# FINITE TIME CONTROL OF REMOTELY OPERATED VEHICLE

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**RESEARCH REPORT SUBMITTED IN FULFILMENT OF THE REQUIREMENTS FOR THE DEGREE OF MASTER OF INDUSTRIAL ELECTRONIC AND CONTROL ENGINEERING**

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## ABSTRACT

The presence of the uncertainties and external disturbances is one of the unavoidable problems in the control system which is addressed in all objectives of this research. In controller design goal of this research, two different sliding surfaces are proposed to deal with trajectory tracking problem by using two control methods, Nonsingular Terminal Sliding Mode Control (NTSMC) and Adaptive Nonsingular Terminal Sliding Mode Control (ANTSMC) for the nonlinear ROV system with one DOF for pitch angle in presence of various uncertainties and external disturbances. Indeed, both adaptive and non-adaptive controllers based on Nonsingular Terminal Sliding Mode Control (NTSMC) are proposed to provide two alternatives which can adjust by changing operating conditions and dynamics. The key features of all four proposed control designs are finite time stability and robustness against uncertainties and external disturbances which provide by using the sliding mode control concept. The finite time stability proofs for all four-controller design are performed by defining a proper candidate Lyapunov function and based on sliding mode control method for the nonlinear ROV system with one DOF for pitch angle. Numerical simulation results are carried out to make a comparison between them and reveal the correctness of fulfilling trajectory tracking goal in all four controller designs. Also, three well-known performance criteria, ISV, IAE, and ITAE are defined to compare these four designed controllers from various aspects. Furthermore, in another objective of this research a class of full order global finite time observers are designed and proposed for a group of nonlinear systems with uncertainties and external disturbances. The studied nonlinear system is a chain form of nonlinear double integrator subsystems that can describe the dynamic behavior of many real systems, including Remotely Operated Vehicle (ROV), gyroscopes, robot manipulators, ships, submarines, and others. In this research, by using modern mathematical analysis

and proofs, the estimation errors between the corresponding states variables of the observer and the nonlinear system converge to a real zero after an adjustable finite time. Also, a mathematical relation is presented for calculating and setting the mentioned finite time. Then, a numerical simulation is carried out on the sample of double integrator nonlinear system of ROV system with one DOF for the pitch angle to determine that the state variables of the proposed observer can accurately estimate the corresponding variables in the nonlinear system. A comprehensive comparison is also made between proposed finite time nonlinear observer and some well-known and recent studies on nonlinear observer design.

Keywords: NSMC, Adaptive, Finite time, Observer, Nonlinear system.

# ABSTRAK

Kewujudan keraguan dan gangguan luar adalah salah satu masalah yang tidak dapat dielakkan dalam sistem kawalan yang ditujukan kepada semua objektif kajian ini. Dalam matlamat merekabentuk pengawal penyelidikan ini, dua permukaan gelongsor yang berbeza dicadangkan untuk menangani masalah pengesanan trajektori dengan menggunakan dua kaedah kawalan, Mod Kawalan Gelongsor Terminal bukan Tunggal (NTSMC) dan Mod Kawalan Adaptif Gelongsor Terminal bukan Tunggal (ANTSMC) untuk sistem ROV tidak linear dengan satu DOF untuk sudut pincangan hadapan dengan kewujudan keraguan dan gangguan luaran. Dengan itu, pengawal (NTSMC) dicadangkan untuk menyediakan dua pilihan alternatif yang boleh diselaraskan dengan menukar keadaan operasi dan dinamik. Ciri-ciri utama dari semua empat reka bentuk kawalan yang dicadangkan adalah kestabilan batasan masa yang terhad dan menghentikan ketidakpastian dan gangguan luaran yang wujud akibat menggunakan konsep kawalan mod gelongsor. Bukti kestabilan batasan masa yang terhad untuk semua empat reka bentuk pengawal dilakukan dengan menentukan fungsi penggunaan Lyapunov yang sesuai dan berdasarkan kaedah kawalan mod gelongsor untuk sistem ROV tidak linear dengan satu DOF untuk sudut pincangan. Hasil simulasi numerik dijalankan untuk dibuat perbandingan dan mendedahkan kesesuaian tujuan pencapaian trajektori dalam semua empat reka bentuk pengawal. Selain itu, tiga kriteria prestasi terkenal, ISV, IAE dan ITAE digunakan untuk membandingkan empat pengawal yang direka dari pelbagai aspek. Selanjutnya, dalam objektif lain penyelidikan ini kelas pemantau batasan masa global menyeluruh dibuat dan dicadangkan untuk kumpulan sistem tidak linear dengan keraguan dan gangguan luaran. Sistem tidak linear yang dipelajari adalah satu bentuk rantaian subsistem integrase berkembar tidak linear yang dapat meramalkan banyak sifat dinamik sistem sebenar, termasuk Remotely Operated Vehicle (ROV), giroskop, manipulator robot, kapal, kapal selam, dan lain-lain. Dalam penyelidikan ini, dengan menggunakan analisis dan bukti matematik moden, kesilapan anggaran antara pemboleh ubah keadaan yang bersamaan dengan pemantau dan sistem tidak linear menumpu kepada sifar selepas batasan masa dilaras. Juga, hubungan matematik dibentangkan untuk mengira dan menetapkan batasan masa yang disebut. Kemudian, simulasi numerika dilakukan pada sampel sistem integrasi sistem tanpa garisan berkembar ROV dengan satu DOF untuk sudut picangan untuk menentukan bahawa pemboleh ubah keadaan pemerhati yang dicadangkan dapat menramalkan secara tepat pembolehubah yang sama dalam sistem tidak linear. Perbandingan komprehensif juga dibuat antara pengamatan tidak linear yang diuji dan beberapa kajian yang terkenal dan baru-baru ini mengenai reka bentuk pemerhati tidak linear.

Kata Kunci: NSMC, Adaptif, Batasan Masa, Pemantau, Sistem tidak linear.

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| | |
|---|---|
| ROV | Remotely Operated Vehicle |
| SMC | Sliding Mode Control |
| NTSMC | Nonsingular Terminal Sliding Mode Control |
| ANTSMC | Adaptive Nonsingular Terminal Sliding Mode Control |
| DOF | Degree Of Freedom |
| ISV | Integral of Square of control input |
| IAE | Integral of Absolute of Error |
| ITAE | Integral of Time multiplied by Absolute of Error |
| $V(x)$ | Lyapunov function |
| $\dot{V}(x)$ | Derivative of Lyapunov function with respect to time |
| $d(t)$ | The model of the uncertainties and external disturbances |
| $\dot{d}(t)$ | Derivative of the uncertainties and external disturbances with respect to time |
| $h$ | The uncertainties and external disturbances in adaptive concept |
| $\hat{h}$ | The estimation of the upper bound of the uncertainties and external disturbances |
| $\dot{\hat{h}}$ | Derivative the estimation of the upper bound of the uncertainties and external disturbances with respect to time (The adaptive law) |
| $h^*$ | The upper bound the estimation of the upper bound of the uncertainties and external disturbances |
| $\tilde{h}$ | The error of adaptive estimation |
| $\dot{\tilde{h}}$ | Derivative the error of adaptive estimation with respect to time |
| $r$ | Adaptive parameter |
| $(|p(\dot{x})|)$ | The derivative of the absolute of the $p(x)$ with respect to time |

| | |
|---|---|
| $\psi(t)$ | Pitch angle of ROV system |
| $e(t)$ | Trajectory tracking error |
| $\dot{e}(t)$ | Derivative of the trajectory tracking error with respect to time |
| $x_d$ | Desired trajectory tracking |
| $\eta$ | The upper bound of the uncertainties and external disturbances (in control design) |
| $s$ | Sliding surface |
| $\dot{s}$ | Derivative of the sliding surface with respect to time |
| $T$ | The settling time (stability time) |
| $\alpha$ | Control parameter |
| $\lvert . \rvert$ | Absolute value |
| $u$ | Control input |
| $x$ | The state variable |
| $v$ | The state variable (The derivative of $x$ with respect to time) |
| $\hat{x}$ | The estimation of the state variable |
| $\dot{\hat{x}}$ | Derivative of the estimation of the state variable with respect to time (the proposed observer) |
| $\dot{\hat{v}}$ | Derivative of the estimation of the state variable with respect to time (the proposed observer) |
| $\tilde{e}$ | The error of observer estimation |
| $\tilde{e}_x^T$ | The transpose of the state variable of dynamic error of observer estimation |
| $\kappa$ | Kappa value (the upper bound $\lvert\lvert v \rvert\rvert$) |
| $\gamma(t)$ | The upper bound of uncertainties and external disturbances (in observer design) |
| $\lvert\lvert . \rvert\rvert$ | Euclidean norm (2-norm) |

# CHAPTER 1: INTRODUCTION

## 1.1 Introduction

Unmanned Underwater Vehicle (UUV) is an underwater vehicle which can operate underwater without a human occupant. Autonomous Underwater Vehicle (AUV) and Remotely Operated Vehicle (ROV) are two types of UUV which are used for exploration in underwater to carry out the risky task for human in challenging environment. The difference of ROVs and AUVs is that ROVs remotely controlled by the human operator which can be wireless or with wire communication on the ground or ship. AUVs, on the other hand, are automatically controlled by computers without any connecting to the surface and can work independently (Soylu, Proctor, Podhorodeski, Bradley, & Buckham, 2016).

As a matter of fact, that some missions in underwater are very complicated and multi-objective which might be possible only with the presence of human operator for a successful mission. Indeed, the nature of the underwater environment is unpredictable and obviously only human can react to some sudden changes in a mission plan. However, AUV can be more suitable for some fixed missions, which are predetermined for data collection as the main aim, and it is not necessary for operator intervention. Moreover, there is still a limitation of the advanced technology of AUVs in both aspects of autonomy and capabilities. For this purpose, a definite choice for a given task is a ROV system (Zain, Noh, Ab Rahim, & Harun, 2016).

In the last decades, because of enormous enhancement of the technology, it is possible to use ROV for some underwater missions. ROV has attracted the interest of people to perform complicated missions in the underwater, which can help people to accomplish their missions in underwater without any risk for their lives and faster. Indeed, ROV has used for many underwater tasks such as military application, oceanographic mapping, inspection of the pipeline, pipeline maintenance, oil and gas exploration, mineral exploration, etc.

According to above paragraphs, the need of ROV system has become increasingly obvious, due to the essential role in the underwater environment. On the other hand, the research of ROV is extremely challenging due to parameter uncertainty and unstructured uncertainty. Lack of the accurate kinematic model of ROV would lead to the highly nonlinear dynamics of the ROV which is called parameter uncertainty.

Furthermore, due to the influence of water on ROV, its weight might be changed which lead to the inaccurate control model, namely, unstructured uncertainty (Jianhua Wang, Song, Zhang, & Liu, 2016). Accordingly, the research of ROV has attracted a great interest of researchers, particularly in control community to overcome the following problems and improve the overall ROV function.

## 1.2 Problem Statement

(a) ***Global infinite time stability:***

Global finite stability is more comprehensive and recent concept than asymptotic stability. In many control efforts, only the global asymptotic stability with infinite stability time has been guaranteed which can be improved by using finite time concept. In fact, finite time concept has been employed to speed up the convergence rate along with high precision.

(b) ***The existence of uncertainties:***

The existence of a variety of the external disturbances and uncertainties in the modeling of many systems can interfere with the process of controlling the systems. These uncertainties may even result in systems instability. These uncertainties can be due to the lack of proper modeling, the neglect of some of the dynamics and even the error of the measuring instruments. On the other hand, in many control investigation, these uncertainties and external disturbances have not been considered in the dynamic of the nonlinear system which is not true in real-time.

(c) ***Robustness and accuracy:***

Preciseness and robustness against all external disturbances and uncertainties are key features of any control efforts, especially in the unstable underwater environment. Some control methods are not robust against these uncertainties and parameter variations and disturbances. Consequently, to deal with these uncertainties, the robustness of the control method would be a key feature to choose proper control scheme.

(d) ***Chattering problem:***

This unwanted phenomenon reduces the useful life control actuators over a long period of using. Also, it causes undesirable and destructive sound in the control input and reducing control accuracy. This destructive problem occurs because of applying some discontinuous control laws in the control method. The undesirable chattering occurrence has been observed in many control inputs in the literature.

(e) ***Unknown parameter uncertainties and external disturbances:***

Uncertainties might exist on most physical systems. Uncertainties refer to the sum of unmodeled dynamics, measured parameter uncertainties, and the error of measuring devices. In addition, external disturbances are one of the unavoidable cases of practical systems. In most cases, precise knowledge of these uncertainties and disturbances is not available, but the upper bound can be assumed. Various methods have been proposed to deal with these unknown external disturbances and uncertainties. One famous and recent method to cope with these unknown external disturbances and uncertainties is to approximate their upper bound and using this estimation in the control input which refers to an adaptive concept.

(f) ***The occurrence of singularity and high sensitivity of the convergence rate to initial conditions:***

These two problems have not subjected to many control designs especially for sliding mode control method. Note that, the singularity occurs due to existing of nonlinear terms with fractional negative powers in sliding surfaces and control laws. This undesirable phenomenon causes in a large control effort when the errors of tracking reach the neighborhood of zero.

(g) ***The attitude of motion control:***

According to the literature survey on the existing researches on the motion control of the ROV system, rarely researchers have investigated the issue of the pitch angle control of the nonlinear ROV dynamics.

(h) ***Velocity sensors:***

The direct and physically measurement of the velocity variable requires additional sensors such as tachometers which increase cost, size, and weight. Also, the measured data are always very noisy in real-time. Estimation this state variable by using observer design is one of the effective solutions for removing this kind of sensors of the system.

(i) ***Nonlinear observer design only for the particular class of the system:***

Most of the nonlinear observer has been designed for a particular class of nonlinear system by using a lot of assumption on the system which may not true for other systems. Therefore, these nonlinear observers are not generalizable.

(j) ***Infinite time observer:***

In most of the observer designs in the literature, only the asymptotic stability for the estimation error between observer state variables and the nonlinear system has been provided. Indeed, the settling time between estimation variables and the real state of the nonlinear system is infinite.

(k) ***Observer design without considering uncertainties:***

Many of nonlinear observer has been designed without considering the external disturbances and uncertainties which is not true for most of the nonlinear system in real-time.

## 1.3 Objective

The goal of this research can be divided into two separate and main parts including controller design and observer design. The aim of designing two different type of controllers, NTSMC and ATSMC methods, in this research is to provide both non-adaptive and adaptive approaches based on finite time stability that can adjust to the changing dynamics and operating conditions and for making a comparison between them. Note that, both non-adaptive and adaptive controller in this research are designed by considering two different sliding surfaces (integral sliding surface and derivative sliding surface) to provide four alternatives of controllers for the control purposes. A comprehensive comparison between four proposed controllers is made in this research by defining three well-known performance criteria, ISV, IAE, and ITAE. The concept of sliding mode control (SMC) is employed in all four control designs to provide a robustness against external disturbances and uncertainties. All four controllers aim to fulfill trajectory tracking goal for the double integrator nonlinear ROV system with one DOF for the pitch angle in presence of matched uncertainties and disturbances. The objectives of this research are listed as follows:

1- To design two class of controller, based on NTSMC approach

2- To design two class of controller, based on ATSMC approach

3- To design a class of full-order global finite-time nonlinear observer for a general form of the double integrator nonlinear system in presence of all external disturbances and uncertainties, while the position state variable is available, and the velocity state variable is estimated. Then, the proposed observer is examined in MATLAB/Simulink on the double integrator nonlinear ROV system with one DOF for the pitch angle. Finally, a comparison is made between the proposed nonlinear observer and some well-known and recent studies on nonlinear observer design.

Note that, the objectives of this research are determined based on the abovementioned problems in the problem statement section. Indeed, all motioned problems are addressed by objectives of this research and they are the motivation for this research.

## 1.4 Methodology

Details methodology for this research is as follows:

1- Conduct through literature survey: this part involves the study of previous research work and investigation details about the proposed control method and nonlinear observer. The literature review of NTSMC and ATSMC includes the studying the historical procedure of improving these control methods and different concept of these two control methods. The investigation of the nonlinear observer focusses on studying the well-known and recent research on nonlinear observer design and finding the gap of some recent studies to improve it.

2- To design a controller by using NTSMC: to achieve this objective, some mathematical theorem and lemmas are used such as the Lyapunov stability theory and Barbalat's lemmas. In fact, the finite time stability proof for the dynamic of the trajectory tracking error is firstly performed mathematically on the nonlinear ROV system with one DOF for the pitch angle. Then the numerical simulation in

MATLAB/Simulink is performed to reveal the validity of proposed design. The mathematical procedure for the finite time stability proof is as follow.

First, a double integrator form of nonlinear ROV system with one DOF for pitch angle is described. Then, the trajectory tracking error is defined to fulfill trajectory tracking error. After that, the dynamic error is considered as a new dynamic of the system. The upper bound of the external disturbances and uncertainties is assumed to be available and known. Then the sliding surface is chosen (which is with integral block) based on NTSMC scheme and then finite time stability proof for the sliding surface is performed. Now, if the state variable of dynamic error by applying control input reaches to this finite time stable sliding surface ($s = 0$), i.e. the state variable of dynamic error reach to zero, the finite time stability of the system will be ensured and proved. Therefore, by designing a proper control input and applying to the system, we will try to converge dynamic error to the sliding surface. For this reason, a proper candidate Lyapunov function is defined by considering the presented conditions of Barbalat's lemmas. Now, by applying the designed control input to the dynamic error and considering the sliding surface, the finite time stability is proved mathematically by using Lyapunov stability analysis. Note that, a mathematical relation for settling time is obtained and presented to ensure that the stability is in a finite time. Finally, the numerical simulation results are performed in MATLAB/Simulink to verify the correctness of mathematical proof. Note that, for the second sliding surface (which is with derivative block) also the same procedure need to perform.

3- To design a controller by using ATSMC: to achieve this objective, some mathematical theorem and lemmas are used such as the Lyapunov stability theory and Barbalat's lemmas. In fact, the finite time stability proof for the dynamic of the trajectory tracking error is firstly performed mathematically on the nonlinear

ROV system with one DOF for the pitch angle. Then the numerical simulation in MATLAB/Simulink is performed to verify the validity of proposed design. The mathematical procedure for the finite time stability proof is as follow.

First, a double integrator form of nonlinear ROV system with one DOF for pitch angle is described. Then, the trajectory tracking error is defined to fulfill trajectory tracking error. After that, the dynamic error is considered as a new dynamic of the system. The upper bound of the external disturbances and uncertainties is assumed to be unavailable and unknown. Then, by using adaptive concept the adaptive law is defined and it is used in designing control input. Subsequently, the sliding surface is chosen (which is with integral block) based on ATSMC scheme and then finite time stability proof for the sliding surface is performed. Now, if the state variable of dynamic error by applying control input reaches to this finite time stable sliding surface ($s = 0$), i.e. the state variable of dynamic error reach to zero, the finite time stability of the system will be guaranteed and proved. Therefore, by designing a proper control input (which is used of adaptive law) and applying to the system, we will try to converge dynamic error to the sliding surface. For this reason, a proper candidate Lyapunov function is defined by considering the presented conditions of Barbalat's lemmas (note that, in order to use of the estimation of adaptive control in control process, only one Lyapunov function should be considered for adaptive law and sliding surface and for the finite time stability proof). Now, by applying the designed control input to the dynamic error and considering the sliding surface and adaptive law, the finite time stability is proved mathematically by using Lyapunov stability analysis. Note that, a mathematical relation for settling time is obtained and presented to ensure that the stability is in a finite time. Finally, the numerical simulation results are performed in MATLAB/Simulink to verify the correctness of mathematical proof. Note that, for the second sliding surface (which is with derivative block) also the same procedure need to perform.

