STUDIES ON Q-SWITCHING AND MODE-LOCKING PULSE GENERATION IN FIBER CAVITY WITH SATURABLE ABSORBER

AHMED NADY EWEIS ALY

FACULTY OF SCIENCE
UNIVERSITY OF MALAYA
KUALA LUMPUR

2017
STUDIES ON Q-SWITCHING AND MODE-LOCKING PULSE GENERATION IN FIBER CAVITY WITH SATURABLE ABSORBER

AHMED NADY EWEIS ALY

THESIS SUBMITTED IN FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF DOCTOR OF PHILOSOPHY

DEPARTMENT OF PHYSICS
FACULTY OF SCIENCE
UNIVERSITY OF MALAYA
KUALA LUMPUR

2017
UNIVERSITI MALAYA

ORIGINAL LITERARY WORK DECLARATION

Name of Candidate: AHMED NADY EWEIS ALY

Registration/Matric No: SHC140130

Name of Degree: DOCTOR OF PHILOSOPHY

Title of Thesis: “STUDIES ON Q-SWITCHING AND MODE-LOCKING PULSE GENERATION IN FIBER CAVITY WITH SATURABLE ABSORBER”

Field of Study: THEORETICAL PHYSICS

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STUDIES ON Q-SWITCHING AND MODE-LOCKING PULSE GENERATION IN FIBER CAVITY WITH SATURABLE ABSORBER

ABSTRACT

Pulsed fiber lasers represent the state of the art in laser technology that hold a great promise for portable and powerful pulsed light sources. Saturable absorber (SA) is a key element in optical pulsed lasers. It enables generation of pulses in one of two possible regimes, passively mode-locked or passively Q-switched. Passively mode-locked fiber lasers are amongst the best pulsed sources available due to their simplicity and ability to create transform-limited optical pulses in the picosecond and femtosecond regimes, whereas passively Q-switched fiber lasers are generally used for generating high-energy pulses at relatively low repetition rates in the microsecond or nanosecond regime. Such lasers offer excellent pulse quality and there is no need for costly modulators as required in actively mode-locked or Q-switched lasers. In this study, numerical simulations of Q-switching and mode-locking operations in fiber cavity with SA have been developed. In the Q-switched laser model, the SA dynamics was taken into account. Peak power, repetition rate, and pulse duration have been calculated as functions of pump power. In the mode-locked laser model, the temporal change in saturable absorption has been taken into account. The effects of each component in the cavity (active fiber, passive fiber, and SA) have been investigated as well as the energetics and pulse properties for different fiber laser arrangements/configurations. Furthermore, novel cobalt oxide (Co$_3$O$_4$), vanadium oxide (V$_2$O$_5$), and nickel oxide (NiO) are introduced as SAs in generating Q-switched and mode-locked EDFL. A Q-switched EDFL is demonstrated utilizing Co$_3$O$_4$ nanocubes, which was embedded into a PEO film. The proposed laser generates a stable pulse train where the pulse repetition rate is tunable from 29.8 to 70.92 kHz and the pulse-width reduces from 10.9 to 5.02 µs as the pump power increases from 55 mW to 165 mW. A V$_2$O$_5$ Q-switched EDFL is established and centered at 1565 nm with 3-dB bandwidth.
of 1.12 nm and pulse duration of 5.6 μs at 165 mW pump power. A mode-locked EDFL based on V$_2$O$_5$-SA has been successfully demonstrated. The generated pulses have centre wavelength of 1559.25 nm with duration and repetition rate of 3.14 ps and 1 MHz, respectively. Another NiO Q-switched EDFL operates at 1561.2 nm with the minimum pulse duration of 5.2 μs at 95 mW pump power. The laser has a pulse repetition rate tunable from 19.57 to 52.18 kHz as the pump power increases from 25 mW to 95 mW.

An ultrashort mode-locked EDFL is demonstrated using NiO based SA to generate optical pulses with 3-dB spectral width of about 2.85 nm centered at 1561.8 nm. The pulses have a duration of 950 fs with a repetition frequency of 0.96 MHz. The results indicate that these new SA materials have a great potential in ultrafast photonic applications.

**Keywords:** Fiber laser, Q-switching, Mode-locking, Saturable absorber, Pulse propagation.
ABSTRAK
meningkat daripada 55 mW hingga 165 mW. Suatu EDFL pensuisan-Q V₂O₅ beroperasi pada 1565 nm dan 3-dB dengan lebar spektrum kira-kira 2.85 nm dengan tempoh denyutan minimum 5.6 μs pada kuasa pam 165 mW. Mod-terkunci daripada rongga laser gentian didopkan erbium (EDFL) berdasarkan V₂O₅-SA telah berjaya ditunjukkan. Denyutan yang dihasilkan mempunyai panjang gelombang tengah 1559.25 nm dengan tempoh dan kadar pengulangan 3.14 ps dan 1 MHz, masing-masing. Suatu EDFL pensuisan-Q NiO beroperasi pada 1561.2 nm dengan tempoh denyutan minimum 5.2 μs pada kuasa pam 95 mW. Laser itu mempunyai kadar pengulangan denyutan yang boleh ditala dari 19.57 hingga 52.18 kHz apabila kuasa pam meningkat daripada 25 mW hingga 95 mW. Suatu denyutan pendek mod-terkunci EDFL ditunjukkan menggunakan SA yang berdasarkan NiO untuk menjana denyutan optik pada 3-dB dengan lebar spektrum kira-kira 2.85 nm yang berpusat di 1561.8 nm. Denyutan-denyutan itu mempunyai tempoh 950 fs dengan frekuensi pengulangan 0.96 MHz. Keputusan menunjukkan bahawa bahan-bahan baru ini mempunyai potensi yang besar dalam aplikasi fotonik pantas.
ACKNOWLEDGEMENTS

First and foremost, I would like to thank my parents and siblings especially Mahmoud for being the most supportive family one could hope for.

Importantly, I would like to express my gratitude to my supervisors, Prof. Raymond Ooi and Prof. Sulaiman Wadi Harun for their guidance and advice. Their patience and encouragement helped me in finishing this work.

Thank you to my inspiring professor, Tarek Ali Mohamed for providing me with the opportunity to engage in this project. I greatly appreciate everything he has done for me.

I also would like to thank my friends, Faisal H. Mathkoor, Mahmoud Hazzaa, Edmund Loh, Numan Archid, and Abdallah Hassaballah for their selfless assistance throughout my study.

Finally, a special thank you to my fiancée, Shrouq. You have made my life worth living again.
DEDICATION

To my immortal brother, Mohammed Nady
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<td>AF</td>
<td>Active Fiber</td>
</tr>
<tr>
<td>AM</td>
<td>Amplitude Modulation</td>
</tr>
<tr>
<td>BP</td>
<td>Black Phosphorous</td>
</tr>
<tr>
<td>CNT</td>
<td>Carbon Nanotube</td>
</tr>
<tr>
<td>CW</td>
<td>Continuous Wave</td>
</tr>
<tr>
<td>EDF</td>
<td>Erbium-Doped Fiber</td>
</tr>
<tr>
<td>EDFL</td>
<td>Erbium-Doped Fiber Laser</td>
</tr>
<tr>
<td>FESEM</td>
<td>Field Emission Scanning Electron Microscopy</td>
</tr>
<tr>
<td>FM</td>
<td>Frequency Modulation</td>
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<tr>
<td>FWM</td>
<td>Four-Wave-Mixing</td>
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<tr>
<td>FWHM</td>
<td>Full Width at Half Maximum</td>
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<tr>
<td>GVD</td>
<td>Group Velocity Dispersion</td>
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<tr>
<td>HRTEM</td>
<td>High-Resolution Transmission Electron Microscopy</td>
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<tr>
<td>LD</td>
<td>Laser Diode</td>
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<tr>
<td>LED</td>
<td>Light Emitting Diode</td>
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<tr>
<td>LP</td>
<td>Linearly Polarized</td>
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<tr>
<td>IR</td>
<td>Infrared</td>
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<tr>
<td>ISO</td>
<td>Isolator</td>
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<td>NA</td>
<td>Numerical Aperture</td>
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<tr>
<td>NIR</td>
<td>Near-Infrared</td>
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<tr>
<td>NPR</td>
<td>Nonlinear Polarization Rotation</td>
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<tr>
<td>OSA</td>
<td>Optical Spectrum Analyzer</td>
</tr>
<tr>
<td>PF</td>
<td>Passive Fiber</td>
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<tr>
<td>PEO</td>
<td>Polyethylene Oxide</td>
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<tr>
<td>RF</td>
<td>Radio Frequency</td>
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<td>Acronym</td>
<td>Term</td>
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<td>-------------------------------------</td>
</tr>
<tr>
<td>SA</td>
<td>Saturable Absorber</td>
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<tr>
<td>SBS</td>
<td>Stimulated Brillouin Scattering</td>
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<tr>
<td>SESAM</td>
<td>Semiconductor Saturable Absorber Mirror</td>
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<td>SMF</td>
<td>Single Mode Fiber</td>
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<tr>
<td>SNR</td>
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<td>Stimulated Raman Scattering</td>
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CHAPTER 1 : INTRODUCTION

1.1 Introduction

Over the past 50 years, fiber lasers have seen progressive developments and become an important device for laser light sources, which has promoted scientific and technological advances in a wide range of areas. In the past 10 years, there has been a revolutionary progress in laser technology and developments in ultrashort pulsed lasers and oscillators with high pulse energy. The developments in fiber lasers have been simplified by the continuous developments in material science. Due to the broad gain bandwidth of rare-earth ions in glass hosts, rare-earth doped fibers are excellent platforms for pulse generation in both nanosecond and ultrashort regimes. Pulsed fiber lasers represent the state of the art in laser technology that carry a great promise for portable and powerful pulsed light sources. Pulsed laser operation can be obtained by modulating the laser cavity using either Q-switching or mode-locking techniques. Q-switched fiber lasers are capable of generating microsecond or nanosecond pulses while mode-locked lasers are capable of generating ultrashort laser pulses with peak powers several orders of magnitude higher than in the continuous wave (CW) mode by locking multiple axial modes in a laser cavity.

1.2 Thesis Motivation

Fiber lasers represent one of the greatest achievements of modern optics and laser physics. Fiber lasers are well suited to versatile applications, including lasers of brute force for industrial applications (cutting and welding) and delicate devices presently being developed for the most precise measurement of time and frequency. They have some intrinsic merits over other types of lasers and have seen a remarkable growth in both research and industry.
Some of the merits of fiber lasers are stated below:

- The intense and concentrated pumping available in rare-earth doped fibers leads to lower threshold three-level laser operation, and improves efficiency and performance.
- Heat removal is much easier than in bulk solid-state lasers and this is due to the long, thin geometry of the optical fiber.
- Several cavity designs and configurations are available, which are less cumbersome and more stable than their bulk optics counterparts.
- The long fiber cavity length leads to a very narrow line width. It also makes continuous wavelength tuning possible since cavity modes are closely spaced.
- High quantum efficiency and large output power handling.
- New transition opportunities, such as in up-conversion fiber lasers.

Fiber lasers are excellent sources for pulse generation via modulating the laser cavity using either active or passive techniques. Compared with active techniques, passively Q-switched and mode-locked fiber lasers have the advantages of simplicity, high efficiency, compactness, and potentially lower cost. Furthermore, the passively Q-switched and mode-locked fiber lasers can be possible in all-fiber design. Among the common passive techniques is inserting saturable absorber (SA) materials into the laser cavity. The SA approach is more preferable than other Q-switching and mode-locking techniques owing to its simplicity and high performance. Therefore, it is very interesting to study Q-switching and mode-locking operation in fiber laser based on SA approach. Numerical simulations are important in order to understand and optimize the laser performance and operation in different cavity configurations. Furthermore, it is essential to look for new SA materials whose fabrication are simple and low-cost and show low saturation intensity with appropriate modulation depth and high damage threshold.
1.3 **Principal Objectives of the Thesis**

The principal objectives of the thesis are as follows:

- To describe and analyze the Q-switching operation in fiber laser cavity with SA through developing numerical simulations taking into account the SA dynamics.
- To develop numerical simulations for passively mode-locked fiber lasers with SA taking into account the temporal change in saturable absorption.
- To introduce new SA materials that can be effective, low-cost, and easily to synthesize for generation of pulsed fiber laser. This objective requires:
  i. Synthesis of the nonlinear optical material.
  ii. Fabrication of the SA device based on the synthesized material.
  iii. Characterization of nonlinear optical absorption of the fabricated SA.
  iv. Validating the fabricated SA device as a Q-switcher or mode-locker in generating pulsed fiber laser.

1.4 **Thesis Synopsis**

This thesis presents theoretical and experimental studies on Q-switched and mode-locked pulse generation in fiber cavity with SA. The content of the work is arranged in 6 chapters as follows: The fundamental principles of fiber laser systems are outlined in Chapter 2, which includes fiber modes, losses in fiber, and fiber dispersion and nonlinearity. The various nonlinear refractive index effects in addition to the other two important nonlinear scattering effects; stimulated Raman scattering and stimulated Brillouin scattering are discussed. Furthermore, the characteristics of the used gain medium and the basis of SA are outlined. In Chapter 3, the basics of laser Q-switching are discussed, and numerical simulations and analysis of Q-switching operation in erbium-doped fiber cavity with SA are presented. Chapter 4 covers the optical pulse propagation in fiber and the general concepts of mode-locking operation in lasers, besides presenting numerical simulations of passively mode-locked fiber lasers based on SA
approach. In these numerical simulations, the influences of the various fiber cavity elements are investigated. Generation of Q-switched and mode-locked pulse in simple erbium-doped fiber cavity using novel cobalt oxide, vanadium oxide, and nickel oxide as SAs, are then introduced in Chapter 5. Finally, the findings of the work are summarized in Chapter 6.
2.1 Introduction

Optical fiber technology started with the growth of the field of telecommunications. The rapid development in the area of communication started when the electrical telegraph was developed and patented in 1837 by Samuel Morse and then the telephone by Alexander Graham Bell in 1878. In 1878, James Maxwell paved the way for discovering radio waves by Heinrich Hertz in 1888. In 1895, Marconi demonstrated the first radio with bandwidth of around 15 kHz and during those days the maximum bandwidth of wireless communication was about a few hundred MHz. This limit of bandwidth is due to the fact that free space propagation of signals is not appropriate for fast and reliable communication links. As a solution for that, using light waveguide was proposed, but the development process took longer time.

In the 1920s, the first optical fiber was fabricated (Baird, 1928; Toth, 1930). But, these optical fibers were fabricated with no cladding, so they were not effective in guiding light because in that case the optical fiber core was not surrounded by material with a lower refractive index which guarantees the total internal reflection (TIR) phenomenon which makes the fiber able to guide light.

After two important developments, fiber optics offered significant progress. First, was the development of the cladded fibers in the 1950s (Hirschowitz, 1961; Hopkins & Kapany, 1954; O'brien, 1958; van Heel, 1954), these types of fibers enhanced guiding light in the fiber core by surrounding it with a silica cladding. The development of low loss fibers in 1979 (Miya et al., 1979) was the second achievement. The host material which is silica has almost perfect purity for which both Rayleigh scattering and material absorption at long wavelengths are at the optical loss limit. The minimum loss (about 0.2 dB/km) is found at wavelength of about 1.55 μm. This is the fundamental reason behind that modern telecommunications use wavelength of 1.55 μm.
The developments in optical fibers opened the door not only for developments in telecommunications, but also for the birth of fiber lasers.

Fiber lasers have been demonstrated in the 1960s by the incorporation of trivalent rare-earth ions such as neodymium ions Nd\(^{3+}\), erbium ions Er\(^{3+}\), and thulium ions Tm\(^{3+}\) into glass hosts (Snitzer, 1961). Soon thereafter Nd\(^{3+}\) ions has been incorporated into the fiber cores (Koester & Snitzer, 1964). Thanks to the high efficiency of the Nd\(^{3+}\) as a laser, early work was concentrated on Nd\(^{3+}\)-doped silica fiber lasers operating at 1.06 µm (Stone & Burrus, 1973). It was not until the 1980s that doping of silica fibers with Er\(^{3+}\) ions was achieved (Mears et al., 1986). Since that time Er\(^{3+}\)-doped fiber lasers have attracted much attention. This is due to its lasing wavelength at 1.55 µm which coincides with the least-loss of silica fibers (as low as 0.15dB/km) which is very suitable for light wave communications. Other rare-earth ions such as holmium ions Ho\(^{3+}\) (Allain et al., 1991; Percival et al., 1992), praseodymium ions Pr\(^{3+}\) (Durteste et al., 1991; Whitley et al., 1993), Tm\(^{3+}\) (Barnes & Townsend, 1990; Hanna et al., 1988; Hanna et al., 1990), samarium ions Sm\(^{3+}\) (Farries et al., 1990), and ytterbium ions Yb\(^{3+}\) (Pask et al., 1995) have been used as dopants or co-dopants in silica or fluoride fibers, generating new lasing wavelengths.

Table 2.1 lists the doped rare-earth ions and the transitions used for the laser operation, the types of the fiber used as host, and the lasing wavelengths. It is observed that fiber lasers cover wide range of wavelengths from about 450 to 3500 nm. This considerable spectral range provides a myriad of potential applications for fiber lasers such as optical communications, photonic switching, data storage, range finding, sensors technology, and medical and military applications.
**Table 2.1**: Lasing wavelengths of rare-earth-doped fiber lasers.

<table>
<thead>
<tr>
<th>Doped Ion</th>
<th>Transition</th>
<th>Host Material</th>
<th>Lasing Wavelength (μm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Er$^{3+}$</td>
<td>$^4S_{3/2} \rightarrow ^4I_{15/2}$</td>
<td>Fluoride</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>$^4S_{3/2} \rightarrow ^4I_{13/2}$</td>
<td>Fluoride</td>
<td>0.85</td>
</tr>
<tr>
<td></td>
<td>$^4I_{11/2} \rightarrow ^4I_{15/2}$</td>
<td>Fluoride</td>
<td>0.98</td>
</tr>
<tr>
<td></td>
<td>$^4I_{13/2} \rightarrow ^4I_{15/2}$</td>
<td>Silica/Fluoride</td>
<td>1.55</td>
</tr>
<tr>
<td></td>
<td>$^4F_{9/2} \rightarrow ^4I_{9/2}$</td>
<td>Fluoride</td>
<td>3.50</td>
</tr>
<tr>
<td>Nd$^{3+}$</td>
<td>$^4F_{3/2} \rightarrow ^4I_{9/2}$</td>
<td>Silica</td>
<td>0.92</td>
</tr>
<tr>
<td></td>
<td>$^4F_{9/2} \rightarrow ^4I_{11/2}$</td>
<td>Silica</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>$^4F_{9/2} \rightarrow ^4I_{13/2}$</td>
<td>Silica/Fluoride</td>
<td>1.35</td>
</tr>
<tr>
<td>Ho$^{3+}$</td>
<td>$^5S_2 \rightarrow ^5I_8$</td>
<td>Fluoride</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>$^5S_2 \rightarrow ^5I_5$</td>
<td>Fluoride</td>
<td>1.35</td>
</tr>
<tr>
<td></td>
<td>$^5I_7 \rightarrow ^5I_8$</td>
<td>Silica/Fluoride</td>
<td>2.08</td>
</tr>
<tr>
<td>Pr$^{3+}$</td>
<td>$^3P_0 \rightarrow ^3H_4$</td>
<td>Fluoride</td>
<td>0.49</td>
</tr>
<tr>
<td></td>
<td>$^3P_1 \rightarrow ^3H_4$</td>
<td>Fluoride</td>
<td>0.52</td>
</tr>
<tr>
<td></td>
<td>$^1D_2 \rightarrow ^3F_4$</td>
<td>Silica</td>
<td>1.05</td>
</tr>
<tr>
<td>Tm$^{3+}$</td>
<td>$^3G_4 \rightarrow ^3H_5$</td>
<td>ZBLAN</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>$^3H_4 \rightarrow ^3H_6$</td>
<td>Fluoride</td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>$^3F_4 \rightarrow ^3H_6$</td>
<td>Silica/Fluoride</td>
<td>1.90</td>
</tr>
<tr>
<td>Sm$^{3+}$</td>
<td>$^4G_{5/2} \rightarrow ^4H_{9/2}$</td>
<td>Silica</td>
<td>0.65</td>
</tr>
<tr>
<td>Yb$^{3+}$</td>
<td>$^4F_{5/2} \rightarrow ^4F_{7/2}$</td>
<td>Silica/Fluoride</td>
<td>1.02</td>
</tr>
</tbody>
</table>
2.2 Optical Fibers

Optical fiber is a circular dielectric waveguide that can carry light and information. It is composed of a core and cladding layer with a slightly lower index of refraction. In the geometrical model, optical fibers wave guiding happens thanks to the TIR of light at the fiber core. Light is trapped as long as the incident angle between the fiber core and cladding achieves the conditions of TIR. This can only happen if the incident light comes from an optically denser medium to a less dense one. Thus to guide light in fibers the index of refraction of the fiber core must be larger than the index of the cladding. This picture which gives an intuitive understanding for guiding light in fibers does not take into account the wave properties of light. Optical fibers are typically made from several transparent materials such as silica, plastic, fluoride or chalcogenide glasses (Yeh, 2013; Zoido, 1998). However, in most cases silica glass is the material that optical fibers are typically made of. This is because of the very low optical loss, the good chemical and mechanical characteristics of silica glass. The difference in index of refraction between the core and the cladding can be achieved through the fabrication process by adding dopants to increase or decrease the refractive index. For example, boron and fluorine doping reduce the index of refraction while germanium and phosphor increase it. The cladding is usually coated with one or two layers of acrylate polymer. This coating is used to protect the fiber from damage and for more environmental protection, depending on the application, several layers of protective sheath are added to form the cable. There are several fiber designs, but the simplest one is a step-index profile whose index of refraction is a constant in the core and in the cladding. Step-index profiles are commonly used because they are easier to fabricate than the complicated index shapes. The cross section of an optical fiber and a single fiber cable are shown in Figure 2.1.
Figure 2.1: Cross section view of (a) an optical fiber and (b) a single fiber cable.

The difference in index of refraction between core and cladding governs the maximum incident angle $\alpha$ needed for TIR. This condition also governs the maximum acceptance angle of the fiber at the end facet shown in Figure 2.2. The sine of this angle is called the numerical aperture (NA) of the fiber and can be expressed as

$$NA = \sin \alpha = \left( n_1^2 - n_2^2 \right)^{1/2}$$

(2.1)

where $n_1$ and $n_2$ are the refractive indices of the core and cladding, respectively, and $\theta_c$ is the critical angle. NA is a measure of the ability of a fiber to gather light. It shows how easy it is to couple light into a fiber.

Figure 2.2: Schematic illustration of determining a fiber numerical aperture.
2.3 Fiber Modes

The concept of mode refers to a specific solution of the wave equation that satisfies the appropriate boundary conditions and has the property that its spatial distribution does not change with propagation. Fiber modes can be categorized into leaky modes, radiation modes, and guided modes (Agrawal, 1997).

Leaky modes (Buck, 2004; Oliner, 1984): They are not true modes of the fiber structure because they refer to propagating waves exhibiting temporary confinement. Leaky waves can be described through the reflecting ray picture as waves that satisfy the transverse resonance condition, but that partially transmit through the interfaces at each reflection. In the symmetric fiber (slab guide), leaky waves can exist if either of two conditions are met: (1) the refractive index of the core is less than that of the cladding, so that the TIR is not possible or (2) the refractive index of the core is greater than that of the cladding, but the incident angle is less than the critical angle.

Radiation modes (Buck, 2004): They transfer light out of the core while guided modes are confined to the core, and carry light along the fiber, so via the guided modes only signal transmission in fiber-optic communication systems take places. In contrast to the leaky and guided, radiation modes have no transverse resonance requirement and always appear in continua at a given frequency (in groups within which the mode propagation constant $\beta$ varies continuously). They have two types: (1) waves that propagate out of the fiber, having angles less than the critical angle, and (2) fields that show exponential decay (but no propagation in z-direction).

