

**A COGNITIVE MODEL OF YEAR FIVE PUPILS'  
ALGEBRAIC THINKING**

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**FACULTY OF EDUCATION  
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A COGNITIVE MODEL OF YEAR FIVE PUPILS' ALGEBRAIC THINKING

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## **Abstract**

This study was carried out to investigate cognitive constructs that could contribute to year five pupils' algebraic thinking and to reveal the web of connection between these constructs. In this study, proposed cognitive constructs are number sense, operation sense, symbol sense and pattern sense. The dependent variable was year five pupils' algebraic thinking. This study applied descriptive research method for data collection and analysis.

Seven hundred and twenty year five pupils from a district of Malacca took part in this study. The samples comprised both female and male pupils from rural and urban schools. Data were collected using two instruments, assessment of number, operation, symbol and pattern senses (ANOSPS) and algebraic thinking diagnosis assessment (ATDA). ANOSPS comprised 15 multiple choice mathematical questions, while ATDA comprised 17 short answer mathematical questions.

The data were analysed using structural equation model (SEM) to derive a model which could predict year five pupils' algebraic thinking. Hence, researcher adopted partial least square-structural equation model (PLS-SEM) method to carry out the evaluation process. Four direct effect relationships were hypothesized from independent variables to dependent variables. Moderating effect of gender and location were also tested. A two-stage approach was employed which involved the evaluation of measurement model followed by an evaluation of the structural model. Findings of this study revealed that number sense, operation sense, symbol sense and pattern sense significantly influence year five pupils' algebraic thinking. This indicated the proposed independent variables play crucial role in the development of algebraic thinking in primary school. The findings also indicated there is no difference in algebraic thinking performance between female and male year five pupils.

However, the algebraic thinking performance of urban school of year five pupils was better than rural school year five pupils. In addition, this study also revealed, symbol sense and pattern sense play mediator role between independent variables and year five pupils' algebraic thinking.

Lastly, the findings showed, gender doesn't moderate all of the hypothesized four direct relationships. Location does moderate two of the hypothesized direct relationships. The direct effect size of operation sense on rural school year five pupils' algebraic thinking is significantly higher than the urban school year five pupils. Followed by this, the direct effect size of symbol sense on rural school year five pupils' algebraic thinking is also significantly higher than the urban school year five pupils.

Findings of this study provided evidence that number sense, operation sense, symbol sense and pattern sense are important constructs in the development of algebraic thinking in primary school level. It also provided an awareness on what is algebraic thinking in primary school level and how the number sense, operation sense, symbol sense and pattern sense are intervened together. Algebraic thinking of primary school pupils can be enhanced by appropriate healthy classroom discussions while teaching arithmetic. Making sense of numbers, underlying properties of arithmetic, generalisation, and conceptual meaning of equal sign should be given more priority compared to getting a correct solution.

# **MODEL KOGNITIF BAGI PEMIKIRAN ALGEBRA MURID-MURID TAHUN LIMA**

## **ABSTRAK**

Kajian ini dijalankan untuk menyiasat pemboleh ubah kognitif yang boleh memberi kesan terhadap pemikiran algebra murid Tahun lima dan mendedahkan hubungan kait antara pemboleh ubah yang berkaitan. Dalam kajian ini, pemboleh ubah kognitif yang dicadangkan adalah peka nombor, peka operasi, peka simbol dan peka corak. Pemboleh ubah bersandar adalah pemikiran algebra murid tahun lima. Kajian ini menggunakan kaedah kajian deskriptif untuk mengumpul dan menganalisis data.

Tujuh ratus dua puluh murid tahun lima daripada satu daerah di Melaka mengambil bahagian dalam kajian ini. Sampel kajian mengandungi kedua-dua murid lelaki dan perempuan dari sekolah dalam bandar dan luar bandar. Data dikumpul menggunakan dua instrumen, penilaian peka nombor, operasi, simbol dan corak (ANOSPS) dan ujian diagnostik pemikiran algebra (ATDA). ANOSPS mengandungi 15 soalan pelbagai pilihan matematik, manakala ATDA mengandungi 17 soalan matematik pendek.

Data dianalisis menggunakan permodelan persamaan berstruktur (SEM) untuk menaksir model untuk meramalkan pemikiran algebra murid tahun lima. Oleh itu, pengkaji telah menggunakan kaedah model separa kuasa dua terkecil (PLS-SEM) untuk proses penilaian. Empat kesan hubungan langsung telah dihipotesiskan dari pemboleh ubah malar kepada pemboleh ubah bersandar. Kesan sampingan dari jantina dan lokasi juga diuji dalam kajian ini. Dua peringkat kajian digunakan untuk penilaian model pengukuran dan diikuti oleh penilaian model berstruktur. Dapatan dari kajian ini mendedahkan bahawa peka nombor, peka operasi, peka simbol dan peka corak memberi kesan yang signifikan terhadap pemikiran algebra murid tahun lima. Ia memberi indikasi bahawa pemboleh ubah malar yang dicadangkan memainkan

peranan yang penting untuk membangunkan pemikiran algebra di sekolah rendah. Dapatan juga memberi indikator bahawa tiada perbezaan pemikiran algebra dalam kalangan murid lelaki dan perempuan tahun lima. Walau bagaimanapun, pencapaian pemikiran algebra dalam kalangan murid tahun lima dalam bandar adalah lebih baik daripada murid tahun lima luar bandar. Sebagai tambahan, kajian ini juga mendedahkan bahawa peka simbol dan peka corak memainkan peranan sebagai perantara antara pemboleh ubah malar dan pemikiran algebra murid tahun lima.

Akhirnya, dapatan kajian juga menunjukkan jantina tidak memberi kesan sampingan kepada semua empat kesan langsung yang dihipotesiskan. Lokasi memberi kesan sampingan kepada dua hipotesis hubungan langsung. Saiz kesan langsung peka operasi ke atas pemikiran algebra murid tahun lima sekolah luar bandar adalah tinggi dan signifikan daripada murid tahun lima sekolah dalam bandar. Sehubungan itu, saiz kesan langsung intuisi simbol ke atas pemikiran algebra murid tahun lima sekolah luar bandar adalah juga tinggi dan signifikan dari murid tahun lima sekolah dalam bandar.

Hasil kajian ini memberi bukti bahawa peka nombor, peka operasi, peka simbol dan peka corak adalah pemboleh ubah yang penting dalam membangunkan pemikiran algebra di peringkat sekolah rendah. Ia juga memberikan kesedaran kepada persoalan apakah pemikiran algebra peringkat sekolah rendah dan bagaimana peka nombor, peka operasi, peka simbol dan peka corak berinteraksi bersama. Pemikiran algebra murid sekolah rendah boleh ditingkatkan dengan perbincangan dalam kelas secara sesuai semasa mengajar aritmetik. Penggunaan nombor secara munasabah, menggunakan asas aritmetik yang betul, generalisasi dan maksud konseptual tanda sama dengan sepatutnya diberi keutamaan berbanding mendapatkan jawapan yang betul.

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This work is dedicated to the memory of my father, Somasundram.

....Appa

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## List of Symbols and Abbreviations

ANOSPS	:	Assessment of number, operation, symbol and pattern senses
ATDA	:	Algebraic thinking diagnostic assessment
$f^2$	:	Unit for effect size
PLS	:	Partial Least Square
PLS- MGA	:	Partial Least Square-Multi Group Analysis
$R^2$	:	Coefficient of Determination
$Q^2$	:	Stone-Geisser's criterion
$q^2$	:	Effect size of $Q^2$
SEM	:	Structural Equation Modelling
TI	:	Tolerance Index
VAF	:	Variance Accounted For
VIF	:	Variation Inflation Factor

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## Chapter 1 Introduction

### Introduction

During the past few decades algebraic thinking has been portrayed as a centrepiece of mathematics education by researchers, educators and policy makers (Blanton & Kaput, 2003; Carraher & Schliemann, 2007; Jacobs, Franke, Carpenter, Levi, & Battey, 2007; Rittle-Johnson, Matthews, Taylor, & McEldoon, 2011). Algebra is generally characterised as the gateway to higher mathematics. Sadly, many students do not ride out this gateway successfully (Swangrojn, 2003). The struggle encountered by many students in the middle and high school mainly because of the blunt introduction of algebra (Kaput, 2008). Arithmetic and algebra are widely treated as two different disciplines (Herscovics & Linchevski, 1994). Arithmetic associated with primary level syllabus while algebra is associated with secondary school syllabus. Primary pupils deal with arithmetic questions from year one and as soon as they enter secondary school, they are exposed to variables, functions and expressions in formal algebra without any introduction. As in Malaysian curriculum, algebraic expression is introduced in chapter seven in the seventh grade (form one) and then bluntly shifts from arithmetic to algebra without any smooth transition. When the students first encountered algebra, which is in abstract form, it led to many difficulties for students who were only exposed to concrete reasoning throughout primary school years (Susac, Bubic, Vrbanc, & Planinic, 2014).

One of the main problems in algebra is the students' difficulties in understanding basic algebraic concept, that is the concept of variables (Lucariello, Tine, & Ganley, 2014; Warren, 2003b). If given an equation such as  $2x + 1 = 3$ , each and every one of us knows the value for  $x$  is 1. Students are taught to find the value

of  $x$  using procedures to bring 1 over to the right side, subtract it from 3 and divide the answer by 2. The major concern here is whether the students know why they are required to do so? What does  $2x + 1 = 3$  mean? How is it derived? What do  $x$  mean? Finally, do they know how to interpret the answer they have found? Students merely memorise the formula and apply it in the questions without knowing the underlying reason and concept. This situation was elaborated by Mariotti and Cerulli "... Algebra, and in particular symbolic manipulation, are conceived as sets of unrelated "computing rules", to be memorized and applied" (Mariotti & Cerulli, 2001, p. 3-343). Teachers will drill them by giving all possible ways of questions which can be asked in the examinations (Davis & Maher, 1990). As a result of excellent drilling, students will achieve outstanding scores in examinations. Same goes to functions. Students hardly know the concept behind  $f(x)$  (Bush, 2011). The poor performance of algebra has been reflected in TIMSS 2011 (Mullis, Martin, Foy, & Arora, 2012). Grade eight (Form two) Malaysian students' average achievement score in Algebra domain was 430. While our neighbouring country Singaporean students' average achievement score in Algebra was 614. Singapore has also been listed as one of the top-performing countries in both fourth and eighth grades (Mullis et al., 2012). Likewise, TIMSS 2015 results had also proven that Singapore is the best in Algebra with average score of 623. Meanwhile, Malaysia had the average score of 467 in Algebra (Mullis, Martin, Foy, & Hooper, 2016). Even though it is slight improvement compared to TIMSS 2011, it is not to deny Malaysia has long way to go in Algebra. Traditionally, algebra has been taught by memorisation, without a basis for algebraic thinking. Nevertheless, the question may arise on whether students know the meaning and connection to derive equation and find the value of  $x$ . This is where algebraic thinking comes into



play. Kieran (1996) defined algebraic thinking without even involving letter-symbolic.

Algebraic thinking can be interpreted as an approach to quantitative situations that emphasizes the general relational aspects with tools that are not necessarily letter-symbolic, but which can ultimately be used as cognitive support for introducing and for sustaining the more traditional discourse of school algebra (Kieran, 1996, p. 275)

She recommended few adjustments to be made in order to develop an algebraic way of thinking from primary school. They are “a) a focus on relations and not merely on the calculation of a numerical answer, b) a focus on operations as well as their inverses, and on the related idea of doing/undoing, c) a focus on both representing and solving a problem rather than on merely solving it” (Kieran, 2004, p. 140). Therefore, the need to foster algebraic concepts in early grades is evident.

More recently, educators and researchers have also noted the need to incorporate algebraic concepts into mathematics instructions starting from primary school (Brizuela & Schliemann, 2004). The motive for introducing algebra in primary school level is because students are only exposed to surface level subject understanding when they study algebra in the beginning of high school (Mason, 2008). Researchers' impression is that the elements of algebra underlie and connect many basic principles in early mathematics. They have highlighted that algebraic thinking in primary school can support how students structure their mathematics knowledge from the beginning.

Various researches have been conducted in examining young students' capability to think algebraically in primary school levels (Blanton & Kaput, 2004;

Houssart & Evens, 2003; Warren, Cooper, & Lamb, 2006). Houssart and Evens (2003) investigated responses of 11-years old students for questions involving patterns and classifying successful strategies and unsuccessful solutions. Their aim was to investigate children's performance on expressing generality. The authors provided the students patterning questions which only involved squares and circles. Then the students were required to figure out the total number of squares for given number of circles according to the pattern given in the question. The findings have shown that students successfully solved the questions when they represented the patterns in table forms. Students also used other strategies such as drawing and observing relationship. The students managed to provide correct answers using different strategies. Therefore, findings of this study illustrate good example to show simple generalisation technique which leads to algebraic thinking.

Blanton and Kaput (2004) focused at functional thinking capacity of primary pupils. Their study was aimed to investigate how pre-kindergarten students to grade five students are able to develop and express functional relationships. They analysed responses of students for the tasks involved analysing and developing functional thinking between the number of dogs and corresponding total number of eyes and tails. According to them, the students were capable of demonstrating functional thinking at an earlier age than expected. Hence, it was then concluded that primary school mathematics education should include and provide opportunity for students to exhibit functional thinking to encourage algebraic thinking.

Similarly, Warren, Cooper and Lamb (2006) also conducted a study to examine development of functional thinking among nine-year-old students. They used a teaching experiment method with lessons comprising function tables. This was to encourage students to develop mental representations by observing the relationship

between input and output numbers. These lessons were actually developed to provoke functional thinking in algebra but at the same time lessons were designed in such by just using arithmetic and without abstract function forms. Again, students demonstrated the ability in developing functional thinking and communicating their thinking both verbally and symbolically. Based on the literature evidence, it is not an exaggeration that young students are capable to think algebraically right from primary school level.

Although researchers have recognised the important role of algebraic thinking in primary school, the mathematics education field has not yet established a wider view to promote algebraic thinking contents at primary school level. Knowing the importance of algebraic thinking, it is crucial to seek the cognitive variables impacting the success of algebraic thinking to better prepare the students to do well in algebra as a prelude to understanding the higher mathematics in later stages of education. In the process of seeking cognitive variables contributing to success in algebraic thinking, researchers have examined the puzzle of what the predictors for accomplishment might be. Studies have been done on such things as the effects of math aptitude and prior achievement, attitudes and beliefs, self-efficacy, and demographic characteristics such as gender, ethnicity and age on performance in algebra and mathematics courses among countless other variables (Hahn, 2008; Lamie, 2014; Liu, 2010). However, there are no studies which have been conducted particularly to identify cognitive variables that might impact algebraic thinking in primary school level.

Even though the importance of algebraic thinking is highly emphasised in the research field of mathematics education, a student's algebraic thinking depends mainly on the experiences that they have gained in classrooms. Students do not develop all of their algebraic thinking and abilities by merely memorising concepts and carrying

out routine procedures in middle and high school. Teaching methods and classroom activities play an important role too (Blanton & Kaput, 2003). On the other hand, teaching methods and classroom activities cannot be improved without a systematic evaluation of students' algebraic thinking and proper identification of the cognitive variables which influence it.

For all of these reasons, the present study has taken a step ahead and in attempting to fill the gap by exploring cognitive variables that are associated with algebraic thinking of year five pupils. These constructs will be discussed further in chapter 2 (Literature review).

### **Problem Statement**

Most students have experienced difficulties in learning algebra when it was due to premature introduction of symbolic mathematical notation (Edwards, 2000). Many students did not understand the symbols, concepts and reasoning skills that are found in algebra (Spang, 2009). The undeniable fact is primary school mathematics mainly focused on numeracy and calculation skills (Van den Heuvel-Panhuizen, Kolovou, & Robitzsch, 2013). Thus, this traditional focus has led students' understanding in algebra to be lacking in seeing the connection and meaning of symbols (Swangrojn, 2003). Learning of basic algebra concepts at early stage is the foundation for understanding of higher level algebra concepts at later stages of education. This is aligned with objective of the Ministry of Education to create youths with higher order thinking skills in all aspects (Ministry of Education [MOE], 2014). However, we may still question how well algebra makes sense to primary pupils today.

Generally, arithmetic and algebra are separated in most of school mathematics curricula (van Amerom, 2002). Whereby arithmetic is often focused in primary school

mathematics, algebra has been given importance in the beginning of middle and high school. As such, this separation caused a cognitive gap between arithmetic and algebra which then made learning of algebra even more difficult for students in later years of school (Kieran, 2007). Early algebra does not mean teaching formal algebra in primary school. Early algebra refers to teaching arithmetic with underpinning algebraic thinking and reasoning. National Council of Teachers of Mathematics (NCTM) (2000) has also recognised importance of exposing primary pupils to algebraic thinking (National Council of Teachers of Mathematics [NCTM], 2000).

In Malaysia, algebra has been introduced formally in grade seven (Form one), after six years of primary education. This is not denying the fact that primary curriculum mathematics actually comprises some elements of algebra. The Malaysian curriculum text books contain basic word problems with missing subtrahend and addend. There is no denying the textbooks do expose students to unknown. At the end of arithmetic operations chapters, there is a small column which requires the students to think about "*anu*", which means unknown. In this section, simple word problems are provided and students are required to identify the unknown. They are not required to solve it. This seems to be a mild introduction to the algebra world. These activities are actually algebraic in nature as they provoke students' understandings on number properties and arithmetic operations (Gan, 2008). However, there is no evidence to show how much importance has given to discuss this section in the classrooms. It is questionable even the teachers are aware of the importance and the necessity of this section.

Incorporating algebra into early mathematics teaching in Malaysia may help to bridge the gap between Malaysia and top performing nations around the world. Students in Malaysia are below average internationally in mathematics, and lag behind

many other countries. Malaysia's rank dropped from 20<sup>th</sup> in 2007 to 26<sup>th</sup> in 2011 in mathematics based on the Trend in Mathematics and Science Study (TIMSS) in 2011 (Mullis, Martin, Foy, & Arora, 2012). The average mathematics score fell from 474 in 2007 to 440. From 1999 to 2011 achievement of Malaysia has been declining (Mullis et al., 2012, p.8). In TIMSS 2011, it was reported Malaysia was one of the countries with the greatest decrement in points having 40 points decreased or more. Malaysia has 2%, 12%, 36% and 65% of students in advanced, high, and intermediate and low respectively in international benchmark in year 2011 (Mullis et al., 2012, Table 2.20).

Malaysia was especially ranked 29 out of 42 participating countries in algebra, mathematical content domain. In TIMSS, Malaysian students' average score in domain of algebra in 2011 was 430 which was much lesser than the score in 2007 which was 455. In addition, Malaysian students' achievement in the algebra questions was also significantly lower than the average score of country for all the four content areas comprised in this international study (Number, Algebra, Geometry, Data and Chance). In TIMSS 2015, the achievement for Algebra was improved. The average scale score for Algebra (467) was even better than the performance in Geometry and Data and Chance. However, this score is still far from top performing countries' scores in Algebra which range from 600 and above (i.e., Singapore, Taipei, Korea). This position implies the seriousness to look into ways to improve algebra teaching and learning in Malaysia.

When discussing about the poor achievement of students in TIMSS, it is also important to take note of the factors affecting the achievement. TIMSS also reports performance differences based on gender and location. In TIMSS 2015, Malaysia's Form Two female students' scores were significantly higher than the male students'

scores in the content of algebra (Mullis et al., 2016). In TIMSS 1999, there was no any difference in the algebra achievement in terms of gender. It was also noted that this gender difference constantly occurred from the 2003 to 2015 TIMSS reports. From year 2003, female students began to outperformed male students. Literature has also provided evidence that gender has a strong association with mathematics achievement (Anjum, 2015; Ethington, 1992; Fennema & Sherman, 1977; Ismail & Awang, 2008). It is evident that gender difference in mathematics achievement among Malaysian students has increased over the last five years (MOE, 2013). It would be more appropriate if researchers considered the factor of gender to be included in their studies to gain more comprehensive view of gender and mathematics association.

Besides location, students' achievements in TIMSS were also influenced by school locations. Likewise, researchers have also provided similar findings that school location affects the achievement too (Haller, Monk & Tien, 1993; Lee & McIntire, 2000; Shuaibu, 2014). The most common contention is rural school students are always disadvantaged by isolated geographical location, inadequate funding from government and hesitation of teachers to relocate to rural areas to teach in rural schools. Thus, school locations often play an important role in determining the achievements of students in mathematics. This situation leads researchers to probe further on whether gender and location are the factors for students' poor performance in algebra? Based on various factors explored in the literature for the students' achievement in mathematics particularly in algebra has motivated the present study. It took an initiative to identify the cognitive curricular constructs which influence that algebraic thinking and the difference of this influence in terms of gender and location. The subsequent section highlights about the possible cognitive variables from previous studies that influence young students' algebraic thinking.

The countries that are ahead of Malaysia have standards aimed at thinking and structuring knowledge. Their standards encourage using reasoning and justification when solving problems. This helps students make cognitive connections between basic and higher-level concepts. For instance, Singapore's Mathematics Syllabus for early grades establishes a progression from all four arithmetic operations, from part-whole numbers to fractions, which builds slowly up from the most basic concepts of the earliest grade (Kieran, Pang, Schifter, & Ng, 2016). The syllabus focuses largely on student thinking around open problems to ensure students understand the underlying concepts by using bar drawing which is also known as Model Method (Cai, 2003). In the TIMSS 2015 study, Singapore was one of the strongest performing countries in both fourth and eighth grades.

Failure to focus on algebraic thinking will lead students to carry out mathematical procedures, without understanding the meaning and connections on how these procedures work and will end up in incorrect results interpretation. This lack of understanding of algebra will especially affect students' abilities to apply mathematical perspectives, concepts, and tools flexibly in real life and workplace in the future (NCTM, 2009). Business and industry require their employees to pose higher levels of thinking that go beyond those acquired in a formal course of algebra (Kieran, 1987). Acquisition of algebraic thinking will produce Malaysian citizens who can make wise decisions involved in their daily life such as managing their personal finances, selecting insurance or health plans. It will also produce workforce which can satisfy the increased mathematical needs in profession areas ranging from health care to small business (NCTM, 2009). In line with this concern, the present study had attempted to investigate the cognitive constructs that would encourage primary pupils' algebraic thinking. The selection of cognitive constructs was based on literature's



findings on potential curriculum based cognitive constructs that would influence primary pupils' algebraic thinking. The following section briefly discusses influential constructs that have been identified in the literature.

Slavit (1999) has drawn attention to operation sense in the development of algebraic thinking. There were 10 aspects defined by Slavit which can help in depicting students' operation sense and contribute to early algebraic thinking. With regard to this, Warren (2003) examined seventh and eighth grade students' understanding of commutative and associative laws and also how they represented these laws symbolically. It was found that majority of the students lacked mathematical structure notion and failed to recognise operations as general processes. Ability to examine the abstract relationships and principles are the fundamental key points in mastering algebra. Students can only acquire this ability if they are probed to examine the group properties and operations as general processes in upper primary school.

Secondly, van Amerom (2002) highlighted problems encountered by students in handling inversion and precedence of operations when they were first exposed to algebra is due to the lack of number sense. Making sense of numbers and properties would be a prerequisite to understand algebra as generalised arithmetic. In another case, Molina, Castro and Mason (2008) analysed 26 eight-year-old students' strategies while working with true/false number sentences to investigate the level of their relational thinking using six sessions of teaching experiment. Relational thinking refers to how the elements in a sentence are related and how the students establish the arithmetic structure. They found that some students who managed to solve the number sentences without calculation by just using relational thinking. When the students attempted by using relational thinking, they made use of number sense and operation

sense to solve it structurally rather than procedurally. For example, when a true/false number sentence given as  $75 - 14 = 340$ ; the students were able to identify the magnitude difference between numbers and also exhibited some knowledge of impact of operations. Hence, they explained it was false as the answer cannot be bigger than 75 when subtracting 14. In this case, it shows how number sense indirectly helps students to have some sense making while dealing with equations.

Concurrently Molina and Ambrose (2008) emphasised on the importance of sense making of symbols. They referred to “equal sign” as a symbol which has always been a stumbling block in analysing expressions in formal algebra. They asserted that conceptual understanding of equal sign would definitely promote algebraic thinking in primary school. Misconception of equal sign as "to do something" or "the answer" signal leads to problems when they are exposed to number sentence in the form of  $c = a + b$  (Molina & Ambrose, 2008).

Lastly, discussion of algebraic thinking in primary school will never be complete without mentioning working with patterns. Pattern-discovery tasks can promote students to generate conjectures about the rule that the creator of the series might have used in order to generate them. In fact, NCTM (2000) included “understanding patterns” from grade pre-K to grade 12. Exploration of visual growth patterns act as a basic approach in introducing algebraic thinking to work with functions (Warren et al., 2006).

These studies show the evidence that primary pupils’ algebraic thinking has been a major concern of mathematics researchers and they have investigated primary pupils’ algebraic thinking in various aspects. It also revealed that the researchers have identified a few cognitive aspects such as number sense, operation sense, symbol sense and pattern sense which would contribute to algebraic thinking. However, these

studies did not explicitly focus on the contribution of these cognitive aspects towards algebraic thinking. Even though importance of algebraic thinking in primary level is highly emphasised by mathematics researchers, to date there is no study to show variables that might be predictors of algebraic thinking in primary school level, especially from cognitive perspective.

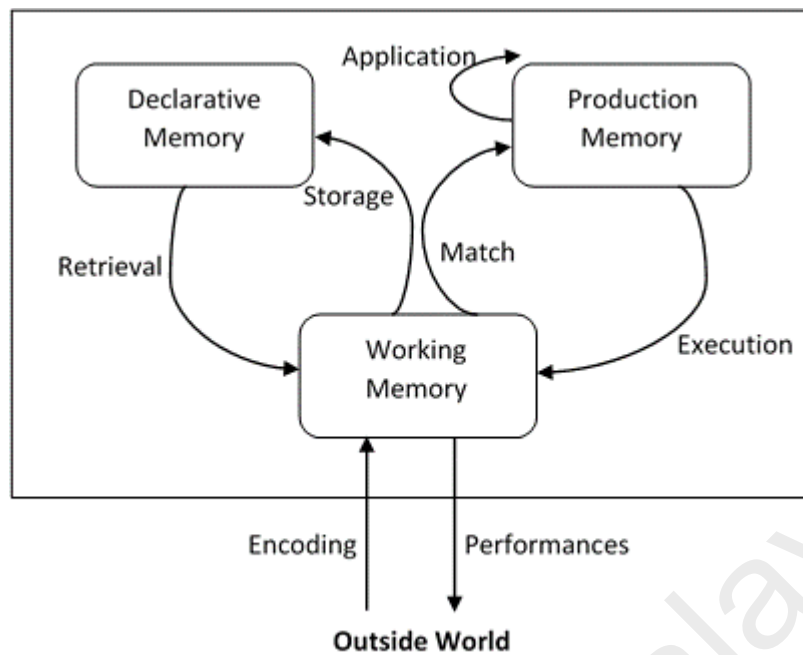
In the process of algebra thinking acquisition, it is crucial to explore the relationship between cognitive variables and algebraic thinking. Researchers as well as educators need to determine the key factors that cause the differentials in algebraic thinking so that they can concentrate their effort in overcoming students' limitations or weaknesses. While talking about this, it is also important to investigate the influence of cognitive variables towards algebraic thinking in terms of gender and location. As mentioned earlier, these two factors also determine the students' performance. Therefore, the researcher has investigated the cognitive variables that might contribute to algebraic thinking of year five pupils with regard to gender and location. It is clear that algebra is a major concern in mathematics education in Malaysia. Investigation on algebraic thinking may address the national concern over poor performance of Malaysian students in algebra high school and tertiary. Identification of the cognitive variables may assist in building strong early foundation in basic algebraic thinking which will lead students to perform well in algebra in later stages of education level.

### **Theoretical Framework**

Theoretical framework of present study is based on Anderson's (1983) ACT-R framework. It is based on information processing theory. ACT-R is acronym for Adaptive Control of Thought - Rational. It provides a framework for cognitive skill

development. Anderson (1983) incorporated declarative knowledge about facts and procedural knowledge about rules into his psychological model of memory. According to his theory, there are three stages in the process of learning namely, declarative stage, knowledge compilation stage and the procedural stage. Knowing verbal rules or facts regarding a task such as learning to do subtraction refers to declarative knowledge. Second stage is knowledge compilation which focuses on making information retrieval more efficient. In the third stage, it is based on condition-action pairs which are called productions. The newly acquired productions become tuned (Anderson, 1983).

Figure 1.1 shows besides declarative and procedural, a third component called working memory has been added. Commonly known as long term memory, it comprises general elements of knowledge which come under the declarative memory. On the other hand, short term memory is referred as the working memory which comprises volatile elements of knowledge. Lastly, compilation of productions is referred as procedural memory. Productions are “condition-action pairs that specify that if a certain state occurs in working memory, then particular...actions should take place” (Anderson, 1987, p. 193). In the present study, rules for handling arithmetic could be referred as productions while actions involved when discussing about the conceptual background of arithmetic would be declarative knowledge. For instance, the conceptual knowledge is about equal sign.



*Figure 1.1.* A general framework for the ACT-R production system, identifying the major structural components and their interlinking processes (Anderson, 1983, p. 19)

According to Anderson's model, the information produced by the environment then goes to the cognitive system through perception. Then it will be encoded and working memory will keep it. In the present study, this model is applied as students recognise the arithmetic operations which are actually encoded in the working memory. However, merely doing this will be meaningless. Hence the information in perception transmits to the declarative memory, where the operations will become a signal for arithmetical tasks. Due to the limited storage capacity of working memory, it leads to temporary storage of perception and enables faster retrieval. At the end, perceptions will be stored in declarative memory for longer duration time. They will be linked to other events and objects, whereby it forms the groundwork to retrieve complex information from the declarative memory.

For example, in figural pattern generalisation, first two or three figural patterns will be given and students will be required to find the  $n$ th or subsequent pattern. When

the first three terms of patterns were given and required finding of the subsequent pattern, this information will be transmitted to production memory via working memory. If the conditions for the pattern match with the subsequent pattern, initiation to figure out the subsequent pattern in the working memory will be activated by production memory. As a result of cognitive activity, the student will finally draw the subsequent pattern on a piece of paper. On the other hand, if the conditions failed to match with the pattern given, more information about sequence of the pattern in general and about the specific task is retrieved from the declarative memory and transmitted working memory and then to the production memory. Until reaching the solution, this process of information retrieval and matching condition-action pairs will be continued.

This model conceptualises algebraic thinking with regard to declarative and procedural components. The present study actually begins from pupils' calculation procedure performed in algebraic thinking diagnostic assessment (ATDA). Mistakes in the procedure signify the conceptual understanding deficiencies. According to Anderson (1983), commonly mathematics education begins with procedural knowledge and figure out the structure of declarative knowledge on this basis. This is most appropriate for the present study as it will assess students' current state of algebraic thinking and will reflect the declarative knowledge (arithmetic).

In the present study, researcher has made some assumptions to explore cognitive variables related to year five pupils' algebraic thinking based on this framework.

1. Year five pupils' algebraic thinking is based on how they process the knowledge of arithmetic.

2. The year five pupils' performance in Algebraic Thinking Diagnostic Assessment (ATDA) reflects their algebraic thinking.
3. Year five pupils have not learned the concept of formal algebra during this study.
4. Year five pupils have tried their best in the algebraic thinking diagnostic assessment (ATDA) and assessment of number, operation, symbol and pattern senses (ANOSPS), answer honestly in the assessments given.
5. There is no assurance that the instruments used for measurement of the variables in the study are accurate. The psychometric measures of the instruments and the use of structural equation modelling (SEM) should help to limit this validity threat.
6. It is assumed that the statistical tests which were used are suitable and possessed the required ability to detect differences in the variables if they are present.

These assumptions have helped the researcher to narrow down the scope of the study to facilitate the review process and facilitate the implementation of the study. These assumptions have also provided guidance to the researcher in data collection and analysing relevant data to answer the research questions and to interpret the results.

### **Purpose and Research Questions of the Study**

This study initiated with a purpose, to determine the cognitive variables (i.e., number sense, operation sense, symbol sense, and pattern sense) that are influential in year five pupils' algebraic thinking. Researcher has included measured variables that are related to each of these areas identified from the literature as influences on primary pupils' algebraic thinking. The objectives of present study are stated as below:

- 1) To determine the year five pupils' performance in algebraic thinking.
- 2) To determine if the hypothesized model valid for year five pupils' algebraic thinking.
- 3) To investigate if the proposed cognitive variables contribute to year five pupils' algebraic thinking.
- 4) To examine role of mediating variable(s) in determining year five pupils' algebraic thinking.
- 5) To examine the relationship between proposed cognitive variables and year five pupils' algebraic thinking in the final model.
- 6) To examine moderating effects of gender and location on year five pupils' algebraic thinking.

In line with the objectives of this study, this study intended to answer the following research questions:

- 1) What is the year five pupils' performance in algebraic thinking?
- 2) Is the hypothesized model valid for year five pupils' algebraic thinking?
- 3) To what extent proposed cognitive variables contribute to year five pupils' algebraic thinking?
- 4) Is there any construct(s) which acts as a mediator in the hypothesized model?
- 5) What is the relationship between proposed cognitive variables and year five pupils' algebraic thinking in the final model?
- 6) Is there moderating effects of gender and location on year five pupils' algebraic thinking?



## **Definition of Terms**

The following definitions will be used for the present study:

**Cognitive model.** The aim of present study is to derive a cognitive model of year five pupils' algebraic thinking. Definition of the cognitive model may differ based on various research areas. Generally, the cognitive model is concerned with how basic cognitive science processes such as learning, remembering, predicting, planning, thinking, and decision making interact (Busemeyer & Diederich, 2009). In the present study, the independent variables comprised curriculum based cognitive constructs such as number sense, operation sense, symbol sense and pattern sense which involve sense making of numbers, operations, symbols, and patterns. These constructs require cognitive science processes mentioned earlier such as learning, remembering, predicting, planning, thinking, and decision making interact.

**Algebraic thinking.** Year five pupils' algebraic thinking is the dependent variable of present study. Algebraic thinking of year five pupils is characterized based on Kaput's (2008) definition. Ability to work with three strands namely generalised arithmetic, modelling and function referred as algebraic thinking in the present study. This variable is measured by year five pupils' achievement in ATDA.

**Modelling.** This variable is measured by the ability to solve the arithmetic-based items which contain ordinary number sentences with letters to represent unknowns in terms of open number sentences and equivalence. These items also aimed to look into the participants' perspective on equal sign's essence.

**Generalised Arithmetic.** This variable is measured by the ability to simplify calculations using number properties like property of zeroes and ones. It also refers to utilizing operation properties like the commutative property, associative and distributive property.

**Functions.** According to Kaput (2008), function is “the study of functions, relations, and joint variation” (p. 11). The present study encompasses two components; a) numerical patterns, b) figural patterns. This variable is measured by the ability to work with numerical and figural patterns in terms of building subsequent terms, building rules and treating it as generalised relationships.

**Generalisation.** This term is used in two different aspects in the present study. First aspect refers to year five pupils’ ability to grasp and demonstrate the understanding on general mathematical properties such as associative, commutative, distributive, and also properties of odd and even numbers. Second aspect is pattern generalisation, whereby it refers to year five pupils’ ability to predict or exhibit “some form of regularity: a ‘rule’ of sorts could be used to define that grouping of numbers, shapes or figures” (Ralston, 2013, p. 26).

*Near generalisation (patterns).* This refers to the year five pupils’ ability in figuring out the subsequent number or figure when given a sequence of numbers or figures. As an example, if the first three numbers were given in a sequence of numbers, the students should be able to figure out the fourth number. Likewise, students should be able to find the subsequent figure based on the first few terms figures given in a series of figures.

*Far generalisation (patterns).* This refers to year five pupils' ability in figuring out the tenth or 15<sup>th</sup> number when first three numbers were given in a sequence of number pattern. They should not attempt to figure out subsequent numbers (i.e., fourth, fifth, and so forth) in order to get 15<sup>th</sup> term. Likewise, they should be able to generate the tenth or eleventh figure when first the figures were given in a figural pattern without looking for subsequent figures.

**Number sense.** Hsu, Yang and Li (2001) defined number sense into five components based on the previous literature. This study used those five components to measure number sense. This variable is measured by the ability to work with a) understanding number meanings and relationships; b) recognizing the magnitude of numbers; c) understanding the relative effect of operations on numbers; d) developing computational strategies and being able to judge their reasonableness; and e) having ability to represent numbers in multiple ways.

**Operation sense.** In the present study, operation sense encompasses direction of change which involves only addition and subtraction Haldar (2014). The ability to identify addition and subtraction are inverse of each other. This aspect involved to dimensions namely symmetric and asymmetric. Symmetric refers to "...addends and subtrahends that are equal to one another" (Haldar, 2014, p. 22). While, asymmetric refers to "students need to reason with unequal addends and subtrahends, which requires them to compare the magnitudes of the addend and subtrahend to determine how the initial number changes" (Haldar, 2014, p. 22).

**Symbol sense.** Arcavi (1994) described symbol sense as "an individual's ability to understand how and when symbols can and should be used to display relationship and generalisations" (p.31). From the perspective of algebra, two symbols are inevitable. They are equal sign and letters (commonly used to represent variables).

*Equal sign.* In the present study, equal sign refers to the mathematical symbol ( $=$ ) which used to indicate equality. It is measured by the conceptual understanding of equal sign.

*Variables.* This is measured by the ability to find the value for the unknowns. Literal symbols (e.g.,  $x$ ,  $y$ , and  $z$ ) are used to represent variables. They serve a variety of roles in mathematics, especially in algebra.  $a$  means for expressing generalised arithmetic (i.e.,  $a + 0 = a$ ),  $a$  means for representing an "unknown" number, an argument of a function, and a constant. The present study used symbols such as  $\nabla$ , and  $\diamond$  in the instrument items to represent unknown quantities.

**Pattern sense.** The ability to work with number and figural patterns is measured by this variable. This means the pupil should able to generate subsequent term from a given pattern series, able to see the general relationship and ability to find any arbitrary terms. The present study includes "growing or irregular pattern, which 'grows' in an irregular but yet generalizable way" (Gan, 2008, p. 17).

*Number patterns.* Numerical irregular patterns, which increase or decrease over time in some predictable way. For an example 2, 4, 6, 8.... or 80, 77, 74, 71, 68....

*Figural patterns.* Figural irregular patterns, "growing or irregular pattern, which 'grows' in an irregular but generalizable way" (Gan, 2008, p. 17). For an example  $\square$   $\square\square$   $\square\square\square$ .

## **Limitations and Delimitations**

This study comprised some limitations with regard to the research design, data collection, sampling technique and theoretical framework. Meanwhile there are also some delimitation related to the algebraic thinking, mathematical content, and settings.

**Limitations.** The first limitation is related to the research design. The present study has utilised quantitative research design and data were analysed using SEM. Even though SEM has become a very popular data-analytic technique, it has several limitations too. One of them is the statistical tests results might be less related than other types of techniques such as ANOVA (Kline, 2011). SEM enables a higher-level perspective to the analysis by allowing entire model evaluation. Although representation of individual effects in models might be the interest of researcher, at final point, one should decide about the whole model whether to accept or modify. Hence, the view of the whole model has more importance than individual effects in SEM.

Another possible limitation is computer estimate statistical significance such as  $p$  values for effects of latent variables. This estimate could be a little different when a different estimation algorithm is used or when different computer tools used (i.e., SmartPLS, WarpPLS) for the same analysis and data set. AMOS, for instance, uses factor loadings for convergent validity however SmartPLS uses AVE (Average Value Extracted) value.

Secondly, data collection procedure was carried out using two instruments which actually look like a mathematics test. The two instruments were administered on the same day in order to locate same students rather than conducting them on two separate days. However, administration of two instruments on the same day might lead pupils to pay less attention in responding the two assessments due to the urgency

to complete it on the same day. It might also lead to students' exam fatigue. Additionally, it ensures same student's response for both assessments. There is another limitation in one of the instruments. ANOSPS test was carried out through paper and pencil written test. Each item in ANOSPS comprises answer and reasons section (discussed further in chapter 3). The pupils were required to select an answer and the most suitable reason for their answers. There is a possibility for the pupils to choose the reason first and then guess the answer based on the reason.

Thirdly, due to the cluster sampling technique external validity could be threatened. The results of present study may not be suitable to generalise to other geographic locations whereby students may be from entirely different cognitive background (i.e., eastern states such as Sabah and Sarawak). Reduction of precision is the main drawback of cluster sampling technique. Hence, generalisation of results is limited to similar groups who are from same general geographic area with similar demographics.

**Delimitations.** The first delimitation of the present study is associated with mathematical construct which is algebraic thinking. The present study examined the thinking of year five pupils from the perspective of cognitive variables contributing to the success of algebraic thinking. Based on the data collected through algebraic thinking diagnostic assessment (ATDA), the researcher can only make assumptions about the year five pupils' thinking because it is impossible for the researcher to know what exactly is in the mind of students. Other aspects of learning such as learning using technology, reading ability and proficiency in counting are not involved in the study. The present study has focused only on the three strands of algebraic thinking classification by Kaput (2008) and four possible predictors of algebraic thinking. The

strands are generalised arithmetic, modeling, and functions and the possible predictors which are number sense, operation sense, symbol sense, and pattern sense. There are many other factors in the literature that were not considered in this present analysis, such as learning styles, working memory, short term memory, family composition, SES, language spoken and reading ability. In addition, this study was only limited to students in national schools from a district in Malacca. The rest of the states were not included due to cost and time constraint. The results also might be different if compared to vernacular schools.

### **Significance of the Study**

Discovering cognitive variables is necessary for a teacher to identify and recognize at earlier stage of education, as students with alternate conception may constantly face difficulties with formal algebra throughout the school years (Ralston, 2013). A significant body of research indicated that algebraic thinking is an essential contributor of success in algebra. Nevertheless, not many studies have been conducted to determine whether or not primary pupils, especially upper primary pupils are capable of these necessary skills and also conceptual understanding of relationship between algebraic thinking content and processes. Investigating combination of all these constructs in relation with each other together contributes to a more comprehensive view and enables an understanding of year five pupils' algebraic thinking. This may raise awareness among mathematics educators of what type of aspects which influence year five pupils' algebraic thinking prior to formal algebra exposure.

The present study will contribute to mathematics education field on the interrelationships of algebraic thinking and associated cognitive variables. One of the

focuses of this study was to clarify the web of connections among primary pupils' algebraic thinking and cognitive constructs which had been identified through the literature search. As these constructs were believed to influence algebra achievement in later years of education, little is known about their connections with primary pupils' algebraic thinking. Practically, in the literature these constructs were examined independently by teaching experiment or clinical interview to show that young students are capable to think algebraically. These studies only focused on only one aspect of algebraic thinking. The present study combined all of these variables in whole to see a model to foster algebraic thinking at early stage of education.

The final model could enable the teachers to plan their instructional practices according to the need of their students. Structural equation models (SEM) would be the best recommendation to identify and examine these complex relationships. As it could produce a coherent overall best fit model to show the connection between the constructs and its link to year five pupils' algebraic thinking, it may shed some light for teachers to identify the algebraic aspects underpinned in arithmetic. The model also shows the cognitive variables that help to foster primary pupils' algebraic thinking.

In addition, this study has informed educational policy makers on algebraic thinking aspects that are effective in promoting primary pupils' algebraic thinking. It could be useful for policy makers to look into these attributes when organising their syllabus and related assessments. Similarly, the findings may also inform the educational policy makers to include algebraic thinking supporting materials in curriculum for better algebra achievement which is the backbone of technology and science. Furthermore, the findings might provide necessary new aspects for



researchers in a way to consider algebraic thinking and associated cognitive variables as a whole and their interaction with each other.

## **Summary**

This chapter has provided an overview of the present study. It has discussed the importance of algebra in school, tertiary level and working environment. However, due to the obstacles faced by many students in grasping algebra in middle and high school level, this chapter has proposed the need of early algebra and fostering algebraic thinking via arithmetic tasks starting from primary school level. Subsequently, this chapter has presented the underpinning theoretical framework for the present study. The purpose of the study justified the selection of the theoretical framework. The research questions clearly guided the objective of this study. Significance of the study has highlighted the contribution of the present study to help the process of arithmetic to algebra transition with ease in primary school students. Preceding section has provided definitions for some important terms used in the present study and touched little on limitations and delimitations of the study. The following chapter will discuss a review of further related past literatures on early algebraic thinking, conceptual framework, constructs and methodology of this study in detail.

