DESIGN AND DEVELOPMENT OF A 3-DEGREE OF FREEDOM PARALLEL MANIPULATOR

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ABSTRACT

Parallel manipulators have gained a lot of interest during the recent five decades. A parallel manipulator is constructed with an upper moving platform and a lower fixed platform, connected by at least two linkages in parallel, hence the name. Parallel manipulators have so many advantages over their serial counterparts. Some of these advantages are high accuracy, high stiffness and low inertia which make them favorable for numerous applications. Parallel manipulators also have several disadvantages such as high cost, small workspace, complex forward kinematics and complicated forms. To alleviate these disadvantages, the design and development of parallel manipulators with less than six degree of freedom was focused on by numerous researchers.

In this paper, the focus is on 3-DOF parallel manipulators. A number of 3-DOF parallel mechanisms were compared and contrasted on the basis of type of configuration, workspace, stiffness and dexterity in order to choose the best manipulator. However, there is no criteria that can be used to choose the best parallel robot. It all depends on desired application. Here, a 3-RPS parallel manipulator was chosen for a massage application due to its proposed suitability for the task. Forward and Inverse Kinematics of the mechanism were studied. The mechanism was designed using CAD software. A prototype was built and analyzed. It has been found that the CAD software both predicted the output and the limitations of the mechanism. The system can be extended to motion platform once the dynamics equation is established.

ABSRAKT

Manipulator selari telah mendapat banyak faedah pada baru-baru ini lima dekad. A manipulator selari dibina dengan platform yang bergerak atas platform dan tetap yang lebih rendah, yang berkaitan dengan sekurang-kurangnya dua hubungan secara selari. Manipulator selari mempunyai begitu banyak kelebihan berbanding rakan-rakan siri mereka. Antara kelebihan ini adalah ketepatan yang tinggi, kekakuan tinggi dan rendah inersia yang membuat mereka baik bagi pelbagai aplikasi. Manipulator selari juga mempunyai beberapa kelemahan seperti kos yang tinggi, ruang kerja yang kecil, kinematik ke hadapan kompleks dan bentuk rumit. Untuk mengurangkan kelebihan ini, reka bentuk dan pembangunan manipulator selari dengan kurang daripada enam darjah kebebasan telah memberi tumpuan kepada.

Dalam kertas ini, tumpuan diberikan kepada manipulator selari 3-DOF. Beberapa mekanisme selari 3-DOF dibandingkan dan berbeza berdasarkan ruang kerja, kekakuan dan ketangkasan untuk memilih manipulator yang terbaik. Walau bagaimanapun, tidak ada kriteria yang boleh digunakan untuk memilih robot selari yang terbaik. Ia semua bergantung kepada aplikasi dikehendaki. Di sini, seorang manipulator selari 3-RPS telah dipilih bagi permohonan urut kerana kesesuaian cadangan untuk tugas itu. Hadapan dan Songsang Kinematik mekanisme yang telah dikaji. Mekanisme ini telah direka dengan menggunakan perisian CAD. Prototaip dibina dan dianalisis. Ia telah mendapati bahawa perisian CAD yang kedua-dua meramalkan output dan batasan mekanisme.

Acknowledgements

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LIST OF SYMBOLS AND ABBREVIATIONS

λ dimension of space

S self-determining loops

m number of connections

l number of links

F number of degree of freedom

f_i degree of freedom associated with i

C_k mobility of limb k

η global condition index

V workspace capacity

P_x x component of P in the upper xyz plane

P_y y component of P in the upper xyz plane

P_z z component of P in the upper xyz plane

Ψ angle of rotation of the moving frame about the x-axis

Θ angle of rotation of the moving frame about the y-axis

Φ angle of rotation of the moving frame about the z-axis

r_a radius of the lower platform

r_b radius of the upper platform

DOF degree of freedom

RPS Revolute-prismatic-spherical parallel manipulator

SPS spherical-prismatic-spherical

UPU universal-prismatic-universal

PRS prismatic-revolute-spherical

CHAPTER 1

1.1 INTRODUCTION

Design and development of parallel manipulators can be traced back to the ‘60s. At the time Gough and Whitehall designed a tire testing machine. Later Stewart used the same concept to design a flight simulator. Parallel manipulators are gaining a lot of interest in the industries, space centers, medical field and commerce. The high popularity of parallel mechanisms is due to their improved high speed and positioning accuracy. There are so many potential applications of these devices that are been explored such as walking machines, mining machines, space docking and planetary explorations, medicine, flight simulation for training, natural disaster simulators, tuning shields, satellite antennas, haptic devices, cable actuated cameras, vehicle suspensions, automation, semiconductor machining, electronic assembly and pointing devices amongst others.

Despite the parallel manipulators having small workspace and dexterous manipulability, they have greater loading capacity as a result of load sharing by the parallel limbs connected to the fixed base. The mechanisms also have the advantages of low inertia, high structural stiffness and high manipulability, sensitivity to errors which can be averaged and high controllability.

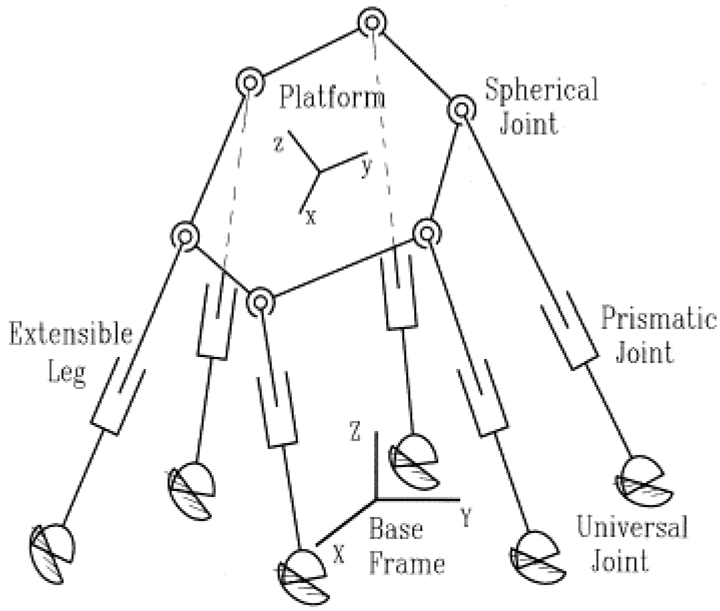


Figure 2.1 6-DOF UPS parallel manipulator

Another application of parallel manipulators is as massage systems. One of the most popular massages in the world is the Chinese massage technique which involves rubbing, stroking, pressing, pinching, flapping and rolling. There is a growing interest in countries such as China, Japan and the USA to mechanize and automate the Chinese massage system (Yonggen, 2010). Massage has proven advantages like; alleviation of pain from a particular part of the body, improves blood circulation, reduces stress, depression and anxiety, relieves severe headache, relieves labor during child delivery, reduces scars and stretch marks, reduces fatigue and sickness and generally prolongs life.

There are a lot of massage devices in the market nowadays ranging from big massage chairs to massage pillows and small handheld devices. The interest in this research is to design and develop body massager that is both portable, small and automated. The topology to use is a 3-DOF parallel manipulator. The structure is chosen due to its numerous advantages including, low cost, portability, manipulability and controllability. A three degree of freedom is enough for the particular application where stretching, contacting and rolling techniques are sufficient. Another benefit of the massager is that it can be placed on the table like a table lamp. A person will lie on a bed and operate the machine just like he is in a massage parlor. This is very important since not all people are comfortable with going to massage parlors or the attendance of a Masseurs due to personal, social or environmental reasons, despite the advantages offered by massage applications. The use of the mechanism will be safe on the body as the end effector will have a rolling ball connected as ball and socket joint with no sharp edges. This simple massage system will involve patting, stroking and rubbing.

1.2 PROBLEM STATEMENT

1. Due to numerous advantages of massage to the human body, a massage system involving the best position for getting the treatment, which is lying down, should be developed.
2. Most effective massage systems are expensive. Safety and the right environmental simulation are not considered.

1.3 OBJECTIVES OF THE RESEARCH

1. To formulate the kinematics equation for the parallel manipulator
2. To design the parallel manipulator in CAD environment and simulate it. This will consider link interferences and how far the strut will move.
3. To develop the parallel manipulator by choosing the proper links and joints to satisfy the kind of motions needed.

1.4 SCOPE AND LIMITATION OF THE STUDY

1. Kinematic modeling and simulation of the parallel manipulator

2. Development and fabrication of the device

Limitation

1. Only one type of Inverse kinematics formulation will be used.
2. The manipulator can only massage one part of the body at a time.
3. Only the main structure will be developed. The stand and the rolling ball on the end effector will be in future research.

1.5 REPORT ORGANIZATION

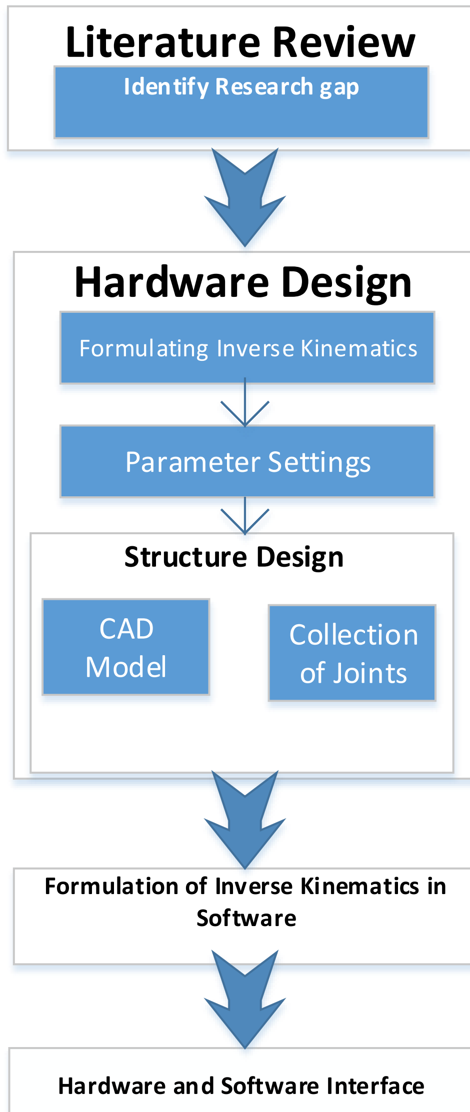
Chapter two presents the robot manipulator. A comparison of the series and parallel manipulators will be made. The advantages, disadvantages and possible applications of 3-DOF parallel manipulators will be discussed. Some concept underlying the classifications of parallel manipulators based on Tsai enumeration method, such as Euler and Grubler criterion will be addressed. The choice of parallel manipulators will be decided upon their kinematics, workspace, stiffness and dexterity. The selection of the most suitable mechanism will be made based on all the highlighted factors and with relevance to its complexity.

Chapter three presents the kinematics and mechanical design of the chosen 3-DOF parallel manipulator. It also considers the control of the mechanism. The kind of joint, actuators, complexity and materials will be evaluated. The Inverse and forward kinematics of the parallel manipulator is derived. The Inverse kinematics was converted into C++. Structure prototyping will be made. A communication method will be derived between the hardware and the software.

In Chapter four, the CAD model of the structure will be made using Creo ProE. Simulation will be made using mechanism and animation to determine the joint angles and the limb lengths of the structure. The degree of freedom, mobility and limitation of the mechanism will be tested for easier extension to the built prototype.

In Chapter five, conclusion will be made about the suitability, degree of freedom and limitation of the structure. Recommendation will be made about how to improve the structure.

1.6 METHODOLOGY OF THE STUDY



First in the methodology is identification of the research problem. The second is hardware design. This involves formulating the inverse kinematics formulae and setting the parameters in the formulae. Then follows the structure design; this entails drawing the CAD model to determine the model parameters and to simulate the model in the CAD environment. From the designed model, the type of joints to be used are considered and chosen.

Subsequently, the hardware and the software will be interfaced by formulating the Inverse Kinematics formulae in the software environment and uploading it to the hardware to achieve the desired objective.

CHAPTER 2 LITERATURE REVIEW

2.1 Introduction

The Robotics Institute of America defined robot as "A Robot is a re-programmable multi-functional manipulator designed to move materials, parts, tools, or specialized devices through variable programmed motions for the performance of a variety of tasks". A parallel mechanism or manipulator comprises of numerous linkages coupled by joints. Allowable movements of the mechanism are determined by the number of links and connections the mechanism possesses (Tsai, 1999).