Guided modes: These modes are known as the bound or trapped modes of the fiber; each guided mode represents a pattern of both electric and magnetic field distributions repeated along the fiber at equal intervals and only a discrete number of modes can propagate along the fiber. The wavelength of the mode and the size, shape, and nature of
the fiber determine which modes can propagate. Guided modes and radiation modes together form complete set that is any field distribution in the guided structure that satisfies Maxwell’s equations can be described as an expansion of radiation and guided modes.

The grouping of the different wave types in fiber is summarized in the \((\omega - \beta)\) diagram shown in Figure 2.3. The key boundaries are lines of constant phase velocities, having the slope of \(\omega/\beta = c/n_1\) and \(c/n_2\). For example, guided modes lie between these boundaries and the forbidden zone contains waves having phase velocities slower than \(c/n_1\).

**Figure 2.3:** Propagation constant grouping for the wave types in an optical fiber.

The following discussion is totally focused on the guided modes of a step-index fiber. Each component of the electric and magnetic fields satisfies Helmholtz equation

\[
\nabla^2 E + n^2 k_0^2 E = 0
\]

Helmholtz equation can be written in a cylindrical coordinate system as follows:
where the complex amplitude $E = E(r, \phi, z)$ refers to any of the Cartesian components of the electric or magnetic fields. Using the method of separation of variables and then substituting

$$E(r, \phi, z) = E(r) \exp(-jl\phi) \exp(-j\beta z), \ l = 0, \pm 1, \pm 2, \ldots$$

into Equation 2.3, the following ordinary differential equation for $E(r)$ is obtained

$$\frac{\partial^2 E}{\partial r^2} + \frac{\partial E}{r \partial r} + \left( n^2 k_0^2 - \beta^2 - \frac{l^2}{r^2} \right) E = 0$$

(2.5)

If the propagation constant $\beta$ is smaller than the wavenumber in the core $\beta < n_1 k_0$ and larger than the wavenumber in the cladding $\beta > n_2 k$, the wave is guided and define

$$k_1^2 = n_1^2 k_0^2 - \beta^2$$

(2.6)

$$\gamma^2 = \beta^2 - n_2^2 k_0^2$$

(2.7)

Then, the above equation can be written in the core and cladding separately as

$$\frac{\partial^2 E}{\partial r^2} + \frac{\partial E}{r \partial r} + \left( k_1^2 - \frac{l^2}{r^2} \right) E = 0$$

(2.8)

$$\frac{\partial^2 E}{\partial r^2} + \frac{\partial E}{r \partial r} - \left( \gamma^2 + \frac{l^2}{r^2} \right) E = 0$$

(2.9)

These equations have the solutions of family of Bessel functions as follows:
where \( J_l(x) \) is the Bessel function of the first kind with order \( l \), and \( K_l(x) \) is the modified Bessel function of the second kind with order \( l \). The function \( J_l(x) \) behaves such as the sine or cosine functions with amplitude decaying. The parameter \( k_i \) determines the rate of change of \( E(r) \) in the core while \( \gamma \) determines the rate of change of \( E(r) \) in the cladding. If \( k_i \) has large value, this indicates faster oscillation of the radial distribution in the core while a large value of \( \gamma \) indicates faster decaying and smaller penetration of the wave into the cladding. The sum of the squares of \( k_i \) and \( \gamma \) is a constant

\[
  k_i^2 + \gamma^2 = (n_i^2 - n_2^2)k_0^2 = NA^2k_0^2 = \frac{V^2}{a^2} \quad (2.11)
\]

If \( k_i \) becomes greater than \( NAk_0 \), \( \gamma \) will be imaginary and the wave will stop being bound to the core.

If \( X = k_ia \), \( Y = \gamma a \) (to normalize \( k_i \) and \( \gamma \)), Equation 2.11 simplifies to

\[
  X^2 + Y^2 = V^2 \quad (2.12)
\]

\[
  V = k_0aNA = 2\pi \frac{a}{\lambda_0} NA \quad (2.13)
\]

where \( k_0 = 2\pi/\lambda_0 \) (the wavenumber) and \( a \) is the fiber core radius. \( V \) number is an important parameter that determines the number of modes of that fiber can support and their propagation constants. For the wave to be guided \( X \) must be less than \( V \).
Most of optical fibers are weakly guided, so that the guided rays are approximately parallel to the fiber axis. Therefore, the transverse components of the electric and magnetic fields are much stronger than the longitudinal components and the guided modes are considered transverse. So the linear polarization in the directions of $x$ and $y$ form orthogonal states of polarization. Linearly polarized $(l, m)$ mode is symbolized as $LP_{lm}$ mode. The two polarization of mode $(l, m)$ travel with the same propagation constant with the same spatial distributions.

For weakly guiding fibers, the characteristic equation (Saleh et al., 1991) turns out to be nearly equivalent to the condition of the continuity of the function $E(r)$ in Equation 2.10 at boundary. This can be satisfied if

$$\frac{(k,a) \frac{dJ}{dx}(k,a)}{J_i(k,a)} = \frac{(\gamma a) \frac{dK}{dx}(\gamma a)}{K_i(\gamma a)}$$

The derivatives $dJ/dx$ and $dK/dx$ of the Bessel functions satisfies the following identities:

$$\frac{dJ}{dx} = \pm J_{l\pm 1}(x) \pm l \frac{J_i(x)}{x}$$

$$\frac{dK}{dx} = -K_{l\pm 1}(x) \pm l \frac{K_i(x)}{x}$$

Substituting these identities into Equation 2.14 and using the normalized parameters $X = k, a$, $Y = \gamma a$, the characteristic equation is obtained as (Saleh et al., 1991)

$$X \frac{J_{l\pm 1}(X)}{J_i(X)} = \pm Y \frac{K_{l\pm 1}(X)}{K_i(X)}$$
For every azimuthal $l$, the characteristic equation gets multiple solutions which gives discrete propagation constants $\beta_{lm}$, $m = 1, 2, \ldots$, and each solution represents a mode. $l = 0$ is corresponding to meridional rays.

By plotting the right and left hand sides versus $X$ and finding the intersections the characteristic equation is solved, as illustrated in Figure 2.4. The figure shows that as $V$ is increased, the number of intersections increases because the right hand side goes to the right with increasing $V$. The number of modes $M$ equals the number of roots of $J_{l-1}(X)$ that is smaller than $V$. When $V$ becomes less than 2.405, all modes are cut off except for the fundamental mode $LP_{01}$ (Agrawal, 2007). Some of the roots values are listed in Table 2.2.

![Figure 2.4: Solution of the characteristic equation for $l = 0$, $V = 10$ (Teich & Saleh, 1991).](image)

<table>
<thead>
<tr>
<th>$l$</th>
<th>$m$</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>3.83</td>
<td>7.016</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2.405</td>
<td>5.52</td>
<td>8.654</td>
<td></td>
</tr>
</tbody>
</table>
The number of modes for fiber lasers with large V-parameters is approximated by (Teich & Saleh, 1991)

\[
M \approx \frac{4}{\pi^2} V^2
\]  
(2.18)

while the propagation constant is estimated by

\[
\beta_{lm} \approx n_l k_0 \left[ 1 - \frac{(l + 2m)^2}{M} \Delta \right]
\]  
(2.19)

in which \( l = 0, 1, \ldots, M^{1/2} \), \( m = 1, 2, \ldots, (M^{1/2} - 1)/2 \).

In a single mode fiber, the mode-width parameter \( w_s \) is described as

\[
w_s = a \left( 0.65 + 1.619 V^{-1.5} + 2.876 V^{-6} \right)
\]  
(2.20)

Figure 2.5 shows the electric near field amplitude profiles for all the guided modes of a step index fiber. The two colours refer to different signs of electric field values. The lowest-order mode (\( l = 0, m = 1 \), called \( LP_{01} \) mode) shows an intensity profile similar to that of a Gaussian beam.
Figure 2.5: Electric near field amplitude profiles for all the guided modes of a step-index fiber (Source: "rp photonics").
**Evanescent field:** Another important phenomenon under conditions of TIR in optical fibers is the form of the electric field in the fiber cladding. It is found that there is still an electric field which penetrates into the fiber cladding, although there is no propagating light in the fiber cladding based on the conditions of TIR. The amplitude of the field in the fiber cladding is observed to decay exponentially in the x-direction as shown in Figure 2.6. This is called an evanescent field. A field of this type has the ability to store energy and transport it in the propagation direction (z), but does not transfer energy in the transverse direction (x). Exponentially decaying evanescent fields can be used for developing various types of intensity modulated fibre optic sensors (Shizhuo et al., 2008).

![Evanescent Field Diagram](image)

**Figure 2.6:** Illustration of exponentially decaying evanescent wave in the cladding region.

### 2.4 Losses in Optical Fibers

Loss or attenuation refers to the power reduction in the output signal as it propagates along the optical fiber (Keiser, 2003). There are various mechanisms related to material characteristic and manufacture parameters, which contribute to the net loss in optical fibers. These mechanisms can be either intrinsic or extrinsic. Intrinsic loss originates from the fundamental properties of fiber material (silica glass) and thus is related to the desired refractive indices and operating wavelengths. On the other hand, extrinsic loss originates...
from imperfections in the fabrication process that could be removed with appropriate refinements.

Basically, if a light wave with an initial power $P_0$ is applied to a fiber of length $L$, the transmitted power $P_T$ will be

$$P_T = P_0 \exp(-\alpha L)$$  \hspace{1cm} (2.21)

where $\alpha$ denotes the attenuation constant, corresponding to a measured coefficient of total fiber loss. Loss is usually described as the ratio of the input to the output powers per kilometer length, expressed in (dB km$^{-1}$) as (Agrawal, 2007)

$$\alpha_{\text{dB}} = -10 \log_{10} \left( \frac{P_T}{P_0} \right)$$  \hspace{1cm} (2.22)

The attenuation constant $\alpha$ and dB loss per unit length are the same. The equivalence is expressed by a conversion factor for the two units:

$$1 \text{ dB km}^{-1} = 2.303 \times 10^{-6} \text{ cm}^{-1}$$  \hspace{1cm} (2.23)

This is a convenient way of measure because the contributions to loss from different sources can be added to obtain the total value by simple summation.

### 2.4.1 Intrinsic Loss

In silica fiber, there are three different mechanisms of intrinsic loss which are important at visible and near-infrared (NIR) regions. These mechanisms include the two resonances centered in the ultraviolet (UV) and mid-infrared (Mid-IR) regions and Rayleigh scattering. The three mechanisms depend on wavelength, and their combined effects govern the basic range of wavelength which is appropriate for signal transmission.
**UV absorption**

The UV resonance is of electronic origin and centered at wavelength of 0.1 µm. The imaginary part of the susceptibility which are associated with this resonance is of sufficient width for the tail of the curve to produce considerable absorption in the visible region, but its effect can be negligible in the NIR region. If some dopant materials are added, the UV tail or Urbach tail tends to shift toward the IR region. For example, the addition of germania produces a loss that is expressed by the empirical relation (Nagel et al., 1982)

\[
\alpha_{UV} = \frac{1.542m}{46.6m + 60} e^{4.63/\lambda}
\]

(2.24)

where \( m \) is the mole fraction of GeO\(_2\). At typical levels of doping (\( m = 0.02 \) for single mode), the addition of germania yields negligible increase in loss at 1.3 µm and above.

**IR absorption**

Lattice vibrational modes of silica and dopant glasses causes the second intrinsic loss mechanism. The vibrational modes yields absorptive resonances centered between 7 and 1 µm, the resonances for silica and germania take place at 9 and 11 µm, respectively. Remarkable broadening happens due to anharmonic coupling between the several vibrational modes leading to an IR absorption tail which extends into the transmission wavelength region in the NIR. As a rule, lighter atomic masses leads to shorter resonant wave length, the effect is to shift the IR tail further into the transmission wavelength range.

The power loss due to IR absorption can be described as

\[
\alpha_{IR} = Ae^{-a_{IR}/\lambda}
\]

(2.25)

For GeO\(_2\):SiO\(_2\) glasses, values of \( A \) and \( a_{IR} \) that provide a reasonable fit to measured data are \( 7.81 \times 10^{11} \) dB/km and 48.48 µm, respectively (Nagel et al., 1982).
**Rayleigh scattering**

This mechanism is classically described by the excitation and reradiation of light by atomic dipoles of dimensions that are much less than the incident wavelength. Due to the silica glass structure during manufacture process the material density exhibits random microscopic variations (local variations of refractive index occur with the changes in density) which act as scattering centers. Additional structural and index fluctuations come from dopant molecules that are introduced into the SiO$_2$ lattice structure, leading to scattering losses that depend on dopant concentration. Index fluctuation arising from concentration or density fluctuations are of dimensions much less than a wavelength and thus meet the Rayleigh condition.

The loss due to Rayleigh scattering is expressed as

$$\alpha_S = \frac{B}{\lambda^4}$$  

where $B$ is the Rayleigh scattering coefficient.

The net loss associated with intrinsic effects $\alpha_{in}$ can be obtained by the sum of all contributions from each mechanism that is

$$\alpha_{in} = \alpha_{UV} + \alpha_{IR} + \alpha_S$$  

At NIR wavelength, $\alpha_{UV}$ can be negligible compared to other terms, so the net loss $\alpha_{in}$ becomes

$$\alpha_{in} \approx Ae^{-\alpha_{IR}/\lambda} + \frac{B}{\lambda^4}$$  

The minimum loss and the corresponding wavelength can be obtained by direct differentiation of Equation 2.28 with respect to $\lambda$. It can be shown that the minimum loss wavelength is approximated by $\lambda_{min} \approx 0.03\lambda_{IR}$ μm (Lines, 1984) and the minimum loss
corresponding to this wavelength is then determined by Equation 2.26 that is Rayleigh scattering is the dominant intrinsic loss process in the vicinity of the lowest loss wavelength.

2.4.2 Extrinsic Loss

Extrinsic loss is the loss sources that are not associated to the fundamental material properties. Generally, it arises from the additional substances existing in the glass compound that are not essential to the light-guiding properties of the fiber and can be eliminated with suitable refinement in the fabrication process. Cation impurities such as Ni, Cu, Fe and Mn comprise one class of the extrinsic loss and show very strong absorption bands in the visible and NIR regions. In addition, rare earth impurities introduce absorption loss that is important in NIR.

The most difficult extrinsic loss source to eliminate is the OH group, which enters the glass in the form of water vapor. The fundamental stretching resonance of OH group is centered at wavelength between 2.7 and 3.0 µm. The OH vibrational mode is somewhat anharmonic, which causes oscillation at overtone frequencies. Figure 2.7 shows the spectral attenuations in GeO₂:SiO₂ fiber having OH concentration of approximately 0.5 ppm. The effects of intrinsic mechanisms and OH absorption are shown. The low loss window centered at 1.55 µm is also shown.
Figure 2.7: Spectral attenuations of a $\text{GeO}_2:\text{SiO}_2$ fiber showing the effects of intrinsic and extrinsic losses (Buck, 2004).

### 2.4.3 Bending Loss

There are two types of bending loss due to two different mechanisms. The first mechanism is macrobending which is associated with axial bends of relatively large radius. The second one is microbending which represents one case of a more general loss mechanism due to microdeformations (Marcuse, 1984) and involves the cumulative loss arising from small magnitude ripples in an otherwise straight fiber, which are formed through small displacements of the fiber in directions that are transverse to its axis, and which have magnitude of order micrometer. This effect can occur when the fibers comes into contact with a rough surface such as a plastic jacket material that would be used, for example, to bind several fibers together in a cable.

Multimode fibers generally experience lower microbending loss, which is essentially independent of wavelength. Generally, macrobending loss becomes stronger for fibers with low NA and at longer wavelengths, although the wavelength dependence is often strongly oscillatory.
2.4.4 Coupling Loss

Coupling light in the fiber requires consideration of the source emission characteristics and the fiber design. Many factors determine the choice of fiber and source for a given system or experiment, the coupling issue is just one of these. Therefore, coupling efficiency that is less than ideal must be tolerated.

The efficiency is defined as

\[ \eta = \frac{P_c}{P_s} \]  

(2.29)

where \( P_s \) denotes the total power emitted by the source, and \( P_c \) refers to the coupled power into the fiber. The power loss from coupling is given in terms of these values as

\[ \alpha_c = 10\log\left(\frac{P_s}{P_c}\right) \]  

(2.30)

The most desirable source is a well-collimated laser, whose beam profile tailored to yield optimum coupling. Although, light emitting diodes (LEDs) are usually the most favorable sources for low-budget, low-band width systems, they are not desirable in terms of coupling efficiency because its broad angular range of emission and large surface area provide substantially lower coupling efficiency.

2.5 Dispersion in Optical Fibers

Dispersion is the spreading of the pulses as they travel along the fiber, they will generally broaden in time. In multimode fibers, the dispersion largely arises from the different propagation speeds for the different modes, which is known as intermodal dispersion. Single-mode fibers have no intermodal dispersion, however they have other sources of dispersion which will be addressed below. The various kinds of dispersion can be distinguished as follows:
• Chromatic dispersion which includes waveguide dispersion, material dispersion, and polarization dispersion.

• Intermodal dispersion

2.5.1 Chromatic Dispersion

Basically, chromatic dispersion means the pulse spreading where different wavelengths is traveling at different velocities. The chromatic dispersion is a specific property of the material which cannot be avoided. The radiation of longer wavelengths has smaller indices of refraction than that of the shorter wavelengths, so the light at different wavelengths travel with different velocities.

The total dispersion for a single mode fiber is the sum of the contributions coming from material dispersion and waveguide dispersion (Agrawal, 2007; Lakoba & Agrawal, 1999). Figure 2.8 shows that the growth of the negative slope of the waveguide dispersion shifts the zero of dispersion towards longer wavelengths. Therefore, the clear way of shifting the dispersion minimum to the third window of transmission is to enlarge the waveguide dispersion.

![Figure 2.8: Effect of wavelength on material, waveguide, and total dispersion.](image)
Material dispersion is the result of the finite linewidth of the light source and the dependence of material index of refraction on the wavelength. This is related mainly to the properties of the core glass and to some extent by the cladding glass. Material dispersion alone can be measured on bulk samples of the glass. Figure 2.9 shows the wavelength-dependent refractive indices and group indices of pure SiO₂ and germanosilicate glass with 10% GeO₂. If germanosilicate glass is used for the fiber core, surrounded by pure fused silica, NA of 0.25 will result.

![Material dispersion of different glass samples](Paschotta, 2010)

The waveguide effect further modifies chromatic dispersion. This is called waveguide dispersion. Essentially, the phase delay per unit length is affected by the optical confinement of the modes, and that effect depends on both the propagation mode and the wavelength. It is generally stronger for waveguides with small mode areas, and it can be affected via the details of the refractive index profile of the fiber core.

Fiber dispersion effects can be calculated mathematically via expanding the mode-propagation constant $\beta(\omega)$ in a Taylor series about the central frequency $\omega_0$ of the pulse as follows:
The first derivative $\beta_1$ is the inverse group velocity (the temporal delay of a pulse maximum per unit length of fiber). The second derivative $\beta_2$ is the second-order dispersion per unit length or group velocity dispersion (GVD) and the third derivative $\beta_3$ quantifies the third-order dispersion. The pulse envelope travels with the group velocity and the parameter $\beta_2$ accounts for dispersion of the group velocity, which leads to temporal pulse broadening (Agrawal, 2007).

Figure 2.10 shows the variation of $\beta_2$ and walk-off parameter $d_{12}$ with wavelength for fused silica and it is obvious that $\beta_2$ goes to zero at a wavelength of 1.27 $\mu$m (the zero-dispersion wavelength $\lambda_0$) and then has negative values at longer wavelengths.

![Graph showing variation of $\beta_2$ and $d_{12}$ with wavelength for fused silica](image)

**Figure 2.10:** Variation of $\beta_2$ and $d_{12}$ with wavelength for fused silica (Agrawal, 2007).
The effects of higher-order dispersion such as third-order dispersion \( \beta_3 \) (Lakoba & Agrawal, 1999) should be taken into account which can distort the ultrashort pulses through asymmetric broadening in both linear and nonlinear regimes. The third-order dispersion effect becomes important only when the wavelength \( \lambda \) approaches \( \lambda_0 \).

Chromatic dispersion has an important property in which pulses with different wavelengths travel inside the fiber at different velocities due to a mismatch in their group velocity values. This property results in a walk-off effect that takes part in the description of the nonlinear phenomena involving two or more closely spaced pulses. The interaction between two pulses stops when the faster pulse completely walks through the slower pulse. The walk-off parameter \( d_{12} \) can be defined as (Agrawal, 2007)

\[
d_{12} = \beta_1(\lambda_1) - \beta_1(\lambda_2)
\]  

where \( \lambda_1 \) and \( \lambda_2 \) are the center wavelengths of the two pulses.

In the field of fiber-optics, it is common to use the dispersion parameter \( D \) instead of \( \beta_2 \) which is related to each other via the relation (Agrawal, 2007)

\[
D = \frac{d\beta_1}{d\lambda} = -\frac{2\pi c}{\lambda^2} \beta_2
\]  

and has the opposite sign, since an increase of frequency implies a decrease of wavelength. Therefore, it is not convenient to use the terms positive and negative dispersion, but rather normal and anomalous dispersion.

### 2.5.2 Normal and Anomalous Dispersion

The sign of GVD parameter is very important because the nonlinear effects in fibers show qualitatively different behaviors according to the sign of the GVD. As illustrated in Figure 2.11, when \( \lambda < \lambda_0 \) and as \( \beta_2 > 0 \) the fiber is in normal dispersion regime. In this
regime, low-frequency components of the pulse travel faster than high-frequency components (Agrawal, 2007), in other words, a negative $D$ implies that longer wavelengths or low-frequency components have a shorter arrival time, that is, they travel faster. While in the anomalous dispersion regime when $\beta_2 < 0$ and $\lambda > \lambda_0$, a positive value $D$ of indicates that low-frequency components have longer arrival times, and therefore travel slower. It is very essential to study the nonlinear effects in the anomalous-dispersion regime because in this regime the fiber supports solitons via balance between the effects of fiber dispersion and nonlinearity.

Figure 2.11: Dispersion in standard single-mode fiber (Agrawal, 2007).

2.5.3 Polarization Dispersion

A birefringence phenomenon (Lin & Agrawal, 2004) does not exist in ideal fiber due to the absence of distinguished optical axis because the materials of the core and cladding are isotropic. In real fibers, the tensions, thickness change, and the accidental changes of shape and core diameter leads to formation of distinguished optical axes and hence local birefringence. This results in two orthogonal components propagating in a fiber as ordinary and extraordinary rays moving in a fiber with different velocities. The different
velocities of the two orthogonal components cause the phase difference changing in time of propagation along the fiber and change of polarization. In addition to the change of polarization with time of propagation, the different velocities of ordinary and extraordinary rays make the rays reach the fiber end in different time.

2.5.4 Intermodal Dispersion

Multi-mode fibers are able to guide several light modes due to their larger core size. Each mode ray enters the fiber at a different angle and therefore travels at different paths. So the rays reach the fiber output at different times. This causes temporal broadening in the pulse at the end of the fiber causing signal overlapping so extremely that you will not be able to distinguish them anymore. This temporal broadening is called modal dispersion. Modal dispersion limits the bandwidth of multimode fibers. Figure 2.12 summarizes the different types of dispersion.

![Diagram of different types of dispersion](image)

**Figure 2.12:** Simple sketch for different types of dispersion.
2.6 Nonlinear Effects in Optical Fibers

The intensity dependence of the medium index of refraction and inelastic scattering phenomenon are responsible for the nonlinear effects in fibers. The intensity-dependent refractive index causes what is called optical Kerr-effect (Agrawal, 2007) while the inelastic scattering phenomenon leads to stimulated effects; Stimulated Brillouin-Scattering (SBS) and Stimulated Raman-Scattering (SRS). Figure 2.13 lists the different nonlinear effects in fibers.