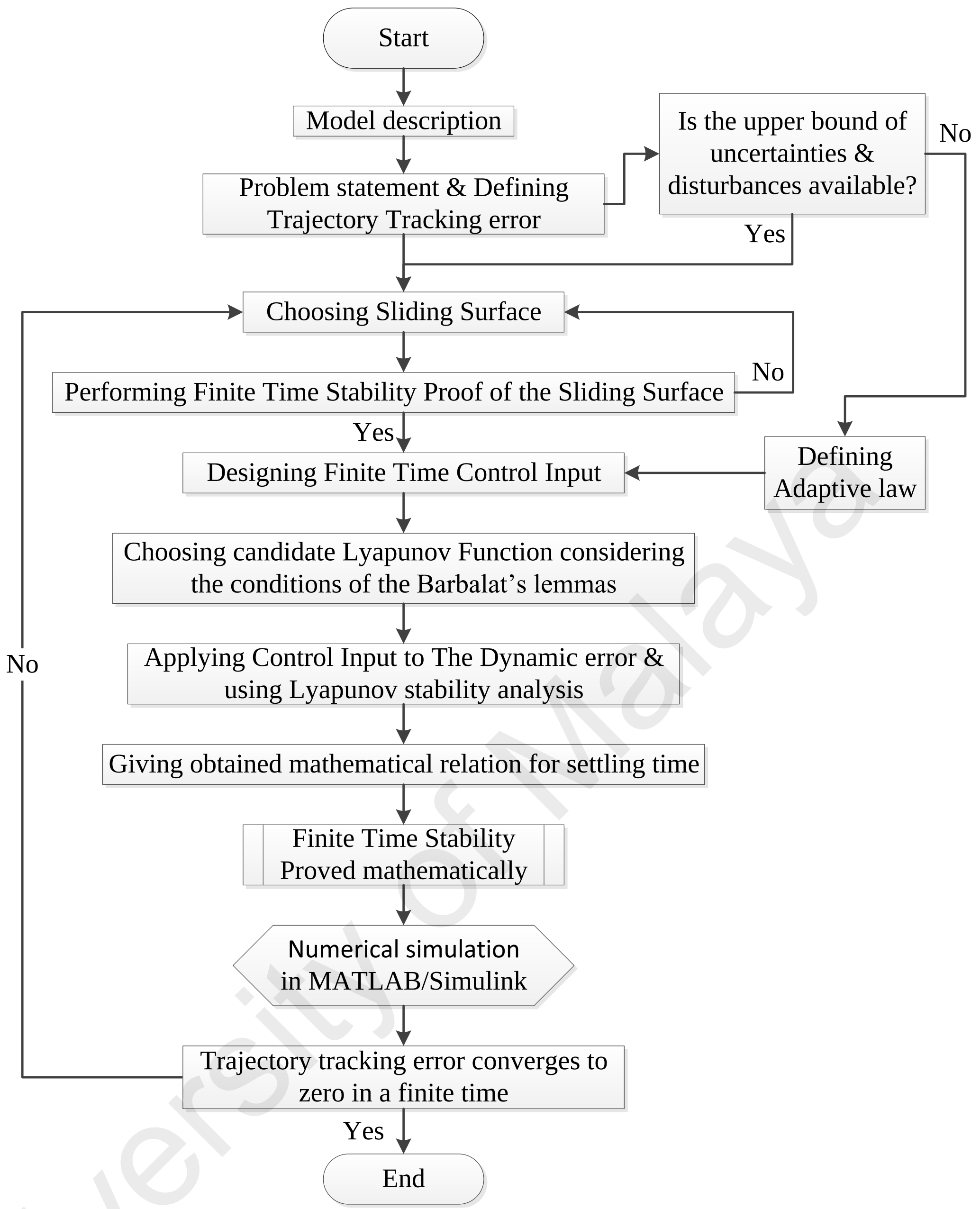



**Figure 1.1: Flowchart of the methodology of both adaptive and non-adaptive proposed controller design**

4- To design a nonlinear observer based on finite time concept: in order to achieve this objective, some mathematical theorem and lemmas are used such as the Lyapunov stability theory and Barbalat's lemmas. In fact, the finite time stability proof for the dynamic of estimation error is firstly performed mathematically on the general form of double integrator nonlinear system. Then the numerical simulation in MATLAB/Simulink is carried out on the sample of double integrator system of nonlinear ROV system with one DOF for the pitch angle to reveal the validity of proposed design. The mathematical procedure for the finite time stability proof is as follow.

First, a general form of double integrator nonlinear system is described. Some assumptions are considered which are always true to proceed with mathematical observer proof. For example, the upper bound of the external disturbances and uncertainties is assumed to be available and known. Then, the estimation error is defined to fulfill the convergence of the time response of the state variables (created by the nonlinear system) to the state variables estimation (estimated by the proposed nonlinear observer), which is achieved by making these estimation error to zero. After that, the dynamic of estimation error is considered as a new dynamic of the system. The nonlinear observer based on finite time concept is designed (which is the derivative of the state estimation) and this proposed observer (derivative of the state estimation) is substituted into the dynamic estimation error. Subsequently, a proper candidate Lyapunov function is defined by using the state variable of the estimation error and by considering the presented conditions of Barbalat's lemmas. Now, by substituting the state variable of the estimation error into this candidate function and its simplification, the finite time stability for the dynamic estimation error is proved mathematically by using Lyapunov stability analysis. Note that, a mathematical relation for settling time of estimation states and real states (the time response of the system) is obtained and presented to ensure that the observer proof is in a finite time. Finally, the numerical simulation results are carried out in MATLAB/Simulink on the sample of double integrator system of nonlinear ROV with one DOF to verify the correctness of mathematical proof.

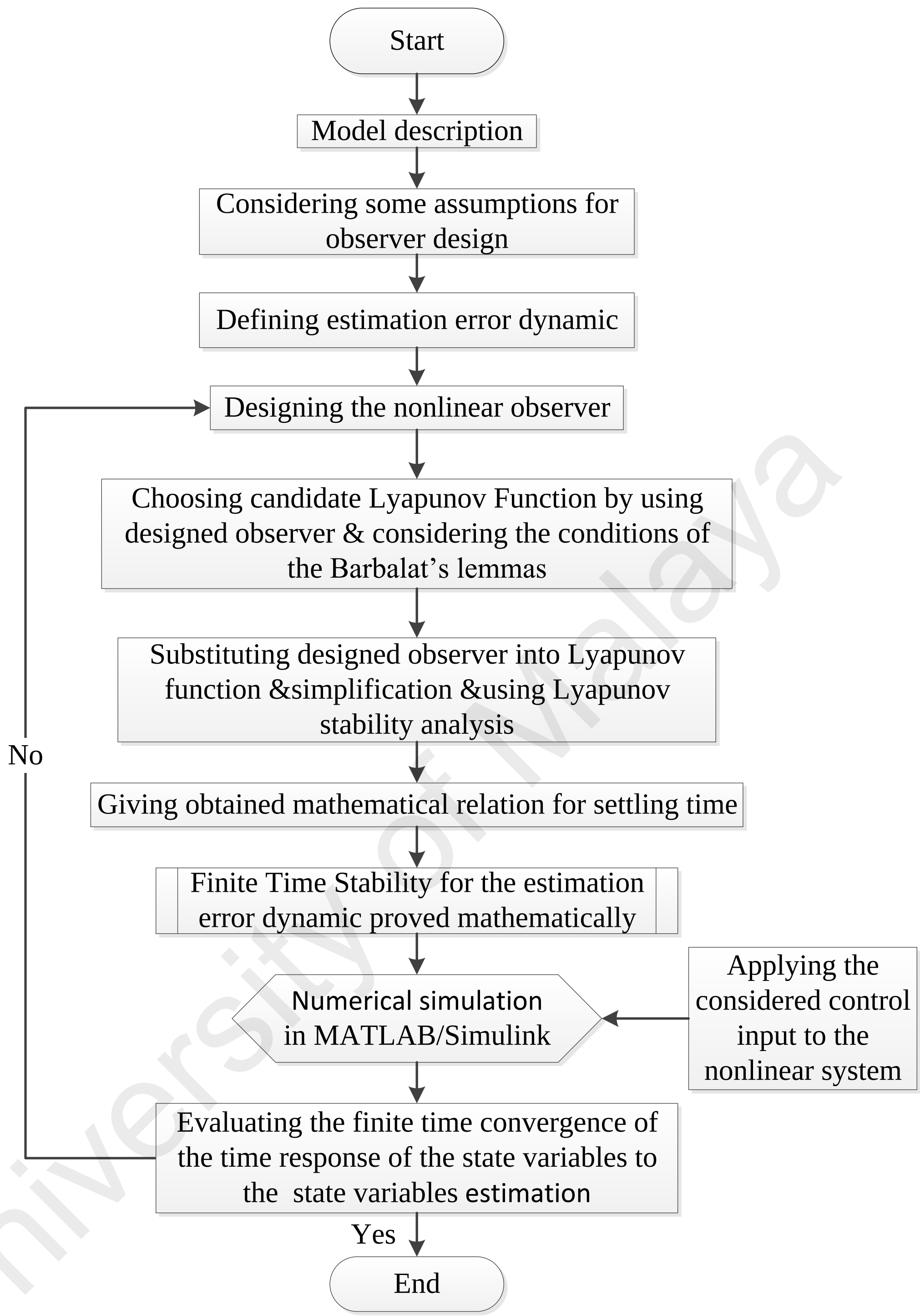



**Figure 1.2: Flowchart of the methodology of the proposed nonlinear observer design**

## 1.5 Research Report Outline

The remaining of this research report is organized in the following manner:

Chapter 2 presents firstly a comprehensive literature review on the historical procedure of improving the sliding mode control method and its combination with different control method to apply on different type of system. A survey on finite time concept and adaptive concept are given and the integration of all of them are investigated in the literature. Subsequently, the nonlinear observer is investigated deeply in the recent literature and its incorporation with finite time concept is studied in many references.

Chapter 3 is dedicated to the methodology of controller design for a nonlinear ROV system with one DOF for pitch angle. Four different controllers are designed by using NTSMC method and ATSMC method. The finite time stability proof is performed mathematically in four separate part for the four proposed controllers in this research and presented in detail in this chapter.

Chapter 4 is dedicated to the methodology of the nonlinear observer for a general form of double integrator nonlinear system of which the presented nonlinear ROV system with one DOF for pitch angle in chapter 3 is a part. The nonlinear observer is designed based on finite time concept and the mathematically proof of the finite time stability for the dynamic estimation error is presented in detail by using Lyapunov theory.

Chapter 5 is divided into two main part as follows. The first part is devoted to represent the numerical simulation results of the four designed controllers in this research. Then, a comparison section of four designed controllers is given by defining three performance criteria. The second part presents the numerical results of nonlinear observer on the presented nonlinear ROV system with one DOF for pitch angle. The discussion of the observer results and comparison between them and previous research is the last section of this chapter.

Chapter 6 is devoted to the conclusion of this research and some suggestion for the future studies as continuous or extension for this study.

# CHAPTER 2: LITERATURE REVIEW

## 2.1 Introduction

Controller design by utilizing the Sliding Mode Control (SMC) scheme is very common for all types of systems. This scheme is known as a robustness control approach against diverse types of external disturbances and uncertainties. The SMC method would guarantee asymptotic stability of the systems. In the Terminal Sliding Mode Control (TSMC) method, in addition to the robustness against disturbances and uncertainties, the system also has a finite time stability. Subsequently, a Nonsingular Terminal Sliding Mode Control (NTSMC) scheme has been introduced to cope occurrence singularity as an unwanted issue. In all these methods, the upper bound of the external disturbances and uncertainties are obtained experimentally, which is considered for controlling the system. When there is no precise knowledge of the uncertainties and external disturbances, it is necessary to approximate the range of them. In this case, Adaptive Nonsingular Terminal Sliding Mode Control (ANTSMC) scheme has been employed where at any moment the upper boundary of external disturbances and uncertainties are approximated by using adaptive control concept.

Many applications require the measurement of the state variables. In many cases, these state variables are measurable physically by using some sensors. The problem arises when they are not measurable physically or measuring them is very costly and noisy. To overcome this problem, the observer concept is used to estimate these state variables without using any sensors which can reduce the size, cost, and weight of the system. This chapter is divided into two main parts which are a control method literature review and nonlinear observer literature review, as follows.

## 2.2 Control method literature review

In this part, the control method sliding mode control and its integration with finite time concept and adaptive control is investigated on the different type of system. Note that, all presented control literature review in this part have been addressed to fulfill trajectory tracking problem.

### 2.2.1 Trajectory tracking problem

Nowadays, considering comprehensive exploitation of ROV in underwater tasks, trajectory tracking goal with fast convergence and high accuracy has attracted interest of many people to study in this challenging area of research. Consequently, many areas of research (Da Cunha, Costa, & Hsu, 1995; Fernandes, Sorensen, & Donha, 2013; Wei, Zhou, Chen, & Han, 2015; Zhu & Gu, 2011) have been carried out on this control problem for ROV system since a few past decades. Additionally, this problem has been investigated for the different system (Bhat & Bernstein, 1998; Cheng, Chien, & Shih, 2010; Hsu & Fu, 2006; Su, 2009) in the literature. Trajectory tracking control is a method to design a controller to guide the vehicle to track an inertial reference trajectory (Do & Pan, 2009).

Two basic shortcomings of the incipient approaches to overcome the trajectory tracking issue are classified as follows: First, they are only able to achieve the globally asymptotic stability (Cheng et al., 2010; Da Cunha et al., 1995; Fernandes et al., 2013; Hsu & Fu, 2006; Wei et al., 2015; Zhu & Gu, 2011) .Second, mostly they are not robust against external disturbances, system's uncertainties (Da Cunha et al., 1995; Fernandes et al., 2013; Hsu & Fu, 2006; Wei et al., 2015; Zhu & Gu, 2011). However, the proposed VS-MRAC method in (Da Cunha et al., 1995) has been reported as a quite robust algorithm with respect to substantial unmodeled dynamics and even delays.

An adaptive integral back-stepping control scheme with Nonlinear Disturbance Observer (NDO) has been proposed in (Wei et al., 2015) of ROV system. The integral terms have been added to the feedback loop to develop the robustness of the ROV system. In (Wei et al., 2015), the results have revealed that the controller can tackle and estimate factors including uncertainties and disturbances model and guarantee trajectory tracking precisely. However, the globally asymptotic stability has been only ensured in this research, which can be improved to the globally finite time stability.

### 2.2.2 Finite time concept

To cope with first weakness mentioned in the last section, finite time control approaches have been presented utilizing the finite time stability concept (Bhat & Bernstein, 1998, 2000; Hou, Zhang, Deng, & Duan, 2016; Parsegov, Polyakov, & Shcherbakov, 2013; Polyakov & Poznyak, 2009; N. Wang, Lv, & Liu, 2016; Zuo & Tie, 2014) that precisely speed up the rate of convergence. In fact, after introducing the theorems and lemmas of the finite time concept (Aimad, Madjid, & Mekhilef, 2014; Parsegov et al., 2013), this finite time stability concept has attracted remarkably the interest of the researchers to use in control community. A finite time stability is a more comprehensive and recent concept than asymptotic stability.

Finite time stability refers to reaching the state variables of the system to the real zero at a finite time. Indeed, the finite time concept guarantees reaching the system state variables to the real zero in the finite time and the upper bound of this finite time should be also presented. The finite time stability concept has been utilized for the control of numerous systems (Fan, Zhang, Wu, & Shi, 2017; X. Li & Mao, 2016; Zong, Ren, & Hou, 2016). SMC scheme has been incorporated with the finite time concept to synchronize the chaotic gyros systems in (Lijian Yin, 2017). The output feedback control method has been integrated with finite time concept in (Junxiao Wang, Zhang, Li, Yang,

& Li, 2017) to control a DC-DC buck converter. The finite time concept has been employed in (S. Li, Wang, & Zhang, 2015; X.-N. Shi, Zhang, & Zhou, 2017; X. Wang, Li, & Shi, 2014) for the controlling the multiagent, quadrotor, and AUV systems, respectively. In (Du, Wen, Cheng, He, & Jia, 2017), a high-order nonholonomic mobile robots system has been controlled by using finite time control scheme.

In (S. Shi et al., 2017) a finite time controller has been proposed for the systems with mismatches uncertainties. In (Yuan, Ma, Li, & Jiang, 2017), a controller has been designed without angular velocity measurements for trajectory tracking of a spacecraft by using the finite time concept. In (Y. Xu, 2017), the concept of robust finite time control has the Autonomous Operation of an Inverter-Based Microgrid has been controlled by employing a robust finite time concept. In (X.-N. Shi et al., 2017), a quadrotor system has been controlled by designing a finite time control input to fulfill trajectory tracking.

In general, recently, three finite time stabilization strategy for a nonlinear system has been introduced (Bhat & Bernstein, 1998, 2000; Healey & Lienard, 1993; Hou et al., 2016; Parsegov et al., 2013; Polyakov & Poznyak, 2009; Tapia, Bernal, & Fridman, 2017; N. Wang et al., 2016; Zuo & Tie, 2014) first method, namely, geometric homogeneity based approach (Hou et al., 2016; N. Wang et al., 2016), can be applied only to the homogenous nonlinear systems with negative homogeneity degree. In the second method, namely, direct Lyapunov like approach (Bhat & Bernstein, 1998, 2000; Parsegov et al., 2013; Polyakov & Poznyak, 2009; Zuo & Tie, 2014) is a complex task to find an appropriate Lyapunov function and there is lack of a systematic method. And the third approach is the second order NTSMC method (Healey & Lienard, 1993; Tapia et al., 2017), which is known as two major features including free-chattering and robustness. In comparison, the second order NTSMC provides a method, which is more systematic and in terms of practical realization is cost-effective and easier.

### 2.2.3 Sliding Mode Control (SMC) method

To overcome second weakness, SMC scheme has been introduced to control different types of systems (Aimad et al., 2014; Elsayed, Hassan, & Mekhilef, 2013; Elsayed, Hassan, & Mekhilef, 2015b; Healey & Lienard, 1993; Tapia et al., 2017; Utkin, 1993). This control method is well known for its main feature which is robustness against external disturbances and uncertainties. However, the SMC approach deals only with the globally asymptotic stability with infinite settling time.

In (Tapia et al., 2017), a novel nonlinear SMC strategy has been proposed for systems with both matched and unmatched disturbances and uncertainties. The proposed scheme has been incorporated with exact convex expressions to provide both the nonlinear surface and reducing chattering problem significantly. Although the proposed method has reduced undesirable chattering problem significantly, it has not eliminated completely. In (Healey & Lienard, 1993), a SMC approach has been proposed for an AUV system with six Degrees Of Freedom (DOF). The main advantage of the proposed method in the mentioned paper is robust performance when designed separately for steering, speed control, and diving activity. However, it does not deal with chattering problem and finite time stability.

Furthermore, in various control efforts, this method has been employed to control the systems (Aamir, Kalwar, & Mekhilef, 2017; Elsayed et al., 2013; Elsayed, Hassan, & Mekhilef, 2015a; Feng & Shi, 2017; Ferrara & Magnani, 2007; Guzman, de Vicuña, Castilla, Miret, & Martín, 2017; Qi, Li, Tan, & Hui, 2018; Tapia et al., 2017). In (Guzman et al., 2017) the robust control input has been designed by using SMC method for a particular class of the nonlinear multi-agent system with time delay. In (Qi et al., 2018) this control method has been employed to control the voltage of a buck converter, and in

(Feng & Shi, 2017) the robust control input has been designed by applying SMC concept for singular stochastic Markov jump systems. In (Jeong & Chwa, 2018), this method has been used to control hovercraft systems in presence of external disturbances for robust trajectory tracking.