## Chapter 2 Literature Review

### Introduction

This chapter includes an in-depth literature on conceptual framework on which this study is based and literature review which provides evidence on the necessity of this study. The importance of algebra in mathematics education and concerns in mathematics education about young students' algebraic thinking are reviewed. This chapter has reviewed algebraic thinking in two aspects. Firstly, content of algebraic thinking assessment based on generalised arithmetic, modeling, and functions are discussed. Secondly, cognitive variables which might contribute to algebraic thinking are discussed. This is where discussion of number sense, operation sense, symbol sense, and pattern sense might influence algebraic thinking. Finally, a summary provides gaps between related literature and the need for current study.

### ACT-R

Theoretical framework adopted in the present study is based on Anderson's ACT-R framework. ACT-R is acronym for Adaptive Control of Thought- Rational. The three stages of ACT-R framework explain in detail on the transition of students' arithmetic knowledge to algebraic knowledge. Arithmetic could be defined as working with straight-forward calculations with known numbers (van Amerom, 2003). According to van Amerom (2003), a calculation that begins with known numbers and proceeds directly to unknown is called as arithmetic. In contrast, reasoning about unknown or variable based on unknown, via the known, to the equations is known as algebra. Therefore, arithmetic differs from algebra with regard to the fact it deals with a specific (arithmetic) or general (algebra) situation (van Amerom, 2003). This view

is also consistent with Usiskin's (1997) two important conclusions about algebra. They are a) algebra is the most suitable tool for expression of arithmetic general properties, and b) "...algebra supports the arithmetic at every juncture; it is not separated from it" (p. 356).

The present study has viewed transition from arithmetic to algebra through Anderson's transition of declarative to procedural knowledge via three stages (Anderson, 1983). The three stages are declarative, associative stages and proceduralization. Information will be stored as facts in the declarative stage for which there are no ready-made activation procedures. Followed by this is the associative stage. Due to its difficulty in using declarative knowledge which has raw information, the learner attempts to sort out the information into more efficient production sets by means of 'composition' by collapsing several discrete productions into one, and 'proceduralization' where by applying a general rule to particular instance. This applies to present study as declaration knowledge (declarative stage) is student's knowledge about arithmetic facts such as addition, subtraction and how to perform it. These are known as facts that will be stored in declarative memory. Procedural knowledge would be how students retrieve (associative stage) back to the arithmetic knowledge facts from long term memory and apply it into algebraic situation (proceduralization). For an example, arithmetic question would be when pupils required to find  $n$  when  $5 \times 7 = n$ . Pupil will retrieve the fact (from declarative memory) that,  $5 \times 7 = 35$  as they have memorised multiplication table, hence  $n$  is 35. However, if the same question asked what number can be replaced to make the statement true would be treated as algebra question (Usiskin, 1997). In this scenario, student's procedural memory would produce 'if-then' statement to get what number

would make the number sentence true. A conceptual change is required in pupils' learning process as they progress from arithmetic to algebra.

Declarative knowledge can have a negative effect on behaviour (Anderson, 1982; 1983). If a learner obtained knowledge incorrectly or not processed correctly, an incorrect procedure can be performed. Children's equal sign interpretation would be a good example to explain this because equal sign is widely observed as important for success in algebra (Knuth, Stephens, McNeil, & Alibali, 2006; Powell & Fuchs, 2010). According to Kieran (1981), the equal sign (=) is a relational symbol, signifying equality of both sides and indicates the interchangeability of both numbers. Despite this relational view, pupils view the equal sign operationally, meaning "add up the numbers" or "the answer" (McNeil & Alibali, 2005). Viewing equal sign operationally may lead children to compute traditional arithmetic problems and derive correct solution (i.e.,  $3 + 5 = \underline{\quad}$ ), but they will fail to succeed in solving equations which are more complex (Byrd, McNeil, Chesney, & Matthews, 2015). Lack of relational thinking lead the children to think the algebraic principle of maintaining equality is insensible and they begin to memorise many arbitrary rules to transform equations (Jacobs, Franke, Carpenter, Levi, & Battey, 2007). Therefore, it is important ensure learner obtained knowledge correctly.

There are several aspects which have been considered in the selection of ACT-R framework which is based on information processing theory as a theoretical framework of the present study compared to other theories. The subsequent section provides the justification for the selection of ACT-R framework from the aspects of samples, data collecting procedures, and data analysis.

From the aspect of samples, ACT-R framework assumes knowledge possessed by mathematics students such as the ability about numbers, belongs to the students'

natural ability and they can build a mental representation that depicts or corresponds to the existing world of reality ontology. In this context, this theory considers the student (i.e., year five pupils) as a tool to process information and all activities (i.e., algebraic thinking questions) can be represented mathematically in a computer language accurately and formally (Nik Azis, 1999). In the present study, the researcher is not focusing on how the knowledge is built. In other words, the present study is not intended to focus on how year five pupils acquired algebraic thinking. Rather it has focused on the measurement of year five pupils' algebraic thinking at current point of state.

In terms of data collection procedures, ACT-R framework has emphasised on how learning occurs while a question is being solved. In addition, the measurement of achievement (i.e., ATDA score) should be based on our knowledge of learning and the course of acquisition of competence in the subject matters that one teaches (Glaser, Lesgold, & Lajoie, 1985). As the researcher has mentioned earlier, the present study is not interested on how year five pupils build algebraic thinking. Thus, learning processes and fact retrieval from long term memory are important to solve the algebraic thinking questions. As such, ATDA score revealed the ability to retrieve fact from long term memory (procedural memory).

In terms of data analysis, ACT- framework only focuses on cognitive processes of students learning mathematics without focusing on providing evidence for mathematical knowledge possessed by students and how they think (Nik Aziz, 1999), unlike radical constructivism, the main focus is on form of knowledge and how students construct it. The radical constructivist position focuses on the individual's construction, hence taking a cognitive or psychological perspective. On the other hand, social constructivists believe that knowledge production is a result of social

interactions. They see higher mental processes as socially mediated. From a social perspective, knowledge resides in the society, which is a system that is greater than the sum of its parts. Thus, the difference lies between the individual construction of knowledge and the knowledge constructed by socially-mediated processes. This would not be parallel with the purpose of the present study. The present study does not collect data on socially-mediated processes. In other words, this study does not focus on the form of knowledge and how students construct it. Overall, ACT-R framework which is based on information processing theory suits best the need of present study compared to other learning theories such as radical constructivism, and social constructivism.

### **Algebra**

Many may consider algebra is very challenging and view it as an unnecessary discipline with little value in their day-to-day lives. However, one should be aware that there are many benefits in mastering it, as algebra is very well known as a gateway to higher education and job opportunities. Algebra is a prerequisite for many studies, such as medicine, engineering, banking, information technology and the social science fields. To be highly successful in today's society and technologically oriented world, it requires the algebraic thinking innate in it. Often algebra is considered as "gatekeeper" in many fields in working environment (Kaput, 2008). NCTM (2000) has been promoting algebra for all students. Locally, this movement has gained further attention with the release of TIMSS 2011 report. The most significant concern is the poor performance of Malaysian students in international assessments compared to other countries. Prior to looking into what can be done to improve the performance of

algebra, the following section will discuss what algebra is according to some mathematicians.

Numerous different descriptions of algebra can be found in the body of literature. For instance, Usiskin (1988) came up with four conceptions of school algebra; a) “algebra as generalised arithmetic (i.e.,  $a + b = b + a$ ), b) algebra as a study of procedures for solving certain kinds of problems (i.e.,  $5x + 3 = 40$ ), c) algebra as the study of relationship among quantities (i.e.  $y = 11x + b$ ), and d) algebra as the study of structures (i.e., factor  $3x^2 + 4ax - 132a^2$ )” (pp. 11-15). Kaput (1995) classified algebra according to five aspects; a) generalisation and formalization, b) syntactically guided manipulations; c) the study of structure, d) the study of functions, relations and joint variations, and e) a modelling language. According to him, generalisation, formalization and syntax manipulation are ones that underlie all others.

On the other hand, Kieran (1996) classified school algebra based on the students’ activities; a) generational activities, b) transformational activities, and c) global meta-level activities. Developing expressions and equations to represent problem situations or generalities refers to general activities. Rule-based activities such as collecting like terms, factoring, and simplifying make reference to transformational activities. The important aspect in this activity is maintaining the equivalence despite transformation of form. Finally, problem solving, modelling and proving activities, which algebra plays a role as a tool, are referred as meta-level activities. However, Lee (2001) characterized algebra as: algebra as a language; algebra as a way of thinking; algebra as a problem-solving activity; algebra as a tool for making thinking more effective and for carrying and transmitting messages; and algebra as generalised arithmetic.

The present study takes the view that algebra is a way of thinking. Thus, year five pupils should have the ability to develop an algebraic way of thinking when working within arithmetic. They should be able to see the relational aspects of operations without focusing on calculation and correctness of solution. With regard to this Kieran (2004) has highlighted the algebraic way of thinking as follows:

1. A focus on relations and not merely on the calculation of a numerical answer;
2. A focus on operations as well as their inverses, and on the related idea of doing undoing;
3. A focus on both representing and solving a problem rather than on merely solving it;
4. A focus on both numbers and letters, rather than on numbers alone. This includes:
  - (i) working with letters that may at times be unknowns, variables, or parameters;
  - (ii) accepting unclosed literal expressions as responses;
  - (iii) comparing expressions for equivalence based on properties rather than on numerical evaluation;
5. A refocusing of the meaning of the equal sign (pp. 140-141).

These ways of thinking algebraically were discussed in detail in subsequent section.

### **Algebraic Thinking**

There are numerous perspectives on how algebraic thinking has actually emerged. However, many agreed that symbolization and generalisation skills are



essential to build strong foundation for algebra (Kaput, 2008). The processes of identifying similarity and dissimilarities, noting differences, classifying and labelling, together with algorithm seeking might be the foundation of algebraic thinking (Mason, 1996). An indication of algebraic thinking is related to students' ability to begin to use particular number to argue a general case (Blanton & Kaput, 2003). The authors claimed that algebraic thinking occurs when the students are engaged in looking for relationships. Algebraic thinking may also be related to using representation, which may eventually guide symbolizing relationships in mathematically efficient ways (Blanton & Kaput, 2003). Algebraic thinking differs from arithmetic thinking which involves a focus on specific numbers and calculation, but algebraic thinking involves the relations between numbers and ideas of generalisation (Carragher & Schliemann, 2007).

Kieran (1996) has defined algebraic thinking as "the use of any of a variety of representations that handle quantitative situations in a relational way" (pp. 274-275). In addition, Driscoll and Moyer (2001) asserted essential ability in algebraic thinking is "the capacity to recognize patterns and organize data to represent situations in which input is related to output by well-defined function rules" (p. 282). These definitions is consistent with Warren's (2003b) claim that algebraic thinking also emerges when students get involved in activities such as looking for relationships between quantities and representing the relationships between quantitative situations.

The fundamental point highlighted in the literature regarding algebraic thinking in primary school is it begins from arithmetic. Working with arithmetic is not distinct entity from algebra. Often questions arise how to teach algebra in primary schools. Early algebra is about instilling algebra way of thinking from primary school.

The algebraic way of thinking listed by Kieran (2004) explains how working with arithmetic provokes children to think algebraically.

*i) A focus on relations and not merely on the calculation of a numerical answer*

When a number sentence is present, firstly the relations should be focused rather than algorithm to achieve a correct solution. The focuses on relations involve the ability to think relationally. Relational thinking is crucial when working with arithmetic (Carpenter, Franke, & Levi, 2003; Napaphun, 2012; Smith, 2008; Stephens, 2008). According to Napaphun (2012), relational thinking skill is crucial element in arithmetic which can improve the understanding of arithmetic while building a foundation for the development of algebraic thinking. He has conducted a study to investigate upper primary pupils' conception of number sentences and characteristics of their relational thinking. This information was later gathered to create an instructional model to ease the process of transition from arithmetic to algebra. His findings imply misconception of equal sign being a major obstacle to acquire relational thinking.

The pupils often perceive the equal sign as a symbol to state the answer rather than a sign that denotes a relationship (Stephens et al., 2013). This misconception can be caused by various aspects. The primary school text books present equal sign as a sign to indicate final answer. Often, the problems presented on the left side of equal sign and children required write correct answer on the right-hand side of equal sign (Rittle-Johnson et al., 2011; van Amerom, 2002). There is no variability in the form of number sentences presented in the text books. In addition, the teachers often did not encourage pupils to think relationally (Jacobs et al., 2007). They rather focused on algorithm to get the correct numerical answer. Hence, in order to develop algebraic thinking in primary schools it is necessary for the curriculum designers and educators

to emphasise the whole structure of number sentences. Teachers should highlight the design of number sentence by encouraging skills such as relating, searching and extending (Napaphun, 2012). This is to ensure some light shed on the algebra elements underpinned in arithmetic.

Likewise, Stephens (2009) also has emphasised the importance of relational thinking in the development of algebraic thinking. The way of thinking algebraically should start from the ability to see a number sentence as a whole structure. When they able to see a number sentence as whole structure, it will lead to students to think beyond the particular situation. Fujii and Stephens (2008) investigated six years old children's justification for their decision on the validity of number sentence;  $173 - 35 + 35 = 173$ .

The validity of this number sentence can be judged effortlessly by relational thinking, subtracting and adding same quantity will not make any difference. However, the authors found, some children performed calculation to find the answer, some began calculation but then realised they have to subtract and add same number so made their decision, some managed to look at the structure and made decision without any calculation. This study shows the importance of relational thinking by moving forward and backward across the bridge connecting number sentences and generalisations that can be derived from them. Therefore, exposing the children to the algebraic nature of number sentences can also furnish a strong bridge to the idea of variable. This idea is discussed further in Modelling section later in this chapter.

ii) *A focus on operations as well as their inverses, and on the related idea of doing / undoing*

The second in the list of Kieran's (2004) way of thinking algebraically is the idea of doing and undoing. The process of doing and undoing has been widely accepted as a good starting point to foster algebraic thinking in primary schools (Cai, 2004; Cai, Ng, & Moyer, 2011; Ng, 2004). From every TIMSS results, it is evident that Singapore is the one of the top performing countries in mathematics achievement for both primary and secondary school levels. According to Ng (2004), the curriculum in Singapore primary level emphasis on three approaches. These approaches encourage the development of algebraic thinking. They are problem solving, generalisation and functions. These approaches are aligned with three types of thinking process namely analysing parts and whole, generalising and specialising, and doing and undoing. The thinking process of doing and undoing highlighted by Kieran (2004) is a basic element of algebraic thinking which can be acquired while doing arithmetic. This could be one of the reasons for Singapore's top performance in mathematics particularly algebra in every international assessment such as TIMSS and PISA. They build foundation of algebraic thinking elements from primary school through model method (Ng, 2004). In Singapore's mathematics curriculum, the information and relationships given in a problem represented by rectangles and numerical values which is called model method. The rectangles are used to represent unknowns. The representation of unknowns by rectangles provides visual appearance of unknowns which can make the students to understand the relationships and information given in the problem more easily. From the rectangles, the process of doing and undoing being presented easily to show the reverse effect of operations such as addition and subtraction or multiplication and division.

The process of doing and undoing provide an in depth understanding of the operations. It is good if primary pupils are exposed to the process of undoing

informally to master the equation-solving activities in the later years of education (Cai et al., 2011). Doing and undoing refers to reversibility which means the ability of undoing a mathematical process. It eventually encourages children to think to work forward and backwards. This is an essential way of thinking to construct foundation for formal algebra activities such as equation solving, factorisation, inverse of functions and also anti derivatives (Cai et al., 2011).

In algebra, there are many instances where one should work forward and backward. In other words, it is about how to undo an operation and work backwards. This process of doing and undoing can be introduced from kindergarten. Children should be exposed to repeating patterns and train them to work forward and backward. This basic fundamental element then can be strengthened in grade one by introducing reverse operations to work forward and backward. Opposite actions show how a quantity remains unchanged. Addition and subtraction can be a good example to show the effect of reversibility (Cai et al., 2011). Therefore, it is not exaggerating to highlight process of doing and undoing plays an important role in the development of algebraic thinking. Furthermore, simplest activities such as working with patterns could be promoted from kindergarten to familiarise the students with reverse operations. When they come to higher education level, they will get used to work forward and backward especially solving equation.

Doing and undoing also been emphasised by Driscoll et al. (2001) in their toolkit for staff development on fostering algebraic thinking. The materials presented in the toolkit focused on the cultivating algebraic habits of mind. Development of algebraic thinking will not occur overnight. It is a slow process over time. With regard to this, the authors have proposed three habits of mind namely doing/undoing, building rules to represent functions, and abstracting from computation. Doing and undoing

has been highlighted for effective way of algebraic thinking. It involves reversibility which allows to using a process to achieve a goal and also to understand the process to well verse to calculate backward from the ending point.

*iii) A focus on both representing and solving a problem rather than on merely solving it.*

Third in the list of Kieran (2004) refers to the emphasis on representing and solving a problem rather than focusing on getting the solution. Majority of the textbooks designed by presenting symbolic activities firstly followed by story problems towards the end of the chapter. This show the authors believe in story problems are greater in difficulty compared to symbolic activities (Nathan & Koedinger, 2000). This sequence creates a myth that symbolic representation is easier than working with story problems. A study conducted by Nathan and Koedinger (2000) asserted that even teachers also belief in "Solving math problems presented in words should be taught only after students master solving the same problem presented as equations" (p. 130).

Koedinger and Nathan (2004) suggested that working with simple story problems first could actually enable students to see the relationships between quantities. When the students exposed to symbolic representations, they were as tough learning a new language. They find it difficult to grasp the concept and see the relationships between symbols. Difficulties in comprehending the symbols lead to problem in acquiring quantitative reasoning. Students tend to work with symbols without knowing the actual meaning. To avoid this, the new chapters should introduce the story problems first and then encourage students to represent the problem in

symbolic forms. This way could provide a chance for students to see the connection between the problem and the symbols (Koedinger & Nathan, 2004).

These results were affirmed by another study conducted by Koedinger, Alibali and Nathan (2008). They have conducted a study using college students to identify the strength and weaknesses of algebraic symbols representation which is more abstract compared to describing the situation verbally which is grounded representations. This study extended to look at the complexity of story problems. The findings showed simple story problems and representing it in symbolic forms enable students to understand better the algebra concepts compared to complex story problems.

This brings to a conclusion that as Kieran (2004) stated, algebraic way of thinking is the ability of representing a problem and find solution. Simple story problems could trigger the students to understand actual situation and lead them to represent it in symbolic form. Which indirectly motivate the students to grasp the elements of algebra. The habit of mind (Driscoll et al., 2001) is matters in cultivating the way of thinking algebraically while learning arithmetic.

*iv) A focus on both numbers and letters, rather than on numbers alone*

There is abundance of studies show students struggle mastering formal algebra. The root cause started begun from how expressing the quantitative relationships which focus on general mathematics relationships are being introduced to children and how they interpret it. It has been widely discussed that basic elements of algebra should be instilled systematically starting from primary school in stages throughout the school education system. The question is how this can be done? In preceding sections, a few ways have been discussed pertaining the way to cultivate algebraic elements while learning arithmetic.

This section also discusses another way of instilling underlying algebraic elements in arithmetic. Algebra can be introduced to children from primary school through various parts while teaching arithmetic. One of the examples is introducing the concept of numbers and letters to children from primary school. In Japan mathematics curriculum, grade three and four pupils introduced with shapes such as  $\square$  and  $\circ$  to replace the unknowns (Fujii & Stephens, 2001). This allow the grade four pupils to recognise that  $\square$  and  $\circ$  are two quantities and able to see the relationships in the expression such as  $\square + \circ = 10$ . Besides this, expressing the word relationships in numerical quantity can be useful building foundation for algebraic thinking. For an example, "A is twice as long as B" can be expressed as "Length of A = Length of B  $\times$  2" (Fujii & Stephens, 2001, p. 258).

Number sentence such as  $\square + 8 = 23$ , and  $63 - \square = 49$ , introduce students to the task of finding the value of unknown numbers (Fujii & Stephens, 2008). Eventually the students can be introduced to literal symbols in the form of  $x + 8 = 23$  and  $63 - y = 49$ . In this case  $x$  and  $y$  are not variables. As explained by Radford (1996), "While the unknown is a number which does not vary, the variable designates a quantity whose value can change" (p. 47). However, an emphasis on single missing value in number sentences in primary school can be an eye opener when they are exposed to variables in the middle and higher school of education.

In the study conducted by Fujii and Stephens (2008), none of the samples called "triangle" or "circle" when they were presented with number sentence such as  $32 + \circ - 10$  or  $32 - \nabla$ . These symbols were used as a placeholder to represent a quantity. All students were able to see the connection between these symbolic representations with the number sentences. This can develop the students' understanding of a variable. Symbolic representation means to them as a way of representing multiple numerical



expressions. However, this symbolic representation does not have a meaning on their own as they will in formal algebra. Further exercises should be given to enable them to work with detaching “symbols” from specific number sentences that give them meaning (Fujii & Stephens, 2008). Eventually they will get to build strong foundation in grasping nature of the variables.

v) *A refocusing of the meaning of the equal sign.*

Role of equal sign in development of algebraic thinking has been widely emphasised by all mathematics scholars. Focus on equal sign often neglected while learning arithmetic (Byrd et al., 2015; Jacobs et al., 2007). Some good arithmetic instructions could provide relational view of equal sign (Matthews, Rittle-Johnson, McEldoon, & Taylor, 2012). Conceptual understanding of equal sign acts as a link between arithmetic and algebra. Conceptual understanding of equal sign refers to realisation that equal sign is an indicator of "...sameness of the expressions or quantities represented by each side of an equation" (Matthews et al., 2012, p. 222). However, students often see it as an indication to write the numerical answer. This issue has been discussed further later in this section. As refocusing of the meaning of the equal sign overlaps with Kaput's (2008) classification of algebraic thinking, further discussion can be found in generalized arithmetic section later in this chapter.

Kaput (2008) defined algebraic thinking as containing two specific aspects namely; generalisation and symbolization. These two aspects form three strands namely; generalised arithmetic, functions and modelling. The aspects of generalizing and symbolizing have required these strands. Nonetheless, the aspect of generalization more appropriate to be classified as generalised arithmetic and the aspect of symbolizing could be referred part of modelling. Kaput's (2008) classification of

algebraic thinking mostly overlaps with the ways of thinking algebraically as stated by Kieran (2004). It can be seen from literature that many elements of the algebraic thinking often comprised or suitable for more than one of these three strands. Thus, Ralston (2013) displayed these three categories as over-lapping circles as shown in Figure 2.1.

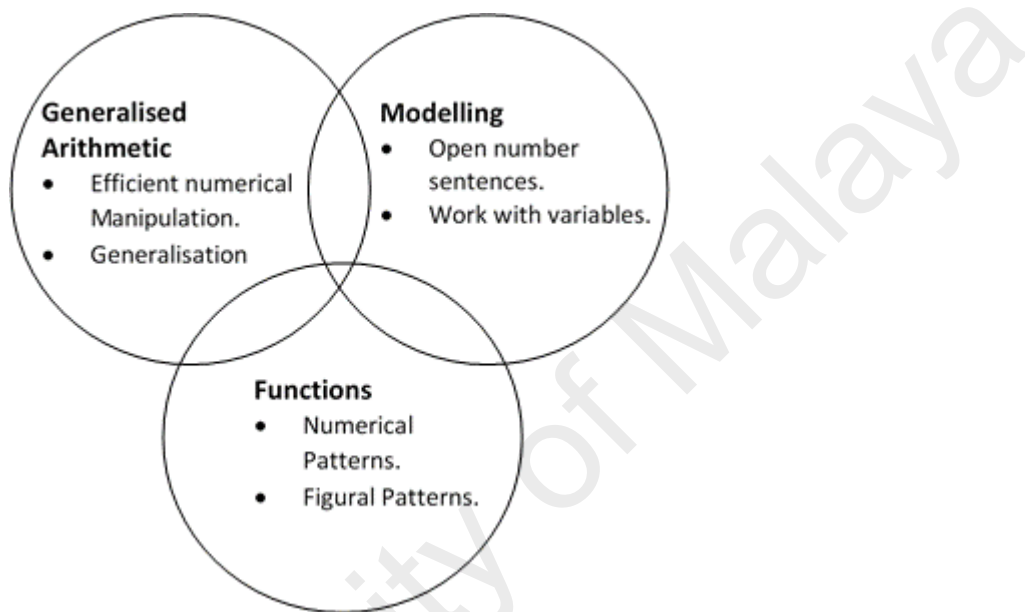


Figure 2.1. Algebraic thinking framework adapted from Ralston (2013)

The present study has used Kaput's (2008) classification of algebraic thinking into three main strands: generalised arithmetic, modelling and functions which were used by Ralston (2013) to developing content of algebraic thinking diagnostic assessment. The subsequent section discusses about these three strands in detail.

### **Generalised Arithmetic**

Generalised arithmetic defined as “the study of structures and systems abstracted from computations and relations, including those arising in arithmetic (algebra as generalised arithmetic) and in quantitative reasoning” (Kaput, 2008, p. 11).

According to Kaput (2008), it encompasses efficient numerical manipulation and generalisation. Generalised arithmetic is one of the four conceptions described by Usiskin (1988). According to interpretation, a variable plays a role of pattern generaliser. The important aspect in this interpretation is to translate and generalise. As an example, the arithmetic expressions such as  $-3 \times 5 = -15$  and  $-1 \times 6 = -6$  can actually be generalised to  $-x \times y = -xy$ . This variable helps to shift from algorithmic computation to generalisation. Generalised arithmetic can be divided further into two sub strands. They are efficient numerical manipulation and generalization.

**Efficient numerical manipulation.** Carpenter et al. (2003) provided evidence that gave a task such as  $67 + 83 = \square + 82$ , the students were able to identify that 82 is one less than 83 and 68 should be placed in the box. The instructional method and selection of tasks enable these students to focus on the equation's underlying algebraic characteristics (i.e.,  $x + y = \_ + (y - 1)$ , so  $\_ = x + 1$ ) and in the area of work with variables (i.e.,  $45 + x - x = 45$  can be developed from  $45 + 12 - 12 = 45$ ). The students did not find it necessary to find the answers of both sides of the equation in order to decide which number to place in the box. Obviously, these problems can be solved by calculation alone. However, solving it with the consideration of relationship involved and thinking algebraically could lead the students to possess the simplifying skill which is more vital for formal algebra lessons in future (Stephens, 2008).

These kind of simplification exercises provide students opportunity to think of conjectures, ability to justify, and come up with generalisation which involve mathematical ideas related to number properties. Further, students who become capable of generalizing and using commutative principle will be able to adapt to learn conventional algebra in formal algebra classes in later years of education (Hunter,

2010). Unfortunately, there is very limited research has been done in the aspect of efficient numerical manipulation at any grade level, although majority mathematics researchers have acknowledged these activities tasks are essential and meaningful (Ralston, 2013). However, it is undeniable fact that acquiring the efficient number manipulation skill is a vital and compelling skill that is needed to develop higher order algebraic thinking skills (Ralston, 2013).

**Generalisation.** Kaput (2008) stated that one of the major strands of algebraic reasoning is generalisation. Generalisation helps the bridge transition from arithmetic to algebra. When the students actually make a general statement that covers many instances, they are able to perform arithmetic generalisation (Kaput, Blanton, & Moreno, 2008) such as identifying that sum off two odd numbers is always an even number is a generalisation about addition.

Carpenter, Levi, Berman, and Pligge (2005) have further emphasized the necessity of operations' properties and meanings of understanding, by elaborating as

The best students have always figured out generalisations, and by doing so they make mathematics easier to learn and apply. Making generalisations explicit so that they are available to all students can address important issues of equity and access to powerful ideas of mathematics (p. 97).

A misconception generally experienced by students in understanding properties of arithmetic is that they have failed to identify that a same mathematical procedure can be applied to two different scenarios (Ralston, 2013).

Stephens (2005) attempted to examine grade six, seven and eight students' understanding on commutative principle by asking "if  $h + m + n = h + p + n$  was always, sometimes, or never true" (p. 96). Majority from the total of 371 participants

answered 'never', at the same time less than half of these participants answered correctly 'sometimes' and significant number of participants responded 'never'. An interview was conducted with a few of these participants and discovered that they have wrong interpretation for variables. Their perception was different variables cannot represent the same value. This finding is not surprising as the concept of two different variables  $m$  and  $p$ , can have the same value "is a mathematical convention, not a notion that is intuitively obvious" (p. 97).

Generalised arithmetic indeed a crucial element for the development of algebraic thinking (Haldar, 2014). According to Kaput (2008), arithmetic and algebra go hand in hand and in fact the two strands of thinking can be developed simultaneously. Generalised arithmetic involves generalizing arithmetic operations and their properties about general relationships and their forms such as properties of zero, commutativity and inverse relationships. Past studies have proved that fostering arithmetic generalisations can lead to better understanding of the equal sign and variables, which have been biggest obstacles for students to learn algebra (Kieran, 2004). Therefore, incorporating arithmetic generalisations in elementary mathematics is essential (Haldar, 2014; Hunter, 2010; Jacobs et al., 2007; Schoenfeld & Arcavi, 1988).

Currently many primary school students have limited opportunities to explore number properties. Eventually, this has led to students to experience arithmetic as a procedural process. The procedural process begins to be stumbling block for students when they need to work with abstract numbers and operation properties (Hunter, 2010). In the classroom setting, students should be engaged to make sense of arithmetic rather than performing computation instrumentally. The sense-making of

arithmetic will fill the cognitive gap between arithmetic and algebra in later years (Carpenter et al., 2003; Kaput, 2008; Mason, 2008).

Literature reveals a range of areas within arithmetic generalisation which are important in developing early algebraic thinking. They are as follows:

- Understanding the properties of operations: The commutative, associative and distributive properties.
- Understanding the properties and relationships of numbers: Odd and even numbers, zero and one.

The following section discusses further about these two in detail.

*Commutative, associative and distributive properties.* Young students are capable in using commutative property implicitly to support them to solve problems involving addition and multiplication (Hunter, 2013). However, majority of the students are still unable to understand this operational law. For example, Hunter (2010) reported findings from a classroom experiment with students aged 9 years to 11 years in New Zealand urban school where students were asked to identify true and false of following number sentences:

$$15 + 6 = 6 + 15$$

$$15 - 6 = 6 - 15$$

$$15 \times 6 = 6 \times 15$$

$$15 \div 6 = 6 \div 15$$

80% of participating students in the study had failed to identify the true number sentences. Many had the assumption that all of them were true. Those five who had identified which sentences were true, were unable to justify their answer. This provides evidence on the upper primary school students' limited knowledge on commutativity of addition and multiplication.

Associative property is equally important as commutative property in fostering algebraic thinking. Encouraging understanding of the associative property and how it could be applied to multiplication help students to work flexibly with number system (Hunter, 2013). Warren (2003b) argued that young children have limited opportunity to explore this property compared to commutative property as they are often asked to solve only two factors multiplication problems rather than three or more factors. In her study with students aged 12 years to 14 years old students, many of the students found difficulty in identifying associative property than commutative property. Meanwhile, most students were able to recognize the commutative property of addition (94%) and multiplication (93%). However, fewer students identified associative law as correct for addition (80%) and multiplication (78%) number sentences. She also highlighted the difference was possibly due to the increased complexity of number sentences involving brackets.

Carpenter et al. (2003) have given emphasis on distributive property along with the commutative and associative properties because they have established a solid foundation for study of number and operation generalisations, and for justification of such generalisations. In other words, these properties are at heart of early algebra. Distributive property is crucial in developing conceptual understanding of multiplication and also for algebra reasoning. Lack of distributive property understanding often leads students to make mistakes in adding variables (Ding & Li, 2010). At the same time, Schifter, Monk, Russell, and Bastable (2008) asserted that students essentially draw on the distributive property within their solution process, but it is often challenging for students to clearly generalise and justify their reasoning.

***Odd and even numbers.*** Identifying odd and even number structures can lead to development of algebraic thinking (Hunter, 2013). Identifying and creating odd and

even number structures and expressing representational material help to strengthen students' understandings on odd and even number structures which will eventually enable them to employ conjectures and generalisations (Carpenter et al., 2003; Hunter, 2010; Schifter et al., 2008). Study by Hunter (2010) shows how odd and even numbers are used to foster algebraic thinking. In her study, representational material and odd and even numbers definitions have helped students to develop solid justification of their conjectures.

Schifter, Russel and Bastable (2009) elaborated that 8 years to 9 years old students' classroom episode where the students were involved in a discussion of factors. When the students in the class were asked whether two was a factor of 156, a student started to provide justification based on previously discussed generalisation that the sum of two even numbers is even. Then the students were able to identify that the generalisation is needed to be extended to include three even numbers and justified verbally. Another student then continued to develop reasoning by referring to the structure of even numbers and used a visual image for justification.

*Properties of zeroes and ones.* The properties of zero and one play an important role in helping students to develop and justify conjectures and generalisations. While teaching arithmetic, active class discussion can help students to make generalisations. For instance, Carpenter and Levi (2000) conducted a case study with 7-years to 8-years old students. At first, the students were given a false number sentence such as  $78 - 49 = 78$ . This type of number sentence aimed to raise an in-depth observation of property of subtraction. Then the teacher showed large number sentences involving addition and subtraction such as  $789\ 564 - 0 = 789\ 564$  and  $0 + 5869 = 5869$ . This was to guide students to generalise about the properties of



zero in addition and subtraction. The findings have shown that young students were capable of identifying if those number sentences were true by justifying their generalisation. Hence, possessing good knowledge about operations' meanings and properties will help children for smooth transition from arithmetic to difficult algebraic problem in the later grades (Ralston, 2013).

## **Modelling**

Kaput defined modelling is "the application of a cluster of modeling languages both inside and outside of mathematics" (p. 11). Activities that can be classified as modeling are refer to the ability to work with equivalence, variables (in terms of unknowns), and open number sentences which exhibit the knowledge on role of equal sign.

**Solving open number sentences.** Generally, elementary school syllabus actually comprises some elements of algebra. For instance, questions pertaining to find missing addend or minuend in some simple word problems; simple mathematical sentence which looks like " $10 + \_ = 14$ ", " $\_ + 5 = 32$ ", and " $54 - \_ = 22$ ". Also, these concepts extended to multiplication and division. Such as " $8 \times \_ = 40$ ", and " $54 \div \_ = 9$ ". These questions which demand to find the unknowns are actually algebraic in nature and provide opportunity for the students to grasp the properties and relationships between arithmetic operations (Carraher & Schliemann, 2007).

However, classifying working with open number sentences as algebraic thinking is debatable among mathematics researchers. Herscovics and Linchevski (1994) disagreed that categorizing missing addend problem such as " $4 + \_ = 0$ " could not be considered as algebra, because this problem does really need algebraic thinking

while it can be solved using purely arithmetic. This means it can be solved by counting procedures or an inverse operation. It is notable that, in the past, these types of non-canonical representations were not exposed until formal algebra classes secondary schools (Wagner & Kieran, 1989).

Evidence shows that young children can reason with open number sentences (Carpenter et al., 2003). Understanding of inverse (addition and subtraction) and commutative property play an important role in tackling the open number sentences. If the students failed to acquire these skills, they might have alternate conceptions to solve the open number sentences (Carpenter et al., 2005). This is typically because students manipulate arithmetic as procedural process. Possessing these skills to work with open number sentences is crucial as this ability builds a strong foundation to algebraic elements such as reasoning and justification and make it more attainable to young students (Carpenter & Levi, 2000). McNeil, Fyfe, Petersen, Dunwiddie, and Brletic-ShIPLEY (2011) have provided evidence that solving such non-traditional formats (i.e.,  $\_\_\_ = 8 + 5$ ) can improve student's knowledge on equivalence and equal sign. At this point, it is necessary to discuss about importance of equivalence knowledge as it acts as a prerequisite skill. As the development of an appropriate conception of the equal sign, open number sentence can actually foster relational thinking (Carpenter et al., 2003).

**Equivalence.** This aspect literally the most widely investigated element of algebraic thinking skills. Rittle-Johnson and Alibali (1999) identified three required components of knowledge namely: “a) the meaning of two quantities being equal, b) the meaning of the equal sign as a relational symbol, and c) the idea that there are two sides to an equation” (p. 177). Generally used items to evaluate the understanding of

equivalence are very closed to the open number sentence type of items; nonetheless, these items have at least two known and one unknown values on each side of the equal sign. As an example, position of the unknowns in these items can be vary, whereby it could be in the second, third or even fifth position (i.e.,  $a + \_ = c + d$ ,  $a + b = \_ + d$ ,  $a + b = c + d + \_$ ) (Ralston, 2013).

The substantial body of research has documented that children often exhibit equal sign understanding as a symbol in announcing an arithmetic operation result rather than as a mathematical equivalence symbol (Rittle-Johnson & Alibali, 1999). “Children in the elementary grades generally consider that the equal sign means to carry out the calculation that precedes it; this is one of the major stumbling blocks when moving from arithmetic to algebra” (Carpenter et al., 2005, p. 84). For example, when third and fifth grade students were asked what does equal sign referring to, and the most of them said it referred to "when you add something, get the total", "end of question" (Byrd, McNeil, Chesney, & Matthews, 2015). Carpenter et al. (2003) reported the following alternate conceptions exist when solving equivalence problems: extending the problem, using all the numbers (i.e., changing the number sentence), and the answer is. These procedural errors possibly resulted from poor conceptual understanding about equal sign. The equal sign often interpreted operationally rather than relationally.

There are considerable numbers of studies which have documented these equivalence conceptions persist among elementary school students. Rittle-Johnson and Alibali (1999) studied relations between children's (grade four and five) conceptual understanding of mathematical equivalence and their procedures for solving equivalence problems (i.e.,  $3 + 4 + 5 = 3 + \_$ ). The authors suggested emphasis on concept of mathematical equivalence problems will be an effective way

to foster flexible problem-solving skill and conceptual understanding rather than merely teaching procedures to solve them.

Knuth, Stephens, McNeil and Alibali (2006) extended their study to grade six, seven and eight students and found that equal sign understanding substantially influences early algebraic thinking and performance. Their study results demonstrated middle school students' equal sign understanding strongly correlated with their performance solving equations, such as  $4m + 10 = 70$ . Interestingly, majority of grade six and eight students provided operational definition whereas slightly more than grade seven students relational definition. The study also revealed as grade level increases, it is unlikely that students demonstrate relational thinking. It is prevalent that students' understanding on equivalence does not develop immensely in overnight when they go to secondary school. Thus, consequence of students who are with inadequate relational understanding of the equal sign have also faced difficulty when it comes to understanding the steps involved in an algebraic strategy because they do not get the point why they do the same thing to both sides. In addition, despite the algebraic or arithmetic strategy used to solve the equations, students with relational view about equal sign outperformed students who solved equations incorrectly. This association has remained even when controlling for standardised mathematics test scores. Therefore, it is evident that equal sign understanding is related for solving equation performance.

Possessing correct knowledge on equivalence early is very necessary skill to master other areas of algebra successfully. Alibali, Knuth, Hattikudur, McNeil and Stephens (2007) discovered that the earlier students acquired the ability to view equal sign relationally enable the students to solve equivalence problems better. They investigated children's capability to determine if the variable value (i.e.,  $n$ ) is the same

in given two equations (i.e.,  $2 \times n + 15 = 31$  and  $2 \times n + 15 - 9 = 31 - 9$ ). Findings have shown that relational understanding precedes and predicts advanced solving. Students who interpreted the equal sign relationally were more likely than those who did not to recognise the equivalence of the two equations, and relational understanding tended to precede this recognition. Furthermore, the students' performance was better at the end of eighth grade if they acquired relational thinking earlier. This suggests the children's relational thinking substantially influenced performance in early algebraic thinking. However, aforementioned studies only focused on relational interpretation of equal sign and its effects on solving equation performance.

Byrd et al. (2015) compared both relational and non-relational interpretations of equal sign in addressing the limitation mentioned in preceding section. In a more recent longitudinal study conducted on grade three and fifth students, findings indicate that understanding of equal sign associated with early algebraic thinking and performance. Measurement on students' equal sign interpretation and solving mathematical equivalence performance was done before (pre-test) and after (post-test) instruction to mathematical equation at the beginning and end of the year. Their results extended existing evidence on how the early algebraic thinking could be developed via looking into children's equal sign misconceptions by demonstrating fifth graders arithmetic-specific equal sign interpretations.

One common point that was constantly emphasized in past studies is the importance of understanding of equal sign as early as elementary school to shape children's mathematics learning. This fundamental understanding will lead students to be successful in other areas of algebra.

**Work with variables.** As expected, the concept of variable has also received extensive attention in the early algebraic thinking research community. The results of those studies asserted that the literal symbols usage in algebra being a major challenge for students learning formal algebra. Variable is quite ambiguous to be explained, if it is just required to be explained in one word (Schoenfeld & Arcavi, 1988). For the reason that “the meaning of variable is variable” (p. 425), the varies range definitions for variable can make the term, difficult to be grasped by students (Schoenfeld & Arcavi, 1988). According to the authors, a variable is central to the transition from arithmetic to algebraic reasoning. Researchers have acknowledged that ability to work with variables is a very vital skill to master formal algebra in later years of education. However, the requirements of the skill itself are not well defined and it changes over time (Usiskin, 1988). A substantial literature on students’ difficulties in understanding and interpreting the symbolic notation used in algebra has accumulated over recent decades. A key finding is that many students struggle to understand fundamental algebraic concepts (Knuth, Alibali, McNeil, Weinberg, & Stephens, 2011; MacGregor & Stacey, 1997; McNeil et al., 2010).

Kuchemann (1978) developed a framework of what he considered the six different student interpretations of variables:

letter evaluated (i.e., the letter is a specific number, for example  $a + 3 = 5$ ),  
letter ignored (i.e., the letter is not given meaning, for example  $a + b = 43$  so  $a + b + 2 = ?$ ), letter as object (i.e., the letter stands for an object, for example  $s$  stands for students), letter as specific unknown (i.e., the letter is a specific yet unknown number, for example add 4 onto  $n + 5$ ), letter as generalised number (i.e. the letter can represent several numerical values), and letter as variable

(i.e., the letter can represent a large range of unknown numerical values) (p. 25).

Kuchemann (1981) documented that the majority of 13 to 15-year-olds are not fully prepared to work with algebraic letters as unknowns or generalised numbers.

MacGregor and Stacey (1997) assessed students' capabilities in identifying operations and structures, the students' interpretation for simple functions, and their ability to construct and solve equations. They obtained data from approximately 2000 students' written assessment (pre and post-tests) aged 11-15 from 24 Australian secondary schools. 14 students were interviewed as they worked on some of the items from the test. The findings were analyzed based on year seven students aged 11-12, who had not been taught any algebra. Then the same students were included again to study the progress made in an eight-week algebra unit that formed part of their normal Year 7 curriculum. Followed by samples from Year 7 to 10 in 22 schools, the authors found that "students frequently base their interpretations of letters and algebraic expressions on intuition and guessing, and on analogies with other symbol systems they know, or on a false foundation created by misleading teaching materials. Students' misinterpretations lead to difficulties in making sense of algebra and may persist for several years if not recognized and corrected" (MacGregor & Stacey, 1997, p. 15).

They discovered the students aged 11-15 possess the following prevailing interpretations of variables such as letter equals one, it is a label for an object, it is an alphabetical or numerical value, use of different letter, letter to stand for an abbreviated word, or lastly letter completely ignored. For instance,  $c$  stands for cake, so  $5c$  might mean 5 cakes. Furthermore, students were often exposed to word problems which usually involved letters which denoted by the initial letters of their names (A for area,

$m$  for mass,  $t$  for time, etc.). In contrast to Kuchemann (1981) findings, the outcome suggested that younger students' misinterpretations did not indicate low levels of cognitive development; they were thoughtful attempts to sense making of a new notation or were caused by transfer of meanings from other contexts.

The discussion of the literature in the preceding section shows there is widespread agreement that primary school pupils might not understand the meaning of true variable but definitely they are capable to comprehend that the symbol represents a number. This may help eliminate common alternative conceptions which always been hindrance to work with true variables in later years of education. Thus, the present study will utilize algebraic letters for work with variable context.

## **Functions**

According to Kaput (2008) function can be elaborated as “the study of functions, relations, and joint variation” (p. 11). Likewise, Smith (2008) explained functional thinking as students’ ability to focus representational thinking especially on the relationship between two varying quantities. Another definition by Warren, Cooper and Lamb (2006) mentioned, “The construction and use of functions is considered to be the central to most mathematical investigations and has been found to be notoriously difficult for most students at all levels of learning” (p. 209). Generally, the researches carried out on functions have only involved middle and high school students by focusing the formal algebra. However, Blanton and Kaput (2011) highlighted that the study of functions should be done from beginning in early elementary school. Previous researches have shown that tasks that were focused toward the development of functional thinking skills assisted children as young as kindergarten to make sense about functions (Blanton & Kaput, 2004; Smith, 2008).



Functional thinking can be developed by class discussions involving activities such as finding and generalising patterns (Blanton & Kaput, 2004).