Figure 2.13: Nonlinear effects in optical fibers.

2.6.1 Nonlinear Refractive Index Effects

In a dielectric material, an optical pulse induces electric dipoles because of the interaction between the bound electrons and the electric field of light. For high intensity field, this motion is anharmonic and the induced polarization P shows nonlinear behavior. Due to the inversion symmetry in silica glass the second order susceptibility $\chi^{(2)}$ vanishes. Hence, fibers do not show second order nonlinear refractive effects. Thus, the third order susceptibility $\chi^{(3)}$ is the lowest order which results in nonlinear behavior of the induced polarization (Agrawal, 2001).
Basically, in nonlinear medium, the linear susceptibility $\chi^{(1)}$ is replaced by an effective susceptibility $\chi''(1)$

$$\chi''(1) = \chi^{(1)} + \frac{3}{4} \chi^{(3)} A^2$$  \hspace{1cm} (2.35)

which is a change $\Delta\chi^{(1)} = \frac{3}{4} \chi^{(3)} A^2$ from the linear susceptibility $\chi^{(1)}$.

Since it is the susceptibility $\chi^{(1)}$ which governs the index of refraction $n = \sqrt{1 + \chi^{(1)}}$, this change in $\chi^{(1)}$ results in a change in the index of refraction

$$\Delta n = \frac{1}{2\sqrt{1 + \chi^{(1)}}} \Delta\chi^{(1)} = \frac{\Delta\chi^{(1)}}{2n} = \frac{3}{8} \chi^{(3)} A^2$$  \hspace{1cm} (2.36)

The change in the index of refraction $\Delta n$ is proportional to the square of the electric field amplitude, and is governed by $\chi^{(3)}$. The electric field amplitude is related to the light intensity $I$ as

$$A^2 = \frac{2I}{nc\varepsilon_0}$$  \hspace{1cm} (2.37)

So the change in the index of refraction is

$$\Delta n = \frac{3\chi^{(3)} I}{4n^2 c\varepsilon_0}$$  \hspace{1cm} (2.38)

Thus, the index of refraction will be written as

$$n = n_o + n_2 I$$  \hspace{1cm} (2.39)

where $n_o$ is the linear index of refraction while $n_2$ is the nonlinear index of refraction

$$n_2 = \frac{3\chi^{(3)}}{4n^2 c\varepsilon_0}$$  \hspace{1cm} (2.40)

This variation of index of refraction with light intensity is called optical Kerr effect or Kerr nonlinearity.
In addition to Kerr effect, Raman response causes also intensity dependence of refractive index and should be added to \( n_2 \) in Equation 2.39, but the contribution of Raman response is not as significant as Kerr contribution.

Kerr effect can cause a slight variation of the mode field distribution and propagation constant. It appears in three important effects, self-phase modulation, cross-phase modulation, and four-wave mixing as discussed below.

### 2.6.1.1 Self-Phase Modulation

When an optical pulse travels through an optical fiber, the higher intensity parts of the pulse meet a higher refractive index of the fiber than the lower intensity parts according to Kerr effect. In fact, a temporal variation in pulse intensity leads to a temporal variation in refractive index in a medium with intensity-dependent refractive index. Therefore, the leading edge of the pulse will experience a positive refractive index gradient while the trailing edge will experience a negative gradient and this leads to a temporally varying phase change. The optical phase varies with time in the same way as the optical signal (Stolen & Lin, 1978).

To see this effect, study a pulse with frequency \( \omega_0 \) propagating in the \( +z \) direction and enters a nonlinear medium of thickness \( L \), as shown in Figure 2.14. Assume the propagating wave in the medium is

\[
E(z,t) = A \cos(\omega_0 - kz)
\]  \hspace{1cm} (2.41)

where

\[
k = (n_o + n_z I)k_0
\]  \hspace{1cm} (2.42)

and \( k_0 = \omega_0 / c \). At \( z = 0 \), the electric field is

\[
E_z(t) = A \cos(\omega_0)
\]  \hspace{1cm} (2.43)

At \( z = L \), the electric field will be
\[ E_2(t) = A \cos(\phi(t)) \]  

where the time-dependent phase of the wave \( \phi(t) \) is

\[ \phi(t) = \omega_0 t - (n_o + n_2 I) k_0 L \]  

The frequency which is defined is

\[ \omega = \frac{d\phi}{dt} = \omega_0 - n_2 k_0 L \frac{dI}{dt} \]  

If the intensity is constant in time \( dI/dt = 0 \), \( \omega = \omega_0 \), and the frequency is simply the coefficient of \( t \) in the cosine function of Equation 2.41. While if the intensity is varying in time, the frequency differs from \( \omega_0 \). Moreover, if \( dI/dt \) is itself varying in time, the frequency \( \omega \) will change in time. Thus, Equation 2.46 is interpreted as giving the instantaneous frequency of the wave. Figure 2.14 shows how this change in frequency affects the pulse in a nonlinear medium. When the pulse leading edge enters the nonlinear medium, \( dI/dt > 0 \), and \( \omega \) becomes less than \( \omega_0 \) while as the trailing edge of the pulse is passing through the medium, \( dI/dt < 0 \), and \( \omega \) becomes greater than \( \omega_0 \). The center of the pulse is unshifted because in this case, \( dI/dt = 0 \) and \( \omega = \omega_0 \). This can be summarized as follows, in a nonlinear medium, the instantaneous frequency of a pulse is raised when the intensity is decreasing in time, and lowered when the intensity is increasing. This leads to a frequency chirp, in which the leading edge of the pulse is shifted to lower frequency (red-shifted) while the trailing edge is shifted to higher frequency (blue-shifted).

This nonlinear phase modulation is termed self-phase modulation (SPM) because it is actually self-induced, which is the most dominant nonlinear effect in single channel system (Agrawal, 2007; Basch, 1987; Boyd, 2003). The primary effect of SPM is spectral broadening of optical pulses (Kikuchi & Sasaki, 1995). SPM effect is very important in solitons and in pulse compression (Singh et al., 2007).
When two or more signals propagate simultaneously inside a fiber with different wavelengths, the nonlinear phase shifts generated as a result of interaction between them is called cross phase modulation (XPM). XPM causes asymmetric spectral broadening of co-propagating signals. For equal intense optical field with different wavelengths, the effect of XPM on the nonlinear phase shift is double strength compared to the effect of SPM. XPM reduces the performance of the system through the same mechanism as SPM (chromatic dispersion and chirping frequency), but XPM can spoil the system performance even more than SPM. The XPM phenomenon can be used for pulse compression and optical switching (Singh et al., 2007).

Figure 2.14: Phenomenological description of SPM effect.
2.6.1.3 Four-Wave-Mixing

Four-wave-mixing (FWM) is a nonlinear phenomenon which originates from the nonlinear optical response of the fiber to high intensity optical field due to the third order susceptibility $\chi^{(3)}$. FWM means that when three optical fields having different carrier frequencies $\omega_1$, $\omega_2$, and $\omega_3$ co-propagate in a fiber at the same time, they produce a new fourth optical field, the frequency of which does not coincide with any of the others. However, the new field of frequency $\omega_4$ is related to the others via the relation

$$\omega_4 = \omega_1 \pm \omega_2 \pm \omega_3$$  \hspace{1cm} (2.47)

An example two waves mixing at frequency $\omega_1$ and $\omega_2$ is shown in Figure 2.15. If these waves are mixed up, they produce sidebands at $2\omega_1 - \omega_2$ and $2\omega_2 - \omega_1$.

In similar way, nine new sideband waves can be generated through mixing of three co-propagating waves. These sidebands travel along with the original waves and will grow to the detriment of signal-strength depletion. Generally, for number of wavelengths $N$ travelled down a fiber, the number of generated mixed products $M$ will be

$$M = \frac{1}{2} N^2 (N - 1)$$  \hspace{1cm} (2.48)

Practically, most of these combinations do not establish due to a phase-matching condition. Frequency combinations of the form $\omega_4 = \omega_1 + \omega_2 - \omega_3$ are often making difficulty for multichannel communication systems as they can become approximately phase-matched when channel wavelengths lie near to the zero-dispersion wavelength $\lambda_0$. Actually, the degenerate FWM process for which $\omega_1 = \omega_2$ is the dominant process. Basically, FWM process can be described as a scattering process in which two photons...
of energies $h\nu_1$ and $h\nu_2$ are vanished, and their energies create two new photons of energies $h\nu_3$ and $h\nu_4$. The phase-matching condition then comes from momentum conservation conditions. Because all four waves propagate in the same direction, the phase mismatch is written as

$$\Delta k = \beta(\omega_1) + \beta(\omega_2) - \beta(\omega_3) - \beta(\omega_4)$$

$$= \frac{1}{c} \left[ \bar{n}_1 \omega_1 + \bar{n}_4 \omega_4 - \bar{n}_3 \omega_3 - \bar{n}_2 \omega_2 \right]$$

(2.49)

where $\beta(\omega)$ denotes the propagation constant, $\omega$ is the wave frequency, and $\bar{n}_j$ represents the effective mode index at $\omega_j$. In the degenerate case, $\omega_1 = \omega_2$, $\omega_4 = \omega_1 + \Omega_j$, $\omega_3 = \omega_1 - \Omega_j$, where $\Omega_j$ denotes the channel spacing (frequency shift). Using the Taylor expansion in Equation 2.31, it is found that the terms $\beta_0$ and $\beta_1$ cancel, and the phase mismatch is simply, $\Delta k = \beta_2 \Omega_j^2$. FWM process is completely phase matched when $\beta_2 = 0$. When $\beta_2$ is small (less than 1 ps$^2$/km) and channel spacing is also small (less than 100 GHz), this process can still happen and transfer power from each channel to its nearest neighbors. Such a power transfer not only produces a power loss for the channel, but also causes interchannel crosstalk which reduces the system performance.

Generally, FWM has to be avoided. However, it provides an effective basic way for measuring the nonlinearity and chromatic dispersion of fibers and can be useful in designing lightwave systems. It can also be useful for wavelength conversion and squeezing (Singh et al., 2007).
Figure 2.15: Mixing of two waves.

Physical mechanisms of phase-matching

The condition of phase-matching $\kappa = 0$ can be written as

$$\kappa = \Delta k_M + \Delta k_W + \Delta k_{NL} = 0$$

(2.50)

where $\Delta k_M$, $\Delta k_W$, and $\Delta k_{NL}$ denotes the mismatch due to material dispersion, waveguide dispersion, and the nonlinear effects, respectively. The $\Delta k_M$ and $\Delta k_W$ contributions are obtained from Equation 2.49 if the effective indices are described as

$$\tilde{n}_j = n_j + \Delta n_j$$

(2.51)

where $\Delta n_j$ is the change in refractive index $n_j = n_M(\omega_j)$ due to waveguiding.

For degenerate FWM ($\omega_1 = \omega_2$), the three contributions in Equation 2.50 will be

$$\Delta k_M = \left[ n_3\omega_3 + n_4\omega_4 - n_1\omega_1 \right]/c$$

(2.52)

$$\Delta k_W = \left[ \Delta n_3\omega_3 + \Delta n_4\omega_4 - (\Delta n_1 + \Delta n_2)\omega_2 \right]/c$$

(2.53)

$$\Delta k_{NL} = \gamma (P_1 + P_2)$$

(2.54)
In order to achieve phase matching, at least one of them is negative.

The contribution of material dispersion $\Delta k_M$ can be described in terms of the frequency shift $\Omega_s$ ($\Omega_s = \omega_1 - \omega_3 = \omega_4 - \omega$) by using the Taylor expansion in Equation 2.31 and note that $\beta_j = n_j \omega_j / c$ ($j = 1$ to $4$). Retaining up to fourth-order terms in $\Omega_s$ in this expansion,

$$\Delta k_M \approx \beta_2 \Omega_s^2 + (\beta_4 / 12) \Omega_s^4$$  \hspace{1cm} (2.55)

where $\beta_2$ and $\beta_4$ represent the dispersion parameters at the pump frequency $\omega_1$. If the pump wavelength $\lambda_1$ is not too near to $\lambda_0$, $\Delta k_M \approx \beta_2 \Omega_s^2$ can be used. As $\beta_2$ is greater than zero for $\lambda_1 < \lambda_0$, $\Delta k_M$ is positive in the visible or NIR region. Phase matching for $\lambda_1 < \lambda_0$, can be achieved if $\Delta k_w$ has been made negative by propagating different waves in different modes of a multimode fiber. In the case of a single-mode fiber, $\Delta k_w = 0$ since $\Delta n$ is approximately the same for all waves.

In order to obtain phase matching in single-mode fibers, there are three techniques used (Agrawal, 2007):

- If the pump wavelength becomes greater than $\lambda_0$ and $\Delta k_M$ becomes negative, phase matching can be achieved for $\lambda_1$ in the vicinity of $\lambda_0$.
- For $\lambda_1 > \lambda_0$, phase matching can be achieved by adjusting $\Delta k_{NL}$ via the pump power.
- For $\lambda_1 < \lambda_0$, phase matching can be realized by polarizing different waves differently with respect to a principal axis of the fiber due to modal birefringence in polarization-preserving fibers.
All these above effects reduce the performance of fiber optic systems. However, they are also beneficial for various applications as mentioned above. Table 2.2 shows comparison between the different nonlinear effects based on nonlinear refractive index.

**Table 2.3:** Comparison between the different effects of nonlinear refractive index.

<table>
<thead>
<tr>
<th>Nonlinear Phenomenon Characteristics</th>
<th>SPM</th>
<th>XPM</th>
<th>FWM</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Origin</strong></td>
<td>The third order susceptibility $\chi^{(3)}$</td>
<td>The third order susceptibility $\chi^{(3)}$</td>
<td>The third order susceptibility $\chi^{(3)}$</td>
</tr>
<tr>
<td><strong>Effects of 3rd order susceptibility</strong></td>
<td>Phase shift</td>
<td>Phase shift</td>
<td>New waves are created</td>
</tr>
<tr>
<td><strong>Shape of broadening</strong></td>
<td>Symmetrical</td>
<td>Maybe asymmetrical</td>
<td>-</td>
</tr>
<tr>
<td><strong>Energy transfer between medium and optical pulse</strong></td>
<td>No</td>
<td>No</td>
<td>No</td>
</tr>
<tr>
<td><strong>Application</strong></td>
<td>Solitons, Pulse Compression, Optically Tunable Delays, and Optical 40 Gb/s 3R Regenerator</td>
<td>Optical Switching, Pulse Compression, and Pulse Retiming</td>
<td>Wavelength Conversion and Squeezing</td>
</tr>
</tbody>
</table>
2.6.2 Nonlinear Scattering Effects

The inelastic scattering of a photon to a lower energy causes nonlinear scattering effects in fibers. In which the energy of an optical wave is transferred to another wave with longer wavelength (lower energy) and the difference in energy appears in form of phonons (molecular vibrations) (Boyd, 2003).

In optical fibers, two nonlinear scattering phenomena are existed and both are associated with silica vibrational excitation modes known as stimulated Raman scattering (SRS) and stimulated Brillouin scattering (SBS). (Bass & Resnick, 2005; Buckland & Boyd, 1996, 1997; Chandran et al., 2007; Shen & Bloembergen, 1965; Singh et al., 2007; Wang et al., 2006). The fundamental difference between them is that, the optical phonons take part in SRS whereas SBS is via acoustic phonons. So, SBS happens only in one direction while SRS can happen in both directions. At high optical power levels, these nonlinear scattering phenomena cause disproportionate loss. They also cause power transfer from one mode to others in forward or backward direction at different frequencies and they can provide an optical gain but with frequency shift.

2.6.2.1 Stimulated Brillouin Scattering

Brillouin Scattering is a nonlinear phenomenon occurring in fibers at high intensity. The high intensity creates compression in the fiber core of fiber via electrostriction process (Boyd, 2003). It causes density-fluctuations in the fiber medium which increases the material disorder, and in turn modulates the linear refractive index of the medium resulting in an electrostrictive-nonlinearity (Buckland & Boyd, 1996). The modulated refractive index acts as an index grating, which is pump-induced. The scattering of pump light via Bragg diffraction by the modulated refractive index is termed Brillouin scattering. This disorder depends on time, so the scattered light is shifted in frequency (Brillouin shift). Figure 2.16 shows the process of Brillouin scattering, for an electric field
at the pump frequency $\omega_p$, the electrostriction process produces a macroscopic acoustic wave at some frequency $\omega_B$.

The Brillouin scattering could be spontaneous (Figure 2.16(a)) or stimulated (Figure 2.16(b)). In spontaneous Brillouin scattering, a pump photon is annihilated leading to generation of Stokes photon and an acoustic phonon simultaneously.

In single mode fibers, the spontaneous Brillouin scattering could also happen in forward direction. This can happen because there is relaxation of the wave vector selection rule because of acoustic waves guided nature. This process is known as guided acoustic wave Brillouin scattering (Agrawal, 2007). In this situation, a small amount of very weak light is created. When scattered wave is generated spontaneously, it interferes with the pump beam. This interference produces spatial modulation in intensity causing amplification of acoustic wave through the electrostriction effect. The amplified acoustic wave in turn increases the spatial modulation of intensity and thus the scattered wave amplitude. Again there is an increase in the amplitude of acoustic wave. This positive feedback dynamics is responsible for the stimulated Brillouin scattering, which eventually, transfers all power from the pump to the scattered wave.

Normally SBS hinders optical communication systems. However, it is beneficial for various applications especially optical devices such as fiber sensors, Brillouin fiber amplifiers, beam Combiner, pulse delaying and advancement, and Pipeline Buckling detection (Singh et al., 2007).
2.6.2.2 Stimulated Raman Scattering

Raman scattering is the inelastic scattering process (Boyd, 2003) of a photon with an optical phonon, which arises from a finite response time of the third order susceptibility of the material (Lan et al., 1981). When a monochromatic light travels in a fiber, spontaneous Raman scattering which is shown in Figure 2.17(a) happens and transfers some of the photons to new frequencies since the scattered photons either gain energy (anti-Stokes shift) or lose energy (Stokes shift). If some photons at other frequencies are already existing, this enhance the probability of scattering to those frequencies. This process is known as stimulated Raman scattering (SRS) which is shown in Figure
In SRS, a coincident photon at the downshifted frequency can obtain a gain. They make use of this feature in Raman amplifiers. Also, SRS process can be useful in many applications such as Raman fiber amplifier and laser, and eye-safe laser (Singh et al., 2007).

Figure 2.17: (a) Spontaneous Raman scattering and (b) stimulated Raman scattering phenomena (Singh et al., 2007).

Despite many similarities between SBS and SRS, they are different in several ways:

- Brillouin scattering happens owing to Bragg type scattering from propagating acoustic wave while Raman scattering occurs due to the individual molecular motion.

- Brillouin scattering can happen only in backward direction while Raman scattering can happen in both directions.
- Brillouin shift arises from the photon-acoustic phonon interaction whereas Raman shift is a result of photon-optical phonon (molecular vibrations) interaction. The stokes shift due to SBS is smaller by three orders of magnitudes than stokes shift due to SRS.

- The gain bandwidth of Brillouin scattering is very narrow compared to that of Raman scattering.

- The threshold power level of SBS is relatively low compared to that of SRS.

2.7 Interaction of Dispersion and Nonlinearity

Dispersion changes pulse duration and shape during propagation in the fiber. Hence the GVD affects the SPM, since SPM depends on pulse intensity and shape. Furthermore, an interaction between these two effects is thanks to the fact that GVD and SPM influence the pulse chirp during propagation. The GVD separates or unites the spectral components of a pulse, while SPM causes a red shift at the leading edge and blue shift at the tailing edge of a pulse.

In order to compare the effect of GVD and SPM on a propagating pulse, the dispersion length \( L_D \) and nonlinearity \( L_{NL} \) length should be defined,

\[
L_D = \frac{t_0^2}{|\beta_2|}
\]

\[
L_{NL} = \frac{1}{\gamma P_0}
\]

The dispersion length corresponds to the propagation distance after which the pulse duration of a band width limited Gaussian pulse has doubled due to GVD. \( t_0 \) denotes the half 1/e pulse width for Gaussian pulse or half sech (1) pulse width for hyperbolic secant pulse. The nonlinear length corresponds to the propagation distance after which the SPM
has introduced a phase shift of 1 at the pulse maximum with peak power $P_0$ (Agrawal, 2007).

If the nonlinear length is much greater than the dispersion length, fiber dispersion dominates over SPM, the influence of SPM on pulse evolution is small and the interaction between GVD and SPM can be neglected. Both effects have the same strength, if the nonlinear length is equal to dispersive length.

In a fiber with normal dispersion, GVD and SPM generate a positive pulse chirp. Thus, the pulse broadens more rapidly than without significant SPM. Finally, after several dispersion lengths, the pulse evolution is dominated by GVD as the pulse broadening reduces the intensity. If the nonlinear length is equal to the dispersive length in a fiber with anomalous dispersion, the chirp induced by SPM and GVD nearly cancel each other along the central part of a Gaussian pulse.

It can be mathematically shown that under these circumstances a complete cancellation occurs or hyperbolic secant pulse is generated. These pulses were called optical solitons, since the shape of these pulse remains constant during propagation in the fiber. Solitons are the only solutions of the propagation equation, which are stable against small perturbations in anomalous dispersive fiber. For this reason, solitons evolve during pulse propagation in the anomalous dispersive fiber. Even if the input signal deviate slightly from a soliton.

SPM dominates over GVD if the nonlinear length becomes much shorter than the dispersive length. SPM therefore induces a chirp onto the pulse and new frequency components are created by SPM. Nevertheless, the GVD cannot be treated as small perturbations, since dispersive effects have a significant influence on pulse shaping. The reason for that is the large SPM induced frequency chirp. For Gaussian pulse, the SPM induced chirp is linear and positive over a large central region and approaches zero in the outer pulse wings. In anomalous dispersive fiber, the interaction of this central chirp with
fiber dispersion results in a pulse compression and the pulse evolves into one or several solitons. On the other hand, in normal dispersive fiber, the GVD induced chirp has the same sign as SPM induced chirp in the central pulse region. Therefore, the stretching caused by GVD is stronger for the central part than for the outer wings and the pulse becomes more rectangular with sharp trailing edges and leading edges. This is accomplished by a linear chirp nearly across the entire pulse width. In addition, during this pulse stretching, SPM generates typically multi peaks in the central part of the spectrum, which were modified and reduced by the additional effect of GVD.

2.8 Fiber Laser Fundamentals

In order to understand the principles of fiber laser operation, it is essential to shed the light on the process of population inversion which plays a main role in laser physics since it provides a means for light oscillation and amplification.

2.8.1 Population Inversion

It is worthwhile mentioning here the basic statistics of the particles’ distribution at thermal equilibrium in order to understand population inversion phenomenon. In thermal equilibrium state, the number of particles at two nondegenerated energy levels obeys the Boltzmann distribution

\[ N_2 = N_1 \exp \left( -\frac{E_2 - E_1}{k_B T} \right) \]

(2.58)

where \( E_2 > E_1 \). Particles can be in either level 1 with population \( N_1 \) or level 2 with population \( N_2 \). Therefore, the total number of particles is

\[ N = N_2 + N_1 \]

(2.59)
Basically, at thermal equilibrium the lower energy level is more populated than the higher level and the atomic system is naturally relaxed (no light emission). Even at higher temperatures, the number of particles in the higher level increases, but it never becomes more than the number of particles at the lower level. Based on the Boltzmann distribution formula in Equation 2.58, levels 2 and 1 may have the same population only at infinite temperatures. Though, in order to obtain an optical gain via stimulated emission, a non-equilibrium situation needs to be achieved where the population of the upper level becomes higher than that of the lower energy level. This non-equilibrium condition is known as population inversion and has been proposed to accomplish electromagnetic waves amplification. Many ways have been proposed to obtain population inversion condition.