The classification of robots is based on certain criteria such as; degrees of freedom, physical configuration, kinematic structure, work space geometry, drive technology and motion characteristics. Considering the kinematic structure, a robot is called; serial robot when its arrangement takes the form of an open loop chain and a parallel manipulator when it takes the form of a close loop. Not all parallel manipulators are closed loop in nature; a single degree of freedom closed loop chain in series is a series manipulator (Qian, 2009). Serial robots have large workspaces and deft of maneuverability like the anthropological hand, nevertheless their cantilever arrangement makes them to have a small load carrying capacity. Hence for applications needing large weight carrying capacity, accurate positioning and good dynamic features, the parallel manipulator should be used. The parallel manipulators are getting a good research consideration due to the numerous advantages relating to their structure. To gain the advantages of both series and parallel manipulators they can be combined together. This configuration is called hybrid manipulators.

2.1.1 Serial architecture

A serial manipulator is a classical anthropomorphic form of robot manipulator (Figure 2.1). It consists of a number of links attached in sequence by joints, typically revolving and linear moving joints. First part of the manipulator is fixed to the ground (called base) whereas the second part is free to move in space (called end-effector). Advantages of serial manipulators are large work volume and dexterous manipulability. The disadvantages are low precision (joint errors are cumulative), low payload-to-work ratio (each actuator supports the weight of successive links), poor force exertion ability, heavy due to motors located along the manipulator and high inertia due to moving parts.

Another disadvantage of serial manipulators is with regards to Inverse kinematics. Inverse kinematics is when the desired positions and orientation of the output links are given; the problem is to find the value of the actuator joints. Multiple solutions for the Inverse kinematics exist for serial manipulators. This complicates the control algorithm.

The disadvantages of serial manipulator make it expensive because accurate gears and powerful motors must be used. It is also another reason for not using the serial robot for high precision applications such as flight simulation and fast pick and place tasks.

Figure* 3.*1* *Serial manipulator (KUKA LWR 4+, courtesy of Aalto University)

2.1.2 Parallel architecture

Parallel manipulator is an un-anthropomorphic closed-loop kinematic mechanism with a compact topology. It has two main components, an end-effector and a base which are linked together by some independent kinematic chains (Merlet, 2001). Mostly the number of actuators corresponds with the degree of manipulability of a mechanism but a parallel manipulator can have redundant actuators making the controlled degree of freedom less. A parallel manipulator that has chains exactly equal to the degree of freedom is called a fully parallel manipulator.

Gosselin categorized completely parallel mechanisms by the relation;

$\mathrm{Q}(r - 6) = 6$, where Q denotes number of chains and r is the quantity of static forms in a chain.

In a parallel robot, the least number of chains supporting the end-effector is two. Each chain has at least one actuator and a sensor which measures the value of the actuated variable (linear motion or rotation angle). When the motors are locked the movement of the manipulator is zero.

Considering the disadvantages of serial manipulators, parallel manipulators are very important since they can provide high accuracy, better dynamic behavior, repeatability, reliability, rigidity, greater bandwidth and manipulability of large loads. In addition parallel manipulators can be made in small package sizes. *Fast* robots are required for tasks such as flight simulation and pick and place. Any slight change in the pose of parallel manipulators will be noticed and measured easily. Parallel platforms experience reduced deformations even under high loads due to abundant use of spherical and universal joints. These joints make the limbs to experience only compressive or tensile loads but no shear force, bending and torsion moments (George, 2012).

In recent years, parallel manipulators have gained popularity in various industrial applications, medical surgeries, nano-manipulation, material handling, planetary exploration, satellite antennas, haptic devices, vehicle suspensions, cable-actuated cameras and precision optics, among others. Parallel ability of these mechanisms makes it liable for the robots to be considered in a way that a movable platform does not carry the burden of the actuators that energize it (Tsai, 1996). Hence huge powerful actuators can drive small configurations enabling faster, stiffer and stronger designs.

Parallel manipulators suffer the disadvantages of lower dexterity due to link interference, constraints due to universal and spherical joints and platform singularities (George, 2012).

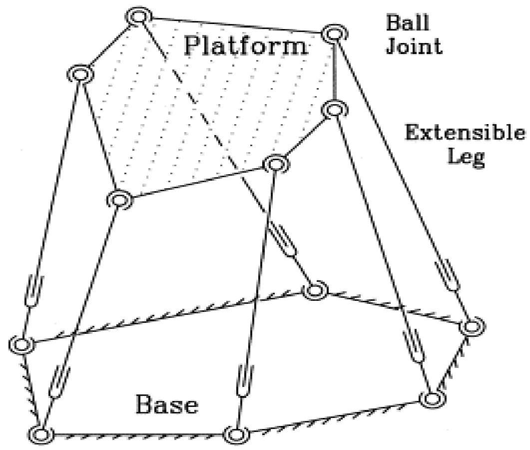


Figure 2.2: Parallel manipulator, 6-DOF SPS Stewart platform

Classification of parallel manipulators

↓ ↓ ↓ ↓

Symmetric Planar Spherical Spatial

Symmetrical parallel mechanisms have total of limbs equivalent to the degree of freedom so also equal to number of loops. Planar manipulator has more than one planar kinematic chain acting together on a common rigid platform. Each joint of a planar parallel manipulator must be revolute or prismatic, each having a mobility of one. Prismatic joints lie in the plane of motion while the revolute are perpendicular to it. Spherical manipulators make the end effector to move according to spherical motions. It has permissible revolute connections crossing at a common point. In spatial, the location and coordination of the movable part is in 3-dimensional space.

2.1.3 Hybrid architecture

A hybrid mechanism consists of both open and close loop chains. It combines the advantages of serial manipulator with that of parallel manipulator i.e; large workspace, dexterity, manipulability, accuracy, rigidness and high inertia. The disadvantages of the two combined manipulators are reduced making the hybrid kinematic manipulator a superior contender for the next generation of machine tools. The *Tricept* is a conventional hybrid kinematic manipulator that is already out for sale.

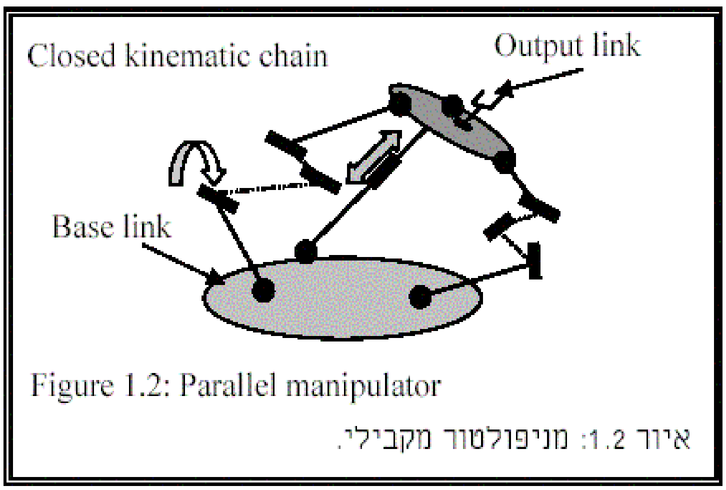


Figure 2.3 Hybrid mechanism

2.1.4 Comparison between Serial and Parallel manipulators.

First, the parallel and serial link manipulators are compared based on mechanism and control in order to apply each in the most advantageous way. Serial link has one actuator for each of its moving fragments hence its weight and sluggishness is large. In parallel mechanisms, the mass can be greatly reduced since all the actuators are placed close to the base. Workspace of serial link manipulator is larger since its links do not obstruct with each other in positioning motions (Arai, July 1990). Inverse calculation is necessary to compute the position control which is why it is easy for parallel manipulators. For the same reason, force location is easy in serial manipulator while force detection is easy in parallel manipulator. Positioning error in serial manipulator is accumulated while it is averaged in parallel. The load capacity of serial manipulator is limited. It is the total number of motors in a parallel mechanism.

Serial and parallel manipulators can also be compared based on other factors. Parallel manipulator is a closed loop manipulator having all its limbs connected to the two platforms in a closed form. Serial is open loop having each of its limbs joined to the next. The end effectors of parallel and serial manipulators are commonly known as platform and gripper respectively. The parallel mechanism is normally described in Cartesian space while serial is described by joint space. Actuators in parallel manipulator are placed near the immobile base while for series is on the links.

The parameters to consider when designing parallel mechanisms are structure, workspace, singularity and link interference. For serial, we need to consider the manipulator strength and stiffness and vibration characteristics. Parallel manipulators are very stiff. Serial manipulators are dexterous. Forward kinematics for parallel manipulator is difficult and complex, the inverse is straight forward and unique. For serial manipulator the reverse of the parallel mechanism is the case. The former are suitable for precise positioning and serial are better when used for gross positioning.

Table 1.1 Comparison of serial and parallel manipulator based on control and mechanism.

		Serial manipulator	Parallel manipulator
Mechanism	Inertia	Large	Small
	Workspace	Large	Small
	Anti-environment	Weak	Isolated easily
	Looks	Human-like	Closed form
Control	Point control in workspace	Difficult	Easy
	End effector location	Easy	Difficult
	Force location in workspace	Easy	Difficult
	Force sensing	Difficult	Easy
	Positioning inaccuracy	Accumulated	Average
	Force control error	Averaged	Accumulated
	Maximum force	Min ($r_i$max)	Σ li
	Near singular point	Degenerate in force control	Decrease of positioning accuracy
		Large motion in actuator	Large force in actuator
	Dynamics	Complicated	Much more complicated

Table 2.2: Basic comparison of Series and Parallel manipulators

	Type of manipulator	
	Parallel manipulator	Serial manipulator
Type of manipulators	Closed loop	Open loop
Hand	Platform	Gripper
Standard depiction	In Cartesian coordinate	In joint coordinate
Actuator position	Close to the immovable platform	On the linkages
Inertia, forces and stiffness	Less and more respectively	High and low respectively
Design considerations	Structure, workspace, singularity, link interference	Strength and stiffness considerations, vibration characteristics
Preferred property	stiffness	Dexterity
Use of direct kinematics	Difficult and complex due to dependent unactuated joints.	Straight forward and unique
Use of inverse kinematics	Straight forward and unique	Complicated
Singularity	Static	Kinematic
Direct force transformation	Well defined and unique	Not well defined; may non-existent, unique or infinite
Preferred application	Precise positioning	Gross motion

2.2 DEGREE OF FREEDOM PARALLEL MANIPULATOR

The leading pioneers of a parallel robot are Gough and Whitehall. They presented a six degree of freedom tire testing machine having two platforms linked by six extensible screw jacks. In 1965, Stewart published his famous paper in the proceedings of IMechE describing a six degree of freedom platform for flight simulation. Later all platform based manipulators are called Stewart-Gough platforms. Potential applications of parallel manipulators as mining machines, pointing devices and walking machines increased their popularity. Hunt proposed the use of parallel machines in lieu of serial manipulators because of their advantages. These marked the early idea of research on parallel manipulators in general and Stewart platforms in particular in robotic fields.

Parallel Kinematic Machines (PKMs) were developed as a result of the need for high-speed machining. PKMs are founded on the parallel mechanism structure. A parallel robot comprises of a stationary member and a moving member coupled by several legs. Typically the number of legs is equivalent to the degree of mobility of the manipulator in a way that each leg is driven by one motor and the motors are placed near the fixed form. For the case when the degree of freedom is more than the sum of limbs, then more than one energizer is needed in some limbs. Most six degree of freedom parallel machines are built upon the Stewart platform. However the huge cost of six degree of freedom, complicated analysis and structure reduced their requirement in machine tools, telescope, motion simulator and other applications. In these types of applications higher mobility is un-called for, the manipulator will be made light weight and very stiff, which minimizes inertia (George, 2012). The lightness of 3-DOF parallel manipulator is due to the motors been fixed to the base and the low inertia is as a result of the motors not contributing to the inertia of the links. Another advantage of 3-DOF parallel mechanisms is that accuracy, velocity and repeatability can be enhanced. The 6-DOF parallel manipulators have the additional drawbacks of challenging direct kinematics, coupled location and angles of the movable form, difficulty in manufacturing accurate spherical connections and so on (Tsai, 1996). However, the study of a 6-DOF mechanism is easier than that of a 3-DOF manipulator in inverse kinematics; this is due to the fact that the rest of degrees of freedoms are limited by restrictions.