In path leading to algebra, generally patterns are emphasized as an approach for transition from arithmetic thinking to algebraic thinking. According to Lee (1996), not only algebra, but all of mathematics is about generalizing patterns. "Expressing generality" is one of the four different roots of algebra described and suggested by Mason (1996). Pattern activities in lessons which can be seen as early as kindergarten level have important roles to form basis of algebra (Tanisli & Ozdas, 2009). A pattern refers to the rule involved in constructing the elements of a series of mathematical objects. Pattern activities begin from kindergarten level. Young children are often exposed to repeating patterns, which is repeated in some generalizable way and those could be either numerical or figural.

Recent studies of early algebra are often centered pattern exploration (Blanton & Kaput, 2011; Lannin, Barker, & Townsend, 2006; Tanisli & Ozdas, 2009). A productive way to develop children's algebraic reasoning is activities involving patterns (Ferrini-Mundy, Lappan, & Phillips, 1997). This shows the ability to work with patterns may indicate the children's early algebraic thinking. For instance, the look for patterns in different situations, the use of symbols and variables that represent patterns and generalisation are important elements of early algebraic thinking. Using patterns is seen as way of approaching algebra (Mason, 1996). He made vast activities involving figural patterns which will encourage pupils to express generality. He advocated four stages in this process; a) looking through, b) looking at, c) seeing a generality through the particular, and d) seeing the particular in the general. He emphasized that students often generalise their world that they live. Generalisation is only successful when it accompanied by making sense (Mason, 1996). As such,

patterning activities are seen as most suitable for young children and therefore there are few studies which investigated various aspects of responses of young children to activities involving patterns (Stacey, 1989; Warren et al., 2006).

Though algebraic thinking has been described in many different ways, the process of "generalisation" has been commonly named an important aspect of it (Blanton & Kaput, 2005; Hunter, 2013; Ishida & Sanji, 2002; Lannin, 2005; Mason, 1996; Stacey, 1989). In context of mathematics, generalisation refers to the process of stepping back from mathematical situations or expressions and identifying commonalities among and rules to describe them (Sorkin, 2011). According to Driscoll and Moyer (2001), generalisation is "abstracting from computation", which is more readily associated with algebra, and perhaps more easily imagined to be feasible for young children. For instance, when one recognises to generalise that the sum of two numbers will always be the same regardless of order ( $a + b = b + a$ ) or that zero added to any number yields that same number ( $0 + x = x$ ). It is clear that understanding these two concepts is very useful and essential, for success in arithmetic, and indeed in many other areas of mathematics.

Numerous studies have used patterning activities and generalisation to investigate various aspects of early algebraic thinking (Booker & Windsor, 2010; Gan & Munirah Ghazali, 2008; Jurdak & El Mouhayar, 2014). Gan and Munirah Ghazali (2008) compared year five pupils' successful and unsuccessful solutions geometric patterns problem solving. Their study highlighted the importance of incorporating the study of patterns in Malaysian primary mathematics classrooms. Booker and Windsor (2010) studied on students' algebraic thinking as they generalised and articulated their solution processes when representing and solving structural problems in numerous contexts. Recently Jurdak and El Mouhayar (2014) focused on students' reasoning

level and developmental trend in pattern generalisation across grade level. The authors also looked into the role of task variables as a mediator in the developmental trend of student level of reasoning within and across tasks.

Numerous researches have shown that pattern exploration supports students in the primary school to develop algebraic thinking (Lannin, 2005; Lannin et al., 2006; Stacey, 1989; Tanisli & Ozdas, 2009). The studies have examined students' abilities with pattern tasks. These studies have been conducted with middle school, and some older elementary school students, and these in general report that students experienced difficulties with such activities, especially at first. However, they were capable to recognize, describe, extend, and create patterns.

Stacey (1989) explored linear patterns, in which the element can be expressed as  $an + b$ . The samples aged nine to thirteen years answered linear generalizing problems generated using three task templates,  $3n + 2$ ,  $4n - 1$ , and  $6n - 2$ . The first two tasks were in pictorial form and the third task was an arithmetic sequence. The presentation of tasks whether pictorial or not did not affect the methods used by students. Stacey categorized students' responses into four methods, namely: counting, difference, whole-object and linear. Only the last method always guides students to correct answers. The counting method is only used when a pattern is based on pictures. She also found that the participants were able to recognize constant difference property, whereby students sought  $n$ th term of a pattern series from the previous term. However, when they undertook generalisation, many students used a misguided whole-object method. Stacey's findings showed students chose different methods for 'near generalisation' tasks (i.e., find the tenth term), and 'far generalisation' tasks (i.e., find the seventieth term): "Nearly one in seven of those who used a linear method for

the near generalisation swapped to a whole-object method for the far generalisation” (Stacey, 1989, p. 155).

Lannin (2005) extended investigations of linear patterns focusing on justification given by the students for their generalisation created. He found that generally students were capable to generalise and justify using generic examples. He categorized participants’ justification according to five level frameworks developed by Simon and Blume (1996). The five levels are namely, “level 0: no justification, level 1: appeal to external authority, level 2: empirical evidence, level 3: generic example, and level 4: deductive justification” (Lannin, 2005, p. 236). The results have shown that the empirical justification and generic examples were the most commonly used types of justification. Generally, students utilized empirical justification to test their rules. However, Lannin could not provide evidence whether the participants were able to differentiate the empirical arguments and the generic examples during whole-class setting discussion. In contrast to Stacey (1989), Lannin reported participants were successful in providing general arguments and valid justification when they used geometric schemes. Participants focused more on particular value (near generalisation) than on general relations (far generalisation). Students were able to seek subsequent element in a series of pattern based on a previous one. This method hindered them from relating each term to its position in patterns given and identifying all elements’ general structure in whole. He pointed out participants’ validity in understanding of their generalisations was crucial when working with patterning activities in the classroom. Thus, it is recommended to investigate different types of tasks that will enable students to examine various types of justifications and generalisation strategies that other students use.

Prior to looking into students' justification for their answers for patterning activities, one should ask how young children generalise these types of tasks. Studies involving the patterning activities have yielded positive information about children's abilities. For instance, Lannin et al. (2006) investigated students' recursive and explicit rules usage by investigating the generalisation developed in patterning activities. In addition, the authors also exposed the students on the use of spreadsheet as a tool for generalisation. This study has found the students' difficulty towards explicit rules when progressing from the successful use of recursive rules. They progressed to numeric strategies from iconic/visual strategies by disregarding the importance of reasoning that would enable to them build connections across the tasks. Students generally, when attempting to construct explicit rules, will concentrate on particular term instead of looking at general relationships. They also faced difficulties to look for the differences between their recursive and explicit rules. Participants' lack of understanding on the meaning and connections of the mathematical operations as the main problem. This includes addition and multiplication as the significant contribution to the struggle. For example, ability to understand repeated addition as multiplication shows the strong understanding of the interconnectivity of the operations.

However, the authors could not identify from the discussion if the students were capable to see the connection between multiplication and addition. In sum, some students were able to construct some in-depth explicit rules understanding as they built and, when the research continued, their focus was on particular element instead of whole relationships to generate the explicit rules. Hence, patterning activities are good to promote students' recursive rules and move to explicit rules and identify the links exist between these two types of rules.

Similarly, Tanisli and Ozdas (2009) investigated grade five students' strategies of using the generalizing patterns. They conducted this study on five Grade 12 students of varying abilities; low, medium and high in mathematics. Similar to many other researches, the authors employed task-based interview to collect data. Unlike Lannin (2005), Tanisli and Ozdas (2009) focused only on visual and numerical approaches and two types of generalisation strategies namely: recursive and explicit. These strategies also investigated in terms of types of generalisation; near generalisation and far generalisation.

The findings have shown that the visual approach is made easy for generalisation when both visual and numerical approaches were adopted in the generalisation of patterns by students from different levels. Furthermore, it was also found that in near generalisation recursive strategies were utilised, while in the far generalisation explicit strategies were utilised. In addition, the students' success level and the use of the visual and numerical approaches relationship were also reported. According to it, students with high-success level adopted both approaches; students in the middle-success level adopted rather the numerical approach, and students with low-success level adopted the visual approach. Likewise, the present study's one of the intentions in investigating the relationship between students' mathematics ability and their justification level in various strands of algebraic thinking.

Aforementioned studies have pointed out the need of teaching and learning of patterns from primary school and provided a few strategies for pattern generalisation especially in developing functional thinking.

## Early Algebra

Early algebra is not teaching algebra early (Carraher, Schliemann, & Schwartz, 2008). Early algebra is different from middle and high school algebra. It acts as a bridge to link arithmetic and algebra where it helps to introduce algebra gradually from arithmetic. Algebra resides quietly within arithmetic and representational systems. The instructional method helps it to emerge and bring algebraic character via class discussion while teaching elementary mathematics. To be more precise, early algebra refers to transition from arithmetic to algebra. van Amerom (2002) has clearly drawn a table to show how arithmetic relates to algebra. Table 2.1 shows the characteristics of arithmetic and algebra which was drawn by van Amerom. As can be seen from the table, connection line between arithmetic and algebra is very mild. Appropriate class room tasks and instructional methods can actually build the bridge to the transition of arithmetic to algebra. The characteristics explained in Table 2.1 eventually fall in one of the three stands of algebraic thinking which was defined by Kaput (2008).

Table 2.1 also shows that arithmetic is direct calculations involving known numbers to get unique solution (van Amerom, 2002). Algebra deals unknowns to generalise a unique solution. School curriculums often separate arithmetic and algebra. Arithmetic is taught in primary school level and algebra begins in middle school level. Early algebra refers to the conceptual bridge between arithmetic and algebra. It is not a new syllabus to be introduced in the curriculum. Early algebra should be cultivated while teaching arithmetic in primary school level.

Table 2.1

*Characteristics of arithmetic and algebra*

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<b>Arithmetic</b>	<b>Algebra</b>
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general aim: to find a numerical solution	general aim: to generalise and symbolise methods of problem solving
generalisation of specific number situations	generalisation of relations between numbers, reduction to uniformity
table as a calculational tool	table as a problem-solving tool

Table 2.1, continued

<b>Arithmetic</b>	<b>Algebra</b>
Manipulation of fixed numbers	Manipulation of variables
Letters are measurement labels or abbreviations of an object	letters are variables or unknowns
Symbolic expressions represent processes	symbolic expressions are seen as products and processes
Operations refer to actions	operations are autonomic objects
equal-sign announces a result	equal sign represents equivalence
reasoning with known quantities	reasoning with unknowns
unknown as end-point	unknown as starting point
Linear problems in one unknown	problems with multiple unknowns: system of equations

*Note.* From "Reinvention of early algebra" by Amerom, B.A. van, 2002, p. 20.

The following section discusses the kind of activities which are involved in early algebra. Growing body of literature have discussed and argued about early



algebra activities and some early algebraic skills and concepts that can be developed right from the beginning in primary grades. It can then build a strong foundation in acquiring algebraic concepts for future success in algebra (Kaput, 2008). Literature shows that three main algebraic concepts can be developed early are equivalence, generalisation, functional thinking and variables (Hunter, 2013; Lannin, 2005; MacGregor & Stacey, 1997; McNeil, 2008).

### **Early Algebraic Thinking**

Traditionally, algebra has always been associated with secondary and tertiary level education. Early mathematics teaching is full of arithmetic and intuitive procedures of "finding the answer". Students often face difficulty in algebra as they are required to use structures that they have previously been able to avoid (Childs, 1995). To overcome this obstacle, researchers have recognised the importance of promoting algebraic thinking in elementary students. Conventionally, students are exposed to arithmetic in elementary grades and only in form one onwards students will be introduced to algebra. There is an increasing concern, that separating arithmetic education and algebraic education makes it challenging for students to gain conceptual knowledge of algebra in higher grades (Cai & Moyer, 2008). They believed the obstacles to master algebra can be more effectively tackled by assisting the students to develop algebraic thinking skills right from the primary school. Researches to date have shown that expecting primary pupils to think algebraically is not an issue these days.

Evidence has been accumulated to show that algebraic thinking can be taught effectively in the elementary level (Gan & Munirah Ghazali, 2014; Mason, 2008; Schliemann, Brizuela, & Earnest, 2006; Swafford & Langrall, 2000). For instance,

Swafford and Langrall (2000) interviewed ten Grade six students with six verbal problem situations and asked them to solve a series of similar tasks for each. The problem situations were complex algebra problems which involves two to three unknown variables. These students had little or no formal instruction in solving complex algebraic type problems. However, the results of this study affirmed young students were able to generalise and describe relationships as well as write equations. Although these equations were not necessarily written in standard notations, this study shed some light as the importance of pre-instructional knowledge that supports algebra.

Carraher, Schliemann, Brizuela and Earnest (2006) analysed if young students could integrate algebraic concepts and thinking. The data obtained from a 30-month longitudinal study conducted in four classrooms ranging from grade two to four a Massachusetts public school. The authors gathered data clarifying the conditions that allow young students to make use of algebra ideas and representations. It was evident that provided proper teaching and support, young children can learn functional relationships and representing numbers with symbols.

Mason (2008) went further in stating algebraic thinking begins at a very early age. He discussed how children's use of power to make sense of mathematics. The powers that Mason described are associated with generalisations. He believed that a student's ability to think algebraically begins shortly after birth. This supports the notion of integrating algebraic thinking into early elementary mathematics curriculum. Babies and toddlers learn to make patterned noises (before talking in words and sentences). Furthermore, he believed that all children who can both walk and talk possess "powers" that can be used to help them develop algebraic thinking. These powers are imagining and expressing, focusing and defocusing, specializing and

generalizing, conjecturing and convincing, and classifying and characterizing. Since at a very young age, children use these powers outside of mathematics every day, their intuitive understanding can be transferred to mathematical situation. This is illustrated when students work with sequencing of blocks to build towers with a sequence of colours, for example green, yellow, blue, green, yellow, and blue. In language arts there is rhyming patterning in poetry. These generalisations grow as the students grow. Gattegno (as quoted in Mason, 2008) states “As soon as they use concepts, as soon as they use language, and that they of course bring this mastery and algebra of classes with them when they come to school” (p. 90).

Since there has been a lot of evidence to show elementary students who are capable to think algebraically, Gan and Munirah Ghazali (2014) studied algebraic thinking of year five pupils. They attempted to infer algebraic thinking among five 11-year-old pupils while solving three early algebraic problems involving geometric patterns. Based on the data collected via one to one interview, the authors suggested year five pupils exhibit ‘look for pattern’, ‘recognize pattern’ and ‘extend pattern’ the most as algebraic thinking skills. The results have supported the past literature that elementary school children are able to think algebraically. In addition, the study revealed participants were able to look for, recognize, describe and extend patterns to solve generalisation problems involving geometrical patterns. These criteria reflected their abilities to detect sameness and differences, as well as to make distinctions. However, this study was limited to five students and three problems. Thus, the results obtained from this study may not be able to generalise about the entire year five student populations. Though, it could not be generalised but the results have pointed out the capability of year five pupils in algebraic thinking.

## Cognitive Variables

The following section describes about the cognitive variables used in the present study that is expected to be influential for success of year five pupils' algebraic thinking. To the best of the researcher's knowledge, there is no study which has been conducted in finding the influencing cognitive variables of algebraic thinking. Hence, the potential variables selected for present study have been from literatures on cognitive aspects which influence or associated with algebraic thinking. Figure 2.2 shows a clear structure of the cognitive variables used in the present study. This structure was designed by researcher to provide a clearer picture of the cognitive variables involved together with its sub-branches. This structure has been used to design assessment of number, operation, symbol and pattern senses (ANOSPS). Subsequent section discusses in detail about each variable.

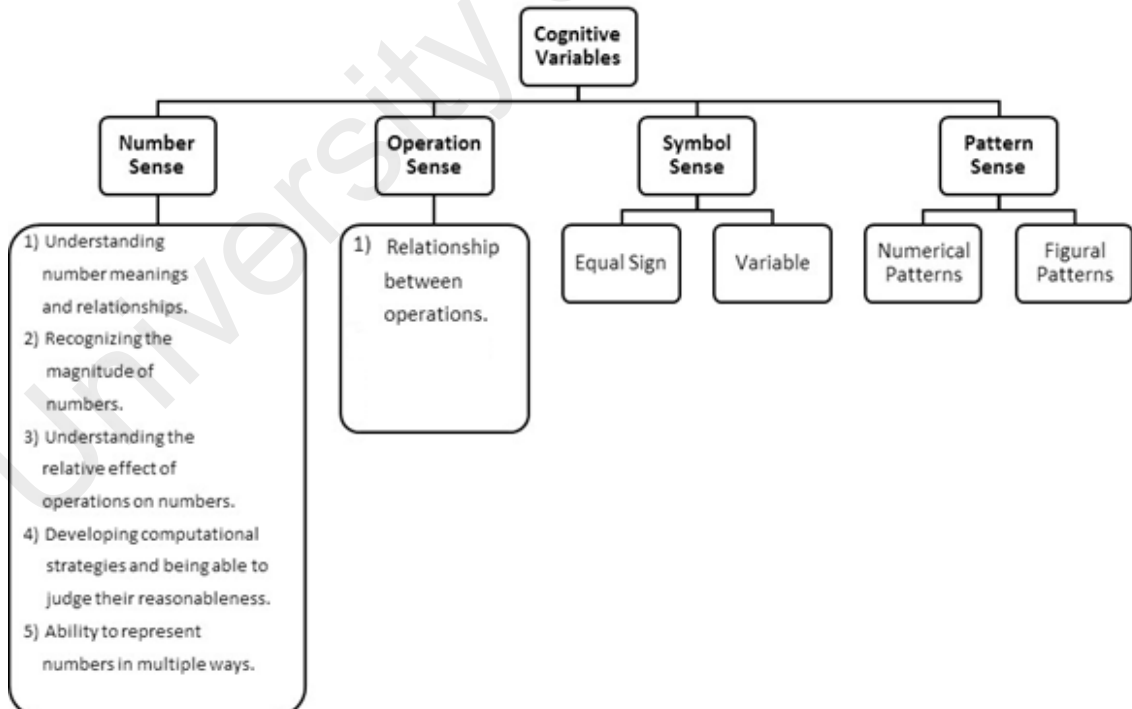


Figure 2.2. Cognitive variables used in the present study

**Number sense.** Generally primary mathematics curriculum deals with real numbers which is known as arithmetic. Often arithmetic is addressed as a prerequisite for algebra. However, school mathematics curriculum is designed in the way that arithmetic and algebra are two disjointed subjects (Cai & Moyer, 2008; Herscovics & Linchevski, 1994). This is why students generally struggle to bridge the cognitive gap between arithmetical and algebraic concepts (Herscovics & Linchevski, 1994). In arithmetic only straight-forward calculations are involved with known numbers (van Amerom, 2003). For instance,  $3 + 5 = 8$ , means that nothing more, nothing less. Particularly, working from known to unknown using computations is traditional arithmetic. Meanwhile, reasoning about unknown when it proceeds from the unknown, via the known, to the equations is formal algebra. Hence, the difference between arithmetic and algebra is that the former involves a specific situation while the latter involves a general solution (van Amerom, 2003).

According to Blanton and Kaput (2003), algebraic thinking can be developed through the use of existing arithmetic activities, transform them from problems with a single correct solution to chances for identifying patterns and generate conjectures or generalisation about mathematical facts and relationships and justifying them. This strategy called as "algebrafying instructional materials" (p. 70). They asserted algebrafying arithmetic tasks enabled children to do many things at once, including practicing number facts, developing number sense, and recognizing and building patterns to model situations.

The above discussion shows how sense-making of arithmetic leads to algebraic thinking. Literature indicated that number sense is difficult to define because it is not a single entity, but rather has many dimensions. Similar to 'common sense', number sense is a valued but difficult notion to characterise (McIntosh, Reys, & Reys, 1992).

Numerous definitions and characterisations of number sense can be found in the mathematics education literature. McIntosh, Reys, and Reys (1992) defined number sense as follows:

Number sense refers to a person's general understanding of number and operations along with the ability and inclination to use this understanding in flexible ways to make mathematical judgments and to develop useful strategies for handling numbers and operations. It reflects an inclination and an ability to use numbers and quantitative methods as a means of communicating, processing, and interpreting information. It results in an expectation that numbers are useful and that mathematics has certain regularity. (p. 3)

The authors described that good number sense refers to proficiency in mental calculation, computational estimation, judgment of the relative magnitude of numbers, recognition of part-whole relationships, and problem solving. Number sense entails an individual's general understanding of numbers and operations, with the ability to develop useful, flexible and efficient strategies for handling numerical problems (Yang, Hsu, & Huang, 2004). Number sense becomes meaningful and valuable to students when teachers believe that developing number sense is more important than mastering the rules associated with written computation (Yang, Reys, & Reys, 2009). Based on literature, Hsu et al. (2001) defined number sense components and developed a Number Sense Rating Scale (NSRS). The components of number sense are as follows:

1. Understanding number meanings and relationships.
2. Recognizing the magnitude of numbers.
3. Understanding the relative effect of operations on numbers.

4. Developing computational strategies and being able to judge their reasonableness.
5. Having ability to represent numbers in multiple ways.

Number sense and algebraic thinking are considered important topics of mathematics education, and the development of both is essential for mathematics learning (NCTM, 2000). Due to the importance of number sense in learning algebra, this topic has attracted a growing amount of attention and mathematics research worldwide. This can be seen from various ministry of education and bodies all around the globe that are striving hard to create a reformation in the learning and instruction of algebra with the emphasis of the number sense component. The importance of connection between number sense and algebra is evident because in many countries (i.e., New Zealand, Singapore and Hong Kong) number and algebra constitute together the strand "Number and Algebra" in their mathematics curricula.

Literature shows a strong understanding of number and operation, solid mental arithmetic strategies, and a deep enough understanding of operations will enable a smooth transition to algebra (Carpenter & Levi, 2000; Chrysostomou, Pitta-Pantazi, Tsingi, Cleanthous, & Christou, 2013). Understanding numbers and counting can encompass knowledge of spatial relationships, patterns, and combinations that coincide with early concepts of algebra. Number sense is algebraic in certain ways. Hence, emphasis on number sense may assist children in acquiring an accurate structural and algebraic understanding of numbers even before they learn to manipulate them (Strother, 2011).

In the present study, number sense construct has been investigated based on the five components defined by Hsu et al. (2001) as discussed in preceding section. Five items used in ANOSPS to investigate each of the components.

**Operation sense.** Herscovics and Linchevski (1994) identified five obstacles in the context of algebra. They were a) failure to perceive cancellation in an expression, b) a static view of the use of brackets, c) the lack of acceptance of the equal sign as a symbol for decomposition, d) an incorrect order of operations, and e) an inability to select the appropriate operation for partial sums. A static view of the use of brackets and incorrect order of operations indicates the lack of operation sense from arithmetic stage. For instance, the authors investigated the ability of seventh grade students to solve an equation like  $6 + 9 \times n = 60$ . 29% of the students failed to solve this because they used wrong order of operations. It was interpreted by them as  $15 \times n = 60$ . This shows how operation sense in arithmetic leads to algebraic thinking. Misconception of operation sense might lead to poor foundation of algebraic thinking in late years of education.

Operation sense plays an important role in algebra and highly associated with early algebraic thinking (Slavit, 1999). According to Slavit (1999), besides describing student development of operations concepts, operation sense can also be used for transition into algebraic ways of thinking. He defined operation sense as "conceptions that involve the operation's underlying structure, use and relationships with other mathematical operations and structures, and potential generalisations" (p. 254). Students should be able to sense the underlying properties the operations and transform it into a representation of symbol system. This understanding of operation basis and representing the basis in symbol system will eventually lead to a better understanding



of variable, makes equation solving more meaningful, and understand the equivalence concept (Slavit, 1999).

Slavit (1999) added that operation sense comprises numerous types of flexible conceptions such as “operation's underlying structure, use, relationships with other mathematical operations and structures, and potential generalisations” (p. 254). Followed by that he has identified ten aspects of operation sense namely; a) conceptualisation of base components of process, b) familiarity with properties of operation, c) relationships with other operations, d) various symbol systems associated with operations, e) familiarity with operations contexts, f) familiarity with operation facts, g) ability to use operation without concrete/ situational referents, h) ability to use operation on unknown/ arbitrary inputs, i) ability to relate the use of operation across difference mathematical objects, and ability to move back and forth between the preceding conceptions.

These ten aspects were then classified into three broader aspects namely; property, application and relational aspects. Property aspects is pertaining to the properties that each operations carry and involves a) the ability to break the operations into its base components, b) knowledge of the operation facts, c) understanding of the properties associated with the operations, and d) understanding of the various symbol systems that represent the operations. Application aspects are the ability to apply the operations in a variety of contexts, in context-free situations and on unknown and arbitrary units. Relational aspect comprises of a) understanding of the relationships between the operations, b) understanding of various representations of the operation across the differing number systems and c) ability to move backwards and forwards between these conceptions.

To add further, Driscoll and Moyer (2001) have also stressed that number and operations knowledge goes hand in hand with the following three aspects of algebraic thinking. They are a) doing and undoing - ability to 'reverse' and undo mathematical processes through working backwards from the answer to the starting point, b) building rules to represent functions - capacity to recognize patterns and organize data to represent situations, and c) abstracting from computation - ability to abstract system regularities from computation and think about computations independently of particular numbers used. According to the authors, when these aspects are used habitually, eventually it will lead students towards the learning outcomes listed by Principles and Standards for School Mathematics (NCTM, 2000). “Building rules to represent functions” (Driscoll & Moyer, 2001, p. 283) is related to NCTM's principles and standards of “represent and analyse patterns and functions, using words, tables, and graphs” (NCTM, 2000, p. 158) as these algebraic habits of mind required the students to do the raw facts organisation, pattern prediction, describe rule, utilise various representations, describe change and justify rule. Where else, “abstracting from computation” (Driscoll & Moyer, 2001, p. 283) is related to “recognize and generate equivalent forms for simple algebraic expressions and solve linear equations” (NCTM, 2000, p. 222) as this habit of mind required the students to justify computational shortcuts, calculate without computing, generalise beyond examples, dealing with equivalent expressions and symbolic expressions.

Warren (2003) investigated primary school leavers' acquisition of associative and commutative laws, of addition and division as general processes. About 672 students ranging from age 11 to 14 took the written test. First two tasks consist of operation sense elements as defined by Slavit (1999) which were addition and division's property and application aspects. The remaining two tasks out of four tasks

given aimed at determining students' understanding of both the commutative and associative properties. She then discovered many of the participants failed to demonstrate an in-depth grasp of addition and division as generalised processes. They faced difficulties in looking for more similar cases. Though they were asked to state two examples, most of them were only able to state one more example. In addition, many students failed to understand the commutative and associative laws in general terms. This study implies that many primary school leavers have limited understanding of the mathematical structure notion and arithmetic operations as general processes. Due to this limited awareness in arithmetic, most of the students have failed to acquire the connection and basics needed for algebra.

As discussed in preceding section, operation sense can have a wide range of view. Slavit (1999), explained ten different aspects of operation sense. However, it is quite tedious to investigate based on each and every aspect of operation sense in the present study. Thus, the present study only measures operation sense based on relationship between operations (i.e., addition and subtraction) (Haldar, 2014). The reason to restrict operation sense to just relationships between operations is, this aspect comprised main conception of operation sense out of ten aspects described by Slavit (1999). Relationship between operations examined in two aspects; symmetrical and asymmetrical. Q6 in ANOSPS aimed to address symmetrical and Q7 and Q8 aimed to address asymmetrical.

**Symbol sense.** Proficiency of algebra can only be achieved with an understanding of letters or what Arcavi (1994) referred to as 'symbol sense'. He asserted that having 'symbol sense' is the main focus to algebra and teaching should be geared towards achieving symbol sense making. According to Arcavi (1994),

symbol sense is, "an individual's ability to understand how and when symbols can and should be used to display relationship and generalisations" (p. 31). This statement seems to be in line with the viewpoint by Slavit (1999) who explained that communication in mathematics is feasible only if symbolic systems are known and relations between systems could be used to strengthen symbolic understanding. An equation,  $y = 70x$ , has a string of symbols including the letters of  $x$  and  $y$ , a numeral 70, and the equal sign that stand for something mathematical. As the above definition, the term, symbols, have been used in the present study to encompass variables and equal sign.

**Variables.** In fact, the most notable attribute of algebra is the use of letters as symbols, which are literally tools in communicating mathematical thinking (Kieran, 2004). Children create their own kind of algebra when they generate general rules and exhibit these connections via symbols to represent operations and variables. Therefore, young students should be encouraged to their own symbols' invention and not necessarily should learn the algebra formal notation (Berkman, 1998). Invention of own symbol system by young students can lead to acquisition of operation sense and consequently the development of algebraic thinking (Slavit, 1999). NCTM (2000) has described understanding what a variable is, using variables in geometric formulas and linear equations, and understanding what algebraic symbols represent as fundamental concepts that are prerequisites for success in algebra.

However, many students struggle to understand fundamental algebraic concepts (Herscovics & Linchevski, 1994; Knuth, Alibali, McNeil, Weinberg, & Stephens, 2011; Kuchemann, 1981; MacGregor & Stacey, 1997; McNeil et al., 2010). Students have difficulties in understanding and interpreting the symbolic notation used

in algebra (Kuchemann, 1981; MacGregor & Stacey, 1997). To overcome this problem, many studies have suggested and proved that young children are actually capable in dealing with unknowns (Brizuela & Schliemann, 2004; Carraher et al., 2006; Stephens, 2005). Students' performances in algebra are impacted by the instruction types that the students are familiar with. If the students are taught to an algebrafied mathematics curriculum right from primary school, eventually, they would be capable to work with more complicated mathematics tasks (Brizuela & Schliemann, 2004).

Introducing variables in primary school level does not mean teaching  $x$  and  $y$ . The pedagogical instructions should be designed to promote conceptual understanding of literal symbols as variables. For example, Carraher, Schliemann and Schwartz (2008) gave a basic comparison problem (i.e., one child having three more candies than another) to third grade students. The students were capable to represent this situation by proposing that if one child has  $N$  candies then the other one would have  $N + 3$  candies. There were students who also came out with answers (3, 6), (9, 12), (4, 7), (5, 8), where all are valid responses as they are different by three. These findings have shown that working with variables is not something that far from young students' mathematical thinking and this definitely achievable if similar types of activities carried out in daily teaching and learning of mathematics in primary school. Another form of algebrafied arithmetic task is " $n + n + n + 2 = 17$ ". The fourth and fifth grade students were able to solve it by first identifying that " $n + n + n$ " must equal 15 and then using the fact that 15 divided by 3 to solve the problem (Jacobs et al., 2007). Hence, in the present study researcher has used symbols such as inverted triangles, and diamond shapes to indicate unknowns in the ANOSPS.

**Equal sign.** Another most important symbol in algebra is the 'equal' symbol (Alibali, Knuth, Hattikudur, McNeil, & Stephens, 2007; Byrd et al., 2015; Knuth et al., 2006; Rittle-Johnson, Matthews, Taylor, & McEldoon, 2011). Students with relational understanding of equality will have a way of representing arithmetic ideas; hence, it will enable them to communicate and further reflect on these ideas. Lack of relational understanding is the major obstacle for students when they progress from arithmetic to algebra. The understanding of equal sign enables students to focus their reasoning on the quantities and operations, not just the numbers. Hence, incorporating algebra while teaching arithmetic can help students to increase their understanding of arithmetic and also get exposed to algebraic concepts. Thus, emphasis on equal sign should be established right from primary school curriculum to expose students to a richer understanding of equations and equal sign understanding also act as a prerequisite of early algebraic thinking (Gan, 2008).

Equal sign has two interpretations, as operational and relational. Young students often misinterpret the equal sign as an operational symbol instead of relational symbol (Sherman & Bisanz, 2009). By right, students should establish an understanding of equal sign as relational, which indicates that a relationship exist between the numbers on both side of the equal sign (Jacobs et al., 2007). Relational understanding enables students to interpret that numbers on both sides of equal sign should have same value. Operational understanding will lead to mistakes in solving equations with missing numbers or non-canonical (Sherman & Bisanz, 2009) equations and difficulties with algebraic thinking (Rittle-Johnson & Alibali, 1999).

Lack of exposure to non-standard or non-canonical equations is one possible reason for misinterpretation of equal sign. Canonical arithmetic problems are equations such as  $a + b + c = \_$ . Non-canonical equations are  $a + b + c = \_ + c$

(Sherman & Bisanz, 2009). These types of non-canonical equations are believed to be crucial in promoting a relational understanding of the equal sign (McNeil et al., 2006). Problems with understanding of equal sign begins when the students are exposed and only deal with equations such as  $2 + 3 = \underline{\quad}$  right from primary school (McNeil, 2008). In this type, equations are in the form of a number, operator symbol, number, equal sign, and blank. Misunderstanding of equal sign has been a stumbling block for students as young as kindergarten age (Carpenter & Levi, 2000). Mathematics education researchers conjectured that as students are exposed to work with typical teacher or textbook presented equations which are in the form of an answer which always needs to be computed after the equal sign, students tend to conclude that the equal sign is an operational indicator directing to perform a calculation (McNeil et al., 2006).

Relational understanding of equal sign is crucial for two reasons. Firstly, it enables students to solve equations. Mostly when students are given canonical type of equations, they are able to solve, and they begin to recognize the equal sign as an operational symbol and still answer the equations correctly. Relational understanding plays an important role when students are required to solve equations such as missing addend, minuend where by the equal sign is not in canonical form (i.e.,  $3 = 4 + \underline{\quad}$ ;  $5 + 10 = 13 + \underline{\quad}$ ). If students think the equal sign is to perform computations, for the equations where the missing part is not the sum or difference, they will most likely answer incorrectly. Secondly, relational understanding of equal sign is important to work on higher level mathematics problems in later years of education such as algebra (Powell & Fuchs, 2010).

**Pattern sense.** There have been many studies that used pattern sense in promoting generalisation as a primary pupils' algebraic thinking activities (Jurdak & El Mouhayar, 2014; Lannin et al., 2006; Stacey, 1989). The emphasis on sense of pattern is often in the recent studies of early algebraic thinking concepts. According to (Kaput, 1999), generalisation process involves:

Generalisation involves deliberately extending the range of reasoning or communication beyond the case or cases considered, explicitly identifying and exposing commonality across cases, or lifting the reasoning or communication to a level where the focus is no longer on the cases or situations themselves but rather on the patterns, procedures, structures and relationships across and among them (which, in turn, become new, higher level objects of reasoning or communication) (p. Kaput 6)

Identifying generality plays an important role in mathematical activity and also seems to act like a connecting bridge to transition students from arithmetic to formal algebra (Kaput, 1998). Figural and numeric generalisation enables a connection to referential context that can assist student understanding of symbolic representations, at the same time links students' prior knowledge of arithmetic (Lannin, 2005). Children are believed able to think functionally at an early age (Lannin, 2005). Lannin et al. (2006) developed a conceptual model to facilitate the potential strategies that a student could probably use to investigate in generalisation. There are three possible ways a student could use to generalise numeric situations (Lannin et al., 2006, p. 302):

- a) determining the general relationship that exists among the output values (i.e., a recursive rule).
- b) examining the relationship that exists between the input and the corresponding output values (i.e., an explicit rule).



- c) considering the relationship between reasoning recursively and reasoning explicitly (i.e., recursive and explicit rule relationships).

In the series of growth patterns, recursive reasoning is when a student is able to examine consecutive output values (see Figure 2.3). By looking at counting the circle in each term, a student could find the possible number of circles for image 5 and 6, and realizes the need to add two circles to form each subsequent term in the series. Thus, student will able to perform near generalisation. This is when the student attempts to examine a general rule. After realizing that to form a new image, one circle is added to vertical and horizontal each, the student identifies total number of vertical circles can be found by  $(n - 1)$  and number of horizontal circles is  $n$  where by  $n$  refers to  $n$ th image. A point to highlight here in this situation is when the student begins to reason recursively, and then investigates how increment of two circles is associated to the figural patterns in the series given, and generally concludes on the recursive relationship.



Figure 2.3. Item 15 from ANOSPS

At the same time, a student could also generate an explicit rule for this series of pattern. As an example, the student could think how to find the total circles in 6<sup>th</sup> image. Noticing that the number of vertical circles is  $(n-1)$  and  $n$  horizontal circles in  $n$ th image, leads to generate explicit rule;  $T = 2n - 1$ , given that  $T$  is the total number of

circles and  $n$  is the  $n$ th image. This shows the student has started to reason by identifying a figural relationship and managed to link this reasoning to develop a general case by using explicit rule (Lannin et al., 2006).

Additionally, the student could link the recursive and explicit rules. Based on the fact that there is an increment of two circles for each term, the student could identify that first term begins with 1 circle. Since each term in the series has an increment of two circles, the student could be able to generate a rule such as  $T = 1 + 2(n-1)$ .

If a student could generalise explicitly, hence s/he would be able to perform far generalisation. Far generalisation is when a student is required to find 10<sup>th</sup> or 15<sup>th</sup> image based on pattern in Figure 2.3. Explicit generalisation enables student to find a 'rule' as discussed in preceding section and will help to find the 16<sup>th</sup> image without finding 15<sup>th</sup> image. If a student only could generalise recursively, then s/he will look for consecutive images from 4<sup>th</sup> image until 16<sup>th</sup> image. The present study will explore on year five pupils' pattern sense based on recursive and explicit strategies used. However, pattern sense items in ANOSPS were provided with multiple choices as it can be a hint for students to figure out pattern structure. This is followed by near generalisation and far generalisation items. Students who reason explicitly will be able to answer far generalisation item correctly. Students who reason recursively may not see the hint given in previous question and probably might attempt to figure out each term.

### **Influence of Demographic variables**

**Gender.** Gender and location have been focal points in many education field studies. However, this section will discuss gender and location studies particularly in

mathematics and algebra field. It has been a general assumption that mathematics is a male domain (Anjum, 2015; Fennema & Sherman, 1977; Hyde, Lindberg, Linn, Ellis, & Williams, 2008). On the other hand, recent TIMSS reports and researchers have shown female students who outperformed male students in mathematics (Ismail & Awang, 2008; Mullis et al., 2012; Şengül & Erdoğan, 2014).

The gender difference has been a major concern since 1977. Fennema and Sherman (1977), investigated grade 9 to grade 12 students' gender difference in mathematics achievement along with spatial visualization and affective factors. The study was carried out to investigate the statement that males are superior in mathematics compared to females. However, they do not find any evidence to show males are superior to females in mathematics achievement. At the same time, the common belief that females are not capable to do well in mathematics also was not supported. This study has explained mathematics is attainable for everybody regardless of gender.

While many studies have focused on gender difference in mathematics achievement and algebra in secondary schools, Fennema and colleagues attempted to study gender difference in mathematical thinking by looking at the strategies used for problem solving (Fennema, Carpenter, Jacobs, Franke, & Levi, 1998). This longitudinal study in 90's observed progression of 82 pupils from first grade to third grade pupils in problem solving and computational strategies used. It was found that there are no gender differences in problem solving from first grade to third grade. Nevertheless, there is a difference between strategies used by girls and boys. Girls are more algorithms oriented while boys tend to use more abstract strategies. This difference in strategies used could lead to difference in conceptual understanding as

they progress through to secondary school. This could also be a reason for gender difference in secondary schools and as reported in TIMSS results.

The gender difference continues to be a major concern in 2000's. A review conducted by Vale and Bartholomew (2008) showed Australia and New Zealand had no problem with gender equity when comes to mathematics. However, international surveys such as PISA and TIMSS have shown results otherwise. The reports have recorded differences in achievement by gender. The gender differences are also apparent when boys tend to prefer higher-level mathematics subjects in secondary mathematics. Therefore, a question arises "Is mathematics born male?" In total, there are some countries which maintain gender equity and some with the gender differences in mathematics achievements. These findings show both females and males may be with similar "innate intellectual potential" (p. 4), end up with differences due to various factors exist in the environment (Hastings, 2013).

Gender difference is not only a concern in mathematics. It has been widely researched in algebra performance as well. Ma (1995) indicated performance of female and male 13-year-old students are the same across four education systems namely; British, Ontario, Hong Kong and Japan. In addition, there is no gender difference found within the education system in each country. The only difference in gender was in the mathematics achievement reported between Canadian and Asian education systems.

Likewise, according to Stites, Kennison and Horton (2004), there is no difference between female and male in solving algebra related word problems. Their study involved 96 college students in the U.S by anticipating male would outperform in problem solving requiring algebraic solutions. Surprisingly, it was reported that

females and males equally performed with less confidence when working with algebraic problems containing irrelevant information.

Recently, Cavanagh (2016) has conducted a study to analyse the primary pupils' algebraic reasoning abilities prior to instruction. The abilities were investigated based on five factors namely problem influence, type of problem, question, grade level, and gender. One of the aims of this study was to fill the gap by looking at the influence of gender in algebraic reasoning particularly functional reasoning. The author carried out a qualitative study by interviewing 60 children from grade one to three. They were required to solve eight problems which involved growing patterns. Participants should find the near and far position shapes by providing a valid reasoning during interview. The differences in reasoning capability were observed with and without assistance. Surprisingly there is no gender differences noted in the performance of all three grade levels. Furthermore, there is no difference in performance by gender even with and without assistance.

The outcome of the study shed some light on the performance and reasoning differences of young children by gender. At the same time, this study has highlighted the importance of function and recommendations to incorporate into primary school curriculum. The author studied the gender factor because Gong, He and Evans found gender to have strong correlation with cognitive differences, such as spatial reasoning (as cited in Cavanagh, 2016, p. 110). However, Cavanagh (2016) did not find any significant differences by gender. The author suggested for future research to consider gender factor while investigating cognitive performance. This is also encouraging researcher to look into role of gender in the relationship between cognitive variables proposed in the present study and algebraic thinking. The findings would definitely contribute towards body of literature.

These findings suggested that female and male students are capable to perform equally well in mathematics and especially in the field of algebra. However, the studies showing gender difference is possibly due to various external factors such as socio-economic level and geographical locations. An education system should look into gender equity in all aspects to avoid any gender to be left out.

**Location.** While discussing about influence of geographical location on mathematics performance, many studies have also been carried out investigating this factor (Haller, Monk & Tien, 1993). Performance in mathematics and conceptual understandings could be affected by geographical location (Abdul Ghagar, Othman, & Mohammadpour, 2011). The schools located in rural areas are commonly disadvantaged by insufficient funding, lack of teachers and the isolated locations.

According to Heller, Monk and Tien (1993), schools in rural area are actually not disadvantaged. They investigated the influence of school location on higher order thinking skills of tenth grade U.S students in mathematics and science. Though the authors anticipated urban school students to outperform rural school students, the findings failed to show a relationship between students' higher order thinking skills and geographical location.

General contention is that rural school said to be disadvantaged by isolated geographical location, funding problem and lack of qualified teachers and resources. However, Lee and McIntire (2000) suggested that schools located in rural area actually have some advantages indeed. The students in rural schools advantaged by better community support, individualized instruction as the number of students is small, with more parental commitment compared to urban school students' parents. These

advantages could balance the rural school students against the impact from isolated location, inadequate funding, and lack of teachers.

There are pros and cons due to the geographical location. According to Considine and Zappala (2002), geographical location is not a predictor of school performance in Australia. Their study was intended to investigate the influence of social and economic in the academic performance. The samples comprised 3329 students from year one to twelve in Queensland, South Australia. Due to vast literature on influence of school on academic achievement, it was conjectured geographical location as one of the predictors of academic performance. However, sex, unexplained absences, ethnicity, parental educational attainment, housing type and student age were found to be the significant predictors of academic performance. On the other hand, family income and school location did not act as predictors of academic performance.

At the same time, it is also evident that geographical location of schools does influence the academic achievements based on a few studies conducted in different countries. The studies conducted investigating the influence of school location in students' achievement found difference between the achievements of rural and urban school students (Alordiah, Akpadaka, & Oviogbodu, 2015; Shuaibu, 2014). Alordiah, Akpadaka and Oviogbodu (2015) attempted to evaluate the effect of gender, school location, and socio-economic (SES) status on secondary school students' mathematics achievement in Nigeria. The findings have shown urban school students outperformed rural school students. The authors argued that the difference of performance between rural and urban school students was due to hesitation of teachers work in rural schools. In addition, students in rural schools often spent their time working in farms to help their parents.

Similarly, Shuaibu (2014) examined the mathematical thinking ability in rural and urban senior secondary school students in Nigeria. The findings have shown rural school students had greater mathematical ability compared to urban school students. These two previous studies were conducted in Nigeria but found contradictory results. First study showed urban school students outperformed rural school students in mathematics but latter showed vice versa. Therefore, the results have shown both rural and urban school students were capable to perform in mathematics. The difference in the achievement and ability could be dependent on individuals and other external factors regardless of geographical location. Further detailed investigation is still needed to make a conclusion about influence of geographical location.