The two basic laser operational schemes for the conception population inversion achievement are four-level and three-level laser systems. The three-energy-level system was the first to establish laser operation. However, the four-energy-level scheme is widely used for most practical applications. The difference between the two schemes lies in the lower laser level position. For the three-level scheme, the ground state represents the lower laser level, while for the four-level scheme the lower laser level is an excited state of the system which is assumed to decay quickly back to the ground state.

In order to obtain population inversion in the three-level system it is required that at least half of the atoms is promoted out of the ground state, which needs much pump energy. On the other hand, the four-level system can accomplish population inversion with only a small number of atoms promoted out of the ground state. It is worthwhile presenting here these two laser operation schemes.
**Four-level laser operation scheme**

It is easier to achieve amplification and lasing with a four-level system since the pump energy required to excite atoms from the ground state is not so much. Figure 2.18 shows the four-energy-level scheme. Pumping process excite atoms from the ground level 1 to the excited level 2. After fast non-radiative relaxation to energy level 3, atoms spontaneously decay to level 4 and then the atoms eventually return again to the ground level 1 via non-radiative relaxation. This is exactly the full excitation cycle of the four-energy-level scheme. The spontaneous decay from level 3 to 4 is exactly the transition where laser action is. Particles begin to accumulate at level 3 after fast non-radiative relaxation from level 2, while level 4 stays unpopulated due to the fast relaxation of its particles to the ground level 1. Thus, in order to achieve population inversion between levels 3 and 4, only one particle needs to occupy level 3 and this can happen provided that the spontaneous decay from level 3 to 4 is slower than the relaxation from level 4 to 1.

![Diagram](image)

**Figure 2.18:** Four-energy-level scheme of laser operation.

The transfer of population between energy levels in a four-level system can be understood by considering the analogy of water flow between holding tanks in a recirculating system, as shown in Figure 2.19. The amount of water in a tank is corresponding to the population of that energy level. Through a water pump, water in the lowest holding tank which
represents the ground state is pumped up into tank 3, which drains quickly into the next-tank 2 via a large hole in the bottom. Tank 2 which represents the upper laser level has a much smaller hole in the bottom, so water can build up there. The water that drips from tank 2 down into tank 1 is quickly drained from tank 1 by a large hole in its bottom, and returns to the lowest tank (the ground state). In this flowing-water analogy, tanks with larger holes in the bottom are corresponding to energy levels with faster relaxation rates. It is observable that there will be little water in tanks 1 and 3 in the steady state. Therefore, population inversion is readily achieved because \( N_2 > N_1 \). It is also obvious that level 2 is in effect directly populated by the pump as any water in tank 3 very quickly goes down to tank 2.

![Figure 2.19: Flowing-water analogy for four-level laser system.](image)

**Three-level laser operation scheme**

In the three-level system, the lower laser level is the ground state as shown in Figure 2.20. Thus, after spontaneous decay, atoms return directly to ground level 1 instead. This means that in the three-energy level scheme, the laser-terminating level is always populated (ground state). And this makes the requirements to achieve population
inversion very strict. So, most of the total atoms must be excited to the laser’s upper level so as to create population inversion. Therefore, much higher excitation intensities are required with special care to get rid of re-absorption of the amplified light during travelling along the laser medium. These factors increase the laser threshold and reduce laser efficiency. However, the three energy-level laser scheme is vital in fiber lasers and amplifiers because of the small core area of fiber which allows the intensity to be very high even for modest optical powers. These high intensities are sufficient to produce steady-state population inversion and lasing.

![Three-energy-level scheme of laser operation.](image)

**Figure 2.20:** Three-energy-level scheme of laser operation.

### 2.8.2 Fiber Laser and Amplifier

Light can be amplified in fiber with laser active atoms doped in the fiber core, if the wavelength of the light signal is inside the gain spectrum. However, this requires a population inversion of these atoms, which is obtained by optical pumping. Under such circumstances, the amplification increases with fiber length until saturation occurs, since the light is guided by the fiber structure. This is an important advantage compared to bulk amplifier where the Gaussian beam propagation limits the interaction length. As a result, the amplification in such fibers can be quite high and is typically limited only by the available pump power or by onset of laser oscillations, if the amplification becomes equal to the optical feedback. Such an optical feedback can be provided by Rayleigh and
stimulated Brillouin scattering and by reflection of fiber connections and fiber ends. The high gain potential of such fibers enables laser operation even for low optical feedback and therefore simplifies the realization of fiber lasers.

Usually, the rare earths elements or lanthanides such as erbium, ytterbium, neodymium or thulium are widely used as active dopants in fibers. Ionization of these atoms in silica or other glass happens to form a trivalent state (i.e. $\text{Er}^{3+}$). One of the 4f electrons and two of the 6s electrons are ejected, but the outer 5s and 5p shells stay intact. Consequently, the remaining 4f electrons are partially shielded from perturbation by external fields. Therefore, the wavelengths of absorption and fluorescence of rare earths ions are less dependent on the external electric field than those of other ions, for example, the ions of the transition elements, which do not have similar electronic shielding.

### 2.8.3 Erbium-Doped Fibers

EDF is an optical fiber doped with erbium ions $\text{Er}^{3+}$. The fluorescence and absorption spectrum of $\text{Er}^{3+}$ in a silica based host is shown in Figure 2.21. It shows that $\text{Er}^{3+}$ have absorption bands of interest at 800 nm, 980 nm and 1550 nm, and fluorescence band at 1550 nm. Thus the lasing can be available at about 1550 nm and pumping can be done via the 800 nm and 980 nm absorption bands. The overlap between absorption and fluorescence bands at 1550 nm points to a three level scheme.
Figure 2.21: Absorption and fluorescence spectrum of erbium in bulk samples of GeO$_2$-P$_2$O$_5$/SiO$_2$ single-mode fiber (Urquhart, 1988).

Figure 2.22 illustrates a simplified energy level diagram of Er$^{3+}$ ion in which the energy levels are broadened because of the Stark effect (Mears & Baker, 1992), which results in a relatively broad bandwidth. When an EDF is pumped by a 974 nm laser diode, Er$^{3+}$ ions are excited from the ground state $E_1$ to the excited higher level $E_3$. The excited Er$^{3+}$ ions at $E_3$ will fast decay to level $E_2$ via non-radiative relaxation. The excited ions on $E_2$ ultimately relax to ground level $E_1$ via spontaneous emission generating photons with wavelength ranging from 1520 – 1570 nm. The number of photons are amplified as they propagate through the fiber, especially when the pump power is increased. Since amplified spontaneous emission covers broad range of wavelength (1520-1570 nm), it can be used as a broadband light source. If a laser pulse of a wavelength between 1520 and 1570 nm is fed into an EDF simultaneously with a 974 pump laser as shown in Figure 2.22, the laser pulse photon will have three possible cases:

- Stimulated absorption: the photon excites an erbium ion from the state $E_1$ to a higher level $E_2$ and become annihilated in the process.
- Stimulated emission: the photon stimulates an erbium ion at state $E_2$ to decay to $E_1$, producing another identical photon (undergoes amplification).
- The photon can travel through the fiber unaffected.

In the same time, there is always spontaneous emission between level $E_2$ and level $E_1$. When the pump power is high enough that the population inversion is accomplished, the input laser pulse going through the fiber is then amplified. In this case, EDF and pump laser can be used to establish an optical amplifier, which is called erbium-doped fiber amplifier. The spontaneous emission also could be amplified by pump laser. So amplified spontaneous emission always exists in erbium-doped fiber amplifier, and it is the main source of noise in these amplifiers.

![Simplified energy levels of Er\(^{3+}\) ions in erbium-doped fiber.](image)

**Figure 2.22:** Simplified energy levels of Er\(^{3+}\) ions in erbium-doped fiber.

The laser is simply the optical amplifier with positive feedback. If the output of erbium-doped fiber amplifier is fed back to its input to create a loop, and laser power is pumped into the fiber loop, the erbium-doped fiber amplifier is converted to a fiber laser, which is called erbium-doped fiber laser (EDFL). EDFLs can operate in several wavelength regions, ranging from visible to far infrared. The 1.55-μm region has attracted much attention because it coincides with the least-loss of silica fibers used for optical
communications. The lasing wavelength changes with the cavity loss. So one may adjust the cavity loss in order to tune the lasing wavelength.

2.9 Saturable Absorber

Saturable absorber (SA) is a key factor in pulsed lasers. It enables the generation of pulses in one of two possible regimes, passively mode-locked or passively Q-switched.

2.9.1 Basis of Saturable Absorber

For many years, the nonlinear optical response of a great variety of materials has been studied and the contributions of absorption and refraction properties to that response have been characterized and measured, and the underlying physical mechanisms have been explained. Most materials show saturable absorption effect: that is, they absorb most of the incident light power when the input fluence is low, and as the fluence increases the absorption decreases. This happens because the absorption cross-section(s) for the excited state(s) is lower than the absorption cross-section for the ground state. The reduction observed in absorption with increasing the fluence comes simply from the depletion over time of the population of ground state as an ever-increasing fraction of the absorber atoms is excited. In other words, if high intensity light is incident, a large number of the atoms will be excited from the ground state to a higher energy state. Because of the reduced number of atoms in the ground state, the probability of absorbing a photon is decreased as illustrated in Figure 2.23. Therefore, the absorption probability decreases as the optical intensity increases. This effect is known as optical bleaching, an intensity-dependent absorption.
In pulse laser generation, SAs are broadly used as Q-switchers or mode-lockers that produce repetitive short optical pulse trains (Herrmann & Wilhelmi, 1987). The SA is an optical material which exhibits an intensity-dependent transmission, in effect, it is a kind of nonlinear optical modulator whose modulation depth is controlled by the optical pulse itself. When placed in a laser cavity, a SA attenuates low-intensity light. This process continues as the light oscillates in the cavity making selective amplification of the high-intensity spikes, and absorption of the low-intensity light. After many round trips, a train of pulses is generated and the suitable SA should be able to continue shortening the pulse by a fixed ratio on each round trip or successive pass, even when the pulse duration becomes very short.

Figure 2.24 illustrates simply the effect of SA, in which the weak pulses are suppressed, and the strong pulse shortens and is amplified as a result of SA.
The factors that determine the performance of SA and consequently govern pulsed laser performance are described below:

- **Wavelength range**: the wavelength range of SA is the range over which it is capable of absorbing power and showing saturation behavior.
- **Modulation depth**: It refers to the maximum change in absorption. For fiber lasers, a larger modulation depth is required, where the round-trip gain and loss are high, and fiber dispersion and nonlinearity have strong effect on the pulse formation.
- **Saturation intensity**: the saturation intensity of SA is the intensity required to reduce the absorption to half of its unbleached value.
- **Non-saturable loss**: the non-saturable loss represents the part of the losses which cannot be saturated. It is an intrinsic material property, but it can be strongly affected by fabrication process.
- **Damage threshold**: the damage threshold constitutes an upper thermal limit of SA.
- **Recovery time**: recovery time of SA represents its intrinsic response time which can strongly affect the pulse duration. Based on its recovery time, the SA can be fast or slow (Kurtner et al., 1998).

**Figure 2.24**: Simple illustration for the effect of SA on light intensity.
Fast SA has recovery time of hundreds of fs or a few ps. Typically, a fast SA is meant to support a short pulse duration (pulse duration < 1 ps). On the other hand slow SA has recovery time of tens of ps to ns. Typically, a slow absorber is strongly saturated and exhibits ease of self-starting. In case of a slow SA, the SA recovery time is comparable or even longer than the final pulse duration.

The pulse shortening in fast SA (pulse width > recovery time) obviously occurs because the stronger central part of the pulse partially burns through the SA and is transmitted with very less absorption, whereas the weaker pulse wings are more strongly absorbed. This shortening occurs, however, only over a limited range of input intensities. If the input pulse is too weak, no saturation and hence no pulse shortening occurs. Also, if the pulse is too strong, essentially all of the pulse burns through the absorber and consequently no pulse shortening occurs. In case of slow SA, in general, the pulse shortening occurs at slightly higher input intensities relative to the saturation intensity. The primary mechanism in the slow-absorber situation is that the SA absorbs the leading edge of the input pulse, but then, if there is sufficient energy in the input pulse, the medium becomes bleached part way through the pulse, so that the trailing edge of the pulse is transmitted more or less unchanged. Thus, the output pulse in this situation could acquire an asymmetry in time, with a faster leading edge, and a more or less unchanged trailing edge.

### 2.9.2 Overview of Real Saturable Absorber Materials

SAs can be categorized into two types:

- **Artificial SAs**: devices that exploit nonlinear effects to induce an intensity-dependent absorption as in mode-locking based on nonlinear polarization evolution technique.

- **Real SAs**: materials that show an intensity-dependent absorption.
For the systems used as real SA, the first demonstrations were in 1964 using colored glass filter and reversibly bleachable dye (Bret & Gires, 1964; Soffer, 1964). Figure 2.25 summarizes the evolution of the real SA technologies over years. In 1990s semiconductor saturable absorber mirror (SESAM) has been proposed (Keller et al., 1992; Zirngibl et al., 1991). SESAMs rapidly became a successful technology in generating energetic Q-switched and ultrashort mode-locked pulses from fiber lasers. But, their operation wavelength ranges are relatively small, the recovery speed is limited to ps, and their fabrication are costly. After that, the field of nanomaterial SAs gained traction as 1D carbon nanotubes (Set et al., 2004) (CNTs) and 2D graphene (Bao et al., 2009; Hasan et al., 2009) arose as promising materials showing saturable absorption and ultrashort recovery times (Martinez & Sun, 2013). Due to its 2D structure and zero bandgap which enable broadband saturable absorption graphene attracted particular interest (Novoselov et al., 2005). However, the absence of band-gap in graphene that enable broad band operation, could restrain some of its applications when strong light-matter interaction is essential. Graphene represents only one of the 2D materials family. There are other 2D materials that can be obtained in monolayer and few-layer crystals from different bulk materials, including topological insulators (TIs), black phosphorous (BP), and transition metal dichalcogenides (TMDs). TMDs materials such as molybdenum disulfide (MoS$_2$) and Tungsten disulfide (WS$_2$) have also attracted much attention as they show good potential for pulse laser applications because of their unique absorption properties and thickness dependent band-gap (Du et al., 2014; Liu et al., 2014; Wu et al., 2015; Zhang et al., 2015). All of these materials show distinct, yet complementary properties (Novoselov et al., 2005; Wang et al., 2012; Xia et al., 2014; Zhang et al., 2010), but the fabrication process of such SAs is sometimes complicated and costly.

In 2016, transition metal oxides nanoparticles such as zinc oxide (ZnO), titanium dioxide (TiO$_2$), and Fe$_3$O$_4$ (Ahmad et al., 2016; Ahmad et al., 2016; Bai et al., 2016; Latiff
et al., 2017) have been demonstrated as SAs in a Q-switched fiber laser with results comparable to the conventional materials used as SA.

Figure 2.25: The evolution of real SA technologies (Woodward & Kelleher, 2015).
CHAPTER 3 : Q-SWITCHING OPERATION IN FIBER CAVITY WITH SATURABLE ABSORBER

3.1 Introduction

Fiber lasers are normally capable of producing output powers of the order of tens to hundreds of milliwatts when operated in a continuous-wave fashion. For some applications, pulses with much higher peak powers and energies are required, and although mode-locking technique may generate pulses with high peak powers, these pulses are not very energetic. If the fiber laser is Q-switched, it is possible to generate pulses with both high peak powers and energies. Currently, Q-switched fiber lasers represent one of the most important and challenging types of fiber lasers due to their highest potential for different applications including laser ranging, laser cutting and drilling, nonlinear frequency conversion, and biomedical applications.

Laser Q-switching was first proposed by Hellwarth (1961) and experimentally established by McClung & Hellwarth (1962). The basic idea of Q-switching operation is that the laser cavity is spoiled (reducing its Q-factor) so that very large population inversions can be obtained by the strong sustained pumping power (without the system lasing). Then, the cavity is returned to its high Q-factor so that the population inversion is well above the threshold value. In this situation, substantial energy is stored in the cavity by the sustained pumping, and when the cavity Q-factor is restored, most of the stored energy is released in a very short time in the form of intense pulse to bring the population inversion back to the threshold appropriate to the high value.

This chapter briefly reviews the basic principles of laser Q-switching and introduces numerical simulations and analysis for Q-switching operation in erbium-doped fiber laser (EDFL) cavity based on saturable absorber (SA).
3.2 General Description of Laser Q-Switching

The basic principles of laser Q-switching are illustrated schematically in Figure 3.1. As shown in the figure, the cavity loss is made to initially be at high value (low Q-factor) with very high threshold population inversion while the population inversion, and hence the gain and the energy stored, in the cavity are pumped up to a high value. Basically, the high loss stops the build-up of oscillation (lasing), while the laser pumping process is allowed to build up a population inversion to a larger value than the normal. Then, at certain time, the cavity Q factor is abruptly switched from a low to a high value (the cavity loss is abruptly reduced to a normal value) and after switching, the round-trip gain becomes much greater than the loss in the cavity where the population inversion becomes well above threshold. At this point, the initial spontaneous emission in the cavity immediately starts to establish at an unusual rapid rate, which is soon developing into a rapidly growing pulse. This pulse rapidly becomes powerful enough to start saturating (depletion) the inverted atomic population. The laser pulse rapidly drives the population inversion down well below threshold and the pulse (light intensity) decays nearly as fast as it grew. The pumping time of the population inversion is generally much shorter than interval of oscillation build-up and pulse duration. Thus, the inversion obtained in a long pumping time is discarded during short pulse duration.
There are two main techniques commonly used to realize Q-switching; active and passive methods. In active Q-switching (Andrés et al., 2007; Wang & Xu, 2007; Williams et al., 2010), the cavity loss is modulated using fast electronics in a time scale as fast as a few ns. In passive Q-switching, the modulation is done by light itself through the process of saturable absorption, which can be obtained by inserting SA material into the laser cavity. The characteristic of such material is that its transmission increases when the light intensity exceeds some threshold.

**Figure 3.1:** Step-by-step action of laser Q-switching operation.
3.3 General Analysis of Q-Switched Operating Regime

In this section, the formulae used to calculate the main parameters of the laser operated in Q-switched regime based on Wagner & Lengyel (1963) work is shown and in their analysis, they introduced the normalized variables as follows:

\[ \bar{n} = \frac{N}{N_0} \]  \hspace{1cm} (3.1)

where \( N \) represents the population inversion per unit volume while \( N_0 \) refers to the number of active ions in the volume element (total concentration).

The normalized initial and final population inversions \( \bar{n}_i \) and \( \bar{n}_f \) (inverted population prior to opening the Q-switch and inverted population after Q-switch pulse) per unit volume of the laser cavity during pumping are related through the following transcendental equation:

\[ \bar{n}_i - \bar{n}_f = \bar{n}_{th} \ln \left( \frac{\bar{n}_i}{\bar{n}_f} \right) \]  \hspace{1cm} (3.2)

where \( \bar{n}_{th} \) is the threshold population inversion density which can be described by

\[ \bar{n}_{th} = \frac{\ln \left( \frac{1}{R_1 R_2} \right) + N_0 \sigma_{ab} L}{2N_0 (\sigma_{es} + \sigma_{ab}) L} \]  \hspace{1cm} (3.3)

where \( R_1 \) and \( R_2 \) represent the mirrors reflectivity of the laser cavity, \( \sigma_{es} \) and \( \sigma_{ab} \) refer to the emission and absorption cross-sections of the laser-active ion at the lasing wavelength, respectively, and \( L \) is the length of the active fiber.
The output energy of the Q-switched pulse can be described as follows:

\[ E_p = h \nu_L \frac{1}{\gamma} V N_0 (n_i - n_f) \]  

(3.4)

where \( V \) is the gain medium volume, \( \nu_L \) is the lasing frequency, \( h \) is Planck’s constant and \( \gamma = 1 + g_2 / g_1 \), \( g_1 \) and \( g_2 \) are degeneracies of the upper and lower laser levels. For a three-energy level system with no degeneracy, \( g_2 / g_1 = 1 \), \( \gamma = 2 \). For a four-energy level system \( \gamma = 1 \).

The Q-switch pulse duration is given by (Bocko, 1989; France, 1991)

\[ t_p = \tau_{\phi} \frac{n_i - n_f}{n_i - n_{th}} \left[ 1 + \ln \left( \frac{n_i}{n_{th}} \right) \right] \]  

(3.5)

where \( \tau_{\phi} \) is the photon lifetime in the cavity. It is evident that both the pulse duration and pulse energy are governed by the initial population \( n_i \). Thus, in order to obtain an energetic pulse from the system, the difference between \( n_i \) and \( n_{th} \) must be large. In this respect rare earth-doped fibers are ideal because they have small-gain cross sections together with long lived upper state lifetimes. This means that the excited state population does not decay via spontaneous emission before the cavity Q factor is changed, leading to large values of \( n_i \). The pulse energy is related to the intrinsic loss in the cavity and it has been demonstrated that this loss is extremely small (Williams et al., 1990). Hence, it should be possible to make use of these features to generate energetic pulses from the laser.

Also the pulse duration depends on the photon lifetime \( \tau_{\phi} \) in the cavity which in turn is governed by the fiber length. As fiber lasers can be constructed from long lengths of fiber,
the possibilities exist for the production of energetic broad pulses with high peak powers per pulse.

3.4 Passive Q-Switching Dynamics

In passive Q-switching technique, the loss modulation is obtained by a nonlinear optical element in the cavity whose loss is intensity dependent. In this case, the intra-cavity signal modulates the intra-cavity loss and vice versa in such passive Q-switched systems. Thus, laser inversion is established until the gain reaches the lasing threshold corresponding to the high cavity loss. Thereafter, the cavity loss decreases and a laser pulse is rapidly built up until the second threshold, corresponding to the reduced cavity loss. Thus, the stored energy is emitted in a form of laser pulse. For passively Q-switching operation, loss modulation is typically quite large relative to the gain of the laser medium. However, even small disturbance of gain or loss can lead to much large oscillations of the laser amplitude. Such relaxation oscillations have been typically observed in solid-state lasers, where the photon lifetime in the cavity is substantially shorter than the life time of the upper laser level. The effect of the upper level decay rate $\tau_g$, the cavity photon decay rate $\tau_{\phi}$, and the pumping rate $r$ on the frequency $f_{\text{ro}}$ and the decay rate $\gamma_{\text{ro}}$ of these oscillations can be described by (Siegman, 1986):

$$f_{\text{ro}} = \frac{1}{2\pi} \sqrt{(r-1)\tau_g \tau_{\phi} - \frac{1}{4} r^2 \tau_g^2}$$ (3.9)

$$\gamma_{\text{ro}} = \frac{1}{2} r \tau_g$$ (3.10)

The dynamics in passively Q-switched laser arise from the interaction of three entities; the light field $P$, the laser gain $g$, and the saturable absorption $q$. This can be described by a set of three coupled differential equations (Haus, 1976; Paschotta, 2008):
\[
\frac{\partial P}{\partial t} = \frac{g - l - q}{t_r} P 
\]  
(3.11)

\[
\frac{\partial g}{\partial t} = \frac{-g - g_0}{\tau_g} - \frac{gP}{\tau_g P_g} 
\]  
(3.12)

\[
\frac{\partial q}{\partial t} = \frac{-q - q_0}{\tau_{sa}} - \frac{qP}{\tau_{sa} P_{sa}} 
\]  
(3.13)

where \( t_r \) denotes the cavity round-trip time, \( \tau_g \) is the relaxation time of the laser medium, \( \tau_{sa} \) is the relaxation time of SA, \( l \) is the nonsaturable losses only, including the output coupler transmission, and \( P_g \) and \( P_{sa} \) are the saturation powers of the laser medium and SA, respectively.