Generally, for fully parallel robot architecture, a 3-DOF parallel manipulator will have three independent chains actuated by three actuators (Merlet, 2001). The three chains will be attached to the base or ground while the other ends will be attached to the end-effector forming a triangular shape. The end effector will have three degree of freedom within the plane. A rotation of angle Θ in the direction of $z -$axis and two transformations along the $y-$ $and\ x - axes.$

Each connection will be characterized by dual static forms connected by a joint making a total of three joints. The chains can represent the subsequent arrangements: RRR, RPR, PRR, RRP, PPR, PRP, SPR, UPU, UPS, SPS, RRC and RPS. We will only consider placing the actuators anywhere other than on the movable form so as to alleviate the weight of the moving system.

One of the objectives of this research is to find the appropriate kinematic structure that can be used for a massage application. It is therefore necessary to study the different kinds of kinematic structures for parallel mechanisms. Only kinematic structures that provide 3-DOF between the two platforms will be studied. Kinematic structure represents the chain with disregard to the geometric details such as link length and link shape.

For a robot to perform a specific task the location of the upper platform relative to the lower platform must be established. This is called position analysis (Tsai, 1999).

The first classification of parallel manipulators based on kinematic structure was by Hunt in 1983. Tsai (1999a) introduced a systematic enumeration method for parallel manipulators. In 1991, Herve and Sparacino wrote a few reports on the structure analysis of parallel manipulators.

However, three-DOF parallel robots have their shortcomings

- Performance of three DOF parallel manipulators depends on their geometry
- Some 3-DOF parallel manipulators produce unwanted movements of the moving form which are called parasitic motions. These parasitic motions reduce the accuracy and quality of the manipulators.
- Loads that are not along the desired degree of freedom must be carried as reactions at the joints which may lead to unwanted behavior.
- Presence of workspace singularities and need for control of actuators simultaneously.

2.1.5 Systematic enumeration method

Tsai (1999a, 2000) proposed the following conditions to classify a group of parallel manipulators

1. A manipulator comprises of a movable platform and an immovable platform coupled by links
2. The moving platform has multiple degree of freedom
3. Each limb is an open kinematic chain

4. The number of actuators in each limb is less than or equal to one
5. Actuators should be placed on or near the fixed platform.

   For actuators to be mounted near the base, each limb will have a prismatic or revolute base connected joint or a prismatic joint adjacent to the base connected joint.

   Euler's equation regarding the link amongst the numbers of self-determining loops S, the number of connections *m*, and the number of links *l* for a closed loop mechanism is:

$$S = m - l + 1 \quad (2.1)$$

   The number of degree of freedom F is given by Grubler criterion:

$$F= \lambda(l - m - 1) + \sum_{i=1}^{m} f_i \quad (2.2)$$

Here λ defines the dimension of the space the mechanism will function and f_i represents the degree of freedom associated with *i*. eliminating *l* and *m* gives the loop mobility criterion.

$$\sum_{i=1}^{m} f_i = F + \lambda S \quad (2.3)$$

The mobility C_k of limb k can be defined as the total joint degree of freedom associated with the limb.

$$\sum_{k=1}^{m} C_k = F + \lambda S \quad (2.4)$$

The dimension of the space should be greater than the mobility of the link and the number of degree of freedom.

$$\lambda \geq C_k \geq F \text{ for } k = 1, 2 \ldots m \quad (2.5)$$

The links in any of the limbs can be any number so far as the total degree of movement in the limb is the same as the required connectivity. Considering manufacturing and maintenance, symmetric limbs are preferred. In this research, only revolute (R), prismatic (P), universal (U) and spherical (S) joints will be applied.

Table 2.3: Configurations for spatial 3-DOF manipulators with 3 legs

Type	Kind
120	PUU, UPU, RUU
201	RRS, RSR, RPS, PSR, RSP, PSR, SPR, PPS, PSP, SPP
310	RRRU, RRPU, RPRU, RPPU, PRPU, PPRU, RRUR, RRUP, RPUR, PRUR, PPUR, RURR, RURP, RUPR, PURR, RUPP, PURP, PUPR, UPRR, UPRP, UPPR
500	RRRRR, RRRRP, RRRPR, RRPRR, RPRRR, PRRRR, RRRPP, RRPPR, RPPRR, PPRRR, PRPRR, PRRPR, PRRRP, RPRPR, RPRRP, RRPRP

The first number in the above table indicates the number of 1-DOF joint; the second represents the number of 2-DOF joints while the third denotes 3-DOF joints.

2.1.6 Choice of Kinematic structures

The following criteria can be used in choosing the right kinematic structure formed through combinations of various types of limbs:

1. Simplicity and practicality: a leg made up three or more links is judged impractical. Also legs with large number of revolute joints tend to fold up and lead to singularity.

2. Elimination of passive prismatic joints: the prismatic joint in the actuated link is always made the actuated joint; therefore it is better to limit the joint to one for better controllability.
3. Symmetry: all legs should have the same kinematic structure to avoid any difficulties
4. Proper type motion: any leg with complicated motions should be avoided.

   The above criteria lead to the choice of PUU, UPU, RUU, RRS, RSR, RPS and PRS for analysis.

2.1.7 Kinematics

Kinematics deals with aspects of motion without regard to the forces/torques causing the motion. Kinematics deals with the analysis of related motions amongst different links in a mechanism. Direct and the inverse kinematics are the two types of kinematic analysis. When a set of desired positions and orientations are given, the problem is to obtain all probable set of joint variables and their time derivatives. This is called Inverse kinematics. In Direct kinematics, the end effector positions and orientations are to be found given the joint variables and their corresponding time derivatives. For a serial manipulator, direct kinematics is simple while inverse is difficult. For parallel, inverse kinematics is straight forward while forward is difficult (Tsai, 1999).

In this segment, the kinematic study of the selected 3-DOF parallel manipulators will be considered. Each parallel manipulator has an immobile base and a movable platform joined by three limbs.

2.1.8 Kinematics of Parallel manipulators without parasitic motions

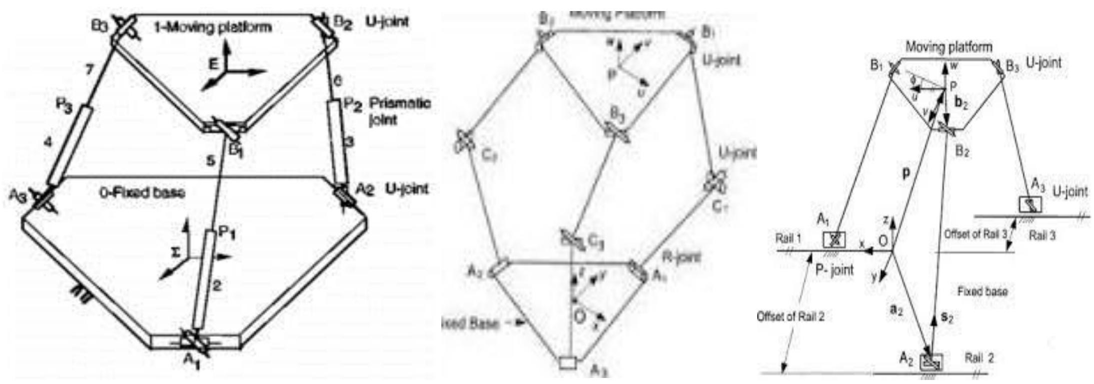


Fig 2.4 shows 3-UPU, 3-PUU and 3-RUU parallel mechanisms (left to right).

a) The first is a 3-DOF 3-UPU parallel manipulator popularly known as Tsai 3-UPU parallel mechanism. Tsai presented the mechanism to generate translational motion (Joshi, 2002). A universal joint connects the two platforms on each end. A linear actuator is used to drive the prismatic joints ($P_{1,}$ $P_{2,}$ P_3).

   Inverse kinematics gives two solutions. The closed form of the mechanism forms a crossing of three spheres resulting in the solution for the direct kinematics examination. There are two answers since the crossing forms a loop that intersects the third sphere in two positions.

b) The second is the RUU manipulator. Di Gregorio presented a parallel wrist called the RUU wrist. The manipulator is over constrained, having same number of legs, including revolving pairs and actuators in line with the coordinate. The revolute joints are driven by rotary actuator. Kinematics gives double solutions for both inverse and direct problems.

c) The third is a PUU manipulator with parallel rails. Each strut joins the fixed platform by a universal connector followed by another universal connector and then a prismatic connector attaches to the moving platform. A linear actuator energizes each of the linear moving joints. Parallel rails are guiding the sliders of the prismatic joints. If the rails are made to be long enough, the manipulator will have a large workspace. At least two solutions are obtained for the inverse kinematics. Two solutions that form two postures, mirror images of each other are found in the forward analysis.

2.1.9 Kinematics of 3-DOF parallel mechanisms having parasitic motions

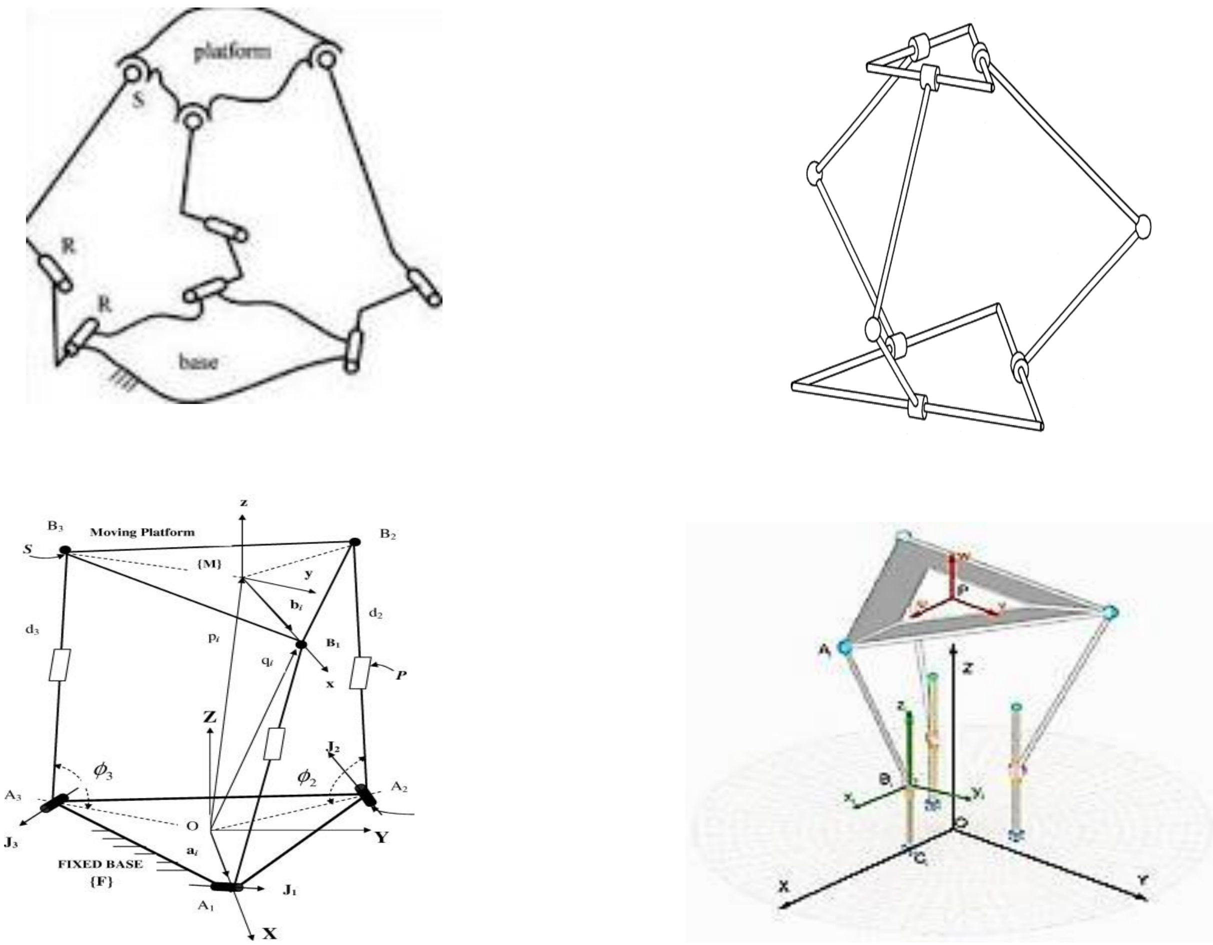


Figure 2.5: 3-RPS, 3-RRS, 3-PRS and 3-RRS parallel manipulators (left to right)

