

CONVECTIVE HEAT TRANSFER IN SQUARE AND CYLINDRICAL POROUS  
ANNULUS

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### **CONVECTIVE HEAT TRANSFER IN SQUARE AND CYLINDRICAL POROUS**

Field of Study: **Heat Transfer**

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## ABSTRACT

The convective heat transfer in porous media has attracted the considerable attention in the past few decades due to its wide applicability in various fields. The present work focuses on investigating the various aspects of convection such as conjugate heat transfer, conjugate double diffusion and mixed convection analysis with thermal equilibrium and thermal non-equilibrium modeling. The finite element method is used to discretize the governing non-linear partial differential equations. Galerkins method has been employed to accomplish the element stiffness matrix. An in-house computer code was developed to solve the resulting equations that describe the behavior of heat transfer in porous medium. The various geometrical and physical parameters have been analyzed with respect to the porous annular duct and annular cylinders embedded with saturated porous medium. The effects of Rayleigh number, radiation parameter, thermal conductivity ratio, Peclet number, duct thickness ratio and other characteristics have been evaluated to determine the variation in the Nusselt number and fluid flow behavior. It has been found that the fluid flow in the considered geometry, exhibit various types of movement due to the enhancement or retardation in the temperature at the vicinity of the solid wall of the annulus. It has also been found that the fluid moves in two symmetric cells due to the isothermal heating and cooling of outer and inner walls of the respective ducts.

## ABSTRAK

Perolakan pemindahan haba dalam media berliang telah menarik banyak perhatian dalam beberapa dekad yang lalu disebabkan penggunaannya yang meluas dalam pelbagai bidang. Penyelidikan ini memberi tumpuan dalam pelbagai aspek, seperti perolakan pemindahan haba konjugat, mengkonjugasikan penyebaran 'double' dan analisis perolakan bercampur dengan keseimbangan terma dan terma model bukan keseimbangan. Kaedah unsur terhingga digunakan untuk 'discretize' persamaan pembezaan separa bukan linear mentadbir. Kaedah Galerkins telah digunakan untuk mencapai matriks kekakuan unsur. Suatu kod komputer di rumah dihasilkan untuk menyelesaikan persamaan yang menggambarkan tingkah laku pemindahan haba dalam medium berliang. Pelbagai parameter geometri dan fizikal telah dianalisis berkenaan saluran anulus berliang dan silinder anulus yang tertanam di dalamnya sesuatu medium berliang tepu. Kesan nombor Rayleigh, parameter sinaran, nisbah kekonduksian terma, bilangan Peclet, nisbah ketebalan saluran dan ciri-ciri lain telah dinilai untuk menentukan perubahan dalam bilangan Nusselt dan tingkah laku aliran cecair. Telah didapati bahawa aliran cecair menunjukkan pelbagai jenis pergerakan disebabkan peningkatan atau penurunan dalam suhu di sekitar dinding pepejal anulus. Telah didapati juga bahawa cecair yang bergerak dalam dua sel yang simetrikal adalah disebabkan pemanasan isoterma dan penyejukan dinding luar dan dalaman saluran masing-masing.

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## LIST OF SYMBOLS AND ABBREVIATIONS

$A$	Area of element ( $m^2$ )
$Ar$	Aspect ratio = $H/L_t$
$c_p$	Specific heat ( $J/kg\text{-}^\circ C$ )
$D$	Wall Thickness ratio = $(r_{sp} - r_i)/(r_o - r_i)$ for cylindrical ratio, and length of the annulus for Cartesian co-ordinates.
$Da$	Darcy number
$D_m$	Mass diffusivity
$D_L$	Thickness ratio of inner solid $(r_{sp1} - r_i)/(r_o - r_i)$
$D_R$	Thickness ratio of outer solid $(r_o - r_{sp2})/(r_o - r_i)$
$g$	Acceleration due to gravity ( $m/s^2$ )
$H$	Inter-phase heat transfer coefficient parameter
$H_t$	Height of cylinder (m)
$L$	Height and length of annulus (m)
$Le$	Lewis number
$L_t$	$r_o - r_i$ (m)
$k$	Thermal conductivity ( $W/m\text{-}^\circ C$ )
$K$	Permeability of the porous medium ( $m^2$ )
$Kr$	Thermal conductivity ratio
$N$	Buoyancy ratio
$N_i, N_j, N_k$	Shape functions at node i, j and k respectively
$n$	Refractive index
$Nu/\overline{Nu}$	Local Nusselt number and average Nusselt number respectively
$\overline{Nu}_f$	Nusselt number for fluid
$\overline{Nu}_s$	Nusselt number for solid
$\overline{Nu}_t$	Total Nusselt number
$Pe$	Peclet Number

$q_r$	Radiation flux ( $\text{W/m}^2$ )
$r, z / \bar{r}, \bar{z}$	Cylindrical co-ordinates/ Non dimensional co-ordinates (m)
$Ra$	Rayleigh number
$R_d$	Radiation parameter
$R_r$	Radius Ratio = $(r_o - r_i) / r_i$
$Sh$	Sherwood number
$T, \bar{T}$	Dimensional ( $^{\circ}\text{C}$ ) and non-dimensional temperature respectively
$u, v$	Velocity components in x and y direction respectively (m/s)
$V_{\infty}$	External Velocity for mixed convection
$W$	Width Ratio
$u, w$	Velocity in r and z directions respectively (m/s)
$x, y$	Cartesian co-ordinates
$\bar{x}, \bar{y}$	Non-dimensional co-ordinates

## GREEK

$\alpha$	Thermal diffusivity ( $\text{m}^2/\text{s}$ )
$\beta$	Coefficient of thermal expansion ( $1/^\circ\text{C}$ )
$\beta_R$	Absorption coefficient ( $1/\text{m}$ )
$\sigma$	Stephan Boltzmann constant ( $\text{W}/\text{m}^2 \text{K}^4$ )
$\mu, \nu$	Coefficient of Dynamic ( $\text{kg}/\text{m}\cdot\text{s}$ ) and kinematic viscosity( $\text{m}^2/\text{s}$ ) respectively
$\rho$	Density ( $\text{kg}/\text{m}^3$ )
$\Phi$	Porosity
$\bar{\psi}$	Non-dimensional Stream function
$\varepsilon$	Viscous dissipation parameter
$\varphi$	Porosity
$\zeta, \eta, \lambda$	Shape functions in local coordinate

### *Subscripts*

$h$	Hot
$c$	Cold
$f$	Fluid
$s1$	Solid 1
$s2$	Solid 2
$L$	Left
$R$	Right
$B$	Bottom
$T$	Top
$Tot$	Total
$i$	Inner
$o$	Outer

$s$	Solid
$p$	Porous
$sp$	Solid-Porous interface
$w$	wall
$\infty$	Conditions at outer radius