### 2.2.4 Chattering problem and SMC

On the other hand, SMC strategy causes the undesirable chattering problem due to applying discontinuous control inputs in this method. Indeed, in the SMC approach, it is necessary to use the sign function to prove the stability and deletion the effect of the external disturbances and uncertainties which creates a destructive chattering phenomenon in the control input. The Chattering problem is a harmful phenomenon, which has devastating effects on control actuators such as reducing the useful life of control actuators over a long period of using. It also reduces control accuracy and causes deleterious sound during the control of the system (Bandyopadhyay, Janardhanan, & Spurgeon, 2013).

Therefore, it is necessary to consider the removal of this issue or its absence in the control input design. Hence, several methods have been presented to reduce the effects of this phenomenon in the literature, such as the derivation of variables and then the use of the integral of the sign function; or the estimation of the sign function with other functions as well as the use of fuzzy logic to reduce the effect of this undesirable phenomenon (Deaecto, Souza, & Geromel, 2014; Elsayed et al., 2013; Saghafinia, Ping, Uddin, & Gaeid, 2015; Xu, 2008) and also it has been thoroughly eliminated in (Elsayed et al., 2013; Khooban, Niknam, Blaabjerg, & Dehghani, 2016; Q. Xu, 2017b). In (N. Wang et al., 2016), (J. n. Li, Su, Zhang, Wu, & Chu, 2013) and (Šabanović, Jezernik, & Wada, 1996), quadrature systems, discrete time delay singular systems and Manipulator robots, respectively have been controlled with the aim of removing chattering phenomenon and by using the SMC method. In (Elsayed et al., 2013), the chattering of control signal has been eliminated by using a third order decouple fuzzy SMC.

### 2.2.5 Terminal Sliding Mode Control (TSMC) method

Consequently, a new Terminal SMC (TSMC) method (Jianhua Wang et al., 2016) has been introduced by combining the finite time concept and SMC strategy. In consequence, TSMC guarantees robustness as well as the trajectory tracking in a finite time. In (Chen, Wu, & Cui, 2013; Jianhua Wang et al., 2016), a super-twisting mode control methodology has been proposed for pitch angle control system of a ROV. Although the undesirable chattering phenomenon has been created in the control input, the finite time stability is a key advantage of TSMC method comparing with SMC method.

Indeed, a TSMC method accomplishes both advantages of SMC, which are the robustness against uncertainties and asymptomatic stability, and additionally provides a global finite time stability. In addition, different systems have been controlled in (Abooee, Arefi, & Abadi, 2017; Gudey & Gupta, 2016; Mojallizadeh, Badamchizadeh, Khanmohammadi, & Sabahi, 2016; Ni, Liu, Liu, Hu, & Li, 2017; Solis, Clempner, & Poznyak, 2017; Van, Ge, & Ren, 2017; Junxiao Wang, Li, Yang, Wu, & Li, 2016; Xiong, Gan, & Ren, 2016; Q. Xu, 2017a; Z. Yang, Zhang, Sun, Sun, & Chen, 2017) by using this concept. For example, the Bearingless Induction Motor system has been controlled in (Z. Yang et al., 2017) by using TSMC method. In (Q. Xu, 2017a), the Piezoelectric Nanopositioning system has been controlled by using the integral sliding surfaces. A finite time Fault Tolerant Control (FTC) strategy has been investigated in (Van et al., 2017) for the robot manipulators by using nonsingular TSMC scheme and time delay approximation.

In (Solis et al., 2017), the van der Pol oscillator system has been controlled by using fast TSMC method. By choosing appropriate control coefficient in the TSMC method can speed up the convergence rate. That is why; the term "fast" is used in some cases, such as (Xiong et al., 2016), where The Fast Terminal Sliding Mode Control (FTSMC) scheme has been employed to control the discretized nonlinear systems. Also, in (Gudey & Gupta, 2016), this method has been used to control the voltage source inverter for a low voltage microgrid system. Furthermore, TSMC methodology has been used in (Abooee, Arefi, et al., 2017; Mojallizadeh et al., 2016; Ni et al., 2017; Junxiao Wang et al., 2016) to control various systems.

### 2.2.6 Nonsingular Terminal Sliding Mode Control (NTSMC) method

Subsequently, three major weakness has been found for TSMC technique in literature survey including unwanted chattering issue, the high sensitivity of the rate of convergence to initial conditions and occurring of singularity. The singularity accrues due to the existence nonlinear terms with fraction negative powers in control lows. Consequently, NTSMC strategy has been introduced (Ghasemi, Nersesov, & Clayton, 2014) to overcome unwanted singularity problem. To solve two other aforementioned problems of TSMC method, the second order NTSMC scheme has been proposed (Mondal & Mahanta, 2014; Yu & Long, 2015; X. H. Zhang, Zhang, & Xie, 2016). This method has been eliminated the chattering problem thoroughly by applying continuous control lows.

### 2.2.7 The existence of uncertainties and using the adaptive concept

The existence of uncertainties in the modeling of many systems can interfere with the process of controlling the systems (Ma, Liu, & Ye, 2017). These uncertainties may even result in systems instability. Indeed, These uncertainties can be due to the lack of proper modeling, the neglect of some of the dynamics and even the error of the measuring instruments (Jeong & Chwa, 2018). Also, external disturbances are one of the inevitable phenomena in many systems (Y. Zhang, Chen, Li, & Zhang, 2018).

Various methods have been introduced to deal with these two phenomena, uncertainties and external disturbances, such as robust control of $H_2$ (Maccari, Montagner, Pinheiro, & Oliveira, 2012) or $H_\infty$ (H. Li, Zhang, Xiao, & Dong, 2015), as well as Quantitative Feedback Theory (QFT) control method (Jinkun & Yuzhu, 2007; Mercader, Åström, Banos, & Hägglund, 2017; Munoz-Mansilla, Aranda, Diaz, & Chaos, 2010). Also, to cope with these external disturbances and uncertainties, and to reduce or eliminate their effects, the functions such as the sign function has been used, which creates a destructive phenomenon, namely, chattering problem. Another method to deal with external disturbances and uncertainties is to approximate their upper bound and using this estimation in the control input, namely, adaptive concept. In fact, adaptive control approach is one of the control scheme to deal with these external disturbances and uncertainties.

In these approaches, the upper bound of the disturbances and uncertainties is approximated, and then this approximation is utilized in the designing the control input. These approaches have been employed to deal with disturbances and uncertainties (Abraham et al., 2017; Al-Dabbagh, Kinsheel, Mekhilef, Baba, & Shamshirband, 2014; Liao, Chen, & Yao, 2017; Pan, Guo, Li, & Yu, 2017; C.-X. Shi, Yang, & Li, 2017; Shin, 2017). The adaptive back-stepping control scheme has been employed in (C.-X. Shi et al., 2017) to provide a controller for the hierarchical multi-agent systems. The adaptive control concept has been integrated with the neural network approach to control a Hypersonic Aircraft system in (Shin, 2017). The adaptive control approach has been incorporated with various control schemes for different practical systems for the controller design in (Abraham et al., 2017; Al-Dabbagh et al., 2014; Liao et al., 2017; Pan et al., 2017). Also, in (Antonelli et al., 2018; Nikdel, Badamchizadeh, Azimirad, & Nazari, 2016; H. Zhang, Yue, Yin, & Chen, 2016), this method has been used to control the system against a variety of external disturbances and uncertainties.

### 2.2.8 Adaptive Sliding Mode Control (ASMC)

On the other hand, the design of the sliding mode control approach requires the knowledge of uncertainties bound, which might be, in real-time, an arduous task, because usually this bound is unknown, only bounded. In other words, dynamic model of external disturbances and parameter uncertainties may be unknown in advance, the only prior knowledge is assumed of their bounds. Adaptive sliding mode controllers have been employed in (Huang, Kuo, & Chang, 2008; Yao & Tomizuka, 1996) to deal with these unknown uncertainties and external disturbances by using adaptive concept. In other words, an adaptive concept has been incorporated into a SMC scheme to approximate the upper bound of these unknown external disturbances and parameter uncertainties.

In (Yao & Tomizuka, 1996), a systematic way of combining SMC approach and the adaptive concept has been developed for trajectory tracking problem of a system in the presence of disturbances and uncertainties. Also, a continuous SMC strategy without unpleasant reaching transient and unwanted chattering phenomenon has been redesigned by utilizing a dynamic sliding mode. Indeed, both non-adaptive and adaptive approaches have been considered to lead a design of robust controllers that are able to adjust to the changing operating conditions and dynamics and for a comparison between them. In (Huang et al., 2008), an Adaptive Sliding Mode Control (ASMC) has been employed for the nonlinear systems in presence of uncertain parameters. It has been guaranteed the tracking performance and interestingly proved the system's robustness against external disturbances and uncertainties. The result of the motioned paper has been shown that the suggested method can be implemented effectively.

In (Rezazadegan, Shojaei, Sheikholeslam, & Chatraei, 2015), a novel method of adaptive trajectory tracking control has been used for an AUV system in presence of parameter uncertainties and assuming an under-actuated system in six degrees of freedom. They suggested adaptive controller based on Lyapunov's direct strategy and a back-stepping method. The proposed method has been provided convergence of the AUV system asymptotically to the desired trajectory and robustness against parameter uncertainties. However, the globally asymptotic stability are provided in three abovementioned papers, they do not deal with a finite time stability concept, which is more comprehensive concept than infinite stability.

### 2.2.9 Adaptive Terminal Sliding Mode Control (ATSMC)

By incorporating the adaptive control and TSMC method, the Adaptive Terminal Sliding Mode Control (ATSMC) has been introduced to adjust with some unknown parameters and disturbances by estimating them in a finite time. In fact, estimating the upper bound of external disturbances and uncertainties is one way to deal with them. Recently, ATSMC scheme has been very much considered in many control efforts. The adaptive concept has been incorporated in (Ullah, Ali, Ibeas, & Herrera, 2017) by fractional order TSMC method to control a doubly fed induction generator based wind energy system. In (Bennehar, El-Ghazaly, Chemori, & Pierrot, 2017) and (X. Yang, Li, & Fang, 2014), an ATSMC has been proposed for parallel manipulators and synchronization of chaotic systems, respectively. In (Wu, Wu, Tan, & Wu, 2013), an ATSMC has been employed to achieve trajectory tracking of the spacecraft and this control method has been used in (Sun & Ma, 2017) to control the linear motor. In (Basin, Yu, & Shtessel, 2017; Liu, Sun, Dong, & Wang, 2017; Qing-lei, Cheng-ping, Zhen-xia, & Ai-hua, 2013; Xue & Zhiyong, 2017), this method has been proposed to control hypersonic missile, nonholonomic wheeled mobile robots, heavyweight airdrop, and spacecraft systems, respectively.

## 2.3 Nonlinear Observer literature review

The problem of an observer design for the nonlinear system is one of the recent and challenging issues in the control community which has many applications in diverse fields. So far, widespread research has been carried out in this regard, and several papers have been published in this area of research (Du, Qian, Yang, & Li, 2013; Kravaris, 2016; Menard, Moulay, & Perruquetti, 2017; Shen & Xia, 2008; Tami, Zheng, Boutat, Aubry, & Wang, 2016; L. Wang, Astolfi, Marconi, & Su, 2017; J. Zhang, Zhu, Zhao, & Wang, 2017; Zhao & Guo, 2017; Zheng, Efimov, Bejarano, Perruquetti, & Wang, 2016; Zheng, Efimov, & Perruquetti, 2016). With careful attention and focus on references and studies on the problem of designing nonlinear observers (Du et al., 2013; Kravaris, 2016; Menard et al., 2017; Shen & Xia, 2008; Tami et al., 2016; L. Wang et al., 2017; J. Zhang et al., 2017; Zhao & Guo, 2017; Zheng, Efimov, Bejarano, et al., 2016; Zheng, Efimov, & Perruquetti, 2016), resulting in that most of these studies have common weaknesses. The most important of these deficiencies can be listed as follows. As the first weakness, each reference has provided a nonlinear observer for a particular class of nonlinear systems along with a number of considered hypotheses on the system, which is not widely available for generalization to nonlinear systems.

As the second deficiency, in the vast majority of references (Kravaris, 2016; Tami et al., 2016; L. Wang et al., 2017; J. Zhang et al., 2017; Zhao & Guo, 2017; Zheng, Efimov, & Perruquetti, 2016), estimation errors between observer state variables and the nonlinear system reach the real zero at infinite-time. In fact, in these references (Kravaris, 2016; Tami et al., 2016; L. Wang et al., 2017; J. Zhang et al., 2017; Zhao & Guo, 2017; Zheng, Efimov, & Perruquetti, 2016), the global asymptotic stability has been proved for the dynamic of the estimation errors. Also, the uncertainties (include parametric uncertainties, model uncertainties, and external disturbances) have not been considered of the nonlinear system during the observer design process which is the third disadvantage that can be seen in many references (Du et al., 2013; Kravaris, 2016; Menard et al., 2017; Shen & Xia, 2008; Tami et al., 2016; L. Wang et al., 2017; Zhao & Guo, 2017; Zheng, Efimov, Bejarano, et al., 2016; Zheng, Efimov, & Perruquetti, 2016).

On the other hand, studying literature survey on finite time control reveals that most of the suggested approaches require the velocity and position measurements of the state variables of the nonlinear system (Galicki, 2015; Healey & Lienard, 1993; Hu, Xiao, & Shi, 2015; Huang et al., 2008; Tapia et al., 2017; Utkin, 1993; Jianhua Wang et al., 2016; Yao & Tomizuka, 1996; Yu & Long, 2015; X. H. Zhang et al., 2016). In these approaches, all state variables have been assumed to be measurable or available which may not be true for many systems in real time. Furthermore, some of these approaches use direct measurements of accelerations or estimations of them as well (Galicki, 2015). Indeed, the measurement of acceleration requires a type of sensor which is extremely expensive, and it is very noisy which is usually not necessary to measurement physically in most of the applications for a control system.

The position of the state variable of the nonlinear system can be measured precisely by utilizing an accurate sensor which is quite cheap and generate less noise. Therefore, the measurement of them by sensors are reasonable, although estimation of them also can help for decreasing weight, cost, and size. On the other hand, the velocity measurement of the state variable requires additional sensors such as tachometers which rise cost, size, and weight of the systems. Also, attained measurements by utilizing tachometers are easily contaminated by noise (Hu et al., 2015). Accordingly, the priority of estimation of velocity variable and using velocity observer in the system instead of the sensor is undeniable. In other words, due to the fact that the direct measurement of velocity variable is an expensive process in real-time and the measured data are always noisy, it is usually preferable to estimate v (velocity) (Abooee, Moravej Khorasani, & Haeri, 2017)

Some researchers have used velocity observer base on trajectory tracking schemes (Abooee, Moravej Khorasani, et al., 2017; Galicki, 2015; Kinsey, Yang, & Howland, 2014; Mallon, van de Wouw, Putra, & Nijmeijer, 2006). In (Kinsey et al., 2014), a nonlinear observer (NLO) and an extended Kalman filter (EKF) approaches have been employed to estimate the position and velocity of the nonlinear dynamic model for ROV in a single degree of freedom. They have provided a comparison of the performance of the EKF and NLO considering precision, convergence, parameter sensitivity, accuracy and robustness as a criterion of comparison to velocity measurement outages. The result has shown that NLO outperformed EKF in all above criteria except accuracy. Therefore, they have stated that the NLO is a superior method. The asymptotically infinite time stability is only addressed in this research which can be improved to finite time stability. Although the suggested approach based on finite time stabilization in (Chu, Zhu, & Yang, 2017) has been proven that the proposed method is feasible and effective, the issue of finite time convergence for the adaptive local RNN estimation error should be paid more attention in the future work.

## 2.4 Summary

This chapter is divided into two parts. In the first part, a comprehensive survey of several types of the sliding mode control method and its integration with some other control concept is presented on a different type of the system. It starts from the fundamental concepts of the SMC, finite time concept and adaptive control, and followed by their modern control method and their integration with together. Also, some other control problem is reviewed briefly in the literature and then applying the sliding mode control or its integration for solving these mentioned problems are investigated in the literature survey. In other words, a historical procedure of improving the sliding mode control by incorporating with some other control concept in the literature are given. In the second part, the nonlinear observer is investigated in the literature survey. The discussion in this part starts with some recent and well-known research in nonlinear observer design and some common weakness of these references are founded and presented. Also, the need for observer design in the nonlinear system is investigated. Additionally, incorporating nonlinear observer and finite time concept are explained by giving some example of the literature.

# CHAPTER 3: FINITE TIME CONTROLLER

## 3.1 Introduction

Most mathematical modeling systems include uncertainties, and in most cases, precise knowledge of these uncertainties is not available. Therefore, many methods have been proposed to reduce the effect of these uncertainties for the control system. In this chapter, two different sliding surfaces are chosen to design controller by using two different control methods, NTSMC and ATSMC for the nonlinear ROV system with one DOF for pitch angle in presence of various uncertainties and external disturbances. Indeed, adaptive and non-adaptive control method is considered to deal with known and unknown uncertainties, respectively. In the following, some basic mathematics and lemmas are given which are used throughout this chapter. Then, the finite time stability proof is performed separately for all four-control design in this research.

## 3.2 Mathematical preliminaries and lemmas

Definition 3.1: The $sgn(x)$ function is defined as (3.1), and the function $sig^{a}(x)$ can be defined as $sig^{a}(x) = |x|^{a} sgn(x)$.

$$sgn(x) = \begin{cases} 1 & ; \quad x > 0 \\ 0 & ; \quad x = 0 \\ -1 & ; \quad x < 0 \end{cases} \tag{3.1}$$

Definition 3.2: The mathematical relation between the absolute function and the $sign(a)$ function is $|x| = xsgn(x)$.

Definition 3.3: In a nonlinear system $x = f(t,x), f(t,0) = 0, x \in \mathcal{E} \subseteq \Re^{n}$, where $\mathcal{E}$ is an open neighborhood of the equilibrium point $x = 0$. If in this system, the equilibrium point has asymptomatic stability in the region $\mathcal{E}$ as well as the time $T$ exists in such a way that,

$$\lim_{t \to T} x = 0 \ and \ x = 0 \ for \ t \geq T \tag{3.2}$$

As a result, the system will be locally stable in a finite time (Bhat & Bernstein, 2000).

Lemma 3.1: In the nonlinear system $\dot{x} = f(x), f(0) = 0, x\epsilon\Re^n$ with initial conditions $x(0) = x_0$, if the candidate Lyapunov function $V(x)$ is globally positive definite, radially unbounded and only at x = 0 is zero, and the time derivative of the Lyapunov candidate function is as $\dot{V}(x) \leq -\rho_1 V^{\rho_2}(x)$, where $\rho_1$ is a positive number and $\rho_2$ is a constant between zero and one; hence the variable $x$ of the system from any initial conditions, it reaches zero in a finite time, and since then it remains exactly equal to zero, i.e. $\lim_{t \to T} x \to 0$ and the upper bound of the settling time, $T$, will be as $T(x_0) \leq \frac{V^{1-\rho_2}(x_0)}{\rho_1(1-\rho_2)}$ (Qiao & Zhang, 2017).

Lemma 3.2: For each value $a_1, a_2, \ldots, a_n \in \Re$ and $0 < q < 2$ we have: $|a_1|^q + |a_2|^q + \cdots + |a_n|^q \geq (a_1^2 + a_2^2 + \cdots + a_n^2)^{\frac{q}{2}}$ (Bhat & Bernstein, 1998).

Lemma 3.3: In the nonlinear system $\dot{x} = f(x) + g(x)u + d$, $d$ is the model of the external disturbances and uncertainties of the system which is approximated at any moment of time as $h \leq \hat{h}$. At any moment of time, the upper bound $h^*$ exists for $\hat{h}$, so that $\hat{h} \leq h^*$ (Qiao & Zhang, 2017).