According to Li and Herve (Herve, 2010), parasitic motion is one of the weaknesses of a 1-translation, 2-Rotation (1T2R) parallel manipulator. A parasitic motion is a kind of unwanted motion that occurs after a desired motion. The most common type of 1T2R manipulator is the 3-RPS PM. It was found to produce three undesirable movements; two translations around the $x - y$ axes of the immobile structure and a single revolution about $z - axis$ of the immovable base.

a) In figure 2.5, the first diagram is of a RPS parallel manipulator. The two platforms are coupled by a revolute joint, in the middle a prismatic joint and then trailed by a spherical joint. The prismatic joint is the actuated joint which is driven by linear motor. For the inverse, out of two solutions obtained only one can be used to maintain the manipulator configuration. Sixteen solutions are obtained for the forward problem.

b) The second diagram shows RRS manipulator. Each limb connects the two platforms via two revolute joints, followed by a spherical joint. R for revolute joint is actuated, driven by a rotary actuator. There are two solutions for the input angles and eight results for the inverse kinematics. On the other hand, there exist sixteen results for the direct problem.

c) The third schematic shows a PRS manipulator. The limbs have prismatic, revolute followed by spherical joints. Linear actuators drive base connected revolute joints. The guiding rails of the prismatic joints are located along the generators of a right circular cone. There exist two outcomes to the inverse kinematic problem, but the solution where limbs $A_iB_{i\,,}$ are inclined inward, from bottom to top is selected. 16 solutions to the forward kinematics analysis of the manipulator are produced.

d) The fourth is an RSR mechanism. Three limbs connect the two platforms by a revolute joint, then a spherical joint followed by another revolute joint. Rotary actuators drive the revolute joints on the base. The actuators are inertial fixed making the actuator well suited for high speed robotic applications. There are two possible solutions for the inverse and only one significant solution of the forward kinematics.

Table 2.4: Comparison of the direct and inverse kinematics of selected PMs.

Manipulator	Inverse kinematics	Forward kinematics
PUU		Has two solutions
UPU	Two solutions acquired	
RUU	Two solutions obtained	Two solutions achieved
RRS	Two solutions for the two input angles and eight solutions for the inverse kinematics	16 solutions
RSR	Two possible solutions for the input angle	Only one significant solution is found
RPS	Two solutions are obtained but only the one with positive limb length is used	16 solutions are produce but can be reduced to eight using Sylvester's dialytic elimination method
PRS	Two outcomes but only one solution is selected	16 solutions are produced

Apart from comparing the kinematics of 3-DOF parallel manipulators, other kinematics related factors have to be considered so as to decide the appropriate parallel mechanism for a particular application. The kinematic related factors are; workspace of the mechanism, its dexterity and stiffness of the manipulator.

2.3 WORKSPACE

Workspace of a mechanism is all the reachable locations the manipulator can cover. The physical limits of passive and active joints determine the workspace. Amongst the prominent shortcomings of parallel manipulators is lower workspace compared to the serial manipulators. For this reason the total covered area of parallel manipulators should be optimized in order to increase the functionality of the mechanisms. However maximizing the workspace might cause adverse effects in some kinematic characteristics such as lower dexterity and manipulability. Workspace optimization is done with the need to determine the parameters that result in largest total workspace (Tsai, 1996). The chosen parameters for a particular architecture define the form, dimension and symmetry of the workspace. If the function is to determine how to maximize workspace with disregard to the quality of the mechanism, then design variables to consider are leg link lengths, sizes of the platforms, limb attachment points and angular position of the legs. Also there are factors that limit the movement of parallel manipulators, which are; interference between the links, mechanical limits of the passive joints and actuator limitations.

- Global conditioning index

A global condition index η takes into consideration the condition number of the Jacobean covering the whole workspace

$$\eta = \int_{w}^{0} \frac{1}{\lambda} dW$$

Where W is the workspace, $\lambda = \|J\|\|J^{-1}\|$ is the state number of the Jacobean at a given point in the workspace and $\|.\|$ is the 2 norm of the matrix.

The Monte Carlo method was taken by (Stamper et al 1997) to evaluate the workspace of the 3-Degree Of Freedom translational platforms and the steps are:

- Overall workspace of the mechanism is described by a hemisphere with a radius equivalent to the entire leg length
- A great number of points within the hemisphere were chosen
- The inverse kinematics of each leg was solved to determine if the points fall within the workspace. If the joint angles are real, the point falls within the workspace
- The number of points that fall within the working area are collected
- The total volume of the workspace is then determined by taking the relation of the points that fall inside the hemisphere to the complete number of points taken and then multiply the result by the volume of the hemisphere.

For unbiased comparison the sizes of the different architectures should be standardized (S.Joshi, 2001). The length of each limb is considered as one. For the parallel manipulators with parasitic motions, a discretization method was used. The whole results can be seen in (Joshi, 2002). In another comparison of three variants of a 3-RPS parallel manipulator (by Tsai et al, Merlet et al and Carretero et al) shows that the workspace of Tsai 3-PRS manipulator is larger than the other two.

Another parameter to consider is the stiffness of the mechanism. When a mechanism executes a task the end-effector exerts force on the environment. This energy will make the end effector to deflect from its intended position. The stiffness of the manipulator relies on numerous factors such as dimension and substance the links are made of, power distribution, energizers and control. Considering the PUU, RUU and UPU parallel manipulators, a study of the stiffness maps by Tsai and Joshi (Joshi, 2001), showed that highest levels were attained by 3-PUU with parallel bars, followed by 3-RUU and then 3-

UPU mechanism. The stiffness of the 3-DOF parasitic motion manipulators was also compared and the stiffness mappings were presented.

2.4 DEXTERITY

One of the most important kinematic parameters is dexterity. It is used as a degree of the kinematic ability of a manipulator. Dexterity is defined as capacity of robot to divert from its location and direction (the two are jointly called 'pose' of the manipulator) or spread forces and torques in random ways (Angeles, 1990). Any measurement of agility can be defined in terms of properties of Jacobean condition. T. Yoshikawa proposed kinematic manipulability indices for measuring the handiness of a mechanism as; square root of the determinant of the Jacobian $\boldsymbol{JJ^T}$. That is $\sqrt{\det(\boldsymbol{JJ^T})}$. A global dexterity index is given by (J.Angeles, 1990) as

$$GDI = \frac{\int_V^0 1/_{\lambda}\, dV}{V}$$

V denotes the workspace capacity, $\lambda = \|J\|\|J^{-1}\|$ is the condition number of the Jacobean and $\|.\|$ is the 2 norm of the medium. A lesser condition number indicates a lower dexterous workspace.

Results presented by (Xu, 2007) indicate that the accessible workspace of a 3-RPS parallel robot falls in the range of 75 degrees of the motor design angle. Comparing 3-RPS, 3-PRS and Tricept manipulators (Carretero, 2007); for applications of high accuracy and stiffness, Merlet's 3-RPS manipulator may be a good choice. For a larger dexterous workspace volume, the Tricept might be the best choice.

2.5 SELECTION OF PARALLEL MANIPULATOR

In the inverse kinematics, required joint variables were found using the given position of the moving platform. Closed form solutions were formulated. The results show that there are two solutions for each leg of all the parallel manipulators studied.

In the forward kinematics, all possible positions of the parallel mechanisms were found given the joint parameters. Each translational parallel manipulator has two solutions for the forward problem. Parallel manipulators with parasitic motions have sixteen solutions, excluding the 3-RSR manipulator which has only one solution for the forward kinematics. The choice of a particular mechanism largely depends on the job intended for implementation. Performance procedures might not be actually significant for a specific process. The structure that meets certain specified requirements should be chosen for a particular task. A manipulator might have a feature which is favorable for a particular task but is unsuitable for another application.

In this research, the 3-RPS parallel manipulator was chosen for the massage application due to fact that the RPS structure allows the upper platform to rotate and translate; needed for the massaging process. There are three mobility abilities: two degrees of freedom orientation and one degree of freedom translation. The origins of the coordinate frames are positioned at the mass centers of the two platforms. The translation will make the mechanism to stretch, the first orientation will allow it to be placed on the skin and the second orientation will make it to roll on the body, thus massaging the body.

The RPS parallel manipulator has also passed the criteria of being a good parallel manipulator. Vis:

- It is simple and practical with only two links in each leg
- It is more controllable because of its prismatic joints being actuated
- The RPS parallel manipulator has symmetry, with all three legs having the same kinematic structure
- It has the proper type of motion since none of its legs has any complicated motions.

2.6 COMPLEXITY OF 3-RPS PARALLEL MANIPULATOR

Present is a problem with the spherical (ball and socket) joints in the 3-RPS manipulator. Usually the structures have a lesser range of movement (usually $\pm$ 15 degrees) or inhibit the rotation of some bodies around the same point needed by several architectures (Merlet, 2006). However, the 3-RPS structure is more simplified than the 3-PRS parallel manipulator; the links between the revolute and spherical joints have been eliminated (Carretero, 2007). This reduces the cost of the manipulator as the prismatic joints will be actuated instead of revolute joints as in the PRS manipulator. The revolute joint joins the base to the prismatic body. The ball and socket joint connects the moving platform to the prismatic joints. The 3-UPU parallel mechanism is the most studied parallel manipulator but it is very sensitive to manufacturing tolerances (Merlet, 2006). The overall topology of the RPS manipulator as seen in figure 2.5 is less complicated. The equations defining the forward and inverse kinematics though similar and ending with identical solutions are easier than for the other manipulators.

The 3-RPS has useful functions such as the orientation of solar panels. It can also be used as a pointing device and as a wrist for adjusting coordinates in space.

2.7 COMPARISON BETWEEN 3-RPS MASSAGER AND OTHER MASSAGE DEVICES

There are lots of massage devices in the market and many more are been invented every day. These devices range from very small ones to very huge ones. Example of small ones are the ‘wooden beaters’ and massage pillows. Examples of big ones are the massage chairs. All these massagers have their advantages. The ‘wooden beaters’ for example; though small and portable, are used in such a way that the stressed area is been plucked. In the end, the particular area is relieved while the hands are stressed. With regards to massage pillows, some are water carrying, hence very heavy and not portable. The greatest disadvantages of massage chairs are their price and size (or bulkiness). Due to these huge costs not many people can afford these chairs. They are normally used in public areas as ‘pay as you go’. In the end, there is no value for money for their usage.

The 3-RPS parallel massager is different especially compared to the massage chairs. The 3-RPS massager is more economical since one can afford to buy it and take it home. The common feature of a massage chair is; it is a recliner, but not everybody is comfortable and can feel relaxed when sitting down. In case of the 3-RPS massager it can be used lying down. The stand can be rotated 360 degrees and can be adjusted up or down according to usage. Massage chairs have complicated motors and gears that turn the rollers which make the massage action. The proposed massager has only three motors and one roller on the end effector to achieve the same effect.

Figure 2.6: A-OTO DANTE ONE-01 massage chair and B-3-RPS parallel massager

Table 2.5: comparison between 3-RPS massager and massage chairs

	3-RPS massager	Robotic Chair
Economics	Comparatively cheap	Very expensive
Complexity	No complexity	Very complex
Portability	Portable	Not portable
Components	Less components are used	Large number of components are used
Usage position	Recline	Lying down (most appropriate position for relaxing)

2.8 SUMMARY

An introduction of the types of robotic manipulators; serial, parallel and hybrid is made. To better understand the advantages of parallel mechanisms, a comparison of the parallel and serial manipulators is made. The target of this research is the analysis and selection of the most appropriate 3-DOF parallel mechanism for a massage application. Analysis of two divisions of 3-DOF parallel mechanisms is made, the first class without parasitic motions and the second with parasitic motions. The different architectures are compared based on their kinematics. In order to normalize the comparison, the same type of coordinate system was used for all the manipulators. The direct and inverse kinematics of all the manipulators are found and presented. Moreover, for a very comprehensive comparison kinematic related features are also analyzed. These factors are workspace, dexterity and stiffness. The advantages and disadvantages of both the proposed 3-RPS massager and other massage devices were enumerated. In the end, the 3-RPS parallel manipulator is chosen for our particular application due to its simplicity, practicality, symmetry and controllability.

