (2007); Ouyang et al., (2010); Maiti (2011); Mahata and Mahata (2011); Soni and Joshi (2013); Guchhait et al., (2014); Yadav et al., (2015); Guchhait et al., (2015); Taleizadeh et al., (2013); Taleizadeh et al., (2011)
7.	EOQEO	13	Liu (2008); Samadi et al., (2013); Sadjadi et al., (2010); Ketsarapong et al., (2012); Bera et al., (2012); Panda et al., (2014); Yadav et al., (2013a); Vijayan and Kumaran (2009); Roy and Maiti (1997); Chou et al., (2009); De and Sana (2013a); De and Sana (2013b); De et al., (2014)
8.	EPQ	4	Lee and Yao (1998); Chang (1999); Lin and Yao (2000); Hsieh (2002)
9.	EPQEQ	7	Mahapatra and Maiti (2006); Chen and Chang (2008); Bag et al., (2009); Das et al., (2011); Pal et al., (2014); Paul et al., (2014); Pal et al., (2015)
10.	EPQW	6	Roy et al., (2009b); Guchhait et al., (2013); Mondal et al., (2013); Mondal et al., (2014); Shekarian et al., (2014b); Shekarian et al., (2014a)
11.	EPQS	7	Halim et al., (2009); Zhang et al., (2009); Wang and Tang (2009b); Hu et al., (2010); Kumar and Goswami (2015b); Kumar and Goswami (2015c); Mahata (2015)
12.	EPQMI	9	Pappis and Karacapilidis (1995); Mandal et al., (2005); Mandal and Roy (2006a); Islam and Roy (2007); Panda and Maiti (2009); Mandal et al., (2011); Björk (2012); Mezei and Björk (2015); Jana et al., (2013)
13.	EPOEQI	11	Maity and Maiti (2005); Mandal and Roy (2006b); Maity and Maiti (2007); Panda et al., (2008); Xu and Zhao (2008); Xu and Zhao (2010); Mandal et al., (2010); Maity (2011a); Das and Maiti (2013); Maity and Maiti (2008); Maity (2011b)

Table 2.1: Continue.

Row	Field of research	No. of Papers	Publications
14.	EPQEO	12	Islam and Roy (2006); Mahapatra et al., (2011); Chang and Chang (2006); Chang et al., (2006); Maity et al., (2008); Pal et al., (2009); Wang and Tang (2009a); Soni and Shah (2011); Chakrabortty et al., (2013); Yaghin et al., (2013); De and Sana (2014); Kumar and Goswami (2015a)

* This paper includes two kinds of models.

- EOQ models: The type of the models that try to determine the optimal quantity from buyer/retailer's perspective by minimizing the inventory costs or maximizing total profit.
- EPQ models: The models that aim to determine the right quantity of a product that should be manufactured through minimizing the inventory costs or maximizing total profit.

As to the subclasses of EOQ and EPQ models, they can be further divided into "basic models", "model with backorders", and "extended models". The basic models refer to a class of models whose consider only the basic costs such as ordering, holding or set up, while extended models cover a set of problem that investigates additional aspects such as multiple products, product quality or process deterioration, delay in payment or their combination. Table 2.1 shows these categorizations according to the gathered papers.

2.5.4 Material Evaluation

Reliability and validity of the gathered papers were evaluated twice where a deductive and inductive method was conducted. In order to derive the structural dimensions which are led to correct classifications, in deductive approach, the materials (i.e. papers) were chosen, and then were analyzed, while in the inductive technique these dimensions are developed from the material by means of generalization (Mayring, 2010). The collected papers were again crosschecked with Scopus that it is one of the most comprehensive academic databases, which contains articles from all fields such as engineering, management, mathematics and operations research and environmental science. These mechanisms could guarantee that all the identified data were sufficient and appropriate. Moreover, applying this method could ensure avoiding error and increasing reliability and validity.

2.6 Economic Order Quantity Model

The EOQ model that is an indispensable and fundamental methodology to overcome some bottlenecks of the supply chain systems, initially and originally proposed by Harris in 1913 (Harris, 1913), deals with finding the optimal order quantity of inventory items at each time that minimizes total inventory cost function or maximizes total profit function which include ordering, holding, and backordering (if any) costs. Because of its necessity to minimize expanse of the inventory systems, the EOQ models have been the subject of extensive extensions, modifications, and investigations by academicians.

Later on, Taft (1918) developed the basic model of EOQ relaxing the assumption of instantaneous replenishment. The concept of the reorder point in the initial EOQ model which showed one of the first attempts to work with probabilistic considerations in inventory management was introduced by Wilson (1934). Thereafter, Whitin (1954) introduced the concept of stochasticity in EOQ model. Other researchers tried to change the assumptions of the basic model with the objective to make it more realistic.

2.6.1 Fuzzy Economic Order Quantity Model

The underlying assumptions in the basic EOQ model is that all the input parameters and decision variables are constant and known in the inventory system. However, demand may vary from time to time, for example. Especially in today's competitive markets and industries, the costs of holding, ordering and backordering are always likely to vary from one cycle to another and are fuzzy in nature.

Therefore, there is a need to develop fuzzy EOQ models to capture the uncertainties accurately. A number of fuzzy inventory models have already been proposed and studied in the literature. The first FEOQ model proposed by Park (1987) who examined an EOQ model by treating ordering and holding costs as trapezoidal fuzzy numbers, used the extension principle for defuzzifying and solved the model with numerical operations. Earlier, many researchers developed other FEOQ models.

2.6.1.1 Fuzzy Economic Order Quantity Model without Backorder (EOQ)

To determine the effect of different approaches to obtain the optimal order quantity, Vujošević et al., (1996) solved an EOQ model with ordering and holding costs and four different solution procedures based on the ranking of fuzzy number and the center of gravity method in a fuzzy environment. It was shown that they give different solutions at which the fuzzy cost function attains its minimum, simply because they handle uncertainty in different ways. There are some problems and shortcomings in this study which were discussed and highlighted by Hojati (2004) and could be improved. He argued that the suggested solution procedures are complicated and time consuming, for example.

Yao and Lee (1999) and Lee and Yao (1999) fuzzified order quantity to trapezoidal and triangular fuzzy number in the total cost of inventory model without backorder, respectively. In both studies, the results showed that after defuzzification the total cost is slightly higher than in the crisp model. In a follow-up study, Yao et al., (2000) investigated an EOQ problem without backorder such that both the order and the total demand quantities are triangular fuzzy numbers. They used a computer program to find the total cost in the fuzzy sense. In order to compare the results of different defuzzification methods (i.e. the centroid and the signed distance) in the total cost of inventory model without backorders, where the total demand and the cost of storing one unit per day is supposed to be triangular fuzzy numbers, another study was conducted (Yao & Chiang, 2003). They proposed some policies to choose the appropriate defuzzification method.

In contrast to previous studies, Wang et al., (2007a) characterized the order and the holding costs as fuzzy variables, and constructed two models using the concepts of possibility/necessity and credibility measures: (1) a fuzzy expected value (FEV) model, and (2) a fuzzy dependent chance programming (FDCP) model while in order to solve these complex models, a particle swarm optimization (PSO) algorithm based on the fuzzy simulation was designed. Syed and Aziz (2007) and Lee and Lin (2011) investigated a fuzzy inventory model with the signed distance method. Similar to the presented approach by Wang et al., (2007a), two variable demand inventory models, with and without backorder, building FEV and FDCP models, were constructed for minimizing inventory cost, treating the holding, ordering and backordering costs and demand as independent fuzzy variables (Samal & Pratihar, 2014). Using genetic and PSO algorithms, they minimized the FEV of the total cost, so that the credibility of the total cost not exceeding a certain budget level was maximized. Comparison of these algorithms proofed when the complexity of the model is increased, PSO has outperform the GA because its particles maintain their memory.

2.6.1.2 Fuzzy Economic Order Quantity Model with Backorder (EOQB)

One of the first attempt to build a FEOQ model with backorder is that of Chen et al., (1996) who considered an inventory model where yearly demand and inventory costs including order, holding, and backorder costs were fuzzified. They used the function principle and the median rule to find the optimal order quantity and the shortage quantity.

Yao and Lee (1996) and Chang et al., (1998) fuzzified the order and the shortage quantity, respectively, as the normal triangular fuzzy number in an EOQ model with backorder. They used two similar approaches to discuss fuzzy sets concepts in the mentioned model. Besides, Yao and Su (2000) used interval-valued fuzzy set to consider the total demand quantity in inventory with backorder for whole of the plan period. To determine the effects of fuzzification of the order and shortage quantities simultaneously in an EOQ model with backorder, Wu and Yao (2003) showed that fuzzification of both of them could give better results than fuzzifying any one variable separately.

Björk (2009) presented the analytical solution to study the effect of uncertainty on backorders and the lead times as well as the demand. In this case, according to a numerical example, the orders were approximately 6% higher than for the crisp case. In addition, comparing the obtained results with those of Chang et al., (1998) showed that they are coherent. In this line of research, a fully fuzzy EOQ model with backorder, where all the input parameters and the decision variables are simultaneously fuzzified in two different cases with both trapezoidal and triangular fuzzy numbers, was conducted by Kazemi et al., (2010). Their results, which were more sensitive to changes in the input parameters when triangular fuzzy numbers were used, contribute that the changes in the values of the decision variables (the maximum inventory level and the batch size) to changes in the costs between the crisp (deterministic) and fuzzy cases have a linear relationship. Similar to the approach used by Björk (2009), a fuzzy EOQ model to the sizing of empty cars on a rail network tested with a real case of the Serbian rail network within a four-day planning period to derive freight car inventory level in station *i* for period *n* of the planning horizon and the ordering quantity for the same station and period *n* as decision variables was proposed (Milenkovic & Bojovic, 2014). They assumed lead time as the time of the freight car traveling between two stations. The work of Björk (2009) was extended by Kazemi et al., (2015) who relaxed the assumption of constant fuzziness by incorporating the concept of learning in fuzziness into the model. They showed that the total cost of the fuzzy inventory model under learning is less than the fuzzy case without learning.

2.6.1.3 Extensions of Fuzzy Economic Order Quantity Model

In this section, the content of the gathered EOQ papers based on the categorizations which are based on the quality of items, the number of items, delay in payment and other extensions are explained.

(a) ***Quality Based Studies (EOQEQ)***

Chang (2003) reformulated the model developed by Porteus (1986) drawing on the concept of imperfect production process with opportunity cost rate as a statistic-fuzzy number. He derived the optimal values of lot size and the process quality level in the fuzzy sense utilizing the logarithmic investment cost function. Building upon the work of Salameh and Jaber (2000) who constructed an EOQ model with imperfect quality, Chang (2004) incorporated the fuzziness of defective rate and the annual demand into the mentioned EOQ model. He employed the signed distance method to find the estimate of total profit per unit time in the fuzzy sense, and then derived the corresponding optimal lot size. Roy et al., (2007) considered a two storage inventory model under bulk release pattern between two warehouses in a stochastic planning horizon with an exponential distribution where demand and deterioration are stock dependent and fuzzy, respectively. Considering the imperfect quality items in each delivered lot, Wang et al., (2007b) developed an EOQ model with the percentage of imperfect quality items in each lot as a random fuzzy variable. They assumed the inspection, the holding, and the setup costs as fuzzy variables and maximized expected long-run average profit by constructing a random fuzzy expected value model which was solved by a PSO algorithm based on the random fuzzy simulation. Rong et al., (2008) introduced a two-warehouse problem for deteriorating items with fuzzy lead time where ordering cost assumed to be partly lead time dependent. Roy et al., (2009a) formulated a fuzzy inventory model of a deteriorating item with displayed stock dependent demand over a stochastic time horizon. Considering the learning effects on percentage of defective items, Yadav et al., (2012a) formulated a

FEOQ model with backorder by taking fuzzified demand rate with dependence upon the frequency of advertisement, while in contrast to previous usual methods, optimal values were obtained by using algebraic method instead of differential calculus. According to some analytically and numerically results, they proved that the number of defective units and shipment size decrease, whereas backordering level and net profit increase as learning increases and as it becomes faster. Therefore, it was recommended to order in smaller lots less frequently. Hsu (2012) explored a fuzzy inventory model with immediate return for defective items. By extending the work of Wee et al., (2007) who focused on the quality of items and full backordering, Mahata and Goswami (2013) developed two fuzzy inventory models in which in the first model the maximum backordering quantity and the order size as decision variables are fuzzified, and in the second one, not only decision variables but also all the parameters are fuzzified. They concluded that the optimal solutions of the second model are same as those under the fuzzy model with the crisp decision variables, and moreover, the decision variables and the annual total profit are highly sensitive due to the fuzziness in the input parameters. It is such that the percentage change due to fuzziness causes approximately double percentage changes in the annual total profit and the backordering quantity is nearly equal of the changes in the components, whereas the order lot size changes marginally.

(b) ***Multi-Item Models (EOQMI)***

A model similar to Roy and Maiti (1998)'s approach where inventory costs were directly proportional to the respective quantities, and unit purchase/production cost was inversely related to the demand was investigated (Das et al., 2000). Besides, fuzzy non-linear programming problem (FNLP) along with genetic algorithm (GA), for the first time, were applied to solve multi-item FEOQ models under fuzzy objective of cost minimization and imprecise constraints on warehouse space and number of production runs with crisp/imprecise inventory costs (Mondal & Maiti, 2002). For two mutually complementary merchandises, Yao et al., (2003) discussed how to determine the optimal ordering policy with fuzzy concept. Applying a new solving approach, Das et al., (2004) formulated a multi-item fuzzy-stochastic inventory model under total budgetary and space constraints. They reduced the model to a corresponding equivalent fuzzy non-linear programming problem, and then, it is solved by FNLP following Zimmermann (1975) technique. Following the work of Roy and Maiti (1995), FEOQ models were developed for multiple items (Yadavalli et al., 2005). The authors showed that among various membership functions, linear membership function provides the best values with the maximum aspiration level. Baykasoğlu and Göçken (2007) solved a fuzzy multi-item EOQ problem with fuzzification of all the parameters as triangular fuzzy numbers except the demand per unit time for the *i*th item which was assumed crisp by employing four different fuzzy ranking methods which are the signed distance, integral value, possibility programming, and expected intervals method. The results showed that models based on different ranking procedures generated different solutions to the problem and, furthermore, the solution of the model based on the integral value ranking approach generated better results than the other ones. In a similar study, Baykasoglu and Gocken (2011) compared the results of the considered model of their previous research (Baykasoğlu & Göçken, 2007) with those of obtained by PSO algorithm as a direct approach. It was concluded by applying a transformation process (i.e. indirect approach) some of the information can be missed; and it highlights the importance of the proper selection of a fuzzy ranking procedure in the direct method. They showed that fuzzy optimization problems can be easily solved using fuzzy ranking and metaheuristics methods without any necessity of transformation into a crisp equivalent.

Fuzzy-stochastic multi-item inventory models with two storage facility and bulk release were solved using a possibility/necessity based optimization technique (Maiti & Maiti, 2007). Maiti (2008) developed a fuzzy two-storage inventory model incorporating simultaneous ordering and transferring, following basic period (BP) approach. Panda et al., (2008a) formulated an inventory model with hybrid inventory costs and fuzzy/fuzzy-stochastic resources where unit cost is demand dependent. Huang (2011) compared the performance of two well-known defuzzification methods (i.e. signed distance and centroid methods) proposing a multi-level lot-sizing problem for multi-item. In an inter-disciplinary research of a fuzzy inventory model and intelligent optimization algorithms, Wang et al., (2013) formulated a FDCP inventory model in a joint replenishment problem (JRP). Mousavi et al., (2014) solved a fuzzy multi-product/period inventory problem using intelligence algorithms under an incremental quantity. According to their results, the performance of the harmony search algorithm to optimize the single and bi-objective inventory control is better than the PSO one.

(c) ***Mix Quality Multi-Item Studies (EOQEQI)***

Roy and Maiti (1998) formulated two fuzzy multi-item multi-objective inventory models with stock dependent demand which were solved by FNLP and fuzzy additive goal programming (FAGP) methods. Later, these models were extended in a fuzzy random environments by Xu and Liu (2008) where a method of solving solution sets of fuzzy random multi-objective programming problems was designed and applied to numerical inventory problems with all inventory costs, purchasing and selling prices in the objectives and constraints assumed as fuzzy random variables in nature. They extended a method to find the membership function of the fuzzy total inventory while the related fuzzy inventory problem is solved via two techniques: (1) fuzzy extension principle and duality, and (2) nearest interval approximations for fuzzy numbers.

Using a similar approach to Liu (2008), a deteriorating multi-item inventory system subject to constraints on fuzzy costs and storage space was proposed (Roy et al., 2008). Wee et al., (2009) developed a fuzzy multi-objective joint replenishment deteriorating items inventory model with fuzzy demand and shortage cost constraint, and maximized the profit and return on inventory investment objectives simultaneously. Regarding the breakable items which get damaged because of the accumulated stress of heaped stocks during storage, Saha et al., (2010) formulated fuzzy multi-item inventory models with stock dependent demand supposing that the number of damaged items depends only on the current stock level linearly and non-linearly. They assumed more options: (1) the demand depends on the current stock but becomes constant when the stock falls below a certain level, and (2) the breakability of the items is up to a certain stock level and thereafter, there is no breaking of the items when the stock falls below the level. In addition, Guchhait et al., (2010) proposed fuzzy inventory models of breakable items, where demand of the items are stock dependent and breakability rates increase linearly with stock and nonlinearly with time. Chakraborty et al., (2013) addressed an inventory model of deteriorating seasonal products for a wholesaler having showrooms at different places under a random planning horizon with different discounting policies. Besides, Jana et al., (2014) formulated a multi-item fuzzy inventory models under random planning horizon and stock-dependent demand.

(d) ***Studies with Delay in Payment (EOQED)***

With a permissible delay in payment, De and Goswami (2006) extended an EOQ model with a fuzzy inflation rate and fuzzy deterioration rate. They discussed three cases where shortage time is smaller, equal or greater than the permissible delay period for settling accounts and the duration of the permissible delay time effectively changes the optimal system cost. These situations were compared with the help of numerical examples. Chen and Ouyang (2006) extended the work of Jamal et al., (1997) considering the rate of the carrying cost, interest paid/earned simultaneously fuzzified based on the interval-valued fuzzy number and triangular fuzzy number. They indicated that there exists a unique interior optimal solution to the proposed model. Another work in this area regarding deteriorating items, is that of Mahata and Goswami (2007) who extended the delay in payments not only for the retailer but also for the customer. Furthermore, the graded mean integration representation (GMIR) method was used for defuzzifying the total variable cost which was derived from fuzzified variables comprising the demand rate, and the holding, ordering, and purchasing costs.

Allowing delay in payments, Ouyang et al., (2010) fuzzified the work of Chang et al., (2003) who linked the supplier credits to ordering quantity. Maiti (2011) introduced customers' credit-period linked dynamic demand with the imprecise planning horizon due to the competitive market.

Under the assumption of having the full trade credit offered by suppliers for the retailer due to their powerful position and just offering a partial trade credit to customers by retailer, Mahata and Mahata (2011) investigated a FEOQ model in a two levels of trade credit. They derived optimal values and proposed some policies. Besides, they obtained some previously published results of other authors as special cases. Their work was extended by Soni and Joshi (2013) adding more realistic assumptions such as imprecise selling price dependent demand rate.

Regarding a variable demand which depends on selling price, stock level and retailer's credit period, Guchhait et al., (2014) developed a FEOQ model of a deteriorating item where not only supplier to the retailer but also retailer to his customer could provide permissible delay in payment and the supplier could consider cash discount to the retailer to pay for their purchases quickly. Yadav et al., (2015) considering the trade credit policy in a fuzzy environment concluded that the retailer can increase the profit by ordering lower quantity. Guchhait et al., (2015) presented an inventory policy for the time-dependent deteriorated goods with different types of imprecise inventory costs, two-level partial trade credit facility and credit period dependent demand. Taleizadeh et al., (2011) and Taleizadeh et al., (2013) extended fuzzy rough joint replenishment multi-product multi-constraint inventory models as mixed integer nonlinear programming to purchase high price raw materials that are solved by meta heuristic algorithms.

(e) ***Other Extensions of EOQ (EOQEO)***

Roy and Maiti (1997) formulated a FEOQ model with limited storage capacity where the demand is related to the unit price and the setup cost varies with the quantity produced/purchased and the fuzziness is introduced in both objective function and storage area. They applied FNLP to solve their model. Later, their analytical procedure was improved by employing formulated solutions based on the max–min operator (Chou et al., 2009). Pointing out the questionable results and resolving the deficiencies to obtain the minimum solution of previous work by Roy and Maiti (1997), with the same inventory model with fuzzy constraints, Chou et al., (2009) proposed a method based on the max–min operator and solved the deterministic inventory model converted from the fuzzy inventory model. Liu (2008) revisited an EOQ model as a profit maximization problem when the demand quantity and the unit cost are fuzzy numbers, and are functions of the price and lot size, respectively. He developed a solution method based on the extension principle where the fuzzy inventory problem is transformed into a pair of two-level mathematical programs to derive the upper bound and lower bound of the fuzzy profit at possibility level α. Then, it is transformed into a pair of conventional geometric programs to solve. The results showed that when the selling price is elastic, the larger the selling price, the smaller the order quantity is. Chen (2003)'s model called economic order time (EOT) was fuzzified when the time period of sales is the decision variable and the components of the model are fuzzified (Vijayan & Kumaran, 2009). It was shown that the solutions of the EOT model with all fuzzy components were the same as those under the fuzzy model with the crisp time period.

Sadjadi et al., (2010) developed a new pricing and marketing planning proposing solutions expressed by membership functions. They assumed the demand is a function of price and marketing expenditure with fuzzy parameters, and the purchasing cost is as a function of the order quantity. Bera et al., (2012) developed an inventory model with an infinite rate of replenishment over a finite but imprecise time horizon considering time dependent ramp type demand. Ketsarapong et al., (2012) developed an uncapacitated single item lot sizing problem in a fuzzy environment that is converted to a crisp model with the help of possibility approach.

Samadi et al., (2013) developed a FEOQ model proposing solutions expressed by membership functions where the demand is a power function of price, marketing and service expenditures and the unit cost is determined as a function of the order quantity. Yadav et al., (2013a) developed a fuzzy version of the model of Lin (2008) who considered a controllable backorder price discount and the reduction of lead-time. De and Sana (2013a) developed an intuitionistic FEOQ where the demand rate is varying with the selling price and the promotional effort (PE). De and Sana (2013b) dealt with a backorder EOQ model while a m-th power promotional index (PI) as a fuzzy decision variable was added to the total profit function along with the order and shortage quantities. They assumed the demand as a function of the PI, and concluded that the value of PI near zero may not give the maximum profit. Following their previous works, with PE dependent demand allowing shortages and assuming the decrease in the demand rate in stock out situation while it comes back to its initial rate since PE continues, they proposed a solution procedure for an intuitionistic fuzzy (IF) backorder inventory model and made a comparative study on Pareto optimality and optimality under Lagrange's interpolating polynomial function (De et al., 2014). They concluded that in contrast to the Pareto optimality test which is valid only for minimization problem, their modified IF programming techniques (IFPT) called interpolation method is valid for any kind of (max/min) problem. Regarding the warehouse inventory systems, Panda et al., (2014) developed a two-warehouse fuzzy-stochastic mixture problem.

2.7 Economic Production Quantity Model

Since the development of EOQ model by Harris (1913), up to now, numerous inventory models have been studied in the literature in order to consider various realistic features. On the other hand, economic production quantity (EPQ) model, which is an extension of the EOQ model, has also been served for over 80 years to determine the optimal lot size in production/inventory systems.

Needless to say that, similar to EOQ model, the results of EPQ models depend largely on a number of basic assumptions such as types of demand, production rates, different cost parameters, replenishment policies and so on (Halim et al., 2009). In the next section, Studies which consider fuzzy set theory into the EPQ model to build a FEPQ model are reviewed.

2.7.1 Fuzzy Economic Production Quantity Model

Similar to its counterpart (i.e. EOQ), economic production quantity model has been developed through the fuzzy set theory. In recent years, many kinds of fuzzy EPQ model were extended. In discussions of production quantity models, it is believed that Sommer (1981) applied a fuzzy dynamic programming approach to solve a production-inventory scheduling problem with capacity constraints. Another earlier work is that of Kacprzyk and Stanieski (1982) who incorporated the fuzzy set theory into production planning and control problems.

2.7.1.1 Fuzzy Basic Economic Production Quantity Model (EPQ)

By fuzzification of both the demand quantity and the production quantity per day, Lee and Yao (1998) investigated a computing schema for the FEPQ deriving the membership function of the fuzzy total cost. The triangular form, extension principle and centroid method were used for fuzzy numbers, calculation and defuzzification, respectively. They found that, after defuzzification, the total cost was slightly higher than in the crisp model. Later, using the procedure used in Lee and Yao (1998), two studies were conducted by Chang (1999) and Lin and Yao (2000) who only fuzzified the production quantity in their model as triangular and trapezoidal fuzzy number, respectively.

Hsieh (2002) constructed two FEPQ models while the first one was fully fuzzified on all the parameters, and in the second one, not only all parameters but also the decision variable (i.e. production quantity) was fuzzified as a trapezoidal fuzzy number. He showed that considered models are executable and useful in the real world.

2.7.1.2 Extensions of Fuzzy Economic Production Quantity Model

In this section, the content of the EPQ papers according to the categorizations which are based on the quality of items, rework, the number of items, and other extensions are explained.

(a) ***Quality Based Studies (EPQEQ)***

Mahapatra and Maiti (2006) addressed a production-inventory model with imprecise preparation time for production aiming to maximize the profit. Chen and Chang (2008) introduced a FEPQ model with defective productions that cannot be repaired. Bag et al., (2009) introduced the fuzzy random variable demand concept to an imperfect production system considering reliability of production system. A production system producing defective items due to the machine failure was addressed by Das et al., (2011). Based on a numerical analysis, they illustrated that there is a direct relation between the production rate and the mean duration of a breakdown. However, an inverse relation was found between the cost of production idle-time and the production rate. Pal et al., (2014) investigated the ramp up demand in an EPQ model with fuzzy setting. They assumed that the demand is a function of time and items in stock could be deteriorated following a Weibull distribution. They showed that the fuzzy model returns a lower total cost value than crisp model for some values of degree of optimism and total replenishment cycles. According to their results, the shorter the production cycle is, the lower the total cost will be. Moreover, Paul et al., (2014) addressed a FEOQ model with fuzzy holding cost and demand in an imperfect production system while reliability of the system was a decision variable. They concluded that their model is more realistic and applicable than traditional production inventory models. Pal et al., (2015) formulated a model with the ramp type demand and the deterioration of the product. They showed the total cost in the fuzzy and the crisp case could be equal when the decision maker is semi optimistic.

(b) ***Rework Based Studies (EPQW)***

Roy et al., (2009b) developed an imperfect production system where a portion of the imperfect items could be remanufactured to as-good-as perfect quality to satisfy customer demand, while the remaining items are irreparable, and consequently are disposed. It was illustrated that the relation between the total profit and the fuzzy confidence level of defective rate are opposite. Guchhait et al., (2013) addressed a FEPQ model with remanufacturing of imperfect quality items using fuzzy differential equation and fuzzy Riemann-integration. Following this line of thought, Mondal et al., (2013) developed a FEPQ model in a rough environment in which the imperfect quality items could be repaired becoming as perfect quality item. They determined that the existence of either uncertainty or inflation has a negative impact on the total profit, and also suggested repairing process should start up from the second cycle when repairing rate is a dynamic control variable. Mondal et al., (2014) formulated a FEPQ problem where units are bulkly transformed from production center to a showroom, and the rework process of defective items starts after the production cycle. Shekarian et al., (2014b) extended a FEOQ model, and showed that formulating the fuzzy model with a trapezoidal membership function leads to a higher total cost comparing to the triangular one. It is due to the fact that trapezoidal fuzzy number increases the dimensions of the system. Shekarian et al., (2014a) extended the model of Shekarian et al., (2014b) without backorder and different fuzzy settings. They illustrated that using the signed distance method leads to a larger lot size in comparison with the GMIR method.

(c) ***Shifting in the Production Status (EPQS)***

Applying a probability function with fuzzy parameters, Halim et al., (2009) examined two different scenarios in a production system that produces imperfect items due to the shifting to out of control state. The authors proposed that although it is difficult to recognize which model (fuzzy and crisp) performs better, it is more appropriate to use the fuzzy model when the fraction of defective items fluctuates. Zhang et al., (2009) formulated a production problem which starts with in control state producing good quality items, and then may change to out of control state during production cycle producing a fixed fraction of defective items. They used fuzzy and random-fuzzy concepts to build their model. In a similar situation, Wang and Tang (2009b) addressed a model with a difference that the time until the production shifts to out of control state is a fuzzy variable instead of being a fuzzy random variable. In contrary to Zhang et al., (2009) and Wang and Tang (2009b), a simpler model was considered by Hu et al., (2010), who assumed setup and holding costs as crisp parameters.

Other works that studied the effect of randomness and fuzziness include Kumar and Goswami (2015b), Kumar and Goswami (2015c) and Mahata (2015), who represented the time in which the system shifts from in control to out of control state as a fuzzy random variable.

(d) ***Multi-Item Studies (EPQMI)***

One of the earliest models dealing with multi-item FEPQ models is that of Pappis and Karacapilidis (1995), who investigated the problem of determining optimal production runs in a batch production system. A fuzzy multi-product multi-objective EPQ system was addressed by Mandal et al., (2005) considering the shortage and the constraints on storage area, production cost and number of orders. Mandal and Roy (2006a) optimized a multi-item inventory model with demand dependent inventory level and shelf-space constraint under three different fuzzy numbers.

Islam and Roy (2007) extended a multi-item version of Islam and Roy (2006)'s model considering different solution procedure. A similar but more complex problem as in Islam and Roy (2007) was studied by Panda and Maiti (2009), where unit production cost was considered to be dependent on stock level as well as demand which was given dependent on unit selling price. A fuzzy production inventory model which the time interval between the decision to produce and the real time of production is variable was studied in Mandal et al., (2011). As the production with preparation time is more costly, they stressed that the decision for production should be made as earlier as possible to reduce the production cost. Björk (2012) investigated a fuzzy multi-item EPQ model with a finite production rate under uncertain cycle time. He recommended cycle time with more flexible interval for such a production system. This work was extended by Mezei and Björk (2015) to account for backorders. Jana et al., (2013) offered FEPQ models assuming the demand and the unit production cost as functions of stock level and the production rate respectively. The authors found that there are not much differences between the result of the model solving with triangular and parabolic fuzzy numbers, and the generalized reduced gradient (GRG) approach gives lower profit than the necessity approach.

(e) ***Mix Quality Multi-Item Studies (EPQEQI)***

One of the first studies considering the deteriorated items with multi-item in a fuzzy production-inventory system is that of Maity and Maiti (2005), who suggested a model with demand and production rate, assumed as a function of time, in which was solved using weighted sum method and simple differential calculus operations. Mandal and Roy (2006b) used hybrid numbers (i.e. numbers that simultaneously contain fuzziness and randomness properties) to present a model with imperfect quality under uncertainty. The author found that when the weight of the objective function increases the value of objective function decreases as a result. Maity and Maiti (2007) suggested fuzzy dynamic production-inventory models when the demand, the production rate and the shortage level are time dependent. A fuzzy multi-item production-inventory system was addressed by Maity and Maiti (2008) with time dependent demand where demand could be decreased or increased under the influence of sale degradation and advertising policy, respectively. Panda et al., (2008b) studied the case of imperfect production process where the constraint is stochastic or fuzzy and concluded that modeling budget and shortage constraints using possibility measure gives the greatest total profit among all possible combinations. Xu and Zhao (2008) formulated a multi-objective fuzzy rough production-inventory model with imperfect quality in which the rate of production is assumed to be the same as the rate of rework. Xu and Zhao (2010) applied the fuzzy rough set theory in a multi-objective programming problem in a manufacturing company in China. Mandal et al., (2010) analyzed an imperfect production system with fuzzy time period under quadratic, linear and constant production rate. They recommended using constant production because of the lower total production cost. The first research in fuzzy EPQ inventory which applied fuzzy inequality as well as fuzzy objective function in modelling the production inventory was the study of Maity (2011a), who addressed a fuzzy EPQ model in a system including a single machine and multiple products. Firstly, Maity (2011a) investigated a FEPQ model applying fuzzy inequality as well as fuzzy objective function with a single machine and multiple products. A multi-item production inventory model with two-warehouse and uncertain constraints was developed by Maity (2011b), who assumed that inventory level, production and demand rates are function of time. Das and Maiti (2013) formulated a FEPQ model with a fuzzy-stochastic constraint on storage space and an optimistic fuzzy equality for budget.

(f) ***Other Extensions of EPQ (EPQEO)***

By introducing fuzziness in objective and constraint goals, a FEPQ model considering investment for reducing setup cost and quality improvement process was proposed by Islam and Roy (2006). Mahapatra et al., (2011) discussed a simpler version of Islam and Roy (2006)'s model but without storage space. Taking a variety of costs into consideration in EPQ inventory systems, Chang and Chang (2006) analyzed a problem by accounting for the relative cost of the inventory system generated from inventory holding and production. Chang et al., (2006) investigated the problem of fuzzy demand in economic lot-size scheduling problem and compared the fuzzy and crisp cases. A production–recycling–inventory system was developed by Maity et al., (2008) where the used items are collected for recycling or disposal, and are treated as-good-as-new. By incorporating the effect of learning on production and setup cost and fuzzification of lifetime of the product, a FEPQ model for a newly launched product was constructed by Pal et al., (2009) where demand depends on time and price during the price discount period. They used GA for optimization of the fuzzy models which are transferred to deterministic ones following possibility/necessity measure on fuzzy goal and necessity measure on imprecise constraints. Wang and Tang (2009a) solved a complex structure of FEPQ problem with fuzzy variable costs, and derived the equivalent value of the fuzzy total cost function. Considering the pre-production time as a fuzzy number in a FEPQ model, Soni and Shah (2011) showed when the demand is taken a trapezoidal fuzzy number into account, the optimal expected total cost, the production quantity and the cycle length are higher than the case that it is considered as an interval fuzzy number. A new solution approach based on intuitionistic fuzzy sets to solve a FEPQ problem was suggested by Chakrabortty et al., (2013). The proposed solution approach was proven to be a strong Pareto-optimal solution using Pareto-optimality test, since the obtained value of the objective function for the test was found to be quite small. Yaghin et al., (2013) formulated a non-linear fuzzy mathematical programming for a production-inventory model confronting different demands from several market sectors.

A FEPQ problem with multiple period and manufacturers/machines was analyzed by De and Sana (2014). They compared general fuzzy optimization and intuitionistic fuzzy optimization methods and showed that the second one performs better than the first one in two out of the three investigated models. Moreover, Kumar and Goswami (2015a) developed a production-inventory model which works under continuous review inventory control policy under the effect of stochastic and fuzzy-stochastic demand.

According to the investigated studies, it should be mentioned that fuzzy inventory management has been used in industries such as services and equipment industry with a EPQEQI model (Xu & Zhao, 2008), garment industry with a EOQEQI model (Jana et al., 2014), arts and crafts industry with a EPQEQI model (Xu & Zhao, 2010) and transportation industry with a EOQB model (Milenkovic & Bojovic, 2014).

2.8 Fuzzified Elements and Characteristics

In this section, more details to highlight the contribution of the proposed models comparing them with the previous literature are provided. These details are gathered in Tables 2.2 and 2.3 where studies are categorized regarding the considered characteristics and the depth of fuzzification of publications and elements. In Table 2.2, gathered papers are classified according to characteristics such as inflation, discounting, screening, rework, learning, and delay in payment. Other characteristics are based on the shortage, quality, and structure of the models which are constraint, objective and the number of items. Besides, Table 2.3 shows the level of fuzzification of the models based on the structural elements of the problem which are parameters, variables, objectives, and constraints. In Table 2.3, the letter “F” shows that the element is fully fuzzified while the letter “P” stands for a status which an element is partially fuzzified. For example, if all the parameters of the model are fuzzified it has been shown with “F”. It is clear that there are a few papers that considered a fully fuzzy status for all the element.

2.9 Chapter Summary

In this chapter, previous studies through a systematic literature review were reviewed. The content of these studies were investigated technically and characteristics of the models were categorized in details. Two main classes of inventory management problems (i.e. economic order quantity and economic production quantity) which are related to the proposed fuzzy models in the next chapters were discussed. Although there is a vast number of works in this field, it is found that there are still some shortcomings. Likewise, little attention has been paid to the FEOQ or FEPQ that are developed in a fully fuzzy environment in the presence of learning. Besides, there are some opportunities to fill the gaps with other important characteristics.

Table 2.2: Classification of the publications according to the considered models in details

Publication	Inf.	Dis.	Scr.	Rew.	Lea.	Del.	Treatment of shortage			Quality of items		Structure of model			Type
							Pb.	Fb.	Los.	Def.	Det.	Cons.	Muo.	Mui.	
1. Park (1987)															EOQ
2. Vujošević et al., (1996)															EOQ
3. Lee and Yao (1999)								√							EOQ
4.1. Yao and Lee (1999)															EOQ
5. Yao and Chiang (2003)															EOQ
6. Hojati (2004)															EOQ
7. Syed and Aziz (2007)															EOQ
8. Lee and Lin (2011)															EOQ
9.1. Samal and Pratihar (2014)															EOQ
10. Chen et al., (1996)								√							EOQB
11. Yao and Lee (1996)								√							EOQB
12. Chang et al., (1998)								√							EOQB
4.2. Yao and Lee (1999)								√							EOQB
13. Yao et al., (2000)								√							EOQB
14. Yao and Su (2000)								√							EOQB
15. Wu and Yao (2003)								√							EOQB
16. Björk (2009)								√							EOQB
17. Kazemi et al., (2010)								√							EOQB
9.2. Samal and Pratihar (2014)								√							EOQB
18. Milenkovic and Bojovic (2014)								√							EOQB
19. Kazemi et al., (2015)					√			√							EOQB
20. Chang (2003)				√						√					EOQEQ
21. Chang (2004)		√	√							√					EOQEQ
22. Roy et al., (2007)											√	√			EOQEQ
23. Wang et al., (2007b)		√	√							√		√			EOQEQ
24. Rong et al., (2008)							√	√	√		√				EOQEQ
25. Roy et al., (2009a)	√	√									√				EOQEQ
26. Hsu (2012)			√							√					EOQEQ
27. Yadav et al., (2012a)		√	√		√			√		√					EOQEQ
28. Mahata and Goswami (2013)			√					√		√					EOQEQ
29. Das et al., (2000)								√				√		√	EOQMI
30. Mondal and Maiti (2002)												√		√	EOQMI
31. Yao et al., (2003)														√	EOQMI
32. Das et al., (2004)								√				√		√	EOQMI
33. Yadavalli et al., (2005)												√		√	EOQMI
34. Baykasoğlu and Göçken (2007)												√		√	EOQMI

Table 2.2: Continue.