## 3.3 Model description of nonlinear ROV system

According to the literature survey on the existing researches on the motion control of the ROV system, rarely researchers have investigated the issue of the pitch angle control of the nonlinear ROV dynamics. Motivated by this research gap and considering maneuvering structure of the studied ROV, the dynamics model of the pitch angle for the ROV is considered in this research to design motion attitude controller. Indeed, in the considered dynamics of the ROV system in this research which has been presented in (Fossen, 2002; Jianhua Wang et al., 2016), only the variable $\psi(t)$ is concerned while other degree of freedom of ROV like $(x, y, z, \theta, \varphi)$ are neglected. Note that the dynamics of pitch motion for the ROV system has been obtained by using stress analysis. Fig. 3.1 represents the sketch of the stress process.

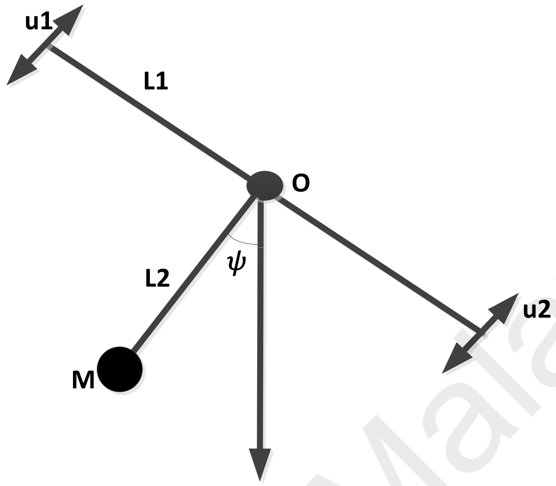



**Figure 3.1: The stress process for pitch angle of ROV**

As shown in Fig. 3.1, the input vector of the two motors are $u_1$ and $u_2$. $o$ is the centre of the buoyancy of ROV vehicle which lies in middle of the two motors. $M$ is the center of gravity of ROV vehicle which is on the vertical line of the ligature of two motors. $L1$ is the distance of centre of buoyancy to one motor of ROV, $L2$ is the distance of center of mass to the buoyancy centre of ROV.

The mathematical dynamics model for the pitch angle of the ROV system has been presented in (Fossen, 2002; Jianhua Wang et al., 2016) as (3.3).

$$\begin{cases} x_1(t) = \psi(t) \\ \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = a_1 x_2(t) + a_2 \sin\big(x_1(t)\big) + bu(t) + d(t) \end{cases} \tag{3.3}$$

Where in this model, $\psi$ represents the pitch angle. $a_1$, $a_2$ and $b$ are positive and constant, and $u(t)$=$u_1$+$u_1$ is the control input, also $d(t)$ is the model of external disturbances and uncertainties of the system.

### 3.4 Problem statement

Since the aim of this section is to track the trajectory tracking of the pitch angle of the ROV system, so trajectory tracking errors are defined as $\begin{cases} e_1 = x_1 - x_{1_d} \\ e_2 = x_2 - x_{2_d} \end{cases}$, and by derivative, we have

$$\begin{cases} \dot{e}_1(t) = e_2(t) \\ \dot{e}_2(t) = a_1\left(x_{2_d} + e_2\right) + a_2 \sin\left(x_{1_d} + e_1\right) + \\ \qquad +bu(t) + d(t) - \dot{x}_{2_d} \end{cases} \tag{3.4}$$

If the error model reaches zero, the system variables will reach the desired trajectory tracking ($e_i \to 0 \implies x_i \to x_{i_d}$). The aim is to track the trajectory tracking in a finite time. Furthermore, in the following sections, control inputs, $u_i$ are designed in such a way that the system in addition to finite time stability is also robustness against various uncertainties and external disturbances.

### 3.5 Two Novel Terminal Sliding Mode Control

In this part, two control inputs are designed by utilizing two different sliding surfaces based on the Nonsingular Terminal Sliding Mode Control (NTSMC) approach for the presented error system in (3.4) to fulfill trajectory tracking goal. Then, the finite time stability is proved by choosing proper candidate Lyapunov function, with considering the conditions in lemma 3.1. To accomplish this goal, it is assumed that the sum of the external disturbances and uncertainties ($d(t)$) and their derivatives ($\dot{d}(t)$) have a high bound as follows

$$\begin{cases} \|d(t)\| \leq \eta_1 \\ \|\dot{d}(t)\| \leq \eta_2 \end{cases} \tag{3.5}$$

### 3.5.1 Theorem1 (NTSMC1)

For the presented system in Eq. (3.4), and the considered conditions for the upper bound of the external disturbances and uncertainties in Eq. (3.5), using the sliding surface in Eq. (3.6) and the designed control input in Eq. (3.7), the finite time stability will be guaranteed. In other words, all state variables of the error system reach zero in a finite time and remain at zero. The settling time is the sum of the two-time including the time of reaching to the stability of sliding surfaces ( $T_r$) as well as the time that the system reaches at the sliding surface ($T_s$). Accordingly, the settling time (stability time) of the system is equal to $T = T_r + T_s$, of which the upper bound of $T_r, T_s$ is presented in the following.

$$s = e_1 + e_2 + \int_0^{\infty} \alpha_1 e_1^{\alpha_2} dt + \int_0^{\infty} \alpha_3 e_2^{\alpha_4} dt \tag{3.6}$$

Where $\alpha_{2i-1}$ are positive constants and greater than one and $\alpha_{2i}$ are constants between one and two.

$$\begin{cases} u = \frac{1}{b}\left(u_r + u_{eq}\right) \\ u_{eq} = -e_2 - \alpha_1 e_1^{\alpha_2} - \alpha_3 e_2^{\alpha_4} - a_1\left(x_{2_d} + e_2\right) - a_2 \sin\left(x_{1_d} + e_1\right) + \dot{x}_{2_d} \\ u_r = -k sig^{\beta}(s) - \eta_1 sgn(s) \end{cases} \tag{3.7}$$

Where $k$ is a positive constant and $\beta$ is a constant between one and zero.

Proof: To prove the finite time stability, it needs first to prove that the control input (3.7) leads the system to reach the sliding surface, $s = 0$. Hence, the candidate Lyapunov function is considered as $V(x) = \frac{1}{2}s^2$, where this candidate function has condition of

Lyapunov function in lemma 3.1. By differentiating this candidate function with respect to time, there comes

$$\dot{V}(x) = s\dot{s} \tag{3.8}$$

By differentiating the Eq. (3.6) with respect to time and by substituting $\dot{e}_2(t)$ into the derivation of Eq. (3.6), followed by, applying the control input (3.7), and its simplification, we have: $\dot{s} = u_r + d(t)$. By substituting the result into (3.8), yields

$$\dot{V}(x) = s\big(u_r + d(t)\big) \Rightarrow \dot{V}(x) = s\left(-ksig^{\beta}(s) \; - \eta_1 sgn(s) \; + d(t)\right) \tag{3.9}$$

According to definition 3.1 and 3.2, we have

$$\dot{V}(x) = -k|s|^{\beta+1} - \eta_1|s| + sd(t) \tag{3.10}$$

Considering $sd(t) \le |sd(t)|$ also $\left||d(t)|\right| \le \eta_1$, one can obtain

$$\dot{V}(x) \le -k|s|^{\beta+1} \tag{3.11}$$

Now, by substituting $|s| = \sqrt{2}(V(x))^{\frac{1}{2}}$ in (3.11) and by choosing the values $\rho_1 = k\big(\sqrt{2}\big)^{\beta+1}$ and $\rho_2 = \frac{\beta+1}{2}$, the result is

$$\dot{V}(x) \le -\rho_1 V^{\rho_2} \tag{3.12}$$

According to Lemma 3.1, the system states reach the sliding surface $s = 0$ in the finite time, $T_s$. The upper bound of $T_s$ is given as below

$$T_s \le \frac{V^{\frac{1}{2}}(e_0)}{\left(\frac{\sqrt{2}\Delta \mathrm{m}}{2}\right)} \tag{3.13}$$

For the second stage of the proof procedure, the finite time stability proof of the sliding surface of $s = 0$ must be performed. First, the Eq. (3.6) is equalized to zero, and then its time derivation is taken to reach the Eq. (3.14), as follows

$$0 = \dot{e}_1 + \dot{e}_2 + \alpha_1 e_1^{\alpha_2} + \alpha_3 e_2^{\alpha_4} \tag{3.14}$$

Numerical solution of (3.14) shows that the variables converge to zero in the finite time, $T_r$, and the upper bound of this time is as below (Qiao & Zhang, 2017)

$$T_r \leq \sum_{i=1}^{2} \frac{\alpha_{2i-1}}{(1-\alpha_{2i}^{-1})} \left| e\left(t_{s_i}\right) \right|^{(\alpha_{2i}-1)} \tag{3.15}$$

As a result, the state variable errors reach zero in the finite time, and its stability time is as $T = T_r + T_s$. ■

Note that, since the $sgn$ function is used in the control input, the undesirable chattering problem is created in the control input in real time. This destructive phenomenon causes many problems in the systems such as an unwanted sound in real time and reducing the useful life of actuators. As a result, a second design is proposed to solve this problem, which is presented below.

### 3.5.2 Theorem2 (NTSMC2)

For the presented system in Eq. (3.4), and the considered conditions for the upper bound of the external disturbances and uncertainties in Eq. (3.5), using the sliding surface in Eq. (3.16) and the designed control input in Eq. (3.17), the finite time stability will be guaranteed. In other words, all state variables of the error system reach zero in a finite time and remain at zero. The settling time is the sum of the two-time including stability time of sliding surfaces ( $T_r$) as well as the time that the system reaches at the sliding surface ($T_s$). Accordingly, the settling time (stability time) of the system is equal to $T = T_r + T_s$, of which the upper bound of $T_r, T_s$ is presented in the following.

$$s = \dot{e}1 + \dot{e}2 + \alpha_1 e_1^{\alpha_2} + \alpha_3 e_2^{\alpha_4} \tag{3.16}$$

Where $\alpha_{2i-1}$ are positive constants and greater than one and $\alpha_{2i}$ are constants between one and two.

$$\begin{cases} u = \frac{1}{b}\left(u_r + u_{eq}\right) \\ u_{eq} = -e_2 - \alpha_1 e_1^{\alpha_2} - \alpha_3 e_2^{\alpha_4} - a_1\left(x_{2_d} + e_2\right) - a_2 \sin\left(x_{1_d} + e_1\right) + \dot{x}_{2_d} \\ \dot{u}_r = -ksig^{\beta}(s) \ - \eta_2 sgn(s) \end{cases} \tag{3.17}$$

Where $k$ is a positive constant and $\beta$ is a constant between one and zero.

Proof: To prove the finite time stability, it needs first to prove that the control input (3.17) leads the system to reach the sliding surface, $s = 0$. Hence, the candidate Lyapunov function is considered as $V(x) = \frac{1}{2}s^2$, where this candidate function has condition of Lyapunov function in lemma 3.1. By differentiating this candidate function with respect to time, there comes

$$\dot{V}(x) = s\dot{s} \tag{3.18}$$

By substituting $\dot{e}_2(t)$ into Eq. (3.16); followed by, applying the control input (3.17), and its simplification, we have

$$s = u_r + d(t) \ \rightarrow \dot{s} = \dot{u}_r + \dot{d}(t) \tag{3.19}$$

By substituting the result into (3.18), yields

$$\dot{V}(x) = s\left(\dot{u}_r + \dot{d}(t)\right) \rightarrow \dot{V}(x) = s\left(-ksig^{\beta}(s) \ - \eta_2 sgn(s) \ + \dot{d}(t)\right) \tag{3.20}$$

According to definition 3.1 and 3.2, we have

$$\dot{V}(x) = -k|s|^{\beta+1} - \eta_2|s| + s\dot{d}(t) \tag{3.21}$$

Considering $s\dot{d}(t) \leq |s\dot{d}(t)|$ also $\left\|\dot{d}(t)\right\| \leq \eta_2$, one can obtain

$$\dot{V}(x) \leq -k|s|^{\beta+1} \tag{3.22}$$

Now, by substituting $|s| = \sqrt{2}(V(x))^{\frac{1}{2}}$ in (3.22) and by choosing the values $\rho_1 = k\left(\sqrt{2}\right)^{\beta+1}$ and $\rho_2 = \frac{\beta+1}{2}$, the result is

$$\dot{V}(x) \leq -\rho_1 V^{\rho_2} \tag{3.23}$$

According to Lemma 3.1, the system states reach the sliding surface $s = 0$ in the finite time, $T_s$. The upper bound of $T_s$ is given as below

$$T_s \leq \frac{V^{\frac{1}{2}}(e_0)}{\left(\frac{\sqrt{2}\Delta m}{2}\right)} \tag{3.24}$$

For the second stage of the proof procedure, the finite time stability proof of the sliding surface of $s = 0$ must be performed. For this purpose, the Eq. (3.16) is equalized to zero, yield

$$0 = \dot{e}_1 + \dot{e}_2 + \alpha_1 e_1^{\alpha_2} + \alpha_3 e_2^{\alpha_4} \tag{3.25}$$

Numerical solution of (3.25) shows that the variables converge to zero in the finite time, $T_r$, and the upper bound of this time is as below (Qiao & Zhang, 2017)

$$T_r \leq \sum_{i=1}^{2} \frac{\alpha_{2i-1}}{\left(1-\alpha_{2i}^{-1}\right)} \left|e\left(t_{s_i}\right)\right|^{(\alpha_{2i}-1)} \tag{3.26}$$

As a result, the state variable errors reach zero in the finite time, and its stability time is as $T = T_r + T_s$. ■

In this design, due to the integration of the $sgn$ function in the control input, the control input does not have chattering phenomenon, and this problem has been eliminated completely.

## 3.6 Two Novel Adaptive Nonsingular Terminal Sliding Mode Control

In this part, the adaptive concept is employed to incorporate with NTSMC method to deal with unknown external disturbances and uncertainties of the presented error system in (3.4) to fulfill trajectory tracking goal. Indeed, the external disturbances and uncertainties are assumed to be unknown and need to approximate the upper bound of them. Accordingly, the upper bound of the external disturbances and uncertainties is estimated and this estimation is used in the two proposed control inputs with two different sliding surfaces. Then, the finite time stability proof is performed by choosing proper candidate Lyapunov function, with considering the conditions in lemma 3.1. The upper bound of the external disturbances and uncertainties is assumed as follows

$$d(t) = h|p(x)| \leq \hat{h}|p(x)| \leq h^{*}|p(x)| \tag{3.27}$$

Where $\hat{h}$ is the estimation of the unknown upper bound of the uncertainties and external disturbances and $h^{*}$ is the upper bound of this estimation, which exists according to Lemma 3.3. Also, $|p(x)|$ is a nonlinear function of the model of uncertainties and external disturbances.

### 3.6.1 Theorem 3 (ANTSMC1)

By considering system (3.4) and the assumed conditions in (3.27) for the external disturbances and uncertainties, by defining the sliding surface in Eq. (3.6), which is repeated in (3.28), and designing control input in Eq. (3.29), as well as considering the adaptive law in Eq. (3.30), not only the system is stable in a finite time, but also the upper bound of the external disturbances and uncertainties is estimated in a finite time and its stability time is $T = T_r + T_s$.

$$s = e_1 + e_2 + \int_0^\infty \alpha_1 e_1^{\alpha_2} dt + \int_0^\infty \alpha_3 e_2^{\alpha_4} dt \tag{3.28}$$

Where $\alpha_{2i-1}$ are positive constants and greater than one and $\alpha_{2i}$ are constants between one and two.

$$\begin{cases} u = \frac{1}{b}(u_r + u_{eq}) \\ u_{eq} = -e_2 - \alpha_1 e_1^{\alpha_2} - \alpha_3 e_2^{\alpha_4} - a_1(x_{2_d} + e_2) - a_2 \sin(x_{1_d} + e_1) + \dot{x}_{2_d} \\ u_r = -sign(s)(\hat{h}|p(x)|) \end{cases} \tag{3.29}$$

And

$$\dot{\hat{h}} = r|s||p(x)| \tag{3.30}$$

Where $r$ is a positive constant and less than one.

Proof3: To prove the finite time stability, it needs first to prove that the control input (3.29) leads the system to reach the sliding surface, $s = 0$. Hence, the candidate Lyapunov function $V(x) = \frac{1}{2}s^2 + \frac{1}{2}\tilde{h}^2$ is considered, where we have $\tilde{h} = \hat{h} - h^*$. This candidate Lyapunov function fulfills the conditions of lemma 3.1. By differentiating the candidate function with respect to time and by considering $\dot{\tilde{h}} = \dot{\hat{h}}$, it is obtained that

$$\dot{V}(x) = s\dot{s} + \dot{\hat{h}}\tilde{h} \tag{3.31}$$

By differentiating the Eq. (3.28) with respect to time and applying the control input (3.29), resulting in

$$\dot{s} = d(t) - u_r \tag{3.32}$$

By substituting into $u_r$ into Eq. (3.32) and followed by substituting Eq. (3.32) into Eq. (3.31), and adaptive law into Eq. (3.31) yields

$$\dot{V}(x) = s\big(d(t) - sign(s)\big(\hat{h}|p(x)|\big)\big) + r|s||p(x)|\tilde{h} \tag{3.33}$$

Since $d(t) = h|p(x)|$, one can obtain

$$\dot{V}(x) \leq |s|(h|p(x)|) - |s|\big(\hat{h}|p(x)|\big) + r|s||p(x)|\tilde{h} \tag{3.34}$$

By adding the term $\pm|s|(h^*|p(x)|)$ to (3.34), yields

$$\dot{V}(x) \leq |s|(h|p(x)|) - |s|\big(\hat{h}|p(x)|\big) + r|s||p(x)|\tilde{h} \pm |s|(h^*|p(x)|) \tag{3.35}$$

Hence

$$\dot{V}(x) \leq |s|(|p(x)|(-(h^*) + h)) - |s|\big(|p(x)|(\hat{h} - (h^*))\big) + r|s||p(x)|\tilde{h} \tag{3.36}$$

By simplification the Eq. (3.36), there is

$$\dot{V}(x) \leq -|s|(|p(x)|\big(h^* - (h)\big)) - |s|(|p(x)|\big(\tilde{h}\big)) + r|s||p(x)|\tilde{h} \tag{3.37}$$

In consequence

$$\dot{V}(x) \leq -|s|(|p(x)|\big(h^* - (h)\big)) - \big|\tilde{h}\big|(|p(x)|\big((1-r)|s|\big)) \tag{3.38}$$

By defining $\Delta_0 = |p(x)|\big(h^* - (h)\big)$ and $\Delta_1 = |p(x)|\big((1-r)|s|\big)$, $\Delta_j$s are non-negative where $j = 0{,}1$, there comes

$$\dot{V}(x) \leq -|s|(\Delta_0) - \big|\tilde{h}\big|\Delta_1 \tag{3.39}$$

If $\Delta_m$ defines as $\Delta_m = \min(\Delta_0, \Delta_1)$, the inequality (3.39) can be rewritten as

$$\dot{V}(x) \leq -\Delta_m(|s| + \big|\tilde{h}\big|) \tag{3.40}$$

According to Lemma 3.2, one can obtain

$$\dot{V}(x) \leq -\Delta_m \left(|s|^2 + \left|\tilde{h}\right|^2\right)^{\frac{1}{2}} \tag{3.41}$$

Hence

$$\dot{V}(x) \leq -\Delta_m\left(2V(x)\right)^{\frac{1}{2}} \Rightarrow \dot{V}(x) \leq -\sqrt{2}\Delta_m\left(V(x)\right)^{\frac{1}{2}} \tag{3.42}$$

By choosing $\rho_1 = \sqrt{2}\Delta_m$ and $\rho_2 = \frac{1}{2}$, yields $\dot{V}(x) \leq -\rho_1 V^{\rho_2}(x)$. Therefore, the system states reach the sliding surface $s = 0$ in the finite time, $T_s$, and the upper bound of the uncertainties is estimated simultaneously. The upper bound of $T_s$ is given as below

$$T_s \leq \frac{V^{\frac{1}{2}}(e_0)}{\left(\frac{\sqrt{2}\Delta_m}{2}\right)} \tag{3.43}$$

For the second stage of the proof procedure, the finite time stability proof of the sliding surface of $s = 0$ must be performed. First, the Eq. (3.28) is equalized to zero, and then its time derivation is taken to reach the Eq. (3.44), as follows

$$0 = \dot{e}_1 + \dot{e}_2 + \alpha_1 e_1^{\alpha_2} + \alpha_3 e_2^{\alpha_4} \tag{3.44}$$

Numerical solution of (3.44) shows that the variables converge to zero in the finite time, $T_r$, and the upper bound of this time is as below (Qiao & Zhang, 2017)

$$T_r \leq \sum_{i=1}^{2} \frac{\alpha_{2i-1}}{\left(1-\alpha_{2i}^{-1}\right)} \left|e\left(t_{s_i}\right)\right|^{(\alpha_{2i}-1)} \tag{3.45}$$

As a result, the state variable errors reach zero in the finite time, and its stability time is as $T = T_r + T_s$. ■

### 3.6.2 Theorem 4 (ANTSMC2)

By considering system (3.4) and the assumed conditions in (3.27) for the external disturbances and uncertainties, by defining the sliding surface in Eq. (3.16), which is repeated in (3.46), and designing control input in Eq. (3.47), as well as considering the adaptive law in Eq. (3.48), not only the system is stable in a finite time, but also the upper bound of the external disturbances and uncertainties is estimated in a finite time and its stability time is $T = T_r + T_s$.

$$s = \dot{e}1 + \dot{e}2 + \alpha_1 e_1^{\alpha_2} + \alpha_3 e_2^{\alpha_4} \tag{3.46}$$

Where $\alpha_{2i-1}$ are positive constants and greater than one and $\alpha_{2i}$ are constants between one and two.

$$\begin{cases} u = \frac{1}{b}\left(u_r + u_{eq}\right) \\ u_{eq} = -e_2 - \alpha_1 e_1^{\alpha_2} - \alpha_3 e_2^{\alpha_4} - a_1\left(x_{2_d} + e_2\right) - a_2 \sin\left(x_{1_d} + e_1\right) + \dot{x}_{2_d} \\ \dot{u}_r = -sign(s)\left(\hat{h}(|p(\dot{x})|)\right) \end{cases} \tag{3.47}$$

And

$$\dot{\hat{h}} = r|s|(|p(\dot{x})|) \tag{3.48}$$

Where $r$ is a positive constant and less than one.