CHAPTER 3 METHODOLOGY

In this chapter, the design of a prototype 3-RPS parallel manipulator for massaging application will be presented. Kinematics design, mechanical design and control of the mechanism will be considered. The kinematics design comprises; number and type synthesis and dimensional synthesis. Mechanical involves selection of actuators, sensors and type of material to use. Basically, the design process will involve material and parts selection, structure dimension, structural design involving the design of links and joints and the selection of actuators (fixed; mechanical stops, limit switches, servo; PTP or CP, Arduino controlled?). The joints will be designed for a maximum of 2rad/sec and an acceleration of 10rad/sec^2.

3-RPS manipulator produces three kinds of motions, a translation about the y-axis, which is called pitch, a rotation about the x-axis called yaw, and a rotation about the z-axis called roll. The revolute and prismatic joints have one degree of freedom each while the spherical has three degree of freedom.

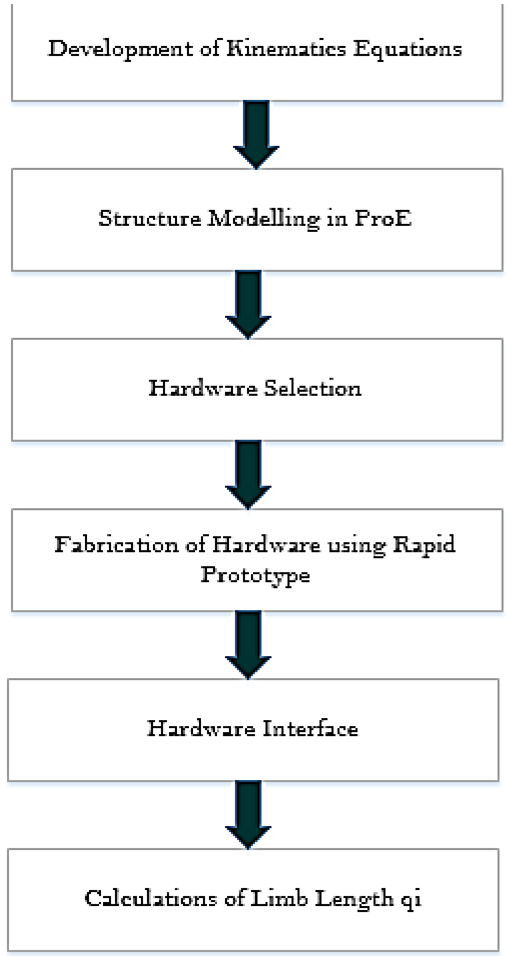


Figure 4.1*: *Flow of Research

3.1 STRUCTURE CONFIGURATION

A 3-RPS parallel manipulator has two platforms; one fixed and the other moving. The mechanism has three limbs/struts, each limb with two links. The limbs are symmetrical, i.e, having the same number and types of links and joints. It consists of three types of joints; a revolute joint, a prismatic joint and a spherical joint. The revolute joints are fixed to the base. The prismatic joints join the two links together, while the spherical joint is attached to the moving platforms. The prismatic joints are the actuated joints, each driven by a base fixed linear actuator.

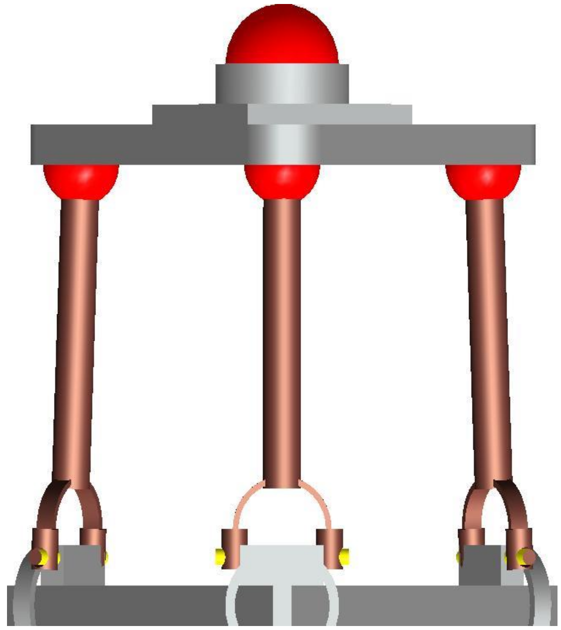


Figure 3.2 CAD model of 3-RPS parallel manipulator

3.1.1 Struts

Struts should be very stiff in order to take on the weight of the mechanism and the acceleration due to gravity in the upside down position. The longer the strut, the higher the flexibility. Hence, the tool accuracy increases and the end effector error increases. It is therefore desirable to maintain a fixed length strut. Moreover, a strut with two attached joints is the weakest element of the manipulator. That is why special consideration should be given to the design of struts. However the link length should be made long enough in order not to impose more constraints on the mechanism. This consideration will make the range of motion of the linear actuators to be fully utilized. Another issue to consider is link interference. In this case, it will be avoided by design and simulation in the computer aided design (CAD) environment.

Figure 3.3: CAD model of one of the struts

The revolute joints are fixed to the base leading to no bending and twisting in static modes. In dynamic modes, small bending and twisting due to vibrations may occur. With these reason materials with low specific density and high axial filling is preferred. A lot of materials from aluminum to steel were studied. The mechanism will be used upside down; therefore its weight is of the utmost concern. Aluminum was chosen because of its low density and corrosion resistance. Low weight aside, carbon composites have excellent thermal, mass and stiffness characteristics.

An outer radius of 10mm and an inner radius of 8mm was chosen. A length of 80mm was chosen from the revolute joint to the end of the first link, the two links been equal.

3.1.2 Joints

Joint stiffness is the most important parameter as it determines the overall stiffness of the mechanism. The spherical joint is compact, straightforward and easy to manufacture. Normally in a mechanism the revolute or the prismatic joints are actuated. In this research, the prismatic joints are actuated with consideration to their ability to achieve a very high level of accuracy and manipulability of heavy loads. Usually, actuated joint selection ensures that, with the general structure, the DOF of the mechanism with the three actuated joints locked is zero.

3.1.3 Platform

The base platform is a triangular shape with all three sides 120 degrees apart. Each side is chosen to be 150mm, having a thickness of 10mm and a radius of 83mm from the middle of the platform labeled point O. The moving platform is also of the same shape, and with each side been 150mm and a radius of 83mm form the platform center, labeled P. As can be seen in figure 3.1, there is a distance between the plane of the revolute joint and the platform.

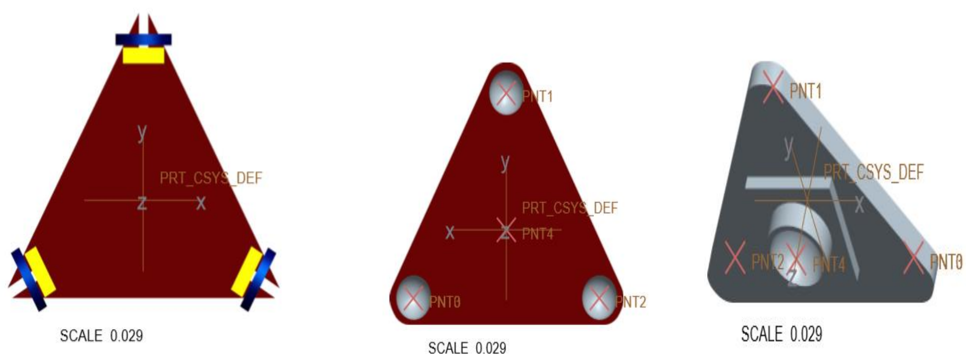


Figure 3.4 CAD model of the top view of the base platform, back view of the moving platform and top view of the moving platform.

3.1.4 Actuators

Linear actuators have the advantages of been precise with increase in motion performance. Actuators with active travel 250mm were chosen. The use of linear actuators has made tremendous changes in terms of simplicity and economics. The actuator has built-in DC motors, thus there is no need for external motors. Also the use of revolute joints has been eliminated. The linear actuator and its mount provide a back and forth motion same as a revolute joint. A 24V DC linear actuator with a speed of 6mm/s was chosen so that the error will be minimized.

3.2 THE 3-RPS PARALLEL MANIPULATOR

The schematic diagram of 3-RPS parallel manipulator is shown in fig 3.4. The two platforms are connected by a revolute joint, in the middle a prismatic joint and then followed by a spherical joint. The prismatic joint is the actuated joint which is driven by linear actuator. Points A_i are assumed to be at a circumradius of r_a from point O while points B_i are assumed to be at a radius of r_b from point p. The distance between point O and point p is assumed to be P_I. The x −axis of coordinate frame A: xyz is in line with OA_1 and the X-axis of coordinate B: XYZ is aligned with PB_1as shown in the figure.

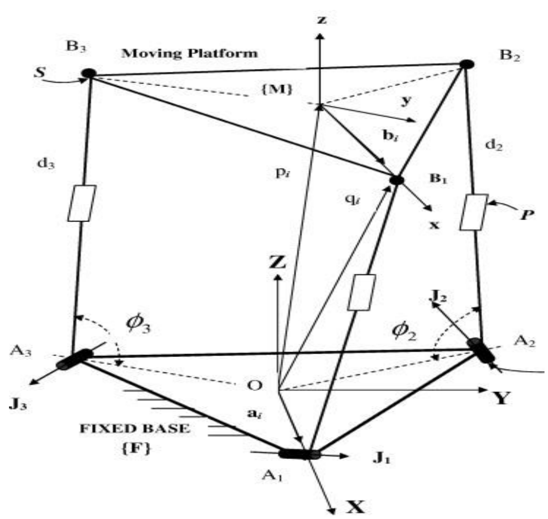


Figure 3.5: 3-RPS parallel manipulator

For a 3-DOF parallel manipulator, the point $\overline{OP}$ is used to define the position of the platform.

$$A_P = [P_x \quad P_y \quad P_z]^T$$

And a rotation matrix $$A_{R_B} = R_z(\Psi)R_y(\theta)R_x(\phi)$$

$$= \begin{bmatrix} c\Psi c\theta & c\Psi s\theta s\phi - s\Psi c\phi & c\Psi s\theta c\phi + s\Psi s\phi \\ s\Psi c\theta & s\Psi s\theta s\phi + c\Psi c\phi & s\Psi s\theta c\phi - c\Psi s\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix}$$

Where ϕ, θ, Ψ are rotation angles about x, y and z-axes, known as roll, pitch and yaw.

Hence, six variables completely define the position and rotation of a moving platform.

$$x = [\,P_x \quad P_y \quad P_z \;\; \phi \quad \theta \quad \Psi]^T$$

The angle for any of the coordinates can be obtained from the transformation

$$\begin{bmatrix} x_i' \\ y_i' \\ z_i' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & cos\phi & -sin\phi \\ 0 & sin\phi & cos\phi \end{bmatrix} \begin{bmatrix} cos\theta & 0 & sin\theta \\ 0 & 1 & 0 \\ -sin\theta & 0 & cos\theta \end{bmatrix} . \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ w \end{bmatrix}$$

$$= \begin{bmatrix} cos\theta & 0 & sin\theta \\ sin\theta sin\phi & cos\phi & -cos\theta sin\phi \\ -cos\phi sin\theta & sin\phi & cos\phi cos\theta \end{bmatrix} \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ w \end{bmatrix}$$

Where $x_i, y_{i,}$ and z_i are the final positions after the transformations. The above denote the positions of the end-effector $b_i(x_i,\ y_{i,}\ z_i)$ for $i = 1,2,3$ and *w* is the translation along the *z-axis* (The extension of the linear actuator).