If the saturation energy of the SA is much smaller than that of the gain medium, the SA is then strongly saturated already at the beginning of the pulse. With a strongly saturated absorber, the net gain during pulse build-up approximately equals the modulation depth of the SA. Ideally, the modulation depth is approximately half the initial gain. In that situation, most of the energy is extracted from the gain medium, and the pulses are temporally close to be symmetric. The pulse duration \( t_p \) and the reduction in stored energy \( \Delta E_{\text{stored}} \) can be estimated as (Paschotta, 2008)

\[
t_p \approx 4.6 \frac{l}{q_0} 
\]  
(3.14)

\[
\Delta E_{\text{stored}} \approx 2q_0 \tau_g P_g 
\]  
(3.15)

(assuming that modulation depth \( \leq \) cavity loss). This provides an estimate of the output pulse energy when taking into account some percentage of laser-internal loss, as can be caused, for example, by the nonsaturable loss of the SA. It becomes apparent that large
pulse energy requires a high modulation depth of the SA, but also a high saturation energy of the gain medium. The pulse repetition rate is essentially governed by the pump power and the extracted pulse energy, because a new pulse is always generated when a certain stored energy (and thus a certain gain) has been reached.

3.5 Numerical Simulations and Analysis of Q-Switching Operation in Fiber Cavity with Saturable Absorber

In this section, the passively Q-switching operation in EDFL cavity based on SA is theoretically studied. The numerical investigations of the Q-switching operation are based on the coupled rate equations of the laser system with the saturable absorption dynamics of the SA material. In this model, the laser system is considered as a ring cavity as shown in Figure 3.2 that consists of a 3-meter-long erbium-doped fiber (EDF) representing the gain medium with 125 μm cladding and 4 μm core diameters, passive fiber (SMF-28), and SA. The cavity is pumped by optical power from a 980 nm Laser Diode (LD) via fused 980/1550-nm wavelength division multiplexor (WDM). To preserve the unidirectional light operation, an isolator (ISO) was used. The output pulsed laser is extracted via a 95/5 fiber output coupler.

![Figure 3.2](image.png)

**Figure 3.2:** Fiber cavity configuration of the Q-switched model.
### 3.5.1 Saturable Absorber Dynamics

Here, consider the SA is considered as a nonlinear optical material (semiconductor thin film). The saturable absorption of such nonlinear optical material can be described by its optically induced carrier densities as (Garmire, 2000)

\[
\alpha(N_a) = \frac{\alpha_s}{1 + N_a / N_{sa}} + \alpha_{NS}
\]  

(3.16)

where \(\alpha_s\) denotes the small-signal saturable absorption (modulation depth of SA), \(N_a\) refers to the photocarrier density of the SA, \(N_{sa}\) refers to the saturation photocarrier density and \(\alpha_{NS}\) denotes the non-saturable absorption coefficient.

The photocarrier density of the SA is described by

\[
\frac{dN_a}{dt} = \frac{I \alpha(N_a)}{h\nu} - \frac{N_a}{\tau_{sa}}
\]  

(3.17)

where \(\tau_{sa}\) is the SA carrier recombination time which can be altered via the fabrication of material.

### 3.5.2 Laser Rate Equations

The dynamics of population in the laser system can be described by the following equations (Bao et al., 2009; Garmire, 2000; Savastru et al., 2012):

\[
\frac{d\phi}{dt} = \frac{\phi}{t_r} \left(2\sigma_{e_s} n_g L - 2\alpha(N_a)L_{sa} - \ln \left(\frac{1}{R}\right) - \delta\right)
\]  

(3.18)

\[
\frac{dn_s}{dt} = W_p - \gamma \sigma_{e_s} c \phi n_g - \frac{n_s}{\tau_g}
\]  

(3.19)
\[
\frac{dN_a}{dt} = c\phi\alpha(N_a) - \frac{N_a}{\tau_{sa}}
\]  

(3.20)

where \( \phi \) denotes the photon density inside the laser cavity and \( n_g \) represents the population inversion density. \( L \) is the length of the active fiber, \( L_{sa} \) is the thickness of the SA, \( c \) is the speed of light, and \( t_r \) refers to the round-trip transit time of the cavity. \( R \) is the output-coupling ratio and \( \delta \) is the dissipative optical loss of the cavity while \( \gamma \) is the inversion reduction factor, \( \sigma_{es} \) is the stimulated emission cross sectional area of the gain medium and \( \tau_s \) denotes the spontaneous decay time of the upper laser level of the gain medium.

The pump rate of the active medium \( W_p \) is approximated as

\[
W_p = \frac{P_p}{h\nu AL}
\]

(3.21)

where \( P_p \) is the pumping power, \( A \) is the effective doping area of the active fiber and \( h\nu \) is the laser photon energy.

The output pulse energy \( E_{out} \) and peak power \( P_{out} \) can be approximated by (Koechner, 2013; Mercer et al., 2007; Miniscalco, 1991; Siegman, 1986)

\[
E_{out} = \frac{h\nu AL}{t_r} \ln \left( \frac{1}{R} \right) \int_0^\infty \phi(t) dt
\]

(3.22)

\[
P_{out} = \frac{h\nu AL}{t_r} \ln \left( \frac{1}{R} \right) \phi_{max}
\]

(3.23)

where \( \phi_{max} \) is the maximum photon density in the cavity.
The Q-switching operation of the proposed laser system was studied by solving the above coupled rate equations using Runge-Kutta method. EDF was chosen as the active medium and the numerical calculations have been done using the following parameters:

\[ \sigma_a = 2 \times 10^{25} \text{ m}^2, \quad \gamma = 1.8, \quad \delta = 0.4, \quad N_{as} = 2.4 \times 10^{27} \text{ m}^{-3}, \quad \tau_{sa} = 0.1 \text{ ns}, \]

\[ \alpha_{ns} = 0.4, \quad \alpha_s = 0.1, \quad L_s = 10 \mu \text{m}, \quad A = 1.26 \times 10^{-11} \text{ m}^2, \quad L = 3 \text{ m}, \quad \rho = 0.95. \]

### 3.5.3 Simulations Analysis of Q-Switching Operation

With pumping the ions population in the excited energy level of the active fiber increases generating photons inside the cavity. A part of these photons is absorbed by the SA which increases the photo carrier density \( N_a \) and this introduces a high loss inside the cavity making the laser system to operate below the lasing threshold. The absorption ability of the SA gets saturated when enough amount of population is accumulated at its excited level (the photocarrier density reaches almost its maximum value). With continuous pumping, the laser system is allowed to overcome the threshold and lasing is realized. The cavity is Q-switched at the time when the inverted population \( n_g \) reaches its maximum value and the photon density \( \phi \) continues to grow up, leading to a pulse whose peak occurs at sometimes after switching. The inverted population density corresponding to the peak of the photon density pulse is approximately \( 7.8 \times 10^{23} \text{ m}^{-3} \) as shown in Figure 3.3(a), which is the same as the critical population inversion \( n_{gc} \). As the photon number in the cavity grows via stimulated emission, the population inversion will drop from its initial value to the final value left after the pulse is over. Simultaneously, the decay rate of the excited energy level of the SA suppresses the absorption and thus the population of the SA excited energy level decreases and when the population drops to a certain level, the absorption (due to SA) dominates again which increases the photocarrier density until the SA excited level becomes saturated again, and consequently
sequential pulses are generated (the recovery of the SA allows Q-switching to start a new round trip). Figure 3.3(c) displays the time behavior of the photocarrier density of the SA.

For every pulse formation, the population inversion falls from its initial value before switching to a final value (after the Q-switching) and then it is restored to its initial value via pumping mechanism before the following Q-switching operation. As the time taken to restore the inversion is approximately equal to the upper state lifetime $\tau_g$, the time between two successive pulses should be equal to or shorter than $\tau_g$. Actually, if the time between two successive pulses is much longer than the upper state lifetime, most of the obtainable inversion will be lost via spontaneous decay. Hence, Q-switched pulsed lasers repetition rate of is typically a few kHz.

![Graphs showing time behavior](image)

**Figure 3.3**: Physical mechanism of forming a single Q-switched pulse: (a) the time behavior of the inverted population density, (b) the time behavior of the photon density, and (c) the time behavior of the SA photocarrier density.

Figures 3.4(a) and (b) show the effect of pump power on pulse duration and repetition rate. As the pump power increases the SA excited energy level needs less time to get into saturation and more time to get out, leading to higher repetition rate and shorter pulse duration. The output peak power has been calculated at different pump powers. As shown in Figure 3.4(c), the output peak power increases with increasing the pump power. This
means that when the pump power increases, the energy stored in the gain medium increases which increases the output peak power.

![Figure 3.4](image)

**Figure 3.4:** (a) Pulse duration, (b) repetition rate, and (c) output peak power versus the pump power of the Q-switched theoretical model.

### 3.6 Summary

In this chapter, the basic principles of laser Q-switching have been presented. Passively Q-switching operation in EDFL cavity has been theoretically studied based on SA taking into account the SA dynamics, which is considered as semiconductor material. In these simulations, peak power, pulse duration, and repetition rate have been investigated as functions of pump power which is considered as a main factor which governs the output performance of the laser system. The presented numerical modelling and simulations could be highly useful for understanding and improving the performance of the passive SA based Q-switched EDFL.
CHAPTER 4 : MODE-LOCKING OPERATION IN FIBER LASER CAVITY WITH SATURABLE ABSORBER

4.1 Introduction

Mode-locking and related forms of mode-coupling in lasers provide important techniques for generating ultrashort light pulses and other useful forms of periodically modulated laser signals, including frequency-swept. In continuous wave (CW) laser, several axial modes are excited when the photons oscillate in the laser cavity. However, the train of photons of each mode are excited in random sequences and each random sequence is independent of each of the others. Thus the phases of the waves in each mode are uncorrelated and random, which means that the intensity of the total output has random fluctuations. The effect of mode-locking is to lock these modes together so that they have a common phase at a certain time and position in the cavity and consequently convert the continuous output laser into a train of pulses.

Up to date, the laser technology has developed so fast and the understanding of mode-locking mechanisms has progressively improved. Many works have been reported in generating ultrashort laser pulses. Various mode-locking techniques have also been developed as reported by Haus (2000). In this chapter, the optical pulse propagation in optical fiber is shown and the general concepts of mode-locking operation in lasers are described besides the different mode-locking techniques. Moreover, numerical simulations of mode-locked fiber lasers with saturable absorber (SA) are presented.

4.2 Pulse Propagation in Optical Fibers

The propagation of optical pulses in fibers is basically described by Maxwell’s equations

$$\nabla \cdot \mathbf{D} = \rho \tag{4.1}$$
\[ \nabla \cdot \mathbf{B} = 0 \quad (4.2) \]

\[ \nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad (4.3) \]

\[ \nabla \times \mathbf{H} = \mathbf{j} + \frac{\partial \mathbf{D}}{\partial t} \quad (4.4) \]

where \( \mathbf{B} \) and \( \mathbf{D} \) refer to magnetic and electric flux densities, respectively, and \( \mathbf{E} \) and \( \mathbf{H} \) denote electric and magnetic field vectors, respectively. Because optical fibers have no free charges, the charge density \( \rho \) and current density \( \mathbf{j} \) are equal to zero. The electric and magnetic flux densities are related to the electric and magnetic fields via the following relations:

\[ \mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \quad (4.5) \]

\[ \mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M}) \quad (4.6) \]

where \( \mathbf{P} \) and \( \mathbf{M} \) represent induced electric and magnetic polarizations, respectively, and \( \varepsilon_0 \) and \( \mu_0 \) are the vacuum permittivity and vacuum permeability, respectively. In a nonmagnetic medium like optical fibers, \( \mathbf{M} \) equals zero. The wave equation that describes the propagation of light in optical fibers is obtained from Maxwell’s equations using the vector identity \( \nabla \times (\nabla \times \mathbf{A}) = \nabla(\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} \) and doing some calculations with assuming that in a homogenous medium, \( \nabla \cdot \mathbf{E} = 0 \) due to no free charge in the medium.

\[ \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\frac{\partial (\nabla \times \mathbf{B})}{\partial t} \quad (4.9) \]

\[ \nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2} - \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2} \quad (4.8) \]
where \( \mu_0 \varepsilon_0 = 1/c^2 \)

\[
\nabla^2 \mathbf{E} - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = \mu_0 \frac{\partial^2 \mathbf{P}}{\partial t^2} \\
\] (4.9)

This wave equation is the starting point in derivation of the basic propagation equation of optical pulses in fibers. The linear and nonlinear terms of the induced polarization are related to the electric field \( \mathbf{E}(r,t) \) via the following Equation:

\[
\mathbf{P}(r,t) = \varepsilon_0 \chi^{(1)} \mathbf{E}(r,t) + \varepsilon_0 \chi^{(2)} \mathbf{E}^2(r,t) + \varepsilon_0 \chi^{(3)} \mathbf{E}^3(r,t) + \ldots \\
\]
(4.11)

It is convenient to separate the rapidly varying part of the electric field in the slowly varying envelope approximation adopted here, by putting it in the following form (Robert, 2003),

\[
\mathbf{E}(r,t) = \frac{1}{2} \hat{x} \{ \mathbf{E}(r,t) \exp(-i\omega t) + c.c. \} \\
\]
(4.12)

where \( \hat{x} \) denotes the polarization unit vector, and \( \mathbf{E}(r,t) \) is a slowly varying field.

The polarization components \( \mathbf{P}_L \) and \( \mathbf{P}_{NL} \) can be described in a similar way as

\[
\mathbf{P}_L(r,t) = \frac{1}{2} \hat{x} \{ \mathbf{P}_L(r,t) \exp(-i\omega t) + c.c. \} \\
\]
(4.13)

\[
\mathbf{P}_{NL}(r,t) = \frac{1}{2} \hat{x} \{ \mathbf{P}_{NL}(r,t) \exp(-i\omega t) + c.c. \} \\
\]
(4.14)

From Equation 4.11, nonlinear polarization is defined by using the fact that \( \chi^{(2)} \) equals zero for optical fibers (Agrawal, 2001; Agrawal, 2007) as
\[ P_{NL} \approx \varepsilon_0 \varepsilon_{NL} E(\mathbf{r}, t) \] (4.15)

The dielectric coefficient is assumed to be locally constant value and define it as (Agrawal, 2007)

\[ \varepsilon_{NL} = \frac{3}{4} \chi^{(3)} |E(\mathbf{r}, t)|^2 \] (4.16)

After substituting Equation 4.12 through Equation 4.14 in Equation 4.10, and applying Fourier transform, the propagation equation is found to satisfy the Helmholtz equation (Agrawal, 2007)

\[ \left( \nabla^2 + \varepsilon(\omega) k_0^2 \right) \tilde{E}(\mathbf{r}, \omega) = 0 \] (4.17)

\[ k_0 = \omega / c, \quad \varepsilon(\omega) = 1 + \chi^{(1)}(\omega) + \varepsilon_{NL} \] (4.18)

where Fourier transform \( \tilde{E}(\mathbf{r}, \omega - \omega_0) \) is defined as (Agrawal, 2007)

\[ \tilde{E}(\mathbf{r}, \omega - \omega_0) = \int_{-\infty}^{\infty} E(\mathbf{r}, t) \exp(i(\omega - \omega_0)t) dt \] (4.19)

Equation 4.17 can be solved by using separation of variables method and the solution is assumed to be in the form (Agrawal, 2007),

\[ \tilde{E}(\mathbf{r}, \omega - \omega_0) = F(x, y) \tilde{A}(z, \omega - \omega_0) \exp(i\beta_0 z) \] (4.20)

where \( \tilde{A}(z, \omega) \) refers to the slowly varying function of \( z \) and \( \beta_0 \) denotes the wave number to be determined later, Equation 4.17 results in the following two equations for \( F(x, y) \) and \( \tilde{A}(z, \omega) \) (Agrawal, 2007)
\[
\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} + \left[ \varepsilon(\omega) \frac{\omega^2}{c^2} - \beta^2 \right] F = 0
\] (4.21)

\[
2i\beta_0 \frac{\partial \tilde{A}}{\partial z} + (\tilde{\beta}^2 - \beta_0^2) \tilde{A} = 0
\] (4.22)

In Equation 4.22, the second derivative \( \frac{\partial^2 \tilde{A}}{\partial z^2} \) has been neglected as \( \tilde{A}(z, \omega) \) is assumed to be a slowly varying function of \( z \).

Equation 4.21 should be solved to obtain the fiber mode distribution and the propagation constant \( \beta_0 \). Additional calculation is found in (Agrawal, 2001; Agrawal, 2007). The dielectric function in Equation 4.21 is approximated by (Agrawal, 2007)

\[
\varepsilon = (n + \Delta n)^2 = n^2 + 2n\Delta n + (\Delta n)^2 \approx n^2 + 2n\Delta n
\] (4.23)

where \( (\Delta n)^2 \) is neglected as \( \Delta n \) is a small perturbation given by (Agrawal, 2007).

\[
\Delta n = n_0 |E|^2 + \frac{i\tilde{\alpha}}{2k_0}
\] (4.24)

where \( \tilde{\alpha} \) is the fiber loss.

The nonlinear refractive index gives a corresponding nonlinear term in the propagation constant, which can be described as (Agrawal, 2007)

\[
\tilde{\beta}(\omega) = \beta(\omega) + \Delta \beta
\] (4.25)

\[
\Delta \beta = \frac{\omega^2 n(\omega)}{c^2 \beta(\omega)} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \Delta n |F(x, y)|^2 dxdy
\] (4.26)
This step completes the formal solution of Equation 4.10 to the first order in perturbation \( P_{NL} \). Using Equations 4.12 and 4.20, \( \mathbf{E}(r,t) \) can be written as

\[
\mathbf{E}(r,t) = \frac{1}{2} \hat{z}\{F(x,y)A(z,t)\exp[i(\beta_0 z - \omega_0 t)] + \text{c.c.}\}
\]  

(4.27)

The Fourier transform \( \tilde{A}(z,\omega) \) of \( A(z,t) \) satisfies Equation 4.22, which can be written as

\[
\frac{\partial \tilde{A}}{\partial z} = i[(\beta(\omega) + \Delta \beta + \beta_0)\tilde{A}]
\]

(4.28)

where Equation 4.25 is used and \( \tilde{\beta}^2 - \beta_0^2 \) is approximated by \( 2\beta_0(\tilde{\beta} - \beta_0) \) (Agrawal, 2007). This equation means that each spectral component in the pulse envelope experiences a phase shift depending on both frequency and intensity. In order to get the propagation equation for \( \tilde{A}(z,t) \), it is necessary to work in time domain by applying the inverse Fourier transform to Equation 4.28. However, as an exact functional form of \( \beta(\omega) \) is rarely known. It is convenient to use Taylor series of \( \beta(\omega) \) in Equation 2.31. Also, \( \Delta \beta(\omega) \) should expand in similar way as

\[
\Delta \beta(\omega) = \Delta \beta_0 + \Delta \beta_1(\omega - \omega_0) + \frac{1}{2} \Delta \beta_2(\omega - \omega_0)^2 + \frac{1}{6} \Delta \beta_3(\omega - \omega_0)^3 + \ldots
\]

(4.29)

where \( \Delta \beta_m \) is defined in similar way as \( \beta_m \) (Equation 2.32).

After some simplifications in Equation 4.28, Equation 2.31 is substituted in Equation 4.28, and then applying the inverse Fourier transform using

\[
A(z,t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \tilde{A}(z,\omega - \omega_0)\exp[-i(\omega - \omega_0)t]d\omega
\]

(4.30)
The resulting equation for \( A(z,t) \) is
\[
\frac{\partial A}{\partial z} = -\beta_1 \frac{\partial A}{\partial t} - i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} + i \frac{\beta_3}{6} \frac{\partial^3 A}{\partial t^3} + i \Delta \beta_0 A \tag{4.31}
\]

The \( \Delta \beta_0 \) term includes fiber loss and nonlinearity effects. By using Equations 4.24 and 4.26, \( \Delta \beta_0 \) can be calculated and substituted in Equation 4.31 which takes the form
\[
\frac{\partial A}{\partial z} + \beta_1 \frac{\partial A}{\partial t} + i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial t^2} - i \frac{\beta_3}{6} \frac{\partial^3 A}{\partial t^3} + \alpha A = i |A|^2 A \tag{4.32}
\]

where the nonlinear parameter \( \gamma \) is
\[
\gamma = \frac{n_2 \omega_0}{c a_{eff}} \tag{4.33}
\]

where \( a_{eff} \) denotes the effective core area and described as
\[
a_{eff} = \frac{\int \int_{-\infty}^{\infty} \int \int_{-\infty}^{\infty} |F(x,y)|^2 dxdy}{\int \int_{-\infty}^{\infty} |F(x,y)|^4 dxdy} \tag{4.34}
\]

By using the retarded time instead of the proper time, the Equation 4.34 will be written as (Agrawal, 2007)
\[
\frac{\partial A}{\partial z} + i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2} - i \frac{\beta_3}{6} \frac{\partial^3 A}{\partial T^3} + \alpha A = i |A|^2 A \tag{4.35}
\]

where \( T = t - z/v_g \equiv t - \beta_1 z \). This is the nonlinear Schrödinger equation (NLSE) which governs the propagation of light in fibers, taking into account the effects of dispersion, nonlinearity, and fiber loss (Robert, 2003).
Including the gain term of the active fiber into the NLSE (detailed derivation is found in (Agrawal, 1991; Milonni & Eberly, 1988)) results in

\[
\frac{\partial A}{\partial z} + i \left( \beta_2 + i \Omega^2 g \right) \frac{\partial^2 A}{\partial T^2} - i \beta_3 \frac{\partial^3 A}{\partial T^3} + \frac{\alpha - g}{2} A = i \gamma |A|^2 A
\]  

(4.36)

The \( \Omega^2 \) term accounts for the decreasing in the gain for spectral components of the pulse far from the gain peak. Including the saturable absorption effect in Equation 4.36 yields the master equation of mode-locking operation (Haus et al., 1991).

### 4.3 Fundamental Principles of Mode-Locking

A laser cavity supports only discrete modes, as a laser oscillates at a number of resonant frequencies whose spacing represents the fundamental frequency \( \omega_k \):

\[
\omega_i - \omega_{i-1} = \frac{2\pi c}{L_c} = \omega_k
\]  

(4.37)

where \( L_c \) is the cavity length. The output electric field of the generated optical pulse is the sum of all the oscillating modes which is described by

\[
E(t) = \sum_n A_n \exp \left[ i (\omega_0 + n\omega_k) t + i \Phi_n \right]
\]  

(4.38)

where \( \omega_0 \) is the referenced center oscillating frequency and \( A_n \) and \( \Phi_n \) refer to the \( n^{th} \) mode amplitude and phase, respectively.

When the laser is in the free oscillating state, \( A_n \) and \( \Phi_n \) can have any value without any bound resulting in generating a CW source. Figure 4.1 shows the typical electric fields and pulse energy as a function of time generated from this source in which electrical fields of five modes with random phase and the power of the total output signal are shown.
Figure 4.1: (a) Electric field amplitudes of five individual modes out of-phase and (b) total output power of a multi-longitudinal mode laser.

The detailed pulse structure inside the cavity relies on the amplitude and phase of the cavity modes. The circulating pulse becomes periodic with the cavity round trip when the amplitude and phase of the various cavity modes are locked. The interference of these modes leads to a bandwidth limited pulse if they are in phase while the phase disturbances increase the pulse duration.

When the modes are locked together, which means that all the modes $N$ are in phase, and $A_n$ and $\Phi_n$ will be constant. For example, when $A_n = 1$, and $\Phi_n = 0$ (the simplest case), using Equation 4.38 results in

$$E(t) = \cos \omega_0 t \frac{\sin(N\omega_r t/2)}{\sin(\omega_r t/2)}$$

This represents an oscillation at frequency $\omega_0$ modulated with the sinc envelope function

$$f(t) = \frac{\sin(N\omega_r t/2)}{\sin(\omega_r t/2)}$$
Thus, the average power can be given by

$$\text{Power}(t) \propto \frac{\sin^2\left(N\omega_R t / 2\right)}{\sin^2\left(\omega_R t / 2\right)}$$

(4.41)

An example of such a pulse sequence is shown in Figure 4.2. This represents a periodic train of pulses having the following characteristics:

- The pulse period is equal to $2\pi / \omega_R$.
- The peak field amplitude equals $N$ times the amplitude of a single mode.
- The pulse width $\tau_p$ can be estimated from Equation 4.41 (Agrawal, 2001) as

$$\tau_p = \frac{2\pi}{N\omega_R} = \frac{1}{N\Delta\nu}$$

(4.42)

which becomes shorter when $N$ increases.

