SYNOPSIS

The Poisson Distribution is one of the few distributions of great universality besides the Binomial and Normal Distribution. It arises, in practice, whenever a process is such that the probability of exactly k events occurring within an interval t is approximately or exactly given by:-

$$e^{-\lambda t} \frac{(\lambda t)^k}{k!}$$

where λt denotes the mean rate of occurrence (of events). Setting $t = 1$,

λ = average no. of occurrence per unit interval.

The Poisson Distribution is said to occur if the following postulates hold:

1. Events concerning changes in non-overlapping intervals are independent.

2. The probability of a given number of changes in an interval depends on the size, not the location of the interval.

3. For any small positive number h, and any time interval of length h, there exists a positive quantity λ such that:

   (a) the probability that exactly are event occurs in the interval is approximately equal to λh. This means it is equal to $\lambda h + r_1(h)$ and

   $r_1(h)/h \to 0$ as $h \to 0$.

   (b) the probability that exactly zero event occurs in the interval is approximately equal to $1 - \lambda h$, in the sense that it is equal to $1 - \lambda h + r_2(h)$, and $r_2(h)/h \to 0$ as $h \to 0$.

   (c) the probability that 2 or more events occur in the interval is equal to a quantity $r_3(h)$ such that the quotient $r_3(h)/h \to 0$, as the length h, of the interval $\to 0$.

(To be Cont'd.)

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The parameter λ is a physical constant determining the density of occurrences. Determination of λ is a statistical problem. This value connot be deduced theoretically, it must be determined empirically. If λ is large the chance of finding no occurrence is small. An estimate of λ is given by:-

$\hat{\lambda}$ = T/N where T = total number of events observed in N experiments.

N = total number of experiments.

This relation enables one to estimate from observations and compare theory with experiments wherever the Poisson Distribution has the unique characteristic of its expected value being equal to its variance, i.e.

$$E(X) = \sigma^2 = \lambda t.$$

The Poisson Distribution can be used as a convenient approximation to the Binomial distribution provided that n is relatively large and p is relatively small. Indeed, the computation is less laborious using the Poisson Distribution. On the other hand, if λ is relatively large, say λ as large as 10 or even greater, then the Poisson Distribution can be approximated by the Normal Distribution.

The Poisson Distribution occurs in a wide range of situations. These situations are those when a large number of observations is observed and the probability of an event taking place in any specific observation is very small. Most of the temporal and spatial distributions follow the Poisson formula.

The Poisson Distribution arises frequently in diverse fields. The Poisson Distribution can be applied to appraise the effect of innoculation, the probability of getting, say, 6 heads in 128 throws of a coin, the number of a certain type of bacteria in samples of water, the responses to an advertisement, the number of defectives in a manufacturing process, etc. It is of great utility in fields of commerce, physics, medicine, management science, quality control, etc.