Publication	Inf.	Dis.	Scr.	Rew.	Lea.	Del.	Treatment of shortage			Quality of items		Structure of model			Type
							Pb.	Fb.	Los.	Def.	Det.	Cons.	Muo.	Mui.	
35. Maiti and Maiti (2007)								√				√	√	√	EOQMI
36. Maiti (2008)	√	√										√		√	EOQMI
37. Panda et al., (2008a)												√		√	EOQMI
38. Baykasoglu and Gocken (2011)												√		√	EOQMI
39. Huang (2011)												√		√	EOQMI
40. Wang et al., (2013)												√		√	EOQMI
41. Mousavi et al., (2014)		√					√		√			√	√	√	EOQMI
42. Roy and Maiti (1998)											√	√	√	√	EOQEQI
43. Roy et al., (2008)											√	√		√	EOQEQI
44. Xu and Liu (2008)											√	√	√	√	EOQEQI
45. Wee et al., (2009)								√			√	√	√	√	EOQEQI
46. Guchhait et al., (2010)										√		√		√	EOQEQI
47. Saha et al., (2010)										√		√		√	EOQEQI
48. Chakraborty et al., (2013)		√									√	√		√	EOQEQI
49. Jana et al., (2014)	√	√			√		√				√	√		√	EOQEQI
50. Chen and Ouyang (2006)						√		√			√				EOQED
51. De and Goswami (2006)	√					√		√		√		√			EOQED
52. Mahata and Goswami (2007)						√					√				EOQED
53. Ouyang et al., (2010)						√					√				EOQED
54. Maiti (2011)	√	√				√						√			EOQED
55. Mahata and Mahata (2011)						√					√				EOQED
56. Taleizadeh et al., (2011)		√				√						√		√	EOQED
57. Soni and Joshi (2013)						√					√				EOQED
58. Taleizadeh et al., (2013)		√				√		√			√	√		√	EOQED
59. Guchhait et al., (2014)	√	√				√		√			√	√			EOQED
60. Yadav et al., (2015)	√	√				√					√				EOQED
61. Guchhait et al., (2015)						√					√				EOQED
62. Roy and Maiti (1997)												√		√	EOQEO
63. Liu (2008)		√													EOQEO
64. Vijayan and Kumaran (2009)															EOQEO
65. Chou et al., (2009)												√			EOQEO
66. Sadjadi et al., (2010)															EOQEO
67. Ketsarapong et al., (2012)												√			EOQEO
68. Bera et al., (2012)							√								EOQEO
69. Samadi et al., (2013)								√							EOQEO

Table 2.2: Continue.

Publication	Inf.	Dis.	Scr.	Rew.	Lea.	Del.	Treatment of shortage			Quality of items		Structure of model			Type
							Pb.	Fb.	Los.	Def.	Det.	Cons.	Muo.	Mui.	
70. Yadav et al., (2013a)		√					√		√						EOQEO
71. De and Sana (2013b)								√							EOQEO
72. De and Sana (2013a)								√				√			EOQEO
73. Panda et al., (2014)								√				√			EOQEO
74. De et al., (2014)								√				√			EOQEO
75. Lee and Yao (1998)															EPQ
76. Chang (1999)															EPQ
77. Lin and Yao (2000)															EPQ
78. Hsieh (2002)												√			EPQ
79. Mahapatra and Maiti (2006)								√			√				EPQEQ
80. Chen and Chang (2008)										√					EPQEQ
81. Bag et al., (2009)										√					EPQEQ
82. Das et al., (2011)								√		√					EPQEQ
83. Pal et al., (2014)	√										√				EPQEQ
84. Paul et al., (2014)			√							√					EPQEQ
85. Pal et al., (2015)	√							√	√		√				EPQEQ
86. Roy et al., (2009b)										√					EPQW
87. Guchhait et al., (2013)			√	√						√		√			EPQW
88. Mondal et al., (2013)	√			√						√					EPQW
89. Mondal et al., (2014)				√						√					EPQW
90. Shekarian et al., (2014b)			√	√				√		√					EPQW
91. Shekarian et al., (2014a)				√						√					EPQW
92. Halim et al., (2009)											√				EPQS
93. Zhang et al., (2009)				√						√					EPQS
94. Wang and Tang (2009b)			√	√						√					EPQS
95. Hu et al., (2010)				√				√		√		√			EPQS
96. Kumar and Goswami (2015b)				√	√		√		√	√					EPQS
97. Mahata (2015)				√	√		√		√	√		√			EPQS
98. Kumar and Goswami (2015c)				√			√		√	√		√			EPQS
99. Pappis and Karacapilidis (1995)														√	EPQMI
100. Mandal et al., (2005)								√				√	√	√	EPQMI
101. Mandal and Roy (2006a)												√		√	EPQMI
102. Islam and Roy (2007)														√	EPQMI
103. Panda and Maiti (2009)												√		√	EPQMI
104. Mandal et al., (2011)								√				√		√	EPQMI

Table 2.2: Continue.

Publication	Inf.	Dis.	Scr.	Rew.	Lea.	Del.	Treatment of shortage			Quality of items		Structure of model			Type
							Pb.	Fb.	Los.	Def.	Det.	Cons.	Muo.	Mui.	
105. Björk (2012)														√	EPQMI
106. Jana et al., (2013)												√		√	EPQMI
107. Mezei and Björk (2015)								√						√	EPQMI
108. Maity and Maiti (2005)											√	√		√	EPQEQI
109. Mandal and Roy (2006b)				√						√		√		√	EPQEQI
110. Maity and Maiti (2007)								√		√		√		√	EPQEQI
111. Maity and Maiti (2008)	√	√									√	√		√	EPQEQI
112. Xu and Zhao (2008)				√						√		√	√	√	EPQEQI
113. Panda et al., (2008b)			√					√		√		√		√	EPQEQI
114. Xu and Zhao (2010)				√						√		√		√	EPQEQI
115. Mandal et al., (2010)										√		√	√	√	EPQEQI
116. Maity (2011a)	√	√								√		√		√	EPQEQI
117. Maity (2011b)										√		√		√	EPQEQI
118. Das and Maiti (2013)								√		√		√		√	EPQEQI
119. Islam and Roy (2006)												√			EPQEO
120. Chang and Chang (2006)															EPQEO
121. Chang et al., (2006)															EPQEO
122. Maity et al., (2008)												√			EPQEO
123. Pal et al., (2009)		√			√										EPQEO
124. Wang and Tang (2009a)								√							EPQEO
125. Mahapatra et al., (2011)															EPQEO
126. Soni and Shah (2011)							√		√						EPQEO
127. Chakrabortty et al., (2013)								√							EPQEO
128. Yaghin et al., (2013)												√	√		EPQEO
129. De and Sana (2014)												√			EPQEO
130. Kumar and Goswami (2015a)								√							EPQEO

** Definition of abbreviation in the Table 1.

Abbreviation	**Inf.**	**Dis.**	**Scr.**	**Rew.**	**Lea.**	**Del.**	**Pb.**	**Fb.**	**Los.**	**Def.**	**Det.**	**Cons.**	**Muo.**	**Mui.**
Definition	Inflation	Discounting	Screening	Rework	Learning	Delay in payment	Partially backlogged	Fully backlogged	Lost sales	Defective	Deteriorating	Constraint	Multi objective	Multi item

Table 2.3: The depth of fuzzification of publications and elements

	Fuzzified element (Level of fuzzification)	Decision variable
	(1) Parameter (F, P); **(2)** Variable (F, P); **(3)** Objective (F, P); **(4)** Constraint (F, P)	
1. Park (1987)	**(1)** Ordering cost, Holding cost (P); **(3)** Total cost (P)	Order quantity
2. Vujošević et al., (1996)	**(1)** Ordering cost, Holding cost (P); **(3)** Total cost (P)	Order quantity
3. Lee and Yao (1999)	**(2)** Order quantity (F); **(3)** Total cost (P)	Order quantity
4.1. Yao and Lee (1999)	**(2)** Order quantity (F); **(3)** Total cost (P)	Order quantity
5. Yao and Chiang (2003)	**(1)** Demand, Holding cost (P); **(3)** Total cost (P)	Order quantity
6. Hojati (2004)	**(1)** Ordering cost, Holding cost (P); **(3)** Total cost (P)	Order quantity
7. Syed and Aziz (2007)	**(1)** Ordering cost, Holding cost (P); **(3)** Total cost (P)	Order quantity
8. Lee and Lin (2011)	**(1)** Demand, Ordering cost, Holding cost (P); **(2)** Order quantity (F); **(3)** Total cost (P)	Order quantity
9.1. Samal and Pratihar (2014)	**(1)** Demand, Ordering cost, Holding cost (P); **(3)** Total cost (P)	Order quantity
10. Chen et al., (1996)	**(1)** Demand, Ordering cost, Holding cost, Backorder cost (P); **(3)** Total cost (P)	Order quantity, Backorder quantity
11. Yao and Lee (1996)	**(2)** Order quantity (P); **(3)** Total cost (P)	Order quantity, Backorder quantity
12. Chang et al., (1998)	**(2)** Backorder quantity (P); **(3)** Total cost (P)	Order quantity, Backorder quantity
4.2. Yao and Lee (1999)	**(2)** Order quantity (P); **(3)** Total cost (P)	Order quantity, Backorder quantity
13. Yao et al., (2000)	**(1)** Demand (P); **(2)** Order quantity (F); **(3)** Total cost (P)	Order quantity
14. Yao and Su (2000)	**(1)** Demand (P); **(3)** Total cost (P)	Order quantity, Backorder quantity
15. Wu and Yao (2003)	**(2)** Order quantity, Backorder quantity (F); **(3)** Total cost (P)	Order quantity, Backorder quantity
16. Björk (2009)	**(1)** Demand, Lead times (P); **(2)** Maximum inventory level (P); **(3)** Total cost (P)	Order quantity, Maximum inventory level
17. Kazemi et al., (2010)	**(1)** Demand, Ordering cost, Holding cost, Penalty cost (P); **(2)** Order quantity, Maximum inventory level (F); **(3)** Total cost (F)	Order quantity, Maximum inventory level
9.2. Samal and Pratihar (2014)	**(1)** Demand, Ordering cost, Holding cost, Backordering cost (F); **(3)** Total cost (P)	Order quantity, Backorder quantity
18. Milenkovic and Bojovic (2014)	**(1)** Demand, Lead times (P); **(2)** Maximum inventory level (P); **(3)** Total cost (P)	Order quantity, Maximum inventory level
19. Kazemi et al., (2015)	**(1)** Demand, Lead times (P); **(2)** Maximum inventory level (P); **(3)** Total cost (P)	Order quantity, Maximum inventory level
20. Chang (2003)	**(1)** Opportunity cost of capital rate (P); **(3)** Total cost (P)	Lot size, Process quality level
21.1. Chang (2004)	**(1)** Defective rate (P); **(3)** Total profit (P)	Order quantity
21.2. Chang (2004)	**(1)** Defective rate, Demand (P); **(3)** Total profit (P)	Order quantity
22. Roy et al., (2007)	**(1)** Deterioration rate (P); **(3)** Total profit (P)	Reorder level at the warehouse which is in the heart of the market place, Length of each cycle
23. Wang et al., (2007b)	**(1)** Holding cost [2], Setup cost [2], Inspection cost [2], Percentage of imperfect quality items [1] (P); **(3)** Total profit (P)	Lot size per cycle
24. Rong et al., (2008)	**(1)** Lead time (P); **(3)** Total profit (P)	Time of placing of an order, Time gap between two shipments from RW* to OW**, Distance of RW from OW, Number of shipments required to transfer the item from RW to OW
25. Roy et al., (2009a)	**(1)** Inflation, Time discounting (P); **(3)** Total profit (P)	Duration of a complete cycle
26. Hsu (2012)	**(1)** Demand, Purchase cost, Perfective rate (P); **(3)** Total profit (P)	Order quantity
27. Yadav et al., (2012a)	**(1)** Demand (P); **(3)** Total profit (P)	Order quantity, Backorder level

Table 2.3: Continue.

	Fuzzified element (Level of fuzzification)	Decision variable
	(1) Parameter (F, P); **(2)** Variable (F, P); **(3)** Objective (F, P); **(4)** Constraint (F, P)	
28.1. Mahata and Goswami (2013)	**(1)** Demand, Holding cost, Purchase cost, Selling price, Screening rate, Ordering cost, Backordering cost, Salvage value of defective item, Screening cost, Good-quality rate (F); **(3)** Total profit (P)	Order quantity, Maximum backordering quantity
28.2. Mahata and Goswami (2013)	**(1)** Demand, Holding cost, Purchase cost, Selling price, Screening rate, Ordering cost, Backordering cost, Salvage value of defective item, Screening cost, Good-quality rate (F); **(2)** Order quantity, Maximum backordering quantity (F); **(3)** Total profit (F)	Order quantity, Maximum backordering quantity
29. Das et al., (2000)	**(3)** Total cost (P); **(4)** Available storage area, Permitted total average shortage cost, Total average inventory investment cost (P)	Order quantity
30.1. Mondal and Maiti (2002)	**(3)** Total cost (P); **(4)** Available storage area, Number of production runs (P)	Order quantity
30.2. Mondal and Maiti (2002)	**(1)** Setup cost, Holding cost (P); **(3)** Total cost (P); **(4)** Available storage area, Number of production runs (P)	Order quantity
31. Yao et al., (2003)	**(1)** Price (P); **(2)** Demand (P); **(3)** Total cost (P)	Order quantity, Demand
32. Das et al., (2004)	**(3)** Total cost (P); **(4)** Available storage space (P)	Order quantity, Demand, Shortage level
33. Yadavalli et al., (2005)	**(1)** Holding cost; Setup cost, Average number of stocked out items (P); **(3)** Total cost (P)	Order quantity
34. Baykasoğlu and Göçken (2007)	**(1)** Holding cost, Setup cost, Total demand of product *i*, Space required by each unit of product *i* (P); **(3)** Total cost (P); **(4)** Maximum available warehouse space, Maximum number of orders placed (P)	Order quantity
35. Maiti and Maiti (2007)	**(1)** Ordering costs, Shortage cost, Replenishment cost, Purchase cost (P); **(3)** Total profit (P)	Units transferred in each shipment, Number of cycles in each time horizon, Reduction rate of successive cycle lengths, Fraction of *i* th cycle length
36. Maiti (2008)	**(1)** Purchase cost (P); **(3)** Total profit (P); **(4)** Investment amount, Storehouse capacity (P)	Number of order (N_M) during the planning horizon (H), Number of times items (M) transferred from second warehouse (W2) to first one (W1) during the basic time interval between orders ($T=H/N_M$); Fraction of W1 allotted for *i*th item, Number of integer multiple of T, Number of integer multiple of L_T, where $L_T= T/M$
37.1. Panda et al., (2008a)	**(1)** Holding cost [6], Setup cost [6] (P); **(3)** Total cost (P); **(4)** Total available space area, Total available budget (P)	Order quantity, Demand
37.2. Panda et al., (2008)	**(1)** Holding cost [6], Setup cost [6] (P); **(3)** Total cost (P); **(4)** Total available space area, Total available budget [1] (P)	Order quantity, Demand
38. Baykasoglu and Gocken (2011)	**(1)** Holding cost, Setup cost, Total demand of product *i*, Space required by each unit of product *i* (P); **(3)** Total cost (P); **(4)** Maximum available warehouse space, Maximum number of orders placed (P)	Order quantity
39.1. Huang (2011)	**(1)** Ordering cost, Holding cost, Lot size (P); **(3)** Total cost (P)	Binary variables on the reception of a lot of an item in a period, Ordering a lot of an item in a period
39.2. Huang (2011)	**(1)** Ordering cost, Holding cost (P); **(3)** Total cost (P)	Binary variables on the reception of a lot of an item in a period, Ordering a lot of an item in a period
40. Wang et al., (2013)	**(1)** Minor ordering cost, Holding cost (P); **(3)** Total cost [***] (P); **(4)** Replenishment (P)	Time-periods between two replenishments, Integer number that decides the replenishment schedule of *i*the item

Table 2.3: Continue.

	Fuzzified element (Level of fuzzification) **(1)** Parameter (F, P); **(2)** Variable (F, P); **(3)** Objective (F, P); **(4)** Constraint (F, P)	Decision variable
41. Mousavi et al., (2014)	**(1)** Discount rate, Storage space (P); **(3)** Total cost, Total required storage space (P)	Number of boxes for *i*th product ordered in period t, Shortage quantity for *i*th product in period t, Ordering quantity of *i*th product in period t, Initial positive inventory of *i*th product in period t
42.1. Roy and Maiti (1998)	**(3)** Total profit, Total wastage cost (P); **(4)** Storage area	Order quantity
42.2. Roy and Maiti (1998)	**(1)** Holding cost, Setup cost, Purchasing price, Selling price (P); **(3)** Total profit, Total wastage cost (P); **(4)** Storage area, Total budgetary cost (P)	Order quantity
43. Roy et al., (2008)	**(1)** Ordering cost, Holding cost (P); **(3)** Total cost (P); **(4)** Storage space, Budgetary cost (P)	Order quantity
44. Xu and Liu (2008)	**(1)** Holding cost [1], Setup cost [1], Purchasing price [1], Selling price [1] (P); **(3)** Total profit, Total wastage cost (P); **(4)** Total budgetary cost [1] (P)	Order quantity
45.1. Wee et al., (2009)	**(1)** Maximum allowed total average shortage cost (P); **(3)** Total profit, Return on inventory investment (P); **(4)** Maximum allowed total average shortage cost (P)	Interval between orders, Period when inventory is positive
45.2. Wee et al., (2009)	**(1)** Demand, Maximum allowed total average shortage cost (P); **(3)** Total profit, Return on inventory investment (P); **(4)** Maximum allowed total average shortage cost (P)	Interval between orders, Period when inventory is positive
46.1. Guchhait et al., (2010)	**(1)** Purchase cost (P); **(3)** Total profit (P); **(4)** Investment (P)	Order quantity
46.2. Guchhait et al., (2010)	**(1)** Purchase costs, Amount of investment (P); **(3)** Total profit (P); **(4)** Investment (P)	Order quantity
46.3. Guchhait et al., (2010)	**(1)** Amount of investment (P); **(4)** Investment (P)	Order quantity
47. Saha et al., (2010)	**(4)** Total budget, Total available space (P)	Initial stock, i.e. replenishment size
48. Chakraborty et al., (2013)	**(1)** Demand, Selling price, Space required (P); **(3)** Total profit (P); **(4)** Budget (P)	Percentage of discount, Replenishment cycles, Length of cycles
49. Jana et al., (2014)	**(1)** Deterioration rate, Inflation rate (P); **(3)** Total profit (P); **(4)** Available budget, Space [1] (P)	Order quantity, Duration of a complete cycle
50. Chen and Ouyang (2006)	**(1)** Holding cost, Interest paid, Interest earned (P); **(3)** Total cost (P)	Order quantity
51. De and Goswami (2006)	**(1)** Inflation rate, Deterioration rate (P); **(3)** Total cost (P)	Order quantity, Shortage quantity, Time from where shortage begins, Length of cycles, Permissible delay period for settling accounts
52. Mahata and Goswami (2007)	**(1)** Demand, Holding cost, Ordering cost, Purchasing cost (P); **(3)** Total cost (P)	Cycle time
53. Ouyang et al., (2010)	**(1)** Deterioration rate, Interest paid, Interest earned (P); **(3)** Total cost (P)	Order quantity, Replenishment time interval
54. Maiti (2011)	**(1)** Planning horizon, Interest rate, Holding cost, Setup cost (P); **(3)** Total profit (P); **(4)** Finite time horizon (P)	Length of one cycle, Time horizon, Credit-period offered by retailer to the customer, Number of the cycles that credit occurred
55. Mahata and Mahata (2011)	**(1)** Demand, Holding cost, Ordering cost, Purchasing cost, Selling price (P); **(3)** Total cost (P)	Order quantity of the retailer, Cycle time
56. Taleizadeh et al., (2011)	**(1)** Demand (P); **(3)** Total cost (P)	Order quantity, Number of packets that have been ordered for the *i*th product, Joint cycle length
57. Soni and Joshi (2013)	(1) Demand, Holding cost, Ordering cost, Purchasing cost, Interest earned, Interest paid (P); (3) Total profit (P)	Retail price, Replenishment cycle time

Table 2.3: Continue.

	Fuzzified element (Level of fuzzification) **(1)** Parameter (F, P); **(2)** Variable (F, P); **(3)** Objective (F, P); **(4)** Constraint (F, P)	Decision variable
58. Taleizadeh et al., (2013)	**(1)** Demand [4] (P); **(3)** Total cost (P)	Order quantity, Number of packets should be ordered for each product, Percentage of demand of each product that will be filled from stock, Period length for joint replenishment
59. Guchhait et al., (2014)	**(1)** Setup cost, Holding cost, Interest paid, Interest earned (P); **(3)** Total profit (P)	Length of each cycle, Length of last cycle, Customer's credit period offered by the retailer, Number of the full cycles during the planning horizon, "m" in Selling price=m× purchase cost, λ in λ× Length of each cycle
60. Yadav et al., (2015)	**(1)** Opportunity cost, Interest earned, Interest paid, Holding cost (P); **(3)** Total profit (P)	Order quantity, Cycle time, Payment delay time
61.1. Guchhait et al., (2015)	**(1)** Purchase cost, Selling price, Selling price of deteriorated units, Holding cost, Setup cost (P); **(3)** Total profit (P)	Order quantity, customer's credit period offered by the retailer
61.2. Guchhait et al., (2015)	**(1)** Purchase cost [4], Holding cost [4], Setup cost [4] (P); **(3)** Total profit (P)	Order quantity, customer's credit period offered by the retailer
62. Roy and Maiti (1997)	**(3)** Total cost (P); **(4)** storage area (P)	Order quantity, Demand
63. Liu (2008)	**(1)** Demand, Product cost (P); **(3)** Total profit (P)	Order quantity, Selling price
64.1. Vijayan and Kumaran (2009)	**(1)** Purchasing cost, Setup cost, Holding cost, Arrival rate (F); **(2)** Selling period (F); **(3)** Total cost (F)	Selling period
64.2. Vijayan and Kumaran (2009)	**(1)** Purchasing cost, Setup cost, Holding cost, Arrival rate[****] (F); **(3)** Total cost (P)	Selling period
64.3. Vijayan and Kumaran (2009)	**(1)** Arrival rate[****] (P); **(3)** Total cost (P)	Selling period
65. Chou et al., (2009)	**(3)** Total cost (P); **(4)** storage area (P)	Order quantity, Demand
66. Sadjadi et al., (2010)	**(1)** Selling price elasticity to demand, Marketing expenditure elasticity to demand, Lot size elasticity to purchasing cost (P); **(3)** Total profit (P)	Order quantity, Selling price, Marketing expenditure
67. Ketsarapong et al., (2012)	**(1)** Demand, Ordering or Setup cost, Unit price or Production cost, Holding cost (P); **(3)** Total profit (P); **(4)** Demand, Purchasing or Production quantity (P)	Ordering or Setup variable, which is 1 when order or setup occurs in period t, and 0 otherwise
68. Bera et al., (2012)	**(1)** Lead time, Time horizon, Inventory level, Duration of the cycle, Shortage level (P); **(3)** Total profit (P)	Length of the last cycle, Time when order is placed
69. Samadi et al., (2013)	**(1)** Demand, Unit cost Ordering cost, Holding cost, Shortage cost, Selling price elasticity to demand, Marketing expenditure elasticity to demand, Services expenditure elasticity to demand, Order quantity elasticity to unit cost (P); **(3)** Total profit (P)	Order quantity, Selling price, Shortage quantity, Marketing expenditure, Service expenditure
70. Yadav et al., (2013a)	**(1)** Demand (P); **(3)** Total cost (P)	Order quantity, Lead time, Backorder price discount
71. De and Sana (2013b)	**(1)** Demand (P); **(2)** Order quantity, Shortage quantity, Promotional index (F); **(3)** Total profit (F)	Order quantity, Shortage quantity, Promotional index
72. De and Sana (2013a)	**(1)** Demand [5] (P); **(2)** Promotional effort [5], Selling price [5] (F); **(3)** Total profit (P); **(4)** Order quantity, Shortage quantity (P)	Promotional effort, Selling price
73.1. Panda et al., (2014)	**(1)** Demand, Lead time demand [1] (P); **(3)** Total cost (P)	Order quantity, Lead time
73.2. Panda et al., (2014)	**(1)** Demand, Lead time demand [1] (P); **(3)** Total cost (P); (4) Budget (P)	Order quantity, Lead time

Table 2.3: Continue.

	Fuzzified element (Level of fuzzification)	Decision variable
	(1) Parameter (F, P); **(2)** Variable (F, P); **(3)** Objective (F, P); **(4)** Constraint (F, P)	
74. De et al., (2014)	**(1)** Demand, Holding cost, Shortage cost, Setup cost, Selling price, Advertisement cost, Promotional index, Power of promotional index, Maximum promotional index (F); **(3)** Total cost (F)	Order quantity, Shortage quantity
75. Lee and Yao (1998)	**(1)** Demand (P); **(2)** Production quantity (F); **(3)** Total cost (P)	Production quantity
76. Chang (1999)	**(2)** Production quantity (F); **(3)** Total cost (P)	Production quantity
77. Lin and Yao (2000)	**(2)** Production quantity (F); **(3)** Total cost (P)	Production quantity
78.1. Hsieh (2002)	**(1)** Demand, Holding cost, Setup cost, Daily production rate, Daily demand rate (F); **(3)** Total cost (P)	Production quantity
78.2. Hsieh (2002)	**(1)** Demand, Holding cost, Setup cost, Daily production rate, Daily demand rate (F); **(2)** Production quantity (F); **(3)** Total cost (F)	Production quantity
79. Mahapatra and Maiti (2006)	**(1)** Preparation time (P); **(3)** Total cost (P)	Time horizon
80.1. Chen and Chang (2008)	**(1)** Storage cost, Setup cost, Production cost of a defective item, Opportunity cost rate (P); **(3)** Total cost (P)	Production quantity
80.2. Chen and Chang (2008)	**(1)** Storage cost, Setup cost, Production cost of a defective item, Opportunity cost rate (P); **(2)** Production quantity (F); **(3)** Total cost (P)	Production quantity
81. Bag et al., (2009)	**(1)** Demand [1], Cycle length [1] (P); **(3)** Total profit (P)	Setup cost, Reliability of the production process, Production period
82. Das et al., (2011)	**(1)** Holding cost, Shortage cost, Coefficients in production cost functions (P); **(3)** Total profit (P)	Production rate
83. Pal et al., (2014)	**(1)** Holding costs, Purchasing cost, Inflation rate (P); **(3)** Total cost (P)	Production time
84. Paul et al., (2014)	**(1)** Demand, Holding cost, Cycle length (P); **(3)** Total profit (P)	Setup cost, Reliability of the production process, Production cycle length
85. Pal et al., (2015)	**(1)** Holding costs, Purchasing cost, Inflation rate (P); **(3)** Total cost (P)	Production time, Production rate
86. Roy et al., (2009b)	**(1)** Defective rate (P); **(3)** Total profit (P)	Cycle length
87. Guchhait et al., (2013)	**(1)** Cycles length, Remanufacturing time, Inventory levels in differed planning times, Production rate, Planning period, demand (P); **(3)** Total profit (P)	Time horizon
88. Mondal et al., (2013)	**(1)** Repairing cost [4], Setup cost [4], Disposal cost [4], Holding costs [4], Selling price [4] (P); **(3)** Total profit (P)	Production function's coefficients, Repairing function's coefficients
89. Mondal et al., (2014)	**(1)** Repairing rate, Defective rate (P); **(3)** Total profit (P)	Production rate, Minimum inventory level at showroom, The period which the units are transported to showroom, Numbers of shipment to showroom
90. Shekarian et al., (2014b)	**(1)** Demand, Production rate, Defective rate, Setup cost, Unit manufacturing cost, Holding cost, Backorder costs (fix and linear) (F); **(3)** Total costs (P)	Production quantity, Maximum backordering level
91. Shekarian et al., (2014a)	**(1)** Demand, Defective rate (P); **(3)** Total cost (P)	Production quantity
92. Halim et al., (2009)	**(1)** Defective rate, Parameters of exponential probability distribution (P); **(3)** Total cost (P)	Planning period, Production time
93. Zhang et al., (2009)	**(1)** Setup cost, Average holding cost, Elapsed time until the production shifts to out of control [1] (P); **(3)** Total cost (P)	Production run length
94. Wang and Tang (2009b)	**(1)** Elapsed time until the production process shifts to out of control, Setup cost, Holding costs (P); **(3)** Total cost	Production run length

Table 2.3: Continue.

	Fuzzified element (Level of fuzzification) **(1)** Parameter (F, P); **(2)** Variable (F, P); **(3)** Objective (F, P); **(4)** Constraint (F, P)	Decision variable
95. Hu et al., (2010)	**(1)** Elapsed time until the production process shifts to out of control (P); **(3)** Total cost (P)	Production run length, Production period during backorder replenishment
96. Kumar and Goswami (2015b)	**(1)** Demand, Lead time demand, Scheduling period (P); **(3)** Total cost (P)	Production quantity, Reorder point
97. Mahata (2015)	**(1)** Holding cost [2], Backorders cost [2], Raw material [2], Labour costs [2], Elapsed time until the production shifts to out of control [4] (P); **(3)** Total cost (P)	Production quantity, Backorder quantity
98. Kumar and Goswami (2015c)	**(1)** Holding cost, Backorders cost, Production costs, Elapsed time until the production process shifts to out-of- control [1], Percentage of defective items [1] (P); **(3)** Total cost (P); **(4)** Maximum available budget, Allowable shortages (P)	Production run length
99. Pappis and Karacapilidis (1995)	**(1)** Demand for ith product (P); **(3)** Total cost (P)	Number of production runs
100. Mandal et al., (2005)	**(1)** Holding cost, Shortage cost, Setup cost (P); **(3)** Total cost (P); **(4)** Total available storage space, Total number of orders, Production cost (P)	Demand, Production quantity, Shortage level
101. Mandal and Roy (2006a)	**(1)** Purchasing price of each product, Holding cost, Display shelf-space cost per unit product, Setup cost, Selling price of each product; **(3)** Total profit (P); **(4)** Total display-shelf space (P)	Number of display quantity, Number of order quantity
102. Islam and Roy (2007)	**(1)** Holding cost, Coefficients of the total cost of interest and depreciation, Coefficients of the production cost function (P); **(3)** Total cost (P); **(4)** Total available storage space (P)	Demand, Setup cost, Production quantity, Production reliability
103.1 Panda and Maiti (2009)	**(3)** Total profit (P); **(4)** Total available space area (P)	Selling price, Setup cost, Order quantity, Production process reliability
103.2 Panda and Maiti (2009)	**(1)** Scaling factor of the unit cost (P); **(3)** Total profit (P); **(4)** Total available storage space (P)	Selling price, Setup cost, Order quantity, Production process reliability
104. Mandal et al., (2011)	**(1)** Holding cost [1], Shortage cost [1] (P); **(4)** Total available storage space [1] (P)	Time horizon
105. Björk (2012)	**(2)** Cycle time (production batch equivalently) (P); **(3)** Total cost (P)	Production quantity, Maximum inventory, level, Cycle time
106. Jana et al., (2013)	**(1)** Selling price, Setup cost, Holding cost, Parameters of the purchase price (P); **(4)** Total available storage space, Total available budget (P)	Production rate, Production cycle length
107. Mezei and Björk (2015)	**(2)** Cycle time (production batch equivalently) (P); **(3)** Total cost (P)	Production quantity, Maximal shortage, Cycle time
108. Maity and Maiti (2005)	**(1)** Holding cost, Production cost (P); **(3)** Total cost (P)	Time horizon
109. Mandal and Roy (2006b)	**(1)** Setup cost, Holding cost, Production cost, Repairing cost (P); **(3)** Total cost (P)	Cycle time
110. Maity and Maiti (2007)	**(1)** Storage space per unit (P); **(3)** Total cost (P); **(4)** Total storage space, Investment capital (P)	Time horizon
111. Maity and Maiti (2008)	**(1)** Inflation, discount rate (P); **(3)** Total cost (P)	Time horizon
112. Xu and Zhao (2008)	**(1)** Selling price [4], Production cost [4], Repairing cost [4], Holding cost [4], Setup cost [4] (P); **(3)** Total cost, Total profit (P); **(4)** Total available budget [4] (P)	Time horizon
113. Panda et al., (2008)	**(1)** Maximum budget, Production cost, Maximum shortage cost (P); **(3)** Total profit (P); **(4)** Screening cost under budget, Maximum shortage (P)	Production quantity
114. Xu and Zhao (2010)	**(1)** Selling price, Production cost, Repairing cost, Holding cost, Setup cost (P); **(3)** Total profit, Total cost (P); **(4)** Total available budget (P)	Time horizon
115. Mandal et al., (2010)	**(1)** Cycle length, Storage capacity, Production cost, Holding cost (P); **(3)** Total cost (P); **(4)** Total available storage space (P)	Time horizon
116. Maity (2011a)	**(1)** Inflation rate, Discount rate (P); **(3)** Total cost (P); **(4)** Total available space area (P)	**Time horizon**

Table 2.3: Continue.

	Fuzzified element (Level of fuzzification) **(1)** Parameter (F, P); **(2)** Variable (F, P); **(3)** Objective (F, P); **(4)** Constraint (F, P)	Decision variable
117. Maity (2011b)	**(1)** Storage area per unit item, Available storage space, Total budgetary capital (P); **(3)** Total profit (P); **(4)** Total available storage space, Total available budget (P)	Production rate, Stock levels
118. Das and Maiti (2013)	**(1)** Production cost, Holding cost, Shortage cost, Storage area per unit item (P); **(3)** Total cost (P); **(4)** Total available storage space [3], Total available budget [7] (P)	Time horizon
119. Islam and Roy (2006)	**(1)** Holding cost, Coefficients of total cost of interest and depreciation, Coefficient of total production cost, Storage area per unit item (P); **(4)** Total available storage space (P)	Demand, Setup cost, Production quantity, Production reliability
120. Chang and Chang (2006)	**(1)** Unit cost, Demand, Number of production cycles, Demand rate, Production quantity, Production cycle time of *i*th process, Percentage of unit cost from initial production scraps for *i*th process, Percentage of unit cost from direct labor cost for ith process, Percentage of unit cost from facility/equipment depreciation and energy consumption for *i*th process, Time of on-line setup for *i*th process, Time of off-line setup for *i*th process, Holding cost (F); **(3)** Total cost (P)	Production quantity
121. Chang et al., (2006)	**(1)** Demand (P); **(3)** Total cost (P)	Cycle time
122. Maity et al., (2008)	**(1)** Holding costs, Serviceable and non-serviceable items (P); **(3)** Total profit (P)	Production quantity, Stock levels
123. Pal et al., (2009)	**(1)** Lifetime of product (P); **(3)** Total profit (P)	Cycle time, Period of discount, Mark ups during discount, Normal period
124. Wang and Tang (2009a)	**(1)** Setup cost, Holding cost, Backorder costs (P); **(3)** Total cost (P)	Production quantity, Maximum backorder level
125. Mahapatra et al., (2011)	**(1)** Holding cost, Coefficients of production, Depreciation cost functions (P); **(3)** Total cost (P)	Demand, Production reliability, Production quantity
126. Soni and Shah (2011)	**(1)** Demand, Production preparation-time (P); **(3)** Total cost (P)	Maximum inventory level, Maximum shortage units
127. Chakrabortty et al., (2013)	**(1)** Demand, Holding cost, Shortage cost, Set up cost (P); **(3)** Total cost (P)	Production quantity, Shortage level
128. Yaghin et al., (2013)	**(1)** Setup cost, Holding cost, Manufacturer's production cost, Manufacturer's purchasing cost, Coefficient of demand rate (P); **(3)** Total profit, The ratio of the profit over the average investment (P); **(4)** Maximum allowed total marketing cost, Demand coefficient	Selling price, Cycle time
129.1. De and Sana (2014)	**(1)** Holding cost, Overtime period, Production level increase and decrease, Regular-time production costs (P); **(3)** Total cost (P)	Predicted demand in *i*th period, Maximum over-time production that can be scheduled, Maximum regular-time production that can be scheduled
129.2. De and Sana (2014)	**(1)** Demand, Capacity mode (P); **(3)** Total cost (P)	Predicted demand in ith period, Maximum over-time production that can be scheduled, Maximum regular-time production that can be scheduled
129.3. De and Sana (2014)	**(1)** Holding cost, Overtime period, Production level increase and decrease, Regular-time production costs Demand, Capacity mode (P); **(3)** Total cost (P)	Predicted demand in ith period, Maximum over-time production that can be scheduled, Maximum regular-time production that can be scheduled
130. Kumar and Goswami (2015a)	**(1)** Defective rate, Time in which the production status changes (P); **(3)** Total cost (P)	Production quantity, Backorder quantity

* Rented Warehouse, ** Own Warehouse, *** Maximizing the credibility of an event that the total cost in the planning periods does not exceed a predefined budget level, **** The number of customers that arrive in the unit time interval which follows a Poisson distribution with mean arrival rate per unit time λ.

1- Fuzzy-random variable
2- Fuzzy variable
3- Fuzzy stochastic
4- Fuzzy-rough variable
5- Intuitionistic fuzzy set
6- Hybrid number
7- Optimistic fuzzy
8- Bifuzzy

CHAPTER 3: METHODOLOGY

3.1 Introduction

In this chapter, the methodology used in this study is explained. The framework of the research is detailed and the techniques and the methods that are used to obtain the fuzzy models are discussed. Furthermore, the optimization methods and the important characteristics of the models are demonstrated. In order to highlight the importance of the research and compare the tools and the methods in each step with the similar ones in previous studies, some tables are provided according to the comprehensive literature review in chapter two.

3.2 Research Methodology

At the first stage, a comprehensive literature review among the previous fuzzy inventory systems dealing with EOQ and EPQ models was conducted. According to these studies, the gap and the shortcoming of the mentioned inventory models are determined. The problems were stated, and the objectives and research questions were identified. Later, the developed fuzzy models with the help of methods introduced in this chapter are tried to be solved.

The first part of our work addresses a fuzzy EOQ (FEOQ) model which is developed in a fully fuzzy environment. This model follows a learning process where it allows the percentage of defective items to be decreased when the decision maker orders more and more. In addition, it is assumed that the holding costs of defective and non-defective (good quality) items are different. The aim is to determine the optimal lot size, and subsequently, the estimation of total profit per unit time.

The second research direction concerns with the developing a fuzzy backward inventory system that integrates an EOQ/EPQ model to make a reverse process while it is affected by the learning process. The fuzzy version of this model is formulated and the effect of different defuzzification methods on optimal policies is compared. The objective is analyzing the behavior of the model under uncertain environment and deriving the optimum recovery lot size and the number of orders for the newly purchased products.

The following diagram in Figure 3.1 shows the general view of the research methodology. In each model, after optimization of the models via proposed methods, analyses are provided according to the arbitrary data through numerical examples and the models are validated comparing their crisp ones with different levels of fuzziness. It is shown that how the crisp models can result in the wrong and bias-optimal quantities in uncertain business environment.

Besides, the applicability of the models is depicted through real scenarios. The first model is applied for a company that is working in the automobile industry. Optimized policies are derived for two important products of this company related to the automobile braking system in an uncertain business environment. The inventory model of the investigated company could be adopted with the first fuzzy model developed in chapter 4. As it is described in chapter 6, it has the characteristics of the first fuzzy model. The defective products could affect the braking system in an automobile causing dangerous accidents. The related information of parameters is gathered for some periods, and then is applied in the model with suggested fuzzy numbers.

As the second model is appropriate to study a reverse logistics inventory models, it is used to make appropriate strategies for a company working in milk manufacturing industry while it produces a product that could be recycled. In fact, the investigated milk company uses this product as a container to pack the produced milk and these containers could be recovered again. Different policies are suggested according to the optimized quantities and they are compared to make the best decision.