The location vectors of points A_i and B_i with regards to frames A and B can be found from

$$A_{\boldsymbol{a}_i} = [a_{iX},\ a_{iY\ ,}\ 0]^T = A_1\left(-\tfrac{1}{2}r_a, \tfrac{\sqrt{3}}{2}r_a,\ 0\right),\ A_2(r_a, 0,\ 0),\ \ A_3\left(-\tfrac{1}{2}r_a, \tfrac{\sqrt{3}}{2}r_a,\ 0\right) \quad (3.1)$$

$$B_{\boldsymbol{b}_i} = [b_{ix}, b_{iy\ ,}\ 0]^T = B_1\left(-\tfrac{1}{2}r_b, \tfrac{\sqrt{3}}{2}r_b, 0\right),\ \ B_2(r_b, 0, 0),\ \ B_3\left(-\tfrac{1}{2}r_b, -\tfrac{\sqrt{3}}{2}r_b, 0\right) \quad (3.2)$$

$$A_{\boldsymbol{b}_i} = A_{\boldsymbol{R}_b} B_{\boldsymbol{b}_i} \quad (3.3)$$

For this mechanism, the independent motion variables are chosen to be $\boldsymbol{x_i} = [\,\Psi, \theta, P_z]^{\boldsymbol{T}}$and the dependent variables are $\boldsymbol{x}_d = \left[P_x,\ P_{y,}\ \phi\right]^T$

Some factors should be considered in the design of a practical manipulator. These factors produce physical constraints such as the limits of the ball and socket joints and the actuating link lengths (Hughes, 1998). The spherical joints are firmly attached to the upper platform such that the axis of symmetry of each socket intersects the normal of the plane. Also the revolute joints on the fixed platform constrain the ball and socket joints on the moving platform to move in a plane generated by the vectors $\overline{OA_\imath}$ and$\overline{A\imath\,B\imath}$.

The x and y coordinates of vector $\overline{OB_\imath}$, r_i are

$$\boldsymbol{r_i} = \boldsymbol{P} + \boldsymbol{A_{R_b}}\boldsymbol{B_{b_i}} \tag{3.4}$$

Substituting the values of r_{ix} and r_{iy} into Eq. (2.48) will yield three constraint equations that relate the independent and dependent variables as:

$$\phi = Atan2(s\Psi c\theta,\ c\Psi + c\theta) \tag{3.5}$$

$$P_y = -r_b s\Psi s\phi \tag{3.6}$$

$$P_x = -\frac{r_b}{2}(c\theta c\phi + s\phi s\theta s\Psi - c\phi c\Psi) \tag{3.7}$$

$A\tan2(x, y)$ Calculates $\tan^{-1}\left({}^{x}\!/_{y}\right)$ but employs the sign of both variables to find the angle quadrant.

Eq. (3.5), (3.6) and (3.7) can be written as

$$\boldsymbol{x_d} = \mathfrak{G}(\boldsymbol{x_i}) \tag{3.8}$$

3.1.5 Inverse kinematics

The independent variables $\boldsymbol{x_i} = [\Psi,\ \theta,\ l_z]^{\boldsymbol{T}}$are given; the problem is to find the limb lengths q_i. The values of $\boldsymbol{x_i}$ are substituted in Eq. (3.8) to find $\boldsymbol{x_d}$**.** Hence the position and orientation of the moving platform will be completely known.

For the moving platform

$$b_1\left(-\frac{1}{2}r_b, \frac{\sqrt{3}}{2}r_b, 0\right),\ \ b_2(r_b, 0, 0),\ \ b_3\left(-\frac{1}{2}r_b, \frac{\sqrt{3}}{2}r_b, 0\right)$$

For the base platform

$$a_1\left(-\frac{1}{2}r_a, \frac{\sqrt{3}}{2}r_a,\ 0\right),\ a_2(r_a, 0,\ 0),\ \ a_3\left(-\frac{1}{2}r_a, \frac{\sqrt{3}}{2}r_a,\ 0\right)$$

From $p_y = \frac{\delta y}{\delta}$ a vector loop equation is written for each limb as

$$q_i s_{2,i} = \boldsymbol{l} + \boldsymbol{b_i} - \boldsymbol{a_i} \tag{3.9}$$

Where q_i is the length of leg i and $s_{2,i}$ is a unit vector in the direction of ith leg. Dot multiplying the Eq. (3.9) with itself gives:

$$q_i^{\boldsymbol{2}} = [\boldsymbol{l} + \boldsymbol{b_i} - \boldsymbol{a_i}]^T[\boldsymbol{l} + \boldsymbol{b_i} - \boldsymbol{a_i}] \tag{3.10}$$

Expanding the right hand side of Eq. (3.10) and taking the square root yields

$$q_i = \pm\sqrt{(P_x + b_{ix} - a_{ix})^2 + (P_y + b_{iy} - a_{iy})^2 + (P_z + b_{iz} - a_{iz})^2}, \text{ for } i = 1, 2, 3. \tag{3.11}$$

Though the equation shows two possible solutions for the manipulator position, only the positive limb length can be obtained without changing the configuration of the manipulator.

3.1.6 Forward kinematics

Forward kinematics requires the limb lengths to be given and the position and orientation of the mechanism to be found. In (L.W, 1999), Tsai introduced a forward kinematics solution of a particular 3-RPS manipulator. The results are used in this section.

The tilt angles of the limbs $A_iB_{i,}$ φ_i are introduced in order to reduce the forward kinematics problem to a 16th degree polynomial in one variable. In this case, we will assume an angle of β_i which is measured from $x - axis$ to OA_1. Angle φ_i is measured from $\overline{A_\imath B_{\imath\,,}}$ to $\overline{A_\imath O}$. The radius vectors of point $B_{i\,,}$ can be calculated from

$$\boldsymbol{r_i} = \begin{bmatrix} (r_a - q_i c\varphi_i)c\beta_i \\ (r_a - q_i c\varphi_i)s\beta_i \\ q_i s\varphi_i \end{bmatrix} \tag{3.12}$$

The position and orientation of the manipulator can be obtained by equating $B_iB_{i+1} = \sqrt{2{r_b}^2 - 2{r_b}^2 c(\beta_{i+1} - \beta_i)}$

That is $[\boldsymbol{r_i} - \boldsymbol{r_{i+1}}]^T[\boldsymbol{r_i} - \boldsymbol{r_{i+1}}] - 2{r_b}^2 + 2{r_b}^2 c(\beta_{i+1} - \beta_i) = 0$ for $i = 1, 2, 3.$

(3.13)

Substituting Eq. (3.13) for $i = 1, 2$ and 3 into eqn above yields

$$\mu_{1i}c\varphi_i c\varphi_{i+1} + \mu_{2i}s\varphi_i\varphi_{i+1} + \mu_{3i}c\varphi_i + \mu_{4i}c\varphi_{i+1} + \mu_{5i} = 0 \text{ for } i = 1, 2, 3. \tag{3.14}$$

Where

$\mu_{1i} = -2q_iq_{i+1}c(\beta_i - \beta_{i+1})$

$\mu_{2i} = -2q_iq_{i+1}$

$\mu_{3i} = 2q_ir_a - 2q_ir_ac(\beta_i - \beta_{i+1})$

$\mu_{4i} = 2q_{i+1}r_a - 2q_{i+1}r_a c(\beta_i - \beta_{i+1})$

$\mu_{5i} = {q_i}^2 + {q_{i+1}}^2 - c^2\beta_i + 2{r_a}^2\big(1 - c(\beta_i - \beta_{i+1})\big) - \quad 2{r_b}^2 + 2{r_b}^2 c(\beta_i - \beta_{i+1})$

The above Eq. (3.14) is converted to a method of polynomial equations so as to eliminate two of the unknowns. Using the trigonometric identities

$$s\varphi_i = \frac{2t_i}{1 + {t_i}^2} \quad \text{and } c\varphi_i = \frac{1 - {t_i}^2}{1 + {t_i}^2}$$

In Eq. (3.14) yields a fourth degree polynomial in $t_{1,} t_2$ and t_3

$$\bar{\mu}_{1i}{t_i}^2{t_{i+1}}^2 + \bar{\mu}_{2i}{t_i}^2 + \bar{\mu}_{3i}t_{i+1} + \bar{\mu}_{4i}{t_i}^2{t_{i+1}}^2 + \bar{\mu}_{5i} = 0 \text{ for } i = 1, 2, \tag{3.15}$$

Where

$\bar{\mu}_{1i} = \mu_{1i} - \mu_{3i} - \mu_{4i} + \mu_{5i}$

$\bar{\mu}_{2i} = -\mu_{1i} - \mu_{3i} + \mu_{4i} + \mu_{5i}$

$\bar{\mu}_{3i} = -\mu_{1i} + \mu_{3i} - \mu_{4i} + \mu_{5i}$

$\bar{\mu}_{4i} = 4\mu_{2i}$

$\bar{\mu}_{5i} = \mu_{1i} + \mu_{3i} + \mu_{4i} + \mu_{5i}$

Sylvester's Dialytic Elimination method is used to further decrease the equations into eight degree polynomial in $t_{1.}$ There is at most eight solutions for t_1 one being the opposite of the other. With these evaluation, values of t_1, t_2 and t_3 can be obtained by back substitution. The angles φ_i can now be obtained as $2\tan^{-1}(t_1)$.

The position vector $\boldsymbol{P}$ on the moving platform has been earlier defined as equidistant from the points B_1, B_2 and B_3 with a radius of r_b, this provides three quadratic equations

$$(l_x - b_{ix})^2 + (l_y - b_{iy})^2 + (l_z - b_{iz})^2 = r_b{}^2 \text{ For } i = 1, 2, 3. \quad (3.16)$$

Subtracting for $i = 1$ from for $i = 2$ and $i = 3$ yields

$$v_{1j}p_x + v_{2j}y + \mu_{3i}c\varphi_i + v_{3j}p_z + v_{4j} = 0 \text{ for } j = 1, 2. \quad (3.17)$$

Where

$v_{1j} = 2(b_{1x} - b_{j+1x})$

$v_{2j} = 2(b_{1y} - b_{j+1y})$

$v_{3j} = 2(b_{1z} - b_{j+1z})$

$v_{4j} = b_{j+1x}{}^2 + b_{j+1y}{}^2 + b_{j+1z}{}^2 - b_{1x}{}^2 - b_{1y}{}^2 - b_{1z}{}^2$

Point ***p*** lies in the plane defined by B_i hence

$$\overline{PB_1}.\,(\overline{B_1B_2} \times \overline{B_1B_3}) \quad (3.18)$$

Solving the above Eq. (3.17) and (3.18) simultaneously yields the position vector ***p.*** The orientation of the mobile platform can be obtained by solving Eq. (3.18)

3.3 COMMUNICATION METHOD

All the base and moving platform parameters were declared. The inverse kinematics equation was converted to C++ in the Arduino environment. The angle was inserted in the Arduino and the calculation was done. The angles and all other parameters were substituted into the inverse kinematics equation to find the coordinate of the end effector. The same set of parameters and equations were inserted in Microsoft Excel in order to find a benchmark for the calculation. This comparison is very important to ensure accuracy in the calculation.

3.1.7 Arduino Uno communication

The Arduino Uno has PWM and DIR pin for the Pulse Width Modulation and for changing the direction of the motors to retract and extend the linear actuators. The Inverse Kinematics equation will be used to control the structure. The Inverse kinematics program is uploaded into the Arduino which sends signals through its output pins to the hardware.