In order to increase the number of locked modes, this mostly is affected by the used mode-locker. For example, this work focuses on the SA approach as mode-locker in which the SA modulation depth is the key factor; the higher the modulation depth is, the larger the number of locked modes and the shorter the pulse will be (the interplay between fiber dispersion and nonlinearity should be taken into account).

Since $N\Delta\nu$ refers to the overall mode-locked bandwidth of all phase-locked modes, the pulse width is inversely proportional to the spectral bandwidth over which phases of several longitudinal modes are synchronized. In other words, the wider the bandwidth is, the shorter the pulse width from the laser. The exact relationship between the gain bandwidth $\Delta\nu_g$ and pulse width and changes according to the nature of gain broadening.

Practically, the actual pulse duration is governed by the shape of each pulse, which is in turn determined by the exact amplitude and phase relationship of each longitudinal...
mode. For example, for a laser generating Gaussian pulse, the minimum possible pulse duration $\Delta t$ is given by

$$\Delta t = \frac{0.441}{N\Delta \nu}$$

(4.43)

where the value 0.441 is the time-bandwidth product (TBP) of the pulse, and this value changes according to the pulse shape. For ultrashort pulse, it is often assumed that the pulse shape is a hyperbolic-secant-squared ($\text{sech}^2$), giving a TBP of 0.315. By using this equation, the minimum pulse duration can be calculated based on the measured spectral width.

![Figure 4.2](image)

**Figure 4.2:** (a) Electric field amplitudes of five individual modes in-phase and (b) the total output power of a periodic pulse train (Ngo, 2010).

### 4.4 Mode-Locking Techniques

Laser mode-locking can be realized by several techniques based on two main categories: Active and passive mode-locking. Figure 4.3 summarizes various approaches of mode-locking.
Figure 4.3: Types of mode-locking techniques.

4.4.1 Active Mode-Locking

The active mode-locking in fiber lasers can be realized by using the same approaches with the solid-state laser counterparts (Chang et al., 2012; Jeon et al., 1998). They can be obtained by modulating amplitude, phase, or frequency of the oscillating light inside the laser cavity. It is normally obtained by modulating the gain or loss of the cavity using an external modulator such as acousto-optic modulator. The active techniques can be realized through amplitude modulation, frequency modulation, synchronous pumping and optical modulation (Scott et al., 1997; Godil et al., 1991; Kinsel, 1973; Morgner et al., 1999).
4.4.2 Passive Mode-Locking

Passive mode locking technique refers to the phase locking mechanism without any external signal to create pulses unlike active mode locking technique, which was achieved by inserting an electrical signal-driven active modulator into the laser cavity. The pulses are generated passively through the internal structure of the laser cavity that gives more advantages to the output pulse signal. Based on this principle various physical mechanisms can be used for passive mode-locking; SA approach, nonlinear polarization rotation (NPR), Kerr lens, additive pulse and colliding pulse approaches (Iqbal et al., 2007; Haus et al., 1994; Haus et al., 1992; Usechak, 2002). Among the different passive mode-locking approaches, the SA approach is more preferable due to its simplicity and high performance. In this approach, a SA device is inserted into a laser cavity to initiate mode-locking. This study focuses on passive mode-locking based on SA approach.

4.5 Numerical Simulations of Passively Mode-Locked Fiber Lasers Based on Saturable Absorber

4.5.1 Modelling and Simulations

The laser is assumed to be a ring cavity that consists of passive fiber (PF) with variable length, 2 m long active fiber (AF), SA device, and output coupler, as shown in Figure 4.4. AF is used to represent the gain medium for the laser. The optical pulse is amplified in the AF and then propagate through the PF which stretches the pulse and slightly decreases its power. The light intensity is then manipulated and modulated by the SA. The output of SA is fed back into the AF after passing through the output coupler to form a loop. The signal then circulates through the optical loop.
Figure 4.4: Schematic of mode-locked laser model.

Propagation of light in the AF can be modelled by Equation 4.36 (generalized NLSE).

It is convenient to rewrite Equation 4.36 in this section;

\[
\frac{\partial A(z,t)}{\partial z} = -\frac{i}{2}[\beta_2 + ig\Omega^2] \frac{\partial^2 A(z,t)}{\partial t^2} + \frac{1}{2}[g - \alpha]A(z,t) + \frac{\beta_3}{6} \frac{\partial^3 A(z,t)}{\partial t^3} + i\gamma |A(z,t)|^2 A(z,t)
\]

where \( A(z,t) \) represents the envelope of the pulse, \( t \) is the retarded time, \( z \) is the propagation distance, \( \beta_2 \) and \( \beta_3 \) account for the second and third order fiber dispersion parameters, respectively, \( \Omega \) is the dipole relaxation time (inverse gain line width), \( \gamma \) is the fiber nonlinearity parameter that accounts for SPM, \( \alpha \) is the fiber loss, and \( g \) is the gain parameter of the AF. The gain dynamics is modelled by a 2-level approximation giving a rate equation for the gain parameter (Becker et al., 1999):

\[
\frac{\partial g(z,t)}{\partial t} = -\frac{1}{\tau_g} \left[ (g(z,t) - g_0) + \frac{g(t)|A(z,t)|^2}{P_{sat-gain}} \right]
\]

(4.44)

where \( g_0 \) is the small-signal gain, \( \tau_g \) is the inversion relaxation time, \( P_{sat-gain} \) is the gain saturation power.
Here, the gain is assumed to have a stationary value \( \frac{\partial g}{\partial t} = 0 \) on a longer time scale. The gain will have saturated over many pulses, or in other words with the average power

\[
\langle |A|^2 \rangle = \frac{E}{T_R},
\]

where \( E \) is the pulse energy and \( T_R \) is the round trip time. The left hand side of Equation 4.44 to set to zero and let \( |A(z,t)|^2 \rightarrow \langle |A|^2 \rangle \), to give (Okhotnikov, 2012):

\[
g(z,t) \rightarrow g(z) = \frac{g_0}{1 + \frac{E(z)}{T_R P_{sat-gain}}}
\]

where \( E(z) = \int_{-\frac{T_{z/2}}{2}}^{\frac{T_{z/2}}{2}} |A(z,t)|^2 dt \). Here, the recovery time is not a factor in Equation 4.45, but the round trip time \( T_R \) is the relevant time scale which governs the effective saturation energy \( T_R P_{sat-gain} \). The PF was simulated using Equation 4.36 by setting the value of \( g = 0 \) as

\[
\frac{\partial A(z,t)}{\partial z} = -i \beta_2 A(z,t) \frac{\partial^2 A(z,t)}{\partial t^2} + \beta_3 A(z,t) \frac{\partial^3 A(z,t)}{\partial t^3} + \frac{\alpha}{2} A(z,t) + i \gamma |A(z,t)|^2 A(z,t)
\]

The dimensionless propagation equations of the optical pulses have been solved using the finite difference with Fourier transform method for both active and passive fibers. The dimensionless parameters are defined as \( z = z/L_c, t = t/t_0 \) where \( L_c = L_{AF} + L_{PF} \) and \( L_c \) is the cavity length, \( L_{AF} \) is the AF length and \( L_{PF} \) is the PF length, and \( t_0 \) refers to the initial conditions of the simulation. Table 4.1 summarizes the parameters used in the simulation.
Table 4.1: Mode-locked laser cavity parameters (Oktem et al., 2010).

<table>
<thead>
<tr>
<th>Cavity element</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>AF</td>
<td>$L_{AF}$</td>
<td>2 m</td>
</tr>
<tr>
<td></td>
<td>$\beta_2$</td>
<td>76.9 fs$^2$/mm</td>
</tr>
<tr>
<td></td>
<td>$\beta_3$</td>
<td>168 fs$^3$/mm</td>
</tr>
<tr>
<td></td>
<td>$\gamma$</td>
<td>9.32 W$^{-1}$km$^{-1}$</td>
</tr>
<tr>
<td></td>
<td>$g_0$</td>
<td>15 dB/m</td>
</tr>
<tr>
<td></td>
<td>$\Omega$</td>
<td>25.5 fs</td>
</tr>
<tr>
<td></td>
<td>$E_{sat_gain}$</td>
<td>10 pJ</td>
</tr>
<tr>
<td>PF</td>
<td>$L_{PF}$</td>
<td>18 m</td>
</tr>
<tr>
<td></td>
<td>$\beta_2$</td>
<td>4.5 fs$^2$/mm</td>
</tr>
<tr>
<td></td>
<td>$\beta_3$</td>
<td>109 fs$^3$/mm</td>
</tr>
<tr>
<td></td>
<td>$\gamma$</td>
<td>2.1 W$^{-1}$km$^{-1}$</td>
</tr>
<tr>
<td></td>
<td>$\alpha$</td>
<td>0.2 dB/km</td>
</tr>
<tr>
<td>SA</td>
<td>$P_{sat}$</td>
<td>3.69 W</td>
</tr>
<tr>
<td></td>
<td>$q_0$</td>
<td>Variable</td>
</tr>
<tr>
<td></td>
<td>$\tau$</td>
<td>Variable</td>
</tr>
<tr>
<td>Output Coupler</td>
<td>$R$</td>
<td>20%</td>
</tr>
</tbody>
</table>
The pulse experiences an action manipulated by the SA. The microscopic characteristics of SA can be modelled to reveal the macroscopic quantities that are important for saturable absorption. Here, the detailed microscopic characteristics of SA is neglected and a general rate equation (Haus, 1975) is assumed:

\[
\frac{dq}{dt} = -\frac{1}{\tau} \left[ (q - q_0) + \frac{|A|^2}{P_{\text{sat}}} \right]
\]  

(4.47)

where \( q(t) \) is the SA loss coefficient whose nonsaturable loss is not included, \( |A(t)|^2 \) is the time-dependent power incident on the SA, \( q_0 \) is the modulation depth of the SA, i.e. the maximum change in absorption that depends mainly on the type of material and can be controlled by fabrication process, and the number of layers of a material (i.e. single layer of graphene has modulation depth of 0.023 while for double layer it equals 0.046). It is a wavelength dependent parameter), \( \tau \) is the SA recovery time, and \( P_{\text{sat}} \) is the SA saturation power. Some of the optical signal is extracted using output coupler. The output coupler action is described by a simple scalar multiplication of the field:

\[
A(t)_{\text{out}} = \sqrt{R} \times A(t)
\]  

(4.48)

where \( R \) is the output coupling efficiency. The output coupler is assumed to be only an amplitude modulation and neglected any phase modulation.

4.6 Simulations Results and Discussion

When the pulse goes through the AF, it is amplified. Then it propagates through the PF that slightly decreases its power due to some loss. The SA shapes the pulse and decreases its width. Finally, the pulse power is reduced by the output coupler. The impact of each component in the cavity is described as follows:
Impact of SA modulation depth and recovery time

Modulation depth and recovery time represent the most important parameters that govern the performance of SA and consequently the output performance of mode-locked lasers in addition to the nonsaturable loss, saturation fluence, and damage threshold (Haiml et al., 2004; Hönninger et al., 1999; Kurtner et al., 1998; Lecaplain et al., 2011; Turitsyn, 2009; Turitsyn et al., 2012; Yarutkina et al., 2015). The pulse duration $t_p$ is inversely proportional to the modulation depth $q_0$ of the SA (Jung et al., 1997).

$$t_p \propto \frac{1}{q_0}$$  \hspace{1cm} (4.49)

where the exponential factor $s$ changes in the different mode locking theories and constantly greater than 1. A high modulation depth always produces shorter pulses, and also with lower energy.

Figure 4.5 shows the pulse temporal waveform for different modulation depths where the pulse shows a shorter duration and lower energy for higher modulation depth.

The physics behind these results can be explained as follows. SA materials have the ability to absorb most of the incident light power when the input intensity is low, and as the intensity increases the absorption decreases because in this case many incident photons will make a large number of atoms excited from the ground state to a higher energy state and because of the reduced number of atoms in the ground state, the probability of absorbing a photon is decreased. Therefore, the absorption probability decreases as the optical intensity increases. The central part of the pulse has high intensity, so it will partially burn through the SA with less absorption while the pulse wings have low intensity which will transmit with high absorption. SA is a kind of nonlinear modulator whose modulation depth or transmission curvature near the peak is controlled by the optical pulse profile itself and the most important property of SA pulse shortening.
is that a suitable SA should continue to shorten the pulse by more or less a fixed ratio on each successive pass, even when the pulse becomes very short. So the SA with high transmission curvature or modulation depth is able to continue to shorten the pulse more than that with low modulation depth. This is illustrated in Figure 4.5 where the pulse shows a shorter duration for higher modulation depth.

![Figure 4.5: Intra-cavity temporal pulse shape for different SA modulation depths.](image)

The SA rate of Equation 4.47 and the gain rate of Equation 4.44 are supposed to be alike. The SA rate equation has specific limits depending on the saturation recovery time as in the gain rate equation case. When the SA recovery time is far shorter than the pulse duration, $dq/dt$ is negligible, the absorption instantaneously will adiabatically follow the incident power $|A(t)|^2$ absorption and Equation 4.47 reduces to steady state case

\[
q(t) = \frac{q_0}{1 + \frac{|A(t)|^2}{P_{sat}}}
\]  

(4.50)

In order to realize the effect of SA recovery time which represents the absorber intrinsic response time which strongly affects the pulse duration especially in a bulk laser
with no dispersion management, the propagation equations have been solved incorporating the effect of SA for $dq/dt \neq 0$ according to Equation 4.47, and compare with the case $dq/dt = 0$ according to Equation 4.50. When the SA has a long recovery time (few ns), it becomes slow SA with strong saturation and shows the ease of self-starting. While fast SA has a relatively short recovery time compared to the pulse duration and thus it responds to the light intensity almost in an instantaneous fashion.

Figure 4.6 shows how the SA recovery time affects the pulse shape and energy by comparing the temporal pulse in cases of Equation 4.47 and Equation 4.50. It is observed that as the absorber recovery time increases, the pulse becomes more stable with asymmetric shape, i.e. the effect of $dq/dt$ becomes substantial and cannot be neglected.

**Figure 4.6:** Intra-cavity pulse shape of different SA recovery times for $dq/dt \neq 0$ plotted in blue color and $dq/dt = 0$ plotted in red color: (a) $\tau = 0.1$ ns, (b) $\tau = 0.5$ ns, (c) $\tau = 1$ ns, and (d) $\tau = 1.5$ ns.
Impact of fiber dispersion and nonlinearity

The effects of fiber dispersion and nonlinearity on the pulse shape and its impact by the SA are investigated. Here, the pulse temporal profile for different values of dispersion and nonlinearity parameters at different values of modulation depths was simulated and the result is presented in Figure 4.7. Basically, the pulse shape and duration are determined by various effects including the action of SA, pulse broadening by the limited gain bandwidth, and the balance between fiber dispersion and nonlinearity. In Figure 4.7, the simulations have been done for slow SA which supports self-starting mode-locking. It shows stable pulses generation because the SA induces slight temporal shift in the pulse and this limits the growth of noise behind the pulse. The fast SA is not ideal for self-starting mode-locking because it disturbs the initial formation of the pulse.

Once the pulses acquire a certain band width which is a consequence of ultrashort pulse duration, they can experience the effects of dispersion. For example, the temporal shape of the pulses broadens as light oscillating in the cavity due to the effects of dispersion. Therefore, it is necessary to study the interplay between fiber dispersion and nonlinearity to control the pulse evolution inside the cavity. Here, the effects of cavity dispersion and nonlinearity on the pulse shape at various modulation depth of SA are investigated. As shown in Figure 4.7, the effects of fiber dispersion and nonlinearity on the pulse shape is relatively small while the SA modulation depth significantly changes the pulse shape. It is also worthy to note that the effect of third order dispersion is very low and can be neglected. Basically, Third order dispersion can distort ultrashort optical pulses by asymmetrically broaden the pulses. However, third order dispersion becomes necessary only when the wavelength approaches the zero-dispersion wavelength to within a few nm. Therefore, it is obvious that the pulse shape is mostly affected by the SA.
Figure 4.7: Intra-cavity temporal pulse shape for (a) different values of PF dispersions at $q_0 = 0.1$, (b) different values of PF dispersions at $q_0 = 0.3$, (c) different values of PF nonlinearities at $q_0 = 0.1$, and (d) different values of PF nonlinearities at $q_0 = 0.3$.

Impact of arrangement of cavity components

The output pulse energy from a ring cavity was compared for different arrangement of components in the cavity. The results are shown for four different arrangements; PF-AF-SA-Output coupler, AF-SA-PF-Output coupler, AF-PF-SA-Output coupler and PF-SA-AF-Output coupler. As shown in Figure 4.8, it is observed that the cavity with (PF-AF-SA-Output coupler) arrangement has higher output pulse energy compared to that of (AF-SA-PF-Output coupler) arrangement. This can be explained via Equation 4.50 where the SA loss becomes minimal when the input power on SA is at maximum and the first configuration shows that the SA is immediate after AF which carries higher input power.
producing low loss at SA. On the other hand, the cavity with (AF-PF-SA-Output coupler) arrangement yields lower energy than that of (AF-SA-PF-Output coupler) arrangement because (AF-PF-SA-Output coupler) cavity arrangement produces a higher loss than that of (AF-SA-PF-Output coupler) arrangement based on the input power on SA, and so is the cavity of (PF-SA-AF-Output coupler) arrangement.

The cavities of arrangements (SA-AF-PF-Output coupler) and (SA-PF-AF-Output coupler) can be considered in a similar way. The difference between the different cavity arrangements grows and becomes distinct when the cavity length increases as shown in Figure 4.9. This happens because when the length of the laser cavity increases, the change in the input power on SA becomes clear and so does the SA loss, and consequently the difference in the output energy for different cavity configurations becomes distinct.

![Figure 4.8: Intra-cavity energy dynamics for different cavity configurations: (a) PF-AF-SA-Output coupler, (b) AF-SA-PF-Output coupler, (c) AF-PF-SA-Output coupler, and (d) PF-SA-AF-Output coupler.](image-url)
Figure 4.9: Intra-cavity energy dynamics for different cavity lengths of two different cavity configurations: (a) PF-AF-SA-Output coupler and (b) AF-PF-SA-Output coupler.

Impact of cavity length

The output pulse energy also varies with variation of cavity length and this is achieved by varying the PF length, while the length of the AF keeps constant. As the length of the cavity increases, the repetition rate $f_r$ decreases and the pulse circulating becomes longer ($f_r = 1/T_R$ and $T_R = nL_z/c$). This leads to in essence longer pulse duration and narrow spectrum (temporal broadening), and consequently the output energy increases as shown in Figure 4.9.

It is obvious that the laser properties (pulse duration, temporal or spectral width and output energy) have less dependence on fiber dispersion and nonlinearity, and cavity configurations at very short cavity length. The modelling and simulations above can be used for design and optimization of passively mode-locked fiber lasers.

4.7 Summary

In this chapter, the basic principles of laser mode-locking have been shown and the output energy and temporal changes of optical pulses have been simulated in a passively
mode-locked fiber laser setup based on SA taking into account the effect of temporal change in saturable absorption. In this numerical study, the energetics and pulse properties for different fiber laser arrangements/configurations have been investigated. The results show that the management of the intra-cavity components is the key factors to increase the output pulse energy. In addition, the SA is the main component and has the highest impact on pulse shaping compared to the fiber dispersion and nonlinearity. Also, the third order dispersion is not effective and becomes necessary only when the wavelength is close to the zero-dispersion wavelength. Slow SA supports self-starting mode-locking and stable pulse formation provided that there is a balance between the fiber dispersion and nonlinearity, while fast SA is not ideal for self-staring mode-locking because it does not support the initial formation of the pulse. Also, it is found that the arrangement of the cavity components in the laser ring resonator affects the output pulse energy. The presented simulations could be highly useful for understanding, optimizing, and improving passively mode-locked fiber lasers with SA.
CHAPTER 5: PULSED ERBIUM-DOPED FIBER LASERS BASED ON COBALT OXIDE, VANADIUM OXIDE, AND NICKEL OXIDE AS SATURABLE ABSORBERS

5.1 Introduction

Saturable absorbers (SAs) represent key elements in pulsed lasers generation. They have been broadly used as Q-switchers and mode-lockers that enable the generation of pulses in one of two possible regimes, Q-switched or mode-locked. Such lasers offer excellent beam quality and there is no need for costly bulk modulators as required in the actively mode-locked or Q-switched lasers.

In this chapter, novel cobalt oxide, vanadium oxide, and nickel oxide-based SAs are introduced for Q-switching and mode-locking operations in simple fiber laser cavity.

5.2 Cobalt Oxide Nanocubes as Saturable Absorber for All-Fiber Passively Q-Switched Erbium-Doped Fiber Laser

Q-switched fiber lasers have gained a growing interest for various applications such as micromachining, fiber optical sensing, medicine and telecommunications due to their high peak power and excellent beam quality (Kumar & Lee, 2013; Nishizawa, 2014). Passive Q-switching methods have been widely employed since they are more useful and cost-effective than active methods.

As discussed in Chapter 2, several types of materials have been introduced for passive Q-switching. Among these materials, transition metal oxides have been reported and used as SAs with good results comparable to conventional SAs. Transition metal oxides represent a class of materials that exhibit nonlinear optical properties. Over the past few decades, they have been intensively investigated not only due to optical nonlinearity but also their mechanical strength, thermal and chemical stability. For instance, several metal oxides films have a large nonlinear optical response with a third-order nonlinear susceptibility in the range between $10^{-8}$ and $10^{-7}$ esu. Among the transition metal oxides, Co$_3$O$_4$ has the largest ratio of nonlinearity over linear absorption. It was found that Co$_3$O$_4$
based thin film possesses refractive index and extinction coefficient that change significantly with laser irradiation. In addition, the two absorption bands of Co$_3$O$_4$ centered at 410 and 725 nm make it promising candidate for fabricating highly nonlinear transmission devices in the visible range. The optical properties of Co$_3$O$_4$ nanoparticles thin film are reported in (Ando et al., 2004; Yamamoto et al., 2002; Zhu et al., 2010; Zhu et al., 2012).

In this section, a passively Q-switched erbium-doped fiber laser (EDFL) was established by employing Co$_3$O$_4$ film based SA.

### 5.2.1 Synthesis of Co$_3$O$_4$ Nanocubes

First, Co$_3$O$_4$ nanocubes were synthesized by the simple technique reported in (Numan et al., 2017). In which, 1 mmol of Co(CH$_3$COO)$_2$·4H$_2$O was dissolved in 45 ml mixture of deionized water and ethanol (3:1) under bath sonication. Then, 15 ml of 1 mmol urea solution was added to the mixture and stirred for 30 min. After that the mixture was placed in a 100 ml Teflon-lined stainless steel autoclave and kept in the preheated oven for 5 hours at 150 °C. The extracted precipitate of Co$_3$O$_4$ nanocubes was cleansed with deionized water and ethanol via centrifugation. Finally, it was dried in a hot air oven at 60 °C.

### 5.2.2 Fabrication of Co$_3$O$_4$-SA

In order to fabricate Co$_3$O$_4$-SA, the Co$_3$O$_4$ nanocubes had to be embedded in a polyethylene oxide (PEO) thin film. The fabrication of thin film itself was done via solution casting route. Firstly, 1 gram of PEO was dissolved in 120 ml of deionized water and put under constant stirring for 2 hours at 50 °C. After that, sufficient amount of synthesized Co$_3$O$_4$ nanocubes was added into the above solution and kept under stirring for further 2 hours. Finally, the uniformly mixed slurry was casted on a Teflon coated Aluminum foil and dried at 60 °C in the oven.
5.2.3 Characterization of Co₃O₄ Nanocubes

The surface morphology of Co₃O₄ nanocubes was investigated by using a field emission scanning electron microscope (FESEM). The high-resolution transmission electron microscopy (HRTEM) was also run to determine the particle size and confirm the cubical shape of the nanocubes. The structural crystallinity and purity of the Co₃O₄ nanocubes were analyzed by X-ray diffraction (XRD) and Raman spectroscopy. The XRD measurement was taken at a scan rate of 0.02 degree per second by using Philips X'pert XRD equipment with copper Kα radiation (1.5418 nm wavelength) while 514 nm green laser was used as a light source for the Raman spectrometer.