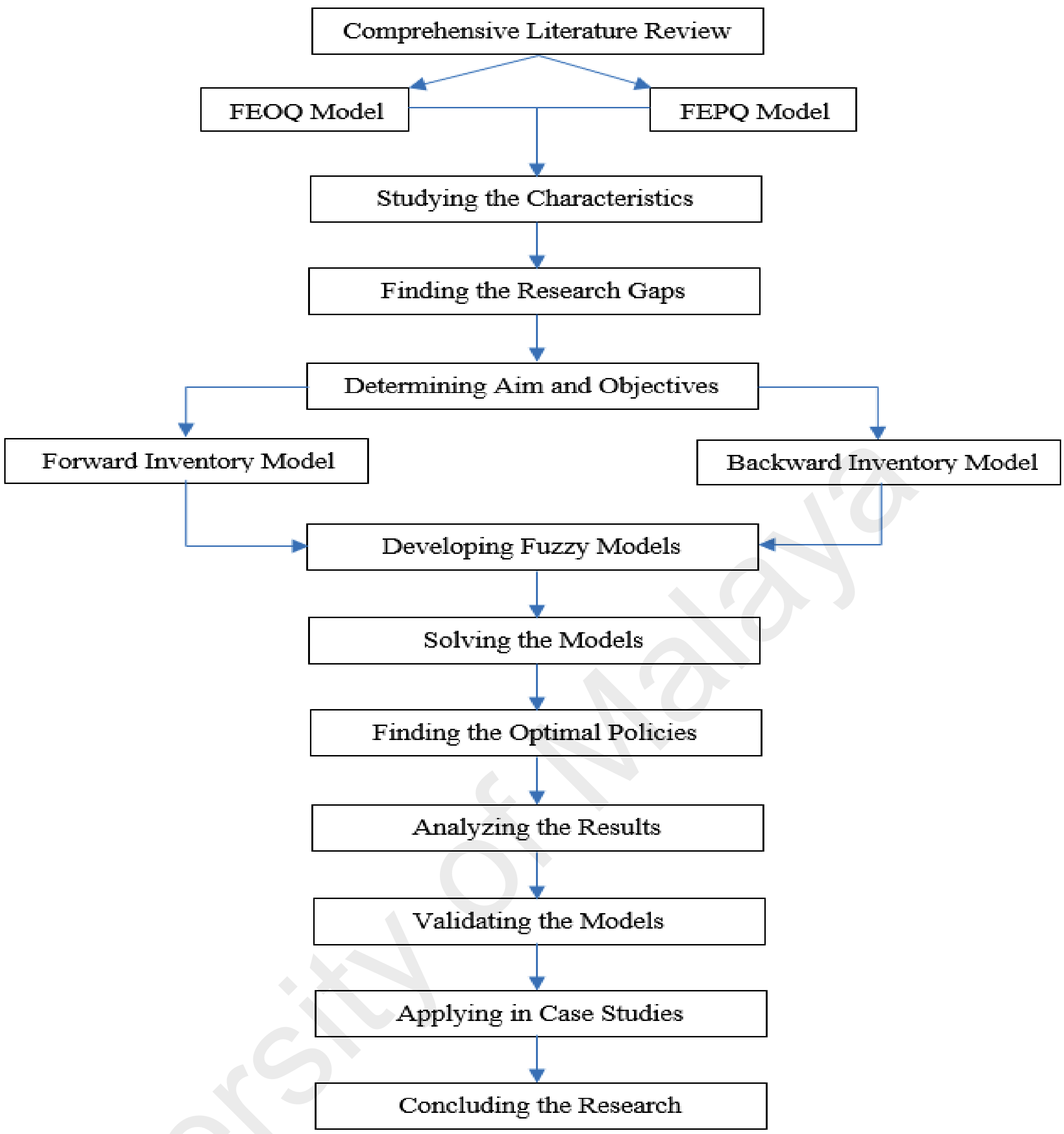

Figure 3.1: Research methodology

3.3 Research Framework in Details

The details of the research framework are presented in Figure 3.2. It is possible to divide them into three phases. In phase one, inventory models are selected according to the research gaps. Then, the elements including parameters and variables that constitute the objective function of the inventory system should be identified to be fuzzified. These factors are discussed when models are explained in more details. Finally, optimization of the defuzzified models is the core part of the third phase. This process is followed to derive the optimal policies for both models (i.e. forward FEOQ, and backward FEOQ).

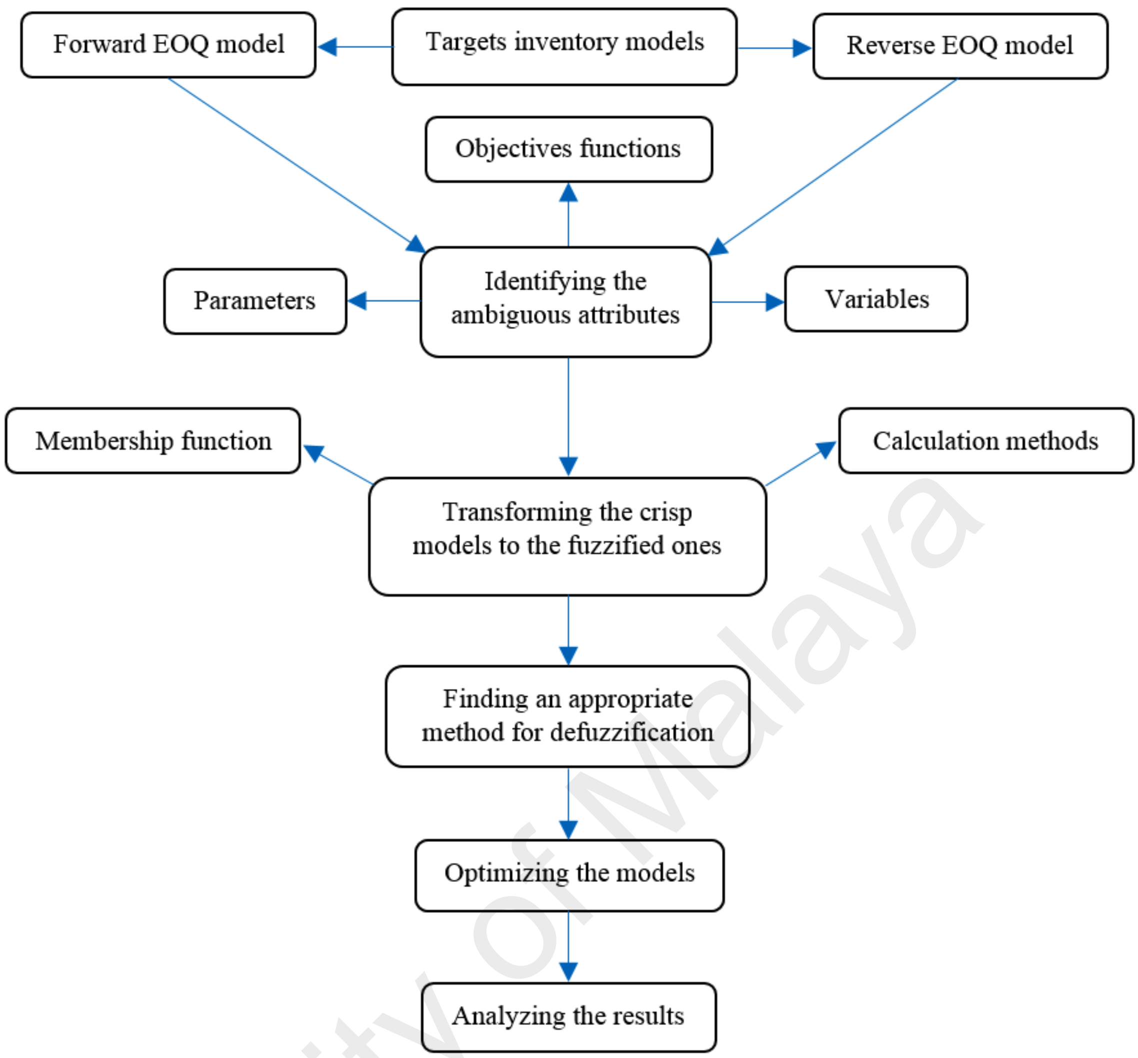

Figure 3.2: Details of research framework

3.4 Fuzzification Process

In this section, methods and techniques that are aimed to develop our proposed fuzzified model are explained. Some basic definitions and principles in which are necessary in the next chapters are introduced.

3.4.1 Preliminaries and Fundamental Definitions

In this section, some basic definitions and principles of fuzzy set theory are provided (Kaufman & Gupta, 1991; Zimmermann, 2001). These are necessary to introduce and treat the FEOQ in later sections.

Definition. 1: Let U be a universe set. A fuzzy set $\widetilde{\mathrm{B}}$ of U is defined by a membership function $\mu_{\widetilde{\mathrm{B}}}(u) \to [0,1]$ where $\mu_{\widetilde{\mathrm{B}}}(u)$, $\forall u \in U$ denotes the grade of membership.

Definition. 2: The fuzzy subset $\widetilde{\mathrm{B}}$ in the universe of discourse is called as a normal set if and only if $sup_{u \in U} \mu_{\widetilde{\mathrm{B}}}(u) = 1$. That is, the largest grade that an element can obtain.

Definition. 3: A fuzzy subset $\widetilde{\mathrm{B}}$ of the universe of discourse U is convex if and only if for all u_1 and $u_2 \in U$ and for $\gamma \in [0,1]$ we have: $\mu_{\widetilde{\mathrm{B}}}(\gamma u_1 + (1-\gamma)u_2) \geq \min(\mu_{\widetilde{\mathrm{B}}}(u_1), \mu_{\widetilde{\mathrm{B}}}(u_2))$.

Definition. 4: A fuzzy set $\widetilde{\mathrm{B}}$ is a fuzzy number if and only if it is normal and convex on U.

Definition. 5: Fuzzy set $\widetilde{\mathrm{B}}_\alpha$ for $0 \leq \alpha \leq 1$ and a range of $u \in R$ is called an *α-level fuzzy point* whose membership function has a form

$$\mu_{\widetilde{\mathrm{B}}_\alpha}(u) = \begin{cases} \alpha, & u = a, \\ 0, & u \neq a. \end{cases} \tag{3.1}$$

It should be noted that if $\alpha = 1$, the membership function of the 1-*level fuzzy point* $\widetilde{\mathrm{B}}_\alpha$ becomes the characteristic function, i.e.,

$$\mu_{\widetilde{\mathrm{B}}_\alpha}(u) = \begin{cases} 1, & u = a, \\ 0, & u \neq a. \end{cases} \tag{3.2}$$

In this case, the fuzzy point $\widetilde{\mathrm{B}}_1$ and the real number $a \in R$ are similar except for their representation.

Definition. 6: For $0 \leq \alpha \leq 1$ and $a < b$, the fuzzy set $[a_\alpha, b_\alpha]$ defined on R is called an *α-level fuzzy interval* if its membership function is given by

$$\mu_{[a_\alpha, b_\alpha]}(u) = \begin{cases} \alpha, & a \leq u \leq b, \\ 0, & \text{otherwise}. \end{cases} \tag{3.3}$$

Definition. 7: The α-*cut* of a fuzzy set $\widetilde{\mathrm{B}}$ which is presented as $\mathrm{B}(\alpha)$ on R for $0 \leq \alpha \leq 1$ includes points u such that $\mu_{\widetilde{\mathrm{B}}}(u) \geq \alpha$, that is $\mathrm{B}(\alpha) = \{u | \mu_{\widetilde{\mathrm{B}}}(u) \geq \alpha\}$.

3.4.2 Generalized Fuzzy Numbers

Generalized fuzzy number $\widetilde{\mathrm{B}}$ is described as any fuzzy subset of the real line R with the membership function $\mu_{\widetilde{\mathrm{B}}}$ which is a continuous mapping from R to the closed interval $[0,1]$ satisfying the below conditions:

- $\mu_{\widetilde{\mathrm{B}}}(u) = 0, -\infty < x \leq \beta_1;$
- $\mu_{\widetilde{\mathrm{B}}}(u) = M(u)$ is a strictly increasing function for $\beta_1 \leq x \leq \beta_2$;
- $\mu_{\widetilde{\mathrm{B}}}(u) = w_{\mathrm{B}}, \beta_2 \leq x \leq \beta_3;$
- $\mu_{\widetilde{\mathrm{B}}}(u) = N(u)$ is a strictly decreasing function for $\beta_3 \leq x \leq \beta_4$;
- $\mu_{\widetilde{\mathrm{B}}}(u) = 0, \beta_4 \leq x < \infty.$

where $0 < w_{\mathrm{B}} \leq 1$, and $\beta_i, i = 1,2,3,4$ are real numbers. This type of generalized fuzzy number could be written as $\widetilde{\mathrm{B}} = (\beta_1, \beta_2, \beta_3, \beta_4; w_{\mathrm{B}})_{MN}$, and it is called a trapezoidal fuzzy number (TPFN). When $w_{\mathrm{B}} = 1$, it easily could be shown as $\widetilde{\mathrm{B}} = (\beta_1, \beta_2, \beta_3, \beta_4)_{MN}$ that is a normalized fuzzy number. If $\beta_2 = \beta_3 = \beta$ then it is transformed to a triangular fuzzy number (TFN) which could be presented as $\widetilde{\mathrm{T}} = (\beta_1, \beta, \beta_4; w_{\mathrm{B}})_{MN}$ and its normal form is shown as $\widetilde{\mathrm{T}} = (\beta_1, \beta, \beta_4)_{MN}$. When $\beta_1 = \beta = \beta_4 = \beta'$, then the TFN $\tilde{T} = (\beta', \beta', \beta')$ is identical to the 1-*level fuzzy point* $\widetilde{\mathrm{T}}_1$.

Definition. 8: For $\alpha \in [0,1]$, the α-*cut* of TFN $\widetilde{\mathrm{B}} = (\beta_1, \beta, \beta_4)$ is $\mathrm{B}(\alpha) = [\mathrm{B}_L(\alpha), \mathrm{B}_R(\alpha)]$, where $\mathrm{B}_L(\alpha) = \beta_1 + (\beta - \beta_1)\alpha$ and $\mathrm{B}_R(\alpha) = \beta_4 - (\beta_4 - \beta)\alpha$.

3.4.3 Overview of the Previous Fuzzy Numbers

Many kinds of fuzzy numbers have been used in the literature that include trapezoidal and triangular fuzzy numbers, Gaussian fuzzy number, interval fuzzy number, statistic-fuzzy number, discrete fuzzy number, parabolic fuzzy number, quadratic fuzzy number, exponential fuzzy number, bell-shaped fuzzy number and other kinds of linear and non-linear fuzzy number. Among them triangular and trapezoidal fuzzy numbers have the most frequency and applicability. In fact, triangular fuzzy numbers are the special cases of the trapezoidal ones. Moreover, Table 3.1 in Appendix A presents the categorization of the literature review based on the publications and related fuzzy numbers.

3.4.4 Justification of the Triangular Fuzzy Number

In this research, triangular fuzzy number $\tilde{\mathrm{T}}$ defined with trio $(\beta_1, \beta_2, \beta_3)$ and membership function $\mu_{\tilde{\mathrm{T}}}(u)$ is applied as depicted in Figure 3.3 for the selected fuzzified form because they have some advantages over other linear and nonlinear membership functions. According to Bansal (2011), trapezoidal fuzzy numbers form the most generic class of fuzzy numbers with linear membership function. They span entirely the widely discussed class of triangular fuzzy numbers. These fuzzy numbers have more applicability in modeling linear uncertainty in scientific problems. They have conceptual and computational simplicity in practice.

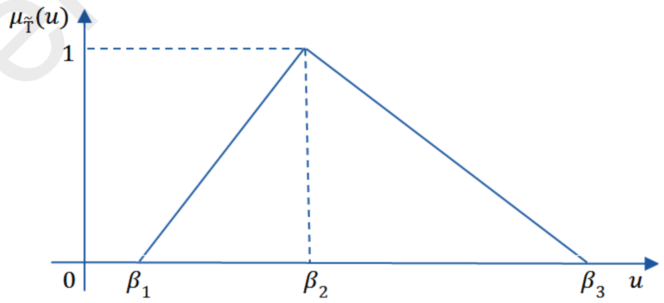

Figure 3.3: Triangular fuzzy number

Besides, triangular fuzzy numbers prevent to obtain a non-convex cost function, which facilitate the optimization process. They are more intuitive and have an easier linguistic interpretation and are easier to handle when comparing with other linear membership functions. It is guaranteed to achieve computational efficiency and ease-of-use to the problem (Shekarian et al., 2014b).

3.4.5 Overview of the Previous Fuzzification Methods

According to the investigated literature, there are different fuzzification methods that are applied to transform the crisp model to the fuzzy environment. Depending on the complexity of the model, these methods may vary. The methods of defuzzification of the previous studies are categorized in Table 3.2 in Appendix B. Among these methods such as possibility/necessity methods, extension principle, interval operations, as it is clear function principle has been used in some studies.

3.4.6 Fuzzy Arithmetic

In order to do the calculations through the proposed fuzzified inventory models, it is preferred to use the function principle method. In the next section, it is explained why this method takes the priority to be applied. Here this method is explained via trapezoidal fuzzy number. However, as the triangular fuzzy numbers are the special cases of the trapezoidal ones, therefore, the derived formula and principle could be easily hold for TFNs.

Chen (1985) suggested arithmetical operations of the function principle method as follows. Consider the below relation in which shows the induction of the fuzzy number $\widetilde{\mathrm{B}}$ from a set of fuzzy numbers belonging to the same family.

$$f_g\left(\widetilde{\mathrm{B}}_1, \widetilde{\mathrm{B}}_2, \dots, \widetilde{\mathrm{B}}_n\right) = \widetilde{\mathrm{B}} = (\mathrm{H}, \Theta, \mathrm{I}, \mathrm{K}; w_{\mathrm{B}})_{MN} \tag{3.4}$$

where g is a mapping from $n-$dimensional real numbers R^n into the real line R and equivalently f_g could be mapped into a fuzzy number from $n-$dimensional fuzzy numbers, $\widetilde{\mathrm{B}}_i = (\zeta_i, \lambda_i, \mu_i, \vartheta_i; w_{\mathrm{i}}); i = 1,2, \dots, n$ and besides we have:

$$w_{\mathrm{B}} = \min\{w_{\mathrm{i}}, i = 1,2,\dots,n\}, \tag{3.5}$$

$$\Theta_i = \min\{u | f_{\widetilde{\mathrm{B}}_i}(u) \geq w_{\mathrm{B}}\}, \tag{3.6}$$

$$\mathrm{I}_i = \max\{u | f_{\widetilde{\mathrm{B}}_i}(u) \geq w_{\mathrm{B}}\}, \tag{3.7}$$

$$T = \{g(u_1, u_2, \dots, u_n) | u_i = \zeta_i \text{ or } \vartheta_i\, , i = 1,2,\dots,n\,\} \tag{3.8}$$

$$T_1 = \{g(u_1, u_2, \dots, u_n) | u_i = \Theta_i \text{ or } \mathrm{I}_i\, , i = 1,2,\dots,n\,\} \tag{3.9}$$

$$\mathrm{H} = \min T \tag{3.10}$$

$$\Theta = \min T_1 \tag{3.11}$$

$$\mathrm{I} = \max T_1 \tag{3.12}$$

$$\mathrm{K} = \max T \tag{3.13}$$

where $\min T \leq \min T_1$ and $\max T_1 \leq \max T$. (3.14)

Let $\widetilde{\mathrm{B}}_1 = (\zeta_1, \lambda_1, \mu_1, \vartheta_1; w_1)$ and $\widetilde{\mathrm{B}}_2 = (\zeta_2, \lambda_2, \mu_2, \vartheta_2; w_2)$ then we have

$$\widetilde{\mathrm{B}}_1 + \widetilde{\mathrm{B}}_2 = (\zeta_1 + \zeta_2, \Theta_1 + \Theta_2, \mathrm{I}_1 + \mathrm{I}_2, \vartheta_1 + \vartheta_2; w_{\mathrm{B}}) \tag{3.14}$$

Figure 3.4 shows the addition of two fuzzy numbers, and for triangular fuzzy numbers $\widetilde{\mathrm{T}}_1 = (t_1, t_2, t_3)$ and $\widetilde{\mathrm{T}}_1 = (t'_1, t'_2, t'_3)$ the membership of the addition is as below:

$$\mu_{\widetilde{\mathrm{T}}_1 + \widetilde{\mathrm{T}}_2}(u) = \begin{cases} 0 & t_3 + t'_3 < u < t_1 + t'_1 \\ M(u) = \dfrac{u - (t_1 + t'_1)}{(t_2 + t'_2) - (t_1 + t'_1)} & t_1 + t'_1 \leq u < t_2 + t'_2 \\ N(u) = \dfrac{(t_3 + t'_3) - u}{(t_3 + t'_3) - (t_2 + t'_2)} & t_2 + t'_2 \leq u < t_3 + t'_3 \end{cases} \tag{3.15}$$

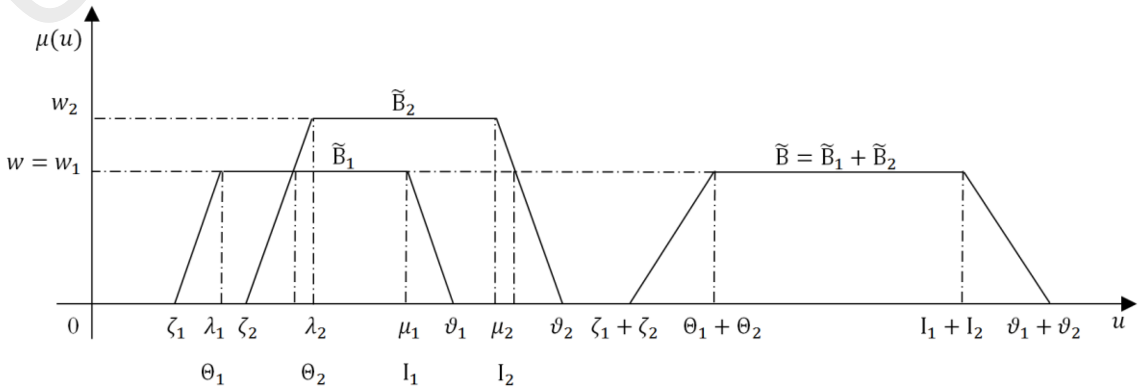

Figure 3.4: Addition operation of two fuzzy numbers

For product of two fuzzy numbers $\widetilde{\mathrm{B}}_1 * \widetilde{\mathrm{B}}_2 = (\mathrm{H}, \Theta, \mathrm{I}, \mathrm{K}; w_{\mathrm{B}})$ we have

$$w_{\mathrm{B}} = \min\{w_1, w_2\}, \tag{3.16}$$

$$\Theta_1 = \min\{u | f_{\widetilde{\mathrm{B}}_1}(u) \geq w_{\mathrm{B}}\}, \tag{3.17}$$

$$\Theta_2 = \min\{u | f_{\widetilde{\mathrm{B}}_2}(u) \geq w_{\mathrm{B}}\}, \tag{3.18}$$

$$\mathrm{I}_1 = \max\{u | f_{\widetilde{\mathrm{B}}_1}(u) \geq w_{\mathrm{B}}\}, \tag{3.19}$$

$$\mathrm{I}_2 = \max\{u | f_{\widetilde{\mathrm{B}}_2}(u) \geq w_{\mathrm{B}}\}, \tag{3.20}$$

$$T = \{\zeta_1\zeta_2, \zeta_1\vartheta_2, \vartheta_1\zeta_2, \vartheta_1\vartheta_2\}, \tag{3.21}$$

$$T_1 = \{\Theta_1\Theta_2, \Theta_1\mathrm{I}_2, \mathrm{I}_1\Theta_2, \mathrm{I}_1\mathrm{I}_2\} \tag{3.22}$$

$$\mathrm{H} = \min T \tag{3.23}$$

$$\Theta = \min T_1 \tag{3.24}$$

$$\mathrm{I} = \max T_1 \tag{3.25}$$

$$\mathrm{K} = \max T \tag{3.26}$$

Figure 3.5 shows the multiplication of two fuzzy numbers, and for triangular fuzzy numbers $\widetilde{\mathrm{T}}_1 = (t_1, t_2, t_3)$ and $\widetilde{\mathrm{T}}_1 = (t_1', t_2', t_3')$ the membership of the product is as below:

$$\mu_{\widetilde{\mathrm{T}}_1 * \widetilde{\mathrm{T}}_2}(u) = \begin{cases} \quad 0 & t_3t_3' < u < t_1t_1' \\ M(u) = \dfrac{u - (t_1t_1')}{(t_2t_2') - (t_1t_1')} & t_1t_1' \leq u < t_2t_2' \\ N(u) = \dfrac{(t_3t_3') - u}{(t_3t_3') - (t_2t_2')} & t_2t_2' \leq u < t_3t_3' \end{cases} \tag{3.27}$$

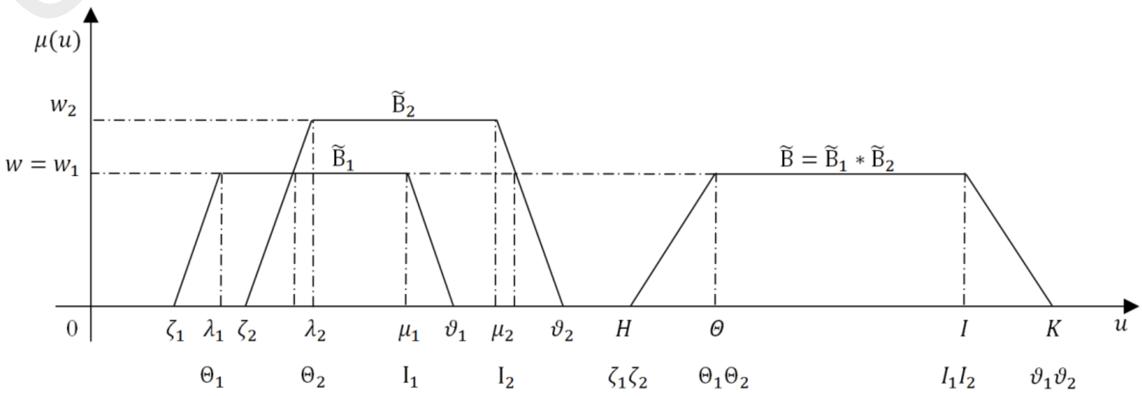

Figure 3.5: Product operation of two fuzzy numbers

The membership of subtraction and the division of the $\widetilde{\mathrm{T}}_1 = (t_1, t_2, t_3)$ and $\widetilde{\mathrm{T}}_1 = (t'_1, t'_2, t'_3)$ can be given as below:

$$\mu_{\widetilde{\mathrm{T}}_1-\widetilde{\mathrm{T}}_2}(u) = \begin{cases} 0 & t_3 - t'_1 < u < t_1 - t'_3 \\ M(u) = \dfrac{u - (t_1 - t'_3)}{(t_2 - t'_2) - (t_1 - t'_3)} & t_1 - t'_3 \le u < t_2 - t'_2 \\ N(u) = \dfrac{(t_3 - t'_1) - u}{(t_3 - t'_1) - (t_2 - t'_2)} & t_2 - t'_2 \le u < t_3 - t'_1 \end{cases} \tag{3.28}$$

$$\mu_{\widetilde{\mathrm{T}}_1/\widetilde{\mathrm{T}}_2}(u) = \begin{cases} 0 & t_3/t'_1 < u < t_1/t'_3 \\ M(u) = \dfrac{u - (t_1/t'_3\ )}{(t_2/t'_2) - (t_1/t'_3)} & t_1/t'_3 \le u < t_2/t'_2 \\ N(u) = \dfrac{(t_3/t'_1) - u}{(t_3/t'_1) - (t_2/t'_2)} & t_2/t'_2 \le u < t_3/t'_1 \end{cases} \tag{3.29}$$

where $t_1, t'_1, t_2, t'_2, t_3$, and t'_3 are real numbers.

3.4.7 Principle of Decomposition Theory

Suppose that $\widetilde{\mathrm{B}}$ is a fuzzy set on R, $0 \le \alpha \le 1$, and its α-*cut* is $\mathrm{B}(\alpha) = [\mathrm{B}_L(\alpha), \mathrm{B}_R(\alpha)]$ which is a closed interval, then we have:

$$\widetilde{\mathrm{B}} = \bigcup_{0\le\alpha\le1} [\mathrm{B}_L(\alpha)_\alpha, \mathrm{B}_R(\alpha)_\alpha] = \bigcup_{0\le\alpha\le1} \alpha\mathrm{B}(\alpha) \tag{3.30}$$

or

$$\mu_{\widetilde{\mathrm{B}}}(u) = \bigvee\nolimits_{0\le\alpha\le1} \mu_{[\mathrm{B}_L(\alpha)_\alpha, \mathrm{B}_R(\alpha)_\alpha]}(u) = \bigvee\nolimits_{0\le\alpha\le1} \alpha C_{\mathrm{B}(\alpha)}(u) \tag{3.31}$$

where

(i) $\alpha\mathrm{B}(\alpha)$ is a fuzzy set with the membership function:

$$\mu_{\alpha\mathrm{B}(\alpha)}(u) = \begin{cases} \alpha, & u \in \mathrm{B}(\alpha), \\ 0, & otherwise. \end{cases} \tag{3.32}$$

(ii) $C_{\mathrm{B}(\alpha)}(u)$ is a characteristic function of $\mathrm{B}(\alpha)$; that is

$$C_{B(\alpha)}(u) = \begin{cases} 1, & u \in B(\alpha), \\ 0, & u \notin B(\alpha). \end{cases} \tag{3.33}$$

For any $a_1, a_2, b_1, b_2, k \in R$, $a_1 < a_2$, and $b_1 < b_2$, the interval operations are as follows:

$$[a_1, a_2](+)[b_1, b_2] = [a_1 + b_1, a_2 + b_2], \tag{3.34}$$

$$[a_1, a_2](-)[b_1, b_2] = [a_1 - b_2, a_2 - b_1]. \tag{3.35}$$

$$k(\cdot)[a_1, a_2] = \begin{cases} [ka_1, ka_2], & k > 0, \\ [ka_2, ka_1], & k < 0. \end{cases} \tag{3.36}$$

Besides, if $0 < a_1$, and $0 < b_1$, then

$$[a_1, a_2](\cdot)[b_1, b_2] = [a_1 b_1, a_2 b_2] \tag{3.37}$$

$$[a_1, a_2](\div)[b_1, b_2] = \left[\frac{a_1}{b_2}, \frac{a_2}{b_1}\right]. \tag{3.38}$$

3.4.8 Justification of the Function Principle Methods

As it was explained, function principle method is used to do operation of addition, multiplication, subtract, division of triangular fuzzy numbers in present research. Comparing with another well-known method (i.e. extension principle) which is not simple in most of the cases, it has some advantages as follows (Shekarian et al., 2014b; Chen & Chang, 2008):

- The function principle is easier to calculate
- The shape of trapezoidal and triangular fuzzy numbers does not change after the multiplication. However, extension principle changes the shape of fuzzy numbers to be drummed.
- In contrast to the other similar methods, it can easily find the results of multiplying more than four triangular fuzzy numbers by pointwise computation.

3.5 Defuzzification Process

Although the inventory system is developed in a fuzzy environment, the results should be transformed to the precise information to be applicable in the real situation. Because it is not possible for the decision maker to draw ultimate conclusions with the fuzzy results. Therefore, in order to interpret the optimal policies, it is necessary to convert the uncertain output to the crisp ones. The method of extracting crisp results from the fuzzy models is known as defuzzification (Mahata & Goswami, 2013). In the following, defuzzification methods used in the literature are reviewed and techniques that are applied in our study are explained.

3.5.1 Overview of the Previous Defuzzification Methods

In this section, defuzzification methods that are used in the previous literature are determined according to the studies in which are gathered in chapter 2. Different direct methods apply to the process of defuzzification such as median rule, centroid method. Also in other models, the process applies indirectly such as reduction of fuzzy inventory problem to a pair of mathematical programs to derive the upper bound and lower bound or converting the problem to a chance constrained programming problem. These methods are categorized in Table 3.3 in Appendix C. In this study, signed distance method and GMIR method are employed to do the mentioned process.

3.5.1.1 GMIR Method

In this section, the GMIR method introduced by Chen (1985) and Chen and Hsieh (1999) that is based on the integral value of graded mean α-level of generalized fuzzy numbers is explained for defuzzifying them.

If M^{-1} and N^{-1} are considered as the inverse functions of M and N respectively in the generalized fuzzy number $\widetilde{\mathrm{B}} = (\beta_1, \beta_2, \beta_3, \beta_4; w_{\mathrm{B}})_{MN}$, then the graded mean α-level value of this number is $\alpha[M^{-1}(\alpha)+N^{-1}(\alpha)]/2$. Figure 3.6 illustrate these relations. The GMIR of $\widetilde{\mathrm{B}}$ shown as $\mathcal{G}(\widetilde{\mathrm{B}})$ with grade w_{B} is defined as:

$$\mathcal{G}(\widetilde{\mathrm{B}}) = \int_0^{w_{\mathrm{B}}} \alpha \left[\frac{M^{-1}(\alpha)+N^{-1}(\alpha)}{2}\right] d\alpha \Big/ \int_0^{w_{\mathrm{B}}} \alpha d\alpha \tag{3.39}$$

where $0 < \alpha \leq w_{\mathrm{B}} \leq 1$.

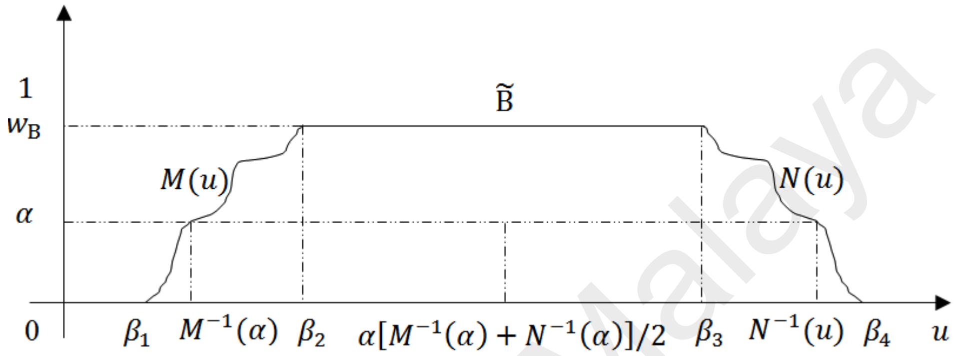

Figure 3.6: The graded mean α-level value of generalized fuzzy number $\widetilde{\mathrm{B}}$

3.5.1.2 Graded Mean Integration Representation of TFN

For the generalized triangular fuzzy number $\widetilde{\mathrm{T}} = (\beta_1, \beta, \beta_2; w_{\mathrm{B}})_{MN}$ we have

$$M(u) = w_{\mathrm{B}}\left(\frac{u-\beta_1}{\beta-\beta_1}\right) \qquad \beta_1 \leq u \leq \beta \tag{3.40}$$

$$N(u) = w_{\mathrm{B}}\left(\frac{u-\beta_2}{\beta-\beta_2}\right) \qquad \beta \leq u \leq \beta_2 \tag{3.41}$$

Therefore,

$$M^{-1}(\alpha) = \beta_1 + \frac{\alpha}{w_{\mathrm{B}}}(\beta-\beta_1) \qquad 0 \leq \alpha \leq w_{\mathrm{B}} \tag{3.42}$$

$$N^{-1}(\alpha) = \beta_2 - \frac{\alpha}{w_{\mathrm{B}}}(\beta_2-\beta) \qquad 0 \leq \alpha \leq w_{\mathrm{B}} \tag{3.43}$$

and

$$S = \frac{M^{-1}(\alpha)+N^{-1}(\alpha)}{2} = \frac{w_{\mathrm{B}}(\beta_1+\beta_2)+(2\beta-\beta_1-\beta_2)\alpha}{2w_{\mathrm{B}}} \tag{3.44}$$

Now, employing the calculated formula for the GMIR, the graded mean integration representation of $\widetilde{\mathrm{T}}$ is derived as

$$\mathcal{G}(\widetilde{\mathrm{T}}) = \int_0^{w_{\mathrm{B}}} \alpha S d\alpha \Big/ \int_0^{w_{\mathrm{B}}} \alpha d\alpha$$

$$= \frac{(\beta_1 + 4\beta + \beta_2)}{6} \tag{3.45}$$

3.5.1.3 Signed Distance Method

For any $a \in R$, the signed distance from a to 0 is defined as $d_0(a, 0) = a$. If a is positive, then the distance from a to 0 is $a = d_0(a, 0)$; if a is negative, the distance from a to 0 is $a = -d_0(a, 0)$. This is the reason why $d_0(a, 0)$ is referred as the distance from a to 0.

Assume that Ψ be the family of all fuzzy sets $\widetilde{\mathrm{B}}$ defined on R with which the α-*cut* $\mathrm{B}(\alpha) = [\mathrm{B}_L(\alpha), \mathrm{B}_R(\alpha)]$ exists for every $\alpha \in [0,1]$, and both $\mathrm{B}_L(\alpha)$, and $\mathrm{B}_R(\alpha)$ are continuous functions on $0 \leq \alpha \leq 1$. Then, for any $\widetilde{\mathrm{B}} \in \Psi$ from the principle of decomposition theory, we have

$$\widetilde{\mathrm{B}} = \cup_{0\leq\alpha\leq1}[\mathrm{B}_L(\alpha)_\alpha, \mathrm{B}_R(\alpha)_\alpha] \tag{3.46}$$

The signed distance of two end points $\mathrm{B}_L(\alpha)$, and $\mathrm{B}_R(\alpha)$ of the α-*cut* of $\widetilde{\mathrm{B}}$ (i.e. $\mathrm{B}(\alpha) = [\mathrm{B}_L(\alpha), \mathrm{B}_R(\alpha)]$) to the origin 0 is $d_0(\mathrm{B}_L(\alpha), 0) = \mathrm{B}_L(\alpha)$, and $d_0(\mathrm{B}_R(\alpha), 0) = \mathrm{B}_R(\alpha)$, respectively.