### **Research Gap**

Firstly, many researches have been conducted examining cognitive variables contributing to algebra and mathematics achievement. Past studies have primarily involved case studies and teaching experiments that highlighted the effectiveness of particular instructional approaches that create contexts to formulate and justify primary pupils' algebraic thinking components (Brizuela & Schliemann, 2004; Carpenter et al., 2003; Lannin, 2005; Stacey, 1989). However, to the best of researcher's knowledge, there are no studies which have been conducted on the inspection of cognitive variables and algebraic thinking. For this purpose, the present study is primarily aimed to identify the cognitive variables (i.e., number sense, operation sense, symbol sense, and pattern sense) and their influence on year five pupils' algebraic thinking and the web of connection among these components which were investigated by using structural modelling techniques.



Secondly, the present study has also revealed the algebraic thinking in the frame of arithmetic. A compilation of items from number sense, operation sense, symbol sense and pattern sense provide an opportunity for educators to get a quick idea what algebraic thinking in primary school is all about. The items in ANOSPS show what type of items can be included in the process of teaching and learning in classrooms. Especially, in the pattern sense items, generation of 'rule' has been emphasised. Thus, with teaching pattern sequence in the classroom, educators could provide more attention for generation of 'rule'. Though the items were from past studies, the present study instrument could provide a compilation of those items in the body of literature.

Thirdly, researcher could hardly locate any studies on examining role of gender and location in algebraic thinking particularly in primary schools. The study involving algebraic thinking of primary school pupils has focused on their ability and thinking strategies. Generally, studies have been conducted on investigating influence of gender and location in secondary level algebra achievement or mathematical thinking. Therefore, even in the literature of present study only discussed the role of gender and location in secondary school level algebra and mathematics achievement. The present study took a step further to study the influence of gender and location on algebraic thinking in the primary school level.

Furthermore, to date there is no studies that have been carried out to investigate the moderating influence of gender on the relationship between cognitive variables and algebraic thinking, although college algebra achievement and young children's algebraic reasoning differences by gender have been studied and reported by many mathematics scholars (Cavanagh, 2016; Susac et al., 2014; Xolocotzin & Rojano, 2015). Likewise, no studies have been conducted to investigate the moderating effect

of location on the relationship between cognitive variables and algebraic thinking. Differences of rural and urban students' performance were only investigated in the perspectives of mathematics achievement and thinking (Chen, 2012; Nepal, 2016).

As in Malaysia, very few local studies have been conducted to investigate Malaysian primary pupils' algebraic thinking. Lim (2007) evaluated nine varying levels of ability from 4 students' ability in solving linear equations. The findings have shown that the low achievers were unable to explain the linear relationship in a linear pattern. However, the moderate achievers, though able to explain the linear relationship verbally or arithmetically, but still were unable to generalise the linear pattern in the form of algebraic expression or linear equation. Finally, the high achievers were able to describe and generalise linear patterns, apply linear concept and then analyze the elements (constant, coefficient and variable) in a linear equation. Gan (2008) studied 13 Year five pupils' early algebraic problem-solving process and inferred their algebraic thinking underlying their solution processes in Kota Samarahan, Sarawak. His findings revealed 13 student's solution processes based on strategy, modes of representation and justification. He has also inferred algebraic thinking underlying the subjects' solution processes. However, the result may not be used for generalisation purpose as the sample size is very small. In addition, the tasks used in the study did not encompass all the three strands of algebra. For instance, the aspects of equality and relation knowledge were not covered in that study.

In addition, Malaysia did not participate in TIMSS for grade four categories. Hence, to date there is no data to show the achievement of upper primary pupils' in algebraic thinking. The findings of present study filled the research gap by providing data on algebraic thinking of year five pupils. The data may provide an insight view of year five pupils' algebraic thinking and may assist to foresee the causes for form

two students' poor performances in TIMSS. The results may also be used to do comparison between Malaysia and other top performing countries in TIMSS.

Beside the aforementioned research gaps, there is no formal evidence to show to what extent algebra is within the reach of primary school students. Hence, policy makers, educators and parents may not be sure of algebraic thinking capabilities of students. Consequence of the unawareness of young students' algebraic thinking may hinder respective parties to incorporate algebraic thinking in the instructional design prior to formal algebra learning. This is especially when there is a gap to understand to what extent algebra can be included in primary school mathematics curriculum. This is because there is not any proper document to show what the elements of algebraic thinking are and how it may be included while teaching arithmetic.

Cognitive variables contributing to success in algebraic thinking aside, the present study has contributed algebraic thinking diagnostic assessment (ATDA) in local context; is an added advantage which completely measures algebraic thinking from all the aspects synthesised from literature. The instrument also designed in dual languages, English and Malay, national language, which is indeed an added advantage as so far there is no any algebra thinking measurement tool available for primary pupils in the national language.

## **Summary**

In summary, review of literature in this section has discussed numerous studies focusing on several aspects including importance of bridging algebra from arithmetic, ability of young children to think algebraically, identifying the mistakes and misconception of middle school students in algebra, performance of young children in

algebraic thinking. To summarise, Table 2.2 shows at a glance of what has been done in literature so far.

Table 2.2

*Summary of algebraic thinking literature focus*

<b>Focus</b>	<b>Source</b>	<b>Area</b>
Capability of young children to think algebraically	Stacey (1989)	Patterns
	Belliso (1999)	Variables
	Swafford & Langrall (2000)	Patterns & variables
	Carpenter et al. (2003)	Generalised arithmetic
	Lannin et al. (2006)	Patterns

Table 2.2, continued

	Gan (2008)	Patterns & variables
	Spang (2009)	Variables
	Hunter (2010)	Generalised arithmetic
	Sorkin (2011)	Patterns
	Slavit (1999)	Arithmetic structure sense
	Jacobs et al. (2007)	Generalised arithmetic
	Sherman & Bisanz (2009)	Equivalence
	Rittle-Johnson et al. (2011)	Equivalence

Evaluation of primary pupils' algebraic thinking	Matthews et al. (2012)	Equivalence
	Ralston (2013)	Generalised arithmetic, variables, patterns
	Halдар (2014)	Generalised arithmetic

However, contribution of cognitive variables towards algebraic thinking has been neglected. The fundamental issue is "What are the elements which influence primary pupils' algebraic thinking?" remains still unanswered from cognitive perspective. In other words, cognitive abilities of students should be questioned in early stages in order to excel in algebra in the later stage of education. To date, there are no studies which have been conducted to determine what cognitive variables influence algebraic thinking of primary pupils.

Although previous studies have shown that young children are capable of thinking algebraically, most studies have analysed qualitatively by focusing on ability and thinking process of young students. Limited studies have been conducted quantitatively in measuring young students' algebraic thinking. There are especially no studies on what cognitive variables influence young students' algebraic thinking. The web of connection between cognitive variables and algebraic thinking are yet to be discovered. This connection is necessary to be studied in order to get a clear picture of what "qualities" will ensure algebraic thinking in early stage of school education.

This chapter also has reviewed cognitive variables of present study. Figure 2.2 summarises the cognitive variables of present study. The purpose of the present study

leads to the choice of research design, data collection methods and data analysis procedures for present study, and will be discussed in chapter three.

University of Malaya

## **Chapter 3 Methodology**

### **Introduction**

This chapter discusses about on the methodology of the study. The discussion consists of eight main sections: research design, location and samples, data collection, instrumentation, validity and reliability, pilot study, and data analysis. The variables to be considered are scores earned in ANOSPS for each construct (i.e., number sense, operation sense, symbol sense, and pattern sense) and also total score of ATDA.

### **Research Design**

Research design is a plan designed by the researchers about how a study to be conducted in obtaining answers to research questions which involve data collection, analysing data and report writing (Creswell, 2009). The decision of choosing a research design depend on three main aspects namely suitability, feasibility, and if it ethical to be carried out (Denscombe, 2010). The present study utilised a descriptive research design which is a cross sectional study as the researcher collected data from a sample of the population that has been identified in advance and carried out in a specific period of time. The method used in this design was collecting data using mathematics tests. It is important to take note that research design does not mean research method (Denscombe, 2010). The method used to collect data was mathematics tests. Descriptive research is most appropriate when the aim of the study is to view the problem comprehensively and also to obtain data for mapping at the current state of time. With regard to this, the aim of the present study was to evaluate year five pupils' algebraic thinking and performance in cognitive constructs and perform mapping at the current state of time. Therefore, descriptive research design is the most suitable of all. The advantages of this design are it provides wide and

inclusive coverage. Secondly, it helps to provide a data of how things are at a specific point in time. This means in the present study, how the performances of year five pupils were in algebraic thinking when the data collection was carried out.

## **Hypotheses**

To answer the research questions listed earlier in Chapter 1, and also based on the literature evidence as discussed in Chapter 2, the following hypotheses were developed:

- H1:** Number sense has positive influence on year five pupils' algebraic thinking.
- H2:** Operation sense has a positive influence on year five pupils' algebraic thinking.
- H3:** Symbol sense has a positive influence on year five pupils' algebraic thinking.
- H4:** Pattern sense has a positive influence on year five pupils' algebraic thinking.
- H5:** Gender moderates the relationships between number sense and algebraic thinking.
- H6:** Gender moderates the relationships between operation sense and algebraic thinking.
- H7:** Gender moderates the relationships between symbol sense and algebraic thinking.
- H8:** Gender moderates the relationships between pattern sense and algebraic thinking.
- H9:** Location moderates the relationships between number sense and algebraic thinking.
- H10:** Location moderates the relationships between operation sense and algebraic thinking.



**H11** Location moderates the relationships between symbol sense and algebraic thinking.

**H12** Location moderates the relationships between pattern sense and algebraic thinking.

### ***A priori* Model**

To address the aims of the present study, structural equation modelling (SEM) is proposed to test a model that consists of cognitive variables influence the algebraic thinking. Specifically, differential influences of selected cognitive variables have been hypothesized. Predictive paths have been hypothesized from operation sense, symbol sense, number sense, and pattern sense to algebraic thinking. All the paths from independent variable to dependent variable predicted as direct influence. This is because there is no previous quantitative study to show any indirect or mediating relationship between these variables. Figure 3.1, shows the hypothesized path model of year five pupils' algebraic thinking.

*A priori* model in Figure 3.1 shows the five constructs and two moderators involved in this study. In the *a priori* model shown in Figure 3.1, there are five measured variables namely: number sense, operation sense, symbol sense, pattern sense and algebraic thinking. Number sense, operation sense, symbol sense and pattern sense are four independent variables or also known as exogenous variables. Four of them are measured using ANOSPS. Algebraic thinking is dependent variable or also known as endogenous variable. The algebraic thinking latent variable has three indicators namely; generalised arithmetic (GA), modelling and function. Due to the space limitation, the three strands of algebraic thinking as well as all the indicators were not shown but included in the measurement. The algebraic thinking score

consists of total scores obtained in ATDA. This total score is based on the three indicators.

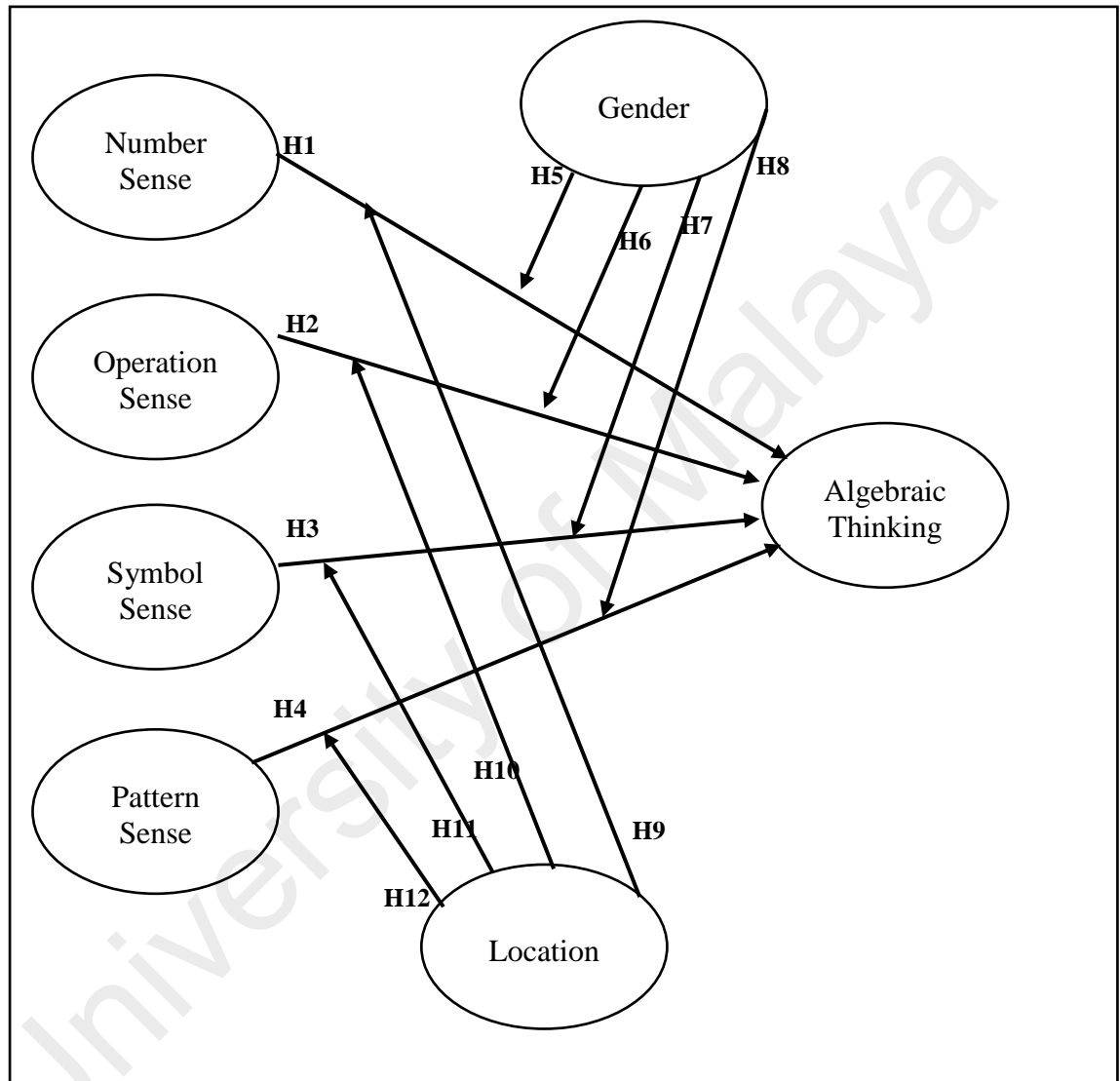


Figure 3.1. A priori model of year five pupils' algebraic thinking

The hypothesized model has attempted to predict algebraic thinking through measured variables of operation sense, symbol sense, pattern sense, and number sense. The four independent variables namely, number sense, operation sense, symbol sense, and pattern sense have indicators from Total01 to Total15iv. These indicators are the

items tested in ANOSPS. The score obtained from each item in ANOSPS represents the value for these indicators. The arrows shown in Figure 3.1 are constructed based on evidence from literature. Gender and location are moderators. These have been included to investigate the moderating effect of gender and location on each direct effect. The subsequent section discusses the reasons for each hypothesized relationship in the *a priori* model.

As explained previously in problem statement section, number sense, operation sense, symbol sense and pattern sense identified by researchers as potential to develop algebraic thinking in early ages. Number sense plays an important role in fostering algebraic thinking (Molina et al., 2008; NCTM, 2000; Warren, 2003b). Number sense eases the process of shifting from arithmetic thinking to algebraic thinking. Flexibility in handling computation procedure is the main element of number sense. The characteristics of number sense will actually lead to work with algebraic structure. This is also clearly visible from The New Zealand Numeracy Project in New Zealand and Algebra for All projects in U.S. Number sense was included as one of the topics in these two major projects. Hence, a direct effect from number sense to year five pupils' algebraic thinking has been hypothesized in the *a priori* model of present study.

In the same way, the role of operation sense too is explored in the studies of early algebraic thinking (Molina & Ambrose, 2008; Slavit, 1999; Warren, 2003a). In algebra, the students are required to analyse expressions by comparing both sides of an equal sign to find the solution (i.e.,  $4x + 8 = 3x + 18$ ). Such analysing skill enhances the students' operation sense which acts as basic element for algebraic thinking (Slavit, 1999). Both number sense and operation sense are used when the students are engaged with analysing expressions. In early years, they probably should be exposed to number sentences which require attentive focus on arithmetic relations rather than computation

and final answer (Molina & Ambrose, 2008). Therefore, operation sense is anticipated to contribute towards year five pupils' algebraic thinking in the present study.

Then again, symbol sense has received equal attention in early algebraic thinking studies (Fujii & Stephens, 2001; Fujii & Stephens, 2008; MacGregor & Stacey, 1997; McNeil et al., 2010). Symbols comprised variables (as in letters) and equal sign. For young children, concept of variables needs to be introduced as “a letter represents a number”. They are not able to grasp the full range of variation, where variables can also represent rational and negative numbers (Fujii & Stephens, 2008). Such introduction referred as “quasi-variable” by Fujii and Stephens (2001; 2008). Similarly, equal sign is another symbol that plays an important role in algebraic thinking (McNeil et al., 2010; Stephens et al., 2013). This is especially for relational view of an equal sign rather than operational view. Lack of relational understanding of equal sign could be the cause for middle school students' difficulties when working with equivalence and variables (McNeil et al., 2006). In line with this, symbol sense has been hypothesized as potential to have direct effect on algebraic thinking.

Lastly, pattern sense is one of the most widely explored areas in the studies of early algebraic thinking (Childs, 1995; Gan & Munirah Ghazali, 2008; Warren et al., 2006). Working with patterns enables children to identify the co-variation relationships, which is a necessary skill to foresee numerous steps involved in the relationships, and also to find a general solution (Warren, 2005). Patterning activities help young children to develop their way of thinking algebraically in suitable ways (Blanton & Kaput, 2004). Working with patterns facilitates an introduction to functional thinking which requires understanding of sophisticated concepts (Zazkis & Liljedahl, 2002). As one of the three strands in algebraic thinking is function as

defined by Kaput (2008), pattern sense is inevitable in the search of influencing cognitive factor of year five pupils' algebraic thinking.

Despite these variables, it would be also interesting to investigate the moderating role of gender and location on each of the proposed direct effect. As stated in problem statement, gender and location also have played an important role in the studies of mathematics. These two factors have also been a vital issue in Malaysia, as many studies and international assessments such as PISA and TIMSS recorded significant differences between gender and location. Hence, these two factors are also included in the hypothesized model to find its moderating effects as shown in Figure 3.1.

Preceding discussion about number sense, operation sense, symbol sense and pattern sense have shown evidence that these cognitive variables have emerged from literature. The researcher does not pick these factors randomly. However, in the studies mentioned earlier, there is no concrete evidence to demonstrate the role of these cognitive variables in the young children's algebraic thinking. Most have been based on observations during teaching experiments, clinical interviews and researchers' own interpretations about these cognitive variables roles in developing algebraic thinking. This gap has brought up the need to investigate the connections between these variables and algebraic thinking. As such, Figure 3.1 shows the hypothesized model based in the synthesis of literature search. As to date, there is no previous model on primary pupils' algebraic thinking. Therefore, the cognitive variables are predicted as has direct relationship on year five pupils' algebraic thinking. However, more paths and connections are anticipated between these cognitive variables and year five pupils' algebraic thinking. The hypothesized model also acts as a conceptual framework of the present study.

The hypothesized model includes all the four independent variables, one dependent variable and together with two moderators. The labels from H1 to H12 show the hypotheses of the present study.

### **Location and Sample**

Generally, sample size calculation involved three approaches namely statistical, pragmatic and cumulative (Denscombe, 2010). The proper approach will be statistical approach. It involves large-scale surveys and probability sampling techniques. Pragmatic approach involves smaller-scale surveys due to cost and time constrain. It involves small-scale surveys and more to non-random sampling techniques. Meanwhile, cumulative approach uses small-scale surveys using non-probability and purposive sampling which mostly used in qualitative researches. In the present study, researcher used statistical approach to get comprehensive data by using large-scale despite the high cost involved. It also took quite a longer time to key in data in SPSS. Bigger sample size provides representative samples and unlikely to be biased.

Thus, the present study used a sample size of 720 year five pupils from random selection of school in *Melaka Tengah*. This large sample size was determined based on two reasons. Firstly, this sample size was more than the required minimum sample size for the power calculations. Secondly, a large randomised sample of students had participated to allow for generalisability of results.

The study took place in a district of Malacca. The districts in Malacca are categorised into Alor Gajah, *Melaka Tengah* and Jasin. The present study's target population was only from *Melaka Tengah* in the interest of cost and time. There are total of 56 *Sekolah Kebangsaan* (SK) schools (clusters) in this district. 27 schools are located in urban area while 29 schools are located in rural area. The total number of

year five pupils in this district was 5347 at the time of the study. There were 2215 (41.4%) from urban schools and 3132 (58.6%) were from rural schools.

The present study utilized a cluster random sampling technique, with students clustered by school. Cluster sampling refers to all people within the cluster/ groups are included in the sample. Denscombe (2010) defined this as, "A cluster sample could be based on a random selection of schools and the inclusion of all students within those schools" (p. 29). This can be carried out based on assumption that each cluster comprises cross-section of the wider population in terms of things like age, sex, ethnicity, social background and academic ability. As such, in the present study school referred to cluster and all year five pupils in the particular school were involved in the study. Cluster sampling is proposed in the interest of cost although there is a disadvantage in reduction of precision associated. Instead of taking greater number of participants within each cluster, it is advisable to take more number of clusters in order to increase the precision (Kalton, 1983). Following that, a simple random sampling was used to choose schools from total of 56 schools. Simple random sampling was conducted using *Rand()* function which was available in Microsoft Excel 2010. All the schools from the list were numbered 1-56. Then random numbers were generated using Microsoft Excel 2010. For the total number of samples, the researcher used Krejcie and Morgan (1970) table (see Figure 3.2) to determine the sample size based on the population size. The sample size in this table was created based on 95% confidence interval.

Based on this table (Figure 3.2), for population of 2215 (urban) and 3132 (rural) desired sample size was around 327 and 341 respectively. However, good result and even precise information was gained when more sample size was taken.

Hence, the researcher had drawn 720 students in total with 360 pupils from urban and rural each. Participants of the present study were year five pupils who had enrolled in 2016 school year. The samples comprised of 370 (51.4%) female pupils and 350 (48.6%) male pupils. The fact that all pupils in year five from a school were included in the study justified the suggestion that results might be generalised to future students in year five in the same zone. In each school all year five pupils involved (i.e., a large number of participants within the cluster), and as many schools (i.e., clusters) as possible were included in this sample. Although it is ideal to include a sample of all the schools in the districts, not all districts were chosen to further reduce travel costs and time consumption. Only randomly selected national (*sekolah kebangsaan*) public schools' year five pupils were included in this sample. Chinese, Tamil and other schools were omitted due to the language factor of the instruments.

The sample size was also more than enough to fulfil the requirement to of PLS-SEM. As the present study utilised PLS-SEM method, it required only smaller sample size. Hair, Hult, Ringle and Sarstedt (2014) observed, "PLS-SEM has higher levels of statically power in situations with complex model structures or smaller sample sizes" (p. 20). With regard to this, PLS-SEM recommends the 10 times rule suggested by Barclay, Higgins and Thompson (1995). This rule requires sample size to be 10 times either the factor that contains the biggest number of formative indicators or 10 times the biggest number of structural paths linked to a specific construct in the structural model. Based on this rule, the proposed model's biggest number of indicators is 6: thus, the required minimum sample size is  $6 \times 10 = 60$  cases.



N	S	N	S	N	S
10	10	220	140	1200	291
15	14	230	144	1300	297
20	19	240	148	1400	302
25	24	250	152	1500	306
30	28	260	155	1600	310
35	32	270	159	1700	313
40	36	280	162	1800	317
45	40	290	165	1900	320
50	44	300	169	2000	322
55	48	320	175	2200	327
60	52	340	181	2400	331
65	56	360	186	2600	335
70	59	380	191	2800	338
75	63	400	196	3000	341
80	66	420	201	3500	346
85	70	440	205	4000	351
90	73	460	210	4500	354
95	76	480	214	5000	357
100	80	500	217	6000	361
110	86	550	226	7000	364
120	92	600	234	8000	367
130	97	650	242	9000	368
140	103	700	248	10000	370
150	108	750	254	15000	375
160	113	800	260	20000	377
170	118	850	265	30000	379
180	123	900	269	40000	380
190	127	950	274	50000	381
200	132	1000	278	75000	382
210	136	1100	285	100000	384

Note: *N* is population size, *S* is sample size

*Figure 3.2.* Table for determining sample size from a given population. Adapted from "Determining sample size for research activities," by R.V. Krejcie, D.W.

Morgan, 1970, Educational and Psychological Measurement, 30, p.608. Copyright

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Besides that, when the sample size is big, it commonly produces higher power for the statistical analysis with respect to the alpha level. In addition, Pallant (2013) asserted that sample size, effect size and alpha level are the three factors that usually

influence the power of test. Consequently, he suggested, the ideal sample size should be more than 150 cases with a ratio of five cases for each indicator. Following this rule, the present study has total of 45 items in both instruments (ANOSPS and ATDA) and according to 5:1 ratio, the minimum sample size would be  $45 \times 5 = 225$  cases. Thus, a sample size of 720 was definitely more than enough to conduct the analysis.

The samples selection process began with acquiring written consent from local Educational Planning and Research Division (EPRD), Ministry of Education to conduct the study in Malacca public primary schools. This letter was shown to Malacca Education Department to get another consent letter. These two consent letters were shown to school principals to obtain their permissions to conduct the study in their schools.

### **Data Collection Procedures**

At first, a comprehensive review of the literature such as journal articles, relevant theses, dissertations, and books on early algebraic thinking were investigated and gathered by the researcher to search the relevant cognitive variables of primary pupils' algebraic thinking. Various algebraic thinking aspects were inspected for the development of ANOSPS. Overlapping features were also taken into consideration from relevant studies. In addition, relevant subscales from literature for the selected cognitive variables also were adapted. Test specifications were derived from synthesis of these relevant literatures. 19 items were developed for ANOSPS. After necessary consent permissions granted, ANOSPS was then pilot tested to year five pupils of year 2015. Second instrument was ATDA which was adapted from Ralston (2013) with her consent. ATDA was specifically created and validated diagnostic assessment tool

for the fifth-grade students (Ralston, 2013). It was developed to measure fifth grade students' algebraic thinking.

After revising the ANOSPS items according to the results of the pilot study and obtaining required permissions from the school, the study was conducted to year five pupils in the mid of 2016 school year. Teachers were advised to administer the ANOSPS and ATDA during the class hours on the same day to locate the same students. The researcher was also present at the schools when the tests were carried out to clarify doubts of teachers and students. The teachers were informed beforehand about the instructions for the administration of the instruments (see Appendix C) and the directions and descriptions for students were included in the instruments. 40-60 minutes were deemed sufficient to complete each instrument. This duration was estimated based on the pilot test.

### **Instruments**

The researcher developed the instruments to be used for data collection with most items which were adapted from previous studies. The medium of instruction for mathematics lessons in classrooms at participated schools was Malay. Some of the schools also had Dual Language Program (DLP). The DLP is where the students learn science and mathematics in English. Considering these issues, the instruments were administered in both English and Malay languages. This is to minimize language constraint as a factor that might impact the result of this study. The content of related evidence was maintained by translating the ANOSPS and ATDA in English into Malay and any differences between the original inventory in English and the translated version were noted in terms of wording by language experts. It was also back translated. In addition, the ANOSPS content and coverage were examined by a panel

of ten experts which comprised local and foreign professors, senior lecturers and lecturers of Mathematics education in terms of face and content validity. Suggestions were noted and required revisions were made. The details of modifications were explained further in pilot study section.

**Assessment of Number, Operation, Symbol and Pattern Senses (ANOSPS).** ANOSPS was developed to assess constructs associated with primary pupils' algebraic thinking. This assessment was developed using four constructs discussed earlier in chapter 2 (section cognitive variables). ANOSPS comprised 19 items in total with four cognitive factors i.e., number sense, operation sense, symbol sense, and pattern sense. Similar to ATDA, this instrument consisted of items developed using only arithmetic and patterning questions. The item structure was slightly different from ATDA. Each item had two sections. In first section, a student should choose the right answer from given four multiple choices (i.e., A, B, C and D). The second section of the same item required the student to choose a valid reason for their answer from given three multiple choices (i.e., A, B, and C). First section of all items from ANOSPS was scored dichotomously that was 1 and 0 for correct and incorrect responses respectively. Second section score for each item ranged from 0 to 2. Final score for each item was based on the combination of the total score for these two sections. The final score ranged from 1 to 6. These figures were used for coding purpose. The reasoning classification and scoring details are shown in Table 3.1. The scores for the combination of two sections are shown in Table 3.2.

Table 3.2 illustrates how the total score for each item in ANOSPS was calculated. If a sample gets 1 for correct answer in the questions section and then a score of 2 for reasoning section, the total score will be 6. Similarly, if the sample gets

0 for correct answer in the questions section and then a score of 1 for reasoning section, the total score will be 2. The total score was calculated based on these pairs. A student will get highest score of 6 when one provides correct answer and conceptual reasoning. It is also notable that higher score was given for the combination of incorrect answer and conceptual reasoning (0, 2) compared to correct answer and incorrect reasoning (1, 0). This is because correct answers are not always reflected a good thinking (Yang, Li & Lin, 2008). Therefore, a correct reason should be given higher priority than a correct answer. Thus, the total scores generated as such that a correct answer without a correct reason was scored 3 while a correct reason without a correct answer scored with 4.

Table 3.1

*ANOSPS reasons classification and its scores*

<b>Constructs</b>	<b>Items</b>	<b>Reasons</b>	<b>Score</b>
Number Sense	Q1, Q2, Q3,	i) Number sense based	2
	Q4, Q5	ii) Rule based	1
		iii) Incorrect/ Inappropriate	0
Operation Sense	Q6, Q7, Q8	i) Conceptual	2
		ii) Computational	1
		iii) Incorrect/ Inappropriate	0

Table 3.1, continued

<b>Constructs</b>	<b>Items</b>	<b>Reasons</b>	<b>Score</b>
Symbol Sense	Q9, Q10	i) Relational	2
		ii) Operational	1
		iii) Incorrect/ Inappropriate	0
	Q11, Q12, Q13	i) Valid reason	2
		ii) Trivial reason	1
		iii) Incorrect/ Inappropriate	0
Pattern Sense	Q14i, Q15ii	i) Deductive	2
		ii) Empirical	1
		iii) Incorrect/ Inappropriate	0
	Q14ii, Q14iii, Q15iii, Q15iv	i) Explicit	2
		ii) Recursive	1
		iii) Incorrect/ Inappropriate	0

Table 3.2

*Total score for each item based on questions and reasoning sections*

<b>Score for questions</b>	<b>Score for reasons</b>	<b>Total score for the item</b>
0	0	1
	1	2
	2	4
1	0	3
	1	5
	2	6

Table 3.3 shows the item difficulty and item discriminant indices for all items in ANOSPS. Item difficulty is the proportion number of samples answered an item correctly by the total number of samples. As a general guideline, item difficulty range is advisable to be within the range of 30% to 80% (Kehoe, 1995). All the items in ANOSPS do fulfilled this requirement except item Q1. The item difficulty index was 0.18 as shown in Table 3.3. This item tests on the pupils' ability to making sense of numbers. Especially on understanding the number meanings and relationships. However, the low item difficulty index could not be interpreted as the item is too tough. As this instrument is aimed for diagnostic in nature in exploring year five pupils' ability in number sense, operation sense, symbol sense and pattern sense. Therefore, this item was not eliminated (Ralston, 2013).

As for item discriminant index, a rule of thumb, “.40 and greater are very good items, .30 to .39 are reasonably good but possibly subject to improvement, .20 to .29 are marginal items and need some revision, below .19 are considered poor items and need major revision or should be eliminated” (Ebel & Frisbie, 1986, p. 232). The

discriminant index for item Q1 considered very low based on rule of thumb range. However, it is not an adequate indication to show this item unable to discriminate from good and weak performing pupils. As the tests used in the present study are diagnostic in nature, again, this index would not be adequate to use in this diagnostic natured instrument (Ralston, 2013).

Table 3.3

Item difficulty and item discriminant indices for ANOSPS items

<b>Items</b>	<b>Item Difficulty Index</b>	<b>Item Discriminant Index</b>
Q1	0.18	0.10
Q2	0.52	0.30
Q3	0.51	0.35
Q4	0.60	0.43
Q5	0.72	0.41
Q6	0.63	0.61
Q7	0.47	0.52
Q8	0.49	0.29
Q9	0.89	0.22
Q10	0.61	0.30
Q11	0.51	0.56
Q12	0.49	0.53
Q13	0.45	0.67
Q14i	0.53	0.38
Q14ii	0.64	0.59
Q14iii	0.47	0.50



Table 3.3, continued

Items	Item Difficulty Index	Item Discriminant Index
Q15ii	0.52	0.42
Q15iii	0.62	0.50
Q15iv	0.48	0.56

**Algebraic Thinking Diagnostic Assessment (ATDA).** The algebraic thinking diagnostic assessment as developed by Ralston (2013) consisted of items to assess year five pupils' algebraic thinking. The ATDA was selected because it was the only assessment tool available in the literature to measure elementary students' algebraic thinking which encompasses all three strands of algebraic thinking defined by Kaput (2008). In addition, it was appropriate for the sample group involved in this study. This assessment is available in five different levels to cater grade one to five pupils. As the present study only focused on year five pupils, the researcher had only requested assessment for grade five from the author.

It was developed and tested using fifth grade students enrolled in both Singapore and U.S and was analysed for internal consistency. In the development of this instrument, Cronbach's alpha coefficient was calculated. For scores on the 27 items, alpha was 0.81 indicating a high degree of internal consistency for group analyses. The inventory has a mean of 17.20 with a standard deviation of 4.83. However, it was reported that there would be better reliability if meaning of equal sign item was deleted. The researcher has excluded this particular item for better internal consistency and also it was an overlapping symbol sense item and this item is more

appropriate to be in ANOSPS as its content wise. Therefore, the total number of items reported in ATDA is 26.

ATDA items were broken into three strands of algebra according to Kaput's (2008) classification of algebraic thinking. They are a) Modelling, which includes work with variables, understanding equivalence, and solving open number sentences; a) Generalised Arithmetic, which includes generalising (i.e., utilising mathematical properties like the associative, commutative, zero properties, etc.) and efficient numerical calculations (i.e., simplify calculations using number compensation strategies and relations); and c) Functions, which includes possessing the "ability to recognize, describe, extend, and create patterns" (Ralston, 2013, p. 54).

ATDA consisted of 26 items. It also comprised two constructed-response items. Constructed-response items were scored with a scoring rubric with 3-levels (0-2) adopted from Ralston (2013). The total score was worth approximately 28 points. Scoring guides are available in Appendix G. This scoring rubric was established by Ralston (2013). Modelling has contributed the majority of the points, with about 42% of the total score, with the remaining points divided up between Generalised Arithmetic (33%) and Function (25%). The number of items per dimension and per dimension strand on each assessment is displayed in Table 3.4

This assessment was designed to be administered in one sitting in approximately 30 minutes (Ralston, 2013). However, during the pilot study the researcher observed students had needed more than 30 minutes to complete it because they were not familiar with the questions structures. Thus, the test was not a timed test so the teacher would be able to choose and allow more than 30 minutes of time or discontinue testing at the end of their double period (which is 60 minutes) as proposed by Ralston (2013).

Table 3.4

*Classification of ATDA items into sub-constructs*

<b>Construct</b>	<b>Sub-construct</b>	<b>Question number</b>	<b>Total items</b>
Modeling	Solving open number sentences.	1,2,3	10
	Understanding equivalence.	4,5,6	
	Work with variables.	7,8,9,10	

Table 3.4, continued

Generalized arithmetic	Efficient numerical manipulation.	11i, 11ii, 11iii, 11iv, 11v, 11vi	8
	Generalization.	12,15	
	Numerical linear patterns	13	
Functions	Numerical nonlinear patterns	16	8
	Figural linear patterns	14, 14i, 14ii	
	Figural nonlinear patterns	17, 17i, 17ii	

Table 3.5 shows the results of item difficulty and discriminant indices for ATDA. The item difficulty index is in the acceptable range of 30% and 80% (Kehoe, 1995) except 3 items (i.e., Q1, Q14ii, and Q17ii). Item Q1 had the difficulty index of 0.97. It fell in the easiest category because majority of the pupils were familiar with the structure of missing addend (i.e.,  $6 + \underline{\quad} = 13$ ). They easily solved the item. On the other hand, item Q14ii and Q17ii fell in the most difficult category. It had tested on the ability of pupils to do far generalisation. The samples had a tough time to do

far generalisation in both numerical and figural type. Only a few managed to solve it. However, these three items were retained as those were developed based on theory-driven conceptualization. Eliminating these items might affect the conceptualization of algebraic thinking strands. Moreover, it is a diagnostic assessment in nature. Therefore, the item difficulty index acceptable range would not be appropriate to determine the adequateness of the item (Ralston, 2013). For the same reason, item discrimination index acceptable range also was calculated but the items with lowest item discrimination index were not eliminated.

Item discriminant index for all the items were in general acceptable range except item number Q1 and Q11iii. These two items fell in the category of to be revised. However, the researcher did not drop or modify the item. The discriminant index was influenced by majority of the samples who were able to answer it correctly (i.e.,  $p = 0.97$  and  $p = 0.92$ ). Hence, it was important to retain these items to examine the ability of samples to work with open number sentences and understandings of properties of operations (Ralston, 2013). These items were created and thoroughly analysed by Ralston (2013) to ensure it is necessary to be included to evaluate year five pupils' algebraic thinking.

Table 3.5

Item difficulty and item discriminant indices for ATDA items

<b>Items</b>	<b>Item Difficulty Index</b>	<b>Item Discriminant Index</b>
Q1	0.97	0.08
Q2	0.46	0.42
Q3	0.86	0.65
Q4	0.43	0.65

Table 3.5, continued

<b>Items</b>	<b>Item Difficulty Index</b>	<b>Item Discriminant Index</b>
Q5	0.43	0.71
Q6	0.48	0.78
Q7	0.62	0.65
Q8	0.56	0.72
Q9	0.49	0.84
Q10	0.53	0.63
Q11i	0.83	0.22
Q11ii	0.76	0.28
Q11iii	0.92	0.13
Q11iv	0.64	0.48
Q11v	0.80	0.28
Q11vi	0.50	0.28
Q12	0.54	0.46
Q13	0.82	0.33
Q14	0.61	0.44
Q14i	0.62	0.56
Q14ii	0.11	0.22
Q15	0.45	0.35
Q16	0.57	0.59
Q17	0.58	0.40
Q17i	0.65	0.48
Q17ii	0.11	0.26

## Pilot Study

A pilot study is essential to refine instruments and to identify any other problems in the design. It is also necessary to pilot the test instrument to make it reliable and valid. In that case, the researcher had to conduct four pilot tests in order to derive final valid instruments. The researcher conducted initial pilot study (Pilot I) with sample size of 102 to gather information about ANOSPS. Issues in the first pilot study were rectified and researcher conducted a second pilot study (Pilot II) to test the modified instrument items. Initially the ANOSPS items were not multiple choices items and also there were no reasoning sections. ANOSPS items had demanded explanations from students for responses of each item. The researcher administered the tests to 102 students in a national school in a district of Malacca. The results of Pilot I showed the year five pupils were not able to answer most of the cognitive factor items and also did not provide any explanation for their responses. The doubts were raised during the test showed that they were not familiar with the question structures and also unsure on how to write explanations. Not only they struggle to answer and write explanations, they were also unable to finish the tests within the allocated time of an hour. The researcher had an informal chat session with their mathematics teachers. The teachers also commented that their students were not used to these types of questions.

For an example, item 11 from ANOSPS was first constructed as shown in Figure 3.3. In this item, pupils had to work on their own and find the answer for  $n$ . The outcome of Pilot I showed the year five pupils struggled to answer these types of questions as they were not familiar with this format. During the test, they raised many questions and doubts on how to solve the questions. The researcher took note of all their doubts and then modified the items by rewording it. It was also translated to

Malay language and each item given was in both English and Malay languages. During the Pilot I, some students had struggled due to the language factor. The section which requires students to write explanation part was excluded. The items were modified to enable the pupils to think algebraically rather asking them for an explanation in words about what they thought. The researcher designed one item for each sub-construct. The total number of items in ANOSPS was reduced to 19 from 24. The reduction in total number items was to mainly avoid students' exam fatigue. In addition, this concern was also raised by one of the content validity experts. Consequently, the number of items was reduced. Non-linear pattern items were removed as the students struggled extremely hard to answer these questions. In initial instruments, more than one item was designed for each sub-construct. During the test modification, these items were reduced 1 or 2 for each sub-construct. After all the modification, the same item was refined to as shown in Figure 3.4.

Find the value of  $n$ . Explain your answer.  
 $n + n + n + 2 = 17$

Figure 3.3. First version of Item 11 from ANOSPS used in pilot I

Find the value of  $\nabla$ .  
*Cari nilai  $\nabla$ .*  
 $\nabla + \nabla + \nabla + 2 = 17$

Figure 3.4. Second version of Item 11 from ANOSPS used in pilot II

Pilot II was conducted with sample size of 104 in a different school. This time, the pupils' performances were better than Pilot I. However, pupils from good class

were able to answer them without any problems. Pupils in average and weak classes still had difficulty in answering the items. One of the main aims of the present study was to test how well year five pupils able to think algebraically and not to grade the pupils. As such, further modifications were done. Multiple choices items were given in ANOSPS to cater more pupils to answer the items in average and weak classes. Since, multiple choices items alone might be misleading, because pupils may just tick any of the options without knowing anything, reasoning section was also included. Reasoning section was also in three multiple choices. The selection of choices for each item was carefully designed by the researcher based on some criteria as shown in Table 3.1.

Thereafter, Pilot III was carried in a different school with a sample size of 154. The outcome of this pilot study was very positive. Many pupils were able to answer the questions and were also able to provide reasons which they had thought were valid. Figure 3.5 shows one of items used in Pilot III.

<p>Find the value of <math>\nabla</math>.  <i>Cari nilai <math>\nabla</math>.</i>  <math>\nabla + \nabla + \nabla + 2 = 17</math></p>
<p><b>Why?</b>  <i>Kenapa?</i></p> <p>A. <math>\nabla</math> is 3 same numbers which give sum of 17.  <i><math>\nabla</math> adalah 3 nombor sama yang memberi hasil tambah 17.</i></p> <p>B. <math>\nabla</math> could be any three different numbers which give sum of 15.  <i><math>\nabla</math> adalah 3 nombor berbeza yang memberi hasil tambah 15.</i></p> <p>C. <math>\nabla</math> is 3 same numbers which give sum of 15.  <i><math>\nabla</math> adalah 3 nombor sama yang memberi hasil tambah 15.</i></p>

Figure 3.5. Item 11 from ANOSPS used in pilot III



The reasons provided in reasoning sections were also sent to some experts in early algebra. They gave some valuable feedback regarding the choice of reasons for each item. Based on the feedback and the researcher's decision, more changes had been made to the items especially the reasoning sections. For example, the word "Find" in the item 11 was suggested to change to "Think of the values". This rewording did make some differences on how the item was designed to let year five pupils to work with variables. In addition, option C was reworded as shown in Figure 3.6 based on an expert's feedback.

**Question 11:**

**Soalan 11:**

Think of the values for  $\nabla$ .  
Fikirkan nilai-nilai bagi  $\nabla$ .  
 $\nabla + \nabla + \nabla + 2 = 17$



**Reason:**

**Sebab:**

- A.  $\nabla$  is three same numbers which give sum of 17.  
 $\nabla$  adalah tiga nombor yang sama dengan hasil tambahnya 17.
- B.  $\nabla$  can represent any numbers.  
 $\nabla$  boleh mewakili sebarang nombor.
- C.  $\nabla$  is three same numbers which give sum of 15 because 15 plus 2 is 17.  
 $\nabla$  adalah tiga nombor yang sama dengan hasil tambah 15 kerana 15 tambah dengan 2 adalah 17.

Figure 3.6. Item 11 from ANOSPS used in pilot IV

The pilot studies involved administering and evaluating the both test instruments (ANOSPS and ATDA). However, validity and reliability measures were not performed for ATDA as it was already a validated instrument. The details of

development and validation processes were discussed in Ralston (2013). ATDA was tested in terms of its content and face validity.