3.4 INVERSE KINEMATICS EQUATION IN C++

First of all, the angles were converted to radians. All the parameters were declared in the Arduino environment. The Inverse Kinematic s equation was converted to C++ language. The equation uses the declared parameters to find the end effector position. Whenever the Arduino receives the new input angle, it calculates the Inverse kinematics according to the figure below.

```
void setup(){
 //manipulator geometry
const float A = 150.0;//side of base
const float B = 150.0;//side of end effector
const float ra = 83.0;// radius of the base
const float rb = 83.0;// radius of the end effector
const float lz = 45.0;// actuator length at home position
int i=0;
//end effector geometry
const float b1x = -41.5;//B(xi, yi, zi)
const float b2x = 83.0;
const float b3x = -41.5;
const float b1y = 71.8;
const float b1z = 0;const float b2z = 0;const float b3z = 0;const
const float b3y = -71.8;

//base geometry
const float a1x = -41.5;const float a3x = -41.5;
const float a1y = 0;const float a2y = 0;const float a3z =
```

Figure 3.6: Variables to be used in the program

During the calculation, the system only receives the input angle which is manually inserted in the Inverse Kinematics equation, in order to change the position of the end effector. There is no provision for error checking and adjustment because no sensor was used. Hence, the system is an open loop system and thus, the actual position of the end effector cannot be established.

```
//trigonometric constants
const float pi = 3.141592653;
const float sin30 = 0.5;
const float cos30 = 0.8660;
const float sin60 = 0.8660;
const float cos60  =  0.5;
const float  sin120  =sqrt(3/2.0);
const float cos120 = -0.5;

//Inverse kinematics
int RPS_calcInverse (float  theta1,float  theta2,float  theta3,float &

 float Angle=atan2(sin(30)*cos(30), cos(30)+ cos(30));
 float ly = -(float)83.0* sin(30)*sin(30);
 float lx = -(83.0/2)*(cos(30)*cos(Angle)+ sin(Angle)*sin(30)*sin(30)-
 Serial.print("Angle is"); Angle;


//calculation of the limb lenght
```

Figure 3.7: Inverse kinematics Equation

3.5 CIRCUIT DESIGN

3.5.1 Schematic Design

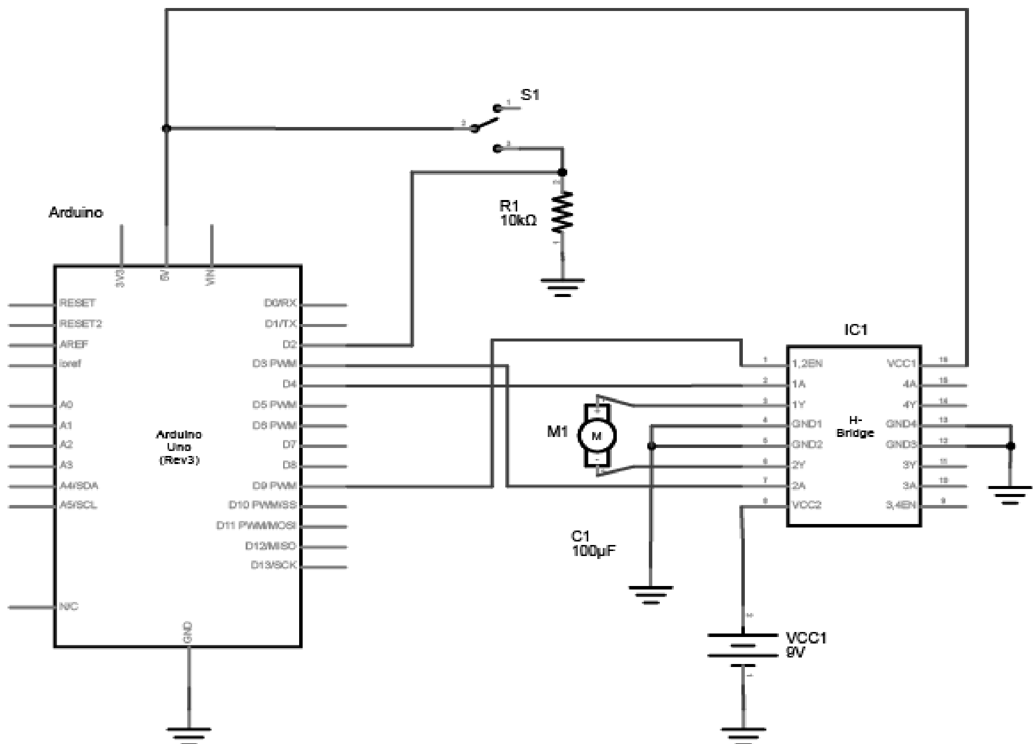


Figure 3.8: Linear actuator DC motor connection to the Arduino and the shield

The circuit in figure 3.8 depicts the connection of one linear actuator DC motor to the Arduino. The Arduino should not be connected directly to the linear actuator as it is not powerful enough to turn on and control the device. That is the reason for using a motor shield. A motor shield is designed to drive inductive loads such as the DC motors in the linear actuators. It can be used alone to control more than one motor or device. It requires external power to operate because the Arduino is not capable of powering it. Motor shield MD 10 is used in this research. It has a bi-directional control for one DC motor, hence, three shield are used, one for each actuator. The external power supply and the linear actuators are connected direct to the terminal block. The board has selectable motor direction and PWM pins.

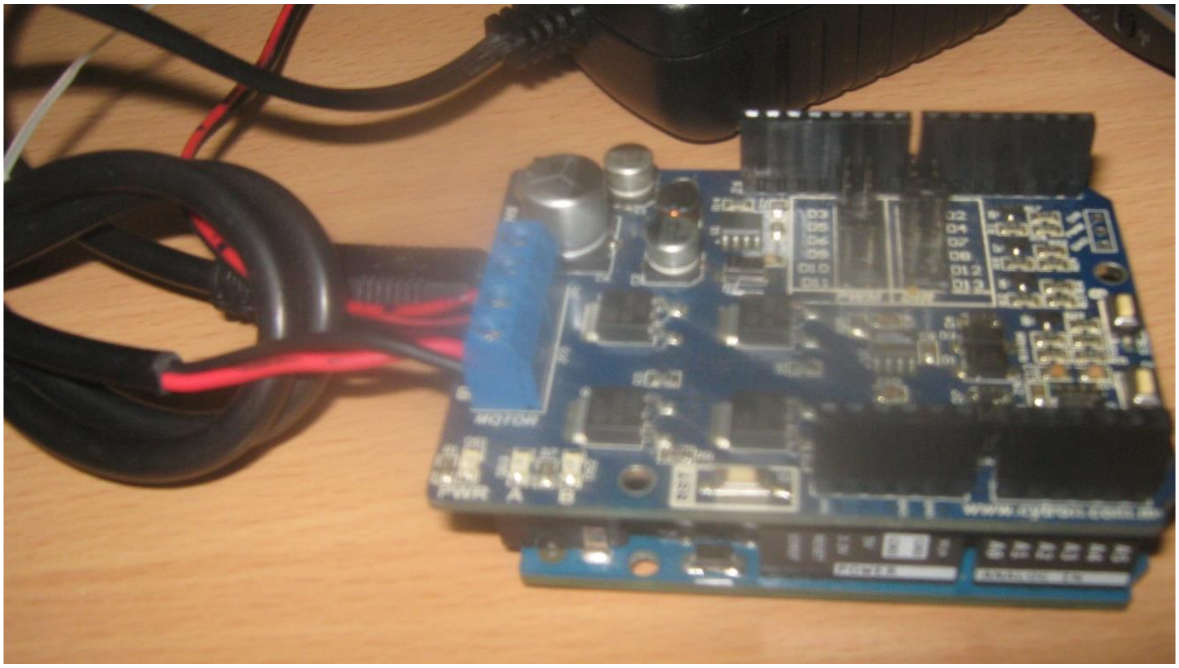


Figure 3.9: Arduino board connected to motor shield

3.5.2 Circuit board layout

i. 24V power supply: an external voltage of 24V provides power for the circuit. The linear actuator voltage is 24V and the current rating is

ii. LED indicator: an LED is connected to each output 13 pin of the board to provide a signal for normal operation condition. When there is no light to the LED, this indicates no power to the board

iii. Arduino Uno board: it facilitates interface between hardware and software. This board can provide serial communication to other devices through its two pins 0(Rx) and 1(Tx) by transmitting or receiving data

3.6 STRUCTURE PROTOTYPING

3.6.1 Structure overview

The parts in the structure were manufactured using different manufacturing processes with various materials, dimensions and tolerances. The top and base platforms were made from Acrylonitrile Butadiene Styrene (ABS) material. ABS has a high impact resistance, durability, light weight and less risk to hazard. The manufacturing process used on the ABS is Rapid prototyping, a process that fabricate the scale model of the three dimensional CAD model. CAD model contains the exact model with the same dimensions as the proposed model. The structure was made to be 10mm ABS. The angles between each linear actuator is 120 degrees.

The linear actuator mounts were fixed to the base platform as per the CAD model. The linear actuators were mounted with screws. The ends of the latter were then attached to the spherical joints, which were then in turn fixed to the upper platform. The most complicated part of the structure is spherical ball joint, which is very hard to get due to consideration for its dimension. The spherical joint used here is 62mm and made of blackened alloy steel.

3.6.2 Part assembly

The structure consists of two platforms, three linear actuators and three spherical ball bearings. The platform was made out of ABS material. The linear actuators come equipped with their mounts which serve as revolute joints. Large linear actuators (300mm, 250mm stroke) were used due to their low speed (6mm/s) in order to minimize error. One end of the linear actuators (the mount) was connected to the base, while the other end was connected to the spherical joints on the upper platform.

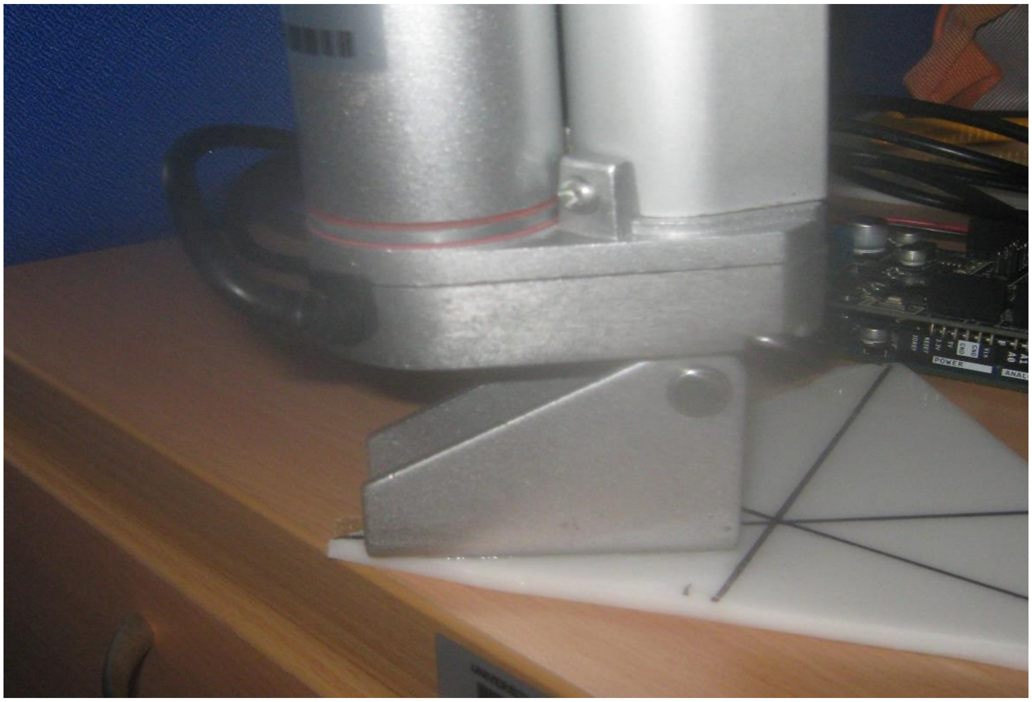


Figure 3.10: Linear actuator and mount to serve as revolute joint

Figure 3.11:Components used in making the manipulator

Table 3.1: list of parts used for the structure

S/N	Type	Number	Materials
1	Platform	2	ABS
2	Linear actuator and mount	3	Aluminum
3	Spherical ball bearing	3	Steel alloy

Figure 3.12: assembled components

CHAPTER 4 RESULTS

4.1 CAD ANALYSIS

The full model of the CAD data was simulated in the CAD environment. The animation window was used to check the angles and the limb lengths of the structure. The exact dimensions for the built model were used for better visualization. All measurements are in millimeters and the angles are in degrees. In the CAD model, the measurements are in cm. At home position, the length of each actuator and its mount is 300mm.