Definition. 7:

$$d_0([\mathrm{B}_L(\alpha), \mathrm{B}_R(\alpha)],0) = [d_0(\mathrm{B}_L(\alpha), 0) + d_0(\mathrm{B}_R(\alpha), 0)]/2 = [\mathrm{B}_L(\alpha) + \mathrm{B}_R(\alpha)]/2. \tag{3.47}$$

The following one-to-one mapping relationship between α-*level fuzzy interval* $[\mathrm{B}_L(\alpha)_\alpha, \mathrm{B}_R(\alpha)_\alpha]$, and the real interval $[\mathrm{B}_L(\alpha), \mathrm{B}_R(\alpha)]$ can be defined, that is

$$[\mathrm{B}_L(\alpha)_\alpha, \mathrm{B}_R(\alpha)_\alpha] \leftrightarrow [\mathrm{B}_L(\alpha), \mathrm{B}_R(\alpha)] \tag{3.48}$$

Because the 1-*level fuzzy point* $\tilde{0}_1$ has a one-to-one correspondence with the real number 0, the signed distance of $[\mathrm{B}_L(\alpha)_\alpha, \mathrm{B}_R(\alpha)_\alpha]$ to $\tilde{0}_1$can be give as:

$$d\big([\mathrm{B}_L(\alpha)_\alpha, \mathrm{B}_R(\alpha)_\alpha], \tilde{0}_1\big) = d_0([\mathrm{B}_L(\alpha), \mathrm{B}_R(\alpha)], 0) = [\mathrm{B}_L(\alpha) + \mathrm{B}_R(\alpha)]/2 \quad (3.49)$$

Furthermore, for $\tilde{\mathrm{B}} \in \Psi$, since the above function is continuous on $0 \leq \alpha \leq 1$, the integration can be applied to obtain the mean value of the signed distance as follows:

$$\int_0^1 d\big([\mathrm{B}_L(\alpha)_\alpha, \mathrm{B}_R(\alpha)_\alpha], \tilde{0}_1\big) d\alpha = \frac{1}{2}\int_0^1 [\mathrm{B}_L(\alpha) + \mathrm{B}_R(\alpha)] d\alpha \quad (3.50)$$

Definition. 8: For $\tilde{\mathrm{B}} \in \Psi$, the signed distance of $\tilde{\mathrm{B}}$ to $\tilde{0}$ can be defined as:

$$d(\tilde{\mathrm{B}}, \tilde{0}_1) = \int_0^1 d\big([\mathrm{B}_L(\alpha)_\alpha, \mathrm{B}_R(\alpha)_\alpha], \tilde{0}_1\big) d\alpha = \frac{1}{2}\int_0^1 [\mathrm{B}_L(\alpha) + \mathrm{B}_R(\alpha)] d\alpha \quad (3.51)$$

For the TFN $\tilde{\mathrm{B}} = (b_1, b_2, b_3)$, the signed distance from $\tilde{\mathrm{B}}$ to $\tilde{0}$ is given as:

$$d(\tilde{\mathrm{B}}, \tilde{0}_1) = \int_0^1 d\big([\mathrm{B}_L(\alpha)_\alpha, \mathrm{B}_R(\alpha)_\alpha], \tilde{0}_1\big) d\alpha = \frac{1}{4}(b_1 + 2b_2 + b_3) \quad (3.52)$$

For two fuzzy sets $\tilde{\mathrm{B}}, \tilde{\mathrm{E}} \in \Psi$ where $\tilde{\mathrm{B}} = \cup_{0\leq\alpha\leq1}[\mathrm{B}_L(\alpha)_\alpha, \mathrm{B}_R(\alpha)_\alpha]$ and $\tilde{\mathrm{E}} = \cup_{0\leq\alpha\leq1}[\mathrm{E}_L(\alpha)_\alpha, \mathrm{E}_R(\alpha)_\alpha]$, and $k \in R$, we have

$$\tilde{\mathrm{B}}(+)\tilde{\mathrm{E}} = \cup_{0\leq\alpha\leq1}[(\mathrm{B}_L(\alpha) + \mathrm{E}_L(\alpha))_\alpha, (\mathrm{B}_R(\alpha) + \mathrm{E}_R(\alpha))_\alpha], \quad (3.53)$$

$$\tilde{\mathrm{B}}(-)\tilde{\mathrm{E}} = \cup_{0\leq\alpha\leq1}[(\mathrm{B}_L(\alpha) - \mathrm{E}_R(\alpha))_\alpha, (\mathrm{B}_R(\alpha) - \mathrm{E}_L(\alpha))_\alpha], \quad (3.54)$$

$$\tilde{k}_1(\cdot)\tilde{\mathrm{B}} = \begin{cases} \cup_{0\leq\alpha\leq1}[(k\mathrm{B}_L(\alpha))_\alpha, (k\mathrm{B}_R(\alpha))_\alpha], & k > 0, \\ \cup_{0\leq\alpha\leq1}[(k\mathrm{B}_R(\alpha))_\alpha, (k\mathrm{B}_L(\alpha))_\alpha], & k < 0, \\ \tilde{0}_1, & k = 0. \end{cases} \quad (3.55)$$

For two fuzzy sets $\tilde{\mathrm{B}}, \tilde{\mathrm{E}} \in \Psi$ and $k \in R$,

$$d\big(\tilde{\mathrm{B}}(+)\tilde{\mathrm{E}}, \tilde{0}_1\big) = d\big(\tilde{\mathrm{B}}, \tilde{0}_1\big) + d(\tilde{\mathrm{E}}, \tilde{0}_1), \quad (3.56)$$

$$d\big(\tilde{\mathrm{B}}(-)\tilde{\mathrm{E}}, \tilde{0}_1\big) = d\big(\tilde{\mathrm{B}}, \tilde{0}_1\big) - d(\tilde{\mathrm{E}}, \tilde{0}_1), \quad (3.57)$$

$$d(\tilde{k}_1(\cdot)\tilde{\mathrm{B}}, \tilde{0}_1) = kd(\tilde{\mathrm{B}}, \tilde{0}_1). \quad (3.58)$$

3.5.2 Justification of the GMIR and SD methods

In this study, the GMIR and SD methods are used as defuzzification methods in the proposed systems. In the second suggested model, the performance of these methods is compared.

The preference for researchers has been shifting towards the signed distance and GMIR methods (Shekarian et al., 2014b). As the membership function does not change under fuzzy arithmetic operations, it is possible to evaluate the defuzzified value directly by graded mean integration method through arithmetic operations. It is more reasonable to discuss the grade of each point of support set of fuzzy number for representing the fuzzy number. GMIR method is effective in the sense that it grades as the degree of each point of support set of fuzzy number and it is possible to measure the degree of similarity between fuzzy numbers in terms of graded mean integration values (Mahata & Goswami, 2013). On the other hand, from the membership grade viewpoint, it will be efficient to defuzzify the fuzzy number by GMIR method instead of the centroid method (Mahata & Mahata, 2011).

Moreover, for example, as the centriod method uses the extension principle, it concludes more complex calculations, especially, when a fuzzy parameter appears in the denominator (e.g., Chang, 2004). Using the SD method and the GMIR method usually do not fail to reach in closed-form solutions. However, if the function contains many multiplicative and complex sentences, GMIR method performs better than the SD method and resolves mathematics cumbersome.

3.6 Optimization Methods

In this section, proposed methods are explained to solve the developed fuzzy models. However, the solvation of the models is discussed after developing the models.

3.6.1 Overview of the Previous Optimization Methods

In this section, optimization methods that are used in previous studies are categorized. These methods include techniques such as simple methods, mathematical theorem and techniques, ranking of fuzzy numbers, non-linear programing techniques, heuristic and meta-heuristic methods and simulation approaches.

All the methods and techniques are classified in Table 3.4 in Appendix D. The first suggested fuzzy models is solved and optimized using KKT method and Intermediate Value Theorem. The second model is optimized applying a suggested algorithm. This algorithm is explained later in chapter 5, section 5.6.

3.6.2 KKT Method

Kuhn-Tucker conditions or KKT conditions deal with the question of how to recognize an optimal solution for a non-linear programming problem (with differentiable functions). Kuhn-Tucker conditions are suitable for situations in which the optimum solution of a non-linear programming problem have to be solved subject to inequality constraints. Taha (1997) and Hillier and Lieberman (2001) have discussed the concept of KKT conditions. According to Hillier and Lieberman (2001), the basic result of KKT conditions is embodied in the following theorem:

Assume that $f(X)$, $g_1(X)$, $g_2(X)$, ... , $g_m(X)$ are differentiable functions satisfying certain regularity conditions. Then $X^* = (x_1^*, x_2^*, \dots, x_n^*)$ can be an optimal solution for the nonlinear programming problem only if there exist m numbers λ_1, λ_2, ... , λ_m such that all the following KKT conditions are satisfied:

$$\frac{\partial f}{x_j} - \sum_{i=1}^{m} \lambda_i \frac{\partial g_i}{\partial x_j} \leq 0, \qquad \text{at } x = x^* \text{ for } j = 1,2, \dots, n.$$

$$x_j^*\left(\frac{\partial f}{x_j}-\sum_{i=1}^{m}\lambda_i\frac{\partial g_i}{\partial x_j}\right)=0, \qquad \text{at } x=x^* \text{ for } j=1,2,\dots,n.$$

$$g_i(x^*)-b_i\le 0, \qquad for\ i=1,2,\dots,m.$$

$$\lambda_i(g_i(x^*)-b_i)=0, \qquad for\ i=1,2,\dots,m.$$

$$x_j^*\ge 0, \qquad for\ i=1,2,\dots,n.$$

$$\lambda_i\ge 0, \qquad for\ i=1,2,\dots,m.$$

3.6.3 Intermediate Value Theorem

The intermediate value theorem is applied for solving the second proposed fuzzy model through an algorithm. This theory is very useful to optimize the convex functions.

According to this theory, if f be a continuous function on the closed interval $[a,b]$ and if $f(a)f(b)<0$, then there exists a number $c\in(a,b)$ such that $f(c)=0$.

3.7 Characteristics of the Developed Models

Regarding the application of the model, inventory systems have different characteristics such as inflation, discount, screening, rework, delay, defective or deteriorating items. To depict and highlight the characteristics of the proposed fuzzy models, characteristics of the previous works in previous chapter are classified and identified. In the following, the important characteristics of the developed models are explained.

3.7.1 Learning Theory

The learning curve concept, originally presented by Wright (1936), is defined as a natural phenomenon that occurs when a worker performs a task repetitively. Investigating the effect of learning through inventory models has been the subject of many studies in recent years. Wright (1936) observed that in airplane assembly when the number of units produced increases, unit production costs reduce. In real production systems, as time goes by, the knowledge and experience of workers about operations and processes increase naturally and lead to improvements in their performance. For example, as a result of learning in a production system, the rate of manufacturing of defective items may reduce. The learning theory in its most popular form states that as the total quantity of units produced doubles, the cost per unit declines by some constant percentage (Jaber et al, 2008).

In this research, the well-known learning curve as power function and 'S'-shaped formulation is used. The earliest learning curve representation is a geometric progression that expresses the decreasing cost (or time) required to accomplish any repetitive operation (Jaber & El Saadany, 2011). It is formulated as below:

$$U_x = U_1 x^{-b} \tag{3.59}$$

where U_x is the time to produce the xth unit, U_1 the time to produce the first unit, x the production account, and b the learning curve exponent. In practice, parameter b is often replaced by another index which is called "learning rate" (LR). This index that is more intuitive occurs each time the production output is doubled. We have:

$$LR = \frac{U_{2x}}{U_x} = \frac{U_1(2x)^{-b}}{U_1 x^{-b}} = 2^{-b} \tag{3.60}$$

$$b = \frac{-\log(LR)}{\log(2)} \tag{3.61}$$

This type of learning curve that is depicted in Figure 3.7 is the most acceptable among practitioners and academicians.

The S-shaped logistic learning curve which is practical in real world is of the form:

$$p(n) = \frac{a}{g + e^{bn}} \tag{3.62}$$

where a, b, and g are the parameters of the model, n is the cumulative number of shipments, and $p(n)$ is the percentage defective per shipment n.

Different phases of S shaped learning curve are illustrated in Figure 3.8. Initial phase is the stage that the worker is getting acquainted with the set-up, the tooling, instructions, blueprints, the workplace arrangement, and the conditions of the process (Jaber et al., 2008). However, the progress in this step is slow. In the next phase, the learning process starts, and reduction in errors and improvement in the environment can be seen. The last phase (maturity) shows the levelling of the learning curve.

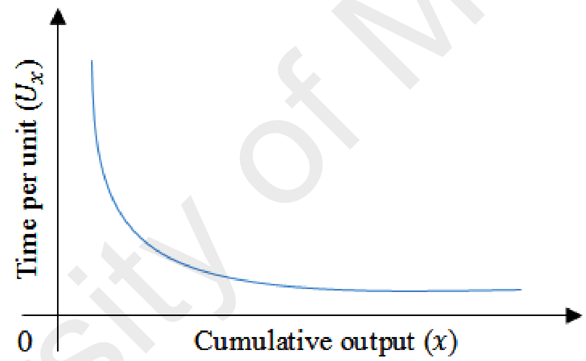

Figure 3.7: Wright's learning curve

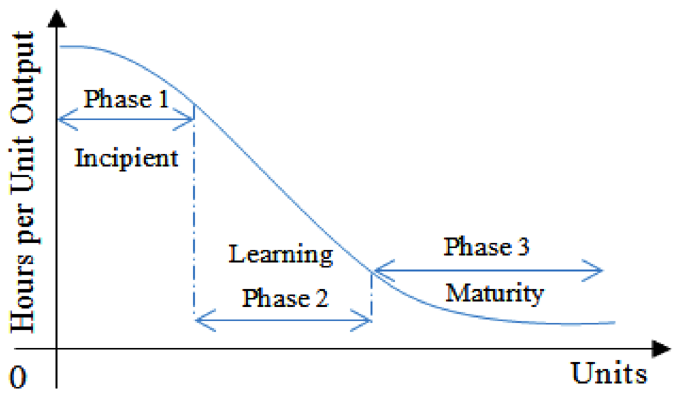

Figure 3.8: The three phases of the learning curve

3.7.2 Imperfect Quality Items

Imperfect items in the raw material and production stages of a supply chain directly impact the coordination of the product flow within a supply chain (Khan et al.,, 2011b). In response to this concern, inventory models have become an important and growing area of research and many kinds of problems have been discussed by researchers.

Adopting the assumption of perfect quality in the EOQ/EPQ models is unrealistic in most industrial applications. It can lead to errors in obtaining the optimal policies such as total cost, total profit, and order size. In the developed fuzzy models of this research, the concept of imperfect quality items is also used. It arises because of machine failure in production process or delivering imperfect items by the supplier.

3.7.3 Holding Cost

The holding cost includes rent for the required space, labor cost to operate the space, opportunity cost, equipment cost, materials cost, insurance and security, and other direct expenses (Wahab & Jaber, 2010). It should be noted that in inventory management, the annual interest rate, I, is computed based on the mentioned costs.

In the real manufacturing environment, the defective and good items are usually stored in the different warehouses. Therefore, the values of the annual interest rate for the good items and the defective items are different. Consequently, the holding cost of good and defective items are different (Paknejad et al., 2005).

3.7.4 Return Process

The process of return occurs in backward supply chain and reverse logistics (RL) when an item can be remanufactured or recycled, and then, can be sold to the customer in the market. Planning for RL is more difficult than forward logistics because of more uncertainty in terms of quantity, time and quality of returned product (Flapper, 1996). As it was cleared from the reviewed literature, there is a shortcoming of developing fuzzified model in this field. In the second proposed fuzzy model, uncertainty of return rate is dealt to fill the gap.

3.8 Chapter Summary

In this chapter, the methodology of this research was explained and illustrated by using some flowcharts. The framework of models was demonstrated. Methods and techniques that are necessary to develop the fuzzy models were discussed. The framework for two proposed fuzzy models was divided in forward and backward conditions.

Fundamental concepts and related theories were explained. Methods of fuzzification, defuzzification, and optimization were reviewed and the used ones were described. The use of the selected methods was also justified. In later chapters, the fuzzy models will be formulated.

CHAPTER 4: FULLY FUZZY FORWARD ECONOMIC ORDER QUANTITY MODEL CONSIDERING LEARNING EFFECTS

4.1 Introduction

In this chapter, a fully fuzzy economic order quantity model in which the number of defective items decreases under the effect of the learning process is formulated. Firstly, the formulation of the base model is reviewed, and then, it is extended through the fuzzy environment. The model is solved and optimized and the crisp results are compared with fuzzy ones. In addition, the results are compared with previous works. At the end of our discussions, some managerial insights are proposed.

4.2 Problem Description

In the following, the base problem is reviewed according to the previous studies (Salameh & Jaber, 2000; Maddah & Jaber, 2008; Wahab & Jaber, 2010). Later, the problem is extended from the crisp environment to the fuzzy one.

Assume a situation where a lot from the supplier is delivered to the manufacturer who places an order with a deterministic purchasing price and ordering cost. In addition, it is assumed that the demand of the manufacturer is a crisp value. Moreover, it is supposed that a part of each lot that is received by the manufacturer includes percentage defectives. The defective items are identified by a 100% screening process with a known rate. After separation of defective items from non-defective ones, both of them are kept in different warehouses with different holding costs. Segregation of these items helps to a better tracking cost as it happens in many industries. These items can be kept in different situation. It is reasonable that the holding cost of defective items is less than the holding cost of good ones. Before shipping a new lot, good and defective items will be sold with different selling prices such that selling price of good items is more than the defective ones. It is assumed that the percentage of defective items decreases under the influence of learning. It is a logical assumption because learning is an inherent part of the mentioned process.

4.2.1 Assumptions

The investigated model in fuzzy environment follows the below assumptions:

i. Demand rate is constant during the planning horizon
ii. Shortages are not allowed
iii. Lead time is zero
iv. Each shipment undergoes 100% inspection process
v. Defective items are sold at a discounted price
vi. Percentage of defectives items follows a learning curve
vii. Time horizon is infinite/finite
viii. All the parameters and variables of the model are assumed to be triangular fuzzy number
ix. It is assumed that percentage of defective items per shipment, follows the below S-shaped logistic learning curve model:

$$p(n) = \frac{\alpha}{\gamma + e^{\beta n}}$$

where α, β, and γ are the parameters of the learning function.

4.2.2 Notations

The following notations are used to formulate the model:

D Demand (unit/time)

y Lot size (unit/order) (Decision variable)

y_n Lot size in nth shipment when learning occurs (unit/order) (Decision variable)

c Variable cost ($/unit)

K	Fixed cost of placing an order ($/order)
$p(n)$	Percentage of defective items per shipment
$f(p)$	Probability density function of defective items
s	Selling price of good quality items ($/unit)
v	Selling price of defective items ($/unit) ($v < s$)
x	Screening rate (unit/time)
d	Screening cost ($/unit)
t_n	Screening time (time/unit)
T_n	Cycle length (time)
h_g	Holding cost for good items ($/unit/time)
h_d	Holding cost for defective items ($/unit/time) ($h_d < h_g$)
Δ_l	Lower bound of the triangular fuzzy number in parameters or variable
Δ_h	Upper bound of the triangular fuzzy number in parameter or variable
TPU	Total profit per unit time ($)

4.3 Model Formulation

The formulation of the models is as below according to the Figure 4.1.

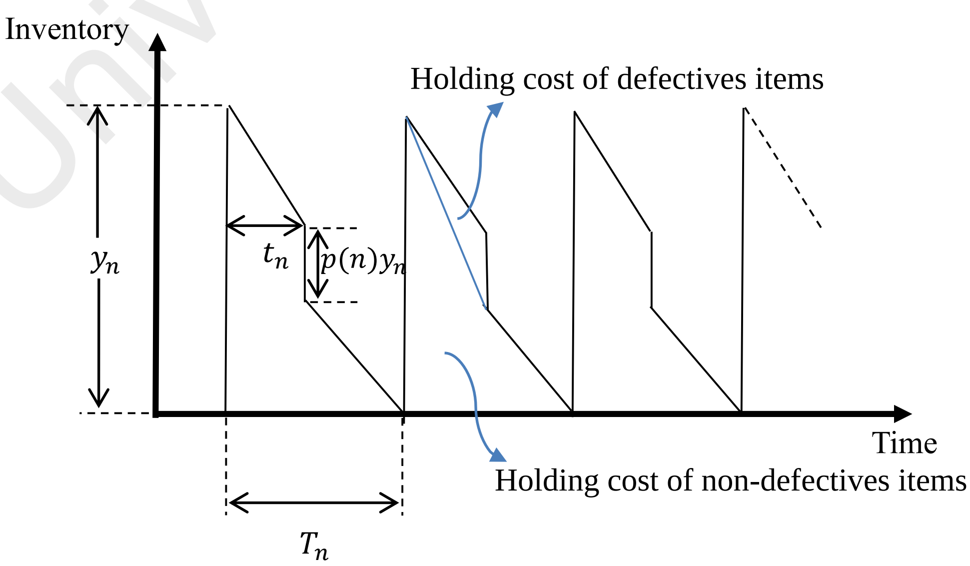

Figure 4.1: Inventory-time plot for an EOQ model with imperfect quality

According to the mentioned studies, assume that the number of good items in each lot size y_n can be given as below:

$$M\left(y_n, p(n)\right) = y_n\left(1 - p(n)\right) \tag{4.1}$$

Because shortage is not allowed, the number of good items should be at least equal to the demand when the screening process is doing and is given by:

$$M\left(y_n, p(n)\right) \geq Dt_n \tag{4.2}$$

If t_n is replace by y_n/x, the below condition can be reached:

$$E[p(n)] \leq 1 - D/x \tag{4.3}$$

where $p(n)$ is a random variable and $D \leq x$. The total revenue can be defined as follow:

$$TR\left(y_n\right) = sy_n\left(1 - p(n)\right) + vy_n p(n) \tag{4.4}$$

As defective and non-defective items are kept separately, holding cost of defective and non-defective items according to the Figure 4.1 can be calculated as below respectively:

$$H_d\left(y_n\right) = h_d\left(\frac{p(n){y_n}^2}{2x}\right) \tag{4.5}$$

$$H_g\left(y_n\right) = h_g\left(\frac{p(n){y_n}^2}{2x} + \frac{y_n\left(1 - p(n)\right)T_n}{2}\right) \tag{4.6}$$

In fact, these equations are the area of the identified places in Figure 4.1.

Regarding the defined notations, the total cost can be given as below:

$$TC\left(y_n\right) = h_g\left(\frac{p(n){y_n}^2}{2x} + \frac{y_n\left(1 - p(n)\right)T_n}{2}\right) + h_d\left(\frac{p(n){y_n}^2}{2x}\right) + K + y(c + d) \tag{4.7}$$

Therefore, the total profit is as:

$$\begin{aligned} TP\left(y_n\right) = sy_n\left(1 - p(n)\right) + vy_n p(n) \\ - \left[h_g\left(\frac{p(n){y_n}^2}{2x} + \frac{y_n\left(1 - p(n)\right)T_n}{2}\right) + h_d\left(\frac{p(n){y_n}^2}{2x}\right) + K \right. \\ \left. + y(c + d)\right] \end{aligned} \tag{4.8}$$

Then, the total profit per unit time, $TPU\left(y_n\right)$ is determined by:

$$TPU(y_n) = \left(\frac{D}{1-p(n)}\right)\left(v - \frac{K}{y_n} - c - d - y_n\frac{(h_g + h_d)}{2x}\right) - \frac{h_g y_n(1-p(n))}{2} + D\left(s - v + y_n\frac{(h_g + h_d)}{2x}\right) \tag{4.9}$$

$$\boldsymbol{E}\left(TPU(y_n)\right) = \boldsymbol{E}\left(\frac{1}{1-p(n)}\right)D\left(v - \frac{K}{y_n} - c - d - y_n\frac{(h_g + h_d)}{2x}\right) - \frac{h_g y_n\left(1-\boldsymbol{E}(p(n))\right)}{2} + D\left(s - v + y_n\frac{(h_g + h_d)}{2x}\right) \tag{4.10}$$

$\boldsymbol{E}\left(TPU(y_n)\right)$ is concave in y_n and thus by derivation we have:

$$y_n^* = \left(\frac{2DK\boldsymbol{E}[1/(1-p(n)]}{h_g(1-\boldsymbol{E}[p(n)]) + \left(\frac{D}{x}\right)(h_g + h_d)\boldsymbol{E}\left[\frac{1}{1-P(n)}\right] - \left(\frac{D}{x}\right)(h_g + h_d)}\right)^{\frac{1}{2}} \tag{4.11}$$

The cycle length T_n depends on the number of defective items which is a random variable. Therefore, the expected cycle length is:

$$\boldsymbol{E}[T_n] = \frac{y_n(1-\boldsymbol{E}[p(n)])}{D} \tag{4.12}$$

Now we have

$$\boldsymbol{E}[TP(y_n)] = sy_n(1-\boldsymbol{E}[p(n)]) + vy_n\boldsymbol{E}[p(n)] - \left[K + (c+d)y_n + h_{\boldsymbol{g}}\left(\frac{{y_n}^2}{2}\left[\frac{\boldsymbol{E}[p(n)]}{x} + \frac{\boldsymbol{E}\left[(1-p(n))^{\boldsymbol{2}}\right]}{D}\right]\right) + h_{\boldsymbol{d}}\left(\frac{\boldsymbol{E}[p(n)]{y_n}^2}{2x}\right)\right] \tag{4.13}$$

Using renewal-reward theorem, we can have:

$$\boldsymbol{E}[TPU(y_n)] = \frac{\boldsymbol{E}[TP(y_n)]}{\boldsymbol{E}[T_n]} \tag{4.14}$$

$$\boldsymbol{E}\left[TPU\left(y_n\right)\right] = \frac{D\{s(1-\boldsymbol{E}[p(n)]) + v\boldsymbol{E}[p(n)] - c - d\}}{(1-E[p(n)])} - \frac{KD}{y(1-E[p(n)]}$$
$$- \frac{yD}{2(1-\boldsymbol{E}[p(n)])}\left[h_g\left(\frac{\boldsymbol{E}[p(n)]}{x} + \frac{\boldsymbol{E}\left[\left(1-p(n)\right)^2\right]}{D}\right)\right.$$
$$\left. + h_d\left(\frac{\boldsymbol{E}[p(n)]}{x}\right)\right] \tag{4.15}$$

By derivation, we can have:

$$y_n^{**} = \left(\frac{2DK}{h_g\boldsymbol{E}\left[\left(1-p(n)\right)^2\right] + \left(\frac{D}{x}\right)\left(h_g + h_d\right)\boldsymbol{E}[p(n)]}\right)^{\frac{1}{2}} \tag{4.16}$$

If the S-shaped logistic learning function is replaced:

$$\frac{\alpha}{\gamma + e^{\beta n}}$$

in Eq. (4.9), the optimal y_n in crisp situation can be obtained by derivation as below:

$$y_n^{***} = \left(\frac{2DK\left[\frac{1}{1-p(n)}\right]}{h_g[1-p(n)] + \left(\frac{D}{x}\right)\left(h_g + h_d\right)\left(\frac{1}{[1-p(n)]}\right) - \left(\frac{D}{x}\right)\left(h_g + h_d\right)}\right)^{\frac{1}{2}} \tag{4.17}$$

4.4 Fully Fuzzy Model

In this section, the crisp model illustrated in previous section is studied in a fully fuzzy environment by fuzzifying all the input parameters and decision variable. It is assumed that all the model is uncertain. The uncertainty of the considered parameters and variable are represented by the concept of the triangular fuzzy number, as explained in chapter 3.

The fuzzy parameters of the model are defined as below:

$\widetilde{D} = (D - \Delta_l^D, D, D + \Delta_h^D)$

$\tilde{s} = (s - \Delta_l^s, s, s + \Delta_h^s)$

$$\tilde{v} = (v - \Delta_l^v, v, v + \Delta_h^v)$$

$$\tilde{h}_g = \left(h_g - \Delta_l^{h_g}, h_g, h_g + \Delta_h^{h_g}\right)$$

$$\tilde{h}_d = \left(h_d - \Delta_l^{h_d}, h_d, h_d + \Delta_h^{h_d}\right)$$

$$\tilde{p}(\mathrm{n}) = \left(p(n) - \Delta_l^{p(n)}, p(n), p(n) + \Delta_h^{p(n)}\right)$$

$$\tilde{x} = (x - \Delta_l^x, x, x + \Delta_h^x)$$

$$\tilde{K} = (K - \Delta_l^K, K, K + \Delta_h^K)$$

$$\tilde{c} = (c - \Delta_l^c, c, c + \Delta_h^c)$$

$$\tilde{d} = \left(d - \Delta_l^d, d, d + \Delta_h^d\right)$$

Moreover, the decision variable is supposed to be a triangular fuzzy number and is given by:

$$\tilde{y}_n = \left(y_n - \Delta_l^{y_n}, y_n, y_n + \Delta_h^{y_n}\right)$$

Besides, we have:

$$\Delta_l^i > 0 \qquad \text{for } i = D, s, v, h_g, h_d, p(n), x, K, c, d, y_n,$$

$$\Delta_h^j > 0 \qquad \text{for } j = D, s, v, h_g, h_d, p(n), x, K, c, d, y_n,$$

$$D > \Delta_l^D \qquad s > \Delta_l^s \qquad v > \Delta_l^v \qquad h_g > \Delta_l^{h_g} \qquad h_d > \Delta_l^{h_d} \qquad p(n) > \Delta_l^{p(n)}$$

$$K > \Delta_l^K \qquad c > \Delta_l^c \qquad d > \Delta_l^d \qquad y_n > \Delta_l^{y_n} \qquad x > \Delta_l^x$$

Moreover, Δ_l^i and Δ_h^j are arbitrary values which can be determined through expert's knowledge and statistical background of the inventory data. For example, the upper bound of the demand in its triangular fuzzy number can be considered as the highest value which is observed for the demand in previous inventory planning horizons and, therefore, Δ_h^D is the deviation of the highest value of the demand from the most promising value. After replacing the above triangular fuzzy numbers in the total profit per unit time formula, the fuzzy total profit per unit time can be obtained as follows:

$$\widetilde{TPU}(\tilde{y}_n) = \tilde{D}\left(\tilde{s} - \tilde{v} + \frac{\tilde{h}_g \tilde{y}_n}{2\tilde{x}} + \frac{\tilde{h}_d \tilde{y}_n}{2\tilde{x}}\right) + \left(\frac{\tilde{D}}{1 - \tilde{p}(\tilde{n})}\right)\left(\tilde{v} - \frac{\tilde{K}}{\tilde{y}_n} - \tilde{c} - \tilde{d} - \frac{\tilde{h}_g \tilde{y}_n}{2\tilde{x}} - \frac{\tilde{h}_d \tilde{y}_n}{2\tilde{x}}\right) - \frac{\tilde{h}_g \tilde{y}_n (1 - \tilde{p}(\tilde{n}))}{2} \tag{4.18}$$

In order to simplify the mathematical computations, the following symbols are defined:

$$\zeta = \tilde{D}\left(\tilde{s} - \tilde{v} + \frac{\tilde{h}_g \tilde{y}_n}{2\tilde{x}} + \frac{\tilde{h}_d \tilde{y}_n}{2\tilde{x}}\right) \tag{4.19}$$

$$\xi = \left(\tilde{v} - \frac{\tilde{K}}{\tilde{y}_n} - \tilde{c} - \tilde{d} - \frac{\tilde{h}_g \tilde{y}_n}{2\tilde{x}} - \frac{\tilde{h}_d \tilde{y}_n}{2\tilde{x}}\right) \tag{4.20}$$

$$\varsigma = \left(\frac{\tilde{D}}{1 - \tilde{p}(\tilde{n})}\right) \tag{4.21}$$

$$\tau = \frac{\tilde{h}_g \tilde{y}_n (1 - \tilde{p}(\tilde{n}))}{2} \tag{4.22}$$

Since the whole parameters are triangular fuzzy numbers, each of the defined symbols is a triangular fuzzy number. Hence, they can be considered as a triangular fuzzy number by the following triplet components:

$$\tilde{\zeta} = (\zeta_1, \zeta_2, \zeta_3) \tag{4.23}$$

$$\xi = (\xi_1, \xi_2, \xi_3) \tag{4.24}$$

$$\varsigma = (\varsigma_1, \varsigma_2, \varsigma_3) \tag{4.25}$$

$$\tau = (\tau_1, \tau_2, \tau_3) \tag{4.26}$$

Using the function principle method described in chapter 3, the process of computations is as below:

$$\zeta_1 = (\mathrm{D} - \Delta_l^D).\left[[(s - \Delta_l^s) - (v + \Delta_h^v)] + \frac{\left(h_g - \Delta_l^{h_g}\right)\left(y_n - \Delta_l^{y_n}\right) + \left(h_d - \Delta_l^{h_d}\right)\left(y_n - \Delta_l^{y_n}\right)}{2(x + \Delta_h^x)}\right] \tag{4.27}$$

$$\zeta_2 = D\left[(s - v) + \frac{h_g y_n}{2x} + \frac{h_d y_n}{2x}\right] \tag{4.28}$$

$$\zeta_3 = (\mathrm{D} + \Delta_h^D) .\left[[(s + \Delta_h^s) - (v - \Delta_l^v)] + \frac{\left(h_g + \Delta_h^{h_g}\right)\left(y_n + \Delta_h^{y_n}\right) + \left(h_d + \Delta_h^{h_d}\right)\left(y_n + \Delta_h^{y_n}\right)}{2(x - \Delta_l^x)}\right] \tag{4.29}$$

$$\xi_1 = \left[(v - \Delta_l^v) - \frac{(K + \Delta_h^K)}{\left(y_n - \Delta_l^{y_n}\right)} - (c + \Delta_h^c) - \left(d + \Delta_h^d\right) - \frac{\left(h_g + \Delta_h^{h_g}\right)\left(y_n + \Delta_h^{y_n}\right) + \left(h_d + \Delta_h^{h_d}\right)\left(y_n + \Delta_h^{y_n}\right)}{2(x - \Delta_l^x)}\right] \tag{4.30}$$

$$\xi_2 = \left(v - \frac{K}{y_n} - c - d - \frac{h_g y_n}{2x} - \frac{h_d y_n}{2x}\right) \tag{4.31}$$

$$\xi_3 = \left[(v + \Delta_h^v) - \frac{(K - \Delta_l^K)}{\left(y_n + \Delta_h^{y_n}\right)} - (c - \Delta_l^c) - \left(d - \Delta_l^d\right) - \frac{\left(h_g - \Delta_l^{h_g}\right)\left(y_n - \Delta_l^{y_n}\right) + \left(h_d - \Delta_l^{h_d}\right)\left(y_n - \Delta_l^{y_n}\right)}{2(x + \Delta_h^x)}\right] \tag{4.32}$$

$$\varsigma_1 = (D - \Delta_l^D)\left[\frac{\left(\gamma + \Delta_h^\gamma\right) + e^{\left(n+\Delta_h^n\right)\left(\beta+\Delta_h^\beta\right)}}{\left(\gamma + \Delta_h^\gamma\right) - (\alpha - \Delta_l^\alpha) + e^{\left(n+\Delta_h^n\right)\left(\beta+\Delta_h^\beta\right)}}\right] \tag{4.33}$$

$$\varsigma_2 = \left(\frac{\gamma + e^{n\beta}}{\gamma + e^{n\beta} - \alpha}\right)D \tag{4.34}$$

$$\varsigma_3 = (D + \Delta_h^D)\left[\frac{\left(\gamma - \Delta_l^\gamma\right) + e^{\left(n-\Delta_l^n\right)\left(\beta-\Delta_l^\beta\right)}}{\left(\gamma - \Delta_l^\gamma\right) - (\alpha + \Delta_h^\alpha) + e^{\left(n-\Delta_l^n\right)\left(\beta-\Delta_l^\beta\right)}}\right] \tag{4.35}$$

$$\tau_1 = \left[\frac{\left(h_g - \Delta_l^{h_g}\right)\left(y_n - \Delta_l^{y_n}\right)}{2}\right]\left[\frac{\left(\gamma - \Delta_l^\gamma\right) - (\alpha + \Delta_h^\alpha) + e^{\left(n-\Delta_l^n\right)\left(\beta-\Delta_l^\beta\right)}}{\left(\gamma - \Delta_l^\gamma\right) + e^{\left(n-\Delta_l^n\right)\left(\beta-\Delta_l^\beta\right)}}\right] \tag{4.36}$$

$$\tau_2 = \left(\frac{h_g y_n}{2}\right)\left(\frac{\gamma - \alpha + e^{n\beta}}{\gamma + e^{n\beta}}\right) \tag{4.37}$$

$$\tau_3 = \left[\frac{\left(h_g + \Delta_h^{h_g}\right)\left(y_n + \Delta_h^{y_n}\right)}{2}\right]\left[\frac{\left(\gamma + \Delta_h^\gamma\right) - (\alpha - \Delta_l^\alpha) + e^{\left(n+\Delta_h^n\right)\left(\beta+\Delta_h^\beta\right)}}{\left(\gamma + \Delta_h^\gamma\right) + e^{\left(n+\Delta_h^n\right)\left(\beta+\Delta_h^\beta\right)}}\right] \tag{4.38}$$

Again, because all of the parameters and variable of the inventory system are triangular fuzzy numbers, the total profit per unit time is also a triangular fuzzy number and could be computed as below:

$$\widetilde{TPU}(\tilde{y}_n) = (C_1, C_2, C_3) \tag{4.39}$$

Using the defined triplet components, we have:

$$(C_1, C_2, C_3) = (\zeta_1 + \xi_1\varsigma_1 - \tau_3, \zeta_2 + \xi_2\varsigma_2 - \tau_2, \zeta_3 + \xi_3\varsigma_3 - \tau_1) \tag{4.40}$$

The GMIV (defuzzified value) of total profit function could be expressed as follow:

$$\begin{aligned}\varphi\left(\widetilde{TPU}(\tilde{y}_n)\right) &= \frac{1}{6}C_1 + \frac{4}{6}C_2 + \frac{1}{6}C_3 \\ &= \frac{1}{6}(\zeta_1 + \xi_1\varsigma_1 - \tau_3) + \frac{4}{6}(\zeta_2 + \xi_2\varsigma_2 - \tau_2) + \frac{1}{6}(\zeta_3 + \xi_3\varsigma_3 - \tau_1)\end{aligned} \tag{4.41}$$

By replacing the obtained terms for above symbols, it could be described as below:

$$
\begin{aligned}
&\varphi\left(\widetilde{TPU}(\tilde{y}_n)\right) \\
&= \frac{1}{6}\Bigg((\mathrm{D} - \Delta_l^D)\Bigg[[(s - \Delta_l^s) - (v + \Delta_h^v)] \\
&+ \frac{\left(h_g - \Delta_l^{h_g}\right)(y_n - \Delta_l^{y_n}) + (h_d - \Delta_l^{h_d})(y_n - \Delta_l^{y_n})}{2(x + \Delta_h^x)}\Bigg] \\
&+ (\mathrm{D} - \Delta_l^D)\left[\frac{\left(\gamma + \Delta_h^\gamma\right) + e^{(n+\Delta_h^n)\left(\beta+\Delta_h^\beta\right)}}{\left(\gamma + \Delta_h^\gamma\right) - (\alpha - \Delta_l^\alpha) + e^{(n+\Delta_h^n)\left(\beta+\Delta_h^\beta\right)}}\right] \cdot \Bigg[(v - \Delta_l^v) - \frac{(K + \Delta_h^K)}{\left(y_n - \Delta_l^{y_n}\right)} \\
&- (c + \Delta_h^c) - (d + \Delta_h^D) - \frac{\left(h_g + \Delta_h^{h_g}\right)(y_n + \Delta_h^{y_n}) + (h_d + \Delta_h^{h_d})(y_n + \Delta_h^{y_n})}{2(x - \Delta_l^x)}\Bigg] \\
&- \left[\frac{\left(h_g + \Delta_h^{h_g}\right)(y_n + \Delta_h^{y_n})}{2}\right]\left[\frac{\left(\gamma + \Delta_h^\gamma\right) - (\alpha - \Delta_l^\alpha) + e^{(n+\Delta_h^n)\left(\beta+\Delta_h^\beta\right)}}{\left(\gamma + \Delta_h^\gamma\right) + e^{(n+\Delta_h^n)\left(\beta+\Delta_h^\beta\right)}}\right]\Bigg) \\
&+ \frac{4}{6}\Bigg(D\left[(s - v) + \frac{h_g y_n}{2x} + \frac{h_d y_n}{2x}\right] \\
&+ D\left(\frac{\gamma + e^{n\beta}}{\gamma + e^{n\beta} - \alpha}\right)\left(v - \frac{K}{y_n} - c - d - \frac{h_g y_n}{2x} + \frac{h_d y_n}{2x}\right) \\
&- \left(\frac{h_g y_n}{2}\right)\left(\frac{\gamma - \alpha + e^{n\beta}}{\gamma + e^{n\beta}}\right)\Bigg) \\
&+ \frac{1}{6}\Bigg((D + \Delta_h^D)\Bigg[[(s + \Delta_h^s) - (v - \Delta_l^v)] \\
&+ \frac{\left(h_g + \Delta_h^{h_g}\right)(y_n + \Delta_h^{y_n}) + (h_d + \Delta_h^{h_d})(y_n + \Delta_h^{y_n})}{2(x - \Delta_l^x)}\Bigg] \\
&+ (D + \Delta_h^D)\left[\frac{\left(\gamma - \Delta_l^\gamma\right) + e^{(n-\Delta_l^n)\left(\beta-\Delta_l^\beta\right)}}{\left(\gamma - \Delta_l^\gamma\right) - (\alpha + \Delta_h^\alpha) + e^{(n-\Delta_l^n)\left(\beta-\Delta_l^\beta\right)}}\right]\Bigg[(v + \Delta_h^v) - \frac{(K - \Delta_l^K)}{\left(y_n + \Delta_h^{y_n}\right)} \\
&- (c - \Delta_l^c) - \left(d - \Delta_l^d\right) - \frac{\left(h_g - \Delta_l^{h_g}\right)(y_n - \Delta_l^{y_n}) + (h_d - \Delta_l^{h_d})(y_n - \Delta_l^{y_n})}{2(x + \Delta_h^x)}\Bigg] \\
&- \left[\frac{\left(h_g - \Delta_l^{h_g}\right)(y_n - \Delta_l^{y_n})}{2}\right]\left[\frac{\left(\gamma - \Delta_l^\gamma\right) - (\alpha + \Delta_h^\alpha) + e^{(n-\Delta_l^n)\left(\beta-\Delta_l^\beta\right)}}{\left(\gamma - \Delta_l^\gamma\right) + e^{(n-\Delta_l^n)\left(\beta-\Delta_l^\beta\right)}}\right]\Bigg)
\end{aligned}
\tag{4.42}
$$

4.5 Finding Optimal Values

In this section, the model is tried to be solved using the KKT theorem described in chapter 3. The conditions of this theory are provided and the related constraints are constructed.