Overall, the ANOSPS was re-designed into multiple choice items with additional reasoning sections. The question structures were closely designed following the format of year six mathematics national examination, *UPSR*. The pupils were very accustomed to *UPSR* kind of questions. Hence, students might feel ‘friendliness’ with the items and structures. Other minor improvement was pupils were told to write the answers in the given separate answer sheet as shown in Appendix E. This was to ease work of the researcher while scoring.

### **Content Validity and Reliability**

The content validity of the instruments was done by a panel of ten experts. The panel of experts in the present study consisted of ten local and foreign university lecturers. The university lecturers specialise in mathematics education. The instruments (ANOSPS and ATDA), objectives of the study, research questions, conceptual framework, definition of terms and items relevance judgment forms were given to each of the panel of experts to determine the face and content validity of the items (i.e., to determine whether the items are relevant to the area of algebraic thinking and the specified constructs). There were some items especially the reasons which were modified according to experts’ feedback.

For instance, option B in item 2 (ANOSPS) reason initially was structured as follows: "I found the answer by comparing each pair". The expert from early algebraic thinking field suggested the explanation could focus on the meaning of multiplication for each product and also the use of commutative property. As such, the researcher modified the option B to "I found the answer by comparing each pair such as  $18 \times 17$

is same as adding 17 for 18 times and  $16 \times 18$  is same as adding 16 for 18 times because the results are the same regardless of the orders." Another feedback was to arrange the items from easy ones to challenging items. Thus, certain items were rearranged from easy to challenging items. Further, another suggestion was to structure the figural patterns item in a way that pupils were guided to what to do next. For this purpose, a guiding question (i.e., item 15i in ANOSPS) was included. This is whereby the figural pattern relationship was displayed in table form and the pupils required to filling in the table. This guides the student to find a relationship between the first term pattern and subsequent terms. However, this item (i.e., 15i) was not included in the data analysis as it meant for guidance. Likewise, patterns questions in ATDA were also restructured to cater Malaysian primary pupils. Especially the patterning tasks were restructured into table form to help the pupils to identify the relationship. As a conclusion, the result of the ANOSPS and ATDA relevance judgment by the panel of experts demonstrated that the assessment items in this study contained a high degree of face and content validity.

The reliability of ATDA was also measured using Kuder-Richardson 20 (KR20) method in statistical package for social science (SPSS) version 22 for internal consistency reliability. Usually reliability is associated with Cronbach's alpha coefficients. Since ATDA items with a range of difficulty and also mostly scored dichotomously, KR20 was used to examine its reliability. The Kuder-Richardson Coefficient of reliability for ATDA with 26 items was 0.83. None of the item was deleted as it was more than 0.70 which was within the acceptable range (Fraenkel & Wallen, 2012). This result reconfirmed the reliability measure reported by Ralston (2013) (26 items,  $\alpha = 0.81$ ).

However, reliability measure is not necessary for ANOSPS. As measurement models of ANOSPS are all formative in nature. They represented the construct's independent causes and not necessarily highly correlated (Hair et al., 2014). In addition, formative indicators are assumed to be error free (Diamantopoulos & Siguaw, 2006). Content validity is given more importance in the place of reliability. In employing formative measurement models, content validity issues rectified by content specification in which researcher has chosen the domain and the indicators intended to measure from literature.

### **Data Analysis**

The evaluation and structural model estimation was performed using structural equation modelling (SEM). SEM is similar to multiple regression, but more powerful data analysis method which enables researchers to assess and modify theoretical models in early stages of theory development (Anderson & Gerbing, 1988). Besides that, it is also allowed to test all the relationships involved in the model as a whole and simultaneously. With a combination of path analysis and factor analysis, part of statistical family is SEM. Hence, it has been chosen as data analysis technique for present study as it serves the purpose. The main purpose of the present study was to investigate the relationships between cognitive factors and algebraic thinking.

It is also notable that SEM is a method that furnishes powerful set of tools to specify, test and estimate mathematical models of the relationship that exist between sets of real world variables. In SEM, the structural model comprises the relationships among the latent variables. These relationships are mainly linear, even though flexible extensions to the basic SEM system allow for the inclusion of nonlinear relations as well (Kline, 2011).

Latent variables are hypothetical constructs which cannot be measured directly. SEM is very useful in addressing these latent variables. It can process non-experimental data and is more advanced than some other multivariate techniques which also takes measurement errors into account and is able to handle redundancy between variables (Schumacker & Lomax, 2010). In addition, SEM is more theoretical-based. When a hypothesized model is built, it is visually seen to be more user-friendly. It also enables regression equations to be tested simultaneously (Schumacker & Lomax, 2010).

Paths between the variables drawn from constructs and primary pupils' algebraic thinking skills were able to be analysed simultaneously by this technique. In other words, SEM serves as an ideal analytical technique in the present study. Besides this, SEM was also chosen as an analytic technique for the purpose of present study. A working hypothesis is that the construct that contributes to year five pupils' algebraic thinking skills which were decided by multiple factors that interacted simultaneously. Viewing each of them separately will compromise the validity of the observation of the whole scenario of the study. Furthermore, algebraic thinking skills are a measure of year five pupils' ability to demonstrate algebraic thinking skills in generalised arithmetic, modelling, and function is a latent variable, which demands the application of SEM.

The data were gathered from the two written assessments on algebraic thinking and cognitive factors were analysed by Statistical Package for the Social Sciences (SPSS) version 22 software and SmartPLS version 2.0.M3. Firstly, the data were coded into a SPSS file. Next, by that, data were scanned for potential improper data entries. For this purpose, descriptive statistics and frequency tables of the items were documented and checked out for unusual values. The ANOSPS data codes ranged

from 1 to 6. As for ATDA, items which did not require explanation were coded dichotomously; item 13 and 16 were coded according to scoring rubric as in Appendix G. The scores range from 0 to 2. Missing values analysis was conducted and the missing values in each indicator were replaced by the mean value of respective indicators. Even though it decreased the variability in the data and likely reduced the possibilities of finding meaningful relationships, it meant value replacement was preferred as the amount of missing data was low (Hair et al., 2014). Otherwise, the entire observation would have been removed. To answer the first research question, descriptive statistics such as mean, standard deviation and frequency distribution was used to describe the trends in the data. Followed by, structural equation modelling techniques were employed to test the hypotheses of the study. Table 3.6 shows the objectives and research questions were addressed in the present study and the method of data analysis.

Table 3.6

*Data analysis method of each research question*

<b>Objectives</b>	<b>Research questions</b>	<b>Data analysis</b>
To determine the year five pupils' achievement in algebraic thinking diagnostic assessment (ATDA) in relation to gender and location.	What is the year five pupils' achievement in algebraic thinking diagnostic assessment (ATDA) in relation to gender and location?	Descriptive & Inferential Statistics (Mann-Whitney <i>U</i> test)

Table 3.6, continued

<b>Objectives</b>	<b>Research questions</b>	<b>Data analysis</b>
To determine if the hypothesized model valid for year five pupils' algebraic thinking.	Is the hypothesized model valid for year five pupils' algebraic thinking?	SEM
To investigate if the proposed cognitive variables contribute to year five pupils' algebraic thinking.	To what extent proposed cognitive variables contribute to year five pupils' algebraic thinking?	SEM
To examine role of mediating variable(s) in determining year five pupils' algebraic thinking.	Is there any construct(s) acts as a mediator in the hypothesized model?	SEM
To examine the relationship between proposed cognitive variables and year five pupils' algebraic thinking in the final model.	What is the relationship between proposed cognitive variables and year five pupils' algebraic thinking in the final model?	SEM
To examine moderating effects of gender and location on year five pupils' algebraic thinking.	Is there moderating effects of gender and location on year five pupils' algebraic thinking?	SEM

Subsequent section explains the reasons for choosing PLS-SEM as data analysis technique.

**Covariance Based SEM (CB-SEM) Versus Partial Least Square SEM (PLS-SEM).** Structural Equation Modelling (SEM) is a statistical technique for hypothesis testing and estimating causal relationships which can evaluate a set of dependent relationships simultaneously. Thus, SEM can be said to be a combination of factor and path analysis (Weston & Gore, 2006). It is because goal of SEM is to evaluate interrelationship among constructs which is similar to goal of factor analysis. Likewise, SEM also enables hypothesis testing which is similar to path analysis. It also caters incorporation of construct when analysing data. Construct refers to a concept that is unmeasurable directly. It can be measured by indicators which contain the raw data (Hair et al., 2014). A hypothesized model called *a priori* developed based on the theory and related literature evident. SEM also plays an important role in instrument validation and testing connection between constructs to determine if the empirical data supports or rejects specified *a priori* model (Henseler, Ringle, & Sinkovics, 2009). This is the strength of SEM which drives to achieve the aim of the present study. Two important components called measurement model and structural model in SEM facilitate the goals of hypothesis testing and variable interrelationship evaluation. SEM has two different techniques to carry out namely; covariance-based (CB) and partial least square (PLS). The obvious difference between CB-SEM and PLS-SEM is the first aimed to theoretical covariance matrix reproduction while, the latter aimed to maximise the explained variance of the dependent latent variable.

Covariance-based Structural Equation Modelling (CB-SEM) is widely used technique in the field of SEM. It can be carried out using tools such as AMOS and



LISREL. CB-SEM involved path models generalisation such as principal component analysis and factor analysis. It validates *a priori* model by goodness-of-fit statistics and provides a best model to represent population. CB-SEM generally requires large sample size and normally distributed data. CB-SEM can have only reflective model measurement in the model. The model was not allowed to have both reflective and formative model. Most importantly, it only catered interval and ratio data to be able for analysis.

Partial Least Square-Structural equation modeling (PLS-SEM) is most suitable in the case of less developed theory involved and the aim of using structural modelling is to predict and explain target constructs (Hair et al., 2014). PLS-SEM consists of measurement and structural model of linear equations. Structural model comprises latent variable relationships while measurement model comprises relationship between latent variables and its indicators. When it comes to measurement models, PLS-SEM can have both reflective and formative model in a model. Reflective measurement model is where the causal relationships are from construct to indicators while formative measurement model is causal relationships are from indicators to construct. Unlike CB-SEM, there is no goodness-of-fit statistics is done in PLS-SEM. On the other hand, fit is determined by reliability measures.

This section discussed about what is SEM and difference between CB-SEM and PLS-SEM. Both CB-SEM and PLS-SEM have its own pros and cons. The subsequent section will discuss why PLS-SEM is chosen as data analysis technique of this present study.

**Why PLS-SEM.** There are a few reasons for selecting PLS-SEM as a data analysis technique for the present study. Firstly, PLS-SEM is able to support both reflective and formative measurement models. Evaluating formative measurement

model in PLS-SEM is easier compared to CB-SEM. In CB-SEM formative measurement model requires a construct to include both reflective and formative indicators to satisfy identification requirements (Hair et al., 2014). As discussed in chapter three, the present study involved two mathematics tests as instruments to collect data from year five pupils. The mathematics questions were from four cognitive aspects namely number sense, operation sense, symbol sense and pattern sense. These aspects were mutually exclusive whereby each indicator measured specific aspect. They were not interrelated. Therefore, formative measurement model is best to represent the constructs involve in the present study. Hence, PLS-SEM will smooth out the process of data analysis.

Secondly, PLS-SEM requires small sample size compared to CB-SEM. Chin (1998) and Barclay et al. (1995) have suggested 10 times rule. This rule requires 10 times the largest number of indicators used to measure a single construct. When it is small sample size, PLS-SEM has the capability to be more appropriate than CB-SEM (Barclay et al., 1995). For an example, a sample size of 100 would be sufficient for moderate effect size in PLS-SEM while CB-SEM may require a sample size of 250 for the same scenario (Reinartz, Haenlein, & Henseler, 2009). The present study consisted of 22 indicators and collected data from 720 year five pupils. The present study opted for PLS-SEM by looking at its sample size requirement.

Thirdly, the flexibility of PLS-SEM when comes to complex model has also been an important aspect to choose PLS-SEM in present study. Though the model in the present study was not too complex, yet PLS-SEM would be more appropriate for data analysis considering the main objective of the present study is for theory development and not to focus on already established constructs' parameter estimation.

## **Summary**

This chapter discussed about the methodology that was used in present study. It explained design of the study which involves data collection and analysing techniques. Detailed explanation of location and sampling techniques of present study were given. Instruments used in the present study were described and validity and reliability issues were discussed. Pilot studies were conducted before the actual study. Issues faced during pilot study were discussed to improve the actual study. Following that, an explanation on how the data of this study was analysed and reported. In summary, this section summarises about the issues discussed in chapter three. The next chapter, chapter four will present and discuss the results of data analysis based on the actual study.

## Chapter 4 Data Analysis

### Introduction

The main objective of the present study was to identify the cognitive variables that contribute to primary school pupils' algebraic thinking. This study especially attempted to provide evidence on how number sense, operation sense, symbol sense and pattern sense are connected to each other to develop algebraic thinking in the early stages of education. The present study revealed the web of connection among these variables. It also has presented the comparison of primary school pupils' algebraic thinking in urban and rural areas.

With regard to these objectives, this chapter presents the statistical analyses that have been carried out. The analysis of the present study performed using partial least square structural equation modelling (PLS-SEM). All analyses were carried out using the Statistical Package for the Social sciences (SPSS) version 22.0 and SmartPLS version 2.0.M3. Subsequent sections discuss the data screening, the descriptive statistics for all variables, the instruments construct validity, and results of each research question.

### Data Screening

Data screening is the first stage in data preparation for researches. It involved the entry of data collected in statistical software. As discussed in preceding sections, the two instruments consisted of 45 mathematics questions. These questions (indicators) were used to form the measurement models of the present study. Each item was given a code as explained in the chapter 3.

The two instruments administered to 800 fifth grade pupils in a district of Malacca which comprises of both urban and rural school area. Each set of instruments was given a letter and three-digit sequel numbers (i.e., R001, U128). The first letter indicated if the data was from urban or rural school and then followed by its sequel number. This is important for later stage of segregation for comparison of urban and rural school pupils. The numbering of data was important to trace errors. Then the researcher entered the answers of all pupils into SPSS software in a systematic way by using predefined items' code. However, only 720 data sets were taken for data analysis purpose which comprised 360 data from urban and 360 data from rural schools. This selection has been done by omitting very poor data out of 800 response sheets. This means data sheets with no response at all or all incorrect responses were omitted to get desired sample size of 720.

As a preliminary analysis, data screened for missing data and outliers which might be caused by human data entry error. Preliminary analysis results showed existence of some missing data. Missing data could be because of the samples did know how to answer the particular question or overlooked and left it blank. Nevertheless, the number of missing data for each indicator was less than 5%. Therefore, mean replacement was used as suggested by Hair et al. (2014). The missing values in each indicator were replaced by the mean value of respective indicators.

A data that is obvious different from other data set is known as outlier. This is especially for, “an extreme response to a particular question or extreme responses to all questions” (Hair et al., 2014, p. 53). Outliers could threaten data analysis output by creating unwanted effect on the correlation coefficient. Thus, it is important to detect outliers in the early stage of data analysis. Outliers were examined in SPSS using minimum and maximum function. A few outliers occurred due to wrong

insertion of data by the researcher. The particular set of papers was then traced using the sequel number given earlier and rectified. The final results confirmed there was not any outlier due to human data entry.

As explained in chapter 3, the data for the present study was collected in a same day using two instruments. Year five pupils sat for ANOSPS before their school recess period and ATDA after their recess period. Since both the instruments were provided on the same day, there could be a possibility for common method bias. Common method bias (also known as common method variance) is referred to “variance that is attributable to the measurement method rather than to the constructs the measures represent” (Podsakoff, MacKenzie, Lee, & Podsakoff, 2003, p. 879). In other words, when the both instruments were administered in the same day, the pupils might be affected psychologically because of exam fatigue. The instruments were in mathematics test format whereby they had to answer each question by calculations. Though, most of the questions need sense making skill without computation, majority of the pupils attempted to do computations.

Harman’s one-factor test was carried out to provide statistical evidence if the common method bias affects the data of the present study. All constructs were included in an unrotated exploratory factor analysis to confirm more than one factor emerged which shows no single factor explained most of the variance observed (Podsakoff et al., 2003). Single factor accounted for only 18.44% (not exceeding 50%) of the variance explained showed that common method bias was not a problem in the data collected for the present study.

Subsequently, the data were tested for multivariate analysis assumptions. Fulfillment of the multivariate analysis assumptions is essential to derive the statistical inferences and results. Hence, satisfying these requirements would lead to further

successful analysis. The shape of data distribution which distributed normally is referred as normality. It is the basic assumption in multivariate analysis. There are three fundamental conditions need to be fulfilled in order to verify the normality of a data distribution (Kline, 2011). They are

1. All the individual univariate distributions are normal.
2. The joint distribution of any pair of the variables is bivariate normal; that is, each variable is normally distributed for each value of every other variable.
3. All bivariate scatterplots are linear, and the distribution of residuals is homoscedastic. (Kline, 2011, p. 60)

Multivariate normality could not be assured from univariate normality. In addition, assessment of multivariate normality needs infeasible procedures to consider all frequency distributions. Nevertheless, in many instances univariate assessment is able to detect violation of multivariate analysis (Kline, 2011). This is also supported by Hair, Ringle and Sarstedt (2011), if all the variables fulfill the univariate normality requirements, then multivariate normality violation is insignificant. Therefore, in the present study univariate normality assessments were conducted to assess the multivariate normality. The next section discusses about the tests have been carried out to assess the assumptions.

### **Assessments of Multivariate Assumptions**

**Normality.** Though PLS-SEM is a nonparametric statistical method which does not require the data to be normal, yet it is important to ensure that the data is not too far from normal. Extreme non-normality would impact the assessment of parameter's significance in the structural model evaluation (Hair et al., 2014). In

accordance to this requirement, the data in the present study were examined for normality.

Normal data refers to the bell-shaped distribution of data. The normality for a single indicator can be examined by two important statistical normality components namely skewness and kurtosis. The asymmetrical mean distribution shape refers to skewness of a data distribution while the peak of that particular distribution refers to kurtosis. The pattern of responses is considered normal when both skewness and kurtosis are close to zero. A general rule of thumb is a distribution of data substantially skewed if the skewness is more than +1 or less than -1 (Hair et al., 2014). Similarly, the distribution is considered too peaked or flat when the kurtosis is not in the range of -1 and +1. Descriptive statistics for all indicators and constructs used in the present study are shown in the Table 4.1.

Based on this information, the skewness and kurtosis of all variables are within the normal distribution range of  $\pm 1$  except variable Total09 and Modelling. Variable Total09 is aimed to investigate pupils' ability to identify symbol '=' and its conceptual meaning. Majority of the students were able to answer this question correctly and provided conceptual meaning. Thus, Total09 is skewed left (-1.23) which denotes most students able to answer it right. Variable modelling's kurtosis value is slightly more than threshold value of  $\pm 1$  (-1.04). This value indicates the distribution is slightly flat. However, both variables were retained in the further analysis as these would not affect the significance test results in the structural model evaluation because non-normality issue is less severe in PLS-SEM (Hair et al., 2014).



Table 4.1

*Descriptive statistics for all variables (N = 720)*

<b>Items</b>	<b>Min</b>	<b>Max</b>	<b>Mean</b>	<b>Std. Deviation</b>	<b>Skewness</b>	<b>Kurtosis</b>
<b>Number Sense</b>	9	29	19.48	3.45	-0.15	0.03
Total01	1	6	3.37	1.09	0.40	0.93
Total02	1	6	3.80	1.45	-0.10	-0.98
Total03	1	6	3.63	1.43	-0.40	-0.99
Total04	1	6	4.06	1.48	-0.81	-0.43
Total05	1	6	4.60	1.56	-0.87	-0.47
<b>Operation Sense</b>	3	18	11.95	3.28	-0.30	-0.55
Total06	1	6	4.52	1.511	-0.97	-0.08
Total07	1	6	3.81	1.57	-0.16	-0.98
Total08	1	6	3.63	1.48	-0.26	-0.96
<b>Symbol Sense</b>	8	30	20.51	4.18	-0.26	-0.43
Total09	1	6	5.02	1.63	-1.23	-0.23
Total10	1	6	4.03	1.59	-0.42	-0.99
Total11	1	6	3.80	1.52	-0.19	-0.98
Total12	1	6	3.65	1.49	-0.06	-0.98
Total13	1	6	3.91	1.47	-0.36	-0.68

Table 4.1, continued

<b>Items</b>	<b>Min</b>	<b>Max</b>	<b>Mean</b>	<b>Std. Deviation</b>	<b>Skewness</b>	<b>Kurtosis</b>
<b>Pattern Sense</b>	10	35	23.74	5.19	0.21	-0.53
Total14i	1	6	4.03	1.53	-0.25	-0.98
Total14ii	1	6	4.4	1.45	-0.92	-0.04
Total14iii	1	6	3.66	1.52	0.03	-0.99
Total15ii	1	6	3.84	1.51	-0.18	-0.99
Total15iii	1	6	4.14	1.59	-0.45	-0.98
Total 15iv	1	6	3.67	1.52	-0.08	-0.98
<b>Algebraic Thinking</b>	1	30	16.82	6.12	0.01	-0.83
GA	0	12	6.44	2.28	0.18	-0.59
Modelling	0	16	6.30	3.13	-0.03	-1.04
Functions	0	8	4.08	2.00	-0.17	-0.70

**Linearity.** Followed by normality assumption, linearity plays an important role in multivariate analysis assumptions. Linearity refers to all variables in the study that are significantly associated linearly to each other. This assumption is to avoid underestimating the actual relationship's strength (Hair, Black, Babin, & Anderson, 2009). As nonlinear impacts will not be shown in correlation and only linear correlation will be shown. The correlation coefficient ranges from -1 to +1. Perfect positive and negative relationships are represented by +1 and -1 respectively while 0 denotes there is no relationship between variables (Hair et al., 2014). To verify this assumption, linearity of the present study data was analysed using Pearson product-moment correlation coefficients computation in SPSS software.

Table 4.2 shows correlation matrix of all variables. Based on the table, all variables are correlated significantly at a 0.01 significance level. The present study data have fulfilled the linearity requirement as it can be seen positive relations between the variables. The strength of relationship among them were moderate (i.e.,  $0.3 < |r| < 0.5$ ) and weak (i.e.,  $0.1 < |r| < 0.3$ ). At the same time, they are not highly correlated because high correlation between two formative indicators may lead to collinearity problem (Hair et al., 2014). Thus, these results also revealed the preliminary evidence for the relationship between the main constructs of the present study. The correlation between the algebraic thinking and pattern sense is the strongest, with correlation of 0.485. Similar association was found between algebraic thinking and number sense, algebraic thinking and symbol sense (i.e., 0.407 and 0.461 respectively). The weakest relationship was found between number sense and symbol sense (i.e., 0.235). Even though weak relationship was found, the analysis can still be carried out as there is no sign of zero correlation and collinearity problems. According to Hair et al. (2014), if the strength is more than 0.9, then the model will face multicollinearity issue. This correlation matrix table provides evidence to proceed with further analysis.

Table 4.2

*Correlations matrix of all variables (N = 720)*

Variable	Number	Operation	Symbol	Pattern	Algebraic
	Sense	Sense	Sense	Sense	Thinking
Number	1				
Sense					
Operation	0.238**	1			
Sense					
Symbol	0.235**	0.259**	1		
Sense					
Pattern	0.317**	0.292**	0.361**	1	
Sense					
Algebraic	0.407**	0.273**	0.461**	0.485**	1
Thinking					

\*\* Correlation is significant at the 0.01 level (2-tailed).

**Homoscedasticity.** Lastly, both univariate and multivariate analysis requires data to be homoscedasticity. Homoscedasticity refers to the relationship between the variables. This requirement refers to an assumption that dependent variables establish equal levels of variance across the range of independent variables (Hair, Black, Babin, & Anderson, 2009). Failure to fulfill this assumption might result in underestimation of correlation measure between the related variables and therefore produces analysis degradation.

Homoscedasticity associated with the normality assumption due to relationships between variables are homoscedastic if multivariate normality is

fulfilled. On the contrary, heteroscedasticity refers to homoscedasticity failure which is not fatal to analysis of ungrouped data. There is a higher chance for predictive if the heteroscedasticity accounted for, even though the linearity between variables is captured by the analysis. If the data is not homoscedasticity, analysis will not be invalidated but it will result in weakened analysis (Hair et al., 2009).

Therefore, the homoscedasticity test was carried out graphically. The residuals scatterplot was used to examine if there is any violation of homoscedasticity using SPSS version 22.0 software. It provides the information homoscedasticity with the associated variables. Figure 4.1 shows the residual scatterplots between the independent variables (cognitive factors) with the dependent variable (algebraic thinking). The visual examination of residual scatterplots points out that the dots were widespread across the graph and formed an approximate rectangle shape. It can be concluded that the data in the present study fulfilled the homoscedasticity condition because there was no noticeable pattern in the scatterplots.

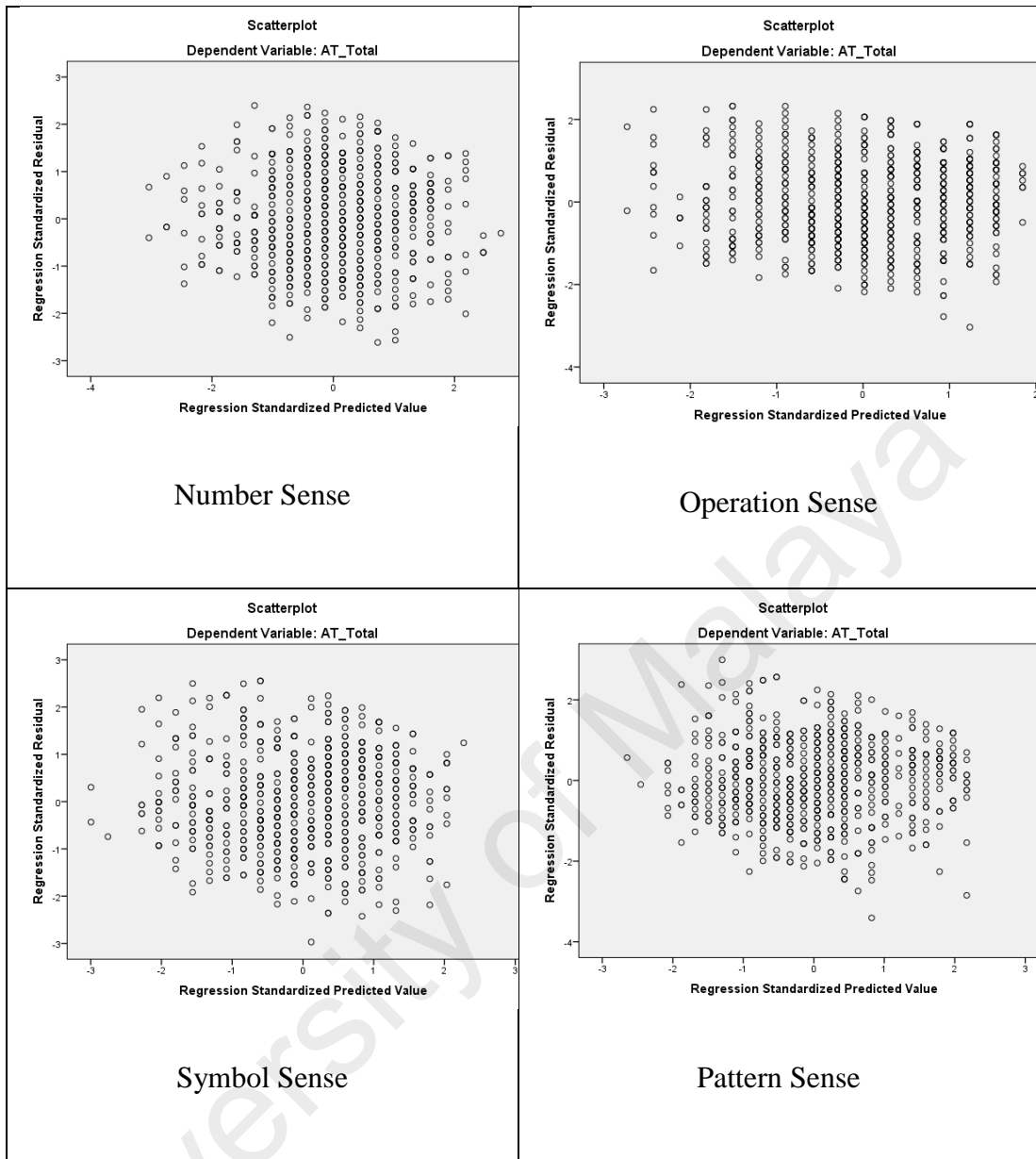


Figure 4.1. Residuals scatterplots for various independent variables (number sense, operation sense, symbol sense, and pattern sense) with the dependent variable (algebraic thinking)

### Demographic Profiles

The samples of present study were year five pupils from National Schools in *Melaka Tengah*, a district of Malacca. There are 56 National Schools with 5347 total students in the particular district. 27 of the schools were categorised as urban and 29

schools were categorised as rural area schools. They were 2215 and 3132 students in urban and rural schools respectively. A sample size of 720 was taken from urban and rural schools with 360 pupils each location. Table 4.3 shows the samples' composition by gender and location. There were 48.6% male pupils and 51.4% female pupils from urban and rural area respectively.

Table 4.3

*Demographic profile of samples by gender and location*

<b>Gender</b>	<b>Urban</b>	<b>Rural</b>	<b>Total</b>
Male	184	166	<b>350</b> (48.6%)
Female	176	194	<b>370</b> (51.4%)
<b>Total</b>	<b>360</b> (50.0%)	<b>360</b> (50.0%)	

Table 4.4

Demographic profile of samples by mid-year examination grades

<b>Grade</b>	<b>Frequency</b>	<b>Percentage</b>
A	118	16.4
B	156	21.7
C	203	28.2
D	122	16.9
E	120	16.7
Missing	1	0.10
<b>Total</b>	<b>720</b>	<b>100.0%</b>

Table 4.4 exhibits the mathematics grades obtained in their respective school mid-year examination. From this table, (719 as 1 indicate as missing data) majority (28.2%) obtained a C for their mid-year mathematics examination. This was followed by 21.7% for grade B. The achievement for grade A, D and E was 16.4%, 16.9% and 16.7% respectively. Grade E indicates the samples failed in their mathematics in mid-year examination. In other words, about 83% of the samples passed in their mathematics subject in their mid-year examination.

### **Measurement Model Specification**

As stated in the beginning of this chapter, one of the major reasons to choose PLS-SEM technique in the present study is the software's ability to handle formative measurement model. The present study involved only formative measurement models. Prior to structural model evaluation, it is necessary to discuss the constructs' nature and the type of measurement models involved (Hair et al., 2014). Constructs can be specified in either reflective measurement model or formative measurement model. In reflective measurement model, all the causal relationships are from construct to indicators. This indicates that the constructs determine the indicators while formative measurement model involves causal relationships from indicators to constructs. This shows that the constructs are described by the indicators.

However, many researchers tend to wrongly specify the constructs due to their lack of concern in specifying measurement model (Jarvis, MacKenzie, & Podsakoff, 2003). This will impact the research model development evaluation. To avoid such problems, Jarvis et al. (2003) have given four decision making guidelines to decide a construct should be reflective or formative. Hence, the present study used the guidelines provided to decide each construct should be formative or reflective. Based



on these four rules displayed in Table 4.5, the present study identified the nature of constructs. The subsequent section discussed the decision made by the researcher for each construct.

Table 4.5

Decision rules to identify construct as formative or reflective

<b>Rules</b>	<b>Formative Model</b>	<b>Reflective Model</b>
1. Direction of causality from construct to measure implied by the conceptual definition	• Direction of causality is from items to construct	• Direction of causality is from construct to items
Are the indicators (items) (a) defining characteristics or (b) manifestations of the construct?	• Indicators are defining characteristics of the construct	• Indicators are manifestations of the construct
Would changes in the indicators/items cause changes in the construct or not?	• Changes in the indicators should cause changes in the construct	• Changes in the indicator should not cause changes in the construct
Would changes in the construct cause changes in the indicators?	• Changes in the construct do not cause changes in the indicators.	• Changes in the construct do cause changes in the indicators.
2. Interchangeability of the indicators/items	• Indicators need not be interchangeable	• Indicators should be interchangeable
Should the indicators have the same or similar content? Do the indicators share a common theme?	• Indicators need not have the same or similar content/ indicators need not share a common theme	• Indicators should have the same or similar content/ indicators should share a common theme
Would dropping one of the indicators alter the conceptual domain of the construct?	• Dropping an indicator may alter the conceptual domain of the construct	• Dropping an indicator should not alter the conceptual domain of the construct

Table 4.5, continued

<b>Rules</b>	<b>Formative Model</b>	<b>Reflective Model</b>
3. Covariation among the indicators.	• Not necessary for indicators to covary with each other	• Indicators are expected to covary with each other
Should a change in one of the indicators be associated with changes in the other indicators?	• Not necessarily	• Yes
4. Nomological net of the construct indicators	• Nomological net for the indicators may differ	• Nomological net for the indicators should not differ
Are the indicators/items expected to have the same antecedents and consequences?	• Indicators are not required to have the same antecedents and consequences	• Indicators are required to have the same antecedents and consequences

(Jarvis et al., 2003, p. 203)

Number sense is aimed at investigating year five pupils' general understanding of number and operations together with the ability to make sensible mathematical judgments while dealing with numbers and operations. In the present study, number sense encompasses five components namely, i) Understanding number meanings and relationships, ii) recognizing the magnitude of numbers, iii) understanding the relative effect of operations on numbers, iv) developing computational strategies and being able to judge their reasonableness, v) ability to represent numbers in multiple ways. Each of these component measures different aspects of number sense. One question for each component is the indicators of number sense. Therefore, these five indicators define the characteristics of number sense. Changes in one of the components (indicators) could alter the definition of number sense (construct). In addition, the indicators definitely are not interchangeable as each one of it measure different aspects. Dropping an indicator may change the conceptual definition of number sense.

Thus, the covariation is not necessary among the indicators. These indicators not required to have same consequences. Table 4.6 summarises the nature of number sense based on the four rules suggested by Jarvis et al. (2003).

Table 4.6

Decision rules to identify the number sense construct as formative or reflective

<b>Rules</b>	<b>Nature of Construct</b>	<b>Decision Made</b>
1. Direction of causality from construct to measure implied by the conceptual definition	Five components define number sense.	
2. Interchangeability of the indicators/items	The five components are not interchangeable	Formative measurement model
3. Covariation among the indicators	The five components are not necessary to covary	
4. Nomological net of the construct indicators	The samples may not able to excel in all the five components	

Operation sense is the ability to understand the property of operations. In detail, it measures the a) understanding of the relationships between the operations, b) understanding of various representations of the operation across the differing number systems and c) ability to move backwards and forwards between these conceptions. With regard to these three aspects, the questions (indicators) were formed for the construct of operation sense. The three questions involve addition and subtraction operations with two types of generalisation questions - i) direction of change, ii)

relationship between addition and subtraction (Haldar, 2014). The construct of operation sense is confined to only these three types of questions. They are not interchangeable. For instance, the question testing on direction of change has no link with the other types of questions. Thus, the changes in one of the indicators are not associated with another. A student who was able to do symmetrical type question would not necessarily be able to do the asymmetrical questions. Table 4.7 summarises the nature of operation sense based on the four rules suggested by Jarvis et al. (2003).

Table 4.7

Decision rules to identify the operation sense construct as formative or reflective

<b>Rules</b>	<b>Nature of Construct</b>	<b>Decision Made</b>
1. Direction of causality from construct to measure implied by the conceptual definition.	Three types of generalisation define operation sense.	Formative measurement model
2. Interchangeability of the indicators/items	The three types are not interchangeable	
3. Covariation among the indicators	The three types are not necessary to covary	
4. Nomological net of the construct indicators	The samples may not able to excel in all the three types.	

Symbol sense is divided into two types of symbols. One is a symbol that denotes equal '=' and another symbol type is shapes (i.e., ♣, ♠) that represent unknowns. Firstly, the present study investigated year five pupils' ability to identify

equal sign and their understanding of its conceptual meaning. Secondly, it looked at year five pupils' ability to work with a symbol which actually represents a number. It played a role of a variable. Five questions were designed for the construct of symbol sense. Among that, two questions involved equal sign and three questions involved variables. The first question for equal sign was to test the knowledge of the symbol and its conceptual meaning. Second question addressed the ability to apply conceptual understanding of equal sign in an equation. Similarly, the three questions for variables tested three different aspects involving variables. First question tested the unwinding strategy and finding unknown. Second question tested on finding the unknown. Third question tested on the ability to work with two unknowns. In sum, all the five questions for the construct of symbol sense measured different aspects of symbols. Thus, they were not suitable to represent as reflective model. Table 4.8 summarises the nature of symbol sense based on the four rules suggested by Jarvis et al. (2003).

Table 4.8

Decision rules to identify the symbol sense construct as formative or reflective

<b>Rules</b>	<b>Nature of Construct</b>	<b>Decision Made</b>
1. Direction of causality from construct to measure implied by the conceptual definition	The equal sign and variable with five questions define symbol sense.	Formative measurement model

Table 4.8, continued

<b>Rules</b>	<b>Nature of Construct</b>	<b>Decision Made</b>
2. Interchangeability of the indicators/items	The five questions are not interchangeable.	
3. Covariation among the indicators	The five questions are not necessary to covary.	
4. Nomological net of the construct indicators	The samples may not able to excel in all the five questions.	

Pattern sense is divided into two types namely numerical and figural. Each type consists of three questions. The three questions for each type test the i) ability to figure out the 'rule', ii) ability to perform near generalisation, and iii) ability to perform far generalisation. There were total of six questions for this construct. Though the aspects were common among the six questions, types of patterns were different whereby they involve numerical and figural. Thus, the six questions played separate roles in measuring pattern sense. Ability to work with pattern investigated how a student identifies the relationship between given first three or four terms and subsequent terms in the aspect of generating a 'rule' and using it to perform near and far generalisation. Therefore, each question (indicator) served its own purpose. As such, it was appropriate to represent pattern sense as formative measurement model too. Table 4.9 summarises the nature of pattern sense based on the four rules suggested by Jarvis et al. (2003).

Table 4.9

Decision rules to identify the pattern sense construct as formative or reflective

<b>Rules</b>	<b>Nature of Construct</b>	<b>Decision Made</b>
1. Direction of causality from construct to measure implied by the conceptual definition	The numerical and figural patterns with six questions define pattern sense.	
2. Interchangeability of the indicators/items	The six questions are not interchangeable.	Formative measurement model
3. Covariation among the indicators	The six questions are not necessary to covary.	
4. Nomological net of the construct indicators	The samples may not able to excel in all the six questions.	

Algebraic thinking is the dependent variable of the present study. The main objective of the study was to investigate the contributions of cognitive constructs towards algebraic thinking. The content of algebraic thinking comprised generalised arithmetic, modelling any function as defined by Kaput (2008). Each of these three strands has a few sub strands. As such, generalised arithmetic consists of items from efficient numerical manipulation and generalisation. Modelling comprises items from open number sentences, equivalence and working with variables. Lastly, function is divided into numerical and figural. These sub strands form each item for algebraic thinking construct. These items are scored dichotomously (i.e., 0- correct; 1- incorrect). Two items were scored using coding rubrics from 0-2. For the analysis purpose, these sub strands' scores were added up. Whereby, each strand's measured

variable is the total score of its sub strands respectively. For example, measured variable for generalised arithmetic in the model is represented by total score of efficient numerical manipulation and generalisation items. The same method applies to modelling and function. This action was taken to maintain the data uniformity between exogenous and endogenous measurement scale. With regard to this, three strands were studied according to Jarvis et al. (2003) four rules to decide if it should be formative or reflective measurement model. The details are summarised in the Table 4.10.

Table 4.10

Decision rules to identify the algebraic thinking construct as formative or reflective

<b>Rules</b>	<b>Nature of Construct</b>	<b>Decision Made</b>
1. Direction of causality from construct to measure implied by the conceptual definition	Generalised arithmetic, modelling and function are the content of algebraic thinking.	
2. Interchangeability of the indicators/items	The three strands are not interchangeable as they are not measuring common theme.	Formative measurement model
3. Covariation among the indicators	The three strands are not necessary to covary.	
4. Nomological net of the construct indicators	The samples may not able to excel in all the three aspects.	



This section clearly explained the reasons behind all the formative measurement models involved in the present study. The next section will discuss the evaluation processes carried out for each of this measurement model.

### **Measurement Model Evaluation**

The measurement model evaluation is the process of identifying if the specified measurement model is acceptable for further data analysis. According to Henseler et al. (2009), common assessments of validity are not applicable for formative measurement models. The internal consistency to assess reliability and convergent and discriminant validities for construct validity is not appropriate as a model is formative in nature. This is due to the fact that formative indicators are not necessary to be correlated with latent variable and also should be error-free. Nevertheless, it is still crucial to establish validity of formative measurement model (Diamantopoulos, Riefler, & Roth, 2008; Hair et al., 2014). This evaluation process followed the guidelines suggested by Hair et al. (2014). Formative measurement model evaluation has two stages as shown in Table 4.11. Firstly, there is a need to ensure there is no collinearity among indicators. Secondly, the significance of the outer weights should be assessed.

Table 4.11

*Systematic evaluation of PLS-SEM results*

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**Step 1: Evaluation of Measurement Model**

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Formative measurement model:

- Collinearity among indicators
- Significance and relevance of outer weights.

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**Step 2: Evaluation of Structural Model**

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- Coefficients of determination ( $R^2$ )
- Predictive relevance ( $Q^2$ )
- Size and significance of path coefficients
- $f^2$  effect sizes
- $q^2$  effect sizes

---

**Collinearity among indicators.** Unlike reflective indicators which facilitate interchangeability and correlation, collinearity issue in formative indicators is considered as a serious problem. The existence of collinearity among formative indicators has potential to influence the significance and weights of the indicators (Diamantopoulos, Riefler, & Roth, 2008). Tolerance index (TI) and variance inflation factor (VIF) are the two indices used to assess the collinearity acceptance level. Based on PLS-SEM context, collinearity issue arises if the tolerance value is 0.20 or less and VIF value of 5.0 or higher (Hair et al., 2014).

As for VIF, each set of formative indicators are examined according to its respective construct. As for the cognitive model of the present study, each construct's (i.e., number sense, operation sense, symbol sense, pattern sense and algebraic thinking) sets of indicators were assessed separately. To obtain the tolerance and VIF

values in SPSS, one of the independent variable should be selected as dependent variable. Therefore, first number sense was used as dependent variable input in SPSS to derive required VIF values to evaluate the collinearity of the formative measurement model. Then, pattern sense was used as dependent variable to get the tolerance and VIF values for number sense. In the present study, all the measurement models were formative type. Thus, VIF values were calculated for all the constructs involved (i.e., number sense, operation sense, symbol sense, pattern sense, and algebraic thinking).

The outcome of both results displayed on the Table 4.12. The rest of the output were discarded and only tolerance and VIF values are considered as shown in Table 4.12 (Hair et al., 2014). If an indicator failed to meet the acceptance level of tolerance and VIF values criteria, construct elimination, combining indicators into a single construct or developing higher-order constructs could be considered at this level. Table 4.12 shows all the tolerance values fell in the range of 0.663 and 0.886 which was above 0.20. All the VIF values were in the range of 1.129 and 1.508 indicating there was no value more than 5.0. Therefore, collinearity is not an issue in the present study.

Table 4.12

Collinearity statistics of number sense, operation sense, symbol sense, pattern sense, and algebraic thinking

Dependent Construct	Independent Construct	Tolerance Value	VIF
Number Sense	Operation Sense	.878	1.139
	Symbol Sense	.750	1.333
	Pattern Sense	.720	1.388
	Algebraic Thinking	.663	1.508
	Number Sense	.816	1.226
Pattern Sense	Operation Sense	.886	1.129
	Symbol Sense	.767	1.304
	Algebraic Thinking	.682	1.466
	Number Sense	.816	1.226

**Significance and relevance of outer weights.** The second assessment of formative measurement model is evaluating the relevance and significance of the indicators by using multiple regressions (Hair, Black, Babin, & Anderson, 2009). The evaluation process involves assessing the indicator's significance by examining the  $t$  - values. If it is more than 1.96, then the indicator is significant. Table 4.13 concluded that the indicators of all the constructs are significant based on the  $t$  values except indicator Total01 (i.e.,  $t = 1.135$ ).

However, Hair et al. (2014) suggested that the item can still be retained if an indicator's weight is insignificant but the corresponding item loading is relatively high (i.e., loading more than 0.50). In the case of indicator's weight which is insignificant

and outer loading is also less than 0.50, the item can still be retained if the outer loading is significant (i.e.,  $p < 0.05$ ). With respect to this, the researcher retained the indicator Total01 in the present study because its outer loading was significant (i.e.,  $t = 2.161$ ).

Table 4.13

Formative indicators' outer weights and significance of number sense, operation sense, symbol sense, and pattern sense

<b>Formative Constructs</b>	<b>Indicators</b>	<b>Outer Weight</b>	<b>Std. Error</b>	<b><i>t</i> Values</b>
Number Sense	Total01	0.085	0.075	1.135*
	Total02	0.199	0.072	2.754
	Total03	0.383	0.070	5.445
	Total04	0.609	0.060	10.225
	Total05	0.521	0.065	7.992
Operation Sense	Total06	0.684	0.105	6.488
	Total07	0.358	0.132	2.718
	Total08	0.273	0.119	2.295
Symbol Sense	Total09	0.179	0.061	2.917
	Total10	0.239	0.062	3.850
	Total11	0.367	0.061	5.983
	Total12	0.402	0.063	6.360
	Total13	0.445	0.067	6.676
Pattern Sense	Total14i	0.454	0.056	8.056
	Total14ii	0.281	0.064	4.354
	Total14iii	0.235	0.063	3.727
	Total15i	0.289	0.057	5.032

Table 4.13, continued

<b>Formative Constructs</b>	<b>Indicators</b>	<b>Outer Weight</b>	<b>Std. Error</b>	<b>t Values</b>
	Total15ii	0.183	0.064	2.879
	Total15iii	0.171	0.061	2.824
	Generalised	0.289	0.061	4.695
Algebraic Thinking	Arithmetic			
	Modelling	0.616	0.060	10.339
	Functions	0.283	0.058	4.876

Note: \*  $p > .05$

This section summarises all formative models used in the present study which are free from multicollinearity issues and the formative indicators are significant and relevant. With that, the subsequent section discusses how the research questions of the present study were answered.

### **Research Question 1**

The first research question of the present study and the sub questions are stated as below:

What is the year five pupils' performance in Algebraic Thinking?

1.1 What is the year five pupils' performance in algebraic thinking in relation to gender?

1.2 What is the year five pupils' performance in algebraic thinking in relation to location?

To address these questions, descriptive statistics and hypothesis test were carried out. The aim of hypothesis test is to examine the effect of gender and location

(i.e., urban and rural) variables on the algebraic thinking. Table 4.14 provides information on performance of year five pupils in each algebraic thinking strand. The maximum score for generalised arithmetic, modelling and function are 12, 10 and 8 respectively. The scores of each strand were then converted to percentage.

Overall, the average correct percentage of the pupils in all items of ATDA was about 54%. The findings have shown that their strongest strand was modelling which they yield about 57% percentage of correct while their weakest strand was function with lowest percentage of correct (about 51%). The year five pupils have performed moderately in the generalised arithmetic strand which yields about 53% correct percentage.

Table 4.14

The descriptive statistics of year five pupils' performance for algebraic thinking strands

Strand	Items	No. of items	Mean	Percent Correct	Std. Deviation
Generalised Arithmetic	Q11i, Q11ii, Q11iii, Q11iv, Q11v, Q11vi, Q12, Q15	8	6.440*	53.7%	2.28
Modelling	Q1, Q2, Q3, Q4, Q5, Q6, Q7, Q8, Q9, Q10	10	6.299**	57.3%	3.13
Function	Q13, Q14i, Q14ii, Q14iii, Q16, Q17i, Q17ii, Q17iii	8	4.081***	51.0%	2.00
Average				54.26%	

\* Measured based on 12 points, \*\* Measured based on 11 points, and \*\*\* Measured based on 8 points

Table 4.15 shows the descriptive statistics of each algebraic thinking strand by gender. As for generalised arithmetic strand, the data has shown that female pupils

outperformed with an average correct percentage about 55% compared to male pupils who have yield about 52% correct percentage. Meanwhile male pupils have performed better in modelling and function strands compared to female with about 59% and 53% percentage of correct respectively. Overall, male pupils' performances were better than female pupils when comparing the means. The results have shown that female pupils' strongest strand was modelling and the weakest strand was function. Meanwhile male pupils' strongest strand was also modelling but the weakest was generalised arithmetic.

Table 4.15

The descriptive statistics of each algebraic thinking strand by gender

<b>Strand</b>	<b>No. of Items</b>	<b>Gender</b>	<b>Percent Correct</b>	<b>Mean</b>	<b>Std. Deviation</b>	<b>No. of Students</b>
Generalised Arithmetic	8	F	55.3%	6.641	2.246	350
		M	51.9%	6.223	2.297	370
		Ave	53.7%	6.440	2.279	720
Modelling	10	F	55.6%	6.119	3.050	350
		M	58.9%	6.489	3.202	370
		Ave	57.2%	6.297	3.128	720
Function	4	F	49.6%	3.965	1.938	350
		M	52.5%	4.203	2.059	370
		Ave	51.00%	4.081	2.000	720

To facilitate the gender comparison further, a hypothesis test was carried out. In practice, generally *t*-test will be carried out if the normality assumption is fulfilled. Otherwise Mann-Whitney *U* test is appropriate for nonparametric data (Field, 2012).



The two most common statistical tests for normality are Shapiro-Wilk test and Kolmogorov-Smirnov test. Each calculates the level of significance for the differences from a normal distribution. However, in the present study to test if the data fulfils the normality assumption, Kolmogorov-Smirnov test was carried out in SPSS. According to Mooi and Sarstedt (2011), Shapiro–Wilk test is best used for sample sizes of less than 50. In addition, it works poorly if the testing variable has many identical values. Therefore, the researcher opted for Kolmogorov-Smirnov test to test the normality. The outcome of Kolmogorov-Smirnov test has shown  $p$  values of 0.00 for each strand. In other words, the data failed to fulfil the normality assumption to carry out  $t$ -test. Hence, nonparametric Mann-Whitney  $U$  test was used to examine further possible effects of gender difference in the algebraic thinking strands.

Mann-Whitney  $U$  test was carried out to investigate if male and female year five pupils differ in their level of algebraic thinking in accordance to three strands. The results are displayed in Table 4.16. It was found that there was a significant difference between means of female and male pupils for generalised arithmetic strand. From this, it can be concluded that female pupils outperformed in generalised arithmetic ( $z = -2.195$ ,  $p = 0.014$ ) than male pupils. Nevertheless, there was no significant difference between female and male in modelling and function strands. Hence, the difference in means as shown in Table 4.15, did not provide evidence to conclude that there was a difference between female and male achievement in modelling and function strands.

As there is a significant difference between means for generalised arithmetic strand, effect size was also calculated to ensure if there is any substantial significance. As nonparametric test was used for hypothesis testing; thus, the effect size was

calculated based on the formula below instead of Cohen's  $d$  value.  $n_x$  and  $n_y$  represent sample size of two groups respectively. In this case it would be 720 (i.e., 360 + 360).

$$r = \frac{z}{\sqrt{n_x + n_y}}$$

Based on Pallant (2013) guidelines, the effect size is small, medium and large if the absolute value of  $r$  is 0.1, 0.3 and 0.5 respectively. The effect size for generalised arithmetic was found to be small ( $\text{abs}[r] = 0.08$ ).

Table 4.16

Significance of the differences between the means of the gender on the algebraic thinking strands

<b>Strand</b>	<b><math>z</math></b>	<b><math>p</math></b>	<b>Outcome</b>
Generalised Arithmetic	-2.195	0.014	$p < 0.05$ , therefore there is a significant difference between the means.
Modelling	-1.656	0.098	$p > 0.05$ , therefore there is no significant difference between the means.
Function	-1.890	0.059	$p > 0.05$ , therefore there is no significant difference between the means.

Table 4.17 shows the descriptive statistics of each algebraic thinking strand by location. Overall, urban school year five pupils were with percentage of correct more than 50% in all three strands. The data has shown urban school year five pupils outperformed with average correct percentage about 58% compared to rural school

year five pupils who have yielded only about 49% correct percentage. Similarly, urban school year five pupils' performance was also better than rural school year five pupils' performance in modelling and function with about 65% and 57% percentage correct respectively. In sum, urban school year five pupils' performance was better than urban school year five pupils' performance when comparing the means. The results have shown that urban school year five pupils' strongest strand was modelling and function was the weakest strand. Likewise, rural school year five pupils' strongest strand was also modelling and weakest was generalised arithmetic.

Table 4.17

The descriptive statistics of each algebraic thinking strand by location

<b>Strand</b>	<b>No. of Items</b>	<b>Location</b>	<b>Percent Correct</b>	<b>Mean</b>	<b>Std. Deviation</b>	<b>No. of Students</b>
Generalised Arithmetic	8	Rural	49.3%	5.914	2.258	360
		Urban	58.1%	6.967	2.178	360
		Ave	53.7%	6.440	2.279	720
Modelling	10	Rural	49.7%	5.469	3.130	360
		Urban	64.8%	7.123	2.901	360
		Ave	57.2%	6.297	3.128	720
Function	4	Rural	45.5%	3.636	1.957	360
		Urban	56.6%	4.525	1.946	360
		Ave	51.00%	4.081	2.000	720

Followed by Mann-Whitney  $U$  test for algebraic thinking strands achievement by gender, Mann-Whitney  $U$  test was carried out to investigate if urban and rural school's year five pupils differ in their level of algebraic thinking. The results are

displayed in Table 4.18. It is found that there was a significant difference between means of rural and urban school year five pupils' performance in all three strands. In all three strands, the  $p$  values were less than 0.05, indicating there was a significance difference between the means. The means of all three strands had shown urban school pupils' results were higher than rural school year five pupils. From this, it can be concluded that urban school year five pupils outperformed rural school year five pupils in algebraic thinking.

Followed by these findings, effect sizes were calculated for each strand. The effect size for generalised arithmetic is found to be medium ( $\text{abs}[r] = 0.23$ ). In other words, the effect of significant difference of means between rural and urban school year five pupils' achievement was medium. The effect size for modelling also fell in the medium category ( $\text{abs}[r] = 0.26$ ). Lastly, function's effect size was 0.20 which was also medium effect.

Table 4.18

Significance of the differences between the means of the location on the algebraic thinking strands

<b>Strand</b>	<b><i>z</i></b>	<b><i>p</i></b>	<b>Outcome</b>
Generalised Arithmetic	- 6.277	0.000	$p < 0.05$ , therefore there is a significant difference between the means.
Modelling	- 7.022	0.000	$p < 0.05$ , therefore there is a significant difference between the means.
Function	- 5.362	0.000	$p < 0.05$ , therefore there is a significant difference between the means.

In sum, the result for research question 1 shows the overall performance of year five pupils from a district of Malacca was moderate. The total correct percentage scores were less than 60%. Though they were literally weak in all three strands, the results have shown function strand was the least performing strand. In addition, there was no difference between female and male pupils' performance in function and modelling. As for difference based on location, results have shown that urban school year five pupils outperformed rural school year five pupils.

## Research Question 2

Is the hypothesized model valid for year five pupils' algebraic thinking?

Measurement model evaluation in previous section clearly discussed and demonstrated the outcome of validity of all constructs involved in the present study. This would be the first step in systematic evaluation of PLS-SEM results as guided by Hair et al. (2014) in Table 4.11. The second step was the evaluation of the structural model. There were a few steps involved in this stage. Structural model evaluation is absolutely necessary to demonstrate how the collected data prove and support the unpinning theories utilised in the study (Hair et al., 2014). With regard to this, it also provides the predictability of the model, the relationships of cognitive factors (constructs) in developing year five pupils' algebraic thinking.

Structural model evaluation in PLS-SEM has four fundamental criteria. They are i) path coefficients' significance, ii)  $R^2$  values levels, iii) the  $f^2$  effect size, and iv) the predictive relevance  $Q^2$ , and the  $q^2$  effect sizes (Hair et al., 2014). Figure 4.2 shows the hypothesized model of the present study. The following sections explained the evaluation process of this structural model according to the four criteria.

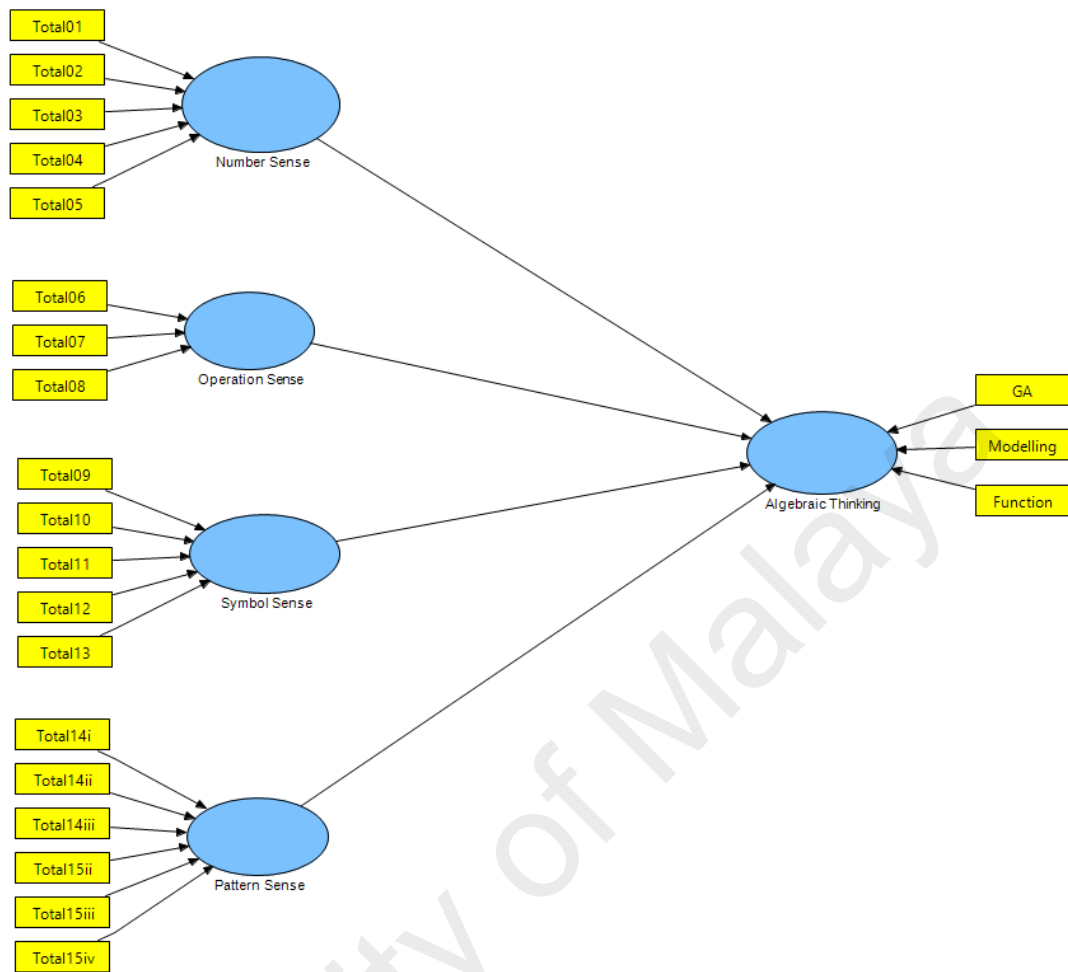


Figure 4.2. Hypothesized model of year five pupils' algebraic thinking

### Assessment of Significance and relevance of the Structural Model

**Relationships.** The combination of all measurement models discussed in the previous section forms the structural model as shown in Figure 4.2. They created path coefficients of the hypothesized model. The present study investigated the relationships of number sense, operation sense, symbol sense and pattern sense towards year five pupils' algebraic thinking.

To assess the significance level of path coefficients, the following settings (shown in Table 4.19) has been carried out in SmartPLS software to execute

bootstrapping function. The cases refer to the sample size used. Upon completion of bootstrapping the results yield is displayed in Table 4.20.

Table 4.19

*Bootstrapping settings in SmartPLS*

	<b>Selected Option</b>	<b>Reference</b>
Sign Changes	No Sign Changes	
Cases	720	(Hair et al., 2014)
Samples	5000	

Table 4.20

Significance testing results of the structural model path coefficients

	<b>Path Coefficient</b>	<b>t values</b>	<b>Significance Level</b>
Number Sense -> Algebraic Thinking	0.248	8.227	0.05
Operation Sense -> Algebraic Thinking	0.085	2.727	0.05
Symbol Sense -> Algebraic Thinking	0.286	7.757	0.05
Pattern Sense -> Algebraic Thinking	0.274	8.935	0.05

Evaluation of the relevance of the significant relationships is equally important as evaluation of significance of the relationship between constructs. In addition, in majority of cases, even though the path coefficient is significant but it deserves very small consideration (Hair et al., 2014). Furthermore, investigating the structural model relationship relevance is useful evidence to interpret the results.



As shown in Table 4.20, all cognitive factors of the present study (i.e., number sense, operation sense, symbol sense, pattern sense) have contributed significantly to the year five pupils' algebraic thinking. In other words, these cognitive factors are significantly important in developing algebraic thinking in primary schools. To be precise, symbol sense played the most important role by contributing the highest ( $\beta = 0.286$ ,  $t$ -value ( $7.757 > 1.96$ )) to the year five pupils' algebraic thinking. Followed by this, pattern sense contributed second highest ( $\beta = 0.274$ ,  $t$ -value ( $8.935 > 1.96$ )) to the year five pupils' algebraic thinking. Contribution of number sense was relatively significant ( $\beta = 0.248$ ,  $t$ -value ( $8.227 > 1.96$ )) towards year five pupils' algebraic thinking. Lastly, the output has shown that operation sense had the very least significant impact on year five pupils' algebraic thinking ( $\beta = 0.085$ ,  $t$ -value ( $2.727 > 1.96$ )).

**Assessment of Level of  $R^2$ .** Measure of a structural model's predictive accuracy can be determined by the assessment of  $R^2$  level which is calculated by the squared correlation between dependent construct and predicted values. Moreover, independent constructs joint effects on the dependent construct reflected by this  $R^2$ . Specifically, it "represents the amount of variance in the endogenous constructs explained by all of the exogenous constructs linked to it" (Hair et al., 2014, p. 175).

According to Henseler et al. (2009),  $R^2$  values more than 0.75 is considered to be substantial for dependent constructs, while it is considered moderate or weak if 0.50 or 0.25 respectively. However, this rule of thumb suggested for studies focusing on marketing issues. This may not be applicable for other fields especially such as present study which involves mathematics items.

Figure 4.3 shows the  $R^2$  value of dependent construct (i.e., algebraic thinking). The  $R^2$  value for algebraic thinking construct is 0.405, which is considered to be moderate. This means that 40.5% of the variance in the algebraic thinking is explained by four cognitive factors investigated in the present study. Number sense, operation sense, symbol sense and pattern sense have contributed to 40.5% of development of algebraic thinking in year five.

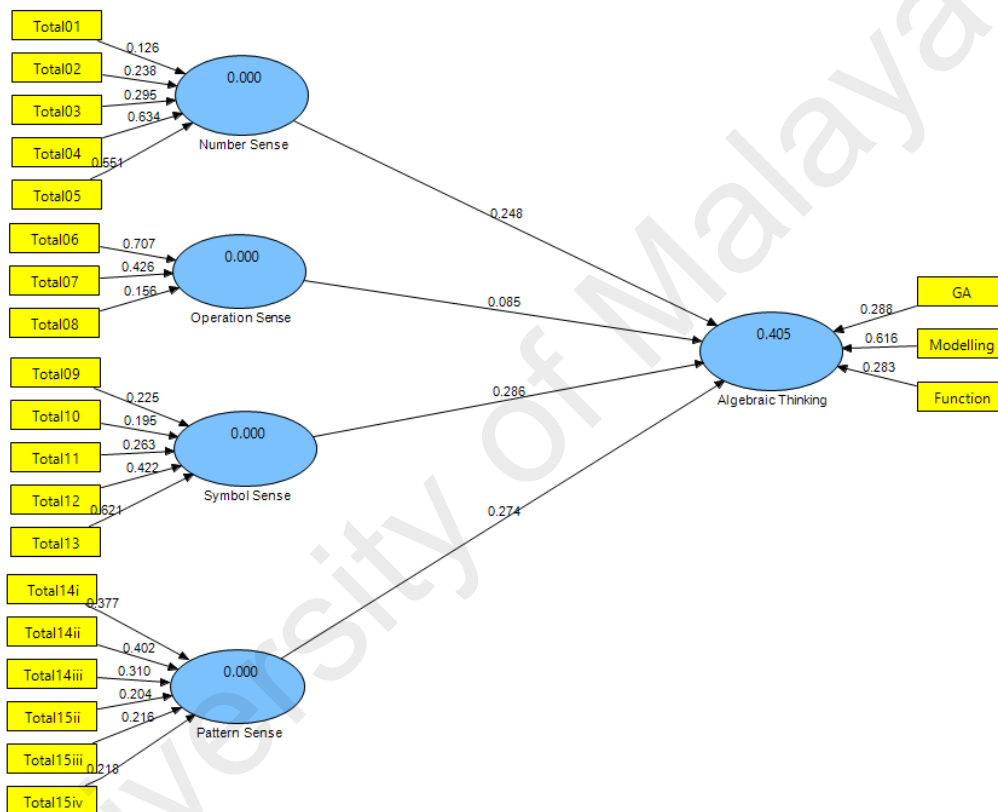


Figure 4.3. The hypothesized model with path coefficients and  $R^2$  value of algebraic thinking

**Assessment of the effect size  $f^2$ .** The effect size ( $f^2$ ) shows what happens if a specific independent construct has been removed from the model when assessing  $R^2$ . In other words, it is to show omitted construct's substantive impact on the dependent construct (Hair et al., 2014). This effect size can be calculated as follows:

$$f^2 = \frac{R_{included}^2 - R_{excluded}^2}{1 - R_{included}^2}$$

The  $f^2$  value can be compared with the benchmark of 0.02, 0.15, and 0.35 to judge the small, medium, and large effects respectively (Cohen, 1988). Based on the Table 4.21, the effect sizes of all cognitive factors on algebraic thinking construct are small as they are less than 0.15. However, it is also notable that an unimportant effect is not always denoted by a small value of  $f^2$ . At the same time, the effects are extremely small to take into account if the values are less than 0.02.

Table 4.21

*Results of  $R^2$  and  $f^2$  values*

<b>Dependent Construct</b>	<b>Independent Construct</b>	<b><math>R^2</math> Included</b>	<b><math>R^2</math> Excluded</b>	<b><math>f^2</math></b>
Algebraic Thinking	Number Sense	0.405	0.354	0.086
	Operation Sense		0.399	0.001
	Symbol Sense		0.339	0.111
	Pattern Sense		0.348	0.096

**Assessment of the Predictive Relevance  $Q^2$  and the  $q^2$  Effect Sizes.** Besides determining  $R^2$  value to evaluate predictive accuracy, it is also important to investigate Stone-Geisser's  $Q^2$  value.  $Q^2$  is used to examine a model's predictive relevance (Hair et al., 2014). This value is an indicator of the model's predictive relevance. When the value of  $Q^2$  is more than zero, it denotes the model has predictive relevance. Using blindfolding procedure for omission distance ( $D = 7$ ) in SmartPLS,  $Q^2$  value of the present study model has been determined. Omission distance can be any value from 5 and 10 provided that the number of observations divided by selected  $D$  should result

in non-integer. As such, 7 has been chosen because 720 (total observations) divided by 7 will result as non-integer. The  $Q^2$  value was 0.243 as shown in Table 4.22.

Similar to effect size ( $f^2$ ) of  $R^2$ , the effect size of predictive relevance can be calculated using the formula below. The effect size values 0.02, 0.15 and 0.35 exhibit small, medium and large impact (Cohen, 1988) from particular exogenous variable towards the endogenous variable. Using the following formula, the present study's final model predictive relevance effect size was calculated. The results are displayed in Table 4.22.

$$q^2 = \frac{Q_{included}^2 - Q_{excluded}^2}{1 - Q_{included}^2}$$

Table 4.22

*Results of  $Q^2$  and  $q^2$  values*

<b>Dependent Construct</b>	<b>Independent Construct</b>	<b><math>Q^2</math> Included</b>	<b><math>Q^2</math> Excluded</b>	<b><math>q^2</math></b>
	Number Sense		0.212	0.041
Algebraic	Operation Sense	0.243	0.249	-0.008
Thinking	Symbol Sense		0.207	0.048
	Pattern Sense		0.197	0.061

Overall, the tests and results have shown that the proposed model is valid and able to proceed with further analysis. The first criterion stated by Hair et al., (2014) to test the validity of a model is the significance and relevance of relationship paths exist in the model. In the proposed model, there are four paths involved. The test results have shown all four paths were significant and relevant. The second criterion is to assess the  $R^2$  values. The proposed model derived a value of 40.5%. This figure illustrates 40.5% algebraic thinking explained by the independent variables. Third

criterion is to assess the effect of all the proposed independent variables. The results have shown all the independent variables have had small effects on dependent variable except operation sense which has an extremely small effect on algebraic thinking. The same result affirmed when calculating the effect of predictive relevance. There was no effect in predictive relevance when operation sense was excluded.

### **Research Question 3**

To what extent proposed cognitive variables contribute to year five pupils' algebraic thinking?

The third research question of present study is as above. To address this question, PLS-SEM technique was used using SmartPLS version 2.0. The path coefficients generated by SmartPLS software was based on 720 samples. The hypothesized model for present study is shown in Figure 4.3 with its path coefficients and  $R^2$  value of dependent variable.  $R^2$  values range from 0 to 1. High variance is explained for the dependent variable if  $R^2$  value closer to 1. It was found that the four cognitive factors explained 40.5% of the variance of dependent variable, algebraic thinking ( $R^2 = 0.405$ ). The remaining about 60% was explained by other variables which were not the interest of present study.

Path analysis is useful to analyse the direct and indirect effects of the independent variables on dependent variable. The strengths among the paths between dependent and independent variables are determined by path coefficients to support the hypothesized relationships (Henseler et al., 2009). Prior to path analysis, it is important to verify the significance of hypothesized relationships (Hair et al., 2014). Hence, bootstrapping procedure was carried out to test the significance of the relationships between cognitive variables and algebraic thinking by using 720 cases

and 5000 samples. These results have been discussed in the previous section, Table 4.20 which displays the results of the bootstrapping procedure. As all the proposed relationships are significant, details of direct effect are discussed subsequently.

In sum, all proposed variables have influence on algebraic thinking to a certain extent. The proposed independent variables were able to explain 40.5% of algebraic thinking. Symbol sense has had the strongest influence on algebraic thinking compared to other variables with the path coefficient value of 0.286. This result is expected as algebra is all about working with symbols. Symbol manipulation is important in learning algebra. Secondly pattern sense was most influential to algebraic thinking with path coefficient of 0.274. This is also expected as algebra also all about working with functions. Working with patterns leads to the ability to work with functions. Thirdly, number sense plays an important role in developing algebraic thinking in year five pupils. Its direct effect value was 0.248. Sense making of numbers is an inevitable element in order to work with algebra in later years of formal algebra. There should be a sense-making ability to work with expressions and problem solving in algebra. Operation sense was the least influential variable in the proposed model according to Figure 4.3. Formal algebra involves a lot of operations manipulation. Yet, the direct effect shown was 0.085. However, the model shows the least path coefficient value which needs further investigation. A complete study which only focuses on manipulation of operations could provide further clarification for its influence on algebraic thinking.

#### **Research Question 4**

Is there any construct(s) which acts as a mediator in the hypothesized model?

The proposed cognitive variables have been hypothesized as direct effect on dependent variable. The fourth research question of the present study aimed to investigate further if there is any mediator involved towards contribution to algebraic thinking. In other words, it aimed to investigate the role of mediator. With regard to this, mediator analysis was used in Smart PLS. The proposed cognitive variables in the present study are based on the review of past literature and researches outcomes. Hence, all the relationships between independent variables and dependent variable are hypothesized as direct effects. However, there is no evidence to show the possible mediating role of these cognitive variables. Thus, researcher has attempted to investigate the mediating role of each cognitive variable in the direct relationship of remaining cognitive variables.

Mediators and moderators are the two inevitable elements in structural equation modelling. A construct acts as mediator when it intervenes between two other related constructs as shown in Figure 4.4. In this case,  $Y_2$  acts as a mediator between  $Y_1$  and  $Y_3$ . Baron and Kenny (1986) claimed three necessary conditions that should be met in order to say mediation exists. They are:

1.  $Y_1$  is significantly related to  $Y_2$ .
2.  $Y_2$  is significantly related to  $Y_3$ .
3. The relationship of  $Y_1$  to  $Y_3$  decline when  $Y_3$  is in the model.

In other words, mediation actually “explains” why there is a relationship between  $Y_1$  (exogenous variable) and  $Y_3$  (endogenous variable). In certain circumstances, a researcher may want to find “why” the relationship between two constructs occur (Hair et al., 2014). Otherwise, to ensure the relationship is the only way between two

constructs. In this case, the role of mediating effect is to provide clarification or explanation of the relationship between two constructs ( $Y_1$  and  $Y_3$ ).

As such, the present study has made an attempt to seek mediation involved in between the relationships of cognitive variables and year five pupils' algebraic thinking. There is no evidence from past literature to show the mediating effect. Thus, the present study made an attempt from zero.

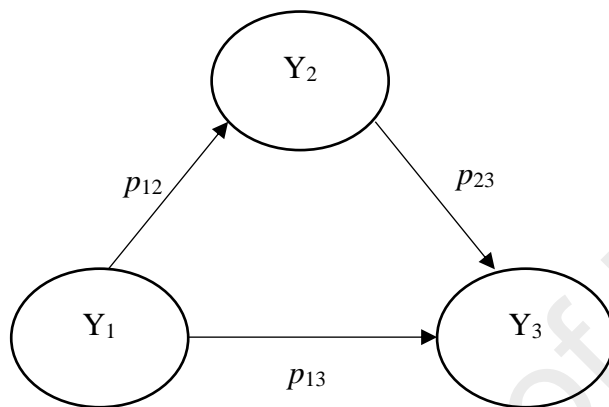


Figure 4.4. General mediator model

Figure 4.4 shows a general mediator model.  $Y_2$  mediates the relationship between  $Y_1$  and  $Y_3$ . The indirect effect is referred to the product of  $p_{12}$  and  $p_{23}$ . The indirect effect should be tested for significance. To check the significance, bootstrapping procedure need to be used. Indirect effects for 5000 samples have been calculated using Microsoft Excel. Subsequently, standard deviation is calculated based on these 5000 samples' indirect effect. The  $t$  value will be the product of indirect effect divided by bootstrapping standard deviation. If the indirect effect is significant, assessment of variance accounted for (VAF) calculation will take place in order to determine the mediation level. VAF can be calculated by the following formula:

$$VAF = \frac{p_{12} \cdot p_{23}}{(p_{12} \cdot p_{23} + p_{13})}$$

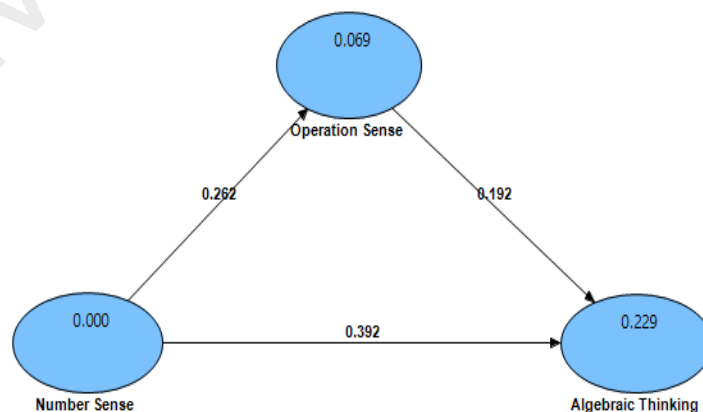


VAF value more than 80% referred to full mediation, between 20% and 80% is categorised into partial mediation and less than 20% considered as no mediation.

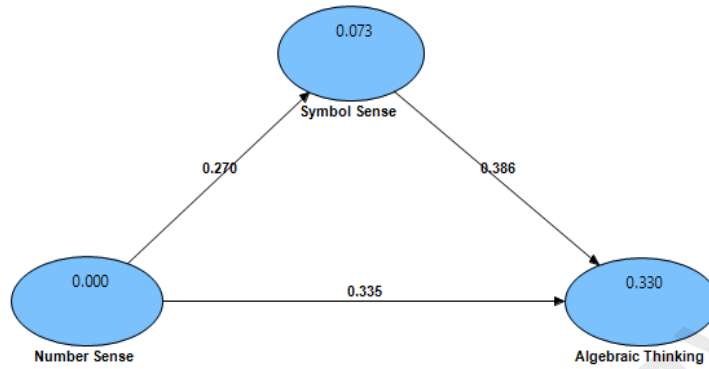
As such, the above procedures have used on all cognitive variables to identify the mediators. Firstly, operation sense, symbol sense and pattern sense mediating effect on the relationship between number sense and algebraic thinking were tested one by one. Then the same procedures were repeated for all the variables to identify the most influencing mediator.

Figure 4.5 shows three different models testing the mediator effect. Model A1 tested the mediating effect of operation sense on the relationship between number sense and algebraic thinking. Model A2 tested the mediating effect of symbol sense on the relationship between number sense and algebraic thinking. In addition, model A3 tested the mediating effect of pattern sense on the relationship between number sense and algebraic thinking. Table 4.23 summarises the significance of test results performed using the procedures explained in preceding section.

### Model A1



### Model A2



### Model A3

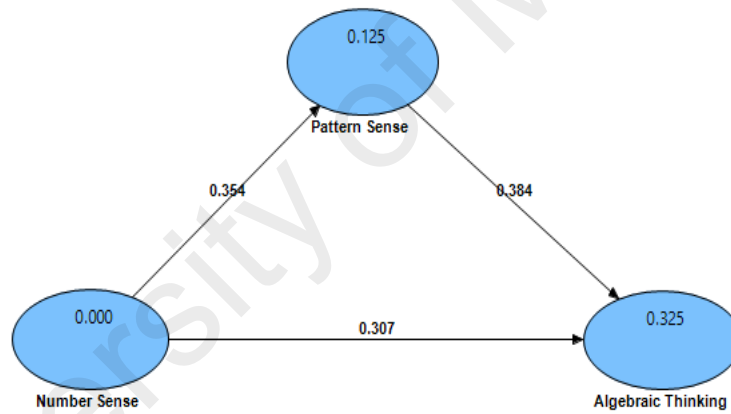


Figure 4.5. Mediating effect of operation sense, symbol sense and pattern sense on the relationship between number sense and algebraic thinking

Table 4.23

Indirect effect, standard deviation and t-values of operation sense, symbol sense and pattern sense on the relationship between number sense and algebraic thinking

<b>Independent Variable</b>	<b>Mediator</b>	<b>Indirect Effect</b>	<b>Standard Deviation</b>	<b>t Value</b>	<b>P Values</b>
Number Sense	Operation Sense	0.050	0.013	4.132	< 0.001
	Symbol Sense	0.104	0.017	6.118	< 0.001
	Pattern Sense	0.136	0.018	7.556	< 0.001

The results have shown that all the three cognitive variables have significant mediating effect. Thus, further VAF analysis was performed to identify the mediator. Table 4.24 summarises the VAF values for each cognitive factor. It is found that there were no mediating effects from operation sense on the relationship between number sense and algebraic thinking. 23.7% of number sense effect on algebraic thinking was explained via the symbol sense mediator. Similarly, 30.7% of number sense effect on algebraic thinking was explained via the pattern sense mediator. Since the VAF was larger than 20% but smaller than 80%, this situation can be considered as partial mediation.

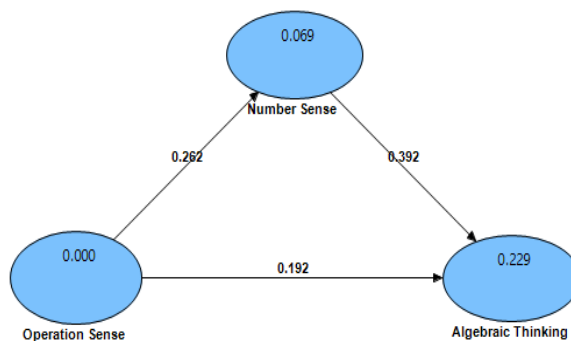
Table 4.24

VAF and mediation type of operation sense, symbol sense and pattern sense on the relationship between number sense and algebraic thinking

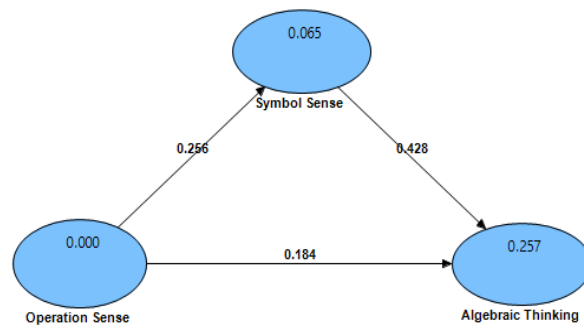
Independent Variable	Mediator	VAF Value (%)	Mediation Type
Number Sense	Operation Sense	11.4	No
	Symbol Sense	23.7	Partial
	Pattern Sense	30.7	Partial

Similarly, Figure 4.6 shows three different models were used to test the mediator effect. Model B1 tested the mediating effect of number sense on the relationship between operation sense and algebraic thinking. Model B2 tested the mediating effect of symbol sense on the relationship between operation sense and algebraic thinking. At the same time, Model B3 tested the mediating effect of pattern sense on the relationship between operation sense and algebraic thinking. Table 4.25 summarises the significance of test results performed using the procedures explained in preceding section.

**Model B1**



### Model B2



### Model B3

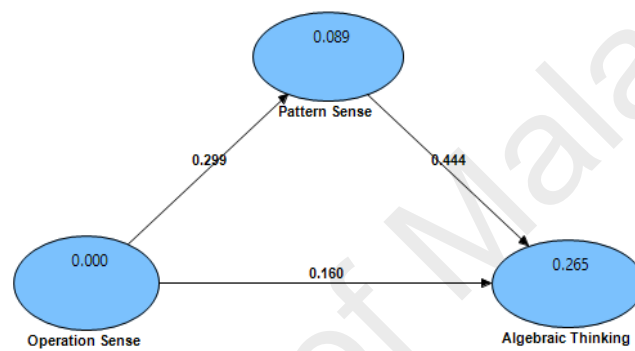


Figure 4.6. Mediating effect of number sense, symbol sense and pattern sense on the relationship between operation sense and algebraic thinking

Table 4.25

Indirect effect, standard deviation and t-values of number sense, symbol sense and pattern sense on the relationship between operation sense and algebraic thinking

Independent Variable	Mediator	Indirect Effect	Standard Deviation	t Value	P Values
Operation Sense	Number Sense	0.103	0.017	6.060	< 0.001
	Symbol Sense	0.110	0.017	6.471	< 0.001
	Pattern Sense	0.133	0.018	7.389	< 0.001

Again, the results have shown that all the three cognitive variables have had significant mediating effect. Thus, further VAF analysis was performed to identify the mediator. Table 4.26 summarises the VAF values for each cognitive factor. It is found that all three predictors have had partial mediating effects on the relationship between number sense and algebraic thinking. 34.8% of operation sense effect on algebraic thinking was explained via number sense mediator. Similarly, 37.3% of operation sense effect on algebraic thinking was explained via the symbol sense mediator. Lastly, 45.3% of operation sense effect on algebraic thinking was explained via the pattern sense mediator. Since the VAF was larger than 20% but smaller than 80%, all three situations can be considered as partial mediation.

Table 4.26

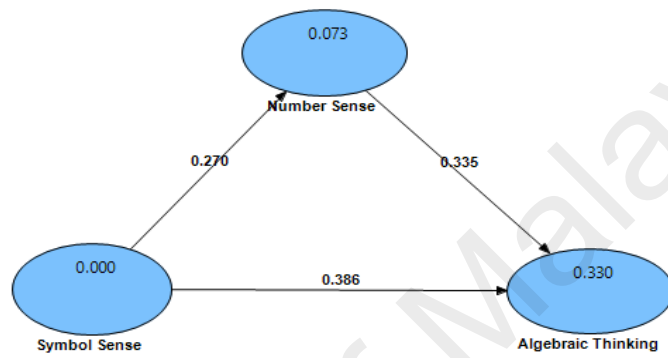
VAF value and mediation type of number sense, symbol sense and pattern sense on the relationship between operation sense and algebraic thinking

<b>Independent Variable</b>	<b>Mediator</b>	<b>VAF Value (%)</b>	<b>Mediation Type</b>
Operation Sense	Number Sense	34.8	Partial
	Symbol Sense	37.3	Partial
	Pattern Sense	45.3	Partial

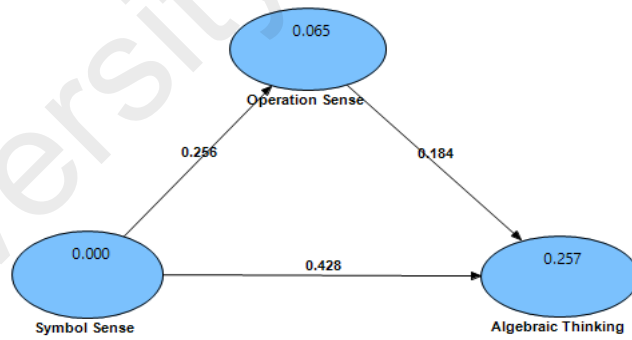
Figure 4.7 shows three different models used to test the mediator effect. Model C1 tested the mediating effect of number sense on the relationship between symbol sense and algebraic thinking. Model C2 tested the mediating effect of operation sense on the relationship between symbol sense and algebraic thinking. At the same time, model C3 tested the mediating effect of pattern sense on the relationship between

symbol sense and algebraic thinking. Table 4.27 summarises the significance of test results performed.

### Model C1



### Model C2



### Model C3

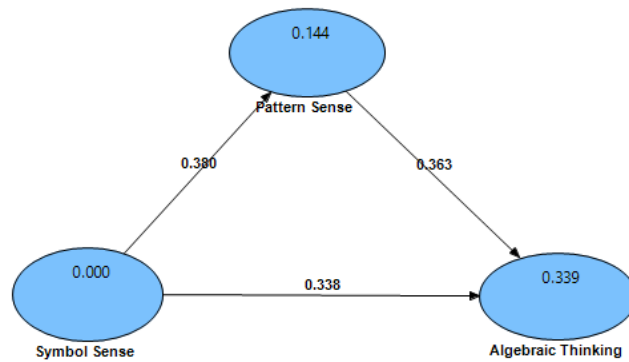


Figure 4.7. Mediating effect of number sense, operation sense and pattern sense on the relationship between symbol sense and algebraic thinking

Table 4.27

Indirect effect, standard deviation and t-value of number sense, operation sense and pattern sense on the relationship between symbol sense and algebraic thinking

Independent Variable	Mediator	Indirect Effect	Standard Deviation	t Value	P Values
Symbol Sense	Number Sense	0.091	0.014	6.500	<0.001
	Operation Sense	0.047	0.011	4.273	<0.001
	Pattern Sense	0.138	0.017	8.118	<0.001

The results have shown all the three cognitive variables have had significant mediating effect. Thus, further VAF analysis was performed to identify the mediator. Table 4.28 summarises the VAF values for each cognitive factor. It is found that there was no mediating effect from number sense and operation on the relationship between



symbol sense and algebraic thinking. 19% of symbol sense effect on algebraic thinking was explained via the number sense mediator. 9.9% of symbol sense effect on algebraic thinking was explained via the symbol sense mediator. Lastly, 29% of symbol sense effect on algebraic thinking was explained via the pattern sense mediator. Since number sense and operation sense VAF values were smaller than 20% it can be concluded that there was no mediation. However, 29% was larger than 20%; thus, it was concluded that there is a partial mediation from pattern sense.

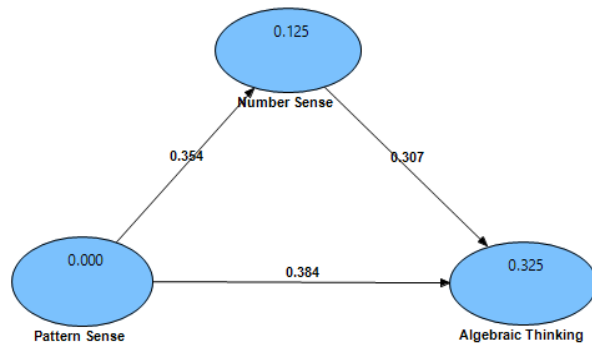
Table 4.28

VAF value and mediation type of number sense, operation sense and pattern sense on the relationship between symbol sense and algebraic thinking

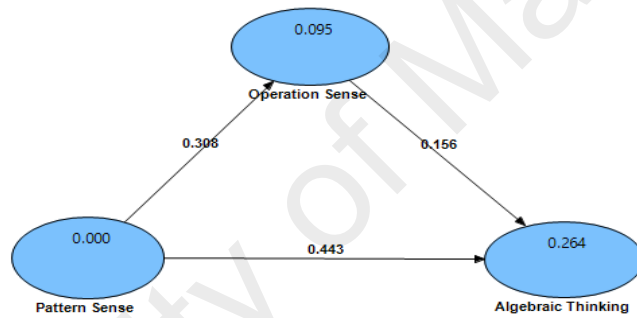
<b>Independent Variable</b>	<b>Mediator</b>	<b>VAF Value (%)</b>	<b>Mediation Type</b>
Symbol Sense	Number Sense	19.0	No
	Operation Sense	9.90	No
	Pattern Sense	29.0	Partial

Figure 4.8 shows three different models used to test the mediator effect. Model D1 tested the mediating effect of number sense on the relationship between pattern sense and algebraic thinking. Model D2 tested the mediating effect of operation sense on the relationship between pattern sense and algebraic thinking. At the same time, model D3 tested the mediating effect of symbol sense on the relationship between pattern sense and algebraic thinking. Table 4.29 summarises the significance of test results performed.