The cubical shape of the Co₃O₄ was also confirmed by HRTEM as shown in Figure 5.1(a). The morphological studies indicated by the field emission scanning electron microscopy (FESEM) reveal that Co₃O₄ nanocubes were successfully synthesized (Figure 5.1 (b)). Moreover, the average particle size was found to be 32 nm.

Figure 5.2(a) shows the XRD pattern of Co₃O₄ nanocubes. The diffraction pattern of pure Co₃O₄ displays prominent crystalline peaks at 20 values of 18.1°, 30.9°, 36.4°, 44.2°, 54.9°, 58.6°, and 64.4° which can be attributed to the (111), (220), (311), (400), (422), (511), and (440) planes, respectively (Numan et al., 2017). Figure 5.2(b) depicts the Raman spectra of Co₃O₄ nanocubes. The strong peaks at 192, 482, 517, 614 and 687 cm⁻¹ observed can be attributed to the B₁g, E₉g, F¹2g, F²2g and A₁g modes of Co₃O₄, respectively (Numan et al., 2016).
Figure 5.1: Co$_3$O$_4$ nanocubes: (a) HRTEM image and (b) FESEM image.

Figure 5.2: Co$_3$O$_4$ nanocubes: (a) XRD pattern and (b) Raman spectrum.

5.2.4 Nonlinear Optical Absorption Characteristics of Co$_3$O$_4$-SA

The nonlinear optical response of Co$_3$O$_4$-SA was determined using twin-detector technique shown in Figure 5.3. A homemade mode-locked fiber laser was used as an illumination source with a repetition rate of 17 MHz, pulse width of 0.90 ps and 1550 nm central wavelength. The output pulse train from the laser was then amplified by an optical amplifier, which was connected to an optical attenuator. Changes in the output power of both detectors were recorded as the attenuation value (via the optical attenuator) was
reduced gradually. A 3-dB coupler was used to split the output signal. One end of the coupler was connected directly to a power meter while the other was connected to the Co$_3$O$_4$-SA before another power meter.

The transmission at various input intensity was recorded and plotted as shown in Figure 5.4 after fitting using the following saturation model (Tian et al., 2015):

$$T(I) = 1 - q_0 \exp\left(-\frac{I}{I_{\text{sat}}}\right) - q_{ns}$$  \hspace{1cm} (5.1)

where $T(I)$ is the transmission, $q_0$ is the modulation depth of SA which represents the maximum change in absorption, $q_{ns}$ is the non-saturable absorption, $I$ is the input intensity, and $I_{\text{sat}}$ is the saturation intensity. The saturation intensity and modulation depth of the Co$_3$O$_4$-SA were determined to be 3 MW/cm$^2$ and 0.35%, respectively.

![Figure 5.3: Modulation depth measurement setup.](image)

University of Malaya
Figure 5.4: Nonlinear optical absorption characteristics of Co$_3$O$_4$-SA.

5.2.5 Q-Switched Laser Experimental Setup

The schematic diagram and real setup of a passively Q-switched EDFL based on SA of Co$_3$O$_4$ are shown in Figure 5.5. The laser cavity is mainly composed of a 3 m long erbium-doped fiber (EDF) (Moritex PureCore MSFER123) as a gain medium and a 5 m long single mode fiber (SMF) as the pigtails of the corresponding components. The EDF has core and cladding diameters of 4 and 125 µm respectively, with numerical aperture of 0.16 and erbium ion absorption of 23 dB m$^{-1}$ at 980nm. The EDF was pumped by a 980nm Laser Diode (LD) via a 980/1550-nm fused wavelength division multiplexing (WDM) coupler. An isolator (ISO) was used to ensure unidirectional operation of the laser. The Co$_3$O$_4$ film is sandwiched between two ferrule connectors via a fiber adapter before it is inserted into the EDFL cavity to act as Q-switcher. The film has an absorption of 3.0 dB at 1550 nm region. An 80:20 fiber coupler was used to extract a 20% intra-cavity power for the output. The total cavity length was about 8 m. A 350-MHz oscilloscope combined with a 1.2 GHz photo-detector, a radio frequency (RF) spectrum analyzer, an optical power meter and an optical spectrum analyzer (OSA) with a spectral resolution of 0.07 nm were used simultaneously to monitor the output pulse train.
Figure 5.5: Experimental setup of the Q-switched EDFL with Co$_3$O$_4$-SA: (a) Schematic configuration and (b) the real setup.

5.2.6 Experimental Results and Discussion

The performance of the presented Q-switched EDFL with a ring configuration was investigated at various power of the 980 nm pump. Initially, a continuous wave (CW) laser was observed to start at pump power as small as 40 mW. As the pump power reached
55 mW, self-started Q-switched pulses were generated and continued to be stable to the maximum pump power of 165 mW. The Co$_3$O$_4$-SA was maintained stably without thermal damage throughout the experiment. Figure 5.6 displays the output spectrum at 165 mW pump power. It shows that the Q-switched pulse centered at wavelength of 1561.6 nm with 3-dB bandwidth of 1.8 nm. Figure 5.7(a) illustrates the typical oscilloscope trace of the Q-switched laser obtained when the Co$_3$O$_4$-SA is added in the laser cavity when the pump power is fixed at 165 mW. The pulse train shows stable Q-switching operation with no fluctuations. The single pulse profile is shown in Figure 5.7(b), which indicates a full width at half maximum (FWHM) of 5.02 μs.

**Figure 5.6:** Spectrum of the output of Q-switched EDFL with Co$_3$O$_4$-SA at pump power of 165 mW.
The population inversion was improved via pumping from 55 mW to 165 mW, which provided more gain to saturate the SA. Consequently, the pulse width dropped from 10.9 to 5.02 µs while the repetition rate of the pulse raised from 29.8 to 70.92 kHz as shown in Figure 5.8. The phenomenon of decreasing pulse width and increasing repetition rate as the pump power increases is a distinct characteristic of passive Q-switching (Svelto, 2010). The average pulse energy and output power versus pump power are shown in Figure 5.9. It shows the maximum output pulse energy of 39.3 nJ at 115 mW pump power.

In order to investigate the stability of our Q-switched pulse the degree of suppression of the adjacent harmonics or side modes through RF spectrum is obtained as shown in Figure 5.10. The RF spectrum shows that the fundamental frequency our fiber laser cavity is 70.92 kHz and the side modes are suppressed by about 60 dB (high signal to noise ratio (SNR)), confirming the stability of the pulse (Von der Linde, 1986).
Figure 5.8: Repetition rate and pulse width as functions of input pump power of Q-switched EDFL with Co$_3$O$_4$-SA.

Figure 5.9: Output power and pulse energy of Q-switched EDFL with Co$_3$O$_4$-SA versus the input pump power.
Figure 5.10: RF spectrum of Q-switched EDFL with Co₃O₄-SA at pump power of 165 mW with 70.90 kHz repetition rate.

5.3 Vanadium Oxide as Saturable Absorber for All-Fiber Passively Q-Switched Erbium-Doped Fiber Laser

Vanadium oxides are one of the most important transition metal oxides. There are series of oxides that form from vanadium which are commonly available as VO, V₂O₃, VO₂, and V₂O₅. The most important of these oxides is V₂O₅ and it is the most stable member of the series (Yan et al., 2009) which shows high nonlinear optical response. The nonlinear optical absorption characteristics of V₂O₅ have been studied in (Molli et al., 2016). In this section, Q-Switched EDFL using a V₂O₅ film as SA is introduced. The V₂O₅ based SA is hosted into PEO film, which is then sandwiched between two fiber ferrules and incorporated into the laser cavity.

5.3.1 Fabrication and Characterization of V₂O₅-SA Thin Film

Vanadium oxide (V₂O₅) was synthesized by chemical reaction reported in (Aslam et al., 2015). In a typical synthesis, 20 g of NH₄VO₃ (Sigma Aldrich, Malaysia) was dissolved in 500 mL of deionized water. After that, 0.1 g of Triton X-100 surfactant was added in the above solution under constant stirring at 90 °C for 60 min in order to
complete the emulsification. Concentrated HNO$_3$ (35% Merck) was then added dropwise in the above mixture to acidify the solution. The dark brown precipitates were formed overnight, which were collected and washed with deionized water in order to removed acid and traces of surfactant. The precipitates were dried in hot air oven at 90 °C for 12 hours. Finally, dried powder was crushed and calcined at 500 °C for five hours in a box furnace at heating and cooling rate of 25 °C/min to obtain yellowish-brown V$_2$O$_5$.

The structural crystallinity and phase purity of V$_2$O$_5$ were investigated by XRD as shown in Figure 5.11(a). As shown in the figure, V$_2$O$_5$ displays sharp peaks at 20 value of 15.4, 20.3, 21.7, 26.2, 31, 32.4, 34.4, 47.4 and 51.3 which correspond to the lattice planes of (200), (010), (110), (101), (310), (011), (301), (600) and (002), respectively. All diffraction peaks for V$_2$O$_5$ can be indexed as an orthorhombic crystal phase (space group P m n 21, PDF 96-101-1226) without showing any impurity. The surface morphology of the prepared V$_2$O$_5$ was then investigated by FESEM. Figure 5.11(b) displays the FESEM image of V$_2$O$_5$ which shows the tubular shape of V$_2$O$_5$.

Synthesized V$_2$O$_5$ was integrated with a polymer in order to fabricate V$_2$O$_5$-SA thin film. 1.0 g of PEO was dissolved in 120 mL deionized water and put under stirring for 2 hours at a constant temperature of 50 °C in order to obtain a uniform transparent solution. Subsequently, an appropriate amount of the prepared V$_2$O$_5$ was added in PEO solution and kept under constant stirring for further 2 hours to obtain homogenous slurry. Finally, the solution mixture containing the V$_2$O$_5$-PEO composite was cast on to the Teflon petri and dried in a vacuum oven at 60 °C for 24 hours to obtained light yellow solid thin film.
Figure 5.11: (a) XRD pattern and (b) FESEM image of the synthesized V$_2$O$_5$.

The nonlinear saturable absorption of the V$_2$O$_5$-SA film was then determined using twin-detector technique with an illumination source of a homemade mode-locked fiber laser with a repetition rate of 15 MHz and pulse width of 0.9 ps centered at 1554 nm. The absorption at various input intensities was recorded and plotted as shown in Figure 5.12, after fitting using the saturation model in Equation 5.1. The saturation intensity and modulation depth of the V$_2$O$_5$-SA were determined to be 71 MW/cm$^2$ and 7%, respectively.

Figure 5.12: Nonlinear saturable absorption of the V$_2$O$_5$-SA film.
5.3.2 Q-switching Performance of the V$_2$O$_5$-SA Based Laser

The schematic configuration of the EDFL system with a V$_2$O$_5$-SA is as the same configuration used in the proposed model in Chapter 3. The total length of the cavity was measured to be about 8 m. A 350-MHz oscilloscope combined with a 1.2 GHz photodetector, an RF spectrum analyzer, optical power meter and an OSA with a spectral resolution of 0.07 nm were used simultaneously to monitor the output laser. Initially, the EDFL cavity started CW laser at 20 mW pump power. A stable Q-switched pulse was observed at pump power greater than 35 mW.

Figure 5.13(a) shows the output spectrum of the Q-switched EDFL at 165 mW pump power with center wavelength of 1565 nm and 3-dB bandwidth of 1.12 nm. The oscilloscope trace of the pulse trains which obtained by V$_2$O$_5$-SA at 165 mW is shown in Figure 5.13(b). It is observed that the typical pulse train shows stable Q-switching operation with no fluctuations. Figure 5.13(d) shows the single pulse temporal profile at pump power of 165 mW with FWHM of 5.6 μs. In order to confirm that the Q-switched pulses are attributed to the V$_2$O$_5$-SA, the FC fiber ferrule with V$_2$O$_5$ film was replaced with a common clean ferrule. In this case, no Q-switched pulses were observed on the oscilloscope even when the pump power was adjusted over a wide range. This has verified that the V$_2$O$_5$-SA was responsible for the Q-switching operation of the EDFL. Figure 5.13(c) illustrates the corresponding radio-frequency (RF) spectrum at fundamental frequency of 60 kHz, showing a SNR of 55 dB indicating that the Q-switched pulse is in a relatively stable regime.
Figure 5.13: (a) Output spectrum, (b) A typical pulse train, (c) a single pulse envelop, and (d) RF spectrum of Q-switched EDFL based V_{2}O_{5}-SA at pump power of 165 mW with 60 kHz repetition rate.

In order to study further the characteristics of the output Q-switched pulses, the repetition rate, pulse duration, and output peak power have been investigated as functions of the pumping power. When more gain was supplied for the SA saturation by increasing the pump power, the threshold energy stored in the cavity to generate a pulse was reached earlier hence, increasing the repetition rate and decreasing the pulse width as shown in Figure 5.14(a) and (b), which show agreement with the simulation results presented in Chapter 3, Figure 3.4. As well, gradually the output peak power increases with increasing the pump power, as shown in Figure 5.14(c) with agreement with Chapter 3 simulation results.
Figure 5.14: (a) Pulse duration, (b) repetition rate, and (c) output peak power versus pump power of Q-switched EDFL with V$_2$O$_5$-SA compared to the theoretical results of the numerical model introduced in Chapter 3, Figure 3.4.

It is found that the experimental results are in agreement with the theoretical results of the proposed Q-switched laser model in Chapter 3. The very little quantitative difference between the experimental and theoretical results is due to the following reasons; In the experimental setup a wavelength division multiplexer and isolator have been used which have not been taken into account in the simulation. In addition, some of the parameters values used in the theoretical model have been estimated (are not exactly equal to the real values) for V$_2$O$_5$ thin film which acts as Q-switcher.

5.4 Vanadium Oxide as Saturable Absorber for All-Fiber Passively Mode-Locked Erbium-Doped Fiber Laser

The performance of mode-locked fiber lasers depends on several factors of the cavity, which include fiber dispersion and nonlinearity, gain, loss of optical components, coupling efficiency of the output coupler, and the SA properties. SAs are widely used as mode-lockers. The main role of the SA is to support the initial formation of the pulse (self-starting) and stabilize the pulse (Jung et al., 1997; Kartner et al., 1996; Kurtner et al., 1998; Okhotnikov et al., 2004). In soliton regime where the total cavity dispersion is anomalous, the SA properties should have strong effect on multiple pulse operations suppression or the co-growth of a continuum wave. A larger modulation depth of SA should have more incentive for self-starting and single-pulse operation stabilizing.
However, larger modulation depth of saturable absorption usually goes with high nonsaturable absorption which might add more loss in the cavity and consequently reduces the laser performance (Jung et al., 1997). Therefore, the absorption of SA should be controlled carefully.

In this section, EDFL using a V$_2$O$_5$ film as SA is introduced. The V$_2$O$_5$ based SA is hosted into PEO film, which is then sandwiched between two fiber ferrules and incorporated into the laser cavity.

### 5.4.1 Mode-Locked Fiber Laser Setup

The experimental configuration of the EDFL system with V$_2$O$_5$-SA is illustrated in Figure 5.15. The EDF was core-pumped by a LD through WDM. A 2.4 m EDF was used as a lasing medium with 2000 ppm erbium concentration, an absorption coefficient of 24 dB/m at 980 nm, and the numerical aperture of 0.4. An ISO was used to ensure unidirectional propagation of light. A 20/80 output coupler was used for laser extraction. The total cavity length is around 201 m after adding 195 m SMF. The group velocity dispersions of the EDF fiber and SMF-28 fiber at 1550 nm are approximately 27.6 ps$^2$/km and -21.7 ps$^2$/km, respectively. In addition, the 0.5 m HI 1060 WDM used has -48.5 ps$^2$/km group velocity dispersion. The estimated net cavity dispersion is about -4.25 ps$^2$.

The output laser performance has been detected using an OSA (Yokogawa, AQ6370B) with a spectral resolution of 0.02 nm and a 350 MHz digital oscilloscope (GWINSTEK: GDS-3352) through 1.3 GHz photodetector (Thorlabs, DET10D/M).
5.4.2 Mode-Locked Laser Performance

Mode-locking operation was started at 81 mW pump power. Figure 5.16 shows the typical optical spectrum of the generated mode-locked pulses centered at a wavelength of 1559.25 nm and in a soliton regime due to the anomalous dispersion in the cavity as the estimated net cavity dispersion was around -4.25 ps².

![Figure 5.16: Output spectrum of mode-locked EDFL with V₂O₅-SA at 166 mW pump power.](image-url)
An autocorrelator and oscilloscope have been used to study the temporal profile of the mode-locked EDFL. Figure 5.17 shows the oscilloscope trace with a stable pulse train showing two adjacent pulses separated by around 1 µs which corresponds to 1 MHz repetition rate. The obtained pulse duration on the oscilloscope is not actual value since the resolution limitation in the oscilloscope. Therefore, an optical Autocorrelator was used to measure the pulse duration. Figure 5.18(a) indicates that the measured autocorrelator pulse trace has sech² pulse profile with FWHM of 3.14 ps. The pulse duration was measured using an optical autocorrelator (HAC-200) shown in Figure 5.18(b). The output pulse energy of the generated mode-locked pulse was found to be around 4.5 nJ.

![Oscilloscope train of mode-locked EDFL with V₂O₅-SA of 1.01 MHz repetition rate at 166 mW pump power.](image)

**Figure 5.17:** Oscilloscope train of mode-locked EDFL with V₂O₅-SA of 1.01 MHz repetition rate at 166 mW pump power.

![Autocorrelation trace of mode-locked EDFL with V₂O₅-SA at 166 mW pump power and (b) image of the used autocorrelator device.](image)

**Figure 5.18:** (a) Autocorrelation trace of mode-locked EDFL with V₂O₅-SA at 166 mW pump power and (b) image of the used autocorrelator device.
The degree of suppression of the adjacent harmonics has been measured to confirm the pulse stability via RF spectrum as shown in Figure 5.19. It shows that the fundamental frequency of the cavity is around 1.01 MHz and the adjacent harmonics are suppressed by 48.6 dB which confirms the stability of the pulse.

![RF spectrum of mode-locked EDFL with V_{2}O_{5}-SA with 1.01 MHz fundamental frequency at 166 mW pump power with 10 MHz span.](image)

**Figure 5.19:** RF spectrum of mode-locked EDFL with V_{2}O_{5}-SA with 1.01 MHz fundamental frequency at 166 mW pump power with 10 MHz span.

5.5 Nickel Oxide Nanoparticles as Saturable Absorber for All-Fiber Passively Q-Switched Erbium-Doped Fiber Laser

In this section, a passively Q-switched EDFL is demonstrated using NiO film as SA. Nickel oxide (NiO) has a good potential as an effective SA with an appropriate modulation depth and low saturation intensity as discussed below.

Z-scan technique on NiO films reveals nonlinear optical properties known as saturable absorption and two-photon absorption (Shablaev & Pisarev, 2003). Many researchers have studied the absorption of photon energy for NiO (Chigane & Ishikawa, 1998; Misho et al., 1988; Pramanik & Bhattacharya, 1990; Šurca et al., 1996; Utriainen et al., 1998;
Varkey & Fort, 1993; Xie et al., 1996). Reported band gap energy value for the NiO is in the range of 3.6 to 4.0 eV (Wang et al., 2015).

### 5.5.1 Synthesis of NiO Nanoparticles

NiO nanoparticles were synthesized by a facile sonochemical method (Duraisamy et al., 2016). At first, nickel chloride hexahydrate (NiCl$_2$.6H$_2$O) was dissolved in 50 ml of deionized water under persistent stirring for about 10 min. After that, a freshly prepared solution of sodium hydroxide (NaOH) was added into the above solution and subjected to sonication using a horn sonicator. The sonication was done using constant amplitude of around 60% for about 1 hour. The product was centrifuged and washed several times with deionized water. Finally, the obtained sample was calcinated at 350 °C for 3 hours in an electrical furnace. The entire synthesis process is shown in Figure 5.20.

![Synthesis of NiO Nanoparticles](image)

**Figure 5.20:** Sonochemical synthesis route for NiO nanoparticles.
5.5.2 Fabrication of NiO-SA

In order to easily integrate the material into a laser cavity, NiO/PEO composite film was fabricated based on solution casting technique. 1.0 gram of PEO was dissolved in 120 mL deionized water and put under constant stirring for 2 hours at a constant temperature of 50°C in order to obtain a uniform transparent solution. Subsequently, appropriate amount of the product of NiO nanoparticles was homogeneously dispersed in the PEO solution along with persistent stirring for 2 hours. Finally, the solution containing the NiO/PEO composite was cast on the Teflon petri and dried in a vacuum oven at 60 °C for 24 hours to prepare the solid film. Figure 5.21 shows the image of the fabricated thin film.

![Figure 5.21: Image of NiO-SA thin film.](image)

5.5.3 Characterization of NiO Nanoparticles

In order to analyze the structural crystallinity, XRD was performed as shown in Figure 5.22. Three diffraction peaks are observed at 37.28°, 43.28° and 62.88°, which are attributed to the (111), (200) and (220) planes respectively. The obtained crystalline planes correspond to the face centered cubic structure of NiO (Duraisamy et al., 2015). Figure 5.23 depicts the FESEM image. Flakes like morphology is observed with particle size of less than 50 nm.
Figure 5.22: XRD pattern of NiO nanoparticles.

Figure 5.23: FESEM image of NiO nanoparticles.

5.5.4 Nonlinear Optical Absorption Characteristics of NiO-SA

The nonlinear optical response of the NiO-SA film was determined using the same twin-detector technique shown in Figure 5.3. But, the illumination source which was a homemade mode-locked fiber laser had a repetition rate of 1 MHz and pulse width of 2.14 ps centred at 1560 nm. The transmission at various input intensities was recorded
and plotted as shown in Figure 5.24, after fitting using Equation 3.24. The saturation intensity and modulation depth of the NiO-SA were determined to be 0.025 MW/cm² and 39%, respectively.

![Figure 5.24: Nonlinear optical transmittance of the NiO-SA.](image)

**5.5.5 Q-Switched Laser Experimental Setup**

Schematic diagram of a passively Q-switched EDFL based on NiO-SA is shown in Figure 5.25. The laser cavity is mainly composed of a 3 m long EDF (Moritex PureCore MSFER123) as a gain medium and 5 m long SMF as the pigtails of the corresponding components. The EDF has core and cladding diameters of 4 μm and 125 μm respectively, with numerical aperture of 0.16 and erbium ion absorption of 23 dB/m at 980 nm. The EDF was pumped by a 980 nm LD via a 980/1550-nm WDM. An ISO was used to preserve the unidirectional laser operation. An 80:20 fiber coupler was used to extract a 20% intra-cavity power for the output. The NiO/PEO film was sandwiched between two fiber connectors via a fiber adapter to form a fiber-compatible NiO-SA, which is then integrated into the laser cavity for the Q-switching operation. The total cavity length of the ring resonator was measured to be about 8 m. A 350-MHz oscilloscope combined with a 1.2 GHz photo-detector, a RF spectrum analyzer, optical power meter and an OSA with a spectral resolution of 0.07 nm were used simultaneously to monitor the output.
Figure 5.25: Schematic configuration of Q-switched EDFL with NiO-SA.