Let assume the following equations to initialize the process of finding the optimal solution for the defuzzified inventory cost function that was obtained in previous section:

$$y_n - \Delta_l^{y_n} = y_1 \tag{4.43}$$

$$y_n = y_2 \tag{4.44}$$

$$y_n + \Delta_h^{y_n} = y_3 \tag{4.45}$$

The defuzzified value of the total profit function is taken as the crisp estimate of fuzzy total profit function where it is optimized subject to the following condition:

$$0 < y_1 \leq y_2 \leq y_3 \tag{4.46}$$

Thus, the optimum solution of defuzzified total profit function can be found by optimizing Eq. (4.42) subject to the following inequality constraints:

$$y_1 - y_2 \leq 0 \tag{4.47}$$

$$y_2 - y_3 \leq 0 \tag{4.48}$$

$$-y_1 < 0 \tag{4.49}$$

Now the Kuhn-Tucker conditions can be employed to find the optimal solution of defuzzified total profit function subject to the below inequalities as imposed conditions.

$$y_1 \left(\frac{\partial \widetilde{TPU}(\tilde{y}_n)}{\partial y_1} - \lambda_1 + \lambda_3 \right) = 0 \tag{4.50}$$

$$y_2\left(\frac{\partial \widetilde{TPU}(\tilde{y}_n)}{\partial y_2} + \lambda_1 - \lambda_2\right) = 0 \tag{4.51}$$

$$y_3\left(\frac{\partial \widetilde{TPU}(\tilde{y}_n)}{\partial y_3} + \lambda_2\right) = 0 \tag{4.52}$$

$$y_1 - y_2 \leq 0 \tag{4.53}$$

$$y_2 - y_3 \leq 0 \tag{4.54}$$

$$-y_1 < 0 \tag{4.55}$$

$$\lambda_1(y_1 - y_2) = 0 \tag{4.56}$$

$$\lambda_2(y_2 - y_3) = 0 \tag{4.57}$$

$$\lambda_3(-y_1) = 0 \tag{4.58}$$

$$y_1 \geq 0, y_2 \geq 0, y_3 \geq 0 \tag{4.59}$$

$$\lambda_1 \geq 0, \lambda_2 \geq 0, \lambda_3 \geq 0 \tag{4.60}$$

where

$$\frac{\partial \widetilde{TPU}(\tilde{y}_n)}{\partial y_1} = \frac{1}{6}\left[(D - \Delta_l^D)\left(A + \frac{(K + \Delta_h^K)}{{y_1}^2}B\right) - A(D + \Delta_h^D)C - \frac{\left(h_g - \Delta_l^{h_g}\right)}{2C}\right] \tag{4.61}$$

$$\frac{\partial \widetilde{TPU}(\tilde{y}_n)}{\partial y_2} = \frac{4}{6}\left[DF + \left(\frac{K}{{y_2}^2} - F\right)DG - \frac{h_g}{2G}\right] \tag{4.62}$$

$$\frac{\partial \widetilde{TPU}(\tilde{y}_n)}{\partial y_3} = \frac{1}{6}\left[\frac{\left(h_g + \Delta_h^{h_g}\right)}{2B} - (D - \Delta_l^D)BE + (D + \Delta_h^D)\left(E + \frac{(K - \Delta_l^K)}{{y_3}^2}C\right)\right] \tag{4.63}$$

The notations of the A, B, C, E, F, and G are given as below:

$$A = \frac{\left(h_g - \Delta_l^{h_g}\right) + \left(h_d - \Delta_l^{h_d}\right)}{2(x + \Delta_h^x)} \tag{4.64}$$

$$B = \frac{\left(\gamma + \Delta_h^\gamma\right) + e^{\left(n+\Delta_h^n\right)\left(\beta+\Delta_h^\beta\right)}}{\left(\gamma + \Delta_h^\gamma\right) - \left(\alpha - \Delta_l^\alpha\right) + e^{\left(n+\Delta_h^n\right)\left(\beta+\Delta_h^\beta\right)}} \tag{4.65}$$

$$C = \frac{\left(\gamma - \Delta_l^\gamma\right) + e^{\left(n-\Delta_l^n\right)\left(\beta-\Delta_l^\beta\right)}}{\left(\gamma - \Delta_l^\gamma\right) - \left(\alpha + \Delta_h^\alpha\right) + e^{\left(n-\Delta_l^n\right)\left(\beta-\Delta_l^\beta\right)}} \tag{4.66}$$

$$E = \frac{\left(h_g + \Delta_h^{h_g}\right) + \left(h_d + \Delta_h^{h_d}\right)}{2(x - \Delta_l^x)} \tag{4.67}$$

$$F = \frac{h_g + h_d}{2x} \tag{4.68}$$

$$G = \frac{\gamma + e^{n\beta}}{\gamma + e^{n\beta} - \alpha} \tag{4.69}$$

From constraints (4.55) and (4.58), it can be deduced that $\lambda_3 = 0$. If $\lambda_1 = \lambda_2 = 0$ in (4.56) and (4.57), then $y_3 < y_2 < y_1$ and this constraint is against the constraint $0 < y_1 \leq y_2 \leq y_3$. Therefore, $y_1 = y_2$ and $y_2 = y_3$, mean that $y_1 = y_2 = y_3 = y^*$. According to this explanation, the solution of the model can be obtained by solving Eqs. (4.50)−(4.60), as follows:

$$\frac{\partial \widetilde{TPU}(\tilde{y}_n)}{\partial y_1} + \frac{\partial \widetilde{TPU}(\tilde{y}_n)}{\partial y_2} + \frac{\partial \widetilde{TPU}(\tilde{y}_n)}{\partial y_3} = 0 \tag{4.70}$$

By taking the derivations and replacing them in Eq. (4.70), the results could be given as follow:

$$y^2\left[A(D+\Delta_h^D)C-(D-\Delta_l^D)A+\frac{\left(h_g-\Delta_l^{h_g}\right)}{2C}-\frac{\left(h_g+\Delta_h^{h_g}\right)}{2B}+(D-\Delta_l^D)BE\right.$$

$$\left.-(D+\Delta_h^D)E-4DF+4FDG+2\frac{h_g}{G}\right]$$

$$=(D-\Delta_l^D)(K+\Delta_h^K)B+(D+\Delta_h^D)(K-\Delta_l^K)C+4DKG \tag{4.71}$$

The solution of the Eq. (4.65) leads to the solution of the model, as follows:

$$y=\sqrt{\frac{\chi_1}{\chi_2}} \tag{4.72}$$

where

$$\chi_1=(D-\Delta_l^D)(K+\Delta_h^K)B+(D+\Delta_h^D)(K-\Delta_l^K)C+4KDG \tag{4.73}$$

$$\chi_2=(D+\Delta_h^D)(AC-E)+(D-\Delta_l^D)(BE-\mathrm{A})+4FD(G-1)+\frac{\left(h_g-\Delta_l^{h_g}\right)}{2C}-\frac{\left(h_g+\Delta_h^{h_g}\right)}{2B}+2\frac{h_g}{G} \tag{4.74}$$

4.6 Numerical Illustrations

In this section, a numerical example is presented to illustrate the application of the suggested fuzzy model. However, the fuzzy model through a real case is discussed later. The relevant information for this example is assumed as $x=170{,}000$ units per year, $D=50{,}000$ units per year, $K=\$3{,}000$ per order, $c=\$100$ per unit, $s=\$200$ per unit, $v=\$50$ per unit, $d=\$0.50$ per unit, $h_g=\$20$ unit per year, $h_d=\$5$ unit per year, $\gamma=819.76$, $\alpha=70.07$, and $\beta=0.79$. In Table 4.1, the mentioned input parameters are fuzzified with arbitrary values. Columns φ and V show defuzzified values and the level of fuzziness (the percentage of difference between crisp and defuzzified values) of the related parameters, respectively. The GMIV method is applied for defuzzification of arbitrary levels of fuzziness.

Table 4.1: Arbitrary fuzzy triangular values for the input parameters

V	Parameter		φ	Parameter		φ
+70		(46,50,264)	85		(2800,3000,15800)	5100
+60		(34,50,246)	80		(2600,3000,14200)	4800
+50		(27,50,223)	75		(2400,3000,12600)	4500
+40	υ	(19,50,201)	70	K	(2000,3000,11200)	4200
+30		(16,50,174)	65		(1900,3000,9500)	3900
+20		(10,50,150)	60		(1600,3000,8000)	3600
+10		(4,50,126)	55		(1300,3000,6500)	3300
+70		(17,20,107)	34		(40000,50000,270000)	85000
+60		(13,20,99)	32		(30000,50000,250000)	80000
+50		(11,20,89)	30		(20000,50000,230000)	75000
+40	h_g	(9,20,79)	28	D	(19000,50000,201000)	70000
+30		(7,20,69)	26		(17500,50000,172500)	65000
+20		(6,20,58)	24		(15000,50000,145000)	60000
+10		(4,20,48)	22		(10000,50000,120000)	55000
+70		(4.55,5,26.45)	8.5		(92.78,100,620)	170
+60		(58.25,5,23.91)	8		(84,100,560)	160
+50		(41.18,5,21.32)	7.5		(71.65,100,500)	150
+40	h_d	(34.94,5,18.46)	7	c	(53.25,100,440)	140
+30		(30.47,5,15.90)	6.5		(40.5,100,380)	130
+20		(25.99,5,13.32)	6		(36,100,320)	120
+10		(19.32,5,11.90)	5.5		(16,100,260)	110
+70		(197,200,1043)	340		(160000,170000,894000)	289000
+60		(165,200,955)	320		(155000,170000,797000)	272000
+50		(139,200,861)	300		(150000,170000,700000)	255000
+40	s	(119,200,761)	280	x	(135000,170000,613000)	238000
+30		(67,200,693)	260		(115000,170000,531000)	221000
+20		(36,200,604)	240		(100000,170000,444000)	204000
+10		(20,200,500)	220		(80000,170000,362000)	187000
+70		(0.46,0.5,2.64)	0.85		(65.25,70.07,369.19)	119.12
+60		(0.41,0.5,2.39)	0.8		(58.25,70.07,334.13)	112.11
+50		(0.38,0.5,2.12)	0.75		(41.18,70.07,309.2)	105.11
+40	d	(0.29,0.5,1.91)	0.7	α	(34.94,70.07,273.38)	98.1
+30		(0.20,0.5,1.70)	0.65		(30.47,70.07,235.79)	91.09
+20		(0.15,0.5,1.45)	0.6		(25.99,70.07,198.21)	84.08
+10		(0.06,0.5,1.24)	0.55		(19.32,70.07,162.88)	77.08
+70		(0.71,0.79,4.17)	1.34		(800.32,819.76,4282.24)	1393.6
+60		(0.64,0.79,3.76)	1.26		(760,819.76,3830.68)	1311.62
+50		(0.60,0.79,3.38)	1.19		(710.9,819.76,3387.9)	1229.64
+40	β	(0.52,0.79,2.98)	1.11	γ	(650,819.76,2956.92)	1147.66
+30		(0.45,0.79,2.57)	1.03		(540,819.76,2521.1)	1056.69
+20		(0.38,0.79,2.16)	0.95		(490,819.76,2133.22)	983.71
+10		(0.27,0.79,1.79)	0.87		(380,819.76,1751.4)	901.74

Table 4.2 in Appendix E presents the results for different values of n. As a full fuzzy model is considered here, n is also fuzzified by the GMIV method. Columns 3 and 4 show defuzzified values and the level of fuzziness, respectively. Both the lot size and total profit per unit time for different levels of learning y_n are calculated. Furthermore, sixth and seventh columns present the difference in the optimal values of the decision variable y_n and related TPU between the fuzzy and the crisp cases.

In the following, three dimensional Figures 4.2 and 4.3 that are plotted using the MATLAB software to investigate the effect of learning and fuzziness simultaneously are analyzed.

Figure 4.2 illustrates the three-dimensional graph of shipment numbers, level of fuzziness and optimal order quantity that are plotted for the first 30 shipments. As the graph shows, for a constant level of fuzziness, the more the number of shipment (n) increases the more the optimal lot size (y_n) decreases. This reduction continues until it is fixed for a particular number of shipments which is about 21 for a 30-percent level of fuzziness. Since the learning increases through the number of shipments. This result conforms to the real environment as whatever the number of shipment increases the knowledge of supplier about the production system intensifies and, therefore, the number of non-conform product (defective items) in a lot decreases.

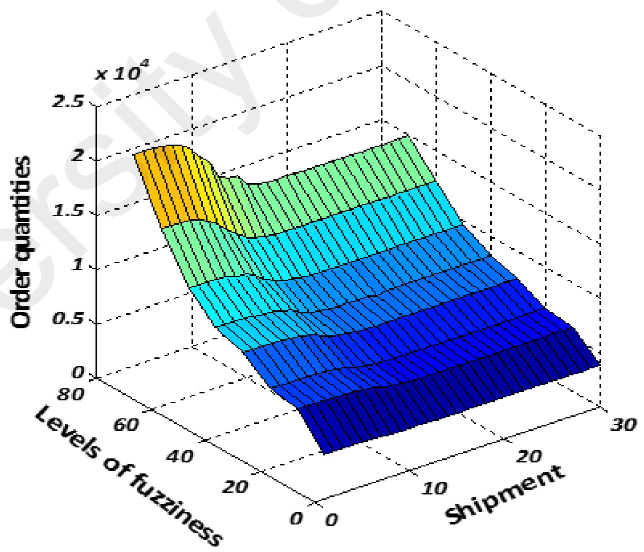

Figure 4.2: Three dimensional-graph of optimal order quantity, level of fuzziness and number of shipments

Also, as it is clear from Figure 4.2, the optimal lot size directly depends on the amount of uncertainty. It increases when the level of uncertainty of a system increases. Therefore, it would merit for practitioners and researchers if they try to decrease the impreciseness of the model to avoid costly inventory policies.

Figure 4.3 depicts the three-dimensional graph of the number of shipments, level of fuzziness and total profit per unit time according to the optimal lot size value plotted in Figure 4.2. This graph also gives the compatible results compared to the figures observed in the graph of optimal lot size (Figure 4.2). The uncertainty has the reverse influence on both the optimal lot size and the total profit per unit time. That is, for a fixed level of uncertainty, when the number of optimal order quantities decreases, the total profit increases. On the other hand, when the level of uncertainty increases, the order quantity and the total profit per unit time increases as well as the quantity of shipments. Therefore, as a managerial insight, the decision maker should expect an increase in the total profit when learning occurs. This strategy justifies some costs that are devoted to the learning process in the long-term.

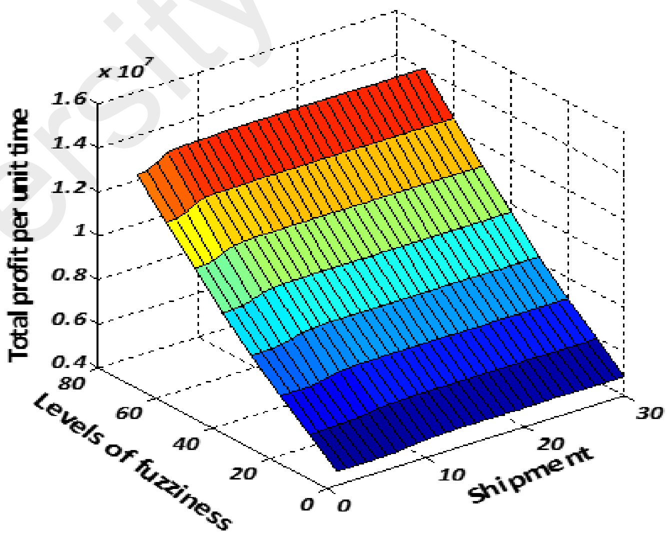

Figure 4.3: Three dimensional-graph of total profit per unit time, level of fuzziness and number of shipments

The values of fuzzy optimal lot size, fuzzy optimal total profit per unit time and their crisp values for each level of fuzziness and the percentage of change between the fuzzy and crisp quantities are represented in Table 4.2 in Appendix E. These values are obtained based on the range of changes that are arbitrarily set from +10% to +70% level of fuzziness for all input parameters, which are depicted in Table 4.2 in Appendix E. Table 4.2 reveals that both optimal lot size and optimal total profit are significantly sensitive to the level of fuzziness. For each shipment, as the uncertainty increases by setting a greater percentage for the level of fuzziness, the difference between the crisp and fuzzy values increases. The changes are from 0 to a maximum percentage of about 199% for optimal total profit and from 0 to a maximum around 424% for optimal lot size. Nevertheless, the comparison of data associated with different shipments also gives some additional results. The optimal lot size shows a descending manner and decreases constantly as the shipment increases; it changes from 1 to 30, as shown in Figure 4.2. In contrast to the optimal lot size, the optimal total profit increases by growth in the number of shipments, as seen in Figure 4.3. As the number of shipments grows and the knowledge about imperfect quality items increases, the order quantity then decreases due to a decrease in the number of imperfect quality in a lot, which leads to more profit.

4.7 Comparing to the Earlier Models

Yadav et al., (2012b) and Yadav et al., (2013b) presented two inventory models that are similar to the EOQ model developed in this research. However, they considered fuzzy models where only the demand rate is fuzzified by TFN, and not only percentage of defective items follows a learning curve, but also a part of ordering and holding costs decreases because of the effect of learning. Besides, Yadav et al., (2013b) considered Type 1 and 2 screening errors which may occur when good items mistakenly taken as defectives ones and defective items mistakenly taken as good ones, respectively. Therefore, in contrast to our fully fuzzy model, their models are developed in a semi-fuzzified system.

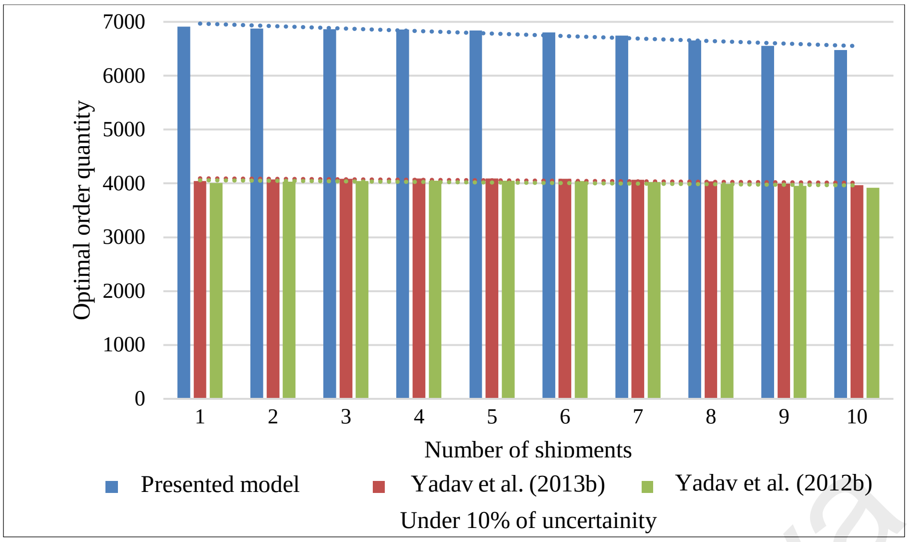

Figure 4.4: Comparison of the behavior of three different models for EOQ under 10 percent of uncertainty for the first 10 shipments

Figures 4.4 and 4.5 compare the optimal EOQ obtained by these models for the first 10 shipments under 10 and 70 percent level of fuzziness, respectively. Moreover, the similar patterns are presented in Figures 4.6 and 4.7 for TPU.

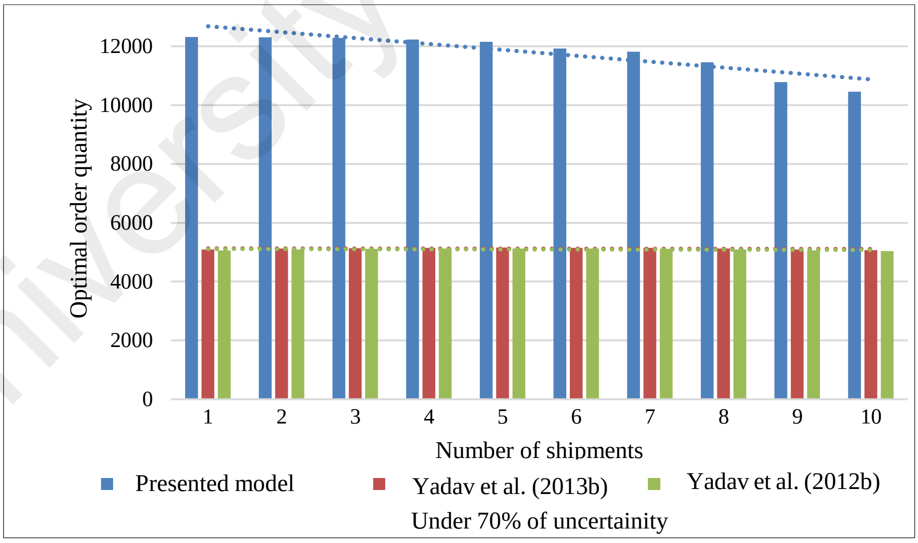

Figure 4.5: Comparison of the behavior of three different models for EOQ under 70 percent of uncertainty for the first 10 shipments

For a better comparison, Table 4.3 shows the average percentage change between the crisp and the approximated defuzzified optimal values of the first 10 shipments for different levels of fuzziness in each model separately. To calculate these results, in addition to the data in the previous section, the penalty cost is considered as $100/unit due to the screening error, and Type 1 and 2 errors 0.02 and 0.03, respectively. The constant part of holding and ordering costs are supposed to be $20/unit/year and $2700 per order while the other part of these costs which is affected by the similar learning rate 0.2 are considered as $5/unit/year and $300/order, respectively (Khan et al., 2011a).

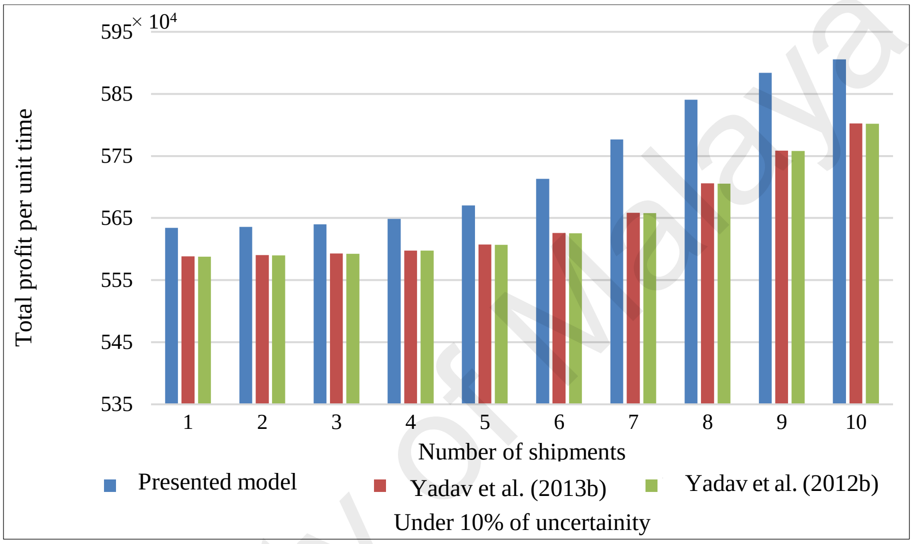

Figure 4.6: Comparison of the behavior of three different models for TPU under 10 percent of uncertainty for the first 10 shipments

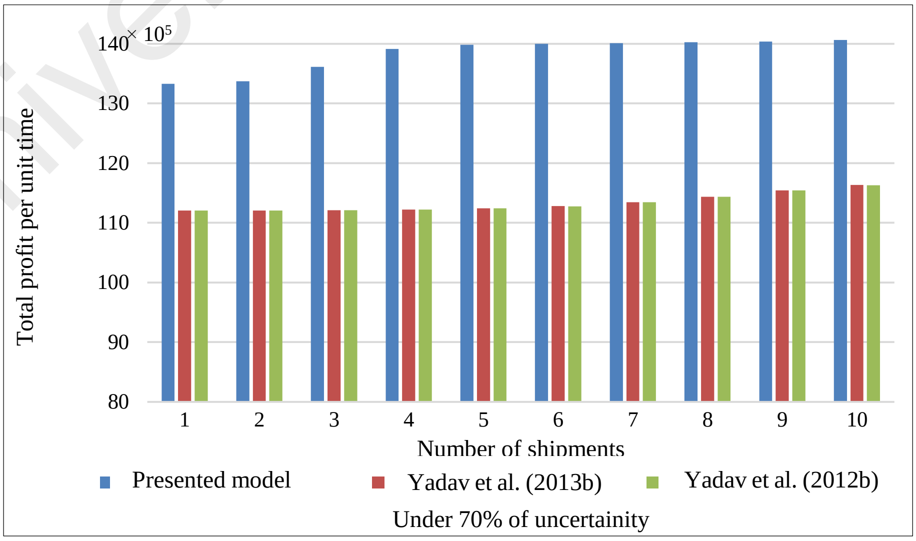

Figure 4.7: Comparison of the behavior of three different models for TPU under 70 percent of uncertainty for the first 10 shipments

The results in Figures 4.6 and 4.7 show that the optimal EOQ proposed by the fully fuzzified model of this paper is the largest of the three models for the mentioned levels of fuzziness. It is clear that the sensitivity of the optimal EOQ for this model is larger than those obtained by the other models.

According to the Table 4.3, it is also evidenced that the increase of average percentage changes of optimal EOQ from the 10% to 70% level of fuzziness for fully fuzzified model is about 8 times more than the other models. Similar results can be extracted regarding the TPU. These results indicate that with increasing the level of fuzziness one of the most effective strategies to capture real inventory situations is handling the induced uncertainty to the inventory system by fuzzifying more parameters. Moreover, the effect of learning is more tangible on the optimal EOQ for the fully fuzzified model when the level of uncertainty increases.

Table 4.3: The variation in optimal EOQ and TPU for three different models

	Average percentage change of optimal (EOQ, TPU) for the first 10 shipments			
Degree of fuzziness	10% uncertainty	30% uncertainty	50% uncertainty	70% uncertainty
Yadav et al. (2012b)	(9.07, 20.10)	(24.88, 60.30)	(38.47, 100.52)	(50.45, 140.75)
Yadav et al. (2013b)	(8.94, 20.10)	(24.45, 60.31)	(37.68, 100.53)	(49.27, 140.76)
Presented model	(65.60, 21.60)	(121.69, 71.30)	(188.32, 129.18)	(402.03, 193.33)

4.8 Managerial Insight

Consider a buyer who sends some orders to a supplier. Each order contains a different lot and the supplier considers separate shipments to meet the buyer's demand. The produced lots are not of perfect quality and contain defective items. They pass the screening process by some workers at the buyer. In this situation, the numbers of defective items per lot may vary from one shipment to another. Therefore, the percentage of defective items per lot conforms to a degree of fuzziness. The buyer screens out the initial lots without previous experience, and, consequently, the number of the optimal lot size increases. As the number of shipments increases, the knowledge of the buyer and supplier about the product quality preferences and production system intensifies. In the later shipments, the buyer could transfer the information about the quality aspects of the received product to the supplier and the supplier could modify their process or adopt some corrective actions, and, simultaneously, the learning process will occur. Therefore, although the number of defective items has an imprecise percentage, it could be reduced by a close collaboration between the buyer and the supplier in gathering and processing quality data.

For example, in foundry industries, the buyer may find some cavity problems in the casting parts and report this quality problem to the supplier, and, consequently, the supplier may add an X-ray operation to its production system in order to detect the cavity in the parts before sending the parts to the buyer. Adding this operation definitely decreases the number of defective items in future shipments. In this scenario, the decision-makers could order smaller batches so that the number of defective items in each batch decreases, and, consequently, they could acquire more profit.

The findings of this research give some insights to managers when they should make a decision. When they face a great deal of uncertainty, adopting inventory policy should be the same as when the available data are exact and accurate. They could raise the gained profit by raising the number of the orders when uncertainty is at a high level. Through this, they could ensure that the number of items, which does not conform with the quality, declines using the learning process, while still meeting the financial target.

4.9 Chapter Summary

Most of the decision makers (DMs) are faced with some kind of uncertainty in making decision as little data are available for making an appropriate decision. To deal with such a situation, the DM has to utilize proper tools to gain reasonable and reliable insights. The

FST has been recognized as the most powerful tool to cope with uncertainty in situations which the DM has less data for making good decisions.

In this chapter, an EOQ model with imperfect quality items under varying holding cost and learning in inspection is reconsidered in a fully fuzzy situation. By applying fuzzy arithmetic operations for obtaining fuzzy total profit function, the GMIV of fuzzy number for defuzzifying fuzzy total profit function and Kuhn Tucker conditions, the optimal order quantity is determined in a fuzzy situation.

The given numerical example showed that both optimal lot size and total profit are highly sensitive to the amount of fuzziness so that they change up to two and four times, respectively. Therefore, managers should notice that removing uncertainties form calculations would lead to a wrong decision that may impose a lot of cost on an inventory system. Moreover, the results reveal that optimal lot size has a descending order and declines continuously as the shipment increases while optimal total profit increases by growth in the number of shipment. These occur because of the learning in the system.

CHAPTER 5: FUZZY BACKWARD ECONOMIC ORDER/PRODUCTION QUANTITY MODEL WITH LEARNING

5.1 Introduction

In this chapter, a fuzzy reverse inventory model that some important factors are assumed to be imprecise is developed. The performance of two defuzzification methods is compared to transform the model to the fuzzy environment. An algorithm is suggested to solve the defuzzified models. The results are explained and discussed through a comprehensive numerical example.

The rest of this chapter is organized as follows: after explaining the problem, the crisp status of the model is overviewed. In Section 5.4, the fuzzy reverse inventory model is developed. The solution and optimization procedures are discussed in Sections 5.5 and 5.6 respectively. Section 5.7 is related to the numerical example. Finally, the chapter is concluded in the last section.

5.2 Problem Description

Consider a supplier or manufacturer who is supplying or producing a product that is recyclable. In this situation, a part of the product can be returned back by the customers to the supplier or manufacturer. In fact, it is assumed that the demand of the market is satisfying by recovered products and newly purchased/produced products simultaneously. Besides, it is assumed that the process follows a multi-order policy for new products and a single setup for recovery process. Moreover, it is supposed that unit production time of the recovered products decreases because of the effect of the learning process.

Figure 5.1 depicts the mentioned processes. The aim is to minimize the fuzzy total cost and obtain the recovery lot size and the number of orders for new products in an uncertain environment.

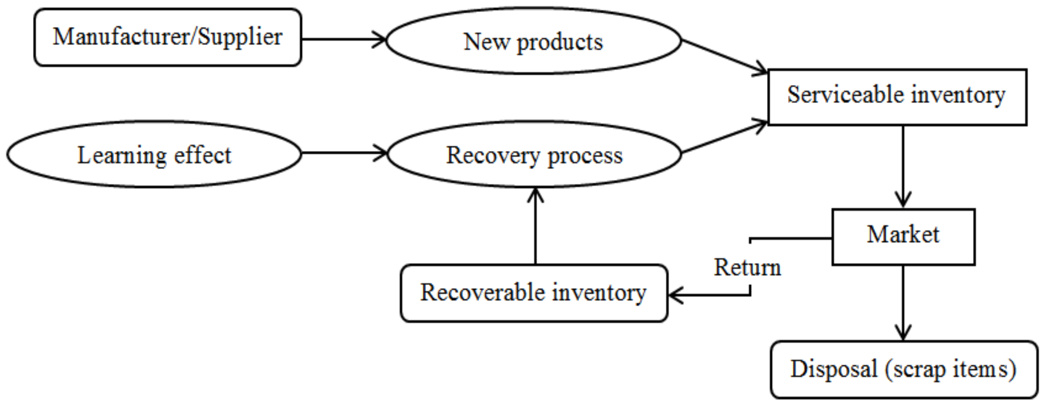

Figure 5.1: Recovery process under the effect of learning

5.2.1 Assumptions

The fuzzy reverse inventory model has the below assumptions:

x. Demand rate of serviceable products is constant during the planning horizon.

xi. Shortages are not allowed.

xii. The demand rate for the serviceable products and the collection rate of the recoverable products from customers are treated as fuzzy numbers and shown by TFNs.

xiii. The time period is infinite.

xiv. All of the collected items can be recovered and made acceptable to customers.

xv. The ordering lots are of equal size through the time.

xvi. The demand rate is greater than the collection rate of the recoverable products.

5.2.2 Notations

To develop the proposed model, the following notations are defined:

y	Recovery lot size for each production run (unit/run) (Decision variable)
n	Number of orders for the newly purchased products during a cycle (Decision variable)
Q	Ordering lot size for the newly purchased products (unit/order)

k	Demand rate of the serviceable products (unit/time) (Fuzzified parameter)
r	Collection rate of the recoverable products from customers (unit/time) (Fuzzified parameter)
C_s	Setup cost for the recovery process ($/setup)
C_o	Ordering cost for the newly purchased products ($/order)
H_r	Inventory holding cost for the collected products ($/unit/time)
H_s	Inventory holding cost for the serviceable products ($/unit/time)
L_r	Learning rate in recovery production
b	Learning exponent
C_l	Labor production cost per unit time ($/time)
C_p	Unit purchase cost for the newly purchased products ($/unit)
C_b	Unit buyback cost for the recovered products ($/unit)
$TCU(y,n)$	Total cost function per unit time ($)
$\tilde{V}(y,n)$	Fuzzified total cost function per unit time ($)

5.3 Reverse Model Formulation

Ordering cost for new items and production setup cost can be calculated as below:

$$nC_o + C_s \tag{5.1}$$

The holding cost for the collected products can be computed according to the Figure 5.2 where T_1, T_2, and T are time of the recovery process, production time of the recovery process, and cycle length of the model respectively. Besides, R is inventory level of the collected products to start the recovery process. It is assumed that time required to recover the xth unit of returned item follows the Wright learning curve explained in chapter 3 as below:

$$t(x) = ax^b \tag{5.2}$$

where a is the time to produce the first unit, and x is the production account, b can be calculated as $b = \log L_r \,/\log 2$ and L_r is the learning rate. It is assumed that $-1 < b \leq 0$.

The cumulative time to produce y units in recovery production run can be given as below:

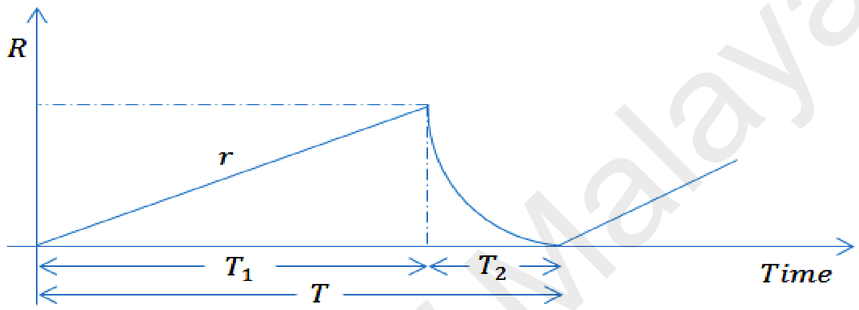

Figure 5.2: Inventory levels of collected items from customers for $n = 2$

$$\begin{aligned} T_2 &= t(1) + t(2) + \cdots + t(y) \\ &= a + a2^b + a3^b + \cdots + ay^b \\ &= a\sum_{x=1}^{y} x^b \approx \int_0^y ax^b\,dx = \frac{ay^{b+1}}{b+1} \end{aligned} \tag{5.3}$$

with solving Eq. (5.3) for y, we have:

$$y = \left[\frac{b+1}{a}T_2\right]^{\frac{1}{b+1}} \tag{5.4}$$

$I(t)$ which is the inventory level of the collected items at time t, can be calculated as below:

$$I(t) = R + rt - \left[\frac{b+1}{a}t\right]^{\frac{1}{b+1}} \tag{5.5}$$

The average inventory level of the collected items for each cycle can be given as:

$$\frac{RT_1}{2} + \int_0^{T_2}\left[R + rt - \left(\frac{b+1}{a}t\right)^{\frac{1}{b+1}}\right]dt$$

$$= \frac{RT_1}{2} + RT_2 + \frac{rT_2^2}{2} - \left(\frac{b+1}{a}\right)^{\frac{1}{b+1}}\frac{b+1}{b+2}T_2^{\left(\frac{b+2}{b+1}\right)} \tag{5.6}$$

It is clear that

$$\frac{R}{r} = T_1 \qquad \frac{ay^{b+1}}{b+1} = T_2 \qquad y - rT_2 = R$$

By substituting these terms, and after some manipulations, Eq. (5.6) is reduced to the following equation:

$$\frac{y^2}{2r} - \frac{ay^{b+2}}{b+2} \tag{5.7}$$

Therefore, the holding cost of the returned items is as below:

$$\left(\frac{y^2}{2r} - \frac{ay^{b+2}}{b+2}\right)H_r \tag{5.8}$$

Holding cost for serviceable items can be formulated as depicted in Figure 5.3 where maximum inventory level of serviceable items is I_{max} and T_3 is elapsed time.

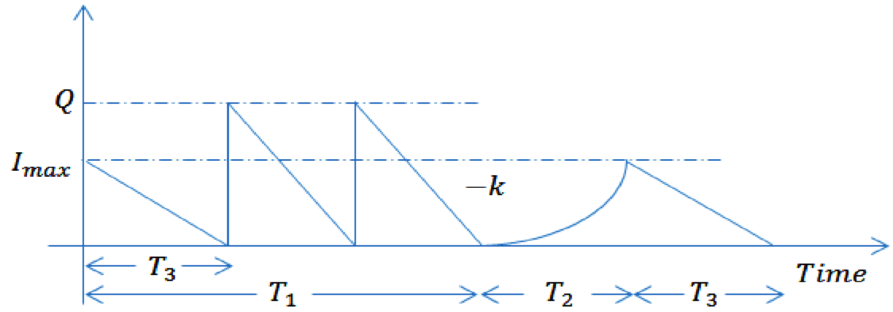

Figure 5.3: Inventory levels of serviceable items for $n = 2$

The following relations can be obtained:

$$T = T_1 + T_2 = \frac{y}{r} \qquad T_3 = \frac{y - kT_2}{k} \qquad nQ = k(T_1 - T_3) = \frac{y(k-r)}{r}$$

$$T_1 - T_3 = \frac{y(k-r)}{rk} \qquad I_{max} = y - kT_2 \qquad Q = \frac{y(k-r)}{nr}$$

The average inventory level of the serviceable items can be derived as:

$$\frac{(y - kT_2)T_3}{2} + \frac{Q(T_1 - T_3)}{2} + \int_0^{T_2} \left[\left(\frac{b+1}{a} t \right)^{\frac{1}{b+1}} - kt \right] dt$$

$$= \frac{y^2 - 2ykT_2 + k^2T_2^2}{2k} + \frac{y^2(k-r)^2}{2nkr^2} + \left(\frac{b+1}{a} \right)^{\frac{1}{b+1}} \frac{b+1}{b+2} \left(\frac{a}{b+1} \right)^{\frac{b+2}{b+1}} y^{b+2} - \frac{kT_2^2}{2} \tag{5.9}$$

After some simplifications, the holding cost of serviceable items can be obtained as:

$$\left(\frac{y^2(k-r)^2}{2nkr^2} + \frac{y^2}{2k} - \frac{ay^{b+2}}{(b+1)(b+2)} \right) H_s \tag{5.10}$$

The labor production cost can be given as:

$$T_2 C_L = \frac{ay^{b+1}}{b+1} C_L \tag{5.11}$$

Cost of purchasing new items and buyback cost for the collected items in a cycle are given by:

$$nQC_p + yC_b = \frac{y(k-r)}{r}C_p + yC_b \tag{5.12}$$

From the above calculations $TC(y,n)$ which is the total inventory cost can be computed as follows:

$$\begin{aligned} TC(y,n) = nC_o + C_s + \left(\frac{y^2}{2r} - \frac{ay^{b+2}}{b+2}\right)H_r \\ + \left(\frac{y^2(k-r)^2}{2nkr^2} + \frac{y^2}{2k} - \frac{ay^{b+2}}{(b+1)(b+2)}\right)H_s + \frac{ay^{b+1}}{b+1}C_L \\ + \frac{y(k-r)}{r}C_p + yC_b. \end{aligned} \tag{5.13}$$

Finally, the total cost function per unit time in a proposed reverse inventory system with a single setup for recovery process and multi-order policy which is termed as $(1,n)$ policy is as below:

$$\begin{aligned} TCU(y,n) = \frac{r(nC_o + C_s)}{y} + H_r\left(\frac{y}{2} - \frac{ary^{b+1}}{b+2}\right) \\ + H_s\left[\frac{y(k-r)^2}{2nkr} + \frac{ry}{2k} - \frac{ary^{b+1}}{(b+1)(b+2)}\right] + \left(\frac{ary^b}{b+1}\right)C_l \\ +(k-r)C_p + rC_b \end{aligned} \tag{5.14}$$

5.4 Fuzzy Reverse Inventory Model

In this section, the reverse inventory model presented in previous section is modified by incorporating the fuzziness of the demand rate of the serviceable products k and the collection rate of the recoverable products r. As demand and return rate are varying in a cycle, the mentioned parameters are very important in the reverse inventory process.

To do so, r and k are fuzzified to be two TFNs $\tilde{r}$ and $\tilde{k}$, respectively, where $\tilde{r} = (r - \theta_1, r, r + \theta_2)$, $0 < \theta_1 < r$, $\theta_2 > 0$, and $\tilde{k} = (k - \theta_3, k, k + \theta_4)$, $0 < \theta_3 < k$, $\theta_4 > 0$. It should be noted that θ_1, θ_2, θ_3, and θ_4 could be determined by decision makers. By fuzzifying the mentioned parameters, the total cost per unit time which is also a fuzzy function can be expressed as

$$\tilde{V} \equiv \tilde{V}(y,n) = \frac{\tilde{r}(nC_o + C_s)}{y} + H_r\left(\frac{y}{2} - \frac{a\tilde{r}y^{b+1}}{b+2}\right) + H_s\left[\frac{y\left(\tilde{k} - \tilde{r}\right)^2}{2n\tilde{k}\tilde{r}} + \frac{\tilde{r}y}{2\tilde{k}} - \frac{a\tilde{r}y^{b+1}}{(b+1)(b+2)}\right] + \left(\frac{a\tilde{r}y^b}{b+1}\right)C_l + \left(\tilde{k} - \tilde{r}\right)C_p + \tilde{r}C_b \tag{5.15}$$

In the next sections, the $\tilde{V}(y,n)$ is defuzzified by using the GMIR and the SD method to investigate the effect of fuzziness on the studied reverse inventory system.

5.4.1 Defuzzification by the SD Method

According to the explanations in chapter 3 related to the signed distance method, signed distance of $\tilde{V}$ to $\tilde{0}_1$ is given by:

$$d(\tilde{V},\tilde{0}) = \frac{(nC_o + C_s)}{y}d(\tilde{r},\tilde{0}_1) + \frac{y}{2}H_r - \frac{H_r a y^{b+1}}{b+2}d(\tilde{r},\tilde{0}_1) + \frac{yH_s}{2n}d\left(\frac{\tilde{k}}{\tilde{r}},\tilde{0}_1\right) + \frac{yH_s}{2n}d\left(\frac{\tilde{r}}{\tilde{k}},\tilde{0}_1\right) - \frac{yH_s}{n} + \frac{yH_s}{2}d\left(\frac{\tilde{r}}{\tilde{k}},\tilde{0}_1\right) - \frac{ay^{b+1}H_s}{(b+1)(b+2)}d(\tilde{r},\tilde{0}_1) + \left(\frac{ay^b}{b+1}\right)C_l d(\tilde{r},\tilde{0}_1) + C_p d\left((\tilde{k} - \tilde{r}),\tilde{0}_1\right) + C_b d(\tilde{r},\tilde{0}_1) \tag{5.16}$$

where $d(\tilde{r},\tilde{0}_1)$, $d(\tilde{k}/\tilde{r},\tilde{0}_1)$, and $d((\tilde{k} - \tilde{r}),\tilde{0}_1)$ are measured as follows. From properties discussed in chapter 3, the signed distance of fuzzy number $\tilde{r}$ to $\tilde{0}_1$ is:

$$d(\tilde{r},\tilde{0}_1) = \frac{1}{4}[(r-\theta_1)+2r+(r+\theta_2)] = r+\frac{1}{4}(\theta_2-\theta_1) \tag{5.17}$$

The left and right end points of the α-*cut* of $\tilde{r}$, and $\tilde{k}$ $(0 \le \alpha \le 1)$ are $r_L(\alpha) = (r-\theta_1)+\theta_1\alpha$, $r_R(\alpha) = (r+\theta_2)-\theta_2\alpha$, $k_L(\alpha) = (k-\theta_3)+\theta_3\alpha$, and $k_R(\alpha) = (k+\theta_4)-\theta_4\alpha$, respectively.

Since $0 < r_L(\alpha) < r_R(\alpha)$, $0 < k_L(\alpha) < k_R(\alpha)$, from the interval operations explained in chapter 3, the left and right end points of the α-*cut* of $\tilde{k}/\tilde{r}$, $\tilde{k}-\tilde{r}$, and $\tilde{r}/\tilde{k}$ are

$$\left(\frac{k}{r}\right)_L(\alpha) = \frac{k_L(\alpha)}{r_R(\alpha)} = \frac{(k-\theta_3)+\theta_3\alpha}{(r+\theta_2)-\theta_2\alpha} \tag{5.18}$$

$$\left(\frac{k}{r}\right)_R(\alpha) = \frac{k_R(\alpha)}{r_L(\alpha)} = \frac{(k+\theta_4)-\theta_4\alpha}{(r-\theta_1)+\theta_1\alpha} \tag{5.19}$$

$$\begin{aligned}(k-r)_L(\alpha) &= k_L(\alpha)-r_R(\alpha)\\ &= (k-\theta_3)-(r+\theta_2)+(\theta_2+\theta_3)\alpha\end{aligned} \tag{5.20}$$

$$\begin{aligned}(k-r)_R(\alpha) &= k_R(\alpha)-r_L(\alpha)\\ &= (k+\theta_4)-(r-\theta_1)-(\theta_1+\theta_4)\alpha\end{aligned} \tag{5.21}$$

$$\left(\frac{r}{k}\right)_L(\alpha) = \frac{r_L(\alpha)}{k_R(\alpha)} = \frac{(r-\theta_1)+\theta_1\alpha}{(k+\theta_4)-\theta_4\alpha} \tag{5.22}$$

$$\left(\frac{r}{k}\right)_R(\alpha) = \frac{r_R(\alpha)}{k_L(\alpha)} = \frac{(r+\theta_2)-\theta_2\alpha}{(k-\theta_3)+\theta_3\alpha} \tag{5.23}$$

respectively.

Thus, from Eqs. (3.51) and (3.52) explained in chapter 3, the signed distance of $\tilde{k}/\tilde{r}$, $\tilde{k}-\tilde{r}$, and $\tilde{r}/\tilde{k}$ to $\tilde{0}_1$ are:

$$\begin{aligned}d\left(\frac{\tilde{k}}{\tilde{r}},\tilde{0}_1\right) &= \frac{1}{2}\int_0^1\left[\left(\frac{k}{r}\right)_L(\alpha)+\left(\frac{k}{r}\right)_U(\alpha)\right]d\alpha\\ &= \frac{1}{2}\left[\frac{r\theta_4+k\theta_1}{{\theta_1}^2}Ln\frac{r}{r-\theta_1}-\frac{\theta_4}{\theta_1}+\frac{r\theta_3+k\theta_2}{{\theta_2}^2}Ln\frac{r+\theta_2}{r}-\frac{\theta_3}{\theta_2}\right]\end{aligned} \tag{5.24}$$

$$d\left(\frac{\tilde{r}}{\tilde{k}}, \tilde{0}_1\right) = \frac{1}{2}\int_0^1 \left[\left(\frac{r}{k}\right)_L(\alpha) + \left(\frac{r}{k}\right)_U(\alpha)\right] d\alpha$$

$$= \frac{1}{2}\left[\frac{k\theta_2 + r\theta_3}{{\theta_3}^2} Ln\frac{k}{k-\theta_3} - \frac{\theta_2}{\theta_3} + \frac{k\theta_1 + r\theta_4}{{\theta_4}^2} Ln\frac{k+\theta_4}{k} - \frac{\theta_1}{\theta_4}\right] \tag{5.25}$$

$$d\left(\left(\tilde{k} - \tilde{r}\right), \tilde{0}_1\right) = \frac{1}{2}\int_0^1 [(k-r)_L(\alpha) + (k-r)_U(\alpha)]\, d\alpha$$

$$= \frac{1}{2}\Bigg([(\theta_2 + \theta_3) - (\theta_1 + \theta_4)]\frac{1}{2} + (k - \theta_3) + (k + \theta_4)$$

$$- (r + \theta_2) - (r - \theta_1)\Bigg) \tag{5.26}$$

respectively. Substituting the results obtained by (5.24)-(5.26) into (5.16), we have

$$V(n, y) \equiv d(\tilde{V}, \tilde{0})$$

$$= d(\tilde{r}, \tilde{0}_1)\Bigg[\frac{(nC_o + C_s)}{y} - \frac{H_r a y^{b+1}}{b+2} - \frac{a y^{b+1} H_s}{(b+1)(b+2)} + \left(\frac{a y^b}{b+1}\right) C_l$$

$$+ C_b\Bigg]$$

$$+ d\left(\frac{\tilde{k}}{\tilde{r}}, \tilde{0}_1\right)\frac{yH_s}{2n} + d\left(\frac{\tilde{r}}{\tilde{k}}, \tilde{0}_1\right)\left[\frac{yH_s}{2n} + \frac{yH_s}{2}\right] + d\left((\tilde{k} - \tilde{r}), \tilde{0}_1\right) C_p$$

$$- \frac{yH_s}{n} + \frac{y}{2} H_r \tag{5.27}$$

By replacing the related terms, we have:

$$V(n, y) \equiv d(\tilde{V}, \tilde{0})$$

$$= \left[r + \frac{1}{4}(\theta_2 - \theta_1)\right]$$

$$\cdot\left[\frac{(nC_o + C_s)}{y} - \frac{H_r a y^{b+1}}{b+2} - \frac{a y^{b+1} H_s}{(b+1)(b+2)} + \left(\frac{a y^b}{b+1}\right) C_l + C_b\right]$$

$$+ \frac{yH_s}{4n}\left[\frac{r\theta_4 + k\theta_1}{{\theta_1}^2} Ln\frac{r}{r-\theta_1} - \frac{\theta_4}{\theta_1} + \frac{r\theta_3 + k\theta_2}{{\theta_2}^2} Ln\frac{r+\theta_2}{r} - \frac{\theta_3}{\theta_2}\right]$$

$$+ \frac{(1+n)yH_s}{4n}\left[\frac{k\theta_2 + r\theta_3}{{\theta_3}^2} Ln\frac{k}{k-\theta_3} - \frac{\theta_2}{\theta_3} + \frac{k\theta_1 + r\theta_4}{{\theta_4}^2} Ln\frac{k+\theta_4}{k} - \frac{\theta_1}{\theta_4}\right]$$

$$+\left(k-r-\frac{\theta_2+\theta_3-\theta_1-\theta_4}{4}\right)C_p \; -\frac{yH_s}{n}+\frac{y}{2}H_r \tag{5.28}$$

$V(n,y)$ is considered as the estimate of the total cost per unit time in fuzzy situation.

The next step is to determine the optimal recovery lot size to minimize the total cost function $V(n,y)$. By setting the first derivative of $d(\tilde{V},\tilde{0})$ with respect to n equal to zero, y can be obtained as follow:

$$\frac{\partial d(\tilde{V},\tilde{0})}{\partial n}=0\rightarrow y=n\sqrt{\frac{2C_o d(\tilde{r},\tilde{0}_1)}{H_s\Delta}}=n\rho \tag{5.29}$$

where

$$\Delta = d\left(\frac{\tilde{k}}{\tilde{r}},\tilde{0}_1\right)+d\left(\frac{\tilde{r}}{\tilde{k}},\tilde{0}_1\right)-2 \qquad \text{and} \qquad \rho=\sqrt{\frac{2C_o d(\tilde{r},\tilde{0}_1)}{H_s\Delta}}$$

By setting the first derivative of $d(\tilde{V},\tilde{0})$ with respect to y, we have:

$$\frac{\partial d(\tilde{V},\tilde{0})}{\partial y}=-\frac{(nC_o+C_s)}{y^2}d(\tilde{r},\tilde{0}_1)+\frac{1}{2}H_r$$
$$-\frac{(b+1)H_r a y^b}{b+2}d(\tilde{r},\tilde{0}_1)+\frac{H_s}{2n}d\left(\frac{\tilde{k}}{\tilde{r}},\tilde{0}_1\right)$$
$$+\frac{H_s}{2n}d\left(\frac{\tilde{r}}{\tilde{k}},\tilde{0}_1\right)-\frac{H_s}{n}+\frac{H_s}{2}d\left(\frac{\tilde{r}}{\tilde{k}},\tilde{0}_1\right)-\frac{ay^bH_s}{(b+2)}d(\tilde{r},\tilde{0}_1)$$
$$+\left(\frac{aby^{b-1}}{b+1}\right)C_l d(\tilde{r},\tilde{0}_1)=0 \tag{5.30}$$

and after substituting y from (5.29) into (5.30), $\tilde{g}(n)$ can be derived as

$$\tilde{g}(n)=-\frac{(nC_o+C_s)}{n^2\rho^2}d(\tilde{r},\tilde{0}_1)+\frac{1}{2}H_r-\frac{(b+1)H_r a n^b\rho^b}{b+2}d(\tilde{r},\tilde{0}_1)+\frac{H_s}{2n}$$

$$+\frac{H_s}{2n}\left(d\left(\frac{\tilde{k}}{\bar{\tilde{r}}},\tilde{0}_1\right)+d\left(\frac{\tilde{r}}{\bar{\tilde{k}}},\tilde{0}_1\right)-2\right)+\frac{H_s}{2}d\left(\frac{\tilde{r}}{\bar{\tilde{k}}},\tilde{0}_1\right)$$

$$-\frac{an^b\rho^bH_s}{(b+2)}d(\tilde{r},\tilde{0}_1)+\left(\frac{abn^{b-1}\rho^{b-1}}{b+1}\right)C_ld(\tilde{r},\tilde{0}_1) \tag{5.31}$$

5.4.2 Finding the Optimal Values for the SD Method

Letting (y^*, n^*) shows the solution for the considered problem. To prove that y^* and n^* are the optimal recovery lot size and the optimal number of orders, respectively, Theorem 1 and 2 are necessary.

Theorem 1: The optimal solution of (y^*, n^*) not only exists, but is also unique. It is clear that it should satisfy $\tilde{g}(n) = 0$, and $y - n\rho = 0$, simultaneously.

Proof. By taking the first derivative of the $\tilde{g}(n)$ with respect to n, we have:

$$\frac{\partial\tilde{g}(n)}{\partial n}=b(\beta'+\delta')n^{b-1}-\frac{2\alpha'}{n^3}+(b-1)\zeta'n^{b-2}-\frac{\gamma'}{n^2}>0 \tag{5.32}$$

where

$$\alpha' = -\frac{(nC_o+C_s)H_s\Delta}{2C_o}$$

$$\beta' = -\frac{(b+1)H_ra\rho^b}{b+2}d(\tilde{r},\tilde{0}_1)$$

$$\gamma' = \frac{H_s}{2}\Delta$$

$$\delta' = -\frac{a\rho^bH_s}{(b+2)}d(\tilde{r},\tilde{0}_1)$$

$$\zeta' = \left(\frac{ab\rho^{b-1}}{b+1}\right)C_ld(\tilde{r},\tilde{0}_1)$$

$$\varepsilon' = \frac{1}{2}H_r+\frac{H_s}{2}d\left(\frac{\tilde{r}}{\bar{\tilde{k}}},\tilde{0}_1\right)$$

It is positive for all value of $n > 0$. Hence, $\tilde{g}(n)$ is a strictly increasing function for $0 < n < \infty$. Moreover, there are the following limitations:

$$\lim_{n \to +\infty} \tilde{g}(n) = \varepsilon' > 0$$

$$\lim_{n \to 0^+} \tilde{g}(n) = -\infty$$

Thus, by the Intermediate Value Theorem introduced in chapter 3, there exists a unique $0 < n^* < \infty$ such that $\tilde{g}(n^*) = 0$.

Theorem 2: $d(\tilde{V}, \tilde{0})$ has a global minimum at (y^*, n^*), where this point is the solution for $\tilde{g}(n) = 0$, and $y - n\rho = 0$.

Proof. From Theorem 1, it is clear that (y^*, n^*) is the only critical point. Therefore, to prove the Theorem 2, the Hessian Matrix of $d(\tilde{V}, \tilde{0})$ should firstly be calculated as follows:

$$\begin{aligned} \frac{\partial^2 d(\tilde{V}, \tilde{0})}{\partial y^2} &= \frac{\partial^2 V(n, y)}{\partial y^2} \\ &= \frac{2(nC_o + C_s)}{y^3} d(\tilde{r}, \tilde{0}_1) - \frac{b(b+1)H_r a y^{b-1}}{b+2} d(\tilde{r}, \tilde{0}_1) \\ &\quad - \frac{bay^{b-1}H_s}{(b+2)} d(\tilde{r}, \tilde{0}_1) \\ &\quad + \left(\frac{(b-1)aby^{b-2}}{b+1}\right) C_l d(\tilde{r}, \tilde{0}_1) \end{aligned} \tag{5.33}$$

For all $y > 0$, $n > 0$, $\partial^2 d(\tilde{V}, \tilde{0})/\partial y^2 > 0$

$$\frac{\partial^2 d(\tilde{V}, \tilde{0})}{\partial n^2} = \frac{yH_s}{n^3} \Delta \tag{5.34}$$

For all $y > 0$, $n > 0$, $\partial^2 d(\tilde{V}, \tilde{0})/\partial n^2 > 0$.

$$\frac{\partial^2 V(n,y)}{\partial y \partial n} = -\frac{C_o d(\tilde{r}, \tilde{0}_1)}{y^2} - \frac{H_s}{2n^2}\Delta \tag{5.35}$$

Substituting $y^* = \rho n^*$ into Eqs. (5.33)-(5.35), and after some simplifications, determinant of the Hessian Matrix of $d(\tilde{V}, \tilde{0})$ at (y^*, n^*) could be given as below:

$$\begin{vmatrix} \frac{\partial^2 V(n,y)}{\partial y^2}\Big|_{(y^*,n^*)} & \frac{\partial^2 V(n,y)}{\partial y \partial n}\Big|_{(y^*,n^*)} \\ \frac{\partial^2 V(n,y)}{\partial y \partial n}\Big|_{(y^*,n^*)} & \frac{\partial^2 V(n,y)}{\partial n^2}\Big|_{(y^*,n^*)} \end{vmatrix}$$

$$= \frac{y^* H_s}{n^{*3}}\Delta\Bigg[-\frac{b(b+1)H_r a y^{b-1}}{b+2} d(\tilde{r}, \tilde{0}_1) - \frac{bay^{b-1}H_s}{(b+2)} d(\tilde{r}, \tilde{0}_1) + \left(\frac{(b-1)aby^{b-2}}{b+1}\right) C_l d(\tilde{r}, \tilde{0}_1)\Bigg] + \frac{{H_s}^2 \Delta^2 C_s}{n^{*5} C_o} > 0 \tag{5.36}$$

Hessian Matrix of $d(\tilde{V}, \tilde{0})$ is positive. Hence, $d(\tilde{V}, \tilde{0})$ has a global minimum at point (y^*, n^*).

Then, the ordering lot size can be estimated for the newly purchased products Q as:

$$d(\tilde{Q}, \tilde{0}) = \frac{y}{n} d\left(\frac{\tilde{k}}{\tilde{r}}, \tilde{0}\right) - \frac{y}{n}$$

5.4.3 Defuzzification by the GMIR Method

By applying the fuzzy arithmetic operations of the function principle method described in chapter 3, the fuzzy total cost per unit time in Eq. (5.15), can be written as follows:

$$\tilde{V}(y,n) = (\chi_1, \chi_2, \chi_3) \tag{5.37}$$

where

$$\chi_1 = \frac{(r-\theta_1)(nC_o+C_s)}{y} + H_r\left[\frac{y}{2} - \frac{a(r+\theta_2)y^{b+1}}{b+2}\right] + C_l\frac{a(r-\theta_1)y^b}{b+1}$$

$$+H_s\left(\frac{y[(k-\theta_3)-(r+\theta_2)]^2}{2n(k+\theta_4)(r+\theta_2)} + \frac{y(r-\theta_1)}{2(k+\theta_4)} - \frac{a(r+\theta_2)y^{b+1}}{(b+1)(b+2)}\right)$$

$$+[(k-\theta_3)-(r+\theta_2)]C_p + (r-\theta_1)C_b \qquad (5.38)$$

$$\chi_2 = \frac{r(nC_o+C_s)}{y} + H_r\left[\frac{y}{2} - \frac{ary^{b+1}}{b+2}\right]$$

$$+ H_s\left(\frac{y[k-r]^2}{2nkr} + \frac{yr}{2k} - \frac{ary^{b+1}}{(b+1)(b+2)}\right)$$

$$+C_l\frac{ary^b}{b+1} + (k-r)C_p + rC_b \qquad (5.39)$$

$$\chi_3 = \frac{(r+\theta_2)(nC_o+C_s)}{y} + H_r\left[\frac{y}{2} - \frac{a(r-\theta_1)y^{b+1}}{b+2}\right] + C_l\frac{a(r+\theta_2)y^b}{b+1}$$

$$+H_s\left(\frac{y[(k+\theta_4)-(r-\theta_1)]^2}{2n(k-\theta_3)(r-\theta_1)} + \frac{y(r+\theta_2)}{2(k-\theta_3)} - \frac{a(r-\theta_1)y^{b+1}}{(b+1)(b+2)}\right)$$

$$+[(k+\theta_4)-(r-\theta_1)]C_p + (r+\theta_2)C_b \qquad (5.40)$$

According to the GMIR method explained in chapter 3, the defuzzified value of $\tilde{V}$ can be given as below:

$$\Phi\left(\tilde{V}(y,n)\right) = \frac{1}{6}(\chi_1 + 4\chi_2 + \chi_3) \qquad (5.41)$$

The next step is to determine the optimal recovery lot size to minimize the defuzzified total cost function $\Phi(\tilde{V}(y,n))$. By setting the first derivative of $\Phi(\tilde{V}(y,n))$ with respect to n equal to zero, y can be obtained as follows:

$$\frac{\partial\Phi(\widetilde{V}(y,n))}{\partial n} = 0 \rightarrow y = n\pi \tag{5.42}$$

where

$$\pi = \sqrt{\frac{C_o[(r-\theta_1)+4r+(r+\theta_2)]}{H_s\left(\frac{[(k-\theta_3)-(r+\theta_2)]^2}{2(k+\theta_4)(r+\theta_2)}+\frac{2(k-r)^2}{kr}+\frac{[(k+\theta_4)-(r-\theta_1)]^2}{2(k-\theta_3)(r-\theta_1)}\right)}}$$

By setting the first derivative of $\Phi(\widetilde{V}(y,n))$ with respect to y, and after substituting $y = n\pi$, $\widetilde{f}(n)$ can be derived as

$$\widetilde{f}(n) = -\frac{\gamma C_s}{6n^2C_o}+\frac{(\beta+\delta)n^b+\zeta n^{b-1}}{6}+\varepsilon \tag{5.43}$$

where

$$\alpha = -\frac{(nC_o+C_s)\gamma}{C_o}$$

$$\beta = -\frac{a(b+1)H_r[(r+\theta_2)+4r+(r-\theta_1)]\pi^b}{b+2}$$

$$\gamma = H_s\left(\frac{[(k-\theta_3)-(r+\theta_2)]^2}{2(k+\theta_4)(r+\theta_2)}+\frac{2(k-r)^2}{kr}+\frac{[(k+\theta_4)-(r-\theta_1)]^2}{2(k-\theta_3)(r-\theta_1)}\right)$$

$$\delta = -\frac{aH_s\pi^b}{(b+2)}((r-\theta_1)+4r+(r+\theta_2))$$

$$\zeta = abC_l\pi^{b-1}\left[\frac{(r-\theta_1)+4r+(r+\theta_2)}{b+1}\right]$$

$$\varepsilon = \frac{1}{2}H_r+\frac{1}{6}H_s\left[\frac{(r-\theta_1)}{2(k+\theta_4)}+\frac{2r}{k}+\frac{(r+\theta_2)}{2(k-\theta_3)}\right]$$

5.4.4 Finding the Optimal Values for the GMIR Method

Letting (y^*, n^*) shows the solution for the considered problem. To prove that y^* and n^* are the optimal recovery lot size and the optimal number of orders, respectively, Theorem 3 and 4 are required.

Theorem 3: The optimal solution of (y^*, n^*) not only exists, but is also unique. It is clear that it should satisfy $\tilde{f}(n) = 0$, and $y - n\pi = 0$, simultaneously.

Proof. The first derivative of the $\tilde{f}(n)$ is positive for all value of $n > 0$.

$$\frac{\partial \tilde{f}(n)}{\partial n} = \frac{2\gamma C_s}{6n^3 C_o} + \frac{b(\beta + \delta)n^{b-1} + \zeta(b-1)n^{b-2}}{6} > 0 \quad (5.44)$$

Hence, $\tilde{f}(n)$ is a strictly increasing function for $0 < n < \infty$. Moreover, there are following limitations

$$\lim_{n \to +\infty} \tilde{f}(n) = \epsilon > 0$$

$$\lim_{n \to 0^+} \tilde{f}(n) = -\infty$$

Thus, by the Intermediate Value Theorem, there exists a unique $0 < n^* < \infty$ such that $\tilde{f}(n^*) = 0$.

Theorem 4: $\Phi(\tilde{V}(y, n))$ has a global minimum at (y^*, n^*), where this point is the solution for $\tilde{f}(n) = 0$, and $y - n\pi = 0$.

From Theorem 3, it is clear that (y^*, n^*) is the only critical point. Therefore, to prove the Theorem 4, the Hessian Matrix of $\Phi(\tilde{V}(y, n))$ should firstly be calculated as follows:

$$\begin{aligned}\frac{\partial^2 \Phi(\tilde{V}(y,n))}{\partial y^2} = & \frac{2(nC_o + C_s)(6r + \theta_2 - \theta_1)}{y^3} \\ & - \frac{ba(b+1)H_r(6r + \theta_2 - \theta_1)y^{b-1}}{b+2} \\ & - \frac{bay^{b-1}H_s(6r + \theta_2 - \theta_1)}{(b+2)} \\ & + \frac{abC_l(b-1)(6r + \theta_2 - \theta_1)y^{b-2}}{b+1}\end{aligned} \quad (5.45)$$

For all $y > 0, n > 0$, $\partial^2\Phi(\tilde{V}(y,n))/\partial y^2 > 0$

$$\frac{\partial^2\Phi(\tilde{V}(y,n))}{\partial n^2} = \frac{yH_s}{3n^3}\left[\frac{(k-r-\theta_3-\theta_2)^2}{2(k+\theta_4)(r+\theta_2)} + \frac{2(k-r)^2}{kr} + \frac{(k-r+\theta_4+\theta_1)^2}{2(k-\theta_3)(r-\theta_1)}\right]$$
$$= \frac{yC_o[(r-\theta_1)+4r+(r+\theta_2)]}{3n^3\pi^2} \quad (5.46)$$

For all $y > 0, n > 0$, $\partial^2\Phi(\tilde{V}(y,n))/\partial n^2 > 0$. Moreover, we have:

$$\frac{\partial^2 V(n,y)}{\partial y\partial n} = -\frac{C_o(6r+\theta_2-\theta_1)}{6y^2} - \frac{H_s}{n^2 6}\left[\frac{(k-r-\theta_3-\theta_2)^2}{2(k+\theta_4)(r+\theta_2)} + \frac{2(k-r)^2}{kr} + \frac{(k-r+\theta_4+\theta_1)^2}{2(k-\theta_3)(r-\theta_1)}\right]$$
$$= -\frac{C_o(6r+\theta_2-\theta_1)}{6y^2} - \frac{C_o[(r-\theta_1)+4r+(r+\theta_2)]}{6n^2\pi^2} \quad (5.47)$$

Substituting $y^* = \pi n^*$ into Eqs. (5.45)-(5.47), and after some manipulations, determinant of the Hessian Matrix of $\Phi(\tilde{V}(y,n))$ at (y^*, n^*) could be given as below:

$$\begin{vmatrix} \frac{\partial^2\Phi\left(\tilde{V}(y,n)\right)}{\partial y^2}\Big|_{(y^*,n^*)} & \frac{\partial^2\Phi\left(\tilde{V}(y,n)\right)}{\partial y\partial n}\Big|_{(y^*,n^*)} \\ \frac{\partial^2\Phi\left(\tilde{V}(y,n)\right)}{\partial y\partial n}\Big|_{(y^*,n^*)} & \frac{\partial^2\Phi\left(\tilde{V}(y,n)\right)}{\partial n^2}\Big|_{(y^*,n^*)} \end{vmatrix} \quad (5.48)$$

$$= \frac{y^* C_o(6r+\theta_2-\theta_1)}{3{n^*}^3\pi^2}\left[-\frac{ba(b+1)H_r(6r+\theta_2-\theta_1)y^{b-1}}{b+2} - \frac{bay^{b-1}H_s(6r+\theta_2-\theta_1)}{(b+2)} + \frac{abC_l(b-1)(6r+\theta_2-\theta_1)y^{b-2}}{b+1}\right] + \frac{C_o(6r+\theta_2-\theta_1)^2(5nC_o+6C_s)}{9{n^*}^5\pi^4} > 0$$

Hessian Matrix of $\Phi(\tilde{V}(y,n))$ is positive. Hence, $\Phi(\tilde{V}(y,n))$ has a global minimum at point (y^*, n^*).

Therefore, ordering lot size can be estimated for the newly purchased products Q as

$$\Phi(\tilde{Q}) = \frac{1}{6}(q_1 + 4q_2 + q_3)$$

$$= \frac{1}{6}\left(\frac{y[(k-\theta_3)-(r+\theta_2)]}{n(r+\theta_2)} + \frac{4y(k-r)}{nr} + \frac{y[(k+\theta_4)-(r-\theta_1)]}{n(r-\theta_1)}\right)$$

5.5 Solution Procedure

Finding a closed-form solution is not possible for $\tilde{f}(n) = 0$ or $\tilde{g}(n) = 0$. Instead of a direct method, n^* can be found by a one-dimensional search procedure. When n^* can be found, y^* can be obtained by $\partial d(\tilde{V}, \tilde{0})/\partial y = 0$. As n^* is a positive integer, the following solution algorithm can also be applied to find the optimal values.

5.6 One-dimensional Search Procedure for Optimization

The obtained defuzzified functions can be optimized through a heuristic mathematical algorithm explained in this section as depicted in Figure 5.4.

Consider a pre-determined error value $\tau > 0$. Set n_l and n_u as suggested guesses of the root such that $f(n_u) > 0$ and $f(n_l) < 0$. The optimal values for both methods (i.e. GMIR method and SD method) could be found by the proposed flowchart in Figure 5.4. In this flowchart, $\lfloor n \rfloor$ and $\lceil n \rceil$ show the nearest integers smaller and larger than n^*.

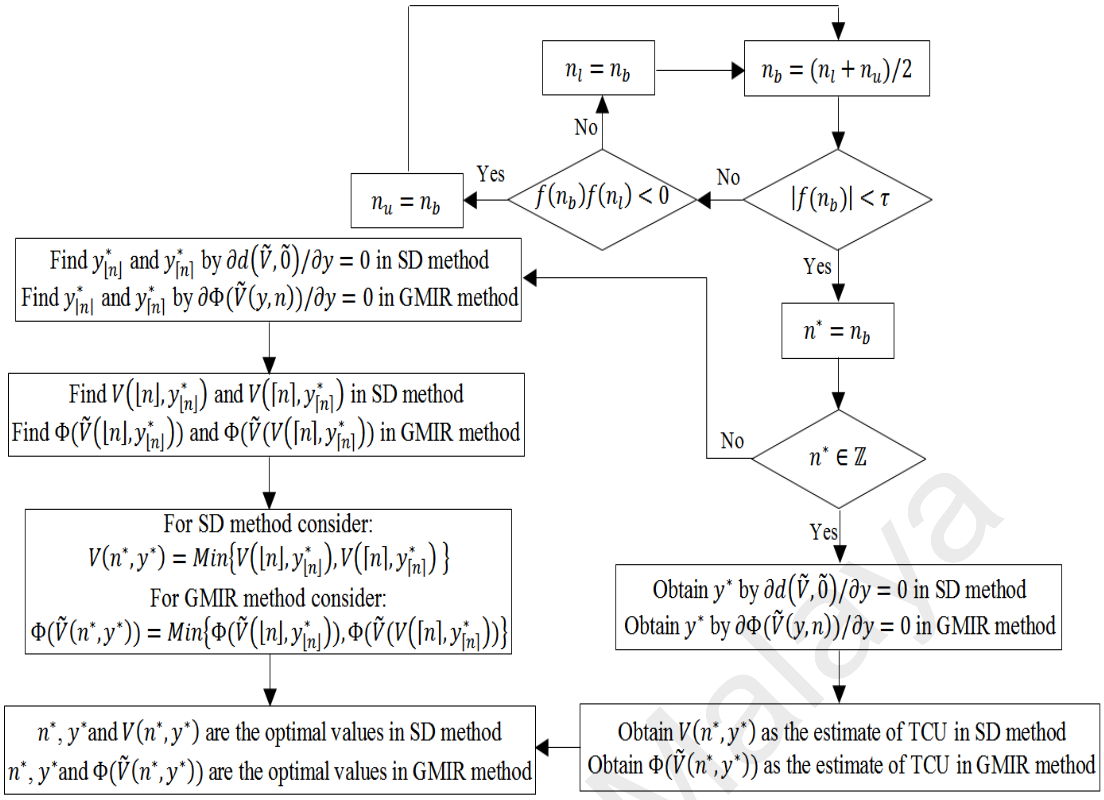

Figure 5.4: Proposed flowchart to find the optimal recovery lot size and the number of orders

5.7 Comprehensive Numerical Example

In this section, the effects of fuzzification on the developed model are analyzed and explained through a comprehensive numerical example. The results of each model are expounded separately. Besides, defining some comparative criteria, these results will be compared with each other simultaneously.

In order to compare the results of the investigated model with those of the crisp one, let us consider the data such that $k = 250$ units/day, $r = 100$ units/day, $C_s =$ \$20,000/setup, $C_o =$ \$2,000/order, $C_p =$ \$200/unit, $C_b =$ \$40/unit, $H_r =$ \$4/unit/day, $H_s =$ \$20/unit/day, $C_l =$ \$1000/day, $a = 0.003$ day/unit, and $b = -0.089$. Furthermore, as it is shown in Table 5.1, assuming some arbitrary sets for $\theta_i, i = 1,2,3,4$, the behaviour of the fuzzified models is examined. These parameters are selected such that $0 < \theta_1 < r$, $0 < \theta_3 < k$, and $0 < \theta_2, \theta_4$.

Due to the uncertainties inherent in the data and lack of existing knowledge about the whole of the inventory system, the mentioned parameters are usually determined according to the experiences of experts as decision makers. The mentioned fuzzy parameters are combined to build 45 iterations. For the fuzzified parameters k and r, five and nine levels of fuzziness are assumed, respectively.

Table 5.1: Considered fuzzy numbers

$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	$\boldsymbol{\tilde{r}}$	$\boldsymbol{\theta_3}$	$\boldsymbol{\theta_4}$	$\boldsymbol{\tilde{k}}$	$\boldsymbol{\theta_3}$	$\boldsymbol{\theta_4}$	$\boldsymbol{\tilde{k}}$
5	55	(95,100,155)	20	14	(230,250,264)	40	50	(210,250,300)
25	50	(75,100,150)	20	20	(230,250,270)	60	40	(190,250,290)
40	40	(60,100,140)	20	26	(230,250,276)	60	60	(190,250,310)
50	25	(50,100,125)	40	30	(210,250,280)	60	80	(190,250,330)
55	5	(45,100,105)	40	40	(210,250,290)			

Table 5.2 presents the results of the GMIR method. From Table 5.2, it is clear that for the fixed values of (θ_1, θ_2), and constant values of the optimal number of orders n^*, when the level of fuzziness increases by varying the values of (θ_3, θ_4), the optimal recovery lot size y^* decreases, but the optimal ordering lot size and the optimal total cost function per unit time increase.

In Table 5.2, for the fixed values of (θ_3, θ_4), as the estimate of the collection rate of the recoverable products $\Phi(\tilde{r})$ decreases by varying (θ_1, θ_2), the optimal recovery lot size y^* decreases for fixed n^*, but the optimal ordering lot size and the optimal total cost function per unit time increase. However, when n^* increases, y^* and TCU^* increase, and Q^*decreases. Besides, when $(\theta_1, \theta_2, \theta_3)$ are fixed, the optimal ordering lot size, and the optimal total cost function per unit time increase for fixed n^*.

Table 5.2: The results of effecting the crisp model by the GMIR method

No.	$\boldsymbol{\theta_1}$	$\boldsymbol{\theta_2}$	$\boldsymbol{\theta_3}$	$\boldsymbol{\theta_4}$	$\boldsymbol{n^*}$	$\boldsymbol{y^*}$	$\boldsymbol{Q^*}$	$\boldsymbol{TCU^*}$	$\boldsymbol{\Phi(\tilde{r})}$	$\boldsymbol{\Phi\left(\frac{\tilde{k}}{\tilde{r}}\right)}$	$\boldsymbol{\Phi\left(\frac{\tilde{r}}{\tilde{k}}\right)}$	$\boldsymbol{\Phi(\tilde{k}-\tilde{r})}$
1	5	55	20	14	4	678	233	41469.90	108.333	2.377	0.439	140.667
2				20	4	677	235	41686.67	108.333	2.388	0.438	141.667
3				26	4	676	236	41904.77	108.333	2.398	0.436	142.667
4			40	30	4	668	231	41480.96	108.333	2.384	0.446	140.000
5				40	5	712	200	41849.07	108.333	2.401	0.444	141.667
6				50	5	709	201	42216.42	108.333	2.419	0.442	143.333
7			60	40	5	702	194	41311.76	108.333	2.380	0.457	138.330
8				60	5	696	197	42059.24	108.333	2.415	0.454	141.667
9				80	5	689	200	42819.22	108.333	2.450	0.451	145.000
10	25	50	20	14	5	701	212	42096.73	104.167	2.509	0.423	144.833
11				20	5	700	213	42319.71	104.167	2.522	0.422	145.833
12				26	5	698	214	42543.94	104.167	2.536	0.421	146.833
13			40	30	5	688	209	42137.25	104.167	2.522	0.430	144.167
14				40	5	684	211	42520.33	104.167	2.544	0.429	145.833
15				50	5	681	213	42906.78	104.167	2.567	0.427	147.500
16			60	40	5	674	205	41998.27	104.167	2.522	0.441	142.500
17				60	5	665	208	42786.72	104.167	2.567	0.439	145.833
18				80	5	656	211	43588.19	104.167	2.611	0.436	149.167
19	40	40	20	14	5	675	226	42744.49	100.000	2.674	0.406	149.000
20				20	5	672	227	42978.14	100.000	2.690	0.405	150.000
21				26	5	670	229	43213.12	100.000	2.707	0.404	151.000
22			40	30	5	659	223	42823.76	100.000	2.694	0.413	148.333
23				40	6	698	200	43225.10	100.000	2.722	0.412	150.000
24				50	6	693	202	43621.69	100.000	2.750	0.411	151.667
25			60	40	6	686	194	42712.89	100.000	2.698	0.424	146.667
26				60	6	676	198	43522.93	100.000	2.754	0.422	150.000
27				80	6	665	201	44345.71	100.000	2.810	0.420	153.333
28	50	25	20	14	6	692	214	43384.83	95.833	2.853	0.389	153.167
29				20	6	689	215	43621.97	95.833	2.873	0.388	154.167
30				26	6	686	216	43860.42	95.833	2.893	0.387	155.167
31			40	30	6	675	212	43474.18	95.833	2.880	0.396	152.500
32				40	6	669	213	43882.84	95.833	2.913	0.395	154.167
33				50	6	664	215	44294.95	95.833	2.947	0.394	155.833
34			60	40	6	658	207	43376.64	95.833	2.887	0.405	150.833
35				60	6	646	210	44220.43	95.833	2.953	0.403	154.167
36				80	7	674	194	45061.16	95.833	3.020	0.402	157.500
37	55	5	20	14	6	669	224	43949.34	91.667	3.010	0.371	157.333
38				20	6	666	226	44190.54	91.667	3.032	0.371	158.333
39				26	6	663	227	44433.11	91.667	3.054	0.370	159.333
40			40	30	6	653	222	44040.65	91.667	3.037	0.377	156.667
41				40	6	647	224	44457.32	91.667	3.074	0.376	158.333
42				50	7	681	205	44871.33	91.667	3.111	0.375	160.000
43			60	40	7	676	197	43940.99	91.667	3.042	0.385	155.000
44				60	7	663	200	44784.16	91.667	3.116	0.383	158.333
45				80	7	650	203	45640.21	91.667	3.190	0.381	161.667

Table 5.3 shows the results of the signed distance method. Based on the columns 7, 8 and 9 of the Table 5.3, for the fixed values of (θ_1, θ_2), and the constant values of the optimal number of orders n^*, the behavior of the optimal recovery lot size y^*, the optimal

ordering lot size Q^*, and the optimal total cost function per unit time TCU^* is similar to that one which is explained in the GMIR method.

Table 5.3: The results of effecting the crisp model by the SD method

No.	θ_1	θ_2	θ_3	θ_4	n^*	y^*	Q^*	TCU^*	$d(\tilde{r},\tilde{0}_1)$	$d\left(\frac{\tilde{k}}{\tilde{r}},\tilde{0}_1\right)$	$d\left(\frac{\tilde{r}}{\tilde{k}},\tilde{0}_1\right)$	$d(\tilde{k}-\tilde{r},\tilde{0}_1)$
1	5	55	20	14	4	700	224	40758.79	112.50	2.278	0.456	136.00
2				20	4	699	226	41067.34	112.50	2.293	0.454	137.50
3				26	4	698	228	41376.43	112.50	2.309	0.452	139.00
4			40	30	4	694	222	40631.95	112.50	2.282	0.464	135.00
5				40	4	693	227	41148.65	112.50	2.308	0.461	137.50
6				50	4	692	231	41666.59	112.50	2.334	0.458	140.00
7			60	40	4	688	219	40213.02	112.50	2.271	0.475	132.50
8				60	4	685	227	41249.79	112.50	2.323	0.469	137.50
9				80	4	682	234	42290.74	112.50	2.374	0.464	142.50
10	25	50	20	14	4	668	243	41669.98	106.25	2.456	0.432	142.25
11				20	5	714	211	41980.99	106.25	2.475	0.430	143.75
12				26	5	713	213	42291.53	106.25	2.493	0.428	145.25
13			40	30	4	662	243	41553.24	106.25	2.467	0.440	141.25
14				40	5	707	212	42072.27	106.25	2.497	0.437	143.75
15				50	5	705	215	42592.09	106.25	2.527	0.434	146.25
16			60	40	4	656	239	41136.63	106.25	2.459	0.451	138.75
17				60	5	698	212	42181.57	106.25	2.520	0.446	143.75
18				80	5	695	220	43225.26	106.25	2.580	0.441	148.75
19	40	40	20	14	5	682	226	42545.29	100.00	2.657	0.407	148.50
20				20	5	681	228	42860.38	100.00	2.677	0.405	150.00
21				26	5	680	231	43175.82	100.00	2.698	0.404	151.50
22			40	30	5	676	226	42430.64	100.00	2.672	0.415	147.50
23				40	5	674	230	42957.30	100.00	2.707	0.412	150.00
24				50	5	672	234	43484.75	100.00	2.742	0.410	152.50
25			60	40	5	670	223	42014.63	100.00	2.667	0.426	145.00
26				60	5	666	231	43069.90	100.00	2.736	0.421	150.00
27				80	5	661	239	44127.79	100.00	2.806	0.417	155.00
28	50	25	20	14	5	651	242	43400.39	93.75	2.860	0.381	154.75
29				20	6	692	217	43716.64	93.75	2.883	0.380	156.25
30				26	6	691	220	44031.79	93.75	2.906	0.378	157.75
31			40	30	5	645	242	43281.22	93.75	2.879	0.389	153.75
32				40	6	685	219	43807.99	93.75	2.917	0.386	156.25
33				50	6	683	223	44334.75	93.75	2.956	0.384	158.75
34			60	40	5	639	240	42855.40	93.75	2.874	0.399	151.25
35				60	6	677	220	43913.43	93.75	2.951	0.395	156.25
36				80	6	673	228	44969.57	93.75	3.029	0.391	161.25
37	55	5	20	14	6	664	226	44180.64	87.50	3.044	0.355	161.00
38				20	6	663	229	44497.05	87.50	3.068	0.354	162.50
39				26	6	662	231	44813.73	87.50	3.093	0.353	164.00
40			40	30	6	659	226	44047.28	87.50	3.061	0.361	160.00
41				40	6	657	230	44575.80	87.50	3.102	0.359	162.50
42				50	6	655	234	45104.92	87.50	3.143	0.357	165.00
43			60	40	6	654	224	43608.03	87.50	3.054	0.370	157.50
44				60	6	650	231	44666.59	87.50	3.136	0.366	162.50
45				80	6	646	239	45727.15	87.50	3.218	0.363	167.50

However, one of the major differences between them is that as θ_1 increases, the decrease in y^* is higher for the GMIR method as compared to the signed distance method. Moreover, for the fixed values of (θ_3, θ_4), as the estimate of the collection rate of the recoverable products $d(\tilde{r}, \tilde{0}_1)$ decreases by varying (θ_1, θ_2), the reduction in the optimal recovery lot size y^* is lower than that of the GMIR method for fixed n^*, and therewith, the optimal ordering lot size Q^*, and the optimal total cost function per unit time TCU^* have the similar trends similar to the GMIR method.

The collection rate of the recoverable products from customers is an important factor in the reverse logistics literature. For positive levels of fuzziness, the estimations of this factor by the signed distance method are higher than those that are obtained applying the GMIR method. Besides, it is observed that, for negative levels of fuzziness, the estimations of the mentioned factor using the GMIR method returns higher value than the signed distance method. These interesting results should be taken into consideration in practical situations. In both methods, the optimal number of orders n^* increases by decreasing θ_2. In other words, the more the estimation of the difference between the demand rate of the serviceable products and the collection rate of the recoverable products $(k - r)$, the higher the optimal number of orders n^* will be.

Regarding a criterion defined in Eq. (5.49), the values of percentage changes for the optimal recovery lot size Q^*, and the optimal total cost function per unit time TCU^* compared to the crisp ones are calculated in Table 5.4. For example, the 3th column of Table 5.4 shows the percentage changes of optimal recovery lot size using the signed distance method (y^*_{SD}%).

$$\left(\frac{\text{Optimal Fuzzy Value} - \text{Optimal Crisp Value}}{\text{Optimal Crisp Value}}\right) \times 100 \tag{5.49}$$

Table 5.4: Comparing the results of the GMIR and SD methods with the crisp ones

No.	n^*	$y^*_{SD}\%$	$Q^*_{SD}\%$	$TCU^*_{SD}\%$	n^*	$y^*_{GMIR}\%$	$Q^*_{GMIR}\%$	$TCU^*_{GMIR}\%$
1	4	-0.709	5.660	-4.263	4	-3.830	9.906	-2.593
2	4	-0.851	6.604	-3.538	4	-3.972	10.849	-2.084
3	4	-0.993	7.547	-2.812	4	-4.113	11.321	-1.571
4	4	-1.560	4.717	-4.561	4	-5.248	8.962	-2.567
5	4	-1.702	7.075	-3.347	5	0.993	-5.660	-1.702
6	4	-1.844	8.962	-2.131	5	0.567	-5.189	-0.839
7	4	-2.411	3.302	-5.545	5	-0.426	-8.491	-2.964
8	4	-2.837	7.075	-3.110	5	-1.277	-7.075	-1.208
9	4	-3.262	10.377	-0.665	5	-2.270	-5.660	0.577
10	4	-5.248	14.623	-2.123	5	-0.567	0.000	-1.120
11	5	1.277	-0.472	-1.392	5	-0.709	0.472	-0.597
12	5	1.135	0.472	-0.663	5	-0.993	0.943	-0.070
13	4	-6.099	14.623	-2.397	5	-2.411	-1.415	-1.025
14	5	0.284	0.000	-1.178	5	-2.979	-0.472	-0.125
15	5	0.000	1.415	0.043	5	-3.404	0.472	0.782
16	4	-6.950	12.736	-3.375	5	-4.397	-3.302	-1.352
17	5	-0.993	0.000	-0.921	5	-5.674	-1.887	0.500
18	5	-1.418	3.774	1.530	5	-6.950	-0.472	0.401
19	5	-3.262	6.604	-0.067	5	-4.255	6.604	0.401
20	5	-3.404	7.547	0.673	5	-4.681	7.075	0.950
21	5	-3.546	8.962	1.414	5	-4.965	8.019	1.502
22	5	-4.113	6.604	-0.336	5	-6.525	5.189	0.587
23	5	-4.397	8.491	0.901	6	-0.993	-5.660	1.530
24	5	-4.681	10.377	2.140	6	-1.702	-4.717	2.462
25	5	-4.965	5.189	-1.313	6	-2.695	-8.491	0.327
26	5	-5.532	8.962	1.166	6	-4.113	-6.604	2.230
27	5	-6.241	12.736	3.650	6	-5.674	-5.189	4.162
28	5	-7.660	14.151	1.942	6	-1.844	0.943	1.905
29	6	-1.844	2.358	2.685	6	-2.270	1.415	2.462
30	6	-1.986	3.774	3.425	6	-2.695	1.887	3.022
31	5	-8.511	14.151	1.662	6	-4.255	0.000	2.115
32	6	-2.837	3.302	2.899	6	-5.106	0.472	3.075
33	6	-3.121	5.189	4.136	6	-5.816	1.415	4.043
34	5	-9.362	13.208	0.662	6	-6.667	-2.358	1.886
35	6	-3.972	3.774	3.147	6	-8.369	-0.943	3.868
36	6	-4.539	7.547	5.628	7	-4.397	-8.491	5.843
37	6	-5.816	6.604	3.774	6	-5.106	5.660	3.231
38	6	-5.957	8.019	4.518	6	-5.532	6.604	3.798
39	6	-6.099	8.962	5.262	6	-5.957	7.075	4.368
40	6	-6.525	6.604	3.461	6	-7.376	4.717	3.446
41	6	-6.809	8.491	4.703	6	-8.227	5.660	4.424
42	6	-7.092	10.377	5.946	7	-3.404	-3.302	5.397
43	6	-7.234	5.660	2.430	7	-4.113	-7.075	3.212
44	6	-7.801	8.962	4.916	7	-5.957	-5.660	5.192
45	6	-8.369	12.736	7.407	7	-7.801	-4.245	7.203
Average		-3.997	7.285	0.809		-3.959	0.073	1.446

In order to have a better comparison, based on this criterion, the behavior of the fuzzified model by both methods is compared for the mentioned optimal values in Figure

5.5 and 5.6, simultaneously. In Figure 5.5, the white stars show the situations that the optimal number of orders n^* increase one unit in the GMIR method, while the black stars show the similar increase in the signed distance method.

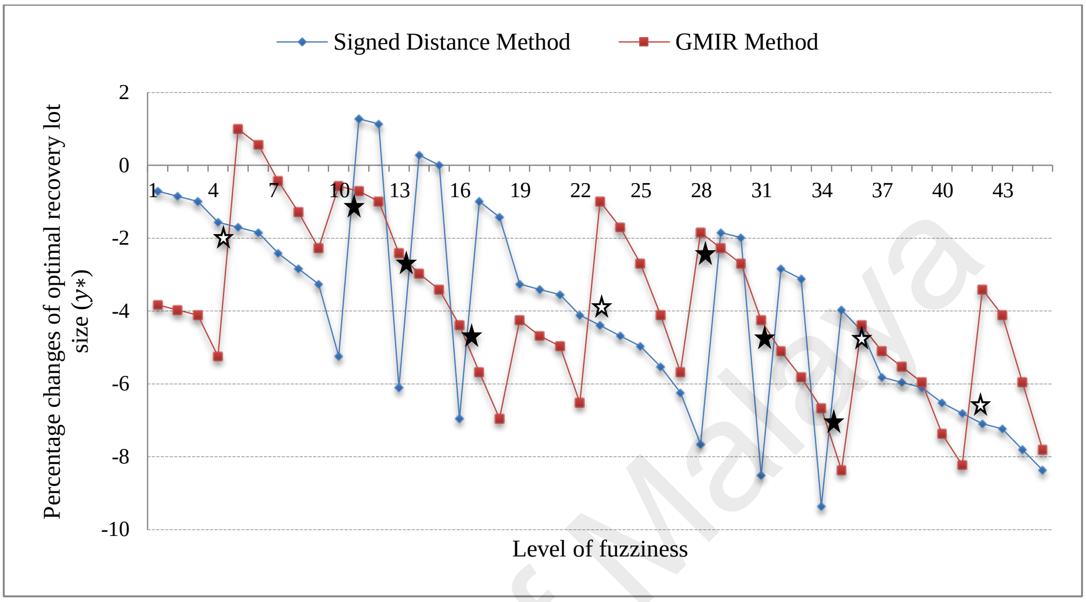

Figure 5.5: Comparing the SD and the GMIR method simultaneously considering the relative variation between the crisp and fuzzy situation for the optimal recovery lot size

According to the Figure 5.5, in states that the levels of fuzziness are similar, and also, the optimal number of orders n^* are equal, the percentage changes of the optimal recovery lot size in the GMIR method are negative $y^*_{GMIR}\% < 0$, and moreover, in these conditions, the difference between $y^*_{SD}\%$ and $y^*_{GMIR}\%$ is always positive $(y^*_{SD}\% - y^*_{GMIR}\% > 0)$. The average of percentage changes for the signed distance and the GMIR method is -3.997, and -3.959, respectively.

Figure 5.6 indicates that taking the percentage changes of the optimal total cost function per unit time into account, in general, there are similar increasing trends for both methods. When $\theta_2 > \theta_1$, except for two cases, the percentage changes of the optimal total cost by the signed distance method $TCU^*_{SD}\%$ are negative. Moreover, when $\theta_2 < \theta_1$, those are positive in all cases. Generally, the GMIR method takes priority than the signed distance method in adopting positive value for the percentage changes of the optimal total cost. Besides, the average percentage change of the total cost function for the GMIR method (1.446) is greater than the similar one for the signed distance method (0.809).

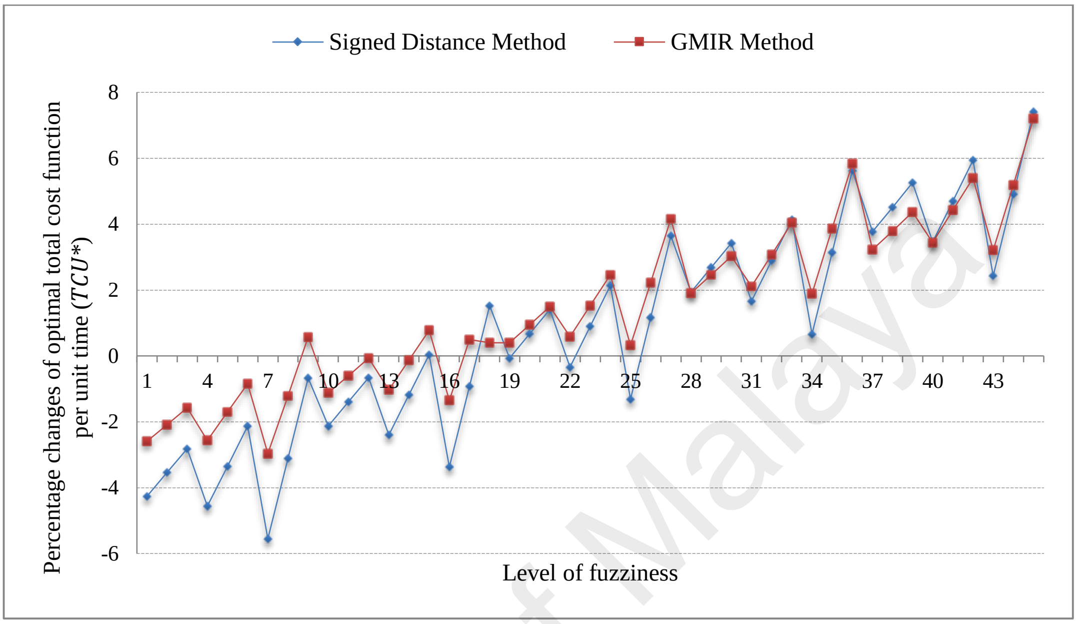

Figure 5.6: Comparing the SD and the GMIR method simultaneously considering the relative variation between the crisp and fuzzy situation for the optimal total cost function per unit time

Table 5.5 presents some descriptive statistics for the optimal values in each level of fuzziness for the collection rate of the recoverable products r, separately. In Table 5.5, "level of fuzziness" is a measure defined as the percentage deviation from the crisp value in each level of fuzziness. It is clear that the mentioned measure for the GMIR method is smaller than the similar one by the signed distance method. Although the overall average value of the total cost by the signed distance method is smaller than the calculated one by the GMIR method, its standard deviation in the GMIR method (1053) is smaller than the similar value (1345) by the signed distance method. It indicates that the GMIR method is more stable than the signed distance regarding the total cost. Considering the average for all levels, both methods lead to the same optimal value (677) for the recovering lot size. However, in this situation, the standard deviation by the signed distance is higher than the one obtained by the GMIR. Therefore, deciding on which method could be used depends on the target strategy that could focus on the total cost, the ordering lot size or the recovery lot size.

Besides, Table 5.6 shows the results of the descriptive statistics for the optimal values according to the criterion defined in Eq. (5.49) by varying θ_1 and θ_2, regardless of whether the percentage change is positive or negative. Unlike Table 5.5, in Table 5.6, these two methods are compared based on the optimal crisp values.

Table 5.5: Comparing the difference between the results of the GMIR and the SD method

	GMIR method				**SD method**			
	Level of fuzziness	$\boldsymbol{y^*}$	$\boldsymbol{Q^*}$	$\boldsymbol{TCU^*}$	***Level of fuzziness***	$\boldsymbol{y^*}$	$\boldsymbol{Q^*}$	$\boldsymbol{TCU^*}$
(θ_1,θ_2)		$(\mu,\sigma)^*$	$(\mu,\sigma)^*$	$(\mu,\sigma)^*$		$(\mu,\sigma)^*$	$(\mu,\sigma)^*$	$(\mu,\sigma)^*$
(5,55)	8.33%	(690,15)	(214,18)	(41866.45,437)	12.50%	(692,6)	(226,4)	(41155.92,571)
(25,50)	4.17%	(683,15)	(211,3)	(42544.21,471)	6.25%	(691,21)	(223,13)	(42078.17,575)
(40,40)	0%	(677,12)	(211,14)	(43243.09,496)	0%	(674,7)	(230,5)	(42962.94,582)
(50,25)	– 4.17%	(673,14)	(211,6)	(43908.60,517)	– 6.25%	(671,19)	(228,10)	(43812.35,582)
(55,5)	– 8.33%	(663,11)	(214,12)	(44478.63,521)	– 12.50%	(657,6)	(230,4)	(44580.13,582)
FOR ALL LEVELS		(677,16)	(212,12)	(43208.20,1053)		(677,19)	(227,9)	(42917.91,1345)

*μ, σ stands for mean, and standard deviation of the optimal values, respectively.

Table 5.6: Comparing the difference between the results of the GMIR and SD method based on the crisp values

	GMIR method			**SD method**		
	$\boldsymbol{y^*}$	$\boldsymbol{Q^*}$	$\boldsymbol{TCU^*}$	$\boldsymbol{y^*}$	$\boldsymbol{Q^*}$	$\boldsymbol{TCU^*}$
(θ_1,θ_2)	$(\mu,\sigma)^*$	$(\mu,\sigma)^*$	$(\mu,\sigma)^*$	$(\mu,\sigma)^*$	$(\mu,\sigma)^*$	$(\mu,\sigma)^*$
(5,55)	(2.52%, 1.70%)	(8.12%, 2.20%)	(1.79%, 0.78%)	(1.80%, 0.84%)	(6.81%, 2.00%)	(3.33%, 1.34%)
(25,50)	(3.12%, 2.12%)	(1.05%, 0.96%)	(0.66%, 0.42%)	(2.60%, 2.54%)	(5.35%, 6.23%)	(1.51%, 0.94%)
(40,40)	(3.96%, 1.72%)	(6.39%, 1.24%)	(1.57%, 1.17%)	(4.46%, 0.95%)	(8.39%, 2.13%)	(1.30%, 1.01%)
(50,25)	(4.60%, 2.03%)	(1.99%, 2.39%)	(3.14%, 1.21%)	(4.87%, 2.72%)	(7.49%, 4.69%)	(2.91%, 1.37%)
(55,5)	(5.94%, 1.54%)	(5.56%, 1.21%)	(4.47%, 1.22%)	(6.86%, 0.81%)	(8.49%, 2.04%)	(4.71%, 1.37%)
Overall	(4.03%,2.19%)	(4.62%,3.18%)	(2.33%,1.67%)	(4.12%,2.53%)	(7.31%,4.01%)	(2.75%,1.75%)

*μ, σ stands for mean, and standard deviation of the optimal values, respectively.

There is an interesting statistical relationship between the summations of the first and second deviation values of the collection rate of the recoverable products from customers and demand rate of the serviceable products in the fuzzy situation, respectively, and the percentage changes of the optimal total cost function by the signed distance and GMIR method. They are calculated with a simple regression in Figures 5.7 and 5.8 for the signed distance and GMIR method, respectively. As it is depicted, $(\theta_1 + \theta_4)$ has been considered

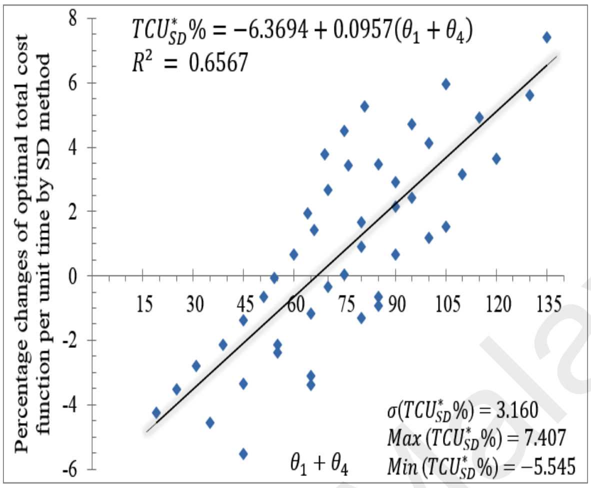

Figure 5.7: Estimating a simple regression relationship between the values obtained by $TCU^*_{SD}\%$ and summation of θ_1 and θ_4.

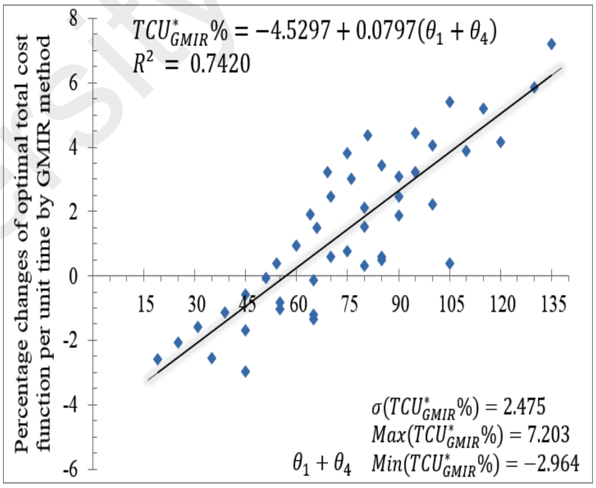

Figure 5.8: Estimating a simple regression relationship between the values obtained by $TCU^*_{GMIR}\%$ and summation of θ_1 and θ_4.

as independent variable; and $TCU^*_{SD}\%$ and $TCU^*_{GMIR}\%$ as dependent variable. The estimation of the percentage changes of the optimal total cost in the GMIR regression is about 10 percent better than the one obtained by signed distance regression because the value of R-square in the first one is about 10 units greater than the second one.

5.8 Chapter Summary

One of the most important issues in the reverse inventory models is the lack of historical data for the demand and return (collection) rate. Accordingly, estimation of the probability distributions of such parameters is not possible. Therefore, these parameters are not determined, and usually it is not logical to decide based on the crisp values while the situation is uncertain. With these perspectives, it is worthwhile to reconsider the reverse inventory system with the learning effect (Tsai, 2012) and provide an alternative approach.

In this chapter, two fuzzy models were proposed for a reverse inventory problem with the learning effect. In both models, the demand rate of the serviceable products and the collection rate of the recoverable products from customers were presented as fuzzy numbers. To estimate the total cost function per unit time in the fuzzy sense, and then the corresponding optimal recovery lot size and the number of orders for the newly purchased products, in the first model, the signed distance method was employed for the defuzzification, while the GMIR was used in the second one. These models were explained and solved by a comprehensive numerical example. The results of both are compared methods. Besides, it is concluded that it is important to decide which method should be chosen regarding the considered strategies.

It is noteworthy that although there are some researches in the forward supply chain literature, which considered fuzzy EOQ/EPQ, there is no similar work in the reverse supply chain literature that compares the performance of the well-known defuzzification methods such as the GMIR and the signed distance method. In this study, the performance of the mentioned methods is compared in the presence of learning in a reverse inventory model.

CHAPTER 6: APPLICATIONS TO CASE STUDIES

6.1 Introduction

In this chapter, the application of the previous developed fuzzy inventory models is introduced through real case studies. However, usually it is difficult to match the theoretical models to the real world scenario completely. Because there are many uncontrollable factors in the business environment that influence the whole of the inventory system.

The first fuzzy model introduced in chapter four is tried to be explained via a case from an automobile industry, and later, the second one suggested in chapter five is discussed through a milk manufacturing company. The most important focus of this chapter is to show that these fuzzy models can improve the policies for decision makers and managers in organization.

6.2 First Case Study

The first case is related to a supply chain network in an automobile industry where there is a supplier and manufacturer who produces some parts of a special automobile to sell to the other manufacturers. Firstly, the manufacturer and the supplier are introduced and the relation of the whole supply chain is depicted.

6.2.1 A Supply Chain in an Automobile Industry

The manufacturer that is studied here is a company called Auto Chassis International Pars (ACI Pars) belonged to the supply chain network of Groupe Renault International in Tehran, Iran. This company is established in 2005 incorporating Robat Machine Co. and Renault by a joint venture as 23% and 77% stock respectively. The company produces and assembles automobile parts specially chassis systems and components of L-90 (Tondar 90) and U-90 products. Figure 6.1 shows a sample of produced L-90.

Figure 6.1: L-90 (Tondar 90)

The plant works based on the Renault Production System (SPR) and has annual production capacity of 200,000 parts for the vehicle. ACI Pars works with other suppliers to produce the parts. In this research, the most important one that is MSTOOS Co. is considered (http:www.acipars.co.ir).

Figure 6.2 depicts the supply chain network and the relationship between the plants. As it is clear, MSTOOS Co. acts as the supplier of ACI Pars and Renault Pars which is another company buying the items and selling them to the two main automobile plants in Iran (i.e. Iran Khodro and Pars Khodro).

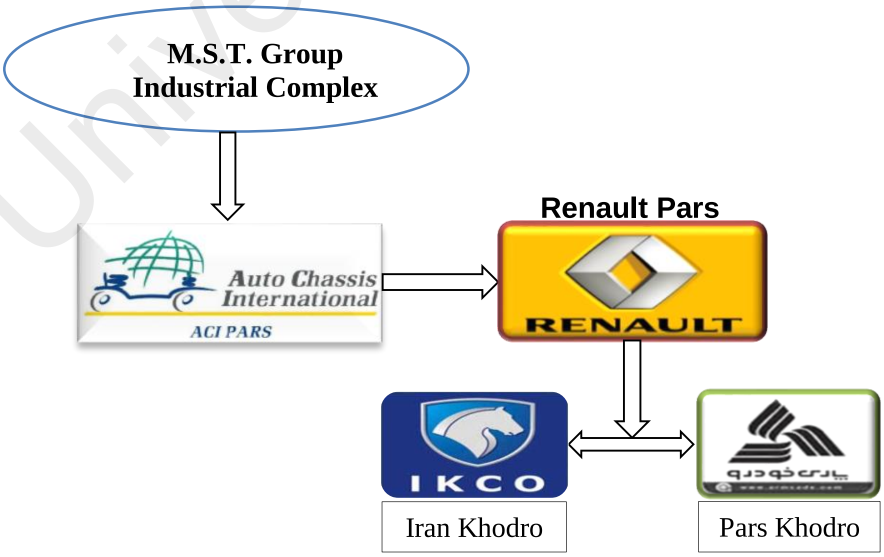

Figure 6.2: Supply chain network of considered automobile industry

6.2.2 Gathering the Information

ACI Pars includes 10 departments which are Production, Supply Chain, Projects, Human Resources, Engineering, Renault Production System, Quality, Purchasing, Finance, and IT&IS. As the warehouse and supplier quality sections are placed in Supply Chain and Quality departments, the required information is gathered from these sections. In fact, Supply Chain department has six sections including customer service, internal logistics, external logistics, customs, PHF, and warehouses. Besides, quality department has two sections which are customer quality and internal quality (http:www.acipars.co.ir), Appendix F/F.1.

The information has been gathered for two main parts that are Brake Disc and Rear Hub Drum as depicted in Figures 6.3 and 6.4 respectively. The mechanism of Brake Disc is briefly shown in Figure 6.5. When the piston (part number 3) is pushed by a class 2 lever (part number 2), it squeezes a hydraulic brake fluid (part number 4). This process causes the force to another piston located in the wider cylinder (par number 5). The Brake Disc is the part number 8 and it causes the wheel stops when the wider piston (part number 5) pushes the brake pad shown as part number 6 and 7.

Figure 6.3: Brake Disc 259*20,6' painted

Figure 6.4: Rear Hub Drum 8' painted

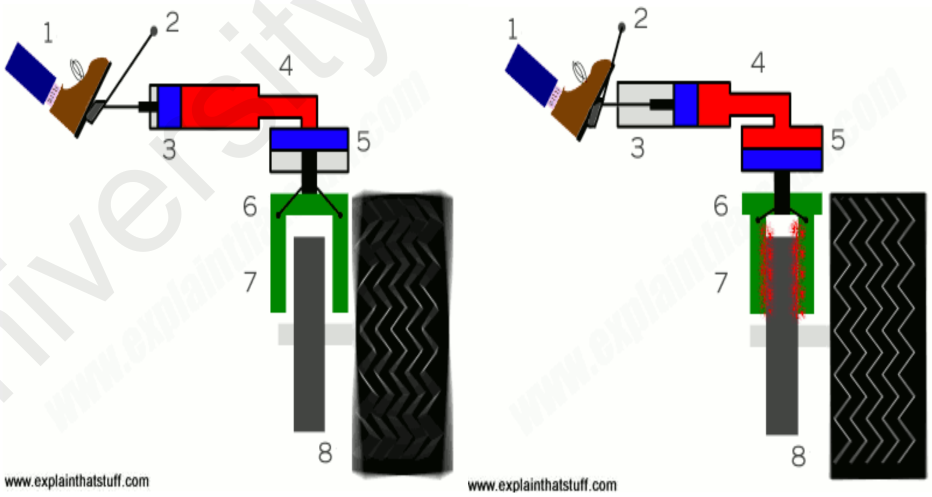

Figure 6.5: Function of the Brake Disc (www.explainthatstuff.com) Woodford (2016)

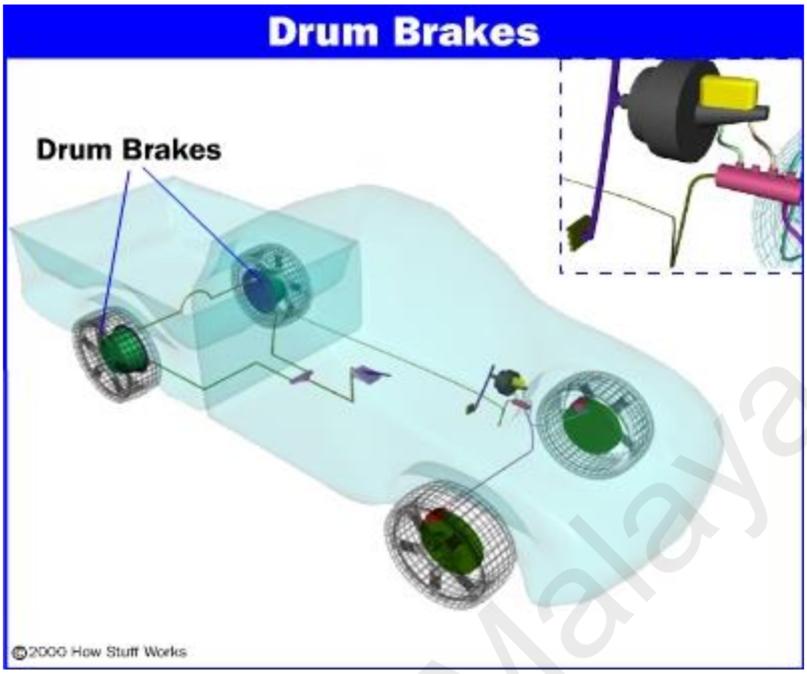

Figure 6.6: Function of the Rear Hub Drum (www.auto.howstuffworks.com)
Nice (2016)

Rear Hub Drum works on the same principle as Brake Disc. In this system, as shown in Figure 6.6, the surface is called a drum. Usually automobiles have Rear Hub Drums (drum brakes) on the rear wheels and disc brakes on the front. Although drum brakes have more parts than disc brakes, they are less expensive to produce. They are harder to service for the manufacturer.

Table 6.1 shows the demand for automobile which is announced by Renault Pars to ACI Pars. Then, according to the available data, eight contracts have been considered between ACI Pars and its supplier (MSTOOS). A period between 2012 till 2015 which the duration of each contract is divided to 6 months is considered.

For more clarifications, lets explain the first contract. This contract is started from January 2012 till Jun 2012 between ACI Pars and MSTOOS. According to this contract, the parts of 7200 automobiles should be satisfied by the ACI Pars that is the supplier of

Pars Renault. As our models are investigated for the Brake Disc and Rear Hub Drum, the demand of these parts are calculated in Table 6.2 and 6.3 respectively. It is clear that each chassis produced by ACI Pars needs two Brake Discs and Rear Hub Drums. The related information of the demand in Table 6.1 has been extracted according to the statistics revealed by the Iran Vehicle Manufacturers Association (Appendix F/F.2).

Table 6.1: Value of the demand for chassis announced by Renault Pars

2012	2012	2013	2013	2014	2014	2015	2015
Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec
7200	8000	9500	12550	12000	14800	16750	18100

Table 6.2: Value of the demand for Brake Disc announced by ACI Pars to MSTOOS

2012	2012	2013	2013	2014	2014	2015	2015
Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec
14400	16000	19000	25100	24000	29600	33500	36200

Table 6.3: Value of the demand for Rear Hub Drum announced by ACI Pars to MSTOOS

2012	2012	2013	2013	2014	2014	2015	2015
Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec
14400	16000	19000	25100	24000	29600	33500	36200

Variable costs include raw materials cost, production wages of workers, sales commissions, packaging supplies, and shipping costs. Tables 6.4 and 6.5 show the variable costs for Brake Disc and Rear Hub Drum respectively. Although variable costs may vary from one month to the other one, it is considered averagely for each contract.

Table 6.4: Variable costs for Brake Disc (Rial*/Unit)

2012	2012	2013	2013	2014	2014	2015	2015
Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec
99000	100000	112000	113000	120000	122500	128500	136200

*Iran's currency (1$ ≈ 34400 Rial)

Table 6.5: Variable costs for Rear Hub Drum (Rial/Unit)

2012	2012	2013	2013	2014	2014	2015	2015
Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec
88000	90000	102000	105000	110000	113000	120500	126200

Fixed costs are those that do not change with fluctuations in production level or sales volume. They usually include such expenses as rent, insurance, equipment leases, payments on loans, depreciation, and other costs for running the business such as management salaries, and advertising. Tables 6.6 and 6.7 show related data for fixed costs of ordering the Brake Disc and Rear Hub Drum for considered contracts.

Table 6.6: Fixed costs for Brake Disc (Rial/Order)

2012	2012	2013	2013	2014	2014	2015	2015
Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec
1089000	1100000	1120000	1135000	1200000	1220000	1385000	1402000

Table 6.7: Fixed costs for Rear Hub Drum (Rial/Order)

2012	2012	2013	2013	2014	2014	2015	2015
Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec
990000	900000	1020000	1025000	1170000	1180000	1300500	1306200

Selling price of good quality Brake Disc and Rear Hub Drum and selling price of defective Brake Disc and Rear Hub Drum are presented in Tables 6.8 and 6.9. These prices are based on the derived information from Saipa (Appendix F/F.3). It is assumed that selling price of defective items is 25 percent of the selling price of the good quality ones.

Table 6.8: Selling price of good and defective quality Brake Disc (Rial/Item)

Year	2012	2012	2013	2013	2014	2014	2015	2015
Month	Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec
G*	199050	200000	222100	232000	250000	275000	300500	320000
D**	49763	50000	55525	58000	62500	68750	75125	80000

*Good items, **Defective items

Table 6.9: Selling price of good and defective quality Rear Hub Drum (Rial/Item)

Year	2012	2012	2013	2013	2014	2014	2015	2015
Month	Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec	Jan-Jun	Jul-Dec
G*	189050	190500	212100	222000	245000	265000	285500	290000
D**	47262	47625	53025	55500	61250	66250	71375	72500

*Good items, **Defective items

Tables 6.10 and 6.11 present costs of screening of items for all contracts. This process is done by the person in charge from the quality department. Each time that a lot is received through the steel pallets, the items are checked and defective items are separated from non-defective ones manually. Figure 6.7 shows a pallet made of steel.

Figure 6.7: Steel pallet for screening

Table 6.10: Screening costs for Brake Disc (Rial/Unit)

2012 Jan-Jun	2012 Jul-Dec	2013 Jan-Jun	2013 Jul-Dec	2014 Jan-Jun	2014 Jul-Dec	2015 Jan-Jun	2015 Jul-Dec
1000	1000	1200	1300	1500	1650	1700	1800

Table 6.11: Screening costs for Rear Hub Drum (Rial/Unit)

2012 Jan-Jun	2012 Jul-Dec	2013 Jan-Jun	2013 Jul-Dec	2014 Jan-Jun	2014 Jul-Dec	2015 Jan-Jun	2015 Jul-Dec
990	1000	1100	1250	1400	1450	1600	1650

Screening rate of Brake Disc and Rear Hub Drum are shown in Tables 6.12 and 6.13. Because Brake Disc has a more complex structure, analyzing the quality of it needs generally more time in comparison with the Rear Hub Drum.

Table 6.12: Screening rate of Brake Disc (Unit/Time)

2012 Jan-Jun	2012 Jul-Dec	2013 Jan-Jun	2013 Jul-Dec	2014 Jan-Jun	2014 Jul-Dec	2015 Jan-Jun	2015 Jul-Dec
28000	32200	30000	50000	45000	60000	70500	75000

Table 6.13: Screening rate of Rear Hub Drum (Unit/Time)

2012 Jan-Jun	2012 Jul-Dec	2013 Jan-Jun	2013 Jul-Dec	2014 Jan-Jun	2014 Jul-Dec	2015 Jan-Jun	2015 Jul-Dec
25000	30000	40000	48000	50000	60600	65500	70200

After separation of the defective and non-defective items, they should be held in different warehouses. The holding price of good and defective quality items are presented in Tables 6.14 and 6.15.

The main costs that should be taken into account for holding costs include rent of the warehouse, cost of the maintenance of warehouse, opportunity cost, equipment cost, insurance and security cost, and other direct expenses. Therefore, it is clear that the holding costs of the good items and the defective items are different as it is supposed in chapter 4. Information related to Tables 6.4-6.7 and Tables 6.10-6.15 were derived according to the dealerships of Iran Khodro (Appendix F).

Table 6.14: Holding costs of Brake Disc items (Rial/Unit/Time)

Quality	2012 Jan-Jun	2012 Jul-Dec	2013 Jan-Jun	2013 Jul-Dec	2014 Jan-Jun	2014 Jul-Dec	2015 Jan-Jun	2015 Jul-Dec
Good	1000	1000	1200	1300	1450	1500	1650	1700
Defective	200	200	250	250	300	300	450	500

Table 6.15: Holding costs of Rear Hub Drum (Rial/Unit/Time)

Quality	2012 Jan-Jun	2012 Jul-Dec	2013 Jan-Jun	2013 Jul-Dec	2014 Jan-Jun	2014 Jul-Dec	2015 Jan-Jun	2015 Jul-Dec
Good	900	950	1100	1250	1350	1400	1550	1600
Defective	150	150	200	200	200	250	350	400

6.2.3 Adapting to the First Fuzzy Model

To apply the provided information into the developed fuzzy model suggested in chapter 4, it should be changed to the fuzzy situation. In order to do this process, the following triangular fuzzy number which is a potential way to change the crisp information to the fuzzy one is built:

$$(Minimum\ of\ parameter, Average\ of\ parameter, Maximum\ of\ parameter)$$

Therefore, the following information is provided. The parameters of learning curve function are assumed as what supposed in chapter 4.

For Brake Disc:

$D = (14400{,}24725{,}36200)$ units per contract, $K = (1089000\ {,}1206375{,}1402000)$ \$ per order, $c = (99000{,}116400{,}136200)$ Rial per unit, $s = (199050{,}249831{,}320000)$ Rial per unit, $v = (49763{,}62458{,}80000)$ Rial per unit, $d = (1000{,}1393{,}1800)$ Rial per unit, $h_g = (1000{,}1350{,}1700)$ Rial per unit per contract, $h_d = (200{,}306{,}500)$ Rial per unit per contract, $x = (28000{,}48838{,}75000)$ units per contract, $\gamma = 819.76$, $\alpha = 70.07$, and $\beta = 0.79$.

For Rear Hub Drum:

$D = (14400{,}24725{,}36200)$ units per contract, $K = (990000{,}1111462{,}1306200)$ \$ per order, $c = (88000{,}106838{,}126200)$ Rial per unit, $s = (189050{,}237393{,}290000)$ Rial per unit, $v = (47262{,}59348{,}72500\ )$ Rial per unit, $d = (990{,}1305{,}1650)$ Rial per unit, $h_g = (900{,}1263{,}1600)$ Rial per unit per contract, $h_d = (150{,}238{,}400)$ Rial per unit per contract, $x = (25000{,}48663{,}70200)$ units per contract, $\gamma = 819.76$, $\alpha = 70.07$, and $\beta = 0.79$.

6.2.4 Calculations

According to the first fuzzy model introduced in chapter 4 and gathered information, the required calculations were done and the following figures were derived. The aim is finding the optimal order quantity and total profit per unit time for Brake Disc and Rear Hub Drum when the company is dealing with uncertain business environment.

In order to avoid tedious and repetitive calculations, using the introduced optimization method, the results for 10 and 70 percent level of fuzziness are obtained. However, interpretation of other levels of uncertainty is similar as well. In fact, the mentioned levels present low and high level of fuzziness.

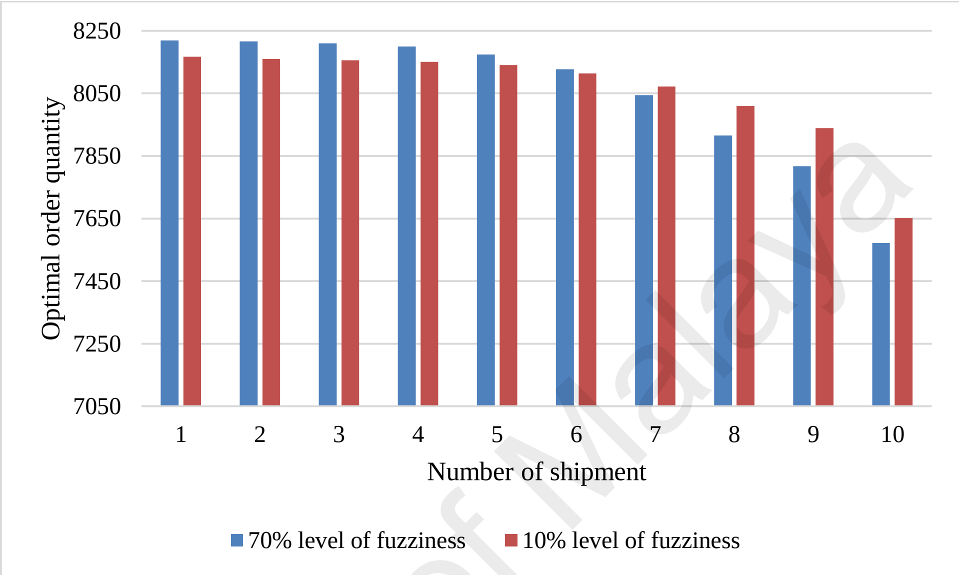

Figure 6.8: Optimal order quantity for Brake Disc for 10 and 70 percent level of fuzziness

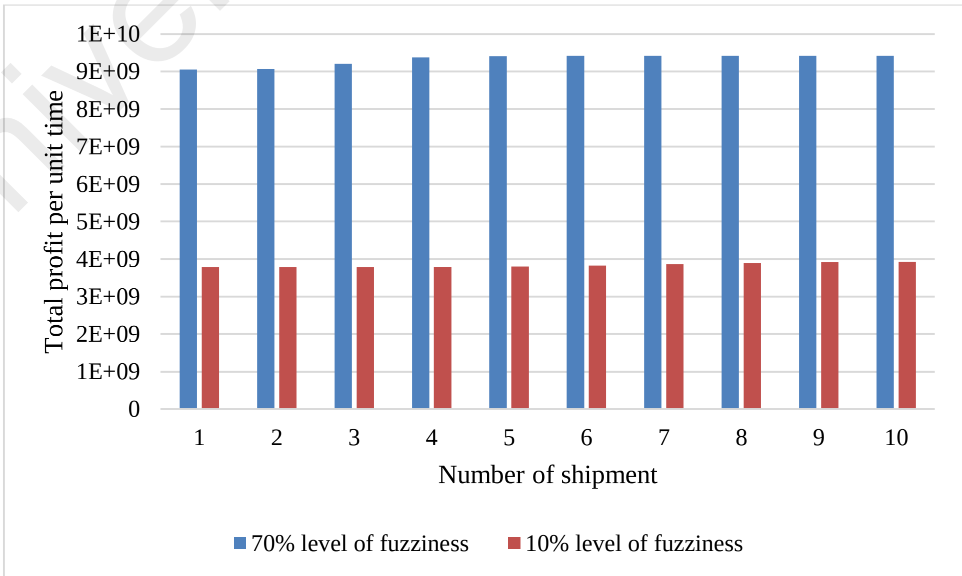

Figure 6.9: Total profit per unit time for Brake Disc for 10 and 70 percent level of fuzziness

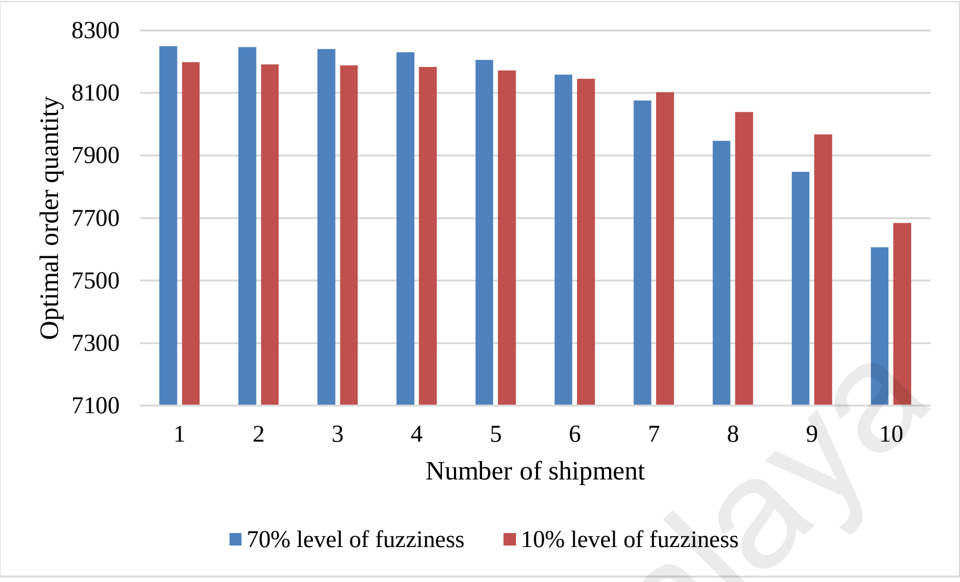

Figure 6.10: Optimal order quantity for Rear Hub Drum for 10 and 70 percent level of fuzziness

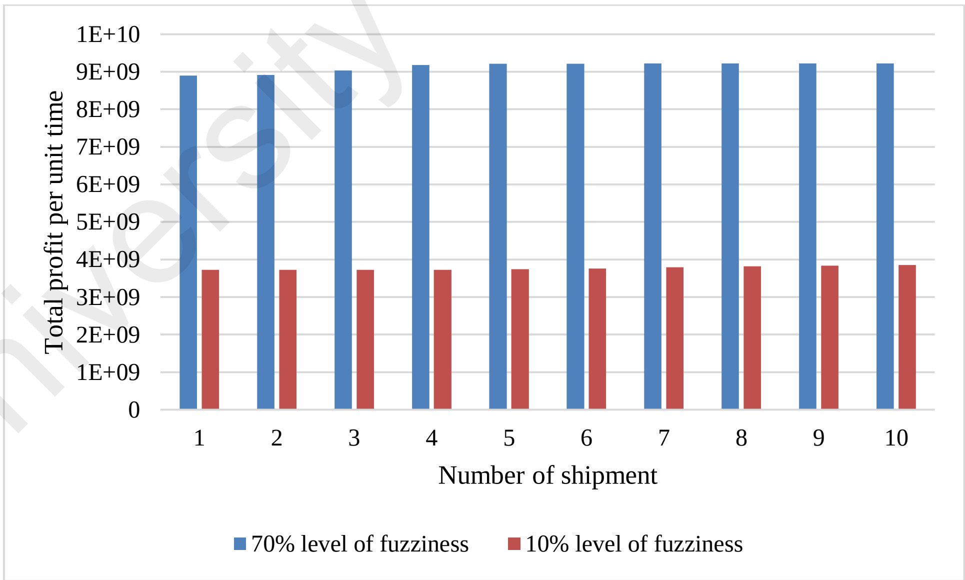

Figure 6.11: Total profit per unit time for Rear Hub Drum for 10 and 70 percent level of fuzziness

6.2.5 Analyzing the Results

Figures 6.8 and 6.10 show the optimal order quantity of Brake Disc and Rear Hub Drum for 10 and 70 percent levels of fuzziness respectively. As it is clear, the trend of both items is similar. For both levels of fuzziness, when the number of shipment increases, the optimal order quantities decrease. This is the result of learning effect during the contracts. The optimal order quantity of Brake Disc and Rear Hub Drum decreases more for 70 percent level of fuzziness in comparison with 10 percent level of fuzziness when the number of shipment increases. Therefore, the more the level of fuzziness, the more the effect of learning. This shows the importance of learning when the uncertainty of business environment increases. It can justify investment on knowledge transfer to increase the effect of learning.

Figures 6.9 and 6.11 present the total profit per unit time for Brake Disc and Rear Hub Drum for 10 and 70 percent levels of fuzziness respectively. It can be seen that by increasing the number of shipment the total profit per unit time increases. It is due to the effect of learning. Therefore, although the optimal order quantities decrease, total profit per unit time increases for both items (i.e. Brake Disc and Rear Hub Drum). However, this improvement is more for 70 percent level of fuzziness in comparison with 10 percent level of fuzziness.

For Brake Disc items, when the level of fuzziness is 10 and 70 percent, this increase is 3.87 and 4.06 percent respectively. For Rear Hub Drum, when the level of fuzziness is 10 and 70 percent, this increase is 3.47 and 3.64 percent respectively. Thus, when a flexible business environment is met with a high degree of impreciseness in information, investment on some parts of the company that amplify the learning can be an appropriate strategy for decision makers. The more the level of uncertainty is increased, the more attention to the effect of learning process is necessary.

6.3 Second Case Study

The second case study is related to the application of the second fuzzy model introduced in chapter five through the milk industry. At first, let's introduce the companies and explain their activities.

6.3.1 Milk Manufacturing Company

In this section, the activity of the factories that are included in our second case study is explained and is briefly introduced. However, this case can be applied to other similar factories with the same conditions.

A local milk manufacturing company which is a part of a big holding is considered. The main factory that manages this holding is Iran Dairy Industries Co. (IDIC) with more than six decades of experience in industrial production and processing of milk and dairy products. The first plan of establishing of this factory was raised by signing agreements on behalf of Planning Organization and Ministry of Health of Iran and UNICEF Institute affiliated to United Nations in the year 1954. Since 2001, this factory has selected *Pegah* brand and introduced it to its customers. In addition to the mentioned local company, Iran Dairy Industries Company has 19 affiliated factories that are active in 13 large province of Iran. They offer 600 pasteurized and sterilized dairy products such as milk, yoghourt, cream, cheese, butter, ice cream, dairy powders, and herbal carbonated drinks to the market in diverse packaging. All the products that are produced under management of this holding, are named as Pegah Pasteurized Milk Companies. Currently IDIC is the biggest dairy factory in Middle East supplying 30% of domestic Iranian market with a population of 70 million people. It produces 1.5 million tons of milk per year.

The local milk manufacturing company investigated in this study is called Pegah Hamedan. It is located in Hamedan Province in the west of Iran. It covers an area of 19,546 km^2 with more than 1.8 million population. Every day, more than 200 tons of raw milk is gathered by this company from local farms. After doing some processes, other dairy products are produced. Moreover, this company acts as the supplier of other food factories in the state.

6.3.2 Polyethylene Terephthalate (PET)

PET was patented by John Rex Whinfield and James Tennant Dickson in England in 1941. As a plastic material, Polyethylene Terephthalate (PET) is a simple long-chain polymer of ethylene glycol with either terephthalic acid or dimethyl terephthalate ($C_6H_4(CO_2CH_3)_2$) that it has increasing application in food and drink packaging. Because of its unique physical properties, including chemical inertness, it is commonly used as packaging material for drinking water, mineral water, cooking oil, edible oils and carbonated beverages in the form of stretched blown bottles (Farhoodi et al., 2009; Farhoodi et al., 2013).

Due to some special characteristics such as safety, strongness, transparency and versatileness manufacturers prefer PET. Also, customers choose it for its light weight and shatter-resistance. The most important factor of PET is that it is made from the recycled material, and these material can be recycled again and again to make new bottles, fiber for carpets, fabric for t-shirts or fleece jackets, non-food containers, winter coats, polyester fiber, strapping, sheet and thermoformed packaging. Moreover, recycled material of PET can be used in parts of automotive industry such as headliners, bumpers, and door panels.

6.3.3 Milk Supply Chain Network

In this section, the milk supply chain network is explained and the relationships and the processes are depicted. At the center of the investigated chain, the local milk company (i.e. Pegah Hamedan) is located. As it is mentioned before, this company is under the management of the IDIC which is the main factory of the holding. Figure 6.12 shows the related network.

According to our scenario, this local company collects and buys raw milk from local farms around the Hamedan province. After doing some particular processes such as testing for antibiotic residues, microbial enumeration, pasteurization, and homogenization, these milks are packed in containers made of PET and can be ordered by other food and dairy factories to provide other products.

As it is discussed in previous section, these PET-made containers are recyclable and highly sustainable. There is another company in Pegah holding that produces PET containers and primary materials of dairy products packaging including multi-layer special bags and special polymer materials. This company is Pegah Packaging Industry shown with yellow color in Figure 6.12. In fact, it is the supplier of PET-containers to the Pegah Hamedan.

However, because of the recyclable characteristics of the PET materials, after doing some repairing processes such as cleaning and testing, some defective containers can be recycled and renewed in Pegah Hamedan and returned back to the network processes. The dairy factories are motivated to take advantage of some discounts if they return the defective containers.

Due to the weather conditions in Hamedan province, production of the milk by local farms fluctuates. It varies from one season to another season. The temperature difference between the coldest and warmest days of a year usually is approximately 40˚ C. In cold winters the production of the milk by local farms decreases while it increases in spring and summer. Therefore, there is an uncertainty in production of the raw milk that indirectly will affect the demand for the PET-made containers.

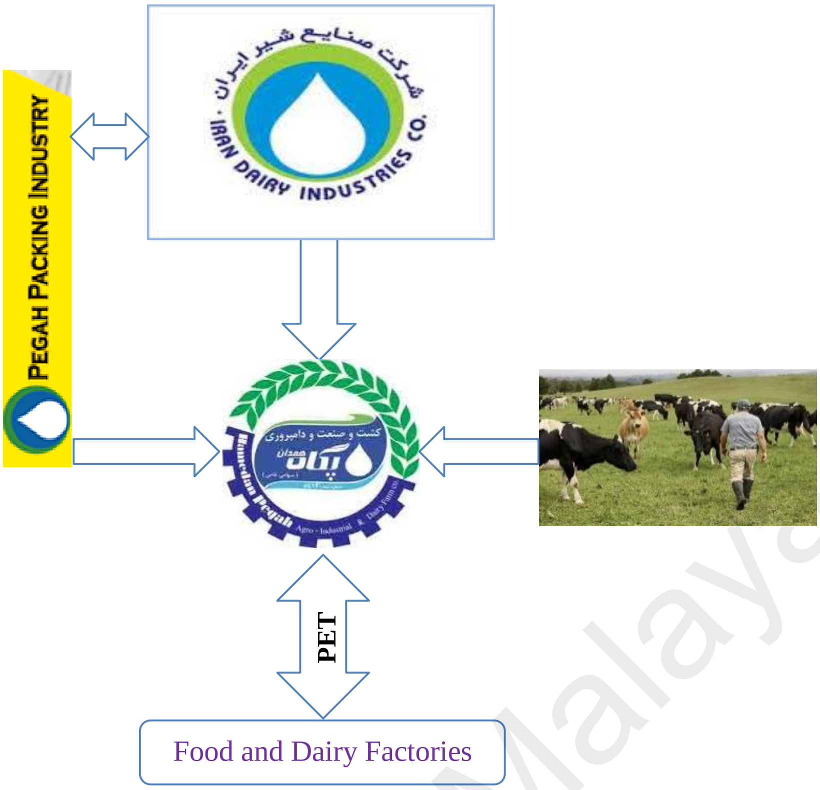

Figure 6.12: Milk supply chain network

6.3.4 Related Information to Adapt the Second Fuzzy Model

In order to meet the demand of PET-made containers, Pegah Hamedan has adopted a policy of one setup for recovery and multi order for new products. This is consistent with the fuzzy reverse inventory model proposed in previous chapter. Based upon their historical data and the notations defined in chapter 5, the following information were considered. As production of containers depends on the production of milk, the official reports of production of milk have been used to estimate the provided information (Appendix F/F.4).

$k = 1500$units/month, $r = 300$units/month, $C_s = \$10{,}000$/setup, $C_o = \$1100$/order, $C_p = 80\$$/unit, $C_b = \$20$/unit, $H_r = \$4$/unit/day, $H_s = \$10$/unit/day, $C_l = \$5000$/day, $a = 0.008$day/unit, $b = -0.152$.

The suggested fuzzy reverse inventory model applied to this scenario is optimized to find the number of orders for the newly purchased PET containers during a cycle, recovery lot size of PET containers for each production run, ordering lot size for the newly purchased PET containers and total cost function per unit time.

6.3.5 Calculations

According to the provided data and the fuzzy reverse inventory model introduced in chapter 5, the results have been calculated for some levels of fuzziness in Table 6.16 for two defuzzification methods (i.e. GMIR and SD methods). Besides, Tables 6.17 and 6.18 show the results according to the average of each parameter in each level of fuzziness. Furthermore, the effect of these methods can be compared based on the Figures. 6.13-6.16.

Table 6.16: The results of affecting the inventory system by two defuzzification methods

(**Parameter**/Level of fuzziness%)	GMIR Method				SD Method			
	n^*	y^*	Q^*	TCU^*	n^*	y^*	Q^*	TCU^*
($\boldsymbol{k}$/10%; $\boldsymbol{r}$/10%)	10	244	113	7497.04	7	265	160	7593.89
($\boldsymbol{k}$/10%; $\boldsymbol{r}$/30%)	10	293	152	9002.76	6	303	227	9116.66
($\boldsymbol{k}$/30%; $\boldsymbol{r}$/10%)	22	234	75	11508.65	10	262	201	10666.23
($\boldsymbol{k}$/30%; $\boldsymbol{r}$/30%)	19	252	110	14705.84	8	283	305	12795.70

Table 6.17: The effect of fuzzification of each parameter separately in GMIR method

	GMIR Method**			
Percent	**10%**		**30%**	
Fuzzified parameter	***k***	***r***	***k***	***r***
$\boldsymbol{n^*}$	10	16	20.5	14.5
$\boldsymbol{y^*}$	268.5	239	243	272.5
$\boldsymbol{Q^*}$	132.5	94	92.5	131
$\boldsymbol{TCU^*}$	8249.90	9502.85	13107.25	11854.30

** The values of the table are calculated averagely in each level according to the results of Table 6.16.

Table 6.18: The effect of fuzzification of each parameter separately in SD method

	SD Method**			
Percent	**10%**		**30%**	
Fuzzified parameter	***k***	***r***	***k***	***R***
n^*	6.5	8.5	9	7
y^*	248	263.5	272.5	293
Q^*	193.5	180.5	253	266
TCU^*	8355.28	9130.06	11730.97	10956.18

** The values of the table are calculated averagely in each level according to the results of Table 6.16.

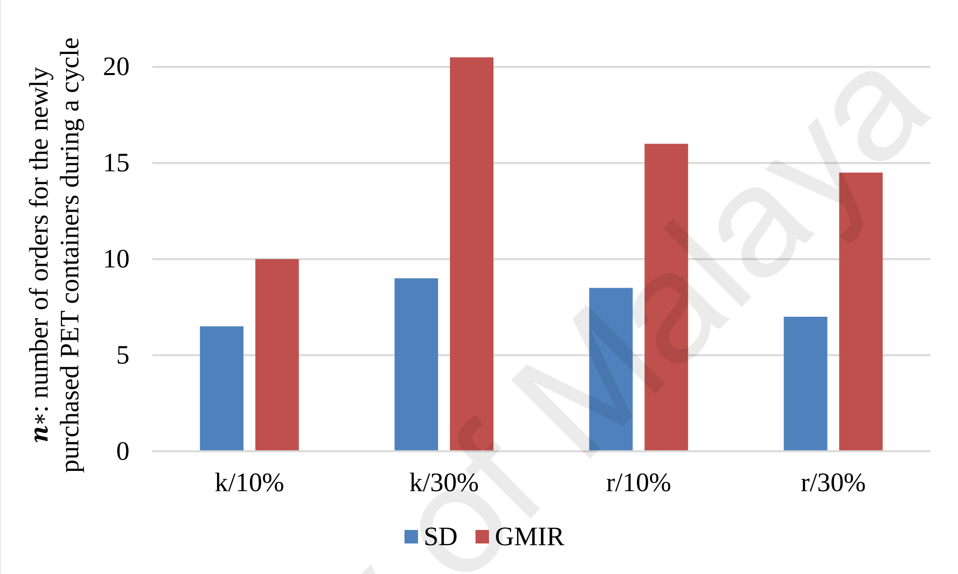

Figure 6.13: Comparison the effect of fuzzification of parameters on the number of orders for the newly purchased items during a cycle in both defuzzification methods

Figure 6.14: Comparison the effect of fuzzification of parameters on the recovery lot size of items for each production run in both defuzzification methods

Figure 6.15: Comparison the effect of fuzzification of parameters on the ordering lot size for the newly purchased items in both defuzzification methods

Figure 6.16: Comparison the effect of fuzzification of parameters on the total cost function per unit time in both defuzzification methods

6.3.6 Analyzing the Results

Table 6.16 shows the results for two different levels of fuzziness. As it is clear, in this case when there is the lowest level of fuzziness in the system, the GMIR method gives the lower total cost per unit time. On the other hand, for the maximum level of fuzziness, the total cost per unit time derived by the SD method is lower.

Tables 6.17 and 6.18 show the results of each parameters separately for the GMIR and SD methods respectively. These results are calculated based on the average of optimal values of the system. As it is clear, in the GMIR method when the uncertainty of demand increases, the optimal number of orders for the newly purchased PET containers increases. However, this increase is less in SD method. The effect of uncertainty of return rate on the optimal number of newly purchased items in both methods is lower as compared to the demand. As can be seen in Table 6.18, increasing in the uncertainty of demand or return rate leads to the increasing in the optimum value of recovery lot size and ordering lot size for the newly purchased items. In GMIR method, by increasing in the fuzziness of demand, the optimum value of recovery lot size and ordering lot size for the newly purchased items, 10 and 30 percent decrease respectively. However, this increase in the fuzziness of return rate cause to 12 and 39 percent increase in the optimum value of recovery lot size and ordering lot size for the newly purchased items respectively. Moreover, an interesting result is that regardless of the value of changes, as the uncertainty increases, total cost per unit time in all level of both parameters increases. Nonetheless the percent change of total costs that resulted from the SD method are less than those that are obtained using the GMIR method.

In Figures 6.13-6.16, the effect of uncertainty has been mutually compared for both defuzzification methods in each level of fuzziness for parameters on considered optimal values.

Figure 6.13 shows that optimum number of orders for the newly purchased PET containers in all levels of uncertainty of both parameters is higher for the GMIR method as compared to the SD method. By analysing Figure 6.14, it can be deduced that the optimal recovery lot size derived by the GMIR method in all level of fuzziness for the fuzzified parameters is more than the ones derived by the SD method. Furthermore, as it is clear from the Figure 6.15, higher ordering lot size can be obtained by the SD method with respect to the level of uncertainty and the fuzzified parameter. If the focus is on uncertainty of one parameter, generally the total cost per unit time obtained by the SD method is lower.

6.4 Chapter Summary

In this chapter, the fuzzy inventory models introduced in previous chapters are applied in real applications. The applicability of the first fuzzy inventory model suggested in chapter 4 is shown in automobile industry. Moreover, the practical application of the second fuzzy reverse inventory model was represented through a real case from the milk industry.

In both cases, the effect of uncertainty of fuzzified parameters is analyzed on the considered optimal values in some level of fuzziness through some figures and tables. According to the results obtained, it can be concluded that based on the strategy of the company which method is more appropriate. For example, if the strategy is decreasing the total cost, using the SD methods is more logical in the second investigated case study.

CHAPTER 7: CONCLUSION

7.1 Concluding Remarks

Inventory management is an integral and important part of a supply chain management in business activities. It plays a vital role in different parts and processes of a company that are related to the finished items and raw materials. Therefore, sufficient attentions should be given to choose a proper inventory control system. Otherwise, organizations suffer from losing the investment in inventory management. Inventory systems can be investigated from many points of view.

In present research, attempts were made to design and formulate inventory systems that are appropriate to deal with the volatile business environment, and at the same time, they take some advantages. To do so, fuzzy set theory is combined with the investigated inventory systems to obtain more robust results. It is shown that these kinds of fuzzy inventory systems are more precious and they work better in real world. Because uncertainty is an inherent part of the information systems especially for data in inventory systems.

Another important characteristic that is taken into account is the learning process. As usually inventory operations are affected by the learning phenomena and can be improved during the time, the influence of uncertainty is studied with the learning theory simultaneously. The results revealed that optimized values could be improved by passing the time due to the effect of learning.

Moreover, other characteristics such as different holding costs for perfect and defective items and return rate for the recovered items are considered. Two inventory systems applying the mentioned theories (i.e. fuzzy set theory and learning theory) were developed and optimized with mathematical procedures.

In order to highlight the contributions of the developed fuzzy inventory systems through the previous researches, a comprehensive literature review was conducted gathering more than 130 papers published in peer reviewed prestigious international journals. The fuzzy inventory models of these researches were analyzed in details analysing their contents. These models are divided in two main categories (i.e. economic order quantity (EOQ) and economic production quantity (EPQ)), and studied these models from some aspects including membership functions and methods and techniques for fuzzification, defuzzification and optimization. It was the first time that such a complete review was done in the fuzzy inventory models literature.

In the first model, a fully fuzzy inventory system was presented with a total cost function including variable and fixed costs, selling prices of good and defective quality items, holding costs of defective and non-defective items, screening rate and screening costs. Besides, it was supposed that percentage of defective items per shipment decreases during the cycles and it follows a S-shaped logistic learning curve model. All parameters and variables of the model are assumed as triangular fuzzy numbers to show an inventory model that is appropriate in an uncertain environment.

The objective was to obtain the lot size in nth shipment when learning occurs minimizing the total cost function. After obtaining the defuzzified total cost function applying the GMIR method, the conditions of KKT theorem were analyzed, and then, the problem was optimized by derivation. In order to test the investigated model, a numerical example is provided and the effect of learning and fuzziness is analyzed simultaneously on the total profit per unit time and order quantity.

It is concluded that the optimal lot size directly depends on the amount of uncertainty. It was shown that it increases when the level of uncertainty of the inventory system increases. To avoid costly inventory strategy, it is necessary to reduce the impreciseness of the model. It is proved that the decision maker should expect an increase in the total profit when learning occurs and it justifies some costs that are devoted to the learning process in the long-term.

The inventory system that is completely fuzzified is compared with the partially fuzzified one. It was shown that the real inventory situation that is affected by uncertainty can be captured if more elements are fuzzified. However, sometimes it causes complex and complicated models which are difficult to be optimized. Moreover, the effect of learning is more tangible on the optimal EOQ for the fully fuzzified model when the level of uncertainty increases.

The third part of this thesis is devoted to study the effect of fuzziness of demand and collection rate of the recoverable products from customers through a reverse inventory model which was partially fuzzified. The effect of two defuzzification methods (i.e. GMIR and SD methods) is compared and analyzed. The defuzzified total cost function of the model including setup cost for the recovery process, ordering cost for the newly purchased products, inventory holding costs of collected and serviceable products, labour production cost, unit purchase cost for the newly purchased products and unit buyback cost for the recovered products was optimized by a one-dimensional search procedure. Recovery lot size for each production run and number of orders for the newly purchased products during a cycle were derived.

Proposing a comprehensive numerical example, the behaviour of the decision variables were analyzed while the upper and lower bound of fuzzified parameters were changed. As the results of these defuzzification methods are different in practice, optimal fuzzy values in considered iterations are compared with similar crisp ones. It is shown that when the levels of fuzziness were similar and the optimal number of orders were equal, the percentage changes of the optimal recovery lot size in the GMIR method were negative as compared to the SD method. Moreover, in the case that upper bound of return rate was greater than the lower bound in triangular fuzzy numbers, the percentage changes of the optimal total cost by the signed distance method were negative. When it was smaller than the lower bound, those were positive in all cases.

In order to show the applicability of the proposed fuzzy inventory models, these models are used in two real cases. The first case was related to investigation of an inventory system in a supply chain for the automobile industry. Optimal order quantities and total profit per unit time of Brake Disc and Rear Hub Drum are found for low and high uncertain levels. It is concluded that the more the level of fuzziness, the more the effect of learning.

Furthermore, the second fuzzy model which was a reverse EOQ inventory system was explained in the milk industry. In the supply chain of a milk manufacturing company, there was a material called PET for packaging in which could be recovered. The number of orders for the newly purchased PET during a cycle, the recovery lot size of PET for each production run, the ordering lot size for the newly purchased PET and the total cost function per unit time for a local milk manufacturing company are found. Regarding the total cost per unit time, it is shown that the behavior of the investigated defuzzification methods are different.

7.2 Contributions and Applications

In this section, the contribution of this research is explained dividing it into two sections that are related to the knowledge and application.

7.2.1 Contribution to the Knowledge

The first fuzzy inventory model that was developed showed a fully fuzzy system. An inventory system that is completely fuzzified generally is very hard to be optimized.

However, the proposed fully fuzzy model was solved using a mathematical algorithm. Although there are many studies in the previous literature, almost all of them considered a partially uncertain system. It means that these models usually target a special parameter or variable and study the effect of uncertainty on the considered element. However, there are models with more than one element that is fuzzified but not to the explained problem.

Learning theory can be observed in the previous inventory literature. In present research, the effect of fuzziness and learning is studied simultaneously where the inventory system is affected by a fully fuzzified learning function. This provides the opportunity to examine the behavior of optimal lot size and optimal total profit in such a situation to make the best strategy.

In the second model, the rate of return was targeted in an uncertain environment where the recovery production of the system is improved due to the learning by passing the time. This was the first time that the effect of fuzzification of two defuzzification methods was analyzed and compared in a partially fuzzified reverse inventory system. This provides the opportunity to select the best decision according to the situation of the company.

7.2.2 Contribution to the Practitioners

As uncertainty is an inherent part of the real world, inventory models that combine fuzzy set theory with the inventory system have many applications in the real world. As almost nothing is constant in real business environment, the developed fully fuzzified model can be very helpful to depict the best strategy. Moreover, learning theory can be applied in each inventory system that the related process could be improved during the cycles. Besides, other investigated characteristics such as imperfect quality items have practical applications in the real inventory systems. The rate of return in a reverse inventory system is a very important factor that can influence the whole of a supply chain.

In any industry that produced or supplied product can be recovered, it plays an important role.

As it is discussed in previous chapter, the developed fuzzy forward and backward models of this thesis can have many applications in real world considering their characteristics. In addition to the discussed applications in chapter 6, first model can apply in semiconductor industries because they usually separate defective and non-defective items. Moreover, second model can have some applications in food and beverage industries. Because they usually supply items in containers that should be recovered due to the environment concerns. Moreover, both models can be used in any industry that deals with defective items.

7.3 Recommendations for Future Research

There are several opportunities to extend this research in the future. Regardless of the complexity of calculations, alternative defuzzification methods such as centroid method could be applied and the results could be compared and analyzed. Besides, other types of fuzzy numbers, such as trapezoidal fuzzy number, could be investigated on the behavior of the fuzzified parameters.

Another room for future research that would be of interest is studying the fuzzy inventory models if learning in fuzziness is occurred. This is for the situation that uncertainty decreases over time because of the effect of learning process. These conditions can on the upper and lower bounds of a triangular fuzzy number.

It may also be of interest to incorporate different types of learning functions to evaluate and compare the results. Also the models can be develop combining the learning with other parameters that have potentials to be improved during the time. In such a scenario, learning curves with different learning ratio could be investigated.

Although this is the first attempt to apply fuzzy set theory in the reverse inventory literature, future works can apply these methods on the other reverse EOQ/EPQ models with other assumptions and conditions.

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LIST OF PUBLICATIONS AND PAPERS PRESENTED

(a) ***Published***

1. Shekarian, E., Olugu, E. U., Abdul-Rashid, S. H., & Kazemi, N. (2016). An economic order quantity model considering different holding costs for imperfect quality items subject to fuzziness and learning. *Journal of Intelligent & Fuzzy Systems*, 30(5), 2985–2997. (ISI, IOS Press)

2. Shekarian, E., Olugu, E. U., Abdul-Rashid, S. H., & Bottani, E. (2016). A fuzzy reverse logistics inventory system integrating economic order/production quantity models. *International Journal of Fuzzy Systems*, 18(6), 1141–1161. (ISI, Springer)

(b) ***Article in press***

1. Shekarian, E., Kazemi, N., Olugu, E. U., & Abdul-Rashid, S. H. (2017). Fuzzy inventory models: A comprehensive review. *Applied Soft Computing*. (ISI, Elsevier)

(c) ***Conference***

1. Shekarian, E., Olugu, E. U., Abdul-Rashid, S. H., & Kazemi, N. (2016). Analyzing optimization techniques in inventory models: the case of fuzzy economic order quantity problems. *Proceedings of the 2016 International Conference on Industrial Engineering and Operations Management*, Kuala Lumpur, Malaysia, March 8-10. (Awarded as the Best Track Paper)