### Model D1



### Model D2



### Model D3

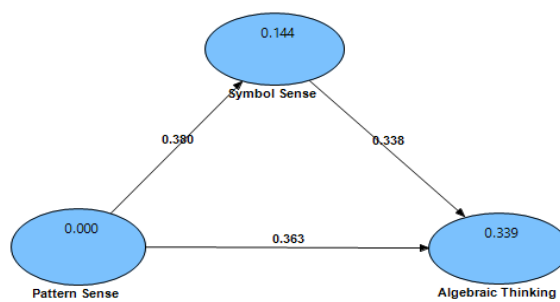


Figure 4.8. Mediating effect of number sense, operation sense, and symbol sense on the relationship between pattern sense and algebraic thinking

Table 4.29

Indirect effect, standard deviation and t-value of number sense, operation sense, and symbol sense on the relationship between pattern sense and algebraic thinking

<b>Independent Variable</b>	<b>Mediator</b>	<b>Indirect Effect</b>	<b>Standard Deviation</b>	<b>t Value</b>	<b>P Values</b>
Pattern Sense	Number Sense	0.109	0.015	7.267	< 0.001
	Operation Sense	0.048	0.013	3.692	< 0.001
	Symbol Sense	0.128	0.016	7.776	< 0.001

The results have shown all the three cognitive variables have had significant mediating effect. Thus, further VAF analysis was performed to identify the mediator. Table 4.30 summarises the VAF values for each cognitive factor. It is found that there was no mediating effect from operation sense on the relationship between pattern sense and algebraic thinking. 22.1% of pattern sense effect on algebraic thinking was explained via the number sense mediator. 9.8% of pattern sense effect on algebraic thinking was explained via the symbol sense mediator. Lastly, 26.1% of pattern sense effect on algebraic thinking was explained via the symbol sense mediator. Since operation sense VAF value was smaller than 20% it is concluded there is no mediation. However, 22.1% and 26.1% were larger than 20%; thus, can be concluded that there is a partial mediation from number sense and symbol on the relationship between pattern sense and algebraic thinking.

Table 4.30

VAF value and mediation type of number sense, operation sense, and symbol sense on the relationship between pattern sense and algebraic thinking

<b>Independent Variable</b>	<b>Mediator</b>	<b>VAF Value (%)</b>	<b>Mediation Type</b>
Pattern Sense	Number Sense	22.1	Partial
	Operation Sense	9.80	No
	Symbol Sense	26.1	Partial

Table 4.31 summarises the results of model A to D together with  $R^2$  values. All the partial mediation type models were selected (i.e., A2, A3, B1, B2, B3, B4, C3, D1, and D3). Models with no mediation type were eliminated (i.e., A1, C1, C2, and D2). Table 4.32 shows the selected models arranged in the descending order based on  $R^2$  values. The highlighted rows indicated the pair of symbol sense, pattern sense (SS & PS) and number sense, pattern sense (NS & PS). This is highlighted in choosing one of them as both pairs involved pattern sense. The best mediator chosen among these combinations was based on VAF values. C3 (29%) gave higher VAF value compared to D3 (26.1%). Hence, C3 has been chosen. Likewise, A3 and D1 is the combination of same constructs (NS & PS). Thus, either one should be chosen. Between A3 and D1, A3 (30.7%) gave higher value of VAF compared to D1 (22.1%). Followed by that, all selected models were rearranged as shown in Table 4.33. Based on this table information, symbol sense and pattern sense have had potential to play mediator roles. The VAF value for pattern sense as mediator was 45.7%, 30.7%, and 29% respectively on relationship between operation sense, algebraic thinking (i.e., Model B3), number sense, algebraic thinking (i.e., Model A3) and symbol sense, algebraic thinking (i.e., Model C3). Similarly, symbol sense also acts as a good

mediator. The VAF value for symbol sense was 37.3% and 23.7% respectively on relationship between operation sense, algebraic thinking (i.e., Model B2) and number sense, algebraic thinking (i.e., Model A2). Model B1 was eliminated because of its lowest value for  $R^2$  compared to the rest. In conclusion, the final models selected were A2, A3, B2, B3 and C3. Based on this new finding, a priori model of the present study was redrawn as shown in Figure 4.9. This information answers the third research question of present study. Symbol sense and pattern sense act as mediator of hypothesized model.

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Table 4.31

Summary of  $R^2$  values, direct, indirect, total effects, VAF values and mediation type of all cognitive variables

<b>Model No</b>	<b>Independent Variable</b>	<b>Mediator</b>	<b><math>R^2</math></b>	<b>IV to Mediator</b>	<b>Mediator to DV</b>	<b>Direct Effect</b>	<b>Indirect Effect</b>	<b>VAF</b>	<b>Total effect</b>	<b>Mediation</b>
A1	Number	OS	0.229	0.262	0.192	0.392	0.050	0.114	0.442	No
A2	Sense	SS	0.330	0.270	0.386	0.335	0.104	0.237	0.439	Partial
A3		PS	0.325	0.354	0.384	0.307	0.136	0.307	0.443	Partial
B1	Operation	NS	0.229	0.262	0.392	0.192	0.103	0.348	0.295	Partial
B2	Sense	SS	0.257	0.256	0.428	0.184	0.110	0.373	0.294	Partial
B3		PS	0.265	0.299	0.444	0.160	0.133	0.453	0.293	Partial
C1	Symbol	NS	0.330	0.270	0.335	0.386	0.090	0.190	0.476	No
C2	Sense	OS	0.257	0.256	0.184	0.428	0.047	0.099	0.475	No
C3		PS	0.339	0.380	0.363	0.338	0.138	0.290	0.476	Partial

Table 4.31, continued

<b>Model No</b>	<b>Independent Variable</b>	<b>Mediator</b>	<b>R<sup>2</sup></b>	<b>IV to Mediator</b>	<b>Mediator to DV</b>	<b>Direct Effect</b>	<b>Indirect Effect</b>	<b>VAF</b>	<b>Total effect</b>	<b>Mediation</b>
D1	Pattern	NS	0.325	0.354	0.307	0.384	0.109	0.221	0.493	Partial
D2	Sense	OS	0.264	0.308	0.156	0.443	0.048	0.098	0.491	No
D3		SS	0.339	0.380	0.338	0.363	0.128	0.261	0.491	Partial

NS-Number Sense, OS-Operation Sense, SS-Symbol Sense, PS-Pattern Sense, IV-Independent variable, DV-Dependent variable

Table 4.32

*Partial mediation models*

<b>Model No</b>	<b>Independent Variable</b>	<b>Mediator</b>	<b>R<sup>2</sup></b>	<b>IV to Mediator</b>	<b>Mediator to DV</b>	<b>Direct Effect</b>	<b>Indirect Effect</b>	<b>VAF</b>	<b>Total effect</b>	<b>Mediation</b>
C3	SS	PS	0.339	0.380	0.363	0.338	0.138	0.290	0.476	Partial
D3	PS	SS	0.339	0.380	0.338	0.363	0.128	0.261	0.491	Partial
A2	NS	SS	0.330	0.270	0.386	0.335	0.104	0.237	0.439	Partial
A3	NS	PS	0.325	0.354	0.384	0.307	0.136	0.307	0.443	Partial
D1	PS	NS	0.325	0.354	0.307	0.384	0.109	0.221	0.493	Partial
B3	OS	PS	0.265	0.299	0.444	0.160	0.133	0.453	0.293	Partial
B2	OS	SS	0.257	0.256	0.428	0.184	0.110	0.373	0.294	Partial
B1	OS	NS	0.229	0.262	0.392	0.192	0.103	0.348	0.295	Partial



Table 4.33, continued

*Selected models after rearrangement*

<b>Model No</b>	<b>Independent Variable</b>	<b>Mediator</b>	<b>R<sup>2</sup></b>	<b>IV to Mediator</b>	<b>Mediator to DV</b>	<b>Direct Effect</b>	<b>Indirect Effect</b>	<b>VAF</b>	<b>Total effect</b>	<b>Mediation</b>
C3	SS	PS	0.339	0.380	0.363	0.338	0.138	0.290	0.476	Partial
A2	NS	SS	0.330	0.270	0.386	0.335	0.104	0.237	0.439	Partial
A3	NS	PS	0.325	0.354	0.384	0.307	0.136	0.307	0.443	Partial
B3	OS	PS	0.265	0.299	0.444	0.160	0.133	0.453	0.293	Partial
B2	OS	SS	0.257	0.256	0.428	0.184	0.110	0.373	0.294	Partial
B1	OS	NS	0.229	0.262	0.392	0.192	0.103	0.348	0.295	Partial

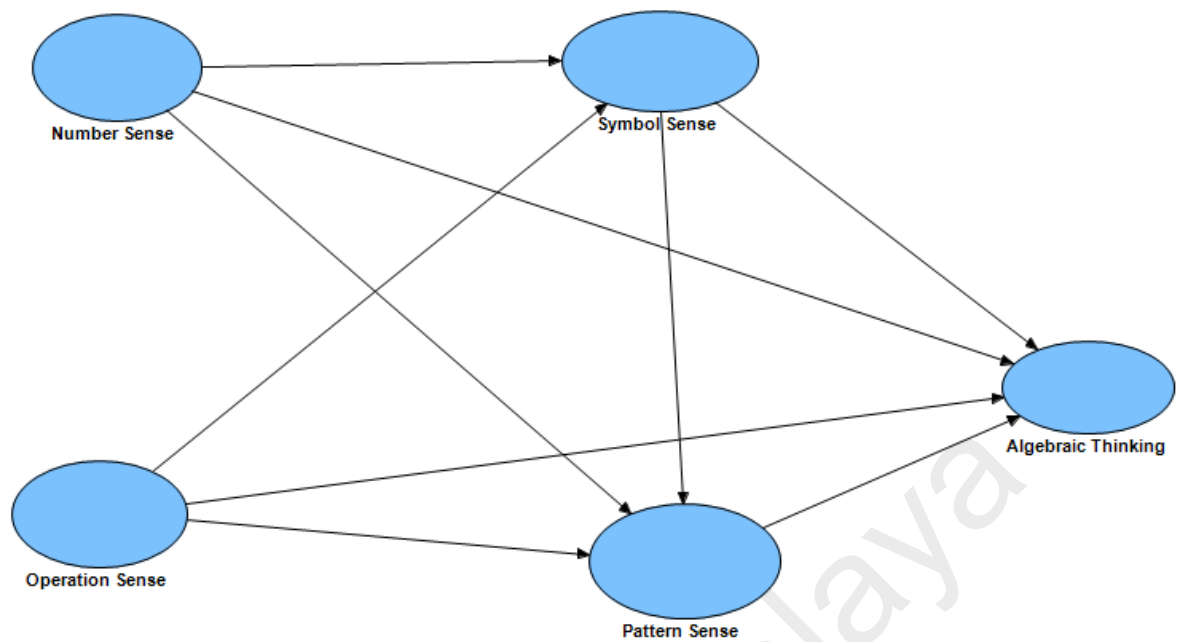


Figure 4.9. Final Model

In order to answer the fourth research question, single mediator analysis was carried out. Each independent construct was treated as a mediator and the VAF values were calculated to identify the best mediator. At the end of the analysis, symbol sense and pattern sense were identified as potential mediator between independent constructs (number sense and operation sense) and algebraic thinking. This interprets knowledge in symbol sense mediates how number sense could be used while working with algebraic thinking tasks. Likewise, it also mediates how operation sense could be used when working with algebraic thinking tasks. With regard to this, sense making of numbers contributes to understanding of variables (symbol sense) and leads to algebraic thinking. In other words, number sense contributes to understanding of variables and equal sign which leads to better performance in algebraic thinking. Sense of operations also involved in working with variables and equal sign (symbol sense) which eventually leads to algebraic thinking. Similarly, pattern sense can influence operation sense, symbol sense and number sense, which in turn can influence

algebraic thinking. For example, working with patterns require some knowledge on operations, symbols involved whether the pattern is growing or shrinking, and sense making of numbers to make a prediction of subsequent patterns or any arbitrary term of patterns.

### **Research Question 5**

What is the relationship between proposed cognitive variables and year five pupils' algebraic thinking in the final model?

As illustrated in Figure 4.3, the main objective of the present study was to investigate the relationship between the cognitive variables and year five pupils' algebraic thinking. However, based on the findings from the third research question of present study, the developed hypotheses tested on the refined model as shown in Figure 4.9. From intense search of literature, number sense, operation sense, symbol sense and pattern sense hypothesized was associated year five pupils' algebraic thinking. Based on previous qualitative research outcomes, it was anticipated that all the four cognitive variables (i.e., number sense, operation sense, symbol sense and pattern sense) explained year five pupils' algebraic thinking. Number sense posited to have a direct effect on year five pupils' algebraic thinking (H1). Similarly, operational sense and symbol sense were inevitable cognitive aspects of algebraic thinking (H2 and H3). Thus, they were expected to be directly influence of year five pupils' algebraic thinking (H4). Lastly, working with patterns was the stepping stone in introducing functions in algebra. Definitely, it was also anticipated to have a positive direct relationship on year five pupils' algebraic thinking. The following sections demonstrate the evidence if the PLS results support the hypotheses below.

- H1:** Number sense has a positive and significant relationship with year five pupils' algebraic thinking.
- H2:** Operation sense has a positive and significant relationship with year five pupils' algebraic thinking.
- H3:** Symbol sense has a positive and significant relationship with year five pupils' algebraic thinking.
- H4:** Pattern sense has a positive and significant relationship with year five pupils' algebraic thinking.

Figure 4.10 illustrates visually the hypothesized direct and indirect paths and the direction of the relationships between the constructs of the proposed model of year five pupils' algebraic thinking.

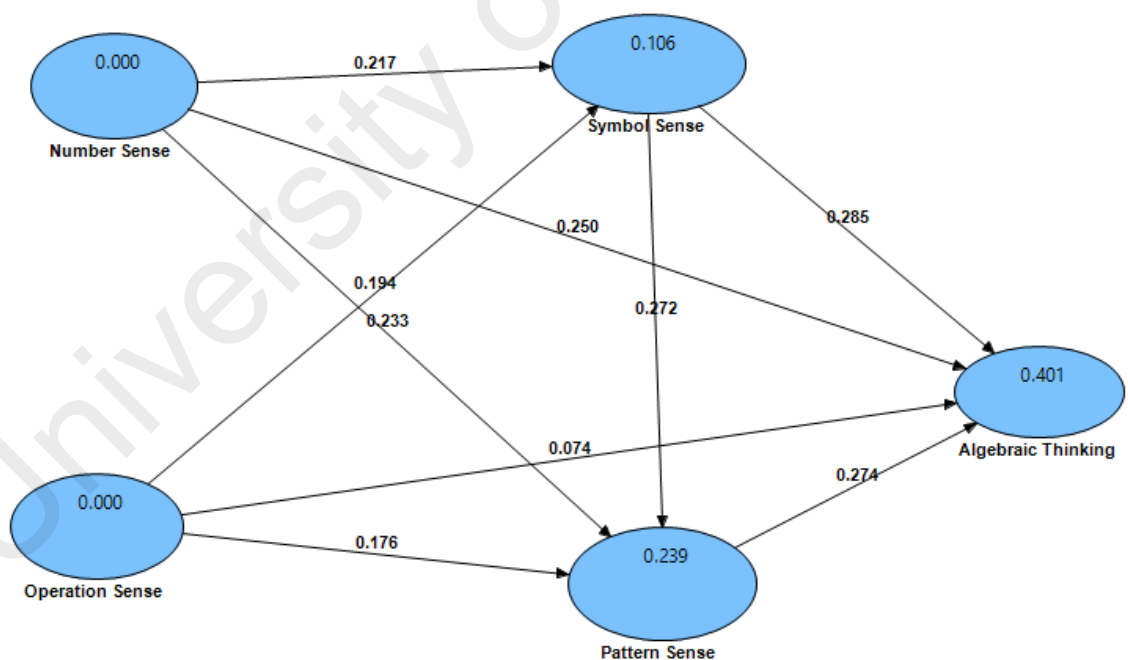


Figure 4.10. Final model with path coefficients and  $R^2$  values

Table 4.34 shows the paths coefficients, the  $t$ -values and significance level of the proposed structural model (Figure 4.10) produced via SmartPLS Software using

sample size of 720. It is found that all the four hypotheses were supported and its path coefficients were statistically significance and in the hypothesized directions. Number sense was positively and significantly associated with year five pupils' algebraic thinking (0.250,  $p < 0.001$ ). Hence, H1 is supported. The outcomes also demonstrate that operation sense significantly and positively associated with year five pupils' algebraic thinking (0.074,  $p < 0.001$ ). Therefore, H2 is also supported. Further, symbol sense has had direct effect on year five pupils' algebraic thinking (0.285,  $p < 0.001$ ). H3 is also supported. Lastly, pattern sense also posited significant direct effect on year five pupils' algebraic thinking (0.274,  $p < 0.001$ ) and supported H4.

The additional information found was, symbol sense mediated the relationship between number sense and algebraic thinking. It also mediated the relationship between operation sense and algebraic thinking. Similarly, pattern sense mediated the relationship between number sense and algebraic thinking. It has had also the potential to mediate the relationship between operation sense and algebraic thinking.

Table 4.34

Standardized paths coefficients, t-values and significance level of the hypothesized model

	<b>Hypothesized Path</b>	<b>Path Coefficients</b>	<b>t-Values</b>	<b>P Values</b>	<b>Results</b>
H1:	NS → AT	0.250	7.950	< 0.001	Supported
H2:	OS → AT	0.074	2.275	< 0.001	Supported
H3:	SS → AT	0.285	8.703	0.02	Supported
H4:	PS → AT	0.274	7.521	< 0.001	Supported

In order to understand the complex interactions of the various cognitive variables of influence on year five pupils' algebraic thinking, all variables were empirically examined for their direct and indirect effects on year five pupils' algebraic thinking. Table 4.35 displays the direct and indirect effects of the cognitive variables in explaining year five pupils' symbol sense, pattern sense and algebraic thinking. Several interesting findings can be noted from the results as discussed.

When a third variable or construct intervenes between two related constructs, mediating effect takes place (Hair et al., 2009). Direct relationship is the relationship between two constructs with a single arrow. At the same time, indirect effect is one of those relationships that involves a sequence of two or more direct effects and represented by multiple arrows. The path that connected number sense and algebraic thinking had a significant coefficient value of 0.250. Hence, it can be concluded that the number sense has a direct positive relationship with algebraic thinking. It is also notable that number sense and algebraic thinking are related indirectly via symbol sense and pattern sense. The direct effect of number sense and symbol sense was 0.217. Thus, indirect effect of number sense on algebraic thinking via symbol sense was 0.06 (i.e.,  $0.217 \times 0.285$ ). On the other hand, the direct effect of number sense on pattern sense was 0.233. It gave an indirect effect of 0.06 for relationship between number sense and algebraic thinking via pattern sense. Likewise, the indirect effect between number sense and algebraic thinking via pattern sense gave value of 0.06 (i.e.,  $0.233 \times 0.274$ ). However, the manual multiplication only gave approximate values because the figure was rounded to three significant numbers. Thus, the researcher used total effects values derived from SmartPLS software for more precise values. The summation of direct and indirect effects of number sense on algebraic thinking gave

total effect of 0.391. The total effects values derived from the SmartPLS software (PLS Algorithm > Quality Criteria > Total Effects).

Similarly, operation sense had direct effect on algebraic thinking with path coefficient values of 0.074. This is to consider small effect and any value less than 0.08 is seldom of interest and hardly provide any substantial information (Hair et al., 2009). However, operation sense effect on algebraic thinking via symbol sense and pattern sense gave higher indirect effect value (0.117) than direct effect value. In sum, the total effect of operation sense on algebraic thinking was 0.191. Followed by this, symbol sense had direct effect value of 0.285 on algebraic thinking. At the same time, its indirect effect value via pattern sense was 0.074. The summation of direct and indirect effects gave a total effect of 0.359 on algebraic thinking. Finally, pattern sense posited a direct effect on algebraic thinking with a value of 0.274.

The results also show that symbol sense ( $R^2 = 0.106$ ) and pattern sense ( $R^2 = 0.239$ ) were explained by number sense and operation sense. The path that connected number sense and symbol sense had significant coefficient value of 0.217 at  $\alpha = 0.05$  ( $t = 5.746, p < 0.001$ ). The direct effect value of number sense on pattern sense was 0.233,  $\alpha = 0.05$  ( $t = 6.424, p < 0.001$ ). In the same way, the direct effect value between operation sense and symbol sense was 0.194 at  $\alpha = 0.05$  ( $t = 4.939, p < 0.001$ ). Meanwhile the direct effect value between operation sense and symbol sense was 0.194 at  $\alpha = 0.05$  ( $t = 4.712, p < 0.001$ ).

Table 4.35

Direct and indirect effects of the number sense and operation sense in explaining the symbol sense, pattern sense and algebraic thinking

<b>Paths</b>	<b>Direct Effects</b>	<b>Indirect Effects</b>	<b>Total Effects</b>
NS →AT	0.250	0.141	0.391
NS →SS	0.217	0	0.217
NS →PS	0.233	0.059	0.292
OS →AT	0.074	0.117	0.191
OS →SS	0.194	0	0.194
OS →PS	0.176	0.053	0.229
SS →AT	0.285	0.074	0.359
SS →PS	0.272	0	0.272
PS →AT	0.274	0	0.274

The model has demonstrated that acquisition of algebraic thinking in year five was a contribution from number sense, operation sense, symbol sense and pattern sense. The compound influence of these constructs shows that they were all significant predictors of year five pupils' algebraic thinking. At the same time, symbol sense and pattern sense also acted as mediators which show understanding of variables, equal sign and pattern series enable to use number sense and ability to work with operation properties efficiently to demonstrate algebraic thinking.



### **Research Question 6**

Is there any moderating effect of gender and location on year five pupils' algebraic thinking?

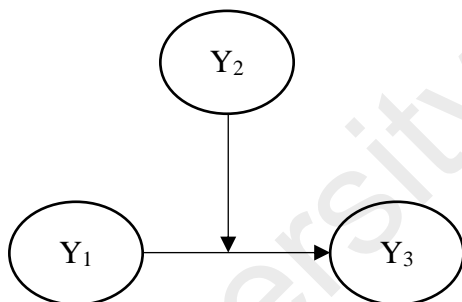
In order to answer the sixth research question of present study, eight hypotheses have been developed to test the moderating role of gender and location in this study. The hypotheses are as stated below.

- H5:** Gender moderates the relationships between number sense and algebraic thinking
- H6:** Gender moderates the relationships between operation sense and algebraic thinking
- H7:** Gender moderates the relationships between symbol sense and algebraic thinking
- H8:** Gender moderates the relationships between pattern sense and algebraic thinking
- H9:** Location moderates the relationships between number sense and algebraic thinking
- H10:** Location moderates the relationships between operation sense and algebraic thinking
- H11:** Location moderates the relationships between symbol sense and algebraic thinking
- H12:** Location moderates the relationships between pattern sense and algebraic thinking

A construct acts as moderator when it changes the strength of a relationship between two constructs in the model (Hair et al., 2014). Figure 4.11 shows the

moderating effects visually. For an example, Y2 could be gender. When the relationship between Y1 and Y3 was significantly different for females and males, it can be concluded that this relationship is moderated by gender.

As such the present study was also interested to investigate the moderating effect of gender and location on proposed algebraic thinking model. Gender and location are the two factors which are most commonly related with mathematical thinking and achievement in the literature. Many studies have been carried out to examine the influence of these factors in the students' mathematical thinking and achievement. With regard to this, researcher had also anticipated some differences between female and male year five pupils' algebraic thinking model and rural and urban school year five pupils' algebraic thinking model.



*Figure 4.11.* General model for moderating effect

According to Baron and Kenny (1986), moderator refers to “qualitative or quantitative variable that affects the direction or the strength of the relation between an independent (predictor) variable and a dependent (criterion) variable” (p. 1174). The variables that causes moderating effects known as moderators. PLS-SEM facilitated the evaluation of moderating effect when the moderator variable was both continuous and categorical. As in the present study, the moderators were gender and

location which was either male or female and urban or rural. Thus, categorical moderator effect approach was used as described by Hair et al., (2014). According to Henseler and Fassott (2010), the moderator variable can be used without any further refinement if the moderator is a categorical variable.

Partial Least Square Multi Group Analysis (PLS-MGA) was carried out (Hair et al., 2014) to investigate the moderating effects of gender and location. Path coefficients based on different samples are always different. “Differences in the model parameters between the different data groups are interpreted as moderating effects,” (Henseler & Fassott, 2010, p. 720). For example, path coefficients of female group will definitely be different from male group. However, the question is whether these differences are statistically different and significant. In order to answer this question, multi group analysis was carried out to test the null hypothesis that the path coefficients were not significantly different. As SmartPLS does not facilitate the multi group analysis by any one single function, the researcher carried out the test by hand by conducting the following guidelines provided by Hair et al. (2014).

- i) The path coefficients and standard error were determined using PLS algorithm and bootstrapping procedures in SmartPLS.
- ii)  $t$ -values and  $p$  values were calculated manually by using the template (PLS-MGA\_Parametric.xlsx ) file available from <http://www.pls-sem.com>.
- iii) Based on  $t$ -values and  $p$  values results of Levene’s test was examined. Then appropriate test statistic selected from whether the standard errors can be assumed to be equal or unequal in the population.

Firstly, the model was estimated for path coefficients ( $p^{(1)}$ ) and standard errors ( $se(p^{(1)})$ ) female group. After that, the path coefficients ( $p^{(2)}$ ) and standard errors

( $se(p^{(2)})$ ) for male group of the model was estimated. With the use of template,  $t$ -values and  $p$  values were calculated for each path in each group. In the comparison between female and male, the path coefficients need to be compared if they are significantly different across the group. Therefore, a bootstrapping procedure was carried out to estimate the  $t$ -values and standard errors. Bootstrapping with resampling size of 5000 was performed to obtain the significant differences of the path coefficients between female and male group. For the number of cases for moderating effect, the model was set to 370 and 350 for female and male groups respectively. It is recommended to use the number of observations in the original sample by Hair et al. (2014).

Figure 4.12 illustrates the template used to calculate these values. It shows the values calculated for NS  $\rightarrow$  AT in each female and male group. The path coefficient and standard error for Group 1 was for female and Group 2 for male. If the result for equality of standard errors value turn out to be between 0.05 and 0.95, the standard errors is assumed to be equal in the population for  $\alpha = 0.10$ . If the value is less than 0.05 or more than 0.95, the standard errors is assumed to be unequal in the population for  $\alpha = 0.10$ . As for NS  $\rightarrow$  AT, the result shown was 0.546 which is within the acceptable range of 0.05 and 0.95; thus, the standard errors are assumed to be equal in the population and respective  $t$ -value (i.e., 1.455) and  $p$  value (i.e., 0.146) were taken from the template. This procedure was repeated for all the paths involved in the model. The PLS-MGA was performed for all the paths in order to identify if there was any moderating effect which existed beside paths mentioned in the hypothesis (H5-H8). The summary of results is shown in Table 4.36.

Please enter values here			Output: Two-tailed test			Test for equality of standard errors
	Group 1	Group 2	Equal standard errors assumed		Unequal standard errors assumed	
Path coefficient	0.201	0.298	t-value	1.455	1.455	0.546
Std. error	0.046	0.048	df	718	713	
Sample size	370	350	p-value	0.146	0.146	

Reject test if value < 0.05 or > 0.95 (for Alpha = 0.10)

Figure 4.12. PLS-MGA\_Parametric.xlsx template

Based on the results shown in Table 4.36, all the paths did not significantly differ across the female and male group except path from operation sense to symbol sense. It shows the effect of operation sense on symbol sense was significantly higher ( $p = 0.316$ ,  $t = 3.241$ ) for year five male pupils at  $\alpha = 0.001$ . Based on these results, it can be concluded that gender does not moderate the relationships between number sense and algebraic thinking, operation sense and algebraic thinking, symbol sense and algebraic thinking and pattern sense and algebraic thinking. This answers the hypotheses from H5 to H8. While Figure 4.13 and 4.14 shows the path model of female and male year five pupils. To generate these diagrams, female and male data were separated. Data for female was used to generate the path model and its beta values for female. Similarly, data for male was used to generate the path model and its beta values for male.

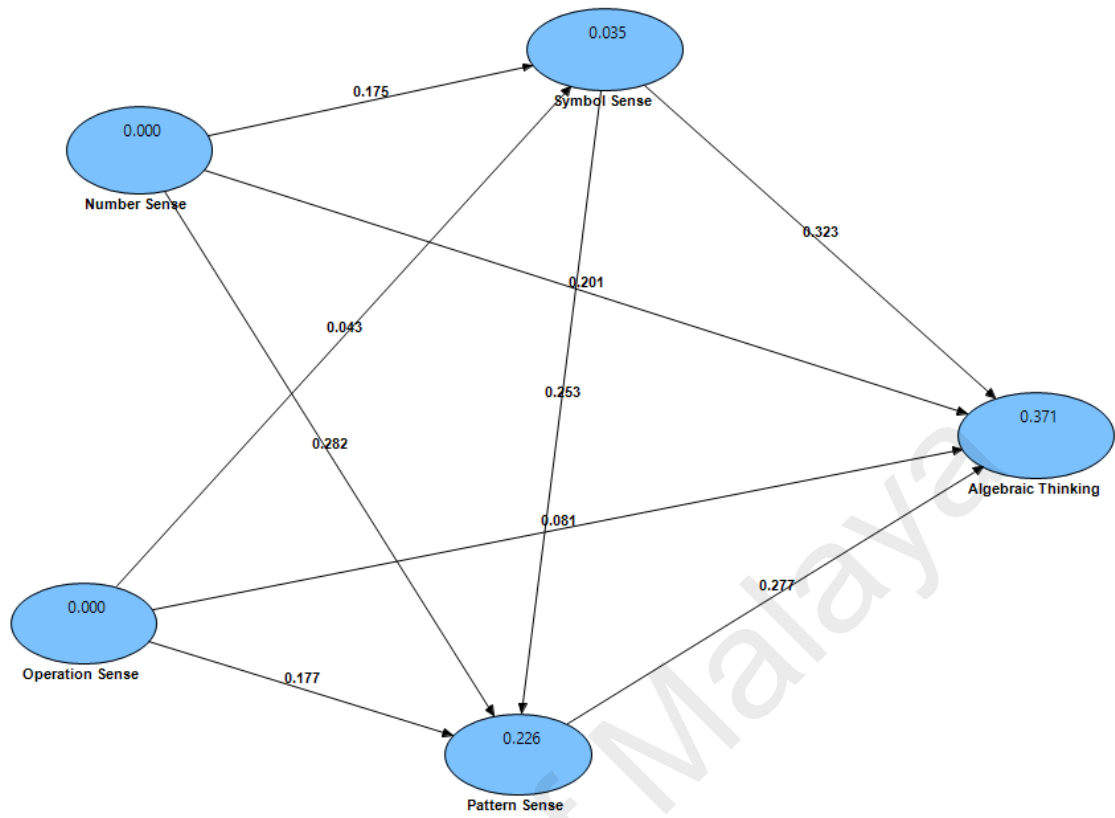


Figure 4.13. Path model of female year five pupils

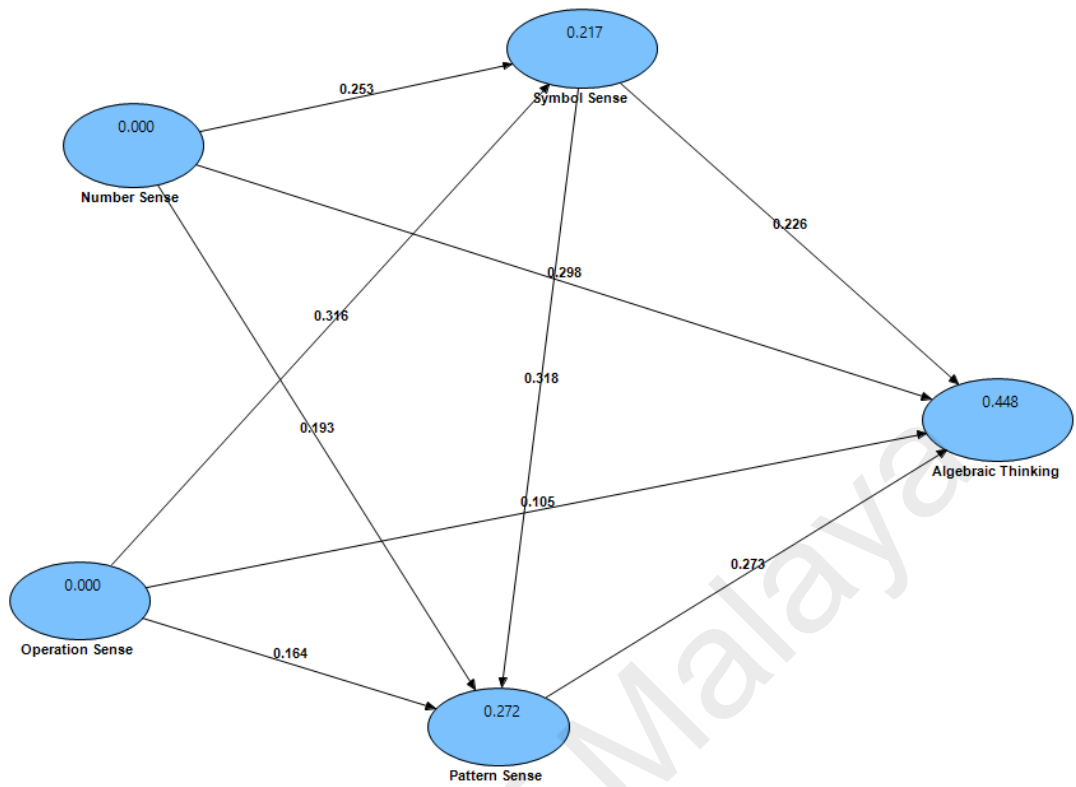


Figure 4.14. Path model for male year five pupils

Table 4.36

*PLS-MGA results for gender*

Paths	Female		Male		Female Vs Male		
	$p^{(1)}$	$se(p^{(1)})$	$p^{(2)}$	$se(p^{(2)})$	$ p^{(1)} - p^{(2)} $	<i>t</i> value	<i>p</i> value
NS → AT	0.201	0.0464	0.298	0.048	0.097	1.455	0.146
NS → SS	0.175	0.0515	0.253	0.0527	0.078	1.06	0.29
NS → PS	0.282	0.0536	0.193	0.053	0.089	1.181	0.238
OS → AT	0.081	0.0478	0.105	0.0479	0.024	0.355	0.723
OS → SS	0.043	0.0686	0.316	0.0491	0.273	3.241	0.001*
OS → PS	0.177	0.0565	0.164	0.0524	0.013	0.169	0.866
SS → AT	0.323	0.0444	0.226	0.0514	0.097	1.43	0.153
SS → PS	0.253	0.0496	0.318	0.053	0.065	0.914	0.361
PS → AT	0.277	0.054	0.273	0.0508	0.004	0.065	0.948
	N = 370		N = 350				

\* $p < 0.001$



Likewise, the same procedures were repeated to test hypotheses H6 to H12. In this procedure, the moderating effect of location was examined. The path coefficients and standard errors of the model were estimated for rural and urban groups. Followed by this, the PLS-MGA\_Parametric.xlsx template was used to calculate the  $t$ -values and  $p$  values. In the bootstrapping procedure, number of cases were set to 360 for each rural and urban as the samples for rural and urban were 360. Table 4.37 summarises the results obtained.

Based on the results, all the paths did not significantly differ across the rural and urban group except paths connecting operation sense to algebraic thinking and symbol sense and algebraic thinking. It shows the effect of operation sense on algebraic thinking was significantly higher ( $p = 0.182$ ,  $t = 2.457$ ) for year five pupils in rural area at  $\alpha = 0.05$ . Similarly, it also shows the effect of symbol sense on algebraic thinking was significantly higher ( $p = 0.358$ ,  $t = 2.191$ ) for year five pupils in rural area at  $\alpha = 0.05$ . Based on these results, it can be concluded that location does moderate the relationships between operation sense and algebraic thinking (H10), symbol sense and algebraic thinking (H11). Meanwhile, location did not have moderate effect between the relationship number sense and algebraic thinking (H9), pattern sense and algebraic thinking (H12). This answers the hypotheses from H9 to H12.

Similar to gender, Figure 4.15 and 4.16 shows the path model of rural and urban school pupils. To generate these diagrams, rural and urban school pupils' data were separated. Data for rural school pupils was used to generate the path model and its beta values for rural schools. Similarly, data for urban school pupils was used to generate the path model and its beta values for urban schools.

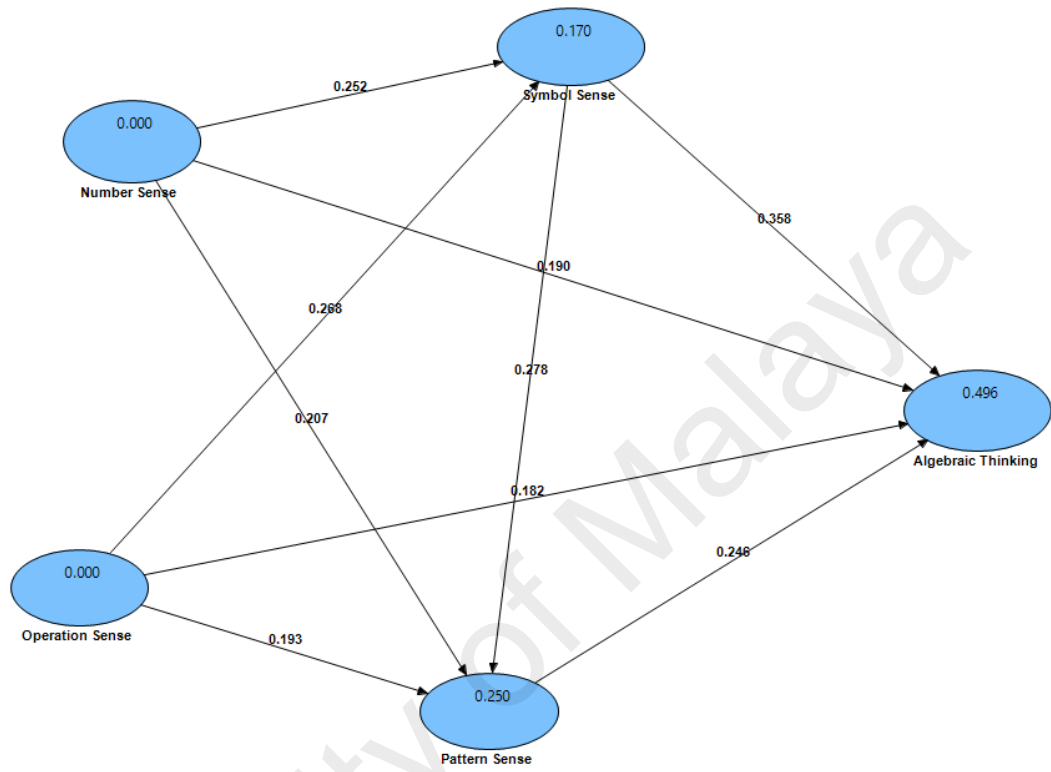


Figure 4.15. Path model for rural school pupils

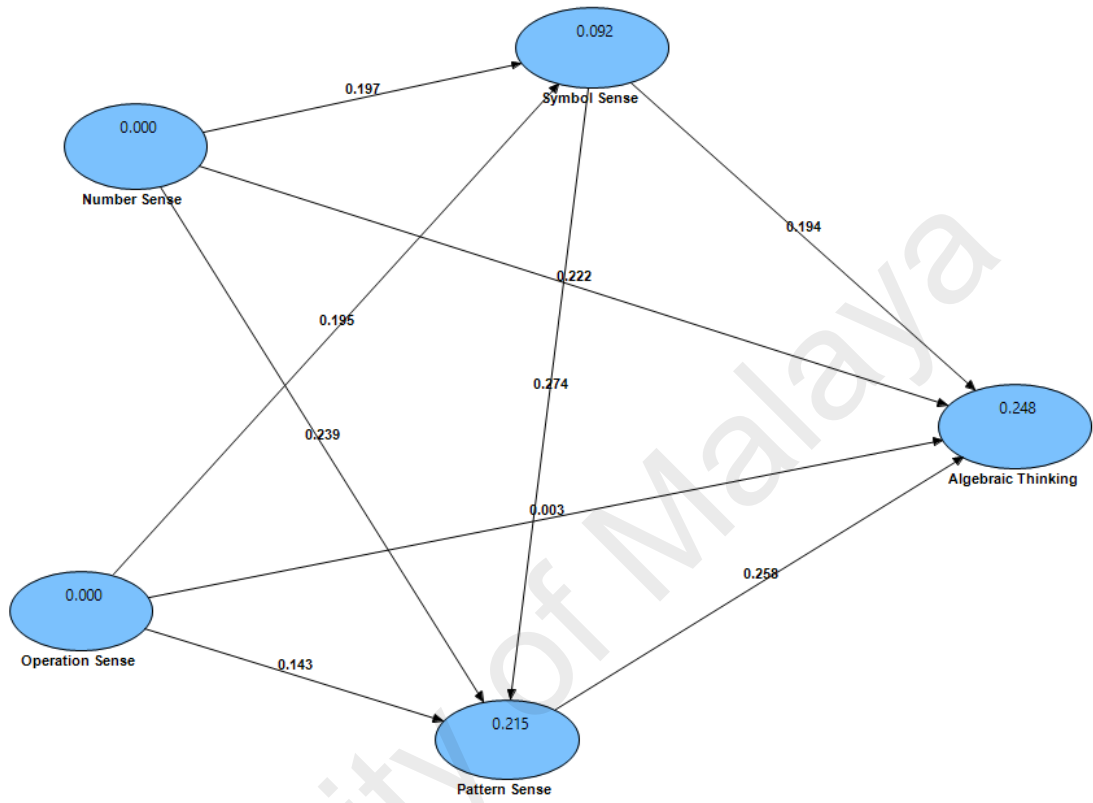


Figure 4.16. Path model for urban school pupils

Table 4.37

*PLS-MGA results for location*

Paths	Rural		Urban		Rural vs Urban		
	$p^{(1)}$	$se(p^{(1)})$	$p^{(2)}$	$se(p^{(2)})$	$ p^{(1)} - p^{(2)} $	<i>t</i> value	<i>p</i> value
NS → AT	0.190	0.042	0.222	0.054	0.032	0.464	0.643
NS → SS	0.252	0.051	0.197	0.059	0.055	0.706	0.480
NS → PS	0.207	0.054	0.239	0.054	0.032	0.418	0.676
OS → AT	0.182	0.041	0.003	0.060	0.179	2.457	0.014*
OS → SS	0.268	0.049	0.195	0.065	0.074	0.907	0.365
OS → PS	0.193	0.055	0.143	0.058	0.051	0.639	0.523
SS → AT	0.358	0.044	0.194	0.060	0.164	2.191	0.029*
SS → PS	0.278	0.052	0.274	0.052	0.004	0.049	0.961
PS → AT	0.246	0.048	0.258	0.057	0.012	0.167	0.868
	N = 360		N = 360				

\* $p < 0.05$

This section shows gender does not moderate any of the relationships in the hypothesized model. This means both female and male year five pupils' algebraic thinking relationships with the independent constructs were the same. The relationship between the proposed cognitive variables and algebraic thinking did not differ. However, as for location, it did moderate the relationships between operation sense and algebraic thinking and symbol sense and algebraic thinking. This means the influence of operation sense on algebraic thinking differed between rural and urban schools' year five pupils. Rural school year five pupils' effect of operation sense on algebraic thinking was higher than urban school year five pupils' effect of operation sense on algebraic thinking. Likewise, the influence of symbol sense on algebraic thinking differed between rural and urban year five pupils. Rural school year five pupils' effect of symbol sense on algebraic thinking was higher than urban school year five pupils' effect of symbol sense on algebraic thinking.

### **Summary**

The hypothesized cognitive model of year five pupils' algebraic thinking was evaluated in this chapter by using PLS-SEM technique. The evaluation was carried out with 720 year five pupils in a district of Malacca using two instruments. The model was evaluated in terms of the measurement and structural model. The evaluation of the measurement model was aimed to examine the indicators of respective constructs. In addition, the evaluation of structural model was aimed to assess the relevance of the model as a whole to represent the year five pupils' algebraic thinking. Formulated hypotheses, H1 to H4 were supported. The model was also compared by gender and location as the moderating effects. The results demonstrated that all the proposed cognitive variables influenced year five pupils' algebraic thinking to certain extent.

Symbol sense and pattern sense also acted as mediators. As for examination of moderating effects, H5, H6, H7 and H8, H9 and H12 were failed to reject. Meanwhile, H10 and H11 were rejected. The findings and implications will be discussed in the subsequent chapter.

University of Malaya

## Chapter 5 Discussion and Conclusion

### Introduction

The present study was based on the influence of number sense, operation sense, symbol sense and pattern sense on year five pupils' algebraic thinking. The cognitive variables of number sense, operation sense, symbol sense and pattern sense were based on previous studies and findings. Past researchers suggested these variables could contribute towards primary pupils' algebraic thinking (Blanton & Kaput, 2004; Fujii & Stephens, 2001; Molina et al., 2008; Slavit, 1999). However, there was not any quantitative evidence provided to support this statement.

Therefore, the present study aimed at producing evidence for the influence of these cognitive variables on year five pupils' algebraic thinking. The main objective of present study was to examine the connection of cognitive variables with the different strands of algebraic thinking of year five pupils in a district of Malacca. This study opted to seek for the web of connection between the proposed cognitive variables and year five pupils' algebraic thinking which involved generalised arithmetic, modelling and function.

With regard to this, subsequent section exhibits the findings and conclusions based on the results and provided some recommendations for future research. The first section of this chapter discusses the findings in this study with respect to the research questions established earlier in chapter 1. Then it was followed by the discussion on the implication for educational practice and recommendations for future researches.

## Discussions

The focus of the present study was to investigate the influence of proposed cognitive variables on year five pupils' algebraic thinking in a district of Malacca. Several research questions were developed to serve as guidelines to perform this study. To facilitate the discussions in order, the subsequent sections organized in three sections as follows.

**Year five pupils' performance in algebraic thinking with respect to gender and location.** As highlighted in past researches, demographic variables such as gender and location could influence primary pupils' performance (Fennema et al., 1998; Ismail & Awang, 2008; Şengül & Erdoğan, 2014). Based on previous studies' findings, some gender differences in year five pupils' performance of algebraic thinking was anticipated. As discussed in chapter 2, Fennema et al. (1998) found that first and second grade male pupils reflected conceptual understanding by using more of abstract solution strategies compared to female pupils. Girls tended to use modelling, counting and concrete solution approaches. Third grade male pupils outperformed female pupils in extension problems by applying their knowledge. Similarly, Sengul and Erdogan (2014) found female sixth grade students outperformed male students in conditional knowledge achievement in solving algebraic problems. However, there was no gender difference in the achievement of declarative knowledge and procedural knowledge. Ismail and Awang (2008) investigated the mathematics achievement differences based on Malaysian eighth grade students' achievement in TIMSS (1999). Based on the secondary data analysis, it was found that gender influenced the students' mathematics achievement significantly. It stated that girls outperformed boys significantly in the mathematics achievement.



This is consistent with van Amerom's (2002) findings. When provided a pre-algebraic open-ended problem to fifth grade pupils, it was found that female pupils made more assumptions than male pupils. Male pupils also showed no interest in answering indeterminate questions. Female pupils performed better than male pupils in algebraic category (i.e., 11% and 4% respectively). Not only that, female pupils were able to provide more written explanation compared to male pupils while male pupils were more inactive. It shows male students were not interested to answer open-ended algebraic problems. The items used in the present study were close-ended. Those with one unique answer and only two items demanded an explanation. Therefore, it could be a reason for no significant difference between female and male achievements in the present study.

On the other hand, Xolocotzin and Rojano (2015), provided evidence that there is no gender difference in the performance of functional thinking. Their findings asserted functional thinking is attainable for both female and male students. This finding goes hand in hand with the present findings, whereby it shows there was no gender difference in the function strand's year five pupils' achievements. These results further affirmed that functional thinking is common for both female and male.

However, when talking about national level, 2003-2015 TIMSS results showed that from two girls outperformed boys in the content area of algebra as discussed in the problem statement. The present study used year five pupils as samples, and found there was no difference between female and male pupils. The TIMSS 2015 results showed girls outperformed boys in algebra in secondary school level. This difference in performance between primary school level and secondary school needs further investigation. Various external factors could affect students' performance when they enter secondary school level. This is especially true because girls are consistent with

their achievement from primary school (MOE, 2013) while boys tend to get distracted from studies once they entered secondary school. This should be overcome immediately to avoid having a cohort of “lost boys” who usually leave school early or leave school with low attainment (MOE, 2013).

Another demographic factor investigated in the present study was location. Location also plays an important role in the performance of students. It is evident with the findings of Ghagar, Othman and Mohammadpour (2011). The authors compared the difference between eighth grade Malaysian and Singaporean mathematics achievements in terms of student-level and school-level factors. Using TIMSS 2003 data, secondary data analysis method was performed. The findings have shown urban school students outperformed compared to rural school students. Besides, many studies also had the same finding that school climate influence students’ academic performance. This is especially because rural area school students were more disadvantaged by family background and school resources (Howie, 2003; Mohd Burhan Ibrahim, 2006). This literature supports the present study’s findings which show urban school pupils performance was better than rural school pupils in all three strands of ATDA. The main reason could be differences in school resources available in rural and urban schools such as library materials, computers and audio-visual materials. Apart from this, inadequate number of teachers in the schools could be a reason for rural school students’ poor performance.

The difference of performance between rural and urban school pupils has been an issue of widespread concern among academic scholars (Marwan, Sumintono & Mislán, 2012; Ting & Tarmizi, 2016). One of the reasons for weaker performance of rural school students is possibly due to less attention from the government during agenda reformation (Marwan, Sumintono & Mislán, 2012). According to World Bank

(2010) report, rural area primary schools lacked in receiving an adequate education. In addition, it added “Disparities within states between rural and urban areas are most prevalent in poorer states like Sabah, Kelantan, and Melaka” (p. 92). As the present study samples were from Malacca, the results were consistent with World Bank’s report. It is evident that the algebraic thinking of urban school pupils was better than rural school pupils.

It is time to look at this disparity. Education department should take more initiative to overcome this problem. One of most worrying drawback of rural area schools is the shortage of teachers. Many teachers in rural schools are required to teach several grades at the same time due to small size of students in these schools. Thus, it leads teachers to struggle in conveying each grade one at a time (Lester, 2005). Apart from this, it is questionable to do the mathematics teaching and learning lessons that really focused on conceptual understanding and properties of operations which eventually leads to algebraic thinking. When the teachers have problems in conducting daily lessons, in schools with inadequate resources, depressed with extra hours of teaching, obviously teachers will tend to focus examination-oriented question and answers while teaching. Therefore, the process of instilling algebraic thinking in primary school pupils does not only depend on teaching and learning process. There are many external factors involved too and it requires immediate focus.

**Cognitive model of year five pupils’ algebraic thinking.** The hypothesized model consists of five variables with four independent and one dependent variable. The independent variables are number sense, operation sense, symbol sense and pattern sense. Cognitive modelling aimed at investigating the interaction between these constructs. The data collected for these variables were from a test instrument

(ANOSPS). The instrument consisted of 19 items which examined various aspects of number sense, operation sense, symbol sense and pattern sense. The dependent variable was year five pupils' algebraic thinking. The data for algebraic thinking was collected via another instrument (ATDA). ATDA consisted of 24 items which evaluated three strands of algebraic thinking (i.e., generalised arithmetic, modelling and function).

Both instruments were pilot tested a few times to identify the reliability, content suitability and validity. The construct validity was examined by two aspects; collinearity among indicators and significance and relevance of outer weights. Then these instruments were administered to 720 year five pupils in a district of Malacca. The data were analysed using PLS-SEM technique.

The overall final model revealed the proposed cognitive variables would contribute 40.1% towards year five pupils' algebraic thinking. This brings to a conclusion that all proposed cognitive variables in the present study associated with year five pupils' algebraic thinking. It has also shown all the indicators were significant to test the proposed model. Further discussions on each variable are as follows.

**Number sense.** Number sense which comprises five elements namely; a) understanding number meanings and relationships; b) recognizing the magnitude of numbers; understanding the relative effect of operations on numbers; d) developing computational strategies and being able to judge their reasonableness; and e) ability to represent numbers in multiple ways have a significant direct effect on year five pupils' algebraic thinking with path coefficient value of 0.250. It has indirect significant effects via symbol sense and pattern sense on year five pupils' algebraic thinking with path coefficient value of 0.06. This finding goes hand in hand with the point

highlighted by Warren (2003a), whereby algebraic thinking develops parallel with number sense. Likewise, development of number sense and algebraic thinking equally gained attention in mathematics learning (NCTM, 2000). In other word, substantial proficiency in number sense is needed in the long-term development of algebraic thinking. The sense of working with number properties eventually will lead to enhancement of pupils' sense making. The ability to make sensible decisions is crucial when working with formal algebra. However, number sense skill alone will not complete the early algebraic thinking literacy.

As discussed in the Chapter 2, so far in the mathematics education field, there are no studies which have been done to investigate the contribution of cognitive variables towards primary pupils' algebraic thinking. Venenciano and Heck (2015), studied the effects of prior mathematics knowledge, age, Measure Up (which consists of topics from area, length and volume), and logical thinking on fifth and sixth graders preparedness of algebra. SEM approach was performed to examine the effects. The findings have shown logical reasoning capabilities and MU curriculum do contribute towards fifth and sixth graders algebra preparedness while prior achievement being an indirect contributor.

The third research question's finding would add on to findings of Venenciano and Heck (2015). It was found that all the four proposed cognitive variables in the present study has had potential in influencing year five pupils' algebraic thinking. Based on the Figure 4.3, 40.5% of year five pupils' algebraic thinking was explained by number sense, operation sense, symbol sense and pattern sense. The direct effect value of number sense on year five pupils algebraic thinking was 0.248. Number sense was the construct that had third highest effect value on year five pupils' algebraic thinking among the four cognitive variables.

With regard to this, teaching and learning activities which could promote number sense eventually will lead to development of algebraic thinking in early years of education. The model provides evidence number sense has an inevitable role in algebraic thinking development. This is also consistent with qualitative studies' discussion on influence of number sense and primary pupils' algebraic thinking (Molina et al., 2008; Warren, 2003b). For instance, when solving a task such as  $6 + \square = 13$  (item no.1 in ATDA), a simple number sense which knowing  $6 + 6 = 12$  then add 1 more will be 13. Hence, the answer would be  $(6 + 1) 7$  for item 1 in ATDA. Solving it in this way encourages students to understand the number sense and properties of operation better rather than memorizing algorithm "6 should be subtracted from 13 to find the missing addend". Therefore, it can be concluded that number sense is important towards infusion of algebraic thinking in primary pupils. It is not exaggerating to have a significant direct effect from number sense to algebraic thinking in the proposed model.

**Operation sense.** Consequently, operation sense which comprised relationship between operations has a significant effect on year five pupils' algebraic thinking with path coefficient value of 0.074. It has an indirect significant effect via symbol sense and pattern sense on year five pupils' algebraic thinking with path coefficient values of 0.06 and 0.05 respectively. This is supported by Slavit's (1999) findings. The elements of operation sense (not limited to the elements investigated in the present study) support the transition of algebraic ways of thinking. This is also supported by Molina et al. (2008). This value signifies the contribution of operation sense towards year five pupils' algebraic thinking literally very small compared to contribution of other cognitive variables. It is quite surprising to see such a small effect derived from the model. Early algebraic thinking comprises operations as one

of the focal themes, “thinking relationally about quantity, number, and numerical operations” (Kieran, 2011, p. 581). Therefore, higher direct effect value was anticipated.

According to Slavit (1999) conception of operation sense, it comprises wide range of aspects such as a) conceptualization of base components of process, b) familiarity with properties of operation, c) relationships with other operations, d) various symbol systems associated with operations, e) familiarity with operations contexts, f) familiarity with operation facts, g) ability to use operation without concrete/ situational referents, h) ability to use operation on unknown/ arbitrary inputs, i) ability to relate the use of operation across difference mathematical objects, and j) ability to move back and forth between the preceding conceptions.

However, this notion of operation sense is too wide to be included in the present study. It is quite impossible to include all the ten aspects of operation sense into the present study. These will require more number of items which lead to longer time duration of test administration in the class. The school administration was concerned about the duration taken for this study as it might affect the school’s regular lessons. Therefore, the present study only looked at year five pupils’ ability to work with operations (addition and subtraction) in terms of relationship between operations (addition and subtraction). Probably, the constraint of operation sense aspects has led to a smaller direct effect value on year five pupils’ algebraic thinking. Due to these delimitations of operation sense investigated, further investigation is required to affirm the result of the present study which has shown operation sense has had very small direct effect on algebraic thinking. Future study with inclusion of all ten aspects of operation sense would be able to confirm further on the direct effect of operation sense on primary pupils’ algebraic thinking. At present, the finding has an indication that

operation sense also has a small direct effect on primary pupils' algebraic thinking. This could shed some light on relationship between operation sense and algebraic thinking development.

Another point to ponder from the result, is the operation sense which refers to working with operations and understanding properties of operations also overlaps with other constructs. Other constructs are also required when working with operations. Most elements of working with operations cannot be a standalone task. It could be a reason for the very small direct effect of operation sense to algebraic thinking.

**Symbol sense.** Working with symbols plays a notable role in developing algebraic thinking in early age. Thus, it has caused highest direct effect value between symbol sense and algebraic thinking. This is supported by findings of Brizuela and Schliemann (2004). They found that ten years old pupils not only were able to work with problems involved unknown amounts but also able to represent it in the equation. The interview outcomes provided evidence that more than 50% of students were able to use letters to represent unknown amounts in the problem given. Therefore, the current findings added value to the body of literature.

Symbol sense with two different elements namely variables and equal sign had a significant effect on year five pupils' algebraic thinking with path coefficient value of 0.285. It also had an indirect effect on year five pupils' algebraic thinking via pattern sense (0.08). Number sense and symbol sense also jointly explained 10.6% of the variance of year five pupils' algebraic thinking. This finding is nothing surprising as symbol sense construct comprises two most important elements of early algebraic thinking; equal sign and variable. These two elements have gained very high attention



among early algebraic thinking studies scholars (Brizuela & Schliemann, 2004; Byrd et al., 2015; Carraher et al., 2006; Knuth et al., 2006; Stephens, 2005).

Symbol sense is an inevitable factor in promoting algebraic thinking in primary level. Conceptual understanding of equal sign especially builds a strong foundation to work with algebraic expression in later years of education (Jacobs et al., 2007; Molina, Castro, & Castro, 2009; Powell & Fuchs, 2010). The inclusion of equal sign as a predictor of year five pupils' algebraic thinking is supported by many studies. Many primary pupils often view equal sign to be operational rather than relational (Jacobs et al., 2007). This always leads to problems in solving equations (McNeil & Alibali, 2005). Besides, Molina and Ambrose (2008), Stephens et al. (2013) advocated that investigating primary pupils' understanding of equal sign has a great influence on their algebraic thinking development. Relational thinking of equal sign leads to a strong foundation to master formal algebra in middle and high schools (McNeil et al., 2006). It is obvious by the symbol sense construct's highest direct effect value on year five pupils' algebraic thinking.

Besides that, symbol sense also mediates relationship between number sense and algebraic thinking, operation sense and algebraic thinking (see Model A2 and B2). Based on Model A2, number sense has a significant effect on symbol sense with  $R^2$  value of 0.073. Meaning, number sense could improve symbol sense by 7.3%, which eventually is able to build strong foundation for algebraic thinking development. This is highlighted by Arcavi (1994), whereby symbol sense and number sense are associated in the development of algebraic thinking. However, to date, there are no quantitative studies which have been done to prove this with evidence. The findings of the present study could be a stepping stone to provoke mathematics researchers to look into these relationships quantitatively.

Furthermore, findings have shown that symbol sense and pattern sense have had significant and stronger effect on enhancing year five pupils' algebraic thinking, which are further supported by studies conducted by Arcavi (1994), and Warren and Cooper (2008). According to Arcavi (1994), one of the symbol sense themes is “Ability to scan a table of function values or a graph or to interpret verbally stated conditions, to identify the likely form of an algebraic rule that expresses the appropriate pattern.” (p. 24). Meaning, symbol sense and pattern sense works hand in hand in promoting primary pupils’ algebraic thinking. Warren and Cooper (2008) provided evidence that primary pupils are capable to work with patterns which could provide a platform to introduce functions.

In most cases, algebra is all about working with variables and equal sign. Symbol sense items which contain working with unknowns are represented by shapes which provoke the capability to demonstrate the understanding of variables. The highest direct effect value derived in the model shows, year five pupils were indeed capable of working with unknowns and equal signs. For an example, when given  $\diamond + \diamond + \diamond = 36$ , year five pupils are able to identify the three numbers should be equal. They have attempted different strategies such as divide 36 by 3 and repetitive addition of 12. It shows the conceptual understanding of unknowns and equal sign.

The question arises here is, what happens to these students when they enter secondary schools? How are they coping with formal algebra in secondary school? The literature has shown evidence that secondary school students’ stumbling block to master algebra is the variables (Edwards, 2000; Swangrojn, 2003). Further investigation could focus on why this cognitive gap occurs between primary school pupils’ capability of working with unknowns and secondary school students’ difficulties in working with variables. The cognitive gap between arithmetic thinking

and algebraic thinking should be investigated further. With those results, Ministry of Education will be able to reform the learning objectives to bridge the cognitive gap.

**Pattern sense.** At the same time, pattern sense has a significant direct effect on year five pupils' algebraic thinking with path coefficient value of 0.274. This value is one of the greatest values compared to other cognitive variables' effect size explored in the present study. Working with patterns and its contribution towards developing early algebraic thinking is one of the most researched areas in the field of early algebraic thinking (Blanton & Kaput, 2003; Blanton & Kaput, 2004; Childs, 1995; Ferrini-Mundy, Lappan, & Phillips, 1997; Smith, 2008). The findings of these studies have provided evidence that ability to work with patterns is strongly associated with foundation for algebraic thinking. Similarly, the findings of present study are also consistent with findings in the literature.

Items in ANOSPS which requires generating a "rule" to perform far generalisation actually evaluate students' capability to work with functions in formal algebra. The ability to work with these items shows the year five pupils' functional thinking of early stage. With appropriate instructional materials and classroom discussion, this functional thinking can be developed into advanced level whereby they will be working with formal algebra functions. The findings yielded in the present study supported by English and Warren (1998). Pattern sense has the potential to develop algebraic thinking via enabling to work with functions (Stacey, 1989; Warren et al., 2006). The authors' study provided evidence that the primary pupils were able to develop functional thinking by identifying patterns provided in table form. The pupils also demonstrated verbal and symbolic communication.

Their findings have established association of pattern sense with algebraic thinking. The finding of present study also affirmed previous studies' findings. Pattern sense also mediates the relationship between number sense and algebraic thinking, operation sense and algebraic thinking, and symbol sense and algebraic thinking. Based on Model A3, number sense has a significant direct effect on pattern sense with  $R^2$  value of 0.125. This means that, number sense could improve pattern sense by 12.5%. Based on Model B3, operation sense has a significant direct effect on pattern sense with  $R^2$  value of 0.089. Based on Model C3, symbol sense has a significant direct effect on pattern sense with  $R^2$  value of 0.144. All relationships within these elements create a web of connection to develop primary pupils' algebraic thinking.

In sum, it can be concluded that all the proposed cognitive variables in present study are associated with early algebraic thinking. The numerical evidence shows they play an important role to certain extent by influencing primary pupils' algebraic thinking. These constructs should be considered while reforming learning objectives in primary school to take algebraic thinking into the next level.

**Symbol sense and pattern sense as mediators.** *A priori* model was developed from literature search which was based on qualitative studies. Therefore, the relationships were predicted as direct. There were no past studies on mediation variables associated with early algebraic thinking. In order to overcome this issue, fourth question in the present study attempted to perform single mediation analysis on each possible combination. Discussion for research question 4 in Chapter 4 has provided detailed findings of mediation analysis. Figure 4.10 depicts the final model derived based on single mediation analysis. Based on the final model, it can be

concluded that symbol sense and pattern sense are mediators between number sense, operation sense and algebraic thinking. The relationship between number sense and algebraic thinking is mediated by symbol sense. The direct effect value of number sense on symbol sense is 0.217 while the direct effect value of operation sense on symbol sense is 0.194. 10.6% of symbol sense construct's variance is explained by number sense and operation sense.

This result is supported by evidence from past literature. It has been shown in the literature that number sense, operation sense and symbol sense are closely knitted with each other in the process of algebraic thinking development from primary level (MacGregor & Stacey, 1997; Molina et al., 2008; Schifter, 1997). According to Arcavi (1994), symbol sense is perceived to be parallel with number sense. Number sense involves:

a sound understanding of their nature and the nature of the operations, a need to examine reasonableness of results, a sense of the relative effects of operating with numbers, a feel for orders of magnitude, and the freedom to reinvent ways of operating with numbers differently from the mechanical repetition of what was taught and memorized. (Arcavi, 1994, p. 24)

In regards to this, symbol sense acts as a medium for the interaction with other "senses" such as number sense, operation sense and pattern sense. The mediation analysis has proved this. Symbol sense acts as partial mediation between number sense and algebraic thinking. It also carries partial mediation role between operation sense and algebraic thinking. Operation sense involves working with operations and understanding of operation properties as discussed in chapter 2. Combination of operations and symbols creates an algebraic expression. Thus, it is not surprising when

the findings show symbol sense has played a partial mediation between operation sense and algebraic thinking. An equation should be with a combination of operations and equal sign. Ability to manipulate operations and symbols is required to find an unknown. Ability of symbol manipulation enables primary pupils to manipulate operation properties too (Slavit, 1999). This is well explained by the model in Figure 4.10. Symbol sense has direct effect on algebraic thinking and also mediates the relationship between operation sense and algebraic thinking.

Likewise, pattern sense also plays a partial mediation between number sense and algebraic thinking, operation sense and algebraic thinking, and symbol sense and algebraic thinking. The direct effect value of number sense on pattern sense was 0.233. The direct effect value of operation sense on pattern sense was 0.176 while direct effect value of symbol sense on pattern sense was 0.272. Number sense, operation sense and symbol sense explained 23.9% of pattern sense construct's variance.

Warren (2005) found that with proper instruction and activities 9 years old children were able to look for the relationships between the repeating patterns and growing patterns. Not only that, they were also able to represent it in abstract form such as  $2 \times N$ , where by  $N$  is the number of times a particular instance repeats. This finding is coherent with the findings of present study's mediation analysis. Pattern generalisation leads children to generate abstract symbol system to express it. Hence, pattern sense mediates the relationship between symbol sense and algebraic thinking.

Besides that, pattern sense also mediates the relationship between number sense and algebraic thinking. Knowing number sense is "a need to examine reasonableness of results" (Arcavi, 1994, p. 24), working with pattern definitely provokes the sense of thought to work with number sense which can lead to algebraic way of thinking (Reys, Lindquist, Lambdin, & Smith, 2009). The qualitative way of

explanation was proven in the present study by presenting a solid model which describes the relationships and interactions involved.

In sum, the present study findings show symbol sense and pattern sense were partial mediators between independent variables and algebraic thinking. These findings have been supported by previous studies. However, there is no evidence which has been shown numerically. The previous findings only discussed the possibility of relationship exists. The model in current study is evident that definitely number sense, operation sense, symbol sense and pattern sense has influenced algebraic thinking. These findings help to overcome the problems identified in the chapter 1. Symbol sense is capable of helping in overcoming the problems students faced in secondary school when dealing with symbols. Based on the final model, symbol sense not only directly influences algebraic thinking, it also acts as a mediator towards algebraic thinking. It means educators should design their lessons and classroom discussions to cater the ability to develop symbol sense. They do not necessarily have to show how to work with  $x$  and  $y$ . While teaching on how to find missing addend or subtrahend, more emphasis on equal sign should be given.

In the first place, teachers should be aware that arithmetic and algebra are not two different subjects. They should not rigidly follow algorithm while teaching. When samples answered item 15 in ATDA (i.e.,  $a - b = b - a$ ), many failed to understand the commutative property. They gave an explanation that it was correct because they both were the same. It shows their mathematics lessons in classroom did not go beyond the curriculum specification and text books. Properties of operations were neglected. This lack of understanding definitely will cause a lot of misunderstandings of symbols and operation as the child progresses to secondary school level.

Another example is when the samples were given an addition/subtraction tasks without numbers (item 7 and 8 in ANOSPS), majority of them were not able to solve it (i.e., 47% and 49 % of correct percentage respectively). More than 50% of the samples failed to answer these items correctly. The reason could be they were only able to work when it was given in the expression form; with numbers and operations ( $a + b =$ ). It possibly revealed that they were only exposed to algorithm-oriented teaching methods. Probably they were not taught beyond finding the correct numerical answer. It may look minor issue at this level, but their cumulative ignorance on each element such as operations properties, symbols, and number sense made a snow ball and ended up in major conceptual understanding problems when learning formal algebra. In order to overcome this, class discussion should be encouraged while teaching arithmetic. For instance, based on simple number sentence such as  $5 + 7 =$ , teacher could probe students to think further rather than just teach how to perform addition in finding the correct answer. Probing further like the answer would be more than 7 or less than 7? Why? What happens if it is the other way around (i.e.,  $7 + 5 =$ ). These types of simple questions before finding the correct numerical solution would help children to think out of the box and not too focused on calculation methods and algorithms.

**Gender and location as a moderator.** There are a few studies done on influence of gender in number sense (Aunioa, Niemivirta, Hautamaki, Shi, & Zhang, 2006). Their findings slightly supported the present study outcomes that there is no influence of gender in the number sense performance. However, there is no study which has been done on moderating effect of gender on relationship between number sense and early algebraic thinking. Thus, the present study findings could contribute a new finding in the field of early algebraic thinking.



Besides the hypotheses (H5 to H8), the findings have also revealed that moderating effect of gender on i) relationship between number sense and symbol sense, ii) relationship between number sense and pattern sense, iii) relationship between operation sense and symbol sense, iv) relationship between operation sense and pattern sense, v) relationship between symbol sense and pattern sense. It was found that gender moderated only the relationship between operation sense and symbol sense. There was no moderating effect of gender on remaining relationship paths.

This means effect of operation sense on symbol sense significantly differed between female and male pupils in year five. The effect size of the relationship between operation sense and symbol sense if 0.043 and 0.316 was for female and male respectively. The influence of operation sense on symbol sense was higher for male compared to female year five pupils. This probably could be because male pupils prefer more abstract strategies (Fennema et al., 1998). Meanwhile female pupils tend to depend on standard algorithms to solve given problems as grade progressed.

As evident from past literature, majority of female students prefer to use modelling and counting strategies as they grow older. They strictly follow the standard algorithm where by performing addition or subtraction by following column rules of ones, tens and hundreds. Meanwhile, male pupils find it easy to work with invented algorithms (Fennema et al., 1998). For instance, when given a task such as  $38 + 26$ , they tend to use strategies such as 30 and 20 is 50, and 8 makes; then 6 more is 64. This supports the findings of the present study on moderating effect of gender on relationship between operation sense and symbol sense.

The items for operation sense in the present study test for conceptual understanding of addition and subtraction (item 6, 7, and 8 in ANOSPS). They could have scored better if computation tasks given. However, there is no strong evidence

to support this claim. In fact, these items were actually aimed to investigate the conceptual understanding of addition and subtraction. Operation sense in the present study investigates the conceptual understanding of addition and subtraction. Hence, the results suggest that the year five male pupils could have demonstrated better conceptual understanding about operations which led to higher direct effect size on symbol sense. Further investigation would be appropriate to affirm these results.

There are many studies in the literature which discussed the influence of rural and urban location of school on the students' mathematics achievement (Cox, 2000; Ma & Klinger, 2000; Mohd Burhan Ibrahim, 2006). Therefore, it is not an exaggeration if the present study sought to find the moderating effect of location on year five pupils' algebraic thinking.

Findings have shown that location moderated the direct effect size of operation sense on algebraic thinking and direct effect size symbol sense on algebraic thinking. As evident from body of literature, rural school students are capable to perform equally. However, due to different factors it has been recorded rural schools' performance always lag behind compared to urban school performance (Cox, 2000). This was also proven in the present study, algebraic thinking of year five pupils in urban schools was better than rural school year five pupils.

Although the overall performance in algebraic thinking favors urban school students, it is worth to take note the direct effect of operation sense on algebraic thinking and direct effect of size of symbol sense on algebraic thinking significantly differed between rural school pupils and urban school pupils. The rural school year pupils' direct effect size of operation sense on algebraic thinking and direct effect of size of symbol sense on algebraic thinking were stronger.

This shows rural school year five pupils' conceptual understanding of operations was probably better than urban school year five pupils. Again, many other factors also could contribute to this result. As a general contention, rural school students are disadvantaged by inadequate number of teachers due to small size number of students. However, this aspect would have contributed to the fruitful classroom discussions to take place due to small number of students in each class. Teachers would have conducted closer discussions with students as the total number of students is small. In urban schools, it would be difficult to conduct classroom discussions with closer contact as the number of students in each class is quite big (Mohd Burhan Ibrahim, 2006). With regard to this, rural school year five pupils' direct effect sizes of operation sense and symbol sense on algebraic thinking was higher than urban school year five pupils. However, the researcher discussed the possible inference could be made based on the findings of the present study. More comprehensive further study could be conducted in order affirm these facts. At the same time, questions may arise why the direct effect sizes of number sense and pattern sense on algebraic thinking are not moderated by the location. Hence, further investigation should be probed to as what factor contributes towards the rural school year five pupils' higher direct effect size of operation sense and symbol sense on algebraic thinking while there was no significant difference between the direct effect size of number sense and pattern sense on algebraic thinking.

### **Implications of Research Findings**

Knowing the importance of cultivating algebraic thinking from young age, understanding the cognitive variables which contribute to develop algebraic thinking is crucial as well. The findings of the present study have made contribution to the

body of knowledge by providing a model to understand better the cognitive variables that influence the year five pupils' algebraic thinking. To be more precise, the present study attempted to develop a model to reveal the connection between number sense, operation sense, symbol sense and algebraic thinking to facilitate preparation of teaching plans. The findings of present study provide essential implications for theory, curriculum designers and educators in primary school mathematics education.

The present study was aimed to develop a model to represent year five pupils' algebraic thinking which can reveal the web of connection between proposed cognitive variables and algebraic thinking. Anderson's ACT-R framework was adopted to guide the development of model. As discussed earlier, in ACT-R framework declarative and production memory works with working memory to store, execute and retrieve information. This particularly describes the process involved in developing algebraic thinking while doing arithmetic in primary school level on how the knowledge of number sense, operation sense, symbol sense and pattern sense are stored in procedural memory and retrieved when solving algebraic problems. The connections are made through proceduralisation. Condition-action pairs enable pupils to apply the cognitive skills such as number sense, operation sense, symbol sense and pattern sense to solve given problems. Hence, it shows how the results of present study provided theoretical implications.

As for mathematics educators, the findings of the present study could provide what is algebraic thinking in primary school level and how the number sense, operation sense, symbol sense and pattern sense are intervened together. Based on the instruments' items, primary school mathematics teachers could get an overview of what are the activities that can promote algebraic thinking. According to Chick and Harris (2007), teachers' knowledge plays an important role in cultivating algebraic

thinking in the daily classroom lessons. On the other hand, the teachers teaching mathematics in primary schools have very limited knowledge of how the daily classroom activities and discussions could lead to development of algebraic thinking and build a strong foundation for later learning of formal algebra. The authors found the teachers were more focused on the correctness of solution, but not looking at the students' reasoning. Too much focus given for computation fluency (Blanton & Kaput, 2005). Therefore, the findings and the items used in the present study could create awareness among mathematics educators in the primary schools on what are the elements and properties that can lead to development of algebraic thinking of primary pupils. Algebraic thinking can be developed in the primary school level, provided teachers select appropriate tasks and encourage discussions in classroom (Jacobs et al., 2007).

Furthermore, outcome of present study also enlightens the primary school mathematics teachers on what to emphasis while doing tasks such as number patterns. As mentioned earlier in chapter 1, the Malaysian Primary Curriculum text books do have activities such as number patterns whereby pupils need to predict the subsequent terms in a series of numbers given. For example, 1000, 2000, 3000, ..... Common numerical patterns given were 'growing' patterns which means it grows in hundreds or thousands. The educators should not stop only at teaching finding the subsequent terms (i.e., near generalisation). They should go beyond that (i.e., far generalisation). Educators should probe students to think of how to find 30<sup>th</sup> term or 50<sup>th</sup> term without working on each term. This will encourage the pupils to think out of the box and make sense of the solutions involved. Eventually they will be able to generate a 'rule' based on the relationships they identified in the given numerical pattern. This ability in

young age will enable them to see connection easily between  $f(x)$ ,  $y$  and  $x$  values later in secondary school algebra.

For instance, the present study has demonstrated how pattern sense will lead to a successful functional thinking. The items (no 14-15) in ANOSPS have shown how the pattern activities should be designed and probe students to generate a 'rule'. With that 'rule' how to perform near and far generalisation. Educators should reform their way of delivering lessons. They should not focus on finding the subsequent term correctly when teaching number series pattern activities. On the other hand, it can be done as group activity to find the 'rule' and work from there. More interaction and discussions should be cultivated among students. This could help in providing more interesting lessons and at the same time provide quality discussions which can enable students to think beyond text books and routine exercises.

Beside this, while teaching arithmetic, it is also important to incorporate sense of numbers. Ability to get correct solution for a task should not be the ultimate aim. Pupils' ability to make sense of numbers is also important. For an example, given a number sentence such as  $2 + 7 = 7$ ; to determine true or false, the pupils should be exposed to evaluate the logic of this number sentence rather than computing 2 and 7 to derive the correct answer. Instead of teaching the sum of 2 and 7 is 9, thus this number sentence is false, the pupils should be encouraged to think when a number is added to 7, definitely the value of 7 should be increased, and thus the number sentence is false. In the latter method, pupils are encouraged to think beyond computation. They will able to make sense of symbols. These are two examples on how educators could cultivate number sense and symbol sense while teaching without any additional introduction of new syllabus.

Next, implications of the present study's findings for curriculum development and professional development are also notable. Curriculum designers could use the findings of the present study to understand the connection and importance of cognitive variables and algebraic thinking. They will be able to look into how the current mathematics curriculum should be reformed to facilitate the cognitive development capacity in algebra learning. Appropriate professional development is necessary to train the teachers to improve the lessons and expose them to elements which can enable pupils to think algebraically. Based on the final model of present study, curriculum developers could consider emphasizing the role of number sense, symbol sense and pattern sense in the curriculum development. The learning objectives could be restructured by giving more emphasis on these cognitive skills. At the same time, instructional designers could prepare activities with incorporating number sense and patterning activities together providing the objective to be achieved. The final model of the present study could be used in the professional development program to provide training for novice pre-service teachers in teaching methods to encourage primary pupils to think algebraically.

The role of professional development is very crucial in making a difference in learner's learning process. Figure 5.1 shows the Guskey's (1986) model on how professional development influences teacher change and change in student learning outcome. According to Guskey (1986), changes in students' learning outcomes occur only when there is a change in teacher attitude and beliefs. This model portrays how appropriate professional development for educators on developing algebraic thinking could also influence the primary pupils' acquisition of algebraic thinking skills. Blanton and Kaput (2005) asserted that "However, most elementary teachers have little experience with the rich and connected aspects of algebraic reasoning that need to

become the norm in schools and, instead, are often products of the type of school mathematics instruction that we need to replace" (p. 414). Thus, it is very crucial to look into from the aspect of professional development for teachers.

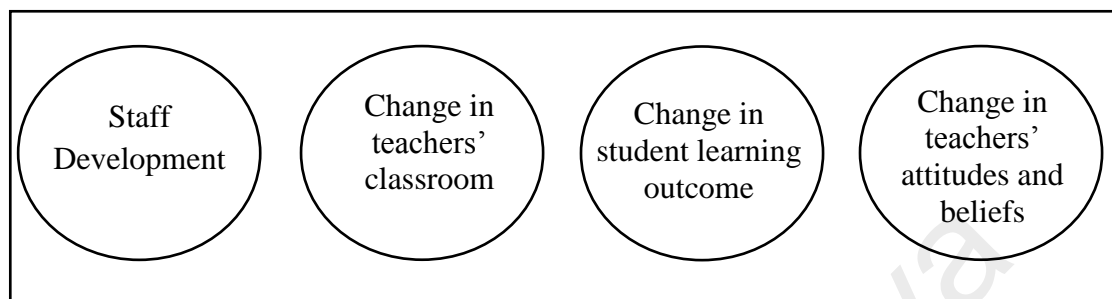


Figure 5.1. Guskey Model

The current primary school text books comprise of numerical figural pattern activities and open number sentence items. However, how the teachers encourage the pupils to solve these problems and to what extent these types of activities given priority and how the classroom discussions being carried out is still questionable. The role of symbol sense and pattern sense as a mediator and direct influence towards algebraic thinking is evident from the model found in the present study. Thus, in future, more attention should be given to these cognitive variables when designing the curriculum.

Another implication of the presents study findings is it has revealed the algebraic thinking level of year five pupils in a district of Malacca. The results have shown the pupils' performance in algebraic thinking is moderate (see Table 4.14). This is a notable contribution of present study to the body of literature. To date, there is are no data to show Malaysian primary pupils' algebraic thinking. Malaysia did not take part in TIMSS for grade four category. Thus, there is no evidence to show if the poor performance of Malaysian eighth grade students' in TIMSS for algebra domain is caused by weak foundation in primary school. The performance of year five pupils in ATDA can be compared with the results reported by Ralston (2013) on fifth grade



students' algebraic thinking level in Singapore and U.S. The comparison shows Malaysian year five pupils' performance fell behind compared to performance of Singapore and U.S year five students. It is essential to take note of this difference, as it is an indication of poor performance of Malaysian eighth grade students in TIMSS could be caused from primary school.

Lastly, policy makers should consider this model to train the pre-service teachers on how to incorporate algebraic thinking in the classroom discussion. They could prepare some sample lesson plans with appropriate activities and guidelines on how to initiate the discussion to solve the problems. Finding the correct solution should not be the ultimate objective. Teachers should not only focus on the correctness of solution. For example, number sense could be infused while teaching the whole numbers and arithmetic topics in the beginning of syllabus. Number sense is not to be included as a new topic in the syllabus. Items 6, 7, and 8 in ANOSPS show an example of how to teach students on the properties of operations. These items show how the understanding of operations properties could be used even without numbers. Same goes to pattern sense. The teachers should be aware that sense of patterns and ability to identify the relationships in either repeating or growing patterns is the foundation to work with functions in the later stage of learning formal algebra. The classroom discussions could encourage pupils to formulate a 'rule' to find the subsequent pattern. It also can be performed by classroom games so that the pupils will learn to think algebraically in more fun filled environment. The sample lesson plans or activities in the form of games could be prepared by policy maker by considering the cognitive variables identified in the present study and strands of algebraic thinking discussed. Teaching variables by introducing shapes such as ♠ and ♣ could let the children think

about unknowns. They don't get frightened to work with unknowns. Eventually they will find it easier to work with variables at later stages of formal algebra.

### **Suggestions for Future Researches**

The final model of the present study is an aid for educators and curriculum developers to foster algebraic thinking from primary school level. It is advisable to develop mathematics lessons based on the final model for primary school syllabus. This would act as an introductory step to prepare primary pupils to face the challenges in later formal algebra. To facilitate infusing the algebraic thinking in primary pupils, the findings could be further refined by looking at more cognitive variables such as logical thinking and more aspects of operation sense as discussed in chapter 2.

The data collection method could be improved by including focus groups to get acquire more rich data from the primary pupils. Body of literature has plenty of studies done qualitatively on investigating primary pupils' algebraic thinking. It would be useful if future studies conducted by including focus groups to evaluate primary pupils' understanding on cognitive variables proposed in the present study such as number sense, operation sense, symbol sense and pattern sense. Besides that, studies on Malaysian pre-service teachers' algebraic thinking are crucial to be identified too. Teachers play an important role to deliver lessons and engaging students in fruitful discussions. Thus, future researches on pre-service teachers' knowledge on early algebraic thinking could provide more data to improve primary pupils' algebraic thinking. Further professional developments could be recommended for the teachers in-service to enhance teaching and learning sessions.

A detailed comparison can be made between the present study findings on year five pupils' algebraic thinking and findings provided by Ralston (2013) on fifth grade

students' algebraic thinking in U.S and Singapore. The ATDA instrument was adapted from Ralston (2013). Thus, the samples of present study and samples of Ralston (2013) answered the same questions.

Besides that, the ANOSPS and ATDA instruments could be designed as online answering tests. By doing this, the research could be done in wider geographic locations with more samples effortlessly. Researcher could communicate with person in-charge for each school throughout Malaysia and carry out the tests to get input on Malaysian year five pupils' algebraic thinking. This will enable evaluation of year five pupils' algebraic thinking in nationwide especially by including Sabah and Sarawak. Not only this, computerized instruments could also overcome the limitation mentioned in chapter 1. Whereby in paper and pencil test, there is a possibility for the students to guess the answer based on the choice of reasons provided in ANOSPS. This limitation could be avoided in computerized test by using two-tier test method.

Lastly, further investigation could be designed to investigate the role of operation sense towards algebraic thinking. As the present study model shows a very small direct effect path value towards algebraic thinking. Different research design should be considered focusing and investigating in depth about contribution of operation sense in the development of algebraic thinking. Probably more elements of operation sense as discussed in chapter 2 could be included to establish a better comprehensive investigation to evaluate role of operation sense.

## **Conclusion**

The present study was motivated by the need to investigate influential cognitive variables towards primary pupils' algebraic thinking. Identification of influential cognitive variables towards primary pupils' algebraic thinking will enable

educators and policy makers to restructure the teaching and learning activities to facilitate children to think algebraically. The data collection especially involved evaluation of year five pupils' algebraic thinking in a district of Malacca and performance in four cognitive variables. The subsequent section discusses the conclusion drawn from the present study.

First the potential cognitive variables that influence year five pupils' algebraic thinking were synthesised from wide search of literature. Based on this, a model was hypothesized to represent the links between the cognitive variables and year five pupils' algebraic thinking. The model was evaluated via PLS-SEM technique. Upon various evaluation stages, the final model was achieved as shown in Figure 4.10. This model has shown all the proposed cognitive variables; number sense, operation sense, symbol sense and pattern sense have had significant direct and indirect effect on year five pupils' algebraic thinking. The findings have also shown symbol sense and pattern sense were mediators between independent variable and year five pupils' algebraic thinking. These findings create awareness among educators that algebraic thinking is not a single standalone construct. It also supports Anderson's ACT-R framework whereby, the cognitive variables such as number sense, operation sense, symbol sense and pattern sense apply to declarative knowledge, which tells about arithmetic facts. Rich declarative knowledge eases the process of retrieval from long term memory through proceduralisation.

The findings have contributed to the body of knowledge on what influences year five pupils' algebraic thinking by enabling policy makers and curriculum designers to find effective and efficient methods to infuse algebraic thinking among primary pupils. Therefore, it can prepare the pupils to make a smooth transition from arithmetic to algebraic way of thinking, which eventually builds strong foundation for

better understanding of formal algebra. The findings have shown that year five pupils were able to solve algebraic thinking items though the achievement was not excellent. They were able to demonstrate some algebraic thinking by solving items involved variables (i.e.,  $c + c + 3 = 15$ ). By providing appropriate classroom discussions and instructional activities could improve the ability of primary pupils to see the connection between arithmetic and algebra when they enter secondary school.

The findings have also revealed that algebraic thinking of year five pupils in rural schools which fell behind compared to algebraic thinking of year five pupils in urban schools. Location of schools should not be a drawback for primary pupils to grasp lessons. Thus, Ministry of Education should look into these differences and leverage the school resources, materials, and improve rural school performance as well.

In sum, algebra is not only essential in all aspects of mathematics such as statistics, calculus and geometry but also in other field like computer programming, engineering, banking sectors and many more. Proficiency in algebra ensures that the child is not left behind in the country and worldwide fast-growing economy. Hence, it is time to prepare each and every child in Malaysia to get ready for algebra.

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