5.5.6 Q-switching Performance of the NiO-SA

CW operation was begun at a low threshold pump power of 18 mW, and stable Q-switched pulses were attained when the pump power was increased to 25 mW. The Q-switching operation is maintained up to the maximum pump power of 95 mW. The typical optical spectrum of the Q-switched pulse at a pump power of 95 mW is illustrated in Figure 5.26, showing a central wavelength of the pulse at 1561.2 nm with a 3-dB spectral bandwidth of about 0.21 nm. The typical Q-switching pulse train with a pulse period of 19.2 μs was stable with no significant pulse jitter seen on the oscilloscope as shown in Figure 5.27(a). Figure 5.27(b) shows a single pulse envelope, which displays a symmetrical Gaussian-like shape with a FWHM of 5.2 μs. To evaluate the stability of the passive Q-switching operation, the RF output spectrum of the Q-switched pulses was measured with a span of 350 kHz. The result is shown in Figure 5.28, which indicates that the pulse repetition rate was 52.18 kHz, matching the pulse period of 5.2 μs. The SNR was about 55 dB, indicating good Q-switching stability.
Figure 5.26: Spectrum of the output of Q-switched EDFL with NiO-SA at pump power of 95 mW.

Figure 5.27: (a) A typical oscilloscope trace and (b) a single pulse envelop of Q-switched EDFL with NiO-SA at 95 mW pump power with 52.18 kHz repetition rate.
To confirm that the Q-switched pulses were attributed to the NiO based SA, the FC fiber ferrule with NiO film was replaced with a common clean ferrule. In this case, no Q-switched pulses were observed on the oscilloscope even when the pump power was adjusted over a wide range. This finding has verified that the NiO based SA was responsible for the Q-switching operation of the EDFL. In order to further study the characteristics of the output Q-switched pulses, the repetition rate, pulse duration, output power, and pulse energy with respect to the incident pump power were investigated. Figure 5.29 shows the pulse repetition rate and pulse duration of the Q-switched fiber laser as functions of the incident pump power. As illustrated in the Figure, it can be found that the repetition rate increases nearly monotonously from 19.57 to 52.18 kHz, while the pulse width decreased from 22.4 to 5.2 µs as the pump power is boosted from 25 to 95 mW. These are typical features of passive Q-switching. By increasing the pump power, more gain was supplied to saturate the SA. As a result, the threshold energy stored in the gain medium to generate a pulse was reached earlier hence, increasing the repetition rate while decreasing the pulse width. The Q-switched pulse output was stable and no significant pulse-intensity fluctuation was observed on the oscilloscope at every specific
pump power and repetition rate. Moreover, as plotted in Figure 5.30, the output power, and correspondingly calculated the single pulse energy are also measured. The output power increases from 0.21 to 1.68 mW on increasing the pump power from 25 to 95 mW. While the pulse energy grew fast in the initial stage, after the pump power was over 66 mW, the pulse energy became saturable evidently. The maximum pulse energy was about 32.2 nJ.

**Figure 5.29:** Repetition rate and pulse width of Q-switched EDFL with NiO-SA as functions of input pump power.

**Figure 5.30:** Average output power and single-pulse energy Q-switched EDFL with NiO-SA as functions of input pump power.
These results demonstrate that the NiO nanoparticles are a potential SA for Q-switched fiber lasers especially at a low pumping strength. Compared to other SAs, NiO based SA is generating pulses with narrower repetition rates than those generated by MoS$_2$ (Li et al., 2015; Luo et al., 2014), TiO$_2$ (Ahmad et al., 2016), and ZnO (Ahmad et al., 2016) based SAs, which are typically in the range of 10.6 to 173.1 kHz, 80.28 to 120.48 kHz, and 41.7 to 77.2 kHz respectively, but a wider range than MoSe$_2$ and black phosphorus based SAs, which typically range from 26.5 to 35.4 kHz and 6.98 to 15.78 kHz, respectively (Chen et al., 2015; Woodward et al., 2015).

5.6 Nickel Oxide Nanoparticles as Saturable Absorber for All-Fiber Passively Femtosecond Mode-Locked Erbium-Doped Fiber Laser

In this section, NiO thin film is proposed as SA in generating mode-locked EDFL. NiO-SA shows larger modulation depth of 39\% with 0.025 MW/cm$^2$ saturation intensity. Such high modulation depth supports the strong shortening of the pulse where the pulse duration is inversely proportional to the modulation depth $q_0$ according to Equation 4.49.

5.6.1 Mode-Locked Fiber Laser Setup

The schematic configuration of our EDFL system with NiO-SA is shown in Figure 5.31. The pump source is LD with emission centered at a wavelength of 980 nm. A piece of 3 m EDF (Moritex PureCore MSFER123) was used as a lasing medium with an absorption coefficient of 23 dB/m at 980 nm and the numerical aperture of 0.16. The EDF was core-pumped by the LD through WDM. A polarization insensitive isolator was used to guarantee unidirectional propagation of light in the laser cavity and suppress the Brillouin backscattering which induces the pulse instability. An output coupler was used to extract only 1\% from the laser. Apart from the active fiber and WDM, the rest of the fiber in the cavity is SMF-28 fiber. The total cavity length is around 208 m after adding 200 m SMF-28 fiber. The group velocity dispersions of the EDF fiber and SMF-28 fiber
at 1560 nm are approximately 27.6 ps²/km and -21.7 ps²/km, respectively. Also, -48.5 ps²/km group velocity dispersion of 0.5 m WDM (HI 1060). The net cavity dispersion was estimated to be -4.37 ps². The output laser performance was observed using an OSA with a spectral resolution of 0.07 nm and a 350 MHz digital oscilloscope via 1.2 GHz photodetector.

![Schematic configuration of mode-locked EDFL with NiO-SA](image)

**Figure 5.31:** Schematic configuration of mode-locked EDFL with NiO-SA.

### 5.6.2 Mode-locking Performance of the Laser

The Mode-locking operation of the laser was started at 98 mW pump power and became stable at around 100 mW pump power. Figure 5.32 shows the typical optical spectrum of the generated mode-locked pulses at pump power of 165 mW. The spectrum is centered at a wavelength of 1561.8 nm with a 3-dB spectral width of about 2.85 nm showing the generated pulse operated in a soliton regime corresponding to the anomalous dispersion in the cavity. Basically, a soliton fiber laser has a pulse duration of a few ps, but for more shorter pulse durations, the soliton periods become very short and the pulse
becomes unstable because of very high nonlinear phase shifts per round trip where the nonlinear phase shift per cavity round trip is inversely proportional to the pulse duration. In the proposed EDFL, the pulse spectrum shows Kelly sidebands as observed in the spectrum where the pulse duration is very short (about 950 fs). Kelly sidebands can happen because there is periodic disturbance in the cavity due to the discrete nature of dispersion and nonlinearity in which the soliton couples to the co-propagating dispersion wave, for some frequencies, the relative phase of soliton and dispersion wave changes by an integer multiple of $2\pi$ per cavity round trip and this leads to formation of narrow peaks superimposed on the soliton spectrum called Kelly sidebands.

![Output spectrum of mode-locked EDFL with NiO-SA at pump power of 165 mW.](image)

**Figure 5.32:** Output spectrum of mode-locked EDFL with NiO-SA at pump power of 165 mW.

The temporal profile of the mode-locked EDFL has also been investigated by using an autocorrelator (HAC-200) and oscilloscope. Figure 5.33 (a) shows the oscilloscope trace which views a stable pulse train with 0.96 MHz repetition rate while Figure 5.33(b) indicates that the measured autocorrelator pulse trace has sech$^2$ pulse profile and FWHM of 950 fs.
The minimum pulse duration can be calculated mathematically using time-bandwidth product formula (TBP) in Equation 4.43 which can be rewritten as

\[ TBP = \Delta t \Delta \nu \]

where \( \Delta t \) in (s) and \( \Delta \nu \) in (s\(^{-1}\)). To convert from \( \Delta \lambda \) in (nm), to \( \Delta \nu \) in (s\(^{-1}\)), the following formula is used,

\[ \Delta \nu = \frac{\Delta \lambda}{\lambda_c^2} c \]  

(5.2)

Here, \( \lambda_c = 1561.8 \) nm (the central wavelength), \( \Delta \lambda = 2.85 \) nm (the spectral width), and \( c \) is the speed of light. The pulse shape is considered as sech\(^2\), so the value of TBP will be 0.315. Therefore, the minimum possible pulse duration \( \Delta t \) will be \( \approx 900 \) fs. The value of pulse energy obtained at a pump power of 165 mW is of 1.14 nJ, although the used output coupler is a 99/1.

To investigate the laser performance and pulse stability, the degree of suppression of the adjacent harmonics or side modes was measured through RF spectrum as shown in Figure 5.34. It indicates that the fundamental frequency of our fiber laser cavity is approximately 0.96 MHz and the side modes are suppressed by about 43 dB confirming the stability of the pulse.
Figure 5.33: (a) Oscilloscope pulse train of 0.96 MHz repetition rate and (b) autocorrelation trace of mode-locked EDFL with NiO-SA at pump power of 165 mW.
Figure 5.34: RF spectrum with 20 MHz span of mode-locked EDFL with NiO-SA with 0.96 MHz fundamental frequency.

Experimentally, NiO-SA is found to be efficient mode-locker only in strong anomalous dispersion (-4.37 ps²) and mode-locking operation can be obtained only with this appropriate cavity length (additional 200 m long SMF-28 fiber). The Q-switching operation can be realized by removing the 200 m long SMF-28 fiber.

Here, numerical modelling and simulations proposed in Chapter 4 are applied in this laser cavity to study how efficient is NiO-SA as mode-locker and how far this study agree with the experiment. The parameters used in the simulation are listed in Table 5.1.

Table 5.1: Mode-locked NiO-SA cavity parameters used in the simulation.

<table>
<thead>
<tr>
<th>EDF</th>
<th>SMF</th>
<th>NiO-SA</th>
<th>Output coupler</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{AF} = 3$ m</td>
<td>$L_{pA} = 205$ m</td>
<td>$q_0 = 39%$</td>
<td>$R = 99%$</td>
</tr>
<tr>
<td>$\beta_2 = 27.6$ ps²/km</td>
<td>$\beta_2 = -21.7$ ps²/km</td>
<td>$\tau = 1$ ps</td>
<td></td>
</tr>
<tr>
<td>$\beta_3 = 150$ ps³/km</td>
<td>$\beta_3 = -120$ ps³/km</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma = 9.3$ 2.1 W⁻¹km⁻¹</td>
<td>$\gamma = 2.1$ W⁻¹km⁻¹</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$g_0 = 15$ dB/m</td>
<td>$\alpha = 0.2$ dB/km</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 5.35(a) shows that in a fiber cavity with cavity length of less than 100 m, the pulse is not stable and NiO SA does not support mode-locking operation. However, as the cavity length exceed 100 m, the SA was observed to support mode-locking and the pulse was formed as shown in Figure 5.35(b) and (c). Figure 5.35(c) shows the simulated mode-locked pulse envelope, which was obtained at 165 mW and 208 m cavity length. It is observed from the figure that output pulse energy is 1.21 nJ with around 0.5 ps pulse width. The observed experimental output pulse energy which was obtained using the same parameters was 1.1 nJ with pulse width of 950 fs.

The simulation results show that NiO-SA is not so efficient mode-locker in normal dispersion and support only mode-locking in a cavity with strong anomalous dispersion (appropriate cavity length > 100 m for the proposed cavity) and the experimental results shows the same but with appropriate cavity length > 200 m for the proposed cavity.

It is worth noting that the dispersions of 0.5 m HI1060 WDM and SA have been neglected in simulations. In addition, as mentioned above some parameters used in simulations are estimated which might affect the degree of agreement between experimental results and simulations. The change in saturable absorption \( \frac{dq}{dt} \) is negligible (the effect of NiO-SA recovery time \( \tau \) has been neglected) and Equation 4.50 was used in the simulations.

![Figure 5.35: Simulated pulse envelope for (a) 80 m cavity length, (b) 100 m cavity length, and (c) 208 m cavity length of mode-locked EDFL with NiO-SA at 165 mW pump power.](image)
5.7 Summary

In this chapter, Co$_3$O$_4$, V$_2$O$_5$, and NiO have been introduced as effective SAs in generation of pulsed fiber laser and the experimental results are summarized as follows:

- A passive Q-switched EDFL was established using Co$_3$O$_4$ film based-SA as a Q-switcher. This is the first demonstration in which a Co$_3$O$_4$ has been used as a SA in a Q-switched fiber laser. The Q-switching operation was successfully achieved with repetition rate increases from 29.8 to 70.92 kHz, and pulse width decreases from 10.9 to 5.02 µs. These results verify that the presented Co$_3$O$_4$ has nonlinear optical properties that make it advantageous for SA applications in cost-effective Q-switched EDFLs.

- Q-switched EDFL in a ring cavity using V$_2$O$_5$-SA has been demonstrated. The generated output pulses based on V$_2$O$_5$-SA whose nonlinear saturable absorption is characterized by modulation depth of 7% with saturation intensity of 71 MW/cm$^2$ have repetition rate ranging from around 20 kHz to 60 kHz, and pulse width decreases from 12 µs to 5.6 µs with high signal-to noise ratio equals about 55 dB when the pump power increases from 55 mW to 165 mW.

- Mode-locked EDFL in a ring cavity using V$_2$O$_5$-SA has been demonstrated. The generated optical pulses are centered at around 1559.25 nm and have a duration of 3.14 ps with a repetition frequency of around 1 MHz.

- A Q-switched EDFL was also successfully presented using a passive SA based on NiO nanoparticles which was synthesized via a facile sonochemical method. The SA film has modulation depth of 39%, and saturation intensity of 0.025 MW/cm$^2$. Stable passively Q-switched fiber laser pulses utilizing the NiO nanoparticles embedded in PEO film were successfully realized at 1561.2 nm with a low threshold pump power of 25 mW and minimum pulse duration of 5.2 µs at 95 mW.
The laser has a pulse repetition rate tunable from 19.57 to 52.18 kHz as the pump power increases from 25 mW to 95 mW.

- An ultrashort mode-locked EDFL in a ring cavity using NiO-based SA has also been successfully demonstrated. The generated optical pulses have a FWHM spectrum of about 2.85 nm centered at 1561.8 nm with a considerable energy of 1.14 nJ at 165 mW pump power through a 99/1 output coupler. The pulses have a duration of 950 fs with a repetition frequency of 0.96 MHz.

The performances of the obtained Q-switched and mode-locked fiber lasers show good prospects of Co₃O₄, V₂O₅, and NiO as excellently and effective SAs for the future and also comparable to other reported material SAs. Again, these new SA materials could contribute to SA family as new potential absorber materials in pulsed fiber laser technology and serve in ultra-fast photonic applications.

In our experimental work, it was found that both V₂O₅-SA, and NiO-SA were able to generate mode-locked laser pulse while Co₃O₄-SA was not able to generate mode-locked pulse and this might be due to the very low modulation depth and high non-saturable loss. Table 5.1 summarizes the SA materials used in our experimental work with comparative analysis with other materials already used as SAs in Q-switched fiber lasers.
Table 5.2: Comparative analysis for SA materials used in Q-switched fiber lasers.

<table>
<thead>
<tr>
<th>SA</th>
<th>Modulation depth/ Saturation intensity (MW/cm²)</th>
<th>Wavelength (nm)</th>
<th>Output power/ Pump power (mW)</th>
<th>Pulse width (μs)/Repetition rate (kHz)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Co₃O₄</td>
<td>0.35%/3</td>
<td>1561.6</td>
<td>2.15/165</td>
<td>5.02/70.9</td>
<td>Ahmad et al., 2016</td>
</tr>
<tr>
<td>V₂O₅</td>
<td>7%/71</td>
<td>1565</td>
<td>1.6/165</td>
<td>5.6/60</td>
<td></td>
</tr>
<tr>
<td>NiO</td>
<td>39%/0.025</td>
<td>1561.2</td>
<td>1.68/95</td>
<td>5.2/52.18</td>
<td></td>
</tr>
<tr>
<td>TiO₂</td>
<td>26.57%/0.2</td>
<td>1558.9</td>
<td>0.148/140</td>
<td>2.05/80.28</td>
<td>Ahn et al., 2016</td>
</tr>
<tr>
<td>ZnO</td>
<td>3.5%/0.016</td>
<td>1561</td>
<td>3.6/360</td>
<td>3/77.2</td>
<td>Ahn et al., 2016</td>
</tr>
<tr>
<td>Fe₃O₄</td>
<td>8.2%/25</td>
<td>1560</td>
<td>0.8/110</td>
<td>3.2/33.3</td>
<td>Bai et al., 2016</td>
</tr>
<tr>
<td>Bi₂Se₃</td>
<td>4.3%/11</td>
<td>1565.0</td>
<td>22.35/360</td>
<td>1.9/940</td>
<td>Yu et al., 2014</td>
</tr>
<tr>
<td>Bi₂Te₃</td>
<td>30%</td>
<td>1543.2</td>
<td>0.045/120</td>
<td>9.5/12</td>
<td>Chen et al., 2014</td>
</tr>
<tr>
<td>MoS₂</td>
<td>2.1%/129.4</td>
<td>1560</td>
<td>1.31/100</td>
<td>12.9/21.8</td>
<td>Chen et al., 2015</td>
</tr>
<tr>
<td>WS₂</td>
<td>2.5%/148.2</td>
<td>1560</td>
<td>6.41/650</td>
<td>4.1/71.2</td>
<td></td>
</tr>
<tr>
<td>WSe₂</td>
<td>3%/270.4</td>
<td>1560</td>
<td>3.16/500</td>
<td>4.8/66.8</td>
<td></td>
</tr>
<tr>
<td>Gold Nanorods</td>
<td>7.5%/35</td>
<td>1549.8</td>
<td>44.1/229</td>
<td>3.2/37.1</td>
<td>Koo et al., 2015</td>
</tr>
<tr>
<td>Silver Nanoparticles</td>
<td>18.5%</td>
<td>1564.5</td>
<td>7.77/139</td>
<td>2.4/58.5</td>
<td>Guo et al., 2016</td>
</tr>
<tr>
<td>PbS Quantum dot</td>
<td>6.4%/4.7</td>
<td>1562.7</td>
<td>19.4/128.7</td>
<td>3.3/24.2</td>
<td>Lee et al., 2016</td>
</tr>
<tr>
<td>ReS₂</td>
<td>0.12%/74</td>
<td>1557.3</td>
<td>0.4/120</td>
<td>5.5/19</td>
<td>Mao et al., 2018</td>
</tr>
<tr>
<td>SnS₂</td>
<td>3.15%/65</td>
<td>1532.7</td>
<td>9.33/637</td>
<td>0.51/233</td>
<td>Niu et al., 2017</td>
</tr>
<tr>
<td>BP</td>
<td>18.5%/10.7</td>
<td>1562.9</td>
<td>1.48/110</td>
<td>13.2/10.42</td>
<td>Chen et al., 2015</td>
</tr>
<tr>
<td>SESAM</td>
<td>18%/70μJ/cm²</td>
<td>1561.9</td>
<td>8.37/46.75</td>
<td>30/5.1</td>
<td>Wang et al., 2014</td>
</tr>
<tr>
<td>CNT</td>
<td></td>
<td>1555</td>
<td>0.23/45.8</td>
<td>7.35/17.1</td>
<td>Zhou et al., 2010</td>
</tr>
<tr>
<td>Graphene</td>
<td></td>
<td>1563</td>
<td>14.6/175</td>
<td>1.85/115</td>
<td>Sobon et al., 2012</td>
</tr>
</tbody>
</table>
6.1 Conclusion

In this thesis, the operations of Q-switched and mode-locked pulse generation in fiber cavity with saturable absorber (SA) have been thoroughly studied. This research involved both theoretical and experimental works. These works aim to develop numerical simulations to describe both Q-switching and mode-locking operations with incorporation of SA in erbium-doped fiber laser (EDFL) cavities as well as to introduce new SA materials that can effectively become Q-switcher and mode-locker for pulse generation.

Firstly, numerical simulations have been successfully developed to describe and analyze the physical mechanism of the Q-switching operation in fiber cavity based on SA. The presented numerical modelling and simulations are the first for EDFL in a ring cavity taking into account the SA dynamics. The simulation results were found to be in highly agreement with experimental results of a passively Q-switched fiber laser demonstrated using V_2O_5 thin film as SA. This could be highly useful for understanding and improving passively Q-switched EDFLs based SA approach.

Passively mode-locked EDFLs have also been numerically simulated based on SA. In this numerical study, the energetics and pulse properties for different fiber laser arrangements/configurations have been investigated. It was found that management of the intra-cavity components is the key to increase the pulse energy. In addition, the SA is the main component in pulse shaping compared to the fiber dispersion and nonlinearity. The third order dispersion is not effective and becomes necessary only when the wavelength is near to the zero-dispersion wavelength. Slow SA supports self-starting mode-locking and stable pulse formation provided that there is a balance between the fiber dispersion and nonlinearity, while fast SA is not ideal for self-starting mode-locking as it gives little
incentive for the initial formation of the pulse. In addition, the arrangement of the cavity components in the laser ring resonator affects the output pulse energy. This numerical study has taken into account the temporal change in the saturable absorption.

Finally, new SA materials have been successfully introduced and proposed for both Q-switched and mode-locked pulse generations. In this work, a Q-switched EDFL has been successfully demonstrated utilizing cobalt oxide (Co$_3$O$_4$) nanocubes film SA as a passive Q-switcher. Co$_3$O$_4$ nanocubes are embedded into a PEO film to produce a high nonlinear optical response, which is useful for SA application. It has saturation intensity and modulation depth of 3 MW/cm$^2$ and 0.35%, respectively. The proposed laser cavity successfully generates a stable pulse train where the pulse repetition rate is tunable from 29.8 to 70.92 kHz and the pulse-width decreases from 10.9 to 5.02 µs as the 980 nm pump power rises from 55 mW to 165 mW. Another passively Q-switched EDFL in a ring cavity has been established using V$_2$O$_5$-SA whose nonlinear characteristics were defined by 7% modulation depth and saturation intensity of 71 MW/cm$^2$. The generated pulses have repetition rate ranging from around 20 kHz to 60 kHz with pulse width decreasing from 12 µs to 5.6 µs. Mode-locked EDFL based on V$_2$O$_5$-SA has been successfully demonstrated. The generated optical pulses have centre wavelength of around 1559.25 nm with duration and repetition rate of 3.14 ps and 1 MHz, respectively. In addition, Q-switched EDFL was also established using a NiO nanoparticles synthesized via a facile sonochemical method as a SA. The SA film has modulation depth of 39%, and saturation intensity of 0.025 MW/cm$^2$. Stable passively Q-switched fiber laser pulses utilizing the NiO-SA embedded in PEO film were successfully realized at 1561.2 nm with a low threshold pump power of 25 mW, high signal to noise ratio of 55 dB, and minimum pulse duration of 5.2 µs at 95 mW. The laser has a pulse repetition rate tunable from 19.57 to 52.18 kHz as the pump power was increased from 25 mW to 95 mW. An ultrashort pulse mode-locked EDFL has also been obtained in a ring cavity using NiO based SA. The
generated optical pulses have 3-dB spectral width of about 2.85 nm centered at 1561.8
nm with a considerable energy of 1.14 nJ at 165 mW pump power via a 99/1 output
coupler. The pulses have a duration of 950 fs with a repetition frequency of 0.96 MHz.

The performance of the obtained pulsed fiber lasers shows good prospects of Co$_3$O$_4$,
V$_2$O$_5$, and NiO as effective SAs. These introduced new SA materials could contribute to
a SA family as a new potential material absorber in pulsed fiber laser technology and
could serve in ultrafast photonic applications.

6.2 Future Work

Fiber lasers have low repetition rate due to the relatively long length of the cavity. However, for various applications such as communications and for quantum experiments, high repetition rate is desirable. Currently, harmonic mode-locking is the most promising way to generate high repetition rates. Future efforts will be focused on the theory of high-repetition rate (few GHz) mode-locked cavities or harmonic mode-locking and experimentally developing a very stable passive harmonic mode-locked EDFL by manipulating the laser cavity.


LIST OF PUBLICATIONS AND PAPERS PRESENTED

List of Publications:


Paper Presented in International Conference: