CHAPTER 4

ANALYSIS AND RESULTS

4.1 Analysis and Results of ARIMA models

The analysis of the BMCI time series starts with the visual inspection of the data plot. Upon examination, it is observed that there exists a slow varying trend in the data. In the first 18 months, it is observed that the mean of the composite index decreases steadily and in the final 20 months, an upward trend can be observed. The plot of the BMCI is shown in Figure 1 below. In order to obtain stationarity the apparent trend has to be removed. Therefore, differencing is performed on the data once. The differenced data is visually inspected and found to be stationary (i.e. without obvious trend or seasonality). The plot of the differenced data is shown in Figure 2.

Once the data is deemed stationary, its autocorrelation and partial autocorrelation plots are examined. It is assumed that 95% of the Autocorrelation function (ACF) and Partial autocorrelation function (PACF) coefficients to fall in the region of $\pm$ $1.96/\sqrt{N}$. This is in accordance with the fact that both the ACF and PACF are functions of normal distribution with mean zero and standard deviation $1/\sqrt{N}$ if indeed the time series has been generated by the a series that resembles white noise (Pindyk and Rubinfeld 1998).

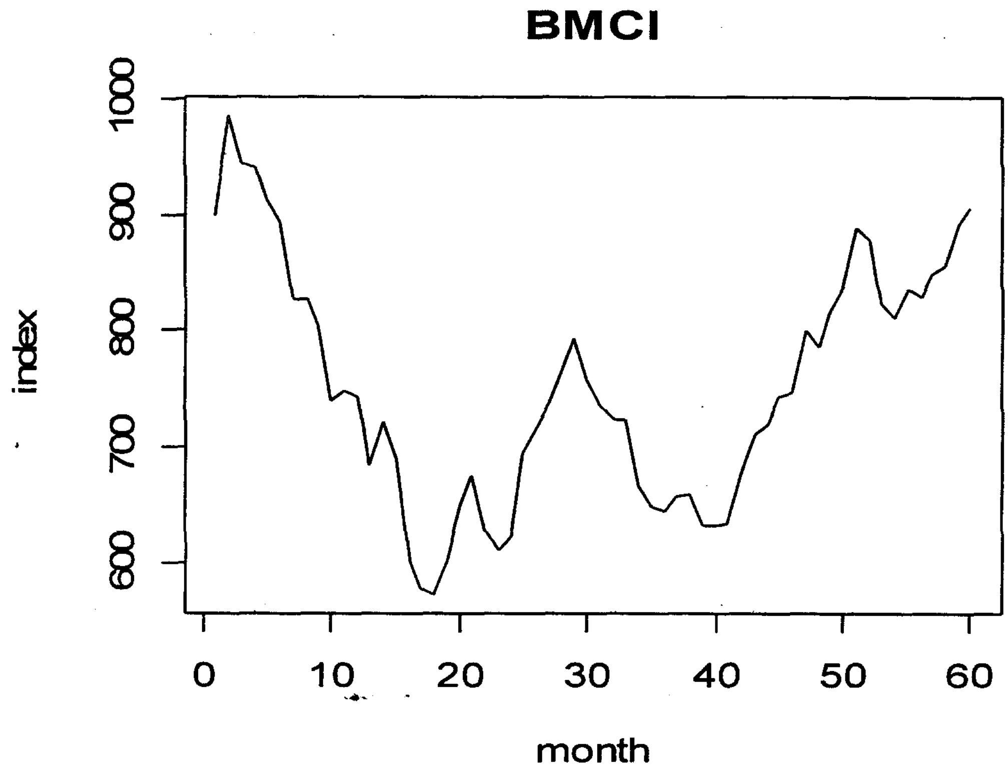

Figure 1. The monthly BMCI from January 2000 until December 2004

According to Box and Jenkins (1970) the shape of the autocorrelation and partial autocorrelation plots could be instrumental in the identification of the order of the AR, MA or ARMA models. Although this approach is somewhat heuristic, it provides a general guideline on how to choose the order of the ARIMA model to be used. The final choice of the model however, is only determined after examining the more accurate Akaike Information Criterion (Makridakis 1998).

The summary of the expected patterns in the ACF and PACF plots are summarized by Table 1.

Model	ACF	PACF
AR(p)	dies down	cut-off after lag p
MA(q)	cut-off after lag q	dies down
ARMA(p,q)	dies down	dies down

Table 1. The summary of expected patterns in the ACF and PACF

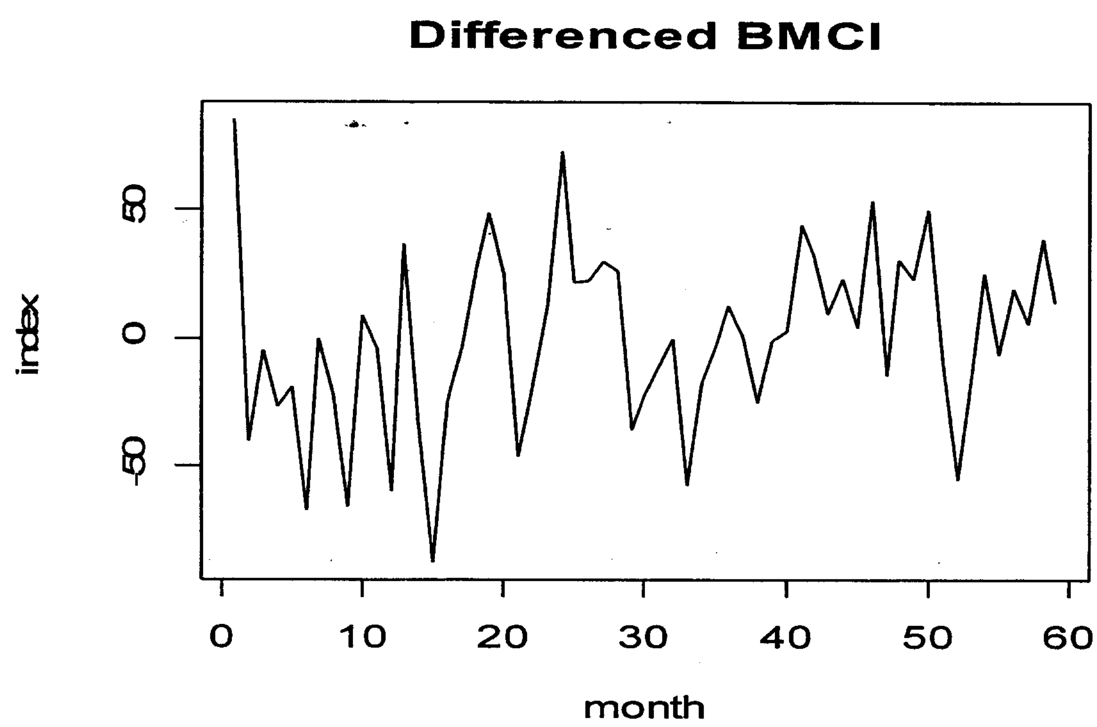

Figure 2. The monthly BMCI after being differenced once

The ACF and PACF plots of the differenced data are shown in Figure 3 and Figure 4 below. It can be seen from the two plots that the magnitudes of most of the coefficients fall between the two dotted lines of 95% confidence interval. They are indeed less than $\pm 1.96/\sqrt{N}$. It should be noted however that there is a theoretical 5% chance that an ACF coefficient might show a value higher than the $1.96/\sqrt{N}$ level when its true value is zero. From Figure 3 it is observed that the value of ACF coefficient of lag zero is very close to one. However, this value should be ignored since lag zero refers to the correlation of each time series datum with itself and certainly it should be one.

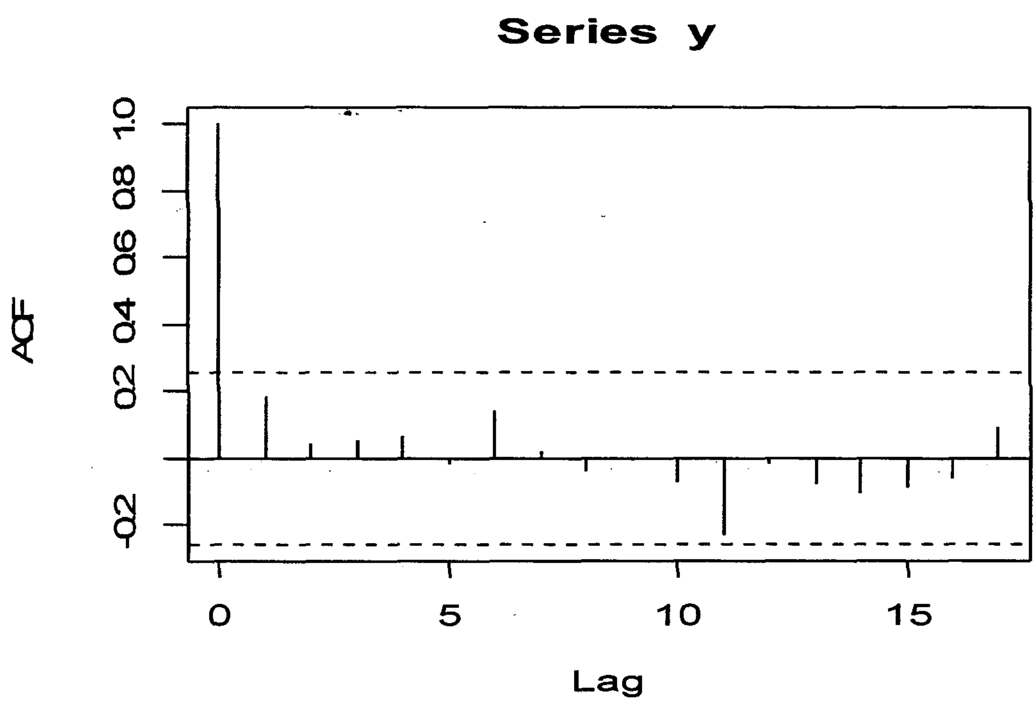

Figure 3. The ACF plot of the differenced data (BMCI)

Ignoring the ACF coefficient of lag zero, there is no other coefficient that is prominently above the dotted line of 95% confidence interval ($\pm 1.96/\sqrt{N}$). Therefore, no obvious conclusion can be made regarding the type and order of the ARMA model to be used yet. From Figure 4, it can be seen that the PACF plot shows a similar pattern to that of the ACF. It is obvious that the values of all of the coefficients are insignificant within the 95% confidence interval. In short, after examining the ACF and PACF coefficients, no conclusion is obtained about a suitable model for the nature of the time series. The values of the first sixteen ACF and PACF coefficients are listed in Table 2.

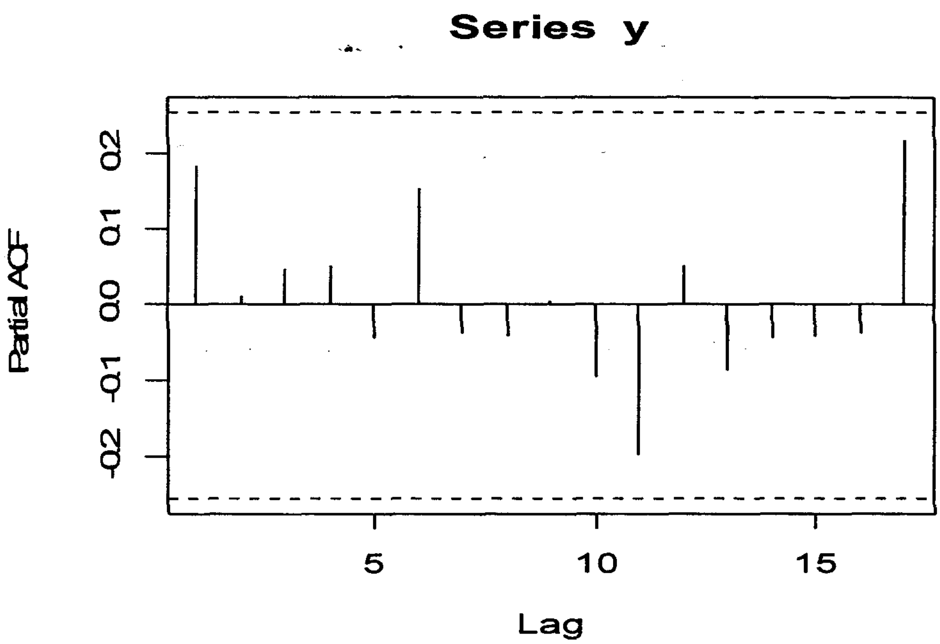

Figure 4. The PACF plot of the differenced data (BMCI)

Lag	ACF	PACF
1	0.184938076	0.184938076
2	0.043947150	0.010090163
3	0.054558556	0.046232813
4	0.068118356	0.051403244
5	-0.019295641	-0.044514510
6	0.141282293	0.154500018
7	0.022576495	-0.037398939
8	-0.037562398	-0.043200456
9	-0.005127988	0.004771521
10	-0.071447480	-0.096233210
11	-0.232535779	-0.199175439
12	-0.018673263	0.050709667
13	-0.078708819	-0.086286866
14	-0.102908148	-0.045170862
15	-0.089369786	-0.043131331
16	-0.059534809	-0.038379502

Table 2. The values of the ACF and PACF coefficients

Since no conclusive pattern is shown by the ACF and PACF coefficients, AIC values of several ARMA models are needed to help determine the right model to choose. Executing the R software for various ARMA(p,q) models using the time series data as input produces the following AIC values as shown in Table 3.

p \ q	0	1	2	3	4
0		589.54	591.52	593.48	594.97
1	589.5	591.5	592.72	594.54	595.94
2	591.5	593.1	594.48	594.88	597.89
3	593.35	594.97	595.67	596.54	598.7
4	595.23	596.11	597.9	599.36	599.28

Table 3. AIC values of various ARMA(p,q) models

Examining the AIC table above, it can be observed that the AR(1), MA(1) and ARMA(1,1) are the three models that provide the minimum AIC values. Since their AIC values are the lowest, these models are chosen for further scrutiny. It is not necessary to analyze other higher order models since it is best to choose models with the fewest number of parameters. Therefore they are rejected on the principle of parsimony (i.e. the lower the order of the model the fewer coefficients it has and thus the better it is). Now that the models have been selected, we are ready to estimate their coefficients as follows.

a) The AR(1) model

The AR model is defined as

$$Y_t = \delta + \Phi_1 Y_{t-1} + e_t$$

where

δ is a constant

Φ_1 is the parameter associated with the past observation of lag 1

e_t is the error term at time t

The expectation on Y gives

$$E(Y_t) = \mu = E(\delta + e_t + \Phi_1 Y_{t-1}) = E(\delta) + E(e_t) + \Phi_1 E(Y_{t-1})$$

$$\mu = \delta + 0 + \Phi_1 \mu$$

$$\mu - \Phi_1 \mu = \delta$$

$$\mu = \delta/(1 - \Phi_1)$$

where μ is the average of Y_t

Let's go back to the original equation

$$Y_t = \delta + \Phi_1 Y_{t-1} + e_t$$

The error term can be written as

$$e_t = Y_t - \delta - \Phi_1 Y_{t-1}$$

and thus the squared error is

$$e_t^2 = (Y_t - \delta - \Phi_1 Y_{t-1})(Y_t - \delta - \Phi_1 Y_{t-1})$$

The total squared error is

$$\Sigma e_t^2 = \Sigma(Y_t - \delta - \Phi_1 Y_{t-1})(Y_t - \delta - \Phi_1 Y_{t-1})$$

The squared error is minimized by taking partial derivatives with respect to δ and Φ_1 and equating them to zero. Solving the two simultaneous equations yields

$$\Phi_1 = \frac{N\sum Y_{t-1}Y_t - \sum Y_{t-1}\sum Y_t}{N\sum Y_{t-1}^2 - (\sum Y_{t-1})^2}$$

and

$$\delta = \mu(1 - \Phi_1)$$

The R software utilizes maximum likelihood estimation technique to obtain the value of the parameters rather than the ordinary least square method above. From the output of the R software, the value for $\delta = 0.428$ and $\Phi_1 = 0.2031$. Therefore the estimated AR(1) model obtained from fitting the time series data and maximizing the likelihood function (which is the same as minimizing the square error for this case) is

$$Y_t = 0.428 + 0.2031 Y_{t-1} + e_t$$

Suppose only the past values of Y_t are known and we want to predict the value of Y_t. The predicted model based on the past values is defined as

$$\hat{Y}_t = \delta + \Phi_1 Y_{t-1}$$

In other words, the value of e_t is set to zero (i.e. set to its mean value, since e_t is a Gaussian noise). Then, the series of predicted values become

$\hat{Y}_1 = \delta$

$\hat{Y}_2 = \delta + \Phi_1 y_1$

$\hat{Y}_3 = \delta + \Phi_1 y_2$

$\hat{Y}_4 = \delta + \Phi_1 y_3$

$\hat{Y}_5 = \delta + \Phi_1 y_4$

and so on.

The predicted values generated by the AR(1) model are given in Appendix A and their plot against the real values of BMCI is shown in Figure 5 below.

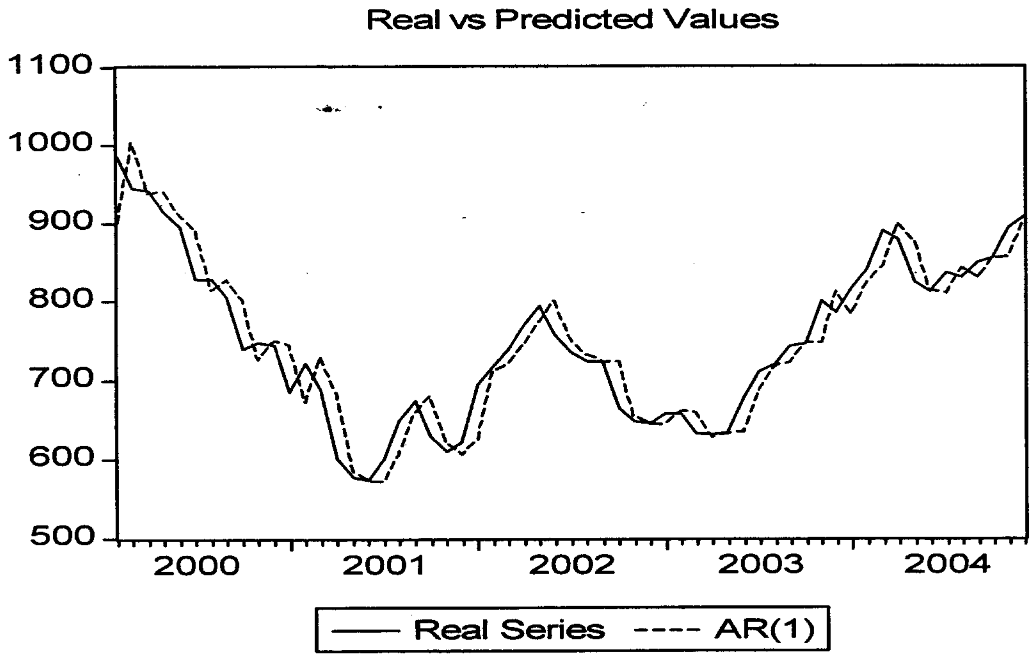

Figure 5. Predicted Values of AR(1) Versus Real Values

Then the ACF and PACF of the residuals are plotted in Figure 6 and 7 to observe if there are any outliers that deviate from the white noise pattern. It is noticed that all ACF and PACF coefficients for lag greater than or equal to 1 can be considered insignificant with 95% confidence since they fall within the dotted lines of $\pm 1.96/\sqrt{N}$. Since the value of the AIC is already given by the R software there is no need to calculate its approximation. For the AR(1) model the AIC value is 589.5.

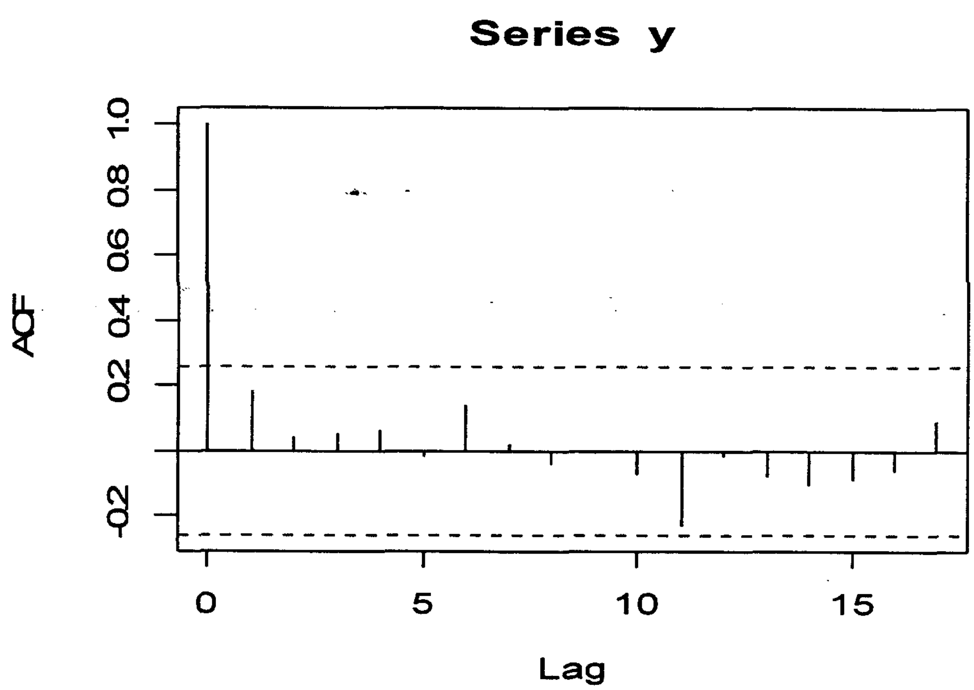

Figure 6. The ACF plot of Residuals for AR(1)

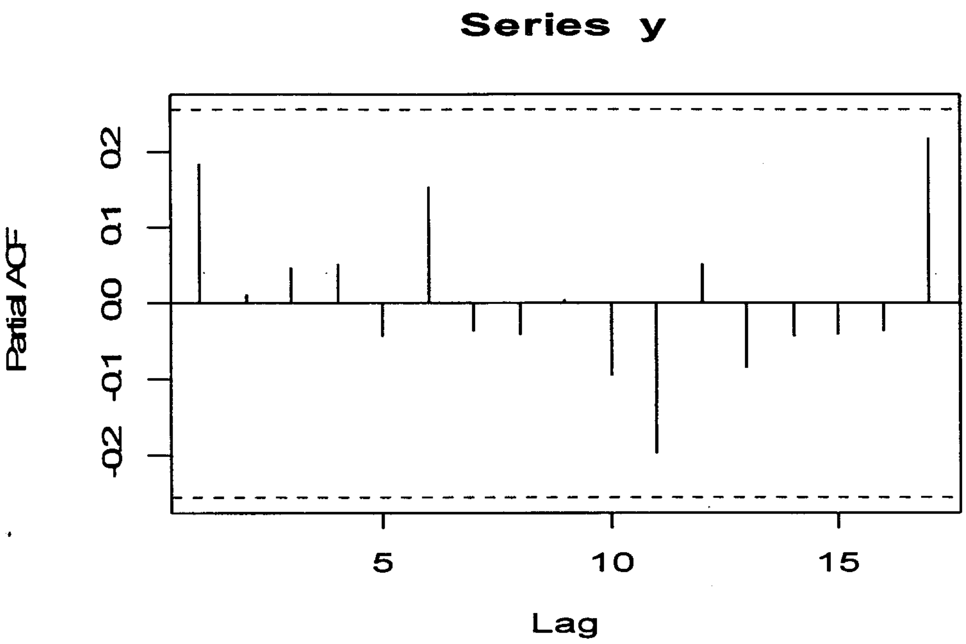

Figure 7. The PACF plot of residuals for AR(1)

The adjusted R squared value and the estimated error variance are calculated as follows.

From the results, we obtained

SSR = 633733.3 SST = 618989.9

So,

R^2 = 1.02382 Adjusted R^2 = 1.06

The sum of squared residuals is 68416.55. Dividing it by the degree of freedom which is equal to the number of data points minus the number of parameters (i.e. 57 for this case) yields the estimated error variance which is 1200.3.

b) The MA(1) model

The MA(1) model is defined as

$$Y_t = \mu + e_t + \theta_1 e_{t-1}$$

where

μ is a constant

θ_1 is the parameter associated with the past error of lag 1

e_t is the error term at time t

The values of the unknown parameters could only be obtained iteratively by minimizing the square error or maximizing the maximum likelihood function since there is no simple formula that can be applied (Pindyck 1998). From the output of the R software, the value for μ = 0.4526 and θ_1 = 0.2026. Therefore the estimated MA(1) model is

$$Y_t = 0.4526 + 0.2026e_{t-1} + e_t$$

Let's go back to the original equation

$$Y_t = \mu + e_t + \theta_1 e_{t-1}$$

The error term can be written as

$$e_t = Y_t - \mu - \theta_1 e_{t-1}$$

$$e_{t-1} = Y_{t-1} - \mu - \theta_1 e_{t-2}$$

$$e_{t-2} = Y_{t-2} - \mu - \theta_1 e_{t-3}$$

Substituting the last equation into the equation before it and working upward we have

$$Y_t = e_t + \mu - \theta_1\mu + \theta_1{}^2 \mu + \theta_1 Y_{t-1} - \theta_1{}^2 Y_{t-2} + \theta_1{}^3 e_{t-3}$$

Further substitution involving e_{t-3}, e_{t-4}, etc. produces the general expression

$$Y_t = e_t + \mu - \theta_1\mu + \theta_1{}^2 \mu - \theta_1{}^3 \mu + \ldots + \theta_1 Y_{t-1} - \theta_1{}^2 Y_{t-2} + \theta_1{}^3 Y_{t-3} - \ldots$$

In our case, since $\mu = 0.4526$ and $\theta_1 = 0.2026$,

$$Y_t = e_t + 0.4526(1 - 0.2026 + 0.2026^2 - 0.2026^3 + 0.2026^4 - \ldots\ldots) +$$

$$0.2026Y_{t-1} - 0.2026^2 Y_{t-2} + 0.2026^3 Y_{t-3} - \ldots$$

Neglecting the error term, or setting its value to zero (its mean), the predicted model is given by

$$\hat{Y}_t = \mu - \theta_1\mu + \theta_1{}^2 \mu - \theta_1{}^3 \mu + \ldots + \theta_1 Y_{t-1} - \theta_1{}^2 Y_{t-2} + \theta_1{}^3 Y_{t-3} - \ldots$$

Then, the series of predicted values become

$$\hat{Y}_1 = \mu - \theta_1\mu + \theta_1{}^2 \mu - \theta_1{}^3 \mu + \ldots$$

$$\hat{Y}_2 = \mu - \theta_1\mu + \theta_1{}^2 \mu - \theta_1{}^3 \mu + \ldots + \theta_1 Y_1$$

$$\hat{Y}_3 = \mu - \theta_1\mu + \theta_1{}^2 \mu - \theta_1{}^3 \mu + \ldots + \theta_1 Y_2 - \theta_1{}^2 Y_1$$

$$\hat{Y}_4 = \mu - \theta_1\mu + \theta_1{}^2 \mu - \theta_1{}^3 \mu + \ldots + \theta_1 Y_3 - \theta_1{}^2 Y_2 + \theta_1{}^3 Y_1$$

$$\hat{Y}_5 = \mu - \theta_1\mu + \theta_1{}^2 \mu - \theta_1{}^3 \mu + \ldots + \theta_1 Y_4 - \theta_1{}^2 Y_3 + \theta_1{}^3 Y_2 - \theta_1{}^4 Y_1$$

and so on.

The predicted values generated by the MA(1) model are also presented in Appendix A and their plot against the real values of BMCI is shown in Figure 8.

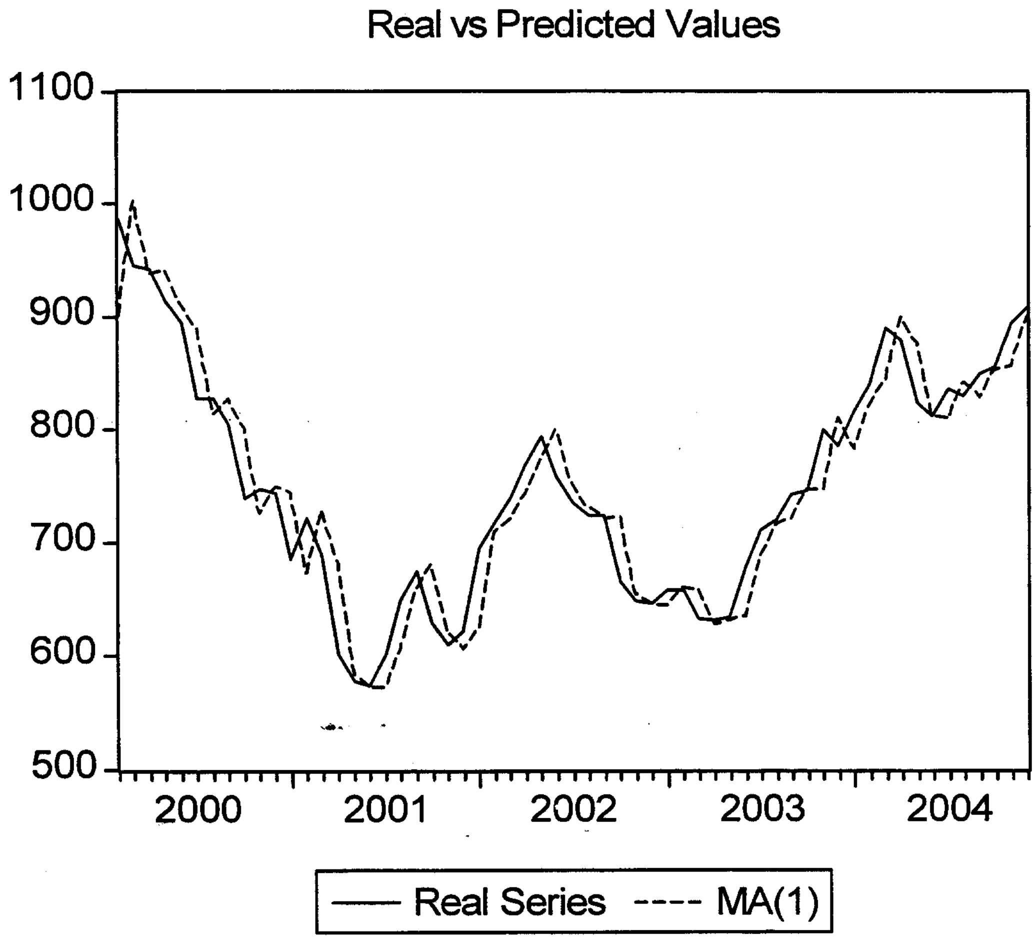

Figure 8. Predicted values of MA(1) versus real values

The ACF and PACF of the residuals are plotted in Figure 9 and 10. All ACF and PACF coefficients for MA(1) model for lag 1 and greater can be considered insignificant with 95% confidence. The AIC value given by the R software is 589.54.

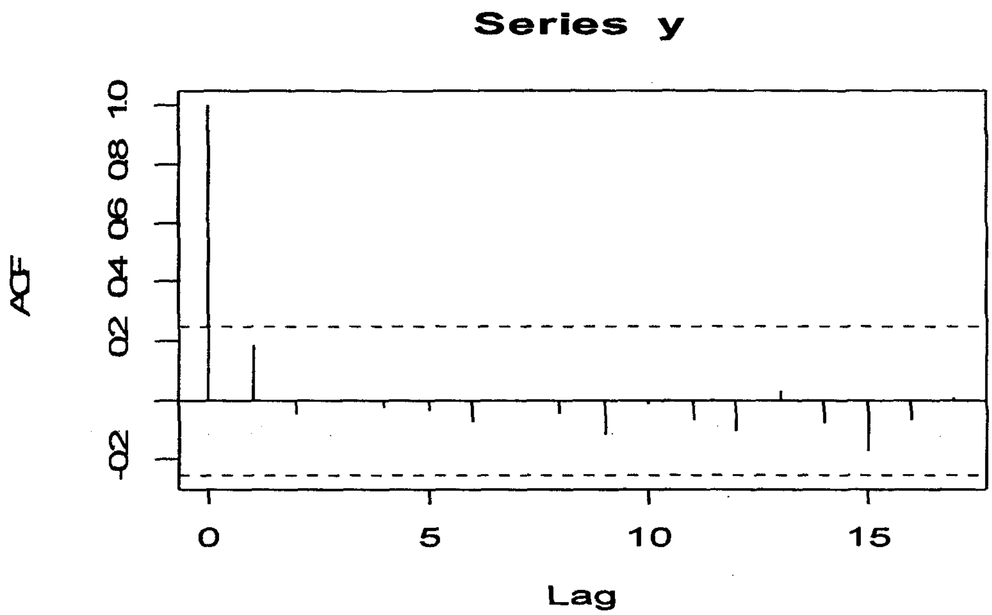

Figure 9. The ACF plot of residuals for MA(1)

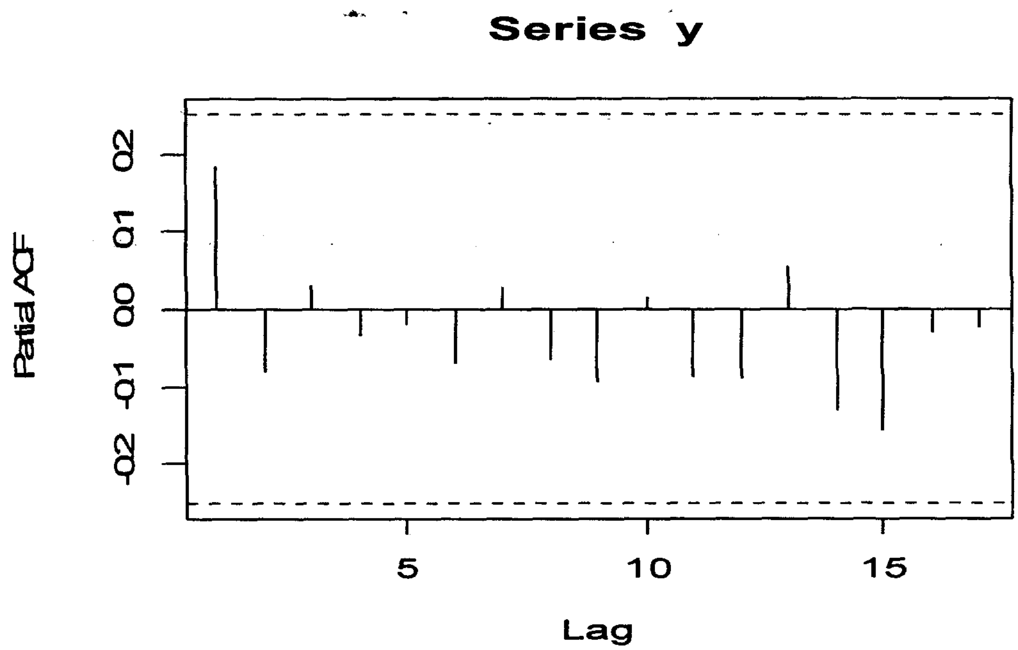

Figure 10. The PACF plot of residuals for MA(1)

The adjusted R squared value and the estimated error variance are calculated as follows.

From the results, we obtained

SSR = 633795 SST = 618989.9

So,

R^2 = 1.024 Adjusted R^2 = 1.06.

The sum of the squared residuals is 68347.8. Dividing it by the degree of freedom (which is 57) yields the estimated error variance of 1199.08.

c) The ARMA(1,1) model

The ARMA(1,1) model is governed by the following formula.

$$Y_t = \delta + e_t + \Phi_1 Y_{t-1} + \theta_1 e_{t-1}$$

where

δ is a constant

Φ_1 is the parameter associated with the past observation of lag 1

θ_1 is the parameter associated with the past error of lag 1

e_t is the error term at time t

Again, as with the MA(1) model, the values of the unknown parameters could only be obtained iteratively by minimizing the square error or maximizing the maximum likelihood function (Makridakis et. al. 1998). From the output of the R software, the value for δ = -0.0949, Φ_1 = 0.1798 and θ_1 = 0.0244. Therefore the estimated ARMA(1,1) model is

$$Y_t = -0.0949 + e_t + 0.1798Y_{t-1} + 0.0244e_{t-1}$$

Looking back at the original equation

$$Y_t = \delta + e_t + \Phi_1 Y_{t-1} + \theta_1 e_{t-1}$$

The error term can be written as

$$e_t = Y_t - \delta - \Phi_1 Y_{t-1} - \theta_1 e_{t-1}$$

$$e_{t-1} = Y_{t-1} - \delta - \Phi_1 Y_{t-2} - \theta_1 e_{t-2}$$

$$e_{t-2} = Y_{t-2} - \delta - \Phi_1 Y_{t-3} - \theta_1 e_{t-3}$$

Substituting the last equation into the equation before it and working upward with further substitution we have

$$Y_t = e_t + \delta - \theta_1 \delta + \theta_1^2 \delta + \Phi_1 Y_{t-1} - \theta_1 \Phi_1 Y_{t-2} + \theta_1^2 \Phi_1 Y_{t-3} + \theta_1 Y_{t-1} - \theta_1^2 Y_{t-2} + \theta_1^3 e_{t-3}$$

Further substitution involving e_{t-3}, e_{t-4}, etc. produces the general expression

$$Y_t = e_t + \delta - \theta_1 \delta + \theta_1^2 \delta - \theta_1^3 \delta + \theta_1^4 \delta - \theta_1^5 \delta + \ldots$$
$$+ \theta_1 Y_{t-1} - \theta_1^2 Y_{t-2} + \theta_1^3 Y_{t-3} - \theta_1^4 Y_{t-4} + \theta_1^5 Y_{t-5} - \ldots$$
$$+ \Phi_1 Y_{t-1} - \theta_1 \Phi_1 Y_{t-2} + \theta_1^2 \Phi_1 Y_{t-3} - \theta_1^3 \Phi_1 Y_{t-4} + \theta_1^4 \Phi_1 Y_{t-5} - \ldots$$

In our case, since δ = -0.0949, Φ_1 = 0.1798 and θ_1 = 0.0244,

$$Y_t = e_t - 0.095(1 - 0.0244 + 0.0244^2 - 0.0244^3 + 0.0244^4 - \ldots\ldots) +$$
$$0.0244Y_{t-1} - 0.0244^2 Y_{t-2} + 0.0244^3 Y_{t-3} - 0.0244^4 Y_{t-4} + \ldots$$

$$+ 0.18Y_{t-1} + 0.18(-0.0244Y_{t-2} + 0.0244^2Y_{t-3} - 0.0244^3Y_{t-4} + 0.0244^4Y_{t-5} - \ldots)$$

With the error term set to zero, the predicted model is defined as

$$\hat{Y}_t = \delta - \theta_1\delta + \theta_1^2\delta - \theta_1^3\delta + \theta_1^4\delta - \theta_1^5\delta + \ldots$$

$$+ \theta_1Y_{t-1} - \theta_1^2Y_{t-2} + \theta_1^3Y_{t-3} - \theta_1^4Y_{t-4} + \theta_1^5Y_{t-5} - \ldots$$

$$+ \Phi_1Y_{t-1} - \theta_1\Phi_1Y_{t-2} + \theta_1^2\Phi_1Y_{t-3} - \theta_1^3\Phi_1Y_{t-4} + \theta_1^4\Phi_1Y_{t-5} - \ldots$$

Then, the series of predicted values become

$$\hat{Y}_1 = \delta - \theta_1\delta + \theta_1^2\delta - \theta_1^3\delta + \theta_1^4\delta - \theta_1^5\delta + \ldots$$

$$\hat{Y}_2 = \delta - \theta_1\delta + \theta_1^2\delta - \theta_1^3\delta + \theta_1^4\delta - \theta_1^5\delta + \ldots + \theta_1Y_1 + \Phi_1Y_1$$

$$\hat{Y}_3 = \delta - \theta_1\delta + \theta_1^2\delta - \theta_1^3\delta + \theta_1^4\delta - \theta_1^5\delta + \ldots + \theta_1Y_2 - \theta_1^2Y_1$$

$$+ \Phi_1Y_2 - \theta_1\Phi_1Y_1$$

$$\hat{Y}_4 = \delta - \theta_1\delta + \theta_1^2\delta - \theta_1^3\delta + \theta_1^4\delta - \theta_1^5\delta + \ldots$$

$$+ \theta_1Y_3 - \theta_1^2Y_2 + \theta_1^3Y_1 + \Phi_1Y_3 - \theta_1\Phi_1Y_2 + \theta_1^2\Phi_1Y_1$$

$$\hat{Y}_5 = \delta - \theta_1\delta + \theta_1^2\delta - \theta_1^3\delta + \theta_1^4\delta - \theta_1^5\delta + \ldots$$

$$+ \theta_1Y_4 - \theta_1^2Y_3 + \theta_1^3Y_2 - \theta_1^4Y_1 +$$

$$+ \Phi_1Y_4 - \theta_1\Phi_1Y_3 + \theta_1^2\Phi_1Y_2 - \theta_1^3\Phi_1Y_1 + \theta_1^4\Phi_1$$

and so on.

The predicted values produced by the ARMA(1,1) model are available in

Appendix A and their plot is shown in the following Figure 11.

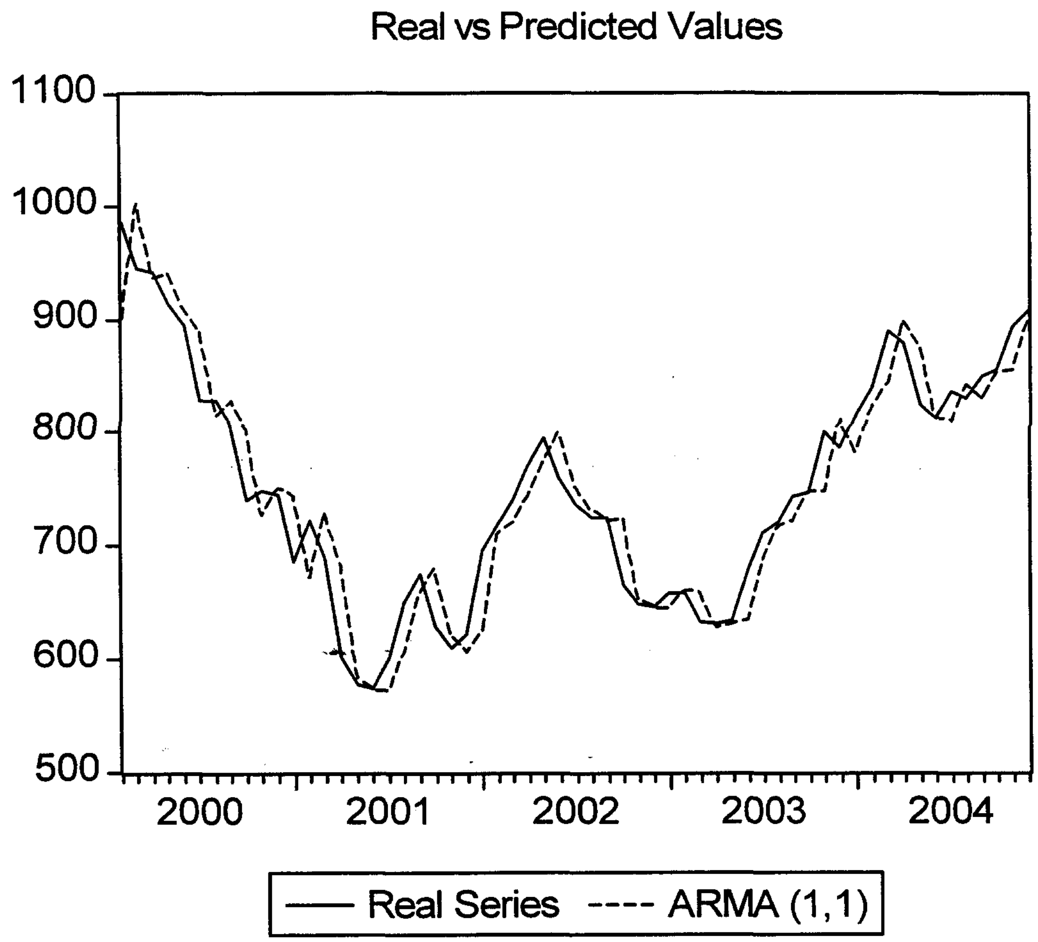

Figure 11. Predicted values of ARMA(1,1) versus real values

The ACF and PACF of the residuals are plotted in Figure 9 and 10. All ACF and PACF coefficients for ARMA(1) model for lag 1 and greater can be considered insignificant. The AIC value given by the R software is 591.5.

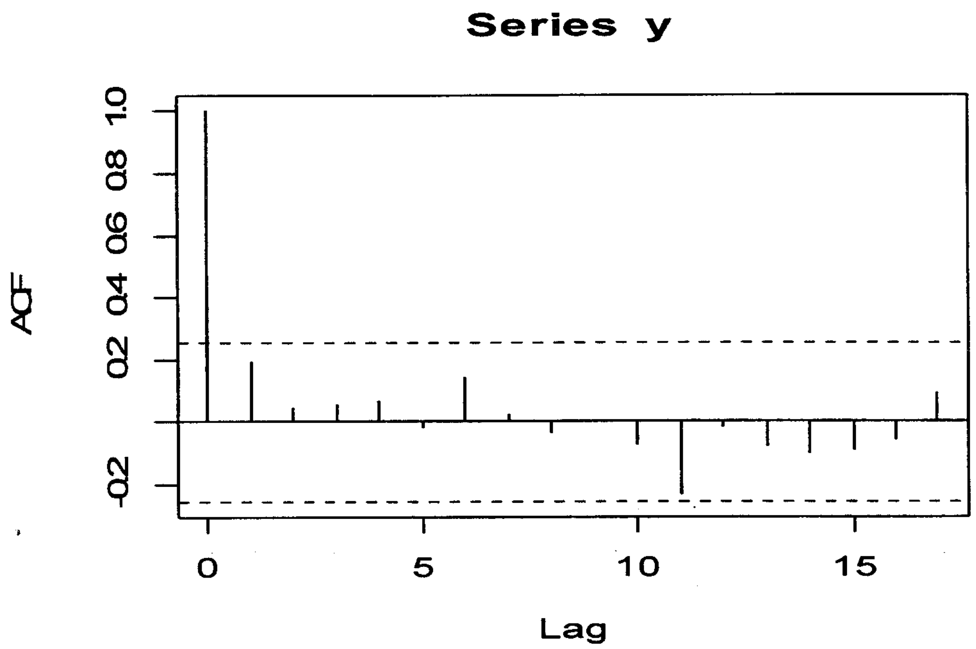

Figure 12. The ACF plot of residuals for ARMA(1,1)

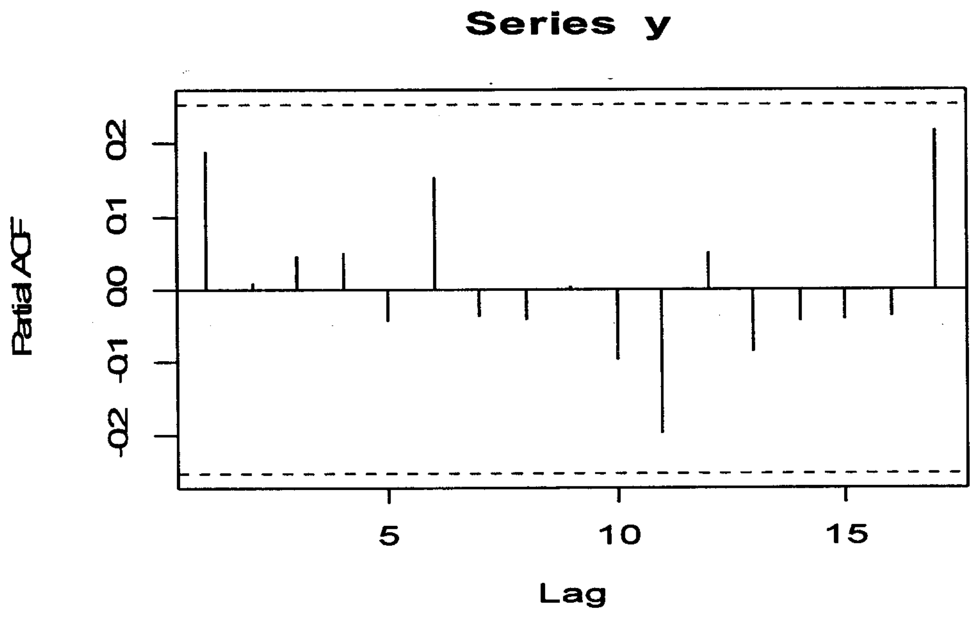

Figure 13. The PACF plot of residuals for ARMA(1,1)

The adjusted R squared value and the estimated error variance for the ARMA(1,1) model are calculated as follows.

From the results, we obtained

SSR = 633404.1 SST = 618989.9

So,

$R^2 = 1.0233$ Adjusted $R^2 = 1.0781$.

Dividing the sum of residuals (68424.72) by the degree of freedom for this model (i.e. 56) produces an estimated error variance of 1221.87.

The results obtained from implementing the three chosen ARIMA models shows that the MA(1) model is slightly better than the other two in terms of its R square, adjusted R square and estimated error variance. The final assessment of the three models is to run a test on their forecasting ability. A forecast is made on the average value of BMCI for January 2005. The values of the forecasts from the three models are compared against the true average value of BMCI for Jan 2005. The forecasting results are as follows.

Recall that the ARIMA models work on the difference of the averages of two consecutive months. The predicted AR(1) model is

$$\hat{Y}_t = 0.428 + 0.2031Y_{t-1}$$

The difference of the monthly average index between November and December 2004 is 13.929. The forecasted value of the difference between December 2004 and January 2005 is

$$\hat{Y}_t = 0.428 + 0.2031(13.929) = 3.257$$

Therefore the AR(1) forecasted BMCI index for January 2005 is the average value for December 2004 (which is 907.233) plus 3.257 which is equal to 910.49.

The predicted MA(1) model is given by the general expression

$$\hat{Y}_t = \mu - \theta_1\mu + \theta_1^{\,2}\mu - \theta_1^{\,3}\mu + \ldots + \theta_1 Y_{t-1} - \theta_1^{\,2} Y_{t-2} + \theta_1^{\,3} Y_{t-3} - \ldots$$

The forecasted MA(1) value for January 2005 must be calculated recursively and the forecasted value generated by computer is 910.056. And finally, the predicted ARMA(1,1) model is governed by

$$\hat{Y}_t = \delta - \theta_1\delta + \theta_1^2\delta - \theta_1^3\delta + \theta_1^4\delta - \theta_1^5\delta + \ldots.$$

$$+ \theta_1 Y_{t-1} - \theta_1^2 Y_{t-2} + \theta_1^3 Y_{t-3} - \theta_1^4 Y_{t-4} + \theta_1^5 Y_{t-5} - \ldots.$$

$$+ \Phi_1 Y_{t-1} - \theta_1\Phi_1 Y_{t-2} + \theta_1^2\Phi_1 Y_{t-3} - \theta_1^3\Phi_1 Y_{t-4} + \theta_1^4\Phi_1 Y_{t-5} - \ldots$$

It is obvious that the forecasted value for ARMA(1,1) for January 2005 must also be calculated recursively and the value generated by computer is 909.8. Indeed the forecasted values of the three models are very similar. When compared to the real average value of 922.5, the MA(1) forecast is the closest.

4.2 Analysis and Results of Kalman filtering

First, it should be noted that BMCI index is a scalar quantity and there is no input to the process. Therefore, the process is governed by the following equations

$$x_k = Ax_{k-1} + w_{k-1},$$

$$z_k = Hx_k + v_k .$$

where A and H are scalar parameters to be estimated. The variances of w_k and v_k (R and Q) are also unknown at this stage. The task at hand is to find the values of these parameters that give the least square errors (residuals) when the values of the estimated state $\hat{x}_k$ are compared to the actual time series Y_t. Therefore the values of A, H, R and Q are changed iteratively to find the least square residuals when the state estimates $\hat{x}_k$ (produced by those parameters) are compared to the actual time series. The process equations are

$$\hat{x}^-_k = A\hat{x}_{k-1}$$

$$P^-_k = AP_{k-1}A + Q$$

and the measurement equations are

$$K_k = P^-_kH/(HP^-_kH + R)$$

$$\hat{x}_k = \hat{x}^-_k + K_k (z_k - H\hat{x}^-_k)$$

$$P_k = (1 - K_k H) P^-_k$$

Assuming A, H, R and Q are static, a C program is written to find the best values for these parameters that would produce the estimated $\hat{x}_k$ that generates the least squared residuals when compared to the BMCI time series. The range for A and H is restricted between 0 and 2 since these two scalars should carry positive correlation. The increment for the values of A and H is 0.1 for each step. The variance of the time series data can be calculated off-line and the range for R and Q is restricted between 0 and the variance of the data. The values of R and Q are incremented by 1 in each step. The initial values chosen for $\hat{x}^-_k$ and P^-_k are usual 0 and 1 respectively.

It is observed that when A, H, R and Q are all equal to 1, $\hat{x}_k$ generates successive predictive values that follow the BMCI time series closely and thus produces the least squared error. In this situation the time update and measurement updated equations become

$$\hat{x}^-_k = \hat{x}_{k-1}$$

$P^-_k = P_{k-1} + 1$

$K_k = P^-_k/(P^-_k + 1)$

$\hat{x}_k = \hat{x}^-_k + K_k\,(z_k - \hat{x}^-_k)$

$P_k = (1 - K_k)\,P^-_k$

The predicted state values produced by the Kalman filter are listed in Appendix A and their plot against the real values of BMCI is shown in Figure 14 below.

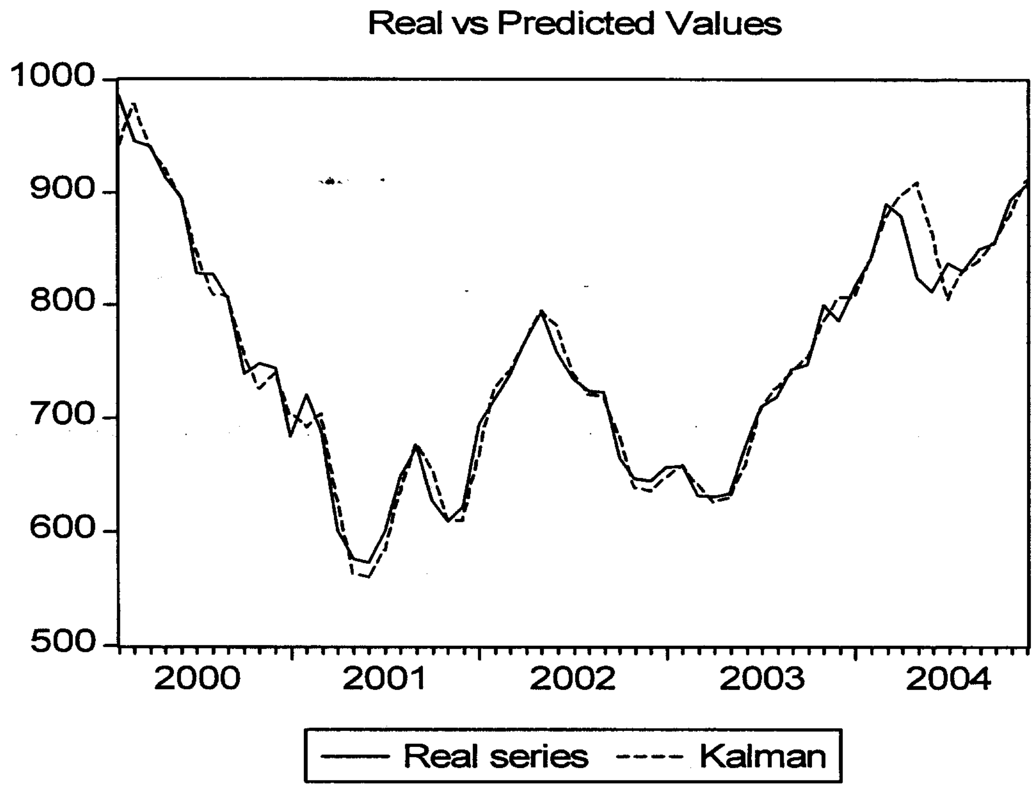

Figure 14. Predicted values of Kalman filter versus real values

The ACF and PACF of the residuals are plotted in Figure 15 and 16. All ACF and PACF coefficients for Kalman filter model for lag 1 and greater are insignificant. No AIC value is generated since the filtering is done in C language directly.

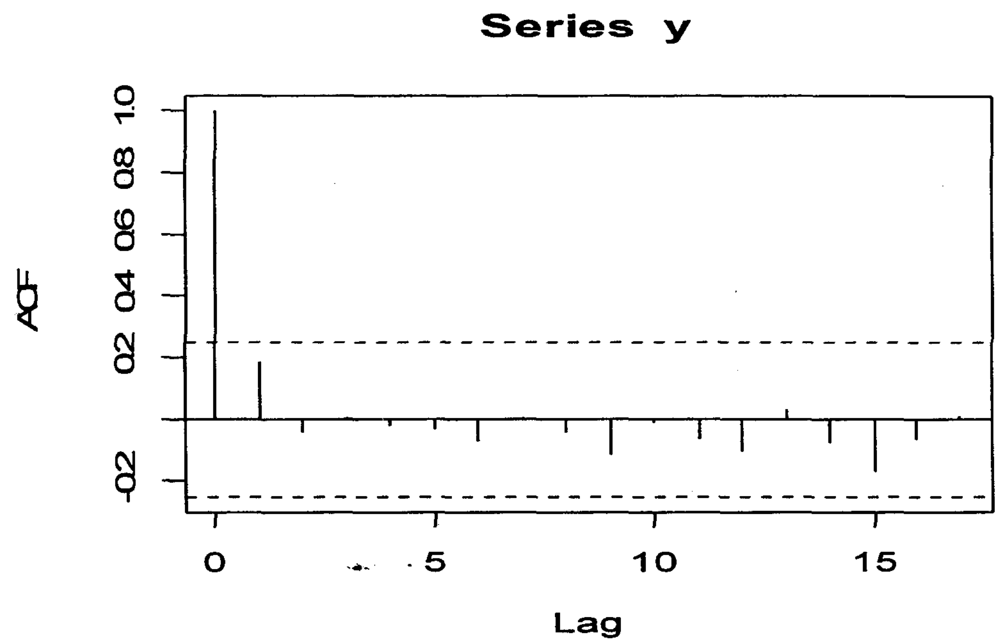

Figure 15. The ACF plot of residuals for Kalman filtering

The adjusted R square and the estimated error variance for the Kalman filter model are calculated as follows.

From the results, we obtained

SSR = 657362.5 SST = 618989.9

So,

R^2 = 1.062 Adjusted R^2 = 1.08.

Dividing the sum of residuals (16749.03) by the degree of freedom for this model (i.e. 58) produces an estimated error variance of 288.8.

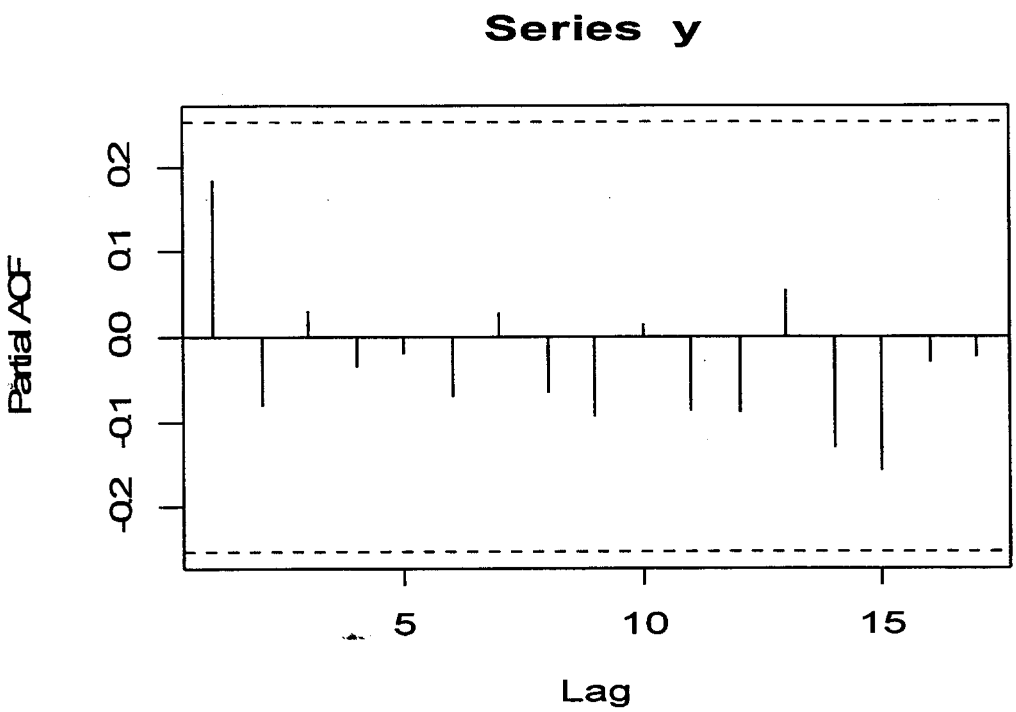

Figure 16. The PACF plot of residuals for Kalman filtering

The January 2005 forecast of Kalman filter is simply equal to its December 2004 predicted value of 912.043 since $\hat{x}^-_k = A\hat{x}_{k-1}$ and A is 1. This is the closest forecast to the real value of 922.5.