Proof3: To prove the finite time stability, it needs first to prove that the control input (3.47) leads the system to reach the sliding surface, $s = 0$. Hence, the candidate Lyapunov function $V(x) = \frac{1}{2}s^2 + \frac{1}{2}\tilde{h}^2$ is considered, where we have $\tilde{h} = \hat{h} - h^*$. This candidate Lyapunov function fulfills the conditions of lemma 3.1. By differentiating the candidate function with respect to time and by considering $\dot{\tilde{h}} = \dot{\hat{h}}$, it is obtained that

$$\dot{V}(x) = s\dot{s} + \dot{\hat{h}}\tilde{h} \tag{3.49}$$

By substituting $\dot{e}_2(t)$ into Eq. (3.46); followed by, applying the control input (3.47), and its simplification, we have

$$s = d(t) - u_r \tag{3.50}$$

By differentiating the Eq. (3.50) with respect to time, resulting in

$$\dot{s} = \dot{u}_r + \dot{d}(t) \tag{3.51}$$

By substituting into $\dot{u}_r$ into Eq. (3.51) and followed by substituting Eq. (3.51) into Eq. (3.49), and adaptive law into Eq. (3.51), yields

$$\dot{V}(x) = s\big(\dot{d}(t) - sign(s)\big(\hat{h}(|p(\dot{x})|)\big)\big) + r|s|(|p(\dot{x})|)\tilde{h} \tag{3.52}$$

Since $\dot{d}(t) = h(|p(\dot{x})|)$, one can obtain

$$\dot{V}(x) \leq |s|\big(h(|p(\dot{x})|)\big) - |s|\big(\hat{h}(|p(\dot{x})|)\big) + r|s|(|p(\dot{x})|)\tilde{h} \tag{3.53}$$

By adding the term $\pm|s|(h^*(|p(\dot{x})|))$ to (3.53), yields

$$\dot{V}(x) \leq |s|(h(|p(\dot{x})|)) - |s|\big(\hat{h}(|p(\dot{x})|)\big) + r|s|(|p(\dot{x})|)\tilde{h} \pm |s|(h^*(|p(\dot{x})|)) \tag{3.54}$$

Hence

$$\dot{V}(x) \leq |s|\Big((|p(\dot{x})|)(-(h^*) + h)\Big)$$

$$-|s|\big((|p(\dot{x})|)(\hat{h} - (h^*))\big) + r|s|(|p(\dot{x})|)\tilde{h} \tag{3.55}$$

By simplification the Eq. (3.55), there is

$$\dot{V}(x) \leq -|s|((|p(\dot{x})|)\big(h^* - (h)\big)) - |s|((|p(\dot{x})|)\big(\tilde{h}\big)) + r|s|(|p(\dot{x})|) \tag{3.56}$$

In consequence

$$\dot{V}(x) \leq -|s|((|\dot{p(x)}|)(h^{*} - (h))) - |\tilde{h}|((|\dot{p(x)}|)((1-r)|s|)) \tag{3.57}$$

By defining $\Delta_0 = (|\dot{p(x)}|)(h^{*} - (h))$ and $\Delta_1 = (|\dot{p(x)}|)((1-r)|s|)$, $\Delta_j$s are non-negative where $j = 0,1$, there comes

$$\dot{V}(x) \leq -|s|(\Delta_0) - |\tilde{h}|\Delta_1 \tag{3.58}$$

If $\Delta_m$ defines as $\Delta_m = \min(\Delta_0, \Delta_1)$, the inequality (3.58) can be rewritten as

$$\dot{V}(x) \leq -\Delta_m(|s| + |\tilde{h}|) \tag{3.59}$$

According to Lemma 3.2, one can obtain

$$\dot{V}(x) \leq -\Delta_m \left(|s|^2 + |\tilde{h}|^2\right)^{\frac{1}{2}} \tag{3.60}$$

Hence

$$\dot{V}(x) \leq -\Delta_m(2V(x))^{\frac{1}{2}} \Rightarrow \dot{V}(x) \leq -\sqrt{2}\Delta_m(V(x))^{\frac{1}{2}} \tag{3.61}$$

By choosing $\rho_1 = \sqrt{2}\Delta_m$ and $\rho_2 = \frac{1}{2}$, yields $\dot{V}(x) \leq -\rho_1 V^{\rho_2}(x)$. Therefore, the system states reach the sliding surface $s = 0$ in the finite time, $T_s$, and the upper bound of the uncertainties is estimated simultaneously. The upper bound of $T_s$ is given as below

$$T_s \leq \frac{V^{\frac{1}{2}}(e_0)}{\left(\frac{\sqrt{2}\Delta_m}{2}\right)} \tag{3.62}$$

For the second stage of the proof procedure, the finite time stability proof of the sliding surface of $s = 0$ must be performed. For this purpose, the Eq. (3.46) is equalized to zero, yield

$$0 = \dot{e}_1 + \dot{e}_2 + \alpha_1 e_1^{\alpha_2} + \alpha_3 e_2^{\alpha_4} \tag{3.63}$$

Numerical solution of (3.63) shows that the variables converge to zero in the finite time, $T_r$, and the upper bound of this time is as below (Qiao & Zhang, 2017)

$$T_r \leq \sum_{i=1}^{2} \frac{\alpha_{2i-1}}{(1-\alpha_{2i}^{-1})} \left|e\left(t_{s_i}\right)\right|^{(\alpha_{2i}-1)} \tag{3.64}$$

As a result, the state variable errors reach zero in the finite time, and its stability time is as $T = T_r + T_s$. ■

## 3.7 Summary

In this chapter, NTSMC and ATSMC methods are employed to design four different controllers for the nonlinear ROV system with one DOF for the pitch angle in presence uncertainties and external disturbances. Two different sliding surfaces are proposed to evaluate the effect of the integral block and the derivative block in the sliding surfaces. Subsequently, for each sliding surface, an adaptive and non-adaptive controller is designed to deal with changing operation condition and dynamics. A mathematical relation for the finite time settling time (stability time) are also obtained for each controller and presented which ensures that all proposed controllers are stable in a finite time with adjustable and settable settling time.

# CHAPTER 4: FINITE TIME OBSERVER

## 4.1 Introduction

One of the unavoidable problems in practical systems is the existence of uncertainties and external disturbances. In this chapter, a general form of double integral nonlinear system with matched and additive external disturbances and uncertainties are considered and presented to design nonlinear observer for it. However, these uncertainties and external disturbances have not been considered in many observer designs in the literature which is practically not true. The finite time concept also is incorporated with a nonlinear design which can speed up the convergence rate of estimation state to the real state. This chapter starts with giving some basic mathematics and lemmas which are used throughout this chapter. Then, the finite time stability proof of the estimation error dynamic is performed by using Lyapunov theory.

## 4.2 Mathematical preliminaries and lemmas

Definition 4.1: The $sgn(x)$ function is defined as (4.1), and the function $sig^{a}(x)$ can be defined as $sig^{a}(x) = |x|^{a} sgn(x)$.

$$sgn(x) = \begin{cases} 1 & ; \quad x > 0 \\ 0 & ; \quad x = 0 \\ -1 & ; \quad x < 0 \end{cases} \tag{4.1}$$

Definition 4.2: The mathematical relation between the absolute function and the $sign(a)$ function is $|x| = xsgn(x)$.

Definition 4.3: In a nonlinear system $x = f(t,x), f(t,0) = 0, x \in \mathcal{E} \subseteq \Re^{n}$, where $\mathcal{E}$ is an open neighborhood of the equilibrium point $x = 0$. If in this system, the equilibrium point has asymptomatic stability in the region $\mathcal{E}$ as well as the time $T$ exists in such away that,

$$\lim_{t \to T} x = 0 \ and \ x = 0 \ for \ t \geq T \tag{4.2}$$

As a result, the system will be locally stable in a finite time (Bhat & Bernstein, 2000).

Lemma 4.1: In the nonlinear system $\dot{x} = f(x), f(0) = 0, x\epsilon\Re^n$ with initial conditions $x(0) = x_0$, if the Lyapunov candidate function $V(x)$ is globally positive definite, radially unbounded and only at x = 0 is zero, and the time derivative of the Lyapunov candidate function is as $\dot{V}(x) \leq -\rho_1 V^{\rho_2}(x)$, where $\rho_1$ is a positive number and $\rho_2$ is a constant between zero and one; hence the variable $x$ of the system from any initial conditions, it reaches zero in a finite time, and since then it remains exactly equal to zero, i.e. $\lim_{t\to T} x \to 0$ and the upper bound of the settling time, $T$, will be as $T(x_0) \leq \frac{V^{1-\rho_2(x_0)}}{\rho_1(1-\rho_2)}$ (Qiao & Zhang, 2017)

Lemma 4.2: Assuming the conditions expressed in lemma 4.1 for the nonlinear system and the candidate Lyapunov function $V(x)$ is globally positive definite, radially unbounded and only at x = 0 is zero, if $\dot{V}(x) \leq -\rho_2 V^{\rho_3}(x) - \rho_1 V(x)$, since $\rho_1, \rho_2 > 0\, , 0 < \rho_3 < 1$ then the system is a globally finite time stable, and its settling time is as $T(x_0) \leq \left(\rho_1(1-\rho_3)\right)^{-1}(ln(\rho_1 V^{1-\rho_3}(x_0) + \rho_2) - ln\rho_2)$ (Abooee, Arefi, et al., 2017).

Lemma 4.3: Assuming the conditions expressed in lemma 4.1 for the nonlinear system and the candidate Lyapunov function $V(x)$ is globally positive definite, radially unbounded and only at x = 0 is zero, if $\dot{V}(x) \leq -\rho_1 V^{\rho_4}(x) - \rho_2 V^{\rho_5}(x)$ since $\rho_1 > 0, \rho_2 > 0, \rho_3 > 1, \rho_4 = 1 - \frac{1}{2\rho_3}, \rho_5 = 1 + \frac{1}{2\rho_3}$, consequently, the equilibrium point $x = 0$ has a globally finite time stability. The settling time of this system is as $T \leq \pi\rho_3\left(\sqrt{\rho_1\rho_2}\right)^{-1}$ (Abooee, Arefi, et al., 2017).

Lemma 4.4: For each value $a_1, a_2, \dots, a_n \in \Re$ and $0 < q < 2$ we have: $|a_1|^q + |a_2|^q + \cdots + |a_n|^q \geq (a_1^2 + a_2^2 + \cdots + a_n^2)^{\frac{q}{2}}$ (Bhat & Bernstein, 1998).

Lemma 4.5: Assume that $i = 1,2,3, \dots, n$ and $a_i$ can be any real numbers and $\sigma_1$ and $\sigma_2$ are two positive real numbers with conditions $\sigma_1 \geq 1$ and $0 < \sigma_2 < 1$. Hence, two following inequality always is true (Hardy, Littlewood, & Pólya, 1952).

$$(i): \left(\sqrt{\sum_{i=1}^{n} |a_i|^2}\right)^{\sigma_1} \leq \left(\sum_{i=1}^{n} |a_i|\right)^{\sigma_1} \leq n^{\sigma_1 - 1} \sum_{i=1}^{n} |a_i|^{\sigma_1}$$

$$(ii): \sqrt{\left(\sum_{i=1}^{n} |a_i|^2\right)^{\sigma_2 + 1}} \leq \sum_{i=1}^{n} |a_i|^{1+\sigma_2} \tag{4.3}$$

## 4.3 Model description and problem statement

Consider a general form of the high order of the double integrator uncertain nonlinear system in Eq. (4.4).

$$\begin{cases} \dot{x} = v \\ \dot{v} = f(x, v) + g(x)u(t, x) + d(t) \end{cases} \tag{4.4}$$

Where $x = [x_1\ x_3 \dots x_{2n-1}]^T$ and $v = [x_2\ x_4 \dots x_{2n}]^T$ are the vector of state variables of the system and $d = [d_1\ d_2 \dots d_n]^T$ are the vector of external disturbances and uncertainties. $f(x, v) \in \mathcal{R}^n$, and $g(x) \in \mathcal{R}^n$ are the vector of the smooth nonlinear functions, i.e. $f = [f_1\ f_2 \dots f_n]^T$ and $g = [g_1\ g_2 \dots g_n]^T$. $u = [u_1\ u_2 \dots u_n]^T \in \mathcal{R}^n$ are the vector of control inputs.

Note that the following four assumptions always is true for any double integrator nonlinear system in the form of Eq. (4.4).

Assumption1. The vector $x \in \mathcal{R}^n$ of the system (4.4) is physically measurable and available, while the vector $v \in \mathcal{R}^n$ is not available from the system (4.4).

Assumption2. For a nonlinear system (4.4) the control input vector $u \in \mathcal{R}^n$ is known and certain. In addition, the vector $u \in \mathcal{R}^n$ is in such a way that the inequality $||v|| \leq \kappa$

is always true and the scalar value $\kappa$ is available. Note that, the symbol $||.||$ refers to the Euclidean norm (2-norm).

Assumption3. The uncertainties vector in the system (4.4) always satisfies the condition $||d(x,v)|| \leq \gamma(t)$ where $\gamma(t)$ is a known scalar function.

Assumption4. For the nonlinear system (4.4) the vector $f(x,v)$ always satisfies the Lipschitz equation which given in Eq. (4.5). Where $\theta_1(x,w)$ and $\theta_2(x,w)$ are two positive scalar functions.

$$\left||f(x,\hat{v})-f(x,v)\right|| \leq \theta_1(x,\hat{v})+\theta_2(x,\hat{v})||\hat{v}-v|| \tag{4.5}$$

In the following, the main objective is to design of a nonlinear observer for a double integrator nonlinear system (4.4) to estimate the vector of the state variables of $v \in \mathcal{R}^n$ in such a way that, the vector of error estimation reaches zero in a finite time. Then, this proposed design observer will be examined on a sample of double integrator system of the double integrator uncertain nonlinear ROV system presented in Eq. (3.3). Finally, the numerical simulation will perform on the ROV system as a case study to verify the correctness of the mathematical proof of the proposed finite time observer.

### 4.4 Finite time Nonlinear Observer design

For the system (4.4), proposed full order finite time nonlinear observer is presented in Eq. (4.6).

$$\begin{cases} \dot{\hat{x}} = \hat{v} - h(x,\hat{v})sgn(\hat{x}-x) - \eta_1 sig(\hat{x}-x) \\ \qquad -\eta_2 sig^{\alpha}(\hat{x}-x) - \eta_3 sig^{\beta}(\hat{x}-x) \\ \dot{\hat{v}} = -\eta_4\hat{v} - \eta_5|\hat{x}-x| + f(x,\hat{v}) + g(x)u(t,x) \end{cases} \tag{4.6}$$

Where $\alpha$ and $\beta$ are as follows $0 < \alpha < 1$ , $1 \leq \beta \leq \infty$. Also, $\eta_1, \eta_2, \eta_3, \eta_4, \eta_5$ are positive constants. $\eta_1$ and $\eta_4$ are determined from two following equality $\eta_1 = \eta_5 + \eta_0$

and $\eta_4 = 1 + \eta_0$, respectively, where $\eta_0$ can be any positive constant. Also, $\hat{x}$ and $\hat{v}$ are the estimation of the vector of $x$ and $v$, respectively.

Note that, $sig(\hat{x} - x) \in \mathcal{R}^n$ and $sig^{\alpha}(\hat{x} - x) \in \mathcal{R}^n$ and $sig^{\beta}(\hat{x} - x) \in \mathcal{R}^n$ and $sgn(\hat{x} - x) \in \mathcal{R}^n$ and $|\hat{x} - x| \in \mathcal{R}^n$, which are used in the nonlinear observer of (4.6), can be express as (4.7) and (4.8), where $sgn(.)$ in the following equations is the known sign function (see definition 4.1).

$$sig(\hat{x} - x) = [|\hat{x}_1 - x_1| sgn(\hat{x}_1 - x_1) \ldots |\hat{x}_n - x_n| sgn(\hat{x}_n - x_n)]^T$$

$$sgn(\hat{x} - x) = [sgn(\hat{x}_1 - x_1) \ldots sgn(\hat{x}_n - x_n)]^T$$

$$|\hat{x} - x| = [|\hat{x}_1 - x_1| \ldots |\hat{x}_n - x_n|]^T \tag{4.7}$$

And

$$sig^{\alpha}(\hat{x} - x) = [|\hat{x}_1 - x_1|^{\alpha}\, sgn(\hat{x}_1 - x_1) \ldots |\hat{x}_n - x_n|^{\alpha} sgn(\hat{x}_n - x_n)]^T$$

$$sig^{\beta}(\hat{x} - x) = \left[|\hat{x}_1 - x_1|^{\beta}\, sgn(\hat{x}_1 - x_1) \ldots |\hat{x}_n - x_n|^{\beta} sgn(\hat{x}_n - x_n)\right]^T \tag{4.8}$$

Also, $h(x, \hat{v})$ which is used in Eq. (4.6), is considered as below

$$h(x, \hat{v}) = \theta_1(x, \hat{v}) + \gamma(t) + \eta_4 \kappa + \left(||v|| + \kappa\right)\theta_2(x, \hat{v}) + \eta_2\left(||v|| + \kappa\right)^{\alpha} + \frac{\eta_3}{n^{\beta}}\left(||v|| + \kappa\right)^{\beta} \tag{4.9}$$

In the theorem 1, it is proved that the time responses of the estimated vectors of $\hat{x}$ and $\hat{v}$ (with starting of any initial conditions and created by observer (4.6)) will converge in a finite time to the vectors of $x$ and $v$ (created by nonlinear system (4.4)), respectively.

Theorem: consider the nonlinear system in Eq. (4.4) with the aforementioned assumptions from 1 to 4, and nonlinear observer in (4.6) and also consider the Eqs. From (4.7) to (4.9).

It will be proved that the time responses of the estimated vectors of $\hat{x}$ and $\hat{v}$ (created by the observer (4.6) and with starting from any initial conditions) will converge in the finite time of $T(\tilde{e}_x(0), \tilde{e}_v(0))$, precisely to the time responses of the two vectors of $x$ and $v$ (created by nonlinear system (4.4)). Also, three different inequality for calculating the upper bound of the finite time $T$ is represented in Eq. (4.10).

$$T \leq \pi\rho_3 \left(\sqrt{2^{1-\beta}n^{-\beta}\eta_2\eta_3}\right)^{-1}$$

$$T(\tilde{e}_x(0), \tilde{e}_v(0)) \leq \left(\eta_2(1-\alpha)\right)^{-1}\left(\left||\tilde{e}_x(0)|\right| + ||\tilde{e}_v(0)||\right)^{1-\alpha} \tag{4.10}$$

$$T(\tilde{e}_x(0), \tilde{e}_v(0)) \leq \left(\eta_0(1-\alpha)\right)^{-1}\left(ln\left(\eta_0\left(\left||\tilde{e}_x(0)|\right| + ||\tilde{e}_v(0)||\right)^{1-\alpha} + \eta_2\right) - ln\eta_2\right)$$

Proof. Define the estimation error vectors $\tilde{e}_x$ and $\tilde{e}_v$ as $\tilde{e}_x = \hat{x} - x$ and $\tilde{e}_v = \hat{v} - v$, then we have

$$\begin{cases} \tilde{e}_x = \hat{x} - x \\ \tilde{e}_v = \hat{v} - v \end{cases} \Rightarrow \begin{cases} \dot{\tilde{e}}_x = \dot{x} - \dot{x} \\ \dot{\tilde{e}}_v = \dot{v} - \dot{v} \end{cases} \tag{4.11}$$

According to the definition of these two error vectors, the dynamic of estimation errors between the observer (4.6) and the nonlinear system (4.4) results in the form of Eq. (4.12)

$$\begin{cases} \dot{\tilde{e}}_x = \tilde{e}_v - h(x,\hat{v})sgn(\tilde{e}_x) - \eta_1 sig(\tilde{e}_x) - \eta_2 sig^{\alpha}(\tilde{e}_x) - \eta_3 sig^{\beta}(\tilde{e}_x) \\ \dot{\tilde{e}}_v = -\eta_4\hat{v} - \eta_5|\tilde{e}_x| + f(x,\hat{v}) - f(x,v) - d(t) \end{cases} \tag{4.12}$$

Now, for the system (4.12), the candidate Lyapunov function is selected as below

$$V = \left||\tilde{e}_x|\right| + ||\tilde{e}_v|| \tag{4.13}$$

By differentiating of this candidate function with respect to time, we have

$$\dot{V} = \left||\tilde{e}_x|\right|^{-1} \tilde{e}_x^T \dot{\tilde{e}}_x + \left||\tilde{e}_v|\right|^{-1} \tilde{e}_v^T \dot{\tilde{e}}_v \tag{4.14}$$

Now by substituting $\dot{\tilde{e}}_x$ and $\dot{\tilde{e}}_v$ of Eq. (4.12) into $\dot{V}(t)$, one can obtain

$$\dot{V} = \left||\tilde{e}_x|\right|^{-1} \tilde{e}_x^T \tilde{e}_v - \eta_1 \left||\tilde{e}_x|\right|^{-1} \tilde{e}_x^T sig(\tilde{e}_x) - \eta_2 \left||\tilde{e}_x|\right|^{-1} \tilde{e}_x^T sig^{\alpha}(\tilde{e}_x) - \eta_3 \left||\tilde{e}_x|\right|^{-1} \tilde{e}_x^T sig^{\beta}(\tilde{e}_x) - \left||\tilde{e}_x|\right|^{-1} \tilde{e}_x^T h(x,\hat{v}) sgn(\tilde{e}_x) - \eta_4 \left||\tilde{e}_v|\right|^{-1} \tilde{e}_v^T \hat{v} - \eta_5 \left||\tilde{e}_v|\right|^{-1} \tilde{e}_v^T |\tilde{e}_x| + \left||\tilde{e}_v|\right|^{-1} \tilde{e}_v^T \big(f(x,\hat{v}) - f(x,v)\big) - \left||\tilde{e}_v|\right|^{-1} \tilde{e}_v^T d(t) \tag{4.15}$$

Eq. (4.15) shows that, $\dot{V}$ is derived from nine terms and phrases. In the following, for each of these nine terms, the inequalities are extracted and at the end, these inequalities will be used.

For the first term $\left||\tilde{e}_x|\right|^{-1} \tilde{e}_x^T \tilde{e}_v$, by using the famous Chuashi-Schwartz inequality, the Eq. (4.16) is obtained

$$\left||\tilde{e}_x|\right|^{-1} \tilde{e}_x^T \tilde{e}_v \leq \left||\tilde{e}_x|\right|^{-1} |\tilde{e}_x^T \tilde{e}_v| \leq \left||\tilde{e}_x|\right|^{-1} \left||\tilde{e}_x|\right| \left||\tilde{e}_v|\right| = \left||\tilde{e}_v|\right| \tag{4.16}$$

For the second term $-\eta_1 \left||\tilde{e}_x|\right|^{-1} \tilde{e}_x^T sig(\tilde{e}_x)$, according to the equality $\tilde{e}_x^T sig(\tilde{e}_x) = \sum_{i=1}^{n} \left|\tilde{e}_{x_i}\right|^2 = \left||\tilde{e}_x|\right|^2$, there comes

$$-\eta_1 \left||\tilde{e}_x|\right|^{-1} \tilde{e}_x^T sig(\tilde{e}_x) = -\eta_1 \left||\tilde{e}_x|\right|^{-1} \left||\tilde{e}_x|\right|^2 = -\eta_1 \left||\tilde{e}_x|\right| \tag{4.17}$$

In the following, consider the third term, $-\eta_2 \left||\tilde{e}_x|\right|^{-1} \tilde{e}_x^T sig^{\alpha}(\tilde{e}_x)$. According to the equality $\tilde{e}_x^T sig^{\alpha}(\tilde{e}_x) = \sum_{i=1}^{n} \left|\tilde{e}_{x_i}\right|^{\alpha+1}$, this term can rewrite as $-\eta_2 \left||\tilde{e}_x|\right|^{-1} \sum_{i=1}^{n} \left|\tilde{e}_{x_i}\right|^{\alpha+1}$. According to $0 < \alpha < 1$ and the second inequality of lemma 4.5, can be claimed that

$-\sum_{i=1}^{n}\left|\tilde{e}_{x_i}\right|^{\alpha+1} \leq -\left||\tilde{e}_x|\right|^{\alpha+1}$ is always true. Now, considering the recent inequality, Eq. (4.18) can be written for the third term

$$-\eta_2\left||\tilde{e}_x|\right|^{-1}\tilde{e}_x^T sig^{\alpha}(\tilde{e}_x) = -\eta_2\left||\tilde{e}_x|\right|^{-1}\sum_{i=1}^{n}\left|\tilde{e}_{x_i}\right|^{\alpha+1} \leq -\eta_2\left||\tilde{e}_x|\right|^{-1}\left||\tilde{e}_x|\right|^{\alpha+1} = -\eta_2\left||\tilde{e}_x|\right|^{\alpha} \quad (4.18)$$

Quite similar to the third term, the fourth term $-\eta_3\left||\tilde{e}_x|\right|^{-1}\tilde{e}_x^T sig^{\beta}(\tilde{e}_x)$ can be expressed as $-\eta_3\left||\tilde{e}_x|\right|^{-1}\sum_{i=1}^{n}\left|\tilde{e}_{x_i}\right|^{\beta+1}$. According to $1 \leq \beta \leq \infty$ and using two inequality of lemma 4.5, resulting in the inequality $-\sum_{i=1}^{n}\left|\tilde{e}_{x_i}\right|^{\beta+1} \leq -n^{-\beta}\left||\tilde{e}_x|\right|^{\beta+1}$ is always true. Now, considering the recent inequality, the fourth term is rewritten as (4.19)

$$-\eta_3\left||\tilde{e}_x|\right|^{-1}\tilde{e}_x^T sig^{\beta}(\tilde{e}_x) = -\eta_3\left||\tilde{e}_x|\right|^{-1}\sum_{i=1}^{n}\left|\tilde{e}_{x_i}\right|^{\beta+1} \leq -\eta_3 n^{-\beta}\left||\tilde{e}_x|\right|^{-1}\left||\tilde{e}_x|\right|^{\beta+1} = -\eta_3 n^{-\beta}\left||\tilde{e}_x|\right|^{\beta} \quad (4.19)$$

According to $\tilde{e}_x^T sgn(\tilde{e}_x) = \sum_{i=1}^{n}\left|\tilde{e}_{x_i}\right|$ and $-\sum_{i=1}^{n}\left|\tilde{e}_{x_i}\right| \leq -\left||\tilde{e}_x|\right|$, the fifth term $-\left||\tilde{e}_x|\right|^{-1}\tilde{e}_x^T h(x,\hat{v})sgn(\tilde{e}_x)$ is rewritten in the form of Eq. (4.20)

$$-\left||\tilde{e}_x|\right|^{-1}\tilde{e}_x^T h(x,\hat{v})sgn(\tilde{e}_x) = -\left||\tilde{e}_x|\right|^{-1}h(x,\hat{v})\sum_{i=1}^{n}\left|\tilde{e}_{x_i}\right| \leq -\left||\tilde{e}_x|\right|^{-1}h(x,\hat{v})\left||\tilde{e}_x|\right| = -h(x,\hat{v}) \quad (4.20)$$

By substituting $\tilde{e}_v + v = \hat{v}$ into the sixth term $-\eta_4\left||\tilde{e}_v|\right|^{-1}\tilde{e}_v^T\hat{v}$ and using Chuashi-Schwartz inequality, the Eq. (4.21) is obtained

$$-\eta_4\left||\tilde{e}_v|\right|^{-1}\tilde{e}_v^T\hat{v} = -\eta_4\left||\tilde{e}_v|\right| - \eta_4\left||\tilde{e}_v|\right|^{-1}\tilde{e}_v^T v \leq -\eta_4\left(\left||\tilde{e}_v|\right| + \left||v|\right|\right) \quad (4.21)$$

According to the Chuashi-Schwartz inequality, for the seventh and ninth terms, two inequalities of Eq. (4.22) is resulted

$$-\eta_5 \left\|\tilde{e}_v\right\|^{-1} \tilde{e}_v^T |\tilde{e}_x| \leq -\eta_5 \left\|\tilde{e}_v\right\|^{-1} \left|\tilde{e}_v^T |\tilde{e}_x|\right| \leq \eta_5 \left\|\tilde{e}_x\right\|$$

$$-\left\|\tilde{e}_v\right\|^{-1} \tilde{e}_v^T d(t) \leq -\left\|\tilde{e}_v\right\|^{-1} |\tilde{e}_v^T d(t)| \leq \left\|d(t)\right\| \leq \gamma(t) \tag{4.22}$$

According to the aforementioned assumption 4 and Chuashi-Schwartz inequality, the eighth term is rewritten as (4.23)

$$\left\|\tilde{e}_v\right\|^{-1} \tilde{e}_v^T \big(f(x,\hat{v}) - f(x,v)\big) \leq \left\|f(x,\hat{v}) - f(x,v)\right\| \leq \theta_1(x,\hat{v})$$

$$+\theta_2(x,\hat{v})||\hat{v}| \tag{4.23}$$

Now, referring to the Eqs from (4.16) to (4.23) and considering the expansion of $h(x,\hat{v})$ in Eq. (4.9), the equality of $\dot{V}(t)$ in Eq. (4.15) is rewritten as (4.24)

$$\dot{V} \leq \left\|\tilde{e}_v\right\| - \eta_1 \left\|\tilde{e}_x\right\| - \eta_2 \left\|\tilde{e}_x\right\|^{\alpha} - \tfrac{\eta_3}{n^{\beta}} \left\|\tilde{e}_x\right\|^{\beta} - h(x,\hat{v}) - \eta_4 \left\|\tilde{e}_v\right\| + \theta_1(x,\hat{v}) +$$

$$\theta_2(x,\hat{v}) \left\|\tilde{e}_v\right\| - \eta_4 \left\|v\right\| + \eta_5 \left\|\tilde{e}_x\right\| + \gamma(t) \tag{4.24}$$

By defining $\eta_4$ and $\eta_1$ as $\begin{cases} \eta_4 = 1 + \eta_0 \\ \eta_1 = \eta_5 + \eta_0 \end{cases}$ and $h(x,\hat{v}) = h_1(x,\hat{v}) + h_2(x,\hat{v})$, where $h_1(x,\hat{v}) = \theta_1(x,\hat{v}) + \gamma(t) - \eta_4 \kappa$, we have

$$\dot{V} \leq -\eta_0 V - \eta_2 \left\|\tilde{e}_x\right\|^{\alpha} - \tfrac{\eta_3}{n^{\beta}} \left\|\tilde{e}_x\right\|^{\beta} + \theta_2(x,\hat{v}) \left\|\tilde{e}_v\right\| - h_2(x,\hat{v}) \tag{4.25}$$

Where $h_2(x,\hat{v}) = \big(\left\|\hat{v}\right\| + \kappa\big)\theta_2(x,\hat{v}) + \eta_2 \big(\left\|\hat{v}\right\| + \kappa\big)^{\alpha} + \tfrac{\eta_3}{n^{\beta}} \big(\left\|\hat{v}\right\| + \kappa\big)^{\beta}$. By substituting $h_2(x,\hat{v})$ into Eq. (4.25) and by simplification, yields

$$\dot{V} \leq -\eta_0 V - \eta_2 \left\|\tilde{e}_x\right\|^{\alpha} - \tfrac{\eta_3}{n^{\beta}} \left\|\tilde{e}_x\right\|^{\beta} - \eta_2 \big(\left\|\hat{v}\right\| + \kappa\big)^{\alpha} - \tfrac{\eta_3}{n^{\beta}} \big(\left\|\hat{v}\right\| + \kappa\big)^{\beta} \tag{4.26}$$

Since, according to the assumption 2, $||\tilde{e}_v|| \leq ||\hat{v}|| + \kappa$, two inequalities $-||\tilde{e}_v||^{\alpha} \geq -(||\hat{v}|| + \kappa)^{\alpha}$ and $-||\tilde{e}_v||^{\beta} \geq -(||\hat{v}|| + \kappa)^{\beta}$ are always true. Now, according to these two inequalities, Eq. (4.26) is rewritten as follows

$$\dot{V} \leq -\eta_0 V - \eta_2 ||\tilde{e}_x||^{\alpha} - \frac{\eta_3}{n^{\beta}} ||\tilde{e}_x||^{\beta} - \eta_2 ||\tilde{e}_v||^{\alpha} - \frac{\eta_3}{n^{\beta}} ||\tilde{e}_v||^{\beta} \quad \Rightarrow \quad \dot{V} \leq -\eta_0 V - \eta_2 (||\tilde{e}_x||^{\alpha} + ||\tilde{e}_v||^{\alpha}) - \frac{\eta_3}{n^{\beta}} (||\tilde{e}_x||^{\beta} + ||\tilde{e}_v||^{\beta}) \tag{4.27}$$

Referring to the second inequality of lemma 4.5, we have $-(||\tilde{e}_x||^{\alpha} + ||\tilde{e}_v||^{\alpha}) \leq -(||\tilde{e}_x|| + ||\tilde{e}_v||)^{\alpha} = -V^{\alpha}$. In the following referring to the first inequality of lemma 4.5, inequality $-(||\tilde{e}_x||^{\beta} + ||\tilde{e}_v||^{\beta}) \leq -2^{1-\beta}(||\tilde{e}_x|| + ||\tilde{e}_v||)^{\beta} = -2^{1-\beta} V^{\beta}$ is obtained. Now, considering two recent inequalities, the Eq. (4.27) can be rewritten in the form of Eq. (4.28), as follows

$$\dot{V} = -\eta_0 V - \eta_2 V^{\alpha} - \frac{\eta_3}{n^{\beta}} \left(\frac{1}{2^{\beta-1}}\right) V^{\beta} \tag{4.28}$$

Now, by considering $\eta_0 = 0$, $\eta_2 = \rho_1$, $\alpha = \rho_4$, $\eta_3 n^{-\beta} 2^{1-\beta} = \rho_2$, $\beta = \rho_5$, and $\rho_3 > 1$, and using lemma 4.3, can be claimed that the vector of estimation errors $\tilde{e}_x$ and $\tilde{e}_v$ reach to zero in the finite time of $T$, and this finite time satisfied the inequality of $T \leq \pi \rho_3 \left(\sqrt{2^{1-\beta} n^{-\beta} \eta_2 \eta_3}\right)^{-1}$.

In the following, by considering $\eta_2 = \rho_1$, $\alpha = \rho_2$, $\eta_3 = 0$, $\eta_0 = \rho_3$ and using lemma 4.2, can be claimed that the vector of estimation errors $\tilde{e}_x$ and $\tilde{e}_v$ reach to zero in the finite time of $T$, and this finite time satisfied the inequality of $T(\tilde{e}_x(0), \tilde{e}_v(0)) \leq \left(\eta_0(1-\alpha)\right)^{-1} \left(ln\left(\eta_0(||\tilde{e}_x(0)|| + ||\tilde{e}_v(0)||)^{1-\alpha} + \eta_2\right) - ln\eta_2\right)$.

Also, by considering $\eta_0 = 0$, $\eta_2 = \rho_1$, $\alpha = \rho_2$, $\eta_3 = 0$, and using lemma 4.1, can be claimed that the vector of estimation errors $\tilde{e}_x$ and $\tilde{e}_v$ reach to zero in the finite time of $T$, and this finite time satisfied the inequality of $T(\tilde{e}_x(0), \tilde{e}_v(0)) \leq \left(\eta_2(1-\alpha)\right)^{-1}\left(||\tilde{e}_x(0)|| + ||\tilde{e}_v(0)||\right)^{1-\alpha}$. Then the proof is completed. ■

## 4.5 Summary

In this chapter, a full order global finite time nonlinear observer is proposed for a general form of double integrator nonlinear system in presence of uncertainties and external disturbances. One of the key features of this design is that the proposed observer is generalizable due to considering a general form of double integrator nonlinear system which includes a wide range of real-time nonlinear systems. Additionally, a mathematical relationship for the convergence between the corresponding states variables of the observer and the nonlinear system is obtained and presented which make this adjustable and settable finite time.

# CHAPTER 5: RESULTS AND DISCUSSION

## 5.1 Introduction

This chapter includes two main parts which are as follows. The first part presents the numerical simulation of four designed controllers in chapter 3. Then, three well-known performance criteria are defined as ISV, IAE, and ITAE. The value of them is calculated by using the Trapezoidal method, implemented in the software Matlab® through command "$\boldsymbol{trapz(t, Xi)}$" and reports in this chapter. A comprehensive comparison is made comparing the numerical simulation and the value of performance criteria between four designed controllers in this research. The second part is devoted to present the numerical simulation of the proposed nonlinear observer in this research. In the discussion section, the numerical simulation result of the observer is analyzed and explained. Then, a comparison is made between the proposed nonlinear observer and some recent and well known nonlinear observer designs from literature.

## 5.2 Numerical simulation of the four proposed controllers

To perform the numerical simulation of this section, the Simulink environment of MATLAB software has been used with the ode4 numerical solution method and with a step size of 0.001. The selected control parameters for this simulation is as follows

$$\alpha_{2i-1_{(ANTSMC1,NTSMC1)}} = 0.6, \alpha_{2i-1_{(ANTSMC2,NTSMC2)}} = 10, \alpha_{2i} = \frac{9}{7},\ k = 1, |p(x)| = |\cos(x_1)|,\ r = 0.5 \tag{5.1}$$

Also, we have

$$a_1 = 0.6, a_2 = 3, b = 2,, d(t) = 0.1\cos(x_1) + 0.1sin(x_2), x_{1_d} = \sin(2t) + \cos(3t)\,,\ \beta = 0.5,\ x_1(0) = \ \psi(0) = 5,\ x_2(0) = \dot{\psi}(0) = -2 \tag{5.2}$$

The upper bound of the uncertainties and external disturbances has been considered for two approaches of NTSMC, as a constant and equal to $\eta_{i_1} = 0.3, \eta_{i_2} = 0.3$ . Fig. 5.1 and Fig. 5.2 represent the state variables, $x_1$ and $x_2$, respectively, of four proposed control methods. Fig. 5.3 shows the designed control inputs of four controllers. Fig. 5.4 and Fig. 5.5 display the trajectory tracking errors with respect to time, $e_1$ and $e_2$, respectively, of four different control methods in this research.

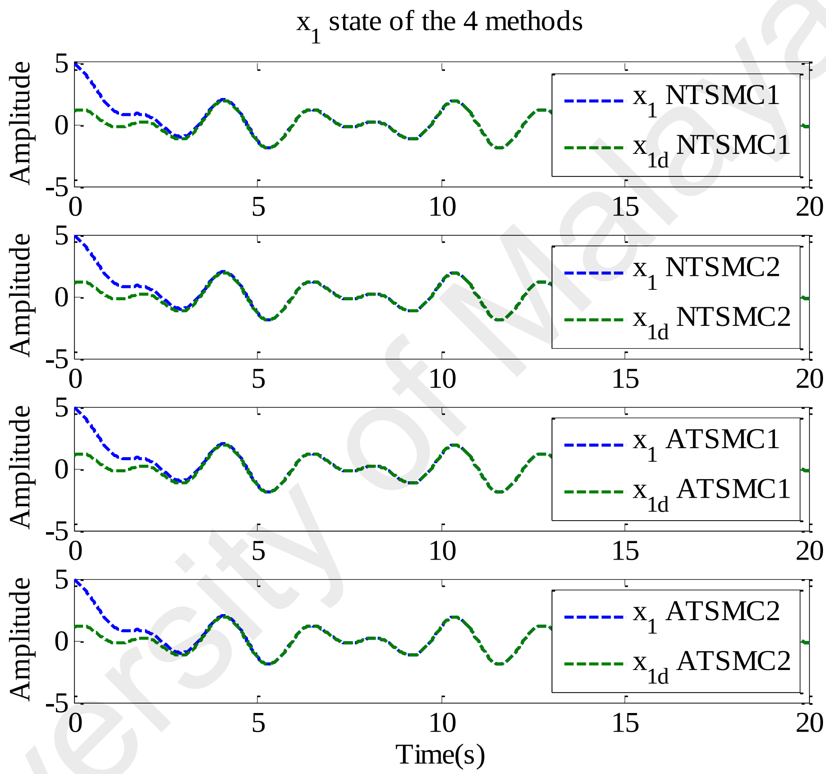



**Figure 5.1: The state variable $x_1$ of four proposed controllers**

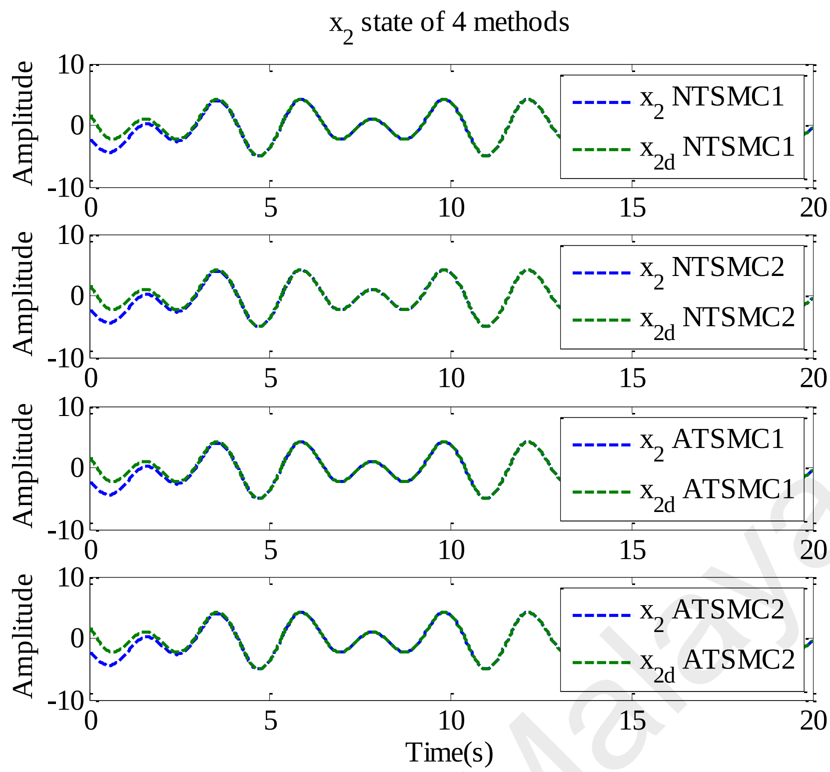



**Figure 5.2: The state variable $x_2$ of four proposed controllers**

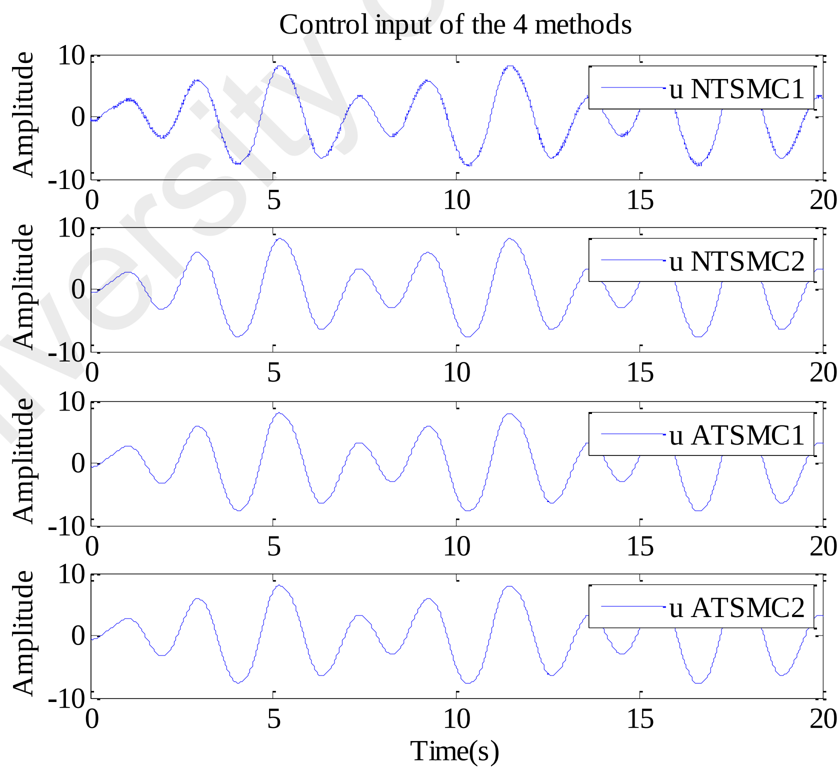



**Figure 5.3: The control inputs of four proposed controllers**

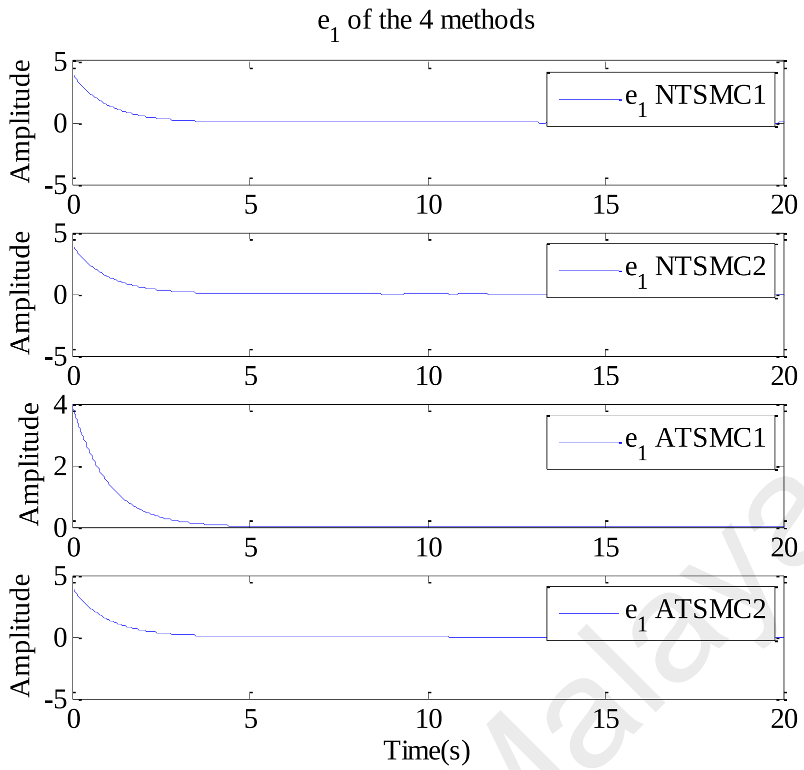



**Figure 5.4: Trajectory tracking error $e_1$ of four proposed controllers**

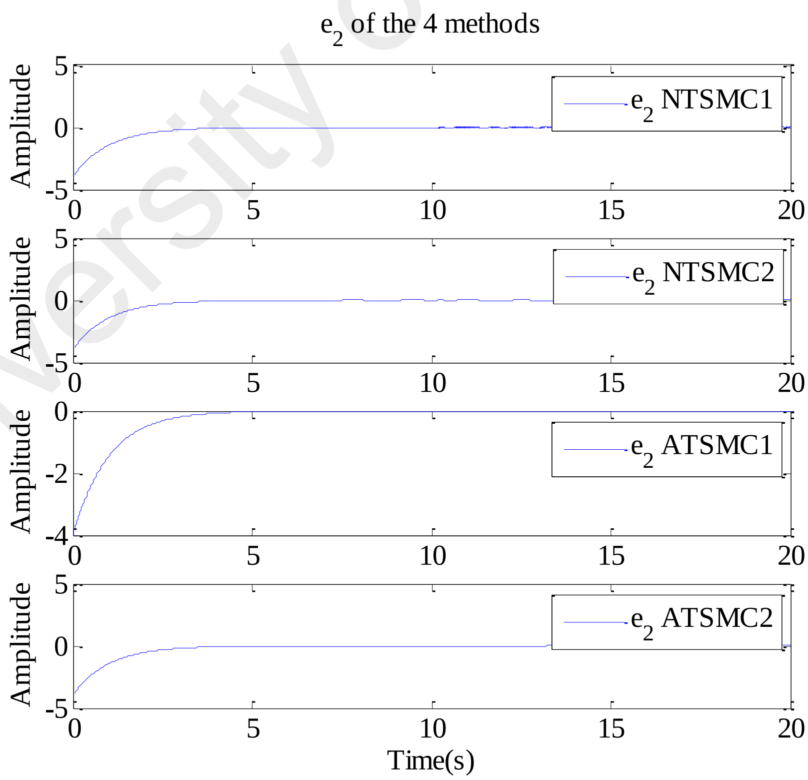



**Figure 5.5: Trajectory tracking error $e_2$ of four proposed controllers**

### 5.2.1 Discussion and Comparison of the four proposed control methods

In this section, a comparison is made between four proposed control designs in this research. To perform an appropriate comparison, the three performance criteria, IAE, ITAE, and ISV are defined and then four proposed methods are compared by using them.

#### 5.2.1.1 Three performance criteria

The performance criteria are defined as Eqs. (5.3) and (5.4) and (5.5). Note that, the control parameters are given in Eqs. (5.1) and (5.2) have been chosen in such way that the stability time of all methods would be approximately equal, to make a logical and correct comparison between four control designs by comparing the value of three performance criteria. Also note that, if the numerical value of each performance criterion is less than another one, it will signify that the method is more appropriate. Performance criteria are defined as follows:

(a) Integral of absolute of error (IAE):

$$IAE = \int_0^{t_f} |e(t)| dt \tag{5.3}$$

(b) Integral of time multiplied by absolute of error (ITAE):

$$ITAE = \int_0^{t_f} t |e(t)| dt \tag{5.4}$$

(c) Integral of the square of control input (ISV):

$$ISV = \int_0^{t_f} u^2(t) dt \tag{5.5}$$

The value of the performance criteria has been calculated by using the Trapezoidal method, implemented in the software Matlab® through command "$\boldsymbol{trapz}(\boldsymbol{t}, \boldsymbol{Xi})$" and the numerical results have been presented in Table 2.1. Also, the mean of each performance criterion has been calculated and shown for four methods in Figs. (5.6), (5.7) and (5.8).

**Table 5.1: Performance criteria for different methods**

| Performance Criteria | NTSMC1 | NTSMC2 | ATSMC1 | ATSMC2 |
|---|---|---|---|---|
| **IAE(e1)** | 4.0001 | 4.0135 | 4.0000 | 4.0012 |
| **IAE2(e2)** | 4.0005 | 4.0438 | 4.0000 | 4.0010 |
| **ITAE(e1)** | 4.0014 | 4.2037 | 4.0004 | 4.0219 |
| **ITAE(e2)** | 4.0094 | 4.6148 | 4.0000 | 4.0138 |
| **ISV** | 386.5615 | 385.9247 | 386.1508 | 386.1445 |

**Figure 5.6: Comparison of the mean of different methods in terms of the IAE criterion**

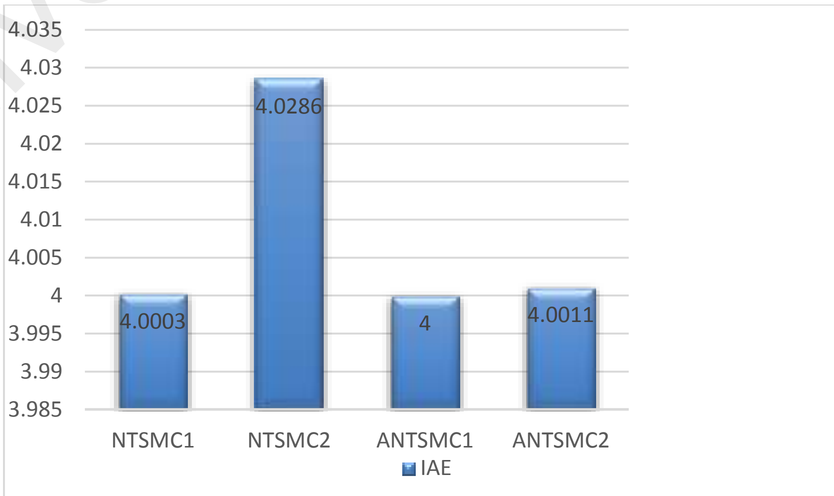


Figure 5.7: Comparison of the mean of different methods in terms of the ITAE criterion

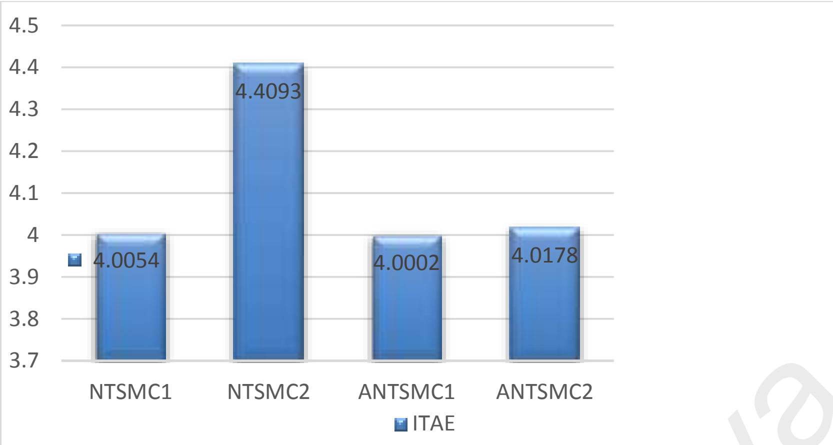
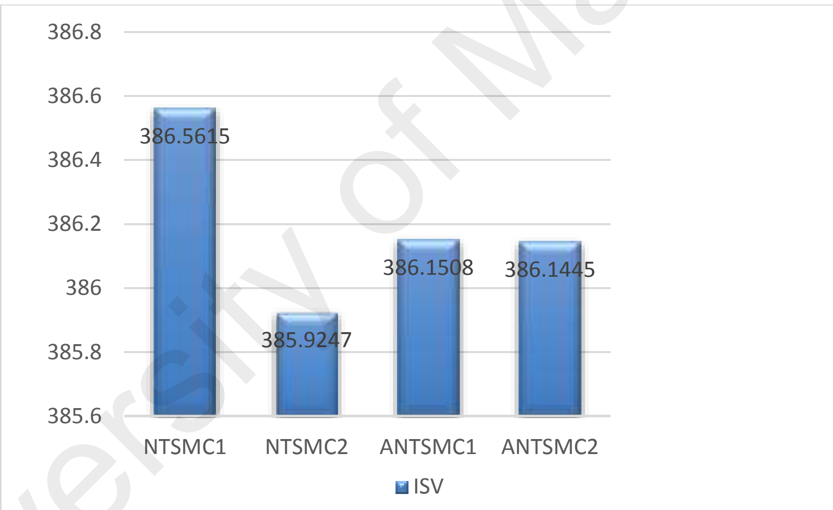



Figure 5.8: Comparison of the mean of different methods in terms of the ISV criterion




#### 5.2.1.2 Key points of compression

In consequence of the comparison of charts, tables and figures, the major points are listed as follows:

- In a comparison of performance criteria, the ISV criterion (which is related to the control input amplitude) within NTSMC1 and ANTSMC1 is greater than two other methods. Note that, The ISV criterion is related to the amplitude of the control input and lower values of ISV can be deduced as lower cost of construction and lower consumption of energy. Accordingly, NTSMC2 and

ANTSMC2 are more cost-effective than two other methods in terms of constructing control inputs. Consequently, using derivative sliding surfaces provides a cost-effective method in terms of constructing control inputs.

- In terms of the value of IAE and ITAE performance criteria, NTSMC1 and ANTSMC1 methods are more appropriate (due to a smaller value). Note that the objective of defining the IAE and ITAE is the numerical measures of the performance of the tracking for the whole error curve, where $t_f$ demonstrates the entire running time. The IAE criterion represents an intermediate result. The ITAE criterion, where time appears as a factor, deeply emphasizes the errors that occur late in time.
- On the other hand, in terms of constructing sliding surfaces, the NTSMC1 and ANTSMC1 methods, since in their sliding surfaces the integral elements have been used, thus, in comparison to the other two methods, NTSMC2 and ANTSMC2, which at their sliding surfaces the derivative elements have been used, are superior because the derivation not only boosts the noise, but it is also expensive and very difficult to construct an ideal derivative element, practically. Consequently, constructing integral sliding surfaces are cheaper and more feasible and easier.
- In terms of control parameters ($\alpha_{2i-1}$), NTSMC2 and ANTSMC2 methods has slightly greater value than two other methods (see Eqs. (5.1) and (5.2)), which cost more to construct greater control parameter. Indeed, NTSMC1 and ANTSMC1 methods reach to stability faster. Therefore, in order to provide almost same settling time for all four controllers, the control parameter of NTSMC2 and ANTSMC2 must choose greater, to make a correct comparison.
- The NTSMC1 method in its control input has created the chattering problem (see Fig. 5.3) as it was predicted (because of using integral sliding surface) and

this is a very destructive phenomenon for control systems. This undesirable problem has been eliminated completely three other proposed controllers. Indeed, by using integral sliding surface, the possibility to face with chattering problem increases sharply in real-time.

- In both ANTSMC methods, the upper bound of the external disturbances and uncertainties is approximated by using the adaptive concept in the finite time and it is used in the control input. In other words, the key point of both ANTSMC methods is that without any knowledge of the external disturbances and uncertainties, the two controllers have been designed which not only provide the finite time stability but also estimate the upper bound of uncertainties and external disturbances.
- All control methods are robust against all the external disturbances and uncertainties, due to the main feature of sliding mode concept.
- All control method provides a global stability in the finite time. The point is that the upper bound of settling time is obtained which is calculable and adjustable. Therefore, the settling time (stability time) can be decreased by changing the control parameters. Note that the obtained settling time $T(x_0)$ in all proposed controller is also depends on initial condition.

**Table 5.2: Comparison of four designed controllers**

| **Based on** | **NTSMC1** | **NTSMC2** | **ATSMC1** | **ATSMC2** |
|---|---|---|---|---|
| **ISV** | High | Low | High | Low |
| **IAE** | Low | High | Low | High |
| **ITAE** | Low | High | Low | High |

| | | | | |
|---|---|---|---|---|
| **Chattering problem** | Create | Eliminate | Eliminate | Eliminate |
| **Constructing sliding surface** | Cost effective (integral block) | Costly (derivative block) | Cost effective (integral block) | Costly (derivative block) |
| **Control parameter** | Smaller (Cost-effective) | Greater (Costly) | Smaller (Cost-effective) | Greater (Costly) |
| **Knowledge of uncertainties** | Available | Available | Unavailable | Unavailable |
| **Robustness and precision** | High | High | High | High |

## 5.3 Numerical simulation of the proposed nonlinear observer

To perform numerical simulation, the second-order nonlinear ROV system as a sample of double integrator system has been considered which has been presented in (Fossen, 2002; Jianhua Wang et al., 2016), in the form of Eq. (3.3) and repeated in Eq. (5.6), as follows

$$\begin{cases} x_1(t) = \psi(t) \\ \dot{x}_1(t) = x_2(t) \\ \dot{x}_2(t) = a_1 x_2(t) + a_2 \sin\big(x_1(t)\big) + bu(t) + d(t) \end{cases} \tag{5.6}$$

Where in this model, $a_1$, $a_2$ and $b$ are positive and constant, and $u(t)$ is the control input, also $d(t)$ is the model of external disturbances and uncertainties of the system.

In this section, the mathematical designed finite time observer of the previous chapter has been applied to the sample of the system of the double integrator ROV system with the one DOF for the pitch angle of Eq. (3.3) (which is repeated in Eq. (5.6)) as a case study. To perform numerical simulation, the Simulink environment of MATLAB has been used with the ode4 numerical solution method using a step size of 0.001.

Also, $a_1$, $a_2$ and $b$ are considered as $a_1 = 0.6, a_2 = 3, b = 2$. The control input is chosen as $u(t) = (\frac{1}{b})(-0.1x - 10v - 1000\sin(10t))$ and the model of uncertainties and external disturbances is considered as $d(t) = -0.15\sin(4t) + 0.1\cos(3t + 2) + 0.15\sin(5t - 5)$. For the uncertainties term always, we have $||d(t)|| \leq 0.15 = \gamma(t)$, hence the assumption 2 of chapter 4 is satisfied. By considering initial condition as $x_1(0) = \psi(0) = 20$ and $x_2(0) = \dot{\psi}(0) = -60$, and choosing $||v|| \leq 11 = \kappa$, the simulation results show that the assumption 3 in chapter 4 is also satisfied for the system in Eq. (5.6). According to equations of the nonlinear system in (5.6), by considering $\theta_1 = 0$ and $\theta_2 = 0.6$, the assumption 4 in chapter 4 is satisfied. The selected observer parameters are as follows

$$\eta_0 = 0.1, \eta_5 = 1, \eta_1 = \eta_5 + \eta_0, \eta_2 = 1, \eta_3 = 1, \eta_4 = 1 + \eta_0, \eta_5 = 1, \alpha = 0.9, \beta = 1.3, n = 2 \quad (5.7)$$

Fig. 1 shows the time responses of the state variables $x_1(t)$ and $\hat{x}_1(t)$ where $x_1(t)$ is created by the nonlinear system in Eq. (5.6) and $\hat{x}_1(t)$ is created by the proposed nonlinear observer.

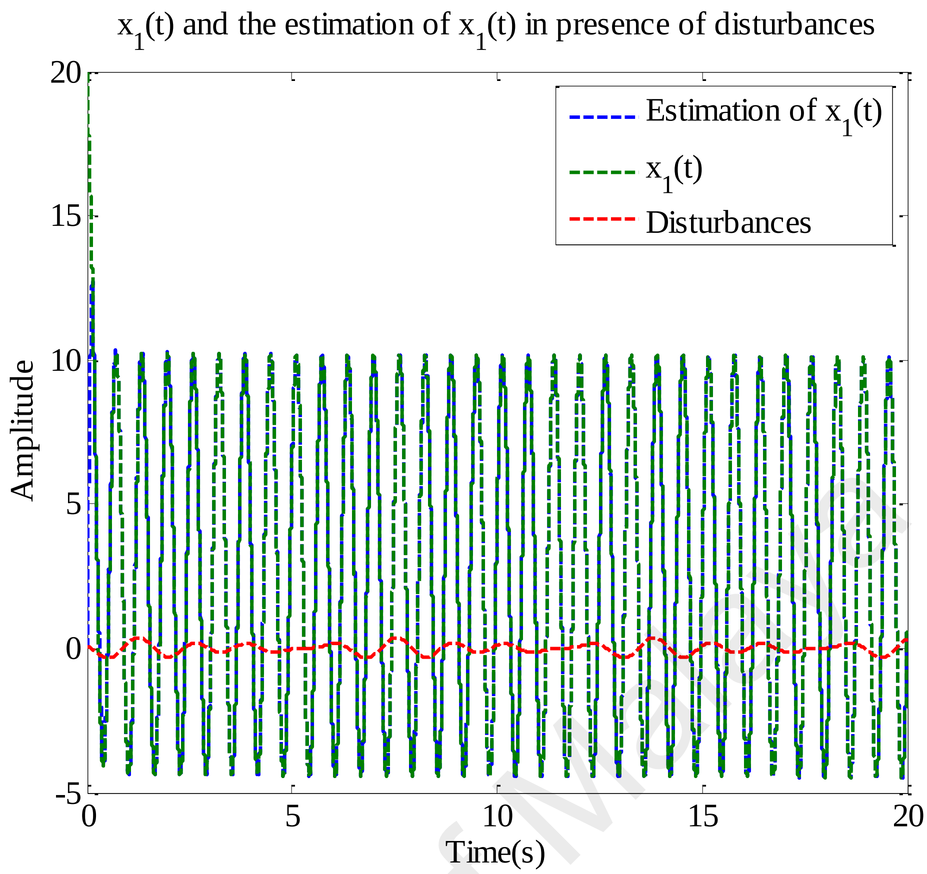



**Figure 5.9: Time response of state variable of $x_1(t)$ (created by the nonlinear system in Eq. (5.6)) and $\hat{x}_1(t)$ (estimated by the proposed nonlinear observer in Eq. (4.6) with selected parameters in Eq. (5.7)) in presence of the variety of uncertainties and external disturbances**

Fig. 2 shows the time responses of the state variables $x_2(t)$ and $\hat{x}_2(t)$ where $x_2(t)$ is created by the nonlinear system in Eq. (5.6) and $\hat{x}_2(t)$ is estimated by proposed the nonlinear observer.

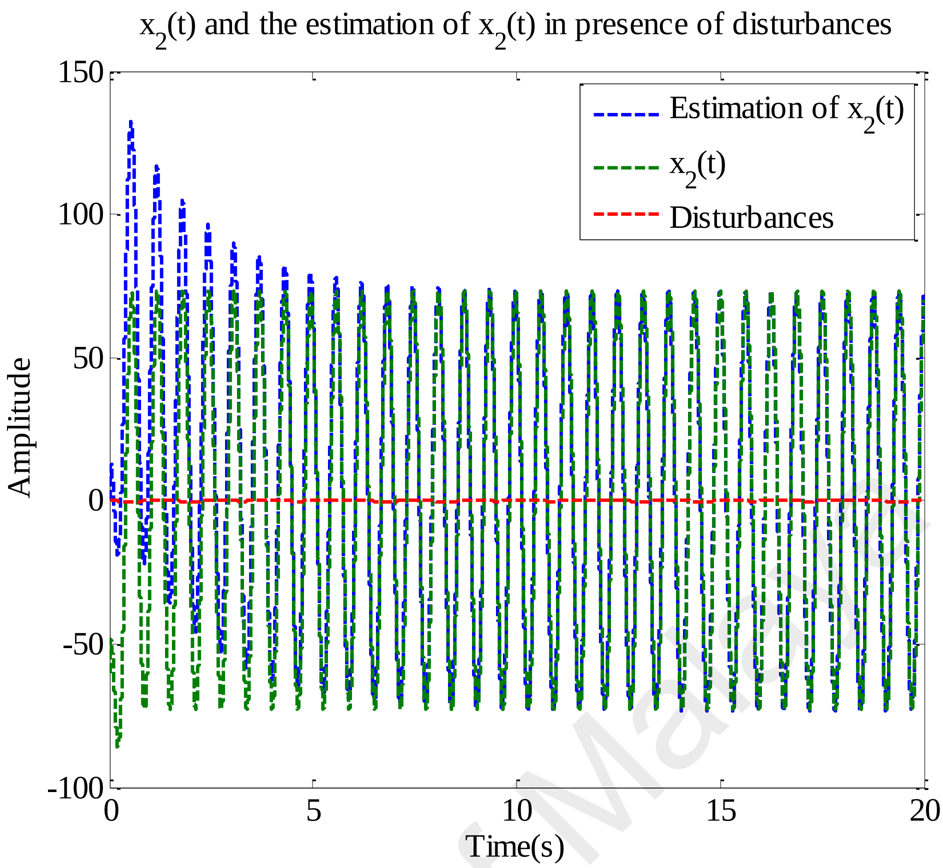



**Figure 5.10: Time response of state variable of $x_2(t)$ (created by the nonlinear system in Eq. (5.6)) and $\hat{x}_2(t)$ (estimated by the proposed nonlinear observer in Eq. (4.6) with selected parameters in Eq. (5.7)) in presence of the variety of uncertainties and external disturbances**

### 5.3.1 Discussion of the simulation results of the observer

As it is shown in Fig. 1, the estimation value of $\hat{x}_1(t)$ state variable converges to the real value of $x_1(t)$ very accurately and almost less than 1 second which is very fast. Also, the variety of uncertainties and external disturbances are shown in this figure which have almost no effect on the precision of the estimation by the proposed nonlinear observer.

As it is shown in Fig. 2, it is clear that the estimation value of $\hat{x}_1(t)$ state variable converges to the real value of $x_1(t)$ almost after 7 second and very accurately, which is fast enough. Also, the variety of uncertainties and external disturbances are shown in this figure which have almost no effect on the precision of the estimation by the proposed nonlinear observer. In both figures of the observer, the estimation has been done in a finite time and just in a few seconds which proves that this proposed nonlinear observer has been used the finite time concept and speed up the convergence rate of the estimation of the state variables.

### 5.3.2 The major points of the proposed finite time observer

- As the first key point of this proposed nonlinear observer, the selected model for a nonlinear system in Eq. (4.4) is in a form of a chain of non-linear subsystems (second-order) double integrator with interactions. This considered class for a nonlinear system includes a wide range of real-world physical and operational systems such as robot manipulators, ships, submarines, gyroscope, reverse pendulum and so on.
- The second and main innovation of this observer is that the designed nonlinear observer is globally and in a form of finite time. That means that the estimation errors between the corresponding states of the observer and the nonlinear system reach to real zero in a calculable finite time (which is adjustable) and after this finite time, time responses of the proposed observer will be completely equal to time responses of the nonlinear system.
- As a third innovation, a mathematical relation is obtained to calculate the finite time of reaching the estimation errors of the state variables to real zero. By using the obtained relation, the relation between the convergence finite time and the parameter of the nonlinear observer can be found and by proper setting, these parameters, the convergence finite time of the estimation errors to zero can be reduced significantly.
- The fourth strong point of the proposed observer design is considering a variety of the uncertainties and external disturbances (in form of match and additive) for the nonlinear system model.

**Table 5.3: Comparison among existing observer and proposed observer**

| Existing nonlinear observer | References | Proposed nonlinear observer |
|---|---|---|
| Ungeneralizable (a nonlinear observer for a particular class of nonlinear systems) | (Du et al., 2013; Kravaris, 2016; Menard et al., 2017; Shen & Xia, 2008; Tami et al., 2016; L. Wang et al., 2017; J. Zhang et al., 2017; Zhao & Guo, 2017; Zheng, Efimov, Bejarano, et al., 2016; Zheng, Efimov, & Perruquetti, 2016) | Generalizable (the nonlinear observer for the class of nonlinear which includes a wide range of real-time nonlinear systems) |
| Infinite time nonlinear observer (the global asymptotic stability has been proved for the dynamic of the estimation errors) | (Kravaris, 2016; Tami et al., 2016; L. Wang et al., 2017; J. Zhang et al., 2017; Zhao & Guo, 2017; Zheng, Efimov, & Perruquetti, 2016) | Finite time nonlinear observer (The global finite time stability has been proved for the dynamic of the estimation errors) |
| Exclude uncertainties and external disturbances in the | (Du et al., 2013; Kravaris, 2016; Menard et al., 2017; Shen & Xia, 2008; Tami et al., 2016; L. Wang et al., 2017; Zhao & Guo, 2017; Zheng, Efimov, Bejarano, et al., | Include uncertainties and external disturbances in the |

| | | |
|---|---|---|
| dynamic model of the nonlinear system | 2016; Zheng, Efimov, & Perruquetti, 2016) | dynamic model of the nonlinear system |
| Infinite time settling time | (Kravaris, 2016; Tami et al., 2016; L. Wang et al., 2017; J. Zhang et al., 2017; Zhao & Guo, 2017; Zheng, Efimov, & Perruquetti, 2016) | Obtained relation for settling time (of estimating states to real states) which is Calculable and adjustable |

## 5.4 Summary

In this chapter, the numerical simulation of adaptive and non-adaptive controllers is represented and the comparison between them is made. The comparison reveals that each controller has some features and in accordance with different applications the superior controller can be different. In fact, two sliding surfaces are chosen to evaluate of the effect of the integral block and the derivative block in the sliding surface. Note that the NTSMC1 and ATSMC1 are designed with integral sliding surfaces and NTSMC2 and ATSMC2 are designed with derivative sliding surfaces. Integral block in the sliding surface usually causes the chattering problem in the control input which occurs in the NTSMC1 (the ATSMC1 is chattering-free which can be due to choosing initial condition). For the ISV (which represents the conception of energy) the derivative sliding surface works better. But in IAE and ITAE, the integral sliding surface give better results. The derivative block in the sliding surface is expensive to construct and hard. Also, in terms of constructing control parameters, the derivative sliding surface provides a costly control method. Overall, if the amplitude of control input and eliminating chattering phenomenon are the key parameter for choosing a controller, NTSMC2 and ATSMC2 will be superior methods which are designed by the derivative sliding surfaces. But if the speed of reaching to the stability and cost of the method in terms of constructing sliding surface and control parameter are two key features to choose a controller, the superior choice will be NTSMC1 and ATSMC1 (which are designed by integral sliding surfaces).

Furthermore, the numerical simulation of the proposed nonlinear observer is presented. The results reveal that the time response of state variable and estimated state variable converge very fast which is only a few seconds. In addition, the comparison between the proposed observer and some previous studies is made. The comparison demonstrates that in terms of considering uncertainties, the proposed control method in this research is better and more realistic. Also, in terms of providing a generalizable and global observer design and high speed of estimating data, the proposed method outperforms of some recent and well-known research in the literature.

# CHAPTER 6: CONCLUSION AND FUTURE WORK

## 6.1 Conclusion

In this study, two robust finite time controllers are designed to achieve the trajectory tracking goal for the nonlinear ROV system with one DOF for the pitch angle by using Nonsingular Terminal Sliding Mode Control (NTSMC) and Adaptive Nonsingular Terminal Sliding Mode Control (ANTSMC) methods. The finite time stability of the proposed techniques is proved, and the performance of the system is evaluated by means of numerical simulations. The control parameters are chosen in such a way that the settling time (stability time) of four designed controllers in this research are almost equal to make a correct comparison on various aspects. Also, three well-known performance criteria are defined including ISV, ITAE, and IAE for the comparison of four designed controllers in this research. The undesirable chattering phenomenon is observed in the control input of NTSMC1 due to using integral sliding surface (see Fig. (5.3)). Consequently, by using derivative sliding surface in NTSMC2, this destructive problem is eliminated thoroughly. This unwanted problem is not observed in control input of both adaptive controllers. In terms of ISV (which is very important performance criteria and shows the conception of energy) NTSMC2 and ATSMC2 which are used derivative sliding surfaces are better. the NTSMC1 and ATSMC1 methods result in lower cost of construction of sliding surface, as it is easier and more feasible to make an integrator than a derivative block. Also, NTSMC1 and ATSMC1 methods yield lower settling time (higher convergence rate). In terms of control parameters, NTSMC2 and ATSMC2 have a greater value which cost more to construct (see Eqs. (5.1) and (5.2)). In conclusion, if a practical derivative block can be made in such a way that cost-effective and noise-resistant, and the cost of constructing control parameters are provided, the proposed NTSMC2 and ATSMC2 controllers can be the superior control methods. The main advantages of these four proposed controllers are robustness against all external disturbances and uncertainties and finite time stability. Furthermore, the upper bound of these uncertainties is approximated in a finite time for both adaptive proposed controllers and these estimations are used in the control inputs.

Furthermore, in the observer design objective of this research, a new global full order finite time observer for a chain of nonlinear systems (second order) double integrator in presence of uncertainties are designed. Since the considered nonlinear system model is widely considered, it is possible to use proposed finite time nonlinear observer for estimating position variables and velocity in many systems such as robot manipulator and submarine and underwater vehicle and especially for ROV system with different DOF and reducing the cost of using velocity sensors. The proposed finite time nonlinear observer is examined on the nonlinear ROV system with one DOF for the pitch angle by performing numerical simulation in MATLAB/Simulink and the numerical simulation results reveal the effectiveness and correctness of proposed observer. Note that if the proposed observer is used in a nonlinear closed-loop system with the existence of controller, the finite time stability of the nonlinear closed-loop system (with the existence of observer and controller) must be proved and verified again.

## 6.2 Recommendation for future work

The recommendations for future are summarized in the following points:

1- Using the proposed finite time nonlinear observer in a nonlinear closed-loop system with the existence of proposed adaptive controller (ATSMC) and proving the finite time stability of the nonlinear closed-loop system (with the existence of observer and controller). Note that it must be proved by defining only one candidate Lyapunov function and using the estimation error and adaptive law in this candidate function.

2- Developing Fuzzy Adaptive Terminal Sliding Mode Control (FATSMC) method with the nonlinear observer of state variables by using fuzzy logic. Indeed, fuzzy logic is employed here to improve the control method.

3- Developing Optimal Fuzzy Adaptive Terminal Sliding Mode Control (OFATSMC) with the nonlinear observer of state variables by using the optimization methods including Linear Quadratic Regulator (LQR) and State-Dependent Riccati Equation (SDRE). These optimization methods work by optimization of the special cost function.

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# LIST OF PUBLICATIONS AND PAPERS PRESENTED

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