a) Moving one of the arms along z-axis

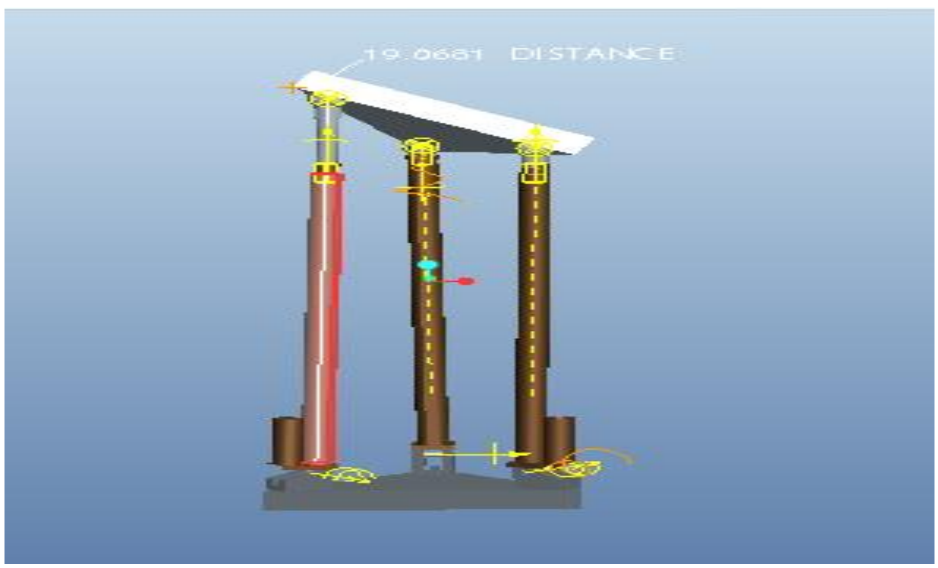


Figure4.1: CAD analysis of one arm

The two other arms were immobile, but the platform tilted to a certain position. The change in x and y-axes of platform coordinate b_i was (0 0 190).

b) Moving two arms up

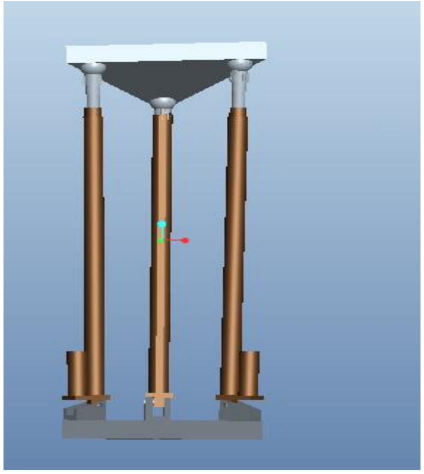


Figure 4.2 CAD model of two arms up

Moving two arms at the same time did not produce as significant a result as moving one arm. The angle formed is smaller than when one arm is moved and the change in platform orientation is less.

c) Moving three arms together

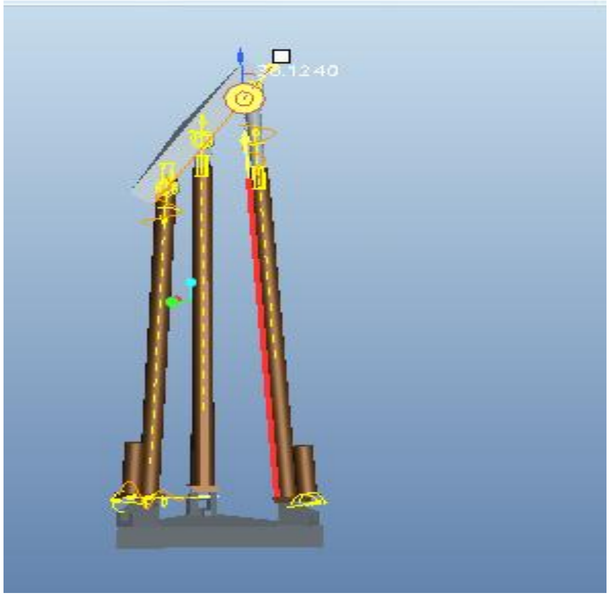
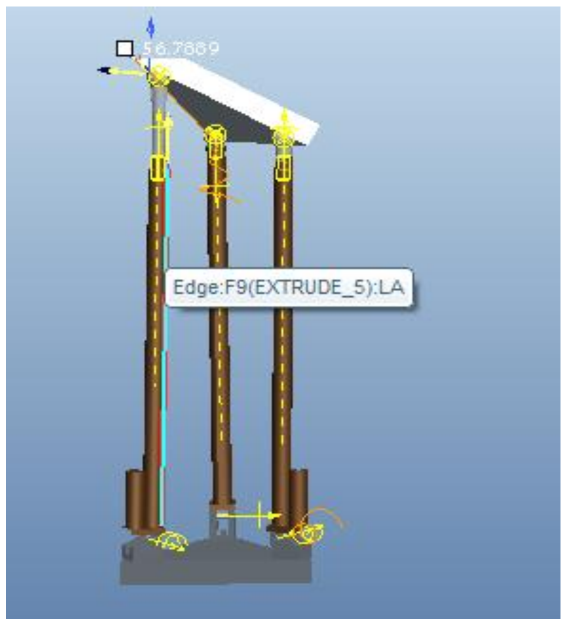


Figure 4.3: Movement of the three arms together

The motion of the three arms together produced a rotation of the platform. The change in the position of the platform was more and the angles produced along each arm were bigger. The change along the z axis is more than the change along the y-axis and it is more than that along the x-axis as verified by the excel calculation.

To check the results of the simulation, the joint angle along the z-axis was changed by tilting the linear actuator at the base.



Figur4.4: Input joint angle

Only a small angle not more than thirty degrees (30 degrees) was used because a larger angle produced a drastic change in the structure. This change by a larger angle even leads to singularity which destroyed the structure. This situation is partly due to the two platforms been of the same shape and dimension, and very small in comparison to the height and width of the linear actuators. So far as the limit of constraint/ angles is not exceeded, the bigger the angle, the higher the extension produced by the actuators.

CHAPTER 5

CONCLUSION

5.1CONCLUSION

A 3-RPS parallel manipulator was chosen for the research report. The structure was designed and developed using various manufacturing processes. The initial designs were made with CAD software; therein it was, visualized, tested and simulated to ensure accuracy in production and application. All measurements and geometry considerations were designed and obtained from the CAD data. The CAD software has dual functions of predicting the output and the limitations of the joints based on the Inverse kinematics formula.

In the Inverse Kinematics problem, the interest is to find the new actuator limb lengths given the independent parameters which consist of the rotation about the y-axis, rotation about the x-axis and the actuator length along the z-axis. This analysis is done in order to find the final actuator position. Hence, the Inverse Kinematics equation was formulated and inserted in the software environment. The parts were assembled and interfaced with the software, and the electronic circuit to implement the whole system.

5.2 RECOMMENDATION

The objective of this research is to design and develop a 3-DOF parallel manipulator for a massage application, but, due to complexity and time limit only the main body of the 3-RPS parallel manipulator was developed. Also, there was difficulty in finding the right spherical joint for the three limbs because of the dimension consideration. Therefore to find a spherical joint that can be used as a rolling member on the end effector platform will be more difficult. Nevertheless, with the bigger spherical joint, the same designed and developed parallel manipulator in this research will be used to simulate the massage procedure.

The research can be improved by:

1. More in-depth analysis of the different types of parallel manipulators should be conducted in order to determine the advantages, disadvantages, complexity and suitability of each structure to a particular application.
2. Different types of joints and their dimension should be studied to determine their effects on the structure movements and limitations in order to obtained the desired number of degree of freedom
3. Other kinds of Arduino boards should be considered in order to reduce the number of boards to be used, and obtain a board that can accommodate several motors individually.

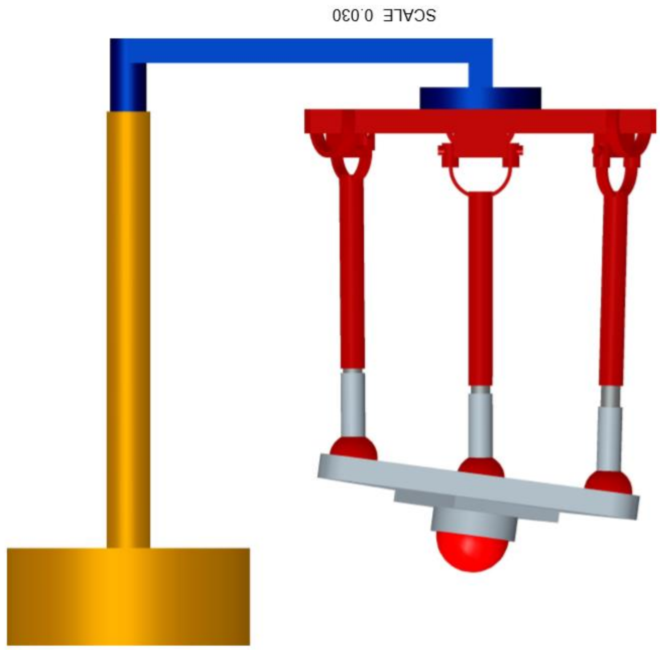


Figure 5.1: Complete 3-DOF, 3-RPS massager

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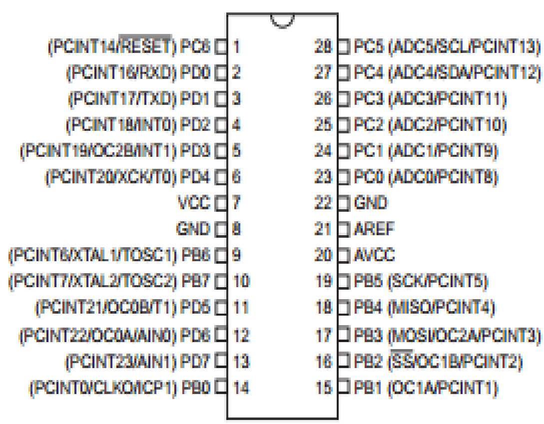
LIST OF APPENDICES

APPENDIX A – HARDWARE DATASHEET

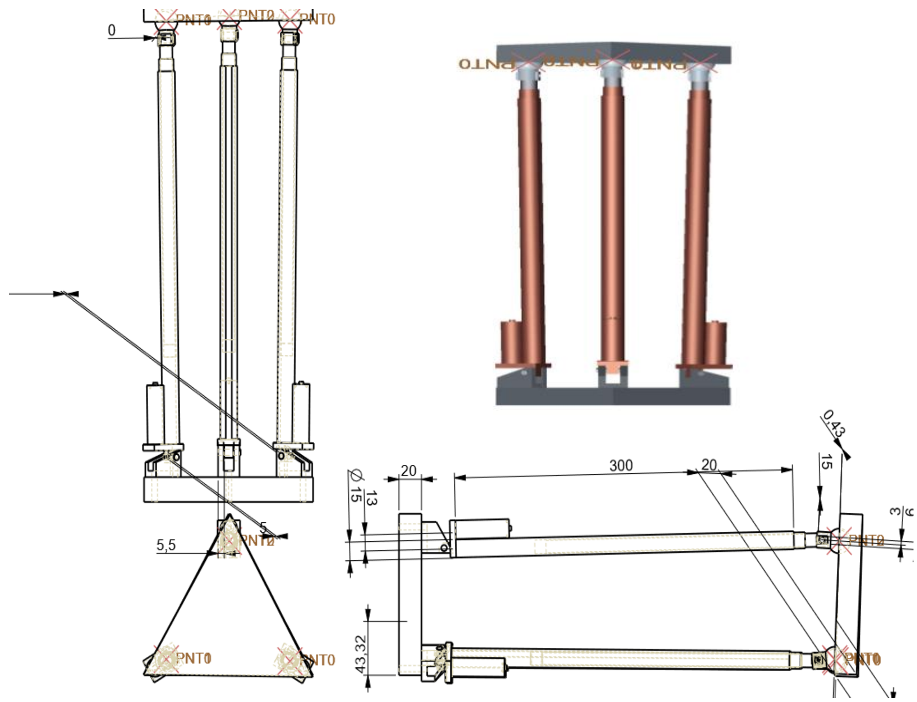
1. Arduino Uno data sheet



Pin no.	Name	Type	Description
0	Rx	Input	Receive serial TTL data
1	Tx	Output	Transmit serial TTL data
2, 4, 7, 8, 12, 13(LED)	I/O	Digital	Digital input/output
3, 5, 6, 9, 10, 11	PWM	Output	PWM Output(AnalogWrite)
14, 19, 20	GND	PWR	Supply Ground
15	AREF	Input	ADC Reference
19	RESET	Input	Reset (Active Low)
20	3.3V	Output	+3.3V output (50mA)
21	5.0V	Output/Input	Output from board
22	Vin	PWR	For external supply Voltage
23 - 27	A0 – A5	I/O	Analog input channel 0 -5

APPENDIX B – CAD DRAWING

2. 3- RPS parallel manipulator CAD design



APPENDIX C – CAD ANALYSIS ANIMATION

AnalysisDefinition12.pbk

AnalysisDefinition13.pbk